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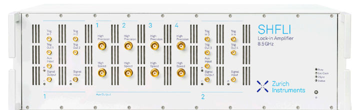

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Noise–dissipation relation for nonlinear electronic circuits

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ABSTRACT

An extension of fluctuation–dissipation theorem is used to derive a "speed limit" theorem for nonlinear electronic devices. This speed limit provides a lower bound on the dissipation that is incurred when transferring a given amount of electric charge in a certain amount of time with a certain noise level (average variance of the current). This bound, which implies a high energy dissipation for fast, low-noise operations (such as switching a bit in a digital memory), brings together recent results of stochastic thermodynamics into a form that is usable for practical nonlinear electronic circuits, as we illustrate on a switching circuit made of an nMOS pass gate in a state-of-the-art industrial technology.

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Electronic circuits operate at an energetic cost—under the form of dissipation in resistive parts of the circuit—which brings technological or economical costs (need for cooling, large batteries, etc.) as well as ecological issues.[1] The search for more energy-efficient devices sometimes runs into the problem of reliability, as the intrinsic noise level may be become non-negligible at low voltages.[2–5] In parallel, there is a quest for theoretical lower bounds on dissipated energy and noise valid for a range of physical systems (electronic or else) fulfilling certain tasks, notably related to computation. One early such bound is Landauer's bound,[6] which states that erasing a bit necessarily dissipates an energy $kT \ln 2$, in whatever technology.

Stochastic thermodynamics[7] offers a recent framework where bounds beyond Landauer's can be formulated and proved rigorously for broad classes of physical systems. Such bounds include the thermodynamic uncertainty relations,[8,9] which state that stationary systems exhibiting a low level of noise must necessarily dissipate a lot—in other words, suppressing noise is costly. Another family of bounds, the (classical) speed limits,[10] provide specific trade-offs between the time of an operation (e.g., writing a bit into a memory) and its dissipation, quantifying precisely the well-known observation that fast operations cost more.

The existing speed limits are not always straightforward to use for electronic circuits, as they rely on assumptions on the form of noise (e.g., discrete jumps) not always satisfied in practice. A recent speed limit[11] is naturally applicable to electronic circuits but does not take noise into account, missing an essential component of the dissipation–time–noise trade-off.

We propose such a trade-off applicable to any linear or nonlinear resistive device, regardless of the noise model. It is a relationship between the total average charge passing through a nonlinear resistive device over a time interval, the energy dissipated in the device, and the total noise (variance) over that interval.

We suppose that the nonlinear device is purely resistive with negligible internal dynamics but is otherwise arbitrary. The device is embedded into an arbitrary circuit. For illustrative purposes and without loss of generality, we assume the simple circuit represented in Fig. 1. This circuit may, for instance, be interpreted as writing a bit from logical 0 to logical 1 by transferring a certain amount of charge into the capacitance.

A (nonlinear) resistive device at temperature T is intrinsically noisy due to the random agitation of the charge carriers (electrons). Indeed, when subjected to a constant voltage difference δv applied externally during a time $\mathrm{d}t$, it is traversed by a *random* charge Δq, dissipating on the way an energy $\Delta E_{\mathrm{dissip}} = \Delta q\, \delta v$. Charges during distinct time intervals are statistically independent, i.e., the noise on current is white (possibly Gaussian, or Poisson, or else).

For a linear device, mean and variance of Δq are related by the celebrated fluctuation–dissipation theorem,[12] which translates into the also famous Johnson–Nyquist formula[13,14] for a linear resistor, holding for all equilibrium ($\delta v = 0$) and non-equilibrium conditions.[15]

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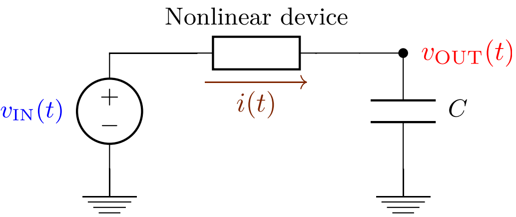

FIG. 1. Charging a capacitance through some (possibly nonlinear) dissipative noisy device.

