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Editors

New Trends in Mechanism Science

Analysis and Design

Springer

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ISBN 978-90-481-9688-3 e-ISBN 978-90-481-9689-0
DOI 10.1007/978-90-481-9689-0
Springer Dordrecht Heidelberg London New York

Library of Congress Control Number: 2010933197

Cover design: eStudio Calamar S.L.

Printed on acid-free paper

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Multi-Objective Optimization of Parallel Manipulators

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Abstract. This paper deals with the optimal dimensional design of Delta robot. The aim is to obtain a manipulator with maximum dexterity and minimum projected area of the occupied volume on the base plane. Two different optimization strategies, which cope with assembly difficulties, have been applied to solve this problem. Comparing optimized Delta robot with and without area reduction, results show an important reduction of area maintaining an acceptable dexterity.

Key words: parallel robots, dimensional synthesis, optimization, closed-loop multibody systems

1 Introduction

Compared with serial robots, parallel robots present advantages in terms of speed, accuracy and stiffness [10]. Nevertheless, they have more singular configurations, smaller workspace and dexterity.

Dimensional synthesis is a basic task in the design of a parallel manipulator, because its performance is highly sensitive to its dimensions. However, dimensional synthesis of closed-loop multibody systems is significantly more complex than serial ones and it is still an unfinished research area.

One of the most important parameters in the design of parallel robots is the dexterity. This parameter is a performance related with the accuracy of manipulator control. It is defined by the condition number of the forward Jacobian matrix. If the value of condition number is 1 over the workspace, the robot is called *fully isotropic* [3]. Achieving isotropic manipulators is a common design objective [2, 4, 6, 8, 9]. However, designing only with respect to a performance index usually leads to a huge space requirements [6] or provides more singular configurations [13].

The aim of this work is to achieve a maximum dexterity and minimum projected area in Delta robot, which is a dimensional synthesis problem. First of all, the multi-objective function is defined. Then, two different strategies are described in order to optimize the manipulator. Finally, results of the optimization process are presented and discussed.

D. Pisla et al. (eds.), *New Trends in Mechanism Science:Analysis and Design*, Mechanisms and Machine Science 5, DOI 10.1007/978-90-481-9689-0_72,

2 Definition of the Synthesis Error Function

The Synthesis Error Function (SEF) or objective function is defined as the sum of two terms:

$$SEF = \eta_1 + \omega\eta_2 \tag{1}$$

Term η_1 is related with the kinetostatic performance of the Delta Robot, and term η_2 is related with the projected area occupied by the robot. Term ω is a weighting factor.

2.1 *Kinetostatic Performance Index: Dexterity*

Dexterity of a manipulator is a kinetostatic performance which is defined by the condition number κ of the forward kinematics Jacobian matrix $\mathbf{J}_f$. It is a measure of the efficiency in motion transmission of the manipulator. $\mathbf{J}_f$ maps the actuated velocities $\dot{\mathbf{q}}$ into the end-effector velocities $\dot{\mathbf{x}}$, as it is shown in Eq. (2)

$$\dot{\mathbf{x}} = \mathbf{J}_f\dot{\mathbf{q}} \tag{2}$$

The condition number κ is computed as the ratio between the largest σ_l and the smallest singular value σ_s of $\mathbf{J}_f$, as it is shown next

$$\kappa(\mathbf{J}_f) = \frac{\sigma_l}{\sigma_s} \tag{3}$$

Note that the values for the condition number as it is defined in Eq. (3) may cover from 1 to ∞. For this reason, it will be used its inverse in this application. Global Dexterity Index (GDI) is a posture-independent index defined as the mean of the inverses of the condition number κ over a volume V in the Cartesian space of the end-effector [1, 9]:

$$GDI = \frac{\int_V \frac{1}{\kappa} dV}{V} \tag{4}$$

Volume V of Eq. (4) is the chosen workspace subset of the manipulator. It is discretized here in eight precision poses which corresponds to the vertices of a 2 cm cube. The aim here is not to compute dexterity at the most critical points, but to obtain the mean value over a given volume. For this reason, the eight vertices of the cube are chosen. Anyway, as the dimensions of the cube are small with respect to the dimensions of the robot, dexterity should not vary too much over the cube. Thus, term η_1 from Eq. (1) will be

$$\eta_1 = -\frac{1}{8}\sum_{i=1}^{8} \frac{1}{\kappa\left(\left(\mathbf{J}_f\right)_i\right)} \tag{5}$$

Negative sign of Eq. (5) is because the aim is to maximize the *GDI*.

