Stability Analysis of Data Driven Complex Systems

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Abstract: In this work we develop a mathematic and algorithmic framework for data-driven stability analysis of black box switching systems by using adaptive sampling and online optimization. Our algorithm learns the optimal sampling strategy to build more accurate stability guarantees with fewer sampling points. This work is an extension of [1], where an algorithm is presented to bound the Joint Spectral Radius (JSR) [2] of an unknown switched linear system within some probability from a finite number of trajectories.

1 Introduction

Many complex systems have dynamical behaviour which is difficult to model (e.g. they have subcomponents of different natures). Without an accurate model, traditional Model-Based Control (MBC) theory has become impractical for many control issues in these kinds of systems. For this reason, the concept of Data-Driven Control theory has recently gained a lot of attention. As a few example, let us cite the optimal energy management of smart buildings, where it is difficult to have a precise model of the thermal behaviour, and the system evolves over time in a stochastic way. Let us also mention the field of protontherapy, where control algorithms are needed in order to reach high accuracy in the proton beam. In view of the complex physical phenomena occuring in the proton beamline, it is believed that future control solutions will rely only on data harvested from the actual system, rather than on an a priori defined model.

2 Problem formulation

As mentioned before, the paper [1] provides a new framework, where firm probabilistic stability guarantees on the dynamical systems can be obtained by randomly sampling the state space, and simulating the behaviour of the switching system (which is a common paradigmatic model for many complex systems) at these randomly sampled points. In the mentioned paper, a uniform sampling of the state space is proposed, but it turns out that once the algorithm has been run, the user can learn from the outcome a better sampling strategy, that would have allowed for a better stability guarantee. This better sampling strategy is represented in Figure 1. Unfortunately, in the initial version of the algorithm presented in [1], the improved sampling can only be obtained a posteriori, and is therefore useless because the points have already been sampled.

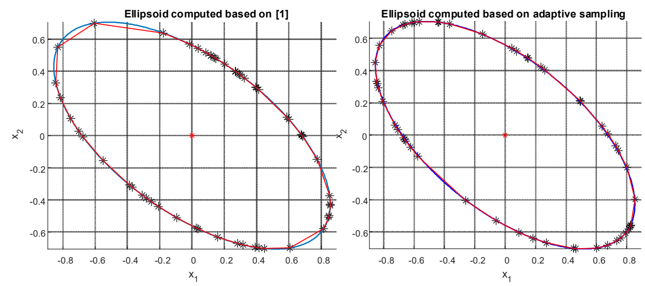

Figure 1: Result of the data-driven algorithm without (left) and with (right) adaptive sampling. The convex hull of the randomly sampled points (black stars) fit much better the unknown ellipsoid with the adaptive sampling approach.

3 Learning of an optimal sampling strategy

We propose an online version of the algorithms, where the points are sampled online, and computations are directly performed, so that the following points to be sampled benefit from the information already available. Adaptive sampling here leads to learning an optimal change of variables. More precisely, we first sample a small batch of points and precompute in initial Lyapunov-like function. From this, we can compute an approximation of the change of variables. Based on the precomputation, we resample and refine the computation.Our algorithm merges techniques from adaptive sampling with concepts from the switching systems literature.

References

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[2] R. Jungers, *The joint spectral radius: theory and applications*, vol. 385. Springer Science & Business Media, 2009.

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