Arbitrary nonlinear devices follow a more general relationship linking all moments, the fluctuation relation,[16] a consequence of which is the thermodynamic uncertainty relation:[8,17]

$$\frac{\mathbb{E}\{\Delta E_{\mathrm{dissip}}\}}{kT} \geq 2\,\frac{\mathbb{E}\{\Delta q\}^2}{\mathrm{Var}\{\Delta q\}}. \tag{1}$$

In (1), both the mean $\mathbb{E}\{\Delta q\}$ and the variance $\mathrm{Var}\{\Delta q\}$ are proportional to Δt (due to whiteness of the noise), hence both sides of (1) scale as $\propto \Delta t$. It was shown that the relation is tight (i.e., is an equality) if the random fluctuations are symmetric around the mean, e.g., for Gaussian fluctuations.[17]

When the voltage across the device is time-varying (non-stationary conditions, for instance, in a switching digital circuit) and random, we may still apply (1) to a small or infinitesimal interval $\mathrm{d}t$ within the whole time interval $[t_0, t_0 + \Delta t]$ and conditionally to a given value $\delta v(t)$. This leads naturally to, for any voltage difference $\delta v(t) = \delta v$ over any infinitesimal time interval $[t, t + \mathrm{d}t]$,

$$\frac{\mathbb{E}\{\mathrm{d}E_{\mathrm{dissip}}|\delta v(t)=\delta v\}}{kT} \geq 2\,\frac{\mathbb{E}\{\mathrm{d}q|\delta v(t)=\delta v\}^2}{\mathrm{Var}\{\mathrm{d}q|\delta v(t)=\delta v\}}. \tag{2}$$

Because we are interested in the unconditional mean dissipation $\mathbb{E}\{\mathrm{d}E_{\mathrm{dissip}}\} = \mathbb{E}_{\delta v}\mathbb{E}\{\mathrm{d}E_{\mathrm{dissip}}|\delta v(t) = \delta v\}$ (where $\mathbb{E}_{\delta v}$ refers to averaging over all values of δv), we may write

$$\frac{\mathbb{E}\{\mathrm{d}E_{\mathrm{dissip}}\}}{kT} \geq 2\,\mathbb{E}_{\delta v}\left\{\frac{\mathbb{E}\{\mathrm{d}q|\delta v(t)=\delta v\}^2}{\mathrm{Var}\{\mathrm{d}q|\delta v(t)=\delta v\}}\right\}. \tag{3}$$

However, we would like to obtain an expression involving unconditional means and variances, more accessible and interpretable than their conditional variants.

To that purpose, we can define a scalar product between arbitrary (square integrable) real-valued functions $f(\delta v)$ and $g(\delta v)$,

$$\langle f|g\rangle = \mathbb{E}_{\delta v}\{f(\delta v)g(\delta v)\}. \tag{4}$$

As all scalar products, it satisfies the well-known Cauchy–Schwarz inequality $\langle f|f\rangle \geq \langle f|g\rangle^2/\langle g|g\rangle$. Let us define $f \equiv \mathbb{E}\{\mathrm{d}q|\delta v(t) = \delta v\}/\sqrt{\mathrm{Var}\{\mathrm{d}q|\delta v(t) = \delta v\}}$ and $g \equiv \sqrt{\mathrm{Var}\{\mathrm{d}q|\delta v(t) = \delta v\}}$ and apply Cauchy–Schwarz inequality. We recognize $\langle f|f\rangle$ as the r.h.s. of (3), up to the factor of 2. Furthermore, we observe from the law of total variance that

$$\begin{aligned}\mathrm{Var}\{\mathrm{d}q\} &= \mathbb{E}_{\delta v}\mathrm{Var}\{\mathrm{d}q|\delta v(t)=\delta v\} + \mathrm{Var}_{\delta v}\mathbb{E}\{\mathrm{d}q|\delta v(t)=\delta v\}\\ &= \mathbb{E}_{\delta v}\mathrm{Var}\{\mathrm{d}q|\delta v(t)=\delta v\} + \mathcal{O}(\mathrm{d}t^2),\end{aligned}$$

the latter term being, thus, negligible. We finally get

$$\frac{\mathbb{E}\{\mathrm{d}E_{\mathrm{dissip}}\}}{kT} \geq 2\,\frac{\mathbb{E}\{\mathrm{d}q\}^2}{\mathrm{Var}\{\mathrm{d}q\}}. \tag{5}$$

