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INFLUENCE OF THE SENSOR LOCATION ON THE PRACTICAL OBSERVABILITY OF A FIXED BED BIOREACTOR

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Abstract. This paper deals with the state observability analysis of a distributed parameter fixed bed reactor. The objective of this paper is to analyze the influence of the position of the "internal" sensor(s) on the observability in order to determine the best observability possible in practice. The criterion is the conditioning number of the observability matrix of the linearized tangent model of the discretized model of the process. It is shown that there is an optimal sensor location for which the conditioning number is minimized.

Key Words. biotechnology, distributed parameter systems, observability, process control, singular value decomposition

1. INTRODUCTION

State observation is certainly a key question in bioprocess monitoring and control. Indeed the implementation of efficient modern control strategies depends highly on the availability of on-line information about the key process components like biomass, substrates or products. But in many instances, there is a lack of on-line sensors for these, or their cost may be high and possibly prohibitive in practice, or the available ones are very fragile. The state observation problem is even more difficult in distributed parameter (e.g. fixed bed or fluidised bed) bioreactors, where the reaction medium is not homogeneous and where the concentration of the different process components depends on the spatial position in the tank. Beside the problem of designing efficient state observers based on the few available data, we are facing the problem of locating sensors at appropriate positions in order to obtain the best information about the process dynamics along the reactor (see also (Kumar and Seinfeld, 1978), (Harris et al., 1980), (Alvarez et al., 1981)).

In this section, we address the following question. We consider a fixed bed bioreactor involving two biochemical reactions (microorganisms' growth and death) and two components (biomass and a limiting substrate) where dispersion is assumed to be negligible. We assume that the substrate concentration is available for on-line measurement via a limited number of sensors. We consider three cases. In the first case, the substrate is measured via one sensor located somewhere along the reactor. In the second and third cases, we consider two and three sensors, respectively, one at the reactor output and one or two somewhere along the reactor.

For the observability analysis and the simulation, the nonlinear distributed parameter model of the plug flow reactor will be reduced, first into a nonlinear lumped parameter model by considering a functional approximation method, more precisely the orthogonal collocation method, then into a linear model by considering the linearized tangent model of the nonlinear lumped parameter model. The observability analysis is carried out on the linearized tangent lumped parameter model by considering the classical linear systems tools.

Here we analyze the influence of the position of the "internal" sensor(s) on the observability in order to determine the best observability possible in practice. The criterion is the condition number of the observability matrix of the linearized tangent model of the discretized model of the process (Damak et al., 1992). It is shown that there is an optimal sensor location for which the condition number is minimized (Tali-Maamar, 1994). Because of the usual uncertainty of the process parameters, a sensitivity analysis is carried out to show the influence of process uncertainty on the optimal location of the "internal" sensor. Similarly, the influence of the parameters of the model approximation (number of collocation points, value of the parameters of the Jacobi polynomial) on the practical observability is analyzed. The practical observability is illustrated via simulation of an extended Luenberger observer combined with an asymptotic observer, based on different conditions (different sensor locations, different process parameter values).

2. DYNAMICAL MODELS

2.1. *Distributed Parameter Mass Balance Model*

Let us consider a plug flow bioreactor with the following growth and mortality/detachment reactions :

$$growth: \quad k_1 S \longrightarrow X \qquad (1)$$

$$death/detachment: \quad X \longrightarrow X_d \qquad (2)$$

where S, X and X_d hold for substrate, living and dead (or detached) biomass, respectively, and k_1 is a yield coefficient. Its dynamics are given by the following mass balance equations :

$$\frac{\partial S}{\partial t} = -u\frac{\partial S}{\partial z} - k_1\frac{1-\epsilon}{\epsilon}r_1 \qquad (3)$$

$$\frac{\partial X}{\partial t} = r_1 - r_2 \qquad (4)$$

$$r_1 = \mu X, \; r_2 = k_d X \qquad (5)$$

and the following boundary conditions :

$$S(z=0,t) = S_{in}(t), \; X_d(z=0,t) = 0 \qquad (6)$$

where u is the fluid superficial velocity, ϵ is the void fraction, z ($0 \leq z \leq$ L) is the spatial coordinate, r_1 and r_2 are the reaction rates of the growth and the death/detachment reactions, respectively, μ is the specific growth rate, k_d is the mortality coefficient, and S_{in} is the influent substrate concentration. In the observability analysis, we have considered a Contois model (see e.g. (Bastin and Dochain, 1990)) for the specific growth rate :

$$\mu = \frac{\mu_{max} S}{K_C X + S} \qquad (7)$$

2.2. *Discretization of the Distributed Parameter Model into a Lumped Parameter Model*

