NOVEL PARAMETRIC-EFFECT MEMS AMPLIFIERS/TRANSDUCERS

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Abstract - A parametric effect amplifier has been built at 200 kHz (input) and 1.84 MHz (output) using a MEMS time-varying capacitor. The capacitor is composed of a thin low stress metallized silicon-nitride diaphragm and is pumped by a large signal voltage at 1.64 MHz. This results in a large change in the capacitance, and parametric amplification of an input signal at 200 kHz. To our knowledge, this device is the first-ever mechanical up-converter parametric-effect amplifier, with an up-conversion ratio of 9:1.

I. INTRODUCTION

Parametric amplifiers, based on non-linear reactances such as capacitors and inductors, have been extensively used in the 1960's for amplification, frequency up- and down-conversion and oscillators at microwave frequencies [1], [2]. Briefly, parametric devices transfer the power from the pump frequency (ω_p) to the signal frequency (ω_s) or to the up-conversion frequency (ω_u), thereby resulting in gain. Parametric amplifiers were commonly built using varactor diodes for microwave applications. Since parametric amplifiers are not resistive based, they do not suffer from the Johnson and *1/f* noise. With the introduction of MESFETs presenting excellent gain and noise characteristics at microwave frequencies, parametric amplifiers were abandoned and to our knowledge, they are not use anymore in any microwave applications. This last decade, the parametric amplification has been used mostly in optical systems [3], [4]. In this paper, we present the interest of a novel MEMS parametric-effect for providing gain at the transducer level.

II. PARAMETRIC AMPLIFICATION THEORY

In 1956, Manley and Rowe derived a pair of equations governing the power balance of passive and lossless one-port devices with an arbitrary nonlinear characteristics [5]. As an example of such a device we may think of a deal varactor diode, of a Josephson junction or of a variable plate capacitor. If such a device forms a part of a parametric circuit is simultaneously fed by a signal source at frequency f_s and a pump source at frequency f_p. Due to its non-linear characteristics, time varying currents and voltages are generated, in principle, at all combination frequencies: $f_{n,m} = n\, f_s + m\, f_p$ with $n,m = -\infty\ldots\ \infty$. In practice, only a single combination frequency (f_2) is kept besides the signal frequency f_s and the pump frequency f_p, the other combination frequencies are filtered. The two most simple choices for f_2 are: *frequency-noninverting case:* $f_2 = f_u = f_p + f_s$ and *frequency-inverting case*: $f_2 = f_l = f_p - f_s$. Manley and Rowe have demonstrated that the available power gain for both cases can be expressed respectively by:

$$G_u = \frac{P_u}{P_s} = -\frac{f_p + f_s}{f_s} \quad (1) \text{ and } \quad G_l = \frac{P_l}{P_s} = -\frac{f_p - f_s}{f_s} \quad (2)$$

Relations (1)-(2) show that a parametric amplifier fed by an input signal f_s and pumped at a frequency f_p can present a large gain if $f_p >> f_s$.
Considering the time-varying capacitance as a linear four-pole, Rowe [6] has calculated the gain, the matching conditions, the bandwidth and the sensitivity for the inverting and noninverting cases. Because the inverting down- and up-converters are potentially unstable and the inverting negative resistance amplifier (degenerate case: $f_p = 2f_s$) requires an isolator (circulator) for operation since the source and the load are at the same port, the noninverting up-converter has been preferred as demonstrator for our study. In fact, as described by Rowe, the noninverting up-converter (signal frequencies f_s and $f_u = f_p + f_s$) is stable device and yields maximum gain with matched source and load. Under matched conditions the power gain is equal to the ratio of output to input frequencies (1); for widely separated signal frequencies the up-converter has substantial gain. Only a relatively small amount of nonlinearity is required to attain a bandwidth equal to the low-signal frequency. Fig. 1 represents the scheme of the parametric up-converter which is analyzed and built using the latest MEMS technologies.
Since we consider ideal input and output filters, all other sidebands are terminated with an open-circuit. In Fig. 1 *C(t)* is the time-varying capacitance, R_c is the parasitic series resistance of the time-varying capacitor, R_G and R_L the source and the load resistances, respectively, and, $R_s + j\omega L_s$ and $R_u + j\omega L_u$ are, respectively, the terminal impedances added for matching purposes. The circuit model of Fig. 1 can be analyzed as a linear four-terminal scheme. The transducer gain is defined as the ratio of the actual power output ($|I_L|^2.R_L$) to the available

power input ($|V_G|^2/4R_G$) and is calculated by the standard linear circuit techniques.

