L'apprentissage des nombres naturels et rationnels

L'apprentissage des nombres naturels et rationnels

Etude comparative entre le Vietnam et la Belgique

Hien Thi Thu NGUYEN

Promoteur

Jacques Grégoire (UCL)

Thèse présentée en vue de l'obtention du grade de Docteur en sciences psychologiques et de l'éducation

UCL PRESSES UNIVERSITAIRES DE LOUVAIN

Collection de thèses de l'Université catholique de Louvain, 2015

Président

Mauro Pesenti (Université catholique de Louvain)

Comité d'accompagnement et jury

Marie-Pascale Noël (Université catholique de Louvain)
Alain Content (Université libre de Bruxelles)
Lieven Verschaffel (Katholieke Universiteit Leuven)
Joëlle Vlassis (Université du Luxembourg)
Aliette Lochy (Université catholique de Louvain)

© Presses universitaires de Louvain, 201x
Dépôt légal : D/201x/9964/xx
ISBN : 978-2-87588-xxx-x
Imprimé en Belgique

Tous droits de reproduction, d'adaptation ou de traduction, par quelque procédé que ce soit, réservés pour tous pays, sauf autorisation de l'éditeur ou de ses ayants droit.
Couverture : Hien Thi Thu Nguyen

Diffusion : www.i6doc.com, l'édition universitaire en ligne
Sur commande en librairie ou à
Diffusion universitaire CIACO
Grand-Rue, 2/14
1348 Louvain-la-Neuve, Belgique
Tél. 32 10 47 33 78
Fax 32 10 45 73 50
duc@ciaco.com

Résumé

Cette thèse étudie le transcodage des nombres naturels et l'apprentissage des nombres rationnels. Afin d'évaluer sur ces derniers l'impact du langage et du curriculum d'apprentissage, notre étude a été conduite simultanément en Belgique et au Vietnam. Nos résultats montrent que les différences linguistiques pour les nombres verbaux ont un impact sensible sur le transcodage des nombres naturels et décimaux. La relation part-tout est importante et joue un rôle fondamental dans la construction et l'enseignement des nombres fraction mais notre analyse montre les limites de cette approche. Les variations de performance observées entre les enfants belges et vietnamiens pourraient s'expliquer par les différences dans les choix didactiques. Cette recherche vise à améliorer notre compréhension de l'acquisition des nombres chez les élèves et propose une série d'implications pour l'enseignement.

Abstract

This thesis studies the transcoding of natural numbers and the learning of rational numbers. To assess the impact on them of language and learning curriculum, our study was conducted simultaneously in Belgium and Vietnam. Our results show that linguistic differences for verbal numbers have a sensitive impact on the transcoding of natural and decimals numbers. The part-whole relationship is important and plays a fundamental role in the construction and teaching of fraction numbers but our analysis shows the limitations of this approach. Performance variations between Belgian and Vietnamese children could be explained by the differences in educational choices. This research aims to improve our understanding of the acquisition of numbers among children and proposes a series of implications for teaching.

Table des matières

Résumé - Abstract **5**

Table des matières **7**

Remerciements **11**

Introduction générale **13**

Chapitre 1. Les nombres naturels et le transcodage **19**

1 Représentations des nombres naturels *21*

1.1 Les premiers systèmes de numération et la représentation analogique 21
1.2 Les systèmes numériques écrits et la représentation arabe de nombre 22
1.1 La représentation verbale du nombre 25

2. Le transcodage et le langage *26*

2.1 Le transcodage 26
2.2 Le transcodage et les spécificités du langage 31

Chapitre 2. Enseignement des fractions et décimaux **37**

1. Enseignement des fractions et décimaux en Communauté française de Belgique *40*

1.1 Les « Socles de Compétences » 40
1.2 Programme des études d'enseignement fondamental 44
1.3 Les manuels scolaires : 4 et 5 « Archi m'aide » 46

2. Enseignement des fractions et décimaux au Vietnam. *50*

3. L'expression verbale du nombre rationnel *53*

3.1 Le langage vietnamien du décimal 53
3.2 L'expression verbale du décimal en Belgique 54

Chapter 3. Rational number construct and whole number bias **57**

1. Rational number construct *60*

1.1 Process of partitioning 61
1.2 The part-whole sub-construct 61
1.3 The ratio sub-construct 64
1.4 The operator sub-construct 65
1.5 The quotient sub-construct 65
1.6 The measure sub-construct 66

2. Whole number bias *67*

2.1 The innate constraint 68

2.2 The Undifferentiated Amount Hypothesis 70
2.3 Learning accounts 72

Conclusion 77

Chapter 4. When 5004 is Said "Five Thousand Zero Hundred Remainder Four": The Influence of Language on Natural Number Transcoding: Cross-National Comparison 83

1. Introduction 85

1.1 The process of number transcoding and the two types of zero 86
1.2 Impact of language characteristics on mathematical thinking and performances 87
1.3 The Vietnamese verbal number 88

2. Method 89

2.1 Participants 89
2.2 Task and stimuli 90

3. Results 90

3.1 Overall analysis 90
3.2 The impact of language 91
3.3 The impact of syntactic zero 93
3.4 Nature of errors and cluster analyses 95

4. Discussion 98

4.1 Impact of structural characteristics of numerical languages on a transcoding task 98
4.2 The impact of the syntactic zero and the nature of errors 99
4.3 Perspectives and future studies 101

Chapter 5. Belgian and Vietnamese children's comprehension of the natural number system: a re-examination of cross-national differences 103

1. Introduction 105

1.1 The impact of language on understanding and performance in mathematics 105
1.2 Vietnamese number word systems and transcoding tasks 106
1.3 Limitation of language influence on cross-national studies and the present study 107

2. Method 109

2.1 Participants 109
2.2 Task 109
2.3 Procedure 110

3. Results 111

3.1 Boys and girls performances 111
3.2 Overall analysis 111
3.3 The zero in the digit identification task 113
3.4 The nature of errors 114

4. Discussion *118*

4.1 The influence of language: the task related to the number-name system 118
4.2 Misunderstanding of the positional number system and the impact of zero. 120

Chapter 6. Arabic representation of the decimal numbers: from natural to decimal numbers **123**

1. Introduction *125*

1.1 The generalization process 126
1.2 Impact of preexisting knowledge on the construction of the decimal number concept 126
1.3 The effect of the curriculum and language systems 130
1.4 Present study 132

2. Method *133*

2.1 Participants 133
2.2 Procedure and Materials 133
2.3 Data analysis 134

3. Results *135*

3.1 Overall analysis (achievement in the conceptual decimal) 135
3.2 The generalization process for decimal number representation 136
3.3 Cluster analysis 139
3.4 The case of the integer part that equaled zero 142

4. Discussion *143*

Chapter 7. The misunderstanding of the part-whole relationship: a didactic obstacle **147**

1. Introduction *149*

1.1 The role of the part-whole relationship in rational number knowledge 150
1.2 Action of partitioning 151
1.3 The present study 151

2. Method *152*

2.1 Participants 152
2.2 Measure and Procedure 153

3. Results *156*

3.1 The first task: to determine the fraction symbol corresponding to the given graphical presentation. 156

3.2 The second task: to determine the graphical presentation associated with the given fraction symbol 157

4. Discussion *161*

Chapter 8. From natural number to decimal number **165**

1. Introduction *167*

2. Method *170*

2.1 Participants 170
2.2 Measures 170
2.3 Procedure 174
2.4 Data analysis 174

3. Results *175*

3.1 Descriptive statistics 175
3.2 Preliminary analyses 176
3.3 Structural equation modeling 178

4. Discussion *180*

General conclusion **183**

Références bibliographiques **197**

Annexes **213**

Remerciements

La réalisation de cette thèse fut pour moi une grande aventure. Cette aventure n'aurait jamais pu se faire sans le soutien et l'aide de tous les gens qui m'ont accompagnée dans mon parcours.

Je remercie tout d'abord mon promoteur, Jacques Grégoire, en premier lieu pour la confiance qu'il a eue en moi en m'acceptant comme doctorante, ensuite pour le regard continu envers mon travail, son aide précieuse dans les moments difficiles, son respect dans le développement de mes propres idées et enfin pour sa patience quand j'essayais de m'exprimer dans un français difficile.

Merci à l'Université Catholique de Louvain de m'avoir octroyé une bourse de coopération au développement. Si l'argent est le nerf de la guerre, il est aussi celui de la Recherche.

Je remercie ensuite les membres de mon comité d'accompagnement, Marie-Pascale Noël et Alain Content. Marie-Pascale pour ses remarques importantes aux moments décisifs et Alain pour son attention soutenue.

Je souhaite exprimer ma gratitude aux membres de mon jury de doctorat pour le temps consacré et leur intérêt pour mon travail : Lieven Verschaffel, Joëlle Vlassis et Aliette Lochy. Merci également à Mauro Pesenti d'avoir accepté de présider le jury.

Comme toutes les histoires ont un début, je dois adresser mes sincères remerciements à Marie-Anne Schelstraete. C'est en effet sur son invitation que je me suis rendue pour la première fois à Louvain-La-Neuve en 2007. Sa gentillesse et ses encouragements m'ont toujours poussée à avancer. Je n'oublie pas Thầy Duy qui, en emportant le projet « Troubles des apprentissages » à l'Université de Pédagogie de Ho Chi Minh Ville m'a permis de rencontrer les professeurs de l'UCL et de l'ULB.

Merci à tous mes collègues en Belgique, Véronique, Catherine, Gaëlle pour le temps de partage que nous avons eu et pour tous leurs encouragements chaleureux. Je remercie particulièrement Laëtitia et Christophe pour leurs conseils pertinents sur la construction de ma recherche, Philippe et Julie du CREM pour leur aide. Toutes les tâches administratives m'ont été grandement facilitées grâce à la disponibilité et la bonne humeur d'Isabelle, de Marianne et de Martine.

Je pense aussi à tous les élèves, enseignants, directeurs d'écoles et étudiants de la Belgique et du Vietnam pour leur participation sur le terrain.

Merci à Chị Van, tu as supporté mon mauvais caractère pendant toutes ces années avec patience et gentillesse. Chị Trang, pour tes encouragements. Toutes les trois, nous avons commencé notre thèse en Belgique dans des conditions très similaires, je suis très heureuse d'avoir partagé tous ces moments avec vous.

Merci à la communauté vietnamienne de Louvain-La-Neuve. Grâce à elle, j'ai pu retrouver une petite partie du Vietnam ici et mes jours ont été plus ensoleillés et plus vivants. Cám ơn My Anh, Viet - Trang, Thang - Hanh, Hang-Diep, c.Vy, em Duc Dung, Ty, Minh, Nga, Duong, Gai, Dung, Hoa, c. Huong, c. Quynh, c.Q.Huong. Merci à Thanh pour son aide à m'exprimer dans la langue de Shakespeare.

Je remercie mes amis d'enfance qui sont toujours les miens aujourd'hui, Chi, Van, Q.Anh, Dung, ils n'ont jamais cessé de m'encourager. Je pense aussi à mes collègues au Vietnam qui m'ont toujours suivie et encouragée, Cô My, Cô Nga et C. Yen.

Je veux également témoigner ma gratitude aux familles qui m'ont accompagnée pendant toutes ces années. Tout d'abord la famille Sacré, ma famille d'adoption, pour leur accueil, leur disponibilité et leur grand cœur. Ensuite la famille Slusarenko, qui est devenue ma belle-famille pendant la réalisation de cette thèse. Ghislaine, Jacques, Patrice, Karine, vous avez toujours été fiers de moi et m'avez donné le réconfort que seule une vraie famille peut donner.

Je remercie évidemment mes parents, ma sœur et mon neveu au Vietnam. Cette thèse nous a séparés géographiquement, mais votre soutien et votre amour indéfectibles ont effacé les kilomètres. Dans un de ses devoirs, mon petit neveu, Bi, écrivait que « sa tante est Docteur ». Je devais obligatoirement finir cette thèse pour que cela ne devienne pas un mensonge. Cám ơn Bố, Mẹ, Chị và Bi.

Et enfin, le plus important, merci Gregory pour chaque seconde.

Introduction générale

Introduction générale

Depuis très longtemps, soumis à la nécessité de compter, de dénombrer, de représenter une quantité, l'Homme a créé des systèmes de numération. La quantité a pu être représentée par plusieurs formes : avec des objets concrets, sous la forme verbale ou avec un symbole écrit. Elles seront ensuite respectivement considérées comme représentations analogique, verbale et arabe des nombres. Les civilisations ont usé de différentes règles pour construire leurs systèmes de numération mais au cours du temps, les systèmes les moins performants ont été adaptés et remplacés par ceux plus efficaces. De nos jours, le système de numération arabe qui symbolise les nombres avec des symboles spécifiques (les chiffres de 1 à 9 et le zéro) est universellement utilisé. Le système de numération verbale, lui, possède des caractéristiques linguistiques propres. La représentation verbale des nombres est dès lors différente en fonction de la langue parlée.

Nous utilisons fréquemment la représentation verbale de nombres (e.g., cinq) ou leur représentation arabe (e.g., le chiffre 5) dans notre vie quotidienne. Nous rencontrons également de nombreuses situations dans lesquelles nous devons passer d'un code à l'autre (e.g., lire à haute voix le prix d'un produit; donner une somme d'argent correspondante au prix énoncé par le vendeur). Transformer la forme verbale des nombres (code verbal) en leur correspondant en chiffres (code arabe), et vice-versa, est appelé le transcodage. Le passage d'une représentation à l'autre n'est pas une simple traduction directe d'un code à l'autre. Pour que le processus transcodage se fasse avec rigueur, uniformément et sans confusion pour tous les nombres, il est nécessaire qu'il respecte certaines conventions. Toutefois, dans le cas des grands nombres (par exemple, les nombres contenant un zéro syntaxique comme 205) ou les nombres avec des formes verbales particulières (par exemple, quatre-vingts en français), les enfants peuvent rencontrer des difficultés de transcodage.

Le système numérique verbal vietnamien est plus transparent et régulier que celui d'autres langages comme le français ou l'anglais. Les nombres verbaux vietnamiens correspondent parfaitement à leurs décompositions (par exemple 35 se dira « trois-dix cinq »). De plus, il possède des particularités lorsque le chiffre des dizaines ou des centaines est égal à zéro. Le vietnamien utilise le mot « zéro » comme primitive lexicale dans la construction du nombre. Par exemple, le nombre arabe 2009 se dira « deux mille zéro cent reste neuf ». Cette caractéristique est spécifique au vietnamien et ne se retrouve pas dans les autres langages, même dans les langages asiatiques comme le chinois, le coréen ou le japonais. Le système numérique verbal vietnamien aide-t-il les enfants Vietnamiens dans la tache de transcodage d'un nombre verbal à un nombre en chiffres ? Les différences dans les systèmes numériques verbaux vietnamien et français conduisent-elles à des variations notables des performances pour les taches numériques? Nous traiterons de ces questions dans notre recherche.

Nous avons étudié l'impact des particularités du système numérique verbal vietnamien dans le cas des nombres naturels, mais aussi dans celui des nombres rationnels (fractions et nombres décimaux). Dans plusieurs situations de répartition, le nombre entier (naturel) ne permet pas de représenter une quantité exacte, par exemple, le résultat d'une division d'un terrain en quatre parts égales. La fraction est alors apparue pour remplir cette fonction. Dans un premier temps, la fraction a été seulement considérée comme l'écriture d'une division (1:4) et n'a été utilisée que dans les situations de fractionnement. Elle a ensuite été considérée comme un nombre et représentée par le symbole « a/b » où a et b désignent des nombres entiers (b étant non nul). Le numérateur « a » et le dénominateur « b » sont liés par le trait de fraction qui équivaut à un signe de division. Pour faciliter les opérations sur les nombres fractionnaires, les fractions peuvent être représentées dans le système décimal (par exemple ¼ = 0,25). Elles sont alors appelées nombres décimaux. Le terme nombre rationnel désigne l'ensemble des nombres fractionnaires et décimaux. Les nombres entiers, qui peuvent s'écrire comme des fractions de dénominateur 1, forment un sous-ensemble au sein de l'ensemble des nombres rationnels.

Les nombres rationnels jouent un rôle majeur dans la théorie de nombres en mathématiques. Ils sont également très importants dans l'enseignement et l'apprentissage des mathématiques ainsi qu'en développement de nombres. Une bonne réussite de l'apprentissage des nombres rationnels est centrale, car les performances des élèves pour les fractions prédisent leur maitrise de l'algèbre et leur réussite en mathématiques par la suite (en contrôlant le Q.I, la capacité de lecture, le mémoire de travail et le revenu familial) (Siegler et al., 2012 et Bailey et al., 2012). Pourtant, l'apprentissage des nombres rationnels est l'occasion de difficultés fréquentes et persistantes (Nesher & Peled, 1986; Resnick et al., 1989 ; Desmet et al.,2010 ; Behr, Harel, Post, & Lesh, 1992; English & Halford, 1995; Grégoire & Meert, 2005; Mack, 1993; Sophian, 1996; Stafylidou & Vosniadou, 2004). L'échec de l'apprentissage des nombres rationnels est un problème majeur de l'éducation mathématique qui peut avoir une influence importante sur le parcours scolaire de nombreux élèves.

Pourquoi les difficultés associées à l'apprentissage des nombres rationnels sont-elles persistantes et fréquentes ? Que vont induire les acquisitions des nombres naturels pour l'apprentissage des fractions et des nombres décimaux ? Quelles sont les approches appropriées pour réduire les obstacles rencontrés lors de l'acquisition des nombres rationnels ? Des recherches sur les nombres rationnels dans l'enseignement des mathématiques ainsi que sur le développement des nombres ont été réalisées à travers plusieurs études avec une diversité de méthodes utilisées. Dans notre recherche, nous étudions l'apprentissage des fractions et décimaux d'un point de vue interculturel. Notre étude a en effet été conduite simultanément dans deux pays, la Belgique et le Vietnam, ce qui nous a permis de traiter la question des différences d'apprentissage des nombres rationnels entre ces deux pays, en fonction de la langue et du curriculum d'apprentissage.

Notre exposé comprend une partie théorique, une partie empirique et une discussion générale. La partie théorique contient trois chapitres et la partie empirique comprend cinq études. Dans le premier chapitre, nous aborderons les trois représentations de nombres :

analogique, verbale et arabe pour laquelle nous décrirons brièvement l'histoire des systèmes de numération. Nous exposerons ensuite le processus de transcodage en nous appuyant sur deux types de modèles de transcodage : sémantique et asémantique. Nous examinerons également le cas du zéro dans la tache de transcodage. Le système de numération verbal vietnamien avec ses caractéristiques linguistiques sera expliqué. Enfin, nous présenterons les systèmes numériques verbaux dans différentes langues et les études sur l'impact des caractéristiques linguistiques sur le processus de transcodage. Dans le deuxième chapitre, nous présenterons tout d'abord l'enseignement des nombres rationnels en Belgique (Communauté Française) et au Vietnam, en analysant les programmes de mathématiques et les manuels scolaires. Dans le troisième chapitre, avec pour objectif de mieux comprendre la complexité et les difficultés dans l'acquisition des nombres rationnels, nous exposerons en détail la construction du concept de nombre rationnel en présentant cinq sous-constructions du nombre fraction. Ensuite, nous aborderons l'origine du biais des nombres naturels ce qui nous donnera un éclaircissement sur la question : pourquoi les connaissances des nombres naturels peuvent-elles provoquer des difficultés lors des manipulations avec les nombres rationnels de manière inévitable, fréquente et persistante.

La partie empirique comprend deux parties. Les deux premières études sont liées à la tâche de transcodage des nombres naturels et les trois derniers chapitres concernent les nombres fractions et décimaux. Dans une première étude, nous examinerons la question suivante : « le système numérique verbal vietnamien est un système transparent pour les zéros syntactiques, aide-t-il les élèves vietnamiens à être plus performants que les élèves belges dans les tâches de transcodage? ». Ensuite, nous étudierons la question suivante dans le chapitre deux : « Quelle est l'étendue de l'impact des caractéristiques linguistiques sur l'acquisition des mathématiques? Cet impact, marquant pour les tâches liées aux systèmes numériques verbaux, influe-t-il pour les tâches concernant la manipulation avec des symboles arabes? ».

Nous orienterons ensuite notre étude vers les fractions et les décimaux. Dans une troisième étude, nous analyserons le développement de la représentation arabe des nombres décimaux en analysant les réponses des élèves de la troisième à la sixième primaire, à partir d'une tâche de traduction de nombres décimaux sous forme verbale en nombres arabes. Cela nous permettra d'explorer l'influence des connaissances antérieures des nombres naturels et des nombres fractions dans l'acquisition des nombres décimaux. Bien que le concept de fraction ait plusieurs facettes, l'approche se basant sur la relation part-tout est fréquemment utilisée dans l'enseignement des fractions. Dans une quatrième étude, nous évaluerons les limites de cette approche ? Enfin, dans une cinquième étude, nous examinerons de manière plus large les relations entre nombres naturels, fractions et décimaux. A cette fin, nous avons utilisé les modèles structuraux d'équations qui permettent de tester des modèles théoriques relatifs aux relations entre les différentes catégories de nombres. Nous verrons quelles relations entre variables peuvent avoir un impact potentiel sur l'apprentissage des nombres décimaux. Nous récapitulerons

l’ensemble des résultats de nos recherches et proposerons des de futures études dans la discussion générale.

Chapitre 1

Les nombres naturels et le transcodage

Les nombres naturels et le transcodage

En pensant aux nombres, nous pensons souvent à leur fonction d'indiquer une quantité. Cette quantité peut être illustrée à travers différentes formes. Par exemple, les cinq doigts de la main peuvent servir à indiquer la quantité 5 et ils sont aussi une représentation analogique du nombre 5. Ce code, en se basant sur les caractéristiques physiques, permet de déterminer la valeur du nombre. La représentation analogique du nombre est très concrète, mais malheureusement, elle est très compliquée et lourde pour manipuler ou réaliser des opérations sur ce code. Il existe aussi d'autres représentations, la représentation auditive-verbale et la représentation visuelle arabe. En effet, nous pouvons aussi représenter un nombre par un code verbal en nommant ce nombre « cinq» ou écrivant «5» pour décrire la quantité où l'ensemble de pommes représentées. Ces représentations sont des symboles et elles sont établies par convention. La vie quotidienne nous impose également de nombreuses situations dans lesquelles nous devons passer d'un code à l'autre, c'est ce que nous appellerons le transcodage. Nous présenterons dans ce chapitre les trois représentations des nombres naturels, la représentation analogique, verbale et écrite arabe et ensuite le transcodage dans différents langages.

1 Représentations des nombres naturels

1.1 Les premiers systèmes de numération et la représentation analogique

Depuis très longtemps, l'Homme a eu la nécessité de compter et de dénombrer un ensemble de diverses choses dans sa vie pratique, par exemple les moutons dans une bergerie, les animaux tués, etc… L'Homme a commencé à compter avec ses doigts, les cailloux et des encoches gravées sur les os d'animaux. En utilisant le principe de la correspondance terme à terme entre l'élément de ces représentations avec l'élément de l'ensemble d'objets, l'Homme pouvait vérifier la numérosité de l'ensemble intéressé. Plus tard, les cailloux ordinaires ont été remplacés par des objets de tailles variées associés à une convention, par exemple un petit caillou pour l'unité, une bille plate pour un groupement, une petite boule pour un groupement plus large, etc. Comme l'utilisation des doigts facilite la tâche de compter, la plupart des civilisations ont choisi le groupement de dix pour le système de numération. Le groupement d'éléments est le fondement initial pour le concept de base dans le système de numération. Les autres civilisations adoptèrent la base vingt (Mayas, Aztèques, Celtes et Basques), base douze et base sexagésimale (Les Sumériens). À ces époques, pour ceux qui adoptèrent la base dix, il y avait des conventions comme un petit caillou pour l'unité, une bille plate pour les dizaines, une petite boule pour les centaines, etc... Pour la base sexagésimale, les représentations étaient les suivantes : un petit cône pour 1, une bille pour 10, un grand cône pour 60, un grand cône perforé pour 600, une sphère pour 3600, etc...(Figure 1).

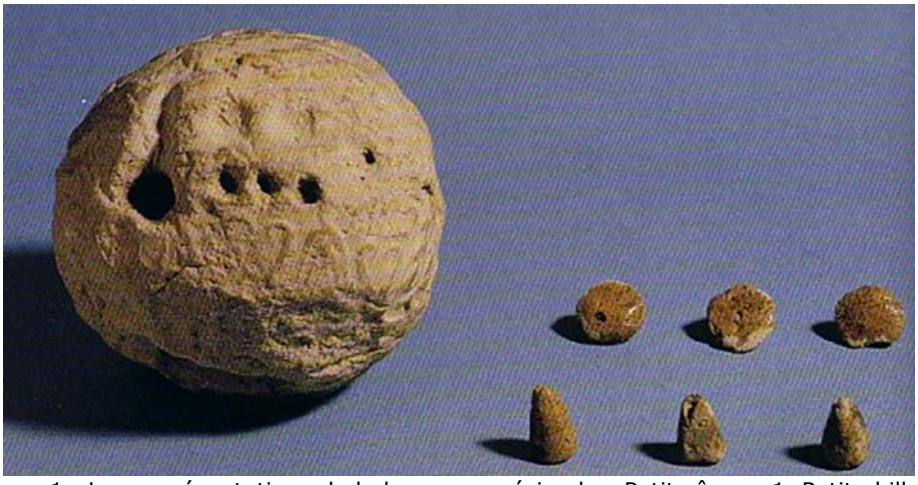


Figure 1. Les représentations de la base sexagésimale : Petit cône = 1. Petite bille = 10. Grand cône = 60. Grand cône percé = 600. Grosse bille = 3600. Grosse bille percée = 36000 (http://www.maths-et-tiques.fr)

De nos jours, l'utilisation des doigts pour les manipulations mathématiques est toujours utilisée, surtout dans le début des années d'acquisition des compétences mathématiques des enfants. Représenter la numérosité d'un nombre par un ensemble d'éléments concrets est considéré comme la représentation analogique des nombres.

1.2 Les systèmes numériques écrits et la représentation arabe de nombre

Après avoir représenté la quantité d'un ensemble d'éléments par des objets concrets, vers 3600 av. J.-C., l'Homme a symbolisé les nombres, par exemple le dessin d'un petit trou pour la bille (qui représente 10), une encoche pour un cône, un cercle pour une sphère, etc. Plus tard, les Grecs, les Juifs, les Chrétiens, les Arabes ainsi que d'autres peuples ont utilisé des lettres de leur alphabet pour représenter les nombres. Par exemple, vers 500 av. J.-C., dans la numération romaine, « le cinq » est indiqué par la lettre V, « le dix » est représenté par X. De plus, la technique d'addition de la valeur des lettres pour obtenir la valeur du nombre est utilisée dans plupart des systèmes de numération de ces époques, ainsi, pour la numération romaine, le nombre 15 s'écrit XV. Vers 400 av. J.-C., dans la numération grecque, les lettres de l'alphabet et des signes supplémentaires ont été utilisés (I pour 1, Γ pour 5, Δ pour 10, H pour 100, X pour 1 000, M pour 10 000). Nous remarquons que la relation de multiplication est intuitivement apparue dans la numération grecque pour écrire certains nombres, par exemple 𐅄 c'est-à-dire (Γ×Δ) → 5 10 → 50 et 𐅅 c'est-à-dire (Γ H) → 5 x 100 → 500 (Ifrah, 1994). Ce système d'écriture de nombre montre une évolution par rapport au système d'addition et se rapproche du système positionnel utilisé de nos jours.

Le système de numération chez les savants Babyloniens, vers le 2e millénaire av. J.-C., est considéré comme le premier à avoir utilisé le système positionnel pour la représentation

écrite de nombres. L'avantage du système positionnel est que nous pouvons exprimer l'infinité des nombres avec quelques symboles graphiques. Le système babylonien est basé sur la base sexagésimale et utilise deux symboles, le « clou vertical » et le « chevron » pour représenter l'unité et la dizaine. Les nombres de 11 à 59 sont écrits sous le principe additif en répétant les symboles. À partir de 60, le principe de position est appliqué. Ce système se montre parfois ambigu. En effet, nous ne pouvons pas distinguer la différence, par exemple, entre les représentations écrites pour 1 et pour 60 (ou bien le nombre écrit pour 61 et 60x60 = 3600). Vers le 3e siècle av. J.-C., ces confusions sont traitées par la présence d'un signe (un espace puis le « 2 tourné ») à une position s'il n'y a rien. C'est aussi la première trace du zéro.

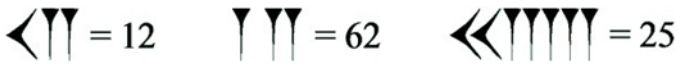


Figure 2. Exemples de nombres écrits en numération babylonienne sexagésimale (http://www.maths-et-tiques.fr)

Le système de numération chinois vers le IIe siècle av. J.-C et celui chez les Mayas vers le Vème siècle après J.C sont aussi des systèmes positionnels. Les Chinois utilisent la base 10 et un espace est utilisé pour marquer l'absence des positions. Ils ont adapté les combinaisons des bâtons en deux directions, verticale et horizontale, pour représenter leur « chiffre ». Les Mayas, quant à eux, utilisent la base vingt, inventant le zéro qui est représenté par un coquillage. L'ordre de position des nombres commence du bas vers le haut par les puissances de 20, sauf au 3e ordre de l'écriture. Au lieu de multiplier le chiffre dans cet ordre avec 20×20, il est multiplié avec 18×20. C'est une irrégularité du système qui est expliquée par le respect de l'année solaire $18 \times 20 = 360$ jours chez les Mayas. Les figures 3 et 4 illustrent les systèmes de numération chinois et mayas.

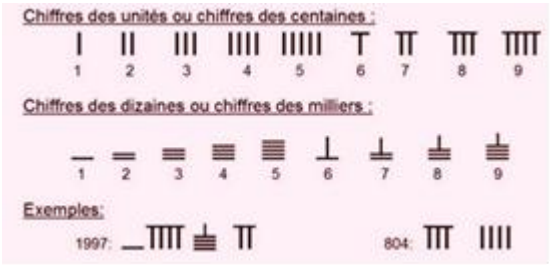


Figure 3. Symboles utilisés dans la numération chinoise antique (http://www.maths-et-tiques.fr)

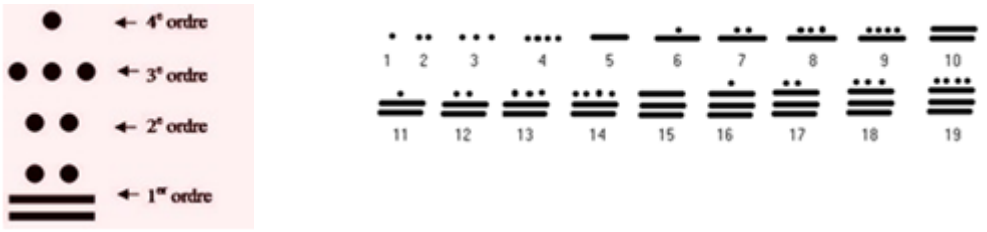


Figure 4. Symboles utilisés dans la numération des Mayas et la représentation du nombre 8332 (http://www.maths-et-tiques.fr)

Ainsi l'écriture ci-contre représente le nombre :

$1 \times 18 \times 20^2$ (4^e ordre) + $3 \times 18 \times 20$ (3^e ordre) + 2×20 (2^e ordre) + 12×1 ($1^{ème}$ ordre) = 8332

Les chiffres « arabes » de 1 à 9 ainsi que le système de numération que nous utilisons de nos jours viennent du système des Indes (Figure 5). La présence des Arabes au VIIe siècle s'étend de l'Inde à l'Espagne, c'est ainsi qu'ils adapteront progressivement le système de numération indien, plus performant que le leur à l'époque. Au cours des siècles suivants, ce système se développera en Europe, en connaissant aussi de nouvelles évolutions, sous l'influence de différents personnages, comme le pape Sylvestre II ou plus tard, le mathématicien italien Fibonacci. Ce développement se fera en opposition au système romain de l'époque, suffisant pour beaucoup aux besoins du commerce quotidien.

Les savants Indiens au Vème siècle ont inventé un système de numération très complet et efficace. Le zéro dans ce système indien occupait le statut de nombre, ce n'est pas le cas pour les systèmes de numération chinois et mayas. Le principe de base, le principe de position ainsi que les neuf chiffres de 1 à 9 et le zéro 0 ont été réunis dans ce système et constituent notre système de numération actuel.

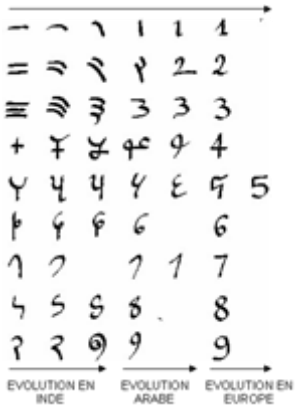


Figure 5. L'évolution des chiffres « arabes » (http://www.maths-et-tiques.fr)

Le système de numération actuel est décimal et positionnel et il est quasiment universel. Nous pouvons représenter l'infinité des nombres à partir des dix chiffres arabes, de 0 à 9, avec le système positionnel de base 10. L'ordre de position des nombres commence de la gauche vers la droite par les puissances de 10. Donc, la valeur d'un chiffre constituant le nombre dépend de sa position. L'absence une puissance est exprimée par le zéro. Par exemple, le nombre 253 est représenté par $2\times10^2+5\times10^1+3\times10^0$ et signifie 2 centaines, 5 dizaines et 3 unités. Dans le nombre 2014, il n'y a pas de centaine donc cette position est exprimée par 0. Cette règle assure le principe que tous les nombres peuvent être représentés à partir de 10 chiffres (0 à 9) et les puissances de 10 $(1,10^1,10^2,10^3...)$ sans exception pour les nombres avec une absence de quantité à certaines positions. En effet, 2014 est représenté par. La représentation d'un nombre par ce système de numération est nommée la représentation arabe de nombre. La représentation analogique est très concrète, mais elle induit des complications pour réaliser des calculs et des opérations arithmétiques. Par contre, la représentation arabe est symbolique, car son apparence physique ne fournit pas d'indications concernant la quantité représentée. La représentation arabe est commode et facilite les manipulations avec les nombres.

1.1 La représentation verbale du nombre

Concernant les représentations du nombre, en plus de la représentation arabe, un nombre peut aussi être représenté sous forme de système verbal. Les systèmes verbaux sont des systèmes conventionnels et sont transmis culturellement. D'après Ifrah (1981), l'évolution des systèmes numériques verbaux a suivi les trois étapes suivantes. À la première étape, le nom d'un nombre est désigné par un terme concret dont l'apparence physique contient une quantité présente dans le nombre (ex. : soleil pour un, yeux pour deux,…). Ensuite, le « un » pourra se dire « auriculaire », le « deux » « majeur » et le « trois » « index » quand les personnes utilisent leurs doigts pour compter. Enfin, les noms des nombres deviennent une symbolisation abstraite, comme les mots « un, deux, trois» en français.

Le système de numération verbal n'est pas le même d'une langue à l'autre, même s'il s'établit en général sur quelques principes communs. En effet, il existe généralement des mots de nombres primitifs dans chaque système verbal (les nombres de un à seize en français, les dizaines ainsi que cent, mille, million et milliard) et la combinaison des primitifs qui forme les autres nombres complexes (ex., « vingt-deux » est composé par « vingt » et « deux »). Selon Hurford (1987), dans la langue française, ces combinaisons des primitifs sont construites en optimisant la différence arithmétique entre les deux termes utilisés (ex., la représentation verbale de « 63 », composée à partir de « soixante » et de « trois », a été choisie au lieu de combinaisons comme « cinquante » et « treize »). Le système verbal actuel est un système « hybride » où une infinité de nombres verbaux sont formés à partir des mots de nombres primitifs et des deux relations d'addition et de multiplication (ex., « vingt-deux » 22 est composé par l'addition de « vingt » et de « deux » ; « cinq cents » 500 est composé par la relation multiplicative de « cinq » et de « cent » et

« cinq cent vingt-deux » 522 est formé à partir des deux relations, « cinq x cent + vingt + deux).

2. Le transcodage et le langage

2.1 Le transcodage

Dans leur milieu social, les gens utilisent régulièrement deux représentations de nombres : orale (verbale) et écrite (arabe). Ils rencontrent aussi des situations demandant une transformation d'un code à l'autre, par exemple, ils doivent noter les numéros de téléphone de la forme orale à la forme écrite en chiffres. Le passage d'une représentation (verbale, arabe et analogique) à une autre représentation du nombre est appelé « transcodage ». Le transcodage permet de passer de l'une à l'autre de ces différentes représentations du nombre. Par exemple, le transcodage du code verbal au code visuel arabe permet d'obtenir le nombre arabe qui correspond à un nombre verbal.

Il convient de signaler que le système de numération verbal n'est pas simplement une traduction directe du système de numération arabe malgré certains points communs dans la construction des deux systèmes. Ainsi, le processus de transcodage peut conduire les enfants à rencontrer des difficultés spécifiques avec ce dernier. Il est nécessaire de maîtriser les codes d'entrée et de sortie ainsi que les règles de transcodage entre ces codes.

L'apprentissage des nombres débute avec la chaîne de comptage et les mots de représentation des nombres. Dans leur contexte social, les enfants ont l'occasion de rencontrer et d'acquérir la numération verbale orale assez tôt, tandis que l'acquisition de la numération arabe exige un enseignement systématique.

La chaîne numérique verbale s'acquiert dès 2 ans et demi et jusqu'à six ans et le cours de son apprentissage comprend plusieurs étapes (Fuson et al., 1982). Les enfants commencent à apprendre par cœur la suite numérique jusqu'à 4 ans. Ensuite, vers 5 ans, ils sont capables d'utiliser les principes concernant la logique combinatoire (relation additive et relation multiplicative) pour produire une suite de chaîne de comptage. Certains peuvent aussi compter à partir d'un nombre « x » quelconque. À partir de six ans, les enfants sont capables de compter « n » nombres à partir de « x » ou de compter de « x » à « y ». Vers quatre ans et demi, les enfants montrent leurs capacités à reconnaître les symboles, les mots de nombres et les chiffres (Noel, 2001). À six ans débute chez eux la capacité de passer d'un code à l'autre. Ces capacités se développent progressivement, avec le transcodage des nombres arabes d'un chiffre vers 6 ans, de deux chiffres vers 7 ans et de quatre chiffres vers 8 ans.

Modèle de transcodage

Nous pouvons faire une distinction entre deux types de modèles de transcodage : les modèles sémantiques et les modèles asémantiques. Pour le modèle de transcodage sémantique, McCloskey, Caramazza et Basili, (1985) et Power et Dal Martello (1990) ont supposé que la procédure de transcodage est produite en passant par la représentation

sémantique abstraite du nombre. En revanche, dans les modèles asémantiques comme ceux de Dehaene (1992), Deloche & Seron (1987) ou Barrouillet et al. (2004), le processus de transcodage est traité par des procédures spécifiques.

Modèles sémantiques

Deux étapes sont distinguées pour ces modèles: l'étape de compréhension du nombre verbal qui permet la construction de la représentation sémantique appropriée et l'étape de production du nombre en chiffres arabes, basée sur cette représentation. La phase de production est considérée plus difficile que celle de la compréhension. Selon McCloskey et al. (1985), la représentation sémantique abstraite est figurée dans la base dix pour indiquer la quantité du nombre (e.g., « quatre cent vingt-cinq »→$[4].10^2$,$[2].10^1$,$[5].10^0$). Les chiffres dans [.] correspondent à la représentation sémantique des quantités de base du nombre. Comme chaque quantité est associée à la puissance de dix correspondante et que le zéro représente une quantité nulle, il n'a pas de représentation sémantique pour le zéro (McCloskey, Sokol & Gooman, 1986). Power et Dal Martello (1990) ont aussi élaboré un modèle sémantique de transcodage des numéraux verbaux en numéraux arabes. Dans ce modèle, la représentation sémantique du nombre retranscrit la structure verbale des numéraux. Sont utilisées les primitives lexicales de 1 à 9 et les primitives sémantiques comme les quantités 10, 100 et 1000. Les autres numéraux résultent de la somme ou du produit de plusieurs primitives. Notons que le préfixe « C » indique la représentation sémantique d'une quantité donnée, les primitives seraient transformées directement dans une représentation sémantique comme « deux » → C2, « cent » → C100 et la représentation sémantique contient les relations additives comme « quinze »→ (C10 + C5). Par conséquent, la représentation sémantique du nombre « trois cent quinze » est donc (C100 x C3) + (C10 x C1) + C5. Lors de l'étape de production, la représentation sémantique est convertie en numéral arabe par l'utilisation de deux règles, sur-écriture et concaténation, règles appliquées respectivement aux relations somme et produit (e.g., <C10> + <C7> = 10 # 7 = 17 pour l'opération sur-écriture, <C100> x <C8> = 8 & 00 = 800 pour l'opération de concaténation). Ainsi, suivant ce modèle, la difficulté d'un enfant qui fait plus d'erreurs sur la somme que sur le produit sera comprise comme une difficulté d'acquisition de l'opération de sur-écriture par rapport à l'opération de concaténation.

Ces deux modèles sémantiques ont fait l'objet de critiques dans la littérature ultérieure, notamment sur leur incapacité à expliquer certaines erreurs, comme celles commises par les enfants français lors du transcodage de nombres complexes à partir de 70 (Seron et Fayol, 1994) ou les limites de modularité et d'activation de la représentation sémantique (Seron & Noël, 1995, Noël & Seron, 1995).

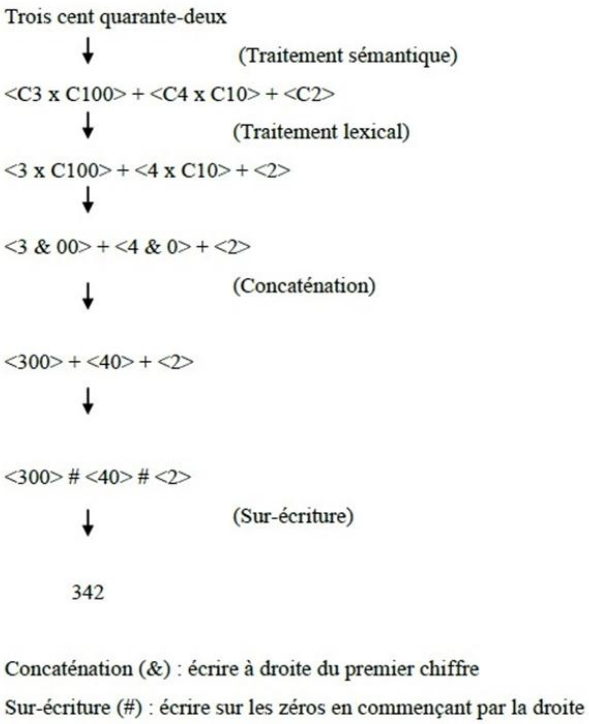


Figure 6. L'écriture des nombres (arabes) sous dictée : algorithme proposé Power & Dal Martello (1990) (d'après Saad, 2011)

Modèles asémantiques

Des modèles asémantiques ont aussi été proposés. Deloche et Seron (1987), ont proposé un modèle du transcodage numérique du code verbal oral vers le code digital suivant une série d'étapes. Ainsi, la chaîne numérique verbale va d'abord être segmentée en primitives lexicales pour que ces dernières soient catégorisées en fonction de leur classe lexicale et de leur position dans la classe (e.g., les nombres « cinq », « quinze », « cinquante » ont la même position dans des classes différentes et « deux », « quatre », « six » la même classe avec des positions différentes). Un échec dans cette série d'étapes, regroupées pour ce modèle sous le nom *d'étape lexicale*, aura pour conséquence la production d'une erreur lexicale (e.g., « quinze » transcodé 50 pour une erreur de classe ou « quinze » transcodé 16 pour une erreur de position.

La seconde étape du modèle, dite de *transcodage*, regroupe la production et le remplissage d'un cadre syntaxique par le biais de règles de réécriture, en se basant sur les données précédentes de position et de classe, c'est-à-dire les bons chiffres à la bonne place. Vient enfin le processus d'encodage pour l'écriture du nombre en chiffres. Les erreurs conséquentes aux atteintes pour cette seconde étape seront dites *syntaxiques*, comme, par exemple, un changement de la longueur du nombre en chiffres. Toujours selon Deloche et

Seron (1987), si les éléments de ne sont pas intégrés dans le cadre syntaxique, un transcodage mot à mot du nombre verbal produira une erreur de « lexicalisation » (e.g., « cinquante-six » transcodé en 506).

Ce modèle est dit asémantique puisque le transcodage passe par des règles d'écriture appliquées aux primitives du code d'origine, sans besoin de recourir à une représentation de la quantité. Il possède l'avantage d'avoir mis en avant une différenciation entre les erreurs lexicales et syntaxiques où les notions de classe et de position, mais il n'est pas exempt de critiques. Barrouillet *et al* (2004), relèvent son caractère limité et inadéquat si l'on se place du point de vue du développement et de l'apprentissage (peu de place accordée à l'apprentissage des connaissances déclaratives par exemple).

Ainsi, Barrouillet et ses collaborateurs (2004) ont proposé le modèle ADAPT (modèle de Transcodage Developmental, Asémantique et Procédural). Lui aussi asémantique, il est par contre le premier modèle développemental, c'est-à-dire qu'il considère que les règles déjà apprises vont moduler l'acquisition des nouvelles règles de transcodage.

Son fonctionnement est le suivant. Une chaîne verbale entendue va être encodée et emmagasinée dans une mémoire tampon. Cette chaîne est ensuite séparée en unités qui pourront être traitées par le système de production, c'est le processus de « parsing », qui se base sur l'identification des primitives lexicales, qui varient d'une langue à l'autre. Comme dans le modèle de Deloche et Seron (1987), les primitives seront associées aux informations lexicales de classe et de position, récupérées dans la mémoire à long terme, puis placées en mémoire de travail pour la construction de la chaîne de chiffres. Le système de production du code numérique final suit la forme « condition-action », où la *condition* est un contrôle des connaissances déjà présentes en mémoire de travail et où l'*action* comprend la récupération et la concaténation des formes arabes en mémoire à long terme pour être utilisées par d'autres règles et construite en une chaîne de chiffres. La chaîne de chiffre est enfin stockée dans un buffer pour sa transformation écrite.

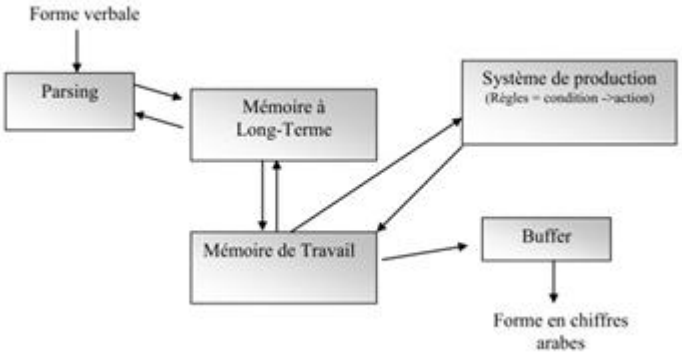


Figure 7. Représentation schématique du modèle ADAPT (Barrouillet *et al*. 2004) (d'après Saad, 2011)

Selon Deloche et Seron (1982), deux types d'erreurs, lexicales et syntaxiques peuvent être distinguées. Les erreurs lexicales indiquent une faute de lexicalisation constituant le nombre tandis que le taille du nombre est conservée (ex., « vingt-cinq » → 28). Les erreurs syntaxiques sont relatives aux règles syntaxiques (ex., « vingt-cinq »→205). Selon une étude de Power et Dal Martello (1990), la plupart des erreurs qui sont commises par les enfants de 7 ans sont syntaxiques. Elles concernent principalement le principe additif (ex., « cent six »→106 au lieu de 1006). D'après les observations d'une étude longitudinale de Seron, Deloche et Noël (1991) chez les enfants de 2e et de 3e primaire, les erreurs syntaxiques proviennent également du principe multiplicatif (ex., « deux cents »→2100 au lieu de 200). Ces observations ont aussi été confirmées dans l'étude de Seron et Fayol (1994) chez les enfants de 7 ans. De plus, les résultats de cette étude ont montré que les enfants ont eu des meilleures performances sur les tâches de compréhension des numéraux oraux que sur les tâches de production des nombres arabes. Les difficultés relatives au transcodage sont également provenues de la taille des nombres (nombre de chiffres) et de la longueur phonologique des formes orales (évaluée en nombre de syllabes) (Noël & Seron, 1995 ; Fayol, Barrouillet & Renaud, 1996).

Le cas du zéro

En se positionnant du point de vue du modèle de Power et Dal Martello (1990) et du modèle ADAPT (Barrouillet et al., 2004) ainsi qu'en examinant le processus de composition des nombres arabes, il apparaît que tous les zéros n'occupent pas le même statut.

Deux types de zéros sont communément distingués. Premièrement, le zéro dit « lexical ». Il est récupéré dans le nombre de façon « directe » pour la chaîne de chiffre en construction. Ce zéro est sémantiquement dérivé et est représenté par une primitive lexicale. Il est considéré comme un concept numérique et ne comprend pas de règle spécifique de production. Selon Grana et ses collègues (2003), ce zéro fait partie du transcodage d'une dizaine comme un zéro final dans le nombre «520» ou comme un zéro interne dans le nombre « 20815 ». Dans la classe des dizaines, les concepts primitifs <C10> à <C90> activent leur forme arabe correspondante. Les zéros inclus dans ces nombres sont ainsi produits comme un tout en association avec le chiffre précédant (vingt → 20).

Nous pensons que les zéros lexicaux sont aussi présents pour les primitives des centaines (100, 200, … 900), comme dans les nombres « 100257 » ou « 5100 », c'est-à-dire que les concepts primitifs <C100> à <C900> suivent le même principe de fonctionnement que décrit ci-dessus. Les zéros inclus dans ces nombres sont produits comme un bloc unitaire en lien avec le chiffre des centaines correspondant.

Deuxièmement, le zéro syntaxique comme dans les nombres 508 ou 7014. Il est inséré dans le nombre par l'application de règles syntaxiques de production. Selon le modèle de Power et Dal Martello, le zéro syntaxique demande une première opération de concaténation puis une seconde de sur-écriture partielle. La sur-écriture est dite partielle, car elle ne recouvre pas tous les zéros formés par la concaténation (e.g., C5 x C1000 + C43 → 5 & 000 # 43 → 5000 # 43 → 5<u>0</u>43). Pour le modèle ADAPT, le zéro syntaxique

s'insère dans les cases qui n'ont pas été remplies par le traitement de la chaîne (e.g., « quatre mille … » → [4 - - -] puis « … trente-sept » → [4 – 37] ; « fin de la chaîne » → [4037]).

À travers ses caractéristiques, nous trouvons que le zéro est considéré comme une source de difficultés pour la production de nombre par sa caractéristique de marqueur positionnel et le zéro syntaxique est plus difficile que le lexical. Citons ici quelques études concernant le zéro. L'étude de Censabella (2000) sur les enfants de 3e année de primaire. L'âge moyen des enfants qui ont participé était de 8 ans et 7 mois, la tâche était d'écrire des nombres verbaux en nombres arabes de 4 à 5 chiffres qui comportaient des zéros des 2 types. Deux résultats sont ressortis de l'étude. Le « zéro syntaxique » conduisait à plus d'erreurs que le « zéro lexical », respectivement 18% et 5%. De plus, la longueur du nombre jouait aussi son rôle dans les erreurs des élèves, le taux d'erreur pour les nombres de 4 chiffres étant de 17%, contre 6% pour ceux de 5 chiffres. Une autre étude de Lochy, Andrés et Seron (2004) sur des enfants plus jeunes de 2e année scolaire, d'âge moyen de 7 ans et 8 mois et sur des adultes, a confirmé que le test de transcodage sur les nombres de 3 chiffres conduisait à plus d'erreurs de type zéro syntaxique que lexical au total. La différence entre les positions des zéros était observée chez les adultes uniquement sur les nombres à 4 et 5 chiffres. Nous pouvons donc conclure que la longueur des nombres et la position des zéros sont deux facteurs influents dans les résultats de la production du nombre.

2.2 Le transcodage et les spécificités du langage

Les nombres verbaux vietnamiens et le cas du zéro

Pour les primitives lexicales vietnamiennes, il n'existe pas de mot particulier pour les nombres onze, douze, treize, quatorze, quinze et seize comme en français. Il est logique de poursuivre la dénomination des nombres allant de onze à dix-neuf avec la relation somme. En vietnamien, les nombres supérieurs s'expriment par une simple combinaison des mots fondamentaux. Par exemple, 11 (onze) est dénommé «dix un». Pour les nombres des dizaines, la langue vietnamienne les dénomme par la relation produit, qui montre bien la structure par la position des nombres. En voici quelques exemples pour les dizaines, «deux dix» pour 20 et «huit dix» pour 80.

La plupart des systèmes de numération verbaux comprennent des primitives lexicales qui sont organisées en classes ordonnées. En exprimant les nombres, nous faisons la combinaison de ces primitives lexicales par les règles de la syntaxe du système au travers de deux types de relations. Ce sont la relation de somme, comme « vingt-cinq » qui signifie « vingt + cinq » et la relation de produit, comme « huit cents » signifie « huit x cent ». Normalement, les nombres sont expliqués par ces deux relations en emboîtement, par exemple « Deux mille cinq cent soixante-huit » signifie {« deux x mille » + « cinq x cent » + soixante + huit ». Dans le langage français, le nombre est composé des trois types d'explications des relations nommées ci-dessus. Dans le langage vietnamien, nous n'avons pas de cas particuliers propres aux dizaines comme dans la langue française. Ainsi tous les

nombres verbaux en vietnamien sont expliqués par les deux relations somme et produit en respectant la priorité de la relation du produit sur celle de la somme. En effet, prenons l'exemple du nombre 23, en français «vingt-trois» est décomposé uniquement par la relation somme. Pour le nombre 523, il est décomposé par deux relations, « cinq x cent + vingt + trois ». Par contre, en vietnamien, 23 (deux dix trois) et 523 (cinq cent deux dix trois) sont décomposés par deux relations en emboîtement. C'est aussi de cette caractéristique du langage vietnamien que découle le fait que les nombres verbaux vietnamiens et leurs décompositions correspondent parfaitement. Prenons l'exemple du nombre 265 avec sa décomposition : 265 = 2×100 + 6×10 + 5. En vietnamien, il est dénommé « deux cent six dix cinq», mais en français son nombre verbal est « deux cent soixante-cinq ». Avec l'avantage de la correspondance entre le nombre verbal et sa décomposition, le système de langage vietnamien peut faciliter les élèves à mieux comprendre la structure du nombre et la position ainsi que la valeur des chiffres dans d'un nombre.

Le langage vietnamien possède des particularités avec les cas du chiffre en position des dizaines ou des centaines égal à zéro. Cette particularité ne se retrouve pas dans d'autres langages asiatiques comme le chinois, le coréen et le japonais. Le vietnamien utilise le mot « zéro » comme primitive lexicale dans la construction du nombre. Par exemple, le nombre arabe 3024, se dira « trois mille zéro cent deux dix quatre ». Il possède aussi une exception quand le zéro se trouve à la position des dizaines, nous le remplaçons par le mot « reste ». Par exemple, le nombre arabe 309 se dira « trois cent reste neuf ». Nous pouvons comprendre le mot « reste » comme le reste de la division de 309 par 100. Par conséquent, pour le nombre arabe 2009, on dira en vietnamien « deux mille zéro cent reste neuf ». Nous constatons que le zéro qui, dans les deux cas, se trouve à la position des dizaines ou des centaines, est explicite dans sa forme verbale.

Donc, pour tous les nombres naturels, la dénomination de ces nombres dans le langage vietnamien est plus longue que dans le langage français. Cependant cette dénomination fait correspondre exactement les formes verbales et arabes du nombre. Les jeunes francophones doivent apprendre par cœur la suite des termes numériques jusqu'à 16, les dizaines (dix, vingt, trente etc.), puis des irréguliers comme soixante-dix, quatre-vingts et quatre-vingt-dix. Comparativement, jusqu'au milliard, la langue vietnamienne comprend seulement 16 mots (contre 29 en français) pour désigner les « mots fondamentaux » à partir desquels elle pourra dénommer tous les nombres par association : Zéro, un, deux, trois, quatre, cinq, six, sept, huit, neuf, dix, cent, mille, million, milliard et « reste ». Cette construction s'effectue selon une logique mathématique qui ne contient pas de cas particuliers. Il n'y a pas de situation ambiguë. Le système numérique verbal vietnamien est totalement régulier et transparent. L'ordre des termes exprimant clairement les relations additives et multiplicatives est équivalant avec la décomposition des nombres entiers en base 10.

