

THE PERMANENT AND TEMPORARY EFFECTS OF STOCK SPLITS ON LIQUIDITY IN A DYNAMIC SEMIPARAMETRIC MODEL

Christian M. Hafner, Oliver B. Linton, Linqi Wang

REPRINT | 2026 / 06

ISBA

Voie du Roman Pays 20 - L1.04.01

B-1348 Louvain-la-Neuve

Email : lidam-library@uclouvain.be

<https://uclouvain.be/en/research-institutes/lidam/isba/publication>

The permanent and temporary effects of stock splits on liquidity in a dynamic semiparametric model

Christian M. Hafner^{*a}, Oliver B. Linton^{†b}, and Linqi Wang^{‡b}

^aLouvain Institute of Data Analysis and Modelling in economics and statistics (LIDAM), and ISBA,
UCLouvain

^bFaculty of Economics, University of Cambridge

Abstract

We develop a dynamic framework to detect the occurrence of permanent and transitory breaks in the illiquidity process. We propose various tests that can be applied separately to individual events and can be aggregated across different events over time for a given firm or across different firms. We use this methodology to study the impact of forward and reverse stock splits on the illiquidity dynamics of the S&P 500, S&P 400 and S&P 600 index stock constituents. Our empirical results show that stock splits have a positive and significant effect on the permanent component of the illiquidity process while a majority of the stocks engaging in reverse splits experience an improvement in liquidity conditions.

Keywords: Amihud illiquidity, DArLiQ, Difference in Difference, Event Study, Nonparametric Estimation, Reverse Split, Structural Change

JEL Classification: C12, C14, G14, G32

^{*}Email: christian.hafner@uclouvain.be.

[†]Email: obl20@cam.ac.uk.

[‡]Corresponding author; Email: lw711@cam.ac.uk; Address: Austin Robinson Building, Sidgwick Avenue, Cambridge, CB3 9DD.

1 Introduction

One commonly held market explanation for why companies split their stock is the theory that this creates “wider” markets.¹ Reducing the price level should make it easier for a bigger pool of retail (uninformed) investors to buy into the stock and allow existing investors to sell part of their holdings more easily to other investors thereby increasing the investor base and the volume of transactions. This in turn should lead to improved liquidity conditions and reduce the cost of capital to companies. However, there are other theoretical arguments presented in [Copeland \(1979\)](#) that may point to a decrease in liquidity following a stock split, such as increases in real transaction costs.² In his empirical study, Copeland found that liquidity worsened following stock splits on average. However, his dataset terminated over fifty years ago and at that time stocks were traded in 1/8ths of a dollar and the trading system was pit based rather than electronic. More recent work on US and international data that we review in the Appendix has found mixed results. We contribute to the evidence on this issue with updated data and new statistical methods.

Many of the empirical studies about stock splits distinguish between short-term and long-term effects but do so in an informal way statistically. We introduce a new econometric methodology based on [Hafner et al. \(2023\)](#) henceforth HLW, to capture this distinction formally and to provide statistical tests of the effects on liquidity of stock splits. We base our analysis on the popular Amihud (il)liquidity measure proposed by [Amihud \(2002\)](#), which is considered an empirical proxy of Kyle’s price impact parameter. But rather than averaging over daily measures as in the usual approach, we feed the daily measures

¹Announcing their 4 for 1 split in June of 2020, the Apple Board of Directors gave the reason “to make the stock more accessible to a broader base of investors.”

²As [Weld et al. \(2009\)](#) say “commissions paid by investors on trading ten \$35 shares are about ten times those paid on a single \$350 share”. Nowadays, trading costs for retail traders would not scale in this way but nevertheless the total costs would be higher.

directly into our model. Our DArLiQ model is a member of the class of multiplicative error models (MEM) that have been applied to many different positive-valued financial time series including volatility, duration between trades, and transaction volume, see e.g. [Engle \(2002\)](#). We allow for a nonparametric time trend to capture the secular improvement in liquidity that has happened for most stocks from the 1960s to the present day. The MEM model and its applications and developments over the last 20 years are reviewed in [Cipollini and Gallo \(2022\)](#).³ Within our econometric model, we test the following hypotheses

$\mathbf{H}_0^1$: *Stock splits have no permanent effect on the level of liquidity against the alternative that they do have a permanent effect.*

$\mathbf{H}_0^2$: *Stock splits have no additional temporary effect on the level of liquidity against the alternative that they do have a temporary effect.*

Our test of $\mathbf{H}_0^1$ is similar to that used in the regression discontinuity literature and the structural break literature - we look for discrepancy between forward- and backward-looking trend estimates. Our estimates are based on nonparametric local linear kernel smoothing, which are well known to be more robust to boundary effects than the local constant method used in HLW, [Fan and Gijbels \(1996\)](#). Our test involves the estimated permanent component of liquidity based on the prewhitened liquidity series, see HLW. We establish large sample properties of our statistic under the null and alternative hypotheses. Since the tests are based on smoothing methods, there is the issue of bias and we use several different methods to account for that in our inference procedures including the recent work of [Armstrong and Kolesár \(2020\)](#). We consider individual event-specific tests, tests that pool across different splits for the same firm, and tests that pool across firms. One issue

³An alternative approach is to fit linear models to the logarithm of liquidity, but we avoid this approach here primarily because taking logarithms emphasizes values close to zero, the inlier problem, and de-emphasizes large positive values of illiquidity, which are of much more interest and concern to policymakers.

is whether they account for major events like the tech bubble and bust of 1999-2001 and the Global Financial Crisis of 2008 that occur at the same time as some forward splits and reverse splits. To control for this we develop a difference in difference (DID) test statistic that estimates the difference in jumps at the specified time relative to the jumps in a control stock (or group of stocks) that did not experience a split at the given time.

Our test of H_0^2 are designed in the spirit of event studies as e.g. in [Fama et al. \(1969\)](#), i.e., we look at abnormal liquidity and cumulative abnormal liquidity and determine whether these quantities are consistent with the null of no change. Our tests are robust to the presence of a permanent break and are looking additionally for short-term adjustments to the new level of liquidity. We propose a nonparametric method for estimating the critical values for these test statistics under the assumption that the shock process is stationary and mixing and show that it is asymptotically valid. Since we focus on liquidity rather than firm valuation, we concentrate on the post split effects rather than the post announcement effects, although our test statistics are computed in some cases over periods including the announcement as well as the split itself.

We next discuss our empirical results. Our results broadly support findings in [Copeland \(1979\)](#) using a more recent sample of the constituents of the S&P 500, S&P 400 and S&P 600 indices. Specifically, we document that stock splits cause significant shifts in the long-term Amihud illiquidity trend while no additional significant effects on short-run liquidity dynamics are detected. This seems to suggest that the market quickly adjusts to the new pricing regime. The change in the long-term trend of illiquidity is predominantly positive, implying that liquidity conditions deteriorate in the long run after stock splits. In [Section 4.1.2](#), we show that our main results are robust to the use of alternative illiquidity measures. An exception arises with the measure based on the quoted bid-ask spread implied by the model proposed in [Li and Ye \(2023\)](#) (henceforth LY). We further elaborate on the potential drivers of these differences in the main text.

We also investigate whether the effects of reverse stock splits on the illiquidity process are symmetric to the ones documented for stock splits. The results suggest that a majority of the low-price stocks engaging in reverse splits experience an improvement in liquidity conditions. However, we find limited effects on the short-run illiquidity dynamics. The fact that reverse splits result in a significant decrease in the illiquidity trend for the majority of those stocks is in line with the results in [Han \(1995\)](#). The pronounced improvement in liquidity conditions for our sample of stocks with low pre-event price levels is consistent with the evidence in [Blau et al. \(2023\)](#). This could be linked to the fact that short-selling activity increases after reverse splits in part because reverse splits ease the constraints on short selling for low-priced stocks ([Kwan et al. \(2015\)](#)).

The remainder of the paper is organized as follows. The following section recalls the Dynamic Autoregressive Liquidity (DARLiq) model introduced in HLW and defines a measure for the permanent effect. Section 3 presents the econometric methodology including estimation of the dynamic model and various tests of permanent and temporary effects. Sections 4 and 5 present an empirical application where we use our framework to analyze the effect of stock splits and reverse splits on the long-run trend and short-run illiquidity dynamics. Proofs of main results and some additional figures for the empirical analysis are collected in the appendices of the supplementary material.

2 The dynamic model for illiquidity

2.1 Liquidity measures

Stock market liquidity is essentially the ability to buy and sell shares quickly at a good price. [Kyle \(1985\)](#), [O'Hara \(1995\)](#), [Hasbrouck \(2006\)](#) identify three aspects of liquidity: trading costs and price impact; depth; and resiliency. Common ways of measuring liquidity using high frequency trade and quote data on limit order book include: quoted bid-ask

spreads, effective spreads, realized spreads, quoted depth and weighted depth, and transaction volume. The SEC mandated in 2005 that trading venues in the US report an effective spread measure of market quality monthly through their rule 605. Many academic studies have used variations on these measures to determine the effect of policy and rule changes, see [Beddington et al. \(2013\)](#). However, high frequency intraday data is now very complex, and requires a lot of computation, and is not available for all markets of interest for a long period of time. For these reasons, low-frequency measures are very popular. There are several methods widely used to measure liquidity using lower-frequency data. The usual approach is to work with daily data and then average to a monthly or longer horizon. Some of these measures are based on the autocovariance function of daily returns such as the Roll measure, see [Li and Linton \(2022\)](#) for a recent contribution. Some are based on the frequency of zero returns in say a month, the so-called LOT model of [Lesmond et al. \(1999\)](#). These approaches are less easy to adapt to create a meaningful daily measure of liquidity in the absence of intradaily data. We focus on the Amihud illiquidity, [Amihud \(2002\)](#), which can be interpreted as a price impact measure of liquidity, and which has a natural daily counterpart. This has been used in countless studies (this paper is cited more than 13500 times) as: a dependent variable measuring market quality; and an independent variable as a priceable factor in asset pricing cross-sectional regressions, [Amihud \(2002\)](#). [Goyenko et al. \(2009\)](#) and [Fong et al. \(2017\)](#) compare a large number of low-frequency liquidity measures, and generally speaking the Amihud measure does pretty well. There are several variations on the Amihud measure, and we present some of them in Table 1.

In the empirical study, our main results are obtained with the measure $\ell_t = \sigma_t/V_t$, where σ_t is the high-low measure of daily volatility. One advantage of this measure is that it avoids the zero problem sometimes encountered with daily returns, see HLW. Furthermore, it has been argued that this version leads to a more efficient low-frequency liquidity measure than the classic daily Amihud proxy, see [Lacava et al. \(2023\)](#). Our main empirical results are

Table 1: Alternative daily liquidity or illiquidity measures.

Paper	Daily Measures
Amihud (2002)	$\frac{ P_t^C - P_{t-1}^C }{V_t \times P_{t-1}^C}$
Amivest	$\frac{V_t \times P_{t-1}^C}{ P_t^C - P_{t-1}^C }$
Lacava et al. (2023)	$\frac{\sigma_t}{V_t}$
Barardehi et al. (2021)	$\frac{ \ln(P_t^C) - \ln(P_t^O) }{V_t}$
Fong et al. (2018)	$\frac{\frac{a\sigma_t}{V_t^{1/2}}}{V_t}$
Li and Ye (2023)	$\frac{2\sigma_t L_t P_t}{V_t}$
Copeland (1979)	V_t
Kyle and Obizhaeva (2016)	$c \frac{V_t^{1/3}}{\sigma_t^{2/3}}$
Hui and Heubel (1984)	$\frac{(P_t^H - P_t^L)}{P_t^L} \frac{mcap_t}{V_t}$
Danyliv et al. (2014)	$\log_{10} \left(\frac{V_t}{P_t^H - P_t^L} \right)$

Note: σ_t is a daily measure of volatility such as the high-low measure $\frac{P_t^H - P_t^L}{P_t^C}$. V_t is trading volume, L_t is round lot size, and $mcap_t$ is the market capitalization. The high, low, close and open prices are denoted P_t^H , P_t^L , P_t^C , and P_t^O .

qualitatively robust to the use of alternative liquidity measures except the LY measure, which gives quite different results. This seems to arise because of the additional presence of the nominal price level.

