

Review Article

Beyond bifurcation: using complex models to understand and predict abrupt climate change

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Received 25 April 2016; Revised 4 September 2016; Accepted 16 November 2016

Abstract

Research on the possibility of future abrupt climate change has been popularized under the term ‘tipping points’ and has often been motivated by using simple, low-dimensional concepts. These include the iconic fold bifurcation, where abrupt change occurs when a stable equilibrium is lost, and early warning signals of such a destabilization that can be derived based on a simple stochastic model approach. In this paper, we review the challenges and limitations that are associated with this view, and we discuss promising research paths to explore the causes and the likelihood of abrupt changes in future climate.

We focus on several climate system components and ecosystems that have been proposed as candidates for tipping points, with an emphasis on ice sheets, the Atlantic Ocean circulation, vegetation in North Africa and Arctic sea ice. In most example cases, multiple equilibria found in simple models do not appear in complex models or become more difficult to find, while the potential for abrupt change still remains. We also discuss how the low-dimensional logic of current methods to detect and interpret the existence of multiple equilibria can fail in complex models. Moreover, we highlight promising methods to detect abrupt shifts and to obtain information about the mechanisms behind them. These methods include linear approaches such as statistical stability indicators and radiative feedback analysis as well as non-linear approaches to detect dynamical transitions and infer the causality behind events.

Given the huge complexity of comprehensive process-based climate models and the non-linearity and regional peculiarities of the processes involved, the uncertainties associated with the possible future occurrence of abrupt shifts are large and not well quantified. We highlight the potential of

data mining approaches to tackle this problem and finally discuss how the scientific community can collaborate to make efficient progress in understanding abrupt climate shifts.

1. Introduction

Linear approaches have been very successful in many fields of science. They often fail, however, to describe discontinuities, abrupt events and persistent catastrophic shifts. Such phenomena occur only from time to time, yet they lurk in many complex systems like financial markets, the human brain, ecosystems (Scheffer et al., 2001) and the climate system (Rial et al., 2004). Climate research faces this challenge in an environment of societal urgency, as the dramatic increase in atmospheric carbondioxide (CO_2) concentrations is driving our climate to a state where the consequences are hard to foresee. An abrupt reorganization of parts of the climate system on top of progressive global warming could be particularly unpleasant if it occurs much faster than society or ecosystems can adapt. To this extent, the political goal to prevent dangerous climate change comprises not only an absolute limit of global warming like the two-degree target but also the avoidance of abrupt events (Smith et al., 2009).

Since the discovery of abrupt climate change in ice cores (Dansgaard et al., 1993; Johnsen et al., 1992), there have been a number of review articles on the mechanisms behind these changes (Broecker, 2006; Clement and Peterson, 2008; Marotzke, 2000; Overpeck and Cole, 2006) and the ramifications for abrupt change in the future (Alley et al., 2003; McNeall et al., 2011). A number of components in the climate system with the potential to undergo abrupt change have been identified mostly in simple models and are now often referred to as climate tipping elements (Lenton et al., 2008; Levermann et al., 2011). Each tipping element is conjectured to undergo abrupt change when an external parameter reaches a certain critical level, the ‘tipping point’. In this paper, we review several climate system components that have been discussed as candidates for tipping points and outline our perspective on some promising research paths to explore the causes and the likelihood of abrupt changes in future climate.

Interestingly, in many cases, abrupt climate change was first hypothesized based on models, before reconstructions had revealed that such changes occurred in reality. Among these models were idealized representations of the Atlantic meridional overturning circulation (AMOC) (Stommel, 1961), global ice coverage (Budyko, 1969; Sellers, 1969), vegetation–atmosphere interaction in North Africa (Brovkin et al., 1998) and the Asian monsoons (Levermann et al., 2009; Zickfeld, 2005). Each of these models describes a positive feedback that results from two processes enhancing each other, e.g. ice loss and warming or vegetation loss and drying. However, positive feedbacks do not necessarily imply a non-linear response or abrupt change, which requires the balance of feedbacks to be state dependent. In the models mentioned above, such state dependence is often due to bounded variables like surface cover fractions or to non-linear processes. If a positive feedback is very strong, alternative stable states can exist under the same external conditions (Fig. 1). The parameter values at which the system’s dynamics suddenly changes are called bifurcation points. Points L_1 and L_2 in Figure 1 represent a very common type of a bifurcation, the saddle-node bifurcation. When a system that is close to a stable equilibrium is driven over one of these points, the current stable equilibrium and an unstable equilibrium annihilate each other, and an abrupt and irreversible shift towards a different stable equilibrium can occur.

It is no coincidence that we find this iconic behaviour in many simple models: If a non-linear system is simplified by neglecting high-order terms, only a small number of generic and structurally robust bifurcations emerge (Guckenheimer and Holmes, 1983; Kuehn, 2013) and only some of them are catastrophic, i.e. allow for abrupt change (Thompson and Sieber, 2011). Despite this generic mathematical behaviour, it is obvious that reality is much more complicated, e.g. because of spatial heterogeneity and temporal variability. In this review, we therefore look at the problem from a different perspective by focusing on the phenomenon of abrupt change itself and the challenge of probing a complex model to understand its occurrence. We review promising research pathways that can help achieve this without falling into the traps of the low-dimensional paradigm.

Not surprisingly, such a phenomenological approach to abrupt climate change is to some extent subjective. In principle, two approaches can be distinguished. The first approach addresses the climatic response to a gradual change in conditions that are considered as external to the system, like orbital forcing or anthropogenic CO_2 increase. A change is considered abrupt if the climatic change suddenly accelerates at a certain point due to mechanisms internal to the climate system and despite an only gradual change in forcing (National Research Council, 2002; Rahmstorf, 2008).

While this definition is useful in the context of anthropogenic climate change, it neglects phenomena that are unforced or where the forcing is not well known. To this end, an alternative purely phenomenological definition is helpful, which classifies abrupt change as an event that separates two rather stationary episodes with a distinctly different climate (Collins et al., 2013). In agreement with this alternative definition, shifts that are due to natural low-frequency climate variability are sometimes also presented as abrupt changes (Narisma et al., 2007; Overpeck and Cole, 2006). Although the term abrupt is not always compelling in the case of natural variability, this alternative definition reminds us that the contributions of forcing and variability cannot be clearly separated (e.g. because external forcings also affect climate variability). We therefore adopt the alternative definition in this article, while noting that our main interest is the anthropogenic influence on the occurrence of abrupt shifts.

We motivate our perspective by reviewing a number of particularly insightful candidates of tipping elements (Section 2), with a focus on terrestrial ice sheets, the AMOC, the Green Sahara and Arctic sea ice. We briefly review how knowledge on these systems has evolved and consider what can be learned from them. We then outline the practical consequences of this perspective by reviewing common methods to find alternative stable states (Section 3), to detect abrupt change (Section 4.1) and to infer the mechanism behind abrupt change (Section 4.2–4.3). Section 5 illuminates the challenges arising from the models' uncertainties and what this means for an assessment of the uncertainty of future abrupt change. Finally, Section 6 outlines how the scientific community may cope with this uncertainty and provides our conclusions.

2. Lessons from suspected tipping elements in the Earth system

2.1. Atlantic meridional overturning circulation

The collapse of the AMOC is arguably the most popular candidate of a climate tipping point. The main positive feedback that makes the AMOC sensitive to freshwater input (e.g. meltwater from the continents) is the salt-advection feedback, identified first in a conceptual box model (Stommel, 1961): As the surface salinity of sea water is larger in the tropics than in the high latitudes, and as salt tends to make the water denser, the transport of salty water to the north enhances the sinking of water in high latitudes, which in turn enhances the circulation and therefore the supply of more salty water. The well-understood ingredients of this feedback (the transport of salt by the circulation, the density dependence on salinity and the fact that the circulation is driven by density differences) cause this to be a robust feedback in the Atlantic Ocean circulation. In relatively simple ocean-only and ocean-climate models, this feedback causes the existence of a multiple equilibrium regime and hence the simultaneous existence of on and off states of the AMOC under the same atmospheric forcing conditions, with a similar bifurcation diagram as Figure 1.

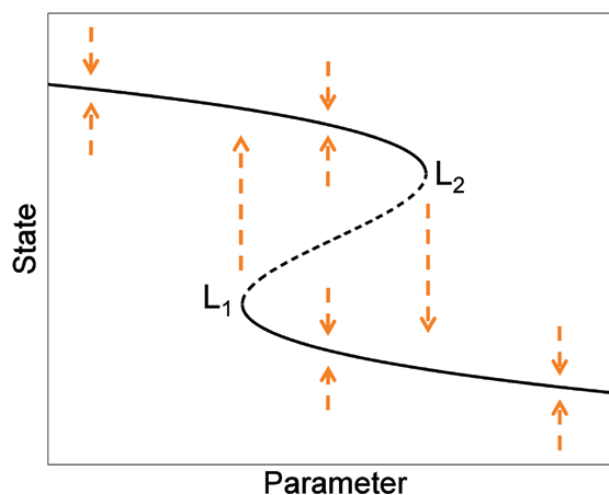


Figure 1. Sketch of the occurrence of multiple states and bifurcation points. The equilibria of the system's state are shown as black lines (continuous: stable; dashed: unstable), in dependence on a certain parameter. The flow towards a stable state is shown as dashed orange arrows; L_1 and L_2 indicate the saddle-node (or fold) bifurcation points, also referred to as limit or turning points.

The saddle-node bifurcations associated with the AMOC transition can be explicitly calculated in ocean-only general circulation models (GCMs) (Dijkstra and Weijer, 2005). Signatures of multiple equilibrium behaviour have also been identified in early coupled ocean–atmosphere GCMs (Manabe and Stouffer, 1988), although the realism of these simulations was questioned because early coupled models did not produce a realistic climate without imposing correction terms. The most sophisticated GCMs where indications for multiple equilibria of the AMOC have been found are the FAMOUS model (Hawkins et al., 2011) and the HadGEM3 model (Mecking et al., 2016). Nonetheless, such multiple steady states have not been found in the state-of-the-art climate models used in the most recent phase of the Coupled Model Intercomparison Project (CMIP5), under greenhouse gas emission scenarios for the 21st century (Collins et al., 2013). However, it may well be that different forcings or improved models are required to find multiple AMOC states. In this regard, the necessary computations have not yet been performed. We will return to the problem of finding multiple states in Section 3.

There are three arguments based on recent results to support the statement that the AMOC may be much more sensitive to freshwater anomalies than the GCMs used in CMIP5 indicate. The first point concerns the bias in the Atlantic freshwater budget of the GCMs. Because of this bias, there is only one stable equilibrium AMOC state, as the AMOC exports salt out of the Atlantic basin and hence no transition to a collapsed state can occur (Drijfhout et al., 2011). However, from observations (and also reanalysis results), the present-day AMOC appears to export freshwater and hence a stable off state may also exist (Bryden et al., 2011; Hawkins et al., 2011). The second argument is that ocean–atmosphere feedbacks are too weak to remove the multiple equilibrium regime (den Toom et al., 2012). The third argument is that when ocean vortices on smaller scales (so-called eddies) are taken into account, the response of the AMOC to freshwater anomalies turns out to be stronger than in the lower resolution (non-eddy) ocean models, such as those used in CMIP5 (Weijer et al., 2012).