Les études comparatives sur les différences des systèmes numériques verbaux

Les mots utilisés pour les nombres sont liés à la langue parlée, ils varient donc d'une langue à l'autre. Comme nous l'avons décrit ci-dessus, la règle de composition linguistique

des mots de nombres diffère entre le français et le vietnamien. Cela implique une différence de niveau de transparence dans les systèmes numériques verbaux. De plus, les systèmes numériques verbaux de certains pays asiatiques, comme la Chine, la Corée, le Japon et le Vietnam sont très semblables. Leurs constructions sont très régulières et reflètent bien la structure décimale. Ces constructions peuvent faciliter les enfants à acquérir la suite verbale des nombres, à mieux comprendre la structure du nombre et la position ainsi que la valeur des chiffres dans un nombre.

Nous remarquons également la similarité entre les systèmes numériques verbaux dans certains langages occidentaux, comme le français, l'anglais, l'espagnol, l'italien, etc. Ils présentent des irrégularités (ex., eleven, twelve en anglais) et manquent de transparence sur la base dix. Ceci peut conduire à un impact négatif sur l'acquisition des conceptions mathématiques ainsi que du système de numération arabe.

À travers plusieurs études comparatives concernant différents systèmes numériques verbaux sur les conceptions et la compréhension des mathématiques, nous constatons qu'un langage différent peut conduire à des facilités ou à des difficultés dans l'exécution de certaines tâches relatives aux mathématiques. Miller et Stigler (1987) ont comparé les capacités de comptage entre les enfants chinois et américains. Grâce à un système plus transparent, en dénommant les nombres naturels, ces auteurs trouvent que les enfants chinois comptent mieux. Ils concluent que le système de dénomination chinois facilite la compréhension de l'aptitude à compter chez les enfants. De plus, l'effet du langage a aussi facilité la compréhension des positions ainsi que des valeurs de chaque chiffre dans le nombre. Miura et al. (1993) constatent que les Chinois, Japonais et Coréens utilisent le canonique de base 10 de façon plus exercée et régulière que les Anglais, Français et Suédois. (Nous développerons plus en détail ces éléments dans la section 3.1 et 4.1 de la thèse).

Notons que le nombre verbal « quatre-vingt-dix » (90) utilisé par les Français est dérivé de deux relations, additive et multiplicative. En revanche, chez les Belges et les Suisses, des noms différents sont utilisés pour les dizaines: « septante » au lieu de « soixante-dix », [« huitante » (canton de Vaux) au lieu de « quatre-vingts »] et « nonante » pour « quatre-vingt-dix ». Cela entraîne plus de régularité pour les dizaines. En effet, sur base de la différence de l'expression verbale des dizaines de 70 et 90, Seron et Fayol (1994) ont montré que les Français font plus d'erreurs que les Belges dans l'écriture des nombres verbaux aux nombres arabes. Par ailleurs, le langage allemand possède la particularité d'inverser la structure dizaine-unité sur plan verbal, par exemple pour 52 (cinquante-deux) qui est dénommé par « deux et cinquante ». L'étude de transcodage de Lochy, Delazer, Domanhs, Zoppoth et Seron (2004) entre les jeunes enfants belges et autrichiens relate que le taux d'erreurs est plus élevé en Autriche (42%) qu'en Belgique (34%) concernant la première au primaire. Ces erreurs correspondaient en grande majorité à des erreurs d'inversion (e.g., écrire « 25 » pour « cinquante-deux », dit littéralement « deux et cinquante »).

La différence de caractéristiques linguistiques dans les systèmes numériques verbaux peut faire varier les performances sur une tache transcodage comme l'a examiné Saad

(2011) dans le cadre de sa thèse. Rappelons que le système numérique allemand a la particularité d'inverser l'ordre d'énonciation des dizaines et des unités ce qui est un renversement par rapport au sens d'écriture de gauche à droite (plus de détails dans la Figure 6). Pour les dizaines exactes (e.g., 40) ou les centaines-dizaines (e.g., 140), le sens d'énonciation est le même que sens d'écriture général (e.g., 140 se dit « cent quarante »). Nous pouvons croire que le transcodage verbal – arabe des nombres dizaines-unités conduira à plus de difficultés que celui des nombres dizaines exacts et des nombres centaines-dizaines pour les enfants Allemands. Le système numérique verbal syrien ressemble à celui de la langue allemande, mais l'écriture des mots et des lettres en Syrien (arabe) se fait de la droite vers la gauche. Par exemple 52 se dit « deux et cinquante » et l'ordre de l'énonciation de ce nombre correspond au sens d'écriture habituel en arabe. Par contre, pour les nombres dizaines exacts (e.g., 30) ou centaines-dizaines (e.g., 140), l'ordre de l'énonciation de ces nombres est contraire au sens d'écriture des chiffres en arabe (plus détails dans la Figure 6). Donc, les enfants Syriens pourraient rencontrer plus de difficultés dans une tache de transcodage avec les nombres dizaines exacts et les nombres centaines-dizaines. Il est dès lors plus difficile pour les enfants allemands et syriens de transcoder des grands nombres. Ces transcodages impliquent plusieurs changements de direction d'écriture afin de transformer un nombre verbal en nombre arabe. Par exemple, pour écrire le nombre 35162 qui se dit « cinq et trente mille cent deux soixante » en allemand et syrien, il est nécessaire de l'écrire dans l'ordre suivant « G ← D, G → D, G ← D » (5, puis 3 à gauche du 5, puis 1 à droite du 5, espace, puis 2 et enfin 6 placé dans l'espace).

	France	Allemagne	Syrie
Sens de l'écriture	---->	---->	<----
Ordre d'énonciation			
Dizaine	---->	---->	---->
Ex :	Trente 30	Trente 30	Trente 30
Dizaine-unité	---->	<----	<----
Ex :	Trente-cinq 35	Cinq et trente 35	Cinq et trente 35
Centaines	---->	---->	---->
Ex :	Cent quarante 140	Cent quarante 140	Cent et quarante 140

Figure 6. Sens d'énonciation vs sens d'écriture des nombres en fonction des langues et des catégories lexicales (Saad, 2011)

Saad (2011) a examiné l'impact de l'irrégularité du système numérique verbal français (pour les nombres de 70 à 99) et l'effet de l'ordre d'énonciation et du sens d'écriture sur le transcodage du code verbal au code arabe. Elle a montré que les enfants Français ont des

performances plus faibles que celles des enfants Allemands et Syriens pour le transcodage des nombres de 70 à 99, illustré dans la figure 7 (56%, 19% et 8% d'erreurs pour trois groupes d'élèves de la première année de l'école primaire chez Français, Allemands et Syriens). De plus, pour les grands nombres (nombres centaines et 1000), les enfants syriens avaient plus de difficultés sur ces nombres que les Français (57%, 41% et 69% d'erreurs pour les trois groupes français, allemands et syriens). Les résultats ont montré que l'inversion de l'ordre, la contradiction entre l'ordre d'énonciation et le sens d'écriture du nombre, a influencé significativement le transcodage. Les enfants syriens ont eu plus d'erreurs sur les nombres Dizaines (D) que sur les nombres Dizaines-unités (DU) et c'est l'inverse pour les enfants allemands. Cette étude a confirmé l'influence des propriétés linguistiques des systèmes numériques verbaux sur le transcodage.

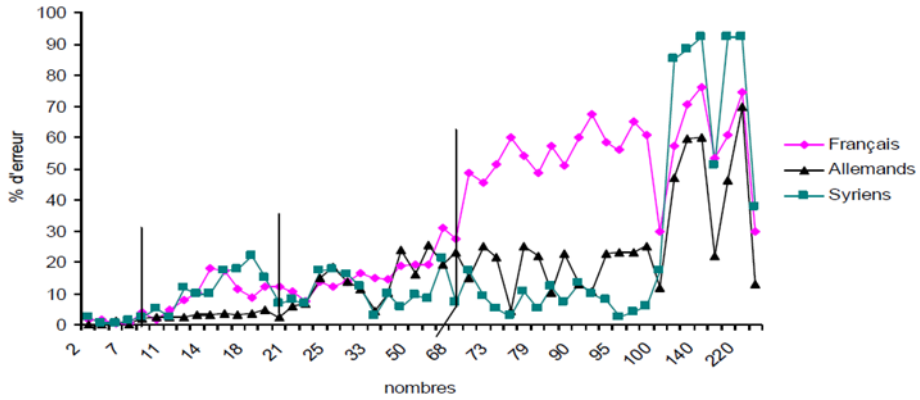


Figure 7. Pourcentage d'erreurs sur les nombres dictés en fonction de la nationalité des enfants: Français, Allemands et Syriens (Saad, 2011)

En résumé

Nous pouvons distinguer trois types de représentations de nombre : la représentation analogique, indépendante du langage, la représentation verbale et la représentation écrite. Ces deux dernières formes de représentation sont basées sur des symboles et instaurées par convention. Alors que la représentation écrite (arabe) est quasi universelle, la représentation verbale est différente en fonction du langage. Le transcodage est le processus qui permet de passer d'une représentation à l'autre, d'un code à l'autre. Les difficultés liées au transcodage peuvent provenir de la taille des nombres (nombre de chiffres), de la longueur phonologique des formes orales (évaluée en nombre de syllabes) ou de la présence d'un zéro syntaxique. Les enfants ont besoin de plusieurs années pour maîtriser le transcodage. Les différences d'un système numérique verbal à l'autre auront un impact sur l'acquisition et la compréhension des concepts mathématiques (comptage, utilisation de la base 10, transcodage…). Les études précédentes ont montré que les caractéristiques linguistiques conduisent à des erreurs spécifiques et des performances différentes dans les taches de transcodage.

Chapitre 2

Enseignement des fractions et décimaux

Enseignement des fractions et décimaux

Les nombres rationnels jouent un rôle important dans l'apprentissage des mathématiques. Ils sont théoriquement importants, car le concept de nombre rationnel possède de multiples facettes et demande une compréhension plus profonde que les nombres entiers. Ils sont aussi pédagogiquement importants, car les difficultés lors des manipulations avec les nombres rationnels sont fréquentes et persistantes chez les élèves de tous les niveaux et même chez les adultes. Les fractions sont substantiellement enseignées en 3ème ou 4ème année de primaire mais une évaluation nationale des progrès éducatifs aux Etats-Unis montrent que 50% d'élèves de 8ème année ne peuvent pas ordonner correctement les trois fractions suivantes 2/7, 1/12 et 5/9 (Martin et al., 2007). La plupart des élèves de 5ème et de 6ème ont choisi 0.274 comme plus grand que 0.83 et moins de 30% des élèves de 11ème année réussissent à transformer 0.029 sous la forme d'une fraction (Rittle-Johnson et al., 2001; Kloosterman, 2010). Des difficultés similaires s'étendent aussi aux étudiants (Schneider & Siegler, 2010; Stigler et al, 2010). Ce phénomène est non seulement présent aux Etats-Unis mais il est également observé dans d'autres pays tels que le Japon, la Chine, Taiwan et la Belgique (Chan al. 2007 ; Ni, 2001 ; Yoshida & Sawano, 2002; Desmet 2012).Cette importance est renforcée par de récentes études qui suggèrent que les performances des élèves dans le domaine des fractions prédisent leur maîtrise de l'algèbre et leur réussite en mathématiques par la suite (en contrôlant le Q.I, la capacité de lecture, le mémoire de travaille et le revenu familial) (Siegler et al., 2012; Bailey et al., 2012).

Le concept de fraction est constitué de plusieurs sous-constructions : relation partie-tout, ratio, opérateur, quotient et mesure. Comme les opérations sur les nombres fractionnaires présentent plusieurs difficultés, les nombres rationnels peuvent être représentés de manière alternative sous une forme décimale, en utilisant le système décimal (base 10). Par conséquent, il existe plusieurs approches possibles des nombres rationnels dans l'enseignement. Chaque approche met l'accent sur des aspects spécifiques des nombres rationnels. L'ordre dans lequel les nombres rationnels sont abordés dans l'enseignement est aussi varié que les différences dans les types d'enseignement des nombres rationnels.

Rappelons que le processus de généralisation d'une conception commence par le recours aux connaissances locales. Donc, le contexte d'enseignement joue aussi un rôle dans le processus de conceptualisation d'une notion chez les élèves. En effet, le contexte d'enseignement d'une connaissance contient le contenu, la façon dont cette connaissance est enseignée, ainsi que le système pédagogique à propos de cette connaissance. Tous ces facteurs de contexte d'enseignement influencent directement et indirectement le processus d'acquisition d'une nouvelle conception. L'enseignement des nombres rationnels au Vietnam et en Belgique est présenté dans ce chapitre à travers des méthodes pédagogiques,

des contenus et de l'ordre dans lequel les notions relatives aux nombres rationnels sont enseignées.

Le programme de mathématiques vietnamien est détaillé pour chaque année scolaire, celui de Belgique est présenté plutôt suivant des cycles de deux années scolaires (Tableau 1). De plus, le programme d'enseignement des mathématiques en primaire au Vietnam est très détaillé avec des indications pour chaque semaine. Les démarches didactiques sont aussi expliquées explicitement aux enseignants par les documents qui leur sont fournis. L'existence d'un manuel scolaire unique rend plus facile l'étude de l'enseignement des nombres rationnels au Vietnam. En conséquence, la partie de ce chapitre dédiée à l'enseignement des nombres rationnels au Vietnam est plus concise que celle consacrée à leur enseignement en Belgique.

1. Enseignement des fractions et décimaux en Communauté française de Belgique

1.1 Les « Socles de Compétences »

Nous rappelons ci-dessous les trois premières étapes de l'enseignement obligatoire belge. Le tableau présente le système d'éducation belge structuré en étapes et en cycles.

Tableau 1. Le système d'éducation belge

Étape 1	1er cycle	de l'entrée en maternelle à l'âge de 5 ans
	2e cycle	de l'âge de 5 ans à la fin de la 2e primaire (5-8 ans)
Étape 2	3e cycle	3e et 4e années primaires (8-10 ans)
	4e cycle	5e et 6e années primaires (10-12 ans)
Étape 3	5e cycle	1ère et 2e années secondaires (12-14 ans)

Les «Socles de Compétences» (Ministère de la Communauté Française, 1999) est le document officiel qui guide la formation mathématique d'enseignement fondamental et de premier degré de l'enseignement secondaire en Communauté française de Belgique. Les compétences relatives aux outils mathématiques de base dans ces socles des compétences sont regroupées en quatre domaines : les nombres, les solides et figures, les grandeurs et le traitement de données. Les nombres rationnels (fraction et décimal) sont abordés à partir de la troisième année de primaire jusqu'à la deuxième année du secondaire. Toutefois, dans le cadre de notre recherche concernant l'enseignement de l'apprentissage des nombres rationnels en primaire, notre analyse met l'accent sur les contenus des fractions et des décimaux en deuxième étape, de la 3eme à la 6eme année de primaire. En effet, ces nombres sont présentés dans la rubrique « les nombres » et « les grandeurs » des socles des compétences.

Nous citons dans le tableau ci-dessous les parties concernant les nombres fractions et décimaux. Il présente les compétences mathématiques dont la maitrise est requise pour les trois premières années de l'enseignement obligatoire. On utilise la lettre C quand la compétence doit être certifiée à la fin de l'étape précisée et la lettre E si la compétence doit continuer à être exercée durant l'étape précisée. Lorsqu'une préparation des élèves au recours à la compétence est nécessaire pendant l'étape précisée, on utilise le symbole↗.

Tableau 2. Compétences concernant des nombres rationnels dans le domaine « les nombres »

Les nombres		**I**	**II**	**III**
Compter, dénombrer, classer	Dire, lire et écrire des nombres dans la numération décimale de position en comprenant son principe.	**C** Des nombres naturels ≤ 100	**C** Des nombres naturels décimaux limités au millième	**E**
	Classer (situer, ordonner, comparer).	**C** Des nombres naturels ≤ 100	**C** Des nombres naturels et des décimaux limités au millième	**C** Des entiers, des décimaux et des fractions munis d'un signe
Organiser les nombres par familles	Décomposer et recomposer.	**C** Des nombres naturels ≤ 100	**C** Des nombres naturels et des décimaux limités au millième	**E**
Calculer	Identifier et effectuer des opérations dans des situations variées.	**C** Avec des petits nombres	**C** Avec des nombres naturels et des décimaux limités au millième.	**C** Avec des entiers, des décimaux et des fractions munis d'un signe. Y compris l'élévation à la puissance.
	Écrire des nombres sous une forme adaptée (entière, décimale ou fractionnaire) en vue de les comparer, de les organiser ou de les utiliser.		**C**	**E**

Tableau 3. Compétences concernant des nombres rationnels dans le domaine « les grandeurs »

Opérer, fractionner	**I**	**II**	**III**
Fractionner des objets en vue de les comparer.	**C** Partager en deux et en quatre.	**C**	**E**
Composer deux fractionnements d'un objet réel ou représenté en se limitant à des fractions dont le numérateur est un (par exemple, prendre le tiers du quart d'un objet).		↗	**C**
Additionner et soustraire deux grandeurs fractionnées.		**C**	**E**

Dans la partie « les nombres », il est souligné que l'inversion de la multiplication et la division conduisent à élargir l'univers des nombres et à accéder aux nombres fractions et décimaux. Toujours dans cette rubrique, nous constatons qu'au niveau du primaire, seulement les nombres décimaux sont abordés avec l'objectif de « dire, lire écrire et classer ces nombres ». Pour le calcul, il est dit « identifier et effectuer des opérations dans des situations variées ». Or, nous ne savons pas, parmi les quatre opérations élémentaires, lesquelles sont choisies et à quel niveau. Toutes les compétences relatives aux décimaux sont limitées au millième pour le primaire. De plus, le développement de ces compétences se poursuit pendant la 1ère et la 2ème année du secondaire avec les nombres fractions et décimaux.

Pour les fractions au niveau de la primaire, elles ne sont présentées que dans la rubrique « les grandeurs » à travers les activités « d'opération et de fractionnement avec les grandeurs ». Dans cette même rubrique, il est souligné que « les opérations de mesurage et de fractionnement conduisent aux nombres décimaux et fractions ». Ces éléments tendent à montrer que la tendance pour aborder les fractions au niveau du primaire est de le faire à travers le travail sur la notion de « grandeur ».

L'activité de fractionnement des objets est abordée pour la première fois en 3ème année et elle est prolongée à un niveau supérieur au cours des années suivantes. Les fractions dont le numérateur est égal à 1 sont également présentées à partir de la 3ème année, mais les compétences sur ces fractions-là restent encore fragiles jusqu'à la fin du primaire pour les élèves. Les opérations comme l'addition et la soustraction sont réalisées sur deux grandeurs fractionnées. Il semble que les fractions ne soient pas abordées comme les nombres au niveau du primaire. Cela expliquerait pourquoi, à travers ces socles de compétences, nous ne pouvons trouver de lien entre les fractions et les décimaux.

En effet, avant la troisième étape, c'est-à-dire les 1ère et 2ème années du secondaire, les fractions sont présentées officiellement avec les compétences de classement et d'opération. Nous analyserons ultérieurement les programmes de mathématique du primaire afin de

pouvoir mieux cerner les rapports grandeurs-nombre fraction dans l'enseignement des fractions en primaire.

En conclusion, nous constatons qu'au niveau du primaire, en ce qui concerne les nombres rationnels, seulement les nombres décimaux limités au millième sont abordés. Les fractions sont présentées implicitement en s'appuyant sur les activités de fractionnement et sur les opérations réalisées sur les grandeurs. Nous remarquons que le décimal joue un rôle plus important que la fraction. Cependant, la simple étude de ces socles de compétences ne nous permet pas d'analyser les séquences d'enseignement des nombres rationnels. Parmi les décimaux, les fractions et les grandeurs, quels sont ceux choisis pour approcher la notion de nombre rationnel et comment ce nombre est-il abordé ? L'analyse des programmes de mathématique de l'enseignement fondamental nous permet de tenter de répondre à ces questions.

1.2 Programme des études d'enseignement fondamental (Administration générale de l'enseignement et de la recherche scientifique, 2002)

Tout comme les socles des compétences pour la formation mathématique de l'enseignement fondamental, le contenu du programme de mathématique en primaire est organisé en quatre parties principales : Les résolutions de problèmes, les nombres, les grandeurs, les solides et les figures. En premier lieu, nous remarquons que ce programme n'est pas vraiment fidèle aux socles des compétences dans les parties relatives aux nombres rationnels. En effet, nous pouvons relever les deux points suivants. Premièrement, les fractions ou bien l'activité de fractionnement apparaissent estompées dans la sous-partie « opérer, fractionner » du domaine « Grandeur ». Il semble que la partie « Grandeur » de ce programme soutienne peu l'approche des nombres fractions. Deuxièmement, les fractions, à travers les compétences de classement et d'opération sont abordées explicitement dans la partie « Les nombres », ceci en alternance avec les nombres décimaux de la troisième année jusqu'en fin de primaire.

Comment les décimaux sont-ils abordés ?

Rappelons que les décimaux et les fractions sont abordés en alternance, mais que les décimaux sont toutefois présentés avant les fractions. Le décimal est approché en premier dans l'étape de codages des mesures : « associer les nombres décimaux au mesurage de grandeur : 1m75cm = 1,75m ».

Le sens de ces nombres est enseigné à travers les activités de dire, lire et écrire en chiffre ainsi que celle « d'exprimer la valeur de chaque chiffre dans un nombre décimal ». Ce programme insiste également sur le rôle de la virgule ainsi que sur l'activité de décomposition d'un nombre décimal en le verbalisant de plusieurs manières.

Après l'approche du nombre décimal, la fraction est présentée en liaison avec l'activité de « partager un objet, une forme géométrique, une collection d'objets en parts d'égale grandeur ». Ensuite, le sens de « fraction », dont le numérateur est égal à 1 est introduit

implicitement par la demande « exprimer qu'une part vaut le … de … » qui est la suite de l'activité « partager en parts égales ».

Le sens d'écriture de la fraction est introduit par le fait de « prendre la fraction de quelque chose par la succession de deux opérateurs ». Citons ici l'explication de ce programme « Multiplier avec ¾ c'est-à-dire multiplier par 3 et diviser par 4 ». Nous constatons que cette fraction est abordée comme « fraction opérateur », comme le symbole d'une chaîne d'opérations. Ainsi le sens de la fraction n'est pas souligné du côté de la conception d'un nombre, mais plutôt comme opérateurs abstraits.

Suite à ces premières approches des nombres fractions et décimaux, les compétences de situer et de comparer ces nombres sont abordées. Notons que ces compétences qui sont relatives aux rationnels prennent une grande place dans ce programme. L'objectif de pouvoir encadrer le nombre décimal et la fraction est tout d'abord réalisé avec des objets concrets (grandeurs, monnaie) puis avec un travail sur la droite graduée et orientée par la suite.

La comparaison des décimaux est abordée en première étape par la manipulation de la droite graduée puis par l'explication des comparaisons par l'écriture des nombres décimaux. Pour les fractions, la comparaison de ces nombres passe par les séquences suivantes. Tout d'abord, l'équivalence des fractions est abordée par la manipulation ou par le dessin. Ensuite la comparaison de plusieurs fractions de même numérateur ou dénominateur. Enfin, la comparaison de fractions de numérateurs et dénominateurs différents. Plus particulièrement, deux fractions sont comparées à travers la comparaison de fractionnements. Par exemple, « comparer le tiers de 60 par rapport au quart de 60 ». Encore une fois, nous notons que le sens et l'utilisation du nombre fraction dans ce programme s'attachent à « prendre la fraction de quelque chose » plutôt que de considérer la fraction dans sa représentation d'un nombre cardinal.

À travers ce programme, nous pouvons noter quelques points qui semblent faire un lien entre ces deux nombres. Premièrement, l'activité de « repérer et situer des nombres fractions ou décimaux sur une droite graduée et orientée », mais nous ne savons pas encore s'il existe une fraction et un décimal qui ont la même valeur et qui serait repéré sur la droite.

Deuxièmement, « l'écriture de nombres sous forme décimale et fractionnaire » pour comparer ou utiliser. Par exemple «1/10 = 0.1». Les fractions décimales ne sont pas présentées explicitement, mais à travers les exemples de cette partie, comme «1/5 = 2/10 = 0,2 et ¼ = 25/100 = 0,25». Nous pouvons penser que ce nombre est utilisé pour faire un pont entre l'écriture fraction et l'écriture décimale. Enfin, dans la partie «calculer», ce programme insiste sur l'utilisation à bon escient de l'équivalence, de l'écriture des nombres décimaux, des fractions et des pourcentages pour effectuer les calculs.

On remarque tout d'abord que l'estimation du résultat par approximation est abordée avant le calcul. Pour les décimaux, le calcul s'effectue toujours en liaison avec des situations de problèmes ou de grandeurs. Il s'agit d'additionner et de soustraire des nombres décimaux usuels (limités au centième), et de multiplier par des nombres décimaux.

C'est seulement en 5ème et en 6ème que la division par un nombre décimal est abordée. Enfin, c'est l'activité de trouver le complément d'un nombre décimal pour arriver à l'unité qui est étudiée.

Pour les fractions, les calculs sont réalisés en s'appuyant sur des fractionnements concrets ou sur des dessins. L'addition et la soustraction de fractions simples qui ont le même dénominateur en 3ème et en 4ème puis, à la fin du primaire, l'addition et la soustraction de fractions de dénominateurs différents et finalement la division d'une fraction par un nombre entier.

1.3 Les manuels scolaires : 4 et 5 « Archi m'aide » (Echterbille et al., 2002)

Pour mieux comprendre l'approche des nombres rationnels, nous avons analysé les manuels qui sont souvent utilisés en primaire en Belgique. Ce sont les manuels 4 et 5 « Archi m'aide ». Nous présentons ci-dessous les activités des élèves correspondant aux contenus mathématiques des nombres rationnels.

Représentation de nombres rationnels selon « ARCHI M'AIDE 4 »

1. Découvrir les nombres décimaux à travers les mesures (p 29-42, p70-71)

La situation introductrice des décimaux est une situation de mesure. Les notions d'échelle 1/10, 1/100 et 1/1000 sont abordées à travers les rapports de multiplication entre les unités conventionnelles. Les nombres décimaux sont utilisés pour exprimer les résultats des mesures.

En effet, une approche des nombres décimaux ou fractionnaires est réalisée à travers les exercices, en demandant aux élèves d'exprimer les résultats des mesures au moyen d'unités différentes. Ce cahier insiste sur le fait que « le changement d'unité nécessite l'introduction de la virgule ».

La comparaison de nombres décimaux est abordée en appuyant le tracé de segments dont la mesure est exprimée par un nombre décimal.

En utilisant des étiquettes représentant un prix ou un poids sous forme de nombre décimal, les exercices proposent d'exprimer les poids dans une autre unité. L'objectif est de donner du sens à l'écriture d'une mesure. Remarquons que les élèves ont l'habitude de rencontrer l'expression du poids en kg sous la forme d'un nombre décimal sur les étiquettes dans la vie quotidienne.

Le sens de la virgule dans l'écriture d'un nombre décimal est approché en demandant « indique ce que désigne chaque chiffre inscrit sur l'étiquette 0,239kg ». Cette séquence d'introduction des décimaux est le codage des mesures.

2. Fractions simples : demi, quart et huitième. Cinquième et dixième. Tiers, sixième et neuvième.

Trois leçons séparées correspondent à l'introduction de trois familles de fractions avec chacune le même d'objectif.

Tout d'abord, les activités avec pour but de partager un carrée en 2, 4 et 8 parts équivalentes sont proposées. Ensuite, les fractions 1/2, 1/4 et 1/8 sont approchées avec la représentation de l'aire de la figure.

Les notions de numérateur et de dénominateur sont introduites avec l'explication suivante « le numérateur indique le nombre de parts prises ou coloriées et le dénominateur indique le nombre de parts faites dans l'unité ». Nous constatons que l'aspect de la fraction relative à ce manuel est centré sur le concept de partage et que l'approche du nombre fraction se fait à partir du fractionnement de l'unité.

Les deux familles des fractions restantes sont abordées avec la même démarche.

Le sens de numérateur et de dénominateur est renforcé par la suite en variant le nombre de parts prises et le fractionnement.

Dans chaque famille de fraction, la comparaison, l'addition et la soustraction de fractions ayant le même dénominateur sont proposées.

La fraction équivalente dans la cadre de chaque famille de fraction est approchée de manière implicite en manipulant des dessins, par exemple : 1/3 = 2/6 = 3/9.

L'écriture opératoire 1/2 × …, 1/4 × … et 1/8 × …, etc., est introduite en complétant des opérations données. Ces leçons insistent aussi sur le fait que prendre la même fraction d'objets différents donne un résultat différent. Par exemple 6/9E est différent de 6/9F donc E et F sont deux objets différents. Nous constatons que la fraction est approchée avec l'aspect d'un nombre.

Les compétences pour comparer et ordonner les fractions sont toujours introduites en manipulant des dessins qui présentent le partage d'une unité en parts égales. Par exemple, le dessin suivant dans le cadre de la famille de fraction demie, quart et huitième.

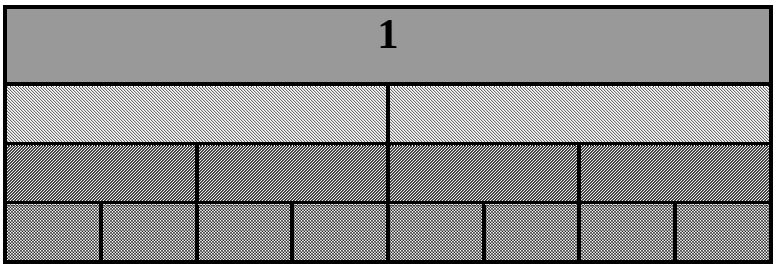


Figure 1. Fractionnement d'une unité en parts égales : demie, quart et huitième

3. Fractionner et comparer (livre B, page 3-6)

Les fractions dont le numérateur est différent de 1 ainsi que les fractions supérieures à 1 avec leurs écritures en lettre sont introduites.

Les fractions qui sont supérieures à l'unité sont proposées à travers les activités de décomposition en deux parties, l'une est l'unité et l'autre est la partie fraction, par exemple 5/3 est l'équivalent d'une unité et de 2/3. Notons que ces activités sont importantes, d'une part pour l'aspect cardinal d'un nombre fraction et d'autre part pour le lien entre la fraction et le nombre décimal.

4. Découvrir, ordonner et décomposer des nombres décimaux jusqu'aux dixièmes (p63-69)

Par l'activité de recherche des nombres décimaux dans tous les travaux de mathématiques antérieurs, l'objectif est de préparer les élèves au passage des nombres décimaux. Cela se fait depuis l'expression d'une mesure jusqu'à la compréhension de l'écriture d'un nombre décimal abstrait.

Le rôle de la virgule est rappelé comme une séparation entre des unités différentes.

Le comptage se faisant par 0,1 et 0,2 sur la droite graduée est présenté.

Le rôle du zéro dans un nombre décimal est proposé ici dans une première approche, en manipulant des euros et des cents. Notons que cette leçon est officiellement l'occasion de présenter et de « fixer » la notion de nombre décimal.

5. Opérations sur les nombres décimaux (p70-73)

L'addition et la soustraction des nombres décimaux « abstraits » limités aux dixièmes sont présentées.

Il y a deux séries d'opérations et les élèves utilisent une forme de décomposition particulière pour les réaliser. Le premier type d'exercice conduit au passage à l'unité supérieure, par exemple :

0,6 + 0,7 = (0,6 + 0,4) + 0,3 = 1,3 et 1,5 + 0,8 = (1,5 + 0,5) + 0,3 = 2, 3

Le deuxième type d'exercice amène au calcul de la partie entière et de la partie décimale séparées : 3,5 + 3,8 = (3 + 3) + (0,5 + 0,8) = 7,3

Nous remarquons que le premier type d'exercice aide les élèves à mieux comprendre la partie entière et la partie décimale dans un nombre décimal et ainsi à éviter les erreurs conduisant à traiter ces derniers comme des nombres entiers lors des calculs. Mais le deuxième type d'exercice au contraire conduit aux erreurs du type suivant:

3,5 + 3,8 = (3 + 3) + (0,5 + 0,8) = 6,13

6. Le lien entre les nombres décimaux et fractions (p77-79)

L'exercice propose aux élèves de placer les nombres décimaux et fractions en même temps sur une droite graduée, l'un au-dessus et l'autre en dessous. En particulier, il existera quelques nombres décimaux qui seront égaux à ceux présentés sous forme de fraction.

Ensuite, le deuxième exercice propose aux élèves de chercher les nombres fractions ou décimaux rencontrés et de les écrire en mots sous deux formes, fraction et décimal.

Remarquons que, jusqu'à ces séquences, les élèves ne rencontrent pas de situations sur le lien entre le nombre décimal et le nombre fraction. Cet exercice suggère alors la liaison entre ces deux nombres.

7. Opérations écrites sur les nombres décimaux (p84-90)

Le calcul est écrit avec les quatre opérations dont un nombre au moins est un nombre décimal limité au centième.

La division avec le reste est abordée en initiant les élèves à la capacité de continuer quelques divisions, il ne fait qu'ajouter une ou plusieurs décimales.

Représentation de nombres rationnels selon « ARCHI M'AIDE 5 »

1. Grandeurs : nombre à virgule : euros-cent (livre 5A p92)

Pour le livre 5A, le sens à l'écriture décimale est abordé. Nous ne trouvons pas beaucoup de contenu relatif aux nombres décimaux et fractions.

2. Situer, encadrer, comparer les nombres décimaux – la valeur de zéro et la place de la virgule (p1-11, livre B)

À travers le compte par 1 ; 0,1 ; 0,01 et 0,001, les activités d'encadrement d'un nombre décimal entre deux nombres sont proposées. Avec la même démarche de décompte par 1 ; 0,1 ; 0,01 et 0, 001, les exercices consistant à situer les nombres décimaux sur la droite graduée sont introduits. Nous constatons que la compétence de pouvoir situer un nombre décimal est soulignée dans ce livre, mais les situations proposées avec la sous-graduation pour introduire la technique de « situer » sont compliquées.

Les nombres décimaux limités aux millièmes en lettres sont abordés à travers l'écriture de ces nombres dans l'abaque. Remarquons que la dénomination des nombres décimaux officielle dans le langage français aide les élèves à mieux comprendre la valeur de chaque chiffre ainsi que sa position dans un nombre décimal.

Le rôle du zéro est renforcé par l'activité « barre les zéros inutiles » dans l'exercice de l'écriture de nombres dans l'abaque et dans la tache de comparaison des nombres décimaux qui ont des zéros à des places différentes.

La place de la virgule est approchée par les taches de comparaison des nombres décimaux et de situer la virgule dans un nombre pour obtenir une égalité indiquée.

La recomposition et décomposition des nombres décimaux est proposée, par l'exemple :

$(7 \times 1000) + (2 \times 100) + (6 \times 1) + (5 \times 0{,}1) + (6 \times 0{,}01) = \ldots\ldots$.

et $7804{,}24 = (\ldots \times 1000) + (\ldots \times 100) + (\ldots \times .10) + (\ldots \times 1) + (\ldots \times 0{,}1) + (\ldots \times 0{,}01)$

Ces exercices sont difficiles, mais ils sont très utiles pour comprendre l'aspect cardinal d'un nombre ainsi que la valeur et la position de chaque chiffre d'un nombre décimal. Ceci

renforce la faculté à éviter beaucoup d'erreurs en utilisant et en traitant les nombres décimaux.

En Résumé

Le Contenu de mathématiques concernant les nombres rationnels en primaire en Belgique est résumé dans le tableau 4.

Tableau 4. Contenu de mathématiques concernant les nombres rationnels en primaire en Belgique

cycle	contenu
2e cycle	-faire connaissance ½ et ¼
3e cycle	-conception fraction (par fractionnement d'une unité) -fractions équivalentes, comparaison et classe des fractions (fraction <1 et numérateur < 10) -opération : l'addition et la soustraction -conception de pourcentage et calculer des pourcentages. - notion : dixième et centième -repérer, situer, comparer des nombres décimaux limités au centième -rôle du zéro et de la virgule.
4e cycle	-les décimaux jusqu'au millième -l'opération d'une fraction dont le dénominateur <= 1000 ou un nombre décimal.

2. Enseignement des fractions et décimaux au Vietnam.

L'enseignement primaire au Vietnam dure 5 ans et les fractions sont introduites officiellement pendant la quatrième année alors que les nombres décimaux sont enseignés en cinquième année. Le programme d'enseignement des mathématiques en primaire au Vietnam est très détaillé avec des indications pour chaque semaine. Les démarches didactiques pour atteindre les compétences des nombres rationnels sont expliquées explicitement aux enseignants par les documents qui leur sont fournis. De plus, l'existence d'un manuel scolaire unique rend plus facile l'étude de l'enseignement des nombres rationnels au Vietnam. Nous présentons ci-dessous les contenus ainsi les approches des fractions et des nombres décimaux en primaire au Vietnam (Tableau 5).

Tableau 5. Contenu de mathématiques concernant les nombres rationnels en primaire au Vietnam[1]

Classe	Contenu
3ème	Faire connaissance avec ½, 1/3, ¼ et 1/n (n < 11)
4ème	-Notion de fraction (le quotient)(dire, lire) -fraction équivalente -la réduction des fractions, la réduction au même dénominateur -comparaison des fractions -les opérations (addition, soustraction, multiplication, division) -l'application pour résoudre des problèmes d'échelle d'une carte
5ème	-fraction décimale -fraction « impropre » -notion décimal (dire, lire, décomposer) -décimaux égaux (le rôle du zéro) -classer, comparer, opérations -application : écriture des grandeurs et résoudre des problèmes avec pourcentages

Les fractions sont présentées implicitement pour la première fois lors de la troisième année. Le terme « fraction » n'apparaît pas encore. Les fractions avec le numérateur égal à 1, comme ½, 1/3, ¼, (1/n avec 2 <= n <= 12) sont présentées avec une représentation graphique et ils sont nommés par « un part deux », « un part trois », « un part quatre », etc. Ces fractions unitaires sont abordées comme un part d'unité quand cette unité est divisée en n parts égales.

En analysant le programme de mathématiques (Bo Giao duc va dao tao Vietnam, 2006), le manuel scolaire et les documents fournis aux enseignants (Do et al., 2006), nous trouvons que le terme « fraction » apparaît officiellement lors de la quatrième année. Le sens des termes numérateur et dénominateur est aussi présenté. La conception de la fraction est introduite en deux étapes. Pour la première étape, les fractions inférieures à 1 (p/n, avec p < n) sont présentées par l'activité de fractionnement d'unité en parts égales combinée avec la représentation graphique. Cette étape est considérée comme l'extension des fractions unitaires qui ont déjà été vues en troisième année. Jusqu'à cette étape, le sens de la fraction est « le nombre de parts qui sont prises à partir d'une unité qui est divisée en n parts égales ». Le numérateur doit être inférieur au dénominateur. En deuxième étape, l'établissement d'une relation entre la division et la fraction conduit à une nouvelle vision

[1] Notons ici que l'expression « fraction impropre » indique la somme d'un nombre entier (naturel) avec une fraction. Nous pouvons l'écrire sous forme d'une fraction, par exemple 10/3 = 3 1/3.

de la conception d'une fraction. En effet, la fraction est abordée comme le quotient d'une division, par exemple « on divise 3 oranges pour 4 enfants, donc chaque enfant recevra 3 : 4 =3/4 d'orange. Le quotient ¾ est présenté par la fraction ¾ ». En approchant la fraction comme le quotient d'une division, la fraction supérieure à 1 est naturellement présentée. Nous constatons que la fraction est abordée comme « fraction quotient ».

Suite à ces approches des nombres fractions, les fractions équivalentes sont présentées. Ensuite est présentée la règle indiquant que quand on multiplie (divise) à la fois le numérateur et le dénominateur avec un nombre entier la fraction ne change pas de valeur. Cette règle prépare pour la réduction des fractions au même dénominateur. Ensuite, l'addition, la soustraction et la comparaison des fractions avec des dénominateurs égaux ou différents sont abordées. Notons ces trois opérations, ainsi que la réduction des fractions au même dénominateur, sont soulignées dans le programme mathématique en quatrième année au Vietnam. Enfin, la multiplication et la division d'une fraction avec un nombre entier et ensuite avec une fraction sont enseignées.

En cinquième année, l'enseignement pour le domaine des nombres commence avec les fractions décimales qui ont 10, 100 et 1000 comme les dénominateurs. Notons que la fraction n'est pas indiquée comme un nombre en quatrième année ainsi qu'à cette étape. Cependant, nous remarquons que ces fractions décimales sont présentées sur une ligne de nombres. Cela peut être interprété comme une préparation pour la découverte du nombre décimal comme un nombre ensuite. Les fractions « impropres » sont aussi abordées avant l'introduction des décimaux.

L'activité d'encodage des mesures au travers des fractions décimales est utilisé pour introduire les décimaux, par exemple 1dm =1/10 m peut s'écrire 0,1 m ; 1 cm =1/100 m peut s'écrire 0,01m, etc. Les fractions décimales, 1/10 ; 1/100 ; 1/1000 s'écrivent comme 0,1 ; 0,01 et 0,001. Les autres fractions décimales, par exemple 5/10, sont généralement écrites par 0,5. Le sens de ces nombres est découvert à travers les activités de dire et de lire des nombres et de représenter la diversité des mesures par les fractions « impropres » par les décimaux (par exemple, 2m7dm = 2 7/10 m = 2,7 m). Les positions de chaque chiffre d'un nombre décimal et sa décomposition sont décrites.

Suite à l'introduction des décimaux, nous remarquons que la règle « la valeur d'un nombre décimal ne change pas quand on ajoute ou élimine le chiffre zéro du côté droit » est enseignée. Ensuite, la comparaison des décimaux est abordée. Les exercices d'écriture des mesures et des grandeurs sous la forme de décimaux sont nombreux. Comme la partie concernant les fractions, l'accent est aussi mis sur les opérations comme l'addition, la soustraction, la multiplication et la division d'un décimal avec un nombre entier puis avec un nombre décimal. Nous observons dans le manuel scolaire les notifications « opérations comme pour les nombres entiers » pour les quatre règles concernant les opérations des décimaux.

3. L'expression verbale du nombre rationnel

3.1 Le langage vietnamien du décimal

Premièrement, nous parlerons de l'expression verbale utilisée au Vietnam. Avant 1998, nous disions « huit unités cinq dix-sept centièmes » pour le nombre décimal « 8,57 ». Nous nommons cette façon de dénommer le décimal par le recours à l'expression « dixième centième millième ». Mais depuis 1998, la dénomination du nombre décimal s'effectue comme son symbole, c'est-à-dire que nous utilisons l'expression « nombre à virgule » pour dénommer le décimal. Prenons un exemple. Pour « 8,57 », nous lirons « huit virgule cinq dix-sept ».

Nous constatons que l'expression « nombre à virgule » illustre la correspondance entre le nombre verbal et le nombre arabe. Il est donc évident que cette expression facilite la tâche de transcodage entre le nombre verbal et le nombre arabe. En effet, les enfants n'ont pas besoin de la compréhension du nombre décimal, ils peuvent quand même réaliser les tâches de transcodage du type ci-dessus. Dans le cadre de notre étude, nous avons eu l'occasion d'effectuer une étude basée sur la tache de transcodage du nombre verbal au nombre en chiffres réalisée sur plus de 2000 élèves de la troisième à la cinquième au Vietnam. Nous demandions aux élèves d'écrire le nombre avec la virgule qui est lu, par exemple « trois virgule cinq ». Nous avons obtenu des scores très élevés aussi chez les élèves de troisième et de quatrième qui n'ont pas encore appris le décimal. En effet, les scores en pourcentage des élèves de la troisième à la cinquième étaient respectivement de 86, 94 et 95. Pourtant, cette expression masque la caractéristique du décimal comme sa signification, le sens de la position et la valeur des chiffres constituant le nombre. De plus, il facilite aussi pour les élèves de faire germer en eux l'idée que le décimal est un nombre naturel plus une virgule. Il est peut-être la source des obstacles de passage du nombre naturel au nombre décimal.

Deuxièmement, nous remarquons que pour le langage vietnamien, les lexicales qui concernent les nombres rationnels contiennent un sens. En effet, le mot indique « dixième » est significatif de « part de dix ». C'est analogue pour les lexicales centième et millième. Nous constatons que les significations de lexicales de nombres rationnels Vietnamiens sont efficacités en conceptualisant de ces nombres. En effet, rappelons que dans l'histoire de la naissance des nombres rationnels, la construction des décimaux était basée sur les nombres fractions. Dans ce cas, l'expression « part de dix » montrera tout d'abord l'action de fractionner et de prendre les parts obtenues et mettra ensuite un sens sur le côté cardinal du nombre. Elle est donc un bon soutien pour conceptualiser les fractions et ensuite les décimaux. Nous pensons que cet effet du langage Vietnamien peut faciliter les élèves, surtout les élèves qui n'ont pas encore appris le décimal, dans le processus de généralisation de conception du décimal.

3.2 L'expression verbale du décimal en Belgique

En étudiant le programme et les manuels scolaires belges, nous constatons qu'en Belgique, les élèves apprennent en même temps deux expressions pour dénommer les décimaux. Il s'agit de « nombre à virgule » et de « dixième centième millième ». A l'école, les élèves apprennent officiellement à lire et à prononcer les décimaux avec l'expression « dixième centième millième ». Dans le cadre de notre étude, un entretien a été réalisé. Nous avons demandé aux élèves de la troisième jusqu'à la sixième de lire à haute voix les nombres décimaux en chiffre qui été écrits sur un papier. Nous leur avons aussi demandé de les lire une deuxième fois, d'une autre façon si possible. Dans un entretien avec 24 élèves de troisième, 96% d'entre eux ont lu les décimaux comme « nombres à virgule ». Par contre, parmi 42 élèves de la quatrième jusqu'à la sixième, nous avons eu 76% d'élèves qui ont lu les décimaux une première fois avec l'expression « dixième centième millième ». De plus, 24% d'entre eux ont lu les décimaux de deux façons qui ne sont pas « nombre à virgule ». Par exemple, 4,32 a été lu par « quatre unités trente-deux centièmes » et « quatre unités trois dixièmes deux centièmes ». Notons aussi qu'en troisième, les élèves n'apprennent pas encore le décimal donc il est clair qu'ils ont tendance à lire ces nombres sous forme de symbole, que nous avons appelé « nombre à virgule ». Les résultats de notre entretien nous conduisent à penser qu'en Belgique, les élèves utilisent en même temps deux expressions verbales pour indiquer le décimal. En réalité, dès que les élèves sont plus mûrs avec les connaissances sur le décimal, comme les élèves à partir du secondaire, ou bien dans quelques contextes particuliers, par exemple les opérations sur les décimaux, ils utilisent l'expression « nombre à virgule » pour indiquer le décimal.

D'un côté, nous avons l'impression que l'expression « dixième centième millième » du nombre verbal du décimal semble compliquée et prolixe en comparaison à celle de « nombre à virgule » qui représente immédiatement le symbole du décimal. En effet, à propos de l'expression « dixième centième millième », elle conduit aussi à certains obstacles dans les réponses des taches de transcodage du nombre verbal au nombre arabe, surtout chez les jeunes élèves. Au Vietnam, nous avons également demandé aux élèves de la troisième à la sixième d'écrire en chiffres des nombres décimaux écrits avec l'expression « dixième centième millième ». Par exemple, l'écriture de « cinq unités trois dixièmes » en chiffres. Le nombre de bonnes réponses était assez faible chez les élèves de troisième et de quatrième. Leurs scores en pourcentages étaient respectivement de 36% et de 51%. Il est compréhensible que pour que les élèves puissent réaliser ces tâches de transcodage, ils aient besoin d'un niveau de compréhension suffisant de la notion et des règles d'écriture et de combinaison des chiffres formant un nombre. Par contre, pour les élèves de cinquième et de sixième Belges, même avec cette expression, les compétences de transcodage étaient bien acquises.

D'un autre côté, cette expression « dixième centième millième » illustre bien le sens du nombre décimal et expose clairement la position et la valeur des chiffres formant le nombre. Cela constitue un avantage et elle est plus efficace pour les enfants qui commencent à apprendre la notion de décimale. De plus, elle renforce la compréhension

des élèves qui ont appris le décimal, prévient les idées fausses de confusion avec les naturels et évite les obstacles épistémologiques.

Au niveau des lexicaux, remarquons aussi que dans le langage Français, les expressions du nombre décimal comme dixième, centième et millième sont susceptibles de conduire certaines confusions respectivement avec dizaine, centaine et milliers pour le nombre naturel. Cette caractéristique peut aussi avoir comme impact chez les jeunes élèves d'identifier les décimaux comme des naturels.

En résumé

En ce qui concerne le système belge, il est difficile de saisir exactement l'ordre dans lequel sont enseignés les nombres rationnels. Il semble se composer d'un mélange de deux contenus « fraction » et « décimal » au travers des cours sur les rationnels en Belgique. En quatrième, les élèves belges apprennent les décimaux jusqu'au centième. Ensuite, en cinquième et en sixième, les élèves apprennent les décimaux jusqu'au millième.

En étudiant les socles de compétences, le programme et une collection de manuels scolaires, nous ne connaissons pas explicitement les démarches didactiques pour l'enseignement des nombres rationnels de la Communauté française en Belgique. Il semble que les fractions sont abordées par l'activité de fractionnement d'unité et les décimaux sont introduits à travers l'encodage des mesures. Le sens de la fraction n'est pas souligné du côté de la conception d'un nombre. Le lien entre les fractions et les décimaux n'est pas souligné avec insistance dans le système

Au travers de l'analyse du programme, du manuel scolaire unique et des documents fournis pour les enseignants, nous pouvons décrire l'enseignement des nombres rationnels au Vietnam par les quelques remarques suivantes. Premièrement, les fractions sont introduites officiellement en quatrième année et elles sont abordées à la fois avec l'activité de fractionnement d'unité et avec l'approche fraction-quotient. Les fractions ne sont pas considérées comme des nombres au niveau du primaire. La découverte des décimaux se fait pendant la cinquième année en rapport à l'encodage des mesures. Le programme mathématique vietnamien met l'accent sur le rôle de la fraction décimale qui joue un rôle intermédiaire comme préacquis pour aborder le décimal. L'addition, la soustraction, la multiplication et la division des fractions et des décimaux sont importantes dans l'enseignement des rationnels au Vietnam.

Chapter 3

Rational number construct and whole number bias

Rational number construct and whole number bias

The domain of rational numbers was investigated by Kieren (1976, 1980, 1988) and Behr et al. (1983). Kieren (1980) proposed that the concept of fractions was multifaceted. It comprised five interrelated sub-constructs: part-whole relations, ratios, quotients, measures and operations. These five sub-constructs of rational numbers received a large consensus of opinion from many researches (Behr, Lesh, Post & Silver, 1983; Freudenthal, 1983; Vergnaud, 1983; Ohlsson, 1988). Despite different interpretations among five sub-constructs, they shared some common elements relating to the identification of the unit and partitioning and the notion of quantity (Carpenter, Fenema and Romberd, 1993). Each sub-construct relates to different cognitive structures and presents the rational number from different perspectives, altogether they constitute a full meaning of the rational number.

Since the mind functions work like a system, the prior and later acquired knowledge or skills have complex interactions. In manipulation with fraction and decimal numbers, children usually tend to use their knowledge of whole numbers. As the properties of the natural numbers are not always valid for rationals, children who use natural numbers rules to manipulate rational numbers will lead to mistakes or build incorrect conceptions. Ni and Zhou (2005) named this bias the natural number bias and it may express an immature understanding of the concept involved. For example, previous studies showed that in a comparison of decimal numbers, children used the rule "the longest number is the largest one" (Nesher & Peled, 1986; Resnick et al., 1989 ; Steinle & Stacey, 2003 and Desmet et al, 2010). It is difficult for children to conceive that the result of the multiplication of two fractions can be a smaller fraction than original ones. Similarly, the division between two fractions may result in a larger fraction (English & Halford, 1995). Children also treat the numerator and denominator independently (e.g., “1/2 < 1/4 as 2 < 4”; “1/2 is the smallest fraction I can imagine because it is composed of small numbers") (Stafylidou & Vosniadou, 2004). When children have to produce fractions symbols from graphical representations, they only perceive the number of parts indicating the numerator or the denominator. For instance, with a circle divided into four parts, three of them grayed out and the fourth one being white, children’s responses are simply "1" or "3" instead of “¾” (Saxe et al., 2005).

To better understand the complexity of the rational number, the acquisition process of the number concept and the difficulties related to the transition from natural numbers to rational numbers, the rational number construct and the origin of the natural number bias will also be presented in this chapter.

1. Rational number construct

Among these five sub-constructs of the rational number (part-whole relations, ratios, quotients, measures and operations), the part-whole sub-construct was considered of the most importance and the foundation of rational numbers knowledge (Kieren, 1988; Freudenthal, 1983). Berh et al. (1983) proposed part-whole sub-construct as fundamental to acquire the other four and suggested a theoretical model linking different interpretations to fractions competencies, as illustrated in Figure 1. They also emphasized the process of partitioning as a point from which the part-whole sub-construct develops. Charalambous and Pitta-Pantazi (2007) conducted an empirical study examining the causal path from five subcontructs of fractions to fraction operations and fraction equivalence, which based on theoretical model of Berh and his colleagues. Data supported the associations included in this model and also proposed some additional links which are described in Figure 2.

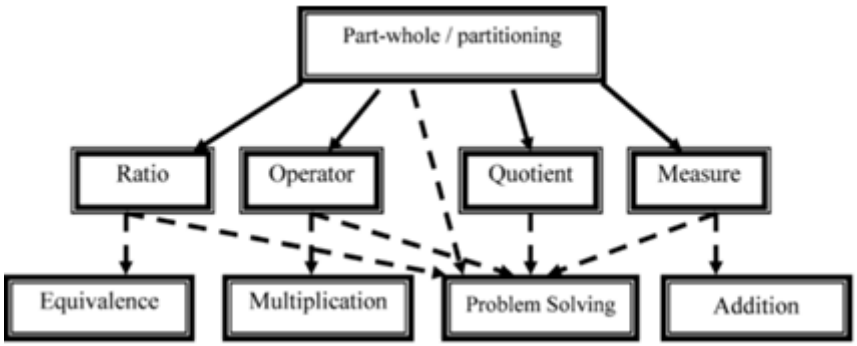


Figure 1. Theoretical model linking the five sub-constructs of fractions to different operations of fractions and to problem solving (Behr et al., 1983)

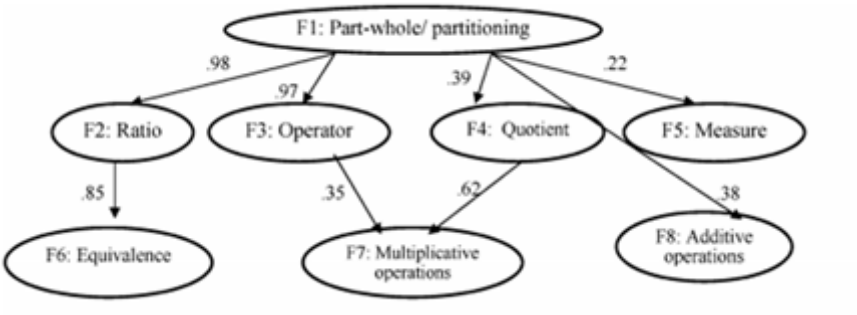


Figure 2. Structural model linking the five sub-constructs of fractions to the operations of fractions and fraction equivalence (Charalambous & Pitta-Pantazi, 2007)

1.1 Process of partitioning

Partitioning indicates the act of dividing in a set S to create a collection of non-empty subsets which satisfy this collection equal to set S. This action can be occurring in two contexts: discrete or continuous. Partitioning schemas is important to establish rational number knowledge (Vergnaud, 1983; Kieren, 1988). It is considered as the key to build initial fraction concepts and to develop a broad understanding of rational numbers (Ball 1993; Mack 1990, 1993; Kieren & al. 1992; Streefland 1991, 1993). In fact, the ability of partitioning allows the awareness of the equality between the sum of parts and the total or the relation between the size of a part and the number of parts ("if the number of parts is bigger, the size of each part is smaller"). The result of the action of partitioning also leads to a new quantity that creates unit-fraction concept.