Let us approximate the distributed parameter model into a lumped parameter model by considering the orthogonal collocation method (see e.g. (Villadsen and Michelsen, 1978)). The first step consists then of writing the finite expansion of each state variable as a weighted sum of their values at the collocation points :

$$\xi_k \cong \sum_{i=0}^{q} \beta_i(z)\xi_{ki}(t) \qquad (8)$$

where $q-1$ is the number of interior discretization points (called *interior collocation points*), z_0 (= 0) and z_q (= L) corresponds to the reactor input and output, respectively, and $\xi_{ki}(t)$ the value of $\xi_k(z,t)$ at the position $z = z_i$:

$$\xi_{ki}(t) = \xi_k(z = z_i, t) \qquad (9)$$

The base functions $\beta_i(z)$ are chosen as *orthogonal* functions with:

$$\beta_i(z = z_j) = \delta_{ij} \qquad (10)$$

The second step in the application of the orthogonal collocation approximation consists of rewriting the space derivatives by considering the above expansion. This gives the following set of differential equations :

$$\frac{d\tilde{S}}{dt} = -u\tilde{C}_1\tilde{S} + u\tilde{c}_1 S_{in} - \tilde{k}_1\tilde{r}_1 \qquad (11)$$

$$\frac{d\tilde{X}}{dt} = \tilde{r}_1 - \tilde{r}_2 \qquad (12)$$

with :

$$\tilde{C}_1 = [c_{1ji}]_{i,j=1\ to\ q} = [\frac{d\beta_i(z_j)}{dz}] \qquad (13)$$

$$\tilde{c}_1 = [\frac{d\beta_0(z_j)}{dz}]_{j=1\ to\ q} \qquad (14)$$

$$\tilde{S} = \begin{pmatrix} S(z=z_1,t) \\ S(z=z_2,t) \\ \vdots \\ S(z=L,t) \end{pmatrix} \qquad (15)$$

$$\tilde{X} = \begin{pmatrix} X(z=z_1,t) \\ X(z=z_2,t) \\ \vdots \\ X(z=L,t) \end{pmatrix} \qquad (16)$$

$$\tilde{k}_1 = k_1\frac{1-\epsilon}{\epsilon} \qquad (17)$$

$$\tilde{r}_i = \begin{pmatrix} r_i(z=z_1,t) \\ r_i(z=z_2,t) \\ \vdots \\ r_i(z=L,t) \end{pmatrix}, \; i = 1,2 \qquad (18)$$

In the simulations and the observability analysis, we have considered 4 interior collocations points (q = 5) located at the zeroes of a Jacobi polynomial with (except when mentioned) $\alpha = 0$, $\beta = 4$. Then the interior collocation points are located at the following positions :

$$z_1 = 0.31L, \; z_2 = 0.58L \qquad (19)$$

$$z_3 = 0.81L, \; z_4 = 0.96L \qquad (20)$$

An extensive simulation study about the application of orthogonal collocation method to the above microbial process example has been carried out in (Tali-Maamar, 1994) by using in particular the ORP (Order Reduction Parameter) notion introduced in (Srivastava and Joseph, 1985).