Integrating (5) over the whole time interval $[t_0, t_0 + \Delta t]$, we obtain a lower bound on dissipation, which is our first main result

$$\frac{\mathbb{E}\{\Delta E_{\mathrm{dissip}}\}}{kT} \geq 2\int_{t_0}^{t_0+\Delta t}\frac{\mathbb{E}\{\mathrm{d}q\}^2}{\mathrm{Var}\{\mathrm{d}q\}}. \tag{6}$$

In (6), $\mathbb{E}\{\Delta E_{\mathrm{dissip}}\}$ is the average *energy dissipation* over the whole time interval $[t_0, t_0 + \Delta t]$, i.e.,

$$\mathbb{E}\{\Delta E_{\mathrm{dissip}}\} = \int_{t_0}^{t_0+\Delta t}\mathbb{E}\{\mathrm{d}E_{\mathrm{dissip}}\} = \int_{t_0}^{t_0+\Delta t}\mathbb{E}\{\mathrm{d}q(t)\,\delta v(t)\}\,\mathrm{d}t. \tag{7}$$

We can relax further this inequality by applying again the Cauchy–Schwarz inequality. We now use the scalar product $\langle f|g\rangle' = \int_{t_0}^{t_0+\Delta t} f'g'\mathrm{d}t$ on real-valued functions $f'(t)$ and $g'(t)$ of time over the interval $[t_0, t_0 + \Delta t]$. We apply it to $f'(t) \equiv \mathbb{E}\{\mathrm{d}q(t)\}/\sqrt{\mathrm{Var}\{\mathrm{d}q(t)\}}$ and $g'(t) \equiv \sqrt{\mathrm{Var}\{\mathrm{d}q(t)\}}$ to obtain

$$\frac{\mathbb{E}\{\Delta E_{\mathrm{dissip}}\}}{kT} \geq 2\,\frac{\mathbb{E}\{\Delta q\}^2}{\int_{t_0}^{t_0+\Delta t}\mathrm{Var}\{\mathrm{d}q\}} = 2\,\frac{\mathbb{E}\{\Delta q\}^2}{\overline{\mathrm{Var}}\{\mathrm{d}q\}\,\Delta t}, \tag{8}$$

which is our second main result. The time-averaged variance $\overline{\mathrm{Var}}\{\mathrm{d}q\}$ is $\int_{t_0}^{t_0+\Delta t}\mathrm{Var}\{\mathrm{d}q\}/\Delta t$. Although this bound is not as tight as (6), it may prove easier to evaluate and interpret in many cases. In particular, $\Delta q = \int_{t_0}^{t_0+\Delta t}\mathrm{d}q$ in the numerator is the total charge passed through the device over the interval, which is usually the quantity of interest.

In summary, for a given Δq, fast charge transfer (low Δt) and/or low-noise process (small $\overline{\mathrm{Var}}\{\mathrm{d}q\}$) implies large dissipation. The relationship (8) can, indeed, be seen as a novel *speed limit* relation: passing a certain charge Δq over a device with a typical level of noise $\overline{\mathrm{Var}}\{\mathrm{d}q\}$ within a duration Δt necessarily dissipates an energy that is inversely proportional to Δt. This speed limit differs from other recent speed limits recently obtained that also obtain a $1/\Delta t$ behavior. For instance, most speed limits[18] only apply to discrete jumps of charges (e.g., shot noise), while we cover all noise sources, discrete or continuous with a single formula. Let us also mention the recent deterministic speed limit[11]

$$\Delta E_{\mathrm{dissip}} \geq \Delta E_{\mathrm{dissip,min}} \equiv \frac{\Delta q^2}{\bar{G}\,\Delta t}, \tag{9}$$

which does not directly take noise into account. In (9), $\bar{G}$ denotes the time-averaged conductance of the driving device.

