

Joint optimization of vehicle dynamics and mechanical
design through a multi-body based strategy

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“L’union fait la force.”

“Eendracht maakt macht.”

“Einigkeit macht stark.”

“Strength lies in unity.”

Belgian motto

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Abstract

In a context of technical, ideological, social and political upheavals, the mobility is, today more than ever, deeply questioned. The automotive industry must constantly reinvent itself and, to do so, develops both innovative technologies and the design methods able to optimize them and to integrate them into new solutions. Thus, considering ever more numerous and diversified aspects, the design of vehicles responding to current and future challenges is an ever more ambitious task.

This thesis aims at proposing a multidisciplinary and integrated design approach, specifically addressed to the conception of road vehicles. Precisely, the strategy consists of the synthesis of automobile suspensions, the optimization of their kinematics in order to obtain the best road performances, while taking the mechanical constraints of design and manufacturing into account.

An algorithm combining several numerical processes based on the multi-body formalism allows, first, to satisfy the constraints to which the vehicle is subjected, then to test this vehicle through a hybrid simulator in which transient simulation and steady-state analysis are juxtaposed, and finally, to evaluate it according to the whole of selected criteria. The method enables obtaining judicious trade-off between the exhaustiveness of the tests and the criteria and the computational time and, thus, handling the design process in its wholeness, in a consistent and efficient way.

This strategy has been successfully applied to the design of a new concept of electric unipersonal three-wheeler dedicated to urban mobility: the Wemoov, characterized by its narrowness imposing its tilting in curve. This particular dynamics has been made possible by the development of an innovative front axle.

Résumé

Dans un contexte de bouleversements techniques, idéologiques, sociaux et politiques, la mobilité est, aujourd’hui plus que jamais, profondément remise en question. L’industrie automobile doit sans cesse se réinventer et, pour se faire, développe autant les technologies innovantes que les méthodes de conception capables de les optimiser et de les intégrer au sein de nouvelles solutions. Ainsi, considérant des facteurs toujours plus nombreux et diversifiés, la conception de véhicules répondant aux défis actuels et futurs est une tâche toujours plus ardue.

L’objectif de la thèse est de proposer une démarche de conception multidisciplinaire et intégrée, spécifiquement dédié à la création de véhicules routiers. Précisément, la stratégie consiste en la synthèse de suspensions automobiles, l’optimisation de leur cinématique en vue d’obtenir les meilleures performances routières, tout en tenant compte des contraintes mécaniques de conception et de fabrication.

Un algorithme combinant plusieurs méthodes numériques sur la base du formalisme multi-corps permet, d’une part, de satisfaire les contraintes auxquelles le véhicule est soumis, d’ensuite tester ce véhicule au travers d’un simulateur hybride dans lequel analyse transitoire et calcul des états stationnaires se juxtaposent, et finalement d’évaluer celui-ci selon l’ensemble des critères retenus. La méthode permet d’obtenir de judicieux compromis entre l’exhaustivité des tests et des critères et le temps de calcul et, ainsi, d’appréhender de façon cohérente et efficace la démarche de conception dans sa globalité.

Cette stratégie a été appliquée avec succès à la conception d’un nouveau concept de tricycle électrique, unipersonnel et dédié la mobilité urbaine: le Wemoov, caractérisé par son étroitesse imposant son inclinaison en courbe. Cette dynamique particulière a été rendue possible par l’élaboration d’un train avant innovant.

List of abbreviations

ATC	Analytical Target Cascading. 78
BMS	Battery Management System. 199
CAD	Computer Assisted Design. 83–86, 138, 139, 145, 184, 185, 199, 206–208, 241
CASCaDE	Computer-Aided Simulation of Car, Driver and Environment. 49
CECI	Consortium des Equipements de Calcul Intensif (from French). 125
CLEVER	Compact Low Emission VEHICLE for uRban transport. 47, 56
CMA-ES	Covariance Matrix Adaptation Evolution Strategy. 73, 74, 77, 122, 124, 163, 239, 240
CPU	Central Processing Unit. 67, 68, 93, 96–98, 100, 101, 111, 124, 125, 127, 129, 143, 182, 183
DAE	Differential Algebraic Equation. 63, 64, 76
DIN	Deutsches Institut für Normung (from German). 87
DLC	Double Lane Change. 223–227, 234
DOF	Degree Of Freedom. 22, 23, 26, 29, 33, 37–39, 43, 44, 51–53, 61, 62, 64, 65, 74–76, 79, 105, 133–135, 137, 141, 152, 181, 187, 188, 190, 196, 201, 243, 245, 247
DTC	Direct Tilting Control. 44, 45, 47–54, 132, 135, 137, 159, 160, 183, 203, 230, 234, 237, 238, 243
EU	European Union. 42, 53, 131, 132
FEM	Finite Element Method. 58, 76
GA	Genetic Algorithm. 75–81
GSL	GNU Scientific Library. 66
GTC	Gyroscopic Tilting Control. 44–46, 48
GUI	Graphical User Interface. 102, 127
HITL	Human In The Loop. 198, 242, 262, 263
HTC	Hybrid Tilting Control. 45, 47, 48, 51–55, 132, 243
HTS	Hydraulic Tilting System. 38

ICE	Internal Combustion Engine. 46
iMMC	Institute of Mechanics, Materials, and Civil Engineering. 59
IMU	Inertial Measurement Unit. 137, 184
IPM	Intelligent Personal Mobility. 54
ISO	International Standardization Organization. 11, 14, 75, 87, 105, 223–226
IT	Information Technology. 58, 64
LQG	Linear Quadratic Gaussian. 75
LQR	Linear Quadratic Regulator. 75
MBS	Multi-Body System. 48, 49, 58–62, 64–67, 75–78, 80, 81, 83–86, 91–95, 98–100, 102, 105, 107, 111, 128, 129, 133, 137, 140–142, 144, 147, 194, 201, 226
MPI	Message Parsing Interface. 125, 127
NR	Newton-Raphson. 65, 67, 95, 97, 103
ODE	Ordinary Differential Equation. 64
OEM	Original Equipment Manufacturer. 201
PD	Proportional Derivative. 103, 226
PID	Proportional Integral Derivative. 103, 104, 135, 137, 155, 226
PSO	Particle Swarm Optimization. 72, 123, 124
RAM	Random Access Memory. 127
RESOLVE	Range of Electric SOLUTION for L category VEHICLE. 42, 53
RMS	Root Mean Square. 75, 80
SDL	Simple Directmedia Layer. 262
SQP	Sequential Quadratic Programming. 78
SS	Steady-state analysis. 165, 172
STC	Steering Tilting Control. 45, 48, 51, 79, 132, 137, 243, 263
TS	Transient simulation. 165
TTW	Three Tilting Wheels. 51
TVA	Tilting Vehicle Australia. 42, 49
UCLouvain	Université catholique de Louvain. 59

Mechanical design index

- 1** Front axle. 186, 192, 197, 199, 204
- 2** Body. 186, 192, 197, 203, 204
- 3** Rear axle. 186, 197, 204, 205
- 4** Top wishbone shaft. 186, 188, 189, 192
- 5** Bottom wishbone shaft. 186, 188, 190–192
- 6** Front axle trunk. 186, 188, 190, 191, 195
- 7** Trunk triangulation bar. 186–189, 194, 195
- 8** Front shock. 187, 188, 190, 191, 195, 196
- 9** Bottom wishbone. 187, 188, 190–192, 194–196
- 10** Tilting system housing. 187–190, 195, 201
- 11** Wishbone shaft reveal ajustement system. 187–192, 195
- 12** Wishbone bushing. 187, 191, 192, 194
- 13** T-pendulum. 187, 188, 190, 191, 194
- 14** Front wheel carrier. 188, 189, 193–196
- 15** Steering column. 188, 198, 199, 201
- 16** Steering rod. 188, 189, 192–195
- 17** Steering fork. 188, 189, 191, 193
- 18** Steering ball joint. 188, 189, 193, 194
- 19** Steering box. 188–194, 201
- 20** Steering universal joint. 188, 189, 193
- 21** Ball joint eccentric mounting. 189, 193, 194
- 22** Suspension ball joint. 188, 189, 192, 194–196
- 23** Steering wheel. 189, 198–201
- 24** Steering box first stage. 189, 190
- 25** Steering box second stage. 189, 190
- 26** Steering box bushing. 189, 190
- 27** Steering box closing plate. 189, 190
- 28** Steering box cover. 189, 190
- 29** Steering box input shaft. 189, 190
- 30** Steering box output shaft. 189, 190
- 31** Steering box intermediate shaft. 189, 190
- 32** Steering box end-of-stroke screw. 189, 190
- 33** Steering angle sensor. 189, 190
- 34** Steering timing pin. 190, 191
- 35** Body nose chassis. 190–192

- 36 Tulip hub. 190, 191, 195
- 37 Tilting shaft bushing. 190, 191
- 38 Tilting gear sector. 190–192
- 39 Tilting actuator. 189, 191, 195, 199
- 40 Tilting planetary gear box. 189, 191
- 41 Tilting sprocket. 191, 192
- 42 Tilting intermediate pinion. 191, 192
- 43 Tilting timing pin. 191, 192
- 44 Tilting housing closer. 189, 192, 195
- 45 Front axle elastomer bush. 189, 191, 192, 195, 199
- 46 Front wheel. 192–195, 204
- 47 Top wishbone. 189, 194–196
- 48 Front rim. 194–196
- 49 Bump stop. 195, 196
- 50 Front brake disc. 194–196
- 51 Front brake caliper. 188, 194–196
- 52 Front spindle. 188, 194–196
- 53 Motorization group. 197, 199, 200, 205
- 54 Body longeron. 198–200, 205
- 55 Battery pack. 198, 199
- 56 Battery closing frame. 198, 199
- 57 Seat. 198–200, 206
- 58 Battery management system. 199
- 59 Propulsion motor driver. 199
- 60 Propulsion motor. 199, 200, 202, 203, 206
- 61 Propulsion motor driver radiator. 199, 206
- 62 Seat slide runner. 198–200
- 63 Front bulkhead. 199
- 64 Swingarm. 200, 203, 205–207
- 65 Roll bar frame. 198–200, 203, 204, 206
- 66 Steering wheel hub. 198, 199, 201
- 67 Steering wheel shaft. 199, 201
- 68 Thrust pedal. 199–201
- 69 Brake pedal. 199–202
- 70 Brake master cylinder. 201, 202
- 71 Brake spreader bar. 201, 202
- 72 Brake spreader trunnion. 201, 202
- 73 Brake spreader rod. 201, 202
- 74 Brake sensor. 201, 202
- 75 Brake sensor arm. 201, 202
- 76 Brake sensor rod. 201, 202
- 77 Propulsion gearbox. 202, 203
- 78 Propulsion gearbox pinion. 202, 203
- 79 Propulsion gearbox output shaft. 202, 203
- 80 Propulsion gearbox cover. 202, 203
- 81 Driving pulley. 202, 203
- 82 Belt. 202, 205

- 83** Anti-tip. 203, 204
- 84** Anti-tip triangle. 203, 204
- 85** Anti-tip wheel. 203, 204
- 86** Anti-tip rod. 203, 204
- 87** Rear shock. 205–207
- 88** Swingarm V reinforcement. 205–207
- 89** Rear axle elastomer bush. 205–207
- 90** Rear wheel. 205–207
- 91** Rear wheel shaft. 205, 207
- 92** Rear wheel shaft support. 205–207
- 93** Belt tensioning screw. 205, 206
- 94** Rear rim. 205–207
- 95** Driven pulley. 205, 207
- 96** Rear brake disc. 206, 207
- 97** Rear brake caliper. 206, 207
- 98** Rear brake caliper support. 206, 207

Contents

Acknowledgements	i
Abstract	v
Résumé	vii
List of abbreviations	ix
Mechanical design index	xiii
Contents	xvii
1 Introduction	1
1.1 Context	1
1.2 Vehicle dynamics optimization	3
1.3 Wemoov project	5
2 Basics	9
2.1 Vehicle dynamics	9
2.1.1 Body general motion	11
2.1.2 Tire dynamics and modeling	12
2.1.3 Suspension kinematics	20
2.1.4 Tilting vehicles	35
2.2 Multi-body dynamics	57
2.2.1 Introduction to Robotran	59
2.2.2 Formalism	60
2.2.3 Generation of dynamics equation	63
2.2.4 Coordinate partitioning	64
2.2.5 Static equilibrium	65
2.2.6 Quasi-static equilibrium	66
2.2.7 Modal analysis	66
2.2.8 Time simulation	67
2.2.9 Conclusion about multi-body dynamics	68
2.3 Optimization	69
2.3.1 Optimization problem	69
2.3.2 Optimization algorithms	71

2.3.3	State of the art of vehicle suspension optimization . . .	74
3	Dynamics optimization methodology	83
3.1	Introduction to methodology	83
3.1.1	General approach	83
3.1.2	Optimization problem formulation	89
3.1.3	Parameter partitioning	90
3.2	General optimization scheme	92
3.3	Data conditioner	93
3.3.1	Algebraic constraints	93
3.3.2	Geometrical constraints	94
3.3.3	Modal constraints	96
3.3.4	Complete data conditioner scheme	98
3.4	Simulator	99
3.4.1	Transient behavior simulation	101
3.4.2	Steady-state cornering simulation	105
3.4.3	Complete simulator scheme	111
3.5	Evaluator	111
3.5.1	Generic criteria	112
3.5.2	Criteria clustering processing	114
3.5.3	Mix law	115
3.5.4	Objective function	117
3.6	Optimizer	121
3.6.1	Parameter normalization	122
3.6.2	CMA-ES use	122
3.6.3	Alternative optimizers	123
3.7	Global optimization scheme	124
3.7.1	Programming language	124
3.7.2	Error management	125
3.7.3	Parallel computation	125
3.7.4	Inequality constraints	128
3.7.5	Pros and cons of the optimization methodology	128
4	Application to the Wemoov	131
4.1	Specifications	131
4.2	Morphology determination	132
4.2.1	First draft	132
4.2.2	Preliminary design	133
4.2.3	MBS model	140
4.2.4	Interests as case study	144
4.3	Development strategy	144
4.4	Wemoov optimization	146
4.4.1	Problem formulation	146
4.4.2	Constraints	151
4.4.3	Simulator	154
4.4.4	Evaluator	158
4.4.5	CMA-ES parametrization	163

4.5	Optimization results	163
4.5.1	Reference selection	163
4.5.2	Optimized parameter evolution	164
4.5.3	Fitness analysis	165
4.5.4	Criterion analysis	171
4.5.5	Time performances	182
4.5.6	Conclusion about optimization	183
4.6	Mechanical implementation	184
4.6.1	General description	185
4.6.2	Front suspension	186
4.6.3	Main body	197
4.6.4	Rear suspension	205
4.6.5	Conclusion about the mechanical design	206
4.7	Semi-recovery	207
4.7.1	Impact of the mechanical design	208
4.7.2	Setup analysis	209
4.7.3	Fitness analysis	210
4.7.4	Criterion analysis	212
4.7.5	Conclusion about semi-recovery	222
4.8	Double lane change assessment	223
4.8.1	Description of the double lane change maneuver	223
4.8.2	Steering controller adaptation	225
4.8.3	Wemoov performances	226
4.8.4	Criterion analysis	227
4.8.5	Conclusion about double lane change assessment	234
5	Conclusion	235
5.1	Contributions	235
5.1.1	Dynamics optimization methodology	235
5.1.2	Wemoov three-wheeler	237
5.2	Prospects	239
5.2.1	Dynamics optimization methodology	239
5.2.2	Wemoov three-wheeler	242
5.3	General conclusion	244
	Appendices	245
A	Other common car suspension arrangements	245
B	Wemoov optimization formulation	249
B.1	Fixed parameters	249
C	Wemoov analysis complementary results	250
C.1	Optimization complementary results	250
C.2	Semi-recovery complementary results	256
D	HITL simulation	262
	Bibliography	265

Chapter 1

Introduction

1.1 Context

“I will build a car for the great multitude. It will be large enough for the family, but small enough for the individual to run and care for. It will be constructed of the best materials, by the best men to be hired, after the simplest designs that modern engineering can devise. But it will be so low in price that no man making a good salary will be unable to own one...”

Henry Ford

From the beginning of the humanity, one of the main concerns is the transport of people and goods, originally from houses to houses, then from villages to villages and finally, from countries to countries, across the continents.

The multiple innovations, as to cite only some of the most impacting ones such as, the wheel, the road, the tire, the internal combustion engine, were enabled to improve endlessly the quantity for which and the distance on which the transports can be envisaged while reducing, at the same time, its painfulness.

The present mobility is the result of a wonderful and amazing evolution of the technology. Each improvement represents a piece of solution facing the past needs. Along the history, we observe that the rate of inventions and developments have been in constant increase. However, paradoxically, the expectations about the mobility have been larger and larger. The progress harbored the expectations and the expectations harbored the progress so that the system get excited and both the progress and the expectation accelerated faster and faster.

Nowadays, the progress is still going on but if we stop a moment the frenetic advancement and review the result of thousands years of improvement, we can notice we enjoy a very high level of mobility. Indeed, there exists a large range of vehicles for all usages. All means have been developed including air, sea and ground transport. Among all of these vehicles, one is particularly versatile and, by the way, highly widespread through the world: the car. Its daily use is so practical that it is presently the first mean of transport for people. There were more than 1.5 billion of cars and assimilated vehicles in 2018. For multiple reasons, cars and other road vehicles have succeeded to overcome all the other

transportation means, at least, over short and middle distances and that, in a few more than a century only.

As too often forgotten, the development of the vehicles depends on an important dual one: the roads. Thanks to the interconnection of multiple level ways from the smallest alleys to the biggest highways, the net is so furnished that the travel between any couple of points on the map can be easily performed. Despite the car has taken a larger part of the game, each road user can find also its own place. Among all the transport nets, the road is, by far, the most open.

Like never before, the people enjoy such a freedom to travel thanks to the current road vehicles and the associated road network. However, the transportation needs have taken such an importance that the large advantages of personal mobility must be balanced with some problems, always more serious.

We can first identify some local problems, essentially due to human densification. The vehicles are mainly motorized by fuel stemmed from oil, so the combustion generates number of toxic pollutants like nitrogen oxides, sulfur oxides and carbon monoxide, as well as a variety of fine particles. Contrary to popular beliefs, these discharges are just a rather short term problem because they naturally degrade into no toxic gases. But in short term, combined to the industrial and household similar discharges, the local concentration of these toxic components in the atmosphere is presently too high, leading to numerous health problems for local population, some respiratory system diseases on the top of the list. This is a major concern in the large cities.

Another consequence of the densification is obviously the numerous traffic jams in which the city drivers lose more and more of their precious time. The reason is that the road network can not be extended and enlarged endlessly, at least no so fast as the demography combined to the increasing accessibility to cars and to other vehicles. The lack of foresight illustrated through the absence of strategic plan from the diverse authorities is also partially liable of the network saturation.

A last local problem refers to the vehicle parking. Because of their number, vehicles need a huge volume to be stored. In the densest areas, the car parks fill some entire buildings despite the rareness and the price of the space. Even worst, the road itself is more and more used as a car park, restricting, hence, the flow section and finally increasing again the problem of traffic jams.

The local problems appear as a kind of limit of the human densification. The global problems are more critical from this point of view. Indeed, it appears that the major part of transport, as for the most of the human activities is based on the oil energy. What was at the origin of the unprecedented humanity evolution remains a limited resource which peters out slowly but certainly. Moreover, the major drawback is that combustion discharges huge amount of carbon dioxide, changing nearly irrecoverably the earth climate. Today, the evidences of the climate change query clearly on the role of the mobility and, in particularity, on the use of car.

Due to urban zone densification, climate change and energy rarefaction, some facets of life habits have to be revised. The mobility does not derogate from this trend and is one of the major future challenges.

There already exist other alternatives to cars, but they are simply less efficient from the practical point of view. Some examples are the diverse public transports across. They are rather unpopular because being naturally more constraining than personal mobility but also because structurally unefficient, for too long, wrongly managed by the incompetent and too little ambitious politic authorities.

The automotive industry is also developing new solutions to cope with the increasing problem of mobility, the need for energy efficiency and the customer requirements. However, some reliable alternatives are slow in coming.

Facing this multiplication of objectives, often conflicting, it is quite unlikely that one particular solution would satisfy all customers for all their daily needs as it was with the car until now. It is no doubt allowed that the solution will come from a better modularity between diverse transport alternatives, that is, the transport mean must be used in their clever utilization, at their best potential, the local and global problems being at the center of the decision-making. So, each of them must be easily interfaced to the others but also intrinsically as efficient as possible.

One can remark that if the personal mobility, and mainly the cars, is more and more decried, but, at the same time, the owning and the use of such vehicles are, on average, still more and more rooted in the individual practice and aspiration. Does this contradiction come from a new split between several groups with different ways of thinking and live or, rather, an illustration of the global hypocrisy? At least, this highlights a deep, serious and complex problem for which the mobility is only one of the components. It is obvious we urgently need both some new mobility concepts and the advanced optimization of these concepts and of those already existing, in order to try to reconcile on one side, the human fundamental need of transport and, from the other side, the mastering of the associated impact on the human live as well as on the overall ecosystem.

1.2 Vehicle dynamics optimization

“Un’auto è soprattutto un motore.”

“A car is above all a engine.”

Enzo Ferrari

We agree that the motion generation is fundamental but this point of view is definitively too reducing. Obviously, the vehicles are several skillful assemblies of systems, each in charge of an indissociable task. The engine, as a part of the power-train, generates the motion as true as the chassis manages this motion and the brakes suppress it. Moreover, with time, the vehicles have been enriched with numerous additional systems to improve performance, efficiency, comfort and security. Although our focus is on the vehicle dynamics, that is, the management of the motion, it must be kept in mind that this concern acts inside a complex whole that can not be ignored. The key of success is to find the right trade-off between abstraction and global consideration.

Research and development are essential in the industry, particular in the case of automotive. History has shown that only the most inventive manufacturers survive and extend. It proves, if it was required, that innovation is imperative. Engineers do not lack new challenges. The vehicle dynamics does not derogate to this rule. For instance, making the most of the tire adhesion is the main concern in order to optimize the control of the vehicle trajectories.

Until recently, the vehicle dynamics optimization was mainly achieved on the basis of experimental plans. The exercise is particularly delicate: the number of parameters is high and, in particular, it is not straightforward to understand their precise action on the global dynamics, all the more, they highly interfere each other. In the same manner, the objectives are numerous, sometimes conflicting and arduous to compare each other.

Beyond the intrinsic difficulties of the vehicles dynamics optimization, the evaluation of the vehicles through the experiments, either on plant test tracks or via motor-sport championships, is extremely expensive and leads to very long development time.

Improving vehicle dynamics is always more and more harder. The context highlights some major difficulties. Because of the increase – in number and quality – of the criteria imposed to tomorrow’s and future vehicles, the industry must propose new types of morphologies, incorporate new technologies and detect a maximum of synergies between these latter. Despite all, the solutions are still more and more complex. The cost reduction objective is not sufficient to keep the vehicles as simple as in the past. We observe a larger particularization of vehicles: there are more and more types declined in more and more versions. Thus, we observe a constant increasing design task complexity while the development times are shorter than ever and the means remain limited. Indeed, the research and development budgets are huge but not endless. Anyway, the technique work forces shortage is glaring everywhere.

Hence, there is a real need for efficient general design methodologies that include, from the earliest stage of the process, a multitude of considerations among which the dynamics takes place.

It appeared that the optimization methodology can be improved by replacing the assessment of the vehicle quality done by experiments, by their evaluation based on some numerical simulations. Indeed, they are cheaper and faster to update with respect to the successive changes.

The multi-body approach is widely exploited to validate and analyze mechanical systems whose functioning requires high dynamic performances. In the field of road vehicle dynamics, it is widely used and considered as able to give correct quantitative predictions of both the global vehicle behavior and the local characteristics like internal efforts. Thanks to the capabilities of the present computers and clusters, and given the maturity of the recent numerical algorithms, an overall optimization of those dynamic systems can be carried out, producing – in a decent simulation time – more satisfactory results than classic approaches like the parametric studies, the experimental plans or the sub-system optimizations.

The present work aims at developing such a design methodology, especially dedicated to road vehicles. We propose a method which takes the dynamic

issues into account from the earliest stages. The optimization method has the particularity to enable to manage the trade-off between dynamic performances and the other considerations as, for example, the mechanical feasibility. To do so, a numerical scheme combines multiple multi-body analyses to face the vehicle dynamics optimization problem which is formulated in a suitable manner.

In the next chapter, we report on the state of the art of the three main fields involved in the development of our methodology. Firstly, in the section 2.1, the current knowledges concerning the vehicle dynamics are summarized. Secondly, the main features about the multi-body approach and the associated tools are introduced in the section 2.2. Thirdly, the relevant optimization methods are presented in section 2.3. This chapter will also be the opportunity to introduce numerous concepts, definitions and nomenclatures.

Finally, the proposed general design approach and the included optimization scheme are described in details in the chapter 3.

1.3 Wemoov project

“Light is right.”

Colin Chapman

In order to face the drawbacks of the today mobility brought out in the context, some actors develop new solutions. The personal mobility is far to be given up. However, rather than developing some new vehicles supposed to be universal, as the car was too long supposed to be, the interesting novelties settle for being credible alternatives to existing widespread vehicles for some particular uses, but with more efficiency. A multitude of new vehicles appears, each of them being able to meet a particular need. Based on the combination of these new vehicles and other transportation means, the multi-modularity will enable to find a solution to the mobility problems.

In this shifting atmosphere, a multi-company consortium decided to create, from nothing, a new concept of vehicle named Wemoov, referring, once again to the hope of a better mobility. The characteristics of the device have been selected to form a coherent and efficient vehicle.

The Wemoov concept is dedicated to dense urban areas with the consequences that the vehicle targets a moderate speed and short distance travels.

The Wemoov is unipersonal. It can seem extreme, even selfish but in fact, lots of studies about mobility show that in urban area, in rush hours, the occupation rate is only 1.1 to 1.2 person per car. Most of the vehicles are only occupied by its driver. Wemoov is thus assigned to travel alone, as for example, the home-to-work daily travels. It is obvious the social activities implying group travels require some alternatives. Without being restrictive, the Wemoov is more suitable as the second vehicle of a household than as the only one. Nonetheless, this radical choice allows to outright decrease the size and the mass of the vehicle. So, the space to store it is reduced, what represents a large advantage in cities. At the same time, the reduction of mass leads to a large energy saving because it is directly proportional to the mass in motion.

Compactness and lightness act also positively on the production cost of the vehicles, through the reduction of the required amount of material.

The Wemoov width is targeted to 85 cm. This extreme reduction of the track classifies it into the family of narrow vehicles, i.e. a class of vehicles sufficiently narrow to be highly nimble in the traffic. It permits to envisage to overcome lanes of classical vehicles in traffic jam, in the same manner than motorbikes. If it appears to be a nice advantage in the daily city travel, it must be balanced with the need to tilt, that is, to incline in the direction of the turn, to equilibrate the vehicle. For bikes and motorbikes, the tilting is said passive: the driver manages the tilt motion through the steering. On the contrary, we prefer to opt for an active tilting. Thanks to such a system, the driver simply manages the steering wheel to get the suitable trajectory while the tilting motion is automatically controlled. The concept aims to give the advantage of the tilt dynamics i.e. the reduction of the width, with the same driving convenience as a car.

As a consequence of the narrowness and the tilting motion, a three-wheeler setup was adopted. As deeply explained in this work, this morphology at midway between a motorbike and a car, offers a good trade-off between the requirement of the tilting action system and the reduction of the suspension complexity. The front suspension is the two wheel axle, this time, for stability reasons.

The Wemoov motorization is electric. In contrast with some common beliefs, this does not solve the problem of energy rarefaction and CO₂ emissions. This can be only achieved if the energetic mix i.e. the means of producing the electrical energy are themselves sustainable, what is absolutely not true, at this time, for most of the countries. Moreover, the massive mobility electrification causes some other problems, as for example the capacity limit and the difficult management of the electric network, and the environmental impacts of the extraction of precious materials involved in the electric motors and batteries, like the neodymium and the lithium.

If the energetic question is not unconditionally solved by the electric motorization, the technique enables to reduce drastically the discharge of toxic components in denser areas. It is the real main advantage of the electrification.

Let us also notice that electric motorization implies the propulsion batteries that are massive and costly components. But, because both the maximum speed, impacting the power, and the travel range, impacting the energy, are moderated due to the target urban use, the battery capacity can also be mastered. Consequently, the impact on the mass and the cost is acceptable.

Wemoov is much more than an electric active tilting motorbike. Thanks to a fully closed cabin within which the driver is seated in a normal car position, the safety and the comfort are largely improved. Additionally, for the acceptance by a larger public, the rain protection consequently obtained is a huge forward step in comparison to motorbikes. At the same time, the seated position reinforces the reassuring feeling of car driving.

Taking this whole of characteristics into account, the Wemmov appears as a good challenger to improve the overall mobility through some targeted but widespread common uses.

Reaching interesting performances correlated with the high level specifications can be only achieved following a rigorous design method. The particularities of this challenging vehicle make it a perfect case study for the optimization strategy developed in this work. This justifies we have been in charge of the design of the first demonstrator, including a functional chassis. The mechanical design and the vehicle dynamics optimization have been achieved jointly to make the most of the possible synergies.

The dynamics optimization methodology which is developed in the chapter 3, is illustrated through its application on the Wemoov concept that is treated in the chapter 4.

Chapter 2

Basics

2.1 Vehicle dynamics

As is well known, the purpose of vehicles is to carry people and goods. For a long time, the industries developed as much variants as particular needs therefore numerous types exist today. All means have been used to fulfill the requirements associated with transport. If there exist (sub)marine, ground or aerospace vehicles, our study only concerns ground vehicles and in particular the rolling vehicles. This last category is itself divided into road and rail vehicles. Although the developed approach could be easily adapted to other machine types, we focus this work on road vehicles. Thus, we will interest in vehicles such as bikes, motorbikes, cars, trucks and some civil or military vehicles. Their common point is the presence, in a variable number, of wheels equipped with tires.

The dynamics is defined as “The branch of physics which treats of the action of force: in earlier use restricted to the action of force in producing or varying motion, and thus opposed to *statics* (which treats of rest or equilibrium under the action of forces); more recently, the name *kinetics* has been introduced for the former, dynamics being taken in a more comprehensive (and more etymological) sense, to include statics and kinetics. But the earlier usage, in which statics and dynamics are treated as co-ordinate, is still retained by some physicists, and has largely influenced the popular and transferred applications of the word and its derivatives”².

Vehicle dynamics relates to the analysis of the motion of the vehicle under the action of forces applied on it. The vehicle dynamics aims to make the most of science knowledges to achieve the transport task in the most efficient way. The suspension, i.e. the liaison between the ground and the vehicle body which plays multiple roles, must be designed and set in that respect. The suspension assembly endorses functionalities as numerous as conflictual. First, it carries the vehicle and its useful load in motion while isolating them from the irregularities of the road pavement and the associated vibrations and shocks. This function which often reduces to some comfort criteria is nevertheless essential

²From the Oxford English dictionary

for the transportation of goods like brittle material. The second function of suspension relates to the determination of the direction taken by the vehicle in motion. The suspension makes the driver able to safely drive the vehicle on the desired trajectory, through the steering mechanism. The last main objectives relate to put in motion or stop the vehicle. In a classical way, the suspensions must transmit the engine torque to the wheels to accelerate while brakes system is responsible for deceleration. To do so, the key idea is that the suspension mechanism must set the tire in the appropriate configuration with respect to the ground in order it could develop forces needed to drive the vehicle. That is the basics of the vehicle dynamics. The desired motion will only be obtained by the corresponding efforts, dictated by the dynamics laws. The efforts will themselves be obtained by a correct work of tires, according to the tribological laws. Despite their outward simplicity, tires are undoubtedly the first key components with respect to vehicle dynamics. It justifies to dedicate the section 2.1.2 to the tire dynamics.

The suspensions are made of mechanical parts organized so that the wheels get some appropriated kinematics with respect to the main body. It can be achieved in many different ways depending on the specifications. The section 2.1.3 gives an overview of the suspension families, points out different key concepts that must be kept in mind for a well designed process, and introduces the specific vocabulary related to suspension components and characteristics.

As already introduced, the road vehicles aggregate significantly different variants that can be classified in various ways. Dynamically speaking, a relevant characteristic refers to the global stability with reference to the roll motion. In particular, one can distinguish vehicles depending on how they deal with the lateral centripetal acceleration occurred when it turns. The distance between the center of gravity and the ground, where the tire forces apply, leads to a torque on the body which tends to rollover the vehicle and must be counteracted to ensure the roll stability. By the simpler manner, the equilibrium can be reached by a self-induced overloading of the outer wheels and unloading of the inner wheels, creating the required balancing torque. The phenomenon appears in most of width vehicles as cars, trucks, etc. On the contrary, for zero-track vehicle as bike or motorbike, such a lateral unloading is obviously impossible. The equilibrium is reached thanks to the main body inclination, towards the center of the curve, which makes the balancing torque unnecessary. Even if it represents the same variable, the inclination angle is called the *roll* in case of cars and the *tilt motion* in case of two-wheelers¹. Unlike for the unloading phenomenon, the inclination of two-wheelers e.g. the tilting must be managed by the driver. If the evolution of ground vehicle leads to two well distinct families, recent developments aim to explore a hybrid roll strategy vehicle class whose Wemoov concept, studied in this work, is part of. The tilting vehicles and the tilting control are the subject of the section 2.1.4.

¹The term "leaning motion" is also used by some authors.

2.1.1 Body general motion

A first reminder concerns the main body motion denomination. According to the International Standardization Organization (ISO) 8855 norm, the frame associated to main body of the vehicle is defined as followed:

- x the longitudinal direction that points in the forward direction, the rolling main motion,
- y the lateral direction that points to the left side,
- z the vertical direction that is outgoing the ground.

The figure 2.1 illustrates the defined frame as well as the rotations associated to the main axes:

- ϕ the *roll* is the rotation along the longitudinal axis [rad],
- θ the *pitch* is the rotation along the lateral axis [rad],
- ψ the *yaw* is the rotation along the vertical axis [rad].

The *bounce* is the translational motion of the main body along the vertical direction. If the roll and the yaw are affected during cornering and associated transients, the pitch is mainly excited by the acceleration and braking. Also, the roll and the pitch are excited by the road irregularities. All these motions could be excited during the normal use conditions directly by road excitation or indirectly, due to the suspension coupling. The three translations and the three rotations are the core of dynamics study, especially for the comfort analysis. The decoupling of motion along the vehicle aligned frame is justified because both the car behavior and the comfort of driver and passengers are anisotropic. Looking at the acceleration level for instance, the comfort threshold is very different in each direction and in each rotation. Moreover, these variables determine the gap between elements that can enter in contact i.e. the distance between wheels and their arches, some important design constraints to keep in mind.

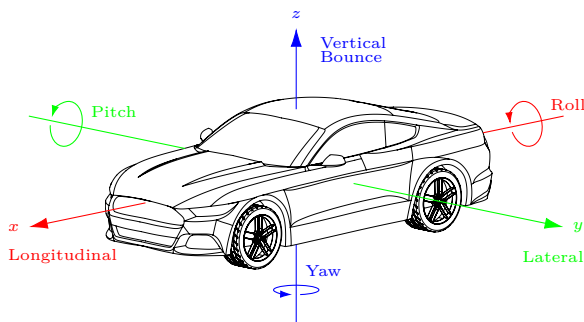


Figure 2.1: Roll, pitch, yaw and bounce motion [167].

The *attitude angle*, β , is defined as the angle between the longitudinal axis of the main body and the speed of the vehicle center of mass as shown in the figure 2.2. Here again, this variable gives information on the vehicle states and, in particular, provides a particular handling feeling to the driver.

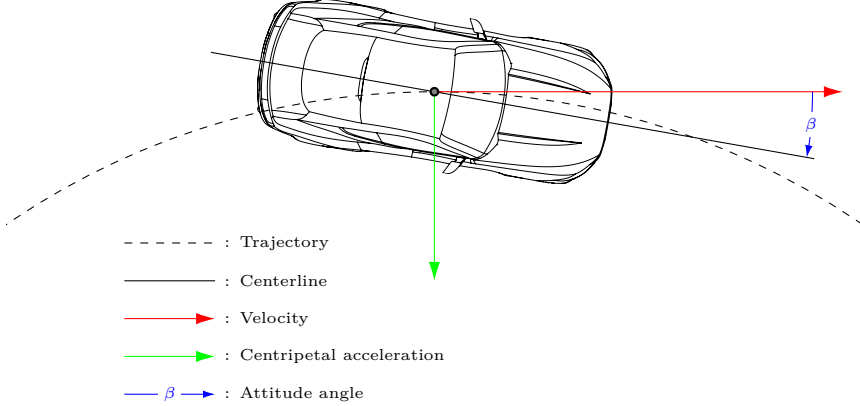


Figure 2.2: Attitude angle [167].

2.1.2 Tire dynamics and modeling

Tire functions and composition

From the wheel invention by wise men convinced by rolling efficiency instead of sliding, 3500 BC, the wheel has been in constant evolution. The historical concerns aimed to make the wheels lighter, stronger, more comfortable, adherent to the ground and trouble-free. Despite large and profitable improvements. These concerns are still current. If the first wheels were made from plain stone or wood stuck, nowadays, the best setup consists of a rubber based tire mounted on a metallic or composite rim and inflated with air, or better, by nitrogen¹. This mounting appears as the best balance between the need of compliance and adhesion².

The tire, as the sole contact with ground, carries the entire vehicle. Its construction provides the stiffness and the damping required to ensure maintaining the contact as well as possible while partially filtering the road irregularities. The tires also provide both longitudinal and lateral adhesion. As one can remark, these main functionalities recalled the overall suspension ones. To fulfill these functions, tire is made of a complex assembly of components. The figure 2.3 shows the typical structure of a standard radial type tire that nowadays equips the most of vehicles.

¹Filling the tire with nitrogen provides better thermal stability and better protection against corrosion.

²Some futuristic developments concern airless tires or spherical wheels.

The *tread* is the area in contact with the ground. The rigidity of the tread, that allows a correct contact despite huge solicitations, is obtained by multiple layer reinforcement. This surface is covered with a net of *grooves* and *sipes* to allow the evacuation of water and, in this way, to limit the risk of aquaplaning¹. The tread design is optimized following the destination of the tire: competition, performance, all-season, rain, snow, all terrain, sand, etc are regularly encountered classes of tires. Both the groove pattern and the rubber composition are adapted to the use in order to find the best trade-off between interpenetration with ground asperities, water drainage, noise generation etc. The goals are the maximization of adhesion and resistance against wear, tear, shear, impact, fatigue, thermic damage, ageing, etc. Storage aptitude enters also into account.

The *side-wall* transfers the effort from the tread to the rim. The stiffness results from a trade-off between driving comfort and adhesion. Here again, the resistance against fatigue, cut and tear are important.

The *bead* realizes the liaison between the tire and the rim and participates to the tightness of the assembly. It consists of a body ply turned around metallic *bundle* sealed into rubber that constitutes the body ply anchorage. This part aspires to avoid unseating.

The tire *armature* ensures the mechanical strength of the whole. The structure type determines the tire shape and performances. The most common structure is the radial one. The bias structure or belted bias structure were used for tilting vehicles as scooter and motorbike but are more and more replaced by the radial tires because more effective.

The rubber is made from natural or synthetic material, vulcanization agents, silicon, carbon black and metallic oxides. The material used for multiple reinforcements are polyester, nylon, aramid and steel.

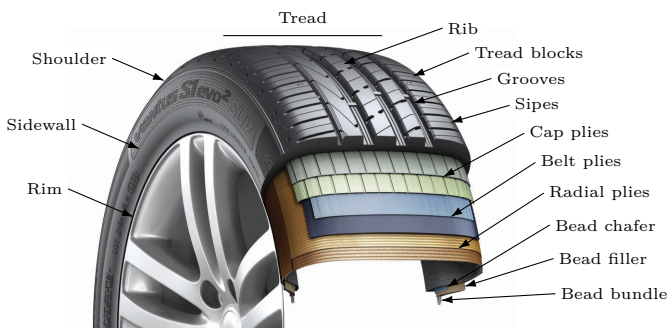


Figure 2.3: Tire structure [166].

¹Undesirable phenomenon while a film of water continuously isolates the tire from the asphalt leading to dramatic drop of adhesion.

General tire dynamics

The tire dynamics response is very complex and difficult to handle. Prof. Pacejka has spent much of his career to develop the fundamental knowledge about tire behavior [66]. An introduction to this science and the associated modeling tools are given in this section.

The dynamic response of a tire is classically formulated as a force and a torque applied on a virtual contact point centered on the actual contact patch. These force and torque depend on the kinematic characteristics of the tire with respect to the ground.

The tire response depends on its kinematic configuration with respect to the surface in contact, to the nature of the contact formed by the pair tire-road and to additional parameters. In this work, the contact with a hard and regular soil is considered. We only are interested in the tire rolling on roads made of asphalt, concrete, pavement or every else equivalent such that the soil can be considered as a rigid surface. For the targeted applications, the study of soft soil rolling is not relevant because not encountered for normal uses of concerned vehicles. Moreover, this kind of analyses requires more complex models based on simplified analytic, continuous, finite elements or granular [56] methods. These last methods have huge computational cost so an optimization scheme based on one of them is, at this time, absolutely unaffordable. In the same manner, it is assumed that the soil is sufficiently regular such that multiple contacts are avoided.

Throughout this work, the motorbike type tires, as used for the studied three-wheeler, is considered. Instead of the classical disc model, a formulation that takes the migration of the virtual contact point along the circular cross-section into account is required for such a tire type [82]. An equivalent formulation for traditional disc model can be found in [78].

The modeling of the tire requires to defines some frame, as depicted in the figure 2.4. As for each body, a frame $\{\hat{\mathbf{W}}\}$ is rigidly associated to the wheel. The frame origin, O , is located at the intersection between the median plane of the wheel and its main axis, which corresponds to the $\hat{\mathbf{W}}_2$ unit vector (figure 2.4). In contrast, the contact force and torque are given according to a frame $\{\hat{\mathbf{R}}\}$ associated to the wheel-ground contact point, G . The normal to the ground surface, $\hat{\mathbf{n}}_g$, fixes the $\hat{\mathbf{R}}_3$ vector. The frame is completely defined by projecting and normalizing the wheel rotation axis vector, $\hat{\mathbf{W}}_2$, on the contact tangent plane that gives the $\hat{\mathbf{R}}_2$ vector. According to ISO norm the $\hat{\mathbf{R}}_1$ or *longitudinal* direction is defined by the intersection of the median plane with the ground, the $\hat{\mathbf{R}}_2$ or *lateral* direction points to the left and the $\hat{\mathbf{R}}_3$ or *normal* direction is outgoing the ground. The contact force vector is projected on the contact frame, giving the longitudinal, $\mathbf{F}_{long}$, the lateral, $\mathbf{F}_{lat}$, and normal force components, $\mathbf{F}_n$, as illustrated in the figure 2.5. Similarly, the pure torque vector is divided into tilting, rolling resistance and self-aligning torques, $\mathbf{M}_n$.

Many experiments have been achieved with tires to determine the most relevant set of kinematic variables to establish the contact force consecutive laws. From a dynamics point of view, it appeared that the problem can be partially split in normal and tangent directions.

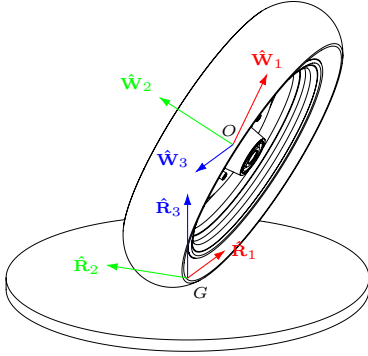


Figure 2.4: Tire frames.

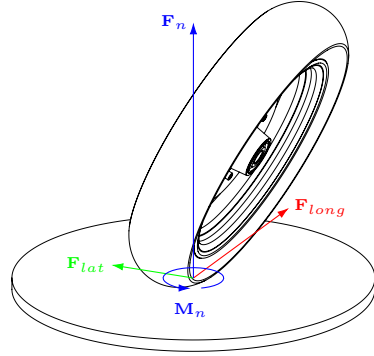


Figure 2.5: Tire forces.

Normal tire dynamics

According to the figure 2.6, the normal tire kinematics is defined by:

- e the *normal penetration* is the maximum distance between the theoretical undistorted external shape of the tire and the ground tangent plane, taken along $\hat{\mathbf{n}}_g$ [m],
- $\dot{e}$ the *normal penetration rate* [m/s],
- γ the *camber angle* between the normal, $\hat{\mathbf{n}}_g$, to the contact and the median plane of the wheel, defined as positive if the top of the wheel is inclined to the right, viewed from rear side [rad].