There are many indications that there have been large-scale reorganizations of both the atmosphere and ocean associated with Dansgaard–Oeschger events. These events consist in large, abrupt shifts identified in ice core records from Greenland and are a prominent feature of the millennial climate variability during the last glacial period. In the sub-polar North Atlantic, Dansgaard–Oeschger events were matched with corresponding sea surface temperature increases of at least 5°C. As discussed in Clement and Peterson (2008) and Crucifix (2012), several different views have been proposed to explain Dansgaard–Oeschger events, but all leading theories involve changes in the Atlantic Ocean circulation. Plausible explanations interpret Dansgaard–Oeschger events as transitions between different AMOC states, and the mathematical phenomena behind these transitions are known as stochastic resonance (Ganopolski and Rahmstorf, 2002), coherence resonance (Timmermann et al., 2003) or simply noise-induced transitions (Ditlevsen et al., 2005).

2.2. Arctic sea ice

The observed loss of Arctic sea ice in recent years, and the fact that this loss was faster than models predicted (Stroeve et al., 2007), has fuelled investigations into the possibility of a tipping point in Arctic sea ice (Lindsay and Zhang, 2005; Notz, 2009). Essentially, two bifurcation scenarios can be distinguished. The first scenario is an abrupt summer ice loss, a transition from a perennial ice cover directly to an ice-free ocean. The second scenario is a gradual loss of summer sea ice to a seasonally ice-covered Arctic, with an abrupt loss of the remaining winter sea ice thereafter. In both cases, an essential positive feedback is the ice-albedo feedback, i.e. the mutual reinforcement of ice loss and warming due to the very different reflectivity of ice and water. In some models, a feedback involving changes in cloud cover is also essential to the existence of bifurcations (Abbot et al., 2011).

Arctic sea ice is a particularly insightful case for understanding the role of spatial and temporal resolution for the system's stability. This is because the physics behind the growth and melt of sea ice are relatively simple, and hence a variety of models of different complexity has been established. In principle, two types of reduced complexity models can be distinguished: energy balance models, resolving latitudinal but no seasonal differences (e.g. Budyko, 1969; North, 1975; Sellers, 1969) and single-column models, often resolving a seasonal cycle but no spatial differences (e.g. Eisenman and Wettlaufer, 2009; Thorndike, 1992). Besides the so-called Snowball Earth instability, where the whole planet becomes ice covered, energy balance models often produced an additional “small ice cap instability” (North, 1975, 1984). Interestingly, Lindzen and Farrell (1977) argued early on that this result was a model artefact arising from the oversimplified nature of heat transport.

Later on, a number of single-column models suggested the scenario of an abrupt summer sea ice loss (Abbot et al., 2011; Flato and Brown, 1996; Merryfield et al., 2008; Thorndike, 1992). However, this behaviour can be attributed

to the limited resolution of space and time in the models. For example, it is important to resolve the annual cycle sufficiently because the balance of feedbacks depends on the season. Hence, the bifurcation in summer sea ice disappears in a model when the annual cycle is better resolved (Moon and Wettlaufer, 2012). Consequently, the transition to a summer without any sea ice is gradual in comprehensive models (Overland and Wang, 2013; Wang and Overland, 2012). Initializing such a comprehensive climate model from an ice-free summer results in a fast recovery due to stabilizing feedbacks in winter (Tietsche et al., 2011).

The second bifurcation scenario, the abrupt loss of winter sea ice, still appeared in a column model with a well-resolved annual cycle (Eisenman and Wettlaufer, 2009). However, in a spatially explicit model version, the diffusive heat transport between different latitudes tends to stabilize sea ice cover and removes the bifurcation (Wagner and Eisenman, 2015b). Wagner and Eisenman (2015b) also showed that the model is much more stable if both seasonal and latitudinal variations are considered, even if one of these variations is unrealistically small. Again, comprehensive climate models are in agreement with these results. In several of these complex models it was explicitly shown that the total loss of Arctic sea ice is reversible (Armour et al., 2011; Boucher et al., 2012; Li et al., 2013; Ridley et al., 2012). There is thus an emerging consensus that no bifurcation-induced abrupt loss of Arctic sea ice is to be expected in the future.

However, the absence of multiple sea ice states in CMIP5 models does not necessarily rule out abrupt sea ice loss. In two of these models, a winter ice area of several million square kilometres disappears within only a few years. In the rest of the models, Arctic winter sea ice area decreases more gradually, but it is still more sensitive to warming than summer ice area (Bathiany et al., 2016a). The reason is that the freezing point introduces a threshold behaviour that can result in a rapid loss of Arctic sea ice. In contrast to the feedback-induced abrupt loss in simple models, the threshold-induced loss is reversible. While complex models agree on the existence of this mechanism, they disagree on how fast Arctic winter sea ice area can disappear. Moreover, multiple sea ice states do not only occur in oversimplified models. For example, Marotzke and Botzet (2007) found a stable, globally ice-covered state in a comprehensive GCM with the current continental distribution. Besides this ‘snowball’ state, two additional stable sea ice states have been found in a GCM with a so-called aqua-planet setup where the whole globe is covered with water (Ferreira et al., 2011; Rose et al., 2013). These states are associated with the meridional pattern of the ocean heat transport, which may suggest the possibility of an interplay between multiple sea ice states and multiple AMOC states. The possibility of multiple states under different continental distributions than today and the large sensitivity of seasonal ice cover mentioned above both indicate that abrupt and large-scale changes in sea ice cover could have occurred in the Earth’s past.

2.3. Ice sheets

The case of Arctic sea ice in the previous section already indicated the importance of external drivers that change over time and question the concept of equilibrium. The great ice ages may be viewed as the manifestation of a particularly dramatic mode of variability of the climate system. The deep sea record analysed by Broecker and van Donk (1970) has revealed the sawtooth character of the latest four ice ages. Each of them followed a cycle of about 100 000 years, with a gradual ice accumulation phase, followed by a catastrophic deglaciation. Spectral analysis of additional records (Hays et al., 1976; Shackleton et al., 1990) subsequently confirmed without ambiguity a signature of the ‘orbital forcing’ (the change in solar insolation due to the variations in the relative position between the Earth and the Sun). More specifically, effects of precession and obliquity on the ice ages dynamics were identified above a continuous background of variability (see also Berger, 1977). Longer deep sea records also highlighted changes in the regime of these oscillations, with a transition around 1 million years ago towards longer, higher amplitude and more non-linear ice age cycles (Lisiecki and Raymo, 2007; Ruddiman et al., 1986). Finally, Antarctic ice core records have revealed that greenhouse gas concentrations varied in concert with temperature, amplifying the cooling during ice ages (Genthon et al., 1987; Luthi et al., 2008; Petit et al., 1999). These six elements (high amplitude glacial variations, asymmetric cycles with abrupt termination, orbital signature, background spectrum, regime change and synchronous greenhouse gas variations) constitute the empirical basis to be addressed by ice age models.

Multistability has been presented as a consequence of the ice-albedo feedback: Extensive ice sheets reflecting sunlight become resilient to small increases in incoming solar radiation (Budyko, 1969; Sellers, 1969). In the works of Budyko and Sellers, the ‘parameter’ of Figure 1 is the solar constant. An improved multistability bifurcation diagram may be obtained by accounting more specifically for the geometry of ice sheets and the parameter is the altitude of

the ‘snow line’ (Weertman, 1976). This framework featuring the two coexisting stable states has remained in use for interpreting experiments with more sophisticated models (Abe-Ouchi et al., 2013; Calov and Ganopolski, 2005; Oerlemans, 1983), except that the control parameter is taken to measure incoming solar radiation during the summer solstice, following Milankovitch’s (1941) theory. However, MacAyeal (1979) warned that the catastrophic character of the deglaciation prompts us to consider at least another control parameter, of which the role is to represent the build-up of ‘potential action’ for a deglaciation, similar to what happens in so-called relaxation oscillations. This kind of non-linearity can easily be captured with continuous dynamical systems (Saltzman and Maasch, 1988) and hybrid dynamical systems (Paillard, 1998). The framework may even be extended to explain regime changes such as the Mid-Pleistocene transition, where the typical frequency of ice age cycles changed (Ashwin and Ditlevsen, 2015).

There is a spread of views about the physical nature of the potential action. Several authors have pointed out various effects associated with ice sheet dynamics, their interactions with the bedrock and ice sheet margins that may be responsible for a runaway deglaciation (Abe-Ouchi et al., 2013; MacAyeal, 1979; Pollard, 1982). Accumulation of dust on ice sheets may also constitute a significant destabilizing effect (Ganopolski and Calov, 2011). Another thread of literature gives a more prominent role to CO₂ dynamics and their coupling with ocean circulation dynamics and sea level (Paillard and Parrenin, 2004; Saltzman and Maasch, 1988). The combination of orbital forcing with relaxation dynamics can be quite complex and many low-order ice age models driven by orbital forcing exhibit a form of ‘strange non-chaotic attractor’ (Mitsui and Aihara, 2014; Mitsui et al., 2015). In essence, the orbital forcing acts as a ‘pacemaker’ synchronizing ice ages (de Saedeleer et al., 2013; Tziperman et al., 2006), but small fluctuations may occasionally shift the sequence of ice ages by one or several insolation cycles (Crucifix, 2012). The concept of ‘tipping points’ has then to be adapted to account for this forcing, and the notion of so-called pullback attractors may constitute a suitable formal basis to this end (Crucifix, 2013). High-resolution ice core records also revealed abrupt changes in atmospheric CO₂ on decadal timescales during deglaciation (Marcott et al., 2014), which suggests that CO₂ dynamics are actively involved in potentially unstable dynamics contributing to the deglaciation process.

Although human intervention may prevent the occurrence of the next glacial inception (Ganopolski et al., 2016; Loutre and Berger, 2000), the mechanisms behind ice age dynamics may play a role for regional abrupt change in the coming centuries. In particular, multiple equilibria have been found in ice sheet models under present-day conditions (Ridley et al., 2010; Robinson et al., 2012), and evidence from observations, reconstructions and model simulations suggests that irreversible ice loss and sea level rise is possible in the future (DeConto and Pollard, 2016; Favier et al., 2014; Mengel and Levermann, 2014).

2.4. The Green Sahara

In addition to the complexity associated with out-of-equilibrium conditions and multiple forcings highlighted in the previous cases, the history of the Sahara is a prime example of the challenge of temporal variability and spatial heterogeneity. The term ‘Green Sahara’ has become popular when vegetation reconstructions revealed that during the Holocene optimum, ~9000–6000 years before present, the vegetation cover of the Sahel was greatly extended to the north (Hoelzmann et al., 1998; Jolly, 1998). The explanation for the Holocene Green Sahara is based on changes in the Earth orbital parameters. In the early to middle Holocene, the Northern Hemisphere received considerably more solar irradiation in summer. In the northern subtropical regions, this led to stronger warming over the continent than over the ocean, an increased temperature gradient between land and ocean and, consequently, intensified monsoon-type circulation in summer which led to increased rainfall over the Sahel/Sahara region. The Holocene greening of the Sahara was likely amplified by a positive feedback between vegetation and rainfall. The Sahara is special due to high albedo of the desert, as up to 40% of incoming radiation is reflected back into space. Charney (1975) pointed out that the heat loss due to the high albedo maintains the sinking motion of dry atmospheric masses and suppresses rainfall over the region. An increase in rainfall leads to more vegetation, and since this vegetation is darker than sand, a lower albedo. Consequently, more radiation can be absorbed over land, which amplifies the monsoon circulation and convection over the continent.