2.2 Area Occupation Index

Previous works [6, 13] show that Delta robot optimization only with respect to Eq. (5) leads to a huge design. In practice, not only a good kinematic performance is expected, but also a space occupation as small as possible [13]. A smaller design could be obtained by reducing the upper bound of a leg length. However, for practical reasons and depending on the application, the projected area is a more suitable index. The goal is not to obtain the smallest area but an acceptable area for a given application. For this reason, another term is introduced here

$$\eta_2 = \frac{1}{8}\sum_{i=1}^{8}\frac{(A_p)_i - A_w}{A_w} \tag{6}$$

where $(A_p)_i$ is the total projected area of the occupied volume on the base plane in each precision poses, i.e. the minimum bounding rectangle around the robot, and A_w is the projected area of the chosen robot workspace subset on the base plane, i.e. 4 cm^2 in this work. This term η_2 is also dimensionless, in order to maintain the dimensional balance on Eq. (1), and it is also independent on the scale of the robot.

3 Kinematic Optimization

Definition of Delta robot and synthesis problem are described in this section. Then, two different strategies are presented to solve the optimization problem.

The Delta robot is shown in Figure 1 invented by Reymond Clavel [5]. Only the 3 translational d.o.f. of the moving platform will be considered in this work.

Design parameters of this problem, grouped in vector **l**, are: the lengths of the upper leg l_A and the lower leg l_B, the distance z_c between the base and the center of the cube and the characteristic radii of the base r_b and the platform r_p

$$\mathbf{l} = \left[\, l_A \; l_B \; z_c \; r_b \; r_p \,\right]^t \tag{7}$$

Fourteen generalized coordinates **q** are defined by means of relative coordinates. There are three revolute joints in each closed-loop and three linear displacement of the end-effector. As the Delta robot is a 3-d.o.f. manipulator in this work, three generalized coordinates are independent (grouped in vector **u**) and nine are dependent (grouped in vector **v**). Nine assembly constraints (**h**) are defined in each pose

$$\mathbf{h}(\mathbf{l}, \mathbf{q}_1(\mathbf{l}), \ldots, \mathbf{q}_8(\mathbf{l})) = \mathbf{0} \tag{8}$$

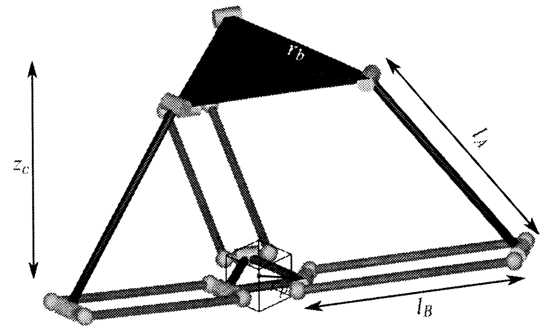


Fig. 1 Delta robot model.

The software tool used to obtain the kinematic equations in a symbolic form is Robotran [12], developed by the CEREM at the Université Catholique de Louvain.

3.1 Definition of the Synthesis Problem

The aim of this work is to reach the maximum *GDI* of the Delta robot when it occupies the minimum projected area. This is an optimization problem, whose solution will be the minimization of the *SEF* (Eq. 1) with respect to design variables $\mathbf{l}$ (Eq. 7). However, *SEF* also depends on the generalized coordinates $\mathbf{q}$ that are constrained by the assembly constraints $\mathbf{h}(\mathbf{l},\mathbf{q}(\mathbf{l})) = \mathbf{0}$. Moreover, design variables are also bound-constrained by means of a set of design inequations as $\mathbf{b}(\mathbf{l}) \geq \mathbf{0}$.

Thus, the *SEF* is evaluated on the eight precision poses and the objective function can be formulated as follows

$$\begin{aligned} &\min_{\mathbf{l}} SEF(\mathbf{l},\mathbf{q}_1(\mathbf{l}),\ldots,\mathbf{q}_8(\mathbf{l})) \\ &\text{subject to } \mathbf{h}(\mathbf{l},\mathbf{q}_1(\mathbf{l}),\ldots,\mathbf{q}_8(\mathbf{l})) = \mathbf{0} \\ &\text{and } \mathbf{b}(\mathbf{l})) \geq \mathbf{0} \end{aligned} \tag{9}$$

In order to guarantee the assembly of the manipulator, two strategies have been applied [6]. The next sections will describe them briefly.

3.2 Artificial *Penalty Optimization Strategy*

In this approach, classical method of Newton–Raphson is proposed to solve the assembly constraints during the optimization process. When the assembly is not possible, a penalization of the objective function is performed.

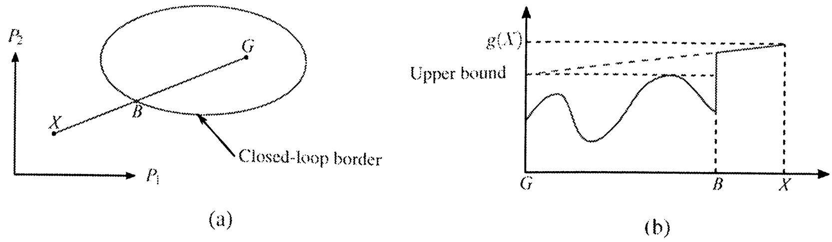


Fig. 2 *Artificial* penalization strategy.