For a motorbike type tire, the penetration is computed according to:

$$\|OC\| = r_{rim} \quad (2.1)$$

$$\|CG\| = r_{tire} - e \quad (2.2)$$

with:

$$r_{rim} = r_{tire} - r_{tire} \quad (2.3)$$

where:

- r_{tire} the radius of the undeformed tire with respect to its main axis [m],
- r_{tire} the transverse radius of the undeformed tire [m],
- r_{rim} the radius of the rigid part of the tire model [m].

If the penetration is positive, the normal force is modeled as being proportional to the deformation and the deformation rate, both influenced by the camber angle, γ .

$$F_n = f_s(e, \gamma) + f_d(\dot{e}, \gamma) \quad (2.4)$$

where f_s and f_d are two nonlinear functions specific to each tire.

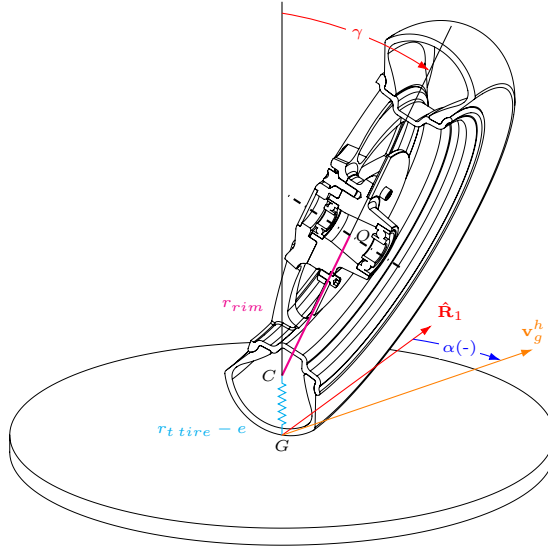


Figure 2.6: Tire kinematic variables.

For motorbike type tire, a reasonable assumption consists in considering the normal force proportional to the penetration and the penetration rate and independent from the camber. This is justified by the circular shape of the tire cross section. It leads to:

$$F_n = K e + D \dot{e} \quad (2.5)$$

where K stands for the vertical stiffness of the tire and D for the vertical damping coefficient.

It is equivalent to a simple linear spring-damper system. It acts as a first suspension stage, isolating the vehicle from ground irregularities. The nonlinearities are neglected as well as other effects as dependencies with the tire inflation pressure, temperature, rotational velocity through the centrifugal efforts, proper deformations of the tire, interpenetrations between tire grooves and ground asperities, etc.

Tangent tire dynamics

In the tangent plane (figure 2.6), the tire kinematics is defined by:

κ the *longitudinal slip*, computed as the ratio of the velocity of the material and unmaterial contact point, both projected in the longitudinal direction, $\hat{\mathbf{R}}_1$:

$$\kappa = \frac{\mathbf{v}_s \cdot \hat{\mathbf{R}}_1}{\mathbf{v}_g \cdot \hat{\mathbf{R}}_1} \quad (2.6)$$

where $\mathbf{v}_s$ stands for the velocity of the tire material contact point and $\mathbf{v}_g$ for the unmaterial one [-],

α the *slip angle* is the angle between $\hat{\mathbf{R}}_1$ and the projection of the velocity vector of the unmaterial contact point on the ground tangent plane $\{\hat{\mathbf{R}}_1, \hat{\mathbf{R}}_2\}$, noted $\mathbf{v}_g^h$ [rad] (figure 2.6).

For a given tire, the force and torque response as functions of the kinematic variables can be measured by experiments which reproduce equivalent rolling conditions. Impressive industrial facilities are required to perform such tests¹. The number of inputs and outputs involved, the interdependence of the responses, the alternation of low and high sensitivities, the large range, the influence of a lot of perturbing parameters and the natural dispersion of tire performances make these tests difficult, long and consequently expensive.

The results of such tests are given in the figures 2.7, 2.8 and 2.9.

The figure 2.7 gives the pure longitudinal response of the tire. As one can observe, for the small longitudinal slip, the longitudinal force is linear with respect to it. Then, the characteristic saturates, pass through a maximum before slowly dropping to an asymptotic plateau. This specific non-linear characteristic is not fully exploited during normal condition. Operating point should be, for obvious safety reasons, kept in the linear region. Conditions beyond the maximum refers to a loss of adhesion due to inherent instability of the system.

As for the other characteristics, the longitudinal force is quite linear with the normal force (figure 2.7). Only a small saturation with respect to the normal force appears such as regarding the polynomial reduction of the curve, the second order term is negative. This light effect involves significant changes in overall vehicle dynamics.

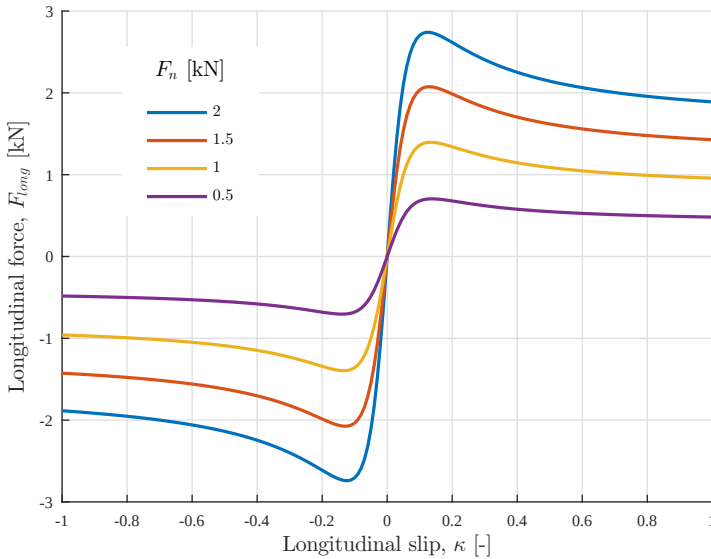


Figure 2.7: Longitudinal response of a 120/70R14 tire.

¹[66] gives a review of the different experimental setups for tire testing.

The pure lateral response is given by the figure 2.8. The curve is similar to the longitudinal one. However, it must be highlighted that the kinematic input is not the lateral slip but the slip angle. The overshoot is reduced or outright vanished, depending on other variables and parameters. One of the most important tire characteristics is the slope of lateral response at the origin, called the *cornering stiffness*. It indicates the "nervous" character of the tire. The curve saturation is the intrinsic reason of lateral acceleration limit of a vehicle. The camber acts on the curve roughly as an offset. This additional force, called *camber thrust* provides an additional contribution to the lateral slip one. Unfortunately, the camber thrust can not be exploited to increase the dynamic capabilities of tire vehicles because the magnitude of the thrust shrinks, so it can be neglected when the slip angle becomes large.

An important consideration must be brought to understand the interrelation between both longitudinal and lateral responses. In general, a solicitation in one direction involves an alteration of the characteristic in the other directions as if the total tangent force magnitude is limited without its behavior being isotropic.

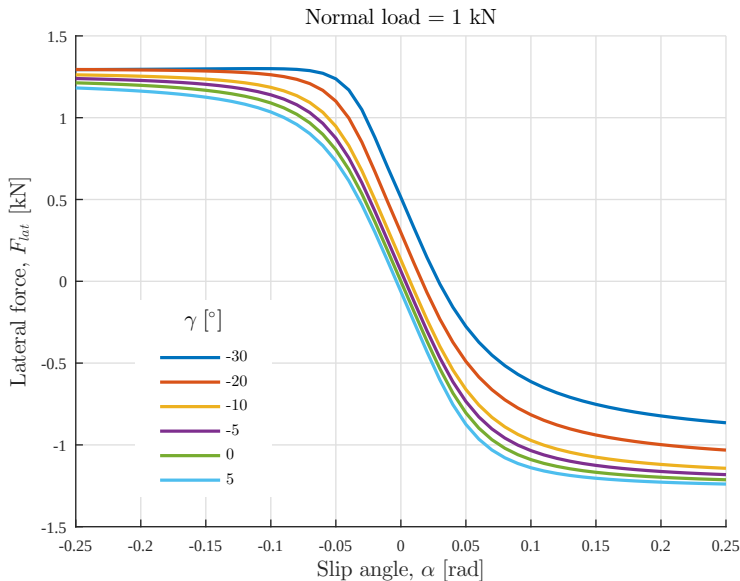


Figure 2.8: Lateral response of a 120/70R14 tire.

The last presented response concerns the self-aligning moment. As clearly expressed by its name, this torque tends to align the wheel with its direction of motion. It is an important phenomenon concerning the overall and steering stabilities. The self-aligning torque is mainly induced by a small offset between the virtual contact point, i.e. the center of the contact patch and the actual lateral force resultant, back located. As shown in the figure 2.9, as a function of the slip angle, after reaching a maximum, the torque decreases until a change of sign occurs and becomes, in this way, destabilizing. Nevertheless, it brings

a very interesting feed-back to the driver. Because the sign change appears approximately at the same level of slip angle as the maximum of lateral force, when the car reaches the maximum of cornering capability, the steering wheel becomes less responsive or "floating", indicating an approximate level of saturation to the driver. Note that the trail, treated in section 2.1.3, can produce a similar effect.

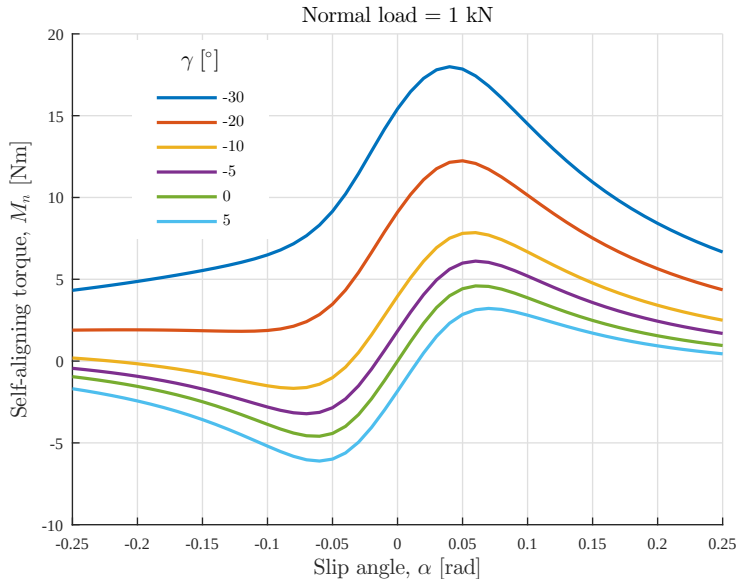


Figure 2.9: Self-aligning characteristics of a 120/70R14 tire.

Despite they could be quantified, the rolling and tilting torques are neglected in this work. Given the followed goals of our study, these hypotheses are fully justified.

It must be noticed that the tire characteristics such as presented here above, is limited to a unique pair tire/surface and a given condition setup. In general, the tire responses are affected by numerous additional parameters: inflation pressure, temperature of different concerned components, tire wear, vehicle speed and acceleration, presence of gravels, dust, water, snow or ice on the road surface, air humidity, etc. The sensitivity with respect to these parameters can be extreme and should not be neglected in practice.

The modeling of the road-tire contact can be achieved in many ways. The most known methods have been developed by prof. Pacejka [66] and consist in fitting a set of parameterized algebraic equations on the experimental measurements. If his first models are rather simple, the last ones include hundreds of equations and parameters. A model is synthesized by assembling mathematical expressions together in order to fit at best with the experimental measurement. In fact, there is no pure physical justification for the selected equations or, at most, a matching between the general curve shape and the mathematical function properties. It is the reason why the latter are called *magic formula* in

the literature. The parameters are determined through a minimization of the error between the model and the experimental results for a specific tire, in specific conditions, with the suitable facilities. According to the needs, the model involves more or less variables and is valid on specific ranges and for specific conditions.

Beside the Pacejka magic formula, there are some other models based on physics. Generally, these models give better results only by admitting very larger computational time that justifies the success of magic formula based modeling. The latter appears to be a tire model suitable for fast vehicle simulations and optimization programs while finite element based methods, for instance, are more suitable for tire development. In this work, only a magic formula based model especially developed for motorbike type tires is used [82].

Being time independent, the tire characteristics presented above refer to the steady state response. Nonetheless, most of time, the vehicle kinematics consist in transient motions. The establishing behavior is generally fast. Depending on condition, one wheel rotation is often considered sufficient to get the new steady state after a variable step. For this work, the frequency content of the tire kinematics variables is low although the transient dynamics can be neglected. If it was not the case, it could be taken into account by treating the outputs of the system through a first order transfer function with a time characteristic inversely proportional to the wheel speed and dependent on tire technical specifications [66, 82].

As one can remark, the tire is an essential and complex component of road vehicles. Its features are hard to handle and its modeling is not straightforward. The tire model formulation used in this work does not have the ambition to give the most accurate results but to be an efficient trade-off between the effect on simulation and the computational time. The deep lack of data available, due to the industrial secret which is quite sacred in the tire industry, is also a major difficulty to achieve correct tire based simulations.

2.1.3 Suspension kinematics

The suspension is the hardware between the vehicle chassis and the wheels. At first glance, it simply hold the wheels, allowing them a relative freedom in particular direction and are locked in the other, but, as it was already introduced, the suspensions are in charge of multiple and conflicting assignments: vertical comfort, lateral dynamics, including steering, and longitudinal dynamics e.g. acceleration and braking. In addition to their intrinsic dynamic performances, they aim to enhance related objectives discussed here below.

The space dedicated to suspension and suspension travel is generally large and enter in conflict with other vehicle systems, like motorization, or with passengers and loading compartments. The required space for the suspension is a central concern in overall vehicle design. In addition to the external conflicts, there are also internal ones simply because lots of functions must be satisfied in the same restricted area.

As for the entire vehicle, and moving parts in general, the mass of suspension system must be as low as possible to affect positively both the dynamic

performances and energy consumption. This goal must be achieved despite the large cyclic efforts transmitted and the consequently fatigue risks, in addition to huge shocks. Furthermore, the structure must resist against and, if possible, absorb the pervasive vibrations encountered during the traveling.

A other requested quality of suspensions is their aptitude to be set, during the design phase or to be tuned, in assembly or reassembly phases to reach optimal kinematic characteristics. Although these two notions are not exactly equivalent, some suspension classes can be easily set while for others, their motions and functions are coupled so a setting variable affects more than one objective that is not convenient for optimization or tuning purposes. The need of tuning arises from the natural manufacturing process geometrical imperfections, leading part not fully respecting the optimal setup, and wear, plasticity and other rolling and time induced damages that can also affect the kinematics. The last group must be, or should be, periodically corrected by tuning. This need is largely enforced by the extreme sensitivity of tire response.

As well, the cost must be as low as possible. The suspension system represents a substantial part of the overall vehicle cost. Reducing the number, the dimension and the complexity of parts are strategies that reduce the cost of the suspension system.

The vehicle makers have been very inventive to propose solutions to this global problem. There exist umpteen suspension arrangements offering proper trade-off between the presented concerns, giving their own advantages and drawbacks mix.

Hereinafter, the major parts involved in suspension assemblies are listed. The main functions of these components are given as well as the advantages and drawbacks of different options. Different ways of suspension system classification are proposed. After, different relevant common suspension arrangements are highlighted and discussed. Finally, the main characteristic parameters of suspensions are defined. Their effects on global dynamics and, in particular, on vehicle stability are exposed. The objective is not to be exhaustive, in fact, it is quite impossible to be, but to fix the base of discussions, to recall main dynamics phenomena, often poorly known or incorrectly understood, and to introduce the specific terminology.

Car suspension part definition

The figure 2.10 illustrates some common car suspensions and labels the main part encountered in such assemblies.

The wheel is made of a tire mounted on a steel or aluminum based alloy rim (figure 2.3). In general, the rim is bolted to the hub of the *wheel carrier*. The wheel carrier is an assembly that has the same kinematics as the wheel except the main rotation of rolling, naturally. The wheel carrier is divided into two parts, the rotating part and non-rotating one. The common interfaces are the ball bearings. The main component of the rotating part is the hub that holds the rim and the brake disc¹. On the other side, the main component of the non-rotating part of wheel carrier is the *knuckle*. This molded part is linked

¹For drum brake system, the hub and the drum is combined into a single part.

to the chassis by means of different links, support the spindle supporting the wheel bearings and hold the brake caliper. It is the classical arrangement for undriven wheel carriers¹.

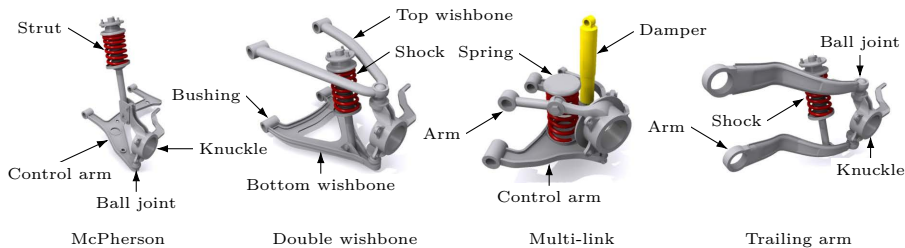


Figure 2.10: Main suspension components [132].

The wheel carrier assembly is linked to the chassis by several structural parts. These links are articulated at both the ends: chassis and carrier sides. The used joint types determine the conferring Degree Of Freedom (DOF). The joints can be rubber based bushing, ball joint, or pivot, ordered from the softest to the hardest. Rubber bushing provides six DOF but, most of time, only the three rotations have significant displacements, the three translations are affected by high stiffness and are used only to filter vibrations. For the ball joint, the stiffness and damping are reduced in the three rotational DOF while they are very stiff in the translational directions. The pivot joint provides a single DOF. Besides the lack of filtering, the main difficult with pivot joints is misalignment. The selection is achieved following the targeted DOF to confer, the related range of motion, and the stiffness and damping expected that all can differ depending on the direction. In general, prismatic joints are avoided for suspension purposes. Sometimes, the links are voluntarily flexible such their proper deformation participates itself to the suspension kinematics. The shape of the link depends on the mechanical solicitation applied and the geometrical interaction encountered during the suspension traveling. The twisted part loaded in bending are generally thick. Those submitted only in translation or compression mode, along their axis, are sleek profiled. In the first case, one speak about *arm* while, in the second, *rod* is used. A useful way to get an equivalent pivot joint without misalignment problem is the simultaneous use of two ball joints between the arm and the chassis. Their combination leaves one DOF as for the pivot. In this case, that can be see as the fusion of two distinct arms, the link take a "A" form what justified the name *A-arm* or *wishbone*².

At this level, the wheel carrier enjoys one or two DOF: the suspension one and, possibly, the steering one. The first corresponds roughly to a transla-

¹For driven wheels, it is a bit different. The spindle does not exist. The hub are bearing mounted on the knuckle while the motorization torque is transmitted by telescopic tripod universal joint shaft through splined connection.

²From the name of chicken bone called like that following an old tradition that consists, after the meal, to break the chicken bone, pulling each at an end. The winner, for who the corresponding branch is not broken, can do a wish.

tion in vertical direction. The second corresponds approximately to a rotation around the vertical axis. The exact kinematics of these motions depends on the geometry of each component: chassis, links and wheel carrier. The exact kinematic or suspension layout is very important because determining the relative position of the tire with respect to the ground during the cornering and other rolling phases.

The suspension DOF is managed by a compliant element, like a coil spring or, formerly, a leaf spring¹, and a damper. Often, the coil is co-axially mounted around the damper tube forming a combined coil-damper classically called *shocks* and illustrated in the figure 2.11. Numerous technologies have been proposed concerning this key elements. Double chamber, simple chamber with gas piston or gas cartridge; hydraulic, pneumatic, electromagnetic or combined; passive, semi active, active; continuously actuated, manual or without tuning on one to four ways are all technologies offering a particular trade-off between handling performance, comfort, durability and price. As for the tire, the game consists in obtaining the suitable force response as a function of displacement and displacement rate. In this case, it is a bit simpler because there is only one direction and because the response is simpler to measure, to characterize and to model. The shock is connected on one side to the chassis and on the other side, to either the wheel carrier or to an arm link. The ends of shocks are realized with the same joint as those used for suspension connection. The top connection, e.g. to the chassis, is sometimes done through a compliant thrust bearing while the bottom connection can also be rigid (see McPherson suspension in the appendix A).



Figure 2.11: Öhlins shocks [134].

If any, the steering DOF is controlled by the steering mechanism. Two main categories exist. The difference comes from the way the rotating motion from the *steering column* is transmitted to the rotation of the wheel carrier. In the first system (figure 2.12), a *steering box* applies a ratio to the rotation signal. A lever, called *fork* or *pitman arm* mounted on the output shaft of the box is linked to a second lever, fixed on the carrier, via a double ball joint rod connection called *steering rod* or *tie rod*. The whole transmits the motion in the same manner as a four bar mechanism. The system is morphologically

¹This technology is still used in small truck due to profitable variable stiffness, self adaptable to the carried mass.

symmetric so the same box lever controls both carrier rotations. In the second arrangement, the box and box lever are replaced by a *pinion-rack* assembly. Such a system is depicted in the figure 2.13 and is also well visible in the figure 2.14. The motion is transformed into a translation through this mean instead of a lever. Due to the intrinsic rack width, the steering rods are shorter than those used with box. Note that both box and rack system can be actuated for convenience reason either by hydraulic or by electric mean. For cars, both options propose similar characteristics. The occupied space shape, length of the steering rods and parasitic motions are the major points of differentiation in the design process.

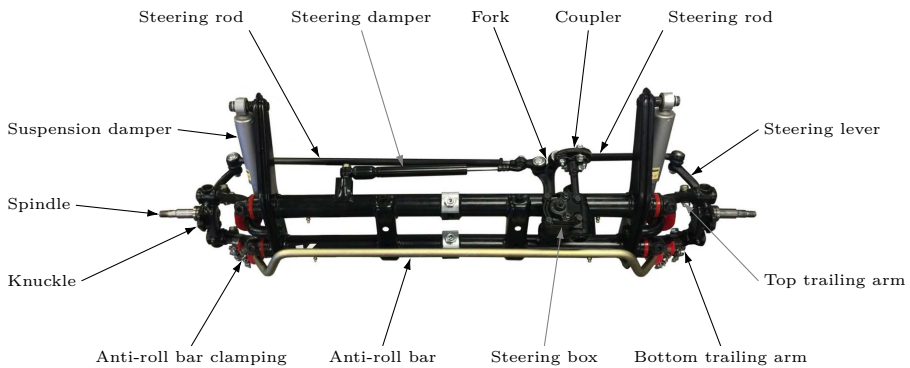


Figure 2.12: Volkswagen 1300: steering box system [139].

The bounce and roll are two distinct motions with particular requirements. Both the motion characteristics are coupled and determined by the single shocks stiffness and damping, what is not suitable. In order to reach the wanted roll stiffness separately from the vertical stiffness, an *anti-roll bar* is added to the wheel axle unit. This compliant U-shaped part is fixed by two compliant pivots to the chassis, and, at the two ends, to the carrier or to a arm, either by a simple compliant mount (figure 2.12) or, better, by anti-roll rods (figures 2.13 and 2.14). The anti-roll bar provides an additional roll stiffness because an asymmetric motion of the two wheel suspensions of the same axle involves an elastic torsion of the anti-roll bar. However, the pure bounce motion is not perturbed at all. The device is very common on cars and trucks even though, sometimes, mounted only on a single axle. Some actuated anti-roll bars exist but the applications are restricted to expensive or very specific vehicles.

Classically, the suspension components are aggregated in several groups according to the corresponding experienced dynamics. The first stage of suspension is provided by the tire compliance. The second stage is obtained by the shocks. The set of components located "between" the tires and the shocks are called *unsprung masses*¹. The set of components located up to the shocks, like main body and all rigidly fixed components, are called *sprung masses*. The classification aims to separate the elements that move like the carrier and those

¹From the German name. Literally translated as "non-suspended masses".

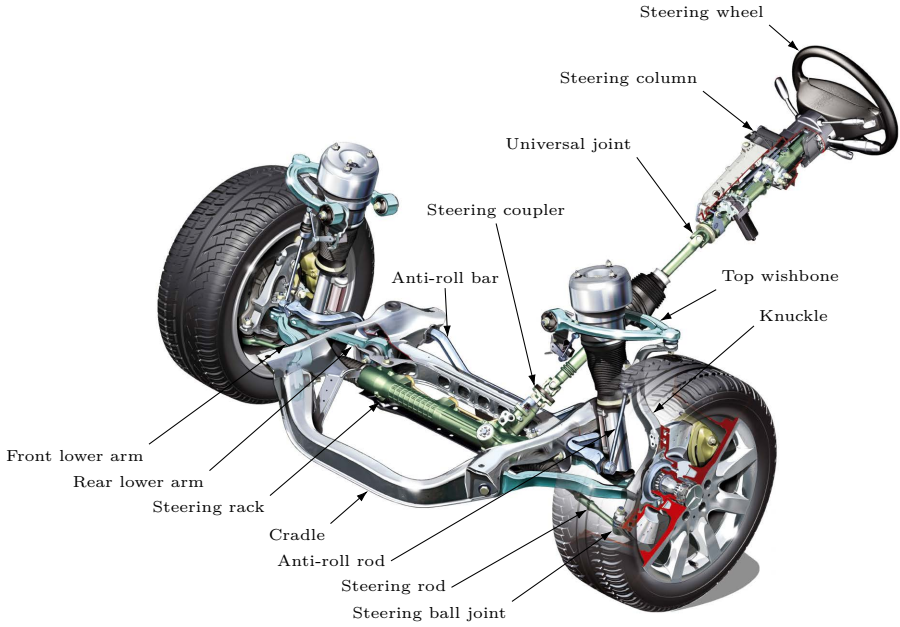


Figure 2.13: Mercedes-Benz S-class: steering rack system [154].

that move like the body i.e. with highly different dynamics. Actually, the division is not so clear. For example, the shock and the link masses must be split and attributed in the categories. In general, all vehicle masses must be as low as possible, the ratio between unprung and sprung masses must be also as low as possible to ensure a correct suspension filtering [39]. The *rotating masses* group the tire, the rim, the brake disc, the hub, the transmission shaft, if any, and associated parts. The rotating masses are, as well, part of the unprung masses. A particular attention must be given to reduce the rotating masses because they participate to unsprung masses and because their inertia have negative effect on dynamics.

For most of cars, the suspension links are fixed directly to the main body through relevant articulations. For comfort and acoustic reasons, it is possible to isolate the body from suspensions by an intermediate sub-frame called the *suspension cradle*. In this case, all the suspension components like arms, wishbones, but also top shocks connection, anti-roll bar pivots are linked to the cradle sub-frame. The cradle supports the body via multiple rubber based bushings (figure 2.13). It forms a third stage of suspension.

Car suspension classifications

Numerous suspension classifications can be done. Here, we propose some design oriented methods of classifications that help to characterize a particular system implementation.

Serial or parallel

As explained above, the suspension is an assembly of articulated parts that provide desired DOF to the wheel carrier. On the one hand, the different function e.g. suspension, steering, can be handled by multiple mechanisms successively mounted one on the other, adding each time a relevant, useful DOF. In that case, the system is *serial* typed. On the other hand, the mechanism can be synthesized such as all required DOF can be provided by a unique and coherent mechanism. It is the *parallel* typing. The classification is regularly applied on industrial robot. The impact on performances are the same for suspensions as robotic devices.

The parallel structure potential advantages are:

- the lightness,
- the smaller deflection,
- the smaller vibration,
- the higher design freedom,
- the higher number of parameters.

while parallel structure drawbacks are:

- the smaller work space,
- the parameter coupling,
- the difficult to design.

The potential adjective suppose that similar optimized systems are compared. The most suitable structure depends on the overall specifications of the suspension to design.

Pushed, trailed or lateral

A suspension can be qualified as *pushed*, *trailed* or *lateral* if the links between the wheel carrier and the chassis are mainly oriented to the rear, to the front, aside with respect to the wheel, respectively. In the two first cases, the articulation rotations are oriented to lateral direction so the lateral displacement and the camber of wheel are less affected by wheel travel than for lateral suspension for which the articulation rotations point mainly to the longitudinal direction. The lateral suspension offers better guidance ability against lateral efforts developed during cornering. In most of cases¹, the trailed suspension is preferred to pushed one because the proposed kinematics help the wheel to

¹If the articulations between the links and the chassis are higher than the articulations between the links and the carrier. In the opposite case, the rule is contrary.

avoid a bump surface irregularities. Passing such obstacles, the trailed suspension turns out to be more comfortable. A hybrid combination between lateral and trailed suspensions is a widespread trade-off.

Dependent or independent

A suspension is *dependent* if the motion of the two wheels of the same axle is coupled by the kinematics of the whole. It arises, for example, when a suspension structure element is common for the two wheels as in the case of rigid or semi-rigid trailed suspension. The independent suspension axles are more efficient regarding to the dynamics performances but dependent are used looking for strength against heavy loading or for cost reduction, possible by sharing of structure element to two wheels. The widespread anti-roll bar, which realizes a dynamics coupling between axle wheels, is considered as insufficient to view the suspension as dependent.

Car suspension arrangements

The engineers were very inventive before the second world war. Amazing old ideas could be founded in automotive museums and collections. The content in this field is very rich. From the 80's, the production has been normalized. The engineering has been more focused on improvement of the components and the parametrization. After all, numerous suspension classes exist and each of them is declined in lots of particular arrangements of which some most common are presented in the appendix A. The possibilities are broad and well exploited by car-makers. Successful designs take advantage of synergies and suitable trade-off regarding the vehicle requirements. With the requirement evolution, historical or new solutions could be used (again).

In this study, we focus on the front suspension at the basis of the front axle of the proposed vehicle: the double wishbone suspension.

Double wishbone suspension

The *double wishbone* suspension is probably one of the easiest suspensions to understand in details. According to the figure 2.14, the carrier is linked to the chassis through two superimposed wishbone arms that ensure very good both longitudinal and lateral guidances. These form a parallelogram of which the carrier is free to displace up and down. Thanks to the ball joints of the two carrier-wishbone connections, the carrier can steer. The shock is generally mounted between the chassis and the lower wishbone, passing through a free space in the upper one, that minimize required space¹. As for the other suspension morphologies the steering can be both box or rack based and the anti-roll bar is optional. In case of unsteer axle, an equivalent steering rod is still present but anchored directly to the chassis that lock the wheel steering. This suspension system is parallel, lateral and independent. The advantages of

¹On hyper car, motorsport car and formula, the shock is deported out of suspension, essentially for aerodynamics reason. The motion is transmitted by rocker lever and rods.

this arrangement are the good dynamics performances, in particular the good guidance of the wheel, the ease to design and implement, ease to understand and set because functions are well decoupled with respect to other classic suspension types. The complexity and then, in final, the cost are middle. The main drawback is the large required space. So the double wishbone suspension applies on coupé, big sedan, limousine, sport car, hyper car, motorsport car, formula, quad-bike, kart-cross, etc.

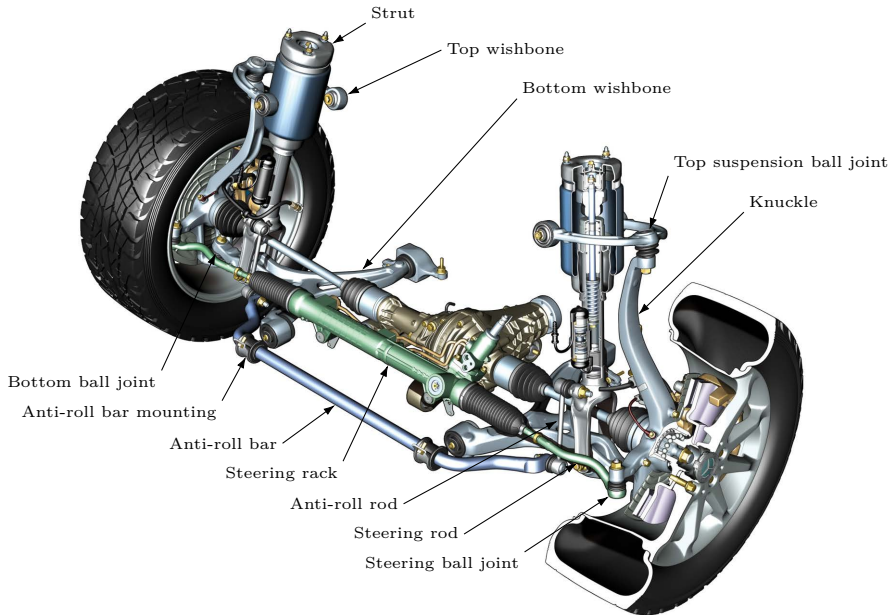


Figure 2.14: Mercedes-Benz R-class: double wishbone suspension [153].

Motorbike suspension

Although some oddities and some specialties¹ exist, motorbike suspensions are, morphologically, all the same.

In general, on motorbike, the rim, spoked or monobloc, supports directly the ball-bearings. So, there is no separated hub. The shaft that does not rotate, is handled at both ends by suspension. One-side mounting exists but is unusual². The rim supports also directly, systematically a brake disk, and the chain or belt sprocket at rear wheel or an possible second brake disk at the front wheel.

¹As example, the BMW R1200GS series uses a front telelever suspension where the steering and suspension are implemented in a parallel arrangement.

²Notable exceptions are scooters of which the swingarm contains the variable transmission and some touring motorbike equipped with universal joint transmission included in a single arm.

On the front, telescopic double shock fork is used (figure 2.15). The wheel shaft is fixed at each side on the ends of shock cartridges. The shock rods are mounted on two *triple trees*. So, the suspension function is completed, the whole is articulated on the motorbike main frame via a rigid pivot giving the steering DOF in series. The arrangements are one of the single applications of translation sliders in vehicle suspensions. Sometimes, the shock direction is inverted to make the system stronger and decrease the unsprung masses. Another performance option is the steering damper, useful on sportive and sport motorbike in order to avoid head shake¹ phenomenon.



Figure 2.15: Motorbike front suspension [150].

At the rear, the suspension is based on a *swingarm* (figure 2.16). The wheel shaft is fixed on the end of the swingarm which is articulated with respect to the chassis through a pivot. Depending on the vehicle weight, engine torque and searched appearance, the swingarm is more or less massive and the chassis is suspended by one, centered or not, or a double shocks articulated between the main frame and the swingarm. An interesting option is the *suspension linkage*, encountered in some cross motorbikes. This mechanism provides progressive stiffness and damping characteristics, advantageous shock placement, larger suspension stroke, as illustrated in the figure 2.16. Some considerations come from the transmission. First, the kinematics of suspension must be compatible with the location of the chain or belt driving sprocket. Secondly, the rear wheel shaft can displace with respect to the swingarm frame such the chain or belt tension is set. The motion is provided by the shaft supports that slide in groves or that rotate eccentrically. The difficulties come from the brake caliper which must be kept concentric with the wheel shaft that is done via a dedicated support. Obviously, the setting system is locked during usage.

If all motorbike suspensions are similar, their responses can be very different depending on the parametrization. As explained in the next section, geometry,

¹Instability of the front suspension steering of motorbike. This phenomenon is dangerous, sudden and brutal.

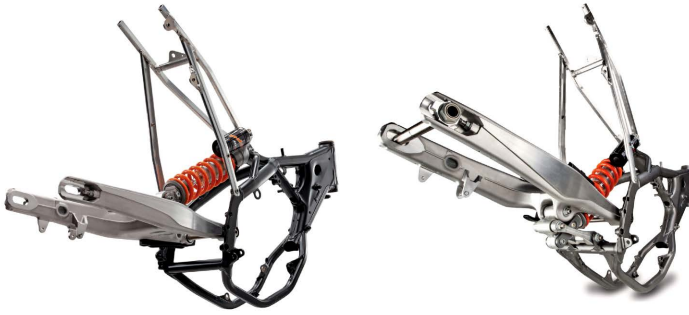


Figure 2.16: Swingarm: KTM PDS and linkage system [145].

and surprisingly small offsets affect the vehicle dynamics dramatically. Design parameters are chosen in accordance with the use and the style: cruiser, touring, cross, hyper-sport etc.

Suspension characteristics

The suspension dynamics is determined by the morphology but above all by the parametrization e.g. multiple dimensions, angles, stiffnesses, dampings that describe components. The number of parameters depends on the arrangement and on the corresponding model. However, some characteristics can be determined for most of suspensions and have straightforward identifiable effects on dynamics. Note that they can be material or immaterial stuffs. Those characteristics are usually defined in some standardized conditions i.e. at rest or at constant speed and for nominal loading etc. But, it could as well be measured for a range i.e. as function of the load, the roll, the wheel travel, the steady-state speed and radius, etc. In this section, we define major suspension characteristics and give their effect on the dynamics.

The four first characteristics concern the position of the wheels on the ground. Their standard definitions relate to the vehicle at rest on a flat, horizontal ground.

Track

The *track* is the width between two wheels of the same axle, taken at the ground level, and with wheel median plane intersections with ground as references (figure 2.17). The track can differ from one axle to another. This characteristic and, in particular, the ratio between the center of masses height and the track plays a significant role in roll dynamics and roll stability. Generally, the track is taken as its maximum under vehicle overall compactness constraints.

Wheelbase

The *wheelbase* is the longitudinal distance between the two axles¹ (figure 2.17). Similarly to the track case, the wheelbase and its ratio with the center of mass height impact the pitch dynamics, which is better for larger wheelbase. In opposite, the wheelbase has to be as short as possible to reduce the minimum turn radius during maneuvers. A compromise must be found between a large attack angle² obtained if the wheel base is large and risk of impact between chassis and ground due to insufficient ground clearance and long wheelbase. Furthermore, the wheelbase impacts the way the wheel and related mechanical elements such as the suspension are introduced in the overall design. As one can see, a suitable wheelbase results from a skillful trade-off.

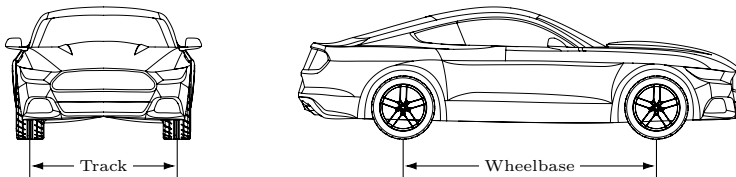


Figure 2.17: Track and wheelbase [167].

Camber

The camber is already defined in section 2.1.2 in the case of a single wheel. Here, we consider a complete suspension on flat, horizontal ground such the camber is the angle between the median plane of the wheel and the vertical vector³. The sign is redefined if the purpose is the suspension camber instead of the wheel one. The camber is defined positive if the median plans cross themselves below the car (figure 2.18). The sign convention remains the same for the left wheels but is inverted for the right ones. Thus defined, the camber is generally set to zero or slightly negative. In case of negative camber, tire camber thrust appears on each wheel but cancel each other in straight line running because being in opposite direction. By contrast, in curve, the self induced overloading increases the camber thrust on outer wheel and the underloading decreases the inner wheel one, therefore the equilibrium between camber thrusts is broken and the force resultant, that oriented to inner direction, help the vehicle to turn. For positive camber, the reasoning is inverted so the phenomenon acts against the cornering. The camber can be initial but also becomes self induced by suspension kinematics: the roll leading to the wheel travel. The effect is so sensitive that normal negative camber does not exceed 1° or 2° for standard setting.

¹For vehicles with more than two axles, multiple wheelbases have to be defined specifically.

²The attack angle is defined as the maximum slope that the vehicle can attack from horizontal ground, without touching any body or suspension element.

³The proposed definition can differ from the one used sometimes in the literature. These second definition considers the angle between the intersection between the wheel median plane and body face plane, and the vertical vector. These two definitions are similar but not equivalent if the toe in angle is not set to zero.

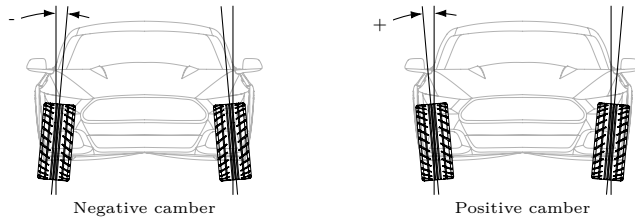


Figure 2.18: Camber [167].

Toe in

The *toe in* is defined as the angle of the intersection of the wheel median plane and the ground with respect to the longitudinal direction (figure 2.19). The toe-in stands for the case for which the median planes intersect ahead of the vehicle. In the opposite case, both *toe out* or negative toe in designations are used. Similarly to the camber angle induced effects, the toe in is equivalent to the slip angle for the left wheel, for a vehicle in straight line on flat horizontal ground. For the right wheels, the sign is inverted. Here again, a pair of opposed lateral force is generated, but due to the slip angle. The unloading breaks the equilibrium so a lateral force results on the corresponding axle. Small toe in leads to the vehicle more responsive while toe out leads to the vehicle handling floppy. As for the camber, the toe in is very sensitive such the setting of these two parameters must be realized carefully. Special equipment allows to set them with suitable precision in the order of $< 0^{\circ}10'$ depending on the makers specifications.

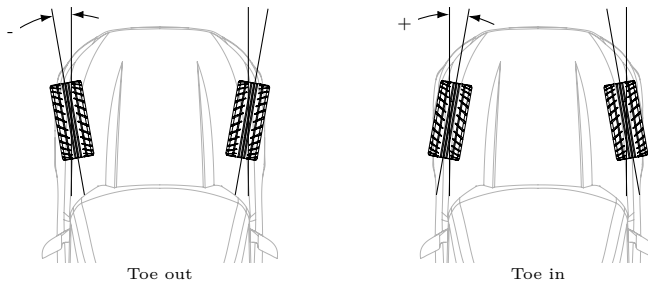


Figure 2.19: Toe out and toe in [167].

The four next characteristics refer to the steering axis (figure 2.20).

Trail

The *trail* or *caster offset* is the distance between the intersection point of steering axis into the ground and the wheel-ground contact point both pro-

jected on the body side plane. The trail is positive if the steering axis is ahead the tire contact point. In this case, the trail stabilizes the steering DOF. Indeed, in case of steering perturbation, e.g. a small undesired steering rotation, the induced slip angle involves a tire lateral force which, combined to the trail, forms a torque around the steering axis, that faces the perturbation. The effect is similar to the self-aligning moment of the alone tire but is higher in magnitude. In this case, the trail can be set in the design phase, what is not possible with self-aligning moment. The trail must be positive to ensure stability but not too large under the risk to face too high restoring torque. The stabilization effect appears only if the vehicle is in motion.

Caster angle

The *caster angle* or *rake*¹ is the angle between the steering axis projected into the side plane, and the vertical (figure 2.20). This is positively defined when the top of steering axis point to rear. The caster angle involves a wheel camber with steering what affects significantly the cornering dynamics.

Scrub radius

The *scrub radius* or *steering offset* is the distance between the intersection point of the steering axis into the ground and the wheel-ground contact point both projected on the front plane, as depicted in the figure 2.20. The scrub radius is positive if the steering axis is inside the tire contact point. Both negative and positive scrub radius can be encountered in suspension but the negative one offers safe restoring torque in case of asymmetric longitudinal shock incurred by steering wheel. Assuming a back longitudinal force applied on the left wheel, due, for example, to a pothole. Naturally, the first effect is a deviation of the vehicle trajectory to the left. However, the longitudinal force is transformed into a torque through the scrub radius lever. This steers the wheel to the right, counteracting smartly the perturbation. On the other hand, in case of unequal grip between two steering wheels of a motorized axle, if large torque is applied on the axle, a severe torque transfer applies on the steering column to the steering wheel and to the driver. Hence, this phenomenon restricts the absolute value of scrub radius, especially for high powered traction cars. At rest or at very low speed, the scrub radius permits the wheel to roll instead of spin during steering. In this way, the required steering torque is reduced and the manoeuvres are easier.

Kingpin inclination

The *kingpin inclination*, or *kingpin angle*, is the angle between the steering axis projected into the front plane and the vertical (figure 2.20). This is positively defined when the top of axis points to the center of the car. As the caster, it involves a wheel camber during steering. But for the general case of

¹The rake appellation is regularly overuse. Someones use it for the axle offset. Attention must be paid concerning this definition.

positive kingpin angle, the induced wheel camber of the outer wheel is wrongly oriented and acts against the cornering. The serious problem is solved thanks to the combination with a positive caster angle. The caster effect corrects the kingpin angle one. Moreover, if the scrub radius is not null, a transfer from gravity applied on the main body and the steering torque applies so a stabilizing or perturbing torque can appear. If both scrub radius and kingpin angle are positive, steering cause small body elevation. Restoring torque is obtained by the gravity in this way. The gravitational centering effect acts even if the vehicle is at rest. Note that a similar effect is due to the combination of trail and caster but in this case, both left and right contributions vanish themselves so there is no torque transmitted to the steering wheel. Because it is very difficult to obtain a negative kingpin angle, it is not possible to benefit from both self centering torque in case of longitudinal perturbation due to a negative scrub radius and a gravitational centering due to positive scrub radius. Here again, a suitable trade-off must be defined.