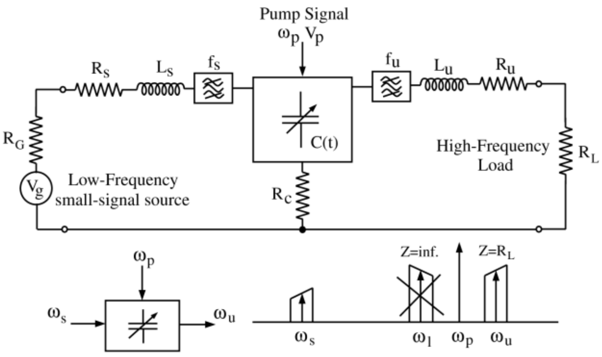

Figure 1. *Equivalent model and amplitude spectra for a generic parametric up-converter.*

Blackwell and Kotzebue in [7] have solved the transducer gain corresponding to equivalent circuit presented in Fig. 1.

$$G_t = \frac{4R_G R_L |Z_{21}|^2}{|(Z_{11}+Z_{T1})(Z_{22}+Z_{T2}) - Z_{12}Z_{21}|^2} \quad (3)$$

with Z_{ij} the two-port Z parameters of the time-varying capacitor $C(t)$, Z_{T1} = total external circuit impedance at f_s: $Z_{T1} = X_s + R_s + R_G + R_c$ and Z_{T2} = total external circuit impedance at f_u: $Z_{T2} = X_u + R_u + R_L + R_c$. Neglecting the losses of the matching inductances L_s and L_u ($R_s = R_u = 0$), the maximum transducer gain and bandwidth are reached under the matched conditions defined as follows:

$$R_G = R_L = R_c \sqrt{1 + \frac{\gamma^2}{\omega_s \omega_u C^2 R_c^{\,2}}} \quad (4)$$

$$X_s = \frac{1}{\omega_s C} \quad (5)$$

$$X_u = \frac{1}{\omega_u C} \quad (6)$$

with $\gamma = \gamma_1\left(1+\gamma_1^{\,2}\right)\left(1-\gamma_2\right)$ and $C = C_0\left(1 - 2\gamma_1^{\,2}\right)$

And it is given by:

$$G_t = \frac{f_P + f_s}{f_s} \frac{x}{\left[1+\sqrt{1+x}\right]^2} \quad (7)$$

where $x = (f_p + f_s)/f_s\ (\gamma Q)^2$ with Q the effective quality factor of the time-varying capacitor, $Q = 1/(\omega_s C R_c)$. It is interesting to note that if we consider a lossless time-varying capacitor ($Q \to \infty$, i.e. $R_c = 0$), the transducer gain tends to the ratio $(f_p + f_s)/f_s$ which is in accordance with the power gain defined from the Manley and Rowe relationship (1) for the noninverting case.

III. MEMS PARAMETRIC UP-CONVERTER

The parametric effect results from a time-varying capacitor which consists of a metallized membrane (top electrode) suspended above a heavily doped silicon bulk (bottom electrode). By pumping the movable top electrode with a large signal voltage at the pump frequency (ω_p), one generates a time-varying capacitance:

$$C(t) = C_0\left(1 + \sum_{n=1}^{\infty} 2\gamma_n \cos(n\omega_p t + \varphi_n)\right) \quad (8)$$

The fabrication process is summarized in Fig. 2 and more details can be found in [8].