Finally, we also see that for a given Δt, a low noise level (low variance) can only be obtained at the cost of high dissipation. Our bounds (8) and (6), thus, express a trade-off between speed, dissipation, and noise for an arbitrary nonlinear resistive device.

Our preliminary application is the case where the driving device in Fig. 1 is a linear resistor of conductance G, which can be covered in detail analytically. From Ohm's law, $\mathbb{E}\{dq(t)|\delta V(t) = \delta V\} = G\delta V dt$. Johnson–Nyquist's formula[13,14] is an electrical equivalent of Einstein's diffusion law,[19] and an avatar of the general fluctuation–dissipation theorem,[12] stating here that $\mathrm{Var}dq(t) = 2kTGdt$.

Note in passing that Johnson–Nyquist's formula is often expressed in the electronic literature for the stationary random current

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$i(t) = dq/dt$ for a constant δV. This current is defined (as a random function of time) only in a weak sense, i.e., if we limit it to a finite frequency bandwidth Δf. The variance is then $\mathrm{Var}\{i(t)\} = 4kTG\Delta f$. In this case, we can interpret the current defined by $i(t) = dq(t)/dt$ (for a small, non-infinitesimal dt) as being limited to the frequency band $[0, 1/2\mathrm{d}t]$.

The system is, thus, a linear system with an external driving $v_{IN}(t)$ and an internal noise, described by a linear stochastic differential equation (a Langevin equation more precisely), which can be solved explicitly. In this context, (2) is satisfied with equality, and our main result (8) becomes

$$\Delta E_{\mathrm{dissip}} \geq \Delta E_{\mathrm{dissip,min}} = \frac{\mathbb{E}\{\Delta q\}^2}{G\Delta t}, \tag{10}$$

which is precisely (9), the minimum dissipation from the deterministic speed limit,[11] for a constant capacitance load and by identifying $\bar{G} = G$. In the linear case, thanks to Johnson–Nyquist's noise model (a specific case of the fluctuation–dissipation theorem) turning (1) and (2) into equalities, our bound coincides with a deterministic treatment.[11] We shall now see it is not the case in a nonlinear case.

Because the scope and the significance of our main theoretical results (8) and (6) are only fully appreciated in circuits involving a nonlinear device, we illustrate their application to the circuit of Fig. 2: a constant capacitor is charged through a pass gate, implemented with a MOS (metal–oxide–semiconductor) transistor. The simulated signals and extracted quantities are depicted in Fig. 3. The charging process is, without loss of generality, here ensured by the linear input voltage ramp of amplitude $V_{\mathrm{DD}} = 1$ V sketched in Fig. 3(a), corresponding to the supply voltage of the used CMOS technology. The bit writing operation is assumed finished when $v_{\mathrm{OUT}}(t)$ reaches $V_1 = 80\%\ V_{\mathrm{DD}}$ [shown in Fig. 3(a)].

In the most advanced technologies favored for digital circuit design,[20] the noise model of the transistor can be very complex (since, in the charge-based modeling approach, it depends on the small-dimension effects part of the deterministic static model[21]) and not known in an insightful closed form. We, therefore, avoided to resort to a simplified and inaccurate analytical model of the transistor and we have performed SPICE noise simulation in the time domain (Fig. 3), compatible with the process design kit provided by the semiconductor foundry.[22]

Industrial simulators, such as Eldo®, provide a transient noise analysis tool.[5] All noise sources are modeled as Gaussian, irrespective of existing discussions of the physics origin of the thermal noise and

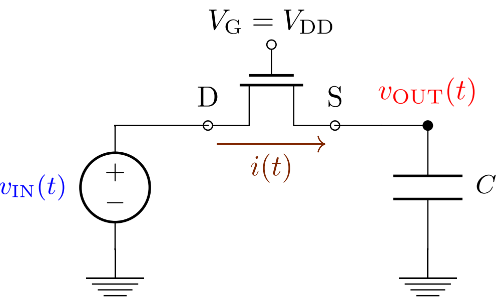

FIG. 2. nMOS pass gate driving a capacitor.