Equal sharing, a partitioning scheme is presented to most children through classroom daily activities, before the formal elaboration of fractions in school (Davis & Pitkethly, 1990). In preschool, children had occasions to halve, which is considered as an early development of the action of partitioning (Hunting & Korbosky, 1990). The repeated folding of a unit and the inversed relation between the size of a part and the number of folds express the conception of partitioning (Kieren, Mason & Pirie, 1992). Five years old children showed an understanding of partitioning into equal parts, inversed relation between a number of people and the size of parts (in which each people receive) and the ability to add and subtract fractional parts of an area (Pitkethly & Hunting, 1996; Sophian, Garyantes & Chang, 1997; Mix, Levine, & Huttenlocher, 1999).

The action of partitioning of an area into fractional parts is a privilege approach in teaching fraction (Corwin, Russell & Tierney, 1991; Saxe et al., 2005). Brissiaud (1998) distinguished two different types of partitioning scheme in the approach of the fraction concept. For example, the number 13 refers to a measure (e.g., 13 mm) and the number 4 refers to a quantity. The fraction 13/4 (e.g.,13mm/4) is read "13 divided by 4" and refers to the share of the total of 13 mm in 4 equal parts. This is a "partitioning-plurality" type. The other type is labeled "unit- partitioning". In this case, the number 13 refers a quantity and we work with "quarters" (e.g., 13 times a quarter (1/4 mm). The fraction 13/4 is then read "13 quarters" and is "a partition of unit followed by a multiplication". This type of partitioning is common in manipulations with graphic representations in early fraction teaching, such like an action of split, "take 3 times a quarter of a unit". These two types of partitioning provide various opportunities for the acquisition of the fraction concept. The "Unit partitioning" derived the unit-fraction concept and the "partition plurality" related to sub-construct ratio and quotient of fraction concept.

1.2 The part-whole sub-construct

Part-whole relation related to the field of multiplicative structure. Considering a "whole" E and a part of its parts "F": E and F are commensurable, it means that there exists a common "unit" S (non-empty set of E and F) in which E and F can be presented E = a.S and F = b.S (a and b are whole number and differ from zero).

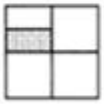
The relation part-whole can be always presented under the form of multiplication: w = n.p in which n is natural number, w measures "whole" and p measures "part". For example, from a problem in a set of natural numbers, how many pizzas are needed to feed 24 children, knowing that a group of 4 children eats one pizza? (Saenz-Ludlow, 1995), we have multiplicative relation between the 'whole"(24) and "part"(4). In the case of rational numbers (Adjiage, 1999), we have the equation 3 = 4. 3/4 in which 3 measures "whole" and ¾ measures "part". Dividing a cake into 5 parts can be expressed by 1 = 5. 1/5, which presents the part-whole relation in the case of a continuous quantity.

As described above, the part-whole sub-construct was considered as a source of rational number (Behr et al, 1983; Kieren, 1976; 1988; Freudenthal, 1983). In a review of rational number, Pitkethly & Hunting(1996) stated that "the learning actions which are fundamental to developing rational number meaning are the equi-division of a unit into parts; the recursive division of a part into subparts, and the reconstruction of a unit. A flexible concept of the unit is important to all later rational number interpretations". In fact, the part-whole relation allows children the opportunity of changing unit. It leads to the awareness of existing plurality of "unit" which is totally different from previous knowledge of natural number.

Regional area, set and number line representations are three types of graphic representations, which are commonly used for teaching and assessing children's conceptual knowledge about rational numbers (Ni, 2001). While regional area and set representations are employed to examine the part-whole relationship, the line number focuses more on the aspects of "rational as a number" and measures the meaning of a fraction (Bright, Behr, Post, & Wachsmuth, 1988; English & Halford, 1995; Behr et al., 1983). In the construction of fraction number, the part-whole relationship was usually associated with the shaded area in which the number of grey part indicates the numerator and the number of part in a whole (unit) indicates the denominator.

To master the part-whole sub-construct and partitioning scheme, the understanding that parts into which the whole is partitioned must be in equal size is required (Baturo, 2004; Boulet,1998, Lamon,1999). Saxe and his colleagues (2005) investigated the relationship between students' uses of fraction notation and their understanding of part-whole relation. Participants comprised 384 children at grade 4, 5 and 6. They were asked to write a notation of fraction to show the grey parts from a shaded area representation. Two types of problem were proposed: Equal area problem (shapes partitioned into equal-sized parts) and Unequal area problem (parts of unequal sizes). All answers are classified into five codes, which are explained in Table 1. The reference code shows the conceptual idea behind the choice of a specific notation to present a fraction. The choice of notation may base on the part-whole relation, part-part relation, etc., illustrated in Table 1.

Table 1. Reference and notation codes for Equal Area and Unequal Area problems (Saxe et al. 2005)

Example of problem type	Reference code	Definition of reference code	Notation codes and sample representations: Unconventional	Notation codes and sample representations: Conventional
Equal Area 	Part-Whole (P-W)	Notation captures the number of gray parts and number of all parts in whole	3-4; 4-3; 3,4; 3 4; 6,8	3/4; 6/8
	Part-Part (P-P)	Notation captures number of gray parts and number of white parts	3-1; 1-3; 3 1; 1 3	3/1
	Integer (I)	Notation captures the number of gray parts	3; 1	—
	Don't Know (dk)	"Don't Know" written as a response	—	—
	Other	None of above	7	7/9
Unequal Area 	Part-Whole: Continuous (P-W:C)	Notation captures unequal size relations in the representation of part whole relation	1-8; 8-1; 8 1; 8,1 16,2; 2-16	1/8; 2/16
	Part-Whole: Discrete (P-W:D)	Notation captures the number of gray parts and number of all parts in whole (without accommodating different sizes of parts)	1-5; 5-1; 5 1; 1,5	1/5
	Part-Part (P-P)	Notation captures number of gray parts and number of white parts	1-4; 4 1; 4-1	1/4
	Integer (I)	Notation captures the number of gray parts	1; 4	—
	Don't Know (dk)	"Don't Know" written as a response	—	—
	Other	None of above	7	7/9

For equal area–problems data document that the part-whole relation is dominant reference to present fraction number (69%). Other references of Not-part-whole such as part-part relation (not emphasizing to the relation between numerator and denominator) and Integer (fraction is identified as a natural number) comprised 31%.

From table 2, we observed that only 9% of children used relation part-whole of continuous quantity to interpret both Equal and Unequal-area problems (Part-Whole on Equal Area and Part-Whole: Continuous on Unequal Area problems). It means that they are aware of the equal-size of parts. A total of 60% students treated Equal Areas as Part-Whole,

but Unequal Areas as incorrect part-whole relation (second line of Part-Whole : Discrete and Not part-whole). It suggested that "their correct responses to Equal Areas were based on whole number counting of shaded parts and total number of parts"(Saxe et al., 2005). They interpreted the factions by comparing the number of shaded parts to the total number of parts but they did not realize the unequal-size of parts. It showed an immature of understanding part-whole.

Table 2. Percentage distributions of students' modal reference categories collapsed cross Equal and Unequal Area Problems (Saxe et al., 2005)

Equal Area problems	Unequal Area problems		
	Not Part-Whole	Part-Whole: Discrete	Continuous Part-Whole
Not Part-Whole	30	1	0
Part-Whole	35	25	9

Reconstructing the whole from its parts and repartition already equipartitioned (e.g., constructing 3/8 from a whole partitioned) being competencies need to understand part-whole sub-construct (Baturo, 2004). When children are asked to identify symbol fraction (e.g., 2/3) from graphic representation, they select a representation depicting 2/5 by counting two times some parts. Thereby, Charalambous et al., (2006) proposed that the idea of inclusion involving the parts which are considered as components of the whole is important to develop the part-whole relationship.

1.3 The ratio sub-construct

In mathematics, a ratio is a relationship between two numbers of the same kind, usually expressed as "a to b" or a:b. Ratio is considered as a dimensionless quotient, indicating a comparative index. Ratio "a to b" represents how many times the first number (a) contains the second (b), the fraction a/(a+b) is usually derived to indicate the quantity of a containing in the total (a+b). For example, there are 4 oranges and 3 lemons in a mix of juice; the ratio of lemons to oranges is 3:4. The ratio of lemons to the total amount of fruits is 3:7 which is considered as a fraction of 3/7 to present how much of the juice lemon is.

To understand the ratio sub-construct of a fraction, children need to be aware of the relative amounts (Lamon, 1993; Marshall, 1993). When two quantities of a ratio change together in some linear way , the relationship between them keeps invariant. For example, if the ratio of A to B is A:B then the value of the ratio remains the same for two quantities 2A to 2B. This property is essentially important to acquire the fraction concept, which includes the equivalent fraction concept (Marshall, 1993).

In primary school, the ratio is widely used in problem solving in which the ratio of two quantities and value of one quantity are known, children were asked to find the remaining quantity.

1.4 The operator sub-construct

Fractions as operators indicate the fractions as relational functions applied to a shape, set or number (Behr et al., 1993; Marshall, 1993). The results of these operators are some forms of the derivative of shape, set or number. For instance, finding a fraction of a quantity is regarded as applying a function to that quantity. To master operator interpretation of fractions, children need to understand the structure of the function underlying of a given fraction (e.g., a 3/4 operator results in transforming an input quantity of 4 into 3) (Behr et al., 1993). In addition, they should be able to identify different ways to apply this operation (Lamon, 1999; Marshall, 1993). For example, 2/5 should be interpreted either as 2 times of [1/5 of a unit] or 1/5 times of [2 units] and then finding 2/5 of a number can be the function where the operation is "multiplied by 2 and then divided by 5" or "divided by 5 and then multiplied by 2". According to Behr et al. (1993), the understanding of fractions as operators provides the advantage and foundation to acquire the multiplication of fractions. In fact, by finding a fraction of a whole number, children who have mastered this operator sub-construct can construct and decompose this act as an application of the numerator of the fraction to the given number, and then the denominator quantity applied to this result. For a multiplication of two fractions (e.g., 2/5 of ¼), in other words finding 2/5 of ¼, prior knowledge of operator sub-construct can help children interpret this multiplication as ¼ multiple by 2 and then the result obtained is divided by 5.

1.5 The quotient sub-construct

Quotient is a content sub-construct of fraction. Any fraction can be considered as the result of a division situation (Kieren, 1993). The fraction a/b can be seen as a form of a being divided by b where a and b are whole numbers and b differs from zero.

Regarding the scheme of action division, besides the scheme of partitioning in which there is a single whole (or measure), there is also the scheme of correspondence division where the dividend and divisor represented two different measures (e.g., two cookies are shared among five children). In this case, the relation size of the dividend and the divisor has no constraint. The divisor can be smaller, equal to or bigger than the dividend. The correspondence division is used for quotient sub-construct. Therefore, there is no constraint regarding the size of the fraction, which is derived from this division. The fraction can be smaller, equal to or bigger than the unit. Unlike the part-whole sub-construct, in the quotient personality of fraction, the numerator can be smaller than, equal to or bigger than the denominator.

Two types of division problems: partitive and quotitive were distinguished in mathematics education literature. The first type of division is partitive division, which is also known as “equal sharing division” and “dealing out division”. Fischbein, Deri, Nello,

and Marino (1985) defined partitive division as a model for situations in which "an object or collection of objects is divided into a number of equal fragments or sub collections. The dividend must be larger than the divisor; the divisor (operator) must be a whole number; the quotient must be smaller than the dividend (operand)… In quotitive division or measurement division, one seeks to determine how many times a given quantity is contained in a larger quantity. In this case, the only constraint is that the dividend must be larger than the divisor. If the quotient is a whole number, the model can be seen as a repeated subtraction" (Fischbein, Deri, Nello, & Marino, 1985, p. 7). The difference between two types of problems is that the partitive division is interested in "the size of parts" (e.g., "if four children equally share eight cookies, how much cookies each of them gets?"), whereas the quotitive division emphasizes the "number of parts" of a given size (e.g., "eight cookies are shared among some children. If each child gets 2 cookies, how many children there are totally?"). To acquire the quotient sub-construct, students are required the understanding of both two types of division, partitive and quotitive (Kieren, 1993; Marshall, 1993). Additionally, the ability to identify the fraction from a division and the understanding of the role of dividend and divisor in this operation are necessary to master the quotient sub-construct (Lamon, 1999).

The scheme of correspondence using in quotient personality of fraction can provide different strategies for sharing activities. For example, if we have to share three chocolate bars between two children. It is not necessary to divide each chocolate bar in two parts (division action in only one measure spaces) and then distributes the halves. Each child could get a whole chocolate plus a half with the correspondence in division situations with two fields of measures. The division in two measure spaces may be also an advantage for students to understand the improper fractions.

1.6 The measure sub-construct

Measure is a content of the sub-construct of fraction (Freudenthal, 1983; Kieren, 1988; Lamon, 2001). This sub-construct is particularly related to the notion of determining a distance with a unit-fraction (Lamon, 2001; Marshall, 1993). For example, 2/3 corresponds to the distance of 2 times (1/3 units) from a given point. This sub-construct is often associated with the use of number lines and the additive operation (1/3 + 1/3).

In the measure of sub-construct, the fraction must be understood as a number representing a numerical value and representing the quantitative aspect of the fraction. In part-whole subcontruct, the fraction is not yet considered as a number. The re-conceptualization of the fractions, which are not part of the counting sequence as a number, presents an evolution in children. The measure of the sub-construct is more difficult than the part-whole sub-construct (Gay & Aichele, 1997; Ni, 2000).

To master the measure of the sub-construct, children should be capable of understanding partition and comprehending the property of the density of rational numbers (Lamon, 1999). The manipulation of number line is often used in this sub-construct. The ability to use a given unit interval to measure any distance from the origin and the

understanding of the order and equivalence of fractions are considered as necessary skills to fully develop the measure personality of fraction (Hannula, 2003 ;Smith, 2002).

In terms of the number lines in measuring sub-construct, children usually get confused with unit (Brown, Carpenter, Kouba, Lindquist, Silver, & Swafford, 1988). They have tendency to count the number of parts derived from partitioning rather than the intervals of the number line. A number line with the length differing from one unit (e.g., a number line is divided into parts equal to a multiple of the denominator of a given fraction) produces more difficulties for children (Ni, 2000; Baturo, 2004).

2. Whole number bias

In manipulation with fraction and decimal numbers, children usually tend to use their knowledge of whole numbers. For example, they confound the number of pieces in a partition with the size of each piece (1/5 is bigger than 1/3 because 5 is bigger than 3) or they consider that numbers between 0 and 1 do not exist. This is considered to be the effect of the whole number bias. According to Weil-Barais and Vergnaud, (1990), these biases may express an immature understanding of the concept involved.

The whole number bias could be defined as a strong inclination to use the single-unit counting scheme when manipulating fractions. Consequently, this bias leads to difficulties in the acquisition of the fraction numbers. In this section, we will present the origin the whole number bias to help us to better understand the process of number concept acquisition and to deduce instructional implications for rational numbers.

According to the nature review of the whole number bias by Ni and Zhou (2005), there are three accounts formulating this bias. The first account, an innate constraint, supposes that the bias is created from an innate mechanism for number concepts, which advantages discrete quantity more than continuous quantity. The second one relates to the undifferentiated amount hypothesis for early quantitative representation. It assumes the whole number bias is the result of a process of development in distinguishing two types of quantities, discrete and continuous. The last one, the learning account, states that a failure of acquisition for rational numbers can be due to the interaction between the instruction on fractions and children's prior knowledge of the whole number. These accounts will be detailed as following.

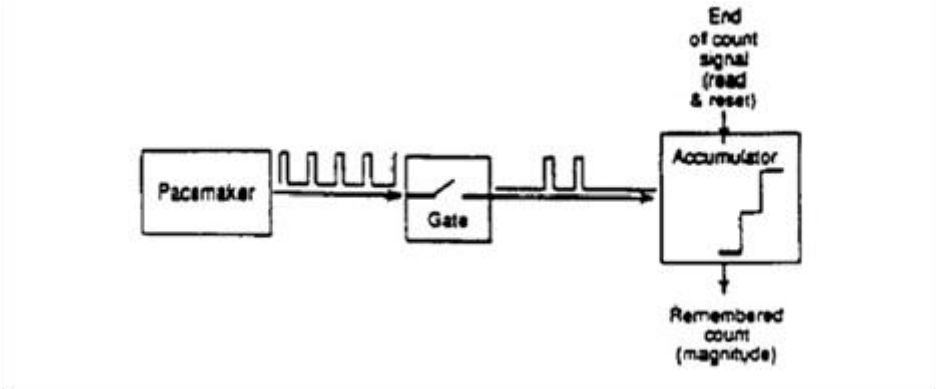


Figure 3. The model for the animal counting mechanism proposed and expremental tested by Meck and Church ; Meck, Church and Gibbon (1985)

2.1 The innate constraint

In the early number development, Gelman and his colleagues (Gallistel & Gelman, 1992; Gelman & Gallistel, 1978) proposed that there is an innate cognitive mechanism that provokes mental magnitudes to represent numerosities. Meck and Church (1983) proposed a model for the animal counting mechanism, which is also called the accumulator model, illustrated in Figure 3. In this model, for each stimulus a burst of impulses will record specific magnitude in a hypothetical accumulator. The total quantity is represented by the resulting fullness of the accumulator. Obviously, it is assumed that the accumulator model also works for infants. According to the accumulator model, it seems that the mechanism for representing numerosities works in a discrete manner. It can be explained by the fact that the gate is opened for the impulse into the accumulator only one at a time and one burst for each item in a set of discrete items. It is reasonable indicating that the children's initial representation of numerosity is discrete mental magnitude (Ni & Zhou, 2005). Noticed that in theory, fractional numerosities can be represented from the numerical magnitude's representation but no recognition of fraction in this innate device is remarked. The innate mechanism for mental magnitude shows advantages for discrete quantity. According to Gallistel and Gelman (1992), the accumulator model is evaluated matching to the preverbal counting mechanism. The implicit counting principles, such as the principle of one-to-one correspondence, stable order of a counting list and the abstraction are assumed to be derived from the preverbal representation of numerical magnitude. From magnitude in preverbal form, through counting, children can obtain number words. Thus, the innate mechanism is considered to assist children in acquiring verbal count words. Noticed that in verbal counting, there is no place for fractional verbal number. For two reasons that the nature of innate mental magnitude is discrete and that the innate mechanism supports the verbal counting process, we consider that the first feature of the whole number bias is innately determined (Ni & Zhou, 2005). It leads us to the first account, the innate constraint.

Studies illustrating that early skill with discrete quantities, such as infant's ability to discriminate small sets of objects and early developing of counting skills, are presented as follows to reinforce the existence of the innate constraint. Many previous studies (Antell & Keating, 1983; Starkey & Cooper, 1980; Strauss & Curtis,1981; Koechlin et al., 1997; Wynn, 1992a) stated that infants of a few months old showed the capacity of discriminating small sets of objects. These studies are designed by relying on the principle that when children have interest in stimuli, they tend to look at them with more attention, meaning in longer time. For example, in the study conducted by Starkey and Cooper (1980), after exposing repeatedly a same set of items to children, they were presented a novel set of items. Children spent more time looking at the new set of objects. In another study (Wynn, 1992a), children were firstly introduced to a small set of objects and then, one or two objects were either added or removed (turning the previous stimuli into new ones) which is assumed as a novel stimuli. Longer looking time for the novel stimuli is considered as an indicator that suggests the capacity of discriminating the numerical differences among small sets of objects. Empirical studies (Gelman &Meck, 1983, 1986, Briars & Siegler, 1984; Fuson, 1988) showed that 4-year-old children could achieve counting a small number of sets of objects in a correct procedure on the condition that these objects were arranged in a straight line and at the same distance from each other. In counting a set of objects putting randomly, they usually do not succeed. At the age of five, children can accomplish counting a single set of objects (Nunes &Byrant, 1996). The assumption that innately-determined magnitude representation links to verbal counting skills also needs to be discussed further. In fact, according to Carey (2001), there are two aspects of analog magnitude representation that gives rise of some discussion about if the children's verbal counting skill is derived from analog magnitude representation. Firstly, the construction of the analog magnitude representation is not based on iterative process. Second, there is a superior limit for the analog magnitude representation of number. It is caused by the discriminability of individuals in a set and the capacity of the accumulator. Whereas the positive integer counting is an iterative process and considered having no upper limit. The analog magnitude representation is assumed to be one of the first types of representation in human's numerical cognition but it does not enable us to map the preverbal counting mechanism onto the verbal counting system. Carey (2001) proposed a developmental discontinuity that plays a mediate role for the passage from the preverbal counting mechanism to the verbal counting system.

In sum, the innate constraint account indicates the existence of number-specific endowment in which children base on to perceive the numerical concept and acquire understanding of the whole number. Recall that this innate account privileges the discrete quantity and connects with the counting skills, which are suited to the whole number domain. As a result, it occasions the learning of concept consistent with early numerical representation, such as addition and subtraction, verbal counting, etc. and it simultaneously provokes constraints for acquiring the concepts of fraction. Recall that the insufficient conceptual reference is considered as the source of children's difficulty with fraction and rational number in this account. Therefore, it is impossible to eliminate the whole number

bias and this innate bias will persist to interfere with children's learning and acquiring of number concepts.

2.2 The Undifferentiated Amount Hypothesis

Mix and colleagues (2002a, 2002b) proposed the undifferentiated amount hypothesis that assumes that early representation of quantities is based on the total-amount. In contrast to previous claims indicating that early quantification is assumed to be number-based and privileges the discrete quantity. The undifferentiated amount hypothesis suggests that infants' quantification is initially based on non-numerical cues, such as area and contour length, volume, or brightness and that infant determine discriminations through total amount terms in an undifferentiated way for two types of quantities, discrete and continuous.

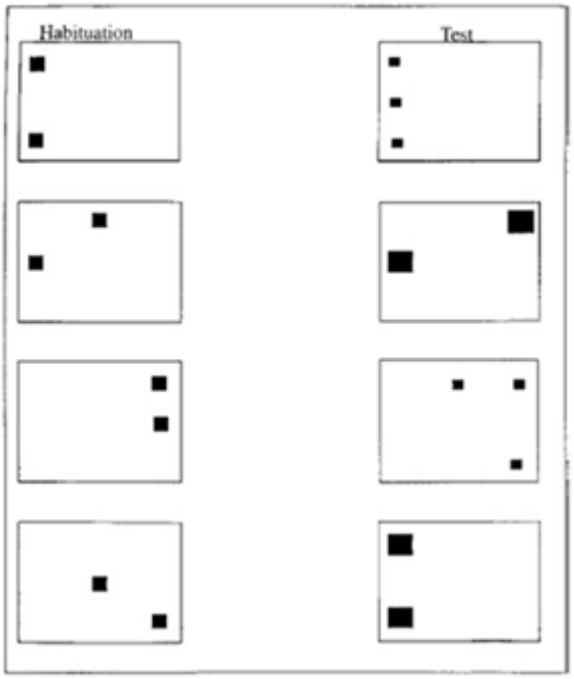


Figure 4. Sample displays used to contrast contour length and number in Clearfield and Mix's (1999) infant habituation study

Evidence that supports the undifferentiated amount hypothesis is derived from Clearfield and Mix's study (1999). Figure 4 presents a sample displayed in the experiment. There are two types of new displays presented in the experiment. The first one had a change in number of squares but the same amount of contour. And in the second new display, the number of squares was held constant but the amount of contour was changed. Results

showed that 6 to 8 month-old infants dishabituated to the novel display with the change of contour length but not to that with the change of number. The similar finding was suggested by Feigenson, Carey, and Spelke (2002) in a replicated and extended study. Results suggested that 6- to 7-month-old infants discriminated to the change of number of objects when number and total front surface area were confounded. However, infants just responded to a variation in front surface area but not to a change in number as two variables were presented separately. In addition, no sensibility to a change of number was observed when front surface area was controlled. The lack of response to a change of number and the sensibility to the dimensions of continuous extent support the assumption that a total – amount based representation, rather than those promoting numerical-specific underlying infants' performance to discriminate small sets of objects. In a review of investigations into the infant's early quantitative representation, Mix et al. (2002a) also confirmed the potential hypothesis that the capacity of discriminating of small sets based on non-numerical information- continuous spatial information.

If the assumption of total-amount is considered right, infants may discriminate continuous amounts. In fact, infants of 6 month old showed the capacity to discriminate between a three-quarter cup full of liquid and a one-quarter in the study conducted by Gao, Levine, and Huttenlocher (2000). It reinforces the undifferentiated way of discriminating two types of quantities, discrete and continuous, in infants' representation of quantities. In addition, results from the study conducted by Mix and her colleagues (Mix et al., 1999) revealed that 4 year old children showed the ability to calculate small fraction amounts. In this study, the child was firstly presented for a few seconds a fractional amount (e.g., a quarter of circular sponge) and then hidden. Semicircles of sponge were shown being added to, or removed from, the hidden amount. The researchers asked the child to indicate the resultant amount from four pictures. These children showed the ability to calculate small fraction amounts. At 6 to 7 year old, children can calculate more complex mixed-numbers. A high equivalence between the development of whole number and fraction calculation is remarked. These results propose that the trajectory of developments for whole number and those for fraction concepts are not really different. It is not in favor with the proposition established in the innate constraint that early representation of quantity helps the acquisition of the whole number but interferes with learning about fraction.

The notion of relative quantity may be considered more primitive than that of exact quantity in human perception (Ni &Zhou, 2005). Recall that the discrete quantity deals with the exact quantity and the continuous mainly relates to the relative quantity. The study by Correa (in Nunes & Bryant, 1996) investigating the young children's understanding of the inverse relationship showed that children at 5-,6-,and 7- year old showed the perception of the consequences of the size of "n" in a n-split . In an equivalent study, Desli (in Nunes & Bryant, 1996) conducted on children at 6-,7-and 8 year old revealed similar findings for the understanding of inverse relationship in dealing with continuous quantities. These results also suggest the capacity to comprehend relational quantity of fractional number of young children.

Based on the previous results of early ability to both discrete and continuous quantity, it leads to the doubt of the assumption that early numerical representation privileges discrete numbers. How to explain this obstruction? Mix et al. (2000a, 2002b) hypothesize that at the starting point the early numerical representation based on the amount of substance for both discrete and continuous quantities. It is an undifferentiated notion of quantity. In the course of development of numerical competence, the differentiation of the two kinds of quantity is perceived. In fact, the development is considered to occur when children develop one- to-one correspondence and then acquire the principle of the determination of the cardinality in counting and use different conventional measurement tools to quantify. These abilities are mainly in contact with discrete quantity and the derivation of the cardinality from counting procedure that promote children to recognize the difference between two types of quantities. Furthermore, it blurs the children' sensibility of continuous quantity as well as representation of relative quantity. The whole number bias is assumed as results of such course of development.

According to Resnick and Singer (1993), children initially have capacity to manipulate the continuous quantity but their achievement tends to decrease when they master using the counting skill to represent discrete quantities. Their performance of tasks related to the change in the target amount (i.e., the numerator) was better than that involved with the change in the reference (i.e., denominator). Results showed that children used a discrete strategy and took no account of relative quantity aspect. It proposes a feature of the original of whole number bias that children's whole number bias toward fraction numbers is a developmental result of the differentiation between discrete and continuous quantities in children.

The available developmental data mentioned above lead to hesitation in the innate constraint that assumed that early quantitative representation is numerical-specific and privileges the discrete quantity. The undifferentiated amount hypothesis is considered as an explanation for this doubt. Notice that it is methodologically difficult to separate the numerical aspect from spatial dimensions of stimuli in the studies. Therefore, the early quantitative representation in the undifferentiated amount hypothesis is assumed total amount-based, in which non-numerical cues (total spatial dimensions of objects) and numerical cues co-vary to discriminate both small set of discrete of objects and different amount of continuous quantity. Moreover, in the course of developing numerical competence, such as counting skills, children begin to differentiate between discrete quantities and continuous quantities. It may lead to unexpected implications for the subsequent development and learning, such as the fraction. It contributes to the formation of the whole number bias; therefore the whole number bias is not innate endowment but rather a result of the development.

2.3 Learning accounts

The third account of the origin of the whole number bias, so-called learning account, indicates that the way of conceiving a new concept may create negative implications for acquiring this new concept, and also for the subsequence knowledge. In our context,

children often project their prior knowledge about the whole number onto the acquisition of rational number. This manner not always provides the reward but also the restriction for children. Moreover, the instruction in school sometime leads to disadvantages for developing and learning rational number. These factors constitute the learning account, which contributes to generating the whole number bias.

In terms of development, the acquisition of discrete quantification is often occurred before the acquisition of continuous quantification and the whole number is introduced before the acquisition of rational numbers. Notice that quantifying discrete amount is easier than quantifying continuous quantities. It can be explained by two different aspects in quantifying discrete and continuous quantities. Since objects in discrete sets are already separated, we need only to count the units. It is not the case in counting continuous amounts; ones need to subdivide into equal part before enumerating. The second point is that in quantifying discrete objects each individual counts without taking into consideration the size aspect and this does not occur in quantifying continuous quantities. Before learning the conventional counting skills, children are assumed to acquire a symbol one to one representation (Huttenlocher, Jordan, & Levine, 1994). However, the one-to–one principle is appropriate only for discrete sets. Children have nearly figured out what is considered as "number", based on the single-unit counting. It leads to the assumption that there is no number between two consecutive whole numbers. It is just natural for children to use their precedent knowledge in the whole number to interpret the fraction and decimal number. However, this prior knowledge does not always deal with the new concept and establish the whole bias number.

Assimilating the existing knowledge and using "familiar" strategy to acquire a new concept do not really facilitate to conceptualize a new notion but it could provide negative implication to learn subsequent concepts. For example, in the instruction on multiplication, this operation is often defined as repeated addition. This initially facilitates the understanding of the multiplication concept but it constraints children to acquire the crucial idea of multiplication as the relation between two measure dimensions (Ni & Zhou, 2005). Similarly, applying the conventional double-count strategy of fractions in classroom may give opportunity for children using acquired strategy, as counting, to approach the fraction number. It helps children to get acquainted with the language about fraction. However, this treatment is unable to help children discriminate fractions and whole number (Davydov & Tsvetkovich, 1991; Kieren, 1994). In addition, the numerator and denominator of fraction can be considered as two unrelated whole number indicating two separate counts. Consequently, it impedes conceiving fraction as a number presenting a quantitative relation between two quantities. The influence of using the counting schema in learning about fraction may extend to the immature concept of fraction as "A thing out of B thing". This interferes in the subsequent acquisition of complete concept and operations involving rational numbers (Ni, 2001; Thompson & Saldanha, 2003). At this point, the instruction in school is considered to contribute to the whole number bias.

The insufficient time for the development and learning of fraction and decimal in school is also assumed as a factor to reinforce the whole number bias. In many classrooms the

concept of fraction is only presented through the part-whole scheme. The quick rule "multiply or divide the numerator and denominator of a fraction by the same number" for calculating fractions and the rule " add the digit of zero on the right of decimal number until all numbers are the same length" for comparing decimal number are also used in instructing of rational numbers. This leads to the difficulty in associating operations and meanings of the new concepts involving to fraction and decimal numbers and unconsciously, promoting using the skills of the whole number to interpret the rational numbers. Consequently, the knowledge and operations involving to the whole number will be persistent for a long time for children, even they had received formal instruction on fraction and decimal numbers before.

Since the mind functions work like a system, the prior and later acquired knowledge or skills have complex interactions. The whole number bias is such example. The strong interplay between the whole number and rational numbers (fraction and decimal) can be explained by many common underlying concepts. The natural way to using the whole number knowledge, innate or learned, to interpret the fraction and decimal number contributes to the whole number bias. This aspect of the whole number bias seems unavoidable. However, the instruction and curriculum in school on teaching rational number also promote the whole number bias.

The ill-conceived treatments for rational number and the lack of opportunities to confront the conceptual conflict between the existing knowledge of the whole number with later knowledge may lead to the fact that children handle rational numbers as a natural extension of whole numbers.

Therefore, a pertinent and structured instruction on rational numbers may alleviate the whole number bias. According to this aspect, the impact of the whole number bias may be unavoidable but can be reduced.

Summary

The concept of fractions is multifaceted and comprises five interrelated sub-constructs: part-whole relations, ratios, quotients, measures and operations. Among them, the part-whole sub-construct was considered of the most importance and the foundation of rational numbers knowledge. Each sub-construct relates to different cognitive structures and presents the rational number from different perspectives. Altogether they constitute a full meaning of the rational number.

This section also presents the investigation of the origin of the whole bias number conducted by Ni and Zhou (2005). There are three accounts, innate constraint account, undifferentiated amount hypothesis account and learning account, which contribute to the response of the origin of the whole number bias. The first account, innate constraint assumes that the early quantitative representation based on numerical-specific and privileges the discrete quantity. Thus, it occasions the learning of concept consistent with early numerical representation, such as addition and subtraction, verbal counting, etc. and it simultaneously provokes constraints for the acquisition of the concepts of fraction. The second account, the undifferentiated amount hypothesis states that the early quantitative

representation is assumed total amount-based and children initially do not differentiate the discrete quantity from the continuous quantity. In the course of developing numerical competence, children begin to differentiate them. The whole number bias is created from this differentiated process and the whole number bias is not innate endowment but rather a result of the development. The third account, the learning account indicates that the complex interaction between the prior knowledge of the whole number and the later acquired knowledge leads to the whole number bias. The instruction on rational number plays a significant role to promote this bias. The three accounts have reported different aspects, which may cause the whole number bias. It provides better understanding about the passage from the whole number to rational number and relevant proposals for the instruction in recognizing fraction and decimal number.

Conclusion

Conclusion

There are three representations of number: analog, verbal and Arabic code. The transcoding process allows translating numbers from one code to another. Two types of transcoding models are distinguished: semantic and asemantic. The difference between them is the requirement of an intermediary semantic representation between the input and the output code. Previous studies have showed that different linguistic have impacts on the performance of transcoding task. Specific errors are observed according to the difference of the system of verbal number. There are comparison studies investigated the impact of language on the understanding and performance of mathematics between Asia and the Occident. These investigations related to the counting, adding mentally, the cognitive representation of a number, and the understanding of the canonical base-10 system, and cardinal and ordinal numbers (Miller et al., 1995; Miller & Stigler, 1987; Geary et al.,1993; Geary et al., 1996; Miura et al.,1993; (Miller, Major, Shu & Zhang, 2000; Ho & Fuson, 1998). However, they have not yet investigated the transcoding task between those systems. In addition, the syntactic zero is considered as a source of difficulty in the transcoding task (Granà et al., 2003; Censabella, 2000). In Vietnamese language, the number-name system has particular characteristics for the syntactic zero, indicating explicitly the position of the zero in a number. It provides an interest occasion to investigate the impact of specific language, with an original angle, on the transcoding task and the function of zero.

As mentioned in section 2, the whole number bias is a result of the development of number and it is unavoidable (Ni & Zhou, 2005). It confirms that the prior knowledge of natural number effects on the acquisition of fraction and decimal number. Most previous studies investigating the difficulties and development of the decimal number in the above direction are usually used the comparison task (Nesher &Peled, 1986; Resnick et al., 1989 ; Desmet et al., 2010). Even students gave a correct response, we don't know if they master the positional decimal system and understand the concept of decimal number thoroughly or they just applied the rule of adding zeros to the shorter number and then compared the decimals the same way that they applied to compare natural numbers. Currently, there is no work to our knowledge examining the notation and the understanding of the value of digits in a decimal number. It is interesting to examine if the whole number bias will affect the transcoding task.

The fraction number is a multi-facet conception, as described in section 2. Thus, there are various approaches in teaching fraction that different approach emphasizes differently on the concept of fraction. Among five sub constructs of fraction number mentioned by Kieren (1980), the part-whole sub construct was considered as the most important and as the foundation of rational number knowledge. In fraction number teaching, an approach using the part-whole relationship is often favored. The visual representation in manipulation contexts, with fair sharing and then association with the part-whole knowledge is a common way to introduce the concept of fractions. In particular, the act of

dividing a whole in parts is dominant in fraction teaching, so children have the tendency to consider a fraction as "a part of a whole" and "a fraction is always smaller than 1 or a whole". Thus, it is difficult to explain the existence and sense of improper fractions in which the number of parts is larger than the number of parts in a whole (Steffe, 2002; Thompson & Saldanha, 2003; Tzur, 1999). This approach poses some limitations for the understating of the fraction concept and restrains the representation of the unit in fraction numbers in different ways (Behr, Harel, Post & Lesh, 1993).

Teaching and learning rational numbers is one of the most challenging domains in mathematical education. It can be explained by the multi-facet conception of fraction number and the complex relation between the natural, fraction and decimal number. A decimal number expresses the fractional quantity but is presented in the decimal number system, which resembles the natural number system. Despites having many common features with decimal numbers, natural number contains numerous properties that can be generated to rational numbers. The prior knowledge of natural number can affect the later acquisition of fraction and decimal numbers in both negative and positive way but the prior knowledge always plays a necessary role, as a foundation, for the later knowledge. Thus, to examine the contributions from the natural numbers to rational numbers and associations between natural, fraction and decimal numbers could lead us the potential implication to improve the rational number teaching.

This thesis aims to examine the impact of language on the transcoding task of natural numbers, the learning of rational numbers and the transition from natural to rational numbers. The empirical part comprises five studies (chapters 4, 5, 6, 7 and 8). The first two studies are focused on the impact of language on the transcoding task and the last three studies investigated the rational number.

In the first study (Chapter 4), we seek to answer the two following questions: "Does the special features of the Vietnamese verbal-number system help children to perform better in the task of transcoding verbal-Arabic numbers?" and "Is the syntactic zero also a source of difficulty for children from grade 3?". A transcoding task from a verbal number into an Arabic number was presented for Belgian and Vietnamese children from grade 3 to grade 6 in this first study. We use a methodology allowing us to explore the impact of language and of the syntactic zero. We analyze the nature of errors by conducting cluster analyses for each sample of children to examine the evolution of the transcoding task related to the grade and country (Belgium and Vietnam).

The difference between children in cross-cultural studies appears in tasks directly related to language and is limited to the symbol system of number names (Nunes, 1992; Miller & Stigler, 1987; Miller, Smith, Zhu & Zhang, 1995; Saxton & Towse, 1998). Does the numerical language have an effect on tasks involving the manipulation of symbols? We investigate this question in the second study (Chapter 5). To determine more specifically the effect of language on the interpretation and performance of children, four tasks assessing comparison of natural number, the understanding of a positional digit in a number, the ability to transcode from a verbal number into an Arabic number and from an

analog representation into an Arabic number were used in the present study. Third Grade French-speaking Belgian and Vietnamese children participated in the study. We compare performances of two samples across four tasks to examine thoroughly the impact of numerical language.

From the third to the fifth study, we direct our investigation toward the development of the rational number and the impact of different curriculums on rational number learning. The third study (Chapter 6) investigates the development of the Arabic representation of the decimal concept of the decimal number. We were interested in the way children reorganize their prior knowledge of natural and fraction numbers to make sense of new mathematical instructions, a symbolic representation of a decimal number. This study may be seen as an investigation of the whole number bias. A large sample of French-speaking Belgian and Vietnamese children enrolled in grades 3 through 6 participated in this study. These children's answers to a transcoding task were measured, with the items being ranked on four levels, which illustrated the generalization process of decimals. The determination of four levels from the answers of children from grade 3 to grade 6 and the distribution of these four levels across the grade illustrate how children generated their presuppositions about decimals and how they reconciled their initial knowledge with the actual notation. The influence of the order of topics in the school curriculum and the various terms used for rational and decimal numbers were also examined by analyzing the answers of both samples, Belgian and Vietnamese children.

The fourth study (Chapter 7) focuses on the fraction number. We investigated the limitation of a part-whole scheme. We used the tasks where children had to associate graphic representations with fraction symbols and vice-versa and in parallel fractions superior and inferior to 1. This study is conducted in Belgium and Vietnam, which offered an opportunity to examine how different approaches to the fraction concept in the school curriculum may influence the acquisition of the part-whole relationship. We predicted that children would use the appropriate reference of the part-whole relation in manipulations with fractions superior to 1. They would overgeneralize the part-whole rule by using "the number of all parts" instead of "the number of parts in a whole" to refer to the denominator of a fraction.

The fifth study (Chapter 8) investigates a more holistic view of the rational concept development by examining the interactions between the whole numbers knowledge with rational numbers. The participants were from a longitudinal study, children make tests of natural and fraction number at fourth grade and then the tests of link fraction-decimal and decimal number in next year (fifth grade). Using structural equation modeling techniques, we examine how children perform in natural and fraction number tasks as well as the understanding of the relation between fractions and decimals which explained children's performances with decimal numbers.

Chapter 4
When 5004 is Said "Five Thousand Zero Hundred Remainder Four": The Influence of Language on Natural Number Transcoding: Cross-National Comparison

When 5004 is Said "Five Thousand Zero Hundred Remainder Four": The Influence of Language on Natural Number Transcoding: Cross-National Comparison

Abstract

The Vietnamese language has a specific property related to the zero in the name-number system. This study was conducted to examine the impact of linguistic differences and of the zero's position in a number on a transcoding task (verbal number into Arabic number). Vietnamese children and French-speaking Belgian children, from grades 3 to 6, participated in the study. The success rate and the type of errors they made varied, depending on their grade and language. At Grade 4, Vietnamese children showed performances equivalent to Grade 6 Belgian children. Our results confirmed the support provided by language to the understanding and performances in a transcoding task. Results also showed that a syntactic zero is easier to manipulate than a lexical zero for Vietnamese children. The relative influence of language and the source of errors are discussed.

Keywords: number transcoding, syntactic zero, lexical zero, cross-cultural, Vietnamese

1. Introduction

In early mathematical activities, such as rote counting of numbers in sequence, children encounter their first code for numbers, the spoken verbal number, for example, "one" or "twenty-three". Later, mainly in primary school, they start to learn other codes. These are the Arabic number (e.g., 23) and the written verbal number (e.g., twenty-three). When they practice mathematics, children not only use numbers in different codes but they progressively have to translate numbers from one code to another. This is called "number transcoding". Number transcoding is a basic and parallel process in learning arithmetical operations and other more advanced mathematical skills.

When children transcode a written verbal number or a verbal number into Arabic code, they regularly make mistakes. This happens particularly when the tens and hundreds in the number are equal to zero. For example, when children transcode the written verbal number "five thousand twenty-four" into an Arabic number, incorrect responses such as "500024" or "5240" are observed. Despite their understanding of place -value, children in primary school often make these errors. This study aimed to examine the effect of the zero's position on a task of transcoding verbal numbers into Arabic numbers.

Additionally, through several studies concerning the influence of language in the conception and understanding of mathematics, researchers found that different languages

can lead to facility or difficulty when performing certain tasks. The Vietnamese language has a special case of denomination for numbers when the digit in the tens or hundreds position is equal to zero. For instance, 5004 is said "five thousand zero hundred remainder four". The purpose of the present study is also to examine the influence of numerical language characteristics on the aforementioned transcoding task.

1.1 The process of number transcoding and the two types of zero

The transcoding process implies the presence of a code input and of an expected code. We consider three different forms of codes for numbers in this study: verbal number, written verbal number and Arabic number. This study specifically focuses on the transcoding of a written verbal number into Arabic code (e.g., "two hundred forty-five" → "245"). We considered two models for number transcoding. The cognitive model of McCloskey, Caramazza & Basili (1985) and an algorithm of number writing to dictation from Power & Dal Martello (1990). For both models, the transcoding process contains two separate stages: the number comprehension stage and the number production stage. In the first stage, a semantic representation is built on the input code (written verbal number). This semantic representation is then converted into the expected code (Arabic number) by the application of lexical and syntactic rules in the production stage. In McCloskey's model, the semantic representation is a base-ten representation, indicating the quantities of the number (e.g., four hundred twenty five → $[4].10^2$,$[2].10^1$,$[5].10^0$). In Power & Dal Martello's model, it is considered as a decomposition of the number (e.g., four hundred twenty five is (4 x 100) + (2 x 10) + 5). The semantic representation is an interpretation of a verbal number. It plays an important and intermediate role in moving from written verbal code to Arabic code. Recall that sum and product relationships are used to express the semantic representation of the number and to transcode a verbal number into Arabic code. A concatenation operation (noted by &) is applied for the product relationship, for example "four hundred" gives 4x 100 = <4> & 00 = 400 and the overwriting operation (noted by #) is used for the sum relationship, for example "fifty two" gives 50 + 2 = <50> # <2> = 52.

According to Deloche & Seron (1982), two types of errors, lexical and syntactic, were distinguished. Lexical errors indicate that wrong lexical elements constitute the number while its syntactic structure (size) is preserved (e.g., "four hundred twenty-five" [425] → 415). Syntactic errors result from an incorrect application of a combination rule that modifies the syntactic structure of the number (e.g., "four hundred five" [405] → 4005). Previous studies conducted on children in grade 2 and 3 have shown that most errors were syntactic (Power & Dal Martello, 1990, 1997; Seron, Deloche & Noeel, 1991; Seron & Fayol, 1994). Syntactic errors are made during the production stage and come from a confusion between the application of the relation sum or multiplication (e.g., "hundred and five" → (100 + 5) → 100&5 → 1005 instead of (100 + 5) → 100#5 → 105). These errors can be explained by the fact that children do not master combination rules. In this case, they transcode each word by their corresponding Arabic counterpart. This kind of error would increase when they work with large numbers or numbers with a zero. Seron and his

colleagues (1991) suggested that these errors come from a wrong generalization of the transcoding rule (e.g., "one hundred and five"→ 105 is correct but "one hundred and thirteen"→ 1013 or "one hundred and thirty"→ 1030 are incorrect).

The zero is considered to be a source of difficulty for children when they have to produce numbers because of its characteristic of having a masked position. The distinction between the two types of zero was mainly emphasized in the transcoding task (Granà, Lochy, Girellid, Seron, & Semenza, 2003). The first type is the lexical zero, as in the numbers "120" or "50145", which is derived from semantics, into the Arabic number. The second type is the syntactic zero, as in the numbers "508" or "7014", which is a production of syntax. The syntactic zero is inserted into the number by application of a rule, to indicate a missing value in a position. Considering their characteristics, the syntactic zero is obviously more difficult than the lexical zero for a transcoding task. This was shown by previous study conducted on children in grade 2 and 3 of primary school (Censabella, 2000). The length of a number and its frequency of production affect the ability to transcode with a syntactic zero.

1.2 Impact of language characteristics on mathematical thinking and performances

The language, which is an element of the cultural context, plays an important role in the processes of acquisition, understanding and performance in mathematics (Steffe, Cobb & von Glasersfeld, 1988). Indeed, numbers and mathematical activities are linked to language. Mathematical activities for children mainly adhere to language and number-name to explain the mathematical representations in a more concrete way.

Through several studies concerning the influence of language in the conception and understanding of mathematics, researchers found that different languages can lead to aptitudes or difficulties when performing certain tasks, presented below. Asian languages (Chinese, Japanese, Korean and Vietnamese) possess a numerical names system in which the verbal number corresponds exactly to its written form. There is no particular case for verbal numbers from eleven to nineteen as in English, for example, 11 is spoken ten-one and 16 as ten-six. Verbal numbers indicating a decade are constituted by the relation of multiplication, for example, 20 as two-ten and 70 as seven-ten. As a consequence, 46 is spoken four-ten-six. Indeed, due to a more efficient numerical names system, Asian children have shown more advanced abilities than Western children in counting (Miller, Smith, Zhu & Zhang, 1995; Miller & Stigler, 1987) and adding mentally (Geary, Bow-Thomas, Fan & Siegler, 1993; Geary et al., 1996). Several studies of Miura and his colleagues (Miura, Kim, Chang & Okamoto, 1988; Miura & Okamoto,1989; Miura, Okamoto, Kim, Chang, Steere & Fayol, 1994; Miura, Okamoto, Kim, Steere & Fayol, 1993) showed that the effect of language also facilitated the cognitive representation of a number, the understanding of the canonical base-10 system (e.g., four ten-blocks and two unit-blocks for the number 42) and the understanding of place-value (e.g., the meaning of the individual digit in a multidigits number) for children from China, Japan and Korea, compared to English, French and Swedish children. The effect of language has also helped

Chinese children to surpass their English and American counterparts in embedded-ten cardinal understanding (Ho & Fuson, 1998) and in the acquisition and use of ordinal numbers corresponding to their ordinal names (Miller, Major, Shu & Zhang, 2000).

Researches conducted on the task of transcoding have shown that the verbal structure also has an influence on the success rate and on the nature of errors. Seron and Fayol (1994) showed that with different verbal names for 70 and 90, the French make more errors than Belgian French-speaking children when writing verbal numbers into their Arabic code. (In Belgium, 70 and 90 are spoken "seventy" (septante) and "ninety" (nonante) while in France, 70 is "sixty-ten" (soixante-dix) and 90 is "four twenty-ten" (quatre-vingt-dix). The German language has the particular feature of reversing the decade-unit structure in the verbal form (e.g. 58 "fifty-eight" is pronounced "eight and fifty"). With regard to this aspect, Lochy, Delazer and Seron (2003) studied the process of transcoding among young French-speaking Belgian and German-speaking Austrian children. They observed that the error rate was higher during the first grade of primary school in Austria than in Belgium.

1.3 The Vietnamese verbal number

According to previous cross-national comparison studies between East Asian and Western countries, the differences can be explained mainly by the transparent verbal number system used in East Asian languages (the regularity of number words). Like other verbal systems of numbers in East Asia, there is not, for the Vietnamese lexical primitives, a word for the particular numbers "eleven", "twelve", "thirteen", "fourteen", "fifteen" and "sixteen". The designation of numbers from eleven to nineteen is done with an addition relationship (e.g., eleven is expressed as "ten one"). The Vietnamese language names the numbers in the tens using a multiplicative relationship, highlighting the structure by the number's position (e.g., "two ten" for 20). This characteristic is not found for tens in French. The absence of special cases for tens in Vietnamese induces a perfect correspondence between verbal numbers and their decompositions. For instance, the number 265 will be decomposed as $265 = 2\times100 + 6 \times 10 + 5$. In Vietnamese, it is expressed as "two hundred six ten five", but in French it is expressed as "two hundred and sixty five". However, the word "sixty" in French does not express "6 x 10" as clearly and straightly as the word sixty in Vietnamese (six-ten). With the advantage of the correspondence between the verbal number and its decomposition, the Vietnamese language system can help children to have a better understanding of the structure of the number and place value.

However, the Vietnamese language has peculiarities when the digit in the tens or hundreds position is a zero. This is not found in other Asian languages like Chinese, Korean or Japanese. Vietnamese uses the word "zero" as a lexical primitive in the number construction. For example, 3024 is said "three thousand zero hundred two ten four". This syntactic zero is not masked in its verbal form like in other languages. There is, however, an exception when the zero is in the tens place where it is replaced by the word "remainder". For example, 309 is said "three hundred remainder nine". We can understand

the word "remainder" as the amount remaining of the division of 309 by 100. Therefore, in Vietnamese, 5004 will be spoken "five thousand zero hundred remainder four".

The semantic representation plays an important and mediate role in the transcoding process from a verbal number into Arabic code. Children have to interpret the verbal representation into a semantic representation and then apply two production rules, sum or product, to produce the corresponding Arabic number. In the Vietnamese language, the semantic representation is nearly identical to the verbal number. Moreover, the particularity of the syntactic zero can erase the differences between syntactic and lexical zeros, which is convenient for Vietnamese children when they are transcoding with a syntactic zero. According to its verbal number characteristics and especially in the case of zero, the Vietnamese language provides a more economic model and better support than other languages for the task of transcoding.

Our first prediction was that Vietnamese children should have higher performances than Belgian children in the task of transcoding from a verbal number into Arabic number (Hypothesis 1). It was also predicted that the syntactic zero would not be a source of difficulty for Vietnamese children in the task of transcoding, because of the clear presentation of this kind of zero in the Vietnamese number name (Hypothesis 2). Concerning the errors, Belgian children should have the tendency to erase the zero in a transcoding task when they manipulate large numbers more often than Vietnamese children, due to the masked position characteristic of the syntactic zero (Hypothesis 3).

The particular property of the syntactic zero in the Vietnamese language provides an important opportunity to extend previous researches about the impact of language on mathematical understanding and competences. Many of the previous studies, examining the effects of the symbolic structure system, were conducted on preschool and young primary children (Grades 1–2). It would be interesting to examine if the use of that system also has an effect on older children (Grades 3–6) in this study.

The current study was aimed at examining the influence of these linguistic differences in relation to the syntactic zero on a verbal-Arabic transcoding task. We examined whether the difference due to the numerical language characteristics between Asian and Western countries is still valid for older children (Grades 3–6) who had already acquired the numbers system at school. The second purpose of this study was to assess the impact of the zero’s position (syntactic and lexical zeros) and to determine the effect of a syntactic zero on older children in both countries, Vietnam and Belgium.

2. Method

2.1 Participants

Two groups of children participated to this research. In Belgium, we assessed 121 children of Grade 3 (55 girls and 66 boys; mean age 107 months), 109 children of Grade 4 (62 girls and 47 boys; mean age 121 months), 143 children of Grade 5 (68 girls and 75 boys; mean age 129 months), and 127 children of Grade 6 (62 girls and 65 boys; mean age

143 months). In Vietnam, we assessed 708 children of Grade 3 (338 girls and 368 boys; mean age 106 months), 690 children of Grade 4 (305 girls and 381 boys; mean age 117 months) and 699 children of Grade 5 (357 girls and 341 boys; mean age 128 months). Our sample included 52 classes from 8 different schools in two developed cities in VietNam, HoChiMinh and BuonMaThuot. In Belgium, we had 28 classes from 14 different schools in the French-speaking Community2. The number of participants in Vietnam was higher than in Belgium. Also, the size of each school and the numbers of children in each class were different because of different school organizations. Each class in Vietnam had about 40 children while in Belgium, there were around 15 children. Despite these differences, both the Vietnamese and the Belgian samples were broad enough to represent the profile of each population.

Transcoding numbers is a basic ability in mathematics so from second grade in primary school, this competence has been taught in both countries. In third grade, children learn to transcode with numbers up to 5 digits.

2.2 Task and stimuli

A paper-and-pencil transcoding task was constructed with eighteen items. The task was for participants to write the natural numbers from a verbal form into Arabic code ("write the number in spoken form into a number with digits", we gave an example "fourteen → 14" in order for children to well understand the task). The test included 18 items with numbers from 3 to 6 digits. There were two categories labelled Syntactic Zero and Non Syntactic Zero. The Syntactic Zero category was used for numbers containing a syntactic zero (e.g. 308 and 35007). Numbers without this type of zero, such as 745 and 700126, were used in the Non Syntactic Zero category. Eighteen items were randomly organized in a list and presented in the same order to all the participants. The test was performed collectively in the classroom during the last month of school year. Children worked individually, without time pressure. The average time to achieve the test was between five and ten minutes. All items are presented in Annexes.

3. Results

3.1 Overall analysis

Age Comparison of both samples

Even though the primary school lasts 5 years in Vietnam when it lasts 6 in Belgium, the age of the children was almost identical. Age of both samples for Grade 3 and 5 did not differ significantly $F(1,762) = 0.041$, $p = 0.840 > 0.05$ and $F(1,802) = 1.770$, $p = 0.184 > 0.05$. Although, Belgian children for Grade 4 were older than Grade 4 Vietnamese children $F(1,749) = 58.407$, $p < .001$.

Gender comparison

For each participant, the mean rate of correct responses for all the items was calculated.

Vietnamese boys' and girls' performances did not differ significantly, $F(1,2092) < 1$, ns. In Belgium, the performances of the boys were better than the girls', $F(1,499) = 6.98$, $p < 0.05$. The mean of correct response in percent of Belgium boys and girls were 74 and 67 respectively.

3.2 The impact of language

Vietnamese Grades 3, 4 and 5 with Belgians Grades 3, 4 and 5

The next table presents the means and standard deviations of scores in percent, according to the grade and the country.