In experiments with the comprehensive atmosphere–vegetation ECHAM3-BIOME model, Claussen (1997) found multiple stable states in the Sahara for present-day conditions: the desert state, if the surface was initialized with a high albedo, and the green state, if the surface initially had a low albedo. For the Holocene case, only the green state was stable (Claussen and Gayler, 1997), while for the Last Glacial Maximum ~21 000 years ago, both green and desert states were stable (Kubatzki and Claussen, 1998). Several other models also revealed multiple stable states for

certain orbital forcings (Irizarry-Ortiz, 2003; Kiang and Eltahir, 1999; Wang and Eltahir, 2000; Zeng and Neelin, 2000). Brovkin et al. (1998) proposed a simple conceptual model for explaining the existence of multiple states in the Sahara due to interaction between vegetation and climate and suggested that the orbital forcing operated as a bifurcation parameter. Experiments with an intermediate complexity model, CLIMBER-2, showed that a combination of orbital forcing changes and the positive climate–vegetation feedback in the model leads to an abrupt decrease in vegetation cover in the Sahara between 6000 and 5000 years ago (Claussen et al., 1999). This is consistent with an abrupt increase in the dust supply ~5500 years before present identified in a sediment core taken off the coast of northern West Africa (de Menocal et al., 2000). These early results seemed to support the hypothesis that today's Sahara desert was born in a climate catastrophe (as represented in Fig. 2). Since then, however, the story has become more complicated.

Current Earth system models do not show alternative vegetation states (Boucher et al., 2012; Brovkin et al., 2009), probably because the atmosphere–vegetation feedback in these models is not strong enough to support alternative states. It was also observed that considering small-scale heterogeneity in the form of subgrid-scale processes also tends to make the transition smoother in model simulations (Claussen et al., 2013; Groner et al., 2015). Moreover, spatial heterogeneity tends to desynchronize changes at different locations and makes a transition gradual on a larger scale (van Nes and Scheffer, 2005). Indeed, reconstructions show that there were large spatial differences in North African vegetation cover and climate and in the speed of the transition to today's desert (Armitage et al., 2015; Kropelin et al., 2008; Shanahan et al., 2015). Hence, these considerations lend support to a more gradual, non-abrupt transition from the Green Sahara to today's desert (represented by the upper right edge of Fig. 2).

A further complication is imposed by natural climate variability which sometimes obliterates multiple states (Guttal and Jayaprakash, 2007) and makes a transition more gradual. On the other hand, climate variability can even enhance an abrupt change. An illustrative example is a simulation of the end of the Green Sahara by Liu et al. (2007) and (2006). As soil moisture fluctuates on longer timescales than rainfall, vegetation can still persist after the start of a drying trend. When the soil water is finally exploited, climate can have changed substantially already, making the

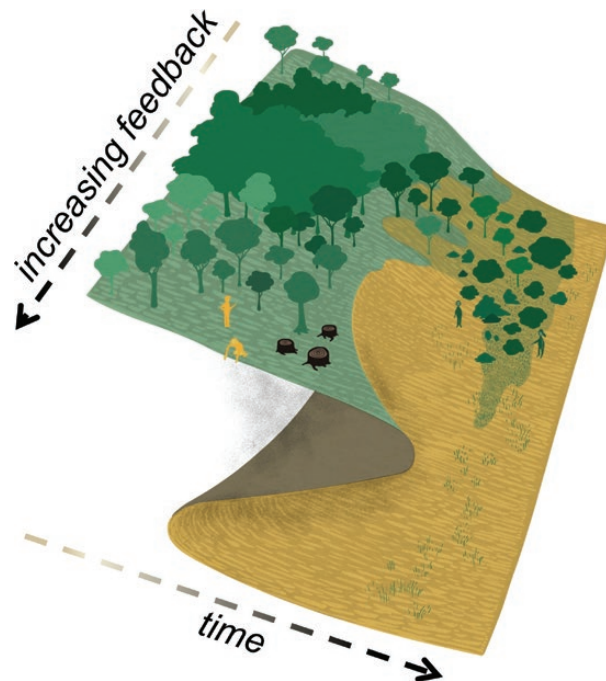


Figure 2. Stability landscape of the Green Sahara and desert. The larger the atmosphere–vegetation feedback (i.e. moving towards the lower left of the figure), the sharper the transition between the two states. (Modified from Model Calendar 2015, designed by Elsa Wikander at Azote, funded by the Beijer Institute of Ecological Economics and the Stockholm Resilience Centre.)

collapse into the desert state more dramatic than the gradual decrease in rainfall. As no strong positive feedback is required, [Liu et al. \(2006\)](#) termed this phenomenon a ‘stable collapse’.

Another important aspect is that the fast fluctuations are usually not independent of the long-term state of the system. This will affect the stability of the system. For example, rainfall fluctuations become small towards the desert state. Once the Sahara becomes too dry, the chances for green spells are very low because the natural variability is also reduced ([Bathiany et al., 2012](#)). Such considerations may be crucial when subgrid-scale variability is introduced in the model by means of stochastic parameterizations. Such parameterizations can be one approach to account for the effects of unresolved temporal variability as well as spatial heterogeneity ([Franzke et al., 2015](#)), and our example highlights that the choice and type of stochastic parameterization may significantly affect the existence and severity of abrupt events.

As the spatial and temporal resolution of reconstructions in North Africa is very limited, and as complex models have limitations in how they capture the interaction between ocean, atmosphere and vegetation, a full picture of the Green Sahara’s demise is still missing. It is clear, however, that a singular, large-scale vegetation dieback at a tipping point is not an adequate description.

2.5. Monsoons

The West African monsoon is not the only monsoon system that has been discussed in the context of tipping points. Observations from several monsoon systems on the planet show a markedly abrupt onset of the monsoonal rainfall in spring ([Ananthakrishnan and Soman, 1988](#); [Sultan and Janicot, 2000](#); [Ueda et al., 2009](#)). Many potential reasons for these abrupt monsoon onsets have been discussed, most of which involve non-linear processes like hydrodynamic instabilities ([Hagos and Cook, 2007](#); [Plumb and Hou, 1992](#)), air–sea interactions and moisture advection ([Minoura et al., 2003](#); [Ueda, 2005](#)). This raises the interesting question whether the same processes can cause tipping points on longer timescales, e.g. a sudden failure of the monsoon due to anthropogenic influence. Interestingly, past abrupt transitions between episodes of strong and weakened (‘failed’) East Asian monsoon have been identified in Chinese cave records ([Wang et al., 2008](#)). Just like the transitions in North Africa, these switches occur due to the oscillations of the Earth’s orbit that modulate summer insolation on the northern hemisphere.

It is still under debate as to which mechanism may have caused these abrupt responses to the gradual change in forcing ([Boos and Storelmo, 2016a, 2016b](#); [Levermann et al., 2009, 2016](#)). Moreover, such records only reveal how climate has changed at a specific location—a problem that also limits our understanding of the Green Sahara. As the amount of rainfall can differ substantially over relatively small distances, a small weakening of the monsoon circulation may make the monsoon rainfall fail to reach a specific location. Therefore, a gradual change of the monsoon circulation might not even be in contradiction to the abrupt shifts in local rainfall or the abrupt monsoon onset in each year. Complex climate models do not predict any abrupt monsoon failure in future scenarios ([Boos and Storelmo, 2016a](#); [Christensen et al., 2013](#)).

2.6. Rainforests and savannas

Another tropical region where the coupling between the land surface and the atmosphere is important is Amazonia. Due to the substantial moisture recycling by the forest, rainfall and vegetation are linked in a positive feedback. Consequently, a dramatic Amazon dieback due to CO₂-induced drying occurred in an early dynamic vegetation model ([Cox et al., 2000, 2004](#)). [Oyama and Nobre \(2003\)](#) even found two alternative equilibria in an atmosphere model coupled to a vegetation model with discrete biomes. However, multiple equilibria in such models can be artefacts of the discretization and often disappear in more realistic, continuous models ([Kleidon et al., 2007](#)). It is now becoming increasingly clear that the Amazon in general will not reach one specific point of no return as a response to greenhouse gas emissions and may be more resilient than previously thought ([Huntingford et al., 2013](#)) but that it will indeed become dryer and more prone to fire ([Malhi et al., 2009](#)).

Even without feedbacks involving the large-scale circulation, such trends may then still trigger tipping points on a local level ([Sternberg, 2001](#)). It has been hypothesized whether the competition between tropical grasses and trees can lead to alternative stable states with the consequence of rapid transitions between them ([Higgins et al., 2010](#)). Fire is often put forward as an essential ingredient of alternative ecosystem states in the form of savanna and closed forests. In a savanna ecosystem, fire occurs frequently and grasses can quickly re-establish in burned areas as they outcompete the slower growing tree saplings. In contrast, where a forest has established, fire is less likely to occur and the high

tree cover can persist at the expense of grasses. Vegetation models indicate that large (sub)tropical land areas on all continents fall into a regime where several stable states are possible (Higgins and Scheiter, 2012; Lasslop et al., 2016). This suggests the possibility of irreversible forest loss, e.g. triggered by climate change or deforestation. Theoretical studies also indicate that fire regimes themselves can undergo an abrupt transition in spatial scale to become ‘spanning clusters’ or so-called ‘megafires’ (Pueyo et al., 2010). As direct evidence for multiple states in real ecosystems is hard to obtain, it is still debated whether fire–vegetation feedbacks are strong enough to support multiple ecosystem states and irreversible transitions (Staal and Flores, 2015; Veenendaal et al., 2015).

2.7. Coral reefs

The same is true about other ecosystems, e.g. coral reefs, which have been found to undergo abrupt and irreversible dieback (Mumby et al., 2007). Convincing proof of alternative states is missing (Dudgeon et al., 2010; Fung et al., 2011; Mumby et al., 2013) because the involved populations are very variable in space and time (Hughes et al., 2010) and exposed to many external conditions that are changing over time themselves (Mumby et al., 2013). It is therefore virtually impossible to identify an ecosystem’s equilibrium from observations and determine how this equilibrium would change with specific external conditions.

2.8. Permafrost

The deviation from equilibrium plays a particularly interesting role in the thawing of permafrost. The positive feedback involved is simple: When the soil in high latitudes thaws, the organic matter in this soil becomes available for decomposition by microbes, and additional greenhouse gases are hence released into the atmosphere. Since they mix over the entire planet, the additional local warming is small. Therefore, the feedback between global warming and permafrost thawing is not considered strong enough to lead to a self-acceleration. The same is true for the climate–carbon cycle feedback in general (Cox et al., 2006). It has to be noted, however, that the uncertainties about the carbon stocks in permafrost that are vulnerable to warming are very badly quantified, with huge uncertainties particularly concerning the organic matter stored in subsea permafrost on the Arctic shelf (Schuur et al., 2015).