A theoretical illustration with two parameters P_1 and P_2 is shown in Figure 2(a). Suppose that the optimizer is calling the objective function outside the closed-loop border, at point X, during the optimization process. A fixed point G is chosen inside the boundaries and the penalization is computed along the direction GX.

As the value of SEF is upper-bounded, a discontinuous penalization can be envisaged. The penalized objective function is straightforwardly computed by summing this upper-bound and a linear extension starting from G along GX for example, as it is shown in Figure 2(b).

In summary, the penalized objective function $g(\mathbf{l}, \mathbf{q}(\mathbf{l}))$ to optimize could be defined as

$$g(\mathbf{l}, \mathbf{q}(\mathbf{l})) = \begin{cases} SEF(\mathbf{l}, \mathbf{q}(\mathbf{l})) & \text{if constraints are satisfied} \\ SEF(\mathbf{l}, \mathbf{q}(\mathbf{l})) + SEF_{pen}(\mathbf{l}) & \text{otherwise} \end{cases} \tag{10}$$

where $SEF_{pen}(\mathbf{l})$ is the penalized or extension term calculated along the direction GX. Design inequations $\mathbf{b}$ play the same role as assembly constraints $\mathbf{h}$ to define de feasible region. This strategy is integrated in a gradient-free direct search algorithm Nelder–Mead Simplex [11].

3.3 Best-Assembly Penalty Optimization Strategy

This strategy is based on the minimization of the assembly constraints instead of their resolution with respect to dependent generalized coordinates ($\mathbf{v}$) [7]. Thus, rather than solving Eq. (8), it is formulated as the following constrained optimization problem

$$\begin{aligned} &\min_{\mathbf{v}} \tfrac{1}{2}\mathbf{h}(\mathbf{l}, \mathbf{u}, \mathbf{v})^t\, \mathbf{h}(\mathbf{l}, \mathbf{u}, \mathbf{v}) \\ &\text{subject to } \mathbf{c}(\mathbf{l}, \mathbf{u}, \mathbf{v}) \geq \mathbf{0}, \end{aligned} \tag{11}$$

where inequalities $\mathbf{c}(\mathbf{l}, \mathbf{u}, \mathbf{v}) \geq \mathbf{0}$ are formulated in order to avoid multiple solutions and singular configurations when assembly problem is solved. Relative coordinates make its formulation easier and an unique and regular assembly solution can be achieved.

Eq. (11) can be seen as an extension of the assembly problem. If the assembly constraints are satisfied, solution of Eq. (11) is zero, as Newton–Raphson in *Artificial* strategy. Otherwise, it is minimized while Newton–Raphson algorithm does not provide a consistent result to the assembly problem. With this strategy, the performance of the robot can be evaluated everywhere in the design space, even if it is not assembled. The penalized objective function g is defined as follows

$$\begin{aligned} & g(\mathbf{l},\mathbf{q}(\mathbf{l})) = SEF(\mathbf{l},\mathbf{q}(\mathbf{l})) + \omega_b f_{pen}(\mathbf{l},\mathbf{q}(\mathbf{l})) \\ \text{such that} \quad & f_{pen}(\mathbf{l},\mathbf{q}(\mathbf{l})) = \textstyle\sum_{k=1}^{8} \min_{\mathbf{v_k}} \frac{1}{2}\mathbf{h}\left(\mathbf{l},\mathbf{u_k},\mathbf{v_k}\right)^t \mathbf{h}\left(\mathbf{l},\mathbf{u},\mathbf{v}\right) \\ \text{with} \quad & \mathbf{c}\left(\mathbf{l},\mathbf{u_k},\mathbf{v_k}\right) \geq \mathbf{0}, \quad k = 1,\ldots,8 \end{aligned} \tag{12}$$

where ω_b is a weighted factor between the SEF and the penalty term f_{pen}.

Its main advantage is that g is always continuous and differentiable over the design space. For this reason, a gradient-based sequential quadratic programming (SQP) algorithm is used with this strategy, which is usually more efficient than direct search algorithms.