Table 1. Means rates (in percent) and standard deviations of scores according to the grade and the country

Grade	Vietnam		Belgium	
	M	SD	M	SD
3	76.5	22.4	35.2	17.0
4	91.2	13.1	65.9	26.9
5	94.1	9.1	85.8	17.0
6			91.3	13.4

We compared the Vietnamese and Belgian samples according to grade (3, 4 and 5). The comparison was conducted by a two-way analysis of variance (ANOVA) with the Grade (3–5) and Country (Belgium and Vietnam) as variables between-subject. The difference between the three grades level was significant, $F(2, 2464) = 472.512$, $p < 0.001$, partial $\eta^2 = 0.28$, showing that correct response rates increased with grades. Post hoc t-tests showed that all grades differed significantly from each other ($p < 0.001$ for all comparisons). The Country effect was also significant, $F(1, 2464) = 705$, $p < 0.001$, partial $\eta^2 = 0.2$ indicating that the response rates of Vietnamese children were higher than those of Belgians. Finally, the interaction between the Country and the Grade was significant, $F(2, 2464) = 109.234$, $p < 0.001$, partial $\eta^2 = 0.08$, which means that the advancement across grades differed according to the country. In fact, the difference between grades 3 to 5 was less stressed in Vietnam than in Belgium. As illustrated in Fig.1, there are striking differences between Grade 3 Vietnamese and Belgians children, but this difference is less important in Grade 5. At the same age, Vietnamese children in the last three years of primary school have a higher ability than Belgians to write numbers from a verbal form in the Arabic form.

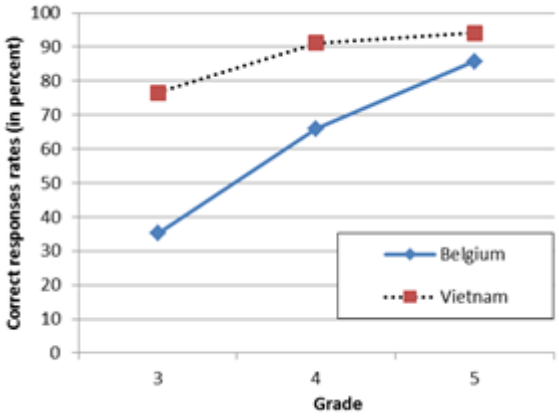


Figure 1. Means of children's scores in percentages according to their Grades and Countries (Grades 3–5 Vietnamese vs. Grades 3–5 Belgian)

Vietnamese Grades 3, 4 and 5 with Belgian Grades 4, 5 and 6

Recall that primary school in Belgium lasts 6 years instead of 5 years in Vietnam. Mathematical program for Vietnamese children in Grade 3 is more advanced than in Belgium concerning large numbers.

Differences in the instruction school between the two countries could explain the significant differences between children of similar age from Grades 3 to 5 as reported above. Consequently, we made a comparison between Vietnamese Grades 3 to 5 and Belgian Grades 4 to 6. Noticed that only in this section of Results, Belgian Grades 3, 4 and 5 used for this analysis (statistic results and in Fig. 2) relate to Belgian Grades 4, 5 and 6 in reality.

There is a one-year lag between Belgian and Vietnamese children, but all effects are still significant according to the two-way analysis of variance with the Grade (3–5) and Country (Belgium and Vietnam) as variables between-subject. The main effect of Grade was significant, $F(2, 2470) = 189.51$, $p < 0.001$, partial $\eta^2 = 0.13$. In addition, performances of Vietnamese children were higher than those of Belgian children. They differed significantly with the effect of Country, $F(2, 2470) = 45.94$, $p < .001$, partial $\eta^2 = 0.02$. The interaction between Grade and Country was also significant, $F(2, 2470) = 5.83$, $p < .05$, partial $\eta^2 = 0.005$.

The advancement of the Vietnamese children still existed across each Grade even with a one–year lag between Belgian and Vietnamese children, as illustrated in Fig.2. For the last three years of primary school, the transcoding performances among Vietnamese children were better than those of the Belgian children. The scores of Grade 4 Vietnamese children (9 years and 9 months) were equal to the scores of Grade 6 Belgian children (11 years 11 months).

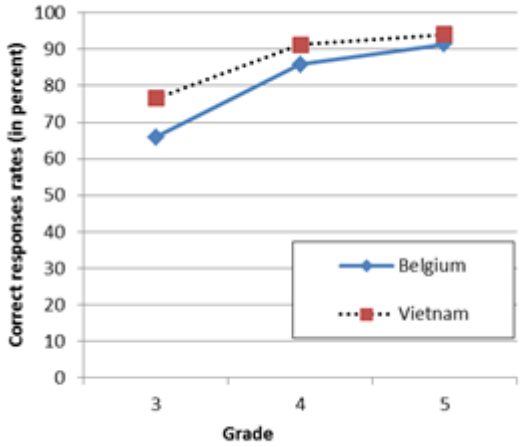


Figure 2. Means of children's scores in percentage according to their Grades and Countries (Grades 3–5 Vietnamese vs. Grades 4–6 Belgian)

3.3 The impact of syntactic zero

The following table presents the means and standard deviations of scores in percentage depending on the category: Syntactic Zero and Non Syntactic Zero.

Table 2. Means rates (in percent) and standard deviations of scores in percentage depending on the type of zero

Grade	Vietnam		Belgium	
	Syntactic Zero	Non Syntactic Zero	Syntactic Zero	Non Syntactic Zero
3	77.5 (23.9)	74.5 (24.1)	35.0 (18.2)	35.4 (19.8)
4	91.7 (14.2)	90.3 (15.4)	64.9 (28.0)	67.7 (28.0)
5	94.8 (9.3)	92.5 (13.2)	86.4 (17.9)	84.4 (19.1)
6			91.6 (13.6)	90.6 (16.9)

We performed a repeated measure ANOVA on the correct response rates with Category (Non Syntactic Zero and Syntactic Zero) as a within-subject variable and Country and Grade (3-5 for both countries) as between-subject variables. The Category main effect was significant, $F(1, 2464) = 4.56$, $p < .05$, showing the higher success rates for the category Syntactic Zero. The main effect of Grade was also significant, $F(2, 2464) = 468.9$, $p < 0.001$. There is a remarkable dissimilarity of the Country main effect as seen in results above. We did find a clear difference between the performances of Vietnamese and

Belgians children, $F(1, 2464) = 679.5$, $p < 0.001$. Indeed, the performances of Vietnamese children were higher than those of Belgian children. The interaction between Category and Grade was also significant $F(2, 2464) = 3.7$, $p < 0.05$, which means the effect of Syntactic Zero was unlike across Grade. The interaction between Category and Country was significant, $F(1, 2464) = 8.4$, $p < .001$ showing that the effect of the type of zero also differed according to the country. Although, the triple interaction Syntactic Zero × Country × Grade was not significant ($p > 0.05$).

To understand more clearly the interactions described above and to examine the effect of the syntactic zero across grade in each country, two analyses of two–way repeated measures ANOVA were conducted separately, with Category (Non Syntactic Zero and Syntactic Zero) as a within-subject variable and Grade as a between-subject variable. In both countries, Belgium and Vietnam, the main effect of Grade was significant (in all cases $p < 0.001$). In Belgium, the Syntactic Zero effect did not affect the transcoding task. Performances of Belgian children in two categories (Syntactic Zero and Non Syntactic Zero) did not differ significantly, $F(1,496) = .001$, ns. In contrast, this main effect was significant for Vietnamese children, $F(1,2094) = 43.21$, $p < .001$, showing that it was easier for Vietnamese children to write verbal numbers in their Arabic form when they had a syntactic zero.

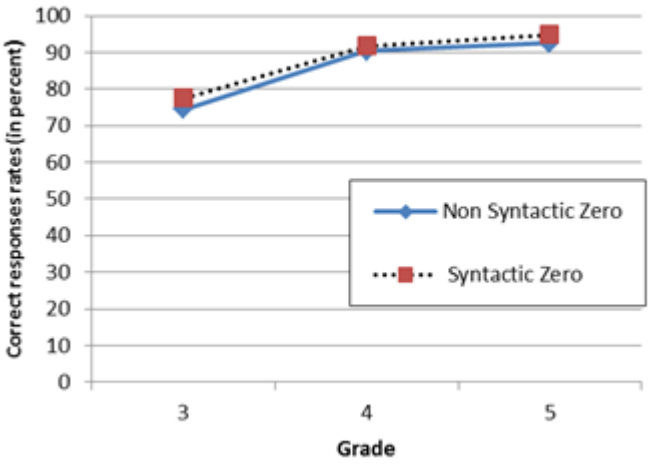


Figure 3. Means of children's scores in percentage by grade in categories Syntactic Zero and Non Syntactic Zero in Vietnam

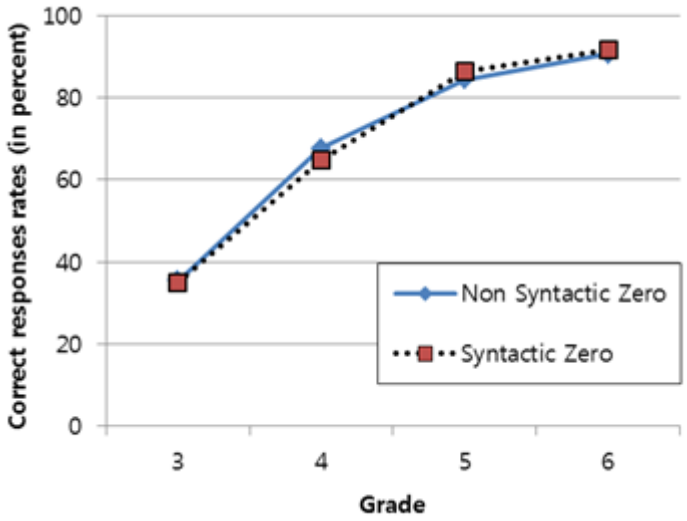


Figure 4. Means of children's scores in percentage by grade in categories Syntactic Zero and Non Syntactic Zero in Belgium

In both countries, the interaction between Category and Grade did not differ (in all cases $p > 0.05$). The same pattern was observed across grades for each country. Indeed, the syntactic zero was a factor facilitating performances for Vietnamese grades 3, 4 and ($p < 0.001$ for grade 3 and 5, $p < 0.05$ for grade 4). Among Belgian children, the syntactic zero wasn't a source of influence for grades 3 and 4 and it was a slight source a difficulty for grades 5 and 6. The Syntactic Zero factor was not significant for every class in Belgium. These results supported Hypothesis 2 (Fig. 3 and Fig.4).

3.4 Nature of errors and cluster analyses

To explore errors for each sample of children, Belgian and Vietnamese, cluster analyses were conducted to group children into homogeneous patterns of responses. Three clusters analyses were performed for three samples: Belgian children, Vietnamese children and a group containing participants in both countries. Four types of responses were determined: the correct response and three types of errors: lexical error (response has the same number of digits than the correct response, e.g., "three hundred eight" (308) → 380), omission error (response has less digits than the correct response, e.g., "twenty-five thousand fourteen"(25014) → 2514) and insertion error (response has more digits than the correct response, e.g., "one thousand two hundred fifty three" (1253) → 1000253).

In the first cluster analysis for Belgian children, three groups of children were identified based on the type of response. The clusters were labeled high-omission (n = 100), omission (n = 95) and expert (n = 305). Children in the cluster "high-omission" had a tendency to provide a response with less digits than the correct one. This tendency was valid for all the items with 5 and 6 digits including 1 or 2 zeros (e.g. 607 029). Cluster "omission" included

children who had the tendency to erase some digits when they transcoded a verbal number into Arabic code. They showed this tendency only for four items that had 6 digits with 2 digits equal to zero. Participants in the cluster "expert" responded almost correctly to all of the items (86% to 99%).

Table 3. Percent of participants per grade of each cluster Belgian

	Grade 3	Grade 4	Grade 5	Grade 6
High-omission	54	22	6	2
Omission	35	28	10	6
Expert	11	50	84	92

The cluster membership distribution in each grade differed significantly from the expected distribution as shown by a chi-square analysis. In Grade 3, $\chi^2(2, N = 121) = 32.34$, $p < 0.001$, the great majority of children belonged to clusters "high-omission" (54%) and "omission"(35%) which means they had a tendency to erase the zero, mainly the syntactic zero, when they transcoded a verbal number into Arabic code. Few children were into the cluster "expert" (11%). In Grade 4, $\chi^2(2, N = 109) = 13.56$, $p < 0.001$, half of the children belonged to the cluster "expert", but the rates of children who belonged to clusters "high-omission" and "omission" were still high (22% and 28% respectively). An evolution was found in Grade 5, $\chi^2 (2, N = 143) = 165.16$, $p < 0.001$ and Grade 6, $\chi^2 (2, N = 127) = 197.73$, $p < 0.001$. Indeed, the majority of children belonged to cluster "expert" (84% for Grade 5 and 92% for Grade 6) and few of them were classed into cluster "high-omission". Noticed that there were children in Grade 5 and Grade 6 who belonged to the cluster "omission" which means they also had difficulties to manipulate a six digits number, with two digits equal to zero. These results supported hypothesis 3, Belgian children have a tendency to erase the syntactic zero in the transcoding task. This tendency is more prevalent among younger children (Grade 3).

The second cluster analysis was run for Vietnamese children. Three clusters were also determined in this sample. The clusters were named "omission" (n = 246), "expert" (n = 978) and "perfect" (n = 876). Children in the cluster "omission" had a tendency to provide a response with less digits than the correct response. This tendency was observed for the five items with 6 digits, 2 of them (digit) equal to zero. Participants in the cluster "expert" responded correctly to almost every item (85% to 98%). Cluster "perfect" included children who gave correct responses to every item (100%).

Table 4. Percent of participants per grade of each cluster Vietnamese

	Grade 3	Grade 4	Grade 5
Perfect	27	45	54
Expert	48	48	43
Omission	25	7	3

The cluster membership distribution in each grade differed significantly from the expected distribution as shown by a chi-square analysis.

In Grade 3, $\chi^2(2, N = 708) = 70.43$, $p < 0.001$, the majority of children belonged to the cluster "expert" (48%), which means they nearly mastered the competence to transcode from a verbal number into Arabic code . A quarter of children in Grade 3 were into the cluster "omission".

In Grade 4, $\chi^2(2, N = 690) = 226.67$, $p < 0.001$, there was the same rate of children who belonged to the cluster "expert" compared to Grade 3. However there were more children in the cluster "perfect" (45%) and fewer children in the cluster "omission" (only 7%). In Grade 5, $\chi^2(2, N = 699) = 297.43$, $p < 0.001$, almost all children belonged to clusters Expert or Perfect (97%).

According to the nature of the separate clusters identified for Belgians and Vietnamese participants and to the distributions in each cluster for each country, Vietnamese children showed more advanced abilities than Belgian children. To examine directly the advance of Vietnamese over Belgian children referring to their errors, a third cluster analysis was run for all participants of both countries. Despite the difference between Belgian and Vietnamese participants, similar errors were observed. Children used to erase some digits, mainly the syntactic zero digit, when they transcoded a verbal number into Arabic code.

Three clusters of children were also identified, based on the type of response in this case. The clusters were labeled "omission" (n = 350), "expert" (n = 1268) and "perfect" (n = 979). Children in the cluster "omission" had a tendency to provide a response with less digits than the correct response. This tendency was observed for items with six digits, two of them equal to zero. Participants in cluster "expert" responded almost correctly to all items (85% to 98% for all items, except 71% for one item). Cluster "perfect" comprised children who gave a correct response to all items (100 %).

The distribution of clusters was influenced significantly by the differences between the two countries and the difference across grades. The chi-square test was the following: χ^2 (2, N = 2597) = 355.63, $p < 0.001$; χ^2 (2, N = 2597) = 256.81, $p < 0.001$ respectively for Country x Cluster and Grade x Cluster. It means that the delivering of children to each cluster depends on the country and the grade level.

The evolution of children across grade and the evolution of Vietnamese children over Belgian children are also illustrated in Table 5.

Table 5. Percent of participants per grade of each cluster according to the country

	Grade 3	Grade 4	Grade 5
Perfect	27	45	54
Expert	48	48	43
Omission	25	7	3

4. Discussion

The present study was motivated by the particular number name feature present in the Vietnamese language and its relation to the syntactic zero. Previous comparison studies between Asian and Occidental children did not use a transcoding task directly related to language. Our study, using this transcoding task on older children, confirmed that different languages lead to variations in the numerical performances between Asian and non-Asian children. The results regarding the impact of a syntactic zero on a verbal-Arabic transcoding task and the nature of errors committed are discussed. Factors influencing performances such as the mathematical program, the numerical experience of children and the perspectives are also discussed below in more detail.

4.1 Impact of structural characteristics of numerical languages on a transcoding task

In a verbal-Arabic transcoding task, Vietnamese children in grade 4 achieved performances equivalent to older Belgian children in grade 6.

Several previous studies on preschool and younger children showed advanced performances for Asian children compared to children from Western countries. Our study presents results from older children at primary school. The findings are consistent with our first hypothesis, the better performances of Vietnamese children are related to a greater transparency of the Vietnamese language compared to the French language. This could be explained by the following two reasons:

First, in Vietnamese, more words are needed than in French to produce a verbal number, particularly for numbers with a syntactic zero. A good example is the number 208004. In French, five words are used to write it (two hundred eight thousand four), while nine words are needed in Vietnamese (two hundred remainder eight thousand zero hundred remainder four). When transcoding a verbal number into an Arabic number, Vietnamese children have to read and translate the words using a term to term correspondence. The semantic representation is embedded in its expression. Belgian children work with fewer words. It seems easier but they have to apply syntactic rules to produce the corresponding Arabic code. The syntactic structure is generally more difficult than the lexical structure to produce

an Arabic number from a verbal number. For a transcoding task, Vietnamese children use the "lexical pattern" and Belgian children use the "syntactic pattern". This can explain results from comparisons between Vietnamese grades 3, 4 and 5 with Belgian grades 4, 5 and 6. Even when the difference in the program between the two countries becomes smaller, the outperformance of Vietnamese children is still highlighted.

Second, according to the Dehaene three codes model (1992), the verbal representation of a number is dependent on the base-10 structure. As the Vietnamese system number name is very transparent with the base-10 structure, it will consequently be a support for Vietnamese children in a verbal-Arabic transcoding task.

Specific errors may emerge from the specific property of a number-name system (Zuber, Pixner, Moeller & Nuerk, 2009). Due to the structure of verbal numbers in French, the French-speaking Belgian children in our study committed a typical error which was not found in the Vietnamese sample. For example, with the number: 542 183 "five hundred forty-two thousand one hundred four-twenty three" (French verbal expression), Belgian children showed many more insertion errors. The percentage of failure was 41% for Belgian children and 9% for Vietnamese. Interestingly, 29% of Belgian children (46% at Grade 3) produced responses ending with "423" (instead of "183") such as 542 100 423, 542 423 or 50 042 423. How could we explain this kind of error? In French, the expression "four twenty" is used to indicate the number 80. In this particular case, it is necessary to combine lexical and syntactic rules to produce the Arabic number from the verbal number. When manipulating large numbers containing a sub-syntactic structure such as "four twenty" (80 in the French language), children have the tendency to provide 423 instead of 83 (four twenty three). When working with large numbers, Belgian children use lexical relations more often than syntactic ones. This error was not observed with Vietnamese children, they also used the lexical relation rather than the syntactic relation or confused the two relations when manipulating large numbers, but their errors were different, for example 50 042 183 or 54 200 183.

4.2 The impact of the syntactic zero and the nature of errors

Our second aim for the current study concerned the influence of the syntactic zero in a transcoding task. By having access to a large sample of children at different grade levels (Grade 3–6), we observed that this influence is less important or non-existent compared to previous results (Censabella, 2000) based on younger samples (Grades 2–3). Indeed, the syntactic zero factor was not a source of influence for the Belgian children from grades 3 to 6. Examining this factor from grade 3 to 6 allowed us to verify the development of children across grades and to correctly understand the location of exertion of the syntactic zero. Unlike previous studies, the syntactic zero was a support for Vietnamese children during a transcoding task. The reasonable explanation for these findings is that the Vietnamese numerical language is transparent for a number with a syntactic zero.

According to the cluster analysis, 89% of Belgian children in grade 3 belonged to clusters "high-omission" and "omission", while 50% in grade 4 belonged to these clusters. It means that the zero would pose a typical difficulty for children in the transcoding process. Notice that we did not find content referring to the lexical and syntactic zeros in mathematical curricula and lessons of both countries. In the Vietnamese language, the difference between syntactic and lexical zeros is not explicit because the notion of syntactic zero is represented in spoken numbers. This is, however, not valid for other languages, such as French. It could be necessary to include the notion of lexical and syntactic zeros in mathematics lessons. Teachers could emphasis the existence of the zero in verbal numbers in order to help children to recognize and understand its role better, at least in verbal representations and transcoding tasks.

Two meanings can be differentiated regarding the function of the zero for the construction of a number. The first relates to the syntactic rule in the production of an Arabic number. When a power of ten is missing in a number, the insertion of a zero is required in order to conserve the positional system of other digits (e.g., "Four thousand twenty-five" is 4025). The second is the opposite. When a digit in a number is equal to zero, even if it is worth null, this digit always occupies a position in the number. For example, in the number 4025, the value of the hundreds position is null but there is always a digit indicating the hundreds, which is 0. From their tendency to omit the digit equal to zero while transcoding a number with a syntactic zero, we can predict that children will encounter difficulties when they manipulate a zero in other skills, such as place-value understanding (e.g., "Determine the digit indicating hundreds in the number 4**0**25"). We expect that for this task, children will respond that "there is no digit indicating hundreds". They might confuse the value of the digit 0 with its role of conservation. In the current or in previous studies, the syntactic zero was analyzed only in transcoding from a verbal number into an Arabic number. The syntactic zero properties might lead to difficulties in other types of presentations, such as symbolic or analog, as well as in other advanced mathematical activities. It would be necessary to explore more precisely the role of the zero in these tasks in a future study.

Previous studies suggested that the most frequent errors in a verbal-Arabic transcoding task were syntactic ones. Children had the tendency to produce a response with more digits than the correct response by inserting extra zeros (e.g. "three hundred and sixty-five" (365) was transcoded by 30065 or 3065). This is called an insertion tendency. Noel and Turconi (1999) explained these errors by the fact that the product relation is normally mastered before the sum relation, children applying the product relation intead of sum relation when they produce an Arabic number (e.g., "four hundred two" → (4x100) + 2 → 400 + 2 → 400&2 → 4002 instead of (4x100) + 2) → 400 + 2 → 400#2 → 402). According to Seron and his colleagues (Seron, Deloche & Noel, 1991; Seron & Fayol, 1994), errors of insertion of extra zeros are common in the second and third grade. Children transcoded each word by its Arabic counterpart (e.g., one thousand nineteen was written 100019). They applied the transcoding rules of addition structures of Units for Decades (e.g., "one hundred three" becomes 103 (correct) but "one hundred thirty" becomes 1030(fault)). In the present study,

children were older in comparison with previous study samples, but they still transcoded each word by their Arabic counterpart (e.g., "one thousand two hundred fifty-three" 1253 → 1000253; 120053 and 100020053). For numbers with a syntactic zero, they mainly erased the zero not present in the verbal form of the number (e.g., the common response for 25014 is 2514 and 503008 is 5308). The tendency to produce a response with fewer digits than the correct response (omission tendency) in this study is explained by the particular characteristic of the syntactic zero. It suggests that children have not yet master the understanding of place-value and the role of a zero in the production phase of an Arabic number.

In short, errors in manipulation with the syntactic zero made by the children were similar across countries. They showed a tendency to erase the syntactic zero when they transcoded a verbal number into an Arabic number. This type of error was more common in Belgium than in Vietnam. This is explained by the fact that the syntactic zero is highly masked in French numerical language, while it is specifically named in the Vietnamese language.

4.3 Perspectives and future studies

In previous cross-national studies and mathematical achievement studies, besides language factor, other factors like parents' behavior, initial arithmetical knowledge, learning context and mathematical methodology could account for the advancement of Asian children over Western children (Ho & Fuson, 1998; Miller, Smith, Zhu, & Zhang, 1995; Saxton & Towse, 1998; Towse & Saxton, 1997). In the present study, the difference between mathematical program and arithmetical experience between Belgium and Vietnam can explain part of the progress of Vietnamese children. In fact, in Belgium, transcoding with numbers up to 5 digits is taught in grades 3 and 4 and with larger numbers in grades 5 and 6. In Vietnam, children have already learnt to manipulate numbers with 6 digits at grade 3. Numbers with 7 digits are introduced at grade 4. In addition, Vietnamese children also have a frequent use of large numbers (starting with 5 digits) in everyday life, in relation to the monetary system whose unit is a hundred.

The task instructions also accounted for the variations in performances in cross-national studies, as reported by Saxton and Towse (1998). We observed that performances in a transcoding task for children at grades 3 and 4 with numbers of 3 and 4 digits seemed less advanced in comparison with the results of previous studies (Seron & Fayol, 1994; Censabella, 2000). Indeed, the test included a large number of items with 5 and 6 digits, including one or two syntactic zeros. So the difficult items may have influenced global performances, mainly for younger Belgian children (grades 3 and 4) who do not have a lot of experience with large numbers. The effect of task instructions was considered as a factor affecting outperformances by Vietnamese children. In order to avoid confounding factors and to focus on the language impact, it would be more useful to select appropriate items for the task instructions.

The influence of language was demonstrated mainly on abilities related to the canonical base-10 system and on linguistic tasks such as counting or transcoding. According to the three codes model of Dehaene (1992), each representation of a number (verbal, Arabic and Analog) is associated with specific manipulations. The Analog (magnitude) representation is independent of the base-10 structure. It could be interesting to examine whether the transparent property of the syntactic zero also emerges in place-value understanding (e.g., identify a digit in a multidigits number) and in non-linguistic tasks, such as Analog-Arabic transcoding. Children would be asked to write down an Arabic number from its analog-representation. The number can be represented with cubes, for example, a big cube represents a hundred and a small cube represents ten. Children would have to give the Arabic number (240) from a presentation with 2 big cubes and 4 small cubes. Therefore, children in both countries would have the same input code. Vietnamese children could no longer rely on the support of the transparent denomination of the syntactic zero. Using non-linguistic tasks is also an efficient way to evaluate the impact of. A future study, not only with verbal-Arabic transcoding but also with Analog representation-Arabic transcoding and place-value understanding is currently being conducted. It will allow a precise understanding of the effect of language on the interpretation and performance of children in a more complete way.

Chapter 5
Belgian and Vietnamese children's comprehension of the natural number system: a re-examination of cross-national differences

Belgian and Vietnamese children's comprehension of the natural number system: a re-examination of cross-national differences

Abstract

Previous researches have demonstrated national differences in mathematical competence are associated with linguistic differences in number-naming systems. The aim of the present study was to examine the influence of language characteristics on performances of number transcoding and place value understanding. In the Vietnamese language, the number-name system has particular characteristics, indicating explicitly the position of the zero in a number (e.g. 2004 is said "two thousand zero hundred remainder four"). We investigated whether the numerical language properties related to zero had an effect on the transcoding from a verbal number into an Arabic number and transcoding from an analog representation and place value understanding. Third Grade French-speaking Belgian and Vietnamese children participated in the study. Results suggest that the impact of language was only marked for the number-name related task and did not affect the one involving the manipulation of symbols. Our results also showed misunderstandings of the meaning of zero meaning when dealing with positional numbers. The relative influence of language and the culture differences in cross-national studies are discussed.

Keywords: language supports, cross-national comparisons, syntactic zero, positional number system, number transcoding.

1. Introduction

1.1 The impact of language on understanding and performance in mathematics

Previous researches have demonstrated national differences in mathematical competence are associated with linguistic differences in number-naming systems. Using a verbal-Arabic transcoding task where children were asked to write the corresponding Arabic number (e.g., 15) of a verbal number (fifteen), previous studies showed that performances and types of errors were related to the different verbal-name system. Seron and Fayol (1994) showed that with different verbal names for 70 and 90, the French make more errors than Belgian French-speaking children writing verbal numbers into their Arabic code. (In Belgium, 70 and 90 are spoken as "seventy" (septante) and "ninety" (nonante) while in France, 70 is "sixty-ten" (soixante-dix) and 90 is "four twenty-ten" (quatre-vingt-dix). Specific properties of a language can also lead to specific errors in a transcoding task. Due to the inversion rule in the German number-word system (e.g. 25 is said "five and

twenty"), inversion errors were observed only with German-speaking children (Zuber et al., 2009). With regard to this aspect, Lochy and colleagues (2004) studied the process of transcoding among young French-speaking Belgian and German-speaking Austrian children. They observed that the error rate was higher during the first Grade of primary school in Austria than in Belgium.

Previous studies conducted between East Asia (Chinese, Japanese and Korean) and Western children showed that the more efficient way of naming numbers in Asian languages was also an important explanation for the differences favoring Asian children. Those differences concerned mathematics skills such as abstract counting (Miller, Smith, Zhu & Zhang, 1995; Miller & Stigler, 1987), mental addition (Geary, Bow-Thomas, Fan & Siegler, 1993; Geary et al., 1996), and understanding of the canonical base-10 system and of place value (Miura, Okamoto, Kim, Steere & Fayol, 1993). The effect of language was also a support for Chinese children to surpass their English and American counterparts in embedded-ten cardinal understanding (Ho & Fuson, 1998), in the acquisition of ordinal numbers and in generating cardinal and ordinal number names (Miller, Major, Shu & Zhang, 2000). The concept of fractional parts is more transparent in the Korean fraction-naming system and the effect of language has also been found for the numerical fraction understanding. Korean children were able to associate numerical fractions with corresponding pictorial representations more efficiently than U.S. and Croatian children (Miura, Okamoto, Vlahovic-Stetic, Kim & Han, 1999).

1.2 Vietnamese number word systems and transcoding tasks

Like other verbal-number systems in East Asia (Chinese, Japanese and Korean), the Vietnamese language possesses an obvious base-10 structure of number-names. The number-name system is constructed with eleven lexical primitives, from "zero" to "ten". The number designation from eleven to nineteen is done with an addition relationship (e.g. 11 is spoken "ten one", 17 "ten seven"). There is no particular word for verbal numbers from eleven to nineteen as in French and other Western languages. Verbal numbers indicating decades are constituted by the multiplication relationship, for example, 20 as "two-ten" and 70 as "seven-ten". The number-name structure in Vietnamese induces a perfect correspondence between verbal numbers and their decomposition. For instance, the number 362 will be decomposed as: $362 = 3 \times 100 + 6 \times 10 + 2$. In Vietnamese, it is pronounced "three hundred six-ten two", while in French it is pronounced "three hundred sixty two". We see immediately that in Vietnamese "six ten" means "6×10", but not in French. With the advantage of the correspondence between the verbal number and its decomposition, the Vietnamese language should facilitate children to understand the structure of the number and the place-value.

However, the Vietnamese language has peculiarities when the digit in the tens or hundreds position is a zero. This is not found in other Asian languages like Chinese, Korean or Japanese. Vietnamese uses the word "zero" as a lexical primitive in the number construction. For example, the Arabic number 2054 is pronounced "two thousand zero

hundred five ten four" in Vietnamese. This zero is not masked in its verbal form as in other languages. There is, however, an exception when the zero is in the ten place where it is replaced by the word "remainder". For example, the Arabic number 205 is pronounced "two hundred remainder five". We can understand the word "remainder" as the amount remaining from the division of 205 by 100. Therefore, the number 3006, is said "three thousand zero hundred remainder six" in Vietnamese.

Two types of zero, depending on its position in a number, were mainly distinguished in the transcoding task (Granà, Lochy, Girellid, Seron & Semenza, 2003). The first type is the lexical zero, such as the zeros in tens such as in numbers "520" or "20145". Lexical zeros originate from a numerical concept and are semantically represented in Arabic numbers. The second type is the syntactic zero, which is a production of a concatenation operation plus a partial overwriting operation, as in numbers "508" or "4019". The syntactic zero is inserted into the number by application of a rule to indicate a missing value in a position. While lexical zeros appear directly in their corresponding Arabic form ("twenty"→20), syntactic zeros are not represented in the verbal number code. Considering their characteristics, Grana et al. (2003) suggested that manipulating a syntactic zero is obviously more difficult than a lexical zero. This was also demonstrated by previous studies conducted on children in Grades 2 and 3 of primary school with a verbal-Arabic transcoding task (Censabella, 2000; Lochy, Andrés & Seron, 2004). Because of the masked attribute of the position of the syntactic zero, children had a tendency to omit the syntactic zero in the Arabic number when they had to produce it from a verbal-number (e.g. "twenty five thousand fourteen" (25014) →2514).

The distinction between syntactic and lexical zeros, proposed by Grana and his colleagues (2003), is more salient for verbal-numbers and semantic representations. In Arabic numbers, lexical zeros (e.g. 20) and syntactic zeros (e.g. 102), both play a role of place-holder. In Vietnamese, due to the regular characteristic of the number names, the semantic representation is nearly identical to the verbal number. Also, the special features of the number-naming system, as previously described, might erase the differences between syntactic and lexical zeros. Syntactic zeros are also lexicalized and represented in verbal code (e.g. 3006, is said "three thousand zero hundred remainder six»). This should be convenient for Vietnamese children when they are transcoding a number with a syntactic zero.

1.3 Limitation of language influence on cross-national studies and the present study

Previous cross-national studies have emphasized the way in which mathematical concepts are represented in the different languages to explain performance variations in some mathematical tasks, not forgetting to consider sociocultural factors (parental expectation and assistance, preschool education, educational practices in schools) that could account for the differences between children. In this area, Asian children are commonly presented as superior to Western children, this superiority being interpreted in relation to the language.

Some authors have developed a different view of the above statement. Nunes (1992) demonstrated that the developed number-name system can restructure mental activity, not involving other basic abilities such as memory and logical reasoning. In addition, the difference between children in cross-cultural studies really appears in tasks directly related to language and is limited to the symbol system of number names. Results from Miller and his colleagues (Miller & Stigler, 1987; Miller, Smith, Zhu & Zhang, 1995) showed that even if Chinese children were better than U.S children in abstract counting, there was no difference between the two groups in object counting and problem solving, those last skills being more symbol related. In the abstract counting task, children were asked to count as high as possible. There was no significant language difference for counting series finishing before 10 and from 20 to 99. The differences were found for the teens decade, where differences between Chinese and English number-naming systems really appear. The limitation of language influence on the cognitive number representation was also remarked in Saxton & Towse's cross-cultural study (1998). They replicated the study of Miura and his colleagues (1993), examining the place-value understanding of English-speaking and Japanese-speaking children. The results suggested that the influence of language is less direct than Miura's study had advanced. Subtle changes in task instructions led to noticeable variations of performances.

According to the triple code model of Dehaene (1992), each number code possesses a specific function. Dehaene presented three basic representations of a number: its verbal representation (e.g. five), its representation of a visual number form (Arabic) (e.g. 5) and its magnitude (analog) representation (e.g. 5 cubes represent the number 5). While the verbal and the Arabic representations are tied to the base 10 structure, the analog representation does not depend on it.

The goal of the present study is to re-examine the influence of language characteristics on performances in manipulations with natural numbers. We investigated whether the syntactic zero characteristics of numerical language have an effect on tasks involving the manipulation of symbols and place value understanding. To determine more specifically the effect of language on the interpretation and performance of children, tasks assessing competences in verbal-Arabic number transcoding, analog-Arabic number transcoding and place-value understanding were carried out in the present study. Therefore, children in both countries had the same input code and the support from the transparent denomination of the syntactic zero was here masked for Vietnamese children. It was hypothesized that the Vietnamese number-naming system would provide support for the verbal-Arabic transcoding task. Vietnamese children should perform better than Belgian French-speaking children. (Hypothesis1). It was also predicted that the linguistic effects would vary with tasks. The advantage of the Vietnamese language should disappear for tasks involving the manipulation of symbols. The particular features of the Vietnamese number-naming system related to zero is a good opportunity to examine the impact of the syntactic zero. We expected that the effect of the syntactic zero would differ according to task and country (Hypothesis 2).

2. Method

As it is difficult to avoid confounding factors in cross-nationals, we decided to choose our sample only among third Grade children and tasks comprised numbers with only four digits, where mathematical programs between two countries on the transcoding ability are the most similar.

Numerical experience could also be considered as confounding variables, influencing children's performance. In fact, Vietnamese children have more experience manipulating numbers up to 5 digits than Belgian children. In addition, Vietnamese children also frequently use large numbers (starting with 5 digits) in everyday life in relation to the monetary system whose unit is a hundred. Therefore, we only used the transcoding tasks with four digit numbers.

2.1 Participants

Two groups of children participated in this research. In Belgium, we assessed 56 children from Grade 3 (26 girls and 40 boys; mean age 106 months). In Vietnam, we assessed 92 children from Grade 3 (51 girls and 41 boys; mean age 105 months). In Vietnam, our sample included 3 classes from a public school. In Belgium, children came from 3 classes of 3 different schools selected from the French-speaking community of Belgium. The study took place during the second half of the school year. The Vietnamese and Belgium children had similar socioeconomic backgrounds.

2.2 Task

Four paper-and-pencil tasks were constructed with twenty four items for each task.

All of the items had four digits. The comparison task consisted of 24 pairs of natural numbers. Half of the items included two digits equal to zero (e.g. 4500 vs. 4050) while the other half consisted of items with one digit equal to zero (e.g. 6302 vs. 6032). Both numbers of each pair had the same digits and the same thousand. They only varied for the position of the digits, which had the following form: $\overline{a0bc}$ vs. $\overline{ab0c}$ or $\overline{ab0c}$ vs. $\overline{abc0}$.

The verbal-Arabic transcoding task was for participants to write natural numbers from a verbal form into an Arabic number. The test included 24 items. There were two categories labeled Syntactic Zero Verbal Transcoding and Non Zero Verbal Transcoding, the former being used for numbers containing a syntactic zero (e.g.36**0**4) whilst numbers without zero, such as 2146, were used in the latter task.

The digit identification task consisted of 24 items of natural numbers, which included none to two digits equal to zero (e.g. 2146; 3064 and 2006). For half of the items, children were asked to circle the digit indicating the tens; for the other half, they were asked to circle the digit of the hundreds. The items fell into two different categories labeled Zero Digit identification and Non Zero Digit Identification. The former comprised the numbers with the digit having to be circled equal to zero (e.g. "Circle the digit indicating the tens in the

number 4205" or "Circle the digit indicating the hundreds in the number 2053"), whilst the latter comprised the numbers with the digit having to be circled different from zero (e.g., "Circle the digit indicating the tens in the number 3264 or 3064").

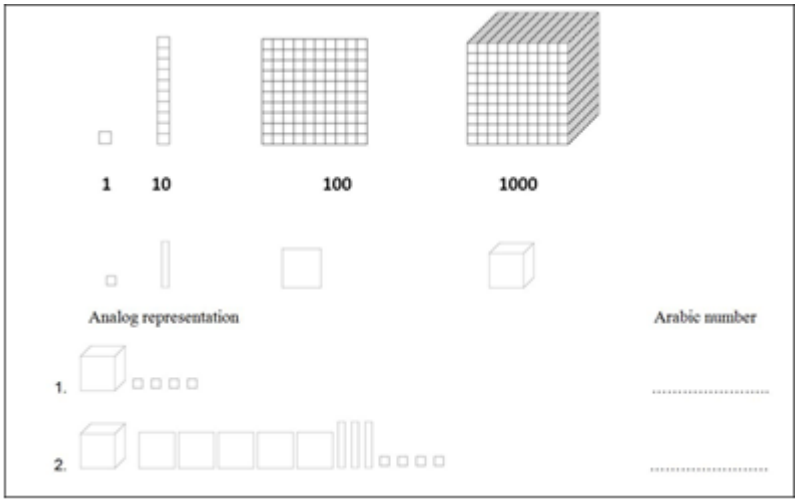


Figure1. Examples of analog-Arabic transcoding tasks

In the analog-Arabic transcoding task, children were asked to produce an Arabic number from a number represented with cubes (e.g. a big cube represents a thousand and a small square represents a unit etc.). Examples are depicted in Figure 1. The same 24 items of the verbal-Arabic transcoding task were used for the analog-Arabic transcoding task. It should be noticed that the item order was changed in the two tasks. There were two categories labeled Syntactic Zero Analog Transcoding and Non Zero Analog Transcoding: the first was used for numbers containing a syntactic zero (e.g.3604) while numbers without a zero, such as 2146, were used in the second. All items are presented in Annexes.

2.3 Procedure

All participants completed the four tasks in the same order. Each task had two versions with a different order of items. The two tasks assessing competences for comparison and verbal-Arabic transcoding were administered during the first session. During the second session, the digit identification and analog-Arabic transcoding tasks, assessing the understanding of the number positional system, were administered. The second session took place three hours after the first. The test was performed collectively in the classroom: children worked individually, without time pressure. The average time to achieve every task was between five and fifteen minutes.

3. Results

3.1 Boys and girls performances

For each participant, the mean rate of correct responses in each task was calculated. Boys and girls performances did not differ significantly for each task in either Vietnam or Belgium (in all cases $p > 0.05$).

3.2 Overall analysis

The mean and standard deviation for each task in each country is given in Table1.

Table 1. Mean rates and standard deviations of correct responses (in percent) in each task by country

	Belgium		Vietnam	
Task	M	SD	M	SD
Comparison	97.0	8.6	89.2	18.9
Digit Identification	77.9	29.8	71.5	41.8
Transcoding Verbal-Arabic	84.5	26.6	90.4	18.6
Transcoding Analog-Arabic	84.9	26.7	65.1	29.5

To investigate whether the performances of children differed across the language and the task, an analysis of variance (ANOVA) was carried on the correct response rates. A two-way repeated measures ANOVA was conducted with the Task (Comparison, Verbal-Arabic Transcoding, Digit identification and Analog-Arabic Transcoding) as the within subjects factor and the Country (Belgium and Vietnam) as a between subjects factor. The main effect of country was significant, $F(1,156) = 7.15$, $p = 0.008$, partial $\eta^2 = 0.44$, which means that scores of Belgian children were higher than those from Vietnam. The main effect of task was also significant, $F(3,468) = 21.01$, $p < 0.001$, partial $\eta^2 = 0.119$, indicating that scores differed across the four tasks. The comparison task had the highest success rate and differed significantly from all other tasks (in all case $p < 0.001$ except $p < 0.01$ in comparison with the verbal-Arabic transcoding task). The verbal-Arabic transcoding task came in second position and also differed significantly from all other categories (in all cases $p < 0.001$ except $p < 0.01$ in comparison with the comparison task).

Scores in the digit identification task did not differ from the analog-Arabic transcoding task.

The interaction between task and country was also significant, $F(3,468) = 6.85$, $p < 0.001$, partial $\eta^2 = 0.042$, as illustrated in Figure 2, which means that the achievement across task differed according to country. For both countries, children had the highest scores in the comparison task. The analog-Arabic transcoding task posed difficulties for Vietnamese children but not for Belgians.

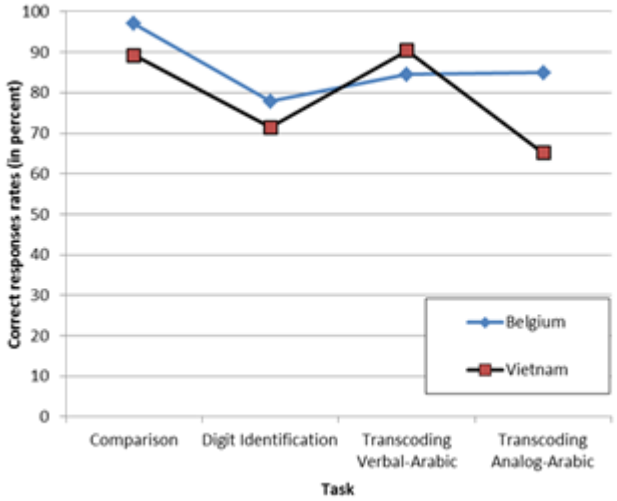


Figure 2. Means of correct responses (in percent) by country in tasks Comparison, Digit Identification, Transcoding Verbal-Arabic and Transcoding Analog-Arabic

To understand the interaction between the type of task and the country more precisely, four one-way ANOVAs with 'country' as a between-subject variable were carried out, separately for each task.

There was significant differences between Belgian and Vietnamese children, $F(1,156) = 9.78$, $p = 0.02$ and $F(1,156) = 18.8$, $p < 0.001$ respectively for the comparison and analog-Arabic transcoding tasks. The results showed that Belgian children's scores were higher than Vietnamese in these tasks. Only in the task of verbal-Arabic transcoding, the scores of Vietnamese children were slightly higher than Belgians but there was no significant difference between them ($p > 0.05$). Also, for the digit identification task, performances in both countries did not differ ($p > 0.05$). The findings were consistent with hypothesis 1, according to which the influence of different mathematical languages changed depending on the tasks which were associated with language in different ways.

The difference between lexical and syntactic zeros in the comparison task not being explicit, we focused the next analysis only on the three remaining tasks: digit identification, verbal-Arabic transcoding and analog-Arabic transcoding. To evaluate the impact of

language related to the syntactic zero (Vietnamese and French), we computed three separate analyses for each task.

3.3 The zero in the digit identification task

The impact of zero

The following table presents the means and standard deviations of scores as a percentage, depending on the category: zero digit identification and non-zero digit identification.

Table 2. Mean rates and standard deviations of correct responses (in percent) in task Identification Digit

	Belgium		Vietnam	
Category	M	SD	M	SD
Zero Digit Identification	69.7	40.5	70.0	43.6
Non Zero Digit Identification	86.1	26.6	72.9	42.7

To investigate the impact of the zero in the digit identification task, rates of correct responses in two categories (zero digit identification and non-zero digit identification) were compared. A two-way repeated-measures analysis of variance (ANOVA) was conducted with the zero identification (zero digit identification and non-zero digit identification) as a within-subjects factor and the country (Belgium and Vietnam) as a between subjects factor. The main effect of zero in the digit identification task was significant, $F(1,156) = 18.9$, $p < 0.001$, which means children had more difficulty identifying a digit in a number when this digit was equal to zero. The interaction between zero identification and country was also significant, $F(1,156) = 9.25$, $p < 0.05$, indicating that the effect of zero differed according to the country. It supported hypothesis 2.

To understand this interaction more precisely, two one-way repeated-measures ANOVA were conducted, separately for each country. As illustrated in Fig.3, the syntactic zero affected Belgian children more than Vietnamese. In fact, this effect was significant for Belgian children, $F(1,65) = 15.3$, $p < 0.001$, which means that the presence of a syntactic zero led to lower scores for them. There was no difference in performance for Vietnamese children between the two categories, zero digit identification and non-zero digit identification. The impact of zero was not significant for the Vietnamese group, $F(1, 91) = 1.64$, $p > 0.05$.

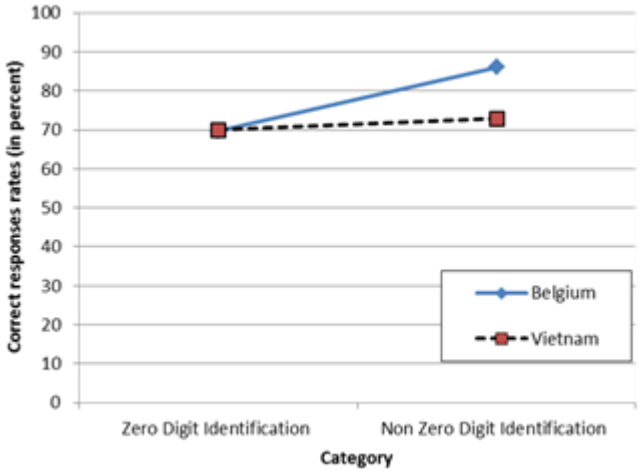


Figure 3. Means of correct responses (in percent) by country in zero digit identification and non zero digit identification categories

3.4 The nature of errors

By observing the testing process, we noticed that some children did not circle any digits for some items. They explained that the digit indicating the tens or hundreds didn't existed. Three types of responses were observed. In addition to the correct response, two types of errors were observed: first, faulty identification which means that children circled faulty digits; second, non-identification which means children did not circle any digit.

A two-step cluster analysis was conducted to group children with a homogeneous pattern of response. Two clusters of children emerged based on the type of response for all participants. Clusters were labeled "impact of zero" (n = 55) and "expert" (n = 102).

Children in the impact of zero cluster had a tendency not to circle any digit in the number when the digit to be identified was equal to zero (e.g. "Circle the digit which indicates the hundreds in the number 2052"). In fact, children in this cluster applied this tendency to all items as illustrated above (47% to 69%). The correct response or faulty identification were well represented (45% -50%) for the rest of the items. Children in the expert cluster responded correctly to all items (98% -100%).

Table. 3 Percent of participants per grade of each cluster in the digit identification task

	Impact by zero	Expert
Belgium	36	64
Vietnam	34	66

In spite of the difference between the two countries, the cluster membership distribution in each country did not differ as shown by a chi-square analysis, χ^2 (1,N = 157) = 0.89, p > 0.05. The pattern of errors across country was similar.

The existence of an impact of zero cluster suggested the influence of the zero on the understanding of the positional system of numbers. It showed children's misconceptions about the function of the zero. When they manipulated a zero, children emphasized more the functions related to its value than to its position. They recognized the value of the digit zero (e.g. in the number 2016, the digit 0 possesses a null value) and forgot its positional function by reasoning that there was no digit to indicate the hundreds.

The zero on the transcoding task

The impact of zero on the verbal-Arabic transcoding task

Table 4. Mean rates of correct responses (in percent) depending on the type of zero in the transcoding task

	Belgium		Vietnam	
Task	Syntactic Zero	Non Zero	Syntactic Zero	Non Zero
Transcoding Verbal-Arabic	85.3	82.0	89.2	93.8
Transcoding Analog-Arabic	83.2	89.9	57.4	88.2

A two-way repeated-measures ANOVA was conducted with task (syntactic zero verbal transcoding and non-zero verbal transcoding) as a within-subjects factor and country (Belgium and Vietnam) as a between subjects factor. The main effect of country was not significant, F(1,156) = 5.3, p > 0.05, partial η^2 = 0.033. The main effect of task in the

verbal-Arabic transcoding task did not differ significantly, F(1, 156) = 0.25, p > 0.05, partial η^2 = 0.002. The interaction between task and country was significant, F(1,156) = 7.4, p < 0.05, partial η^2 = 0.045, which means that the effect of the syntactic zero differed according to the country (Figure 4). These results were not consistent with hypothesis 1.

To understand the impact of the syntactic zero more precisely, two two-way repeated ANOVA were conducted, separately for each country. The results confirmed that the impact of the syntactic zero was not significant for Belgian groups, F(1, 65) = 2.41, p > 0.05, partial η^2 = 0.036 but was significant for the Vietnamese, F(1, 91) = 5.7, p < 0.05, partial η^2 = 0.059 which means that the verbal-Arabic transcoding task with a number containing a syntactic zero was more difficult than without a zero.

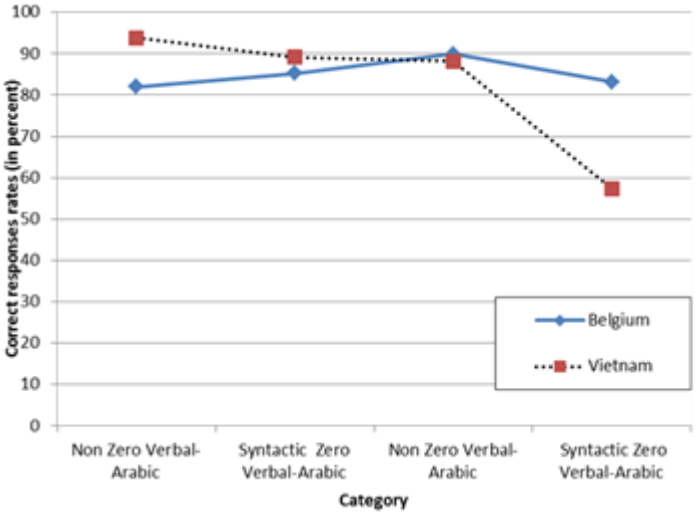


Figure 4. Means of correct responses (in percent) by country in the following categories: Non Zero Transcoding Verbal, Syntactic Zero Transcoding Verbal, Non Zero Transcoding Analog and Syntactic Zero Transcoding Analog

The impact of zero on the analog-Arabic transcoding task

A two-way repeated-measures ANOVA was conducted with task (syntactic zero and non-zero) as a within-subjects factor and country (Belgium and Vietnam) as a between subjects factor. The main effect of country differed significantly, F(1,156) = 13. 06, p < 0.001, partial η^2 = 0.077, which showed that the performance of Belgian children was better than the Vietnamese. The main effect of task in the analog-Arabic transcoding task differed significantly F(1,156) = 44.77, p < 0.001, partial η^2 = 0.223, indicating that scores of the syntactic zero task are lower than scores of the non zero task. The interaction between task and country was also significant, F (1,156) = 18.65, p < 0.001, partial η^2 = 0.107, meaning that the effect of the syntactic zero differed according to the country. It reinforced hypothesis 2.

To understand the impact of the syntactic zero more precisely, two one-way repeated ANOVA were conducted, separately for each country. The results confirmed that the impact of the syntactic zero was significant for both countries, $F(1, 65) = 4.86$, $p < 0.05$, partial $\eta^2 = 0.07$ and $F(1,91) = 53.38$, $p < 0.001$, partial $\eta^2 = 0.37$ respectively for Belgian and Vietnamese samples. The presence of a syntactic zero led to poorer performances for the analog-Arabic, transcoding task, specifically for Vietnamese children. The impact of the syntactic zero was more highlighted for Vietnamese children than for Belgians. We observed a greater difference between performance in the syntactic zero and non-zero categories in the Vietnamese group (Figure 4). These results supported our prediction that the impact of language was limited to the task related to symbolic system. The explicit number naming system in Vietnamese for the syntactic zero did not support Vietnamese children in transforming an analog representation of a number into Arabic code.

Nature of errors and cluster analysis for the analog-Arabic transcoding task

To explore children's errors in the analog-Arabic transcoding task, a cluster analysis was conducted to group children into homogeneous patterns of responses. Four types of responses were identified. In addition to correct responses, there were three error types: first, lexical errors (the response has the same number of digits as the correct response, e.g. "two thousand four hundred five"→ 2404); second, omission errors (the response has fewer digits than the correct response, e.g. "two thousand four hundred five"→ 245); and, third, insertion errors (the response has more digits than the correct response e.g. "two thousand four hundred five"→ 24005).

Two clusters of children were identified based on the type of response, for all participants, labeled "Omission" (n = 35) and "Expert" (n = 123). Children in the omission cluster had a tendency to produce a response with fewer digits than the correct response. This tendency applied for all items including one or two zeros (e.g. 1034 or 3001). Participants in the expert cluster responded correctly to almost all the items (74% to 94%). Children in both clusters gave a correct response for almost all items in the non-zero task (analog-Arabic transcoding).

Children in the two clusters were clearly distinguished by the items in the syntactic zero task. Children in the omission cluster showed syntactic errors, producing many omission and some insertion errors, whereas children in the expert cluster only made lexical errors.

Table 5. Percent of participants per grade in each cluster in the transcoding analog-Arabic task

	Omission	Expert
Belgium	11	89
Vietnam	30	70

The cluster membership distribution in each country differed significantly from the expected distribution as shown by a chi-square analysis, $\chi^2(1, N = 158) = 8.76$, $p < 0.01$. More Vietnamese children belonged to the omission cluster, which means they were more affected by the syntactic zero for the analog-Arabic transcoding task than Belgian children. The aforementioned advantage related to the syntactic zero in Vietnamese seems to disappear in this case.

4. Discussion

The goal of the present article was to examine the influence of language differences on mathematical understanding and performances. The motivation of this study came from the particular case of the zero in the Vietnamese number-name system. We investigated the role of the zero in manipulation with natural numbers across different language. Variations in performance between two samples, Vietnamese and Belgian children, across tasks involving the number-name system in different ways and misunderstandings of the positional number system within the zero are discussed.