2.3. *Linearized Tangent Model*

For the observability analysis, we further transform the nonlinear lumped parameter model (11)(12) into its linearized tangent version around some operating point. If we define :

$$\begin{aligned} \tilde{s} &= \tilde{S} - \tilde{S}_{ss},\ \tilde{x} = \tilde{X} - \tilde{X}_{ss} & (21) \\ m &= \begin{pmatrix} u - u_{ss} \\ S_{in} - S_{in,ss} \end{pmatrix} & (22) \end{aligned}$$

(where the index "ss" holds for steady state values), then it is written as follows :

$$\begin{aligned} \frac{dx}{dt} &= Gx + Dm & (23) \\ y &= Cx + Hm & (24) \end{aligned}$$

with :

$$\begin{aligned} x &= \begin{pmatrix} \tilde{s} \\ \tilde{x} \end{pmatrix} \\ G &= \begin{pmatrix} -u\tilde{C}_1 - \tilde{k}_1(\frac{\partial \tilde{r}_1}{\partial \tilde{S}})_{ss} & -\tilde{k}_1(\frac{\partial \tilde{r}_1}{\partial \tilde{X}})_{ss} \\ (\frac{\partial(\tilde{r}_1}{\partial \tilde{S}})_{ss} & (\frac{\partial(\tilde{r}_1 - \tilde{r}_2)}{\partial \tilde{X}})_{ss} \end{pmatrix} \\ D &= \begin{pmatrix} \tilde{c}_1 S_{in} - \tilde{C}_1 \tilde{S}_{ss} & u\tilde{c}_1 \\ 0 & 0 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} |\frac{\partial(\tilde{r}_1)}{\partial \tilde{X}}|_{ss} &= [(rX)_{ij}] = [|\frac{\partial(\tilde{r}_1(z = z_j))}{\partial \tilde{X}(z = z_i)}|_{ss}] \\ |\frac{\partial(\tilde{r}_1)}{\partial \tilde{S}}|_{ss} &= [(rS)_{ij}] = [|\frac{\partial(\tilde{r}_1(z = z_j))}{\partial \tilde{S}(z = z_i)}|_{ss}] \end{aligned}$$

and C is a px2q matrix (with p the number of sensors (p = 1, 2 or 3)). For p = 2 and 3, the last row of C corresponds to the output sensor and is composed of 0's except at the position $z = L$ where the entry is equal to 1. For the other rows, two situations can be encountered :

1. If the measured substrate concentration is located at a collocation point, then the entries of the corresponding row C are all zero except at the collocation position, where it is 1.
2. If the measured substrate concentration is not located at a collocation point, we use the formula (8) and the entries of C are the values of the functions $\beta_i(z)$ (i = 1 to q) at the sensor position. Because of (8), we have therefore to consider a "direct transmission" in the output equation, $\beta_0(z)S_{in}$: this justifies the introduction of the term Hm in (24).

3. THE OBSERVABILITY MATRIX AND THE CONDITION NUMBER

The observability matrix of the linearized tangent model (23)(24) is equal to :

$$O = \begin{pmatrix} C \\ CA \\ \vdots \\ CA^{2q-1} \end{pmatrix} \tag{25}$$

It has been found that for the three cases (1, 2 and 3 sensors) for all the considered sensor configurations, the observability matrix is full rank. However it may happen that in practice the system may be difficult to observe, i.e. it is difficult, if not practically impossible, to assign the observer dynamics even in the presence of a perfect dynamics knowledge. This happens for instance when although the observability matrix is full rank, it is nearly singular, i.e. its singular values cover a large range of values. Therefore we decided to consider the condition number, i.e. the ratio of the largest singular value over the smallest one, to evaluate the *practical* observability of the system dynamics (23)(24). In that context, the condition number can be viewed as a tool for analyzing the sensitivity of the process observability with respect to the sensor locations.

Moreover, in order to simplify the numerical computations, we have decided to concentrate the practical observability analysis on the substrate concentration dynamics by neglecting the biomass concentration dynamics. Indeed, if we note the matrix G as follows:

$$G = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} \tag{26}$$

we have noted ((Tali-Maamar, 1994)) that in our case the non-diagonal blocks G_{12} and G_{21} have a negligible influence on the values of the eigenvalues of G, i.e. that the system is almost decoupled.