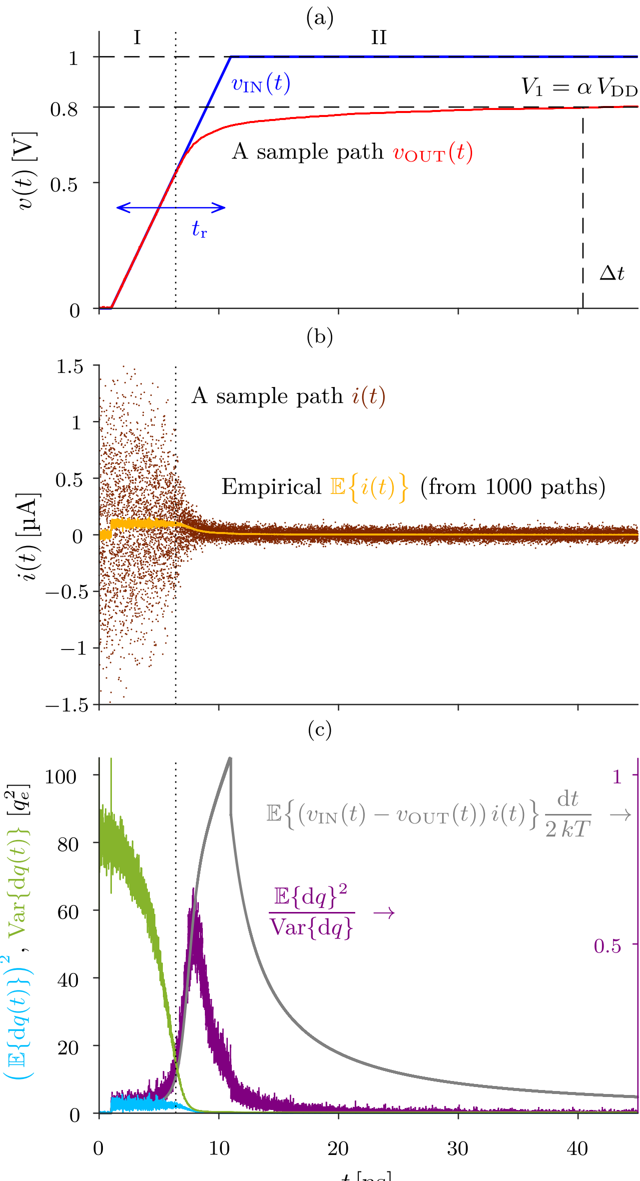

FIG. 3. Transient noise simulation of the bit writing. Circuit parameters: 28 nm FD-SOI regular-V_{th} nMOS of minimal dimensions $L = 30$ and $W = 80$ nm; $T = 25\,°$C; $V_{\mathrm{DD}} = 1$ V; $C = 1$ fF; and input rise time $t_r = 10$ ns. Bandwidth of generated noise: $f_{\max} = 200$ GHz and $\mathrm{d}t = 2.5$ ps. (a) Voltage signals; (b) current trace; (c) statistics of $\mathrm{d}q(t)$ empirically *estimated* from 1000 sample paths (left); and normalized dissipation vs ratio $\mathbb{E}\{\mathrm{d}q\}^2/\mathrm{Var}\{\mathrm{d}q\}$ of (6) (right).

the thermodynamic inconsistency of the pure Gaussian model.[15,17] Individual noise source are dynamically generated in the time domain according the compact model of the device, and the noise is then handled as any other electrical signal during a transient simulation.