4.1 The influence of language: the task related to the number-name system

Regarding the results of Vietnamese children in the digit identification task, we observed a wide variation between them ($M = 71.5$, $SD = 41.8$), while performance across zero digit identification and non-zero digit identification tasks was similar. If Vietnamese children achieved poor performance in this task, it means they failed to grasp the basics from the positional number system, not from the presence of a syntactic zero. For Belgians, the impact of the zero was significant on the digit identification task and variations between children were smaller than for Vietnamese. In addition, the performance of Belgian children in this task was also better than what they achieved in other tasks involving the positional number, such as analog-Arabic transcoding. This suggests that Belgian children's difficulties were due to a specific misunderstanding of the zero more than a general failure to master the positional number. Even if Belgian children surpassed the Vietnamese in this task, they were more affected by the impact of the zero than Vietnamese. Vietnamese children may be able to better detect the role of the zero, namely to occupy a position in the

number. This can be explained by a greater transparency support in the Vietnamese number-name system, where the position of the zero position is explicit.

The influence of language has been closely examined by many studies of Miura and his colleagues. Looking in depth at the impact of languages, Saxton and Towse (1998) replicated a study of Miura (1993) with some subtle changes in task instructions. The results suggested that the impact of language on the cognitive representation of a number was less direct than previously suggested and that numerical experience played an important role. In our previous study (Nguyen & Gregoire, 2014), in the verbal-Arabic transcoding task with numbers from 3 to 6 digits, Vietnamese children at Grade 3 were more advanced than Belgians, particularly with large numbers such as 5 and 6 digits. This difference corresponds to the Vietnamese experience of large numbers in school and daily life and the effectiveness of the Vietnamese number-name system compared to the French system. In this study, we used only 4 digit numbers to avoid the dissimilarity of experience between the two samples. Vietnamese children's performance was higher than their Belgian counterparts, however it was not significant. These results were not consistent with hypothesis 1 and suggested a limitation of the support of the Vietnamese language for the verbal-Arabic transcoding task. In addition, in the analog-Arabic transcoding task, Belgians largely surpassed Vietnamese children.

The explanation is that the base ten structures, appearing in the analog-representation of a number, is independent from the verbal number (Dehaene, 1992), so the advantage of the Vietnamese number-name system was not active in this case. We observed that Vietnamese children achieved almost the same performance in the task without a zero, but the presence of syntactic zero really pulled their scores down. It appears that the particular property of the syntactic zero in the Vietnamese number-name system was not an advantage in this case but rather a constraint. While Belgian children always use syntactic rules to produce an Arabic number containing a syntactic zero, Vietnamese children can benefit from their language advantage, using mainly lexical rules. The difference between syntactic and lexical zeros in a verbal number is not distinguished in the Vietnamese language but it appears in the analog-representation. For the first time in our tests, the presence of syntactic zero was not explicit. Vietnamese children failed to grasp the transcoding process from the analog-representation into the Arabic code. These results showed the limitation of the language influence on tasks which are less related to the number-name system and emphasised the numerical experience role in cross-national performance differences.

The results supported our hypothesis 2. On one hand, children from the two countries have similar performances for verbal-numbers transcoding tasks. Vietnamese children were less affected by the syntactic zero while trying to identify a digit in a number and it can be explained by the greater transparency support in the Vietnamese number-name system. On the other hand, this support showed its limitation when the task was less related to the number-name. For tasks related to Arabic and analog-representation numbers, the effect of language did not appear. Other factors like mathematical experience and differences in mathematics teaching approaches might be a plausible explanation for the differences between the two countries, which favored Belgian children. Using varied tasks

was useful to underscore the strong effect of the different competences related to numerical language. In particular, it also showed how the performance of children in each country changed, according to the relation between the number-name system and the corresponding tasks. It provided a larger understanding of mathematical performances and understanding in cross-cultural studies.

4.2 Misunderstanding of the positional number system and the impact of zero.

The present study examined the performance of children across different tasks related to the digit value understanding, to the positional number system and to the production of an Arabic number. Results showed it was more difficult for children in both countries to master the positional number system and the Arabic number production than the digit value understanding. Children had the highest score on the comparison task, where their competences in the positional number system weren't necessarily active. In fact, children can just apply the rule of comparing each digit respectively, in two numbers with the same number of digits. This task can be considered as an examination of the digit value understanding. In Grade 3, children had no problem to compare numbers with four digits. Regarding the number production task, Belgian children achieved similar performance from two types of input code: verbal number and analog representation. In contrast, Vietnamese showed a large difference between their scores for those two tasks, performing better in the task with the verbal number input code. The first reason is the Vietnamese number-name system, processing by itself the semantic representation of the number. The verbal number is nearly identical to the arithmetical decomposition of the number. It allows Vietnamese children to apply the term-to-term translate process in the transcoding task, from the verbal number into the Arabic number. Regarding the analog representation number, Vietnamese children lack experience in manipulation with this kind of number representation. On the other hand, it showed that these children lacked the competence to handle abstract representations of numbers and had difficulty applying the syntactic rule to produce Arabic numbers.

The presence of a zero led to positional number system difficulty, particularly during manipulations with analog representation numbers and digit identification tasks. Even if children in both samples were different in terms of number-name system, mathematical approaches and initial arithmetical knowledge, they made the same errors, suggesting that children's poor performance may be due to some particular misconception. A cluster analysis, examining the nature of errors, had provided evidence of the impact of the zero. Two functions of the zero in the construction of a number can be distinguished. The first is related to the syntactic rule in production of an Arabic number. When a power of ten is absent in the number, the insertion of a zero is required in order to conserve the positional system of the other digits (e.g. "two hundred five"→2**0**5). The second function is correlated to the first. When a digit in a number is equal to zero, even if it is worth nothing, this digit always occupies a position in this number (e.g. in 205, the value of tens is zero but there is always a digit indicating the tens - 0). The existence of the omission cluster in the analog-

Arabic transcoding analysis and of the impact of zero cluster in the digit identification task analysis testify to the misconception of, respectively, the two functions of the zero mentioned above. In formal mathematics teaching, it would be necessary to focus more on a clear explanation and providing practice of these two functions.

Regarding mathematics teaching in Vietnam, the introduction of analog number representations, in formal mathematics teaching, would improve the capacity to understand the abstract representation of numbers. Having a transparent number-name system is an advantage for Vietnamese children, but it is not enough to master the semantic representation of numbers. An exaggerated dependence on verbal-numbers may sometimes lead to misunderstanding of the positional number system. Children could have a tendency to profit from the transparent characteristics of verbal-numbers and establish technical rules in mathematics, without understanding their original abstract representation and conceptual knowledge. A combination of two codes at the same time, verbal and analog numbers, and an utilization of the benefits of verbal numbers would be recommended to fully provide the preparation needed for Vietnamese children to learn the positional number system.

Chapter 6
Arabic representation of the decimal numbers: from natural to decimal numbers

Arabic representation of the decimal numbers: from natural to decimal numbers

Abstract

Our study aims to describe the development of the Arabic representation of the decimal number concept and how a trace of the natural number is significant in the evolution of students' knowledge. We also examine the influence on this process of different languages and teaching contexts regarding the decimal number. A large sample of French-speaking Belgian and Vietnamese children (n = 1768) enrolled in grades 3 through 6 participated in this study. These children's answers to a transcoding and place-value understanding task were measured, with the items being ranked on four levels with regard to the generalization process. Participants' success rates and their types of errors varied depending on their ages and countries. This result illustrates the students' evolution and the impact of language and learning contexts on children's construction of the decimal.

Key words: generalization process, decimal number, decimals representations, transcoding task.

1. Introduction

Fuson (1998) emphasized a general law of numerical knowledge development such that any conceptualization starts as local first before becoming generalized. The acquisition of a new conception is always based on initial knowledge and on reorganization of existing knowledge. Natural and fraction numbers constitute the prior knowledge before the learning of decimal numbers. These numbers can support and interfere with the conception of decimal numbers given the similar and differing characteristics between them. Tables 1 and 2 from Resnick and his colleagues' paper (1989) describe the comparison of knowledge between decimals and naturals, as well as between decimals and fractions. The similarities and dissimilarities are primarily regarding knowledge about values, as well as notational and naming conventions. A number of previous studies have investigated obstacles related to moving from natural numbers to constructing the concept of rational numbers (Desmet, Gregoire & Mussolin 2010; Sackur-Grisvard & Leonard 1985; Nesher & Peled 1986; Resnick et al. 1989; Stafylidou & Vosniadou 2004; Vamvakoussi & Vosniadou 2004). These studies have primarily focused on value, using comparison tasks to assess their hypotheses. Currently, there are no works to our knowledge examining the notation and naming of decimal numbers. The purpose of this paper is to investigate the development of name and symbolic representations of decimal numbers with regard to the generalization process.

1.1 The generalization process

Generalization is typically regarded as progressive development, yet it should be noted that this process does not unfold without risk (Gauderat-Bagault & Lehalle 2002). Indeed, the application of old treatments to new situations may sometimes lead to overgeneralization. Gauderat-Bagault and Lehalle (2002) explained two types of generalization, known as "extension" and "coordination", that correspond to the two types of generalization, known as inductive and constructive, included in Piaget's theory (Piaget et al. 1978).

Generalization-extension refers to the application of the same treatment to new content, for example, when comparing decimal numbers, children respond that 12.4 < 12.17 because 17 is bigger than 4. No conceptual evolution appears at this level of generalization, as the conceptual evolution always builds new knowledge at a more abstract level. Generalization-coordination describes a novel way to analyze new information by coordinating it with previous knowledge, in accordance with new structural constraints. For example, children respond that 12.94 < 12.7 because a long decimal sequence means smaller parts of a lower value. This type of generalization appropriately fulfills the requirements of conceptual evolution.

Thus, the term "generalization" primarily refers to generalization-coordination, yet most of the generalization process interleaves the two types of generalization. The two types, rather than alternatively, do not occur separately. Additionally, the generalization process varies greatly. It is dependent on the involved domains and on the relationship between the two types of generalization.

Normally, new knowledge is based on previous knowledge, yet, sometimes, previous knowledge is an obstacle with regard to the process of conceptualizing something new. It serves as an epistemological obstacle (Bachelard 1938; Duroux 1983; Brousseau 1998), with the term "obstacle" referring to "a knowledge, a concept, not a difficulty or a lack of knowledge" (Duroux 1983). When learners are aware that their previous knowledge is inaccurate, they are forced to reconceptualize, yet the previous knowledge still exists and may appear in an inopportune and stubborn manner. Therefore, studying the generalization process for a concept leads us to examine the obstacles arising from errors encountered during the conceptualization of a new concept. In the present study, responses, which were primarily errors, were classified into various levels of conceptual evolution to illustrate the generalization process with regard to the two types of generalization, which are extension and coordination.

1.2 Impact of preexisting knowledge on the construction of the decimal number concept

Using an ordering task with three decimal numbers with children from grades 4 through 7, Sackur-Grisvard and Leonard (1985) showed that many children, especially those in grades 4 and 5, used three incorrect rules to form their responses. Each set of three decimals had the same whole number digit (e.g., 12.94, 12.24 and 12.7). According to the authors,

Rule 1 posited that the number with more decimal digits was the larger one (e.g., 12.17 > 12.4 because 17 > 4). Rule 2 stated the opposite, the number with more decimal digits was the smaller one (e.g., 12.94 < 12.7 because the shorter number has tenths and the longer number has hundredths, and tenths are bigger than hundredths). Rule 3 posited that the number with a zero immediately after the decimal point is the smaller one (e.g., 4.08 < 4.7). Rule 3 is a version of Rule 1. These rules represent traces of the rules that are already constructed, which are based on earlier knowledge of the number system, whole-numbers and fractions (Nesher & Peled 1986; Resnick et al. 1989). The authors also examined the conceptual sources for these systematic but incorrect rules. Children who used Rule 1 believed that the longer number was bigger based on their knowledge of natural numbers. In this case, the decimal number was considered as two natural numbers connected by a decimal point, with the decimal part (i.e., to the right of the decimal point) treated as a natural number. When the decimal part started with a zero, Rule 1 became Rule 3, showing an evolution over Rule 1. Rule 2 was shaped by knowledge about fractions, given that children compared the decimal parts as parts of fractions (e.g., tenths, hundredths, etc.).

From the generalization process perspective, Rule 1 corresponds to generalization-extension, whereas Rules 2 and 3 correspond to generalization-coordination. The developmental trend for this system of rules was that the percentage of children using Rules 1 and 3 (natural consideration) decreased from grades 7 to 9, whereas the percentage of Rule 2 users increased at grade 8 and then decreased at grade 9 (Nesher & Peled 1986). Previous studies related to these rules (Swan 1983; Resnick & Nesher 1983) have also suggested that children use the "natural" rule more than the "fraction" rule. Desmet and colleagues (2010) investigated the impact of digit value independently from the impact of length (e.g., the response 0.03 < 0.004 could be due to the idea that "0.004 is longer than 0.03" or "the digit 4 is bigger than the digit 3"). Results showed that the impact of digit value was stronger than the impact of length.

Most of these studies interpreted and classified responses, primarily errors, to examine the underlying concepts regarding the decimal number. Unfortunately, these studies typically use an ordering task with decimal numbers. It is important to consider that some children who were classified as "experts" in this task did not necessarily acquire a conceptual understanding of the decimal number. In fact, they could simply have applied a rule in which they added zeros to the shorter number until both of them had the same number of digits. Then, children could have compared decimal numbers as if they were natural numbers.

Table 1. Comparison of Decimal Fraction and Whole Number Knowledge (Resnick et al. 1989)

Elements of decimal fraction knowledge	Corresponding elements of whole number knowledge	Similar (+) or different
A. Column values	*A. Column values*	
Values decrease with left to right moves	Values decrease with left to right moves	+
Each column is 10 times greater than the column to its right	Each column is 10 times greater than the column to its right	+
Zeros serve as place holders	Zeros serve as place holders	+
Zeros added to the rightmost column do not change the total value	Zeros added to the leftmost column do not change the total value	-
Values decrease with movement away from the decimal point	Values increase with movement away from the decimal point	-
B. Column names	*B. Column names:*	
End in -ths	End in -s	-
Start with tenths	Start with units	-
Naming sequence (e.g., tenths, hundredths, etc.) moves from left to right	Naming sequence (e.g., tens, hundreds, etc.) moves from right to left	-
Reading sequence is from tenths to hundredths to thousandths	Reading sequence is from thousands to hundreds to tens to ones	-
C. Reading rules:	*C. Reading rules:*	
The units must be explicitly specified, and they vary across cases	The ones implicitly serve as units in all cases	-

The difference between natural and rational numbers lies not only in value, as described in Tables 1 and 2, but also in the domains of name and notation. The dissimilarity between names and notations with these types of numbers could shape the construction of the decimal number concept. Natural and decimal numbers are presented with the decimal position system, which has a base-ten structure. These numbers use the same syntactic rules for sum and multiplication relationships to produce an Arabic number. In a given number, a digit is always ten times greater than the digit to its right. This property constitutes both an advantage and an obstacle when learning decimal numbers.

The similarity in lexical words for natural and decimal numbers (e.g., tenths with tens, and hundredths with hundreds) could lead to children identifying elements of the decimal system as ones from the natural system. Thus, they could consider that "tenths" is ten and "hundredths" is ten times greater than "tenths". We predict that children in the early stage of learning rational numbers will conserve the digit order in the sequence "thousands-hundreds-tens" for the sequence "thousandths-hundredths-tenths" in the symbolic representations of the decimal numbers.

Having gained a symbolic representation of natural and decimal numbers, children can easily reorganize the difference between these types of numbers using the presence of the decimal point (comma). In daily language and even in mathematical language at school in some countries (e.g., Vietnam), the decimal is referred to by its symbol, for example, 3.5 is pronounced "three comma five". The decimal point is considered a sign of the decimal number. Children with an immature conception of the decimal number may consider the decimal point to be a point of symmetry between the two sequences, as in "thousands-hundreds-tens-units" and "thousandths-hundredths-tenths". As a result, the decimal part is treated liked a natural number, which could explain children's use of incorrect rules, such as "the longer number is the bigger one", when comparing decimal numbers in previous studies. Using the decimal point and the correct sequence "tenths-hundredths-thousandths" in a symbolic representation may be a marker of the decimal concept's evolution.

An important difference between Arabic representation of naturals and decimals is the digit starting the number (the first digit on the left). For natural numbers, the digit 0 cannot be found at the position starting the number (e.g., natural numbers such as 025; 04 do not exist). On the contrary, this situation occurs for the decimal number. When the decimal part indicates a null quantity, the digit 0 plays a role as place-holder and we have decimal numbers like 0.25 or 0.4. We will investigate this difference in the current study.

Terms, such as tenths, hundredths and thousandths, and their significations are used for both fractions and decimals. They are connected by a decimal fraction, which is a fraction with a denominator that is a multiple of ten (e.g., 1.35 = 135/100). However, their notation conventions are totally different. After having learned fractions, children may use their existing knowledge about fraction notations when they try to represent a decimal number, for example, they may write "two units, three tenths, five hundredths" as2, 3/10, 5/100.

Table 2. Comparison of Decimal Fraction and Ordinary Fraction Knowledge (Resnick et al. 1989)

Elements of decimal fraction knowledge	Corresponding elements of ordinary fraction knowledge	Similar (+) or different(-)
A. Fraction values:	*A. Fraction values:*	
Expresses a value between 0 and 1	Expresses a value between 0 and 1	+
The more parts that a whole is divided into, the smaller each part is	The more parts that a whole is divided into, the smaller each part is	+
There are an infinite number of decimal fractions between 0 and 1	There are an infinite number of ordinary fractions between 0 and 1	+
B. Fraction notation:	*B. Fraction notation:*	
The number of parts that a unit is divided into is represented given implicitly by the column position	The number of parts that a unit is divided into is given explicitly by the denominator	-
The number of parts included in the fractional quantity is the only visible numeral	The number of parts included in the fractional quantity is the numerator of the fraction	-
The whole is only divided into powers of 10 parts	The whole is divided into any number of parts	-
The ending -th (e.g., "tenth") is typical for a fractional part	The ending -th (e.g., "fourth") is typical for a fractional part	+

1.3 The effect of the curriculum and language systems

Local knowledge plays an important role in the generalization process of a concept, which is why the learning context should be examined to better understand how children's existing knowledge affects learning of the decimal number. In grade 3, Belgian and

Vietnamese children have not yet learned decimal numbers. In grade 4, Belgian children learn decimals up to the hundredths place, whereas Vietnamese children have not yet begun to learn decimals. In grade 5, children from both countries learn decimals up to the thousandths place.

In Vietnam, there is a unified textbook for all Vietnamese children, and Vietnamese mathematical instruction emphasizes the role of fractions and the relationship between fractions and decimals. This instruction constitutes the prior knowledge that is gained before learning the decimal system. Thus, in Vietnam, fractions are taught in grade 4 prior to learning decimals. In the Belgian system, it is difficult to determine the exact order in which decimals and fractions are taught, as they are taught in parallel from grades 4 through 6. However, it seems that decimals are taught slightly before fractions. Thus, Vietnamese children may be more influenced by their fraction knowledge than Belgian children when conceptualizing decimals.

A study by Resnick et al. (1989) examined American, Israeli and French children and showed that different curriculums for decimal teaching lead to different appearances of the error rules that children constructed when comparing decimal numbers. In fact, American and Israeli children showed more use of the fraction rule (Rule 2) than French children. In America and Israel, fractions are taught before decimals, whereas it is the opposite in France. When American and Israeli children attempted to acquire new knowledge, they applied their existing knowledge about naturals and fractions to construct their concept of decimals. Although this study investigated the impact of different curriculums, the sample sizes were small and participants were not in similar grades across the samples (grade 5 in the United States, grade 6 in Israeli and grades 4 and 5 in France). A study using a reliable database and more complete statistical analysis is necessary. This is one of the goals of the present study.

Many cross-national studies have shown that different languages can lead to variations in the processes of acquisition, understanding and performing in mathematics. However, these studies have focused on tasks such as counting, addition, understanding of the canonical base-10 system and knowledge of the place-value of natural numbers (Miller, Smith, Zhu & Zhang 1995; Miller & Stigler 1987; Geary, Bow-Thomas, Fan & Siegler 1993; Geary et al. 1996; Miura, Okamoto, Kim, Steere & Fayol 1993; Ho & Fuson 1998; Miller, Major, Shu & Zhang 2000). A study by Miura et al. (1999) investigated the influence of language on the understanding of numerical fractions. The results showed that Korean children performed better then Croatian and U.S children. This better performance was explained by an advantage with regard to the Korean language, which provides specific mathematical terms for fractions. In fact, the concept of fractional parts is included in the verbal number within the Korean language (e.g., 1/3 is called “of three parts, one”). Thus, differences between the terms used for decimal numbers between French (i.e., Belgium’s French community) and Vietnamese languages can be investigated in the current study.

In Vietnamese, the terms used for decimal numbers are consistent with their representations. The expression “number with a comma” (in both French and Vietnamese

languages, we use a comma instead of a period to indicate a decimal number) is used to name the decimal (8,57 is read “eight comma five ten seven” in the Vietnamese language). This expression shows a perfect correspondence between the verbal number and the Arabic number. However, it masks the decimal number characteristics, the meaning of the position and the value of the digits. For children, this language also reinforces the idea that a decimal is a natural number plus a comma. This is a source of difficulty in the passage from the natural number to the decimal number.

Vietnamese lexical words for rational numbers have explicit meanings, for example, "tenths" is said "part ten", such that “four tenths” is called “four part ten”. The same is true for "hundredths" and "thousandths", unlike in French and English languages where terms dixième (tenths) and dix (ten) differ only by the word’s ending. The meaning of the words used for rational numbers in the Vietnamese language is effective for the conceptualization of these numbers. Terms used for fractions (e.g., "part ten") explicitly show the part-whole relation and help children to recognize more easily the difference between natural and rational numbers. We hypothesize that this feature of the Vietnamese language helps children during the generalization process of the decimal concept, especially children who have not yet learned decimals.

In Belgium, children have two expressions that refer to decimals. The first one is the "tenths, hundredths and thousandths" type (e.g., 5, 23 is read as "five units twenty-three hundredths"). This expression is the one that is officially taught in school from grades 4 to 6. The second expression is "number with a comma" (e.g., 5,23 is read as "five comma twenty-three"). This expression is typically used by older children (from grade 6) or when children need to quickly name a decimal number for operations. The expressions "tenths, hundredths and thousandths" seems complicated compared to “number with a comma”, yet this expression illustrates the meaning of the decimal number and clearly outlines the position and value of the digits that form the number. This is an advantage for Belgian children, as this expression is more helpful for children who are beginning to learn the decimal concept. In addition, this expression enhances the understanding of children who have learned the decimal, as it prevents misconceptions and confusion with naturals and eliminates some epistemological obstacles. However, elements of the rational number, like tenths, hundredths and thousandths, may lead to some confusion with tens, hundreds and thousands from the natural number, given the similarity between these words. This may influence Belgian children (from a French-speaking community) who identify decimal numbers as natural numbers.

1.4 Present study

The purpose of the current paper is to examine the development of Arabic representations of decimal numbers with regard to the generalization process. We predict that in both countries, most children will be affected by their natural numbers knowledge when producing a symbolic representation of a decimal number. Vietnamese children will be more influenced by their fraction knowledge than Belgian children. The influence of the order of topics in the school curriculum and the various terms used for rational and decimal

numbers were also examined. Given the support from the use of transparent terms in Vietnamese language to refer to rational number in comparison with the terms used in the French language, we expect Vietnamese children who have not yet learned decimals (grades 3 and 4) to be more advanced than Belgian children (grades 3 and 4) even if at grade 4, Belgian children already started to learn them. But, by grade 5, when children have learned decimals in both countries, Belgian children should outperform Vietnamese children when writing the numbers from the column names (e.g., "two units and six tenths") into Arabic numbers given the Belgian's efficient system for naming decimal numbers.

2. Method

In the present work, we used a transcoding task in which children were asked to write an Arabic number from its written-verbal form. We investigated how children constructed their conceptual sense of a symbolic representation of a decimal number from their preexisting knowledge with regard to the generalization process. Moreover, the transcoding task from a verbal number into an Arabic number allowed us to examine children's place-value understanding of the decimal number. It is interesting to investigate differences between when the available knowledge is only with regard to natural numbers (grade 3) and when it includes fraction knowledge (grade 4). Therefore, the current study was conducted with younger children than those examined in previous studies, our participants being enrolled in grades 3 through 6.

2.1 Participants

Our Belgian sample included 87 children in grade 3 (39 girls and 48 boys; mean age = 108 months, SD = 6.6), 68 children in grade 4 (34 girls and 34 boys; mean age = 120 months, SD = 6.0), 138 children in grade 5 (66 girls and 72 boys; mean age = 129 months, SD = 7.7) and 123 children in grade 6 (62 girls and 61 boys; mean age = 143 months, SD = 5.0). Our Vietnamese sample included 224 children in grade 3 (110 girls and 113 boys; mean age = 105 months, SD = 3.7), 435 children in grade 4 (192 girls and 241 boys; mean age = 118 months, SD = 6.1) and 693 children in grade 5 (354 girls and 338 boys; mean age = 128 months, SD = 4.0). The children belonged to several schools that were randomly selected from Vietnam and a French-speaking community in Belgium[2].

2.2 Procedure and Materials

A "paper/pencil" test was designed to examine the development of children's ideas about the representation of the decimal number. We asked pupils to write a number in

[2] There are three official languages taught in Belgium: Dutch, German and French. Consequently, Belgium shelters a French speaking community, a Dutch speaking community, a German speaking community and a mixed French/Dutch speaking community. To avoid potential bias, we chose our samples exclusively in areas where French is the native language of people and the official language used in school. Children of our sample were learning Dutch and English as second languages, just like most French children learn English, Spanish or German in French schools.

Arabic form from a written-verbal number, for example, they were asked to write an Arabic number containing "5 units, 2 tenths and three hundredths" (5.23).

The task included 12 items that were divided into two categories. The first category (comprising 0.5;0.7;0.08;0.06;0.002 and 0.007), entitled the "Integer part equaled zero", comprised items with an integer part that equaled zero and had a single primitive element of the decimal, which were the tenths, hundredths or thousandths (e.g., "five tenth"→ 0.5). In the second category (comprising 5.9; 63.8; 35.08; 12.05; 15.008;63.005), entitled the "Integer part differed from zero ", items had an integer part that was different from zero and a single decimal element (e.g., "two units and six hundredths"→ 2.06). Together, these 12 items were ordered randomly and then presented in the same order for all participants. The test was presented collectively in each classroom at the end of the school year. Children worked individually without time constraints. Test completion required approximately ten to fifteen minutes.

2.3 Data analysis

Correct responses were marked with 1, and incorrect ones with 0 but all responses were also kept in their original form for the qualitative analysis.

First, we conducted an analysis of variance (Anova) to investigate whether there was a difference of performances across grade and country and the interaction between them.

Then, the qualitative techniques were also use to examine the generalization process. We diagnosed children's transcoding errors from grades 3 to 6 and then classified the errors into various levels of conceptual evolution with regard to the decimal and with the goal of illustrating the two types of generalization: extension-generalization and coordination-generalization.

Next, we used the cluster analysis. It is a task of grouping individuals or objects with similar features in the same group (called a cluster). This classification is based on the distance connectivity of each objects. The elements in the same cluster are more similar to each other than to those in other groups (clusters). The choice of number of groups is often ambiguous. The number of clusters can emerge automatically from some rules or can be fixed by the user. It depends on the shape and scale of the distribution of points in a data set and the desired clustering resolution of the user. In the present study, the cluster analysis was run on the six types of responses obtained from the qualitative analysis mentioned above. Children with similar responses patterns were classified in the same cluster. From the clusters obtained, we analyzed the distribution of children in each cluster to examine the evolution of children across grade according to the types of response and also to examine the difference between the two samples, Vietnamese and Belgian children.

Finally, an analysis of variance was run to verify whether a decimal number starting with a zero led to difficulty for children.

3. Results

3.1 Overall analysis (achievement in the conceptual decimal)

The means and standard deviations for the correct responses for the whole task according to grade and country are reported in Table 3. We first investigated children's global performances across grades and the differences between Belgian and Vietnamese children. A two-way analysis of variance (ANOVA) with grade (3-5) and country (Belgium and Vietnam) as the between-subjects variables was conducted. The effect of country was significant, $F(1, 1639) = 44.90$, $p < .001$, partial $\eta^2 = 0.027$, indicating that the Vietnamese children had higher scores than the Belgian children. The difference between the three grade levels was also significant, $F(2, 1639) = 293.43$, $p < .001$, partial $\eta^2 = 0.26$, showing that correct response rates increased with grade levels. Post hoc t-tests showed that the performances for each grade differed significantly from each other (in all cases, $p < 0.001$). Finally, the interaction between country and grade was significant, $F(2, 1639) = 31.20$, $p < .001$, partial $\eta^2 = 0.037$, suggesting that the advancement across grades differed according to the country. In fact, the difference between grades 3 to 5 was less pronounced in Vietnam than in Belgium. The Vietnamese children in grades 3 and 4 outperformed the Belgian children, but this effect was reversed in grade 5, as illustrated in Figure 1.

Table 3. Means correct responses and standard deviations (in percentages) by country

	Belgium(n=416)		Vietnam(n=1352)	
Grade	Means	SD	Means	SD
3(n=311)	11.92	13.06	45.66	39.55
4(n=503)	41.23	34.03	56.87	42.35
5(n=831)	89.85	18.40	84.44	24.86
6(n=123)	92.31	14.31		

To better understand the improvement in performances across grade levels in each country, we conducted two one-way ANOVA, one for each country. For both countries, the effect of grade was significant, $F(3, 412) = 382.88$, $p < .001$ for Belgium; $F(2,1349) = 152.31$, $p < .001$ for Vietnam. A multiple-comparisons test showed that the difference was not significant between grades 5 and 6 in Belgium ($p > .05$). In all others cases, the performance differed significantly for each grade (in all cases, $p < 0.001$

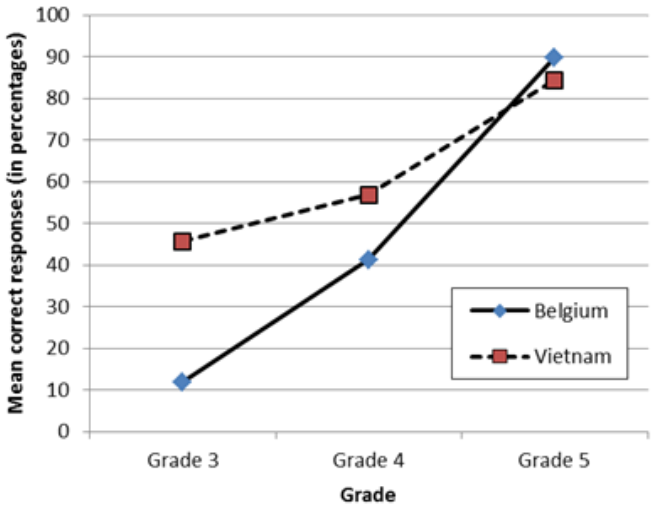


Figure 1. Mean correct responses (in percentages) by country

3.2 The generalization process for decimal number representation

We studied the development of decimal number representation with an emphasis on the distinction between the following two types of generalization: extension and coordination. Then, we classified children's responses through an errors analysis from grades 3 to 6 into four levels. These four levels were based on the conceptual evolution of the decimal, which clearly illustrated the two types of generalization that were previously described.

Level 1: The decimal number is identified as a natural number

Children treated decimal numbers as naturals, even in the presence of new lexical words, such as tenths, hundredths and thousandths, in the verbal expression. From the perspective of the generalization process, this level corresponds to the generalization type known as extension. The decimal number is considered a natural number, without any conceptual development necessary for the new content. Students did not take into account the existence of the expression of the decimal number. Indeed, for all of the verbal numbers, such as "seven tenths" or "five units and nine tenths", the most common responses were 7 or 59, which could be explained according to a familiarity reflex with the natural number. We also recorded responses like 17, 70 and 710 for "seven tenths" and 590, 5910, 95 and 950 for "five units and nine tenths". We can interpret these responses as being influenced by the confusion induced between the words "tenths" and "tens" in the French language and "seven part ten" (0.7) with "seven ten" (70) in the Vietnamese language. Similar patterns were observed for items such as "seven hundredths" or "two thousandths".

From grades 3 to 6, children gain an understanding of the meaning and cardinality of natural numbers. Yet, it seems that the natural number is the only type of number presented

to children at this level. When children produce answers, they tend to respond according to natural numbers, despite the presence of new elements such as the tenths, hundredths or thousandths place.

Level 2: The utilization of the comma (period) to write the decimal number

Level 2 differed from Level 1 in the appearance of a comma to represent the decimal number. Children recognized the role of new lexical words, such as tenths, hundredths and thousandths, and showed a conceptual change by using the comma to produce an Arabic number. Although they seemed to have conquered an evolution in their conceptualization of decimal numbers, a trace of natural numbers was still evident, mainly for cardinal number and the position order in a number. For example, for the item "seven tenths", responses such as 7.0, 7.1, 7.10, 7.0.10, 7.01, 1.7, 10.7, 70.1 and 70.0 were observed. In a similar pattern, such responses as 2.0, 2.00, 2.000, 2.1, 2.10, 2.100, 2.1000, 2.001, 2.0001, 100.2, 1000.2 and 200.0 were recorded for the item "two thousandths".

When transcoding a natural number from a verbal-number into an Arabic number, children typically transcode each word according to its corresponding Arabic counterpart (e.g., "two hundred five" (205 in French) □2005, "seven ten" (70 in Vietnamese)□710). For the item "seven tenths", children had not yet acquired the concept of decimal or they had misconceptions about the syntactic rules when writing an Arabic number. It seems that children interpreted tenths as .0 (point zero), .1, .01 or 0.10 and "seven" as "seven units" of a natural number. According to the order of appearance of "seven" and "tenths" in the verbal-expression, responses such as 7.0, 7.1, 7.10, 7.0.10 and 7.01 were observed.

Responses such as 1.7 or 10.7 represent an overgeneralization of the relational order in the natural number. The tens-ones order of the natural number was applied for the tenths-ones in the decimal number.

The idea that a number cannot start with a zero was consistent in children's responses at this level. Many children answered 7.10 or 7.0 for "seven tenths" and 2.000 or 2.1000 for "two thousandths", yet simultaneously responded 5.9 (correct response) or 5.910 for "five units and nine tenths". This supports the observation that children tried to combine their previous knowledge with new information to solve a novel problem. For some of these children, we observed contradictory responses with items having an integer part that was equal to zero or different from zero. The cluster analysis described in the next section will provide a better understanding of this situation.

In sum, responses classified as Level 2 showed a conceptual evolution to fit new information about decimal numbers. Children, for the first time, distinguished the difference between the symbolic representations of the natural number and the decimal number by using a comma to write the decimal number. The influence of their initial knowledge about natural numbers still had an important impact on many aspects of this process. First, children identified the decimal terms (i.e., tenths-hundredths-thousandths) with tens-hundreds-thousands in the natural number. The confusion between tens and tenths, hundreds and hundredths, and thousands and thousandths was still very strong. Second, the relational order of the natural number system (i.e., thousands-hundreds-tens-

units) was directly applied to the decimal number (i.e., thousandths-hundredths-tenths-units) during an overgeneralization process. Finally, the idea that a number cannot start with a zero was consistent among children. Misconceptions about the decimal number and systematic incorrect responses classify Level 2 as a "coordination" generalization type. This level constitutes a transitional phase in the process of acquiring the Arabic representation system and the concept of the decimal number.

Level 3: Partial acquisition of the decimal number

Responses grouped at this level showed progress in acquiring the concept of the decimal number. Children's responses were divided into the following two subcategories: subcategory 3A and subcategory 3B.

Responses classified in subcategory 3A presented an integration of fraction knowledge when writing decimal Arabic numbers. Given that terms such as thousandths, hundredths and tenths were simultaneously used to describe both fraction and decimal numbers, children in this subcategory showed confusion when choosing the right notation for writing decimals. They combined their prior knowledge of fractions with their presuppositions about decimals to produce responses, such as 6% , 6/100, 6/1000, 6/10, 6/1, 6/0, 6/000 and 6c for "six hundredths" (note that the French unit is "Unité" and the hundredths is "Centième"; hence, the letters "u" or "c" were evident among children's responses). For "thirty-five units and eight hundredths", which is an item that has an integer part that differs from zero, responses included " 35. 8/100", "35 and 8/100", 35.8% and "35 u,8c". We can see that even when the last item responses were incorrect, children correctly represented the quantity aspect of the number.

Subcategory 3B represented a radical conceptual change in the symbolic representation of the decimal. Children used the decimal point (comma) to present a decimal's notation and improved their understanding of cardinal numbers. This subcategory was divided into two groups, which were 3B1 and 3B2. Responses such as 0,0,0,2, 0,2,0,0 and 000,2 for "two thousandths" were placed in category 3B1, which included children who used many decimal points simultaneously or many zeros before the decimal point to produce an Arabic number. These children assimilated their understanding of natural numbers as related to the positional system by using 4 digits to present a number in the "thousandths". Category 3B2 included responses that indicated an almost complete acquisition of the decimal number notation. Children in this subcategory had not yet acquired the syntactic rules to produce an Arabic number and were still affected by their natural knowledge when trying to determine the number of digits. This included such errors as using four digits following the decimal point to write "thousandth", which is similar to using four digits to write "thousands" in the natural number system, for example, "two thousandths" led to responses such as 0.0002 and 0.2000 and "six hundredths" led to responses such as 0.006 and 0.600. A similar pattern of responses was observed for items with integer parts that were different from zero. For instance, "thirty-five units and eight hundredths" produced 35.800 or 35.008 as responses. Children used three digits to indicate the "hundredths" place. The use of these incorrect rules suggests a misunderstanding with regard to the role of the decimal point. Children

considered the decimal point to be a symmetric point consisting of two parts of the decimal number, an integer part and a decimal part, instead of the units' position.

Level 4: Only correct answers

Only correct answers were produced at this level. Children showed a profound place-value understanding and an advanced conceptual evolution of the decimal number.

3.3 Cluster analysis

A cluster analysis was conducted to determine the groups of children with homogeneous response patterns. Based on the four levels of the generalization process previously described, we identified the following six types of responses: Natural (Level 1; responses are based on natural number knowledge), Naïve decimal (Level 2), Fraction (subcategory 3A; responses are based on fraction number knowledge), Partial decimal (subcategory 3B), Correct responses (Level 4) and Not-categorized (includes mixed responses that could not be classified according to an internally consistent category). Five clusters emerged that were based on these response types.

Cluster 1 (n = 117) corresponded to children whose main idea was to assimilate the decimal number as a natural number. They showed weak or nonexistent knowledge of the decimal number's notation and borrowed the natural number's notation to represent the decimal number. For the items with an integer part that equaled zero, most responses belonged to Level 1 (70% to 92%). Responses for the others items were primarily of the Partial decimal type. Cluster 2 (n = 147) comprised children who mainly responded with Naïve decimal (67.5% to 94.9%) and Partial decimal (66% to 78%) types. In this cluster, responses for Level 2 (Naïve decimal) were evident only on items with an integer part that equaled zero, which expresses the natural number trace given that a number cannot start with a zero. Cluster 3 (n = 114) included children who had a symbolic representation of the decimal that was mainly based on their knowledge of the fraction notation. They combined the fraction notation with the decimal point to transcribe the Arabic representation of the decimal number. Cluster 4 (n = 221) included children who provided correct responses for some items (36.7% to 49%) and the remaining responses were of the Partial decimal type (40% to 45%). These children displayed a developing understanding of decimals, yet they had difficulty representing the decimals, which suggests a misunderstanding of the syntactic rules for producing an Arabic number. Finally, Cluster 5 (n = 1169) was comprised of children who had developed a complete understanding of the decimal concept in its Arabic representation. Children in this cluster produced mostly correct responses for all of the items (93.2% to 97.5%).

Table 4. Means of correct responses (in percentages) and the percentage of children distributed in each cluster

Cluster	Cluster1	Cluster2	Cluster3	Cluster4	Cluster5
Means	15.9	15.5	4.2	43.14	96.1
Grade 3	26.7	21.9	4.8	18.6	28
Grade 4	8.3	8.3	17.7	14.9	50.7
Grade 5	2.5	0.8	1.2	9.9	85.6
Grade 6	0.8	0	0	4.9	94.3

Table 4 presents the percentages of children distributed across the five clusters. Children who belonged to Cluster 5 increased with each grade level, whereas they decreased for Clusters 4, 1 and 2. Except for Cluster 3, the highest percentage was evident in grade 4. Obviously, older children (grades 5 and 6) were grouped into the most advanced clusters. A chi-square test showed that grade was a statistically significant factor, $\chi^2(12, N = 416) = 331.29$, $p < 0.001$, which means that the distribution of children across the clusters depended on grade level.

A separate analysis of the distribution of the clusters was conducted to examine the influence of the different curriculums and languages between the Belgian and Vietnamese samples on the generalization process for the decimal number.

Table 5. Percentage of children distributed in each cluster per grade across country (BE = Belgium; VN = Vietnam)

	Cluster 1		Cluster 2		Cluster 3		Cluster 4		Cluster 5	
	BE	VN	BE	VN	BE	VN	BE	VN	BE	VN
Grade 3	62.1	12.9	6.9	27.7	1.1	6.3	28.7	14.7	1.1	38.4
Grade 4	20.6	6.4	4.4	9.0	2.9	20.0	39.7	11.0	32.4	53.6
Grade 5	1.4	2.7	0	1.0	0	1.4	2.2	11.4	96.4	83.4
Grade 6	0.8		0		0		4.9		94.3	

Table 5 presents the percentages of children distributed across each cluster per grade for each country. A chi-square test confirmed the influence of grade on the distribution of the clusters for both countries, $\chi^2(12,N = 416) = 331.29$, $p < 0.001$ for Belgium and χ^2 $(8,N = 1352) = 366.38$, $p < 0.001$ for Vietnam.

In Belgium, the cluster membership distribution for each grade differed significantly from the expected distribution, as shown by a chi-square analysis. In grade 3, $\chi^2(4, N = 87) = 118.69$, $p < 0.001$, the majority of children belonged to Cluster 1 (62%) and Cluster 4 (28.7%), suggesting that these children considered the decimal number to be a natural number. Few children were in Cluster 5 (1.1%) and Cluster 3 (1.1%). In grade 4, the number of children belonging to more advanced clusters increased, $\chi^2(4, N = 68) = 36.60$, $p < 0.001$. It is important to note that Belgian children in grade 4 have officially learned the decimal system at school, yet only 20.6% of children belonged to Cluster 5. An evolution was found in grade 5, $\chi^2(4, N = 138) = 246.83$, $p < 0.001$, and in grade 6, χ^2 $(4, N = 123) = 206.10$, $p < 0.001$. Indeed, almost all of the children in these grades belonged to Cluster 5 (96.4% for grade 5 and 94.3% for grade 6). There were not many Belgian children who belonged to Cluster 3.

A similar pattern was observed in the Vietnamese sample. Children who belonged to Cluster 5 increased with grade level, whereas they decreased for Clusters 4, 2 and 1, with the exception of Cluster 3. The distribution of cluster membership differed significantly from the expected distribution for each grade. In grade 3, $\chi^2(4, N = 244) = 74.35$, $p < 0.001$, there were more Vietnamese children in the advanced clusters than was evident in the Belgian sample. More than 50% of children belonged to Clusters 4 and 5. In grade 4, $\chi^2(4, N = 435) = 329$, $p < 0.001$, the highest percentage of children in Cluster 3 was evident, which confirms that available knowledge plays a substantial role in the construction of new knowledge. In grade 5, $\chi^2(4, N = 693) = 1766$, $p < 0.00$, most children responded correctly (83.4% in Cluster 5 and 11.4% in Cluster 4).

The cluster analysis confirmed our description of the Overall analysis for the Results part that was previously discussed. In grades 3 and 4, Vietnamese children were more advanced than Belgian children, and the difference between grades was more drastic in Belgium than in Vietnam, as illustrated in Table 5.

3.4 The case of the integer part that equaled zero

Table 6 presents the means of correct responses and standard deviations (in percentages) for each cluster for the category Integer part equaled to zero and Integer part differed from zero.

Table 6. Means correct responses and standard deviations (in percentages) for each cluster for the category Integer part equaled to zero and Integer part differed from zero

	Integer part equal to zero		Integer part differed from zero	
Cluster	Means	SD	Means	SD
Cluster 1	1.67	6.72	29.29	15.73
Cluster 2	2.92	8.34	27.32	15.81
Cluster 3	0.9	4.42	7.27	16.97
Cluster 4	48.13	29.12	47.73	21.43
Cluster 5	96.58	6.41	95.81	9.05

Items with an integer part that equaled zero were difficult for children in the less advanced clusters of our previous analysis, such as those in Cluster 2. A number starting with a zero is new and contradictory to children's existing knowledge of natural numbers. Some children could provide a correct response, such as 5.9 for "five units nine tenths" and then respond with 5, 5.10 or 5.0 for "five tenths". To investigate the effects of having the integer part equal to zero, a repeated measures ANOVA on the correct response rates with category (Integer part equaled to zero and Integer part differed from zero) as the within-subjects variable and the cluster (Cluster 1 to Cluster 5) as the between-subjects variables was run. The main effect of category was significant, $F(1,1670) = 348.74$, $p < 0.001$, partial $\eta^2 = 0.173$, which means that a decimal number with an integer part that equaled zero was difficult for children. The effect of cluster was also significant, $F(4,1670) = 6632.66$, $p < 0.001$, partial $\eta^2 = 0.941$. The difference in performances across the five clusters was highly marked. A pairwise comparison showed that all of the pairs differed significantly from each other (in all cases, $p < 0.001$), except for the performances of Clusters 1 and 2,

which were similar ($p = 0.7$). The interaction between category and cluster was significant, $F(1, 1670) = 131.59$, $p < 0.001$, indicating that the influence of an integer part that equaled zero affected children differently across the clusters. An ANOVA examining this impact of unit equaled zero was conducted on each cluster individually. The results showed that this effect was significant only for the three less advanced clusters, which were Clusters 1, 2 and 3 (in all cases, $p < 0.001$) which means that a decimal number with an integer part that equaled zero lead to lower performances for these children. Although this effect was not significant ($p = 0.23$ for Cluster 5 and $p = 0.89$ for Cluster 4), as illustrated in Figure 2.

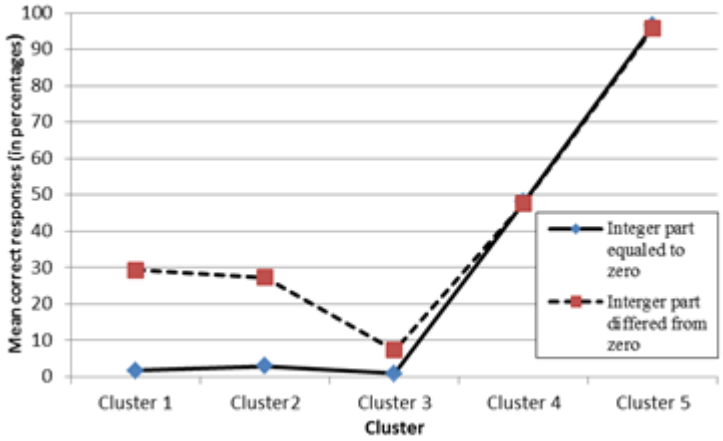


Figure 2. Mean correct responses (in percentages) by cluster for the categories of integer part equaled to zero and integer part differed from zero

4. Discussion

The present study describes the transition from a naïve to a complete understanding of the decimal number's symbolic representation through the generalization process. We demonstrated how children generated their presuppositions about decimals and how they reconciled their initial knowledge with the actual notation. Many previous studies have investigated the development of children's understanding of the rational number in such domains as the comparison of decimal numbers (Desmet et al. 2010; Nesher & Peled 1986; Sackur-Grisvard & Leonard 1985; Stacey 2005; Resnick et al. 1989), the numerical value of fractions (Stafylidou & Vosniadou 2004) and the concept of density (Vamvakoussi & Vosniadou 2004). All of these studies have argued that the acquisition of the rational number was a gradual process and not an immediate one. Our study focusing on the acquisition of the symbolic representation of the decimal number confirmed this idea. Five groups of participants were identified based on different response categories. Children's responses illustrated the development of symbolic representation of the decimal number. Children in Clusters 1 and 2 were clearly influenced by their knowledge of the natural number system. In these groups, we observed difficulties with numbers starting with a digit

that was different from zero and with conservation of the ordering sequence "thousands-hundreds-tens-units" for "thousandths-hundredths-tenths-units" (e.g., "seven tenths"→7, 17 or 10.7). Cluster 3 emerged from children's attempts to access their knowledge about fractions and their willingness to present a decimal number (e.g., "two units seven tenths"→2, 7d or 2, 7/10). Cluster 4 showed a radical conceptual change yet children were still affected by syntactic rules when they wrote decimal Arabic numbers (e.g., "eight tenths"→0.80, 0.08 or 0.18). Finally, Cluster 5 comprised children who had a good conceptual understanding of the decimal number.

According to Steinle and Pierce (2006), the task of comparing decimal numbers is very helpful to identify students with an incomplete or erroneous conceptual understanding of decimal number. In aforementioned studies, using comparison tasks, it was not possible to determine whether this understanding was present. Children could have applied the technical rule of adding zeros to the shorter number and then compared the decimals the same way that they compared natural numbers. This rule is easy to learn, simple to apply and mostly leads to correct responses, but it does not help children to develop a conceptual understanding of the decimal. We suggest limiting the use of this rule in instruction, especially during early stages of learning the decimal numbers. Roche and Clarke (2006) conducted a study with students from grades 3 to 6, interviewed individually on a rank of tasks related to the understanding of the relative size of decimals. Results showed that students classified as "apparent expert" used a rule of adding zeros to numbers in order to obtain decimals with the same length to compare them. These students did not demonstrate a stable understanding of the relative size of decimals, contrary to those with a good place value understanding. This suggested the limitation of a comparison task.

In our study we used a transcoding task from written-verbal "column names" into Arabic numbers. This provided an occasion to access children's place-value understanding and a reliable tool to assess their conceptual understanding of decimal numbers. Based on all the different types of errors produced on this transcoding task, we were able to provide some implications, described below, on rational numbers teaching. We believe that they could help children to avoid some overgeneralization mistakes.

This study assessed children from grade 3, who did not have any formal knowledge of the decimal, to the last year of primary school (grade 5 in Vietnam and grade 6 in Belgium). Children in Clusters 1, 2 and 4 decreased from grades 3 to 6 and the percentage of children in Cluster 5 increased with the grade level increase. The percentage of children in Cluster 3 increased from grades 3 to 4 and then decreased in grade 5. This pattern of changes was consistent with the results of previous studies (Nesher & Peled 1986; Resnick et al. 1989; Sackur-Grisvard & Leonard 1985). This particular change in the percentage of children classified in the group that was influenced by fraction knowledge (i.e., Cluster 3) can be explained by the fact that fractions are taught in grade 4, which is prior to the official introduction of the decimal number in grade 5. The differing order of topics in decimal teaching, which directly affects children's existing knowledge, also influences the distribution of the types of responses. Fractions are taught in grade 4 and then decimals in grade 5 in Vietnam, whereas, in Belgium, fractions and decimals are taught in parallel in

grades 4 and 5. The percentage of responses that were guided by fraction knowledge was higher in Vietnam than Belgium. These results are consistent with the results of the study by Resnick et al. (1989) that was conducted with French, American and Israeli children. These results also confirm our hypothesis about the impact of different curriculums. The highest rate of children belonging to Cluster 3 was found in grade 4 of the Vietnamese sample. This emphasizes the important role of available knowledge when children are attempting to use their existing knowledge to construct a symbolic representation of a new concept. In previous studies (Resnick et al. 1989; Nesher & Peled 1986), the use of a natural-rule was more common than a fraction-rule in the comparison of decimals. In this study, we found that more incorrect responses were based on natural number conceptions than fraction knowledge. These points to the substantial influence of natural knowledge and the temporary leverage of fraction understanding in the conceptualizing process of the decimal number. These influences lead to incorrect responses that are unavoidable during the process of moving to a new concept. Despite differences in language and curriculum between our two samples, we observed similar features in children's systematic incorrect responses. This confirms that the shift from natural to rational numbers consists of many intermediate phases and that the generalization process for the decimal number has many transitive levels. Our results also emphasize the concept of an epistemological obstacle (Brousseau 1998) that is derived from natural and fraction knowledge in the learning of decimal numbers.

In the early state of decimal number learning (grades 3 and 4), Vietnamese children outperformed Belgian children when writing the decimal Arabic numbers from verbal code. This was evident in the advanced performance results (Table 3) and the higher percentage of children in the advanced clusters (Table 6). A difference in mathematical language could explain the differences between the two countries. The terms used for rational numbers in the French language are similar to those used in the natural number system, such as "tenths" and "tens", for instance, which may reinforce the potential symmetry between naturals and decimals. In Vietnamese, "tenths" is pronounced "part ten", which may help children distinguish the difference in quantities between the two terms and aid in their conceptualization of the meaning. These subtle differences could affect the ability to produce an Arabic number from a verbal expression, particularly for younger children who have not yet learned the decimal concept in school. A study by Miura and colleagues (1999) examining the influence of language on numerical fractions suggested that young Korean children (6- to 7-year-olds) outperformed American and Croatian children in a task in which they had to associate a fraction's notation with its corresponding graphical representation. The authors attributed their results to the explicit meaning of the words used for fractions in Korean, which is similar to the Vietnamese language.

In grade 5, when children had acquired the decimal in school, Belgian children outperformed Vietnamese children. These results could be due to many factors, such as differing instruction, approaches and practices in teaching and learning, or they may be due to varying numerical experiences with rational numbers between the two countries. The Belgian curriculum choice with regard to the denomination of the decimal number in

primary school could be an advantage for Belgian children when writing decimal Arabic numbers. The decimal is pronounced with the expressions of "tenths, hundredths and thousandths" (e.g., 5.92 is pronounced "five units nine tenths two hundredths"), which provides the opportunity to better acquire place-value understanding and favors the presentation of the decimal Arabic number from the verbal number. This suggests a superiority in the Belgian curriculum when compared to other countries, such as Vietnam, where the decimal is pronounced with its symbolic representation (5.92 → "Five point ninety-two"). It would be interesting to examine this supposed efficiency in a future study. During the early stages of decimal learning, teaching with the expressions "tenths, hundredths, and thousandths", which explicitly present the position of each digit in the verbal-number, might limit the previously mentioned epistemological obstacle.

As Nesher and Peled (1986) stated, the process of moving from the most naïve to the most expert state when mastering decimals depends on specific pieces of knowledge and is related to the full acquisition of basic concepts. This process can be applied to the acquisition of the symbolic representation of decimals in the current study. Initially, children need to master place-value understanding and the syntactic rules linked to sum and multiplication relationships in the decimal number system (base-10) in natural number system. Such responses as 7, 17, 710 and 70 for "seven tenths" show a weak understanding of the natural number positional system. Vietnamese children make these errors, despite "seven tenths" being pronounced as "seven parts (of) ten". In addition, the moving process from natural to rational numbers consists of many intermediate levels that require a reorganization of prior knowledge structures. Thus, during the teaching of decimal numbers, instructions that are designed to emphasize adjacent knowledge and present similarities and differences between the natural, fraction and decimal number systems could help children to understand more deeply the structural and coherent aspects of these mathematical principles. Encourage the use of fractional language to refer to decimal number names rather than naming decimal numbers with their notation (e.g., "three units and five tenths" rather than "three point five"). This facilitates a clearer conception of the decimal numeration system for decimal numbers. Teachers should pay more attention to the similarity and differences between terms used for natural and decimal numbers (tens, hundreds, thousands and tenths, hundredths, thousandths). This could facilitate the appropriate use of existing knowledge to acquire a conceptual understanding of decimal numbers and avoid confusion between the different sets of numbers.

Chapter 7
The misunderstanding of the part-whole relationship: a didactic obstacle

The misunderstanding of the part-whole relationship: a didactic obstacle

Abstract

The part-whole subconstruct of the rational number is considered as the foundation of rational number knowledge and the approach using the part-whole relationship is privileged in teaching the fraction number. This study examines children's misunderstanding of the part-whole relationship. A total of 203 of Grade 4 children from Belgium (French-speaking Community) and Vietnam were tested, using tasks in which they had to associate a graphic representation with its corresponding fraction symbol. The data document that a majority of children used an incorrect part-whole understanding in manipulation of fractions greater than 1. The number of all parts was used instead of the number of parts in a whole to indicate the denominator. It showed the limitation of the part-whole scheme in the instruction of the fraction number. Different curricula affected the acquisition of the part-whole knowledge. Implications for the teaching of fractions are proposed in an attempt to correct this misunderstanding.