2.2 Time series model

Suppose that non-negative time series ℓ_t follows a multiplicative dynamic stochastic process

$$\ell_t = g(t/T)\lambda_t\zeta_t, \quad B(L)\lambda_t = \omega + C(L)\ell_{t-1}^*, \quad (1)$$

where $g(\cdot)$ is a positive but unknown function of rescaled time while $\ell_t^* = \ell_t/g(t/T) = \lambda_t\zeta_t$ is the rescaled liquidity, and ζ_t is a sequence of non-negative random variables with conditional mean one and finite unconditional variance denoted by σ_ζ^2 . We here consider the special

case where the lag polynomials satisfy $B(L) = 1 - \beta L$ and $C(L) = \gamma$, where $\beta, \gamma > 0$ and $\beta + \gamma < 1$ in which case the process λ_t is mean stationary. The component $g(\cdot)$ captures the long-term slowly evolving trend in the process, which is necessary to include for many stocks and indices due to the longer term improvements in liquidity that we have observed. The dynamic process λ_t represents short-term stationary predictable variation around this trend, which is also needed for liquidity time series that have this moderately persistent deviations from the trend as documented in HLW. The last component ζ_t represents the new information, the shock driving the process. In HLW we investigated the tail thickness issue and found Pareto type tails but tail thickness parameters in the range five to ten. When it comes to estimation, there is an identification issue because we can multiply and divide the two components g, λ by constants and obtain the same value of liquidity. To resolve this we suppose that $E(\lambda_t) = 1$, which is achieved by setting $\omega = 1 - \beta - \gamma$. The properties of this model and the overall estimation strategy are discussed in HLW.⁴

The [Amihud \(2002\)](#) liquidity measure is formally defined as $A_t = \sum_{s \in I_t} \ell_s / n_t$. Usually, A is measured monthly, quarterly, or annually by averaging the daily values of ℓ_s within period I_t of length n_t days.⁵ Note that our trend function $g(t/T)$ can be interpreted as the model-specific counterpart of the low-frequency measure A_t . The key feature of our model is that we allow the detrended liquidity $\ell_t^* = \ell_t / g(t/T)$ to have nontrivial short-term predictability through the process λ_t so that shocks to liquidity have persistent effects on the level of liquidity relative to trend. In practice, both short-term and long-term predictability

⁴A more general approach could be based on [Inoue et al. \(2021\)](#) where all the parameters are allowed to change over time in a smooth fashion. This would be a purely nonparametric model that includes ours as a special case (at least when the shocks are i.i.d.). Our model leans on the old idea of trend+cycle decomposition and imposes more structure with the benefit of faster convergence rates for some quantities.

⁵[Acharya and Pedersen \(2005\)](#) modify A_t by multiplying it by lagged market price to take care of nonstationarity and then winsorizing to reduce the effect of outliers. We instead explicitly model the trend to account for nonstationarity. In HLW we discussed the issue of heavy tails and how to deal with them.

is present in this series, and it is important to take account of both features when carrying out inferential procedures and in predicting future liquidity.

Our model implies that $E(\ell_t \mid \mathcal{F}_{t-1}) = g(t/T)\lambda_t$ and $E(\ell_t) = g(t/T)$, which is the basis of our estimation strategy. It also implies that $\text{var}(\ell_t \mid \mathcal{F}_{t-1}) = g(t/T)^2 \lambda_t^2 \sigma_{\zeta_t}^2$, where $\sigma_{\zeta_t}^2 = \text{var}(\zeta_t \mid \mathcal{F}_{t-1})$. If we additionally assume that ζ_t is i.i.d., then $\text{var}(\ell_t \mid \mathcal{F}_{t-1})$ is, apart from a constant, the square of $E(\ell_t \mid \mathcal{F}_{t-1})$, which may be rather restrictive. Therefore, since we do not need this for estimation, we shall not require this, preferring the weaker assumption that $\zeta_t - 1$ is a martingale difference sequence.

We suppose that the liquidity trend function g is almost everywhere a smooth function of (rescaled) time, specifically, it has two continuous derivatives. The function g captures the long-run variation of liquidity, which is generally smoothly varying (and for most assets shows improvements over time). However, we allow the possibility of permanent shifts in liquidity (structural change) by allowing the function g to be discontinuous at a finite set of known points $u_1 = t_1/T, \dots, u_m = t_m/T \in (0, 1)$. For a given point $u \in (0, 1)$ define the left and right limits of the function and its first two derivatives

$$\lim_{u \uparrow u} g^{(r)}(u) = g_-^{(r)}(u), \quad \lim_{u \downarrow u} g^{(r)}(u) = g_+^{(r)}(u), \quad r = 0, 1, 2,$$

which we assume are well defined and finite. We allow that $g_-^{(r)}(u_i) \neq g_+^{(r)}(u_i)$ for $i = 1, \dots, m$ and $r = 0, 1, 2$, but for all $u \notin \{0, u_1, \dots, u_m, 1\}$ we maintain that $g_-^{(r)}(u) = g_+^{(r)}(u) = g(u)$, for $r = 0, 1, 2$.⁶ We adopt the convention that $g^{(r)}(\cdot)$ are CADLAG (continuous from the right with limits from the left), that is, $g^{(r)}(u_i) = g_+^{(r)}(u_i)$. One question of interest is whether a break has occurred, and, if one has occurred, how big is the effect in terms of the level of g . The size of the jump at point u_i is measured in level terms and percentage terms, respectively, by:

$$\mathcal{J}_i = g_+(u_i) - g_-(u_i), \quad \mathcal{J}_{\%i} = 2 \frac{g_+(u_i) - g_-(u_i)}{g_+(u_i) + g_-(u_i)}. \quad (2)$$

⁶Note that $g_+^{(r)}(0)$ and $g_-^{(r)}(1)$ are also assumed to be well defined finite quantities.

This is the magnitude of the permanent effect of the split on the level of Amihud illiquidity of a given firm at time u_i (the effect that remains in the absence of further changes).

3 Econometric methodology

3.1 Assumptions

We suppose that ℓ_t^* is stationary and alpha mixing. This can be shown to hold under the parameter restrictions provided ζ_t is i.i.d. It may also hold when ζ_t itself is only described as a stationary mixing process although this can be difficult to establish. We next give the main technical assumptions we use with regard to the nonparametric part of our methodology.

ASSUMPTION A1. *We suppose that $g(\cdot) \in \mathcal{G}$, where for $0 < c, C < \infty$*

$$\mathcal{G} = \left\{ g : g : [0, 1] \rightarrow \mathbb{R}_+, g_{\pm}(x) \geq c, g_+(x) = g_-(x) \text{ except for } x \in \{u_1, \dots, u_m\} \subset (0, 1), \right. \\ \left. \left| g_{\pm}^{(j)}(x) \right| \leq C \text{ for all } x \in [0, 1], \text{ for } j = 0, 1, 2 \right\}.$$

ASSUMPTION A2. *Suppose that $\{v_t\}$, where $v_t = \lambda_t \zeta_t - 1$, is a stationary alpha mixing sequence with $E(v_t) = 0$ and $E(|v_t|^{\varkappa}) \leq C < \infty$ for some $\varkappa > 5/2$ for $t = 1, 2, \dots$ such that*

$$\sum_{k=1}^{\infty} \alpha(k)^{\frac{\varkappa-2}{\varkappa}} < \infty. \quad (3)$$

ASSUMPTION A3. *Suppose that K is a fuction that is symmetric about zero with compact support $[-1, 1]$ such that $K(\pm 1) = 0$ and K is twice differentiable where $K^{(2)}$ is continuous on $[-1, 1]$. Let $\|K\|_2^2 = \int_{-1}^1 K(s)^2 ds$, and $\mu_j(K) = \int_{-1}^1 s^j K(s) ds$, $j = 0, 1, 2$.*

The main difference from HLW is in our Assumption A1, which allows for a more general class of functions with discontinuities at a finite number of known points. Assumption A2 is a standard requirement that combines moment conditions with mixing conditions sufficient to drive a CLT. Assumption A3 is a standard condition on the kernel sufficient for our

arguments. We also assume conditions A4-A6 of HLW, which together with our assumptions A1-A3 ensure the root-T consistency of the GMM estimator of the finite dimensional parameters defined below. Since our emphasis in this paper is on the nonparametric part and since conditions A4-A6 of HLW are themselves high level conditions about the moment function (and in the interests of space limitation) we do not repeat these conditions here. In the supplementary material to this paper we show that our local linear estimator defined below is uniformly consistent at the required rate under our conditions, which is the key new step required in the argument to establish root-T consistency of the GMM estimator.

We need one additional assumption for the test of temporary effects given in Section 3.4 below.

ASSUMPTION A4. Define $e_{t,\tau} = \sum_{s=0}^{\tau} (\zeta_{t+s} - 1)$ for each t and $\tau = 0, \dots, E$. Suppose that for each $\tau = 0, \dots, E$, $\{e_{t,\tau}\}$ is stationary and mixing with mixing coefficient that satisfies (3) and has Lebesgue density $f_{e_{t,\tau}}$ that satisfies $\sup_x f_{e_{t,\tau}}(x) < \infty$ and $\inf_x f_{e_{t,\tau}}(x) > 0$.

3.2 Estimation of the model

We observe a sample of daily illiquidities $\{\ell_t, t = 1, \dots, T\}$ for a given firm. We first estimate the trend process by a local linear kernel smoother designed to be robust to potential breaks at known distinct points $u_0 = 0 < u_1 < u_2 < \dots < u_m < 1 = u_{m+1}$. Specifically, we define $\hat{g}(u) = F(\hat{\alpha}_0(u))$ and $\hat{g}'(u) = F'(\hat{\alpha}_0(u))\hat{\alpha}_1(u)$, where for $u \in [u_i, u_{i+1})$,

$$(\hat{\alpha}_0(u), \hat{\alpha}_1(u)) = \arg \min_{\alpha_0, \alpha_1} \sum_{t=1}^T K_h(t/T - u) \left\{ \ell_t - F(\alpha_0 + \alpha_1(t/T - u)) \right\}^2 1(u_i \leq t/T < u_{i+1}). \quad (4)$$

Here, $K_h(\cdot) = K(\cdot/h)/h$, where K is a kernel satisfying condition A3 and h is a bandwidth, while F is a known function. The choice of $F : \mathbb{R} \rightarrow \mathbb{R}_+$ imposes non-negativity on $\hat{g}(u)$, albeit at the cost of nonlinear optimization. For most series this step may be unnecessary and the choice of $F(y) = y$ yields the local linear regression, which has an explicit formula

given in the Appendix. We continue the presentation with the case $F(y) = y$ for simplicity. HLW used a local constant estimator with an additional explicit boundary adjustment. Our method provides an automatic boundary adjustment that preserves the rate of convergence of the estimator at all points $u \in [0, 1]$. At interior points of the interval (u_i, u_{i+1}) the estimator is the standard local linear with two-sided smoothing, but for points $u_i + ch$ with $c \in [0, 1]$ only time points to the right of u_i are included and at points $u_{i+1} - ch$ with $c \in [0, 1]$ only points to the left of u_{i+1} are included. This is exactly what happens with the local linear estimator at a boundary point of the covariate support, see [Fan and Gijbels \(1996\)](#). The estimator $\widehat{g}(u)$ is CADLAG everywhere on $[0, 1]$ and is continuous everywhere except at the points $\{u_0, u_1, \dots, u_{m+1}\}$, consistent with the posited behaviour of $g(u)$. [Robinson \(1989\)](#) established large sample properties of local constant kernel estimators in a similar time series setting without structural breaks, and [Francisco-Fernández et al. \(2003\)](#) established uniform consistency. [Kristensen \(2012\)](#) and others have extended these results to settings with abrupt changes like ours. We carry out the estimation of the dynamic parameters of λ_t using this jump robust estimator.