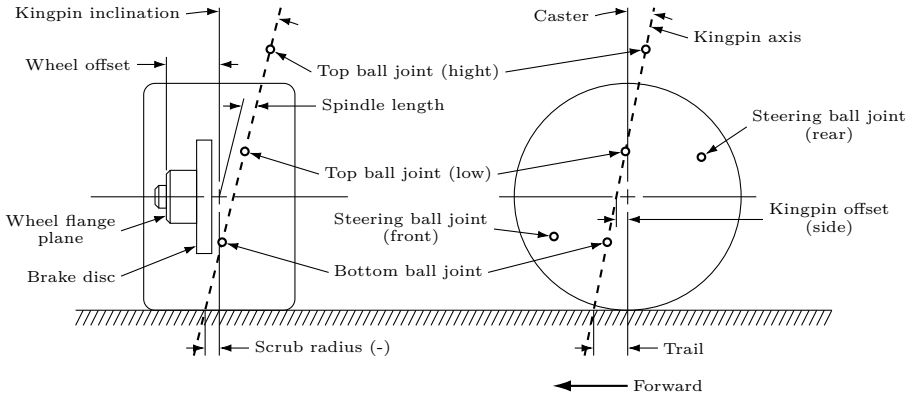


Figure 2.20: Trail, caster angle, scrub radius, kingpin inclination.

Wishbone pivot angles

The wishbone pivot angles are the angles between the projection of the pivots on the lateral plane and the horizontal one. If non-null, it induces a coupling between wheel longitudinal dynamics and suspension one. With an appropriate selection, it allows to attenuate the effect of the acceleration and braking on the main body pitch. Acting on the front and rear suspension pivot angles, it is possible to get respectively an anti-dive and an anti-squat effect.

Olley's rule

The Olley's rule specifies the rear suspension have to be stiffer than front one because in case of obstacle crossing, it allows the pitch response of the car is more absorbed to the detriment of the bounce, justified because body pitch is considered to be more uncomfortable than bounce.

The major suspension geometrical characteristics have been presented. All of them impact significantly differently, often complementarily the effects that must be mastered to get optimal cornering capabilities and correct handling response. Note that the links between the geometrical dimensions and the dynamics behavior are treated for the single case of vehicle at constant speed on a straight line, the basic and easiest one. If any, corresponding behaviors including, for example, stability and steering torque feed-back should be satisfied for the entire range of rolling configurations despite high geometrical changes due to roll, wheel travels, braking or acceleration torque etc. This is especially true for the tilting three-wheeler for which the suspension component displacement are extreme as explained in the next section.

There exist numerous other complementary suspension characteristics. The axle offset, the spindle offset, for examples, are different but redundant with those already presented. Moreover, the axle parallelism, the axle alignment, the thrust angle, are considered as building imperfections and will not be treated in the work.

2.1.4 Tilting vehicles

As explained in this introduction section, two main classes of vehicles exist depending on the way the centrifugal effect is countered. The tilting vehicles, for which the track is reduced or null, are unable to do it by self-unloading of wheels but rather incline themselves in the direction of curving. The most known examples of tilting vehicles are bike and motorbike. This class is deeply different from untilting vehicle because the driver must manage the roll, or tilting, motion in addition to the steering one. Hereinafter, we compare from a simplest point of view, the manner a car and a motorbike turn.

To turn right with a car, the driver turns the steering wheel to the right and by the way, steers the front wheels and their carriers. The current difference between speed and direction of front wheels i.e. a slip angle, generates a lateral force oriented to the right. This force induces an acceleration to the right and a negative yaw acceleration which induces curving. So the rear wheels are also subjected to slip angle and generate a lateral force. The system reaches the equilibrium when the front and rear wheel forces equilibrate in rotation around the car center of mass and their sum generates a centripetal acceleration corresponding to the desired curve.

The mechanism of turning is a bit more complex for a motorbike. To turn to the right, the biker don't turn the handlebar to the right but to the left! The induced slip angle generates a lateral force, to the left, applied on the tire at the ground level. This force destabilizes the roll equilibrium of the bike so a positive roll acceleration appears. The acceleration is reinforced by gravity. At this time, the biker quickly but softly turns the handlebar to the right in order to inverse the direction of the lateral force that allows the driver to control the roll. The transient stops when an equilibrium is reached in vertical direction: the normal forces on tires counteract the weight of motorbike, in lateral direction: the tire lateral forces due to both slip angle and camber equal,

in magnitude, the centrifugal effect, and in roll direction: torque induced by the tire normal and lateral force equilibrate. The cornering radius is a result of the inclination. In fact, the driver manages the trajectory through the inclination in the manner of a two level controller. To close the turn, the bike must incline more what is done by a light redress of handlebar to break the equilibrium before turn more to recover another stable motion. To open the curve, the opposite maneuver is engaged: the driver turns more the handlebar before to steer less than in the beginning. The need to light turn the handlebar in the direction opposed to the desired motion is called the *counter-steering*. Note the present phase explanation is simplified to two dimensions instead of actual full three dimension dynamics. We assume this simplification for introduction purposes. The yaw dynamics is similar to car.

The trajectory control practice of motorbike has several consequences. First of all, despite most of people are able to learn to bike, the bike and motorbike use is not at all instinctive. On the contrary, in case of emergency, the reflex of beginner brings regularly accidents. Moreover, the two levels of control method participate to the slowness of bike control. Because the change of trajectory involves a roll motion, the ability to sharply change the direction is lower compared to a car. This increases again the inherent dangerousness of motorbike driving, already high because of the large exposure in case of crash.

To face some drawbacks and to take advantages from both sides, some new solutions appear on the bridge between the bike and the car worlds. In general, the narrowness of motorbike is an interesting characteristic that has to be maintained what involves to tilt in curve. Despite this tilting requirement, a main objective of the developments is to propose vehicles easier to control than a motorbike. Stability must be improved. Ideally, the instinctive control characterizing cars could be recovered. Based on these requirements, the vehicles can be equipped with two, three or four wheels. A second classification concerns the level of actuation of both suspension and steering systems. Then, we give some notable examples of such tilting vehicles. Here again, the existing concept set is so rich that it is impossible to cover all in this text. The goal is not to be exhaustive but to pull off some highlight conclusions about specific relations between morphologies, dynamics, design and control.

Passive tilting vehicles

With two wheels, the only possible unactuated configuration is motorbike. At this time, no one can do better in this class.

In case of unactuated suspension, the only way to improve handling and dynamics is adding an extra wheel. Based on it, the three-wheeler are divided into two main categories: One front wheel and two rear ones or, on the contrary, two front wheels and one rear. These configurations are respectively named *normal*¹ or *delta*, Δ three-wheeler and *reverse* or *nabla*, ∇ three-wheeler. It is admitted, in a certain extent, that the reverse configuration is the most suitable. The gain in overall stability during braking is the main justification.

¹By convention, we call the one front, two rear wheel three-wheeler the normal configuration by opposition to the reverse appellation.

It does not prevent numerous projects to adopt normal configuration.

For several years, the three wheel scooters have been a commercial success, in particular in some European cities as Paris. In comparison with the traditional two wheel scooters, the first real improvement is the ability to lock the front suspension at rest that allows to avoid to touch the foot on the ground. It is very convenient at cross light or in similar null speed conditions. Three makers share the most of the market. All propose passive, reverse three-wheeler. If all adopt a traditional swingarm based rear suspension, very similar to the scooter one, the front suspension is widely different depending on the makers.

The first available model was Piaggio MP3 [102]. It has been commercialized since 2006. The front suspension is a very inventive serial arrangement, as illustrated in the figure 2.21. From the chassis to the wheel, the first stage is made from a double parallelogram. The central pivots of the main horizontal beams are fixed on the chassis and the ends are articulated with steering sleeves at each side. The parallelogram arrangement allows the wheels to move up and down, inversely synchronously, over a large travel that corresponds to tilting motion which is limited to 40° . The tilting brake is made from a disk sector fixed on the lower main beam and engaged in a brake caliper mounted to the chassis so the action of those caliper locks the tilting motion. The second stage concerns the steering DOF. Each side, the rest of the suspension leg is articulated via a pivot with respect to the steering sleeves i.e. the fourth, outside bar of the parallelogram. The steering mechanism is simple: a third horizontal beam, ahead offset with respect to the main parallelogram plane, articulated via a universal joint between two leg levers and driven at its center by a steering column mounted lever. At the bottom of the leg, a pushed arm is used in combination to a shocks to provide the purely independent suspension DOF. Each wheel, mounted on the end of the pushed arm, is equipped with its disk brake system.

Note that the Yamaha Tricity [103] used a morphology derived from the Piaggio MP3. Its suspension keeps the same architecture as the Piaggio MP3 but the pushed arm suspension stage is replaced by a dual shock fork each side.



Figure 2.21: Piaggio MP3 500 Sport and its front suspension [159, 160].

The suspension of the Peugeot Metropolis [104] is based on a double wishbone suspension (figure 2.22). To allow the tilting motion, several adaptations have been implemented. First, the two shocks are replaced by a single one, attached on the top of the upper wishbones such the roll stiffness drop and the space between the two wishbones is free. To help the shocks to absorb the road induced vibrations, a mixer¹ is rigidly fixed on each upper wishbone. Second, the joints between wishbones and carrier, and at the steering rod ends have been adapted to accept large wheel travel imposed by tilting motion. The lower wishbone ball joints are reoriented such the main axis corresponds to tilting motion instead of steering rotation limiting this last one. The upper wishbone ball joints are replaced by some compliant x double pivots. Third, the structure parts are relatively twisted such the suspension motion is possible on an extended range. They are relatively robust. Note that the upper wishbones are mounted on the same shaft that obliges to place them in tandem what disrupts their symmetry whereas the bottom wishbones are articulated on parallel, close but distinct shafts.



Figure 2.22: Peugeot Metropolis 400 RX-R and its front suspension [157, 156].

The Quadro QV3 [105], for which the front suspension is depicted in the figure 2.23, uses also a double wishbone based suspension at the front axle. The bottom wishbone is close to the wheel axis so it ensures the major the holding of the wheel. The upper wishbone is smaller and provides only the camber maintaining. The suspension is completed thanks to the Hydraulic Tilting System (HTS) system. The HTS is made from three cylinders connected together by a three ways valve. The two first cylinders replace traditional shocks between the bottom wishbones and the chassis. The pistons of these cylinders face to hydraulic oil free to flow, in normal mode, from one cylinder to the other trough the main valve. This transfer allows the tilting motion. A third cylinder is connected at the main valve. Its rodless piston isolates the oil from the main valve and a compressed gas. By this way, the oil can escape from the two main cylinders to the last one in order to propose a suspension DOF.

¹A mixer is a device consisting in a cylinder filled with oil, and wherein a massive piston can move freely between two springs. The device aims to reduce absolute vibration on the contrary of damper which face to relative motion.

The initial gas pressure in the auxiliary cartridge sets the global stiffness so the shocks are not equipped with spring or other mechanical stiffness. A particular interest of this system comes from the main valve which can be closed to lock the suspension and tilting DOF. The wheels steer thanks to separated pivot carriers and a fork based mechanism. The overall system appears like a simple and coherent passive front suspension for three-wheelers.



Figure 2.23: Quadro QV3 and its front suspension [162, 146].

The three described vehicles use some very different arrangements to propose the tilting motion of the front axle. From a performance point of view, although all of them allow to control more or less easily the stiffness and damping associated to tilting DOF, what is not possible for a motorbike. The three-wheelers do not bring a major contribution to the cornering capabilities. The pilot strategy is very similar to a corresponding scooter or motorbike. Moreover, the required additional DOF of tilting leads to complex and heavy front suspensions. Those suspensions have a tilting limited range, dictated by the ball joint or other connection limits and by risk of interactions between moving components like arms, carriers and steering rods. However, the extra wheel helps to face the local losses of grip which are very serious if occurred by the front motorbike wheel. From experiments, it seems the brake distance is reduced without any pure physical explanation justifying this gain. Probably the stability provided by roll stiffness and damping convinces and helps the driver to brake harder while keeping overall stability. In definitive, the effect of the additional wheel decreases the cornering capabilities with respect to a normal motorbike but corrects two of major drawbacks of this last one: risk of local slipping and feeling of imperfect control. Most of all, the three-wheeler do not require to touch the foot on the ground at zero speed.

At that time, the tilting suspension technology is successful in the scooter range, but some vehicles also exist in a powerful range. One of the earlier trial to pass from two wheels to three is an adaptation kit develops for Harley Davison motorbikes: Tilting Motor Works [106]. The result is a nabla three-wheeler. Because the track is larger, the main objective is not to be narrow and nimble but more improve the stability, and have an exceptional style. In the same idea, the Brudelli Leanster [107] is a rebuilt reverse three-wheeler based on a KTM

690 Supermoto. As shown in the figure 2.24, the mechanical quality of this vehicle is very good and involves some unusual devices especially developed for tilting purposes: special ball joints for the double wishbone suspension and a peripheral brake system as examples. The corresponding patent was bought by Yamaha in 2017, probably not to be exploited but in order to lock this alternative to their own technology.



Figure 2.24: Brudeli Leanster [136].

Presented in Tokyo 2017 exposition, the Yamaha Niken [108] is the first true three-wheel motorbike produced in series (figure 2.25). The front axle uses a technology similar to the Yamaha Tricity, excepted the double twice shock forks, which support wheel carriers, are mounted outside the wheels, giving an impressive massive front suspension. Thanks to the very low track of 410 mm, the agility of the motorbike is saved. The maximum tilting angle is 45° what allows a good cornering performance. According to the expert testers, the behavior is equivalent to a the Yamaha MT-09, the two-wheel version taken as base for the Niken. The front suspension adhesion and braking stability are improved but the addition of masses impacts negatively the longitudinal dynamics. The vehicle gives a very secure feeling despite the 85 kW power provided by the engine. In final, the impact of the extra wheel is the same as for the scooter range.

Padova University has developed a delta three-wheeler [21]. The rear axle unit does not tilt and is articulated via a linkage mechanism to the scooter-typed main body. This four bar mechanism is smartly designed to reduce, as possible, the load transfer from one wheel to another. If the results demonstrate the stability of the vehicle over a range of speeds and radii, we do not understand the interest of this layout in comparison with the passive nabra



Figure 2.25: Yamaha Niken [141].

three-wheeler. Indeed, the single front wheel is subjected to the same momentary and local leak of adhesion.

At the opposite the upright bike and three-wheeler, the very low center of gravity of recumbent trike permits to do not tilt. Nevertheless, some prototypes implement tilting for such light vehicles [65].

To be complete, it must be noticed that there exists numerous three-wheeler human powered i.e. most of time powered via pedals. Such category targets ultra low masses and very simple mechanics so this kind of three-wheeler is, at our best knowledge, always passive.

The Sway scooter [109] is developed in USA. A scooter like structure is articulated over a lower untilting base-plate that contains battery and other technical elements. The front suspension is double wishbone based, as depicted in the figure 2.26. The rear one is simply a motorized swingarm articulated on the upper part of the chassis. We classify this three-wheeler as passive because it does not involve any tilting actuator: the roll is not managed via the steering but by driver legs pushing on the base-plate on which the foot stand. The driver is thus responsible of the tilting and steering coordination. The vehicle seems rideable and stable. But, despite a fun aspect, we find it does not bring a major practical improvement with respect to classical scooters or European commercial three-wheelers. The large track of the reverse scooter could be inefficient in traffic.

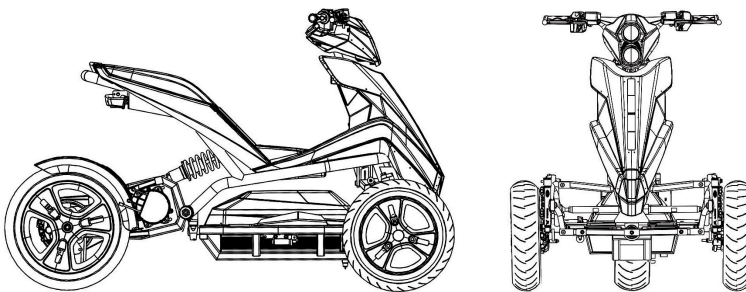


Figure 2.26: Sway [165].

Still keeping a pure passive suspension, the vehicle trajectory can be controlled by the tilting manner i.e. via the steering wheel which do not control the steer of carriers but the roll of the body. In this case, the wheel steering is free, suitably oriented in the direction of motion and stabilized by appropriated geometry of the steer axis and eventually by an additional torsional stiffness and a damping applied inside steering sleeves. At high speed, the caster and caster angle self-steer the wheels while at low speed, like during maneuver, nothing steers correctly the wheels. For the same reason, the system is totally unstable in reverse direction. It is the reason why this technology does not find any commercial application. The Tilting Vehicle Australia (TVA) [123] is a vehicle illustrating this kind of technology (figure 2.27). It is developed by a private, passionate Australian. There exist three and four wheel declination of the prototype.



Figure 2.27: Tilting Vehicle Australia [123].

Naturally, it is still possible to add one more wheel. It exists some models of tilting four-wheelers. The Quadro 4 [105], the four wheel declinations of the Quadro 3 is a good example of this class (figure 2.28). In this case, the advantages and drawbacks of the three-wheeler with respect to motorbike are sufficiently reinforced. It is possible to set the roll stiffness and damping at both front and rear axles but the suspensions involve more parts leading to a rise of weight, complexity and finally of cost. The stability is enough increased but the roll inertia also rises. The evolution seems fit for calm driver who looking more for safety than for dynamics. The main problem concerns the transmission which can not be achieved through a simple swingarm based drive belt transmission or an equivalent chain. The rear suspension is made from two trailer arms, each equipped with a transmission belt plugged on differential gearbox. The overall is similar to the assembly of to symmetric one-side swingarm. On the rear, motorized axle, the use of a wishbone or similar lateral suspension is not possible because the universal joint transmission can not encounter the range of motion due to the tilting and the steering. The belt transmission fits well with swingarm because kinematically compatible that justify the use here.

The recent European Union (EU) founded Range of Electric Solution for L category VEHICLE (RESOLVE) project [113] has been leaded by Piaggio and KTM, and involves several universities and industrial actors. It aims to develop several vehicles improving the mobility capabilities. D1 and D2 prototypes are illustrated in the figure 2.29. They are both passive tilting four-wheeler scooters. The morphology is very classical: double wishbone at the front and trailed



Figure 2.28: Quadro Qooder [161].

arms at the rear. The report [14] discusses the design of the proposed morphology and the motorization. The simulation of the vehicle dynamics is realized with Adams¹. The paper [13] performs a deeper stability analysis and proposes some additional criteria and representation techniques. The questionable hypothesis comes from the model used for the analysis. This considers frozen shocks and tires such that all suspension DOF are locked. Despite these unconservative hypotheses, the study allows to validate the stability over a range of speed but only in straight line. A sensitivity analysis allows to refine the kinematic parameters of the mechanical design.



Figure 2.29: RESOLVE D1 and D2 [143].

The present state of the art about passive tilting vehicle shows how the unactuated addition of one or more wheels changes the dynamics, potentially positively, but does not allow to control the vehicle so easily and so instinctively as for a car. The active tilting suspensions are developed in this objective research.

¹Adams is a multi-body software based on absolute coordinates. Adams Car is widespread in the automotive industry. The software is edited by MSC Software.

Active tilting vehicles

Activation consists in adding one or several actuators in the suspension mechanism to introduce forces or torques controlled by an appropriate algorithm. The goals are the improvement of cornering and handling capabilities as well as to improve the ease for the driver to control the vehicle, restoring if possible a behavior as instinctive as possible.

Before illustrating the application of such actuation on the vehicles, we start describing, from a theoretical point of view, the different approaches of the tilting suspension actuation. In order to control the vehicle tilting, a large amplitude torque must be applied on the chassis in the roll direction. A first distinction refers to the origin of the torque which can be intern or extern. While internally, the torque can be generated by an inertia effect, externally, the torque can be generated by the only controllable, in a given extent, forces applied on the vehicle: the tire forces. In this second case, the control can be based on normal or lateral or both tires forces.

The Gyroscopic Tilting Control (GTC) consists in acting on high speed spinners to benefit the reaction torque, the spinner having the tend to keep its rotational orientation constant with respect to the inertial frame. If the system could be implemented and gives very impressive results, it suffers from several drawbacks, each difficult or impossible to suppress because inherent to the technology. First, the rotation must be applied during all the use/rolling time, what is energy consuming. Low friction journals and bearings, like air or magnetic bearings are used to reduce the influence of this effect. Second, to be efficient, the spinners must have a large inertia i.e. a large mass or a large diameter, and/or turn at a high revolution speed such the stored energy is huge what is relatively dangerous in case of system failures or road accidents. The spinner system must be enclosed in reinforced compartment isolated from the driver. Third, the generated gyroscopic torque can not be maintained constant over more than a few seconds. So the roll had to be kept close to the equilibrium roll depending on the configuration. The gyroscopic torque can induce a roll unbalance to produce required motion and to stabilize the roll around the local quasi-static equilibrium. Four, the system implementation is complex. It consists of the main spinner rotation actuation and the bearing management system, the mechanism of tilting torque generation and the associated controller and sensors, the main controller which manages the overall vehicle dynamics according to the pilot, the road, the suspension and the tilting mechanism.

The Direct Tilting Control (DTC) uses the normal tire forces to generate the roll torque required to control the tilting motion. The tilting DOF of one or more non zero-track axle is actuated so the system overloads one wheel while unloads the opposite wheel of the axle. The difference of normal forces generates the roll torque. The actuator can be electric, hydraulic, pneumatic, etc, linear or rotational. In addition to its insertion into the chassis, the major difficulty consists in transferring the motion to wheel carriers through a transmission compliant with the suspension and steering DOF. The advantages of this technique are the direct effect of the actuation on the roll dynamics, here comes from the name of DTC. The roll torque is limited by the most unloaded wheel take-off so the dynamic load on the axle, i.e. mass distribution on the

chassis and the axle track are the limiting aspects. The drawback comes from the high torque required: several hundreds Nm for small unipersonal vehicle, leading to the need of a robust implementation. Because the roll speed is low, the actuation power is maintained low but has to be minimized for energy consumption purposes.

The Steering Tilting Control (STC) uses the lateral tire forces. The working principle is based on the counter steering like practiced by motorbike riders and explained earlier in this section. In existing implementations, the true steering effector is decoupled from the steering wheel. The steering column is disrupted so a first actuator controls the steering mechanism and a second actuator fits to the steering wheel or the handle bar to capture the intention of the driver and to provide artificial torque feed-back to this last one. The system transfers the static rotation of the steering wheel but superimposes a transient counter-steering to the steering mechanism. The advantage of this system is that it allows to set independently and freely a non-constant transmission ratio, the counter-steering and the torque feed-back. Moreover, the tires which have high sensitive response, act as a torque multiplier such the steering torque needed to control the tilting is one or more orders of magnitude below the torque used by the DTC system. At low speed, the system encounters the same problem as an unactuated tilting vehicle: the effect of speed disappears so the tilting becomes uncontrollable. An additional locking device has to be implemented. It must be mentioned that the method of control required two actuators and related controllers and transmissions. The system is relatively complex and expensive. The design freedom offered by the disrupting of the steering column is high but it causes also a security risk in case of electronic controller shut-off. Until recent and modest change in regulations, the road vehicle legislation imposed a mechanical link between the steering wheel and the carriers. Added to the technical difficulties, it explains why the STC have no commercial application at this time.

The Hybrid Tilting Control (HTC) aims to mix several tilting control strategies to make the most of each of them. This refers generally to a combination of the DTC and STC systems. So the overall benefits the perfect functionality of the DTC at low speed and the significant energetic efficiency of the STC at high speed. The HTC is potentially the most performing one but also more complex than exclusive DTC and STC, without, however, reaching the complexity of the GTC system. In particular, a difficulty comes from the synthesis of a unique controller managing the tilting motion with an appropriate combination of the use of the two well different phenomena.

Two-wheeler

The active tilting two-wheeler can use exclusively the GTC or the STC. Naturally it is impossible to use DTC and then HTC because no axle has a non zero track¹.

The most successful GTC based two-wheeler is developed by LITMotors

¹Here, we consider only the two-wheeler with a tandem wheel configuration, excluding the gyropods and other similar vehicles, too different from the considered class of vehicle.

company [110] in USA. The vehicle, which is illustrated in the figure 2.30, includes two spinners launched in opposite directions to get pure roll torque. The tilting control system generates a huge torque of over 800 Nm what allows a vehicle roll rate over 1 rad/s. The quiescent power is below 100 W what could be considered as reasonable. In contrast, the system [18] suffers from the very long spin up time: 70 s and from a disabling mass: 95 kg. Two retractable legs are deployed to stabilize the vehicle when the GTC is shut off, for parking. Despite impressive road capabilities of prototype and a true will to commercialize vehicles from this concept, it seems the technology is too complex and then too expensive to find the expectations of the market. The over price with respect to a motorbike is not justified compared with the advantage of self stability and the comfort of a full closed cabin.

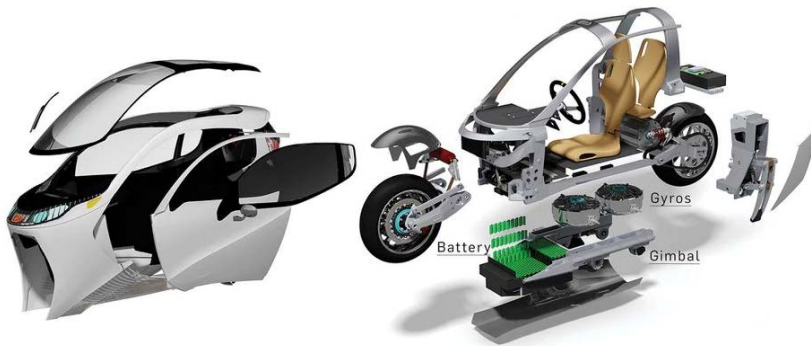


Figure 2.30: LITMotors C1 [135].

Three-wheeler in delta configuration

We start the overview of existing active tilting three-wheelers by the delta configuration category. Despite it is not the most promising morphology, it exists several very serious attempts in this direction supported by well published and high quality academic research.

The Carver [114] is one of the first active three-wheeler (figure 2.31). The concept is developed in the Netherlands from the beginning of the 90's. The main idea is simple: the comfort and the handling of a car associated with the motorbike dynamics. The morphology is based on a delta configuration. The cabin, which receives two persons in tandem tilts by rotation around a main revolute joint linking the rear axle unit. This unit does not tilt so the rear wheels do not take more camber than for a car. This justifies that car type tires are used for this axle. The independent McPherson suspension is used as rear suspension. The unit involves also the Internal Combustion Engine (ICE) motorization. The front suspension is a superimposed double pushed arms. The ball joints are used for arm-carrier articulations so the front wheel steering is possible. A longitudinal steering rod transmits the motion to carrier lever. The

whole permits tilting up to 45° for a width of 1.15 m. The paper [95] describes in details two successive versions of the hydraulic tilting system apperanted to DTC. The last vehicle assumes good results in several rolling tests: lane change, braking, high speed handling, fast cornering, slippery roads, side winds and sloped road. The academic paper and promoting test demonstrate the possibility of this vehicle. The paper [94] presents in detail the tilting control system updated to HTC. The paper [93] gives additional technical informations but overall, a small business plan. In production, the project suffers from its too higher price: approximately 30000 euros. After a bankruptcy in June 2009 and years of mediatic silence, the website of the company [114] newly announces possible pre-ordering for a production planned in 2018. Except the visible change of front suspension which adopts a double shock fork arrangement, the technical details about this new version are not available. We do not know if the tilting control strategy is the same as the previous one.



Figure 2.31: Carver [140].

The Compact Low Emission Vehicle for uRban transport (CLEVER) project was initiated by the Berlin University of technology and largely developed thereafter by the University of Bath. The project was supported by BMW. The vehicle is very similar to the Carver One with which it shares the global morphology: a tilting cabin and front suspension articulated on a rear untilting axle unit (figure 2.32). The tilting angle is limited to 45° and the overall width is 1 m. In a first version [29], only the hydraulic DTC was used. The next works [10, 4] aim in determining the optimal tilting axis inclination to reach the neutrality in case of steady-state cornering. The tilting inclination is used to induce tilting dependent and self induced rear wheel steering. The Berote thesis works [7, 8, 9] proposed a first DTC controller synthesized on the basis of a specifically developed model of the three-wheeler. The work concluded on a vehicle with very good steady-state stability but poor transient behavior. The van Poelgeest thesis [97] explored the model non linearity, the uncertainties and the environmental factors effects on the dynamics. He explored the counter-steering effect on tilting vehicles and proposed some paths to improve the tilting control strategy. The Robertson thesis [76] has updated the DTC to an HTC by adding a steering control. In this second version of the CLEVER, the load transfer is reduced from 40 % thanks to the steering control that improves transient stability. The system could be enough improved if the steer angle signal was sufficiently clean and if the steering stroke was larger.



Figure 2.32: BMW Clever [138].

Another tilting three-wheeler in delta configuration was developed at the Vienna University of technology [31] [30]. It is a full HTC vehicle. The front suspension is based on a motorbike fork and steer-by-wire to offer STC, as illustrated in the figure 2.33. The rear suspension is made from a suspended swingarm supporting the DTC system. The two linear actuators act on two trailed arms supporting the wheels. A transverse lever and four associated rods constrain the motion of the two trailed arms. The whole appears as a very interesting motorized and active tilting rear suspension. The study uses the Multi-Body System (MBS) approach to analyze the stability and the handling characteristics through instructive handling diagrams.



Figure 2.33: Vienna University of technology prototype [31].

A team of the university of Tokyo studies the tilting three-wheeler vehicles. Firstly, they are interested in combination of a DTC in combination with a GTC [90]. The addition of the GTC part of the tilting controller decreases the wheel load difference on the actuated axle and, by the way, reduces the risk of wheel lifting or, worst, general over-rolling. The possible improvement is interesting but severe difficulties of implementation must be faced. Secondly, the team is interested in the free steering control. In this case, the vehicle tilting is controlled by DTC and the front wheel is free and self steered by the

caster and tire self aligning moment effects. It is an actuated version of the TVA. After the vehicle steering characterization [89], MBS simulations are used to prove the stability of the control method [88, 87]. The difficulty of control at low speed and in particular during maneuver remains the same as for the TVA. The concept could not be industrialized and commercialized without the addition of a device to solve the low speed controllability.

The paper [35] presents a research followed by the Nagoya University in collaboration with the University of science and technology of Wuhan and the Okayama University. The targeted vehicle looks like the Honda Gyro [119] but takes advantage of the parallelogram rear suspension to implement a DTC system. In final, the vehicle can be considered as a scooter with an improved stability

Three-wheeler in nabla configuration

The Mercedes F300 Live Jet prototype [115] has been presented at the 1997 Frankfurt motor show. It is a development vehicle for the Daimler-Mercedes group. The project goal was already “Feel and cornering dynamics of a motorcycle combined with the safety and comfort of a car”. The result is a functional tilting three-wheeler for two persons seated in tandem (figure 2.34). The tilt range is extended to 30° and managed by an active tilting system: a hydraulic piston act on the spring struts leading to a pure DTC system. The most interesting information given by the firm is that the F300 Live Jet is the first vehicle entirely designed by Daimler-Mercedes via a simulation tool. It was done with Computer-Aided Simulation of Car, Driver and Environment (CASCaDE), the in-house software developed by Daimler-Benz to optimize the design and the vehicle dynamics. It seems that advanced simulation tools open the way to develop new, more complex but more efficient vehicles.

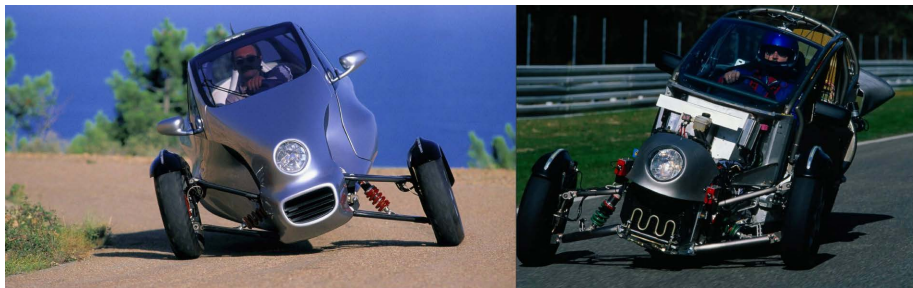


Figure 2.34: Mercedes F300 Live Jet [148, 155].

The Toyota i-Road [131] is the most known tilting three-wheeler dedicated to improving the urban mobility. The three-wheeler in ∇ configuration is illustrated in the figure 2.35. The vehicle morphology is unconventional because the front suspension is motorized and actuated but it is the single rear wheel that steers. By the way, thanks to the front suspension, which is based on longitu-

dinal pushed arms and to the un-steerable wheels, the space between the front wheels is free and can be used for the cabin purposes. In final, the vehicle can be occupied by two persons with a very compact volume. The vehicle driving is relatively straightforward and the stability is not particularly questionable. A feed-back from the DTC system allows the driver to limit the dynamics to the capability of the vehicle. The most perturbing effect comes from the rear wheel steering that brings a particular handling feeling. The vehicle behavior seems similar to a fork-lifter what is sufficiently perturbing without adding the tilting motion. Clearly, the steering is un-intuitive. The concept have arrived to the pre-series production step, so from 2014 to 2017, a fleet of i-Road was proposed with sharing location scheme in the French city of Grenoble. The results of this experience show the vehicles do not fit the expectations of the consumers. The specific steering is probably one of the most important factors to explain such failure.



Figure 2.35: Toyota i-Road [149].

The Moragette [71] is a nabla three-wheeler proposed by the Bochum University of applied sciences, in Germany. The front suspension is based on a double wishbone assembly. The DTC is implemented via a common arrangement used in many other DTC suspensions. It consists of an articulated part, called the T-pendulum, on the same axis as a pair of wishbones while the suspension shocks are connected between this T-pendulum and the wishbones. The actuator plays a torque on the T-pendulum so the shocks transmit the efforts to the carriers through the wishbones. The by-wire steering is fully independent. Each side, a linear actuator acts on the wheel carrier lever through a double rock arm and rod system in series. The arrangement is particularly complicated but compatible with the suspension and tilting kinematics and

suitable to implement a STC. So, the vehicle uses HTC strategy. The front suspension is compatible with a tilting angle up to 45° and presents a track of 1 m. The rear suspension is simply a swingarm. The paper indicated that the vehicle tests conclude by some problematic interactions between the tilting T-pendulum and shocks DOF. There is no known following research.

The Three Tilting Wheels (TTW) [2] project is developed by TTW srl company in collaboration with the Politecnico di Torino since 2005 (figure 2.36). Despite the geometry is not the same, the mechanical morphology of the vehicle is equivalent to the Moragette. The bottom shock connections are linked to the upper wishbones. The tilting is limited at the same range i.e. 45° but only controlled by DTC. The track is larger, 1.16 m, what is too much to be considered as nimble as a motorbike. Despite this drawback, the prototype is mechanically interesting because, thanks to the peripheral brake disc and the specially developed in-wheel electric motor, a full hybrid motorization is harmoniously incorporated to the suspension [34]. The remaining question concerns the unsprung masses which must be sufficiently low to offer an efficient suspension. The paper [2] presents the modeling of the three-wheeler done by both multi-body softwares: SimMechanics and Adams Motorcycles. A speed dependent modal analysis is performed. The study gives off several general conclusions about the three-wheeler dynamics. Considering parallelogram kinematic, the behavior and, in particular, the self-stability, i.e. without tilting control, of such three-wheeler is equivalent to the motorbike one.



Figure 2.36: TTW [2].

The Universidade federal de Santa Catarina, in Brazil and the Universitat politècnica de Catalunya, in Spain, have developed a simulation model to determine the behavior of a nabra three-wheeler face to perturbations from the ground or due to the wind [77]. It seems it uses the DTC system. Because these phenomena are too often excluded, we consider as interesting a study centered on the resistance against perturbation. Unfortunately, the authors do not give any technical information about the suspension and the tilting system implementations.

The University of Minnesota works currently on a new nabra three-wheeler. Multiple tilting control strategies have been tested in simulation and on a first scaled prototype [70]. They found a non-linear controller with linearized feed-

back as the best option to reduce the tilting actuation torque. In a second paper [53], the authors describe briefly a real prototype. The front pure parallelogram suspension and the scooter rear swingarm defined the morphology. The rear wheel is located in a very forward position, so much that it seems useless to lose footprint without increase the compactness of the vehicle. The paper is also focused on the synthesis of the control strategy. A HTC is combined to modulated tilting braking to control the vehicle. The study also explores the free-steering and DTC. Unfortunately, the paper does not give any mechanical implementation details.

The University of Waterloo develops another nabla three-wheeler [54]. The study aims to compare the un-actuated and actuated tilting vehicle performances in a curve. The simulations are performed with the help of Maple software. A simplified 12 DOF model is developed. In the paper, the authors do not give any technical details about the mechanical implementation of the suspension system and the tilting actuation.

Coming from the same university, the paper [91] describes an active DTC hydraulic system. The two pistons that replace the shocks are connected together. Two hydraulic pumps control the cross flow between independent wheel suspension. The highlighted advantages are the simplicity of the arrangement compared to other implementations and the capability to set the suspension height, what is useful in case of variation of relative mass as it is the case for such targeted vehicle. Indeed, the difference of mass between one thin person and two heavy persons can be substantial in comparison to the reference mass of the vehicle. The paper presents the hydro-dynamical model of the suspension and gives the evolution of variables for multiple scenarios: height setting and cornering. In a second paper [92], the author describes the integration of this system in the actual suspension. The nabla configuration is selected. The front suspension, which includes the hydraulic piston, is based on un-steering, trailing arm suspension while the rear suspension is steerable. The morphology is similar to this of Toyota i-Road and probably suffers from the same problem.

Four-wheeler

The Lumeneo Smera [120] is a four-wheeler tilting vehicle developed by the French company Lumeneo. This narrow vehicle is illustrated in the figure 2.37. The Smera was developed in collaboration with the Université Nantes Angers Le Mans. The vehicle, able to handle two persons in tandem, measures only 860 mm width. It inclines up to 20° and, by the way, allows moderate cornering performance in term of maximum lateral acceleration. The DTC is realized by a pair of T-pendulum¹ system implemented on both double wishbone front suspension and double trailing arm rear suspension linked together with a mechanical transmission coupling. The Maakaroun thesis [58] aims to develop a simulation tool specially adapted to the vehicle morphology. It develops a multi-body model for the analysis of the Smera behavior. The dynamic equation set is obtained by hand what is justified by the incapacity of commercial software to handle such unusual morphology. Some simplifications have been

¹The T-pendulum is called “lyre” in the French text from authors.

done to ease the manual process. From our point of view, it is useless to engage so much pain to develop dynamic equations by hands because it exists, from long time, several codes and softwares able to automatically generate the equations of motion based on a morphological description of the suspension kinematics (section 2.2). Obviously, the simulations and in particular the stability analyses done in the Maakaroun thesis keep their interest. The capacity to guarantee the stability with only 20° if the dynamics remains moderate is a key conclusion. The Mourad thesis [61] focuses the development of the tilting control with the objective of reduction of the actuation torque required by the DTC system. She does not use the model early developed by Maakaroun but a simplified 5 DOF model in order to simplify the synthesis of the tilting controller. She uses the flatness characteristic of the model to derive non-linear, progressive, combined HTC controller. The method is successful to improve the vehicle stability while decreasing the needed actuation torque. The industrial project was very complete and reached the pre-serie step. The EU certification had been obtained so the vehicle was proposed to sold. Unfortunately the Smera never encountered the expectations of the consumers. Certainly that the huge prize put the project out of the market. In final, the Lumeneo company went bankrupt in 2013 [120].



Figure 2.37: Lumeneo Smera [164].

The EU-LIVE project [111] is another EU founded project with objectives equivalent to the RESOLVE project. The project is led by the Peugeot scooter and the PSA group and implies numerous technical and academical partners. The four-wheeler result of the project shares some main characteristics with the Smera in spite being wider: two persons seated in tandem, double wishbone front suspension, double trailed arm rear one (figure 2.38). Despite a first prototype is already built, no technical information about the tilting control is available.

Nissan Land Glider [124] is a development prototype of a two seat tilting four-wheeler (figure 2.39). It is equipped with a steering-by-wire that could allow a hypothetical HTC. The very limited range of tilting i.e. 17° implies the low capability in term of lateral acceleration and, by the way, we consider this vehicle suitable neither in urban or in extra urban circulation.



Figure 2.38: PSA EU-LIVE [144].



Figure 2.39: Nissan Land Glider [147].

Toyota i-TRIL [117, 118] is one of the last development vehicles from Toyota corporation. It enters in the field of optionally autonomous vehicle. Nothing allows to determine the type of suspension selected and the associated tilting system. However, it appears the front axle is steerable, what switch with prior development concerning the i-Road, probably as a consequence of the lukewarm results of the i-Road sharing project in Grenoble, France.

The Naro [126] from The Narrow Car Company is a conceptual two seat four-wheeler. There is no technical information about dynamics, excepted it tilts and is supposed to be active.

College of Engineering, Chennai, India, has been developing a suspension morphology for tilting four-wheeler application [3]. The front and rear axle used respectively double and simple trailed arm on each side. The DTC torque is provided by step-motors and transmitted to shocks through transverse levers.

The Intelligent Personal Mobility (IPM) project [17] is developed at the National Chiao Tung University. The concept is very particular concerning the general morphology: it adopts a four wheel, diamond footprint. It is equivalent to add a tilting axle on a motorbike. The additional axle benefit the DTC in a global combined HTC. Despite this incongruous morphology, the design allows to sit two persons despite an overall length of 2 m, what is particularly compact. The very low scooter seated position seems not sufficiently secure and comfortable from our point of view. The paper presents some promising results concerning handling and stability, in simulation and in real test. Nevertheless,

we do not catch the interest of such morphology in comparison to a regular rectangular footprint.

Lots of other industrial and private projects exist over this census which does not have the ambition to be exhaustive but simply complete enough to give an overview of promising vehicles and to extract the main technological issues that must be solved to be able to take advantage of the tilting technology in a daily-use vehicle.

The Sindha paper [84] gives a review of the major developments in the field of the tilting vehicles. The proposed classification is different with respect to the present one. For each of these vehicles, it describes the methodologies chosen, the control strategies adopted, the types of prototypes chosen for test/simulation, the dynamical models applied, the key findings and future work proposed to address the dynamics-related issues about tilting vehicles. We agree with the general conclusions emerging from this state of the art paper. To be an industrial and commercial success, a proposed tilting vehicle should be simple, small, robust and low-cost. It seems that the efficient strategy to control the tilting motion, if required, is the HTC system.

Some websites contain more or less exhaustive listing of existing vehicles. Interested reader could consult the following websites. The blog [130] identifies and refers tens of both pedal powered and motorized tilting vehicles, active or not. The proposed classification is based on the vehicle building century. The French website [121] lists tens of tilting vehicles, indicating the used configurations, the actuation or not, the type of propulsion, the project initiators and the current state of the project. The website [122] treats a whole of miscellaneous vehicles including tilting three-wheelers. The author separates the motorized and human powered vehicles before sorting the vehicle by year of creation. The Wikipedia article [125] lists numerous trikes, but mix tilting and untilting vehicles.

Critic and conclusion about the tilting vehicles

Starting from the observation that the causality between the number of wheels i.e. the topology and the choice between tilting or untilting and, alike, its influence on the overall dynamics are confused, it seems essential to abolish, once and for all, some popular misconceptions.

In the beginning of this section, we remind the interest of the tilting in case of a narrow vehicle. It is clear the tilting motion leads to a more complex suspension kinematics and problem of controllability solved by the skill-full driver or by an automatized tilting controller system. The drawback of this complexity justifies itself only if a very low track has to be obtained. The Renault Twizy [127] is an untilting four wheeler suitable for urban use. This micro car is compared with the Quadro 4 in the figure 2.40. With only 1234 mm width, the vehicle can handle two persons in a comfortable seated position and in security face to the over-roll risk up to 90 km/h. Face to these performances, it appears completely useless to develop a tilting vehicle for urban human transportation with a width over 1100 mm. It is only below this critical limit that the tilting involved complexity could be warranted. This kind of conclusion is established

as a result of multiple studies as for example the CLEVER project [76]. A notable exception of this rule is the vehicles for which the fun factor is a pillar feature of the value proposition. The difference would be made between the leisure and the mobility objectives. Apart of that, the tilting technology is thus beneficial only for very narrow vehicle.



Figure 2.40: Quadro 4 and Renault Twizy [158].