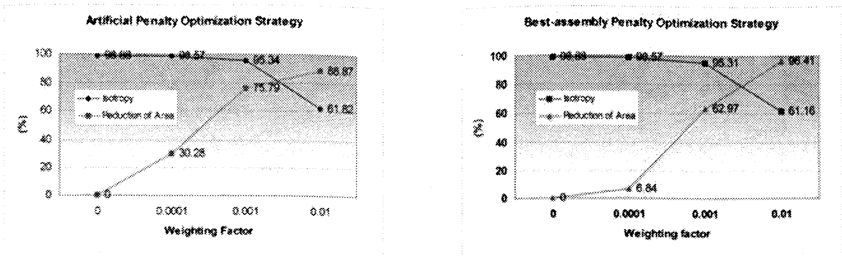
Two major drawbacks appear on this strategy. The first one is the choice of the ω_b. However, the final optimal solution must be an assembled configuration and then, this term vanishes. The value of ω_b in this work is 10^4. The second inconvenient of this strategy is the computation of the sensitivity analysis of the objective function. More information about this issue can be found in [6].

4 Application to Delta Robot: Results of Optimization

An initial simulation, *Sim. 1*, which does not take into account the area occupation index (η_2), i.e., $\omega = 0$ in Eq. (1), has been performed. Then, several optimization process have been computed corresponding to different values of weighting factor, as can be seen in Figure 3, showing the sensitivity analysis of the problem with respect to weighting factor. As the goal of this optimization problem is to keep an acceptable *GDI* reducing as much as possible the area, *Sim. 2* has been chosen as a second result with a weighting factor of $\omega = 0.001$ in Eq. (1).

Table 1 shows initial and optimal values of GDI and projected area when the end-effector is located at the center of the cube. All projected areas in this table are computed at this position. Percentages of optimal area (A_p^{opt}) variation are computed with respect to both initial area (A^{ini}) and optimal area when $\omega = 0$ $(A_p^{opt})_{\omega=0}$. Note that a slightly decrement of GDI, i.e., from 98.88% to 95.3%, entails a reduction of the occupied area more than 72% in both simulations.

Table 2 shows values of the design variables: initial guesses and their upper and lower values and the results of the two simulations. In *Sim. 1*, optimal values of design variables are similar in both strategies. There is only one difference: radii of platform and base. However, it demonstrates that GDI does not depend on these values but their ratio, which tends to unit value.



(a) (b)

Fig. 3 Sensitivity analysis of weighting factor.

Table 1 Optimization results.

		GDI [%]	A_p^{opt} [cm^2]	$\frac{A_p^{opt}-A^{ini}}{A^{ini}}$ [%]	$\frac{A_p^{opt}-(A_p^{opt})_{\omega=0}}{(A_p^{opt})_{\omega=0}}$ [%]	Fnc. evals
	Initial	36.11	353.81			
Sim.1: $\omega = 0$	Artificial	98.88	900.04	154.39		957
	Best-assembly	98.88	801.35	126.49		122
Sim.2: $\omega = 0.001$	Artificial	95.34	217.93	-38.40	-75.79	919
	Best-assembly	95.31	216.47	-38.82	-72.99	199

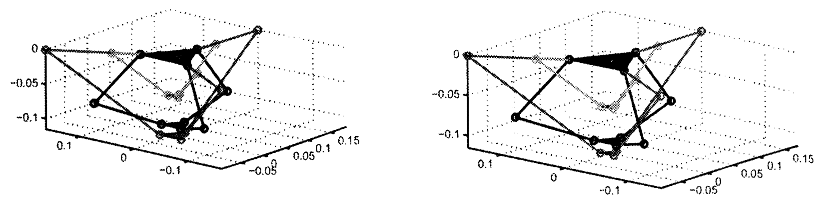
Table 2 Values of the design variables.

		l_A [cm]	l_B [cm]	z_c [cm]	r_b [cm]	r_p [cm]
Initial guesses	Min.	2	2	-20	1	1
	Initial	10	10	-10	5	2
	Max.	20	20	-5	10	10
Sim.1: $\omega = 0$	Artificial	16.39	20	-11.5	2.22	2.25
	Best-assembly	16.39	20	-11.5	1.1	1.2
Sim.2: $\omega = 0.001$	Artificial	8.16	9.93	-5.7	1	1
	Best-assembly	8.13	9.89	-5.7	1	1

Figure 4 shows schematic initial guess and optimal solutions for Delta robot graphically. It can be observed the huge reduction of occupied area. Note that results obtained with both strategies are practically similar.

5 Conclusions

An optimization method for designing parallel manipulators is presented in this work. The objective function is composed by two terms, namely, dexterity and area



(Artificial penalty) (Best-assembly penalty)

Fig. 4 Delta robot: initial guess (black), results of *Sim. 1* (red) and results of *Sim. 2* (yellow).

occupation. The main problem which appears during the optimization of closed-loop multibody systems is its assembly. Two different strategies are applied: *Artificial* and *Best-assembly* penalty optimization strategies. The first one is combined with Nelder–Mead optimization algorithm, and the second with SQP. Results show optimized Delta robot with area reduction occupies up to 75% less than optimized Delta robot without area reduction while the dexterity index is only 4% worse.

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