We discuss how we select the bandwidth h in the supplementary material. The optimal bandwidth in mean squared error (MSE) for $\widetilde{\mathcal{J}}_i$ and $\widetilde{g}_{\pm}(u_i)$ are respectively given by:

$$h_{\Delta, opt}(u_i) = C_K \left(\frac{(g_+(u_i)^2 + g_-(u_i)^2) \sigma_{\zeta}^2}{\left(g_+^{(2)}(u_i) - g_-^{(2)}(u_i)\right)^2} \right)^{1/5} T^{-1/5}, \quad h_{\pm, opt}(u_i) = C_K \left(\frac{g_{\pm}(u_i)^2 \sigma_{\zeta}^2}{g_{\pm}^{(2)}(u_i)^2} \right)^{1/5} T^{-1/5},$$

where $C_K = (\|K^+\|^2 / \mu_2^2(K^+))^{1/5}$ depends only on the kernel.

Define the detrended liquidity $\widehat{\ell}_t^* = \ell_t / \widehat{g}(t/T)$, $t = 1, \dots, T$. We use GMM to estimate the dynamic parameters $\theta = (\beta, \gamma)^\top$ from the conditional moment restriction $E(\ell_t^* | \mathcal{F}_{t-1}) = \lambda_t$, where $\ell_t^* = \ell_t / g(t/T)$, $t = 1, \dots, T$. We work with the residual $\ell_t^* / \lambda_t(\theta) - 1$, which is a martingale difference sequence at the true parameter values $\beta = \beta_0, \gamma = \gamma_0$. In practice, we use $\widehat{\ell}_t^* / \widehat{\lambda}_t(\theta) - 1$, where $\widehat{\lambda}_t(\theta) = 1 - \beta - \gamma + \beta \lambda_{t-1} + \gamma \widehat{\ell}_{t-1}^*$. Then define $\rho_t(\theta, \widehat{g}) =$

$z_{t-1}(\widehat{\ell}_t^*/\widehat{\lambda}_t(\theta) - 1)$ and

$$\widehat{\theta}_{GMM} = \arg \min_{\theta \in \Theta} \|M_T(\theta, \widehat{g})\|_W, \quad M_T(\theta, \widehat{g}) = \frac{1}{T} \sum_{t=1}^T \rho_t(\theta, \widehat{g}), \quad (5)$$

where W is a weighting matrix, while $z_t \in \mathcal{F}_t$ are instruments, for example lagged values, while for a vector a and matrix W , $\|a\|_W^2 = a^\top W a$. This provides initial root-T consistent estimators of θ . The argument is an extension of the arguments presented in HLW, which are based on [Chen et al. \(2003\)](#), to allow for discontinuities in the function g at known points. We show in the supplementary material that our estimator of g is uniformly consistent over the interval $[0, 1]$ at the same rate as if g were continuous and smooth everywhere. The conditions for the root-T consistent estimation of θ include undersmoothing (relative to what would be optimal for estimation of $g(u)$), that is, the first round bandwidth sequence h satisfies $Th^4 \rightarrow 0$.

Given consistent estimates of $\theta, g(\cdot)$ one can improve $\widehat{\theta}_{GMM}$ and $\widehat{g}(\cdot)$ for efficiency gains or simplicity of standard errors by working with the "pre-whitened" liquidity, $\ell_t^\dagger = \ell_t/\lambda_t$. We have $E(\ell_t^\dagger) = g(t/T)$, which provides an alternative local moment condition for estimation of g , and one that is purged of the short-run variation induced by the dynamic process λ_t . We let $\widetilde{g}(u) = \widetilde{\alpha}_0(u)$, where $(\widetilde{\alpha}_0(u), \widetilde{\alpha}_1(u))$ are defined as the minimizers of (4) with ℓ_t replaced by $\widehat{\ell}_t^\dagger = \ell_t/\widehat{\lambda}_t$, where $\widehat{\lambda}_t = \widehat{\lambda}_t(\widehat{\theta}_{GMM}, \widehat{g})$, and $\widehat{\theta}_{GMM}, \widehat{g}(\cdot)$ were estimated in the previous procedure. HLW show that the preliminary estimation of λ_t by $\widehat{\lambda}_t$ has no effect on the large sample distributions of the "pre-whitened" estimator $\widetilde{g}(u)$. As discussed in HLW, the benefit of working with $\widetilde{g}(u)$ is that its large sample variance is much simpler to estimate than the large sample variance of $\widehat{g}(u)$ (which requires long-run variance estimation), which tends to benefit inference in finite and large samples.

To estimate the jump size at points u_i we compute two different estimates of $g(u_i)$, a left sider and a right sider. Specifically, we let $\widehat{g}_+(u_i) = \widehat{\alpha}_{0,+}(u_i)$ and $\widehat{g}_-(u_i) = \widehat{\alpha}_{0,-}(u_i)$, $i = 1, \dots, m$, where: $(\widehat{\alpha}_{0,+}(u_i), \widehat{\alpha}_{1,+}(u_i))$ minimize (4), while $(\widehat{\alpha}_{0,-}(u_i), \widehat{\alpha}_{1,-}(u_i))$ minimize

(4) with $1(u_i \leq t/T < u_{i+1})$ replaced by $1(u_{i-1} \leq t/T < u_i)$. We further let $\tilde{g}_+(u_i) = \tilde{\alpha}_{0,+}(u_i)$ and $\tilde{g}_-(u_i) = \tilde{\alpha}_{0,-}(u_i)$, $i = 1, \dots, m$, where: $(\tilde{\alpha}_{0,+}(u_i), \tilde{\alpha}_{1,+}(u_i))$ minimize (4) with ℓ_t replaced by $\ell_t/\hat{\lambda}_t$, while $(\tilde{\alpha}_{0,-}(u_i), \tilde{\alpha}_{1,-}(u_i))$ minimize (4) with ℓ_t replaced by $\ell_t/\hat{\lambda}_t$ and $1(u_i \leq t/T < u_{i+1})$ replaced by $1(u_{i-1} \leq t/T < u_i)$. Note that $\hat{g}_-(u_i), \tilde{g}_-(u_i)$ consistently estimate $g_-(u_i)$.

The raw and percentage sizes of the jump can be estimated by the first round or second round estimates, and for preserving space we just present here the second round versions

$$\tilde{\mathcal{J}}_i = \tilde{g}_+(u_i) - \tilde{g}_-(u_i), \quad \tilde{\mathcal{J}}_{\%i} = \frac{2(\tilde{g}_+(u_i) - \tilde{g}_-(u_i))}{\tilde{g}_+(u_i) + \tilde{g}_-(u_i)}. \quad (6)$$

3.3 Test of permanent effect

3.3.1 Single split

We first consider the null hypothesis that $g_-(u_i) = g_+(u_i)$ versus the alternative that $g_-(u_i) \neq g_+(u_i)$ for a given u_i . There is a large literature on testing for structural change (at an unknown point) in parametric models, [Perron \(1989\)](#), and in nonparametric regression, [Muller \(1992\)](#), [Delgado and Hidalgo \(2000\)](#), and [Vogt \(2015\)](#). In our case the change point(s) are known in advance so the methodology is more similar to that of the regression discontinuity literature, see [Imbens and Lemieux \(2008\)](#), [Cattaneo and Titiunik \(2022\)](#). The main difference here is that we have a fully specified dynamic model in the background that provides a framework for the construction of standard errors appropriate for the type of dependence found in this type of data, and the model also suggests alternative estimates of the jumps based on the prewhitened liquidity. We test for the presence of a discontinuity at u_i by computing an adjusted t-statistic based on the final stage one-sided local linear estimators. We employ different ways of conducting inference in this nonparametric setting that take account of the smoothing bias, including some recently proposed methods.

Define the standard error and bias and the infeasible test statistic where for simplicity

here we suppose that a common bandwidth h (in practice our estimate of $h_{\Delta,opt}(u_i)$ defined above) is used:

$$SE(u_i) = \sqrt{||K^+||^2 (\sigma_+^2(u_i) + \sigma_-^2(u_i)) / Th}, \quad b(u_i) = \frac{1}{2} h^2 \mu_2(K^+) \left(g_+^{(2)}(u_i) - g_-^{(2)}(u_i) \right),$$

$$\tau(u_i) = \frac{\tilde{\mathcal{J}}_i - \mathcal{J}_i}{SE(u_i)}. \quad (7)$$

Here, K^+ is the right boundary kernel derived from K , which is defined in the supplementary material. Note that the estimators subscripted $+$ are asymptotically independent of the estimators subscripted $-$, because $K^+ \times K^- = 0$, no matter what the correlation structure of the errors (this explains why there is no covariance term in the standard errors). In practice, we replace $\sigma_{\pm}^2(u_i)$ by estimates $\tilde{\sigma}_{\pm}^2(u_i)$, where $\tilde{\sigma}_{\pm}^2(u_i) = \tilde{g}_{\pm}(u_i)^2 \times \hat{\sigma}_{\zeta}^2$, with $\hat{\sigma}_{\zeta}^2 = \frac{1}{T} \sum_{t=1}^T (\hat{\zeta}_t - \bar{\zeta})^2$. Let $\tilde{\tau}(u_i)$ denote the feasible statistic with $SE(u_i)$ replaced by the estimated version $\tilde{SE}(u_i)$.

THEOREM 1. *Suppose that the conditions A1-A3 hold and that conditions A4-A6 of HLW hold. Furthermore, suppose that $Th^5 \rightarrow \gamma$, where $\gamma \in [0, \infty)$. Then, we have as $T \rightarrow \infty$*

$$\tilde{\tau}(u_i) \implies N(\rho_i, 1), \quad \rho_i = \lim_{T \rightarrow \infty} \frac{b(u_i)}{SE(u_i)}. \quad (8)$$

Note that $\gamma = 0$ corresponds to the usual case of undersmoothing, whereas $\gamma > 0$ would allow the case that the bandwidth sequence has been chosen optimally in terms of the mean squared error. We test the hypothesis that $\mathcal{J}_i = 0$ by comparing the value of $\tilde{\tau}(u_i)$ (with $\mathcal{J}_i = 0$) with some critical value that we discuss below.

Stock splits are not the only reason why the liquidity process can experience discontinuities, there may be other firm specific reasons as well as macroeconomic effects and regulatory changes. These other breaks do not impact $\hat{g}(u_i)$ or $\hat{g}_{\pm}(u_i)$ when they occur at sufficiently different timepoints, since our kernel estimators are local, but they may affect estimation of θ (although since the distortion is only affecting a small fraction h of the observations, this may not be catastrophic) and hence $\tilde{g}(u_i)$ and $\tilde{g}_{\pm}(u_i)$. One solution to

this is to identify the other break points using the methodology of [Delgado and Hidalgo \(2000\)](#) and then to implement $\hat{g}(u_i), \tilde{g}(u_i)$ using all the identified breakpoints in (4). Since the break points are estimated at a faster rate than the break magnitudes, their estimation does not affect the limiting behavior of the test statistic $\tilde{\tau}(u_i)$. The alternative solution is to work with $\hat{g}_{\pm}(u_i)$ throughout, although the test statistics associated with this require long run variance estimation.