Moreover, a local positive feedback has recently shifted into focus: The microbial decomposition of the soil causes an additional warming, just like the warmth in one’s compost heap outside. This self-heating effect and the thawing of the soil amplify each other (Jenkinson et al., 1991; Khvorostyanov et al., 2008a, 2008b) and can result in a ‘compost bomb’ whose fuse is lit when the rate of local warming exceeds a threshold value (Luke and Cox, 2011). Such rate-induced tipping points have also been hypothesized in case of the ocean circulation Stocker (Stocker and Schmittner, 1997) and several ecosystems (Scheffer et al., 2008). The rate dependency introduces time into the equations (making it a non-autonomous dynamical system). Rate-induced tipping points cannot be represented with a bifurcation diagram like Figures 1 or 2, since the system is never close to an equilibrium and cannot catch up with the change in forcing. A rate-induced tipping occurs when the system crosses a threshold beyond which the internal dynamics do not act to follow the moving equilibrium anymore, but suddenly drive the system even further away (although it may finally return to its original state on a different trajectory). Recent studies outlined the mathematical theory behind rate-induced tipping points (Ashwin et al., 2012; Wiczeorek et al., 2011).

2.9. From simple tipping points to abrupt change in complex systems

As different as the cases discussed above are in terms of the scientific disciplines required to investigate them, they highlight how common aspects of complexity challenge the conceptual simple view of (alternative) equilibria and bifurcations as depicted in Figure 1. In principle, these challenges relate to the heterogeneity of space, the diversity of processes on multiple timescales and the fact that climate is neither in equilibrium with its forcing nor are different components of the climate system in equilibrium with each other. It also becomes apparent in many cases that adding complexity seems to remove the existence of tipping points. The previous sections could therefore be seen as a tale of how bifurcations in sea ice have melted away in the light of new knowledge, how an abrupt termination of the Green Sahara was revealed as a mirage and how multiple AMOC states in comprehensive models may be deeply hidden under the surface. However, the above examples also show that it would be very premature to rule out the possibility of abrupt change, even in systems without multiple states.

First of all, there can be natural thresholds in the Earth system that may promote rapid change even without any feedback, like the freezing point of water or the wilting point of plants. Surface cover fractions tend to be particularly

prone to non-linear change, because they are bounded variables (between 0 and 1) and thus only vary in a specific climate regime. Of course, this fact can be a problem when models oversimplify a continuous system using cover fractions or even discrete classes, hence introducing artificial tipping points (Kleidon et al., 2007). However, there are plausible cases where the Earth's surface properties can indeed vary in a non-linear way, e.g. for geometrical reasons: The surface area of sea ice or lakes can decrease rapidly even in the absence of a rapid mass loss if the mass is spread out uniformly over a large area. Altered surface properties can have a large influence on surface fluxes and thus affect the atmosphere above. On a local level, horizontal shifts of spatial patterns with sharp geographical gradients, like the desert boundary or the monsoonal rainbelt, also cause abrupt change. Although such shifts are not abrupt on a large scale, they can have similarly dramatic consequences for people and ecosystems which are bound to a certain location.

In light of the Earth system's complexity, it is obvious that the idea of tipping points as catastrophic bifurcations is far too limited. Instead, a more general view on tipping points is required. The original notion that 'little things can make a big difference' (Gladwell, 2000) is a good working definition. A mathematical formalization of this definition is given in the supplementary information of Lenton et al. (2008) and used in that study. Although this definition may appear rather vague from a physics perspective, it is useful because it describes the phenomenology that we are interested in. Of course, when analysing the reasons behind such phenomena, more specific and mathematically well-defined terms will be required. This general view on tipping points moves away from the question whether there are alternative stable states but asks how smoothly the distribution of a climate variable changes under a change in forcing.

As the conceptual models that supported the idea of tipping points are much simpler than complex models, let alone the real world, it will have to be investigated which type of tipping point survives the step into a complex model environment. Ultimately, the aim is to understand large-scale transitions in high-dimensional stochastic dynamical systems (Chekroun et al., 2011; Dijkstra, 2013) and how robust they are in comprehensive, process-based models. An interesting example from fluid mechanics is the wind reversal in the Rayleigh-Bénard flow (which is the flow in a layer of liquid heated from below). When the heating is weak, the first flow transitions occur through elementary bifurcations, such as pitchfork, saddle-node and Hopf bifurcations (Koschmieder, 1993). When the heating is increased, the flow becomes turbulent, first in the boundary layer and then spreads into the bulk of the flow. In experiments under very strong heating, large-scale transitions between turbulent flow patterns appear, the so-called wind reversals (Sugiyama et al., 2010). This is one of the first examples where at least two different large-scale statistical equilibrium flow states occur in what is called the 'ultimate turbulence' regime. The boundaries in parameter space where such states occur are not bounded by saddle-node bifurcations but more generally are characterized as attractor 'crises' and methods of ergodic theory and statistical physics are often used to tackle these type of problems (Eckmann and Ruelle, 1985; Gaspard et al., 1995; Tantet et al., 2015).

Hence, it is not guaranteed that results found in conceptual models are also found in complex models. It is also a challenge to anticipate how model behaviour will change when new processes are added. Shall we therefore put all efforts into improving these models in all aspects, until a robust consensus on future abrupt change emerges? Although such a bottom-up approach may in principle work, it would not be an adequate strategy in practice. Due to the complexity of the climate system, scientific progress would be slow and hence lose the race against the actual manifestations of climate change (a possible strategy to at least explore the uncertainties will be discussed in Section 5). Even if it was possible to build a perfect process-based model in a feasible time, how would we know the stability properties of its states? Could we find out how and why such a model would respond to a certain forcing? To make such inferences, alternative methods of analysis are required. We review some of them in the following section.

3. Methods to detect multiple equilibria

Comprehensive climate models are built from fundamental laws and parameterizations of specific processes. The strength of feedbacks, spatial interactions and the nature of variability are mostly emergent properties. Although models are also 'tuned' to stay within the envelope of observed climate (Mauritsen et al., 2012), their stability properties are generally not known very well. In contrast, the model's phenomenological behaviour is directly accessible by running the model and analysing its output, from which we attempt to infer the model's stability properties. In this section, we illustrate what conclusions different methods of analysis allow regarding the existence of multiple equilibria. These methods can be divided in two groups: interventional and diagnostic methods. With interventional methods, the model itself is altered or used to run specific simulations, whereas diagnostic methods can be applied to

model output as well as observations or reconstructions. Given that such methods are a major tool in many scientific studies, it is surprising that their limitations are rarely discussed (see [Schroeder et al., 2005](#), for an insightful review on ecology models). We will attempt a concise and non-exhaustive assessment in this section.

3.1. Choosing extreme initial conditions

One popular interventional method is to start a model from very different initial conditions—e.g. a forest versus a grass world ([Brovkin et al., 2009](#)) or an ice-free versus an ice-covered Arctic ([Tietsche et al., 2011](#)). If both simulations do not approach the same steady state after a long time, but end up in two different steady states, we have found multiple solutions. Of course, the identified equilibria may not be the only ones, and they could consist of several independent equilibria ([Dekker et al., 2010](#)). What, however, can we conclude if both simulations approach the same steady state? It could in fact be that this is the only possible solution for any initial condition. It could also be that we have chosen the initial conditions in an unsuitable way. In contrast to simple conceptual models, there are many more degrees of freedom in complex models, and different state variables approach an equilibrium on different timescales. These problems suggest another possible reason why multiple equilibria cannot be detected in complex models: they are more difficult to find.

In practice, one has to decide what the spatial pattern of the initial state is, which variables should be perturbed and by how much. This is an experiment design problem. In order to find hidden states that are usually very different from the already known state, it suggests itself to choose perturbations that are much larger than the natural variability of the system. In the case of bounded variables like cover fractions, or ice mass, it is a natural (though perhaps not always the best) choice to pick the most extreme initial conditions. A more difficult problem is what aspects of the climate system should be perturbed. If we pick only fast variables, even an extreme perturbation will not have a large effect on the rest of the system because it decays too quickly. For example, should we simply take away the sea ice at one point in time, or should we let the ocean below adjust to the lack of ice, and only then allow sea ice to grow again? Should we cover the whole world with vegetation, or would vegetation in certain places suppress rainfall in remote locations, potentially hiding a green equilibrium in that region? This last example shows that choosing globally uniform initial conditions implies the assumption that the sign of the response to a certain perturbation does not depend on the location too much. Although this is a plausible assumption in the examples above, we should be aware that we make this assumption. The problem how to decide on the spatial pattern of a perturbation is particularly evident in freshwater hosing simulations with GCMs to address the stability of the AMOC. When the model is in a unique regime, the location where the freshwater is released will not matter much for finding an additional (collapsed state), as it is not there. However, in a multiple equilibrium regime, it definitely matters where to release the freshwater and with some choices the collapsed state may not be found. This issue is discussed in [Cimattoribus et al. \(2012\)](#) who also provide a recipe to systematically find collapsed states in GCMs.

3.2. Hysteresis

Another common method that is related to the choice of initial conditions is to look for hysteresis ([Boucher et al., 2012](#); [Li et al., 2013](#); [Marotzke and Botzet, 2007](#); [Rahmstorf et al., 2005](#)). This is done by imposing a change in forcing, simulate the resulting climate change and then gradually return the forcing to its starting conditions. If the sweep of the forcing encloses a catastrophic bifurcation point, and if one waits long enough for the system to catch up with the forcing, the system will not return to the initial state ([Thompson et al., 1994](#)). In similarity to choosing extreme initial conditions, a negative result can be due to several reasons, e.g. that there are no multiple states, or that the nature or pattern of the forcing was not appropriate to find them. Conversely, it can be difficult in practice to see whether hysteresis is caused by multiple stable states (static hysteresis) or only by the inertia of the system that lags the change in forcing (dynamic hysteresis). This can pose a practical challenge when the system's adjustment time to the change in forcing is very slow, as is the case for the deep ocean that roughly needs a thousand years to warm after atmospheric CO₂ is increased. Trying to sweep the CO₂ concentration back and forth slowly enough would require a huge amount of computational time with a complex model. Moreover, as we will explain in Section 4.2, the return time of a system usually slows down substantially when the system becomes very sensitive to perturbations and essentially becomes infinite at bifurcation points. In addition, the system with a parameter varying in time may not behave in the same way as the autonomous system ([Tredicce et al., 2004](#)). This makes it practically difficult to prove the existence of multiple states with hysteresis experiments. Nonetheless, they have value in showing

if a system is reversible below a certain time horizon and highlighting dynamical regimes where the system becomes unusually slow.

3.3. Continuation methods and control schemes

There are also direct methods to solve the steady-state equations of a model versus parameters without using any time integration (Krauskopf et al., 2005). One of the prominent methods used is pseudo-arclength continuation (Keller, 1977), and it has been applied to very sophisticated fluid flow problems (Dijkstra et al., 2014) including coupled ocean–atmosphere models (den Toom et al., 2012) having millions of degrees of freedom (Thies et al., 2009). The limitation of these techniques is that they are only able to compute steady solutions (and their linear stability) and hence can only detect elementary bifurcations (saddle-node, transcritical, pitchfork and Hopf) when one parameter is varied. An advantage compared to other methods is that even unstable equilibria can be found (or, more generally speaking, so-called unstable manifolds that separate different basins of attraction; in case of Fig. 1, the dashed black line). Another approach to find unstable manifolds is to introduce a control loop that stabilizes the unstable manifold (Sieber et al., 2014). This requires to run a model forward in time but does not require to alter its equations. The unstable manifold may thereby be traced and explored, which is not possible in hysteresis experiments.