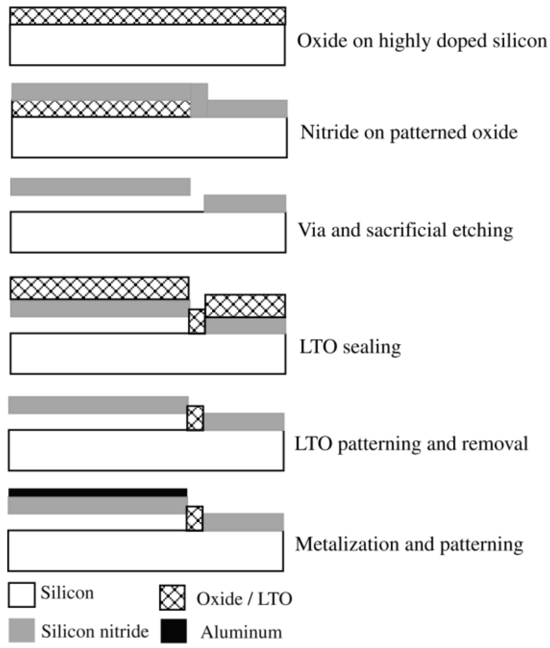

Figure 2. *Fabrication steps of the MEMS capacitor array.*

A. Static Analysis

The external load in that case is an electrostatic force F_e induced by the source voltage V_{dc} applied between the two electrodes of the MEMS capacitor. Fig. 3 shows the capacitance variation of the MEMS capacitor versus dc bias conditions. The suspended nitride membrane is 0.6 µm thick, 50 µm radius and with a residual stress of 170 MPa. The total device area is 1 cm^2. The calculated collapse potential for the measured structure is 85 V. It is important to work under that limit dc potential to avoid the collapse of the membrane.

B. Small-Signal Analysis

In the linear case, we assume that the microstructure displacements (x) are small compared to the physical gap dimensions of the structure (g_0). With other words, if the applied external force has a sine wave form, the movement of the mass will also be sinusoidal at the same frequency (linear system). The linear analysis is useful for determining the mechanical resonant frequency and quality factor of the movable structure versus frequency.

The device resonance can be detected by sensing the device impedance. In fact, by analogy the spring-mass-damper system can be represented by the equivalent small-signal electrical circuit in Fig 4.

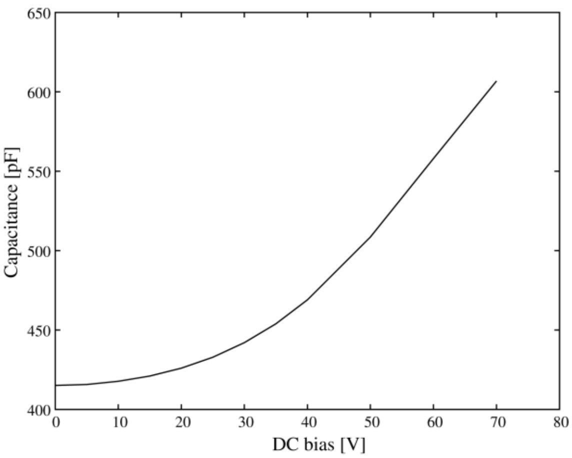

Figure 3. *Measured capacitance variation versus dc voltage.*

In this scheme R_c represents the conductor losses in the capacitor plate. The capacitor current is expressed as $\hat{i}_{ac} = (Y_0 + Y_x)\hat{v}_{ac}$ where

$$Y_0 = j\omega C_0 \tag{9}$$

$$Y_x = \frac{j\omega C_x}{1 - \omega^2 / \omega_0{}^2 + j\omega C_x R_x} \tag{10}$$

From the measurement of its electrical transfer function, the mechanical properties of the resonant MEMS capacitor such as the quality factor *Q* and the resonant frequency are extracted. Under a dc bias of 60 V, the capacitor membrane has a resonant frequency of 1.64 MHz and a *Q* of 18. From the plot of the real (Fig. 5) and imaginary (Fig. 6) parts of the parameter Y_{12}, the series parasitic resistance R_c, the parallel capacitance C_0 and the equivalent electrical lumped elements of the resonator L_x, C_x and R_x are extracted (see Figs. 5 and 6).

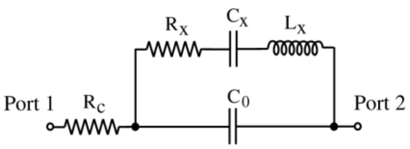

Figure 4. *Small-signal electrical model of the MEMS capacitor.*

C. Large Signal Analysis

If a large pump signal voltage is applied, the displacements of the suspended diaphragm become non-negligible compared to the default gap (g_0), then non-linear behaviors appear and a time-varying capacitance *C(t)* is generated. In order to minimize the magnitude of the pump signal voltage and then the power consumption, the pump frequency f_p is chosen equal to the nature resonant frequency of the capacitor plate. More the displacements are large more the values of the Fourier series coefficients γ_n are important. Their extraction is an essential step for designing a parametric amplifier/up-converter. Function *C(t)* can be extracted by the measurement of the voltage *v(t)* across the time-varying capacitor and the current *i(t)* flowing through it with a digital scope as explained below.