3.3.2 Single split with control group

In some cases, the event of interest takes place at the same time as other structural changes affecting all stocks, such as during decimilization or the Global Financial Crisis. In this case, we propose to include a control group to eliminate common trends at the change time. This amounts to a diff in diff test, [Angrist and Pischke \(2009\)](#). Specifically, suppose that we have a “treatment” stock labelled with an S subscript and a “control” stock labelled with a C subscript. We suppose that the univariate dynamic model holds for both stocks separately but that the corresponding shocks ζ_{St} and ζ_{Ct} may be correlated. We define $\tilde{g}_{S,\pm}(u_i)$, $\tilde{g}_{C,\pm}(u_i)$, $\tilde{\mathcal{J}}_{C,i}$, $\mathcal{J}_{C,i}$, $\tilde{\mathcal{J}}_{S,i}$, and $\mathcal{J}_{S,i}$ for the given period u_i , and then define the diff-in-diff statistic as

$$\tau_{did}(u_i) = \frac{(\tilde{\mathcal{J}}_{S,i} - \tilde{\mathcal{J}}_{C,i}) - (\mathcal{J}_{S,i} - \mathcal{J}_{C,i})}{SE_{did}(u_i)} \quad (9)$$

$$SE_{did}(u_i) = \|K^+\| \sqrt{\frac{\left(\sigma_{S,+}^2(u_i) + \sigma_{C,+}^2(u_i) - 2\sigma_{S,C,+}(u_i)\right) + \left(\sigma_{S,-}^2(u_i) + \sigma_{C,-}^2(u_i) - 2\sigma_{S,C,-}(u_i)\right)}{Th}}$$

$$b_{did}(u_i) = \frac{h^2}{2} \times \mu_2(K^+) \times \left(\left(g_{S,+}^{(2)}(u_i) - g_{S,-}^{(2)}(u_i) \right) - \left(g_{C,+}^{(2)}(u_i) - g_{C,-}^{(2)}(u_i) \right) \right)$$

$$\tilde{\sigma}_{S,C,\pm}(u) = \tilde{g}_{S,\pm}(u_i) \times \tilde{g}_{C,\pm}(u_i) \times \hat{\sigma}_{\zeta_S, \zeta_C}, \quad \hat{\sigma}_{\zeta_S, \zeta_C} = \frac{1}{T} \sum_{t=1}^T (\hat{\zeta}_{St} - \bar{\zeta}_S)(\hat{\zeta}_{Ct} - \bar{\zeta}_C).$$

Define $\tilde{\tau}_{did}(u_i)$ like $\tau_{did}(u_i)$ but with $\tilde{SE}_{did}(u_i)$ in place of $SE_{did}(u_i)$.

THEOREM 2. *Suppose that the conditions A1-A4 hold and that conditions A4-A6 of HLW hold. Furthermore, suppose that $Th^5 \rightarrow \gamma$, where $\gamma \in [0, \infty)$. Then, we have as*

$T \rightarrow \infty$

$$\tilde{\tau}_{did}(u_i) \implies N(\rho_{did}, 1), \quad \rho_{did} = \lim_{T \rightarrow \infty} \frac{b_{did}(u_i)}{SE_{did}(u_i)}.$$

We test the hypothesis that $\mathcal{J}_{S,i} = \mathcal{J}_{C,i}$ by comparing the value of $\tilde{\tau}_{did}(u_i)$ (with $\mathcal{J}_{S,i} - \mathcal{J}_{C,i} = 0$) with some critical value that we discuss below. We comment that the control group approach heavily relies on being able to find stock(s) that are not influenced by the effect on the treatment group, i.e., where spillover effects from S to C are not anticipated. One may use a single stock as control or construct a synthetic control, see [Abadie \(2021\)](#).⁷

3.3.3 Inference

We present below three different approaches for inference. The usual approach, under-smoothing, is convenient, but rules out optimal bandwidths and so can deliver poor power. Nevertheless, it is by far the most widely used approach in practice. We consider two other approaches based on recent contributions that have the promise to deliver better power. The issue of inference in nonparametric settings is surprisingly still not settled and we think it is worth to spell out these three different approaches. Some users may value convenience, whereas others may want to align more with recent developments. We focus on the statistic $\tilde{\tau}(u_i)$, but the same arguments apply to $\tilde{\tau}_{did}(u_i)$.

1. **Undersmoothing:** The central approach is to assume that bias terms e.g., $b(u_i)$, are of smaller order such that one does not need to account for it in the CLT and one does not need to estimate $g_{\pm}^{(2)}(u_i)$. This holds when $g_+^{(2)}(u_i) = g_-^{(2)}(u_i)$, i.e., the curve has a level shift but the curvature from the left and the right are equal. It also holds when this condition is violated provided that $\gamma = \lim_{T \rightarrow \infty} Th^5 = 0$, which is referred to as the undersmoothing case. In this case we carry out the test of $\mathcal{J}_i = 0$ by comparing the statistics $\tilde{\tau}(u_i)$ with normal critical values $\pm z_{\alpha/2}$ for a size α test.

⁷We adopt the synthetic control approach in the empirical analysis of reverse splits; see Section 5.

2. **Bias correction:** An alternative approach is to consistently estimate the bias and subtract it from the estimator, which can be done in a number of ways either explicitly or implicitly but for consistent estimation of the bias another bandwidth has to be used. A popular method called jackknife is due to [Schucany and Sommers \(1977\)](#) in which we replace the estimator $\tilde{g}_{\pm}(u_i; h)$ computed with whatever bandwidth h by $2\tilde{g}_{\pm}(u_i; h/2) - \tilde{g}_{\pm}(u_i; h)$, which removes the bias effect from the limiting distribution but raises the variance by making the implicit kernel of higher order and hence raising its L_2 norm, see [Härdle and Linton \(1994\)](#). [Calonico et al. \(2014\)](#) advocate an explicit bias correction with the same bandwidth used in the estimation of g . This bias correction eliminates the bias asymptotically but leads to an additional contribution to the variance, which needs to be accounted for. In the application we consider a parametrically guided bias correction based on the pilot parametric model that has been used for bandwidth selection as outlined in the supplementary material. Suppose that $g_+(u), g_-(u)$ are globally polynomial with parameters $\sum_{j=0}^p a_j^i u^j$ on the interval $[u_i, u_{i+1})$, in which case the bias is $b(u_i) = h^2 \mu_2(K) r(u_i)/2$, where $r(u_i) = \sum_{j=2}^p j(j-1)(a_j^{i+1} - a_j^i) u_i^{j-2}$, and we estimate the parameters a_j^i by segmented global polynomial regression on the separate regimes. One does not need to adjust the standard error in our case because the estimates of a_j^i are root-T consistent.⁸ In this case we carry out the test of $\mathcal{J}_i = 0$ by comparing the statistics $\tilde{\tau}(u_i) - \tilde{b}(u_i)/\widetilde{SE}(u_i)$ with $\pm z_{\alpha/2}$ for a size α test.

3. **Honest confidence interval:** [Armstrong and Kolesár \(2020\)](#) suggest an alternative approach that provides “honest” confidence intervals where one estimates an upper bound on the bias terms. Specifically, let $\rho_{\max} = \max_{1 \leq i \leq J} \sup_{\mathcal{G}} \lim_{T \rightarrow \infty} (|b(u_i)| / SE(u_i))$ be the upper bound over the class of functions $\mathcal{G}$. We let $\tilde{\rho}_{\max} = \max_{1 \leq i \leq J} (|$

⁸Note that in our case there are also bias terms from $\hat{\theta}_{GMM}$, which are of smaller order since we required that $Th^4 \rightarrow 0$ for that theory.

$\tilde{b}(u_i) \mid / \tilde{SE}(u_i))$ be the estimated upper bound described below. In this case we carry out the test of $\mathcal{J}_i = 0$ by comparing the statistics $\tilde{\tau}(u_i)$ with $cv_{1-\alpha}(\tilde{\rho}_{\max})$, where $cv_{1-\alpha}(\tilde{\rho}_{\max})$ is the $1 - \alpha$ critical value of the folded normal distribution $\mid N(\tilde{\rho}_{\max}, 1) \mid$ for a size α test. In one implementation [Armstrong and Kolesár \(2020\)](#) use a global polynomial model for $g_+(u), g_-(u)$ on the interval $[u_i, u_{i+1})$ to estimate the bias $b(u_i)$, as we have done above for bandwidth selection and bias correction.

We next discuss briefly the properties of our tests. First, they are consistent against all fixed departures for which $\mathcal{J}_i \neq 0$, since $\tilde{\tau}(u_i) \xrightarrow{P} \infty$ (with $\mathcal{J}_i = 0$) in this case. They also have power against some small departures, specifically, power lies between zero and one for alternatives such that $\mathcal{J}_i = \delta_i \Delta_T$ for some sequence $\Delta_T \rightarrow 0$ such that $\sqrt{Th} \Delta_T \rightarrow \Delta \neq 0$. The difference between inference methods will show up here: specifically, the undersmoothing method will only be able to detect larger departures, see [Calonico et al. \(2014\)](#), [Armstrong and Kolesár \(2020\)](#), and others for further discussion. We comment on a further property of our tests that has particular relevance here. Specifically, our tests also have power against local alternatives regarding the timing of the break (when there may be some small anticipation or delay in the effects), such as the break point occurring at $u_i \pm ch$ for some $c \in [0, 1]$, see [Hidalgo and Seo \(2013\)](#) for some discussion and specific results. This is relevant for applications since a number of authors include quarantine periods in their before and after analysis, see e.g. [Copeland \(1979\)](#) and [Li and Ye \(2023\)](#). One can also construct confidence intervals for $\mathcal{J}_i$ and $\tilde{\mathcal{J}}_{\%i}$ using the above approaches without imposing $\mathcal{J}_i = 0$, since the distribution theory is stated in this general case.

To save space, we do not report the simulation results here, but keep them available upon request. We note that both the undersmoothing method and the honest confidence interval approach work well overall, especially when the sample size is large. In smaller samples (e.g. $n=1000$), the honest confidence interval approach performs slightly better than the undersmoothing approach. For the bias correction method, we observe that there

is an over-rejection issue when the true trend does not have a break and this does not improve as the sample size increases. Perhaps a more sophisticated bias correction method would resolve this, but we do not pursue this direction in our paper.

3.3.4 Multiple splits

We provide a joint test of the null hypothesis of no breaks at any of the u_i versus the alternative of one or more breaks. We may also average across a sample of firms $j = 1, \dots, n$ with breaks at times u_{ji} , $i = 1, \dots, m_j$, so we suppose that N is the total number of events being considered, where $N \leq \sum_{j=1}^n m_j$. We present the approach for the generic statistic $\tilde{\tau}(u_i)$ but it also applies to the DID statistic $\tilde{\tau}_{did}(u_i)$. We consider either of the statistics

$$W = \sum_{i=1}^N \tilde{\tau}(u_i)^2, \quad M = \max_{1 \leq i \leq N} |\tilde{\tau}(u_i)|, \quad (10)$$

Provided the splits occur at different times the above arguments regarding the asymptotic variances follow. Under the null hypothesis of no breaks anywhere, W is asymptotically distributed as $\sum_{i=1}^N (Z_i + \rho_i)^2$ and M is asymptotically distributed as $\max_{1 \leq i \leq N} |Z_i + \rho_i|$, where Z_i are i.i.d. standard normal random variables (the individual t-statistics are mutually independent in large samples given the physical separation between u_i and u_j).