The intrinsic noise is the noise generated by the MOS transistor of Fig. 2. Within SPICE formalism, the intrinsic noise of the MOS transistor, which is the noise produced by the device itself, is modeled

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as a current Gaussian fluctuation. One sample path of $i(t)$ (among 1000 generated and processed) is represented in Fig. 3(b), as well as the empirical mean. Whereas the physical white noise is, neglecting quantum effects, of infinite bandwidth, the noise is generated within a specified finite bandwidth, denoted f_{max}, for the transient simulation. The f_{max} must be selected in order to capture the dominant effect of the thermal noise on the charge transfer, extracted as about 200 GHz, beyond the bandwidth of the circuit of Fig. 2 for different v_{IN} from 0 to 1V.

We have recorded 1000 sample paths (or *trajectories* or *traces*) of the voltage and current signals simulated in the time domain. A specified f_{max} imposes a constraint on the (maximum) simulation time step (dt) to be used:[5] $\mathrm{d}t = 1/(2f_{max})$, according to Shannon–Nyquist sampling theorem. The time step $\mathrm{d}t = 2.5$ ps is small enough to be numerically assimilated to an infinitesimal time interval, both to compute the charge increment

$$\mathrm{d}q(t) = i(t)\,\mathrm{d}t = C\,(v_{\mathrm{OUT}}(t+\mathrm{d}t) - v_{\mathrm{OUT}}(t)),$$

and the different integral quantities. The sample mean and variance [of $i(t)$ in Fig. 3(b) and of $\mathrm{d}q(t)$ in Fig. 3(c)] are unbiased empirical estimators,[23] and the variability observed for these statistics in Figs. 3(b) and 3(c) could of course be reduced at the expense of an increased number of simulated paths.

In Fig. 3(c), we first depict the empirical estimators of the two important quantities of (6), $\mathbb{E}\{\mathrm{d}q(t)\}^2$ and $\mathrm{Var}\{\mathrm{d}q(t)\}$ (left y axis). Generally, we expect from the relations (8) and (6) that the ratio $\mathbb{E}\{\mathrm{d}q\}^2/\mathrm{Var}\{\mathrm{d}q\}$ predicts the trend of the evolution over time of the dissipation, i.e., the dissipation is locally large when the intrinsic noise fluctuations (relative to the average instantaneous charge transfer) is low (and conversely). The discrepancy over time between the actual energy dissipation and the ratio $\mathbb{E}\{\mathrm{d}q\}^2/\mathrm{Var}\{\mathrm{d}q\}$ (right y axis) tells us how tight we can expect the lower bounds (8) and (6) to be (see coming results) and thereby assesses the energy efficiency of the charge transfer through the nonlinear dissipative device (of some intrinsic noise level) for the considered input signal and speed.

By inspecting first Fig. 3(a), we can basically distinguish two regimes of the different energy efficiency for the charge transfer, characterized by a larger or lower conductance of the nonlinear driving device. For the (important) special case of the MOS transistor (Fig. 2), the regime may be referred to as *inversion level* (or *region*).[21] In the first (I) regime, the charging process is efficient in the sense that $v_{\mathrm{OUT}}(t)$ closely follows $v_{\mathrm{IN}}(t)$ (the so-called quasi-adiabatic conditions[24,25]) and the dissipation is low. For the case illustrated in Fig. 3, the speed of the charging process, $\mathrm{d}q/\mathrm{d}t$, is low and constant during regime I, which simplifies the interpretation of the general results (8) and (6). The transistor is in strong inversion, meaning that it is highly conductive and also that the intrinsic current noise is large.[21] This is, indeed, what we observe in Figs. 3(b) and 3(c): $\mathrm{Var}\{i(t)\}$ [or $\mathrm{Var}\{\mathrm{d}q(t)\}$] is large and drops gradually (while $\mathbb{E}\{\mathrm{d}q(t)\}^2$ does not vary much). In the second (II) regime, the transistor falls in weaker inversion (low conductance) and $v_{\mathrm{OUT}}(t)$ painfully follows $v_{\mathrm{IN}}(t)$[11] ("the nMOS is not good at passing a 1"[20]). The dissipation is large [see the gray shark-fin-shaped peak in Fig. 3(c)] as evidenced by the high-amplitude intrinsic noise fluctuations in this regime. We believe that this circuit application highlights the existing correlation between fluctuation and dissipation trend, yet in dynamic and nonlinear conditions. Finally, let us emphasize that, in Fig. 3(c), the discrepancy between actual dissipation and $\mathbb{E}\{\mathrm{d}q\}^2/\mathrm{Var}\{\mathrm{d}q\}$ becomes significant after 10 ns, precisely when the charge transfer gets even more inefficient.