4. INFLUENCE OF THE SENSOR LOCATION ON THE PRACTICAL PROCESS OBSERVABILITY

The practical observability analysis has been performed by considering the following steps :

1. Computation of the condition numbers for one approximation configuration ($\alpha = 0$, $\beta = 4$) and by covering the values of z between 0 and L, for 1, 2 and 3 sensor(s);
2. Analysis of the influence of the approximation parameters α and β;
3. Analysis of the influence of the process parameters values on the practical observability.

The observability has been carried out for the steady-state corresponding to the following set of parameter and input values :

$$\begin{aligned} A_L &= 0.02\,m^2,\ F = 2\,l/h,\ S_{in} = 5\,g/l, \\ L &= 1\,m,\ \tilde{k}_1 = 0.4,\ \mu_{max} = 0.35\,h^{-1}, \\ K_C &= 0.4,\ k_d = 0.05\,h^{-1} \end{aligned}$$

The results of step 1 are illustrated in Figures 1 and 2. Figure 1 gives the condition number as a function of the internal sensor(s) position for the three cases (1, 2 and 3 sensor(s)). In the latter case, the condition number with respect to the one "moving" sensor position is drawn in the best case of 2 internal sensors (one sensor is "fixed" in position z = 0.8 L). The minimum value of the condition number is decreasing with the increasing number from 60 to 40) : this tends to show that, as it may be expected, we may have better observability properties with three sensors than with only two. Yet these minima are close to each other (the optimal sensor locations are around $0.7L$ for one sensor, around $0.4L$ for the internal in the two sensor case, and in the three sensor case between $0.25L$ and $0.3L$ for the first internal sensor and between $0.75L$ and $0.8L$ for the second one).

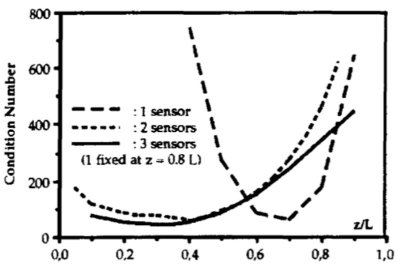

Figure 4.1. Conditioning number with respect to z for 1, 2 and 3 sensors

This latter case is more precisely illustrated in Figure 2 which shows the three-dimensional results of the practical observability analysis for 2 internal sensors (with the condition number as a function of the two internal sensor positions), with the two "valleys" in the above-mentioned spatial regions.

Note that the condition number is lower when there are 3 sensors : this tends to show that in practice, one may expect more robust observation results by using three sensors, instead of two.

The sensitivity of the condition number with respect to the parameters α and β of the Jacobi polynomial used in the orthogonal collocation approximation method and with respect to the model parameters (k_d and μ_{max}) has been analyzed. We found out that the minima in the three cases (1, 2 or 3 sensors) remain located at the same positions, with just a slight difference in the values of the minimum condition number (For instance, in the 1 sensor case, when the value of the parameter k_d is changed by 5%, the minimum is still located in z = 0.7 but the condition number is equal to 59.8 instead of 60.6).

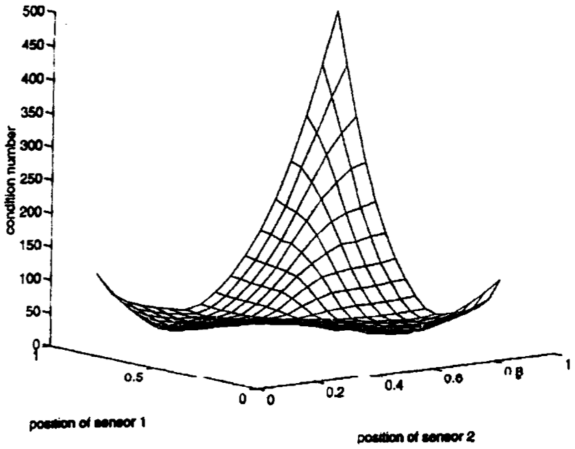

Figure 2. condition number for three sensors

5. STATE OBSERVER

In order to illustrate the influence of the sensor location on the practical observability, a state observer has been designed, constituted of a partial state Luenberger observer for the substrate concentration combined with an asymptotic observer for the biomass concentration. The design of the asymptotic observer is based on the following auxiliary variable (see (Dochain and Perrier, 1993)):

$$\zeta = \tilde{k}_1 X + S \tag{27}$$

The observer has then the following form :