The current *i(t)* is obtained by measuring the voltage across a external resistor R_{ext} connected in series with the time-varying capacitor. Knowing the value of the parasitic resistor R_c (extracted from the small-signal analysis) the voltage across *C(t)* is given by:

$$v(t) = v_t(t) - i(t)(R_{ext} + R_c) \tag{11}$$

Then the time-varying capacitance can be expressed as:

$$C(t) = \frac{\int i(t)dt + C_0 V_{dc}}{v(t)} \tag{12}$$

Finally, considering the extracted numerical function *C(t)* as a periodic time function, it can be decomposed into Fourier series for determining the coefficients γ_n and φ_n. Fig. 7 shows the measured harmonics level at the 50 Ω input of the spectrum analyzer for the MEMS time-varying capacitance pumped with an ac large signal $v(t) = 60 + 15 \cos(\omega_p t)$ with ω_p = 1.64 MHz. From the measured characteristic *C(t)* shown in Fig. 8, the extracted coefficients γ_n and φ_n are: $\gamma_1 = 0.22$, $\gamma_2 = 0.05$, $\gamma_3 = 0.01$, $\varphi_1 = 7°$, $\varphi_2 = 3°$ and $\varphi_3 = 0.85°$.

From the measured characteristics (ω_0, Q, γ_n, φ_n, etc.) of the time-varying capacitor, the input and output matching networks and filters are designed to maximize the gain of the built MEMS parametric up-converter. Fig. 9 shows the measured and simulated transducer gain of the parametric up-converter versus the load resistance. A good agreement is obtained between measurements and simulations. It is interesting to note that the optimum transducer gain achieves effectively the available power gain G_u=1.84 MHz/200 kHz = 9.6 dB when R_c tends to zero. The measured optimum load of 140 Ω is in accordance with the value (133 Ω) calculated with the analytical relationship (4). The transducer gain has been measured as a function of the coefficient γ_1 which depends on the pump signal magnitude. For these measurements, the matching networks and input/output filters has been optimized for γ_1 = 0.22. As predicted by the simulations, the transducer gain of the MEMS parametric up-converter increases with γ_1.

IV. CONCLUSION

A parametric effect amplifier has been built at 200 kHz (input) and 1.84 MHz (output) using a MEMS time-varying capacitor. It can be built for sensors (thermal, acceleration, chemical, etc.) without any CMOS electronics. Its main advantages are the absence of any resistive and *1/f* noise, and most importantly, it provides gain at the transducer level. Some advantages over CMOS based electronic amplifiers are the possibility to operate at very high temperatures (200-600°C), under high particle bombardment (nuclear applications), and very low noise operation due to the absence of thermal and shot noise in parametric amplifiers. This means that a pre-amplification stage can be provided for non CMOS sensors before sending the signal to a CMOS wafer for more gain and processing.

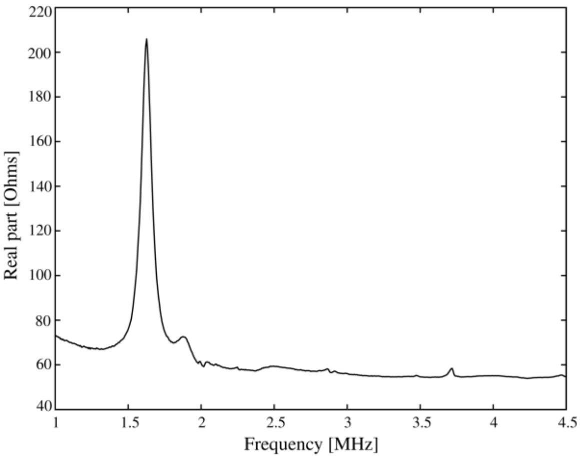

Figure 5. *Measured real part of* $1/Y_{12}$ *versus frequency. Extracted value of* R_c *is 52* Ω.