3.4. Stability diagrams

Another method is to attempt a graphical reconstruction of a bifurcation diagram or a diagram showing the dependency between two state variables that define a positive feedback. Figure 3 illustrates this for the example of vegetation (V) and rainfall (P) in North Africa. Each curve shows the dependency of one variable on the other variable. The intersection points mark the stable and unstable equilibria of the system. If at least one of the dependencies is non-linear [here $V(P)$], there can be multiple equilibria. Diagrams like Figure 3 are supposed to illustrate the nature of multistability, but they have also been used to infer the equilibria in a particular region in complex models (Brovkin

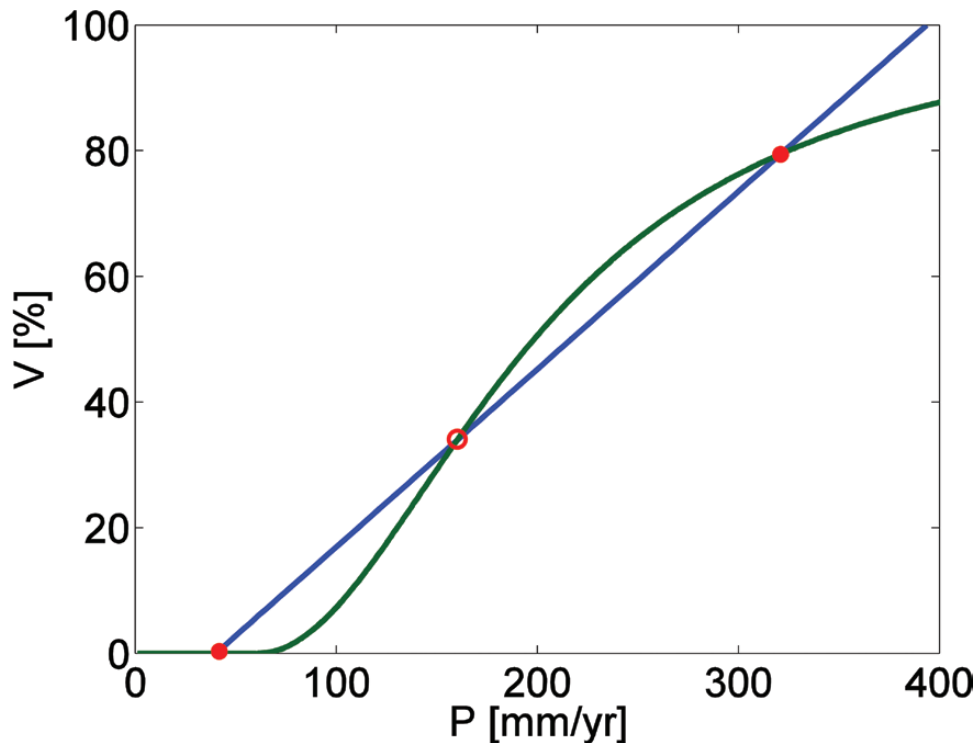


Figure 3. Phase diagram of atmosphere–vegetation interaction in North Africa after Brovkin et al. (1998). The green line shows how vegetation cover depends on rainfall, and the blue line shows how rainfall depends on vegetation cover. Intersection points represent the stable (closed red dots) and unstable (open red dot) equilibria of the system.

et al., 2003; Levis et al., 1999; Renssen et al., 2003, 2006; Wang, 2004). The specific method to construct the diagram differs among publications, but they have in common that one variable in the model is prescribed to obtain the time and spatial mean response of the other variable. This method was suggested for cases when two variables originate from complex model components developed independently from each other, e.g. the vegetation dependence on rainfall is obtained from the dynamic vegetation model, while the effect of vegetation on rainfall is estimated from the atmospheric model. When the model components are tightly coupled within the model code, finding the response curves from the diagnostic output could be difficult. Even if one manages to construct the whole curves properly and assuming that no other variables than the two play a role, this procedure has two problems. First, the time mean of the coupled model's state does not necessarily correspond to a deterministic equilibrium. Second, the effects of spatial differences on the stability of the whole system cannot be captured. For example, prescribing a desert state results in a certain spatial mean rainfall. This rainfall value could not sustain vegetation if it occurred at a single grid cell. Therefore, the phase diagram indicates a desert equilibrium. In the spatially resolved model, one location may still be wetter than the other and maintain vegetation, which in turn could increase rainfall also at other locations. Hence, the diagnosed stable desert state is actually non-existent in the coupled model (Bathiany et al., 2012). Conversely, multiple states that do exist in the complex model may not be seen in the diagram. This can happen when too many stable locations may have been included when choosing a region of analysis or when some locations that are important for the bistability have been left out. This spatial aspect, as well as an effect of noise present in the coupled system, limits the use of stability diagrams to infer the properties of complex models.

3.5. Frequency distributions

Diagnostic methods are generally easier to apply, but they tend to be even less conclusive. For example, a popular diagnostic is the frequency distribution of a state variable that tells us how often a system is in a certain state. This is motivated from a simple system consisting of a deterministic bistable part and an additive white noise term of intermediate magnitude (stochastic motion in a double-well potential). If the random fluctuations are of intermediate magnitude, they sometimes push the system from one basin of attraction into the other (note that this system would not show any long-term dependency on initial conditions or hysteresis). Hence, the system is close to the deterministic equilibria most of the time, but rarely found in between. The deterministic equilibria then reveal themselves as distinct 'modes' in the frequency distribution. This logic has been applied to understand the above-mentioned Dansgaard–Oeschger events in palaeoclimate reconstructions (Kwasniok and Lohmann, 2009; Livina et al., 2011) and to detect savanna and forest states in satellite observations (Hirota et al., 2011; Staver et al., 2011). In the latter case, the additional assumption is made that the different locations represent realizations of one and the same system. Even without this problem of exchanging space for time, the interpretation of frequency distributions in the context of multiple equilibria relies on the applicability of the simple stochastic system framework mentioned above (Lucarini et al., 2012). The less the system is understood, the more difficult this is to justify. From the perspective of the simple stochastic model mentioned above, this has two aspects: noise level and noise source. If the noise level of the stochastic model is very small, we see a unimodal and Gaussian distribution of states. The alternative state would not be detected because it is never sampled. If the noise level is very large, the modes merge into a single one, and the multiple deterministic states become invisible. These cases still assume that the noise is added on the state's evolution, i.e. independent of the model's deterministic part. If we consider a different source of the noise, it becomes state dependent (so-called multiplicative noise). For example, fluctuations in vegetation cover may occur due to processes that affect the vegetation's growth or retreat directly, like grazing or planting trees. But they can also occur indirectly, e.g. due to fluctuations in rainfall. These so-called multiplicative noise sources propagate through the non-linear equations and can often have non-intuitive effects on the state variable. In the case of vegetation modelling, there are examples that a bistable system can show a unimodal distribution (Bathiany et al., 2012) and a monostable system can show a bimodal distribution (Liu, 2010).

4. Methods to detect and interpret abrupt change

4.1. Detecting abrupt change

Due to the large number of variables and grid cells of complex climate models, the phenomena that are encoded in their output are not immediately visible to the eye. In particular, regional abrupt changes may remain unnoticed when results are integrated over large areas or time periods or when several simulations or even models are averaged.

It therefore seems appealing to apply automatic algorithms to detect abrupt change in models. A straightforward approach in this regard is a simple thresholding where a shift is counted as abrupt if it exceeds some rate of change defined by the natural variability (Drijfhout et al., 2015). More sophisticated approaches involve statistical tests in order to detect sudden changes in a certain property (e.g. sudden shifts in the mean, variance or trend) and to identify the most appropriate statistical model (Beaulieu et al., 2012). The problem of detecting abrupt change has a long history in the analysis of observations where it is important to remove artificial change points in the data that are introduced by the measurement instruments (Ducré-Robitaille et al., 2003; Lund et al., 2007; Reeves et al., 2007). Such algorithms are mostly applied to univariate time series, and related methods have been used to detect and interpret real shifts in climate (Beaulieu et al., 2012; Nikolaou et al., 2014), but also in ecosystems (Andersen et al., 2009; Beaulieu et al., 2013; Wooster and Zhang, 2004), the economy (Silva and Teixeira, 2008) and other systems. Another more recent line of research concerns network approaches that have mostly been applied to palaeoclimate reconstructions, as we will discuss in Section 4.3.1.

In the case of climate models, it is an additional constraint that we prefer to detect abrupt change on a certain spatial scale or with a certain spatial pattern. Therefore, methods are needed that take the spatial connections between grid cells into account. Detection algorithms are already successfully used to detect edges in images (Canny, 1986), specifically in satellites images showing the distribution of vegetation (Kent et al., 2006; Sun, 2013), clouds (Dim and Takamura, 2013), sea ice (Mortin et al., 2012) and ocean eddies (Dong et al., 2011). Moreover, spatio-temporal methods can track the evolution of events in observations (Zscheischler et al., 2013), e.g. the development of cyclones (Neu et al., 2013) or ENSO events (McGuire et al., 2014). It is therefore conceivable that related methods can be developed that are tailored to the problem of finding abrupt change in complex climate models. As the definition of abrupt change is subjective, the performance of detection methods will depend on the context, e.g. the spatial and temporal scale of interest.

4.2. Statistical stability indicators

4.2.1. Motivation and advantages

Identifying an abrupt change in a model simulation, reconstruction or in observations is only the starting point of investigating why it occurs. After all, the ultimate goal is to predict abrupt shifts and prevent them from happening or at least reduce the damage. An attractive idea in this context is to observe the natural climate variability and infer certain stability properties from this variability, e.g. how the system would respond to perturbations in general and how an intervention at some location would affect other locations. In statistical mechanics, this idea is at the heart of the fluctuation–dissipation theorem (Kubo, 1966), which connects the internal variability of a system close to equilibrium with the system’s response to external perturbations. Interestingly, climate models have been found to show such a link (Cionni et al., 2004; Gritsun and Branstator, 2007; Leith, 1975). A very simple approach that has become popular in many disciplines is to interpret a system’s variance and autocorrelation as indicators of linear stability.