An alternative approach is to work with a directional test. Suppose that we pool the jumps across the splits (either for a given firm or across firms) as follows

$$\tilde{\tau}_w = \frac{\sum_{i=1}^N w_i \tilde{\tau}(u_i)}{\sqrt{\sum_{i=1}^N w_i^2}}, \quad (11)$$

where w_i is a (possibly stochastic) weighting scheme such as market cap or equal weighting. Then we may show that under the null hypothesis, we have (as $T \rightarrow \infty$ for N fixed) $\tilde{\tau}_w \implies N(\rho_w, 1)$, where $\rho_w = \sum_{i=1}^N w_i \rho_i / \sqrt{\sum_{i=1}^N w_i^2}$. The individual statistics are uncorrelated across distinct points u_i . We may test the null hypothesis by comparing $\tilde{\tau}_w$ with the normal critical values in the undersmoothing case or by comparing $\tilde{\tau}_w$ with critical values $cv_{1-\alpha}(\tilde{\rho}_{w,\max})$ in the ‘‘honest’’ confidence interval case, where $cv_{1-\alpha}(\tilde{\rho}_{w,\max})$ is the

$1 - \alpha$ critical value of the folded normal distribution $| N(\tilde{\rho}_{w,\max}, 1) |$ for a size α test. Here, the worst case ratio is $(\sum_{i=1}^N w_i / \sqrt{\sum_{i=1}^N w_i^2}) \times \max_{1 \leq i \leq N} \sup_{\mathcal{G}} \lim_{T \rightarrow \infty} (| b(u_i) | / SE(u_i))$. This test is more directional in its intent, and will not reject all null hypotheses, only those for which the discontinuities tend to go in the same direction, i.e., for which $\sum_{i=1}^N w_i \mathcal{J}_i \neq 0$. This is similar to the principle underlying the variance ratio tests and the usual way in which event studies are conducted through cumulative abnormal returns.

3.4 Test of temporary effects

We consider how to allow for temporary effects or short-term adjustments that eventually die out. One approach is to include dummy variables in the dynamic equation, that is, let $\lambda_t = 1 - \beta - \gamma + \beta \lambda_{t-1} + \sum_{j=1}^J \delta_j D_{jt} + \gamma \ell_{t-1}^*$, where D_{jt} is a dummy variable that is one in period t_j and zero otherwise. To allow for anticipation effects and slow transmission one could focus on times around the known intervention point, that is, if t_j is a stock split day, include dummy variables for $t_j - E, \dots, t_j + E$ for some event window $\mathcal{E}$ of length $J = 2E + 1$. With multiple splits one could include dummy variables around all the key dates. Under this modelling assumption the level of the process λ_t is affected for all $t \geq t_1$, with a flexible effect between t_1 and t_J , but after t_J the effect decreases rapidly as $t - t_J \rightarrow \infty$ and the long run effect of the intervention is zero. One can estimate the parameters δ by GMM jointly with β, γ , but the estimates of δ_j so-formed are not consistent.

We propose a test of the null hypothesis that $\delta_1 = \dots = \delta_J = 0$ against the general alternative under the assumption of i.i.d. shocks ζ_t . In fact our test is also valid when $\zeta_t - 1$ is only a stationary mixing martingale difference sequence. Our test is based on the residuals from the null estimation, which is the common practice in stock market event studies. Here, we just present the single event setting and take a simple approach. We suppose that the event window is given by $\{t_1 - E, \dots, t_1 + E\}$. Define the residuals $\hat{\zeta}_t = \ell_t / \tilde{g}(t/T) \hat{\lambda}_t$, $t = 1, \dots, T$, where the estimation of $\hat{\theta}$ and $\tilde{g}(\cdot)$ are described above.

Under our conditions, these residuals are asymptotically equivalent (as $T \rightarrow \infty$) to the true unobserved ζ_t . We define abnormal illiquidity and cumulative abnormal illiquidity at times $\tau = 0, \dots, 2E$ as follows:

$$\widehat{AIL}_\tau = \widehat{\zeta}_{t_1 - E + \tau} - 1, \quad \widehat{CAIL}(\tau) = \sum_{s=0}^{\tau} \widehat{AIL}_s. \quad (12)$$

We do not use the usual normal critical values here because this is not likely to be a good assumption in view of the fact that $\zeta_t \geq 0$ and that the event window is typically short, i.e., E is finite so that a CLT does not apply. We use instead nonparametrically estimated critical values. Suppose that $\zeta_t - 1$ is a stationary mixing process with marginal distribution F that is unknown and let F_{e_τ} denote the marginal distribution of the stationary mixing series $\{e_{r,\tau}\}$, where $e_{r,\tau} = \sum_{s=0}^{\tau} (\zeta_{r+s} - 1)$. We estimate the distributions F and F_{e_τ} based on the data not including the event window, $S = \{1, \dots, T\} \setminus \{t_1 - E, \dots, t_1 + E\}$. Specifically, letting $\widehat{e}_{r,\tau} = \sum_{s=0}^{\tau} (\widehat{\zeta}_{r+s} - 1)$, we define

$$\widehat{F}_{\widehat{e}_\tau}(x) = \frac{1}{T_S} \sum_{t \in S} 1(\widehat{e}_{t,\tau} \leq x), \quad x \in \mathbb{R},$$

where T_S is the cardinality of the set S , and $\widehat{F}(x) = \widehat{F}_{\widehat{e}_0}(x)$. Then define the critical values $\widehat{F}_{\widehat{e}_\tau}^{-1}(\alpha/2)$, $\widehat{F}_{\widehat{e}_\tau}^{-1}(1 - \alpha/2)$. We reject the null hypothesis (for a given $\tau = 0, \dots, 2E$) if $\widehat{CAIL}(\tau)$ is outside the interval $[\widehat{F}_{\widehat{e}_\tau}^{-1}(\alpha/2), \widehat{F}_{\widehat{e}_\tau}^{-1}(1 - \alpha/2)]$. There is a large literature about estimation of distribution functions of residuals in time series settings, see for example [Koul and Ling \(2006\)](#) and [Escanciano \(2010\)](#).

THEOREM 3. *Suppose that conditions A1-A4 hold. Then, as $T \rightarrow \infty$, $\widehat{F}_{\widehat{e}_\tau}^{-1}(\alpha/2) \xrightarrow{P} F_{e_\tau}^{-1}(\alpha/2)$ and $\widehat{F}_{\widehat{e}_\tau}^{-1}(1 - \alpha/2) \xrightarrow{P} F_{e_\tau}^{-1}(1 - \alpha/2)$, and therefore the rejection frequency of our test converges to α under the null hypothesis.*

We may average across events (same firm different events or across firms) and obtain a CLT when the number of events being averaged across is large. Provided the timing of the stock splits across firms does not coincide very much, the standard errors can be based on

the “as if independence” assumption. Specifically, let

$$\widehat{AAIL}(\tau) = \sum_{i=1}^N w_i \widehat{AIL}_{i\tau}, \quad \widehat{ACAIL}(\tau) = \sum_{i=1}^N w_i \widehat{CAIL}_i(\tau) \quad (13)$$

denote the averaged abnormal and cumulative abnormal illiquidity across N events. For large N and T , these statistics are asymptotically normal and compared with critical values $\pm z_{\alpha/2} \sqrt{\sum_{i=1}^N w_i^2 \widehat{\sigma}_{\zeta_i}^2}$ and $\pm z_{\alpha/2} \sqrt{(\tau + 1) \sum_{i=1}^N w_i^2 \widehat{\sigma}_{\zeta_i}^2}$, respectively, where $\widehat{\sigma}_{\zeta_i}^2$ is the estimated variance of the corresponding ζ for the particular firm event. In the supplementary material we discuss event window based standard errors and corrections for the multiple testing issue. Our tests are applied using the jump consistent estimate of the trend and so the null hypothesis here includes the possibility of a permanent change to liquidity at the specified points. We should expect to see that adjustments, if any, should be relatively quick if markets are efficient and so there should not be much of a role for slow dynamic responses.

4 Empirical study: stock splits

In our application, we use the proposed framework to analyze whether stock splits have permanent and temporary effects on the illiquidity process. We consider historical daily price and volume data for component stocks of the S&P 500 (large caps), S&P 400 (mid caps) and S&P 600 (small caps) indices.⁹ The sample period starts from each asset’s first available data point (after June 15, 1992) until December 31, 2023. We plot in Figure 1 the number of splits by year. Stock splits happen more often during periods of strong market performance. For example, numerous splits took place during the build-up phase of the dot-com bubble between 1992 and 2000, and a pronounced drop in the occurrence of splits can be observed across most stock indices in the aftermath of its collapse. Likewise, there were hardly any splits during the Global Financial Crisis. This empirical evidence suggests

⁹The price and volume data as well as the stock split information are retrieved from the CRSP database.

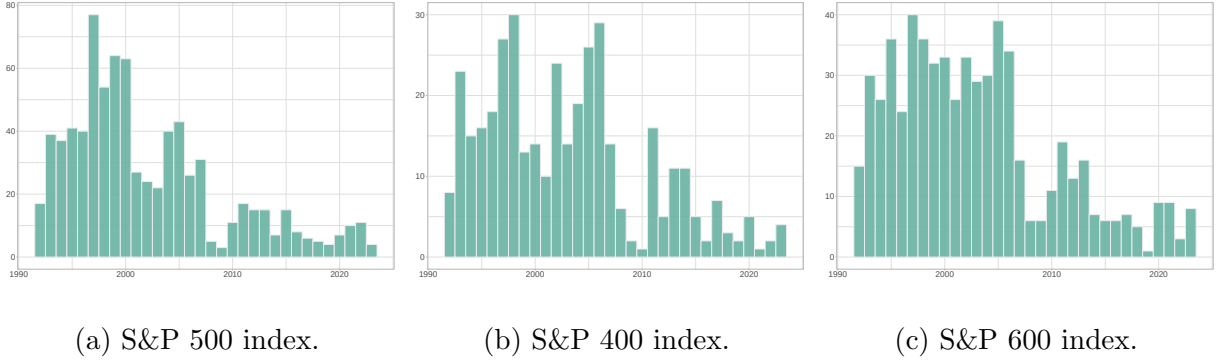


Figure 1: Number of stock splits per year. Panels (a), (b), and (c) respectively show the yearly count of stock splits among the S&P 500, S&P 400, and S&P 600 index constituents from 1992 to 2023.

that periods of high price appreciation could be one of the factors motivating firms' decision to split their stocks, in line with [Fama et al. \(1969\)](#).

Table 2: Distribution of different split sizes by index (1992-2023).

Split Size	<1	1 – 1.25	1.25 – 1.5	1.5	2	3	4	>4	Total
S&P 500	17	2	14	167	518	42	11	13	788
S&P 400	18	2	47	94	201	14	6	1	383
S&P 600	61	8	47	204	263	17	4	1	611

We summarize in [Table 2](#) the frequency of different split sizes. The vast majority of the splits in the sample are two-for-one or lower, and larger stock splits occur less frequently. We focus on the most common split factor in our empirical study, i.e. the two-to-one stock splits, leaving us with 519 distinct index constituents and 943 splits for the analysis below.

Before presenting the estimation results, [Figure 2](#) summarizes the estimation and testing procedure used to assess the permanent and temporary effects of stock splits on illiquidity.