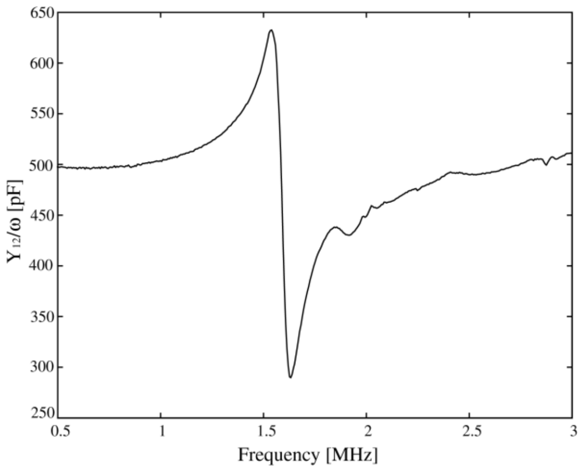

Figure 6. *Measured equivalent capacitance (*Y_{12}/ω*) around the resonant frequency. Extracted values of* C_0, C_x, L_x *and* R_x *are, respectively, 500 pF, 32.5 pF, 0.29 mH and 166* Ω.

V. REFERENCES

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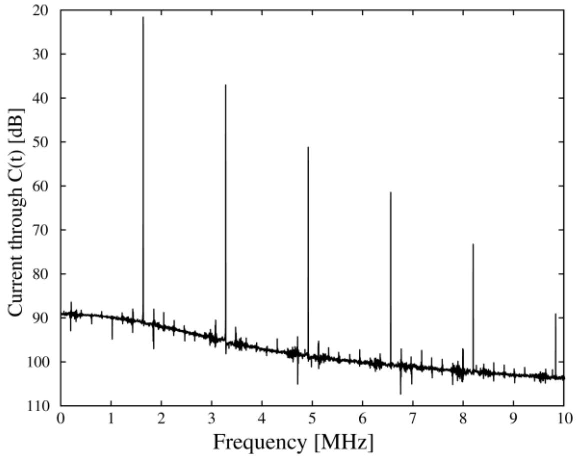

Figure 7. *Measured spectrum of the current flowing through the MEMS time-varying capacitor for* V_{dc} *= 60 V and* v_{ac} *=15 cos(*$2\pi f_p$ *t), with* f_p *= 1.64 MHz.*

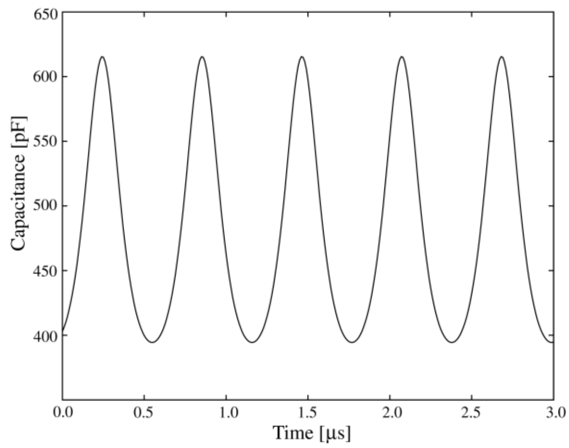

Figure 8. *Extracted C(t) function versus time.*

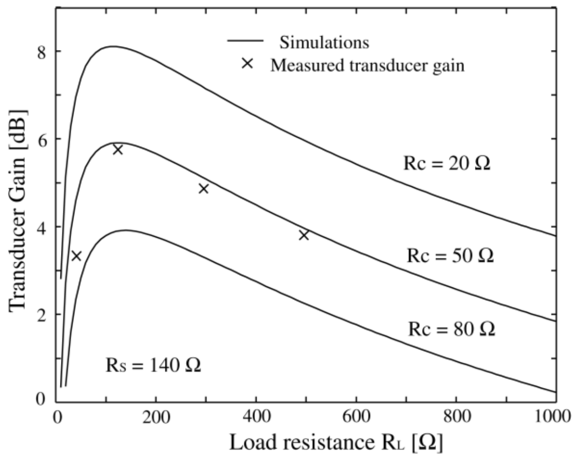

Figure 9. *Measured parametric up-converter transducer gain versus the load resistance value (*f_s *= 200 kHz and* f_p *= 1.64 MHz).*