1. Introduction

The fraction concept allows the realization of any division a:b ($b \neq 0$) in which a and b are two whole numbers, even if the dividend a is inferior to the divisor b. This is impossible for an Euclidean division in an ensemble of natural numbers. The fraction concept is always tied to in the action of partitioning and to the part-whole relationship. Fractions smaller than 1 are usually presented in the early teaching of this concept (e.g. the notation 3/4 represents a quantity composed of three parts that are each one fourth the size of the unit whole). On one hand, this approach renders necessary an apparent and explicit sense of the fraction concept which is consistent with the part-whole relationship. However, it can lead to immature presuppositions "a fraction is always inferior to 1" or "a fraction is a part of a unit". The consistent perception of a fraction as part of a whole shape can lead to difficulty for children in adapting and acquiring later fraction interpretations (Pitkethly & Hunting, 1996; Kerslake, 1986). Children could overgeneralize their understanding of the part-whole relationship, originally derived from the manipulation of graphic representations of fraction smaller than 1, for all fractions. During this manipulation, they counted "the number of shaded parts" and "total number of parts" in the graphic representation to determine the numerator and denominator of the fraction symbol. This method is only correct for fractions smaller than 1, when the "total number of parts" is identical to "the number of parts in a whole". This incorrect rule obstructs the development of the unit-fraction concept that was considered as a foundation for developing rational number knowledge (Kieren, 1993abc; Steffe & Olive, 1993). Much of the existing research on the part-whole relationship has mainly investigated fractions smaller than 1 (Ni, 2001; Saxe, Taylor,

McIntosh & Gearhart, 2005). The current study investigates the misunderstanding of the part-whole relationship. Fractions inferior and superior to 1 were used to examine the incorrect rule of this relationship embodied in the graphical representation.

In the following sections, we will discuss successively the role of the part-whole relationship with rational numbers, the action of partitioning and the different curricula in the fraction number construction.

1.1 The role of the part-whole relationship in rational number knowledge

The study of rational numbers is complex because these numbers can be introduced and approached through a wide variety of subconstructs. Kieren (1980) suggested five subconstructs: part-whole relations, ratios, quotients, measures and operations, which have met with a wide consensus of opinion from many research studies (Behr, Lesh, Post & Silver, 1983; Freudenthal, 1983; Vergnaud, 1983; Ohlsson, 1988). If each subconstruct relates to different cognitive structures and presents the rational number from different perspectives, together, they constitute a full meaning of the rational number. This is consonant with the Vergnaud's theory of conceptual fields of which suggests that the meaning of a mathematical concept always concerns a variety of situations "whose analysis and treatment require several kinds of concepts, procedures and representations that are connected with one another"(Vergnaud, 1990). Among these five subconstructs of the rational number, mentioned by Kieren (1980), the part-whole subconstruct was considered as the most important and as the foundation of rational number knowledge (Kieren, 1988; Pitkethly & Hunting, 1996). The part-whole subconstruct was also suggested as fundamental in acquiring the other four (Behr & al., 1983; Charalambous & Pitta-Pantazi, 2007). In fact, the part-whole relationship is a dominant reference code for the representation of fraction numbers. The relation between a part and a whole is the source of the unit fraction concept, which is a basis for subconstruct measures. In the case of discrete quantity, this relation represents a fundamental construct for the ratio concept.

Concerning the four stages of rational concept apprehension (i.e. the action of partitioning, the part-whole relationship, measure-rational and number-rational), researchers have suggested that the part-whole relationship plays an intermediate and important role for rational number instruction. In fraction number teaching, an approach using the part-whole relationship is often favored. The visual representation in manipulation contexts, with fair sharing and then association with the part-whole knowledge is a common way to introduce the concept of fractions. In particular, the act of dividing a whole in parts is dominant in fraction teaching, so children have the tendency to consider a fraction as "a part of a whole" and "a fraction is always smaller than 1 or a whole". This approach poses some limitations for the understating of the fraction concept and restrains the representation of the unit in fraction numbers in different ways (Behr, Harel, Post & Lesh, 1993).

1.2 Action of partitioning

Before the formal introduction of fractions in school, most children have already experienced fair sharing through classroom daily activities. In preschool, children have occasions to halve, which is considered as an early development of action partitioning (Hunting & Korbosky, 1990). The repeated folding of a unit and the inverse relation between the size of a part and the number of folds express the conception of partitioning (Kieren, Mason & Pirie, 1992). Five year old children show an understanding of partitioning into equal parts, the inverse relation between a number of people and the size of parts and the ability to add and subtract fractional parts of an area (Pitkethly & Hunting, 1996; Sophian, Garyantes & Chang, 1997; Mix, Levine & Huttenlocher, 1999).

Partitioning is fundamental to construct the concept of fraction and this action is usually associated with part-whole interpretation to introduce the representation of fractions. The action of partitioning an area into fractional parts is approach often adopted (Corwin, Russell & Tierney, 1991; Saxe et al., 2005). Many previous studies (Ball 1993; Mack 1990, 1993; Kieren & al. 1992; Streefland 1991, 1993) have hypothesized that partitioning is the key to building initial fraction concepts and to developing a broad understanding of rational numbers. In fact, the ability to partition allows an awareness of the equality between the sum of parts and the total or the relation between the size of a part and the number of parts ("if the number of parts is bigger, the size of each part is smaller"). The result of the action of partitioning also leads to a new quantity that creates the unit-fraction concept.

Brissiaud (1998) distinguished two different types of partitioning schemes in the approach to the fraction concept. For example, the number 13 refers to a measure (e.g. 13mm) and the number 4 indicates a quantity. The fraction 13/4 (e.g., 13mm/4) is read "13 divided by 4" and refers to the share of the total of 13 mm in 4 equal parts. This is a "partitioning-plurality" type". The other type is labeled "unit- partitioning". In this case, the number 13 is a quantity and we work with "quarters" (e.g., 13 times a quarter (1/4 mm). The fraction 13/4 is then read "13 quarters" and is "a partition of unit followed by a multiplication". This type of partitioning is common in manipulations with graphic representations in early fraction teaching, such as the action of splitting, "take 3 times a quarter of a unit". These two types of partitioning provide different opportunities for the acquisition of the fraction concept.

1.3 The present study

Regional area, set and number line representations are three types of graphic representations which are commonly used for teaching and assessing children's conceptual knowledge of rational numbers (Ni, 2001). While regional area and set representations are employed to examine the part-whole relationship, the line number focuses more on the aspects of "rational as a number" and measures meaning of a fraction (Bright, Behr, Post, & Wachsmuth, 1988; English & Halford, 1995; Behr et al., 1983). The set representation presents a discrete quantity and easily leads to confusion of the understanding of unit-

fraction concept because each part of "the whole" is also a unit-whole. Only area graphic representation was presented in the current study.

Saxe and his colleagues (2005) investigated the development of children's notation of fractional parts of an area. Their results showed that the three following references (mathematical ideas) were mostly used: part-whole relation, part-part relation and integer. Part-part reference indicates the answers where the numerator and denominator matched the number of grey and white parts of a shaded area (e.g. the three-sevenths shaded area may be chosen for the symbol l3/4 or 4/3). Integer references were answers based only on the number of grey or white parts (e.g. the three-sevenths shaded area may be written as 3). In addition to graphic representations from the part-whole relation, those derived from the part-part relation were also used in this study.

The present study was interested in Grade 4 children who already had formal knowledge of fractions, so they were aware of the need of two numbers to indicate the notation of a fraction.

We conducted this study in Belgium and Vietnam which offered an opportunity to examine how different approaches to the fraction concept in the school curriculum may influence the acquisition of the part-whole relationship. Conceptual knowledge such as part-whole understanding and manipulations with analog representations are stressed more in Belgium than in Vietnam. The Vietnamese curriculum devotes much time to teaching procedural knowledge such as addition, multiplication and division of fractions. The approach by "unit-partitioning" (unit-dividing) is favored in the teaching of fractions in Belgium (e.g ¾ is considered as "3 times of a quarter"). In Vietnam, the fraction is introduced as a quotient so it is more related to the "plurality-partitioning" approach.

The purpose of the present study is to investigate the part-whole relationship embodied in graphical representations and its limitation in fraction numbers teaching. We used in parallel fractions superior and inferior to 1 with their graphic representations to examine the understanding of the part-whole relationship. We predicted that children would use the incorrect reference of the part-whole relation in manipulations with fractions superior to 1. Fractions superior to 1 were expected to provide more difficulty for children because they have less experience with their graphic representations (Hypothesis 1). At Grade 4, the part-whole relation was the dominant reference and the part-part relation was not a source of difficulty for children (Hypothesis 2). Finally, we expected Belgian children to outperform Vietnamese children in part-whole understanding and "unit-dividing" (unit-partitioning) representation (Hypothesis 3).

2. Method

2.1 Participants

Participants were children from Grade 4. Our sample included 65 children in Belgium (37 girls and 28 boys; mean age 120 months, SD = 4.4). The children came from three classes in three schools in the French-speaking Community. We had 138 children in

Vietnam (72 girls and 66 boys; mean age 117 months, SD = 3.3) who belonged to three different classes in a school. The difference in how schools are organized in the two countries accounted for the different size of two samples. The children all had a similar socio-economic status. They had been introduced to fractions during the fourth Grade. The testing was conducted during the last month of the school year.

2.2 Measure and Procedure

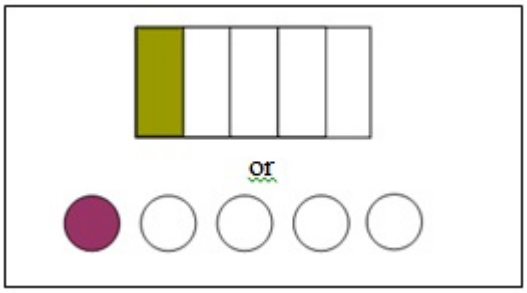


Figure 1. Fraction 1/5 is presented by shape (continuous feature) and by a set of countable objects (discrete feature)

Two "paper/pencil" tasks were designed to tap the understanding of the part-whole relation embodied in the area representation of a fraction number.

For all fractions, the universal notation convention is the bottom number (denominator) indicating “how many equal parts make up the whole”. The top number (numerator) tells about “how many of these parts of interest”. A part-whole understanding of fraction relates to determining what fraction of a shape (region) or a set of countable objects. Fractions of shape deals with the continuous feature and fractions of set relates to discrete feature, as illustrated in Figure 1. Thus, the graphical representation in Figure 2 corresponds to the fractional notation of 4/3 since there are 3 equal parts to make up the whole (denominator) and 4 parts of interest. A whole is considered by mathematical conventions as one unit (one circle). If we consider two circles as a new unit, the graphical representation indicates 4/6 of 2 meaning 4/3. We can interpret the above graphical representation as 4/6 of a total (2 circles), however this representation shows a quantity equal to 4/3. Thus, there is only a notation, 4/3, corresponding to the above graphical representation. We can see the accuracy of this knowledge in an examination of the part-whole representations of Empson (1999) and the courses of fraction in Vietnamese textbook (see Annexes).

The first task included 8 items in which all the fractions were superior to 1. We asked children to choose the fraction symbol corresponding to the graphical presentation (see examples are in Figure 2). For each graphical presentation, three numbers, only one of which was correct, were given (e.g. B. 4/3). The number of grey parts determined the numerator and the total number of parts in each circle (e.g. 3) indicated the denominator. The first distractor was derived from the relation part-part (e.g. C. 2/4): the number of white

parts and number of grey parts indicated the numerator and the denominator respectively. The second distractor came from an incorrect part-whole relation (e.g. A. 4/6): the idea behind this incorrect response was a notation determined by the number of grey parts and number of all parts. All items were found in Annexes.

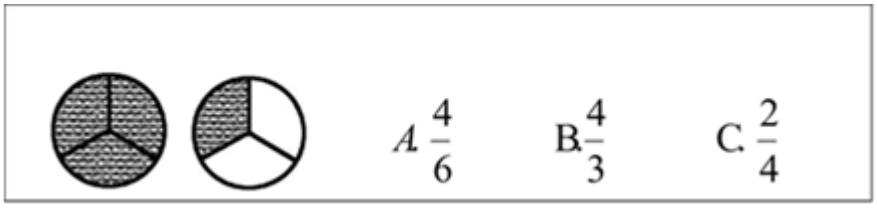


Figure 2. An item of task 1- Choose the fraction number corresponding to the graphical representation

The second task was the opposite of the first. We asked children to circle the representations referring to the given fraction symbol (see figures 3 and 4 for the display used for fractions 3/4 and 3/2). We noticed that they can choose many representations between right ones. This task comprised 6 items, divided into two subcategories: a fraction inferior to 1 (3/4; 4/6 and 5/8) and superior to 1 (3/2; 4/3 and 5/4).

In the test, six different fraction symbols were placed at the top of six different pages. Below each symbol, a set of 9 graphic representations was randomly presented.

For each fraction symbol smaller than 1, there were three correct representations, labeled "Inferior 1"; three distractors derived from an incorrect part-whole relation, labeled "Inferior 1_distractor_Part-whole" and three distractors from the part-part relation, labeled "Inferior 1_ distractor _Part-part". Between the three distractors from the part-part relation, the first one concerns children interested only by the parts indicating the numerator, the second one involves the case where shaded and non-shaded areas correspond to the numerator and denominator respectively, the last one relates to responses not paying attention to equal-part. Examples are given in Figure 3.

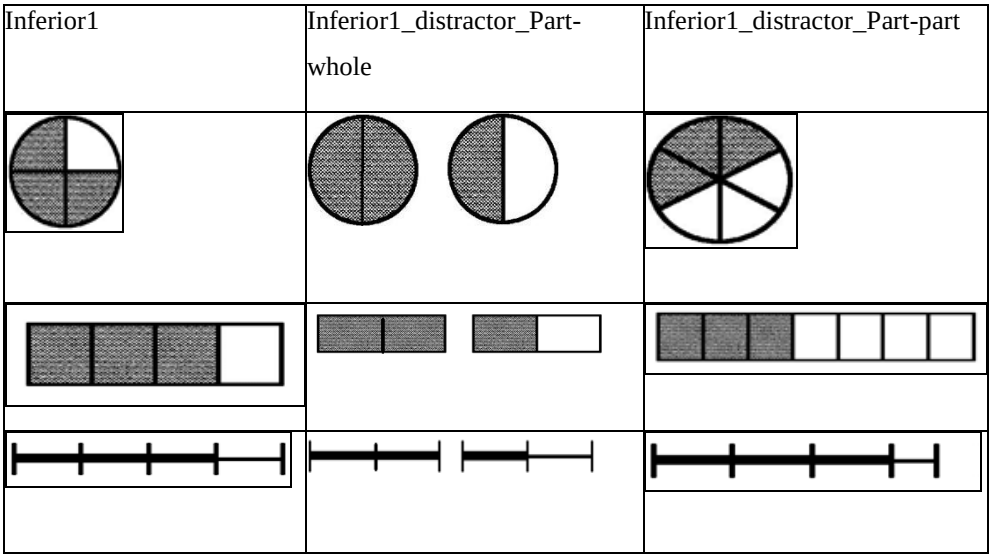


Figure 3. Graphical representations used for fraction 3/4 (Fraction less than 1)

For each fraction symbol greater than 1, there were six correct representations and three distractors. Only distractors derived from the part-part relation were used, labeled "Superior 1_distractor_Part-part". The six correct representations were divided into two categories. Three representations associated with the relation part-whole, labeled "Superior 1", three representations based on unit-partitioning ideas (e.g. 3/2 was considered as three times 1/2), labeled "Superior 1_Unit-Partitioning". Finally, three distractors labeled "Superior 1_distractor_Part-part in which the construction is similar to the above case with the fraction less than 1. Examples are given in Figure 4.

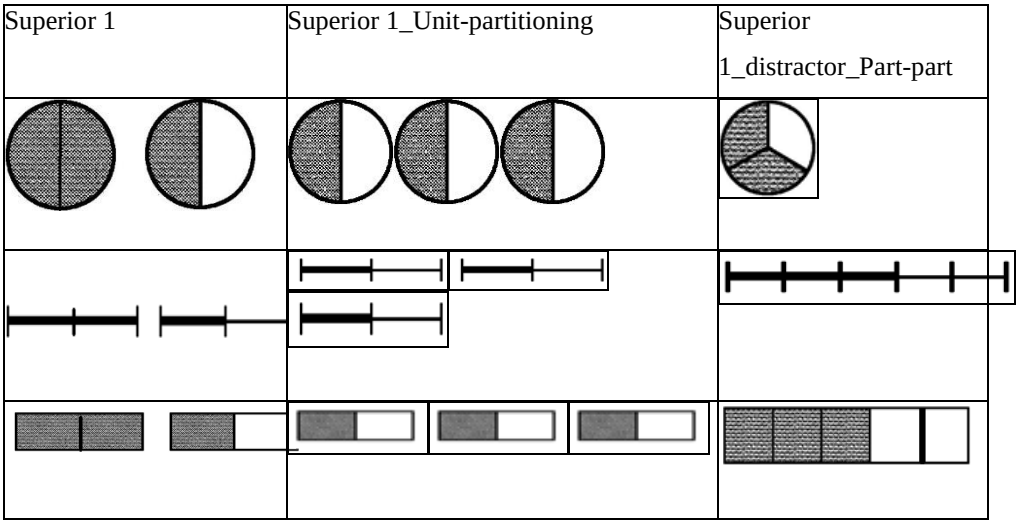


Figure 4. Graphical representations used for fraction 3/2 (fractions superior to 1)

The selection of fraction numbers in the inferior to 1 category (3/4 ; 4/6 ; 5/8) and the superior to 1 category (3/2 ; 4/3 and 5/4) was based on the constitution of countervailing pairs. We can see that, for example, in the pair ¾ and 3/2, the graphic representation of "Inferior 1_distractor_Part-whole" of fraction3/4 was identical to the one of "Superior 1" of fraction 3/2. The assumption that "the number of grey parts indicates the numerator and the number of all parts determines the denominator" is only correct for representations of fractions smaller than 1. If children overgeneralize this rule by applying it for representations of fractions superior to 1, they will chose the distractor3/2 to refer to ¾.

The tasks were carried out without time constraints: the first one took approximately 10 minutes and the second one took less than 30 minutes. The two tasks were conducted separately within 3 days. There were two versions of each task, with a different ordering of items.

3. Results

3.1 The first task: to determine the fraction symbol corresponding to the given graphical presentation.

To examine whether children overgeneralize the part-whole rule in manipulation with fractions greater than 1, only graphic representations of fractions greater than 1 were presented. The scores of Belgian and Vietnamese children are illustrated in Table 1. In both samples, low performance was observed, showing that children had difficulty manipulating fractions superior to 1. The Belgian children were more advanced than the Vietnamese.

Table 1. Mean and standard deviations (in percent) of scores by country

Country	Mean	SD
Belgium	37.3	45.8
Vietnam	10.8	27.3

Table 2 has more details of the ideas behind the responses of children. The majority of incorrect responses derived from an incorrect part-whole relationship. The part-part relation did not have a large impact on children. The distribution of responses for each graphic representation differed significantly from the expected distribution as shown by a chi-test, χ^2 (2, N = 203) = 19.23, p < 0.001. These results supported our expectations about the use of the incorrect rule of part-whole relation in the case of fractions inferior to 1, the less common use of the relation part-part and the better performance of Belgian children.

Table 2. Percent of responses per country of each representation.

Country	Distractor_ Part-whole	Correct symbol	Distractor_Part-part
Belgium	45.4	37.3	17.3
Vietnam	65.1	10.8	24.1

3.2 The second task: to determine the graphical presentation associated with the given fraction symbol

For each item, if children chose the correct graphical representation or did not choose the distractor, they marked a point. The scores of each student is the mean of performances of all items which is later calculated in percent.

Fractions inferior to 1 and incorrect understanding of the relation part-whole

To examine whether children used the incorrect part-whole relation in the construction of the fraction concept, a measure-repeated Anova was conducted with category (Inferior 1_Correct, Inferior1_distractor_Part-whole, Inferior 1_distractor_Part-part) as within variable, separately for each country. In both samples, Belgium and Vietnam, the effect of category was significant, F(2,132) = 4 9.2, p < 0.001, η^2 = 0.427; F(2, 274) = 134.9, p < 0.001, η^2 = 0.496, meaning that the performance of categories Inferior 1 and Inferior 1_distractor_Part-part were higher than those of Inferior 1_distractor_Part-whole.

Table 3. Means and standard deviations of scores (in percent) by types of items and by country

	Belgium		Vietnam	
Fraction inferior to1	Mean	SD	Mean	SD
Inferior 1	85.4	22.1	82.4	26.7
Inferior 1_distractor_Part-whole	38.6	33.0	31.6	30.0
Inferior 1_distractor_Part-part	76.1	25.7	76.9	16.3
Fraction superior to 1				
Superior 1	64.7	30.5	37.6	33.1
Superior 1_Unit-Partitioning	69.0	36.5	29.0	36.2
Superior 1_distractor_ Part-part	74.1	24.0	64.7	26.0

Post hoc t-tests showed that the three performances differed significantly from each other ($p < 0.001$) for all comparisons, except for the comparison between Inferior1_correct with Inferior1_distractor_part, $p = 0.41$ and $p = 0.45$, respectively for Belgian and Vietnamese groups). The lowest score for Inferior 1_distractor_Part-whole showed that children used the incorrect rule of the part-whole relation to determine the fraction symbol. These results supported our hypothesis 1, as illustrated in Figure 4.

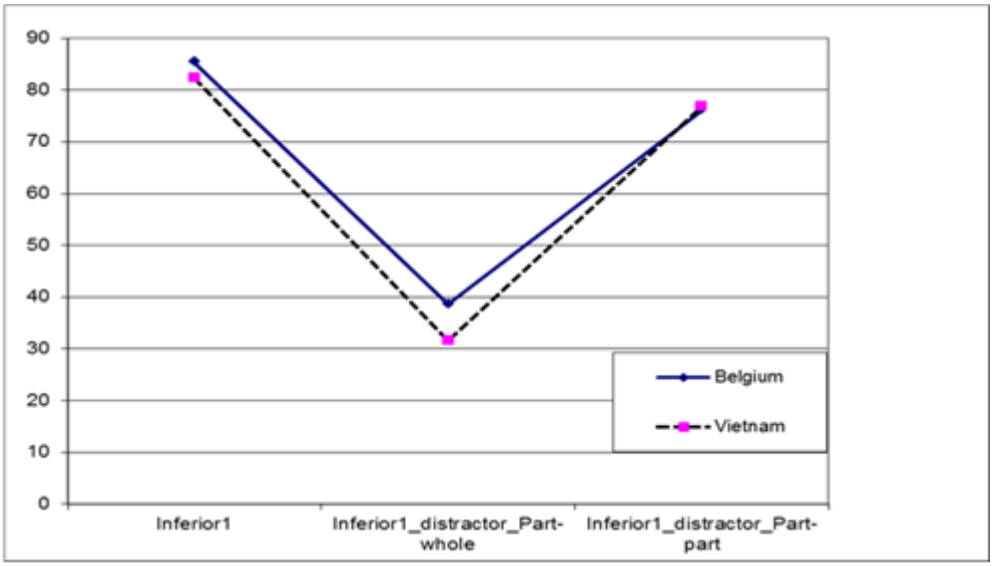


Figure 4. Means of scores on fractions smaller than 1 of three representations by country

Children in both countries achieved good performance in category "Inferior 1" which can be explained by the familiarity of these representations with the learning context. In fact, the inferior to 1 fraction is usually used to approach the concept of fraction in teaching. Children simply applied the knowledge of the part-whole relation under the rule that "the number of grey parts indicates the numerator and the number of parts in a whole indicates the denominator" to determine the correspondent fraction symbol. It becomes more complex in the case of fractions superior to 1, referring to the distractor derived from category “Inferior 1_distractor_Part-whole”. Children overgeneralized the idea above, leading them to incorrect responses. They applied the immature rule in which "the total number of parts" indicate the denominator instead of "the number of parts in a whole". It should be noticed that ,in the case of fractions inferior to 1, if children applied this incorrect rule, it leads to a correct response anyway, because the total number of parts is identical to the number of parts in a whole. The fact that children used this incorrect rule with the distractor built on the part-whole relation confirmed their misconception of this relation.

The distractor made from the part-part relation seemed to pose less of a problem for children than the other distractors. It suggested the children's attention to the relation between numerator and denominator. Children in Grade 4 had formal knowledge of fractions so they considered the fraction as a number and did not treat it as two separate natural numbers any longer. The part-whole relation seems to constitute privileged knowledge for the construction of the fraction concept. Similar patterns of performances were observed in both countries, illustrated in Figure 4. These results supported our hypothesis 2.

Belgian children slightly outperformed the Vietnamese but their performances did not differ significantly in the three categories of fraction inferior tho1, $F(1,203) = 0.65$, $p = 0.42$; $F(1,203) = 2.3$, $p = 0.13$; $F(1,203) = 0.07$, $p = 0.79$ respectively for categories Inferior1_Correct, Inferior1_distractor_Part-whole and Inferior1_distractor_Part-part.

Fraction superior to 1

To investigate if the different types of representation had an impact on children performances (determining the right symbol), an measure repeated Anova was carried out with Category (Superior1, Superior1_Unit-Partitioning and Superior1_distractor_Part-part) as within variable, separately for each country.

In Belgium, the effect of Category was not significant, $F(2,132) = 2.54$, $p = 0.83$, $\eta^2 = 0.037$ which means that the performances did not differ significantly between the three types of representations. The score of the category Superior1_distractor_Part-part was the highest, which is consistent with the results of the first task. The graphic representation derived from the unit-partitioning seems to have provided a slight advantage for Belgian children. This can be explained by familiarity with the learning of the fraction concept in Belgium. These results supported hypotheses 2 and 3.

In Vietnam, the effect of category was significant, $F(2,274) = 75.26$, $p < 0.001$, $\eta^2 = 0.496$, showing that the scores varied according to the type of graphical presentation. Post hoc t-tests showed that the three performances differed significantly from each other ($p < 0.001$) for all comparisons. We observed much lower scores for Vietnamese children in the Superior 1 category in comparison with the Inferior1 category. Vietnamese performance was best on the part-part relation, as was the case with the Belgian samples. Children showed their mastery of this relation for the fraction concept. In contrast to the Belgian sample, the graphic representation related to unit-partitioning led to lower scores. A plausible explanation for this is that Vietnamese children do not have experience of unit-partitioning activities in their approach to the fraction number. These results suggested the impact of different curricula on the understanding of the fraction concept and reinforced hypothesis 3, as illustrated in Figure 5.

For all types of representation with fractions superior to 1, Belgians children $F(1,203) = 54.8$, $p = < 0.001$; $F(1,203) = 6.2$, $p = 0.01$.

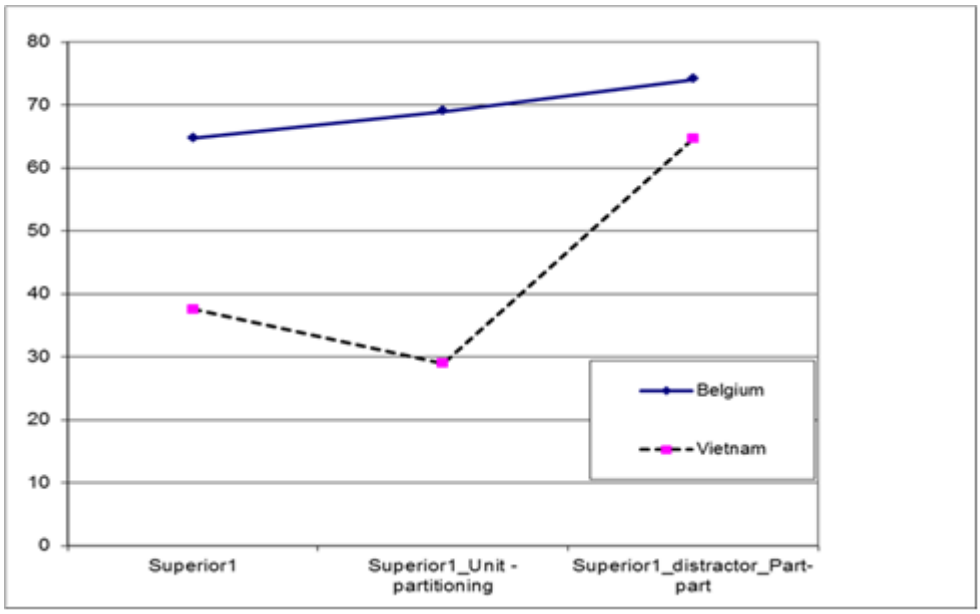


Figure 5. Means of scores on fractions superior to 1 of three representations by country

Fractions inferior to 1 and fractions superior to 1.

To investigate if children had difficulty manipulating fractions greater than 1 regarding the part-whole relation, we compared two subcategories, Inferior 1_Correct and Superior 1 (with correct graphic representations related to the part-whole relation). A repeated measure ANOVA was conducted separately for each country, with category (Inferior 1 and Superior 1) as the within variable. In both samples, the effects were significant $F(1,66) = 31.95$, $p < 0.001$, $\eta^2 = 0.326$ and $F(1,137) = 247.12$, $\eta^2 = .643$ for Belgium and Vietnam respectively, showing that fractions greater than 1 led to lower correct responses, particularly Vietnamese children. We observed a wide difference between the achievements related to fractions inferior and superior to 1 in the Vietnamese sample, as illustrated in Table 3. In addition, their poor performance in the first task also stressed Vietnamese difficulty in the manipulation of fractions greater than 1 relative to part-whole relation. These results supported our first hypothesis.

4. Discussion

The present study investigated the part-whole relation of fraction numbers embodied in graphic representation with Belgian and Vietnamese children at Grade 4. To the best of our knowledge, the current study is the first examination of the misunderstanding of the part-whole relation derived from manipulations with fractions greater than 1. The results were consistent with our hypothesis.

The first task was conducted to examine children's references in determining a fraction symbol. The data demonstrated that the majority of them used the incorrect rule of part-whole relation. "The number of all parts" was mistaken for "the number of parts in a whole" in the determination of the denominator. These two expressions are equivalent only in the case of a fraction smaller than 1. The results obtained from the first task can be explained by two reasons: first, the children usually dealt with fractions smaller than 1 to build their part-whole relationship knowledge and they overgeneralize the rule only valid for fractions smaller than 1 on all fractions; second, the children chose fraction symbols inferior to 1 for representations greater than 1 because of their immature understanding of the numerical value of fractions. Even at the end of Grade 4, when children had many opportunities to do operations with fractions greater than 1, they still believe that "a fraction is a part of a whole", which is considered as an intermediate phase of the understanding of the numerical value of fractions (Stafylidou & Vosniadou, 2004).

The poor performance in the Inferior1_distractor_part-whole subcategory in the second task confirmed that this incorrect rule was consistent with the children. In the current study, fractions smaller than 1 and bigger than 1 formed countervailing pairs. This means that we chose the same graphic representations both for the notation of fractions inferior to 1 as distractors (Inferior1_distractor_Part-whole) and fractions superior to 1 (Superior1) as correct representations. We observed that children in both samples had lower scores in the Inferior 1_distractor_Part-whole subcategory (M = 38.6 for Belgians and M = 31.6 for Vietnamese). In the Superior 1 subcategory, it seemed that Belgian children achieved good performance (M = 64.7) although their results for the Inferior 1_distractor_Part-whole subcategory (M = 38.6) could be interpreted as an effect of their immature understanding of the part-whole relation. In fact, they used alternatively two different applications of the part-whole relationship: this confusion reflects an intermediate stage in the development of the part-whole relation understanding. Vietnamese children had still lower scores (M = 37.6) which reinforced the fact that they had difficulty manipulating fractions greater than 1. They also had lower scores for the Inferior1_distractor_Part-whole subcategory (M = 31.6). It seemed they applied only one incorrect rule for fractions greater than 1. When they worked with a correct representation of a fraction greater than 1, they were unable to apply their incorrect rule. They may have believed that there was no correct representation associated with the notation of the given fraction.

We saw the different impacts of the representations derived from unit-partitioning on Belgian and Vietnamese samples. This confirmed our hypothesis that different choices of curriculum produce different ways of associating fraction symbols with their representations. In addition, the lower scores achieved by the Vietnamese children in the current study could be explained by the emphasis in the Vietnamese curriculum on procedural knowledge and by the fact that the fraction concept is introduced as a quotient. The part-whole relation is conceptual knowledge and the tasks used to measure it typically refer to adding a fraction symbol to shaded parts of objects (Byrnes & Wasik, 1991; Hiebert & LeFevre, 1986; Ni, 2001; Hecht, Close & Santisi, 2003). These tasks and the action of partitioning are rare in the Vietnamese curriculum. A strong emphasis on the procedures for

common denominator, operations from addition to division of fractions rather than part-whole relation was observed in the teaching of fractions in Vietnam. Many existing studies have demonstrated that conceptual and procedural knowledge cannot develop separately (Hallett, Nunes & Bryant, 2010; Hecht, 1988; Canobi & Bethune, 2008; Rittle-Johnson & Koedinger, 2009; Rittle-Johnson, Siegler & Alihali, 2001; Schneider, Rittle-Johnson & Star, 2011). They provide mutual support and both contribute to a complete understanding of a piece of knowledge. It would be appropriate to devote more time to partitioning and the part-whole relation during teaching practice in Vietnam since the latter provides the foundation of rational number knowledge.

The results of this study confirmed the obstacles encountered in part-whole understanding and in the manipulation of fractions superior to 1. These obstacles could be considered to be didactic, deriving from the choices or projects in the educational system (Brousseau, 1988). In early teaching of the fraction concept, part-whole schemes associated with numbers smaller than 1 are regularly used. If children build an incorrect rule of the part-whole relation, they have no opportunity to detect their errors in manipulation with fractions superior to 1. It is necessary to introduce fractions superior to 1 in the approach to fraction numbers. Thus, multiple, complementary approaches are required to help children absorb the concept of fractions.

The part-whole scheme associated with equal sharing is often favored in teaching fractions. This approach, however, has its own limitations. The fraction is derived from a situation of equal sharing and the number of parts (the numerator) is usually smaller than the whole being divided into the parts (the denominator). Thus, it is difficult to explain the existence and sense of improper fractions in which the number of parts is larger than the number of parts in a whole (Steffe, 2002; Thompson & Saldanha, 2003; Tzur, 1999). Our finding reinforces the limitation of a part-whole scheme.

In addition to the part-whole approach, teaching based on spatial ratios may provide an efficient way of conceptualizing fractions (Sophian, 2000). The quotient approach also offers children the opportunity to manipulate fractions larger than 1. Since fractions are a multi-faceted concept, alternative teaching approaches may be used simultaneously for the best understanding. The part-whole scheme could be used first to introduce fractions followed by alternative approaches. This could provide a better understanding of the part-whole relationship and deeper acquisition of the representation of fraction numbers.

Chapter 8
From natural number to decimal number

From natural number to decimal number

1. Introduction

Teaching and learning rational numbers is one of the most challenging issue domains in mathematical education. It derives from two principal reasons. A decimal number is able to express fractional quantities but presented in the decimal number system, which resembles the natural number system. The second reason came from the multifaceted notation included in the fraction concept. Fractions can be conceived from five subconstructs: part-whole, ratio, operator, quotient and measure (Kieren, 1976). It leads to different ordering of topics and challenges in teaching rational numbers.

The prior knowledge and the later acquired knowledge have a complex interaction, as illustrated in aforementioned studies. The existence of the whole number bias reinforces the complex interactions between natural and rational numbers (Ni & Zhou, 2005). When the prior and later acquired knowledge share many common underlying concepts, their interactions seem more involved, such as addition and multiplication, whole numbers and rational numbers. The present study aimed to evaluate the links in the theoretical model illustrated in Figure 1. The model depicts links between natural, fraction and decimal numbers. Links in this theoretical model have been derived from two aspects, mathematical front and development point of view.

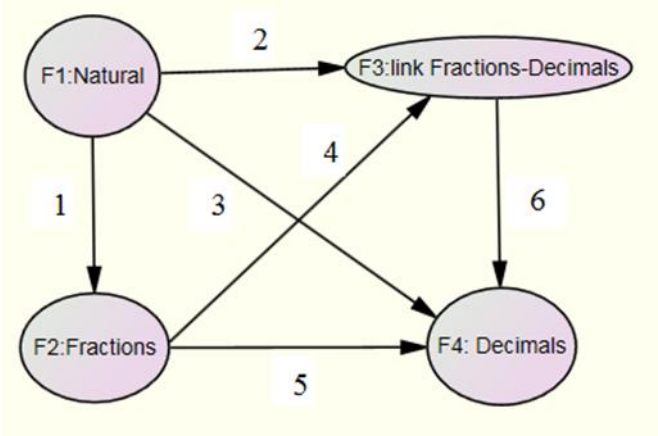


Figure 1. The theoretical model about associates between natural and rational number

Theoretical evidence of mathematics and previous empirical works about relation between naturals and rational numbers

From a mathematical view, rational numbers, presented in the form of fraction numbers, allows an exact presentation of the measurement of continuous quantities and then, all divisions between two whole numbers with the divisor different from zero can be realized. This is impossible the ensemble of natural numbers. Only one unique unit is used for all operations with natural numbers while many various units exist for fraction numbers. The number of parts in a whole from the process of partitioning decides the unit for fractions (e.g., Divide one cake in five parts then the unit-fraction can be 1/5). Consequently, if two fractions have different "units", it poses difficulties for operations. The common fraction with a power of tens as a denominator (e.g., 3/10) and the decimal number which is considered as a different version of a fraction number (e.g., 0.3) were introduced to facilitate calculations. From the mathematical front, relation between fractions and decimals is solid and can provide deep understanding of the numerical value of rational numbers. It explains the existence of path 5 (Figure 1) between fraction and decimal numbers.

The pathway from fractions to decimals goes through decimal fractions (e.g., 3/10 = 0.3). Thus, the link between fractions and decimals related to the decimal fraction (path 6) may play a substantial role in the construction of the rational number concept. Although decimal numbers are derived from fractions, the connection between them is implicit. It is masked by the representation of decimals in the base-10 notation. The associations between decimal fractions and decimals seem stronger than those between fractions and decimals.

Decimal fractions represent a confluence of fractions and whole numbers (Hiebert et al., 1991). In addition, decimal fractions with the decimal representation (e.g., 0.3) are expressed in the base-10 notation. It leads to fractional quantities, measured by a type of unit which is not the whole unit, tens of unit, etc… but tenths of unit, hundredths of unit, etc… Thus, the principles used to present decimal quantities are the same with the notation used to express whole-number quantities. Fractions and decimals have a continuous character but decimal fractions expressed in base-10 blocks the presentation of the discrete character. It marks the link between decimal fractions and natural numbers (path 3).

Regarding the relation between natural numbers and rational numbers and considering the distinct notations and properties between naturals and fractions, we expect a weaker association between them than the one between natural and decimal numbers. In a division with a constant dividend, if the divisor increases then the quotient decreases. This property is transformed for fraction numbers as a larger denominator makes smaller fractions (under condition that the numerator does not vary). This implicit transition from natural numbers sometimes does not provide the support needed for the acquisition of fractions but can led to incorrect thinking such as the one for two fractions with the same numerator, "the bigger number is the one with larger denominator". Natural and decimal numbers are presented with the decimal position system, which is based on a base-ten structure. They use the same syntactic rules for sum and multiplication relationships to produce an Arabic number. So they both process many common properties such as a digit is always ten times greater than the digit to its right in a number and the values decrease as you move from left to right. As

a result, the knowledge of natural numbers, especially the place-value understanding, can be expected as an important contribution for the acquisition of decimal numbers.

From the development point of view, many previous studies investigated the influence of the existing knowledge of natural numbers for the instruction of rational numbers (Desmet et al., 2010; Resnick et al.,1989; Nesher & Peled, 1986; Sackur-Grisvard & Leonard, 1985; Stafylidou & Vosniadou, 2004; Vamvakoussi & Vosniadou, 2004). These studies indicated that the natural number knowledge and the fraction knowledge affected the acquisition of the decimal number. It is showed by paths 1, 5 and 6 in the theoretical model (Figure1).

Using an ordering task of three decimal numbers on children from Grade 4 to 7, Sackur-Grisvard and Leonard (1985) showed that many children, mainly in Grade 4 and Grade 5, used three incorrect rules in their responses. The ideas behind rule 1 was that the number with more decimal digits was the larger one (e.g., 12.4 < 12.17 because 4 < 17). Rule 2 referred to the opposite, the number with more decimal digits was considered smaller (e.g., 12.94 < 12.7). Rule 3 was to select a smaller number with a zero immediately after the decimal point (e.g., 4.08 < 4.7). The conceptual sources of these systematic but incorrect rules derived from earlier knowledge of the number system, the whole-number and fractions (Nesher & Peled, 1986; Resnick et al., 1989). Rule 1 and Rule 3 was considered as the "natural rule" and Rule 2 the "fraction rule" which was shaped by the knowledge about fractions. These empirical studies demonstrated the impact of an existing knowledge in the development of the rational number acquisition.

Later, using the conceptual change as a reference point, many studies suggested that children need to reorganize their prior knowledge of natural numbers for the conceptualization of rational numbers (Desmet et al., 2010; Stafylidou & Vosniadou, 2004; Vamvakoussi & Vosniadou, 2004). Results from our previous study (Chapter 6) examined the construction of the symbol representation of a decimal number, reinforced by the contribution of natural numbers in the acquisition of rational numbers. In addition, the whole number bias also demonstrated the link between natural and rational numbers (Ni & Zhou, 2005). Results supported the paths 1 and 3 between these numbers.

Saxe and his colleagues (2005) examined the relationship between students' uses of fraction notations and their understanding of the part-whole relation. We observed the notations which were correlated to the natural number knowledge. For example, children gave a symbol "3" or "1" to express a graphic representation indicating a circle divided into four equal parts with three parts colored in gray. It confirmed the correlation between the natural knowledge and the fraction knowledge.

Notice that in previous studies (Resnick et al., 1989; Nesher & Peled, 1986), the use of the natural-rule was more common than the fraction-rule in the comparison of decimals. The data of our previous study relative to the representation of decimal numbers (Chapter 6) documents that the majority of incorrect responses derived more from natural numbers conceptions than from fractions ones. It suggested the substantial influence of the natural knowledge and the temporary leverage of fraction understanding in the conception process

of the decimal number. It is consistent with the similarity between natural and decimal number system and distinct from the properties of naturals and fractions which were described from a mathematical point of view.

Some studies already investigated the associations and predictors related to rational numbers, such as the associations between the subconstructs of the fraction concept (Charalambous & Piita-pantazi, 2007); relations between mathematical knowledge, children characteristics and fractions outcomes (Hecht et al., 2003); links between fraction skills, working memory, classroom behavior, conceptual knowledge of fractions and simple arithmetic (Hecht & Vagi, 2010) and the competences of fractions with later mathematics achievement (Bailey et al., 2012).

This study investigates a more holistic view of the rational concept development by examining the interactions between the whole numbers knowledge with rational numbers. We examine if our observations support assumptions about the theoretical model presented in Figure 1. We look if relationships between these numbers can have a potential impact on the learning of decimal numbers.

2. Method

2.1 Participants

The sample consisted of 60 fourth grade Belgian children (26 boys and 34 girls; mean age = 9 years 10 months). Children came from three classes of three schools of the French-speaking Community. There was 127 fourth grade Vietnamese children (60 boys and 67 girls; mean age = 9 years 5 months). They belonged to three different classes in a school. The difference of two sample sizes are caused by the diversity in the schools in Belgium and Vietnam. The schools from which the samples were drawn were socioeconomically similar.

2.2 Measures

The percentage of correct answers was recorded for all tests.

Natural numbers knowledge

Natural numbers transcoding

This test was composed of 36 items. Children were asked to write natural numbers from written number words into an Arabic numbers. There were two types of number, 4 digits numbers and 5 digits numbers. All items comprised from one to two syntactic zeros (e.g., "three thousands four"→3004; "four thousands twenty five"→4025).

Natural numbers comparison

This task comprised two categories, items with four or five digits. Each category consisted of 18 pairs of natural numbers. Half of the items included two digits equal to zero (e.g., 12050 vs. 12005) and the other half consisted of items with one digit equal to zero (e.g., 4105 vs. 4015). Both numbers of each pair had the same number of digits and the same thousand. They only varied for the position of the digits, with the following form: $\overline{a0bc}$ vs $\overline{ab0c}$ or $\overline{ab0c}$ vs $\overline{abc0}$ for category of four digits.

Place-value understanding

The digit identification task consisted of 36 items of natural numbers. Half of the items had 4 digits and the other half had 5 digits. The items included one to two digits equal to zero (e.g., 3064 and 46005). For half of the items, children were asked to circle the digit indicating the tens and for the other half, they were asked to circle the digit of the hundreds. The items fell into two different categories. The first one comprised the numbers with the digit needed to be circled equal to zero (e.g. "Circle the digit indicating the tens in the number 25**0**3" or "Circle the digit indicating the hundreds in the number 3**0**64"). The second category comprised the numbers with the digit needed to be circled different from zero (e.g. "Circle the digit indicating the tens in the number 1034").

Fraction numbers knowledge

Comparison of fractions

For each of the 36 items, children were presented a pair of numbers. They were asked to indicate which one of the two numbers was the larger one or to place an equal sign if the numbers had the same value. Half of the items were related to comparison between fraction numbers and whole numbers (e.g., 2/7 and 1). The other half comprised pairs of two fraction numbers with the same numerator and different denominators (e.g., 2/5 and 2/7).

Words problems

This test was composed of 8 two-steps word problems involving the numerical of fractions. For example, "In a classroom, each table can allow 5 pupils. There are 12 pupils, how many tables do we need for all of them?"

Representation of fractions

For each of 6 items, children were presented a fraction symbol (e.g., 3/4) with a set of 12 graphic representations. Children were asked to indicate the representation referring to the given fraction symbol (an example in Figure 2). There are two types of representations, regional area and line number. This task focused on the part-whole relationship of fraction knowledge.

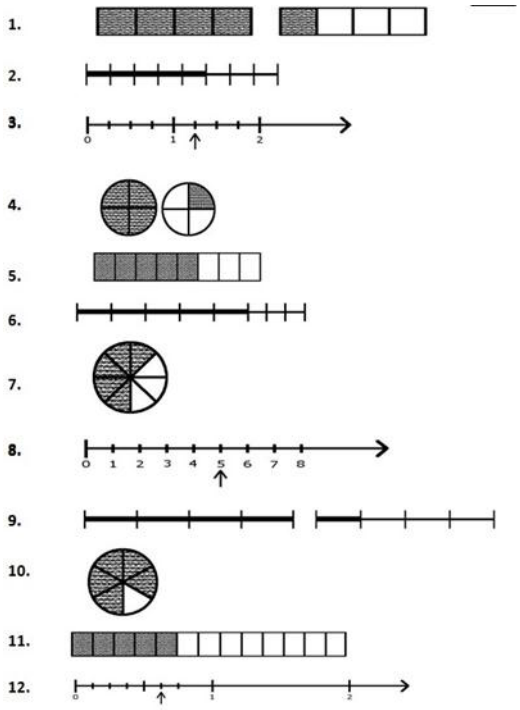


Figure 2. Choose the corresponding representations of the fraction number 5/8

Link between Fractions and Decimal numbers

Measures

In recall the relation between measures, such as 1m = 10 dm; 1m = 100 cm and 1m = 1000 mm, we asked children to represent a measures by a decimal number (e.g., 8m 5 dm 6 cm = ...m or 34 dm = …m). This task comprised 22 items.

From fraction symbol to decimal symbol and vice versa

Fractions to decimals: for each of 12 items, children were presented a fraction symbol. They were asked to write the decimal number which is equal to the given fraction (e.g., 3/10 = 0.3; 237/100 = 2.37) There were two types of items, fractions with a denominator equal to multiples of 10 (e.g., 3/10) and fractions with a denominator who was not a multiple of 10 (e.g., 1/5 = 0.2).

Decimals to fractions: for each of 12 items, children were asked to write a decimal number under the notation of a fraction number (e.g., 0.7 = 7/10; 2.5 = 10/4). There were two types of items, decimals smaller than 1 (e.g., 0.7 and 0.26) and decimal number greater than 1 (e.g., 2.5 and 1.2).

Decimal numbers

Transcoding Decimals

We asked pupils to write a number in Arabic form from a written-verbal number. For example, they were asked to write an Arabic number containing "5 units, 9 tenths " (5.9).

The task included 24 items that were divided into two categories. The first category, "Unit equaled zero", comprised items with an integer part that equaled zero and had one to three primitive elements of the decimal, which were the tenth, hundredth and thousandth (e.g., "five tenth"→ 0.5). In the second category, "Unit differed from zero ", items had an integer part that was different from zero (e.g., "two units, three tenths and six hundredths" → 2.36).

Comparison of Decimals

This comparison task on pairs of decimal numbers comprised 24 items. Children were presented with two decimal numbers (e.g., 4.3 vs. 4.20; 8.6 vs. 8.60; 8.6 vs.8.07) and asked to indicate which one of the two numbers was the larger one or to place an equal sign if the numbers had the same value. 24 items were equally divided into three categories. In the first category, for example 4.3 vs. 4.20, the longer number (two digits and the digit in the hundredths position equal to 0) had the digit in the tenths position smaller than the digit in the tenths position of the shorter number. Thus, the longer is the smaller number. In the second category, for example 8.6 vs.8.07, the longer number is also smaller. The digit in the tenths position of the longer number is 0. The digit in the hundredths position is larger than the digit in the tenths position of the shorter number. For the third category, for example 8.6 vs. 8.60, the two numbers were equal but they differed by their length.

Density

This task assessed the understanding of decimals density (24 items). Children were presented two decimals (e.g., 3.5 and 3.7) and asked to circle the numbers among a given list which were included between the two mentioned decimal numbers (Figure 3). They were also asked to write a number included between the two given numbers (e.g., 72 and 75; 48.7 and 49.7).

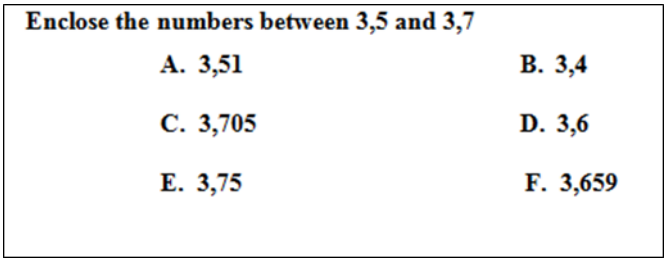
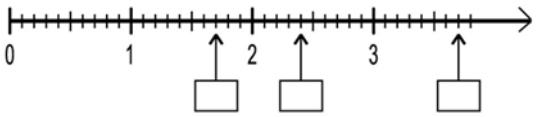


Figure 3. An example of density task for decimal numbers

Line number

Children were asked to determine the decimal number in number lines. This task



comprised 12 items. An example was found in Figure 4.

Figure 4. Write the appropriate decimal numbers in the empty cases

Place-value understanding

The digit identification task consisted of 44 items of decimal numbers. The items fell into four categories in which children were asked to circle the digit indicating the unit, tenth, hundredth and thousandth respectively for four categories. For each category, there were numbers with the digit needed to be circled equal to zero (e.g. "Circle the digit indicating the tenth in the number 4.09 and other numbers with the digit needed to be circled different from zero (e.g. "Circle the digit indicating the tenth in the number 4.19").

2.3 Procedure

The participants were from a longitudinal study, only children who participated in all sections of grade 4 and grade 5 were comprised in this study.

All tests were the paper-and-pencil task. The natural and fraction tests were administered in April of fourth grade and the link fraction-decimal and decimal number test were administered in April in next year (fifth grade) in two countries. Children have acquired the fractions and decimal numbers at the testing moment, respectively for fourth grade and fifth grade. The test was performed collectively in the classroom at their school site. Children worked individually without time pressure.

2.4 Data analysis

The analysis was conducted using the AMOS18. The parameters of the models were estimated using the maximum likelihood method. The chi-square compares the sample covariance matrix with the theoretical model covariance matrix in structural equation modeling. Thus, a non-significant chi-square indicates a satisfactory model fit. Beyond the chi-square, numerous fit indices were used to evaluate the fit of the data to the theoretical model under investigation. Three frequent fit indices and their optimal values were: the Comparative Fit Index (CFI), the values of which should be equal to or larger than 0.90 (or close to 0.95), the Root Mean Square Error of Approximation (RMSEA) with acceptable values less than or equal to 0.08 and P the test of close fit (PCLOSE) with a good fit is higher than 0.05 (Hu & Bentler, 1999; Schreiber et al., 2006 and Byrne, 1998).

Notice that confirmatory factor analysis (CFA) is usually used to assess the proposed measurement model in a structural equation model. CFA is distinguished from structural equation modeling by the fact that in CFA, it does not allow the arrows between latent factors. In structural equation modeling (SEM), factors are presumed to directly cause one another. Thus, CFA is often considered as a first step for structural equation modeling. The CFA is often called "the measurement model", while the relations between the latent variables (with directed arrows) are called "the structural model". In the current study, we conducted firstly a confirmatory factor analysis and then a structural equation modeling.

3. Results

3.1 Descriptive statistics

Table 1 presents the means and standard deviations of children's performances on each task measuring competences for naturals, fractions, decimal numbers and the link between fractions and decimals. Results showed that performances of Belgian children were significantly higher than Vietnamese in the majority of tasks. There was no significant difference between them in the scores of transcoding natural numbers and the Measures tasks. Vietnamese children were better for only two tasks: Comparison of fraction numbers and fraction-decimal symbols. As the current aims to examine the associations between four latent variables: naturals, fractions, decimal numbers and the link between fractions and decimals of the general theoretical model (Figure 1), we used simultaneously observations of two samples, Belgian and Vietnamese in the next analysis.

Table 1. Means of correct responses (in percent) and standard deviation for each measure

	Mean (SD)		t-tests
Measures	Belgium	Vietnam	p-values
Natural performances			
Comparison	**97.9** (4.5)	95.6 (9.4)	p=0.027<0.05
Place-place understanding	**92.54** (20.8)	76.7 (39.2)	p<0.001
Transcoding	56.6 (22.1)	63.4 (22.7)	p=0.056>0.05
Fractions performances			
Comparison	54.7 (28.9)	**65.5** (26.1)	p=0.016<0.05
Words problems	**92.5** (17.4)	51.1 (43.9)	p<0.001
Representation of fraction	**62.0** (12.5)	51.9 (11.5)	p<0.001
Link Fractions-Decimals			
Fraction and decimal symbol	60.8 (35.2)	**83.4** (21.3)	p <0.001
Measures	78.4 (33.5)	83.0 (24.3)	p=0.34>0.05
Decimal number			
Transcoding	**89.5** (22.1)	71.9 (33.4)	p <0.001
Density	**64.7** (34.4)	43.9 (27.7)	p <0.001
Line number	**89.4** (14.5)	63.0 (38.1)	p <0.001
Comparison	**98.3** (6.6)	94.8 (13.1)	p=0.016<0.05
Place-value understanding	**95.5** (15.3)	71.1 (40.6)	p <0.001

3.2 Preliminary analyses

A factor analysis was firstly conducted to study the dimensionality of a set of variables. Results suggested four components with eigenvalues larger than one: 2.3; 2.0; 1.8 and 1.1. They explained 55, 9% of the total variability (17.7%; 15.4%; 14.1% and 8.7%, respectively). Table 2 presents the factor loading after varimax rotation for each factor from observed variables. Observing these high factor loading(e.g., > 0.5), we can see that four factors (component 1, 2, 3 and 4) can be considered to measure the decimals number, link between fractions and decimals, fractions and natural number competencies respectively. It supported our theoretical model including four latent variables: Natural, Fractions, Link fractions-decimals; Decimals.

As stated by Hecht and colleagues (2003), "Confirmatory factor analysis (CFA) was also used as a further check concerning whether the data supported the current assumptions about the underlying structure of the current tasks", a confirmatory factor analysis was

conducted which showed that the four-factor model had a good fit to the data. Table 3 presents the correlations between exogenous variables. These correlations were significantly different from zero (in all case $p < 0.001$ and $p < 0.05$). Theoretical statistics propose that in case of these correlations are sufficiently high (e.g., > 0.8), consolidating the corresponding factors into a single factor should be considered. As can be seen in Table3, the correlations between Decimals and Naturals, Decimals and Fractions were high (0.95 and 0.93 respectively) but the purpose of the current study is to examine the causal path between them so we didn't join them into a single factor. These factors are correlated but separated. We can explain these high correlations by the indicators which were used to measure Naturals, Fractions and Decimals competencies sharing a common variability. In the measurement model, some covariance between errors of the observed variables was also estimated to get a good fit as described above (Table 3).