The partial state Luenberger observer

$$\begin{aligned} \frac{d\hat{S}_i}{dt} &= -(\hat{\zeta}_i - \hat{S}_i)\frac{\mu_{max}\hat{S}_i}{K_C(\hat{\zeta}_i - \hat{S}_i) + \hat{S}_i} \\ &\quad -u\sum_{j=1}^{q} c_{1ji}\hat{S}_j + uc_{10i}S_{in} \\ &\quad +\sum_{k=1}^{p}\lambda_{ik}(S_k - \hat{S}_k),\ i = 1\ to\ q \end{aligned} \tag{28}$$

The asymptotic observer

$$\frac{d\hat{\zeta}_i}{dt} = -u\sum_{j=1}^{q} c_{1ji}\hat{S}_j + uc_{10i}S_{in} - k_d(\hat{\zeta}_i - \hat{S}_i)$$

$$\hat{X}_i = \frac{\hat{\zeta}_i - \hat{S}_i}{\tilde{k}_1}, \; i = 1 \; to \; q$$

In the above equation (28), the index k refers to the measured variable(s). The values of the gains λ_i (i = 1 to q) are computed so as to assign the poles of the observer on the basis of the linearized tangent model (23)(24) (at the initial steady-state), i.e. of the matrix :

$$G_{11} - \Lambda C_1 \tag{29}$$

with :

$$\Lambda = [\lambda_{ik}]_{i=1 \; to \; q, \; k=1 \; to \; p} \tag{30}$$

and C_1 contains the first q columns of C.

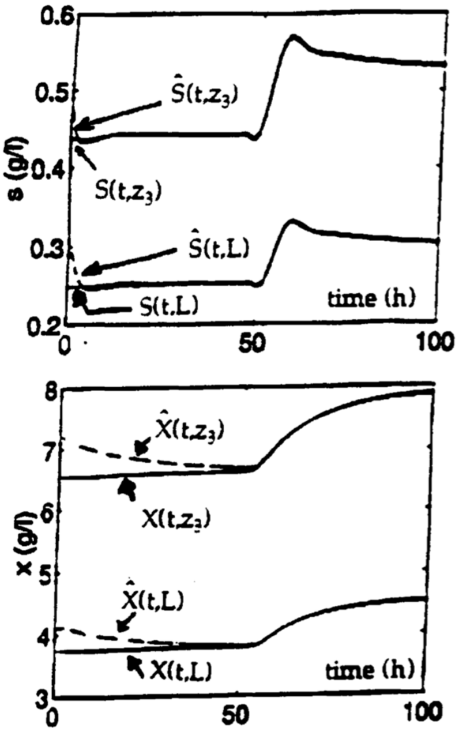

Figure 3. State observation with one sensor at the "optimal" position z = 0.7 L

The observer has been implemented under the following conditions:

Initial values :

$$\begin{aligned}
X_1(0) &= 29.4026g/l, \; S_1(0) = 1.9602g/l \quad (31)\\
X_2(0) &= 13.2069g/l, \; S_2(0) = 0.8805g/l \quad (32)\\
X_3(0) &= 6.5457g/l, \; S_3(0) = 0.4364g/l \quad (33)\\
X_4(0) &= 4.1758g/l, \; S_4(0) = 0.2784g/l \quad (34)\\
X_5(0) &= 3.734g/l, \; S_5(0) = 0.2489g/l \quad (35)\\
\hat{S}(0) &= 1.1S(0), \; \hat{X}(0) = 1.1X(0) \quad (36)
\end{aligned}$$