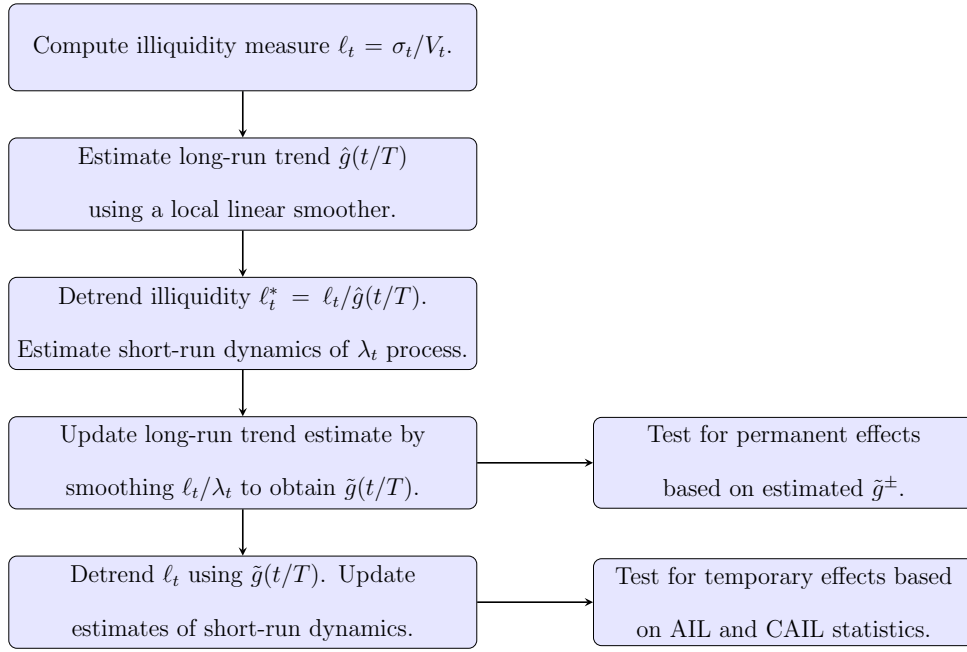


Figure 2: Summary of the estimation and testing procedure.

4.1 Test for permanent effects

We assess whether a permanent shift in illiquidity occurs at the time of the i^{th} split for stock j , denoted as u_i^j . This is achieved by testing for potential discontinuities in the long-run trend function g at u_i^j . More specifically, we estimate $g^\pm(u_i^j)$ using the local linear approach and construct the test statistics $\tau(u_i^j)$ as described in Section 3.3. To facilitate the computation of the variance of the two estimates $\hat{g}^\pm(u_i^j)$, we work with the improved estimator obtained by smoothing out ℓ_t , i.e. $\ell_t/\hat{\lambda}_t$. We first look at the size of the jump in percentage terms for each split as defined in (2), i.e. $\widetilde{\mathcal{J}}_\%(u_i^j)$. Panel (a) of Figure 3 presents the distribution of $\widetilde{\mathcal{J}}_\%(u_i^j)$ obtained using the undersmoothing method. The results show that majority (85.58%) of the distribution lies in the positive domain, indicating significant increases in the long-run illiquidity trend level and a corresponding negative effect of stock splits on long-run liquidity conditions. Additionally, we find that 82.18% of the splits yield a positive $\widetilde{\mathcal{J}}_\%(u_i^j)$ statistic when we consider the bias correction method.

We compute the test statistic $\tau(u_i^j)$ using the undersmoothing (τ^{US}), bias correction

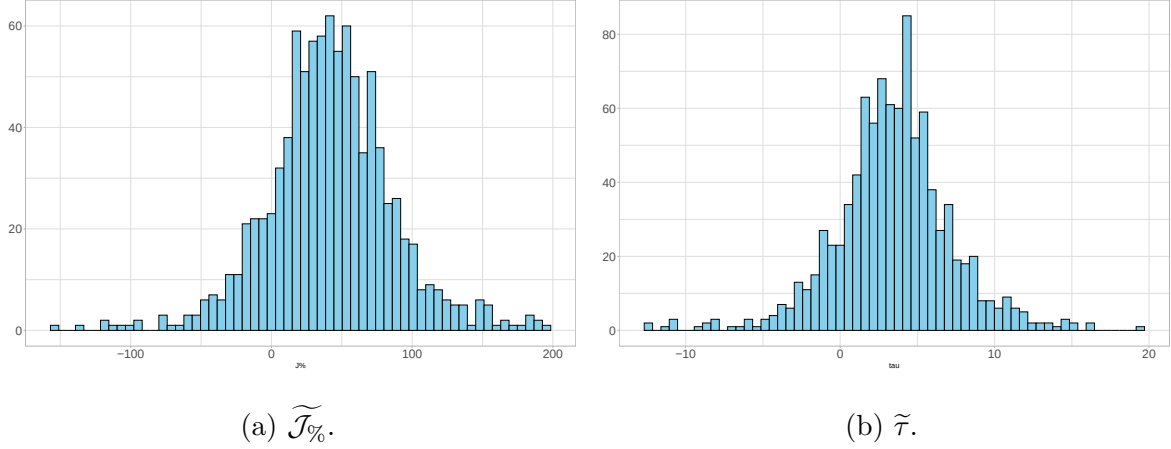


Figure 3: Distribution of permanent illiquidity jump statistics across all the stock splits in the sample. Panel (a) shows the distribution of the estimated percentage change in the long-run illiquidity trend $\widetilde{\mathcal{J}}_{\%}$. Panel (b) plots the corresponding test statistics $\widetilde{\tau}$ from the undersmoothing approach.

(τ^{BC}) and honest confidence interval (τ^{HCI}) approaches developed in Section 3.3.¹⁰ Panel (b) of Figure 3 presents the distribution of $\tau(u_i^j)$ obtained with the undersmoothing method. Majority (68.29%) of the test statistics are positive and exceed 1.96, suggesting a significant deterioration in liquidity conditions following stock splits. This result is robust across different inference approaches, with respectively 66.81% and 59.60% of the test statistics being significant when one uses the bias correction and honest confidence interval approaches.

The timing of stock splits might sometimes coincide with other market events affecting all stocks simultaneously, for example, the Global Financial Crisis. To control for these significant events, we use the stock market index ETF (SPY) as a control group to eliminate common trends at the event date and isolate the effects of stock splits from other

¹⁰In the undersmoothing and bias correction cases, the τ_w statistics should be compared to standard normal critical values (± 1.96 for $\alpha = 5\%$). For the honest confidence interval approach, we should compare the τ_w statistics with the critical values from the corresponding folded normal distribution obtained based on the worst-case-scenario bias.

market-wide events. We report in Table 3 the difference-in-difference (DID) test statistics computed as in (9). We observe that 80% of the test statistics are positive and around 55% are significantly so. This confirms that stock splits negatively impact long-run liquidity conditions even when accounting for potential confounding effects from concurrent market-wide events. We note that using SPY as a control may not be sufficient to account for events that happened over a longer period and impacted different stocks in a staggered fashion, such as the introduction of decimalization between mid-1997 and April 2001. However, Figure 11 in the supplementary material shows no clear shift in the distribution of the τ statistics before and after decimalization. We thus leave a systematic treatment of this type of event for future research.

Table 3: Summary of difference-in-difference test statistics (τ^{DID}) for stock splits.

	Min	25%	Median	75%	Max	$\tilde{\mathcal{J}}_{\%w}/\tau_w > 0(\%)$	significant (%)
τ_w^{DID}	-13.49	0.39	2.29	4.36	15.68	79.74%	54.85%

Note: The “significant (%)” column shows the percentage of positive and significant τ_w statistics.

4.1.1 Aggregating multiple splits by firm

We first look at the average jump size in percentage terms aggregated across all splits for each stock. The values of the statistics are summarized in the second and third rows of Table 4. The first row provides the number of splits.¹¹ We observe that stock splits increase the illiquidity trend level by 41% in the median case. The interquartile range suggests a high degree of variability around this central tendency.

We aggregate the individual test statistics $\tau(u_i^j)$ for each company j to provide a joint test of the null hypothesis of no breaks at any point of the illiquidity series for this stock.

¹¹Our analysis includes a total of 519 stocks: 257 stocks with one two-to-one split, 154 stocks with two, 71 stocks with three, and 37 stocks with more than three splits within the sample period.

Table 4: Summary of average jumps and joint test results for stock splits aggregated by firm.

	Min	25%	Median	75%	Max	$\tilde{\mathcal{J}}_{\%w}/\tau_w > 0(\%)$	significant (%)
# of splits	1	1	2	2	6		
$\tilde{\mathcal{J}}_{\%w}^{US}$	-152.48%	19.54%	41.31%	63.07%	185.16%	88.25%	
$\tilde{\mathcal{J}}_{\%w}^{BC}$	-187.29%	15.88%	38.25%	63.49%	195.32%	84.20%	
p_W^{US}	0.00	0.00	0.00	0.00	0.93		84.78%
p_W^{BC}	0.00	0.00	0.00	0.03	1.00		75.92%
τ_w^{US}	-12.47	1.99	4.56	7.17	20.95	87.86%	75.72%
τ_w^{BC}	-19.09	1.72	5.02	8.01	19.04	84.01%	73.41%
τ_w^{HCI}	-19.58	1.89	5.18	8.10	19.11	85.74%	59.34%

Note: The statistics are aggregated by firm. $\tilde{\mathcal{J}}_{\%w}$ is the average jump in percentage as defined in (2). p_W is the p-value of the aggregated statistic $W = \sum_{i=1}^m \tau(u_i)^2$. τ_w is the statistic computed as in (11) and is asymptotically $N(0, 1)$ under the null hypothesis. US stands for undersmoothing and BC stands for bias correction. HCI stands for honest confidence interval where we compare the τ_w statistics with $cv_{1-\alpha}^{HCI}$. The “significant (%)” column shows the percentage of positive and significant τ_w statistics.

The fourth and fifth rows of Table 4 report the p-values for the statistic $W^j = \sum_{i=1}^m \tau(u_i^j)^2$ for the undersmoothing and bias correction approaches. In the undersmoothing case, the p-values associated with the W^j statistics are below 5% for 85% of the stocks considered in our analysis. When considering the bias correction approach, the p-values suggest that the effects are significant for 76% of the stocks. These results indicate strong evidence against the null hypothesis of no long-term effect on liquidity from the stock split events.

The last three rows of Table 4 provide the directional average statistics (τ_w) defined as in (11) and computed using an equal weighting scheme. Each row reports the summary results using the undersmoothing, bias correction and honest confidence interval inference approaches. Most τ_w statistics are positive – 88% for undersmoothing, 84% and 86% for bias correction and honest confidence interval. In addition, 76% of the statistics are positive and significant, with a median value of 4.56 for τ_w . This indicates a significant difference in pre- and post-split long-run illiquidity trends on average. For the honest confidence interval method, we observe a lower percentage of significant statistics but a majority of the companies still experience a significant increase in the illiquidity trend level (59%).

4.1.2 Alternative illiquidity measures

This section evaluates the robustness of our main results to the use of the alternative illiquidity measures introduced in Section 2.1. The results are summarized in Table 5 and confirm that our main empirical findings remain robust across the different measures considered. The only exception is the LY implied bid-ask spread measure in the last row, which yields notably different results.

Table 5: Summary of the aggregated directional statistics τ_w for alternative illiquidity measures.

ℓ_t	Min	25%	Median	75%	Max	$\tau_w > 0(\%)$	significant (%)
$\frac{\sigma_t}{V_t}$	-12.47	1.99	4.56	7.17	20.95	87.86%	75.72%
$\frac{ R_t^{CC} }{V_t}$	-8.27	3.91	5.89	7.60	13.17	95.18%	90.17%
$\frac{ R_t^{OC} }{V_t}$	-13.12	0.88	2.78	4.52	12.98	84.01%	61.08%
$\frac{\sigma_t}{\sqrt{V_t}}$	-9.13	2.57	5.41	8.06	18.81	88.82%	77.65%
$(\frac{\sigma_t^2}{V_t})^{\frac{1}{3}}$	-12.63	2.66	5.60	8.48	19.11	88.05%	79.19%
$\frac{\sigma_t}{V_t} P_{t-1}^M$	-19.56	1.57	4.78	7.57	20.27	85.16%	72.64%
$\frac{\sigma_t}{V_t} \text{mcap}_t$	-10.92	2.31	5.15	7.86	17.85	90.75%	78.23%
$\frac{\sigma_t}{V_t} P_t$	-15.84	-6.90	-3.63	-1.32	7.40	13.68%	68.40%

Note: R^{CC} is close-to-close return while R^{OC} is the open-to-close intra-day return. P^M is the price of the market index. The “significant (%)” column shows the percentage of positive and significant τ_w statistics for the first seven measures and the percentage of negative and significant τ_w statistics for the last measure.