Table 2. Factor loading of each variable observed after rotation

Measures	Component			
	1	2	3	4
Natural performances				
Comparison	,360	,153	,249	,188
Place-place understanding	,020	,077	,211	**,877**
Transcoding	-,030	**,686**	,127	,213
Fractions performances				
Comparison	-,120	**,517**	**,508**	-,410
Words problems	,453	-,080	**,514**	,076
Representation of fraction	,000	,004	**,733**	,096
Link Fractions-Decimals				
Fraction and decimal symbol	,103	**,782**	-,026	-,091
Measures	,441	**,689**	,002	,055
Decimal number				
Transcoding	**,582**	,292	,318	,087
Density	**,380**	,221	**,562**	,204
Line number	**,519**	,064	,481	,104
Comparison	**,672**	,068	-,033	,048
Place-value understanding	**,736**	,000	,076	-,123

Table 3. Correlations between exogenous variables

			Estimate	p-value
Fractions	<-->	Link F&D	,428	,003
Fractions	<-->	Decimals	,947	***
Natural	<-->	Fractions	,792	,007
Link F&D	<-->	Decimals	,537	***
Natural	<-->	Link F&D	,659	***
Natural	<-->	Decimals	,928	,001

Note: *** = p < 0.001.

3.3 Structural equation modeling

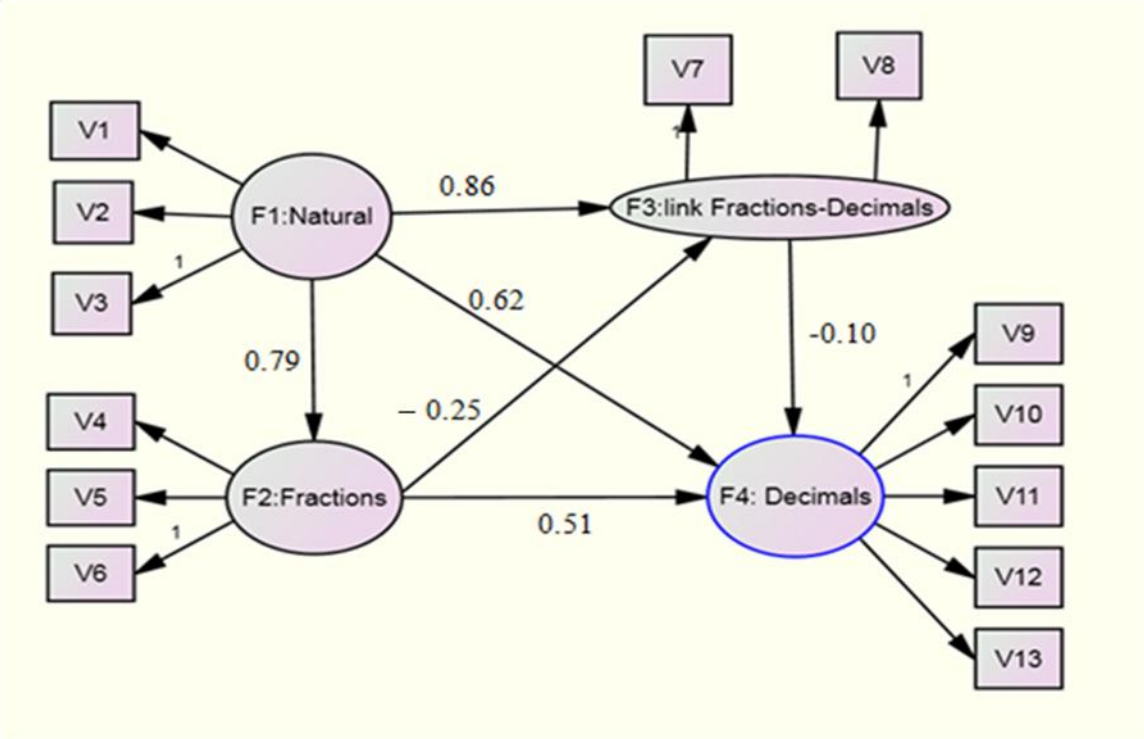


Figure 5. Equations -Standardized structural coefficients for paths:

V1 = Comparison_Natural = 0.43F1***;

V2 = Place-value_Natural= 0.28F1*** ;

V3 = Transcoding_Natural =0.35F1;

V4 = Comparison_Fractions=0.28F2**;

V5 = Words Problem =0.57F2***;

V6 = Representation_Fractions =0.4F2;

V7 = Measures=0.92F3;

V8 = Fractions_Decimals=0.54F3***;

V9 = Transcoding_Decimals =0.66F4;

V10 = Density_Decimals =0.68F4***;

V11 = Number_line =0.65F4***;

V12 = Comparison_Decimals=0.39F4***;

V13 = Place-value_Decimals=0.4F4***;

Notes: Errors were omitted. *** = p < 0.001;** = p < 0.01

As can be seen in Figure 5, each one of the 13 tasks observed variable used in the test was significant in the prediction for latent variables respectively in which they were expected to measure. Notice that in this model, the regression weight was fixed at 1, not estimated for V3; V6; V7 and V9 respectively for Naturals, Fractions, Link Fractions-Decimals and Decimals factor. Recall that all results in Figure 5 were standardized estimates. There were five correlations errors differing significantly from zero also estimated in the current model.

The model representing the associations included in the theoretical model had a good fit to the data (CMIn/df = 1.15;CFI = 0.98;RMSEA = 0.028 and PCLOSE = 0.89). The chi-square was not significant (= 62; df = 54; p = 0.11), suggesting that the sample of the covariance matrix did not differ significantly from the theoretical model of the covariance matrix.

Although the data fitted the theoretical model, all the paths between Naturals, Fractions, Link Fractions-Decimals and Decimals were not significant, except the path between Naturals and Fractions ($p < 0.05$). Notice that in the previous step, the confirmatory factor analysis, Decimals correlated highly with Naturals and Fractions factor. In addition, the model contained many covariances (significantly different from zero) between errors terms which presented unexplained variance. It suggests that the observed variables (indicator) did not measure the four factors, Natural, Fractions, Link Fractions-Decimals and Decimals separately. For example, from the correlations between the tasks related to Fractions and Link Fractions-Decimals, we can use the Fractions-Decimals as an indicator for Fractions. It can improve the regression weight between the four factors but it seemed not consistent with the purpose and design of the current study.

Based on the theoretical model for the mathematics and development point of view, we decided to keep the non-significant paths of the model in Figure 5. We can examine the effect between four variables in the model. As can be seen in Table5, R-square represent the amount of variance in each endogenous variable (second column) explained by the exogenous variables (first column). In many cases, the exogenous variables captured a substantial variance in the endogenous variables.

We observed the high total effects from Natural to Fractions, link Fractions-Decimals and Rational numbers. The data supported the theory that the acquisition of rational numbers is based on natural number knowledge. The factor Natural explained 63%, 74.3% and 38.4% of the variance of children' performance on fractions, link between Fractions-Decimals and decimal numbers respectively. It confirmed our expectation about the force of links between Natural, Fractions and Decimal numbers. The factor Fraction explained a small part (6.4%) of the variance of performance on the link Fractions-Decimals. The link Fractions-Decimals explained only 1% of the variance of performance on decimal numbers. It is interesting to note that the Fractions have a negative effect on Link Fractions-Decimals and the same goes for Link Fractions-Decimals to Decimals. It suggested that the Fractions knowledge may disturb acquisition the fraction decimal and the mastering of the relation between fractions and decimals did not contribute to better performances for decimal numbers.

Table 5. Proportion of variance explained in endogenous variables and standardized direct and total effects

Independent variable	Dependent variable	R-square	Direct effects	Total effects
Natural	Fractions	63%	0.79	0.79
Natural	Link Fractions_Decimals	74.3%	0.86	0.66
Natural	Decimals	38.4%	0.62	0.96
Fractions	Link Fractions_Decimals	6,4%	-0.25	-0.25
Fractions	Decimals	25.7%	0.51	0.53
Link Fractions_Decimals	Decimals	1%	-0.1	-0.1

4. Discussion

The results add empirical support to the theory regarding the strong associations between the natural, fractions, decimal fraction and decimals numbers. As illustrated in Table 3, all cross correlations between them significantly differed from zero. The highest correlation between natural and decimal numbers can be explained by similarity of these system numbers. They are both represented in the notation 10-base. The correlation between decimal fraction and decimal number is higher than the correlation between decimal fraction and fraction. It suggested the larger point common between decimal numbers and decimal fraction represented using the decimal system (e.g., 0.5) than the fractions and decimal fraction represented by fractions (e.g., 1/5). The results of current

study reinforce the assumption about the unavoided associates between prior and later acquired knowledge and the more they are similar, the stronger their correlations are.

It is naturally that children used to project their number theory ground in natural number to new kind of numbers, such as fractions and decimal number. The prior knowledge may lead to a negative influence on the instruction of later knowledge and children may need a conceptual reorganize (Resnick et al., 1989; Nesher & Peled, 1986, Desmet et al., 2010; Stafylidou & Vosniadou, 2004; Vamvakoussi & Vosniadou, 2004; Chapter 4). Our findings showed that the natural number competence drives the gains in fractions, decimal fractions and decimal numbers. The prior knowledge and their learning experience (e.g., natural number) also help to build the conceptual ground needed for children to acquire the later knowledge (e.g., fractions and rational numbers). The prior knowledge can leads to appreciable effect on learning the subsequent concept in either a positive or a negative way.

According to Desmet (2010), the findings indicated that the competencies in whole numbers and decimal system understanding contribute more for the leaning of decimal number than fraction knowledge. It is also in accordance with the majority of errors derived from prior knowledge about natural number than fraction number in construction of decimal number (Resnick et al., 1989; Nesher & Peled, 1986). In writing the Arabic decimal number from verbal from, children were affected more by knowledge of natural numbers than fraction knowledge (Chapter 4). Regarding instruction of rational number, Brousseau and colleagues (2007) suggested the substantial approach to teaching rational number indicating that it is not necessary to learn fractions in order to learn decimals. These previous studies can lead to a prospective suggestion of teaching the decimal number in which the decimal number is being taught after the natural number and before the fraction number. Our data showed the higher total effects from Natural to Decimal number than from Natural to Fraction number. It is suggested that the role of natural numbers is more important than fraction numbers to acquire the decimal numbers, although from historical view and mathematical front, decimal number is just another format of fraction number. Therefore, teaching the decimal number precedes the fraction number would benefit by the similarity of natural number and decimal number. The instructional method for teaching is not always the same as the historical process of a concept (Brousseau et al., 2007).

The Link Fractions-Decimals variable related to the decimal fraction which represents a bridge connecting the fractions and decimal number. Under mathematical point of view, decimal fraction plays an important role to explain the connection and the existence of two kinds of numbers, fractions and decimals. However, decimal fractions have two distinct representations, by fraction notation (e.g., 5/10) and decimal system (e.g., 0.5). If decimal number is represented as fraction notation, it shows the continuous character because it can represent quantity continuously (e.g., 5/10; 51/100; 511/1000). If decimal number is represented in decimal system (which is similar to natural number), it shows the discrete character (e.g., 0.5; 0.51; 0.511). Thus, under point of view of developpement, fraction knowledge can be not support for decimal fraction acquisition and the decimal fraction can disturb the instruction on decimal number. Our findings supported these assumptions. We

observed the negative effect from performance of fractions on decimal fraction and from decimal fractions to decimals, -0.25 and -0.1, respectively. It leads to the supposition that children may deal separately fractions and decimal numbers, not necessary go through the concept mediate, fraction decimal. These results boost the assumption that the historical way is not always appreciated way and provide the support for an alternative approach to teaching rational concerning the teaching decimal numbers precedes the fractions.

Reminding that in teaching the rational numbers, the fraction is being taught before decimal number in Vietnamese curriculum while in Belgium, the decimal number seems to be taught before fraction number. We observed that Belgian children perform decimal number better than Vietnamese counterparts. We cannot yet just base on the Belgium choice of curriculum to explain these results. Further experimental researches should be conducted to investigate or to have better understanding about the choice of the approach in rational. The difference of curriculum, language, cultural between Vietnam and Belgium can lead to variation of associations between natural, fraction and decimal number. However, in this current study, the number of participants was not enough to conduct two analyses in each country that helps better understanding about associations between natural, fraction and decimal number and lead to more practical implications in teaching number in primary school.

General conclusion

General conclusion

The overall aim of this thesis was to improve the comprehension of the impact of language on the transcoding task of natural numbers and the acquisition of rational numbers, covering the differences of language and instruction on numbers between Belgium and Vietnam. Each study contributed to some understanding, but also leaves some discussions to this topic and ideas for further research. This section addresses to such discussions and possibilities for further research.

The impact of language characteristics on transcoding task and the presence of zero in number

Children usually use numbers in different representations in practicing mathematics and daily activities. They also have many occasions to translate numbers from one code to another. This transcoding process requires mastering the input and the output codes. Two types of transcoding models are distinguished: semantic and asemantic. In the semantic model, the process has to pass an intermediary semantic representation between the input and the output codes. The asemantic model proposes that we use a procedural system and do not require access to the meaning of numbers. Children encounter difficulties with the transcoding task at early stages of learning and they need some years to acquire the transcoding competence. The length of a number (the number of digits), the infrequency of the number, the phonological length of verbal numbers (evaluated by the number of syllables) and the presence of a syntactic zero are factors leading to difficulties for the transcoding task. Previous studies showed that linguistic differences have an impact on the performances of a transcoding task. Specific errors are observed in relation to the verbal number system differences. Comparison studies previously investigated the impact of language on the understanding and performance of mathematics between Asia and Occident. Our first study (Chapter 4) was motivated by the particular number name feature present in the Vietnamese language and its relation to the syntactic zero. Previous comparison studies between Asian and Occidental children did not use a transcoding task directly related to language. Our studies carry out a new approach and address a different aspect by using the transcoding task and examining the difficulties of the syntactic zero. It confirmed that different languages lead to variations in the numerical performances between Asian and Occidental children. The better performances of Vietnamese children are related to the effectiveness of the Vietnamese number-name system compared to the French system. In addition, specific errors may emerge from the specific properties of a number-name system (Zuber et al., 2009; Saad, 2011). For example, French children have difficulties for transcoding with numbers from 70 to 99, Germans and Syrians often make the inversion errors in transcoding. The syntactic zero is considered as a source of difficulty for children of grades 2 and 3 (Censabella, 2000). The current study showed that this influence is less important or non-existent for Belgian children. The effect of the presence of a syntactic zero in a number for a transcoding task was not significant for every grade

from 3 to 6 in Belgium. It can be explained by the fact that participants of our study were older and that the items we used had more digits compared to Censabella's study (2000). When children have more experience in transcoding, the syntactic zero would affect them less. In addition, our task comprised many items with 6 digits so children may generally have troubles in manipulation with large numbers on top of their difficulties with items comprising a syntactic zero. In particular, the syntactic zero was a factor facilitating performances for Vietnamese grades 3 and 4. Even if syntactic and lexical zeros are distinguished in the Arabic form, the difference between them is not explicit for a transcoding task from verbal to Arabic numbers in Vietnam. In addition, Vietnamese children might make less lexical errors with items having a syntactic zero since all syntactic zeros are explicitly pronounced in the Vietnamese language.

The influence of language was demonstrated mainly on abilities related to the canonical base-10 system and on linguistic tasks such as counting or transcoding. According to the three codes model of Dehaene (1992), each representation of a number (verbal, Arabic and analog) is associated with specific manipulations. The analog (magnitude) representation is independent of the base-10 structure. It could be interesting to examine whether the transparent property of the syntactic zero also emerges in place-value understanding (e.g., identify a digit in a multi-digit number) and in non-linguistic tasks, such as analog-Arabic transcoding. The second study (Chapter 5) was conducted to explore more precisely this influence. Results suggested that the impact of language did not affect the tasks involving the manipulation of symbols. Belgian children had better performances in comparison, place-value understanding and analog-Arabic transcoding task than Vietnamese children, except in verbal-Arabic transcoding. The explanation is that the ten-base structure, appearing in the analog-representation of a number, is independent from the verbal number (Dehaene, 1992), so the advantage of the Vietnamese number-name system was not active in this case. We observed that Vietnamese children benefit from the transparent number-name system for the verbal-Arabic transcoding task, but this advantage vanishes with the digit identification task and analog-Arabic task, suggesting an immature representation of the natural number. Having a transparent number-name system is an advantage for Vietnamese children, but an exaggerated dependence on verbal-numbers and establishing technical rules in learning may prevent an in-depth understanding. Regarding instructional mathematics in Vietnam, more emphasis on place-value understanding and the manipulation with analog number representations would be recommended to fully provide the preparation needed for Vietnamese children to learn the positional number system.

Using diverse tasks, which relate differently to the number-name system, the second study contributed to a better understanding of cross-cultural mathematical performances regarding the number-name system. To appraise more precisely the influence of language on mathematical performances, further researches examining the appropriateness of tasks used in previous studies (such as Saxton and Towse, 1998) should be conducted.

Concerning verbal-arabic transcoding models, we can distinguish two main types: the semantic models such as McCloskey (McCloskey et al.,1985; Power & Dal Martello, 1990) which indicates that we have to use the semantic representation (number sense) in the

process of transcoding; or the asemantic model like those of Dehaene (1992) ; Deloche et Seron, (1987); Barrouillet et al., (2004, p368) that assumes that "*the transcoding process shifts from an algorithmic strategy to the direct retrieval from memory of digital forms*". The debate between two models is not yet solved (Noël, 2005; Seron & Noël, 1995; Lochy, Pillon, Zesiger & Seron 2002; Lochy, Domahs, Bartha & Delazer, 2004), but the investigation of Jarlegan and colleagues (1996) supported the asemantic model. Moreover, the errors of French children in transcoding complex numbers from 70 to 99 cannot be explained by the semantic model (Seron & Fayol, 1994). According Barrouillet and colleagues (2004), the asemantic model (ADAPT) is considered relatively independent from linguistic characteristics. ADAPT can account for the transcoding in other languages with some modifications, for example the Japanese language with the transparency in the number-name system (Saad, 2011). It is clear that the difficulty of the transcoding of a number depends on the number of rules (actions) required for its transcoding. Since the Japanese number-name system reflects better the ten-base structure, the transcoding model becomes simpler (Saad, 2011) and it can be accounted for the better performances of Asian children (Vietnamese children in our study) over Belgian children in verbal-arabic transcoding (Chapter 4). The results in the second study (Chapter 5) supported the asemantic model. If we have to use the semantic representation (analog representation or number sense) in the process of transcoding, we cannot have higher performances of verbal-Arabic transcoding task than one of analog-Arabic transcoding task, as observed in this study.

The zero in the Arabic number representation and the misconception about the function of the zero in a number

The digit zero "0" plays a role of place-holder in the Arabic decimal system. It means that a zero have two correlated functions. The first is related to the syntactic rule in production of an Arabic number. When a power of ten is absent in the number, the insertion of a zero is required in order to conserve the positional system of the other digits (e.g., "two hundred five" □ 205). The second function reflects the opposite side of the first. When a digit in a number is equal to zero, even if it is worth nothing, this digit always occupies a position in this number (e.g., in 205, the value of tens is zero but there is always a digit indicating the tens - 0). The syntactic zero leads to difficulties for children at grade 3 in the transcoding task (Censabella, 2000). Could the zero lead to difficulties in other types of presentations, as well as in other advanced mathematical activities? The second study (Chapter 5) used varied tasks to investigate the impact of zero. Since the distinction between syntactic and lexical zeros is proposed and related to the transcoding process (Granà et al., 2003; Power & Dal Marteloo, 1997), we used, at the same time, two transcoding tasks: from verbal to Arabic code and from analog to Arabic code. To a larger extent, the functions of the zero in the Arabic representation were also investigated by a task of digit identification. For a verbal-Arabic transcoding task, syntactic zero led to lower performances for Vietnamese children. This observation was the opposite of the results of the first study (Chapter 4), but we should take into account that in the second study, we used only items of 4 digits while in the first one, most of the items had 5 or 6 digits. For the

Belgian samples in the first study, the effect of syntactic zero was also not significant. For the analog-Arabic transcoding task, the syntactic zero led to difficulties for both countries. Concerning the digit identification task, results showed that the impact of zero was no found in the Vietnamese sample but the presence of a zero led to lower scores for Belgian children. Data suggested that the presence of a zero led to positional number system difficulty, particularly during manipulations with the analog representation of numbers and digit identification tasks for both samples of Belgians and Vietnamese, even though they were different in terms of number-name system, mathematical approaches and initial arithmetical knowledge. In formal mathematics teaching, it would be necessary to focus more on a clear explanation and to emphasize the practice of the zero in a number.

Limits and perspectives

Our first study had some limitations related to the tasks. The verbal-Arabic transcoding included 18 items. Two categories were distinguished: with a syntactic zero and without syntactic zero. However, the distribution of the items into the two categories was not equivalent (12 items for syntactic zero category and 6 for non-syntactic zero category). This problem was solved in the second study.

The better performances of Vietnamese children over Belgians children in the first study was interpreted by the transparency of language. It can be also explained by differences in mathematical program and arithmetical everyday experience. Vietnamese children received a more advanced program mathematic related to large number. In addition, they have more experience of large number in everyday life since the monetary system use units of "hundred". However, the task in the first study included 9 items of 6 digits and only 9 items for the numbers of 3, 4 and 5 digits. Is it possible that the lower performance of Belgians children due to the large numbers? We cannot examine this assumption in context of this thesis since there are not sufficient to carry out such analysis. Further study could investigate this aspect. Recall that in previous cross-national studies and mathematical achievement studies, besides language factor, other factors like parents' behavior, initial arithmetical knowledge, learning context and mathematical methodology could account for the advancement of Asian children over Western children (Ho & Fuson, 1998; Miller, Smith, Zhu, & Zhang, 1995; Saxton & Towse, 1998; Towse & Saxton, 1997). For further researches that focus on the language impact, it would be more careful when constructing the task in order to avoid confounding factors.

Whole number bias and the acquisition of the rational number

Mastery of the rational number is an important mission of mathematical literacy since the acquisition of these numbers is essential for later learning of algebra and for its strong predictive value for later mathematics achievement (Bailey et al., 2012; Booth & Newton, 2012; Siegler et al.,2012; Siegler, Thompson, & Schneider, 2011; Torbeyns et al., 2014). A good understanding of rational numbers is considered as a large challenge for teaching and learning mathematics. Children usually receive a formal instruction on rational numbers from fourth grade. Previous studies observed that older children and adults often have

difficulties to understand and use rational numbers. They make systematic errors, which are considered as a derivation of the "natural number bias" (Vamvakoussi et al., 2012).

Why understanding and using rational numbers represent such a difficulty? One explanation is that natural numbers shares some common properties with the rational numbers, but many features, which are only applicable for natural numbers, are erroneously used for rational numbers. Thus, when children deal with fractions and decimals, it leads to systematic errors.

Among previous and recent studies (study 3 was conducted in 2010) that investigated the difficulties of rational numbers, those difficulties can be sorted in some aspects which are related to the number's size, density, arithmetic operations and representation of rational numbers (Vamvakoussi et al., 2012, Hoof et al., 2014). Concerning the number's size aspect, the studies using a comparison task of decimal numbers (Resnick et al., 1989; Desmet, Gregoire & Mussolin, 2010; Sackur-Grisvard & Leonard, 1985; Nesher & Peled, 1986) showed that errors were related to the assumption "longer decimals are larger" and "shorter decimals are smaller". The first assumption is related to the knowledge of natural numbers is that the number with more decimal digits is the larger one (e.g., 12.17 > 12.4 because 17 > 4). The second one is derived from the knowledge of fractional numbers and is that the number with more decimal digits is the smaller one (e.g., 12.94 < 12.7 because the shorter number has tenths and the longer number has hundredths and tenths are bigger than hundredths). In addition, the length of the fractional part, the digit value and the position of zero in decimal numbers also affected the comparison of decimal numbers (Desmet et al., 2010). For the comparison of fractional numbers, the wrong assumption "a fraction's numerical value increases when its denominator, numerator, or both increase" which is derived from the interpretation of the symbol a/b as two independent natural numbers, leads to difficulties in comparing fraction numbers (Mamede, Nunes, & Bryant, 2005; Meert, Grégoire, & Noël, 2010, Vosniadou & DeWolf, 2014; Hoof et al., 2014; Vamvakoussi et al., 2012). The relations among fraction arithmetic procedures are considered to be a source of difficulty in learning fractions. In fact, beliefs about "multiplication makes bigger" and "division makes smaller" established with natural numbers are resistant and generate obstacles in dealing with operations of fraction numbers (Greer, 1994). These obstacles also affect adults (Graeber, Tirosh, & Glover, 1989; Hoof et al., 2014). The difference between natural and rational numbers involving the property of density leads to the frequent and false assumption that there are no numbers between two pseudo-successive rational numbers. This kind of mistake is difficult to overcome and was examined by many studies (Hannula et al., 2006; Hartnett & Gelman, 1998; Merenluoto & Lehtinen, 2002; Smith et al., 2005; Vamvakoussi & Vosniadou, 2004, 2010; Vamvakoussi, Christou, Mertens, & Van Dooren, 2011). Naturals have a unique symbolic representation to indicate a quantity while a fraction has infinity of possible representations. In addition, even if a fraction number representation differs from a decimal number representation, they can both present the same quantity. Studies that examined this aspect showed that this difference increases the difficulties in dealing with rational numbers (Vamvakoussi et al., 2012; Smith et al., 2005; Stafylidou & Vosniadou, 2004).

Many studies investigated various aspects of the difficulties and the development of rational numbers. The erroneous understanding can be derived from the natural number bias. Each investigation may provide a better understanding about the natural number bias and the development of numbers. Natural and decimal numbers are both represented in decimal positional system, but there transcoding in Arabic numbers follow partly different rules. Does the natural number bias occur in this process? To the best of our knowledge, there is no study investigating this aspect of the natural number bias. The third study (Chapter 6) in this thesis examined the development of the decimal number's symbolic representation through the transcoding task and the impact of natural knowledge on it. Children responses observed for a transcoding task of decimal numbers can be explained by different conceptions. One of these conceptions is derived from the knowledge of natural numbers in which *the decimal is identified as a natural number*, for example, responses such as 7, 17, 70 or 710 for the item "seven tenths". These responses showed that children tend to respond according to their natural number knowledge, even in the presence of elements related to rational numbers such as the tenths, hundredths or thousandths. An evolution in the decimal number conceptualization was observed through children's responses. They distinguished the difference between the symbolic representations of the natural number and the decimal number by using a comma to write the decimal number. However, the influence of their knowledge about natural numbers still had an important impact on many aspects of this process. First, children identified the decimal terms (i.e., tenths-hundredths-thousandths) with tens-hundreds-thousands in the natural number. The confusion between tens and tenths, hundreds and hundredths, thousands and thousandths was still very strong. Second, the relational order of the natural number system (i.e., thousands-hundreds-tens-units) was directly applied to the decimal number (i.e., thousandths-hundredths-tenths-units) during an overgeneralization process, for example responses such as 1.7 or 10.7 for "seven tenths". Finally, the idea that a number cannot start with a zero was consistent among children. This study was as an investigation of the natural number bias from a new angle. Unlike previous studies, which often used simultaneously congruent and incongruent items to detect the natural number bias, we analyzed all possible responses from transcoding tasks to examine the impact of the natural number bias. .This study suggested that the natural number bias occurred with the understanding of the positional number system.

Concerning the evolution of rational numbers representation and relations between natural and rational numbers.

The third study assessed children from grade 3 to the last year of primary school (grade 5 in Vietnam and grade 6 in Belgium). A cluster analysis was conducted to assemble children who had a homogeneous pattern of responses. It allowed us to examine the pattern of the conceptual change across the grades. Despite the difference between our two samples, we observed similar features in children's systematic incorrect responses. At grade 3, children did not have any formal knowledge of the decimal number. The majority of their responses were affected by the knowledge of natural numbers. They had difficulties with numbers starting with a digit that was equal to zero and with the conservation of the

ordering sequence "thousands-hundreds-tens-units" for "thousandths-hundredths-tenths-units". At grade 4, Belgian children officially learned the decimal numbers at school. At the same time, Vietnamese children did not learn decimal numbers, but were already taught about fraction numbers. They were also affected by the acquisition of natural numbers, but most of them showed a radical conceptual change about their representation of decimal Arabic numbers. Vietnamese children in grade 4 were particularity disturbed by their knowledge about fractions to present a decimal number (e.g., "two units seven tenths" 2, 2/10). It shows the importance of existing knowledge when children are constructing a symbolic representation of a new concept. At grade 5, children in both countries were instructed about decimal numbers. Children in grade 5 in both countries had a good understanding about the representation of decimal Arabic numbers.

From the mathematical point of view and from the development of rational number point of view, relations between natural numbers, fractions, decimals and decimal fraction are complex. As it has been showed, the knowledge of natural numbers make difficult the acquisition of fraction and rational numbers. According to our third study (Chapter 5), errors made by children in dealing with decimal numbers are more affected by naturals knowledge than fractions knowledge. It is consistent with the results of previous studies that children often used the natural-rule over the fraction-rule when they compare decimal numbers (Resnick et al. 1989; Nesher & Peled, 1986). Therefore, natural numbers knowledge would hinder the acquisition of decimal numbers stronger than fraction numbers knowledge. In addition, based on the history of rational numbers, the fraction knowledge was expected to support the learning of decimal numbers. However, the competencies in natural numbers and mainly the decimal system understanding strongly contributed for acquisition of decimal numbers (Desmet, 2012). While prior knowledge often plays an important role as foundation, for the later knowledge, it does not always have the expected effect on learning of new concepts, in a positive or a negative way.. The fifth study (Chapter 8) was conducted to investigate a larger view of relations between natural and decimal numbers. Results showed that natural number competences drive gains in fractions, decimal fractions and decimal numbers. The natural number is suggested as the strongest precursor for the decimals numbers. Its prediction is even larger than the contributions of fraction numbers for decimal numbers. Regarding instruction of rational number, our findings supported the substantial approach of teaching rational number, indicating that it is not necessary to learn fractions in order to learn decimals (Brousseau et al., 2007). The important contribution of this study is that even through the essential associations between fractions, decimal fractions and decimal numbers, under the point of view of development, both fractions and decimal fractions affected the decimal number in a negative way. It opens an alternative approach in teaching rational numbers.

The part-whole schema should be a privilege in teaching the fraction?

The fraction number is complex since it can be interpreted through a wide variety of subconstructs. The partitioning action associated with the part-whole relationship is considered as a foundation for developing rational number knowledge and essential in construction of fraction. In fraction number teaching, an approach using the part-whole

relationship is often favored. In particular, the act of dividing a whole in parts is dominant in fraction teaching, so children have the tendency to consider a fraction as "a part of a whole" and "a fraction is always smaller than 1 or a whole". Thus, it is difficult to explain the existence and sense of improper fractions in which the number of parts is larger than the number of parts in a whole (Steffe, 2002; Thompson & Saldanha, 2003; Tzur, 1999).

The purpose of our fourth study was to investigate the misunderstanding of fraction through the part-whole approach. We used parallel fractions superior and inferior to 1 with their graphic representations to tackle this question. The findings suggested that the majority of children had a tendency to overgeneralize the rule used for fractions inferior to 1 to fractions superior to 1. "The number of all parts" was mistaken for "the number of parts in a whole" in the determination of the denominator. This study, with its new approach, reinforced the limitation of the part-whole scheme in the teaching of fraction numbers. Results gave some implications for the instruction of fractions. Children would have the occasion to manipulate fractions superior to 1, even in the context of a part-whole relationship. The quotient approach also offers children the opportunity to manipulate fractions larger than 1. Since fractions are a multi-faceted concept, alternative teaching approaches may be used simultaneously for a better understanding. The part-whole scheme could be used first to introduce fractions followed by alternative approaches. This could provide a better understanding of the part-whole relationship and deeper acquisition of the representation of fraction numbers. It is consistent with the suggestion that an over-reliance on the part whole aspect of fraction number may lead to an obstacle to further understanding rational numbers concepts (Mamede, Nunes, & Bryant, 2005; Moss, 2005).

Difference between Vietnam and Belgium: effect of the curriculum and language systems

From the third study (Chapter 6), in the early state of decimal number learning (grades 3 and 4), Vietnamese children outperformed Belgian children when writing decimal Arabic numbers from verbal code. Belgian children were influenced more by natural knowledge than their counterparts. In addition, the responses were based on the fraction knowledge that observed for Vietnamese children at grade 4. In grade 5, when children have learned the decimal in school, Belgian children outperformed Vietnamese children. These differences between the two countries could be explained by the difference in mathematical language and curriculum.

The terms used for rational numbers in the French language are similar to those used in the natural number system, such as "tenths" and "tens", for instance, which may reinforce the potential symmetry between naturals and decimals. In Vietnamese, "tenths" is translated "part (of) ten", which may help children to distinguish the difference in quantities between the two terms and also help them in their conceptualization of the meaning. For children who have not yet learned the decimal concept at school (grade 3 in Belgium and grade 3 and 4 in Vietnam), these subtle differences could affect the ability to produce an Arabic number from a verbal expression. A study examining the influence of language on numerical fractions suggested that young Korean children (6 to 7 years old) outperformed

American and Croatian children in a task in which they had to associate a fraction's notation with its corresponding graphical representation (Miura et al., 1999). Their results were also explained by the explicit meaning of the words used for fractions in Korean, which is similar to the Vietnamese language. However, this assumption is challenged by the study of Paik and Mix (2003). These authors showed that the superiority of Korean children was not linked to the linguistic transparency, but to other cultural factors (e.g., time devoted to mathematics). It is a challenge to separate the impact of language from other factors in a national comparison study, in particular when children have received the formal education in school. In our third study, Belgian children outperformed Vietnamese children in grade 5, unlike in grades 3 and 4. The denomination of decimal number in primary school in Belgium (e.g., 5,92 is pronounced "five units nine tenths two hundredths" instead of "five point ninety-two") could partly explain the superior results of Belgians children. These results can be linked to many factors, such as different instruction, approaches and practices in teaching and learning; they may also be due to various numerical experiences with rational numbers between the two countries. In Belgium, the system of currencies comprises hundredths (1 cent of euro), tenths (10 cents of euro) and one euro. In Vietnam, units are hundred (100 Vietnam dongs), thousand (1000 Vietnam dongs). The fifth study (Chapter 8) confirmed the superiority of Belgians children for competencies of decimal numbers. This finding seems to be due to a more effective curriculum in teaching the decimal number to Belgians.

The impact of topic ordering for the instruction of rational numbers was also remarked on the understanding of the fraction concept and representation of decimal numbers. Vietnamese children at grade 4 produced more responses based on fraction numbers knowledge (Chapter 6) and they had the lowest performances in manipulating with items related to unit-partitioning (Chapter 7). These observations could be clarified by the fact that in Vietnamese curriculum, fractions are taught at grade 4 and the fraction concept is emphasized by a quotient sub-construct.

Limits and perspectives

In the third study (Chapter 6), we used the transcoding task in which the decimal part of all items has a single element (tenths, hundredths or thousandths). We observed responses such as 0,2000 for "two thousandths"; 35,800 for "thirty five units and eight hundredths". It suggested that the decimal part "thousandths" and "hundredths" are treated as "thousand" and "hundred" from natural numbers. In previous studies using comparison tasks (Resnick et al., 1989; Desmet, Gregoire & Mussolin, 2010; Sackur-Grisvard & Leonard, 1985; Nesher & Peled, 1986), incorrect responses were usually associated with the rule "longer decimals are larger", derived from natural numbers knowledge. Is it possible that children are not simply using this rule when they compare decimal numbers? Could they misunderstand the decimal system application for the decimal part of decimal numbers? It would mean that when comparing decimal numbers, for example 3,258 and 3,6, children could interpret the digit "2" in the first number as "hundred" and digit "6" in the second number as "unit". Thus, 3,258 is supposed bigger than 3,6. Future researches should investigate if the source of error in a comparison task is linked to a misunderstanding of the

decimal system. A verbal-Arabic transcoding task of decimal numbers with items comprising two and three elements (e.g., "three units, five tenths and two hundredths") associated with a post-testing interview of participants could be used as a reliable tool to evaluate those questions.

Does the Belgian choice of curriculum concerning the denomination of the decimal number in primary school provides the opportunity to a better understanding of rational numbers? In the context of this thesis, a study examining the effectiveness of expressions "tenths, hundredths, and thousandths" was conducted only in Belgium. Children from grade 4 to 6 (74 children in grade 4, 137 in grade 5 and 121 in grade 6) fulfilled two comparison tasks with verbal decimal numbers: one task with the numbers represented with their symbolic representation (e.g., "eight point fifty-three" vs. "eight point nine") and the other with the expressions "tenths, hundredths, and thousandths" (e.g., "eight units five tenths and three hundredths" vs. "eight unites nine tenths"). It should be noticed that two tasks had the same pairs but different expressions (see Annexes). Results showed that for each grade from 4 to 6, performances with expressions "tenths, hundredths, and thousandths" were significantly higher. It showed the appropriateness of the expressions "tenths, hundredths, and thousandths" which explicitly present the position of each digit in the verbal-number for the acquisition of decimal number. Should the Vietnamese curriculum use expressions "tenths, hundredths, and thousandths" in teaching the decimal number? It would be interesting to examine the efficiency of such a choice in a future study. A future research, using a didactical intervention using these expressions should investigate the impact of mathematical language in the acquisition of rational numbers.

The third study was conducted with participants from grade 3 to grade 6. We classified all responses into different levels to illustrate the evolution of the decimal number representation and then we used a cluster analysis to determine the groups of children who had a homogeneous pattern of responses. It allowed us to examine the pattern of the conceptual change across grades. However, we don't know when the conceptual change occurs for each individual. A latent profile analysis and latent transition analysis in a longitudinal way would allow this assessment (McMullen et al., 2014). For a further study, using the approach mentioned above could be recommended for a better understanding of the developmental process of the rational number.

The teaching of the rational number in Vietnam perfectly follows its historical way of development. The fraction is elaborated in grade 4 and decimal number is taught in grade 5. A great emphasis is placed on the decimal fraction as a bridge connecting fractions and decimal numbers. In various tasks related to fractions, decimal fraction and decimal numbers, Vietnamese children had higher performances for decimal fraction numbers than Belgian children. However, in all the other tasks, Belgian children outperformed Vietnamese. Could it be explained by the effectiveness of the approach in teaching rational in Belgium? A deeper investigation is needed to examine this question. Although the fifth study was conducted on both samples, Belgian and Vietnamese children, the number of participants was not sufficient to carry out two structural equations separately. It allows us

to examine the associations between natural and rational numbers in two different contexts of teaching the rational number. We leave this door open for further investigations.

Références bibliographiques

Références bibliographiques

Administration générale de l'enseignement et de la recherche scientifique. (2002). *Programme des études d'enseignement fondamental.* Bruxelles : Ministère de la Communauté française.

Administration générale de l'enseignement et de la recherche scientifique. (2002). *Programme des études d'enseignement fondamental.* Bruxelles : Ministère de la Communauté française.

Adjiage, R. (1999). *L'expression des nombres rationnels et leur enseignement initial. Mathematics*. Thèse non publié, Université Louis Pasteur - Strasbourg I. French.

Antell, S. E., & Keating, D. P. (1983). Perception of numerical invariance in neonates. *Child Development, 54*, 695–701.

Bachelard, G. (1938). *La formation de l'esprit scientifique.* Paris: Vrin.

Bailey, D. H., Hoard, M. K., Nugent, L., & Geary, D. C. (2012). Competence with fractions predicts gains in mathematics achievement. *Journal of Experimental Child Psychology*, *113*, 447-455.

Ball, D. (1993). Halves, pieces and twoths: Constructing and using representational contexts in teaching fractions. In T. P. Carpenter, E. Fennema and T. A. Romberg (eds.), *Rational Numbers: An Integration of Research* (pp. 157-195) Hillsdale NJ: Lawrence Erlbaum Associates.

Barrouillet, P., Camos, V., Perruchet, P., & Seron, X. (2004). ADAPT: A developmental, asemantic, and procedural model for transcoding from verbal to arabic numerals, *Psychological review, 111*, 368-394.

Baturo, A.R. (2004). *Empowering Andrea to help year 5 students construct fraction understanding*. Proceedings of the 28th PME Conference, Vol. 2, pp. 95–102, Bergen University College, Bergen.

Behr, M. J., Harel, G., Post, T. R., & Lesh, R. (1992). Rational number, ratio, and proportion. In A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning. A project of the national council of teachers in mathematics* (pp. 296-333). New York, NY: Simon and Schuster Macmillan.

Behr, M., Harel, G., Post, T. & Lesh, R. (1993). Rational numbers: Towards a semantic analysis - Emphasis on the operator construct. In T. P. Carpenter, E. Fennema & T. A. Romberg (eds.). *Rational Numbers: An Integration of Research* (pp. 13-47), Hillsdale, NJ : Lawrence Erlbaum Associates.

Behr, M. J., Lesh, R., Post, T. R. & Silver, E. A. (1983). Rational number concepts. In R. Lesh & M. Landau (eds.), *Acquisition of Mathematics Concepts and Processes* (pp. 91-126). Academic Press, New York.

Bo giao duc va dao tao. (2006). Chuong trinh Toan tieu hoc.

Booth, J. L., & Newton, K. J. (2012). Fractions: could they really be the gatekeeper's doorman? *Contemporary Educational Psychology*, *37*, 247-253.

Boulet, G. (1998). Didactical implications of children's difficulties in learning the fraction concept. *Focus on Learning Problems in Mathematics 20*(*4*), 19–34.

Briars, D. J., & Siegler, R. S. (1984). A featural analysis of preschooler's counting knowledge. *Developmental Psychology, 20,* 607–618.

Bright, G. W., Behr, M. J., Post, T. R., & Wachsmuth, I. (1988). Identifying fractions on number lines. *Journal for Research in Mathematics Education, 19(3)*, 215–232.

Brissiaud, R. (1998). *Les fractions et les décimaux au CM1. Une nouvelle approche.* XXV Colloque des formateurs et professeurs de mathématiques chargés de la formation des maîtres. COPIRELEM. Loctudy, (pp 147-171).

Brousseau, G. (1998). *La théorie des situations didactiques: Didactique des mathématiques 1970-1990*. Grenoble, France La Pensée Sauvage.

Brousseau, G., Brousseau, N., & Warfield, V. (2007). Rationals and decimals as required in the school curriculum. Part 2: From rationals to decimals. *Journal of mathematical behavior, 26,* 281-300.

Brown, C., Carpenter, T., Kouba, V., Lindquist, M., Silver, E. & Swafford, J. (1988). Secondary school results from the fourth NAEP mathematics assessment: Algebra, geometry, mathematical methods, and attitudes. *Mathematics Teacher*, *81*(5), (337-347).

Byrnes, J.P., & Wasik, B.A. (1991). Role of conceptual knowledge in mathematical procedural learning. *Developmental psychology, 27,* 777-786.

Canobi, K. H., & Bethune, N. E. (2008). Number words in young children's conceptual and procedural knowledge of addition, subtraction and inversion. *Cognition, 108,* 675– 686.

Carey, S. (2001). On the very possibility of discontinuities in conceptual development. In E. Dupoux (Ed.), *Language, brain, and cognitive development: Essays in honor of Jacques Mehler* (pp. 303–324). Cambridge, MA: MIT Press.

Carpenter, T. P., Fennema, E. & Romberg, T. A. (1993). Towards a unified discipline of scientific enquiry. In T. P. Carpenter, E. Fennema, and T. A. Romberg (eds.), *Rational Numbers: An Integration of Research* (pp. 1-11), Hillsdale, NJ : Lawrence Erlbaum Associates.

Censabella, S. (2000). *L'écriture des nombres sous dictée chez l'enfant. Etude qualitative et implication de la mémoire de travail*. Unpublished master thesis, Université de Liège, Liège, Belgique.

Chan, W.-H., Leu, Y.-C. & Chen C.-M. (2007). Exploring group-wise conceptual deficiences of fractions for fifth and sixth graders in Taiwan. *The Journal of Experimental Education*. *76*, 26–57.

Charalambous, C. Y. & Pitta-Pantazi, D. (2007). Drawing on a Theoretical Model to Study Students' Understandings of Fractions. *Educational Studies in Mathematics*, *64*, 293-316.

Cipolotti, L., Butterworth, B., & Warrington, E. K. (1994). From “one thousand nine hundred and forty five” to 1,000,945. *Neuropsychologia, 32,* 503-509.

Clearfield, M. W., & Mix, K. (1999). Number versus length in infants’ discrimination of small visual sets. *Psychological Science, 10*, 408–411.

Corwin, R. B., Russel, S. J., & Tierney, C. (1991). *Seeing fractions: A unit for the upper elementary grades.* Sacramento, CA: California Department of Eduction.

Davis, G. E. & Pitkethly, A. (1990). Cognitive aspects of sharing. *Journal for Research in Mathematics Education, 21*(2), 145-154.

Davydov, V. V., & Tsvetkovich, Z. H. (1991). The object sources of the concept of fraction. In V. V. Davydov (Soviet Edition Editor) & L. P. Steffe (English Language Editor), *Soviet studies in mathematics education: Psychological abilities of primary school children in learning mathematics* (pp. 86–147). Reston, VA: National Council of Teachers of Mathematics.

Dehaene, S. (1992). Varieties of numerical abilities. *Cognition 44*, 1–42.

Deloche, G., & Seron, X. (1982). From one to 1: An analysis of a transcoding process by means of neuropsychological data. *Cognition 12*, 119–149.

Deloche, G., & Seron, X. (1987). Numerical transcoding: A general production model. In G. Deloche & X. Seron (Eds.), *Mathematical disabilities: A cognitive neuropsychological perspective* (pp. 137–170). Hillsdale, NJ: Erlbaum.

Desmet L. (2012). *L’apprentissage des nombres décimaux, des nombres rationnels représentés par le système décimal de position*. Thèse non publié, Université catholique de Louvain, Belgium.

Desmet, L., Grégoire, J., & Mussolin, C. (2010). Developmental changes in the comparison of decimal fractions. *Learning and Instruction, 20,* 521-532.

Do, H.D., Nguyen, A., Dang, A.T., Vu, C.Q., Do, D. T., Do, H. T., Dao, L.T., Tran, L.V., Pham, T.T., Kieu, T.D., Le, T.T., & Vu, T.D. (2006). *Sach giao khoa toan 5*. Hanoi: Nha xuat ban giao duc Vietnam.

Do, H.D., Nguyen, A., Vu, C.Q., Do, D.T., Do, H. T., Tran, H.D., Dao, L.T., Pham,T.T., Kieu, T.D., Le, T.T., & Vu, T.D. (2006). *Sach giao khoa toan 4*. Hanoi: Nha xuat ban giao duc Vietnam.

Do, H.D., Nguyen, A., Vu, C.Q., Do, D.T., Do, H. T., Tran, H.D., Dao, L.T., Kieu, T.D., Le, T.T., & Vu, T.D. (2006). *Toan 4 Sach giao vien*. Hanoi: Nha xuat ban giao duc Vietnam.

Do, H.D., Nguyen, A., Dang, A.T., Vu, C.Q., Do, D. T., Do, H. T., Dao, L.T., Tran, L.V., Pham, T.T., Le, T.T., & Vu, T.D. (2006). *Toan 5 Sach giao vien*. Hanoi: Nha xuat ban giao duc Vietnam.

Duroux A. (1983). La valeur absolue : Difficultés majeures pour une notion mineure. *Petit x, 3,* 43-87.

Echterbille, G., Elsevier, L., Marchal, J., Schram, W., & Vandenbergh, A. (2002). *Archi m’aide 4.* Belgium: Van In.

Echterbille, G., Elsevier, L., Marchal, J., Schram, W., & Vandenbergh, A.(2002). *Archi m’aide 5.* Belgium: Van In.

Empson, S.B. (1999). Equal sharing and shared meaning: the development of fraction concepts in a first- grade classroom. *Cognition and instruction. 17*(3), 283-243.

English, L., & Halford, G. S. (1995). *Mathematics education: Models and processes*. Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Fayol M., Barrouillet P. & Renaud A. (1996). Mais pourquoi 1'Ecriture des Grands-Nombres est-elle aussi difficile? *Revue de Psychologie de l'Education, 2*, 87-107.

Feigenson, L., Carey, S., & Spelke, E. (2002). Infants' discrimination of number vs. continuous extent. *Cognitive Psychology, 44*, 33–66.

Fischbein, E., Deri, M., Nello, M., & Marino, M. (1985). The role of implicit modles in solving problems in multiplication and division. *Journal for Research in Mathematics Education, 16*, 3-17.

Freudenthal, H. (1983). Didactical Phenomenology of Mathematical Structures. *Mathematics Education Library*, D. Reidel, Boston.

Fuson, K. C. (1988). *Children's counting and concepts of number*. New York: Springer-Verlag.

Fuson, K. C., Richards, J., & Briars, D. J. (1982). The acquisition and elaboration of the number word sequence. In C. J. Brainerd (Ed.), *Children's logical and mathematical cognition: Progress in cognitive development research* (pp. 33-92). New York: Springer-Verlag.

Gallistel, C. R., & Gelman, R. (1992). Preverbal and verbal counting and computation. *Cognition, 44*, 43–74.

Gao, F., Levine, S. C., & Huttenlocher, J. (2000). What do infants know about continuous quantity? *Journal of Experimental Child Psychology, 77*, 20–29.

Gauderat-Bagault, L., & Lehalle, H. (2002). La généralisation des connaissances numériques avant et après 7/8 ans. In J. Bideaud & H. Lehalle (Éd.), *Le développement des activités numériques chez l'enfant* (pp. 103-125). Paris : Hermès.

Gay, S. A., & Aichele, D. B. (1997). Middle school students' understanding of number sense related to percent. *School Science and Mathematics, 97*, 27-36.

Geary, D. C., Bow-Thomas, C.C., Fan, L., & Siegler, R.S. (1993). Even before formal instruction, Chinese children outperform American children in mental addition. *Cognitive Development, 8*(4), 517-529.

Geary, D. C., Bow-Thomas, C.C., Fan, L., & Siegler, R.S. (1996). Development of arithmetical competencies in Chinese and American children: Influence of age, language, and schooling. *Child Development, 67*(5), 2022-2044.

Gelman, R., & Gallistel, C. R. (1978). *The child's understanding of number.* Cambridge, MA: Harvard University Press.

Gelman, R., & Meck, E. (1983). Preschoolers' counting: Principles before skill. *Cognition, 13*, 343–360.

Gelman, R., & Meck, E. (1986). The notion of principle: The case of counting. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 29–57). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Geoffrey B. Saxe, G. B., Edd V., Taylor E. V., McIntosh C., & Gearhart M. (2005). Representing Fractions with Standard Notation: A Developmental Analysis. *Journal for Research in Mathematics Education*, *36*, 137-157.

Gordon, P. (2004). Numerical cognitive without words: Evidence from Amazonia. *Science, 306,* 496-499.

Graeber, A. O., Tirosh, D., & Glover, R. (1989). Preservice teachers' misconceptions in solving verbal problems in multiplication and division. *Journal for Research In Mathematics Education*, 20(1), 95–102.

Granà, A., Lochy, A., Girellid, L., Seron, X. & Semenza, C. (2003). Transcoding zeros within complex numerals. *Neuropsychologia 41*, 1611–1718.

Greer, B. (1994). Extending the meaning of multiplication and division. In G. Harel, & J. Confrey (Eds.), *The development of multiplicative reasoning in the learning of mathematics* (pp. 61–85). Albany: State University of New York Press.

Grégoire, J., & Meert, G. (2005). L'apprentissage des nombres rationnels et ses obstacles. In M. P. Noël (Ed.), *La dyscalculie. Trouble du développement numérique chez l'enfant.* (pp. 223-251). Marseille: Solal.

Hallett, D., Nunes, T., & Bryant, P. (2010). Individual differences in conceptual and procedural knowledge when learning fractions. *Journal of Educational Psychology, 102*(2), 395-406.

Hannula, M. S. (2003). Locating fraction on a number line. In N. A. Pateman, B. J. Dougherty and J. Zilliox (eds.), *Proceedings of the 2003 Joint Meeting of the PME and PMENA*, Vol. 3, (pp. 17–24) CRDG, College of Education, University of Hawaii, Hawaii.

Hannula, M. S., Pehkonen, E., Maijala, H., & Soro, R. (2006). Levels of students' understanding on infinity. *Teaching Mathematics and Computer Science, 4(*2), 317–337.

Hartnett, P. M., & Gelman, R. (1998). Early understandings of number: Paths or barriers to the construction of new understandings? *Learning and Instruction, 8*, 341–374.

Hecht, S. (1998). Toward an information-processing account of individual differences in fraction skills. *Journal of Educational Psychology, 90*, 545-559.

Hecht, S., Close, L., & Santisi, M. (2003). Sources of individual differences in fraction skills. *Journal of experimental child psychology, 86*(4), 277-302.

Hecht, S., & Vagi, K. (2010). Sources of group and individual differences in emerging fraction skills. *Journal of Educational Psychology, 102(4),*843-859.

Hiebert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 1-27). Hillsdale, NJ: Lawrence Erlbaum Associates.

Hiebert, J., Wearne, D., & Taber, S. (1991) Fourth graders' gradual construction of decimal fractions during instruction using different physical representations. *Elementary School Journal, 91,* 321-341.

Ho, C. S.-H., & Fuson, K. C. (1998). Children's knowledge of teen quantities as tens and ones: Comparisons of Chinese, British, and American kindergartners. *Journal of Educational Psychology, 90* (3), 536–544.

Hoof, J. V., Vandewalle, J., Verschaffel, L., & Wim Van Dooren, W. V., (2014). In search for the natural number bias in secondary school students' interpretation of the effect of arithmetical operations. *Learning and Instruction*.

Hunting, R. P., & Sharpley, C. E. (1988). Fraction knowledge in preschool children. *Journal for Research in Mathematics Education, 19* (2), 175-180.

Hunting, R. P., & Korbosky, R. K. (1990). Context and process in fraction learning. *International Journal of Mathematical Education in Science and Technology*, *21*, 929-948.

Hurford, J.R. (1987). *Language and number. The emergence of a cognitive system*. New York: Blackwell.

Huttenlocher, J., Jordan, N., & Levine, S. C. (1994). A mental model for early arithmetic. *Journal of Experimental Psychology: General, 123,* 284–296.

Ifrah, G. (1981). *Histoire universelle des chiffres. Lorsque les nombres racontent les hommes*. Paris : Seghers.

Ifrah, G. (1994). *Histoire universelle des chiffres. L'intelligence des hommes racontée par les nombres et le calcul*. 2 vol., Ed. Robert Lafont, Paris.

Jarlegan, A., Fayol, M., & Barrouillet, P. (1996). De soixante-douze à 72, et inversement: Une étude du transcodage chez les enfants de 7 ans. Revue de Psychologie de l'Education, Numéro spécial "Le Nombre", 3, 109-131.Une étude du transcodage chez les enfants de 7 ans. *Revue de Psychologie de l'Education, 1*, 109-131.

Kerslake, D. (1986). *Fractions: Children's strategies and errors: A report of the strategies and error in secondary mathematics project.* Windson, England: NFER-Nelson

Kieren, T.E. (1976). On the mathematical, cognitive, and instructional foundations of rational numbers. In R. Lesh (ed.), *Number and Measurement* (pp. 101–144). Papers from a Research Workshop, Columbus, OH, ERIC/SMEAC.

Kieren, T. E. (1980). The rational number construct its elements and mechanisms. In T. E. Kieren (ed.), *Recent Research on Number Learning*, (pp. 125-150). Columbus, OH: ERIC/SMEAC.

Kieren, T. E. (1988). Personal knowledge of rational numbers - Its intuitive and formal development. In J. Hiebert and M. Behr (eds.), *Number concepts and operations in the middle grades* (pp. 162-181). Reston, VA: Lawrence Erlbaum.