In both tilting and untilting cases, for stability reason, the footprint of a particular configuration should be maximized as allowed by the overall dimension. It is useless to decrease the track of an axle if the general width can not be reduced. For this reason, we do not find any interest of a trapeze four wheeler whether tilting or not. In the same idea, the three-wheeler configuration is suitable only if a tilting dynamics is envisaged. For an untilting vehicle, the lost footprint of a three-wheeler with respect to an equivalent four-wheeler induces a dramatic drop of stability [33]. The gain in term of mass and number of components, often spotlighted by makers, is, for our point of view, ridiculous in comparison with the lost in roll stability. The ECAR333 [112] is a very illustrative counter-example of good practice. An untilting three-wheeler configuration was early selected what divested the vehicle of its full potential stability. Note that the current version of the project is equipped with a fourth wheel. The final trapeze four-wheeler configuration is not more that a wrong trade-off between minimal stability requirement and original characteristic of the ECAR333 project (figure 2.41).

Here again, the fun factor can constitute an exception to the rule. As examples, the Can-Am Ryker and Spyder [116] are both untilting three-wheelers (figure 2.42). The very low center of mass associated to a front track equivalent to a small car allow this configuration, an essential characteristic of such recreational vehicles. For the tilting three-wheeler, it is a bit more complicated. The use of one single wheel axle which is naturally tilting compliant is often used to avoid the implementation of a double wheel tilting axle, which is relatively complicated, at both front and rear axles. In this case, the three-wheelers appear as a possible trade off between the complexity and the roll

stability. Despite more and more recent tilting projects select the four wheeler configuration, numerous tilting vehicle projects select a three-wheeler one for this reason.



Figure 2.41: ECAR333 [142].



Figure 2.42: Can-Am Ryker [137].

As one can see, the field of tilting vehicle is largely open. Lots of proposals have been done either from the academic, industrial, private or mix environments. The projects have varying technical quality. If several passive commercial three-wheelers i.e. the simplest tilting vehicles after the motorbike and scooter, encounter a relative commercial success, the active tilting vehicles never penetrated the market until now, despite intensive research in this field from more than 30 years. It can be simply explained: the innovation and differentiation are not unconditionally vectors of the performance improvements. Tilt to tilt is not a successful approach because the results do not meet the exacting expectations from the consumers. The creation of tilting or untilting vehicles that deeply and positively impact the mobility starts by ambitious but realistic specifications in phase with market expectations, differentiated from the concurrence and in accordance with regulations. It is manifest that the dynamical issues associated with the multiple target problem are highly complex. The development of such efficient vehicles requires thus some adapted simulation tools of which the fundamental aspects are exposed in the next section.

2.2 Multi-body dynamics

The dynamics is the science which studies the interaction between the forces and the motion. Historically, the first attention concerned the kinematics of the bodies. In 1687, Newton have conciliated for the first time, both the kinematics and the forces in action. Thereafter, Euler has extended the laws to rotational motions. The dynamics of system made of several *bodies* articulated together by *joints* can be determined via the exploitation of the equations of motion.

For a particular mechanical system, this set of equations is obtained by the very classical methodology of which the main steps are:

- the identification of the bodies,
- the choice of the reference frames,
- the choice of the generalized coordinates,
- the formulation of the constraints,
- the computation of the kinematics,
- the identification of the applied forces and torques,
- the application of Newton and Euler laws that give the equations of motion.

From this time, the bases of dynamics are fixed and can be applied on mechanical systems made of a finite number of rigid bodies. This is a first limitation of the multi-body dynamics methodology. The method suffers for an important other limitation: the computation capabilities limit the system complexity i.e. the number of bodies and the complexity of the joints involved. This limitation has partially disappeared with the development of the modern computers. On the one hand, the numerical methods allow to handle the equations of motion which present high non-linearity and which are difficult to solve or to integrate, without any numerical method. On the other hand, the Information Technology (IT) algorithms allow to automate the process giving the dynamics equations and, thus, to be able to treat the mechanisms involving larger number of bodies. The modern MBS which is developed from 60'S, solves dynamics for the poly-articulated systems. Based on a formalism, the system morphology is described and an appropriate algorithm generates the equations of motion which are thereafter handled by some numerical methods to extract the relevant information depending on the problem. Here is the base of the multi-body dynamics.

There exist tens of codes and softwares concerning the MBS. They can be commercial or free, open or fully closed, stand alone or as an add-on of a larger tool, as a CAD software for example. The MBS software can be general or, on the contrary, very specialized to a particular field of application such e.g. the vehicle suspensions, the industrial robots, or the biomechanics elements. The MBS is often associated to other simulation tools to extend the capability of the tool. As already exposed, the first limitation of MBS is the hypothesis of rigidity of the involved bodies. This limitation can vanish by the extension of the basic formalism. Stiffness lumping, finite segments or Finite Element Method (FEM) are some technics used to treat a flexible body. FEM or particle method are frequently coupled with MBS for the overall simulations of sets involving flexible structures, granular or fluid medias in combination with poly-articulated mechanism. In this work, all bodies, in the meaning of MBS formalism is considered as rigid so the flexible MBS technics are neither used nor exposed.

2.2.1 Introduction to Robotran

Robotran [128] is a MBS software dedicated to poly-articulated assemblies of rigid bodies. The software is divided into three main parts:

MBSysPad is the graphical editor used to describe the morphology of the model in the respect of a suitable formalism.

MBSysTran is the generator which, based on the morphological description, derives the set of dynamics equations.

MBSysC is a set of modules which, with the help of the appropriate numerical methods, handle the dynamics equations to perform particular analyses: equilibrium, modal analysis, time simulation, etc.

The software is developed in-house in the Institute of Mechanics, Materials, and Civil Engineering (iMMC), Université catholique de Louvain (UCLouvain). It is the result of contribution from multiple PhD students and researchers. As a generalist MBS tool, it was applied in the fields of research as various as:

- the generation and parallelization of the dynamics equations via the symbolic approach [74],
- the symbolic modeling of multi-body systems including electromechanical devices [81],
- the simulation of redundantly actuated parallel manipulators [37],
- the optimization of closed-loop mechanisms [20],
- the modeling of pneumatic suspensions for railway applications [25],
- the sensitivity analysis of multi-body systems via the symbolic approach applied on the subjective evaluation of the cyclist effort [73],
- the synthesis and the optimization of medical rehabilitation robots [36, 80],
- the quantification of joint and muscle efforts in the human body during motion [75],
- the quantification of intervertebral efforts with application to scoliosis surgery [1],
- the analyses of early piano action mechanisms [11],
- the optimization of locomotion control for humanoid robots [96],
- the gaze stabilization of humanoid robots [41],
- the coupling of multi-body and granular media dynamics with application to railway track [56].

As one can see, Robotran is very versatile. By its main approach which requires to come back to basic kinematic elements, Robotran is also particularly appropriate to model unusual and innovative vehicle suspension as done in this work. Indeed, the inflexibility of some commercial software has often be decried. The limitation pushed some authors to come back to some hand-made models what is absolutely not at all optimal. Moreover, the code is relatively open, especially for the development team we are part of. So, it is easier to build advanced optimization algorithm using the existing numerical modules as blocks for larger construction.

2.2.2 Formalism

The formalism defines how the mechanics is described as well as how the elements are established to simulate the actual device. It also specifies the manner the equations are written.

The MBS is declined in three distinct formalisms following the manner the kinematic is described by the coordinates. There exist absolute, relative and natural coordinates. Each of them brings advantages and drawbacks, less or more significant depending on the concerned problem, so there is no universal best method. In this work, we consider the formalism adopted in the Robotran software, that opt for the relative coordinates.

The mechanism is described by a succession of pair joint-body, each adding a possible motion. The structure starts obligatorily from the base, that is supposed to be inertial. The chains can split but never converge so the general structure can be related to a tree structure: starting from the ground, the trunk is divided in multiple branches, themselves divided into smaller branches and so on until the final leaves. It is the reason why the multi-body structure is widely called a *tree-like* structure. The downstream elements are called children elements while, on the contrary, the unique direct upstream element is called the parent element. The last bodies of the chains are leaf elements.

In Robotran, there are only two types of joints, the prismatic one and the revolute one, each declined in the three main directions. All more complex joints can be built by a combination of these basic elements. Each joint is associated to a coordinate. They are designated as generalized because they can be a displacement or an angle depending the type of the joint. The reference configuration is defined by those for which all coordinates are set to zero. By convention, a body is associated to each joint. The body can be fictive or materialized. A fictive body is simply made of a reference frame between two consecutive joints as the manner as an intermediate kinematic element. The reference frame origin is located at the arrival of the parent joint and the frame is aligned with the inertial frame in the reference configuration. A materialized body involves a reference frame but also a mass, a center of mass given in the body frame with respect to the body origin and an inertia matrix defined in the body frame with respect to the center of mass. The body description is completed by a list of anchor points on which the children joints can be plugged, the forces or torques can be applied or for which the kinematics can be computed and/or constrained. These are defined in the reference frame associated to the body.

The whole of the actual mechanisms can not be represented by a tree-like structure. Indeed, in the general case, there are multiple paths to pass from a body to another. The presence of such *kinematic loops* differs with the formalism requirement of tree-like structure that had to be restored for equation generation purpose (see below). So, each loop must be open by introduction of a *cut*. From a general point of view, a cut is a set of kinematic constraints which, when satisfied, guarantee kinematic equivalence between the tree-like model and the actual closed loop mechanism. In Robotran, there are three cuts available:

Solid cut is used when there is no possible motion between the two related bodies. It introduces six algebraic constraints: three in translation, three in rotation. This is equivalent to a rigid assembly.

Ball cut is used when the two related points are constrained to be confused. It introduces three algebraic constraints in translation. This cut can be used to open a loop instead of a perfect ball joint i.e. friction free and clearance free.

Rod cut is used when the two related points are maintained to a given constant distance. It introduces a single algebraic constraint. This cut can be used in replacement of a perfect rod: a rod without mass and with perfect joints at its two ends.

The selection of the appropriate cut result a trade-off between computation complexity and underlying hypothesis. As example, the rod cut is very efficient because it allows to suppress a body (the rod itself) and up three joints and their associated generalized coordinates, and introduces a single algebraic constraint. However the rod cut is the one that involves the more substantial hypothesis.

Note that the constraints can be introduced for multiple other reasons that for loop cutting purposes.

In comparison with the tree-like structure, the closed loop mechanism enjoys less DOF due to introduced kinematic constraints. So, all the generalized coordinates do not correspond to real DOF. A first distinction must be operated between the *independent* and *dependent* coordinates. The independent coordinates are "true" DOF and are integrated or solved based on the *reduced* dynamics equations (see below). The dependent coordinates are computed to satisfy the constraints. In the standard case, the number of dependent coordinates is equal to the number of constraints. In some particular case, the constraints are redundant or a priori satisfied so the number of effective constraints is reduced.

A second distinction has to be done between independent or dependent and *forced* or *driven* joints. While the first group are computed following the equation of dynamics and constraints, the second one is specified by the user. The forced joints are used in case of joints locked in a given position or angle. The main use is a change of reference frame orientation, for example, to implement joints that are no aligned with a reference frame direction. In this case, the coordinate value is a constant. For each driven joint, the user specifies the position, speed and acceleration of the coordinate for the entire the simulation. It is a way to control the kinematics of a particular joint.

In final, there are 4 natures of joints: independent, dependent, forced and driven.

The kinematics of a particular point can be obtained by the *sensor* tool. It provides the position, the speed and the acceleration of the point as well as the rotation, angle, speed and acceleration of the associated body frame in the initial frame.

The MBS model is completed by the introduction of forces that can be internal or external. The internal forces act between bodies with direct parenting, through the joints. Only the generalized joint forces, i.e. the forces or

torques aligned with the joint can be controlled. The generalized forces are a force in case of a prismatic joint and a torque in case of a revolute joint. The typical application are joint internal friction, motor torque, piston force etc.

The external forces are applied on particular points via a dedicated tool. For each, the force and/or the torque is computed by an user model on the basis of kinematic information identical to those provided by a sensor. It is used to take into account the effort from the environment on the model such as, for example, the contact effort of the ground to a wheel, the aerodynamic forces and torques on a main body etc.

Sometimes, it is relevant to replace an internal element by an equivalent effort. In this case, both the action and the reaction have to be inserted into the model. For convenience purpose, the *link* allows to insert, with the same tool, the action and reaction, in the correct and coherent direction, based on the information of relative kinematics, the most suitable one for such type of application. Compared to a double external force, the operation is obviously easier. For the unidirectional version of the link¹, the only one used in this work, the user synthesizes the force law based on the displacement and speed between the two anchor point involved. In vehicle dynamics analyses, the springs and dampers and the combined shocks of suspension are typical application candidates of such links. Note than, in case of a such replacement, the mass of the device has to be sufficient low to be neglected.

Example: a quarter car model

In order to illustrate the different elements of the formalism through a practical example, we give the description of a quarter car model implementing the double wishbone suspension. A description of such a type of suspension could be found in the section 2.1.3.

The model is mainly based on the topology description of the MBS assembly what is given in the figure 2.43.

Starting from the *base* to which is fixed the inertial frame origin, the *body* of the car is articulated through a single prismatic joint in the z direction, q_1 . This motion corresponds to the bounce of the main *body*. The other DOF of the *body* are not considered in this simple model.

The *wishbone_dn* and *wishbone_up* are articulated with respect to the *body* through two revolute joints for which q_4 and q_{12} are the coordinates, respectively. The joint axes are oriented in the space, firstly in the z direction and secondly in the *relative* y directions, thanks to the forced joints q_2 , q_3 respectively, for the *wishbone_dn*, and thanks to q_{10} , q_{11} for the *wishbone_up*.

The *carrier* is linked, at its two extremities, to the wishbones through suspension ball joints considered as perfect. The bottom one is modeled by a series of three rotational joints: q_5 , q_6 and q_7 . In order to recover a MBS tree-like structure despite the suspension kinematic loop, the upper ball joint is taken into account through a ball cut, which allows to express the associated loop constraints.

¹The general three dimensional link exists also in Robotran.

The *wheel* is articulated w.r.t. the *carrier* through a revolute joint, q_9 , for which the axis is inclined around the relative x direction by the forced joint q_8 .

The *shock* takes place between the *wishbone_dn* and the main *body*. The shock is considered as perfect, i.e. massless and frictionless (at its ends articulation). It is modeled via a link i.e. a point-to-point force.

A F sensor is fixed on the center of the wheel. It aims to manage the ground-tire contact: it allows to measure the kinematics of this body and to apply the force and torque resulting from the tire contact model.

The S sensor located on the carrier allows to measure its kinematics.

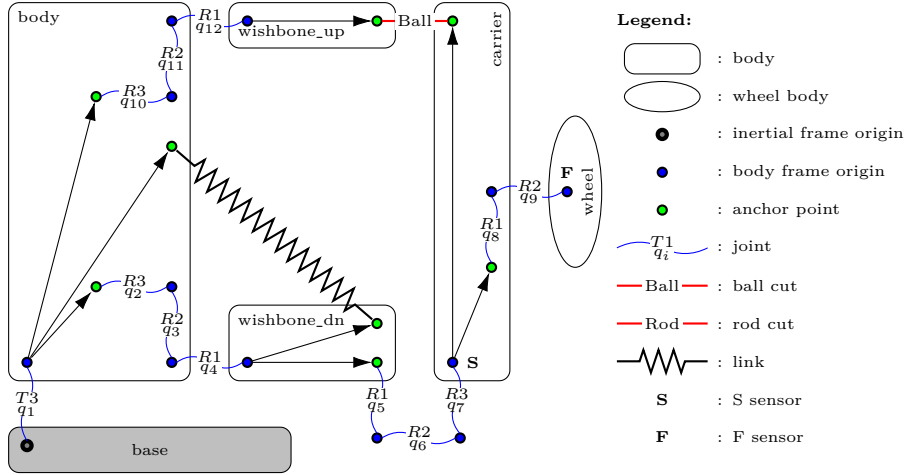


Figure 2.43: Formalism elements: a quarter car multi-body model example.

2.2.3 Generation of dynamics equation

The Robotran generator [26] is used to generate dynamics equations based on the description of the mechanism in the formalism just introduced. The generator algorithm proceeds in two recursive steps: a forward kinematics computation and a backward Newton-Euler dynamic formulation. In the first step, the algorithm starts from the base and progresses, joint by joint, through the tree to farthest elements i.e. the leaf bodies. The local kinematics is computed by addition of the motion associated to the relative coordinate of parent joint to the kinematics of the parent body. For the second step, because the kinematics is thus fully known, the Newton-Euler equations can be written for the leaf bodies, and so on, in backward direction, to the base. The recursive characteristics of the algorithm justify the requirement of a unique possible path from the base to a particular body, a tree like structure which, if it is not natural, must be restored with the help of cuts.

The equations of constraints and their associated contributions are added to the tree-like structure set of equations to take the kinematic loops and other constraints into account. At the end, we get the following Differential Algebraic Equation (DAE) system:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{c}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{frc}, \mathbf{trq}, \mathbf{g}) = \mathbf{Q}_q + \mathbf{J}^T \boldsymbol{\lambda} \quad (2.7)$$

$$\mathbf{h}(\mathbf{q}) = \mathbf{0} \quad (2.8)$$

$$\dot{\mathbf{h}}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}} = \mathbf{0} \quad (2.9)$$

$$\ddot{\mathbf{h}}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) = \mathbf{J}(\mathbf{q})\ddot{\mathbf{q}} + \dot{\mathbf{J}}\dot{\mathbf{q}}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{0} \quad (2.10)$$

where $\mathbf{q}$ are the generalized coordinates of the multi-body system, $\mathbf{M}$ is the mass matrix, $\mathbf{c}$ is the vector which consists of centripetal, gyroscopic and Coriolis effects, gravity $\mathbf{g}$, external forces $\mathbf{frc}$ and torques $\mathbf{trq}$, $\mathbf{Q}_q$ is the vector of the joint generalized forces, $\mathbf{J}$ is the Jacobian matrix of the constraints, $\mathbf{h}$, $\boldsymbol{\lambda}$ are the Lagrange multipliers associated with the constraints.

The Robotran generator adopts the symbolic approach to generate the constrained equations of motion and to simplify them, which is relatively specific to this software. This simplification procedure allows to save significant part of the computation needs, during the numerical use in modules, without any lost of precision. The method uses the algebraic rules to simplify the expression that reduces the computation complexity but is possible only thanks to the symbolic approach. From a practical point of view, the generator produces IT functions that output one the matrix of the system: $\mathbf{M}$, $\mathbf{c}$, $\mathbf{J}$, $\mathbf{h}$ or one of their derivatives. The symbolic approach allows, also, among others, to get easily the function file in multiple possible programming languages like C, Matlab, Python, Fortran etc.

2.2.4 Coordinate partitioning

A DAE system such the (2.7-2.10) are not straightforward to handle. There exist special methods able to master them but they suffer multiple possible drawbacks: lost of precision, low robustness, poor efficiency etc... For these reasons, we prefer the use of the coordinate partitioning method which, based on the partitioning of the generalized coordinates according to their nature, allows to simplify the initial DAE systems into a reduced Ordinary Differential Equation (ODE) systems. This last one has better properties that make it easier to handle by the numerical method.

The coordinate partitioning method separates the coordinates in three categories: the independent coordinates $\mathbf{q}_u$, the dependent coordinates $\mathbf{q}_v$ and the forced and driven coordinates $\mathbf{q}_c$. Here, the forced and driven coordinates are treated together because there is no mathematical reason to proceed otherwise. The number of DOF of the MBS is given by the number of $\mathbf{q}_u$. The partitioning allows to reorganize the equation 2.7. In a first reduction procedure, the lines of the system corresponding to dependent coordinates are isolated, so the Lagrange multipliers can be computed and re-injected in the lines corresponding to the other coordinates. In a second reduction, the lines corresponding to the forced and driven coordinates are simply turned aside because the dynamics is already known. After some matrix manipulations, the initial DAE system is reduced to a simpler ODE system, in terms of the independent coordinates:

$$\mathbf{M}_r \ddot{\mathbf{q}}_u + \mathbf{F}_r(\dot{\mathbf{q}}_u, \mathbf{q}_u) = \mathbf{0} \quad (2.11)$$

For a particular closed loop system, there are several possible partitionings. The determination of the amount of dependent coordinates and their selection can be done by the user or by a numerical method. The most important objective is to avoid the numerical singularity which corresponds to a mechanical one. In a second time, the precision and the convergence of the constraints resolution can be maximized. In case of numerical selection, the algorithm prefers the dependent coordinates which give the highest pivot values during the triangulation of the Jacobian matrix of constraints $\mathbf{J}$, which is the mathematical expression of singularity avoidance. More information about the two steps reduction and the numerical coordinate partitioning can be found in [78].

2.2.5 Static equilibrium

About the vehicle dynamics analysis as for other purposes, the first interest concerns the position of the model at rest i.e. the determination of the configuration adopted by the model without any action or perturbation. From a MBS point of view, the static equilibrium analysis consists in searching the independent coordinates $\mathbf{q}_u$ that satisfy the equilibrium equation:

$$\mathbf{F}_r(\mathbf{q}_u) = \mathbf{0} \quad (2.12)$$

which is the simplified form of the equation 2.11 where all accelerations and velocities are forced to zero.

From a vehicle dynamics point of view, this process can be used to compute the configuration of the vehicle at rest as a starting point for a time integration simulation or as reference conditions for modal analysis. Moreover, the results can be used for itself e.g. the computation of the body position at rest: height, roll, pitch, etc.

It must be noticed that some variables can be *ignorable*. As a reminder, a variable is defined as ignorable if it has no influence on equilibrium equations. For example, in the case of equilibrium for a vehicle at rest on a flat ground, the DOF for longitudinal position, lateral position, yaw angle and the wheel rotations are completely ignorable. Despite the solving algorithm could detect such ignorable variable, it is preferable if the user indicates those which he has the knowledge, improving, by the way, the convergence of the solver.

Solving

The solution of equilibrium equations (equation 2.12), are straightforwardly found via an iterative solver e.g. Newton-Raphson (NR) based algorithms. Because these equations are fully non-linear, some numerical tricks are implemented to allow or to improve the convergence. Initial values for equilibrium variables are roughly precomputed according to the target configuration. Then, some relaxations are applied during the solving. In some cases, they are only applied on some variables and, in other cases, according to the relative numerical stiffness, variable by variable.

2.2.6 Quasi-static equilibrium

The quasi-static equilibrium aims to analyze the steady-state behavior of MBS model. Concerning the vehicle dynamics, it relates to the straight line travelled at constant speed and the steady-state cornering. Here again, the results can be used as the input for a time simulation or a modal analysis or for itself i.e. for vehicle characterization purposes. Indeed, despite the pure steady-state configurations are rarely encountered in practical rolling, a such quasi-static analysis allows to capture a substantial part of dynamics behavior.

The quasi-static equilibrium analysis is the extension of the static one where the position *and* the velocity of the generalized coordinates can take non null values. Only the generalized coordinate accelerations are maintained to zero.

Note that the speed of a generalized coordinate which is ignorable is not necessarily ignorable. For example, the wheel rotation of a car is ignorable, as it was already mentioned, but the speed of the wheel rotation is not because it acts in force equilibrium through the tire response. In case of quasi-static equilibrium, the set of variable, $\mathbf{x}$, must be specify, involving either non ignorable independent coordinates and non ignorable derivatives of them.

$$\mathbf{F}_r(\mathbf{x}) = \mathbf{0} \quad (2.13)$$

The kinematic chain must be such it is possible to describe the targeted steady-state motion for which all the independent coordinates have a constant value either for the position or for the speed. The particular requirement needs some specific developments explained, in details, in section 3.4.2.

The quasi-static equilibrium problem is solved with the same iterative solver as for the static one.

2.2.7 Modal analysis

The modal analysis gives the information on the eigen vibration modes of the poly-articulated system.

This linear study is always performed around an equilibrium configuration which can be both static or quasi-static. The $\mathbf{F}_r$ vector must be first linearized with respect to independent coordinate speeds, $\dot{\mathbf{q}}_u$ and positions, $\mathbf{q}_u$ around the equilibrium solution $\mathbf{q}_u^{eq}$. Robotran uses an iterative parabolic fitting method to get a maximum precision in this operation. We must notice the quality of the final results depends on both the linearization precision and the reference equilibrium result quality. The system of equation 2.11 is transformed into the linearized form:

$$\mathbf{M}_r \Delta \ddot{\mathbf{q}}_u + \mathbf{G}_r \Delta \dot{\mathbf{q}}_u + \mathbf{K}_r \Delta \mathbf{q}_u = \mathbf{0} \quad (2.14)$$

Then, this second degree linear system is transformed to a first degree double-sized system and the GNU Scientific Library (GSL) library is used to extract, for each eigenmode, the eigenvalues and the normalized eigen vectors. Based on that, the modes are classified following their types: oscillating or not,

stable or not. The angular frequency ω or the frequency ν ¹ and the damping ratio ζ are computed for oscillating modes.

In vehicle dynamics, the modal analysis helps to characterize a vehicle in terms of comfort and/or of stability. On the one hand, the frequency and damping rate of suspension associated modes indicate the quality of the suspension stages and, in final, define the vertical dynamics comfort characteristic of a vehicle. In this case, the modal analysis is performed around a static equilibrium or a quasi static equilibrium representing a straight line rolling. On the other hand, the modal analysis highlights and quantifies potential instabilities arisen from, for examples, the steering mechanism or of the overall roll dynamics, especially in the case of tilting vehicle. Such studies are mainly done around the straight line rolling and the steady-state cornering.

2.2.8 Time simulation

A time simulation aims to capture the transient behavior of a targeted mechanism. The objectives is then the getting of the kinematics of a particular mechanical system submitted to efforts. To be precise, the time simulation module computes the value of coordinates, at position, velocity and acceleration levels, as functions of the time, by numerical integration of the MBS equations of motion. In addition to the equations which derive from the morphological description, the process needs the description of effort either as predetermined functions of time or as functions of currently, i.e. at each time step, available data like kinematics coordinates. The integration requires also the initial conditions for the generalized coordinate positions and speeds, which, without this being mandatory, can be the result of a static or a quasi-static equilibrium.

The time simulation consists of two embedded numerical processes. The main one is the time integration of both the mechanical dynamics equations 2.11 and optional user model state equations. The variables of this time integration are the independent coordinates $\mathbf{q}_u$ and the user model states $\mathbf{z}$. The direct dynamics module process is depicted in the figure 2.44. The integrator must be selected depending on the nature of the physical problem. A Runge-Kutta integrator is suitable for regular problem. In case of stiff problem i.e. when quick changes of force magnitude occur as for impacts, a specific integrator has to be adopted. In this case, Gear and Adams-Bashforth methods are, for examples, good candidates.

The constraint solving is the embedded second numerical method acting in the time simulation. The constraint equations are solved at each time step of the integration. As reminder, this loop closure is realized by a NR based algorithm.

As for all the numerical integration processes, an attention must be given to the time step length. In a fixed time step scheme, this must be set by the user as it must be done for the maximum time step in variable time step scheme with adaptation following precision criteria. The optimal - maximum - time step is a trade off between the precision of the results and the Central Processing Unit

¹The angular frequency ω [rad/s] and the frequency ν [Hz] are linked with the relation $\omega = 2\pi\nu$.

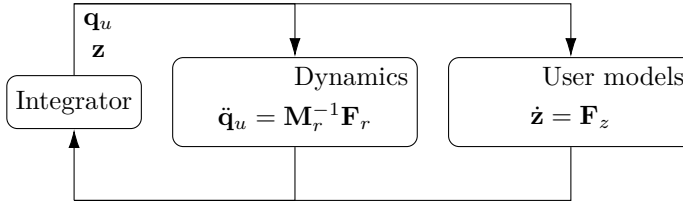


Figure 2.44: Time integration simulator.

(CPU) time requirement. To capture the motion with an acceptable precision, the maximum time step must be, as a minimum, one order of magnitude lower than the period of the highest frequency mode. The modal analysis can thus help to determine the suitable time step. Moreover, the time step must be sufficiently low to allow the loop closure process to converge¹.

Because the constraints are solved, the forces are computed following the user models, the equations are reduced according to the coordinate partitioning and the independent coordinates are solved, all at each time step, the time integration is a particularly CPU time consuming process. Despite this, thanks to a recursive formulation in relative coordinates and to the algebraic simplifications achieved by the Robotran symbolic generator, a dynamics simulation of a simple vehicle model can be done lightly faster than real time². By this way, a real time simulation including in simulator is also possible.

The time simulation is the most complete analysis but the most difficult to interpret. The output is very rich and the possibilities of simulation are very wide so an efficient vehicle characterization is achieved with the time simulation tool only defining suitable comparison framework and using well defined performance criteria.

2.2.9 Conclusion about multi-body dynamics

The multi-body dynamics is used to determine the behavior of polyarticulated mechanisms. While this technique is suitable for a large application field, it is particularly adapted to the simulation of road vehicles, one of the major applications.

The present description has been relatively oriented to Robotran, the used software and to the relative coordinates formalism.

We have presented the way the equations of motion are generated on the basis of the morphological description and how the equations are exploited by the numerical modules. These modules complete each other. Each one gives particular outputs, all fully interesting in the case of a vehicle analysis. Likewise,

¹In case of specific problem concerning this requirement, it is possible to implement additional numerical stuffs like the integration of the dependent coordinates in order to use the outgoing results as better initial conditions for the closure algorithm.

²The simulation of the WEMOOV rolling on a test track run in 14.1 s for 16.7 s of real time. It is programmed in C, compiled by the standard compiler of Visual Studio from Microsoft with optimization of speed performances and run on a single core of an Intel i7.

the module can be used sequentially, the result of the first being used as the input of the follower. Their combination enriches the vehicle characterization and is thus fully recommended.

The interested reader can found deeper information about the multi-body technique and related formalisms in [78].

2.3 Optimization

Once a technique is relatively mastered, that is, it is well understood, successfully implemented and/or precisely simulated, it becomes quite natural to hope going ahead to the next step in the continuous research of efficiency and performances: to optimize it.

Optimization is “the action or process of making the best of something; (also) the action or process of rendering optimal; the state or condition of being optimal”¹. Depending on the objects, objectives and then criteria are different. It is or should be the main target for every privates and professionals whatever their activity fields. Technicians and engineers do not except to the rules. An important part of the work consists in improving endlessly the methods, processes and products.

To help to face such an arduous task, the mathematician has conceived an arsenal of tools able to handle the optimization problems when it can be represented through a mathematical model.

In this section, we first start defining a framework for optimization problem. Then, we describe the most important optimizer families relevant for the problem of suspension and give their main properties. In final, we summarize the state of the art of the optimization of the road vehicle suspensions on which we focus.

2.3.1 Optimization problem

From the mathematical point of view, an optimization problem is based on a model which depends on a set of variables so called the *parameters*, $\mathbf{p}$. They represent the settings on which it is possible to act on the object, in order to improve it. In general, the variables present different natures, physical quantities and by consequent different units that complexifies the problem formulation. Similarly, they can be continuous or discrete. For each of them, an interval of admissible value is specified. The boundaries of these intervals define the optimization work-space of which the dimension is equal to the number of parameters. On the other side, the object must be characterized by a set of criteria which quantify the efficiency and performance regarding each a particular objective. The *criteria* are functions of the parameters through the model. If for the simplest model, the criteria are given by simple explicit expressions of the parameters, for the complex model the criteria can be the results of multiple computation procedures including numerical solvings and integrations. For

¹From the Oxford English dictionary.

this work, we limited the scope to a deterministic model i.e. the model gives always the same criteria value for a particular set of parameters.

$$\mathbf{E} = \mathbf{E}(\mathbf{p}) \quad (2.15)$$

Despite it is the easiest case, the optimization involves rarely a single criterion. An arbitration problem appears with multiple criteria. Indeed, if for a mono objective problem, the optimal solution is unique; for a multiple objective problem, the whole of undominated¹ solutions forms a space of dimension equal to $n_p - 1$, called the Pareto front. Two possibilities are offered to the user, either he is interested in knowing the whole of optimal solutions or he is able to arbitrate the different criteria. Therefore it is possible to reduce the problem to a mono objective one with a unique solution. In the second case, the user must also provide the *mix law*, the mathematical expression of the arbitration. The mix law is a combination of the different criteria. The heterogeneity of the criterion natures, the physical quantities and the units have to be managed. In the simplest way, a weighted sum of criteria forms the global objective but a more advanced mix can be synthesized depending on the problem.

$$F_o = F_o(\mathbf{E}) \quad (2.16)$$

The optimization problem is often completed by a set of optimization constraints, expressed as a set of equalities or inequalities:

$$\mathbf{C}_=(\mathbf{p}) = \mathbf{0} \quad (2.17)$$

$$\mathbf{C}_>(\mathbf{p}) > \mathbf{0} \quad (2.18)$$

$$\mathbf{C}_\geq(\mathbf{p}) \geq \mathbf{0} \quad (2.19)$$

These expressions of constraints can be also diversified as those of criteria, despite there must not confuse these two elements. So a solution is acceptable only if it respects the constraints, leading to a reduction of the optimization domain.

There exist multiple manners to deal with optimization constraints.

First, it is possible to reduce the work domain by excluding the area where the constraints are not satisfied. The inconvenient of this method is that the precomputation of the domain can be resource consuming and complex and that it can lead to discontinuous domain what is inappropriate for some optimization algorithms.

Secondly, the constraints can be treated as objectives. In addition to the increase the number of objectives and the requirement of a multi-status mix law, the method suffers some major drawbacks: the constraints are never fully satisfied and it complicates the global objective function. This is a very simple method but very poor in term of performances.

Third, in the same manner as the constrained multi-body equations are solved, the constrained optimization problem can be solved by parameter partitioning. The parameters are divided into independent and dependent parameters. The dependent parameters are used as variables to solve the constraints

¹A solution is dominated if it exists another which is better for each criterion.

while the independents remain driven by the optimizer to find the optimal solution. By the way, the size system is reduced and the constraints are exactly satisfied. This is the method used for the optimization in this work (section 3.1.3). The advantages and drawbacks of such a method are treated in detail below.

There are also other advanced methods. Some of them suffer limitations, especially in the possible expressions of constraints. This subject is not treated in this work.

Globally, the complexity of the problem depends on the number of parameters, the number of objectives and constraints and their complexity. The resolution time is also influenced by the computation time of criteria and constraints. There are numerous properties that define an optimization problem. The most important is the convexity of objective function. As reminder, a domain is convex if there is not any straight line between a pair of arbitrary points intersecting the boundary of the domain. The property comes on the front of the scene because it guarantees the absence of local optimum. The contiguity of the parameter domain and the continuity of the criterion and constraint functions are other important characteristics that influence the choice of the optimization algorithm.

As explained later, the vehicle suspension optimization problem involves a large number of parameters and objectives. In the approach developed in the next chapter, we select the mix law method to reduce the problem to a simple objective function problem. The criterion, the mix law and the constraint functions are so complex and irregular that no interesting property can be assumed. Only the parameters domain is continuous and contiguous. By choice, we restrict the analysis to continuous i.e. not discrete parameters. Some preliminary tests have shown that such global objective functions are not convex and consequently involve local extrema.

2.3.2 Optimization algorithms

Except for the simplest models, that is never the case for the real vehicle suspension concerns, the optimization problem find its solution with the help of an iterative numerical algorithm. The multiple methods are classified according to they are based on deterministic or stochastic algorithms and depending on requirements of some properties like convexity or contiguity of different domains and continuity, linearity or quadratic behavior of different functions. Note that if the satisfaction of the required properties ensures, for example, either the convergence of the method or the existence of the solution, the absence of satisfaction does not necessary lead to the failure of the method, so some methods are regularly used without the adjacent problem properties.

Because, for the suspension optimization problem, there can not take advantage of positive properties, the range of suitable optimization algorithm is reduced. Indeed, the following classical method families have to be rejected for the main reasons:

- The direct analytic methods because the objective function is not and can be not approximated by a simple analytic function.

- The zero of gradient researches because the objective function is not continuous and because the method is highly sensitive to the local extremum.
- The combinatorial optimization techniques because the parameter domain is mainly continuous and not discrete.

In final, only the heuristics and meta-heuristics method ranges are fully adapted to the problems involving numerous local extrema. Among this category and subcategories, the most common are:

- the particle swarm optimization,
- the evolutionary algorithms:
 - the genetic algorithms,
 - the covariance matrix adaptation evolution strategy,

In general, these optimization algorithms are based on a large number of *particles* for which the objective function is computed. Following the method prescribed by the type of algorithm, the results allow to determine the parameters for a set of particles for the next iteration and so on until the convergence. There is no direct handling of optimization constraints.

Particle swarm optimization

The Particle Swarm Optimization (PSO) [51] is based on the displacement of a large whole of particles which, under conditions, converge to the minimum of the objective function. At an iteration step i , the next speed of a particle, $\mathbf{V}_{i+1}$ is given as a weighted sum of three terms:

- an inertia term based on the current speed $\mathbf{V}_i$,
- the error between the current particle position and the best historical position of the particle, $\mathbf{P}_p$,
- the error between the current particle position and the best historical position of the neighbor particles or for the entire swarm, $\mathbf{P}_s$.

$$\mathbf{V}_{i+1} = \omega \mathbf{V}_i + a_1(\mathbf{P}_p - \mathbf{X}_i) + a_2(\mathbf{P}_s - \mathbf{X}_i) \quad (2.20)$$

$$\mathbf{X}_{i+1} = \mathbf{X}_i + \mathbf{V}_{i+1} \quad (2.21)$$

where ω is the inertia and a_1 and a_2 are respectively two weights gotten randomly in $[0, \phi_1]$ and $[0, \phi_2]$. The convergence criteria relate to the speed of the particles, tending to zero at the end of the optimization. The method is very simple to understand and to implement but the performances are conditioned by the selection of the weights which determine the convergence and the tendency to be subjected to the local minima.

Genetic algorithms

The genetic algorithms are based on the transcription of the particle optimization parameter values into a genetic description: the chromosomes. From a theoretical point of view, the method is inspired from the Darwinian evolution theory. For the basic algorithm, between the initialization and termination steps, the iteration loop repeats three main operations:

The selection: The objective function is computed for each particle that allows to determine a classification of the candidates. The algorithm selects only or preferentially a given amount of particles among the best ones.

The crossover: During this operation, pairs of selected particles, the parents, are formed and exchange some genomes of their chromosomes either randomly or following some deterministic rules. The obtained chromosomes define the particles for the next generation, the children. Thanks to the selection and the crossover, the children are supposed to be, on average, more efficient with respect to the objective function, than the previous generation elements.

The mutation: The optional step affects randomly a part of the chromosomes of some particles. The operated change fights against segregation. It aims to avoid premature convergence and thus the risk of local minima finding.

The options and variants of such a genetic algorithm are unlimited, as well as the possible algorithm parameters and the manner to parametrize them. It is a very rich class of algorithm why it has found numerous applications in the technical and scientific fields as in many other applications. Likewise, the literature on this subject is very large.

Covariance matrix adaptation evolution strategy

The Covariance Matrix Adaptation Evolution Strategy (CMA-ES) [43] is an algorithm especially developed for the non-convex optimization problem characterized by a large number of parameters and a lack of the objective function determinant or a numerical estimation of this one. The objective function is assumed to be continuous.

The main idea stands the evolutionary strategy could be improved if the stochastic exploration of the parameter space follows some rules with more theoretical bases than the motion of the swarm or the genetic crossover. It aims a better convergence and to improve the resistance against the local minima.

The process is based on the iterative incrementation of state variables. The main ones are:

- the object parameter vector,
- the covariance matrix of mutation distribution,
- the evolution path,
- the conjugate evolution path,
- the step size.

At each time step, each variable is computed as a weighted sum of its previous value and an expression involving independent realizations of stochastic distribution and combination of the other variables. Practically, it consists in exploring the parameter domain with particles generated aleatory but in a controlled manner i.e. such as the equipotential surface center, shape and size shift regularly. Indeed, the use of the incremental variables and, in particular,

the covariance matrix and both main and conjugate evolution paths guarantee the mutation distribution best evolution. The method is said derandomized not because it is free of aleatory generation of particles but because the mutation distribution evolution is mastered and realized via a set of deterministic equations where for other evolution strategies, the crossover and the mutation are partially or fully randomized.

The CMA-ES in its more advanced version [42] is characterized by a high level of robustness and a very good convergence rate, two primary characteristics essential for an efficient optimization algorithm. It has been already applied with success over a wide range of industrial and scientific applications characterized by equivalent properties and complexity as a complete suspension optimization.

2.3.3 State of the art of vehicle suspension optimization

In this section, we focus on the vehicle suspension optimization methodology. We identify the main academic developments about the optimization problem formulation especially developed for the improvement of both road and rail vehicles. These two categories are treated sequentially. We close this section highlighting the main conclusions of the state of the art.

Road vehicle suspension optimization

Vertical comfort optimization

A large part of the identified studies cover the restricted problem of the pure vertical comfort optimization. We start the state of the art by the studies on this concern.

It is possible to find a very elegant solution for the problem of vertical comfort but in this case, lots of simplifications must be admitted. The simplest models use the quarter car model. It consists in considering the vertical dynamics as equivalent to a double mass-spring unidirectional system. From the ground, the first spring represents the stiffness of the tire; the first mass, the unsprung masses; the second spring, the actual suspension shocks and the second mass, the quarter of the sprung masses. In the same manner, the half car model study only an isolated axle in order to analyze the roll motion, or, on the contrary, the longitudinal version to study the pitch motion over the pure vertical vibration.

The Nakhaie study [64] is interested in optimizing both the comfort and the working space of the suspension. By using a simple quarter car model, they extract an analytic model that finds an analytic solution. Indeed, the Pareto front function is computed and, with the introduction of the second degree cost function i.e. the mix law, an optimal function is found without time integration and numerical solving. Despite the elegance of the approach, the result application is restricted by the deep hypothesis, including the poverty of the excitation function and the numerous missing chassis DOF.

In order to cut partially these limitations, Baumalr [6] proposes a planar, longitudinal, half car model. The use of genetic algorithm based on discrete

genomes allows to find optimal solutions for a more complex problem: more realistic excitation, more unknowns, more complex objective function. In the same team, He extended the work to the active suspension and the associated controller of which the algorithm parameters need to be tuned aside the mechanical ones. In a first paper [48], they come back to a quarter car model to be focused on the active suspension before extending again the model to a half car model in a second paper [46]. The main objective is the reduction of the body vertical acceleration. The active suspension controller involves a Kalman filter in combination with a Linear Quadratic Gaussian (LQG) algorithm put in the center of the optimization task. A multi-level optimization scheme is adopted. It converges in 12.53 hours for a total of 6936 fitness evaluations what is very long. It must be noticed that the algorithm does not take advantage of the possible parallel computation offered by the Genetic Algorithm (GA) strategy what explains this counter-performance.

Quarter car model is also used for the truck applications. Sun [86] optimizes the suspension stiffness and damping with the objective of reducing the dynamics load on the pavement, probably to reduce the deterioration due to the heavy vehicles. The optimal solution of these two parameters problem is easily solved by a genetic algorithm for which the convergence is analyzed with respect to different optimizer parameters. Tens of generations are sufficient to converge.

Similarly, the Georgiou study [38] combines the normal force magnitude criterion with the reduction of the body acceleration and the wheel travel, always using a quarter car model and a rough road profile description.

A real three dimensional model with simplified suspension appears as the minimum model to successfully synthesize suspension with an attention to the pitch and roll dynamics as is done by Bouazara in [12]. The Root Mean Square (RMS) body acceleration and the tire deflection are optimized in a comparison between passive, semi active and active suspensions. From our point of view, the study offers a good trade-off between the outgoing conclusions and the involved model complexity. On the contrary, the use of the tire deflection as a measure of the handling capability seems a doubtful extrapolation.

With 8 DOF, the model described in [83] gives a comfort optimization with the same level of modeling. The objective function satisfies the ISO 2631 standards: it is found on a weighted sum of RMS acceleration in multiple directions. The controller of the active suspension works with a Linear Quadratic Regulator (LQR) algorithm. The exhaustive problem includes some optimization constraints handled with the help of the penalty strategy that allows to use the free GA to find an optimum.

Increasing enough the quality of the analysis, the Gonçalves paper [40] explains how a flexible MBS model can be used in an optimization scheme despite the inherent simulation cost. The study is interested in comfort optimization, again according to the ISO 2631 standards. It is focused on the vibration modes with frequencies from 0 to 25 Hz and considers the higher frequencies as noise. To enforce the quality of the approach, the vertical excitation is given by a rough track defined by splines from the real pavement measurements. Multiple scenarios are used to compute the objective function which aims to minimize

the suspension travel and to improve the comfort. The spring and damper coefficients are the optimization parameters. The scheme couples flexible and rigid bodies what is made easier by the choice of Cartesian Coordinates. For the flexibility of concerned body, the lumping technique is applied. It consists in restricting the possible deformations to linear combination of a given number of the main self vibration modes. In this case, the modes are precomputed via the Ansys FEM code.