Desired Luenberger observer poles :

$$-1.1, \; -1.2, \; -1, \; -0.9, \; -1.5 \tag{37}$$

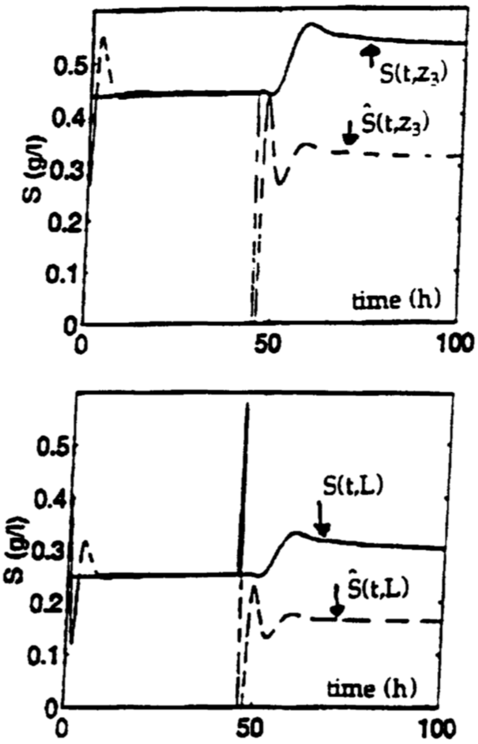

Figure 4. State observation with one sensor at the "non-optimal" position z = 0.1 L

Figures 3 and 4 show the performance of the observer with one sensor at two collocation points (z_3 and L). In Fig.3, the sensor is located at the "optimal" position z = 0.7 L while in Fig.4, the sensor has been located in z = 0.1 L (for which the order of magnitude of the condition number is 10^4). Note the significative deterioration of the observer performance in the second case (The figure tends to show that there is a bias in the state estimation, but which will tend to zero in the long term).

A behaviour similar to the one observed in the 1 sensor case has been observed in the two other cases. As a matter of illustration, Figure 5 shows the observation results with three sensors located at z_1 = 0.37 L, z_2 = 0.8 L and z_3 = L (i.e. not very from the "optimal" positions (0.3 L, 0.75 L, L). The robustness of the observer with respect to parameter uncertainties has been evaluated in simulation, and it was found that the observer can be sensitive to erroneous parameter values especially for μ_{max} (Figure 6, where the value of μ_{max} has been set to 0.36 h^{-1} in the observer (instead of 0.4 h^{-1} in the model)), see also (Tali-Maamar, 1994).

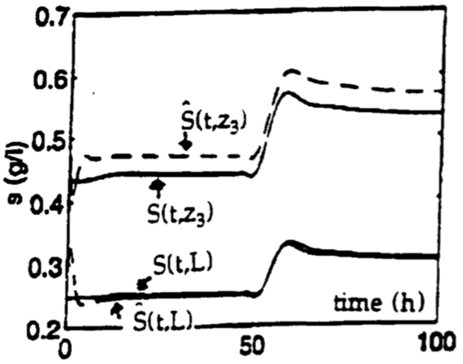

Figure 5. State observation with three sensors located at the "non-optimal" positions

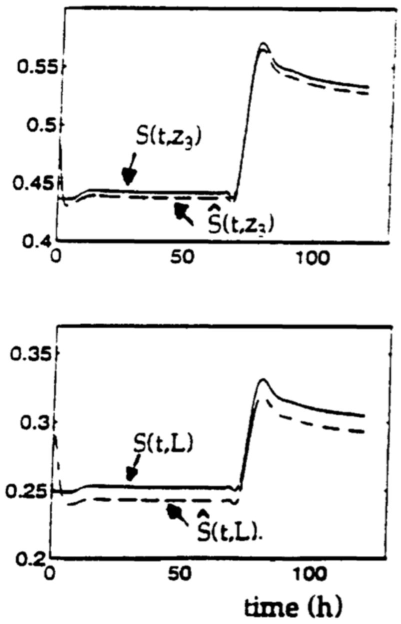

Figure 6. State observation with one sensor at z = 0.67 L
and uncertainty on μ_{max}

6. CONCLUSIONS

This paper has dealt with the state observability analysis of acdistributed parameter fixed bed bioreactor. The objective of this paper was to analyze the influence of the position of the "internal" sensor(s) on the observability in order to determine the best observability possible in practice. The criterion was the conditioning number of the observability matrix of the linearized tangent model of the discretized model of the process. It has been shown that there is an optimal sensor location for which the conditioning number is minimized.

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