Kieren, T. E., Mason, R. & Pirie, S. (1992). *The Growth of Mathematical Understanding in a Constructivist Environment*. Paper presented at the CSSE Meeting, P.E.I. Charlottetown.

Kieren, T. E. (1993a). *Bonuses of Understanding Mathematical Understanding*. Paper presented at the International Study Group on the Rational Numbers of Arithmetic, University of Georgia, Athens, GA.

Kieren, T. E. (1994).Multiple views of multiplicative reasoning. In G. Harel & G. Confrey (Eds.), *The development of multiplicative reasoning in the learning of mathematics* (pp. 387–397). Albany, NY: State University of New York Press.

Kloosterman, P. (2010). Mathematics skills of 17-year-olds in the United States: 1978 to 2004. *Journal for Research in Mathematics Education*, *41*, 20–51.

Koechlin, E., Dehaene, S. J., & Mehler, J. (1997). Numerical transformations in five-month-old human infants. *Mathematical Cognition, 3*, 89–104.

Lamon, S.J. (1993). Ratio and proportion: Children's cognitive and metacognitive process In T.P. Carpenter, E. Fennema & T.A. Romberg (eds.). *Rational Numbers: An Integration of Research* (pp. 131–156), Lawrence Erlbaum Associates, New Jersey.

Lamon, S.J. (1999). *Teaching Fractions and Ratios for Understanding*. Lawrence Erlbaum Associates, New Jersey.

Lamon, S.L. (2001). Presenting and representing: From fractions to rational numbers. In A. Cuoco & F. Curcio (eds.). *The Roles of Representations in School Mathematics-2001 Yearbook* (pp. 146–165) Reston: NCTM.

Lochy, A., Pillon, A., Zesiger, P., & Seron, X. (2002). Verbal structure of numerals and digits handwriting: New evidence from kinematics. *The Quarterly Journal of Experimental Psychology, 55A* (1), 263-288.

Lochy, A., Delazer, M. & Seron, X. (2003). *Influence of language in the early acquisition of numbers writing: A comparison of French and German*. Paper presented at the Third Aachen–Gent Brain and Number Workshop, Aachen, Germany.

Lochy, A., Andres, M., & Seron, X. (2004). Status of 0 in Arabic numerals writing: The story of transcoding told by kinematics data. *Manuscript submitted for publication.*

Lochy, A., Delazer, M., Domahs, F., Zoppoth, S., & Seron, X. (2004). The acquisition of Arabic number writing by children: A cross-linguistic study of French vs. German. *Manuscript submitted for publication.*

Lochy, A., Domahs, F., Bartha, L., & Delazer, M. (2004). Specific order impairment in Arabic number writing: A case study. *Cognitive Neuropsychology, 21*, 555-575.

Mack, N. K. (1990). Learning fractions with understanding: Building on informal knowledge. *Journal for Research in Mathematics Education, 21* (1), 16-33.

Mack, N. K. (1993). Learning rational numbers with understanding: The case of informal knowledge. In T. P. Carpenter, E. Fennema, & T. A. Romberg (Eds.), *Rational numbers. An integration of research* (pp. 85-105*)*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Mamede, E., Nunes, T., & Bryant, P. (2005). The equivalence and ordering of fractions in part-whole and quotient situations. In H. L. Chick, & J. L. Vincent (Eds.), *Proceedings of the 29th conference of the International Group for the Psychology of Mathematics Education* (pp. 281-288). Melbourne, Australia: PME.

Marshall, S.P. (1993). Assessment of rational number understanding: A schema-based approach. In T.P. Carpenter, E. Fennema and T.A. Romberg (eds.). *Rational Numbers: An Integration of Research* (pp. 261–288), Lawrence Erlbaum Associates, New Jersey.

Martin, W. G., & Strutchens, M. (2007). *Learning Mathematics, 2007 Yearbook of the National Council of Teachers of Mathematics.* Reston, VA: National Council of Teachers of Mathematics.

McCloskey, M., Caramazza, A. & Basili, A. (1985). Cognitive mechanisms in number processing and calculation: Evidence from dyscalculia. *Brain and Cognition 4*, 171–196.

McMullen, J., Laakkonen, E., Hannula-S M. & Lehtinen E., (2014). Modeling the developmental trajectories of rational number concept(s). *Learning and Instruction*.

Meck, W. H., & Church, R.M. (1983). A mode control of counting and timing process. *Journal of Experimental Psychology: Animal Behavior Processes, 9*, 320–142.

Meert, G., Grégoire, J., & Noël, M.-P. (2010). Comparing the magnitude of two fractions with common components: which representations are used by 10- and 12-year-olds? *Journal of Experimental Child Psychology*, *107*, 244-259.

Merenluoto, K., & Lehtinen, E. (2002). Conceptual change in mathematics: Understanding the real numbers. In M. Limon, & L. Mason (Eds.), *Reconsidering conceptual change: Issues in theory and practice* (pp. 233–258). Dordrecht, The Netherlands: Kluwer.

Miller, K., Major, S. M., Shu, H. & Zhang, H. (2000). Ordinal knowledge: Number names and number concepts in Chinese and English. *Canadian Journal of Experimental Psychology 54* (2), 129–140.

Miller, K. F., Smith, C. M., Zhu, J. & Zhang, H. (1995). Preschool origins of cross-national differences in mathematical competence: The role of number-naming systems. *Psychological Science 6*, 56–60.

Miller, K. F. & Stigler, J. W. (1987). Counting in Chinese: Cultural variation in a basic cognitive skill. *Cognitive Development 2* (3), 279–305.

Miura, I. T., Kim, C. C., Chang, C. M. & Okamoto, Y. (1988). Effects of language characteristics on children's cognitive representation of number: Cross-national comparisons. *Child Development 59*, 1445–1450.

Miura, I. T. & Okamoto, Y. (1989). Comparisons of American and Japanese first graders' cognitive representation of number and understanding of place value. *Journal of Educational Psychology 81*, 109–113.

Miura, I. T., Okamoto, Y., Kim, C. C., Steere, M., & Fayol, M. (1993). First Graders' Cognitive Representation of Number and Understanding of Place Value: Cross-National Comparisons-France, Japan, Korea, Sweden, and the United States. *Journal of Educational Psychology, 85* (1), 24-30.

Miura, I. T., Okamoto, Y., Kim, C. C., Chang, C. M., Steere, M. & Fayol, M. (1994). Comparisons of children's cognitive representation of number: China, France, Japan, Korea, Sweden and the United States. *International Journal of Behavioral Development* 17, 401–411.

Miura, I. T., Okamoto. Y., Vlahovic-Stetic, V., Kim, C. C. & Han, J. H. (1999). Language supports for children's understanding of numerical fractions: Cross-national comparisons. *Journal of Experimental Child Psychology, 74* (4), 356-365.

Mix, K. S., Levine, S. C., & Huttenlocher, J. (1999). Early fraction calculation ability. *Developmental Psychology*, *35*, 164–174.

Moss, J. (2005). Pipes, tubes, and beakers: New approaches to teaching the rational-number system. In M. S. Donovan, & J. D. Bransford (Eds.), *How students learn: Mathematics in the classroom* (pp. 121–162). Washington, DC: National Academic Press.

Nesher, P., & Peled, I. (1986). Shifts in reasoning. The case of extending number concepts. *Educational Studies in Mathematics, 17*, 67-79.

Nguyen, H.T.T., & Grégoire, J. (2014). When 5004 is Said "Five Thousand Zero Hundred Remainder Four": The Influence of Language on Natural Number Transcoding: Cross-National Comparison. *Journal of the Korea Society of Mathematical Education Series D: Research in Mathematical Education*, *18* (2).

Ni, Y. J. (2000). How valid to use number lines to measure children's conceptual knowledge about rational numbers? *Educational Psychology*, *20*, 139–152.

Ni, Y. J. (2001). Semantic domains of rational numbers and the acquisition of fraction equivalence. *Contemporary Educational Psychology, 26*, 400–417.

Ni, Y. J., & Zhou, Y. D. (2005). Teaching and learning fraction and rational numbers: The origin and implications of whole number bias. *Educational Psychologist*, *40*(1), 27-52.

Noël, M.-P., & Seron, X. (1995). Lexicalization errors in writing Arabic numerals: A single case study. *Brain and Cognition*, *29*, 151-179.

Noël, M-P. & Turconi. E. (1999). Assessing number transcoding in Children. *European Review of Applied Psychology, 49* (4), 295–302.

Noël, M. (2001). Le transcodage chez l'enfant. In A. Van Hout & M.C (Eds.), *Les dyscalculies* (p. 109-117). Masson.

Noël, M.P. (2005). Le transcodage chez l'enfant. In Van Hout, A., Meljac, C., & Ficher, J.P. *Troubles du calcul et dyscalculies chez l'enfant* (pp. 111-122). Paris: Masson, 2ème Edition.

Nunes, T. (1992). Cognitive invariants and cultural variation in mathematical concepts. *International Journal of Behavioral Development*, *15*, 433-453.

Nunes, T., & Bryant, P. (1996). *Children doing mathematics.* Cambridge, MA: Blackwell.

Ohlsson, S. (1988). Mathematical meaning and applicational meaning in the semantics of fractions and related concepts. In J. Hiebert & M. Behr (eds.), *Research agenda for mathematics education: number concepts and operations in the middle grades* (pp. 55-92), Reston: VA, NCTM.

Paik, J. H., & Mix, K. S. (2003). U.S and Korean children's comprehension of fraction names: A reexamination of cross-national differences. *Child Development, 74*, 144-154.

Piaget, J. (1978). *Recherches sur la généralisation.* Paris : Presses universitaires de France.

Pica, P., Lemer, C., Izard, V., & Dehaene, S. (2004). Exact and approximate arithmetic in an Amazonian indigene group. *Science, 306* (5695), 499-503.

Pitkethly, A., & Hunting, R. (1996). A review of recent research in the area of initial fraction concepts. *Educational Studies in Mathematics*, *30*, 5-38.

Power, R. J. D. & Dal Martello, M. F. (1990). The dictation of Italian numerals. *Language and Cognitive Processes 5*, 237–254.

Power, R. J. D. & Dal Martello, M. F. (1997). From 834 to eighty thirty four: The reading of Arabic numerals by 7-year-old children. *Mathematical Cognition 3* (1), 63–85.

Resnick, L. B. & Nesher, P. (1983). *Learning complex concepts: The case of decimal fractions*. Paper presented at the Twenty-Fourth Annual Meeting of the Psychonomics Society, San Diego, California.

Resnick, L. B., Nesher, P., Leonard, F., Magone, M., Omanson, S., & Peled, I. (1989). Conceptual bases of arithmetic errors: the case of decimal fractions. *Journal for Research in Mathematics Education, 20* (1), 8-27.

Resnick, L. B., & Singer, J. (1993). Protoquantitative origins of ratio reasoning. In T. P. Carpenter, E. Fennema, & T. A. Romberg (Eds.), *Rational numbers: An integration of research* (pp. 107–130). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Rittle-Johnson, B., & Koedinger, K. R. (2009). Iterating between lessons concepts and procedures can improve mathematics knowledge. *British Journal of Educational Psychology, 79*, 483-500.

Rittle-Johnson, B. Siegler, R. S., & Alibali, M. W. (2001) Developing conceptual understanding and procedural skill in mathematics: an iterative process. *Journal of Educational Psychology, 93* (2), 346–362.

Roche, A., & Clarke, D. M. (2006). When successful comparison of decimals doesn't tell the full story. In Novotná, J., Moraová, H., Krátká, M. & Stehlíková, N. (Eds.), *Proceedings 30th Conference of the International Group for the Psychology of Mathematics Education* (pp. 425-432). Prague: PME.

Saad, L. (2011). *Transcodage des nombres chez l'enfant. Approche développementale inter-linguistique et différentielle.* Thèse non publié, Université de Bourgogne.

Sackur-Grisvard, C., & Leonard, F. (1985). Intermediate cognitive organization in the process of learning a mathematical concept: the order of positive decimal numbers. *Cognition and Instruction, 2* (2), 157-174.

Saenz-Ludlow, A. (1995). *Ann's fraction schemes*, ESM vol. 28, (pp. 101-132), Kluwer Academic Publishers, Dordrecht, Netherlands.

Saxe, G.B., Taylor, E.V., McIntosh, C., & Gearhart, M. (2005). Representing fractions with standard notation: A developmental analysis. *Journal for Research in Mathematics Education, 36*(2), 137-157.

Saxton, M., & Towse, J. N. (1998). Linguistic Relativity: The Case of Place Value in Multi-Digit Numbers. *Journal of Experimental Child Psychology, 69* (1), 66-79.

Schneider, M. & Siegler, R.S. (2010). Representations of the magnitudes of fractions. *Journal of Experimental Psychology: Human Perception and Performance*. *36*, 1227-1238

Seron, X. Deloche, G. & Noël, M-P. (1991). Number transcoding by children: Writing Arabic numbers under dictation. In: J. Bideaud, C. Melijac & J. P. Fisher (Eds.), *Pathway to Number* (pp. 245–264). Hillsdale NJ: Lawrence Erlbaum Associates.

Seron, X. & Fayol, M. (1994). Number transcoding in children: A functional analysis. *British Journal of Developmental Psychology 12*, 281–300.

Seron, X., & Noël, M.-P. (1995). Transcoding from the arabic code to the verbal one and *vice versa* : how many routes ? *Mathematical cognition, 1*, 215-243.

Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., Susperreguy, M. I., & Chen, M. (2012). Early predictors of high school mathematics achievement. *Psychological Science*, 23, 691-697.

Siegler, R. S., Thompson, C. A., & Schneider, M. (2011). An integrated theory of whole number and fractions development. *Cognitive Psychology*, *62*, 273-296.

Smith, J.P. (2002). The development of students' knowledge of fractions and ratios. In B. Litwiller and G. Bright (eds.), *Making Sense of Fractions, Ratios, and Proportions* (pp. 3–17). Reston: Virginia, NCTM.

Smith, C. L., Solomon, G. E. A., & Carey, S. (2005). Never getting to zero: Elementary school students' understanding of the infinite divisibility of number and matter. *Cognitive Psychology, 51*, 101–140.

Schneider, M., Rittle-Johnson, B., & Star, J. R. (2011). Relations between conceptual knowledge, procedural knowledge, and procedural flexibility in two samples differing in prior knowledge. *Developmental Psychology, 46,* 1525-1538.

Sophian, C. (1996). *Children's numbers*. Boulder, CO: Westview Press.

Sophian, C. (2000). Perceptions of proportionality in young children: Matching spatial ratios. *Cognition, 75*, 145–170.

Sophian, C., Garyantes, D., & Chang, C. (1997). When three is less than two: Early developments in children's understanding of fractional quantities. *Developmental Psychology*, *33*, 731–744.

Stacey, K. (2005). Travelling the road to expertise: A longitudinal study of learning. In H. L. Chick & J. L. Vincent. (Eds.), *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education* (pp. 19-36). Melbourne: Psychology of Mathematics Education.

Stafylidou, S., & Vosniadou, S. (2004). The development of students' understanding of the numerical value of fractions. *Learning and Instruction*, *14*, 503-518.

Starkey, P., & Cooper, R. G., J. (1980). Perception of numbers by human infants. *Science, 210*, 1033–1035.

Steffe, L. P. (2002). A new hypothesis concerning children's fractional knowledge. *Journal of Mathematical Behavior, 20*, 267–307.

Steffe, L. P., Cobb, P. & Von Glasersfeld, E. (1988). *Construction of arithmetical meanings and strategies*. New York: Springer-Verlag.

Steffe, L. P., & Olive, J. (1993). *Children's construction of the rational numbers of arithmetic.* Paper presented at the International Study Group on the Rational Numbers of Arithmetic, University of Georgia, Athens, GA.

Steinle, V., & Stacey, K. (2003). Grade-related trends in the prevalence and persistence of decimal misconceptions. In N. Pateman, B. Dougherty, & J. Zilliox (Eds.), *Proceedings*

of the 2003 Joint Meeting of PME and PMENA (Vol. 4, pp. 259-266). Honolulu: International Group for the Psychology of Mathematics Education.

Steinle, V., & Pierce, R. (2006). Incomplete or incorrect understanding of decimals : An important deficit for student nurses. In Novotná, J., Moraová, H., Krátká, M. & Stehlíková, N. (Eds.), *Proceedings 30th Conference of the International Group for the Psychology of Mathematics Education* (pp. 161-168). Prague: PME.

Stigler, J.W., Givvin, K.B. & Thompson, B. (2010). What community college developmental mathematics students understand about mathematics. *The MathAMATYC Educator, 3* (1), 4-16.

Strauss, M. S., & Curtis, L. E. (1981). Infants perception of numerosity. *Child Development, 52*, 1146–1152.

Streefland, L. (1991). *Fractions in Realistic Mathematics Education: A Paradigm of Developmental Research*, Kluwer Academic Publishers, Dordrecht.

Streefland, L. (1993). Fractions: A Realistic Approach. In T. P. Carpenter, E. Fennema &T. A. Romberg (eds.), *Rational Numbers: An Integration of Research* (pp. 289-325), Hillsdale, NJ: Lawrence Erlbaum Associates.

Swan, M. (1983). *The Meaning and Use of Decimals.* Shell Centre for Mathematical Education, Nottingham University, U.K.

Thompson, P. W., & Saldanha, L. A. (2003). Fractions and multiplicative reasoning. In J. Kilpatrick, W. Gary Martin, & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 95–113). Reston, VA: The National Council of Teachers of Mathematics.

Torbeyns, J., Schneider, M., Xin, Z., & Siegler, R. S. (2014). Bridging the gap: fraction understanding is central to mathematics achievement in students from three different continents. *Learning and Instruction, 37*, 5-13.

Towse, J. W. & Saxton, M. (1997).Linguistic influences on children's number concepts: Methodological and theoretical considerations. *Journal of Experimental Child Psychology, 66*, 362–374.

Tzur, R. (1999). An integrated study of children's construction of improper fractions and the teacher's role in promoting that learning. *Journal for Research in Mathematics Education, 30*, 390–416.

Vamvakoussi, X., Christou, K. P., Mertens, L., & Van Dooren, W. (2011). What fills the gap between discrete and dense? Greek and Flemish students' understanding of density. *Learning and Instruction*, *21*, 676–685.

Vamvakoussi X., Dooren W. V., Verschaffel L., (2012). Naturally biased? In search for reaction time evidence for a natural number bias in adults. *Journal of Mathematical Behavior, 31*, 344– 355.

Vamvakoussi, X., & Vosniadou, S. (2004). Understanding the structure of the set of rational numbers: a conceptual change approach. *Learning and Instruction*, *14*, 453-468.

Vergnaud, G. (1983). Multiplicative Structures. In R. Lesh and M. Landau (eds.), *Acquisition of mathematics concepts and processes* (pp. 127-174). New York: Academic Press.

Vosniadou, S., & DeWolf, M. (2014). The representation of fraction magnitudes and the whole number bias reconsidered. *Learning and Instruction*, *37*, 39-49.

Weil-Barais, A., & Vergnaud, D. (1990). Students' conceptions in physics and mathematics: Biases and helps. In J-P. Caverni, J-M. Fabre, & M. Gonzalez (Eds.), *Cognitive biases* (pp. 69–84). Amsterdam, The Netherlands: North-Holland.

Wynn, K. (1992a). Children's acquisition of the number words and the counting system. *Cognitive Psychology, 24*, 220–251.

Yoshida, H. and Sawano, K. (2002). Overcoming cognitive obstacles in learning fractions: equal-partitioning and equal-whole. *Japanese Psychological Research*, *44*, 183–195

Zuber, J., Pixner, S., Moeller, K. & Nuerk, H.-C. (2009). On the language specificity of basic number processing: Transcoding in a language with inversion and its relation to working memory capacity. *Journal of Experimental Child Psychology, 102*, 60–77.

Annexes

Annexes

Chapter 4: Transcoding task used for study 1

Ecrivez les nombres écrits-verbaux suivants sous leur forme en chiffres

	Nombre verbal en français	Nombre en chiffre
0	**Cent quatorze**	**114**
1	Sept cent quarante cinq	
2	Trois cents huit	
3	Deux cent cinq mille quatorze	
4	Neuf cent milles six cent quarante cinq	
5	Huit mille cinq	
6	Sept mille cinq cent trois	
7	Deux cents cinq milles cent soixante trois	
8	Vingt cinq milles quatorze	
9	Cinq cents quarante deux milles cent quatre vingt trois	
10	Trente cinq milles sept	
11	Sept cent milles cent vingt six	
12	Mille deux cents cinquante trois	
13	Cinq cents neuf milles sept cents cinquante deux	
14	Cinq cents trois mille huit	
15	Six cent sept mille vingt neuf	
16	Quatre mille cinquante sept	
17	Treize mille cinq cents soixante sept	
18	Neuf cents sept mille six	

Chapter 5: Comparison task used for study 2

Entoure LE PLUS GRAND ou place”=” en cas d’égalité.

4105		**4015**	**2503**		**2530**
4005		**4500**	**6032**		**6320**
2503		**2053**	**3700**		**3070**
4105		**4150**	**2053**		**2530**
4050		**4005**	**3406**		**3046**
6004		**6400**	**8002**		**8200**
4015		**4150**	**6004**		**6040**
4500		**4050**	**3406**		**3460**
6400		**6040**	**8020**		**8002**
6302		**6032**	**3046**		**3460**
3007		**3700**	**8200**		**8020**

Chapter 5: Analog-Arabic Transcoding task used for study 2



Nombre sous la forme en cubes

Nombre sous la forme en cubes

1. .....

2. .....

3. .....

4. .....

5. .....

6. .....

7. .....

8. .....

9. .....

10. .....

11. .....

12. .....

13. .....

14.  .....

15. .....

16. .....

17. .....

18. .....

19. .....

20. .....

21. .....

22. .....

23. .....

24. .....

Chapter 5: Verbal-Arabic transcoding task used for study 2

Ecrivez les nombres écrits-verbaux suivants sous leur forme en chiffre

0	Nombre verbal en français	Nombre en chiffre
1	Trois-mille-six-cent-quatre	
2	Quatre-mille-vingt-cinq	
3	Trois-mille-quatre	
4	Trois-mille-deux-cent-soixante-quatre	
5	Deux-mille-quatre-cent-six	
6	Trois-mille-cinquante-et-un	
7	Deux-mille-six	
8	Deux-mille-cent-quarante-six	
9	Mille-trois-cent-quatre	
10	Deux-mille-cinquante-trois	
11	Mille-quatre	
12	Mille-cinq-cent-trente-quatre	
13	Deux-mille-cinq-cent-trois	
14	Mille-trente-quatre	
15	Deux-mille-trois	
16	Deux-mille-quatre-cent-cinquante-trois	
17	Trois-mille-cinquante-et-un	
18	Deux-mille-quarante-six	

	Nombre verbal en français	Nombre en chiffre
19	Trois-mille-un	
20	Trois-mille-deux-cent-cinquante-et-un	
21	Quatre-mille-deux-cent-cinq	
22	Trois-mille-soixante-quatre	
23	Quatre-mille-cinq	
24	Quatre-mille-six-cent-vingt-cinq	

Chapter 5: Digit identification task used for study 2

Dans chacun des nombres suivants, entourer **le chiffre des dizaines** :

4005	3064	3001
4205	2146	3501
1531	2003	3264
1034	2503	2046

Dans chacun des nombres suivants, entourer **le chiffre des centaines**:

1004	2053	2006
1304	3251	2406
4625	3004	2453
4025	3604	3051

Chapter 6: Transcoding task used for study 3

Ecrivez les nombres décimaux en chiffre qui aient :

		Nombre en chiffre
1	Huit centièmes	
2	Cinq unités et neuf dixièmes	
3	Sept millièmes	
4	Trente-cinq unités et huit centièmes	
5	Six centièmes	
6	Deux millièmes	
7	Soixante-trois unités et huit dixièmes	
8	Douze unités et cinq centièmes.	
9	Sept dixièmes	
10	Soixante-trois unités et cinq millième	
11	Cinq dixièmes	
12	Quinze unités et huit millièmes	

Chapter 7: Task 1 used for study 4

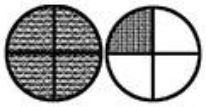
Pour chaque groupe de figures, indiquez le nombre fraction correspondant



A. $\frac{3}{2}$ B. $\frac{3}{4}$ C. $\frac{1}{3}$



A. $\frac{8}{9}$ B. $\frac{8}{3}$ C. $\frac{1}{8}$



A. $\frac{3}{5}$ B. $\frac{5}{4}$ C. $\frac{5}{8}$



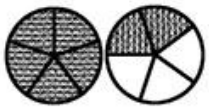
A. $\frac{12}{5}$ B. $\frac{12}{15}$ C. $\frac{3}{12}$



A. $\frac{4}{6}$ B. $\frac{4}{3}$ C. $\frac{2}{4}$



A. $\frac{5}{2}$ B. $\frac{1}{5}$ C. $\frac{5}{6}$



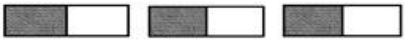
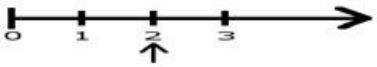
A. $\frac{7}{10}$ B. $\frac{3}{7}$ C. $\frac{7}{5}$

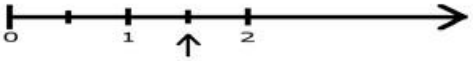


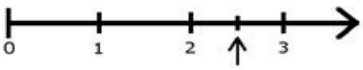
A. $\frac{3}{9}$ B. $\frac{9}{4}$ C. $\frac{9}{12}$

Chapter 7: Task 2 used for study 4 (6 items) and for study 5 (Chapter 8) for grade 4

Choisis les représentations qui correspondent au nombre fraction $\frac{3}{2}$

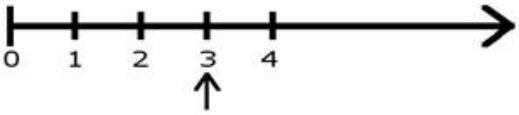
1. 
2. 
3. 
4. 
5. 

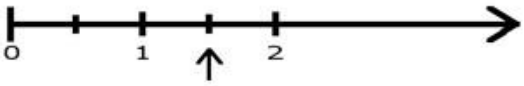
6. 
7. 
8. 

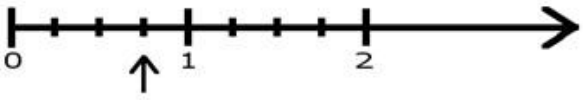
9. 
10. 
11. 

12. 

Choisis les représentations qui correspondent au nombre fraction $\frac{3}{4}$

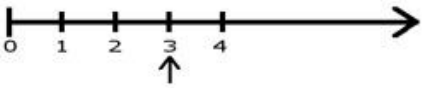
1. 
2. 
3. 
4. 
5. 

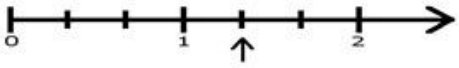
6. 
7. 
8. 

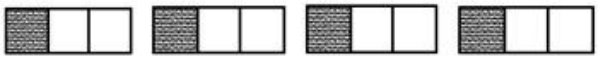
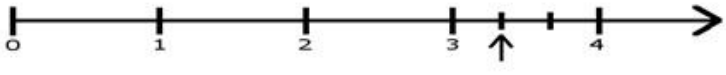
9. 
10. 
11. 

12. 

Choisis les représentations qui correspondent au nombre fraction $\frac{4}{3}$

1. 
2. 
3. 

4. 
5. 
6. 
7. 
8. 

9. 
10. 
11. 

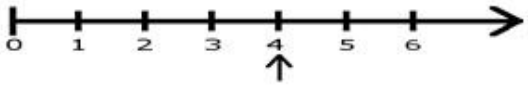
12. 

Choisis les représentations qui correspondent au nombre fraction $\frac{4}{6}$

1. 

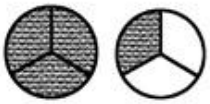
2. 

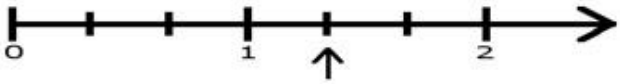
3. 

4. 

5. 

6. 

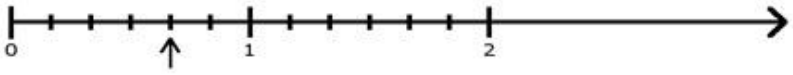
7. 

8. 

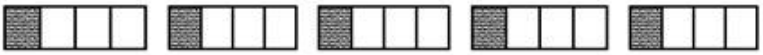
9. 

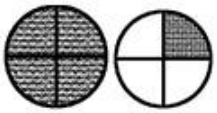
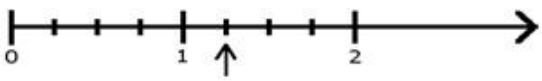
10. 

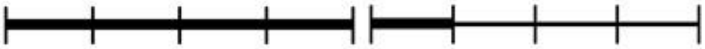
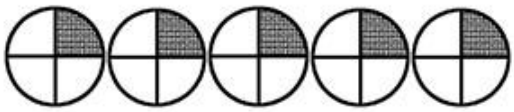
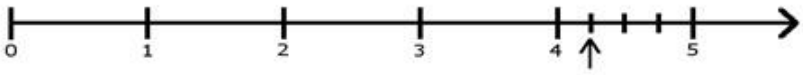
11. 

12. 

Choisis les représentations qui correspondent au nombre fraction $\frac{5}{4}$

1. 
2. 
3. 

4. 
5. 
6. 

7. 
8. 
9. 
10. 
11. 
12. 

Choisis les représentations qui correspondent au nombre fraction $\frac{5}{8}$

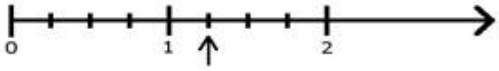
1.



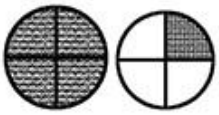
2.



3.



4.



5.



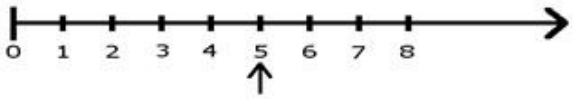
6.



7.



8.



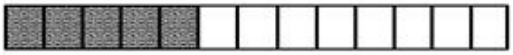
9.



10.



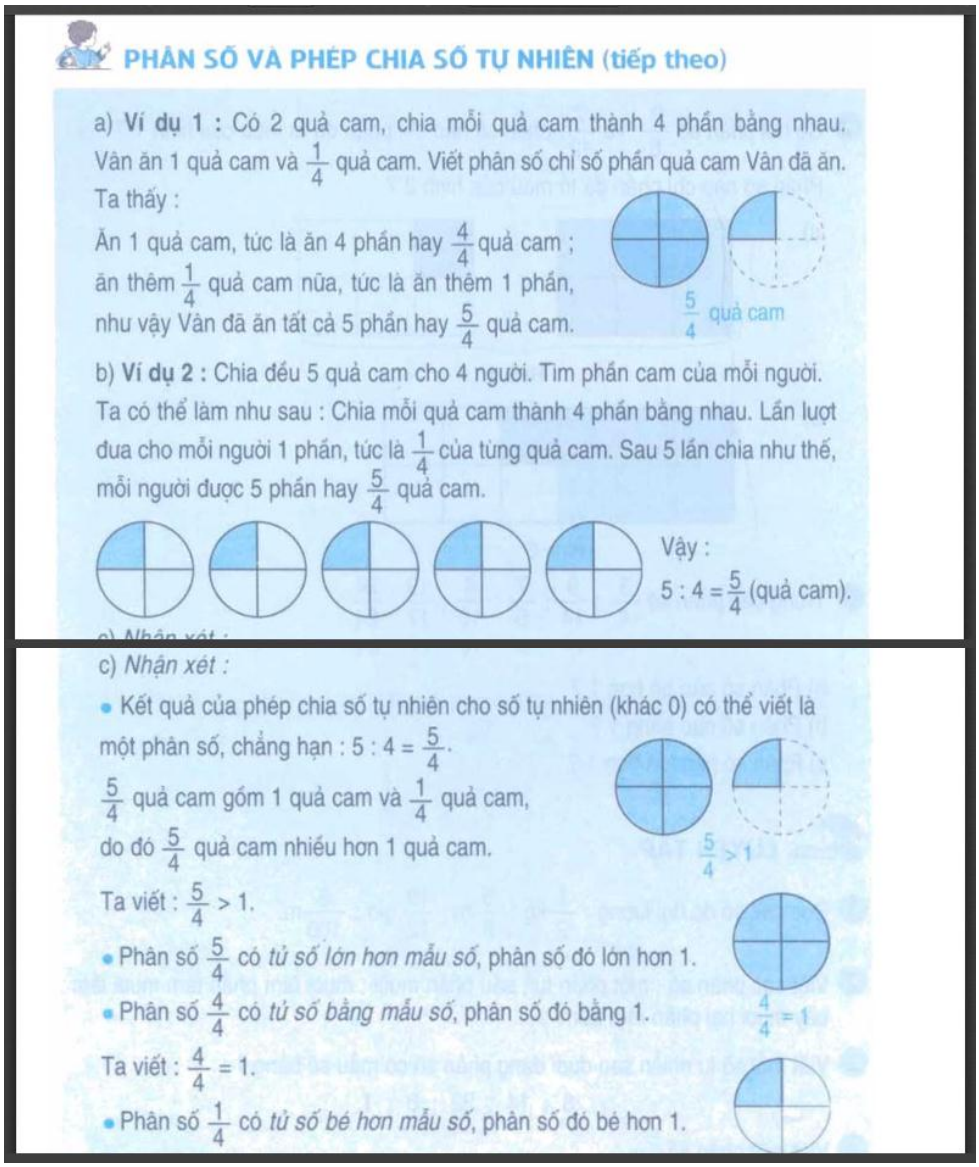
11.



12.



Vietnamese textbook used for Chapter 7 concerning the graphical representation of fraction inferior than 1



PHÂN SỐ VÀ PHÉP CHIA SỐ TỰ NHIÊN (tiếp theo)

a) **Ví dụ 1 :** Có 2 quả cam, chia mỗi quả cam thành 4 phần bằng nhau. Vân ăn 1 quả cam và $\frac{1}{4}$ quả cam. Viết phân số chỉ số phần quả cam Vân đã ăn.

Ta thấy :

Ăn 1 quả cam, tức là ăn 4 phần hay $\frac{4}{4}$ quả cam ;

ăn thêm $\frac{1}{4}$ quả cam nữa, tức là ăn thêm 1 phần,

như vậy Vân đã ăn tất cả 5 phần hay $\frac{5}{4}$ quả cam.

$\frac{5}{4}$ quả cam

b) **Ví dụ 2 :** Chia đều 5 quả cam cho 4 người. Tìm phần cam của mỗi người.

Ta có thể làm như sau : Chia mỗi quả cam thành 4 phần bằng nhau. Lần lượt đưa cho mỗi người 1 phần, tức là $\frac{1}{4}$ của từng quả cam. Sau 5 lần chia như thế, mỗi người được 5 phần hay $\frac{5}{4}$ quả cam.

Vậy :

$5 : 4 = \frac{5}{4}$ (quả cam).

c) *Nhận xét :*

- Kết quả của phép chia số tự nhiên cho số tự nhiên (khác 0) có thể viết là một phân số, chẳng hạn : $5 : 4 = \frac{5}{4}$.

$\frac{5}{4}$ quả cam gồm 1 quả cam và $\frac{1}{4}$ quả cam,

do đó $\frac{5}{4}$ quả cam nhiều hơn 1 quả cam.

$\frac{5}{4} > 1$

Ta viết : $\frac{5}{4} > 1$.

- Phân số $\frac{5}{4}$ có *tử số lớn hơn mẫu số*, phân số đó lớn hơn 1.
- Phân số $\frac{4}{4}$ có *tử số bằng mẫu số*, phân số đó bằng 1.

$\frac{4}{4} = 1$

Ta viết : $\frac{4}{4} = 1$.

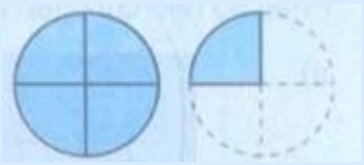
- Phân số $\frac{1}{4}$ có *tử số bé hơn mẫu số*, phân số đó bé hơn 1.

English translation of the previous annex

FRACTION AND THE DIVISION OF NATURAL NUMBERS

a) Example 1: there are 2 oranges, we divide each of them into 4 equal parts. Van eats 1 orange and ¼ orange. Write the fraction indicating number of parts that Van has eaten.
We may see:

Ate 1 orange means ate 4 parts or 4/4 orange;
Ate ¼ more means ate 1 part more.
Hence Van ate 5 parts in total or 5/4 oranges.



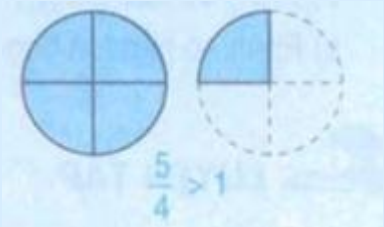
b) Example 2: divide 5 oranges for 4 persons. Find each part for them.
We can do like this: divide each orange into 4 equal parts. Share to each of them one part, means ¼ of each orange. After 5 times dividing like that, each of them will have 5 parts or 5/4 oranges.
So, 5:4 = 5/4 (orange)



c)Comment:

- The result of the division of natural number and natural number (differ from 0) could be written as a fraction, for example: 5:4 = 5/4.

  5/4 orange including 1 orange and ¼ orange, therefore 5/4 orange is more than 1 orange.
  We write: 5/4>1



- Fraction 5/4 has numerator superior to denominator, this fraction is superior to 1.
- Fraction 4/4 has numerator equal to denominator, this fraction is equal to 1. We write: 4/4=1.



- Fraction 1/4 has numerator inferior to denominator, this fraction is inferior to 1.



Chapter 8: tasks used for study 5

Comparison task for naturals numbers for grade 4

Entoure le plus grand nombre ou place “=” en cas d’égalité

4105		4015	25306		25360
4005		4500	25060		25006
12507		12057	6032		6320
12005		12500	3700		3070
4105		4150	25036		25360
4050		4005	25600		25060
12507		12570	3406		3046
12050		12005	8002		8200
4015		4150	35402		35042
4500		4050	35002		35200
12057		12570	3406		3460
12500		12050	8020		8002
6302		6032	35402		35420
3007		3700	35020		35002
25306		25036	3046		3460
25006		25600	8200		8020
6302		6320	35042		35420
3070		3007	35200		35020

Entoure le plus grand nombre ou place "=" en cas d'égalité

25006		25600
6302		6320
3070		3007
6302		6032
3007		3700
25306		25036
4500		4050
12057		12570
12500		12050
12507		12570
12050		12005
4015		4150
12005		12500
4105		4150
4050		4005
4105		4015
4005		4500
12507		12057

8200		8020
35042		35420
35200		35020
35402		35420
35020		35002
3046		3460
35002		35200
3406		3460
8020		8002
3406		3046
8002		8200
35402		35042
3700		3070
25036		25360
25600		25060
25306		25360
25060		25006
6032		6320

Verbal-Arabic transcoding task for naturals numbers for grade 4

Ecrivez les nombres écrits-verbaux suivants sous leur forme en chiffre

	Nombre verbal en français	Nombre en chiffre
0	**Cent quatorze**	**114**
1	Quatre mille deux cent cinq	
2	Vingt - cinq mille quatre cent huit	
3	Quatre mille cinq	
4	Douze mille trente-quatre	
5	Trois mille soixante-quatre	
6	Vingt-cinq mille huit	
7	Trois mille cinq cent et un	
8	Vingt- huit mille cinquante-trois	
9	Trois mille un	
10	Quarante -six mille deux cent cinq	
11	Deux mille quarante-six	
12	Quarante -six mille cinq	
13	Deux mille cinq cent trois	
14	Mille trente-quatre	
15	Quinze mille vingt-sept	
16	Trente -cinq mille quatre cent six	
17	Deux mille trois	
18	Trente -cinq mille six	

Ecrivez les nombres écrits-verbaux suivants sous leur forme en chiffre

	Nombre verbal en français	Nombre en chiffre
19	Mille trente-quatre	
20	Trente -cinq mille quarante-six	
21	Mille quatre	
22	Quinze mille deux cent sept	
23	Douze mille quatre	
24	Trois mille cinquante et un	
25	Deux mille cinquante-trois	
26	Vingt -huit mille trois	
27	Deux mille quatre cent six	
28	Vingt-huit mille cinq cent trois	
29	Quarante -six mille vingt-cinq	
30	Deux mille six	
31	Trois mille six cent quatre	
32	Quatre mille vingt-cinq	
33	Douze mille trois cent quatre	
34	Trois mille quatre	
35	Vingt-cinq mille quarante- huit	
36	Quinze mille sept	

Digit identification task for naturals numbers for grade 4

Dans chacun des nombres suivants, entoure le chiffre de la position dizaine

4005	25048	3001
35006	3064	46005
4205	15207	3501
35046	2003	46025
12304	2503	28503
1034	25008	2046

Dans chacun des nombres suivants, entoure le chiffre de la position centaine

1004	15007	2006
12034	2053	28053
1304	25408	2406
12004	3004	28003
35406	3604	46205
4025	15027	3051

Comparison task for fraction numbers for grade 4

Entoure le nombre le plus grand. Si les deux nombres sont égaux, inscrit le symbole « = » dans le case du milieu.

$\frac{1}{5}$		$\frac{1}{9}$
$\frac{3}{5}$		1
$\frac{2}{5}$		$\frac{2}{7}$
$\frac{5}{2}$		**3**
$\frac{9}{7}$		$\frac{9}{5}$
$\frac{5}{4}$		2
$\frac{1}{3}$		$\frac{1}{7}$
$\frac{8}{5}$		**1**
$\frac{3}{7}$		$\frac{3}{4}$
$\frac{7}{2}$		**3**
$\frac{7}{6}$		$\frac{7}{4}$
$\frac{11}{4}$		**2**
$\frac{1}{8}$		$\frac{1}{13}$
$\frac{3}{4}$		**1**
$\frac{5}{11}$		$\frac{5}{13}$
$\frac{11}{5}$		**3**
$\frac{11}{8}$		$\frac{11}{6}$
$\frac{3}{2}$		**2**

$\frac{1}{7}$		$\frac{1}{11}$
$\frac{7}{4}$		**1**
$\frac{7}{10}$		$\frac{7}{16}$
$\frac{16}{5}$		**3**
$\frac{13}{6}$		$\frac{13}{8}$
$\frac{5}{2}$		**2**
$\frac{1}{12}$		$\frac{1}{15}$
$\frac{2}{7}$		**1**
$\frac{11}{14}$		$\frac{11}{18}$
$\frac{15}{7}$		**3**
$\frac{16}{13}$		$\frac{16}{11}$
$\frac{8}{7}$		**2**
$\frac{1}{16}$		$\frac{1}{11}$
$\frac{9}{7}$		**1**
$\frac{13}{16}$		$\frac{13}{18}$
$\frac{22}{7}$		**3**
$\frac{18}{15}$		$\frac{18}{11}$
$\frac{17}{5}$		**3**

Transcoding task for decimal numbers used for grade 5

Ecrivez les **nombres décimaux** en chiffre qui aient :

		Nombre en chiffre
0	**Une dizaine et 4 unités**	**14**
1	Cinq dixièmes	
2	Une unité, quatre dixièmes, huit centièmes et deux millièmes	
3	Sept millièmes	
4	Une unité, cinq dixièmes, quatre centièmes	
5	Cinq dixièmes et deux centièmes	
6	Cinq unités, neuf dixièmes	
7	Cinq unités, deux dixièmes, quatre centièmes et six millièmes	
8	Trois unités, cinq dixièmes et sept millièmes	
9	Six centièmes	
10	Huit unités, trois dixièmes, cinq centièmes	
11	Deux dixièmes, huit centièmes et cinq millièmes	
12	Cinq unités et deux millièmes	
13	Sept dixièmes	
14	Deux unités, cinq dixièmes, un centièmes et quatre millièmes	
15	Quatre unités, six centièmes et cinq millièmes	
16	Trois unités et huit dixièmes,	
17	Trois unités, deux dixièmes, cinq centièmes et sept millièmes	
18	Deux unités et huit centièmes	
19	Deux unités, trois dixièmes et six centièmes	
20	Quatre unités, une dixièmes, cinq centièmes et trois millièmes	
21	Sept dixièmes, cinq centièmes et trois millièmes	
22	Quatre unités, trois dixièmes et deux centièmes	
23	Huit dixièmes et sept centièmes	
24	Deux unités, cinq dixièmes, trois centièmes et huit millièmes	

Density task for decimal numbers used for grade 5

1. Ci-dessous, entoure les nombres compris entre **3,5 et 3,7**

A. 3,51 **B. 3,4**

C. 3,705 **D. 3,6**

E. 3,75 **F. 3,659**

2. Ci-dessous, entoure les nombres compris entre **6,382 et 6,405**

A. 6,389 **B. 6,37**

C. 6,3 **D. 6,392**

E. 6,4 **F. 6,39**

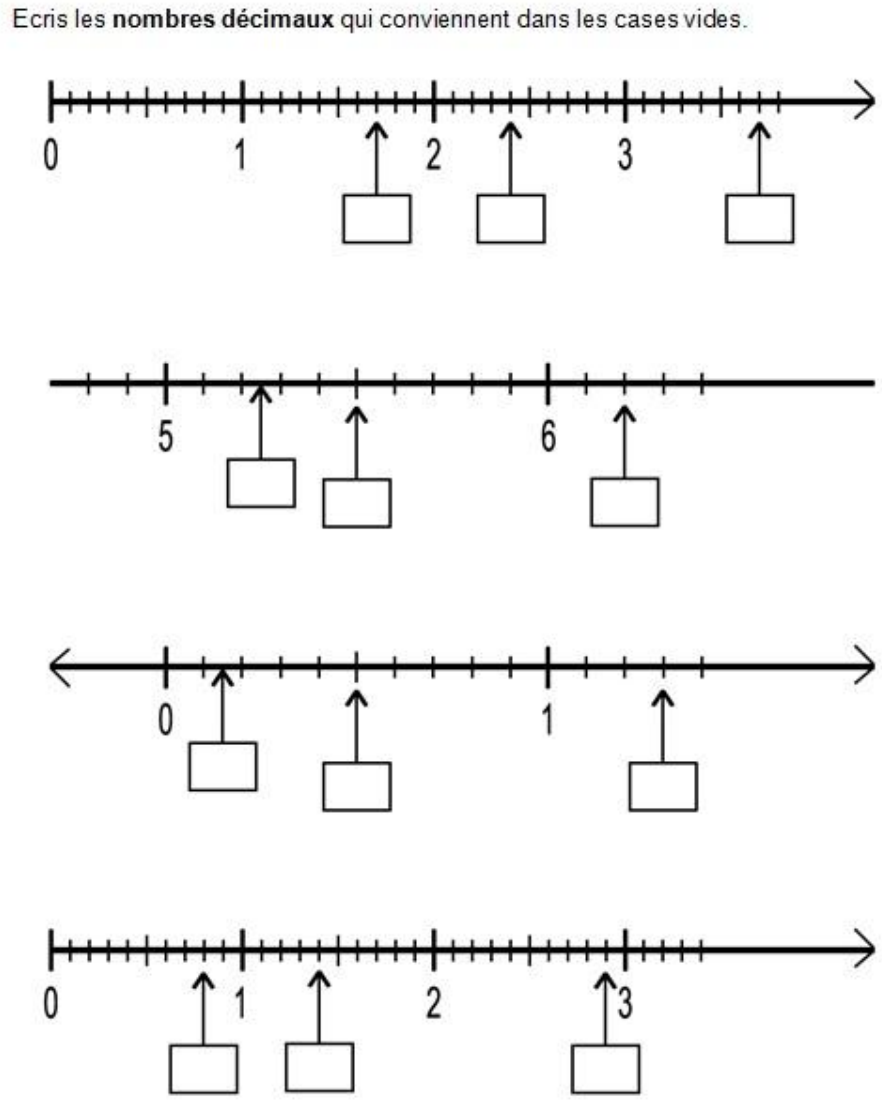
3. Complète le tableau suivant

Nombre entier qui vient juste avant	Nombre décimal	Nombre entier qui vient juste après
	631,2	
	42,75	
	83,5	
	1000,09	

4. Dans la case vide, écris **un nombre** compris entre les deux nombres donnés.

72		75
48,7		49,7
72,4		72,5
125		129
15,3		16,3
27,8		27,9

Number line task for decimal numbers used for grade 5



Comparison task for decimal numbers used for grade 5

Entoure le nombre LE PLUS GRAND ou place "=" en cas d'égalité.

1,3		1,30
2,5		2,40
1,5		1,06
5,70		5,8
6,80		6,9
2,4		2,40
7,6		7,50
3,1		3,10
7,06		7,5
8,60		8,6
4,3		4,20
4,2		4,03

5,70		5,7
2,4		2,05
5,08		5,7
6,09		6,8
1,4		1,30
6,80		6,8
3,2		3,10
7,50		7,5
3,1		3,02
4,2		4,20
8,60		8,7
8,07		8,6

Digit identification task for decimal numbers used for grade 5

Entoure le chiffre des unités dans le nombre suivant :

4,13	**1,9**	**0,19**	**0,7**
0,127	**9,831**	**3,52**	**0,65**

Entoure le chiffre des dixièmes dans le nombre suivant :

4,19	**9,174**	**4,07**	**13,24**
5,013	**82,351**	**5,72**	**6,09**
25,192	**2,074**	**18,37**	**3,218**

Entoure le chiffre des centièmes dans le nombre suivant :

7,12	7,20	119,24	215,142
4,205	4,217	8,27	6,70
210,12	123,415	9,502	12,835

Entoure le chiffre des millièmes dans le nombre suivant :

4,172	4,740	1250,715	2130,412
1281,115	6,193	8,120	9,115
3,240	3235,314	3,514	3,110

Measure task used for grade 5 (Link fraction-decimal numbers)

Rappel: 1m = 10dm ; 1m = 100cm et 1m = 1000mm.

1. Ecris dans chaque cas **le nombre décimal** qui convient.

4m 7 dm 5 cm 2 mm =.....m	**9dm =.....m**
7 mm =.....m	**3dm 7cm =..... m**
8cm 2 mm =.....m	**2m 7 dm =**
3m 9 mm =.....m	**8m 5 dm 6 cm =.....m**
5m 9cm 1 mm =.....m	**5cm =.....m**
5dm 2 mm =.....m	**5dm 7 cm 3 mm =.....m**
3m 8 dm 3 mm =.....m	**3m 5 cm =.....m**

2. Ecris dans chaque cas **le nombre décimal** qui convient.

425cm =.....m	**2m 36 cm =.....m**
58 dm =.....m	**73 cm =.....m**
46 cm =.....m	**34 dm =.....m**
5m 37cm =.....m	**506cm =.....m**

Fraction symbol to decimal symbol and vice versa used for grade 5 (Link Fraction-decimal numbers)

Complète le tableau suivant avec le **nombre décimal :**

Ecriture fractionnaire		Ecriture décimale
$\frac{3}{10}$	=	.....
$\frac{237}{100}$	=	.....
$\frac{1}{2}$	=	.....
$\frac{7}{4}$	=	.....
$\frac{8}{1000}$	=	.....
$\frac{6}{5}$	=	.....
$\frac{17}{10}$	=	.....
$\frac{1}{5}$	=	.....
$\frac{5}{100}$	=	.....
$\frac{1258}{1000}$	=	.....
$\frac{5}{2}$	=	.....
$\frac{1}{4}$	=	.....

Complète le tableau suivant avec le **nombre fraction:**

Ecriture décimale		Ecriture fractionnaire
0,7	=	.....
0,125	=	.....
2,456	=	.....
2,5	=	.....
5,14	=	.....
1,518	=	.....
0,2	=	.....
0,05	=	.....
1,2	=	.....
0,003	=	.....
0,26	=	.....
1,25	=	.....

Tasks discussed in discussion for general conclusion

Comparison task of verbal decimal numbers with expression "tenths, hundredths, thousandths"

Entourer l'expression représentant le plus grand nombre.

En cas d'égalité, inscrivez le signe « = » dans la case du milieu.

Six unités vingt six centième		Huit unités cinquante sept centième
Huit unités cinquante trois centième		Huit unités vingt neuf centième
Trois unités cinq centièmes		Trois unités cinq millièmes
Trois unités cinquante neuf centièmes		Huit unités vingt cinq centième
Trois unités vingt sept centièmes		Trois unités cinquante quatre centièmes
Sept dixièmes		Quatre cents soixante trois millièmes
Quatre unités neuf millièmes		Quatre unités neuf dixième
Sept unités vingt neuf centième		Quatre unités cinq cents quarante neuf millièmes
Trois dixièmes		Cinquante neuf centièmes
Trois unités trois cents quinze millièmes		Trois unités six cents douze millièmes
Six unités huit centièmes		Six unités trente neuf millièmes
Cinq unités cinquante sept centièmes		Cinq unités quatre dixième
Quarante huit millièmes		Sept dixième
Sept unités trois cents quinze millièmes		Sept unités cinq cents vingt neuf millièmes
Trois unités quatre dixième		Trois unités vingt un centièmes

Entourer l'expression représentant le plus grand nombre

En cas d'égalité, inscrivez le signe « = » dans la case du milieu.

Cinq unités neuf cents vingt six millièmes		Huit unités trois dixième
Quatre unités deux cents quinze millièmes		Quatre unités cinquante sept centièmes
Huit cents quinze millièmes		Cinquante trois centièmes
Deux unités cinq cents quatorze millièmes		Deux unités quarante sept centièmes
Sept unités huit dixièmes		Sept unités huit centièmes
Cinq unités quatre cents dix neuf millièmes		Cinq unités soixante deux centièmes
Huit unités vingt cinq millièmes		Huit unités sept dixièmes
Quatre unités sept cents douze millièmes		Quatre unités cinquante trois centièmes
Cinq unités six dixièmes		Cinq unités soixante centièmes
Six unités trois dixième		Six unités soixante deux centièmes
Soixante deux centièmes		Quatre cents trente cinq millièmes
Sept unités six dixième		Sept unités six millièmes
Huit unités cinquante neuf centièmes		Huit unités sept dixièmes
Quatre dixièmes		Sept cents vingt huit millièmes
Huit unités trente centièmes		Huit unités trois dixièmes

Comparison task of verbal decimal numbers with their symbolic representation

Entourer l'expression représentant le plus grand nombre.

En cas d'égalité, inscrivez le signe « = » dans la case du milieu.

Huit virgule cinquante trois		Huit virgule vingt neuf
Trois virgule zéro cinq		Trois virgule zéro zéro cinq
Six virgule vingt six		Huit virgule cinquante sept
Trois virgule vingt sept		Trois virgule cinquante quatre
Zéro virgule sept		Zéro virgule quatre cents soixante trois
Trois virgule cinquante neuf		Huit virgule vingt cinq
Cinq virgule cinquante sept		Cinq virgule quatre
Quatre virgule zéro zéro neuf		Quatre virgule neuf
Trois virgule quatre		Trois virgule vingt et un
Sept virgule vingt neuf		Quatre virgule cinq cents quarante neuf
Trois virgule trois cents quinze		Trois virgule six cents douze
Zéro virgule zéro quarante huit		Zéro virgule sept
Six virgule zéro huit		Six virgule zéro trente neuf
Sept virgule trois cents quinze		Sept virgule cinq cents vingt neuf
Zéro virgule trois		Zéro virgule cinquante neuf

E. Entourer l'expression représentant le plus grand nombre.

En cas d'égalité, inscrivez le signe « = » dans la case du milieu.

Huit virgule zéro vingt cinq		Huit virgule sept
Quatre virgule deux cents quinze		Quatre virgule cinquante sept
Zéro virgule huit cents quinze		Zéro virgule cinquante trois
Deux virgule cinq cents quatorze		Deux virgule quarante sept
Cinq virgule neuf cents vingt six		Huit virgule trois
Sept virgule huit		Sept virgule zéro huit
Cinq virgule quatre cents dix neuf		Cinq virgule soixante deux
Quatre virgule sept cents douze		Quatre virgule cinquante trois
Huit virgule cinquante neuf		Huit virgule sept
Cinq virgule six		Cinq virgule soixante
Six virgule trois		Six virgule soixante deux
Sept virgule six		Sept virgule zéro zéro six
Zéro virgule quatre		Zéro virgule sept cents vingt huit
Zéro virgule soixante deux		Zéro virgule quatre cents trente cinq
Huit virgule trente		Huit virgule trois