This diverging result may stem from the presence of the nominal price level in the LY measure, which changes by the split size between the pre- and post-split dates. This feature may introduce a mechanical downward effect in the tests. Let P^+ (ℓ^+) and P^- (ℓ^-) be the price (illiquidity) right after and before a split so that $P^+ = \varsigma P^-$, where ς is (one over) the split factor. The percentage effect of a split at u_i on the implied percentage bid-ask spread, can be measured by $2(L\ell^+P^+ - L\ell^-P^-) / (L\ell^+P^+ + L\ell^-P^-)$. Assuming that the lot size L does not change over the split period, we replace $\ell^\pm$ by the corresponding permanent component $g^\pm$ and simplify the expression which yield an alternative LY measure of the

permanent effect of a split at u_i on liquidity within our framework:¹²

$$\mathcal{J}_{\%i}^{LY} = 2 \frac{\varsigma_i g^+(u_i) - g^-(u_i)}{\varsigma_i g^+(u_i) + g^-(u_i)}.$$

For forward splits, $\varsigma_i < 1$ and so $\mathcal{J}_{\%i}^{LY}$ could be negative when $\mathcal{J}_{\%i}$ is positive - a split could worsen the Amihud illiquidity but improve the LY implied percentage bid-ask spread.

4.2 Test for temporary effects

We plot in Figure 4 the aggregated test statistics for the temporary effects across all splits for all considered firms as defined in (13). The blue line represents the Aggregated Cumulative Abnormal Illiquidity (ACAIL) test statistics at horizons ranging from 45 days before to 45 days after the event, i.e. $\widehat{ACAIL}(\tau)$, $\tau = 0, \dots, 90$. The red lines are the 2.5% and 97.5% quantiles, i.e. $z_{\alpha/2} \sqrt{\tau \sum_{i=1}^N w_i^2 \widehat{\sigma}_{\zeta_i}^2}$. We observe a mild increase in short-term illiquidity on average during the event window but there is no supporting evidence for a statistically significant temporary effect of stock splits.

Figures 12 and 13 in the supplementary material provide the event-specific results for a selection of stocks. The directions of the temporary effects are mixed, with most firms exhibiting insignificant increases and decreases in liquidity across all split events. These unclear patterns are consistent with the mixed evidence in previous literature on the effect of stock splits on market liquidity, with some authors reporting liquidity improvements following a split (Lamoureux and Poon (1987)) and others documenting a reduction in liquidity or muted effect in the post-split period (Lakonishok and Lev (1987)). We observe a few occurrences of short-lived significant increases in cumulative abnormal illiquidity during the event window.¹³ This is in line with prediction from signaling theory of Brennan and Copeland (1988) who model a firm's decision to split its stock as a costly signal about its prospects, which is associated with a (at least) temporary decrease in the stock liquidity.

¹²If the round lot size changes in the same proportion, the effect cancels out and $\mathcal{J}_{\%i}^{LY} = \mathcal{J}_{\%i}$.

¹³See e.g. the 1993 split of Cisco Systems Inc. and Netflix.

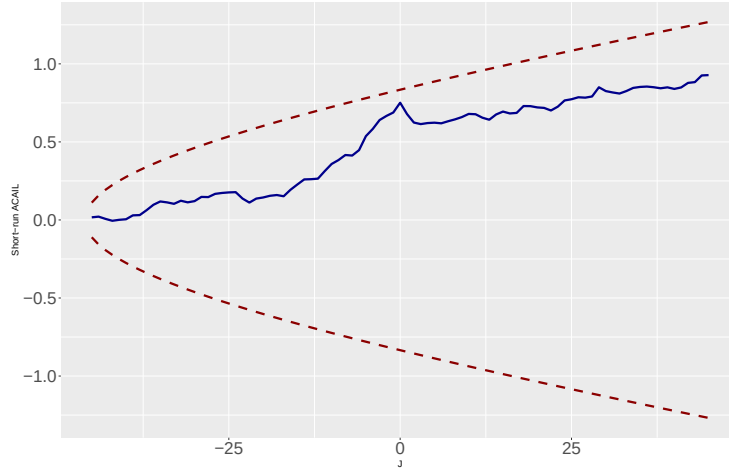


Figure 4: Test for temporary effects of stock splits. The blue line is the aggregated cumulative abnormal illiquidity (ACAIL) statistic and the red lines are the corresponding 2.5% and 97.5% quantiles.

To summarize, our empirical evidence suggests that stock splits have an overall significant permanent effect on the long-run trend level of illiquidity but very limited effects on the short-run illiquidity dynamics. The documented increase in stocks' long-run illiquidity following a split challenges somewhat the predictions from the optimal price range and optimal tick size ([Angel \(1997\)](#)) theories that splitting firms should experience an increase in the liquidity of their stock in the long run. See also [Goyenko et al. \(2006\)](#) for an in-depth discussion of the short- and long-run liquidity effects of stock splits.

5 Reverse stock splits

We investigate whether the effects of reverse stock splits on illiquidity are symmetric to the ones documented for stock splits in [Sections 4.1 and 4.2](#). The analysis in the literature shows that the reverse splits have a more pronounced improvement in liquidity conditions for low-price stocks. This could be because reverse splits ease the constraints on short selling for low-priced stocks, see e.g. [Kwan et al. \(2015\)](#). Therefore, in this part of the analysis, we focus our attention on the constituents of the S&P 500, S&P 400 and S&P

600 indices with a pre-event price level below \$5. In total, we have 53 stocks with only one reverse split for each stock in our sample.

Table 6: Directional test statistics τ_w for reverse stock splits.

	AAON	ACLS	AIG	ARWR	ASRT	BANR	BCOR	BKNG	C	CAR	CBB
Split size	1-4	1-4	1-20	1-10	1-4	1-7	1-10	1-6	1-10	1-10	1-5
τ_w	-2.32	-1.73	-27.12	-10.19	5.00	-3.81	-6.16	-13.98	8.71	-7.19	-0.33
$\tau_{w,SC}$	1.46	-2.23	-14.89	-8.89	1.96	5.17	-6.17	-5.02	4.73	-9.64	-0.73
	CCOI	CIEN	CPE	CPF	CSII	CYTK	DXPE	EHC	EPAC	EXPR	FBP
Split size	1-20	1-7	1-10	1-20	1-10	1-6	1-2	1-5	1-5	1-20	1-15
τ_w	-4.55	-13.91	-1.30	-9.24	-8.43	-26.35	-3.44	-2.06	-7.92	-0.33	-5.92
$\tau_{w,SC}$	-3.89	-0.19	-1.03	-4.64	-6.08	-3.80	-4.58	-1.44	-3.70	-0.50	-2.48
	FTR	HAFC	HPR	HSKA	KEM	KLXE	LCI	LPI	MSTR	MTH	NEU
Split size	1-15	1-8	1-50	1-10	1-3	1-5	1-4	1-20	1-10	1-3	1-5
τ_w	-1.51	-5.64	-5.21	-6.08	-8.16	1.57	-2.44	-7.48	-12.31	-0.52	-3.49
$\tau_{w,SC}$	-1.94	-6.31	-3.04	-6.02	-7.97	1.32	-1.72	-4.26	-6.90	1.29	-4.92
	ODP	OPCH	PFBC	PPBI	RRC	SANM	SBCF	SNV	SPPI	SRPT	SSP
Split size	1-10	1-4	1-5	1-5	1-15	1-6	1-5	1-7	1-25	1-6	1-3
τ_w	-3.00	1.54	-5.81	-1.65	-1.18	-9.55	-2.66	-3.99	6.95	-15.69	13.08
$\tau_{w,SC}$	-2.43	3.14	-3.54	-1.83	-1.29	-5.69	-4.30	-2.65	7.98	-11.83	10.69
	THRM	TISI	UCBI	UFI	UIS	UVE	VIAV	WLL	XPO		
Split size	1-5	1-10	1-5	1-3	1-10	1-4	1-8	1-75	1-4		
τ_w	0.96	-2.38	0.96	-7.32	-19.60	-15.88	-1.84	-5.31	0.22		
$\tau_{w,SC}$	0.93	0.21	-1.29	-3.99	-10.42	-7.50	0.90	-2.78	0.30		

Note: SC stands for synthetic control.

We estimate the $g^\pm(u_0)$ functions using the local linear approach and construct the test statistics $\tau(u_0)$ for the permanent effect introduced in Section 3.3. The test statistics $\tau(u^j)$, using an undersmoothing approach, are reported in the second row of Table 6. We observe for 44 out of 53 stocks that there is a negative effect of the reverse splits on the illiquidity trend – corresponding to an improvement in liquidity conditions. Among those, there are 35 stocks for which the decrease in the illiquidity trend after a reverse split is statistically

significant.¹⁴ The fact that reverse splits result in a significant decrease in the illiquidity trend for the majority of the stocks considered is in line with the results in Han (1995). The pronounced improvement in liquidity conditions for our sample of stocks with low pre-event price levels is consistent with the evidence in Blau et al. (2023) that short-selling activity increases after reverse splits.

We also consider the robustness of our results to the inclusion of a control group to account for other events that can potentially impact all stocks at the event time. We use the synthetic control approach of Abadie (2021). For each reverse stock split event, we use the constituents from the same index that did not undergo a reverse split as the donor pool for the control group. We use the pre-event trend to construct the weights, i.e. $\min_w \sum (g^S - w'g^C)^2$ subject to $w \geq 0$, and $1'w = 1$. The superscripts “S” and “C” refer to the “treatment” stock and “control” group respectively. The test statistics are reported in the third row of Table 6. Our conclusion that reverse stock splits significantly decrease the illiquidity trend level is robust to controlling for common changes in stock trends around the event time via the synthetic control approach.

We plot in Figure 5 the average test statistics for the temporary effects of the reverse splits aggregated across all firms. The results echo our analysis in Section 4.2 for stock splits, with limited evidence supporting a significant effect of reverse splits on short-term illiquidity. However, we do observe a short-lived worsening in liquidity conditions around the execution date although it is not significant. Figures 16 to 17 in the supplementary material provide the event-specific results for a selection of stocks. A few stocks exhibit significant changes in cumulative abnormal illiquidity during the event window and, as in

¹⁴We also consider the robustness of our results to the use of the bias-corrected and the honest confidence interval approaches. Using bias correction, 42 out of 53 stocks experienced improved liquidity conditions after a reverse split, and it is statistically significant for 35 of them. When adopting the honest confidence interval, 42 out of 53 stocks experienced improved liquidity conditions with 32 being statistically significant.