The reasoning behind this is based on the phenomenon of critical slowing down that affects the dynamics of systems in the vicinity of local bifurcation points, like the fold bifurcation encountered in simple climate models (Fig. 1). This phenomenon has first been known in statistical physics (Gaspard et al., 1995; Solé et al., 1996; van Kampen, 1981) and has then been studied in the context of ecology (Oborny et al., 2005; Wissel, 1984) and the climate (Held and Kleinen, 2004; Kleinen et al., 2003). Critical slowing down implies that a system becomes slow at recovering back to equilibrium after a perturbation, i.e. its relaxation time increases (Fig. 4). One can thus observe this effect by performing perturbation experiments to measure the recovery rate (van Nes and Scheffer, 2007). More importantly, one can hope to infer critical slowing down indirectly from the natural fluctuations around equilibrium that are usually interpreted as stochastically induced. This makes the approach diagnostic and hence attractive for analysing complex model output and observations. The dynamics of a slowing, destabilizing system tends to resemble a red noise process: each state starts to look more like its state in the past or, in statistical terms, autocorrelation increases (Wiesenfeld, 1985). At the same time, slowness leads to an increase in variance when subsequent random perturbations add up due to the slow decay (Wiesenfeld and McNamara, 1986). Both variance and autocorrelation then increase in a continuous fashion before a system crosses a bifurcation (Fig. 4D and F). This theory has motivated the use of rising variance and autocorrelation as ‘early warning signals’ of

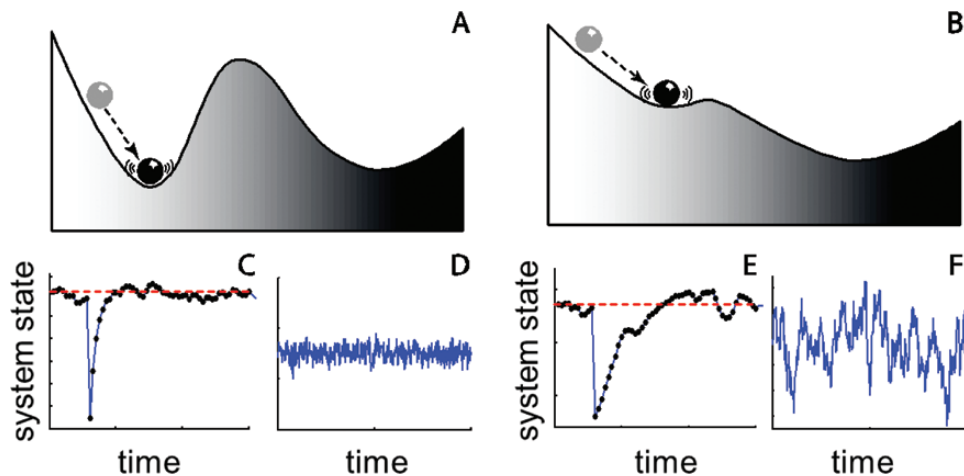


Figure 4. Slowing down and its indicators in a destabilizing system. Here, the position of the ball indicates the state of the system which is subject to random perturbations. The minima in the stability landscape are stable states. If the system is far from the bifurcation (A) it is more stable than close to the bifurcation (B). Far from the bifurcation, the system recovers quickly from a perturbation (C), and its variability in the steady state is small and fast (D). Close to the perturbation, the recovery is slow (E), and as a consequence, the variability shows long excursions from the mean and has larger total magnitude (F) (adapted from [Scheffer et al., 2012](#)).

approaching bifurcations ([Scheffer et al., 2009](#)). Other, related indicators have also been suggested, e.g. higher statistical moments ([Guttal and Jayaprakash, 2008a](#)) and spatial correlations in spatially extended systems ([Dakos et al., 2010](#); [Donangelo et al., 2010](#); [Guttal and Jayaprakash, 2008b](#)).

Besides the prediction context, statistical indicators of stability change can also be used to understand why an abrupt change occurs, because they can only be expected in some scenarios (like local bifurcations) but not others (like random shifts to an alternative state or threshold-induced shifts without positive feedbacks). For example, [Dakos et al. \(2008\)](#) found early warning signals prior to some abrupt shifts in palaeorecords, and changing indicators prior to Dansgaard–Oeschger events may support that these transitions were associated with a bifurcation point, though the evidence in this case is still controversial ([Cimadoribus et al., 2013](#); [Ditlevsen and Johnsen, 2010](#)).

The strength of statistical stability indicators lies in the generality of the theory. Consequently, it has been applied to essentially all tipping point candidates described above and in a variety of other fields like ecology, economics, sociology and medicine ([Scheffer et al., 2012](#)). Extensive practical guides ([Dakos et al., 2012a](#); [Kefi et al., 2014](#)) and a suite of methods ready to use in freely available toolboxes ([Dakos et al., 2012a](#)) for analysing trends in statistical properties have been facilitating their application.

4.2.2. Limitations and challenges

The increasing popularity of statistical indicators in different applications makes it necessary to raise awareness for their limitations ([Dakos et al., 2015](#)). Most importantly, increases in autocorrelation and variance do not imply the existence of a catastrophic bifurcation but merely reflect current changes in a system's timescale. This means that using these metrics as early warnings for upcoming tipping points can only be justified in systems where there already is evidence for the existence of catastrophic bifurcations. Therefore, single estimates of stability indicators do not have predictive value. Conversely, even when abrupt change is imminent, it is not always preceded by statistical precursors. A trivial but common practical problem is that climate time series are often far too short to detect slowing down. For example, detecting meaningful autocorrelation changes requires roughly 1000 data points ([Ditlevsen and Johnsen, 2010](#)), i.e. 1000 years for annual resolution. In shorter records, it is difficult to draw any conclusions. For example, the lack of early warning signals in palaeorecords of monsoon transitions between high and low rain periods could imply no stability change or a destabilization that is too fast to become statistically evident ([Thomas et al., 2015](#)). Moreover, random fluctuations can push a system to a different state long before the bifurcation is reached (although one may be

able to calculate the probability of such a transition). It is therefore important to also develop methods that explore the whole basin of stability and not only the linear stability of the local state (Menck et al., 2013; Mitra et al., 2015).

In spatially coupled systems, an abrupt change at one location can also induce an abrupt change at another location, where it would come as a surprise (Bathiany et al., 2013a). Methods have been developed to apply the essentially one-dimensional concept in systems with many dimensions (Held and Kleinen, 2004; Kwasniok, 2015; Williamson and Lenton, 2015). However, it can be difficult in practice to determine the necessary transformation and to specify the boundaries of the system which is embedded in the rest of the climate system. For example, Held and Kleinen (2004) projected the natural variability of the AMOC on its critical mode to be able to reduce the problem to the one-dimensional framework of Figure 4. For this projection, they required knowledge on the critical mode, which becomes apparent only close to the bifurcation. Bathiany et al. (2013a, 2013b) applied this method to the Green Sahara transition and showed that knowledge on the spatial boundaries of the tipping element is required. Without such knowledge, the variability of the global climate system would obscure the signals that exist in the critical region. Using an iterative process, one can however search for the region that maximizes an early warning signal and identify the causal origin of an abrupt change. Hence, statistical indicators can also contribute to the problem of identifying causality described in the previous section.

Moreover, it is important to note that it always has to be established if stability indicators can be applied at all to a particular system, because the conceptual stochastic framework outlined above may not be an adequate description (Lucarini et al., 2012). For example, the concept assumes that the noise is independent from the rest of the system and simply adds on its deterministic evolution. In a more complex situation, noise can be multiplicative and interact with the slower dynamics in a complicated way. As a result, variance may as well decrease instead of increase towards a bifurcation (Dakos et al., 2012b). Although the theory may still apply infinitely close to the bifurcation point (Ditlevsen and Johnsen, 2010), such cases hinder its application to practical purposes. Similarly, an increasing recovery time is not necessarily associated with a higher vulnerability to a change in external conditions. It can also arise if the system's effective mass increases, e.g. a larger amount of ocean water or a thicker layer of sea ice respond more sluggishly to heat flux perturbations (Bathiany et al., 2016b; Boulton and Lenton, 2015; Wagner and Eisenman, 2015a). Moreover, the timescale separation in the form of noise and slow dynamics of the system is often an oversimplification because in reality spectra rather tend to be continuous. If the separation of timescales is not a good approximation, other ways of detecting critical slowing down may be possible. For example, when the forcing oscillates with a timescale that is similar than the timescale of the system, one can infer slowing down from an increased lag of the system behind the forcing (Williamson et al., 2016) or from the power spectrum (Wiesenfeld, 1985).

Given the above complications, it becomes clear that autocorrelation and variance do not need to increase prior to abrupt change nor does abrupt change always imply the existence of early warnings. Insofar, the limitations of the approach reflect the limits of the simple bifurcation logic discussed in the previous sections. It is therefore not surprising that attempts to identify statistical precursors of tipping points in complex climate models have often produced negative results. For example, the Amazon rainforest dieback in a complex model shows no precursors as suggested by the theory because of multiplicative noise and a too rapid change in forcing (Boulton et al., 2013). Terrestrial systems can also be a dimensionality challenge because all land points are connected via the atmosphere, which is why approaches to assess the stability from local information can fail (Bathiany et al., 2013a; Menck et al., 2013). Another example is the loss of Arctic sea ice which is preceded by rising autocorrelation due to the thermal inertia of the ocean, an effect that is active even in the absence of a tipping point (Bathiany et al., 2016b; Wagner and Eisenman, 2015a). Probably the most convincing case of slowing down in complex models concerns the AMOC (Boulton et al., 2014; Held and Kleinen, 2004; Kleinen et al., 2003).

It is sometimes still argued that no physical understanding of the system under consideration would be required in order to apply early warning signals. In the light of the above limitations, this promise cannot be kept. Nonetheless, exploiting the link between variability and system stability and looking for precursors of abrupt change remain very powerful ideas. They live on in various approaches, some of which we highlight in the next subsections.

4.3. The causality of abrupt change

4.3.1. From time series analysis to network approaches

Although the detection of abrupt shifts outlined in Section 4.1 is only based on phenomenological criteria, it can be the starting point in understanding why these shifts occur. This is particularly useful in the case of palaeoclimate

reconstructions or observations, where no experiments are possible. For example, in the case of several abrupt events occurring one after the other at a certain location, or at different locations, a good chronology of events can give insight into the causal chain of events and into the role of external drivers. A prominent example are Dansgaard–Oeschger events, where the exact timing is important to infer whether they are triggered by an external regular forcing or by the internal, much more erratic, climate variability (Braun et al., 2010, 2011; Braun and Kurths, 2011; Rahmstorf, 2003; Rasmussen et al., 2014). A similar motivation underlies attempts to understand changes in the reconstructed North African climate and how they coincide with important stages of human evolution (Donges et al., 2011b; Trauth et al., 2009).

The analysis of such palaeoclimate records poses the challenge that each record usually represents only a single location. Interestingly, the theory of dynamical systems shows that it is possible to unfold a multidimensional system from a single time series (Packard et al., 1980; Takens, 1981). It has become popular in recent years to apply this approach to reconstructions and to analyse the resulting system with methods derived from the theory of complex networks (Donges et al., 2011a; Donges et al., 2011a, 2015; Marwan and Kurths, 2015). Technically, this is done by interpreting the temporal structure of a time series (in particular, how similar states repeatedly occur over time) as the geometrical properties of a complex network that can be depicted as a web of nodes and links (Marwan et al., 2009). This network's geometrical structure can then be analysed using different quantifiers of local and global properties. By analysing how the network's architecture changes over time, it is possible to detect abrupt transitions that can appear very subtle in the original climate record but may indicate a major reorganization of climate dynamics at a certain time. Applying network approaches to climate reconstructions, several known transitions can indeed be detected, e.g. the beginning of the Pleistocene and the Mid-Pleistocene transition (Donges et al., 2011b). Abrupt shifts in the dynamics of the Asian monsoon system in the Holocene have been identified with similar approaches and have been discussed in the context of societal conflict and crises (Donges et al., 2015).