The choice of the best MBS formulation is discussed in [101]. These scientists are interested in finding a robust method to optimize the mechanisms including some kinematic loops and possibly subjected to kinematic singularities depending on the progress of the optimizer. From the authors, the DAE direct solving integration is not suitable because being unstable while the coordinate partitioning¹ reduction method is subject to singularities. The penalty formulation is used instead of solving the loop constraints. A gradient based optimization is applied on the problem of a 5 bars mechanism which presents the singularities and on the Iltis all-road vehicle suspension. The method is very efficient. Applied to the Iltis problem, the mono-objective evaluator is made of the time integration of the fourth power of the vertical acceleration. The vehicle is described by a 14 DOF model for which 6 parameters are optimized. The process needs only 40 iterations to converge with 15 s for each iteration. The optimization is fast thanks to the gradient based optimizer but subjected to local minima for the same reason. The sensitivity analysis is joined to the optimization to testify the optimum is robust.

The concern of the optimization robustness is the focus of the study described in [5]. It aims to improve the comfort feeling of the driver and the passengers. To do so, the full model of the chassis is completed by the seat vertical dynamics including stiffness and damping, where the accelerations are measured. Only two parameters of the suspension are optimized following an objective function made of both the ride comfort index and the robustness captured via a sensitivity analysis. The hybrid objective function aims to avoid the setups very efficient but particularly sensitive to the precision of the parameters. Such a precision could be impossible to reach or to maintain for lots of technical reasons as the natural dimensional dispersion of the manufacturing, the creep, the wear, or the other tampering. For this study, a Cartesian coordinates MBS is combined to a Monte Carlo optimizer.

Handling performances optimization

The use of the GA algorithm is also used for the concern of the handling. In a first paper [22], Datoussaïd applies this strategy on two problems: the kinematics optimization of the steering angle of a double wishbone suspension depending on the mechanical link geometry and the dynamics optimization of a railway urban vehicle for which, among others, the relative yaws of elements are minimized. In a second paper [23], the authors detail the optimization process and in particular, the change of optimizer from GA to an evolutionary

¹The coordinate partitioning is also called the Maggi's formulation in the literature and in this paper.

algorithm based on the step length control, a primitive version of the CMA-ES explained in the section 2.3.2. The penalty strategy is chosen to manage the optimization constraints. They also detail the applications on the double wishbone suspension of an Iltis vehicle and on the city-runner rail vehicles but also on a shock absorber for the railway applications. The convergence of the method is analyzed and seems good. The advantages and drawbacks of the method are listed. They justify notably the use of GA in a first time and the evolution strategy in a second time by the impossibility to compute the gradient of objective function due to the numerical instabilities.

When the number of concerns increases as it can be the case for the optimization of a complete chassis, the technique of hierarchical optimization is often proposed. In [16], a multi-level optimization combines optimization of both performances and robustness. At each level, the model is simplified corresponding to the local concerns and the trade-off between performance and robustness is examined through a Pareto analysis, a very original idea unlike the classic mixed objective function. The study deals with all major aspects of vehicle dynamics, including, as well, the comfort and the handling. Multi-scenario, multi-criteria are used. Criteria as specific as the under/oversteering coefficient are joined to more classical ones in a quantification based on an elasto-kinematic model. In definitive, we found this article impressively exhaustive.

Unlike a true optimization, the suitable dynamics capabilities of a car can be also obtained by the solving of design constraints that, if relevant, ensure a correct car behavior. On the same principle, the paper [59] proposes a separated setting process of the front McPherson suspension and the rear compound crank axle. For each, the axle characteristics like caster, camber, kingpin, roll, steer, Ackermann geometry, are found as functions of the time and of the input target acceleration of main motion. The use of inverse kinematic and inverse dynamics¹ determines these setting without neither time integration nor optimization. The paper does not precise how final values of parameters are fixed.

In [79], the optimization of a double wishbone suspension aims to fit the curves of axle pure kinematic characteristics with optimal solutions by modification of a set of 14 design parameters including, for examples, tread width, camber, toe-in, kingpin inclination, kingpin offset, caster angle, caster offset, etc... The difficulties are well highlighted by authors: a large number of parameters, a strong coupling between parameters, a model, and so, an objective function highly non linear. Facing this, despite the risk of finding local minima, they simply opt for a deterministic gradient based optimizer. The strategy sound interesting except they provide nor theoretical justification of the pretended optimal kinematic characteristics nor the method to obtain such data.

The complexity of the model, reinforced again by the uncertainty treatment, leads to a large optimization process time. A possible solution is proposed in [100]. The MBS model of the double wishbone suspension realized with Adams Car is reduced to meta-model based on the Chebyshev functions

¹The interested readers can find all required information about the inverse kinematics and the inverse dynamics in [78].

in order to save time in the objective function evaluations time during the optimization phase. The Chebyshev functions are used instead of the high order polynomials because they offer a better convergence in the meta-model parameters determination and, by the way, decrease the number of evaluations in the parametrization phase. Despite the interesting properties of the Chebyshev based meta-model, the selected optimizer is a multi-island GA.

In his PhD thesis [57] done in collaboration with Renault, Loyer develops a multi-criteria, multi-scenario, robust optimization scheme for the active and semi-active automobile suspension improvement. The full model which involves more than 400 parameters is derived to different models for each specific scenario. For example, a longitudinal half car is used for the comfort and braking test while a 20 parameter bicycle model is deployed for the cornering analysis. A simplified four wheel model is also needed for advanced purposes. The author argues these simplifications speed up the model by a factor up to 500000. Multiple tests confirm that the simplified models conform the full one. The scenario computations are based on both time integrations and modal analyses. The equation is handmade despite the MBS model could be easier and more efficient. The optimization scheme approach involves some interesting choices. First, the method removes some redundant objectives based on a statistical pre-computation. Secondly, the solutions are sorted according to the Pareto efficiency and the Pareto optimality. Thirdly, the hybrid optimizer is a mix between GA and Sequential Quadratic Programming (SQP). On the contrary, we doubt the insight of some choices. The models do neither take into account nor measure the over-understeering, so some strange correction coefficients are applied to correct this lack. The satisfaction of Ackermann condition¹ is adopted as a main hypothesis but not as an objective to satisfy or to optimize. The four slope damper model is not compatible with a linearized modal analysis. We are also interested in checking if the use of a simplified model is compatible with the robust optimization. Here again, the design constraints are treated by a soft method i.e. considering their satisfaction as objective criteria. It causes the need of a large population in the heuristic optimization and by the way, an efficient objective function evaluation, justifying the resort to a simplified model.

Facing the diversity of the vehicle dynamics concerns, Hazare [44] proposes to attack the problem via a multi-layer approach. The multi-objective, multi-scenario analysis implemented for this study is so complete that it involves this multitude of tests: steady-state handling, transient handling, on-center handling, emergency handling, parking, steering feed-back, handling on different road surfaces, coupled dynamic cornering, disturbance sensitivity, and straight-line stability. According to the authors, managing such a large quantity of data to formulate a unique objective function is complicated and is not fully relevant. The optimization scheme is thus structured via the Analytical Target Cascading (ATC) method. The overall problem is split into multiple local ones. The main division stands the levels also called the layers. The first, second, third level relate respectively to overall vehicle, subsystems and compo-

¹In this thesis as in France in general, the Ackermann geometry and condition are called respectively Jeantaud geometry and condition.

nents. Two surimposed levels exchange together, on the one hand, descending specifications and on the other hand, ascending resulting performances. Such exchanges are also possible between elements of a same level. The net allows to organize the global optimization. A GA optimization is applied at each local problem. In his PhD thesis [44], Hazare underlines the problem of the driver status. Indeed, the open loop simulations are relatively artificial while in case of closed loops, it is very difficult to separate the car and driver performances, why both the analyses are implemented in the evaluation tests. In [45], they push ahead the problematics related to the driver. In addition to the difficulty of interpretation of simulation results, the difference between individuals, the conflict between the different cases and the different design functions, the study is also interested in the link between the subjective quantification and the objectives measurements, achieved through statistical correlations.

Driver controller for optimization purpose

As previously introduced, an efficient optimization of the handling behavior can not be achieved without a driver model. The controller manages both longitudinal acceleration i.e. acceleration and braking through the wheel torque, and the direction of travel through the steering mechanism.

The paper [68] is focused on the development of a driver controller for a Formula One model. The dynamics model is extremely reduced. It involves only three DOF and includes very simple sub-models for aerodynamics, brake, transmission etc... In fact, all stiff dynamics like the knuckle motions are excluded from the analysis. The model is so simplified that it considers the same steering angle for the two front wheels. The optimal control theory is applied to determine the driver outputs to ensure the following of a track described with a curvilinear coordinate system and the getting of maximum performances.

The importance of the driver model is again reinforced if the study involves the optimization of the steering mechanisms and even more in case of active devices. As observed in section 2.1.4, the STC is often implemented on the tilting vehicles. The paper [63] analyzes the interactions between a driver and an active steering system where the angle control of both elements act in series. It aims to develop a driver controller that reflects the human behavior in their interactions with active steering, what can be complex. The proposed controller reacts to both lateral and yaw derivation to improve the behavior. The target applications of such driver controller are obviously the simulation and the optimization of road vehicles equipped with an active steering device.

State of the art papers on suspension optimization

In our search, two promising state of the art papers appear. We don't summarize all the information contained in these two very complete documents but we simply indicate the majors elements that have not or not sufficiently been outlined.

First, the paper of Cole [19] is concerned by the fundamental issues in suspensions for heavy road vehicles. It reminds the main considerations for the vehicle design: ride, suspension working space, energy consumption, rollover stability, yaw stability, braking and traction. Let us remind that, on the contrary of cars, for some vehicles, the rollover limit is reached before the adhesion limit. Specific criteria are thus required. Concerning the vertical dynamics, the criteria based on the frequency weighted RMS chassis body acceleration are basic for comfort evaluation but can be improved by the biological human dynamics and completed by considerations that do concern the only comfort like the road load criteria and the tire correct working for the adhesion quality. As conclusion, he diagnoses there are not enough studies in the suspension tuning and the suspension configuration. A lack has to be filled.

Secondly, the authors of paper [15] comment the large amount of development realized in the multitude of fields in relation with the vehicle dynamics. These developments are particularly motivated by the introduction of new motorizations like the electric and hybrid groups. From the comfort optimization point of view, they claim the right trade-off must be found between the simplicity of the model and the targeted goals. They notice “the quarter-car vehicle model that cannot capture the wheelbase filtering effect would overestimate the bounce acceleration responses of the sprung mass, when compared with a pitch-plane vehicle model”. The implementation of full model is required to study the interdependence between roll and pitch motion, a fundamental concern. The authors promote the semi-active suspension that seems a suitable trade-off between the resulting riding performances and the energy consumption.

Both papers insist on the prime importance of the Olley’s rule that, as reminder, consists in increasing the natural frequency of rear axle with respect to the front to ensure better pitch behavior in case of obstacle crossing.

As in the first one, the authors conclude that the suspension kinematics impact on the overall dynamics is under-investigated.

Rail vehicle suspension optimization

If, from a first view, the rail vehicles appear very apart from passenger road vehicles essentially due to the difference of masses and dimension, in fact, these two categories share numerous characteristics and techniques including simulation and optimization tools. It is very interesting to check how this community deals with the suspension optimization problem.

The design optimization and, in particular, the passive or active suspension of rail vehicle is the concern of [47]. The optimization scheme stands on a MBS approach for the simulation and a GA for the optimizer. Different models with variable complexity are implemented for the multiple purposes. To simplify the modeling, the whole of the kinematic loops and the resulting kinematic constraints are omitted. The different objectives namely, the lateral stability, the vertical RMS acceleration and the curving performances are mixed together in a multi-criteria unique objective function through a linear weighted sum of criteria. The Pareto analysis is furthermore applied on conflicting objectives.

It allows to maximize the travel speed but keeping a given stability level.

The maximum curve speed increase is also the main goal followed in [62]. The simulation is performed thanks to a co-simulation, Simpack computing the overall dynamics and Simulink solving the magneto rheological yaw damper equations. In a first time, the better set of parameters is selected by a sensitivity analysis based on the Monte-Carlo method. In a second time, the multi-objective optimization problem is handled by a GA using a multi-step strategy. Unfortunately, despite the whole of the means implemented for the optimization strategy, the study handles only 3 parameters, but well chosen.

As this state of the art attests, there exist lots of approaches for vehicle suspension optimization. The approaches, techniques and methods are so various as the targeted objectives can be. In general, the studies tend to increase the amount of elements taken into account and also to increase the number of parameters, and, for that reason, become more sophisticated: multi-criteria, multi-scenario, robust, etc. The MBS approach appears as widely used and efficient for such optimizations. The heuristic optimizer seems to also give good results for that purposes. Surprisingly, the design constraints remain systematically ignored or treated with soft methods. It becomes apparent that the influence of the suspension kinematics on the vehicle dynamics is insufficiently taken into account as well as the handling performance evaluation and the corresponding criteria. The strategy explained in the next chapter proposes an original optimization scheme as a solution to these major preoccupations.

Chapter 3

Dynamics optimization methodology

3.1 Introduction to methodology

3.1.1 General approach

The dynamics optimization integrates a larger process which is the vehicle design and what must be spelled out to correctly capture the expectations in terms of dynamics performances. We derive a conceptual general approach able to drive the design process of a prototype from scratch to the manufacturing plans. This involves through three main steps (figure 3.1):

- the morphology determination,
- the dynamics optimization,
- the mechanical implementation.

Morphology determination

The first step of the process, the *morphology determination* aims to determine the topology of the vehicle suspension which can satisfy the expected *functions*. The topology means the number and the manner diverse mechanical components are arranged together to ensure the whole satisfies the expected features. For the case of vehicles, it notably consists in synthesizing the suspension mechanism that allows to guide the wheels while ensuring some possible wheel travels, or the steering mechanism that steers the front wheels transmitting the effort from the steering wheel, if any.

From a methodological point of view, the morphology determination is carried out in parallel in the two study tools used here. On one side, a skeleton scheme is drawn with a CAD software, SolidWorks in this case. On the other side, the kinematically corresponding tree-like structure is defined in the MBS software, Robotran. Thanks to the skeleton scheme or the tree-like structure which are, in definitive, two representations of the same reality, we fix the

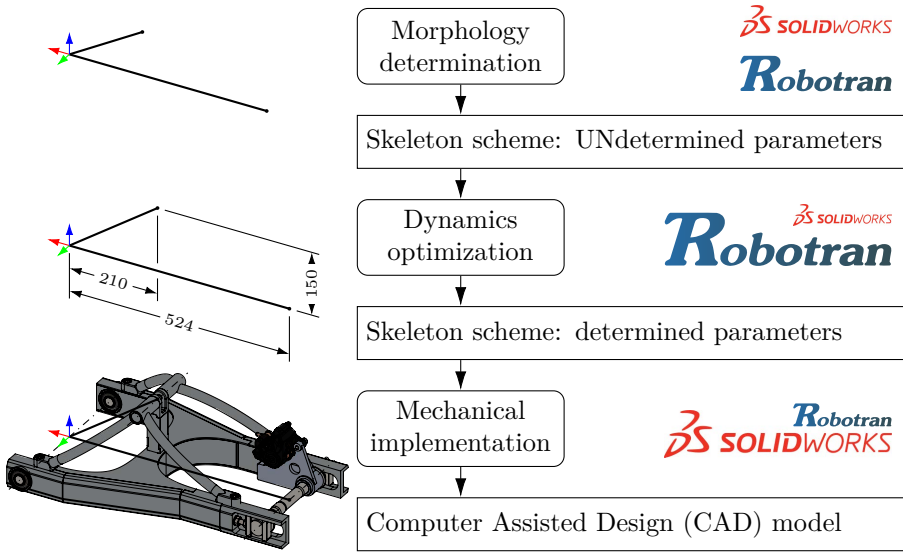


Figure 3.1: General approach.

structure of the suspension. In addition to the pure mechanical kinematics, the model must include the specific devices and the modeling of their behavior. The process aims to generate a parametrized model. The parameters can be geometric like the distances and the angles that characterize a mechanical component, or the parameters of the specific component model as for example the stiffness of a spring or the damping coefficient of a shock. At this step, the parameters of both representations are listed but still only predefined. Each parameter receives an initial value from either a rough estimation, a standard value in practice or the result of the common rules of thumbs. At the end of this step, the MBS model must be sufficiently complete to allow the simulation of the vehicle and the evaluation of its performances.

One can notice that SolidWorks uses the absolute coordinates and Robotran the relative coordinates, as already mentioned. Thus, this step must be performed with care to ensure the coherence between the two models that have to be maintained along the study.

At our best knowledge, there is currently no method to automatize successfully this part of the process. There exist, at most, some difficult attempts to synthesize mechanisms on the basis of a collection of selected components. Furthermore, these methods are unable to handle problems as complex as the complete vehicle suspension kinematics. Thus, this task has to be done "by hand" or, more exactly, by brain storming, with the help of a vast knowledge of existing technical possibilities or, in lucky moments, the inventiveness. The major difficulty of the work comes from the arbitration between multiple concurrent solutions. This process is hard because their performance evaluation can only be achieved after the realization of the next steps: the dynamics opti-

mization. In fact, after its generation, each solution has to be optimized what appears as a tedious task. Despite it is not the best case, the optimization of all possible configurations is not practicable. Only the most promising topology follows the rest of the process.

Dynamics optimization

The second step, which is central in this work, is the *dynamics optimization*. Basically, the process aims at determining the value of the model parameters which give the best behavior. In particular, the optimization determines the optimal parameters to enhance the dynamic capabilities, while considering the mechanical constraints. Because the problem involves numerous and various parameters as well as numerous, various and conflicting considerations, its statement must be well defined and formalized. As a solution facing this problem, we propose a new optimization scheme that handles all the elements involved in the problem. Such scheme development is based on a *parameter partitioning*. The formulation of the problem, the parameter partitioning and the development of this complete scheme are treated in details in the next sections of this chapter.

At the end of this step, all the parameters are tuned so that the behavior of the related vehicle is supposed to be optimal. The kinematics and dynamics properties of the vehicle are definitively fixed and known.

Mechanical implementation

Based on the previous skeleton model for which all parameters have been determined, the *mechanical implementation* materializes the different bodies with different assemblies of parts. This operation is essentially realized in the CAD software, SolidWorks in this study.

The mechanical design follows a top-down conception strategy. The CAD model assembly is organized in five levels:

1. The head assembly contains the overall vehicle.
2. The assemblies of the main divisions of the vehicle. Different splits can be elaborated depending on the scope and the organization of the work. We have established the division between three main assemblies:
 - the front axle,
 - the main body,
 - the rear axle.

For some very complex vehicles, an additional level could be inserted with the equivalent structuring objectives.

3. The body level is the one for which the elements strictly correspond to the bodies of the MBS description. Once assembled, such assemblies are rigid from the vehicle dynamics point of view. For example, at this level, a steering rack must be divided into two parts: a moving slider and a fixed housing. In the same manner, the suspension or the steering ball joints

are split such the pivot and the housing are part of different assemblies. On the contrary, the main body¹ is clearly a rigid body despite the doors can be open because this kind of motions is un-relevant and thus excluded from the dynamics analysis.

4. At this level, the elements can be an assembly organized with more or less depth, following the complexity of the considered implementation, or directly a part.
5. The parts that form the last level, are the elements that can not be divided anymore. A part consists of either a piece of continuous and homogeneous matter or a supplier assembly that must be considered² as a whole from the point of view of the chassis designer³. This way, supplier components on the shelf as the electric motors and their gear reducers and controllers, are considered as indivisible parts despite they are definitively mechanical assemblies.

The three first level structure is determined during the morphology determination step including the kinematic skeletons of the body assemblies. The mechanical implementation starts by defining the structure of inferior levels until the parts that can be materialized. To do that, the kinematic skeletons are completed by the mechanical skeletons, what involves the definition of functional surface references as for example the contact surfaces or the bearing surfaces. A reference mechanical skeleton is associated with each assembly. The common interfaces that imply interactions with elements outside the current assembly are defined in the upper level assembly reference skeleton, while the interface that only concerns the elements of the current assembly are defined in the associated reference skeleton. As an example, for a revolute joint, the shaft and the bore diameter are determined by a unique variable stored inside the reference skeleton of the upper assembly. The unique value is thereafter transferred to parts by some references. The use of the reference system is fully justified to ensure the inner coherence of the model.

Component categories

Progressively going downwards in the design tree⁴, the complexity of the elements are split and therefore reduced. At the part level, the function of the part is completely defined such that all functional surfaces are known. Basically, two choices are offered to the designer: re-use an existing part or design a new one. In fact, there are several possible trade-offs between these two extremes:

¹As an exception, the main body is part of multiple levels because it is, as well, a major part of the vehicle (level 2) and a body within the meaning of MBS formulation (level 3).

²Such situations are regularly encountered under commercial and protection policies.

³These situations are however sometimes conflicting with the requirement of equivalence between level 3 assemblies and MBS bodies. A supplier motor is a good example. Its internal structure is under discretion of the manufacturer but the rotor and stator have to be considered separately from the dynamics point of view.

⁴The metaphor of the tree is also used for characterizing the structure of the CAD model. However, it must not be confused with the MBS tree-like structure.

The *standard parts* consist of components available on the market for which all the technical data are determined. These elements are, in general, available in discrete range so the selected part fits more or less to the initial requirements. These standards part ranges can correspond to standards like the ISO and the Deutsches Institut für Normung (from German) (DIN) ones or, on the contrary, are specific to the manufacturer.

The *parameter configurable parts* that, contrary to the standard parts, give the opportunity to set one or more parameters what increases quickly the number of possibilities and make the incorporation in the design easier.

The *morphology configurable parts*, in addition to the standard parts, give the opportunity to add some optional features to the part. These functions are often themselves parametrizable.

The *in-manufacturing customized parts* are components of the first three categories which, to ensure the fully satisfaction of the requirements, need some additional or specific manufacturing operations. This kind of parts are out of the standard commercial program of the supplier but, all the same, proposed by the company under given conditions subjected to discussions.

The *out-manufacturing customized parts* are the solution if the original suppliers refuse or are unable to apply the envisaged complementary operations. In this case, these operations are handled by other companies.

The *fully customized parts* are used if the part is and must be so specific that the other methods do not give satisfactory results.

There exist plentiful types of standard and configurable parts from thousands of manufacturers, so a lot of needs are already encountered. For the automation industry, it is possible to design a machine exclusively with parts from the three first categories or, at least, with a high rate of these parts. On the contrary, in the automotive industry, more customized solutions are adopted for multiple reasons.

Obviously, in term of design potential, the customized parts offer more freedom. The customization allows the best fitting to the specifications. The components are more complex but, in the same time, integrate much more functions, allowing the reduction of the part number and, by that way, a reduction of the assembly complexity and time, the weight, the space, and, under some conditions, the cost. In general, the assemblies involving customized parts are potentially more efficient.

The standard parts offer the advantage that they are built for others in a massive production what dramatically reduces their cost. On the contrary, for fully customized parts, the fixed cost as the production synthesis must be handled only for the present need over an intrinsic lower production efficiency, what increases the unit cost.

Moreover, if the standard parts are, in general, made for stock and quickly available, the parameter and morphology configurable parts are produced on

demand with determined and ready manufacturing method, that extents the delay. The customized parts need a customized production analysis and its implementation, which elongates again the delay.

The best choice consists in a trade-off between the three main considerations: the performances, the logistic constraints and the cost. We should raise awareness of the need to consider the total cost including the engineering cost over the unit cost that involves several cost sources in the production chain and in the logistics. This trade-off depends on the target. If for a demonstrator, a clever mix between parts categories is the best option, for the commercial production phase, the series that must be envisaged to reach a production cost compatible with the market are so large that the deal is different[98]. The volume is so large that low costs are reachable without sharing the fixed costs with other consumers and by the way, allows a better protection of the intellectual property [99]. The need of performances and especially the weight saving overtakes the lead time problem which is solved by an impressive logistical management. The elements justify why, in the automotive industry, the most of the parts are in-manufacturing or fully customized parts.

Notice that it is also possible to re-use or to modify a custom part already implemented in the company either for another purpose in the same project or for another project. The practice is widely spread in the automotive industry.

Part embodiment

The embodiment of a part is a skillful work where the best combination between multiple possible operations from numerous manufacturing methods must be found in order to create the component which satisfies, in the best possible way, the technical and the non-technical requirements.

Whether it is to integrate an existing part or to design a new one, some back and forth are required between the design levels. Local and global upstream flows are joined the main downward design process that make this last one more cyclic and iterative than previously hinted.

First, the mass properties of bodies are the results of their complete embodiment, but, at the same time, represent the essential inputs of the dynamics optimization that gives the skeleton model. This design loop has to be cut. Despite the cumbersomeness of the process, the design optimization and the embodiment must be iterated starting from a rough estimation to the "more or less" converged result.

Similar problems can be encountered when the mechanical embodiment is not possible due to either the mechanical interactions between parts in the useful motion range of kinematics or a lack of the mechanical strength of the parts that, both, cannot be reasonably overcome. These two limitations often appear together because of their coupling. Here again, the solution requires a come-back to dynamics optimization through the implementation of some additional design constraints or objectives related to the issues of the mechanical interactions and the part strength.

Detail design

Once the part embodiment is finished and coherent with the dynamics optimization, the engineering process leading to the production documentation must be completed by the *detail design* phase that consists of a series of precisions and verifications such as:

- to draw the detail elements as chamfer, fillet, offset, thread start-up, run-out, etc...,
- to determine the shape and geometric tolerances, the surface conditions, etc...,
- to select the materials and, the exact alloy grades, the state, etc...,
- to check the manufacturability of the parts and determine the manufacturing procedures including the operation sequences,
- to check the assemblability of the parts,
- to choose the fasteners.

The detail design step is achieved in conjunction with the drawing of the technical plans for the manufacturing and the assembly, and the preparation of the manufacturing listings and files.

3.1.2 Optimization problem formulation

Now that the environment of the optimization is clarified and the underlying links with upstream and downstream processes are established, we are interested in the problem formulation, which is depicted in the figure 3.2.

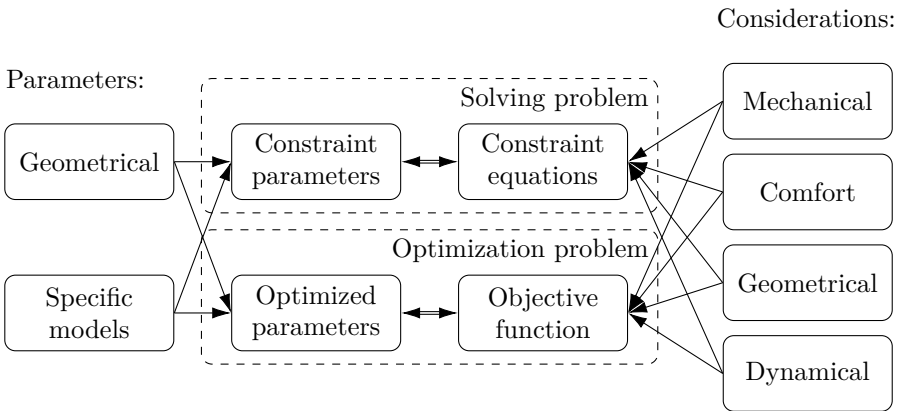


Figure 3.2: Optimization problem formulation.

From one side, the model is characterized by a set of parameters. There can consist of geometric parameters as for example the length of a component, the orientations of some axes, or some specific model parameters, as for example

the stiffness of a spring or the gain of a controller. On the other side, the considerations about the vehicle come from different concerns including: mechanical and comfort considerations, geometric limits and dynamics capabilities. Of course, these considerations are numerous and can be conflicting. For example, stiffening the suspension shocks gives better handling performances but leads also to an increase of the vertical body acceleration and, in final, the drop of the comfort. In the same manner, the curving performance optimization tends to increase endlessly the wheelbase and the axle track which are however limited by the overall geometry.

For each consideration, it should be determined if it is recognized as a constraint or, on the contrary, as an objective. It is not a definitive way to handle a particular consideration but the choice depends on the problem context and on the point of view adopted by the designer. A constraint formulation is recommended if the concern is dual i.e. binary, satisfactory or not. On the contrary, an objective formulation is more suitable if the improvement of the criterion contributes to a constant improvement of the global satisfaction. A consideration is never fully and unquestionably member of either the constraint set or the objective set but a choice must be done in order to get a correct problem formulation.

For simplification reasons, we have decided to only consider the constraints which express equalities¹. This limitation does not affect the capability of the method in the case of the Wemoov optimization. Hence, the problem is double: a constraint solving one and an optimization problem, highly coupled together.

3.1.3 Parameter partitioning

To tackle the problem of dynamics optimization, we propose to apply a partitioning method on both considerations and parameters. The considerations expressed as constraints result in a set of constraint equations to solve, related to a set of selected parameters. The residual parameters are used to minimize or maximize some criteria associated with the other considerations.

For management reasons, the parameters are classified into four families, according to their type into the model:

- the components of the geometrical vectors as the components of anchor points,
- the orientation angles i.e. the forced coordinates,
- the lengths associated with distance constraints i.e. the length of the rod cuts,
- the specific user-model parameters.

The inertia elements of the bodies i.e. the mass, the center of mass and the inertia matrix components are not considered as parameters in this phase of the study because they are the result of the mechanical implementation, the step following the dynamics optimization in the general approach, and not as true setting options of the model (figure 3.1).

¹Moreover, a way to, nonetheless, treat the inequalities constraints as developed in the section 3.7.4.

The complete set of parameters is partitioned in:

- $\mathbf{p}_n$ The *null* parameters are those whose value has been voluntarily set to a zero value. This type differs from the next one, the fixed type because, thanks to their zero value and the symbolic multi-body approach, it is not required to create a corresponding symbol. So, the expressions are automatically simplified and the dynamics computation is more efficient.
- $\mathbf{p}_f$ The *fixed* parameters are defined to their final values by the designer himself, prior to the optimization process. Their value results from considerations which are external to the optimization process, elsewhere in the global design approach. This constitutes a form of simplified self-satisfying constraints. Naturally, the use of such parameters type reduces the complexity of the process.
- $\mathbf{p}_c$ The *constrained* parameters are in charge of the satisfaction of the design constraints. Their number is identical. As explained below, this category is even split into different groups according to the proposed optimization scheme.
- $\mathbf{p}_o$ The *optimized* parameters are those that the optimizer algorithm determines to reach the best improvement of the selected criteria through the minimization of a unique global objective function.

The vigilant reader will have noticed a large similarity between the classical MBS problem formulation and the proposed optimization one. A parallel is done between the forced and driven, dependent and independent coordinates and respectively the fixed, constrained, optimized parameters, as well as between the kinematic constraints and the design constraints, or between the dynamics equations handled by the time integrator and the objective function managed by the optimizer. Despite this deep similarity, it consists of two completely different processes. It is essential to not confuse the concepts of the two formulations, both coexisting together in the embedded global scheme.

This parameter partitioning is achieved by the designer prior to the optimization process. There are multiple possible partitionings but each must satisfy the following obvious requirements:

- The number of constrained parameters must be equal to the number of the design constraints.
- Each constrained parameter must influence the constraint equations. Mathematically speaking, the Jacobian matrix of those constraints with respect to the involved parameters must be of full rank.
- The number of optimized parameters is free but at least equal to one.
- Each optimized parameter influences, at least, one criterion included in the objective function.

Despite it would be possible to implement a numerical process that automatically performs the parameter partitioning via, for example, a sensitivity analysis, a "by hand" partitioning is practiced.

Thanks to the parameter partitioning which is just introduced, it is now possible to synthesize an optimization scheme that treats both the objectives and design constraints in a differentiated manner, suitable for vehicle optimization. This methodology is introduced in the next section.

3.2 General optimization scheme

In order to find a solution to the global optimization problem, we have derived a numerical process based on the exchange between an iterative optimization algorithm and a *trial*. According to the figure 3.3, the optimizer generates a set of values for parameters related to the objectives and transmits it to the trial which returns the quality evaluation of the corresponding vehicle. The evaluation of multiple particles of a generation allows the optimizer to improve the quality of the next generation particles themselves evaluated by the trial, and so on until the convergence of the process. The general optimization scheme is depicted in the figure 3.3.

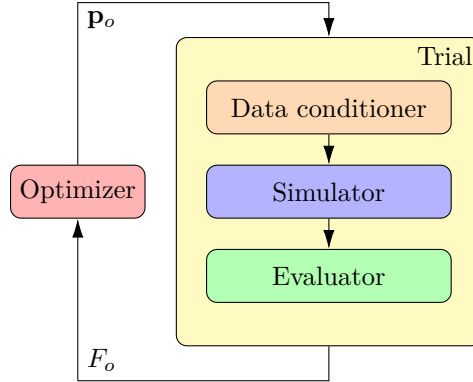


Figure 3.3: General optimization scheme.

The trial, which, for each set of optimized parameters, computes the evaluation of the corresponding vehicle, is divided into three main steps:

The *data conditioner* ensures that all types of constraints are satisfied: geometrical, frequency-based and analytic one, setting the non-optimized parameters to a suitable value. At the end of this process, all the parameters are set, making a model complete and coherent. By this way, the MBS model is ready to be run by the simulator.

The *simulator* tests and records the dynamics behavior of the vehicle in a hybrid procedure made of both a complete steady state analysis and a transient simulation.

The *evaluator* computes the objective function for the current vehicle on the basis of the data arising from the simulator. The global objective function, F_o , allows the optimization algorithm to iterate.

The parameter partitioning allows the implementation of such a process that offers multiple advantages.

First, thanks to the data conditioner, the optimizer does not deal with any constraints whatever their nature: geometrical, kinematic or dynamic. Indeed, the optimizer only manages the optimized i.e. independent parameters, p_o , what offers a wider choice for the selection of the optimization algorithm. As already mentioned in the section 2.3.1, the algorithms able to handle the constraints are few in number and not especially efficient. All the other parameters are computed by the data conditioner to satisfy the constraints leading to a significant reduction of the optimized parameters, what improves the efficiency and the convergence of the method.

Secondly, each vehicle simulated in the hybrid test, which is the most costly part of the process in term of CPU requirement, is viable since it satisfies all the constraints. In other words, no time is lost to simulate impossible or unsuitable vehicles.

In the next sections, we deeply detail the inner working of the data conditioner, the simulator and the evaluator. We also specify how the optimizer is introduced in the process before discussing pros and cons of the overall method.

3.3 Data conditioner

The proposed data conditioner aims at completing the model after the generation of the optimized parameters by the optimizer. It is made of an assembly of several numerical methods coming from the MBS toolbox. Each one is dedicated to the solving of a particular nature of constraints. In this formalism, we identify three sub-natures, each associated to a sub-set of the constrained parameters, p_c :

- C_a the *algebraic constraints* associated with the *algebraically constrained parameters*, $\mathbf{p}_{ac}$,
- C_g the *geometrical constraints* associated with the *geometrically constrained parameters*, $\mathbf{p}_{gc}$,
- C_m the *modal constraints* associated with the *modal constrained parameters*, $\mathbf{p}_{mc}$.

This division achieves a parameter sub-partitioning of the main one, described in the section 3.1.3. The dictated rules about the equality of the number of constraints and the associated parameters and the sensitiveness applied distinctly on each type. In the rest of this section, we describe the way the different types of constraints are solved.

3.3.1 Algebraic constraints

The first constraints treated by the data conditioner are the algebraic ones which are written, in the general case:

$$\mathbf{C}_a(\mathbf{p}_f, \mathbf{p}_{ac}, \mathbf{p}_{gc}, \mathbf{p}_{mc}, \mathbf{p}_o) = 0 \quad (3.1)$$

Although all kinds of algebraic relations could be used to determine the *algebraically constrained parameters*, $\mathbf{p}_{ac}$, we only use them, in the case of vehicle optimization, to maintain symmetry and equality relations in the model. Indeed, from a kinematic point of view, the vehicles are relatively symmetric with respect to the median plane. This symmetry has to be maintained during the optimization despite the changes operated by the optimizer and by the other numerical processes. By convention, all data associated with the left components are considered as the references and those with the right component data as the slaves. The user must indicate the symmetry characteristic of the right components and the references to the corresponding left component items. So, the algebraic relations are automatically generated by the code in a pre-run before the optimization process. The operations are simple: the left parameters are simply copied to the right, with a possible change of sign in function of the nature of the parameters i.e. in function of the vector components.

Due to the limited use of algebraic constraints, they express by explicit equations that do not require a solver to be computed as it would be in case of complex implicit expressions.

$$\mathbf{p}_{ac} = \mathbf{F}_{ac}(\mathbf{p}_f, \mathbf{p}_{gc}, \mathbf{p}_{mc}, \mathbf{p}_o) \quad (3.2)$$

3.3.2 Geometrical constraints

The numerous interdependencies between mechanical design and vehicle dynamics are studied simultaneously in the present work. Most of these are mechanical design constraints appearing as geometrical constraints that the optimization must consider. A priori, they cover the global arrangement of the vehicle taken in its reference position, as well as the interactions that could occur in kinematic displacements induced by suspension and steering strokes. In a first time, we only focus on the geometry of the vehicle at rest on a flat and horizontal ground, naturally chosen as the reference position. Here, we aim in reconciling the position of some parts for which the location has been pre-determined either by conventions or by choices of the designer. But, on the other hand, from the MBS point of view, these locations are the results of the relative i.e. serial kinematic chains from the base to these particular bodies. These chains include multiple parameters of the model among which some of them are selected as the *geometrically constrained parameters*, $\mathbf{p}_{gc}$.

For example, in a common racing suspension setting procedure, the shock neutral lengths of the vehicle are set to ensure the respect of the target body height, roll and pitch more an additional condition of symmetry, generally based on the equality of the tire loading, axle by axle. The geometrical constraints concern the reference frame of the main body, while the selected parameters are the neutral length of shocks. Because the change of parameters from another nature affects the satisfaction of constraints, the geometrically constrained parameters must be freely adjustable by the numerical process all along the optimization. In our example, if the shocks neutral length are determined otherwise, i.e. classified as fixed parameters, other parameters have to be selected, for example, a component of the anchor point vectors of the shocks support brackets, body side or suspension side. In the same manner, some constraints

can be imposed on the wheel position, a target axle track for example, to avoid the optimization to enlarge endlessly the width of the car researching the increase of dynamics capability.

The geometric constraint equations have to be satisfied for the reference configuration, so, jointly with the static equilibrium equations. In order to solve the whole, we start from the equilibrium computation process that initially finds values of independent coordinates, q_u^{eq} , thanks to the NR iterative solver (section 2.2.5). The constraint equations are added to the equilibrium subset so that all the equations are solved simultaneously, by the same solver. This is called the *extended static equilibrium* process. There exist two complementary ways to do this:

Variable addition

This method consists in adding the geometrical constraint equations, C_g , to the standard MBS equilibrium equation 2.12 and also, in the same time, an equivalent number of *added geometrically constrained parameters*, p_{agc} to the solver variable list, x_{eq} .

For example, to impose the wheelbase, wb , of a motorbike, we can set

$$C_g(i) = wb_{current}(x_{eq}) - wb_{imposed} \quad (3.3)$$

and incorporate the length of the rear fork, or any other sensitive parameter, into p_{agc} , and so, into x_{eq} .

Variable exchange

This second method consists in replacing some *exchanged independent generalized coordinates*, q_{eu} , in x_{eq} by some *exchanged geometrically constrained parameters*, p_{egc} . This exchange involves some implicit geometrical constraints: the exchanged geometrically constrained parameters are maintained to their initial values, determined by the designer, instead of being computed by the equilibrium process.

To illustrate the interest of this second method, let us consider a standard equilibrium to determine the chassis height of a vehicle, given the neutral length of the front suspension springs. In reality, from a mechanical design point of view, it is preferable to determine the neutral length as a result of the equilibrium, with the chassis height fixed at a desired value. The exchange between the neutral length and the generalized coordinate related to the chassis height in x_{eq} permits to take this constraint into account in the simplest way.

In sum, the two methods can be used together. In such a case, the solver S1 solves the system:

$$\begin{cases} F_r(x_{eq}) = 0 \\ C_g(x_{eq}) = 0 \end{cases} \quad (3.4)$$

$$(3.5)$$

with

$$x_{eq} = q_u \setminus (q_{iu} \cup q_{eu}) \cup p_{agc} \cup p_{egc} \quad (3.6)$$

The synthetic scheme of the extended static equilibrium is illustrated below:

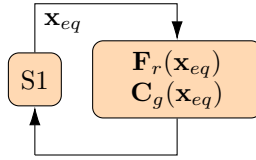


Figure 3.4: Extended static equilibrium scheme.

The described method for the management of the geometric constraints features multiple advantages. In case of variable addition, the use of the same solver for both the equilibrium computation and the constraints solving allows to be particularly efficient in comparison to other strategies like e.g. a two embedded loop process. Under a correct variable sensitivity, the convergence of the joined problem is robust. In the case of variable exchange, it is even better: the constraints solving does not at all increase the CPU requirements.

3.3.3 Modal constraints

As explained in the section 2.2.7, the modal characteristics of the vehicle suspensions can be computed in a very efficient way. In vehicle dynamics, these characteristics are regularly used as a measure of the comfort and the stability. Hence, it is natural that these characteristics can be subject to some objectives and constraints. Here, let's start by the processing of *modal constraints*. The modal constraints $\mathbf{C}_m$ are equation that fix each, for a particular mode, an outgoing result of the modal analysis. In this study, these constraints refer to, for an oscillating eigen-mode, either the frequency, ν , or the damping ratio, ζ . Despite more complex relations are possible, we simply fix the modal characteristics to some pre-defined values which express, for examples:

$$\mathbf{C}_m(\mathbf{p}_{mc}) = \begin{bmatrix} \nu_{i,current}(\mathbf{p}_{mc}) - \nu_{i,imposed} \\ \zeta_{j,current}(\mathbf{p}_{mc}) - \zeta_{j,imposed} \\ \vdots \end{bmatrix} = \mathbf{0} \quad (3.7)$$

Here again, each modal constraint requires the selection of a related *modal constrained parameter*, $\mathbf{p}_{mc}$.

Based on an equilibrium solution, the modal analysis straightforwardly finds the eigen-modes and the related characteristics so the constraints can be computed. However, the sorting and the selection of the targeted modes is not that easy. Extracting the suspension modes, which are the most interesting for our analysis, requires to compare each mode to the others based on the content of normalized eigen-vectors. A whole of conditions on the relative amplitude of joint motions is implemented to isolate several interesting modes based on their expected shapes. The synthesis of such rules is delicate because

the modes evolve, sometimes quickly, change their nature¹ or mutate² with the change of parameter values along the optimization process. Once the modes are isolated, the constraint current values can be computed. As shown in the figure 3.5, the solver S2 drives the *modal constrained parameters* $\mathbf{p}_{mc}$ to meet modal constraints $\mathbf{C}_m = 0$. Despite the fact that modal analysis implied a linearized model, the equations 3.7 are completely non-linear with respect to the modal constrained parameters, $\mathbf{p}_{mc}$ and their continuity can only be assumed on a range restricted by the nature changes or the mutations which both can occur on the targeted modes. To solve these equations, we have implemented the S2 solver on the basis of the NR algorithm with relaxation if required. The needed gradient, $\frac{\partial \mathbf{C}_m}{\partial \mathbf{p}_{mc}}$, is computed by finite differences around the current point. Because the action of the solver on modal constrained parameters, it is needed to refresh again the algebraic and geometric constraints and because the computation of the modal analysis must be performed around an exact equilibrium, the extended equilibrium must be performed at each iterative step, before the modal analysis, the mode extraction and the evaluation of the modal constraints. This is illustrated in the figure 3.5.

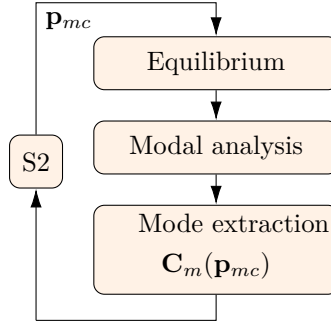


Figure 3.5: Extended modal scheme.

Imposing the frequency and the damping ratio for both front and rear suspensions mode enables to optimize the vehicle without authorizing the optimizer to stiffen or soften the suspensions that could be favorable for the objective function but would have a detrimental influence on the comfort. By this way, we avoid to add comfort criteria to the objective function and to implement a tire model coherent with uneven ground, which is CPU time costly but required to study the comfort. This simplification is fully efficient to save computational time with respect to a complete time simulation process.

The frequency and damping ratio of main suspension modes are determined, by the designer, as some trade-offs between comfort and pure handling performances depending on the character targeted for the vehicle under study. Regarding the frequency, the range extents from values below 1 Hz for very spacious limousines, to 6 Hz or more for sport car. Following the Olley's rule

¹The mode nature can switch to another like: oscillatory or not, stable or not, astable.

²For example, the car combined suspension modes corresponding to the bounce and the pitch can mutate to distinct front and rear hopping modes.

(section 2.1.3), it is suitable that the natural frequency of the front suspension is lower than the rear one what can be early taken into account in the equations 3.7.

3.3.4 Complete data conditioner scheme

By combining the three previously described methods, it is possible to satisfy the three types of constraints in the complete data conditioner scheme, depicted in figure 3.6. It consists of three embedded algorithmic loops:

- the modal constraint loop, solved by S2,
- the geometrical constraint loop, solved by S1,
- the MBS kinematic loop constraint loop, solved by NR (not represented).