TABLE I. Actual energy dissipation compared to the different lower bounds derived or referred in this paper. All the quantities were extracted from the simulation of Fig. 3, notably $\Delta t \approx 40$ ns defined at $V_1 = 80\%\ V_{\mathrm{DD}}$.

Energy dissipation	Value (kT)
Actual $\mathbb{E}\{\Delta E_{\mathrm{dissip}}\}$	
Equation (7)	6919
Speed limit bounds of this article	
Integral Eq. (6)	1568
Equation (8) over $[t_0, t_0 + \Delta t]$	375
Deterministic speed limit[11]	
Equation (9)	213

In Table I, we summarize the dissipation-related quantities extracted from the transient simulation presented in Fig. 3. The expected or average dissipation, computed according to the definition (7), is the reference value to which we compare our different lower bound. These are listed in the ascending order in Table I, which is consistent with their order of appearance in the text.

The integral bound (6) is lower than the actual dissipation only by a factor of 4.4. At the light of Fig. 3(c), we have attributed such discrepancy to the second regime of the charging process, where the nMOS transistor becomes highly inefficient in fully passing a logical 1. The worsened conductivity (larger dissipation) is correlated with a lower noise level, reality reflected in the relationship (6).

Importantly, the reported discrepancy reveals the non-Johnson–Nyquist nature of the noise fluctuation–dissipation process of the nonlinear device[17] (as opposed to the linear resistor). We know that the shot noise model is consistent with a MOS transistor operation in weak inversion.[21] In the case of shot noise,[17] the relation (1) is loose by a factor of 4 for a voltage difference $\delta V \approx 200$ mV (i.e., about eight times the thermal voltage kT/q_e, see Fig. 1 in Ref. 17) which is broadly consistent with our numerical observations. This shows that the fact that even though our bound holds for any model of noise, different models will make the bound more or less tight.

The lower bound (8) is, as announced earlier and here verified experimentally, less tight than (6) as lying more than one order of magnitude below the dissipation.

To conclude, we have exploited the theoretical framework of the stochastic thermodynamics to propose noise–dissipation relations valid for in non-equilibrium and non-stationary conditions, relevant for switching digital circuits that are strongly nonlinear. Two different lower bounds were provided for the energy dissipation. The relations involve the time-varying statistics of the noise over the charge transfer process. We have applied and discuss them over linear and nonlinear dipoles. The quantitative analyses deduced from the simulations are insightful about the physical origin of the noise, that cannot merely be pure Johnson–Nyquist noise for nonlinear dissipative devices.[17] Further work would deepen this aspect, in link with recent noise measurement and modeling work,[2,5] and would extend the mathematical

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formalism to more complex circuits with multiple dissipative and dynamic components (e.g., CMOS logic gates).

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Léopold Van Brandt: Conceptualization (equal); Data curation (equal); Formal analysis (equal); Funding acquisition (equal); Investigation (equal); Methodology (equal); Software (equal); Validation (equal); Writing – original draft (equal); Writing – review & editing (equal). **Jean-Charles Delvenne:** Conceptualization (equal); Data curation (equal); Formal analysis (equal); Funding acquisition (equal); Investigation (equal); Methodology (equal); Project administration (equal); Resources (equal); Supervision (equal); Validation (equal); Writing – original draft (equal); Writing – review & editing (equal).

DATA AVAILABILITY

The data that support the findings of this study are available within the article.

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