In the case of model results, of course, we have direct access to essentially the whole state of the system and can directly access what happens at all locations. In this case, the interpretation of the system as a network is more straightforward by identifying each model grid cell as a node in the network (Donges et al., 2009a, 2009b; Tsonis and Roebber, 2004; Tsonis et al., 2006). Links between the nodes are then established based on statistical relationships between the observables at the grid cells. Although abrupt changes of the mean climate in a spatially resolved model would be easy to detect without any network approaches, these approaches can complement the information obtained with more traditional methods. The framework of networks not only allows to highlight the global structure of links (e.g. teleconnections that relate distant locations with each other). It can also reveal non-linear transitions that are not directly apparent to the eye, e.g. transitions between two chaotic attractors. Such transitions can still affect humans and ecosystems in fundamental ways, and they can cause abrupt shifts in components of the Earth system and human systems that are not represented in the climate model. It therefore seems promising to apply network approaches not only to past climate reconstructions but also to future scenarios. In particular, as the phenomenon of slowing down described in Section 4.2.1 affects the correlations between different grid cells, network approaches can be a useful tool to devise early warning indicators of abrupt shifts. This idea has recently been explored in models simulating an AMOC collapse (Feng et al., 2014; van der Mheen et al., 2013) and desertification events (Tirabassi et al., 2014; Yin et al., 2016).

4.3.2. Other approaches to causality

Another important question from the network perspective concerns the causal chain of events during an abrupt change. Which location is the origin of an abrupt change and which locations do just passively respond? The same can be asked about different variables at one and the same location. For example, does Arctic sea ice collapse in a model because too much sunlight is suddenly absorbed to balance the heat losses? Or is this increased absorption mainly the result of the low albedo of the ocean that is exposed when the ice is lost? The networks described above cannot provide such information since the statistical relationships that determine the links of the network do not allow direct conclusions about causality. A starting point is to consider networks with directed and weighted links, in order to capture the different strengths and directions of the interactions. For example, Marwan and Kurths (2015) built a network based on the timing of severe rain events in South America and show how this approach even allows for predictions. Moreover, causal discovery algorithms that have recently been introduced to climate research can follow the flow of information in a complex model (Ebert-Uphoff and Deng, 2012, 2015) and identify regions with a

particularly large influence on other regions (Runge et al., 2015). An essential advantage of such methods is that they do not only consider two variables at a time but use conditional independence tests that involve more than two variables (Ebert-Uphoff and Deng, 2012). This allows to distinguish direct cause-and-effect relationships from spurious correlations that can arise from a common driver.

Unfortunately, several contradictory causal explanations can be in agreement with the data. It therefore seems adequate to apply several complementary methods of analysis, and all additional knowledge about the system one may have, to eliminate as many (incorrect) causal explanations as possible. This will at least reduce the number of interventional experiments one may have to perform with a complex model to determine the true causal chain of events. It poses a problem in this respect that abrupt changes tend to be singular events that do not provide enough data themselves to infer causality, especially when all variables (at all locations) change in synchrony. The logic behind the statistical indicators of stability, or early warning signals, presented in Section 4.2, may provide an interesting approach to this problem: Could knowledge on the causal flow associated with the natural climate variability also contribute to an understanding of what causes an abrupt change? In principle, this is not guaranteed, since abrupt change is an inherently non-linear behaviour with much larger amplitude than the natural variability. Therefore, it is probably worthwhile to explore how causal discovery methods might be applied to detect what the causal origin of an abrupt event is and by what perturbation it is most likely triggered.

Another recent method to infer causality from the internal fluctuations of complex systems is convergent cross mapping (Sugihara et al., 2012). The method builds on the approach to reconstruct a multidimensional system from a single time series (Packard et al., 1980; Takens, 1981) mentioned in Section 4.3.1 and detects whether two time series are independent or belong to the same dynamical system. Convergent cross mapping has been applied to infer the effect of temperature on atmospheric CO₂ during glacial–interglacial cycles (van Nes et al., 2015). In combination with our knowledge on the effect of CO₂ on global temperature, this result highlights the importance of carbon cycle feedbacks to understand ice age cycles.

4.3.3. Radiative feedback analysis

Regarding the climate feedbacks active in global warming, it is not possible to detect positive feedbacks directly with the causality detection methods mentioned above, because the involved causal graphs do not allow for closed cycles. To unravel such feedbacks and quantify their importance, it may be beneficial to consider methods that have been designed to understand climate change in general, in particular related to climate sensitivity. Such methods comprise a number of approaches that are interventional to different degrees. A common interventional method is to suppress certain feedbacks in a model to quantify their contribution to global warming (Hall and Manabe, 1999). As such an approach is very cumbersome in complex models, diagnostic methods have become popular (Bony et al., 2006). For example, one can calculate how a certain property like surface albedo or cloud distribution alters the radiative balance of the Earth. The processes involved in these sensitivities are relatively well understood and do not need to be repeated with different models. With this knowledge, one can then infer the strength and spatial pattern of radiative feedbacks from global warming simulations where the property under consideration (albedo, clouds,) is changing (Soden et al., 2008; Wetherald and Manabe, 1988). As the climate is a non-linear system, feedbacks are not independent from each other (Aires and Rossow, 2003), a factor that could be particularly relevant in the context of abrupt change. In close analogy to the methods to detect multiple steady states in models outlined in Section 3, it is obvious that tools to infer the cause of abrupt changes suffer no less from ambiguities. It is therefore only a combination of methods that can allow the most thorough autopsy of a certain abrupt change.

5. How likely is future abrupt climate change?

In the previous sections, we have discussed how one can hope to detect and understand an abrupt change once it occurs. But how likely is it to happen? As we have outlined above, most candidates of tipping points so far involve very complex and uncertain mechanisms have arisen from very idealized models (Eisenman and Wettlaufer, 2009; Levermann et al., 2009; Stommel, 1961) and involve a substantial component of expert elicitation (Kriegler et al., 2009; Lenton et al., 2008). In a first systematic attempt to scan the available complex climate models for abrupt change, Drijfhout et al. (2015) focused on a selection of two-dimensional key variables and identified 37 abrupt events in simulations of future climate change scenarios. Although the regions where these abrupt shifts tend to occur

do show some similarity to previous expectations (Fig. 5), the current catalogue of abrupt shifts involves a huge uncertainty, since every abrupt shift only occurs in a few models. Given the immense ecological, social and financial implications of an abrupt change, it would be beneficial to constrain the possibility of abrupt change even more systematically. The rapidly increasing amount of data from climate models and observational sources (Fig. 6) offers the opportunity to gain insight from data mining approaches (Overpeck et al., 2011).

Nonetheless, the currently available climate model simulations are not a sufficient basis to reduce the uncertainty of abrupt change in a significant way. The prediction of climate (or, more precisely, the modelled response to a given forcing scenario) is always hampered by the fact that many relevant processes are not captured well in comprehensive climate models. Some of these processes are simply not sufficiently understood, while others cannot be modelled with sufficient detail, often due to the vast range of temporal and spatial scales. Therefore, when building a comprehensive Earth system model, there are many choices to be made on how to model certain processes. Often, it is not clear which equations are most appropriate. This source of uncertainty is called structural uncertainty. Moreover, after having decided on a certain structure, additional choices have to be made about what values to choose for parameters appearing in the equations. This source of uncertainty is called parameter uncertainty. All these different choices of model structure and parameter values lead to a certain model, which needs several days to weeks to run on a state-of-the-art supercomputer, only to give a single result. What are the consequences of all these choices for the outcome of the prediction?

Given the complexity of the model in combination with the huge effort it takes to make a single simulation, it seems a hopeless enterprise to robustly quantify the structural and parameter uncertainties. In this regard, we are lost in a high-dimensional space, feeling our way forward in the dark, with nothing more than a tiny torch. An important achievement towards illuminating uncertainties is the project climateprediction.net (Allen and Stainforth, 2002), where the problem of limited resources was tackled by using otherwise idle computing time on thousands of personal computers. A central research goal in this context is to quantify the climate sensitivity, the response of global mean temperature to anthropogenic forcing. Stainforth et al. (2005) showed that by only varying six different parameters in a model with fixed structure, the distribution of climate sensitivity is already much broader than the range covered by standard climate models.

In principle, this approach of a large perturbed-physics ensemble could also be applied to explore the uncertainty of abrupt changes in climate models. However, this task may be even more difficult than quantifying the uncertainty of climate sensitivity for several reasons. First, the timing of abrupt events can be subject to chance. The reason is that natural climate variability can make changes more or less abrupt, and it can trigger abrupt events when the system is close to a tipping point or in a state of self-organized criticality (Jones, 2000). As it therefore depends on the realization if an event occurs, even in the same model, the uncertainty of such events is more difficult to quantify than the uncertainty of climate sensitivity. Second, the cases of abrupt changes in climate models we have seen so far often rely on processes that are particularly uncertain, involving crude parameterizations of processes that are not well understood. Third, abrupt change rarely occurs on the global scale but is most pronounced on a regional scale, where model biases can be very large. These biases will probably tend to increase the uncertainty of the timing or even the existence of abrupt changes. Results from climateprediction.net show that the regional pattern of climate change differs for the different parameter choices (Murphy et al., 2004). Hence, knowledge about the distribution of possible global mean warming cannot be simply translated to distributions of abrupt change on the local scale. Moreover, it is probably not adequate to assess all regional tipping elements together in one global model because the modelled climate is often realistic in some regions but unrealistic in others.