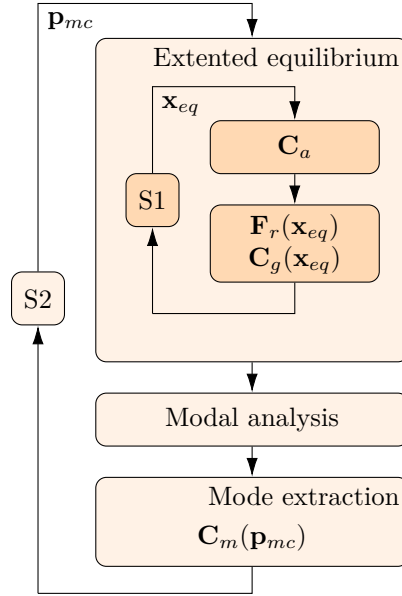


Figure 3.6: Complete data conditioner scheme.

Above all, the arrangement of these processes inside the data conditioner follows the sequencing logic rules. First, the equilibria must be performed for vehicles that are symmetric. This explains why the algebraic constraints computation is embedded in the extended equilibrium solving loop. Secondly, in the same manner, the modal analyses must be performed for vehicles at equilibrium, what justifies the extended equilibrium is embedded in the extended modal analysis loop. Besides these rules determining the structure of the data conditioner, we observe a beneficial property. The processes are organized such they are as deeply embedded as they have low CPU needs. From the point of view of the global efficiency, this arrangement is optimal.

Having multiple embedded iterative solving loops allows to solve all the constraints in a consistent and efficient way but it could lead to some convergence problems. This point is discussed in the section 3.7.2.

Once the constraints are satisfied, that is, the numerical iterative solver, S2, has converged, the dependent parameters i.e. the constrained ones are all definitively determined, with respect to this particular trial. Because the independent parameters i.e. the optimized parameters, and the null and fixed parameters are also determined upstream of the data conditioner run, respectively by the optimizer and by the designer, the entire MBS model is defined. In addition, thanks to the equilibrium process, the vehicle is already at rest, ready for its dynamic behavior evaluation.

3.4 Simulator

In the optimization process, the evaluation of the vehicle dynamics performances is a key element. As it is already introduced, we have chosen to divide the task into two main steps. The vehicle model starts by undergoing multiple tests while the relevant information is recorded. This part is so-called the simulator which is developed in this section. Thereafter, based on the recordings, the evaluator computes the value of the global objective function that measures the overall quality of the vehicle.

Ideally, the vehicle should be tested through the largest number of scenarios in order to capture the entire behavior in a complete characterization. All the relevant characteristics should be brought out by one or more simulations. However, the computation cost of such simulations is very high so a trade-off must be found between the evaluation quality and, oppositely, the optimization process needs. It is especially true because the multiplication of scenarios and of criteria complicates the synthesis of the global objective function.

In the beginning of the development of the optimization process, we tried to evaluate the three-wheeler on the basis of a single scenario: a transient simulation involving a time integration of the equations of motion. This test is described in the sections 3.4.1 and 4.4.3. It consists of a rolling simulation of the vehicle on a reference trajectory traveled at a specific speed. The optimization has allowed to downright improve the performances of the vehicle in this unique scenario. Each criterion present in the global objective reached an acceptable level. However, we observed that these good results were unfortunately only valid over a small range around the reference conditions used for the evaluation simulation and that they degraded considerably when conditions were changing. More precisely, the results strongly depend on the prescribed speed and on the reference trajectory. In particular, the stability of the vehicle was strongly affected. Of course, this major drawback was unacceptable and must be solved.

After diagnosing a large dependency of the optimization results with respect to the tested speed and trajectory, we have imagined several solutions to face the problem. The most apparent one was simply to perform a multi-scenario evaluation made of multiple trajectories with various characteristics, each traveled, at different speeds in the usage range. In order to cover the entire

range of use of the vehicle, too many combinations must be considered. This solution was thus immediately rejected because its CPU cost was completely prohibitive. Rather than a costly multi-scenario evaluation such implemented by several authors [40, 44, 45, 57], we propose a hybrid simulation. Indeed, we propose to enrich the evaluation with the results from other simulations realized with the help of other MBS tools than the time integration. If all the available numerical process are susceptible to offer information relevant for the evaluation purposes, the quasi-static equilibrium seems particularly interesting. In fact, as explained deeply in the section 3.4.2, it allows to perform a complete steady-state analysis, giving a large vehicle characterization for a marginal CPU cost.

In the proposed hybrid simulator, the steady state curving analysis which is computed by a quasi-static equilibrium method over a range of radii and a range of speeds, forms the base of the evaluation. It allows to test the vehicle performances regardless of the test conditions of speed and trajectory. This part of the simulator provides all information we need about the steady state vehicle behavior over its range of use. Nevertheless, the time integration simulation still remains in the simulation because it provides the significant but missing information about transient behaviors. Both methods are enrolled in the scheme because each of them provides different informations that complete each other. Moreover, thanks to the high efficiency of the equilibrium method, each information is obtained at its optimal cost so the total cost of the global evaluation remains acceptable. Indeed, the hybrid scheme allows to get the best of these two numerical processes without any major drawback.

The evolution of the global scheme is depicted in the figure 3.7. With respect to the figure 3.3, the hybrid simulator is detailed in its two components.

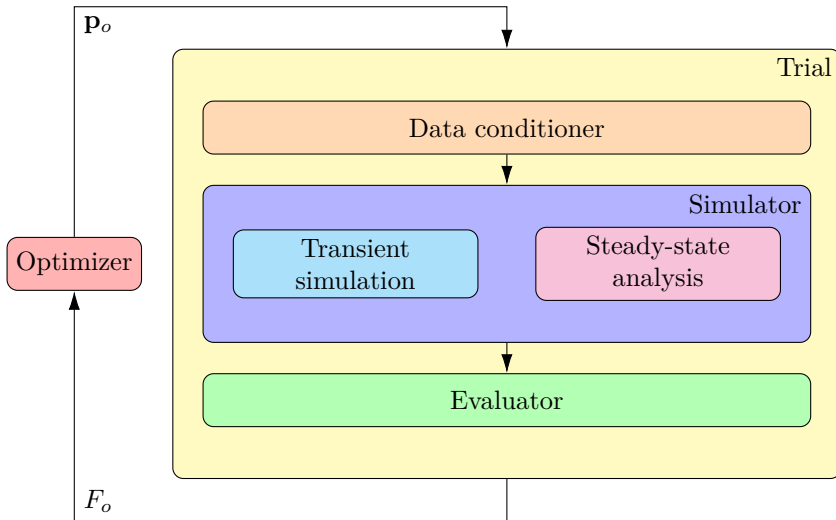


Figure 3.7: Optimization schema with hybrid simulator.

As one can remark, the execution of both processes is fully independent. For both, the inputs are the configuration at rest computed in the data conditioner while the outputs are different sets of data transmitted to the subsequent evaluator. The computation of these processes could theoretically be parallelized, but, due to the marginal advantage of parallelization at this level, the steady-state cornering analysis is realized before the transient simulation.

3.4.1 Transient behavior simulation

The transient behavior of the vehicle is computed thanks to time integration simulations. The scheme of this process, already presented in the section 2.2.8 is reminded in the figure 3.8

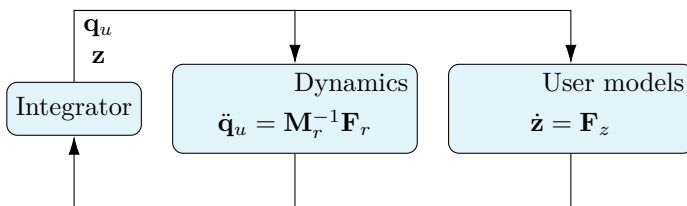


Figure 3.8: Time integration simulator.

The simulations involve the vehicle to optimize in some practical situations for which the measurement of some interest values allows to evaluate the quality of the vehicle. It is very important that the tests are representative of the real-life uses of the targeted vehicle at the risk to make it very good in some tested situations that are never encountered in practice but inefficient in the daily travels. However, the selected test must be challenging enough, in order to hope a true optimization and not a simple verification of the vehicle ability in very simple case-studies. In other words, the simulations must push the vehicle at the limits of its expected range of applications.

In our study, for both the CPU cost economy and the simplicity, we have implemented a single scenario transient simulation. The S-shape trajectory especially developed for the Wemoov testing is described in the section 4.4.3. The path is traveled in free-wheel with a given initial speed. According to the previous considerations, we agree this setup is insufficient and could be improved in term of trajectory representativeness. This is discussed in the section 5.2.1.

In order to evaluate the vehicle, the value of criteria must be recorded to allow the evaluator to compute the objective function. Each criterion is evaluated at each time step of the integration but not necessarily recorded. For the optimization run, only the minimum, maximum and mean values of the criteria are needed. At each time step, the minimum or maximum values are possibly updated according to the current value and this last one is recorded and sent to the evaluator. At the end of the simulation, the mean of the criteria can be computed as the integrated criteria divided by the elapsed time.

Because it is not straightforward to determine the initial conditions for the MBS tree joints and for the related model variables that allows to start the simulation without any glitch in signals, we have chosen to launch the simulations with approximative initial conditions while removing a short lapse of time from the recorded data. Whereas a few fractions of a second are sufficient, considering the cost of the time simulation, the improvement of the initial conditions could be an advantageous improvement of the method.

Steering controller

During a given transient simulation, the steering fork rotation, controlled in position as a driven joint, is set by a dedicated controller. It aims to control the vehicle to follow a predefined trajectory which depends on the objective of the simulation.

Track description

The predefined targeted trajectory is described by a series of cubic splines in the horizontal plane. This mathematical tool has been chosen for its good properties: continuity, high degree of differentiability, easy to manipulate, fast to compute, light to store, etc. It allows to envisage both open and closed trajectories.

The spline is defined by parts. With i , the index of the spline segment, the global curvilinear coordinate is noted $S \in [0 \rightarrow N]$ in the case of a simple lap or $\in [0 \rightarrow \infty]$ in case of multiple laps, and the local coordinate is $s \in [0 \rightarrow 1]$ such:

$$i = \text{floor}(S) \quad (3.8)$$

$$s = S - i \quad (3.9)$$

For segment i , the spline is defined by two polynomials of the third degree that give the Cartesian coordinates as function of the local coordinate s :

$$x_i(s) = p_{x_{i0}}s^3 + p_{x_{i1}}s^2 + p_{x_{i2}}s + p_{x_{i3}} \quad (3.10)$$

$$y_i(s) = p_{y_{i0}}s^3 + p_{y_{i1}}s^2 + p_{y_{i2}}s + p_{y_{i3}} \quad (3.11)$$

A special tool has been developed to generate the track description. In a Matlab based Graphical User Interface (GUI)¹, the designer introduces, by simple clicks in a free area, a given number of ordered points. The points correspond to the integer values of the global coordinate S . So, a cubic spline can interpolate the points selected by the user while ensuring the continuity up to the second degree i.e. the curvature level. If the spline is open, some boundary conditions for the initial and final tangencies and curvatures must be applied. By the way, the spline is entirely defined. The polynomial coefficients

¹The GUI part of the tool has been developed by Nicolas Docquier.

are computed and stored in matrices: one for x and one for y . Finally, the matrices are recorded into a xml file in order to be loaded by the optimization code.

Once the spline description is loaded in the optimization code, some functions are available to obtain the data about a particular point described by its curvilinear coordinate S :

x, y	the point coordinates [m],
$\frac{\partial x}{\partial s}, \frac{\partial y}{\partial s}$	the first derivatives of the coordinates [m],
$\frac{\partial^2 x}{\partial s^2}, \frac{\partial^2 y}{\partial s^2}$	the second derivatives of the coordinates [m],
σ	the orientation of the trajectory tangent, w.r.t. the x axis [rad],
ρ	the trajectory curvature [1/m].

Steering controller algorithm

The controller is inspired from the natural human behavior. Firstly the driver turns the wheel according to the appreciation he has about the curvature of the trajectory. Secondly, he tries to align the direction of traveling of the vehicle with the direction of the trajectory. Thirdly, he endeavors to minimize the distance between a reference point of the vehicle and the ideal trajectory. The numerical controller is built on the same principle. The good drivers anticipate the trajectory during their driving so the curvature and the orientation of the trajectory taken into account are not measured at the nearest point, N , but rather evaluated at a target point, T , ahead on the trajectory, at a distance, L_{target} , from the nearest point which depends on its speed. Hence, in the present work, the motion of the steering effector i.e. the rotation of the steering fork, μ , is given by the sum of three components:

- a non-linear function of the trajectory curvature ρ which depends on the vehicle steering mechanism and of the driver,
- a Proportional Derivative (PD) term on the difference, $\Delta\sigma$, between the current orientation of the vehicle σ_v and the curve orientation σ_t ,
- a Proportional Integral Derivative (PID) term on the difference, ϵ , between the current position of the vehicle and the spline.

The steering effector displacement is thus computed as follows:

$$\mu = f_{\rho\mu}(\rho) + PD(\Delta\sigma) + PID(\epsilon) \quad (3.12)$$

The different geometrical variables are illustrated in the figure 3.9.

At each time step, the nearest point on the curve, N , is the solution of a minimization problem which is handled by a NR algorithm. Then, the target point, T , is computed via the anticipation length equation, solved, here again, via an iterative solver.

The differential parts of the controllers, $\dot{\sigma}$, $\dot{\epsilon}$, are computed analytically whereas, on the contrary, the integral of the error position, $\int \epsilon dt$, is realized by the simulation integrator jointly with the acceleration and the velocity of the independent coordinates, and other variables from the specific model state equations.

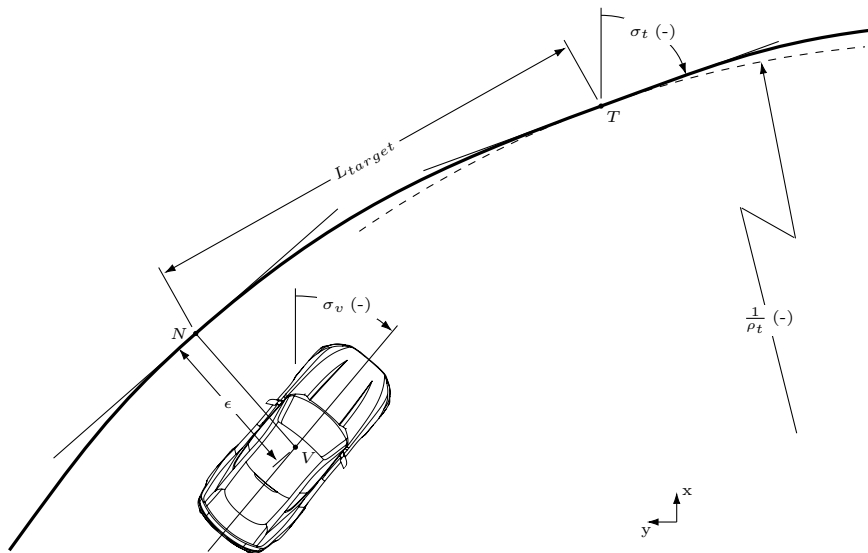


Figure 3.9: Steering control strategy [167].

The parameters of the controller are set, in a first time, following the proportionality and stability rules usual in PID setting. In a second time, they are refined by an optimization¹ for which the integral of the position error is minimized. Once the controller is parametrized, it is able to maintain the virtual vehicle on the trajectory selected for the vehicle dynamics optimization.

Speed controller

In this work, the scenario is relatively simple. The vehicle starts with an initial speed and freely rides along the trajectory up to the finish line. In other words, there is no speed control for the transient simulation. The implementation of a speed controller, that is not done, could be an interesting improvement of the framework in order to better evaluate the vehicle. Such a speed controller, and in particular, its introduction into the optimization process is not easy because it must take into account the target trajectory and the dynamics capabilities of the vehicle, notably in terms of longitudinal and lateral dynamics. In fact, it is not possible to know and to use these limits for the controller particularization and its action during the simulation while this process aims precisely to determine them. At our best knowledge, there do not exist yet an elegant solution to this problem. So, this question remains open.

¹For this purpose, the general optimization scheme was used. In this case, only the tilting controller coefficients have been selected as parameters. The objectives correspond to the mean and maximum values of the trajectory following error.

3.4.2 Steady-state cornering simulation

The quasi-static equilibrium analysis is used to compute the steady-state cornering behavior of the vehicle. This numerical process is able to compute the state of the vehicle for a given curve configuration: a given speed and a given radius. Repeating this computation over a range of speeds and a range of radii allows to characterize the vehicle on the entire range of use but only for the steady-state cases. The quasi-static computation requires several specific implementations described here above.

Original quasi-static equilibrium formulation

In a first formulation, the quasi-static equilibrium aims to find the equilibrium of the MBS tree following the equation:

$$\mathbf{F}_r(\mathbf{x}_{eq}) = \mathbf{0} \quad (3.13)$$

with

$$\mathbf{x}_{eq} = \mathbf{q}_u \setminus \mathbf{q}_{iu} \cup \dot{\mathbf{q}}_{iu} \quad (3.14)$$

As already mentioned in the section 2.2.6, the system has n_u , the number of independent coordinates, degrees of freedom, without being necessarily the independent coordinates. The possible variables are the position of the independent generalized coordinates or their velocity, via a variable exchange. All accelerations are forced to zero.

Let us apply this to the case of vehicle dynamics. In order to compute a steady-state curving as a quasi-static equilibrium, the problem must be formalized so that all generalized coordinates and their velocities are either constant, forced to zero or ignorable as the equilibrium is reached. Therefore, the kinematic chain between the inertial frame and the main body of the vehicle must be adapted to correspond to these specifications. This chain comes down to articulating the vehicle at the end of an arm in rotation around a vertical axis, in the manner of a carousel. The figure 3.10 illustrates the 7 DOF chain used in this work according to the ISO norm for the reference frame.

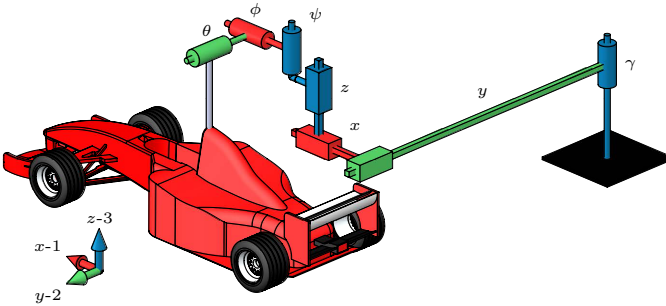


Figure 3.10: Kinematic chain for the quasi-static equilibrium.

γ	(R3) corresponds to the absolute angle of rotation of the vehicle in the described curve [rad].
y	(T2) corresponds to the radius of the trajectory [m].
x	(T1) is the longitudinal displacement, perpendicularly to the carousel arm [m].
z	(T3) corresponds to the height of the vehicle chassis [m].
ψ	(R3) corresponds to the attitude angle [rad].
ϕ	(R1) corresponds to the roll angle [rad].
θ	(R2) corresponds to the pitch angle [rad].

Note that for this kinematic chain, ψ exactly corresponds to the attitude angle¹ and not to the absolute yaw as it is the case for a transient simulation.

Because there are some ignorable variables, $\mathbf{q}_{iu}$, they must be exchanged with their velocities, $\dot{\mathbf{q}}_{iu}$. If the vehicle yaw rate $\dot{\gamma}$ takes a leading role in the equilibrium, the angle value γ is ignorable under the hypothesis of an horizontal flat ground with an homogeneous adherence characteristic, an indispensable condition to formulate a consistent steady-state problem. In the same manner, due to their predominant roles in the tire contact forces computation, the wheel rotational velocities must be considered instead of their angles which are dynamically ignorable, reason why they are exchanged.

By this mean, it is possible to compute the vehicle configuration for a given steering angle and a given torque applied on the driven wheels. So, this method allows to compute efficiently the exact steady state curving equilibrium.

Generalized quasi-static equilibrium formulation

As mentioned above, the static and quasi-static equilibria are some useful numerical processes that compute the generalized coordinates as functions of parameters, for different cases of study. In fact, for design and optimization purposes, the opposite problem is more relevant: finding parameters which correspond to a given equilibrium configuration i.e. to some given coordinates values. The interest for such a method is illustrated with two examples.

As taken as an example in the geometrical constraint computation via a variable exchange presented in the section 3.3.2, the goal of suspension tuning is to compute the neutral length of shocks springs as a function of given the body height and the pitch, and not the opposite.

In the case of the steady state curving computation for optimization purposes, the goal is to compute the vehicle configuration, the required steering wheel angle and the required wheel torques for an imposed speed and an imposed curve radius. This significantly differs from the natural results of a standard quasi-static equilibrium analysis i.e. a computed radius as a function of the steering angle and a speed resulting from the applied torque.

We have generalized the classical quasi-static equilibrium again via the variable exchange method which consists in exchanging the role of the so-called *exchanged independent generalized coordinates*, $\mathbf{q}_{eu}$, with some *exchanged quasi-static parameters*, $\mathbf{p}_{qse}$, into the solving process.

¹The attitude angle is defined as the angle between the speed and the median plan of the body.

$$\mathbf{F}_r(\mathbf{x}_{eq}) = \mathbf{0} \quad (3.15)$$

with

$$\mathbf{x}_{eq} = \mathbf{q}_u \setminus (\mathbf{q}_{iu} \cup \mathbf{q}_{eu}) \cup \dot{\mathbf{q}}_{iu} \cup \mathbf{p}_{qse} \quad (3.16)$$

Once more, the variable exchange method allows to invert some inputs and outputs of the numerical process what makes it more suitable for the optimization purposes.

From another point of view, this method can be considered as a mean to apply some constraints to the independent coordinates, but without introducing any associated forces and torques.

For the purpose of the steady state curving computation, we do the following exchanges. The length of the carousel arm, y , is exchanged with the steering column rotation, here classified as a forced coordinate and so considered as a parameter. In this way, the steering wheel angle is computed to get the required curve radius which is set to the y variable. In order to compute the driven wheel torques, this parameter is exchanged with the longitudinal chassis displacement, x , which is an independent coordinate in this formulation but locked to zero in the exchange, for obvious reason.

In addition to being exact and efficient, the generalized quasi-static equilibrium allows to formulate the problem so the results are more relevant from the point of view of the designer.

Tire model linearization

As the other MBS equation systems in general, the generalized quasi-static equilibrium equations 3.15 are strongly non-linear. This is even reinforced by the enclosed tire models themselves severely stiff and non-linear as shown in the figure 3.11 illustrating one characteristic curve of a tire model: the longitudinal force as a function of the longitudinal slip. The problem is encountered even with the simple implementation of the tire model via the steady-state "Magic formula" [66], for which, all transient components are omitted for the equilibrium computation (section 2.1.2).

This manifest high stiffness and the non-linearity affect the convergence of the solving process. To ensure a good convergence despite these problems, it is required to linearize the tire model, at each iteration of the equilibrium solver and, individually, for each tire. The proposed implemented algorithm is illustrated in the figures 3.11 and 3.12.

According to the figure 3.12, the iterative process starts with the computation of a first linearization of the tire model. The reference point (p_0) is the result of the static equilibrium performed earlier. In this configuration, the longitudinal and lateral forces are zero but it is not the case for the normal loads. Based on this first linearization (l_1), a first equilibrium is computed (p_1) and is projected on the actual tire characteristics (p_1'). Then, a second linearization (l_2) is computed around the new reference point (p_1') in order to restart the loop until the convergence is reached. The projections from the equilibria points on the actual tire characteristics are achieved following the

paths for which the kinematics variables, i.e. the inputs of the tire model, remain constant (vertical magenta lines in the figure 3.12) rather than the paths normal to the curve or those for which the tire forces, i.e. the outputs of the model, are constant, because this offers more numerical stability avoiding to surpass the saturation point. In general, only 4 or 5 iterations are sufficient to reach a relative precision of 10^{-5} on the equilibrium equations.

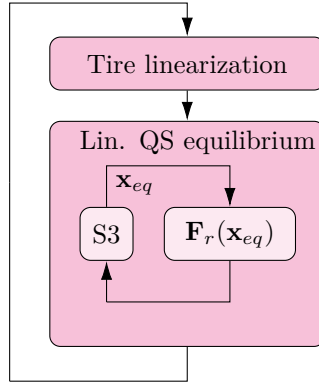


Figure 3.11: Quasi-static equilibrium analysis.

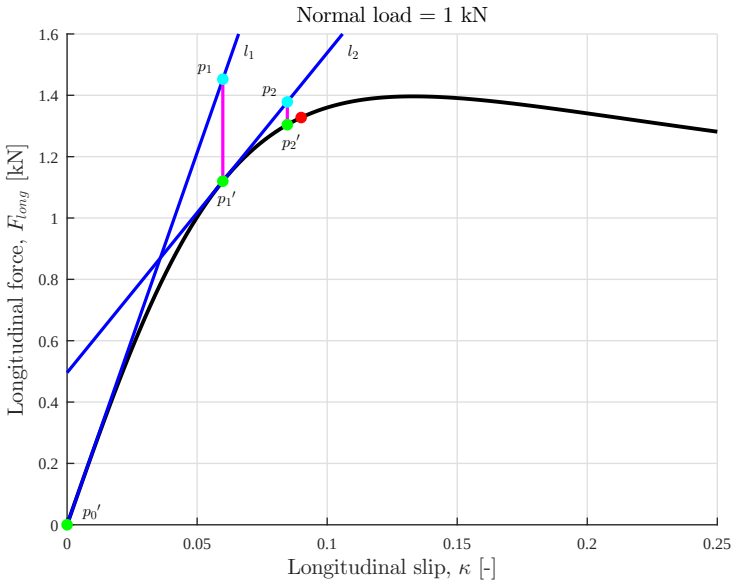


Figure 3.12: Tire model linearization algorithm.

The linearized tire model corresponds to the equation:

$$\begin{bmatrix} F_{long} \\ F_{lat} \\ M_n \end{bmatrix} = \frac{F_n}{F_{n_{(i-1)'}}} \left(K_i \cdot \begin{bmatrix} \kappa - \kappa_{(i-1)'} \\ \alpha - \alpha_{(i-1)'} \\ \gamma - \gamma_{(i-1)'} \end{bmatrix} + \begin{bmatrix} F_{long_{(i-1)'}} \\ F_{lat_{(i-1)'}} \\ M_{n_{(i-1)'}} \end{bmatrix} \right) \quad (3.17)$$

In this expression, the index $(i-1)'$ refers to the reference points around which the successive linearizations are realized. K_i is the matrix of the linear coefficients for the iteration i . The coefficients of the matrix are numerically evaluated, at each iterative step, by the two points centered finite difference method. Among the inputs, the normal force receives a particular processing: the ratio between the current normal force and the reference normal force multiplies the rest of the model. This choice is inspired from the Coulomb's law that indicates the prime importance of this variable in the friction behavior. Moreover, the practical experiences show that this model gives the best convergence properties. We have not found any theoretical justification for this result.

Here, we want to remind that the linearization of the model only aims to improve the convergence of the quasi-static equilibrium process. Hence, in this case, the performance of the linearization is not related to the precision of the linearized model with respect to the Magic formula or to the actual tire behavior but only to the convergence properties. Thus, the precision of the equilibrium results is not affected by the linearized model because the linearization is embedded in an iterative loop which is supposed to converge.

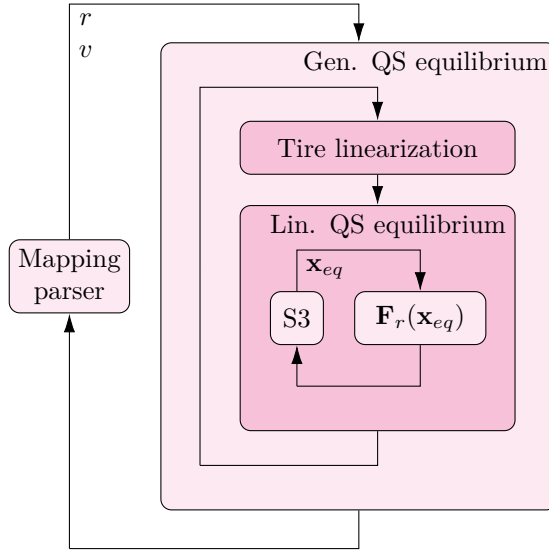


Figure 3.13: Steady-state analysis.

Steady-state cornering analysis

Independently of the used formulation, the quasi-static equilibrium solving computes a particular vehicle steady-state which is obviously insufficient to characterize the vehicle. Its dynamics must be probed in multiple points well spread over the range of use. The mapping can be coordinated by multiple couples of variables as for examples:

- the original formulation: the steering angle and the driven wheel torque,
- the improved formulation: the vehicle speed, v , and the curve radius, r .

In this study, the second formulation is used. The net points are regularly spaced on these two axes. The range and spacing define the number of points that must be selected as a trade-off between the mapping roughness and the cost of the analysis. The points which each corresponds to a generalized quasi-static equilibrium computation are computed one after the others following the routing scheme illustrated in figure 3.14. The lines and rows respectively correspond to the radius, r , and the speed, v .

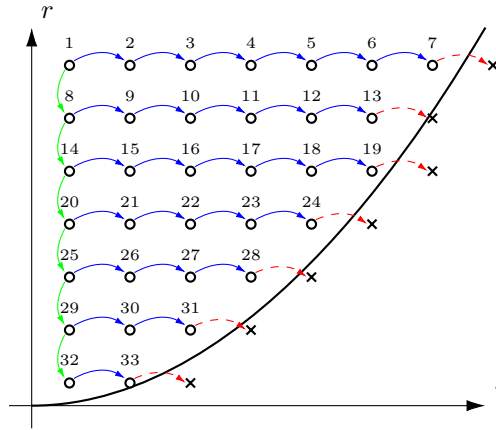


Figure 3.14: Steady-state analysis map routing.

The routing is chosen to optimize the convergence of the equilibria as well as several tips and tricks are implemented for the same reason. The routing starts at the highest radius and the lowest speed. The initial conditions of this first equilibrium is simply the pure static equilibrium results. On the first line, the speed is increased incrementally and an equilibrium is computed at each step (blue arrows in the figure 3.14), until the convergence of the solver can not be achieved (red arrows), what indicates, if the problem is well formulated, that the dynamics limits of the evaluated vehicle are reached. A second reason of the line stop is the reach of the centripetal acceleration limit fixed by the designer (dark parabola). The initial condition of each equilibrium is the solution of the previous one. At the end of the line, the results of the first line equilibrium of the current row, that have been preparatorily saved, are restored as the initial

conditions for the computation of the first point of the next line (green arrows). All the lines i.e. all the radii are computed following this strategy.

In order to help the convergence even more, the initial conditions can be improved: the value of several independent coordinates are roughly pre-computed by a simplified model. Such pre-computations can be implemented either for the first equilibrium, at the beginning of a new line or at each equilibrium. According to the type of pre-computations, the MBS geometrical and/or kinematics loop closures are required to update the dependent coordinates at the position and the velocity levels.

Thanks to this whole of provisions, the number of iterations required to reach the tolerance threshold is reduced. In general, between two successive equilibria, only one or two iterations are sufficient which makes the whole set of equilibria very efficient.

As for the transient behavior simulation, the dynamics characteristics of the vehicle have to be recorded to be used by the evaluator. At each equilibrium, the counters of successful and of failed equilibria are updated according to the reach of convergence. For the successful equilibria, the criteria values are compared to the minimum and maximum current bounds, which are updated, if any. Because the equilibrium analysis is based on a discrete probing over the $\{v, r\}$ map, the means are computed by the cumulative sums of the criteria values divided by the number of the successful equilibria. This constitutes the matter useful for the evaluator.

3.4.3 Complete simulator scheme

The complete hybrid simulator scheme is depicted in the figure 3.15. Both the transient simulation and the steady-state analysis are juxtaposed: there is not interrelation between these two processes. They could be parallelized but because of the large difference of CPU time needs between them, the interest of the approach is very low and not implemented.

3.5 Evaluator

The evaluator aims to compute, for each vehicle candidate, a unique scalar that quantifies its dynamics qualities. By convention, the problem is formulated as a minimization, that is, the objective function has to be as lower as the vehicle performs well. This function is built on the basis of the evaluation of some criteria recorded along the simulations. As explained in the section 3.4, the vehicle is evaluated on a hybrid simulator only made of a transient simulation and a steady-state analysis. In the general case, the evaluator could also exploit the information coming from the static equilibria, from the modal analyses, or from other numerical processes, either already present in the data conditioner or added inside the hybrid simulator. For the Wemoov project (chapter4), the need of such integration did not appear to be mandatory.

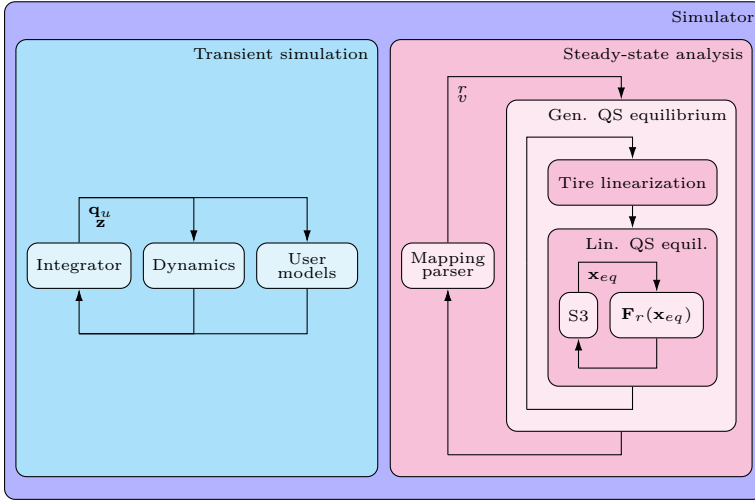


Figure 3.15: Complete simulator scheme.

For any objective, the corresponding criterion must be formulated such the ultimate i.e. unreachable, best performance corresponds to the zero value (green) and degrades progressively toward either negative (blue) or positive values (red).

3.5.1 Generic criteria

In this section, some generic criteria related to the vehicle dynamics are introduced. They can be applied on a broad range of road vehicles. The criteria especially dedicated to the Wemoov are developed separately in the section 4.4.4.

Under-oversteering coefficient

The under/over-steering behavior is a major concern in the field of the vehicle dynamics. It is related to the adherence balance between the front and the rear suspensions. It is computed by the difference between the mean slip angle of the front suspension and of the rear one:

$$E_{\alpha} = |\overline{\alpha_{ft}}| - |\overline{\alpha_{rr}}| \quad (3.18)$$

for which a negative, null and positive values respectively refer to the oversteering, neutral steering and understeering.

A good performance for this criterion allows to reduce the risk of loss of control in case of a high lateral acceleration cornering. This improves the lateral dynamics in general.

The criterion is influenced by tens of parameters as varied as the location of the center of mass of the body, the location of the roll axle, the stiffness of the anti-roll bar, the camber, the toe-in toe out, the suspension kinematics, the

tire characteristics and particularly the tire inflation pressures, to cite only a few of the most influential ones. This criterion is obviously the source of lots of couplings in the optimization problem.

Pseudo-Ackermann criterion

The pseudo-Ackermann criterion expresses the transverse adherence balance between the two wheels of the steering axle. It is a dynamic criterion similar to the geometrical Ackermann condition, hence the name of "pseudo-Ackermann criterion". From another point of view, this criterion can be interpreted as the dynamical extension of the toe-in/toe-out characteristic for the entire suspension and the steering motion range.

It is computed by the difference between the slip angles of the two wheels of the steering axle i.e., in general, the front wheels:

$$E_{Ack} = \alpha_{lt} - \alpha_{rt} \quad (3.19)$$

where α_{lt} and α_{rt} are respectively the slip angles for the left and the right front tires.

If minimized, this criterion improves the lateral dynamics. Indeed, large values for this criterion, positive as well as negative, involve the tire lateral force saturation and reduce the cornering capability. At the same time, the criterion aims also to minimize the tire wear.

The pseudo-Ackermann criterion is governed by the kinematics of both the steering axle suspension and the steering mechanism.

Steering wheel torque

This criterion aims in limiting the effort needed to control the steering wheel, whilst providing the suitable torque feed-back to the driver.

The first consideration is all the more important if the vehicle is free of steering assistance. The torque must not only be physically acceptable for the average driver but also be such the drive feels comfortable.

However, as explained earlier, the driver needs the whole information got from the torque feed-back. Optimally, the steering wheel torque, T_{sw} , must be as close as possible to the ideal torque feed-back, T_{fb} , as a function of variables: at least, the steering wheel angle, μ , and the speed, v :

$$E_{T_{sw}} = T_{sw} - T_{fb}(\mu, v) \quad (3.20)$$

Unfortunately, the ideal feed-back function, T_{fb} , is not so easy to synthesize. Basically, it depends on the type of vehicle and the spirit of the manufacturers want to give to these vehicle models. The ideal feed-back varies as a function of the temperament, the level of driving ability, the physical force, the gender, the community etc. The expectations of the particular individuals are very different. Moreover, the function could also depend on the road conditions. Finally, the transients need to be specifically treated.

The determination of such an ideal feed-back function being utterly out of our fields of competence, we have decided to consider a criterion simplification. In this case, the steering wheel torque is simply taken as a criterion.

$$E_{T_{sw}} = T_{sw} \quad (3.21)$$

In that way, we only consider the first goal and pass over the management of the torque feed-back despite its importance. Given this simplification, the status of this criterion is questionable. Indeed, it can not be reduced too much, what is contrary to the classical characteristic of the criteria. Thus, at the end of the process, the results of the optimization must be reviewed to ensure the compatibility with the controllability specifications.

The most influencing parameters are the steering box ratio, the geometry of the steering mechanism and the characteristics of the steering axes: the trail, the caster angle, the scrub radius and the kingpin angle.

Track following error

This variable, here considered as a criterion, has been already introduced in the description of the steering controller in the section 3.4.1. As a reminder, it is measured by the distance between a reference point on the vehicle and the nearest point on the target trajectory (figure 3.9).

$$E_{\epsilon} = \epsilon \quad (3.22)$$

In a first time, this criterion is used alone for the preliminary optimization of the steering controller. In a second time, it allows to improve the controllability of the steering mechanism from a mechanical point of view. The deviations of the trajectory, intense and punctual or, conversely, regular, are harmful and sanctioned via such a criterion.

Although the steering controller parameters are the most sensitive, the most of the design ones impact on the criterion.

3.5.2 Criteria clustering processing

The criteria introduced in the previous section are quantities that can be evaluated at each time of the transient simulation and for each equilibrium point of the steady-state analysis. So, for each criterion, the data measured along the simulator represent a large amount of information which must be agglomerated in order to be compared with the other criteria by the mix law. In this section, the clustering processing of the data is given. This task is performed separately, criterion by criterion contrarily to the mix law whose purpose is to combine the multiple criteria. The latter is the subject of the next section.

In order to agglomerate the data, we have decided to work, for each criterion, along two different axes:

Firstly, the data from the transient simulation and the steady-state analysis are treated separately because they concern different process but also because they refer to continuous and discrete computations respectively.

Secondly, we propose to focus the criteria analysis on two distinct clustered meta criteria. From one hand, we found an interest for the mean value of the sample. On the other hand, the maximum value over the sample also seems certainly interesting to take into account.

In definitive, for each criterion, four *meta-criteria* are computed. Their definition are summarized in the following table:

	Transient simulation (TS)	Steady-state analysis (SS)
Mean value	$\bar{E}_i = \frac{1}{t_{end}-t_{go}} \int_{t_{go}}^{t_{end}} E_i(t) dt$	$\check{E}_i = \frac{1}{n_k} \sum_{k=1}^{n_k} E_i(k) $
Max value	$\hat{E}_i = \max_{t \in [t_{go} \rightarrow t_{end}]} (E_i(t))$	$\tilde{E}_i = \max_k (E_i(k))$

where:

- $\bar{E}_i$ the mean value of the criterion in the transient simulation,
- $\hat{E}_i$ the maximum value of the criterion in the transient simulation,
- $\check{E}_i$ the mean value of the criterion in the steady-state analysis,
- $\tilde{E}_i$ the maximum value of the criterion in the steady-state analysis,
- i the index of the criterion,
- k the index of the successful equilibrium in the steady-state analysis,
- n_k the number of the successful equilibria in the steady-state analysis.

Note that the meta-criteria have respectively the same units as the related criteria.

As already mentioned in the section 3.4.1, the values of each criterion over the time span of the simulation are not really recorded. In the optimization runs, only the current integral and the current maximum value of each criterion are maintained. In this way, large amount of memory is saved and the simulation is hastened. Similar techniques are implemented for the steady-state analysis.

3.5.3 Mix law

The mix law, which has been introduced in the section 2.3.1, aims to combine the multiple meta-criteria used to assess the vehicle quality into a unique scalar. This mix law is at the basis of the computation of the objective function detailed in the next section. Getting a coherent mix is complex for several reasons.

Firstly, each of the criteria measures one of the transversal characteristics which are fundamental in the study of the vehicle dynamics. So, the nature of the criteria are very different and, in the same way, also their units. For example, the comparison between the track following error and the steering torque, expressed in meters and in Newton-meters respectively, requires some harmonization rules.

Secondly, although their nature can be the same, the criteria values may present unequal orders of magnitude, making some criteria sometimes disproportionate with respect to the others.

Thirdly, the criteria do not have the same relation to the global vehicle quality, taken from the point of view of the designer. For some criteria, an improvement of one unit constitutes an unhoped great enhancement, but for some others, such gain is considered as insignificant. For example, a change of 0.1 Nm on the steering torque feed-back corresponds to the upper bound for the sensitiveness threshold whereas a 0.1 rad shift for the pseudo-Ackermann or the under-oversteering coefficients indicates a radical conversion of tire work mode e.g from linear to complete saturation. So, the non-linearity of the function between the particular criteria and the global vehicle quality has to be carefully taken into account.

The well-known classic mix law is made of the weighted sum of the n_E criteria evaluation:

$$F_o = \sum_{i=1}^{n_E} w_i E_i \quad (3.23)$$

According to the previous discussion, this method is rather ineffective because incomplete. The presence of the weights, w_i , in the formula allows to take the two first concerns i.e. the difference of nature and order of magnitude of the criteria into account but, on the contrary, the non-linearity is completely omitted. Thus, we propose to upgrade the common classic law to the so-called *advanced mix law*:

$$F_o = \sum_{i=1}^{n_E} w_i E_i^{z_i} \quad (3.24)$$

where the weights w_i and the exponents z_i are the setting coefficients of this new mix law. By this way, the non-linearity is introduced in a given extent. We do not claim that this mix law is perfect and that it responds definitively to the difficult task of the scalar objective function synthesis. Actually, we consider such a choice is more suitable than the classic law for the technical optimization purposes.

At this stage, the sensible question of the choice of the law setting coefficients is still open and even revived by the introduction of the exponents z_i . In this regards, we suggest a simple but efficient and universal method. The determination of the coefficients is based on the equivalence of satisfaction between all involved criteria: for each criterion, the designer gives a *satisfaction* level, E_s , which correspond to a "10 points" score and an *acceptance* level, E_a , which corresponds to a "1000 points" score. This scoring results in two equations for which the two unknowns are the mix law coefficients, w_i , z_i . Those can be easily extracted:

$$\begin{cases} w_i E_{s_i}^{z_i} = 10 \\ w_i E_{a_i}^{z_i} = 1000 \end{cases} \Rightarrow \begin{cases} w_i = 10^{\frac{3 - \frac{\log(E_{a_i})}{\log(E_{s_i})}}{1 - \frac{\log(E_{a_i})}{\log(E_{s_i})}}} \\ z_i = \frac{2}{\log\left(\frac{E_{a_i}}{E_{s_i}}\right)} \end{cases} \quad (3.25)$$

The weights, w , have the inverse units of the corresponding criteria, at the z power, while the exponents, z , are unitless. The advanced mix law is a non-linear adimentionalization method.

As a practical example, the driver steering torque is part of the objective function as discussed before. We consider that, physiologically speaking, the maximum torque that a common driver is able to develop is 16 Nm, that constitutes the acceptance limit. We assume too that, for driving comfort reason, a maximum torque of 4 Nm is suitable. It fixes the satisfaction level. Finally, we can infer the coefficients:

$$w_{T_{sw}} = 10^5 \quad (3.26)$$

$$z_{T_{sw}} = -3.322 \quad (3.27)$$

These coefficients are useful in the evaluation of the objective function, computed on the basis of the mix law following the equation 3.24.

3.5.4 Objective function

This section explains how the complete characterization of the vehicle derived from the input set of optimized parameters is summarized and sent to the optimizer through a single scalar with the help of the advanced mix law (equation 3.24). This extreme reduction of the entire trial work, while maintaining vehicle robustness, efficiency and consistence, is a big challenge.

The convergence of the entire process depends on a fundamental principle about the objective function: the returned value must be regularly as high as the vehicle candidate is wrong. Moreover, it has to be enough sensitive so that each improvement is noticed.