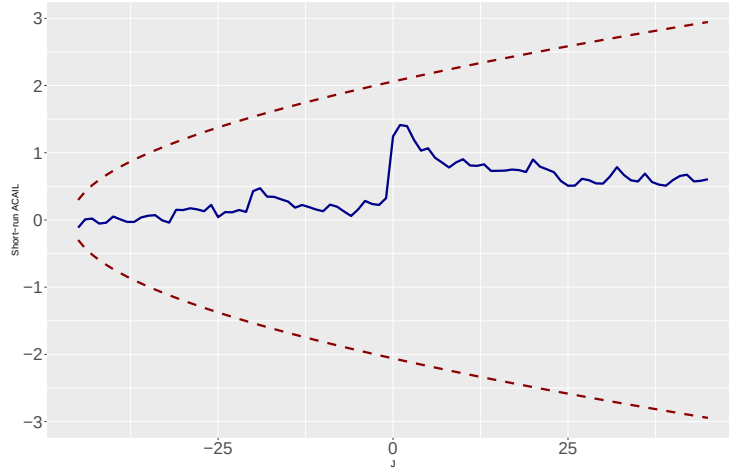


Figure 5: Test for temporary effects of reverse stock splits. The blue line is the aggregated cumulative abnormal illiquidity (ACAIL) statistic and the red lines are the corresponding 2.5% and 97.5% quantiles.

the stock split case, no clear pattern emerges for the impact of reverse splits on short-term illiquidity. We note however that the worsening in liquidity conditions observed for some stocks before the execution date is consistent with reverse splits signaling a lack of confidence from executives in the prospects of their firm (Han (1995)).¹⁵

To summarize, our empirical evidence suggests that reverse stock splits have an overall significant permanent effect on the long-run trend level of illiquidity but limited effects on the short-run illiquidity dynamics. The documented decrease in stocks' long-run illiquidity following a reverse split is the symmetric image of our results in Section 4.1 for stock splits and confirms earlier evidence in the literature such as Han (1995).

6 Conclusions

We have developed a new methodology that can be used to conduct event studies about stock market liquidity based on low frequency data. We showed that according to our approach stock splits appear to reduce permanently the aspect of stock liquidity we con-

¹⁵See e.g. HighPoint Resources Corp. and Avis Budget Group Inc.

sider. There are several extensions that may be worthwhile. First, we can extend the methodology to the case where the break points are unknown, which would require some new theoretical results. Second, we may try to reconcile our empirical results with those derived from high frequency measures and those targeting other aspects of liquidity.

Acknowledgements

We are grateful for helpful comments and discussions with seminar participants at the Tinbergen Institute, the Janeway Institute Cambridge, the German Econometric Association, the University of Sheffield, the Oxford-Man Institute and the participants at the 2024 SoFiE conference, the Asian and European summer meetings of the Econometric Society and the SoFiE summer school at Northwestern University. We thank two referees and the Editor for excellent feedback that helped us improve the paper substantially. The first author gratefully acknowledges financial support by the Belgian Federal Science Policy (contract ARC 18/23-089). The second and third authors acknowledge financial support from the Janeway Institute.

References

- ABADIE, A. (2021): “Using synthetic controls: Feasibility, data requirements, and methodological aspects”. *Journal of Economic Literature* 59.2, pp. 391–425.
- ACHARYA, V. V. AND PEDERSEN, L. H. (2005): “Asset pricing with liquidity risk”. *Journal of Financial Economics* 77.2, pp. 375–410.
- AMIHUD, Y. (2002): “Illiquidity and stock returns: Cross-section and time-series effects”. *Journal of Financial Markets* 5.1, pp. 31–56.
- ANGEL, J. J. (1997): “Tick size, share prices, and stock splits”. *The Journal of Finance* 52.2, pp. 655–681.

- ANGRIST, J. D. AND PISCHKE, J.-S. (2009): *Mostly harmless econometrics: An empiricist's companion*. Princeton University Press.
- ARMSTRONG, T. B. AND KOLESÁR, M. (2020): “Simple and honest confidence intervals in nonparametric regression”. *Quantitative Economics* 11.1, pp. 1–39.
- BARARDEHI, Y. H., BERNHARDT, D., RUCHTI, T. G. AND WEIDENMIER, M. (2021): “The night and day of Amihud’s (2002) liquidity measure”. *The Review of Asset Pricing Studies* 11.2, pp. 269–308.
- BEDDINGTON, J., FURSE, C., BOND, P., CLIFF, D., GOODHART, C., HOUSTOUN, K. AND LINTON, O. (2013): *Foresight: the future of computer trading in financial markets*. Tech. rep. Accessed: 2025-01-10. The Government Office for Science.
- BLAU, B. M., COX, J. S., GRIFFITH, T. G. AND VOGES, R. (2023): “Daily short selling around reverse stock splits”. *Journal of Financial Markets*, p. 100832.
- BRENNAN, M. J. AND COPELAND, T. E. (1988): “Stock splits, stock prices, and transaction costs”. *Journal of Financial Economics* 22.1, pp. 83–101.
- CALONICO, S., CATTANEO, M. D. AND TITIUNIK, R. (2014): “Robust nonparametric confidence intervals for regression-discontinuity designs”. *Econometrica* 82.6, pp. 2295–2326.
- CATTANEO, M. D. AND TITIUNIK, R. (2022): “Regression discontinuity designs”. *Annual Review of Economics* 14, pp. 821–851.
- CHEN, X., LINTON, O. B. AND VAN KEILEGOM, I. (2003): “Estimation of semiparametric models when the criterion function is not smooth”. *Econometrica* 71.5, pp. 1591–1608.
- CIPOLLINI, F. AND GALLO, G. M. (2022): “Multiplicative Error Models: 20 years on”. *Econometrics and Statistics*, forthcoming.
- COPELAND, T. E. (1979): “Liquidity changes following stock splits”. *The Journal of Finance* 34.1, pp. 115–141.
- DANYLIV, O., BLAND, B. AND NICHOLASS, D. (2014): “Convenient Liquidity Measure for Financial Markets”. Available at SSRN <https://ssrn.com/abstract=2385914>.
- DELGADO, M. A. AND HIDALGO, J. (2000): “Nonparametric inference on structural breaks”. *Journal of Econometrics* 96.1, pp. 113–144.
- ENGLE, R. (2002): “New frontiers for ARCH models”. *Journal of Applied Econometrics* 17.5, pp. 425–446.

- ESCANCIANO, J. C. (2010): “Asymptotic distribution-free diagnostic tests for heteroskedastic time series models”. *Econometric Theory* 26.3, pp. 744–773.
- FAMA, E. F., FISHER, L., JENSEN, M. C. AND ROLL, R. (1969): “The adjustment of stock prices to new information”. *International Economic Review* 10.1, pp. 1–21.
- FAN, J AND GIJBELS, I. (1996): *Local polynomial modelling and its applications*. Chapman and Hall.
- FONG, K. Y., HOLDEN, C. W. AND TOBEK, O. (2018): “Are volatility over volume liquidity proxies useful for global or US research?” *Kelley School of Business Research Paper* 17-49.
- FONG, K. Y., HOLDEN, C. W. AND TRZCINKA, C. A. (2017): “What are the best liquidity proxies for global research?” *Review of Finance* 21.4, pp. 1355–1401.
- FRANCISCO-FERNÁNDEZ, M., VILAR-FERNÁNDEZ, J. M. AND VILAR-FERNÁNDEZ, J. A. (2003): “On the uniform strong consistency of local polynomial regression under dependence conditions”. *Communications in Statistics-Theory and Methods* 32.12, pp. 2415–2440.
- GOYENKO, R., HOLDEN, C. W. AND UKHOV, A. (2006): “Do stock splits improve liquidity?” *EFA 2006 Zurich Meetings Paper*.
- GOYENKO, R. Y., HOLDEN, C. W. AND TRZCINKA, C. A. (2009): “Do liquidity measures measure liquidity?” *Journal of Financial Economics* 92.2, pp. 153–181.
- HAFNER, C. M., LINTON, O. B. AND WANG, L. (2023): “Dynamic autoregressive liquidity (DArLiQ)”. *Journal of Business & Economic Statistics* in press, pp. 1–26.
- HAN, K. C. (1995): “The effects of reverse splits on the liquidity of the stock”. *Journal of Financial and Quantitative Analysis* 30.1, pp. 159–169.
- HASBROUCK, J. (2006): *Empirical market microstructure: The institutions, economics, and econometrics of securities trading*. Oxford University Press.
- HIDALGO, J. AND SEO, M. H. (2013): “Testing for structural stability in the whole sample”. *Journal of Econometrics* 175.2, pp. 84–93.
- HUI, B. AND HEUBEL, B. (1984): “Comparative liquidity advantages among major US stock markets”. *DRI Financial Information Group Study Series* 84081.
- HÄRDLE, W. AND LINTON, O. (1994): “Chapter 38 Applied nonparametric methods”. Vol. 4. *Handbook of Econometrics*. Elsevier, pp. 2295–2339.

- IMBENS, G. W. AND LEMIEUX, T. (2008): “Regression discontinuity designs: A guide to practice”. *Journal of Econometrics* 142.2, pp. 615–635.
- INOUE, A., JIN, L. AND PELLETIER, D. (2021): “Local-linear estimation of time-varying-parameter GARCH models and associated risk measures”. *Journal of Financial Econometrics* 19.1, pp. 202–234.
- KOUL, H. L. AND LING, S. (2006): “Fitting an error distribution in some heteroscedastic time series models”. *The Annals of Statistics* 34.2, pp. 994–1012.
- KRISTENSEN, D. (2012): “Non-parametric detection and estimation of structural change”. *The Econometrics Journal* 15.3, pp. 420–461.
- KWAN, A., MASULIS, R. AND MCINISH, T. H. (2015): “Trading rules, competition for order flow and market fragmentation”. *Journal of Financial Economics* 115.2, pp. 330–348.
- KYLE, A. S. (1985): “Continuous auctions and insider trading”. *Econometrica: Journal of the Econometric Society*, pp. 1315–1335.
- KYLE, A. S. AND OBIZHAIEVA, A. A. (2016): “Market microstructure invariance: Empirical hypotheses”. *Econometrica* 84.4, pp. 1345–1404.
- LACAVA, D., RANALDO, A. AND MAGISTRIS, P. SANTUCCI DE (2023): “Realized Illiquidity”. *Swiss Finance Institute Research Paper* 22-90.
- LAKONISHOK, J. AND LEV, B. (1987): “Stock splits and stock dividends: Why, who, and when”. *The Journal of Finance* 42.4, pp. 913–932.
- LAMOUREUX, C. G. AND POON, P. (1987): “The Market Reaction to Stock Splits”. *The Journal of Finance*, pp. 1347–1370.
- LESMOND, D. A., OGDEN, J. P. AND TRZCINKA, C. A. (1999): “A new estimate of transaction costs”. *The Review of Financial Studies* 12.5, pp. 1113–1141.
- LI, S. AND YE, M. (2023): “Discrete prices, discrete quantities, and the optimal price of a stock”. *R&R at Journal of Finance*.
- LI, Z. M. AND LINTON, O. (2022): “A ReMeDI for microstructure noise”. *Econometrica* 90.1, pp. 367–389.
- MULLER, H.-G. (1992): “Change-points in nonparametric regression analysis”. *The Annals of Statistics* 20.2, pp. 737–761.
- O’HARA, M. (1995): *Market microstructure theory*. John Wiley & Sons.

- PERRON, P. (1989): “The great crash, the oil price shock, and the unit root hypothesis”. *Econometrica* 57.6, pp. 1361–1401.
- ROBINSON, P. M. (1989): *Nonparametric estimation of time-varying parameters*. Springer.
- SCHUCANY, W. R. AND SOMMERS, J. P. (1977): “Improvement of Kernel Type Density Estimators”. *Journal of the American Statistical Association* 72.358, pp. 420–423.
- VOGT, M. (2015): “Testing for structural change in time-varying nonparametric regression models”. *Econometric Theory* 31.4, pp. 811–859.
- WELD, W. C., MICHAELY, R., THALER, R. H. AND BENARTZI, S. (2009): “The nominal share price puzzle”. *Journal of Economic Perspectives* 23.2, pp. 121–142.