One particular assessment of a climate tipping element focused on the prediction of a possible AMOC collapse in an intermediate complexity model (Challenor, 2004, 2007). In contrast to climateprediction.net, the model was only run 100 times, and the results were used to build an emulator. Essentially, the emulator interpolates the model results and provides a statistical approximation of the original climate model. An alternative approach of empirically capturing the relation between parameter input and model output is to use a neural network (Knutti et al., 2003). Such methods can provide an efficient way to obtain results for a large number of parameter settings, without having to run the climate model each time. Challenor et al. (2006) obtained a probability of roughly one-third for an AMOC collapse until 2100 in the model they used. Their definition of an AMOC collapse relied only on the total circulation change until 2100 and not on how abruptly the circulation changed over time. While this makes sense in the case of the slowly responding ocean circulation, a more refined definition of abrupt change might be required for analyses of other tipping elements. Exploring how prone a model is to produce abrupt changes would of course still not reveal

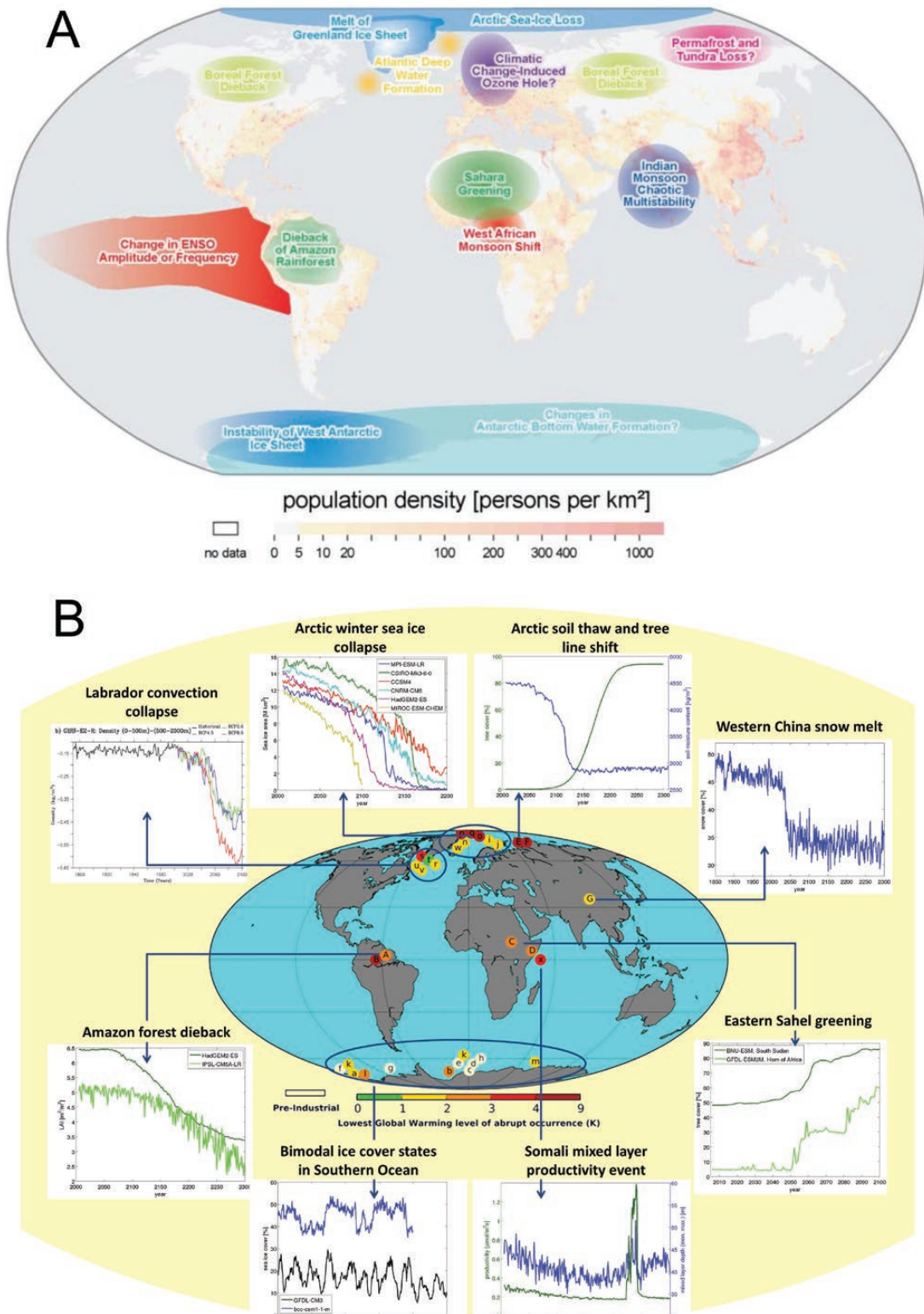


Figure 5. Regions with the potential for abrupt shifts in two previous studies. (A) Potential tipping elements in Lenton et al. (2008) [Copyright (2008) National Academy of Sciences]. (B) Abrupt shifts in complex climate model simulations modified from Drijfhout et al. (2015).

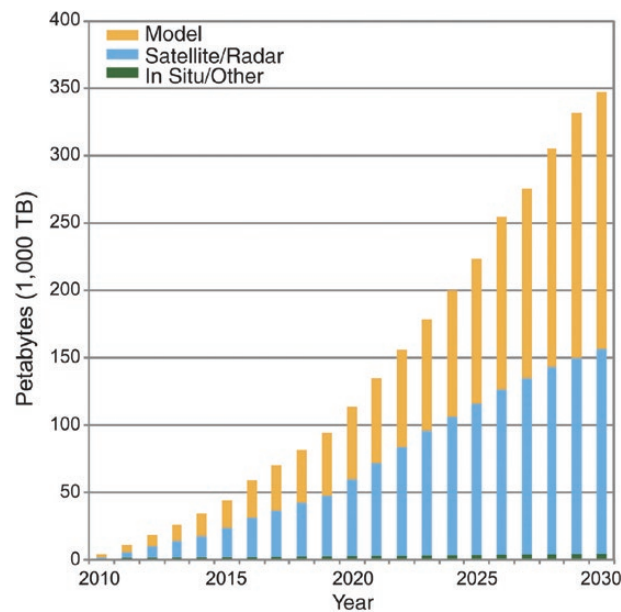


Figure 6. Projected increase in the amount of data from climate models, remote sensing and in situ observations (from [Overpeck et al., 2011](#)).

how likely abrupt change is in the real world. After all, we are stuck with the imperfect models we have. However, given that the candidates of tipping points currently explored mostly originate from very crude conceptual arguments or analogies to events in the Earth's past, a systematic sensitivity analysis of complex models could provide a clearer picture of the risks that are ahead. This picture could then be a fruitful starting point for more refined, process-based studies to eliminate or substantiate different candidates of tipping points.

6. Conclusions

We have reviewed several cases of potential tipping elements in the Earth system, and we have discussed the implications of these examples for investigations into future tipping points. The picture that emerges is far from complete, but it indicates that the existence of multiple steady states is often more elusive than the manifestation of abrupt change. For example, multiple AMOC states in general circulation models have rarely been identified, but there is little doubt that the AMOC is sensitive to freshwater forcing and that the abrupt Dansgaard–Oeschger events are associated with AMOC changes. A hierarchy of sea ice models has demonstrated that multiple states in sea ice disappear once spatial differences and the seasonal cycle are accounted for, but they also show that the freezing point introduces a threshold that can make sea ice loss very non-linear. The few reconstructions we have from the Sahara and current climate models indicate that the Green Sahara was transformed to today's desert in a spatio-temporally complex transition that was gradual on a large scale, but single palaeorecords also indicate abrupt transitions on a local scale. A similar picture emerges for the evolution of Asian monsoons under orbital forcing and the future of permafrost. Finally, no conclusive evidence of alternative stable states in ecosystems like rainforests, savannas and coral reefs has been found, but the non-linearities in these complex systems certainly allow for rapid and catastrophic shifts.

As different as these systems are from each other, they do indicate some common features that promote the occurrence of abrupt change. For example, the meridional overturning circulation in the Atlantic is a system that extends over a huge distance and closely couples the climate in remote regions. Moreover, it contains powerful feedbacks associated with the advection of heat and salt, and there are regionally confined hotspots of deep water formation. This structure of the system makes it susceptible to perturbations, and the prospects for low-order approaches to the problem are relatively good. A similar argument may be applied to ice sheets which involve two strong positive feedbacks associated with albedo and elevation and that can have a large extent with relatively uniform surface properties.

In contrast, the terrestrial vegetation in the Sahara is a much more heterogeneous system. Although the monsoon circulation plays the role of a globalizer, the heterogeneity and the limited strength of local atmosphere–vegetation feedbacks does not seem to allow a large-scale collapse. Sea ice falls in between these extremes. Although a tipping point is very unlikely to occur in the future, the freezing point as a non-linear threshold, the relative homogeneity of the Arctic Ocean and the ice-albedo feedback make the Arctic prone to rapid ice loss. In our review, we have pointed out that abrupt change goes beyond the question of multiple equilibria. In this regard, the hunt for alternative stable states in models is not always the most meaningful endeavour. Instead, the development and application of new methods to detect and understand abrupt change in general deserves more research emphasis. As we have outlined above, traditional methods that arise from a low-order logic can fail to capture the mechanisms behind abrupt change.

Despite the huge progress that has been made since the pioneering studies on multiple equilibria and abrupt change, it often remains unclear whether tipping points exist and how they can manifest themselves in the future. This uncertainty leaves ample room for interpretation that one may be tempted to fill with overconfident claims or promises. In principle, one can distinguish two approaches to this uncertainty in the scientific community. The first approach is to follow a bottom-up strategy by observing and modelling certain processes with as much realism as possible until a robust result finally emerges (e.g. [Dijkstra, 2013](#)). The second approach is to adopt a top-down perspective (as we have done in this paper) by exploring mere possibilities of tipping points and devising methods to analyse and categorize them. In our experience, these two perspectives sometimes manifest themselves in two camps of scientists that are separated by a cultural gap. To illustrate this cultural gap, we may describe a stereotype representative of the first camp as a diligent tinkerer who permanently strives to study processes in more and more detail, e.g. by increasing the complexity of models. In contrast, his counterpart, the creative dreamer, tries to simplify, unify and categorize the world. While the tinkerer often rejects the idea of tipping points as it does not compellingly follow from the data, the dreamer sees no factual proof against these ideas, and both of them consider the burden of proof to be on the other one. In the worst case, it occurs that the tinkerer perceives the dreamer's work as speculative at best and that the dreamer sees the tinkerer's approach as unimaginative and pedantic. In these situations, the scientific debate appears like a reinstallment of the 1970s dispute about the 'catastrophe theory' ([Zahler and Sussmann, 1977](#); [Zeeman, 1976](#)). Given the great deal, we have learned since then, the scientific community should not make the same mistake twice. Instead, it should be easy to acknowledge that it is the tinkerer who lays the foundation of all our progress by pushing the limits of our process understanding. Conversely, there are uncountable examples of how the imagination of the dreamer opens up new perspectives, provokes new and fruitful hypotheses and helps to build inspiring links between disciplines. Neither of them can excel without the other, since the dreamer can hardly create new knowledge, while the tinkerer can be blinded by complexity.

Putting these oversimplified stereotypes aside, we note that an emphasis on the linkage between the two perspectives appears as the most promising strategy to make progress. Additionally, closer collaborations with the developers of tools to study complex spatio-temporal systems are needed, in order to make efficient use of their expertise and prevent the proverbial reinvention of the wheel. In practice, this highlights the importance of close exchange and focused collaborations between disciplines. It is also an argument to strive for a continuous and systematic problem-specific model hierarchy that is often called for but rarely realized. The division of labour we address here is particularly important for the assessment of the risk of abrupt climate change in the future. As we have outlined above, a robust assessment of the uncertainty of abrupt change poses considerable scientific challenges. Hence, we can probably best tackle this challenge by an iterative process: identifying the most likely candidates of climate tipping elements in the form of regional hotspots of non-linear processes, formulating important key questions whose answers would reduce the uncertainty in an efficient way, performing process-based studies that use observations and models of different complexity, update the risk assessment and reiterate. Of course, this process is already happening as it emerges from scientific interactions. We can help to intensify it by better integrating both, bottom-up and top-down perspectives, in scientific projects. The alliance of the two camps will unleash the energy we need to tackle the challenges of abrupt climate change.

Acknowledgements

This article emerged from a project workshop within the program of the Netherlands Earth System Science Centre (NESSC, www.nessc.nl), financially supported by the Ministry of Education, Culture and Science (OCW). We acknowledge the very constructive and helpful comments we received from two anonymous reviewers. We are also grateful to Peter Cox and Appy Sluijs for fruitful discussions over beers and balls.

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