Admittedly, in a final phase, the objective function is based on a mix of the different meta-criteria evaluated along the simulator. This allows to refine the parameters up to the best vehicle from the point of view of the criteria taken into account. But, such a comparison supposes that the particles are able to be evaluated through the tests implemented in the simulator. Such a capability is not trivial, in particular at the beginning of the optimization for which the volume of search is maximal. The success of the trial regroups several conditions which are as many exams for which the candidate can succeed or fail. In every case, the global objective function has to satisfy the fundamental principle of sensitivity. In order to respect this feature, all the potential causes of failure must be identified, listed, ordered and associated with a proper penalty score inversely proportional to the progression along the overall trial. In this way, the optimizer is able to promote the particles which are evaluable or, at least, as close as possible to this condition.

The computation of the global objective function is split in different successive phases according to the progress into the trial process:

- Phase 1: failure in the data conditioner.
- Phase 2: failure in the simulator.
- Phase 3: success in the simulator.

The computation methods of these 3 phases are detailed here below.

Phase 1: failure in the data conditioner

First of all, the vehicle must be kinematically feasible, i.e. able to satisfy the kinematic loop closure constraints. Then, it must satisfy the three types of design constraints. In this regard, the potential causes of failures that can occur inside the data conditioner are classified, from the worst to the less harmful, in the following table:

id	Data conditioner failing processes	Penalty scores
1	Kinematic loop closure constraints	10^{15}
2	Algebraic constraints	10^{14}
3	Geometrical constraints	10^{13}
4	Modal constraints	10^{12}

Table 3.1: Potential causes of failures in the data conditioner.

Deeper the process is embedded more the associated failure is considered as detrimental for the overall convergence.

Note that due to the form of the algebraic constraints used in this work, a failure in this operation is impossible. It is not the case for the general formulation, for which, this kind of constraints must also require to be solved with a numerical iterative solver.

In the process, the scale of progress is discrete. In fact, for a non-realistic vehicle, it is difficult to assess the gap with respect to a possible one. Indeed, the progressive penalties based on the residue or some other indicators, are not robust because subjected to the local minimum. Moreover, in case of divergence of the solving, these penalties are not properly defined. So, despite the fact that we largely penalize the failure in the data conditioner, if too many particles encounter convergence failure, the optimization can not converge.

Phase 2: failure in the simulator

Once the data conditioner is passed, the vehicle is ready for the hybrid simulator (figure 3.3). In this case, a more progressive marking is possible. From this phase, the global objective function combines the contribution of the steady-state analysis and the transient simulation. It is simply the sum of the two components:

$$F_o = F_{oTS} + F_{oSS} \quad (3.28)$$

Transient simulation

The vehicle is launched on the test trajectory with the objective to reach the point of arrival. Unfortunately, some different problems can occur on the travel and stop the progress to this goal. In the general case, the most common troubles are either the rollover or the lost of control due to excessive tire sliding (understeering or oversteering depending on the setting). Some other issues can be detected specifically depending on both the vehicle and the test trajectory.

Such pitfalls are on the responsibility of the settings of the vehicle candidates and must be penalized.

Specifically to each simulation, the designer must formulate the conditions of failure that avoid the possibility to continue with the expectation of relevant results. At each time step of the simulation, the failing conditions are evaluated and, in case of overrun of the predetermined threshold, a marker is switched on, what definitively stops the integration loop. In this case, the evaluation of the objective function is chosen to be inversely proportional to the elapsed time before the failure detection and weighted in order to be higher than the worst vehicle which is able to reach the finish line. In view of the above, the transient simulation part of the objective function is computed by:

$$F_{oTS} = 10^9 + 10^7(t_{ref} - t_{fail}) \quad (3.29)$$

where t_{fail} is the time when the failure is detected, t_{ref} is the reference time for which the travel of the entire test trajectory is possible. We do not take advantage of the possibility to treat differently the diverse failure condition.

In that way, the vehicles able to travel longer before failing are promoted. So, the optimization steers the candidates through the success in the test trajectory. In these cases, the particles are evaluated in phase 3.

Moreover, throughout the transient simulation, the kinematic loop closure solver can fail to converge. At this level, it can be considered that the problem comes from a bad geometrical or mechanical parametrization and not from a numerical issue. In all cases, this is viewed as critical as the failures detected by the designer conditions. So, the loop closure divergence is affected to the following penalty score:

id	Simulator failing process	Penalty score
5	Kinematic loop closure constraints	10^{11}

Table 3.2: Potential cause of failures in the transient simulation.

Steady-state analysis

According to the quality of the vehicle candidate, the number of quasi-static equilibria that converge is more or less high. Nevertheless, because, in the data conditioner, the static equilibrium convergence has been obtained, there is a very low risk that absolutely none of the quasi-static one converges. This was never encountered in practice. So, in general, we have, for the first time, the opportunity to assess the quality of the vehicle with the dynamics criteria. In this case, we use both the clustering and the mix law to compute this part of the objective function:

$$\begin{aligned}
F_{OSS} &= \left(\frac{n_k}{n_{kref}} \right)^{-z_{SS}} \left(\sum_{i=1}^{n_E} \check{w}_i \check{E}_i^{\check{z}_i} + \sum_{i=1}^{n_E} \tilde{w}_i \tilde{E}_i^{\tilde{z}_i} \right) \\
&= \left(\frac{n_k}{n_{kref}} \right)^{-z_{SS}} \left(\sum_{i=1}^{n_E} \check{w}_i \left(\frac{1}{n_k} \sum_{k=1}^{n_k} |E_i(k)| \right)^{\check{z}_i} \right. \\
&\quad \left. + \sum_{i=1}^{n_E} \tilde{w}_i \left(\max_k (|E_i(k)|) \right)^{\tilde{z}_i} \right)
\end{aligned} \tag{3.30}$$

where:

- i the index of the criterion,
- k the index of the successful equilibrium in the steady-state analysis,
- n_k the number of successful equilibria,
- n_{kref} the reference of the number of successful equilibria,
- z_{SS} the exponent of equilibrium success ratio.

The term $\left(\frac{n_k}{n_{kref}} \right)^{-z_{SS}}$ multiplies the rest of the evaluation in order to promote the vehicles that succeed in a maximum of points in the mapping. Naturally, the condition $z_{SS} > 0$ must be respected. The experience has shown that, thanks to this choice, the number of successful equilibria quickly reaches the reference what vanishes itself its own effect.

The global objective function, made of the sum of equations 3.29 and 3.30, fosters the success in the transient simulation. In this case, the vehicle is evaluated in the phase 3.

Phase 3: success in the simulator

When the vehicle is able to follow the full trajectory, its performances during the transient simulation are evaluated on the basis of the criteria merged together, as for the steady-state analysis, through the clustering processing (section 3.5.2) and the mix law (section 3.5.3):

$$\begin{aligned}
F_{OTS} &= \sum_{i=1}^{n_E} \bar{w}_i \bar{E}_i^{\bar{z}_i} + \sum_{i=1}^{n_E} \hat{w}_i \hat{E}_i^{\hat{z}_i} \\
&= \sum_{i=1}^{n_E} \bar{w}_i \left(\frac{1}{t_{end}-t_{go}} \int_{t_{go}}^{t_{end}} |E_i(t)| dt \right)^{\bar{z}_i} \\
&\quad + \sum_{i=1}^{n_E} \hat{w}_i \left(\max_{t \in [t_{go} \rightarrow t_{end}]} (|E_i(t)|) \right)^{\hat{z}_i}
\end{aligned} \tag{3.31}$$

In this phase, the steady-state part of the evaluation remains the same and is thus given by the equation 3.30. The global objective function is still formed by the sum of the both parts following the equation 3.28.

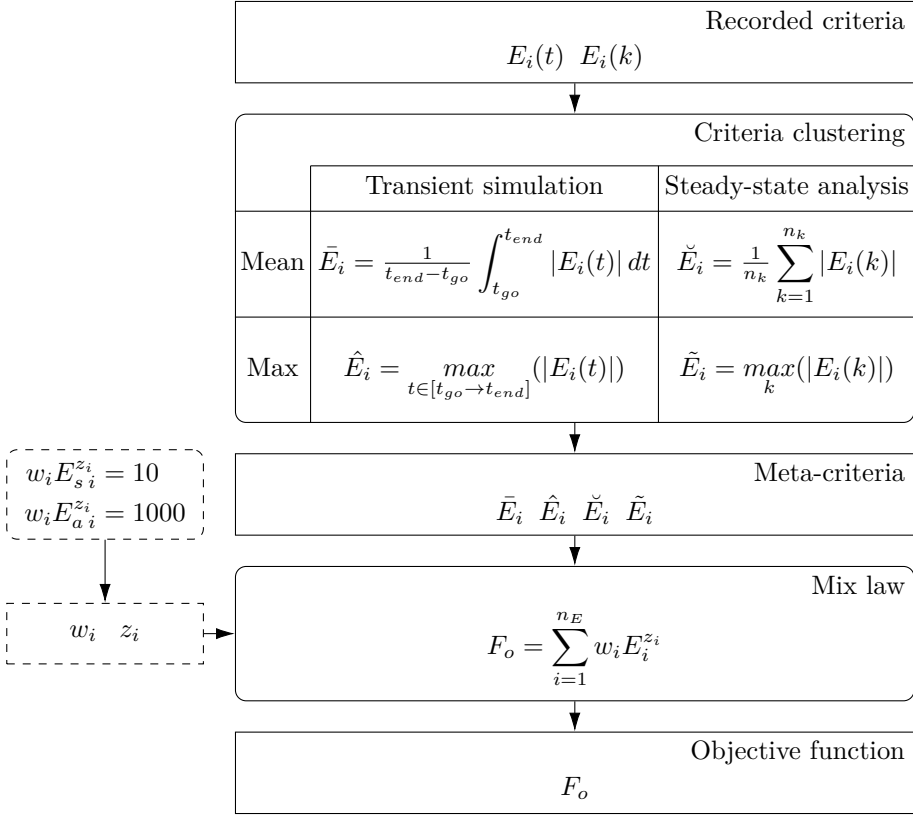


Figure 3.16: Evaluator process.

3.6 Optimizer

The optimizer is the optimization process engine. It generates the vehicle candidates under the form of the optimized parameter vectors transmitted to the trial for evaluation. This trial assesses the vehicle quality and computes an objective function on this base. Obviously, the optimization algorithm ignores that the concern is a vehicle optimization. It blindly drives the parameters with the objective function as single feed-back. The whole forms a loop which iterates until reaching the convergence of the algorithm.

Such an optimization problem is challenging because the number of independent parameters is high and they have various natures. Moreover, the ranges on which the parameters extent are large. It is clear that the vehicle dynamics optimization problem as formulated in this work do not enjoy interesting mathematical properties: the relation between the optimized parameters and the objectives function is neither linear, nor convex, nor continuous. In fact, the continuity can be observed only by parts and so, is not guaranteed. The partial derivative of the objective function are unknown and can not be

computed by a numerical method because the continuity is not guaranteed on whole the working space. However, thanks to the proposed scheme, the optimizer does not deal with the design constraints, which are, furthermore, solved internally by the data conditioner.

Eventually, as already discussed in the micro state-of-the-art concerning the optimization algorithms (section 2.3.2), the only class of optimizer able to handle this kind of optimization problem is the heuristics and meta-heuristics methods.

3.6.1 Parameter normalization

In order to enlarge the possible choices for the optimization algorithm, we decide to normalize the parameters. This is a common practice in the field of optimization and even required by some particular optimizers.

In other words, the optimizer does not directly deal with the optimized parameters $\mathbf{p}_o$ but with a normalized parameter vector $\mathbf{x}_o$ which follow theses conversion rules¹:

$$\mathbf{x}_o = \frac{\mathbf{p}_o - \mathbf{p}_{o \min}}{\mathbf{p}_{o \max} - \mathbf{p}_{o \min}} \quad \Leftrightarrow \quad \mathbf{p}_o = (\mathbf{p}_{o \max} - \mathbf{p}_{o \min}) \cdot \mathbf{x}_o + \mathbf{p}_{o \min} \quad (3.32)$$

where $\mathbf{p}_{o \min}$ and $\mathbf{p}_{o \max}$ are the range limits for $\mathbf{p}_o$, given by the designer. So, the optimizer only deals with $[0 \rightarrow 1]$ range parameters and all of them are treated equally despite of their natures, units and ranges.

3.6.2 CMA-ES use

The CMA-ES optimization algorithm [42, 43] is used in the context of this work. Its working principle is briefly introduced in the section 2.3.2. It is classified among the non-deterministic evolutionary algorithms. The CMA-ES specifications requires that the objective function is continuous to guarantee the convergence. However, despite the present problem is not fully continuous over the entire work space, the optimizer converges very well and gives full satisfaction. For this reason, we have never tried to switch to another optimizer that could be more compliant with such discontinuous problems.

Convergence criteria

The CMA-ES code gives multiple options for the choice of the termination conditions i.e. those that, if respected, stop the process. A first group of conditions is based on the convergence thresholds on:

- the objective function value,
- the variation of objective function (between two consecutive generations),
- the variation of objective function (considering historical best candidate),
- the step size in $\mathbf{x}$ -space.

¹For these two expressions, the scalar multiplication and division apply.

Note that only the objective function value of the best particles of its generation is considered. In addition, the use of joint criteria is also possible.

There are also some safeguards enable to avoid an endless process, what forms a second group of conditions:

- the maximum number of objective function calls,
- the maximum number of generations,
- the threshold for the increase of the standard deviation, σ .

Number of particles

The algorithm uses a predetermined number of particles n_{part} , which represent as many vehicle candidates. At each generation, the algorithm produces n_{part} vector parameters $\mathbf{p}_o$ and evaluates each of them via the trial. The number of particles by generation is an important setting of the optimizer. On one hand, the required computer resources are directly proportional to this number, which must be limited. On the other hand, if the number of particles is insufficient, the optimization may exhibit difficulties to converge. By experience, we have found that the condition $n_{part} > 2n_{\mathbf{p}_o}$ must be respected to ensure a correct convergence.

3.6.3 Alternative optimizers

Although it was not detrimental for the realized optimizations, the discontinuity of the objective function could bring some convergence problems. Here, we propose some potential solutions if such a problem happened.

In order to improve the adequacy between the selected optimizer and the suspension optimization problem, it is possible to opt for an algorithm more compliant with the encountered discontinuities. PSO, which has been introduced in the section 2.3.2, does not require such a property and thus appears as a good candidate to overcome the problem. It is the simplest way but the efficiency could be affected.

To go further, several different discontinuities of the objective function can be distinguished:

- The first source comes from the fact that the computation is split in three phases, all of them with some very different computation rules.
- In the phase 1, the penalty scores are discrete and so, discontinuous by nature.
- In the phase 2, the non-convergence of the loop closure algorithm leads to an isolated penalty score.
- In phases 2 and 3, the number of converged equilibria in the steady-state analysis participates in the objective function. Here also, this is a discrete measure that induces some discontinuities.
- In phases 2 and 3, a small change of the design parameters can lead to a large upheaval of the vehicle behavior and so to some large discontinuous impacts on the objective function result. The origin of such bivalent behavior stems from the discontinuities inside the model itself like, for example, the unilateral bump stops contact of the shocks.

As one can remark, the worst discontinuities appear at the beginning of the optimization run. If, in the early generations, some discontinuities of the objective function are certainly encountered, over a milestone, the objective function is more regular. Thus, a hybrid optimizer could be implemented. In a first stage, a PSO or every else other discontinuity compliant optimizers is used. In a second stage, that is initiated once the loss of discontinuity is tracked, the process switches to CMA-ES and benefits from these advantages. This hybrid solution have not been implemented despite it could be a good trade-off between the discontinuity handling and the convergence efficiency.

3.7 Global optimization scheme

The global optimization scheme is illustrated in the figure 3.17. It is made of a complex tangle of numerical processes in which act up to five levels of embedded solving and optimization loops.

3.7.1 Programming language

For the creation of a such optimization method, the choice of the programming language is very important. Indeed, it has a large impact of the development and on the computational efficiency.

Above all, we have chosen to work with a unique language for all the optimization process. Firstly, it is simpler than to juggle with multiple languages. Secondly, the interfaces between different languages represent a large development time. Also, their run leads to a prohibitive execution time.

We have chosen to implement the code in the C language. It is a widespread language with large amount of material available: large basic function set, multitude standard libraries for every specific purposes, abundance of documentation, multitude of forums, tutorial and pre-coded examples. It is a rigorous language that suits perfectly with the technical and scientific programming. It offers a very good memory and CPU efficiency, in particular, using the best compilers. These compilers are able to optimize the code and, in the way, to reduce the CPU time by a factor 3 or 4 with respect to a standard compiler. Note also that the C codes can be developed nearly free of charge: numerous editors, compilers and debuggers are available for free in the their basic versions, what are largely sufficient for our utilization. Finally, C is cross-platform: fully compatible by Window and Linux environment. It must be admitted that the language is not full friendly but it is the price of the operation control and the efficiency.

In addition to the intrinsic advantage of the C programming, some useful tools required for this work are available and fully compatible with C. Robotran was yet partially implemented in C. A C version of CMA-ES is available and free. Moreover, the Linux based hosting cluster is fully compliant with the C code.

3.7.2 Error management

The evaluation of a vehicle candidate implies tens of numerical processes and each of them is potentially deficient. Moreover, the overall optimization involves thousands particle evaluations, so, the probability to encounter a problem is very high. Thus, an error management system must be implemented to avoid that a single small local problem leads to the total optimization failure.

Without entering into details, we have implemented an error management based of the C function return, int typed. The true outputs of the functions are forwarded through the pointers in arguments and never as the return. In the normal execution, the return of each function is 0, while, if something wrong appears, a specific code is returned. This code depends on the type of error and the location and the context of the occurrence. Once an error is generated, the parent functions skip the normal execution and transmit the errors through the call stack until the error is caught and treated. In general, the errors are associated with a penalty score instead of the criteria based objective function, as explained in the section 3.5.4.

3.7.3 Parallel computation

Because the objective function evaluation via the trial is completely independent between particles, it is possible to run the optimization with a natural parallelization of the tasks. Obviously, it allows to dramatically drop the global optimization lead time. In this respect, a CPU core is dedicated to the evaluation of each particle. These processes are called the slaves. An additional core, the master, is reserved to the optimizer task itself. From a C code implementation point of view, the Message Parsing Interface (MPI) library is used as the communication channels between the master and the slaves: to send the vectors of parameters under their normalized form $\mathbf{x}_o$ and to get the objective function results.

The code execution was entrusted to the Lemaitre3 cluster of the Consortium des Equipements de Calcul Intensif (CECI). The CPU type of the cluster is SkyLake at 2.3 GHz. With up to 1920 cores available, the cluster easily allows to allocate between 50 to 100 cores i.e. more than twice the number of the treated optimized parameters $n_{\mathbf{p}_o}$. The cluster uses a Linux environment. It proposes a pre-installed C compiler and usual libraries. The cluster environment is compared to that of the development laptop in the table 3.3.

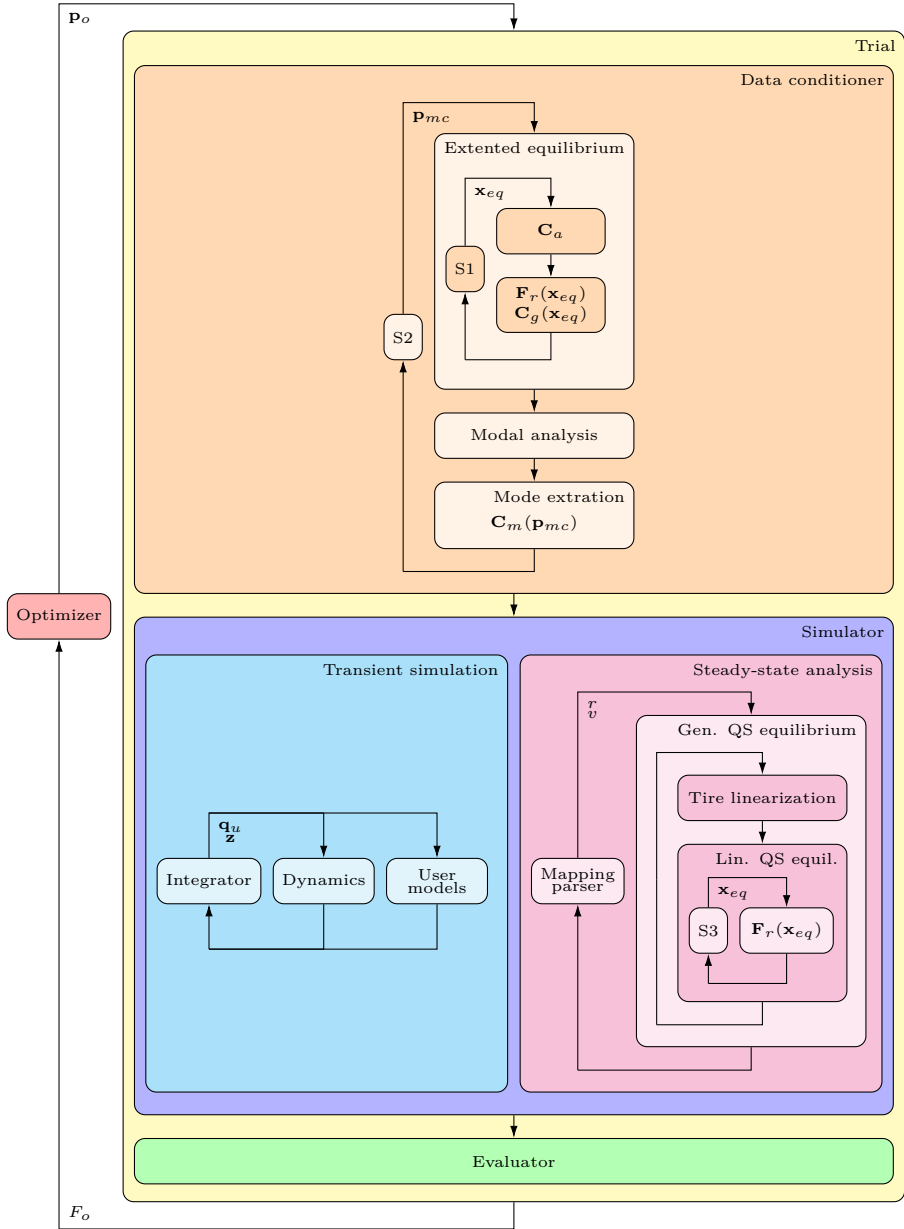


Figure 3.17: Global optimization scheme.

	Laptop	Cluster
Model	Dell Precision M6700	Lemaitre3
Processor	Intel Core i7-3540M CPU @ 3.0 GHz	80 nodes with two 12-cores Intel SkyLake 5118 CPU @ 2.3 GHz
Random Access Memory (RAM)	16 GB DDR4	95 GB (3970 MB/core)
Operating system	Window 7 Professional SP1 64 bit	Linux
Project manager	CMake 3.8.1	CMake
Development environment	Microsoft Visual Studio Express 11.0 2012	/
Compiler	MSVC 17.0 /O2	GCC 6.4.0
MPI version	Microsoft HPC Pack 2012	OpenMPI 2.1.1
Secure shell client	PuTTY 0.63	/
File transfer protocol GUI	MobaXterm v11.1	/
Uses	Development, debugging, trial runs, real time runs, test optimization runs	Optimization runs

Table 3.3: Comparison of the programming environments: laptop and cluster.

3.7.4 Inequality constraints

A priori, the formalism does not handle inequality constraints expressed as:

$$\mathbf{C}_{\geq}(\mathbf{p}) \geq \mathbf{0} \quad (3.33)$$

But, in fact, this limitation can be easily overcome. It is enough to introduce some fictive optimized parameters $\mathbf{p}_{o\geq}$ and to constrain the values subjected to the inequality constraint to these new fictive parameter values through some additional constraints $\mathbf{C}_{=(\geq)}$:

$$\mathbf{C}_{\geq}(\mathbf{p}) = \mathbf{p}_{o\geq} \quad \Leftrightarrow \quad \mathbf{C}_{=(\geq)}(\mathbf{p}) = \mathbf{C}_{\geq}(\mathbf{p}) - \mathbf{p}_{o\geq} = \mathbf{0} \quad (3.34)$$

In this case, the boundaries of the optimized parameter ranges have to be fix such as the inequality constraints are respected:

$$\mathbf{p}_{o\geq \min} = \mathbf{0} \quad (3.35)$$

$\mathbf{p}_{o\geq \max}$ must be chosen sufficiently large to allow free and so optimal work of the optimizer.

By this combination of both optimized parameters and constraints, each kind of inequality constraint is henceforth practicable. This technique was not used in this work because not required in the Wemoov development.

3.7.5 Pros and cons of the optimization methodology

The global optimization scheme that has been developed is able to handle the vehicle optimization problems as illustrated in the next chapter. Before to present the application of this approach, the main advantages and drawbacks of the method have to be summarized.

The main interest of the proposed approach is that both the constraints and the criteria can be taken into account in the same numerical process. Thanks to the developed data conditioner, all the constraints are not handled by the optimizer itself, but fully managed inside the trial function. This enlarges the choice of optimizers and enables to decrease the number of optimized parameters with the associated gain in terms of convergence.

Thanks to the efficiency of the MBS tools used in the data conditioner, the computation time for the constraint solving is negligible with respect to the simulation time (Simulator), leading to a really efficient optimization process.

Moreover, each simulation in the simulator block is run on a vehicle which satisfies all types of constraints. So, there is not any mean spent to characterize unsuitable vehicle candidates.

Notice that the large reduction of the dynamics optimization problem is possible thanks to the comfort considerations that we have decided to take into account through the constraints and not through some objectives. The comfort characterization is based on the frequency and on the damping ratio of the suspension modes. Such approach must be restricted to vehicles for which the comfort is not primordial or to first optimization attempt purposes.

This extremely simplified approach, furthermore very efficient, could be enriched by additional criteria based on additional scenarios especially dedicated to comfort.

A drawback of the method is that the complete set of parameters must be correctly and manually partitioned into predefined subsets to ensure that the data conditioner can solve the design constraints. This task must be done once per problem formulation, what is not so inconvenient, if we exclude a morphological optimization anyway out of the reach at this time. Nonetheless, a correct and deep understanding of the model and of the formulation is required. Let us note that a numerical process could take over this task via a sensitivity analysis of the constraints equations with respect to the parameters. This improvement is still to be done.

Another main feature of the optimization scheme is the hybrid simulator made of both a steady-state cornering analysis and a transient simulation. The combination of these two numerical processes enables to provide each dynamics characterization needed to assess the quality of the vehicles, at its optimal cost. Indeed, the complementary of the simulations is extensively used to reduce the amount of time simulation, the most CPU consuming MBS tool.

In the same way, in order to reduce as much as possible the optimization time, a unique scenario is used to characterize the entire transient dynamics. However, ensuring to capture all the aspects of the behavior would require to enrich the transient analysis with several other scenarios, each associated with some additional criteria.

In addition to the extensive steady-state cornering analysis and the upgraded transient simulation, we found a large interest in the option to complete the simulator with data arising from a steady-state stability analysis. This could be made of some modal analyses on the basis of the quasi-static equilibria used for the steady-state analysis. Indeed, the stability is also a major concern in the vehicle dynamics, especially for the tilting vehicles.

The evaluator, in combination with an error management system, is able to provide a coherent objective function for each candidate in spite of all the problems that can arise in the complex tangle of numerical processes. The multi-phase evaluator calls first some penalty scores to switch, if any, to evaluation on the basis of the result of the simulator computed through the newly developed clustering and mix law rules. The whole allows to respect the principle of sensitivity of the objective function and to ensure the convergence of the global optimization process. Despite the management of the discontinuities could be more elegant and rigorous, this problem has been also overcome.

Although it is not intrinsically a consequence of the proposed optimization scheme, we want, all the same, to highlight the damageable simplification of the processing of the steering wheel torque feed-back consideration. It is obvious that in this work, facing a lack of knowledge in this very delicate aspect of the vehicle dynamics, we have been constrained to sweep away the use of a really suitable function for the steering wheel torque feed-back. An improvement of this criterion is essential to claim achieving a general and coherent vehicle dynamics optimization.

The last remark applies to the modularity of the proposed optimization scheme. Thanks to its construction, a full optimization is allowed as well as more targeted uses of parts: design constraint solving, simulations of the vehicle with or without evaluation separately. The created environment is sufficiently versatile to be particularized to various concerns along an overall vehicle development.

Chapter 4

Application to the Wemoov

As already introduced, the Wemoov project was initiated by a multi-company consortium with the objective to propose an innovative solution to the current mobility problems. The proposed vehicle is an urban three-wheeler taken as the main case study of the design methodology exposed in the previous chapter. Indeed, the consortium requested our laboratory for the development of the Wemoov. This assignation included, the morphological study, the vehicle dynamics optimization and the electro-mechanical design of a first functional demonstrator.

4.1 Specifications

The project started from scratch with a few specifications addressed to the final results.

The original specifications² are:

- target zone: dense area, cities,
- three-wheeler morphology,
- overall width of 80 cm,
- electric motorization,
- unipersonal,
- driving without any particular skills: as close as possible to car driving,
- complete rain protection,
- EU vehicle regulation compliant.

This specification list has been completed by some general guidelines. Firstly, the vehicle must fit in the context of the ecological concerns without specifying the implications of such a policy. Secondly, the project must respect the KISS principle, standing for "Keep It Simple, Stupid". The principle aims to avoid useless complexity and, at the same time, to reduce the production cost.

²From a fundamental point of view, the specifications of the Wemoov are wrongly posed. Indeed, the specifications must always be expressed in term of target performances and constraints, but never in term of characteristics. This is however partially the case. The infringement was imposed to us.

Due to the funding of the project, the commercial relations must be firstly developed with companies and organizations seated in Wallonia.

Starting an automotive project from a blank sheet and with so very brief specifications has advantages as drawbacks. On the one hand, it opens the possibility to propose some deeply innovative solutions but, on the other hand, the risk is high to derive to a product that does not meet the real expectations of the ordering consortium.

4.2 Morphology determination

4.2.1 First draft

As the first step of the overall design process, we started by determining the morphology of the Wemoov. Although the above specifications are elementary, some characteristics can be already deduced at this step.

After multiple reflections on the driving position, it has resulted the standard seated position appeared as the best option to combine a utilization close to the car driving and the implementation of a closed cabin ensuring rain protection, comfort and security. Indeed, each significant difference with this position would lead to either more risk of injuries in case of crash, or a significant reduction of comfort, or an impossibility to form a coherent cabin. This is a crucial choice, leading to serious implications on the rest of the development. Moreover, this driving position suggests the control of the steering through a wheel instead of through a handle bar.

Due to its low target width, the vehicle stability requirement imposes to tilt it when it turns. Directly linked to the maximal lateral acceleration, the maximal tilting angle must be chosen as large as possible. To fix an idea, a 1 g lateral acceleration roughly corresponds to a 45° tilting angle, what is pretty unreachable taking account the whole of considerations. Added to the suspension own motion, it leads to an extremely large wheel travel and to unacceptable geometrical interactions between different parts. In the same manner, the geometry of the lower part of the chassis must be such that the tilting motion can take place without any contact with the ground.

Unlike the various tilting vehicles available on the market, we want to confer an intuitive driving, close to that of a car. As a consequence, the driver must not be in charge of the control of the tilting motion. It justifies why an active tilting system has been adopted. Anyway, the closed cabin and the seated driver position do not enable to put the foot on the ground at rest: this greatly argue for the active tilting. As discussed in the section 2.1.4, such actuation can be performed by several systems: DTC, STC, HTC. In case of the Wemoov, we preferred to opt for the DTC because it is, even now, the only system matching with the EU vehicle regulation rules. Indeed, the pure steering by wire, required in case of STC and thus HTC, is still prohibited.

The active tilting dynamics justifies also the three-wheeler morphology. Indeed, the DTC requires, at a minimum, one axle with a non-zero track. The one wheel trackless axle is intrinsically self-tilting compliant. So, the three-wheeler morphology seems adapted. In our case, a reverse-trike or ∇ configuration

(i.e. two front wheels, one rear) has been adopted: the main justification relates to braking deceleration capabilities as compared with the acceleration produced by the motorization. Indeed, in case of braking, the longitudinal load transfer is directed towards the two-wheel axle and not to a single wheel such as for a motorbike or a Δ configuration three-wheeler.

Consequently, the front suspension is the most challenging part of the vehicle and needs an important development. The rear suspension is simpler from the morphological point of view: the single wheel rear axle implies, in the manner of a motorbike, a swing-arm suspension, which naturally tilts.

Considering this whole of deductions and decisions, a first draft of the Wemoov could be sketched. This is illustrated in figure 4.1.

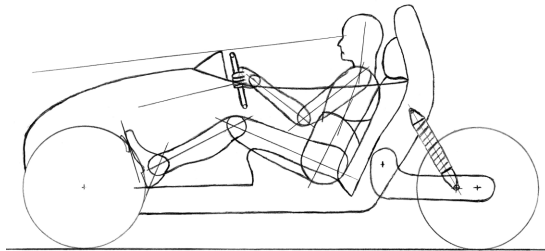


Figure 4.1: First draft of the Wemoov concept.

4.2.2 Preliminary design

The preliminary design aims at making decisive choices impacting the rest of the development. It covers the type and the topology of the suspensions but also the tilting actuation method, the type of steering, the motorization, the brake system etc. At the end of this step, the designer must dispose of sufficient information to build the MBS model and to formulate the optimization problem. The design considerations that overcome these objectives are postponed to the mechanical implementation phase.

Front suspension

The front suspension is more specific than e.g. that of a car because, in addition to the regular DOF that are the steering and the suspension, the Wemoov front suspension must also allow the active tilting. The main challenge is the synthesis of this suspension that must provide a large range of motion in each direction i.e. for each DOF. Kinematically speaking, these objectives are conflicting so a trade-off must be found.

Another major concern of the morphological determination is the reduction of the deflections and the vibrations of the front axle. This is all the more important given the model does not take these phenomena into account. Indeed, the used optimization method is based on the dynamics simulations of rigid MBS. A rigid body formalism has been selected to keep the optimization

computational time acceptable. The deflections of bodies under efforts and the associated high frequency vibrations are thus out of scope of this modeling. However, the selected morphology and the mechanical implementation of the optimization results have to target to highest stiffness for suspension components in order to maintain the deflections and vibrations below an admissible level. There are multiple reasons to consider it. First, the deflections induced by efforts throughout the suspension can modify the orientation of the wheels with respect to the direction of travel, leading, in general, to a loss of handling capabilities. Secondly, the low frequency vibrations are transmitted to the vehicle interior and to the driver whereas the high frequencies are partially transformed into noise. Both of them dramatically affect the comfort of the driver. Worse still, the vibrations that are transmitted everywhere bring, along their path, multiple mechanical damages whose most known are screws and nuts loosening, fatigue cracking and breakings. Beside the own stiffness of suspension parts, a key decision in the axle design is its general morphology. A serial structure provides a decoupling between the three main functions of the front axle: suspension, steering and tilting, whereas a parallel structure is lighter and is less subjected to deflections and vibrations. In reference to the market, the three-wheeler Piaggio MP3 uses a serial structure. For this implementation, one can observe parasitic vibrations, visible to the naked eyes. The Peugeot Metropolis and Quadro 3 use a parallel structure. We have opted for the same choice.

Our solution to implement the front tilting unit of the Wemoov is comparable to a traditional double wishbone except for the shock top connections, which are not directly linked to the chassis but to an intermediate component: an articulated T-shaped pendulum, consequently named *T-pendulum*. This part is articulated on an axis which is parallel to those of the bottom wishbones. The system confers the additional DOF corresponding to the tilting motion. To ensure the vehicle roll stability, this DOF is actuated.

The figure 4.2 illustrates each of the front axle DOF: the left picture shows the reference position, the three others refer to the hop, steering and tilting motions respectively.

The topology is similar to several existing tilting front suspensions, as for example the Brudelli Leanster (section 2.1.4). The double wishbone system, improved with the T-pendulum, appears as the simplest solution to combine efficiently the three required DOF over extended ranges in each direction. In this case, the range limits result more from the mechanical materialization than from the topology itself. In that respect, this challenge is partially postponed to the mechanical implementation phase.

Tilting actuation

The tilt motion is actively controlled via a torque applied between the chassis and the T-pendulum. From the road side, the torque leads to a load transfer between the two front wheels. On the other side, the torque is applied on the chassis in order to control its roll motion. Let us note that the actuation very simply explicated here above, involves a long and complex kinematic chain

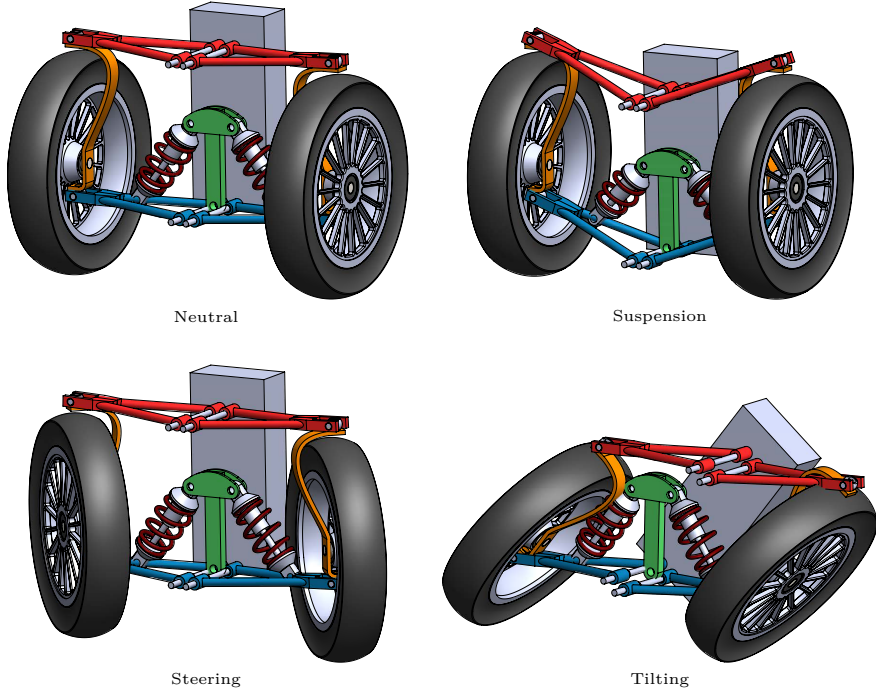


Figure 4.2: Three main front suspension DOF.

including backlashes and compliant elements. In particular, the tilting forces go through both the suspension shocks and the tires, what has unavoidable consequences on the comfort, as discussed below.

If the double wishbone with T-pendulum improvement has been already implemented in the past, the way we implement the actuation of tilting motion is, at our best knowledge, innovative. We propose to act on the T-pendulum through a gear sector, driven by a sprocket whose axis is stationary with respect to the chassis. The sprocket is itself driven by an electric motor through a planetary gearbox. A motor brake, normally closed, enables the vehicle to be stable at rest without any active control. This brake is activated, i.e. open, when the controller switches on. This preliminary implementation of the tilting actuation system is illustrated in the figure 4.3.

The tilting torque, T_{tilt} , applied to the T-pendulum axis is determined by a dedicated DTC controller. It is computed via a PID on the three-wheeler roll angle, ϕ :

$$T_{tilt} = PID(\phi_{msr} - \phi_{cmp}) \quad (4.1)$$

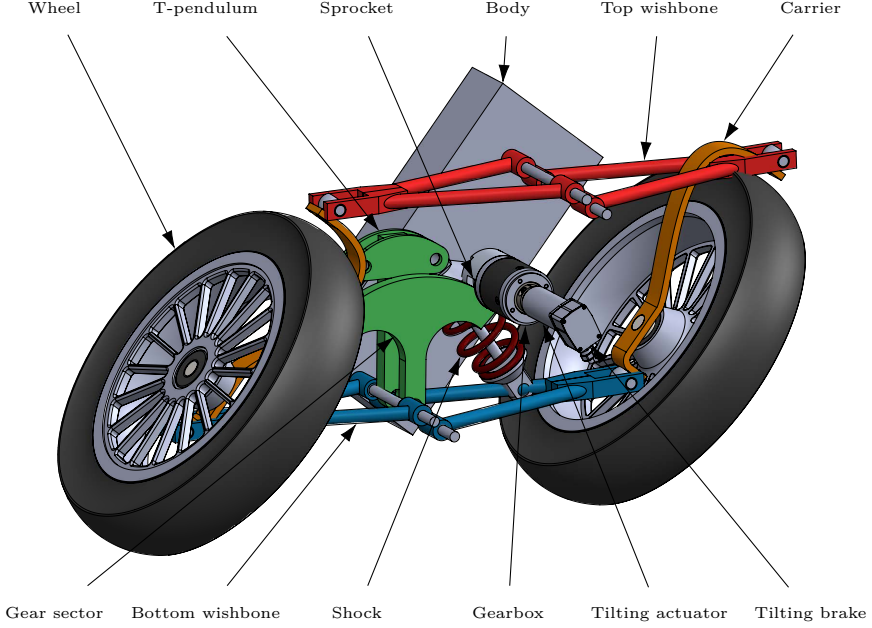


Figure 4.3: Preliminary implementation of tilting system.

The computed roll, ϕ_{cmp} , is based on two main current states: the speed, v and the steering fork rotation, μ ¹:

$$\phi_{cmp} = -\arctan\left(\frac{v^2 \rho}{g}\right) \quad (4.2)$$

where the speed, v , is evaluated thanks to the rotational speed of the rear wheel, and the curvature, ρ , is computed thanks to:

$$\mu = f_{\rho\mu}(\rho) \Leftrightarrow \rho = f_{\mu\rho}(\mu) \quad (4.3)$$

where $f_{\rho\mu}$ is a non-linear function which depends on the vehicle steering mechanism and other states of the vehicle. In this work, $f_{\rho\mu}$ has been roughly estimated through a very simple geometrical model giving:

$$f_{\rho\mu} = \arctan 2 \left(D_{wb}, \sqrt{\frac{1}{\rho^2} - D_{cm\ rr}^2} \right) \quad (4.4)$$

for which D_{wb} is the wheelbase and $D_{cm\ rr}$ is the longitudinal distance between the center of mass of the vehicle and the rear wheel axis. This takes into account neither the non-linearity due to the steering mechanism kinematics nor the effect of the tilting. The estimation could be largely improved considering the

¹The steering fork angle is linked to the steering wheel angle through the steering transmission ratio, r_{st} .

results of the steady-state analysis. The parameters of the PID controller are given in the table 4.7.

The measured roll, ϕ_{msr} , easy to obtain in simulation, is however very difficult to estimate in real operations. The data arising from an Inertial Measurement Unit (IMU) rigidly fixed on the main body could be on the basis of the estimation. The sensor must be correctly calibrated that is not straightforward. Moreover, the signal should be processed, in real-time, with a Kalman filter or something equivalent, using the suitable dynamics model. For the case of banked or uneven road, both the estimation of the body roll and the computation of a suitable tilting angle is more complex. This generalization and the numerous issues related to the correct roll estimation that are really challenging developments, are out of the scope of this work.

The selected controller algorithm is very simple and robust. In contrast, some different advanced DTC controllers, possibly coupled with STC, have been developed in the context of the Wemoov project [27, 28]. They are not exploited in this work despite they are potentially more efficient. We hypothesize that the present controller is sufficient to allow an effective optimization of the Wemoov chassis.

Steering mechanism

As the space between the wheels is completely full (figure 4.3), the steering mechanism must be pushed up. Two different morphologies have been envisaged: a rack system based on a translational motion and a steering box based on a rotational motion. As explained in the section 4.6.2, the optimization tries to maximize the steering rod lengths by promoting the steering box option. Independently of the rod lengths, the rotation based mechanism gives better results than the translation based one. Therefore, a steering box is finally preferred to a rack.

At this stage, the topology of the Wemoov is practically defined. Indeed, the functional DOF are known and listed in the table 4.1. More details on the topology can be found in the description of the MBS model given in the section 4.2.3.

Body kinematics:	
Free motion	6
Front axle DOF:	
Wheel rotations	2
Suspensions	2
Steering	1
Tilting	1
Rear axle DOF:	
Wheel rotation	1
Suspension	1
Total	14

Table 4.1: Wemoov DOF.

Motorization and braking

The rear wheel is driven by an electric motorization. In any case, it was pretty impossible to motorize the front axle: the transmission by universal joints or similar devices is impossible due to the envisaged range of wheel displacement. The in-wheel motorization has also been rejected because being too bulky in this area and also because detrimental to the sprung/unsprung mass ratio. Thus, we have opted for the very classical rear motorbike motorization, electric in this case.

Each wheel is equipped with its own disc brake for braking performances and security reasons. This standard hydraulic brake system acts jointly with the electric regenerative braking.

Overall geometry

In this task, we try to precise the position, the space and the mass of the main functional elements:

- the wheels,
- the main suspension parts,
- the driver and his interfaces:
 - the steering wheel,
 - the seat,
 - the thrust and brake pedals,
- the propulsion motor,
- the battery packs,
- the main electric components.

All these elements must be located in such a way that the whole is coherent and compact, and there is no interaction, including possible one due to the combined displacements. The structural parts are also taken into account when a sufficiently precise idea is at our disposal, as for example, the swingarm. The secondary elements and the rest of the structural parts are taken into account through some ghost elements. They have an approximated shape and mass in order to represent the missing parts. Obviously, there is no complex computation at this step. The analysis is mainly based on the relative position of the parts in the 3D CAD model.

As already mentioned, a key specification of this vehicle is the overall width, targeted to 0.8 m as it must be able to go through a standard door. This imposes a lot of severe design constraints to the front axle. Moreover, because of the tire width, the track is reduced to 0.68 m. In general, the width between the wheels is split in four main types of zones:

The *structure* consists of the frame on which other elements are fixed.

The *useful space* is dedicated to the driver, passengers and interfaces with them.

The *auxiliary space* is used to place engine and fuel tank, in case of a fuel car, or electric motor and battery, in case of an electric car, as well as all other auxiliary components.

The *suspension zone* is reserved to all moving parts of the axle in their motions due to degrees of freedom: suspension motion, steering and tilting in this case.

In contrast, in the case of the three-wheeler, the track is so reduced that it is impossible to envisage having some space between wheels, available as a useful or an auxiliary space. Thus, the cockpit strictly comes behind the front suspension unit leading to a significant increase of the wheelbase, compared to a motorbike for example. The gain in term of narrowness is partially compensated by the loss in total length.

The figure 4.4 illustrates the CAD model at this step. The resulting main design characteristics are given in the table 4.2.

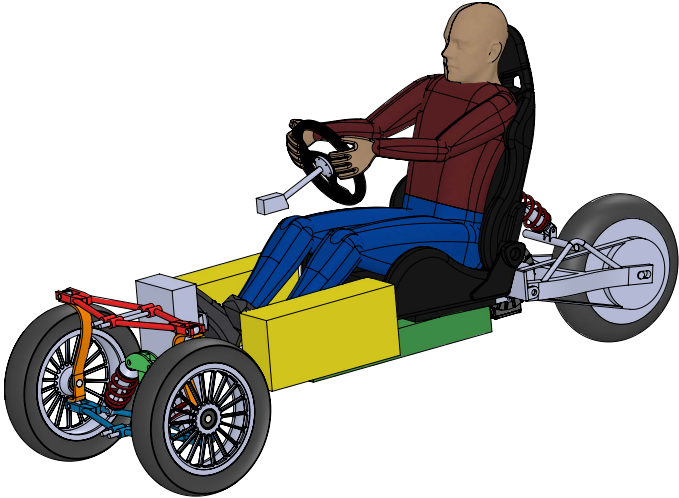


Figure 4.4: Preliminary overall geometry.

Wheelbase	2.0 m
Track	0.68 m
Mass	300 kg
Mass distribution	50/50
Power	10 hp
Speed	90 km/h

Table 4.2: Preliminary design characteristics.

4.2.3 MBS model

MBS scheme

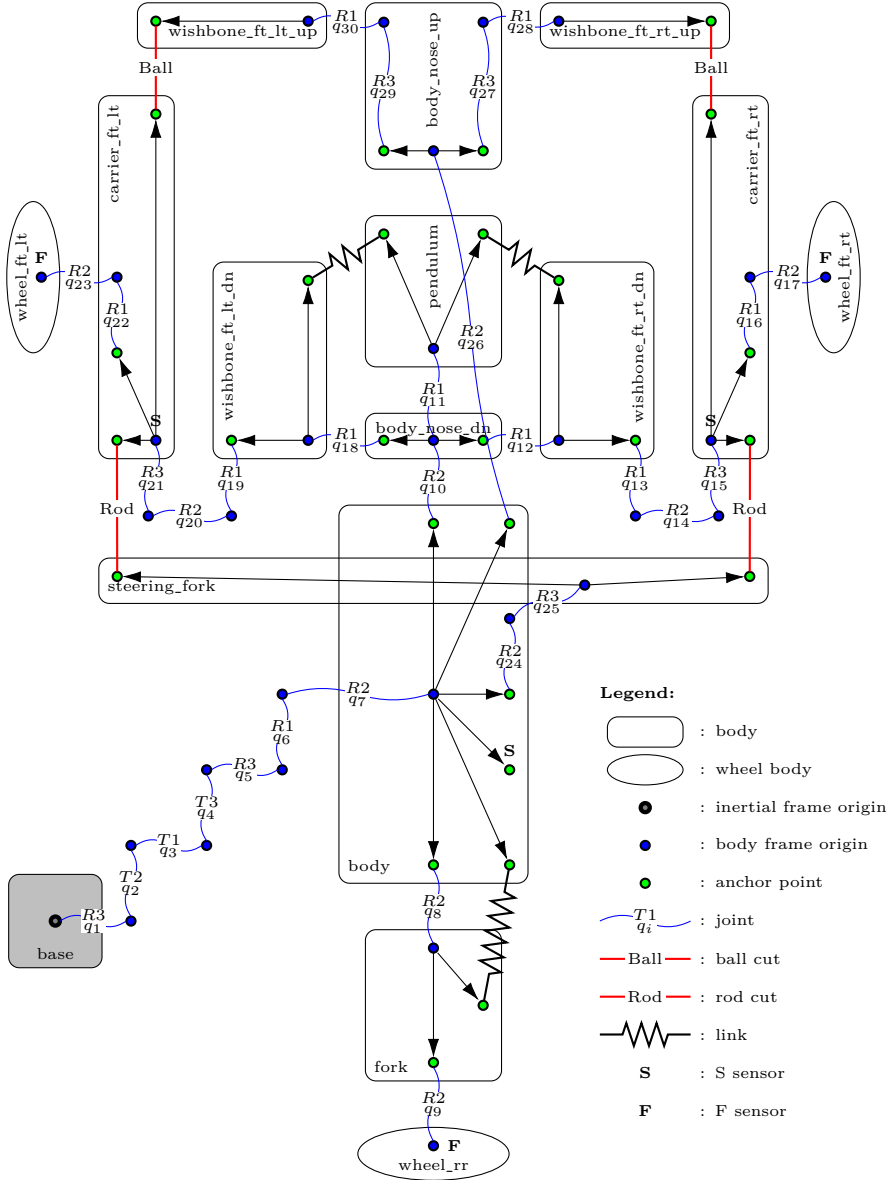


Figure 4.5: Wemoov MBS scheme.

As the basis of the modeling, the MBS scheme represents the topology of the Wemoov such as developed until here. The scheme is depicted in figure 4.5

according to the Robotran conventions. Here, it must be reminded the MBS modeling uses the relative coordinates.

Starting from the *base* on which is fixed the inertial frame origin, a kinematic chain with 7 generalized coordinates links the main *body*. They correspond to 6 DOF of the main *body* free motion plus an additional DOF, the first R3, which is only used for the quasi-static equilibrium computations. The reference point of the *body* is defined as the point at the vertical of the heel contact of the driver, in the x direction, on the median plane, in the y direction and at 200 mm from the ground level, in the z direction, in this reference position.

The *body_nose_dn* and *body_nose_up* are not articulated with respect to the *body*. The joints q_{10} and q_{26} are forced joints and aim simply to incline the corresponding reference frames to incline the axle of the up and down wishbones in the y direction. In the same manner, the orientation of the upper wishbones axes, in z direction, is allowed by the forced joints, q_{27} and q_{29} . A similar orientation of the bottom wishbones is not allowed because it would create a damageable coupling between the suspension and the tilting in addition to being quite impossible to be implemented.

The left suspension loop is made of the *wishbone_ft_lt_dn*, the *carrier_ft_lt* and the *wishbone_ft_lt_dn*, plus the chassis. The wishbones are articulated with respect to the chassis through simple revolute joints, q_{18} and q_{30} . The carrier is linked, at its two extremities, to the wishbones through ball joints considered as perfect. The bottom one is modeled by a serie of three rotational joints: q_{19} , q_{20} and q_{21} . In order to maintain a MBS tree-like structure despite the suspension kinematic loop, the upper ball joint is taken into account with a ball cut, which allows to express the associated loop constraints.

The *steering_fork* is articulated through a revolute joint, q_{25} , itself oriented, in y direction, by the forced joint q_{24} . In order to close the steering kinematic loops, the steering rods, considered as perfect, are modeled by a pair of rod cuts to establish the associated constraints.

The *pendulum* is articulated via a revolute joint, q_{11} . The axis is parallel to those of the down wishbones because all these elements are based on the *body_nose_dn*.

The shocks are considered as perfect, i.e. massless and frictionless. They are modeled via links¹. The shock modeling details are given later in this section. The front shocks take place between the bottom wishbones and the pendulum.

The *wheel_ft_lt* axis is the joint q_{23} which is inclined, in x direction with respect to the carrier via the forced joint q_{22} .

The front right suspension is symmetric to the front left one.

The rear suspension is much simpler. The *fork* is articulated with respect to the *body* through a revolute joint, q_8 . The *wheel_rr* axis is the joint q_9 .

A F sensor is fixed on the center of each wheel. They aim to manage the ground-tire contacts as explained later in this section.

The S sensor located on the *body* aims to compute the data for the reference point used in the steering controller. The S sensors located on the front carriers help to compute various design constraints.

¹In the Robotran formalism, the *link* corresponds to a point-to-point force (section 2.2.2).

The joints, the anchor points, the rods, and the user models of the Wemoov model are respectively listed in the tables 4.3, 4.4, 4.5, 4.6 and 4.7.

The masses, the centers of mass and the inertia properties of the bodies, which are computed based on the rough geometry, are given in the table 4.33.

Main modeling assumptions

In order to evaluate the vehicle through the hybrid simulator, some assumptions must be adopted to allow the MBS modeling. The main assumptions are listed here below:

- Some bodies are considered as rigid:
 - The part deformations under stresses are neglected.
 - The suspension bushes are frozen (sections 4.6.2, 4.6.3 and 4.6.4).
 - The suspension settings are frozen (section 4.6.2).
 - The belt tensioning system is frozen (section 4.6.4).
- The mass and inertia of some elements are neglected:
 - the mass of shock elements, allowing their modelings as links,
 - the mass of lighter bodies in the front suspension.
- The articulations are supposed to be perfect:
 - backlash free,
 - frictionless.
- The dimension shift induced by the manufacturing tolerances are not taken into account.
- All the aerodynamics forces are omitted.

Two exceptions of the body rigidity assumption have to be noticed: the shock springs and the tires are specifically treated to take account of their deformations. These developments are explained here above.

Obviously, the assumptions lead to some simulation error. However, they are judged acceptable with respect to the objectives.

Shock model

The actual shock characteristic could be very sophisticated in order to improve the vehicle dynamics. Nevertheless, in the developed model, the characteristics are simplified. The spring and the damper forces are considered as linear, respectively to the shock length and the shock velocity. Additional bump stops which limit, each side, the shock stroke, are also treated via a linear model. The computation of the shock forces are performed separately, for each shock, at each time step, following the algorithm 1.

```

 $F_{spring} = K(L - L_0)$ 
 $F_{damper} = D\dot{L}$ 
if  $L < L_{min}$  then
  |  $F_{stop} = K_{stop}(L - L_{min})$ 
end
if  $L > L_{max}$  then
  |  $F_{stop} = K_{stop}(L - L_{max})$ 
end
 $F_{shock} = F_{spring} + F_{damper} + F_{stop}$ 

```

Algorithm 1: Shock force computation.

with:

K	the spring rate [N/m],
D	the damping rate [N.s/m],
L	the current length [m],
L_0	the neutral length [m],
L_{min}	the minimum length before lower bump stop contact [m],
L_{max}	the maximum length before upper bump stop contact [m],
K_{stop}	the bump stop stiffness [m],
F_{spring}	the spring force [N],
F_{damper}	the damper force [N],
F_{stop}	the bump stop force [N],
F_{shock}	the total shock force [N].

The parameters are classified among the user model parameters. Their values are given in the table 4.7.

Tire model

The vehicle is equipped with scooter tires because it exhibits a similar dynamics. Thus, the simulator needs a tire model especially adapted to the scooter-motorbike type. The road-tire contact forces are computed according to the reduced steady-state model of Pacejka [66] which excludes all asymmetric terms. The parameters of this model have been identified by Sharp and al. [82]. More information on the Magic formula based tire models can be found in the section 2.1.2.

A well-suited road-tire contact model is the key point to produce quality results. In fact, if a "rolling without slipping" constraint would be sufficient to study the stability or the control of the three-wheeler, a more refined model is needed to consider issues like under-oversteering. In our study, the very widespread Magic formula is a suitable trade-off between the needed information, the required precision and the CPU efficiency.

In this study, we limit the possible test track to a flat and horizontal ground. To improve the kinematics contact model to a more universal one could be an interesting future development. Indeed, the dynamics and, in particular, the stability of the Wemoov is not tested and therefore not guaranteed on sloping roads. Due to its very low track, it is clear that the three-wheeler dynamics is

largely affected by the transverse slopes. At this time, this point is still to be developed.

In the same manner, only the steady-state tire response is taken into account in the contact model, even for the transient stimulation. In other words, the tire relaxation is completely omitted. From our point of view, this hypothesis is coherent with the selected steering controller and the selected reference test track (section 4.4.3). The encountered dynamics is moderate with respect to that of tires. The improvement of such a concern could be done following the indications given by Sharp [82], that provides all the information to complete the model with a relaxation length term in order to take the internal dynamic of tires into account.

4.2.4 Interests as case study

The complexity, the number and the interdependence of issues involved make this vehicle design a hard optimization problem to handle. This three-wheeler application appears to be very suited to the proposed methodology because it introduces numerous and varied difficulties:

- The front suspension is kinematically complex, with 3D loops of bodies.
- The vehicle is naturally unstable: the associated difficulties are multiple.
- The design must consequently include a reliable tilting controller.
- The steering behavior must be simulated by a driver controller.
- The expected performances are very different in nature.
- The mechanical design and specifications involve very diversified design constraints.
- The optimization parameters are also numerous and different in nature. Moreover, they are strongly interdependent in their relations with the targeted performances.

Therefore, the use of the developed optimization scheme such as exposed in the chapter 3 is fully justified.

4.3 Development strategy

Once the preliminary design and the complementary MBS model has been established, the optimization process can start. It aims to determine the best set of parameters regarding the dynamics performances and the design constraints. For the Wemoov development, we have chosen to follow a multi-step strategy illustrated in figure 4.6.

First, a complete optimization is performed according to the method explained in the chapter 3. This includes sub-steps such as: the problem formulation, the constraint expressing, the determination and parametrization of the enclosed simulation conditions, the evaluator and the optimization algorithm settings. These points are treated in the section 4.4. The result of this step, the first optimized configuration, denoted as CF1, is compared with some unoptimized reference configurations in the section 4.5. The CF1 configuration

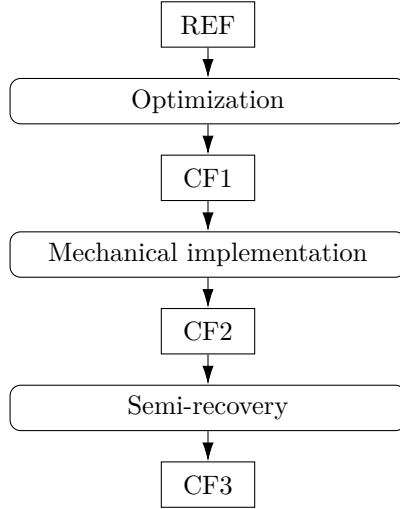


Figure 4.6: Development strategy.

respects obviously all the constraints and is supposed to be optimal from the point of view of the applied criteria.

In a second phase, the mechanical implementation takes place on the basis of the fully determined skeleton scheme resulting from the optimization process. This implementation is summarized in the section 4.6. This important step obviously implies some alterations of the optimized configurations. Some anchor points are shifted to avoid mechanical interpenetrations. Another consequence of the mechanical design is that the inertia properties of all the bodies are better known than before. So, the initial model that involves simplified estimated inertia properties can be corrected with the much more precise values outgoing from the CAD software. The second configuration, CF2, refers to the so corrected three-wheeler model. Because of the applied changes, this second configuration is neither constraint compliant nor optimal from the point of view of the objectives. However, the CF2 model is closer to the true vehicle than CF1.

The changes leading to CF2 could justify to process again the complete optimization leading thereafter to a need doing again the mechanical implementation and so on, iteratively until convergence. Because these two steps are very time consuming and the research and development means at our disposal are limited, it is unfortunately not possible to iterate the process manually. From another way, it is not yet possible to automatize such the process. However, it is still possible to apply a sub-part of the data conditioner which acts only on the non-geometrical parameters. In this way, some constraints are satisfied again. We can already notice that the recovery of some constraint satisfactions can affect positively as well as negatively the performances with respect to the objectives. This semi-recovered configuration, CF3 is the final setup for this work (figure 4.6).

A reference and the three main configurations, CF1, CF2 and CF3 are characterized in the section 4.7. These configurations are compared together from the point of view of the optimization criteria. It helps us to measure the impact of the mechanical implementation through the associated changes and the semi-recovery benefits. The quality of the final configuration can also be assessed.

4.4 Wemoov optimization

4.4.1 Problem formulation

In order to formulate the problem, the first task is to list all the parameters of the model. As a reminder, in this work, the variables that can be considered as the possible parameters are:

FJ the forced joints,
 PC the anchor point vector components,
 RL the rod lengths,
 UP the user model parameters.

The tables 4.3, 4.4, 4.5, 4.6, 4.7 list the parameters for each of these type categories. The parameter partitioning is realized at this step, according to the constraints listing, given in the section 4.4.2. The parameter natures are:

N the null parameters,
 F the fixed parameters,
 A the algebraically constrained parameters,
 G the geometrically constrained parameters,
 M the modal constrained parameters,
 O the optimized parameters.

The tables gives the nature, i.e. the parameter partitioning, and the value for the optimized configuration CF1.

The list of joints, given in table 4.3, contains the coordinate type and the coordinate nature¹ i.e. the coordinate partitioning of the joints, following the formalism described in the section 2.2.4:

I the independent coordinates,
 D the dependent coordinates,
 C the driven coordinates,
 F the forced coordinates.

The model involves 30 joints whose only 7 are forced and consequently considered as parameters. In a preliminary analysis, the optimal value of the joints 27 and 29, i.e. the azimuth orientations of the front top wishbone axes, are judged incompatible with the downstream mechanical implementation. So,

¹The label "Q" and "P" allow to distinguish the coordinate and parameter nature and type respectively.

they are fixed¹ to a zero value and are consequently useless. They are not removed from the model despite the potential gain in term of performances. The other forced joints are either constrained or optimized.

The list of points, given in tables 4.4 and 4.5 contains a half of null parameters. This is due to the MBS formulation. Thanks to the symbolic manipulation capabilities of Robotran, the performances are not affected by these numerous useless parameters. A large part of the remaining parameters are fixed, either because their value must be determined independently of the optimization process or because they have been fixed via preliminary optimizations (e.g. the anchor points of `body_nose_dn`).

ID	Name	Q type	Q nature	P nature	CF1
1	R3_body_qs	R3	C		0.0000
2	T2_body	T2	I		0.0000
3	T1_body	T1	I		0.0010
4	T3_body	T3	I		0.2000
5	R3_body	R3	I		0.0000
6	R1_body	R1	I		0.0000
7	R2_body	R2	I		0.0000
8	R2_fork	R2	I		-0.1064
9	R2_wheel_rr	R2	I		0.0000
10	R2_body_nose_dn	R2	F	O	0.1233
11	R1_pendulum	R1	I		-0.0000
12	R1_wishbone_ft_rt_dn	R1	I		0.1000
13	R1_carrier_ft_rt	R1	D		-0.3316
14	R2_carrier_ft_rt	R2	D		-0.4136
15	R3_carrier_ft_rt	R3	D		-0.0945
16	R1_wheel_ft_rt	R1	F	A	0.2118
17	R2_wheel_ft_rt	R2	I		0.0000
18	R1_wishbone_ft_lt_dn	R1	I		-0.1000
19	R1_carrier_ft_lt	R1	D		0.3316
20	R2_carrier_ft_lt	R2	D		-0.4136
21	R3_carrier_ft_lt	R3	D		0.0945
22	R1_wheel_ft_lt	R1	F	G	-0.2118
23	R2_wheel_ft_lt	R2	I		0.0000
24	R2_steering_fork	R2	F	O	0.1500
25	R3_steering_fork	R3	C		0.0000
26	R2_body_nose_up	R2	F	O	0.2000
27	R3_wishbone_ft_rt_up	R3	F	A	-0.0000
28	R1_wishbone_ft_rt_up	R1	D		-0.0905
29	R3_wishbone_ft_lt_up	R3	F	F	0.0000
30	R1_wishbone_ft_lt_up	R1	D		0.0905

Table 4.3: Wemoov joints.

¹The joint 27 is fixed to zero through the associated symmetry algebraic constraint.

ID	Name	Comp.	P nature	CF1
1	body			
1	1 body_to_fork	x	F	-1.0300
		y	N	0
		z	F	0.1100
	2 body_to_shock_rr	x	F	-1.0900
		y	N	0
		z	G	0.5048
	3 body_to_sensor	x	F	-0.0500
		y	N	0
		z	F	0.6000
1	4 body_to_body_nose_dn	x	G	0.5473
		y	N	0
		z	F	0.0100
	5 body_to_steering_fork	x	O	0.5998
		y	N	0
		z	O	0.3677
	6 body_to_body_nose_up	x	O	0.4323
		y	N	0
		z	O	0.3041
2	fork			
2	1 fork_to_wheel_rr	x	F	-0.5250
		y	N	0
		z	N	0
	2 fork_to_shock_rr	x	F	-0.2100
		y	N	0
		z	F	0.1500
3	wheel_rr			
3	1 origin	x	N	0
		y	N	0
		z	N	0
4	body_nose_dn			
4	1 body_nose_dn_to_wishbone_ft_rt_dn	x	N	0
		y	A	-0.0000
		z	A	0.0000
	2 body_nose_dn_to_wishbone_ft_lt_dn	x	N	0
		y	F	0.0000
		z	F	0.0000
5	pendulum			
5	1 pendulum_to_shock_ft_rt	x	A	-0.1700
		y	A	-0.0200
		z	A	0.1600
	2 pendulum_to_shock_ft_lt	x	F	-0.1700
		y	F	0.0200
		z	F	0.1600
6	wishbone_ft_rt_dn			
6	1 wishbone_ft_rt_dn_to_carrier_ft_rt	x	N	0
		y	A	-0.3112
		z	N	0
	2 wishbone_ft_rt_dn_to_shock_ft_rt	x	A	-0.0300
		y	A	-0.1776
		z	A	0.0100

Table 4.4: Wemoov points (1/2).

ID	Name	Comp.	P nature	CF1
7	carrier_ft_rt			
1	carrier_ft_rt_to_wishbone_ft_rt_up	x	N	0
		y	N	0
		z	A	0.3700
	carrier_ft_rt_to_wheel_ft_rt	x	A	0.0284
		y	A	-0.0450
		z	A	0.0650
	carrier_ft_rt_to_steering_rod_ft_rt	x	A	0.1200
		y	A	-0.0542
		z	A	0.4200
8	wheel_ft_rt			
1	origin	x	N	0
		y	N	0
		z	N	0
9	wishbone_ft_lt_dn			
1	wishbone_ft_lt_dn_to_carrier_ft_lt	x	N	0
		y	G	0.3112
		z	N	0
	wishbone_ft_lt_dn_to_shock_ft_lt	x	F	-0.0300
		y	G	0.1776
		z	F	0.0100
10	carrier_ft_lt			
1	origin	x	N	0
		y	N	0
		z	N	0
	carrier_ft_lt_to_wishbone_ft_lt_up	x	N	0
		y	N	0
		z	O	0.3700
	carrier_ft_lt_to_wheel_ft_lt	x	G	0.0284
		y	F	0.0450
		z	O	0.0650
	carrier_ft_lt_to_steering_rod_ft_lt	x	F	0.1200
		y	O	0.0542
		z	O	0.4200
11	wheel_ft_lt			
1	origin	x	N	0
		y	N	0
		z	N	0
12	steering_fork			
1	steering_fork_to_steering_rod_rt	x	A	0.1200
		y	A	-0.0400
		z	N	0
	steering_fork_to_steering_rod_lt	x	F	0.1200
		y	O	0.0400
		z	N	0
13	body_nose_up			
1	body_nose_up_to_wishbone_ft_rt_up	x	N	0
		y	A	0.0000
		z	N	0
	body_nose_up_to_wishbone_ft_lt_up	x	N	0
		y	F	0.0000
		z	N	0
14	wishbone_ft_rt_up			
1	wishbone_ft_rt_up_to_carrier_ft_rt	x	A	0.0000
		y	A	-0.2328
		z	A	0.0000
15	wishbone_ft_lt_up			
1	wishbone_ft_lt_up_to_carrier_ft_lt	x	F	0.0000
		y	G	0.2328
		z	F	0.0000

Table 4.5: Wemoov points (2/2).

The two steering rods that are modeled as the rod cuts (section 2.2.2) are inventoried in the rod list, given in the table 4.6. Obviously the right steering rod length is the symmetry of the left one, which, it-self is a geometrically constrained parameter determined by the data conditioner.

ID	Name	P nature	CF1
1	steering_rod_lt	G	0.3064
2	steering_rod_rt	A	0.3064

Table 4.6: Wemoov rods.

The user model parameters are listed in table 4.7. The tire characteristics have been determined out of the optimization process such all the associated parameters are fixed. This is discussed in the perspectives. In contrast, the shock parameters are very sensitive and useful in the satisfaction of the geometrical and modal constraints. The tilting controller parameters have been optimized in a preliminary optimization and are fixed for this problem formulation.

ID	Symb.	Name	P. nature	CF1
1	wheel_ft			
	1 $K_{tire\ ft}$	K_tire	F	200000
	2 $D_{tire\ ft}$	D_tire	F	10000
	3 $r_n\ tire\ ft$	r_n_tire	F	0.2615
	4 $r_t\ tire\ ft$	r_t_tire	F	0.0600
2	wheel_rr			
	1 $K_{tire\ rr}$	K_tire	F	200000
	2 $D_{tire\ rr}$	D_tire	F	10000
	3 $r_n\ tire\ rr$	r_n_tire	F	0.2615
	4 $r_t\ tire\ rr$	r_t_tire	F	0.0700
3	shock_ft			
	1 $K_{sh\ ft}$	K	M	38640
	2 $D_{sh\ ft}$	D	M	3277
	3 $L_0\ sh\ ft$	L_0	G	0.3186
	4 $L_{min\ sh\ ft}$	L_min	F	0.2350
	5 $L_{max\ sh\ ft}$	L_max	F	0.3040
	6 $K_{stop\ sh\ ft}$	K_stop	F	1000000
4	shock_rr			
	1 $K_{sh\ rr}$	K	M	66011
	2 $D_{sh\ rr}$	D	M	4202
	3 $L_0\ sh\ rr$	L_0	G	0.3536
	4 $L_{min\ sh\ rr}$	L_min	F	0.2650
	5 $L_{max\ sh\ rr}$	L_max	F	0.3640
	6 $K_{stop\ sh\ rr}$	K_stop	F	1000000
5	tilting_ctrl			
	1 $K_p\ \phi$	Kp	F	3000
	2 $K_i\ \phi$	Ki	F	800
	3 $K_d\ \phi$	Kd	F	200

Table 4.7: Wemoov user model parameters.

4.4.2 Constraints

In this section, the constraints are inventoried at the same time as the associated parameters. Because the constraints are solved, it is required that the number of parameters and constraints corresponds for each category. The fixed parameters are given in the appendix B.1.

Algebraic constraints

For the Wemoov project, the algebraic constraints (table 4.8) are only used to maintain the geometrical symmetry of the model. Following the adopted convention (section 3.3.1), the left parameters are the masters and the right ones are the slaves. The use covers the joints, points and rod lengths but not the user model parameters because this makes no sense. Notice that such a symmetrization by constraints could be advantageously replaced by a symbolic symmetrization, much more efficient from every point of view.

For convenience reasons, the generation of the algebraic constraints is partially automatized. This takes places in a pre-run before the optimization run. The process detects the symmetry and chooses to apply an equality or a sign change in function of the direction of the joint or the point component, the rods being equal in any case. A symmetry based on the median plane is supposed to be applied.

ID	Slave name	Type	Master name	CF1
1	body_nose_dn_to_wishbone_ft_rt_dn_y	PC	body_nose_dn_to_wishbone_ft_lt_dn_y	-0.0000
2	body_nose_dn_to_wishbone_ft_rt_dn_z	PC	body_nose_dn_to_wishbone_ft_lt_dn_z	0.0000
3	pendulum_to_shock_ft_rt_x	PC	pendulum_to_shock_ft_lt_x	-0.1700
4	pendulum_to_shock_ft_rt_y	PC	pendulum_to_shock_ft_lt_y	-0.0200
5	pendulum_to_shock_ft_rt_z	PC	pendulum_to_shock_ft_lt_z	0.1600
6	wishbone_ft_rt_dn_to_carrier_ft_rt_y	PC	wishbone_ft_lt_dn_to_carrier_ft_lt_y	-0.3112
7	wishbone_ft_rt_dn_to_shock_ft_rt_x	PC	wishbone_ft_lt_dn_to_shock_ft_lt_x	-0.0300
8	wishbone_ft_rt_dn_to_shock_ft_rt_y	PC	wishbone_ft_lt_dn_to_shock_ft_lt_y	-0.1776
9	wishbone_ft_rt_dn_to_shock_ft_rt_z	PC	wishbone_ft_lt_dn_to_shock_ft_lt_z	0.0100
10	carrier_ft_rt_to_wishbone_ft_rt_up_z	PC	carrier_ft_lt_to_wishbone_ft_lt_up_z	0.3700
11	carrier_ft_rt_to_wheel_ft_rt_x	PC	carrier_ft_lt_to_wheel_ft_lt_x	0.0284
12	carrier_ft_rt_to_wheel_ft_rt_y	PC	carrier_ft_lt_to_wheel_ft_lt_y	-0.0450
13	carrier_ft_rt_to_wheel_ft_rt_z	PC	carrier_ft_lt_to_wheel_ft_lt_z	0.0650
14	carrier_ft_rt_to_steering_rod_ft_rt_x	PC	carrier_ft_lt_to_steering_rod_ft_lt_x	0.1200
15	carrier_ft_rt_to_steering_rod_ft_rt_y	PC	carrier_ft_lt_to_steering_rod_ft_lt_y	-0.0542
16	carrier_ft_rt_to_steering_rod_ft_rt_z	PC	carrier_ft_lt_to_steering_rod_ft_lt_z	0.4200
17	steering_fork_to_steering_rod_rt_x	PC	steering_fork_to_steering_rod_lt_x	0.1200
18	steering_fork_to_steering_rod_rt_y	PC	steering_fork_to_steering_rod_lt_y	-0.0400
19	body_nose_up_to_wishbone_ft_rt_up_y	PC	body_nose_up_to_wishbone_ft_lt_up_y	-0.0000
20	wishbone_ft_rt_up_to_carrier_ft_rt_x	PC	wishbone_ft_lt_up_to_carrier_ft_lt_x	0.0000
21	wishbone_ft_rt_up_to_carrier_ft_rt_y	PC	wishbone_ft_lt_up_to_carrier_ft_lt_y	-0.2328
22	wishbone_ft_rt_up_to_carrier_ft_rt_z	PC	wishbone_ft_lt_up_to_carrier_ft_lt_z	0.0000
23	R1_wheel_ft_rt	FJ	R1_wheel_ft_lt	0.2118
24	R3_wishbone_ft_rt_up	FJ	R3_wishbone_ft_lt_up	-0.0000
25	steering_rod_rt	RL	steering_rod_lt	0.3064

Table 4.8: Algebraically constrained parameters.

Geometric constraints

The geometric constraints, which are listed in table 4.9, pursue multiple purposes. The target settings of the front axle are given in the table 4.10.

A first set of constraints aims to impose the geometry according to the specifications from the preliminary design. For instance, the constraints 2 impose a suitable position of the front axle unit with respect to the reference point. The constraint 3 sets the track of the front axle to its maximum value, that is the target maximum width, 800 mm, minus the tire width, 120 mm, that gives 680 mm. This very low track is a big challenge of this work. In the same manner, the constraints 9 and 10 force the main body to be located following the design convention. The right body height and pitch are obtained by setting the shock neutral lengths in the same way as each motor-sport technician would do in a practical operation.

A second set of constraints aims to improve the potential of shock stroke. An important setting is the shocks equilibrium length in their total range. In general, the designers prefer to promote the bump with respect to the rebound. 2/3 bump, 1/3 rebound is an usual setup. For the Wemoov application, a centered position (1/2 bump, 1/2 rebound) has been judged preferable because a large part of the shock displacement is due to the tilting actuation that is symmetric unlike of the road imperfection absorption. The constraints 1 and 4 deal with the shock anchor point locations to satisfy the centered equilibrium position.

The third set of constraints helps to obtain a suitable overall dynamics. The constraints 5, 6, 7 fix respectively the trail, the scrub radius and the camber of the front axle to some values that should ensure stability in case of right line rolling. The selection of these values is an attempt to reach a proper trade-off between multiple considerations. This trade-off is as important as delicate, as explained concisely in the section 2.1.3. In particular, the scrub radius must be lightly negative to ensure the steering DOF stability. This geometrical requirement leads furthermore to some conflicts in the mechanical implementation as explained in section 4.6.2. We chose to use some constraints instead of objectives measured on the basis of dedicated simulation, jointly to strictly take into account these important requirements and to save simulation time. Instead of this set of constraints, the enrichment of the global objective function with additional criteria joined to the addition of some related simulations in order to take all these aspects into account constitutes a possible promising improvement for the future. This could allow to determine precisely the best value for the trail, the scrub radius and the camber.

Finally, the steering rod length is used to set the toe of the front axle as usual (constraint 8). Surprisingly, the target value is simply fixed to zero. We know that this value is not optimal from the dynamics point of view. In fact, this is, once again, an admission of the limits of the model. The actual toe strongly depends on the suspension loading. Yet, the infinite body stiffness and the backlash free joints assumptions makes the model unable to determine the impact of the deformation on the toe. Despite its relatively small value, the elastic toe impacts the dynamics, at least sufficiently to lose all the interest of an optimized value based on the rigid model. Under the adopted assumptions,

the only option to determine a correct toe is the experiment.

Notice that because the optimal front axle settings have to be refined, the mechanical implementation of the front suspension involves an independent setting for the toe and two combined settings for the trail and the camber. The settings are designed as large as possible to compensate the lack of certitude about their value. Unfortunately, these settings impact also the other front axle characteristics.

ID	Symb.	Name	Type	Associated equation: C_g	CF1
1		body_to_shock_rr_z	PC	$L_{sh\ rr} - \frac{L_{min\ sh\ rr} + L_{max\ sh\ rr}}{2}$	0.5048
2		body_to_body_nose_dn_x	PC	$D_{axle\ ft} - D_{axle\ ft\ tg}$	0.5473
3		wishbone_ft_lt_dn_to_carrier_ft_lt_y	PC	$D_{track} - D_{track\ tg}$	0.3112
4		wishbone_ft_lt_dn_to_shock_ft_lt_y	PC	$L_{sh\ ft} - \frac{L_{min\ sh\ ft} + L_{max\ sh\ ft}}{2}$	0.1776
5		carrier_ft_lt_to_wheel_ft_lt_x	PC	$D_{trail} - D_{trail\ tg}$	0.0284
6		wishbone_ft_lt_up_to_carrier_ft_lt_y	PC	$D_{scrub} - D_{scrub\ tg}$	0.2328
7		R1_wheel_ft_lt	FJ	$D_{camber} - D_{camber\ tg}$	-0.2118
8		steering_rod_lt	RL	$D_{toe} - D_{toe\ tg}$	0.3064
9	$L_{0\ sh\ ft}$	shock_ft/L_0	UP	T3_body exchanged	0.3186
10	$L_{0\ sh\ rr}$	shock_rr/L_0	UP	R2_body exchanged	0.3536

Table 4.9: Geometrically constrained parameters.

Symb.	Description	Unit	Value
$D_{axle\ ft\ tg}$	Distance from the reference to the front wheel axle (target)	[mm]	550
$D_{track\ tg}$	Front axle track (target)	[mm]	680
$D_{trail\ tg}$	Front axle trail (target)	[mm]	50
$D_{scrub\ tg}$	Front axle scrub (target)	[mm]	-10
$D_{camber\ tg}$	Front axle camber (target)	[°]	0
$D_{toe\ tg}$	Front axle toe (target)	[°]	0

Table 4.10: Targeted geometrical setup.

Modal constraints

The modal constraints are listed in the table 4.11. They simply consist in imposing the frequency, ν , and the damping ratio, ζ , of both the front and rear suspension hopping modes. As a reminder, these suspension characteristics are measured thanks to the modal analysis embedded in the outer loop of the data conditioner (section 3.3.4). The spring stiffness and damping rate of both the front and rear shocks are the four parameters associated with these four constraints.

The target values for the modal setup are given in the table 4.12. The adopted frequencies are in the upper range for a public vehicle. This is justified because the stiffness of the front suspension has a direct influence on the energy needed to control the tilt motion. The higher the stiffness, the higher the energy efficiency of the tilting system is. Moreover, the tilting energy being a criterion embedded in the global objective function, it is needed to impose the frequency constraint to avoid a too large stiffening that would be detrimental

to the suspension work and finally to the comfort. Despite this runs counter the previous consideration, the rear suspension is significantly stiffer than the front suspension following the Olley's rule (section 2.1.3). The setting has been optimized for a speed of 12 m/s, a common travel speed in urban area.

The damping ratios are identical and take a very common value. Notice the constraints corresponding to the damping ratios are weighted because the experience has shown that such a trick is necessary to meet a good convergence of the modal constraints solving.

ID	Symb.	Name	Type	Associated equation: C_m	CF1
1	$K_{sh\ ft}$	shock_ft/K	UP	$\nu_{ft} - \nu_{ft\ tg}$	38640
2	$D_{sh\ ft}$	shock_ft/D	UP	$w_\zeta(\zeta_{ft} - \zeta_{ft\ tg})$	3277
3	$K_{sh\ rr}$	shock_rr/K	UP	$\nu_{rr} - \nu_{rr\ tg}$	66011
4	$D_{sh\ rr}$	shock_rr/D	UP	$w_\zeta(\zeta_{rr} - \zeta_{rr\ tg})$	4202

Table 4.11: Modal constrained parameters.

Symb.	Description	Unit	Value
$\nu_{ft\ tg}$	Front suspension natural frequency (target)	[Hz]	1.55
$\zeta_{ft\ tg}$	Front suspension damping ratio (target)	[-]	0.40
$\nu_{rr\ tg}$	Rear suspension natural frequency (target)	[Hz]	2.00
$\zeta_{rr\ tg}$	Rear suspension damping ratio (target)	[-]	0.40
w_ζ	Damping ratio weight	[-]	10.00

Table 4.12: Targeted modal setup.

4.4.3 Simulator

In this section, we give some details of the hybrid simulator and in particular, the specificity of the Wemoov evaluation.

Reference track

During the transient simulation, the vehicle follows a reference track that must sufficiently excite the vehicle to evaluate all required performances. For the three-wheeler optimization, the chosen track is a "S" trajectory depicted in the figure 4.7. The first turn, to the right, has a minimum radius of 10 meters and is followed by a second turn, to the left, with a minimum radius of 8 meters. The curvature change between the two turns of the S trajectory is extremely challenging for a tilting vehicle such the three-wheeler.

The Wemoov travels this track in free rolling. Its initial speed is 5 m/s (18 km/h). For this simulation, we have chosen not to apply torque on the rear wheel so as not to perturb the test, which is already sufficiently challenging, with the associated reactions. Also, we have judged this scenario more consistent than a constant speed test, considering the natural driver behavior who brakes before the curve, passes through it in free rolling before accelerates once the curvature drops. On the contrary, this last behavior, very complete, is also

not used because too complex in view of the vehicle assessment. Furthermore, the dynamics reactions under wheel torque are partially taken into account in the steady-state analysis.

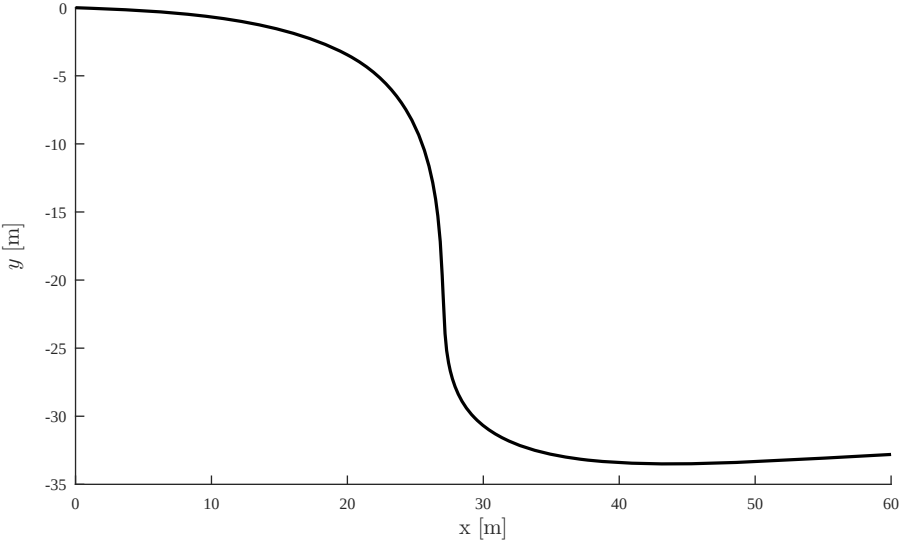


Figure 4.7: Reference track.

Steering controller

The steering controller, whose operation is described in section 3.4.1, is parametrized following the table 4.13. The target point is located at just 2 m ahead of the vehicle. The driver gaze being pointed much beyond, it is surprising that the values of this order of magnitude give the best results. The PID coefficients have been determined by some similarity rules and corrected by a trial and error approach. The result is very satisfactory. Indeed, along the reference trajectory, the maximum following track error is only about 35 mm. The mean error is far below this peak value as depicted later, in the figure 4.24.

Symb.	Description	Unit	Value
L_{target}	Distance between nearest and target points	[m]	2.0
$K_{p\epsilon}$	Proportional coefficient for following error	[rad/m]	15.0
$K_{i\epsilon}$	Integral coefficient for following error	[rad/m.s]	0.3
$K_{d\epsilon}$	Differential coefficient for following error	[rad.s/m]	0.1
$K_{p\sigma}$	Proportional coefficient for orientation	[-]	0.8
$K_{d\sigma}$	Differential coefficient for orientation	[s]	0.2

Table 4.13: Steering controller setup.

Termination conditions

For the Wemoov application, three termination conditions have been retained for the transient simulation.

The first condition is the take-off of one of the front wheels. A negative penetration, e , in the computation of the wheel forces indicates the take-off:

$$e_{lt} < 0 \quad \text{or} \quad e_{rt} < 0 \quad (4.5)$$

This condition, in addition to be intrinsically problematic, forecasts a probable rollover what must be avoided by all means. It is the most often encountered condition when the success of the transient simulation can not be reached.

The second condition is the detection of a roll threshold, being a byword for an imminent fall. For the Wemoov, the limit is fixed to 1 rad (approximately 60°) because over this value, normal rolling can absolutely not be restored, the normal tilting range extending up to about 38° .

$$\phi = q(6) > 1.0 \quad (4.6)$$

A more restrictive threshold could be applied but, in fact, this condition is rarely used. It is mainly encountered in case of falling in the direction of the turn, a pretty rare case in our optimization approach.

These two conditions are failures and are treated in the same manner. The rules of evaluator in phase 2 are applied. So, the transient part of the objective function is computed through the equation 3.29 (section 3.5.4).

The last condition, the only one considered as a success, is the completion of the reference track travel. It is detected on the basis of a threshold on the x coordinate of the chassis, namely:

$$x = q(3) > 60.0 \quad (4.7)$$

In this case, the rules of evaluator in phase 3 are applied (section 3.5.4).

Steady-state analysis mapping

The steady-state analysis is performed in accordance with the strategy described in section 3.4.2. The settings proper to the Wemoov analysis are given in the table 4.14. The complete routing involves 228 points but only 159 of those can be reached by the Wemoov. For the others, the lateral acceleration limit is applied or some convergence problems occur.

Axis	Unit	Start	Increment	End	Steps
Speed	[m/s]	2.50	1.25	16.25	12
Radius	[m]	50.00	-2.50	5.00	19

Table 4.14: Steady-state analysis mapping range.

For example, the figure 4.9 illustrates the steady-state curving configurations of the vehicle for a radius of 10 m and three different speeds: 2.5, 5.0 and 7.5 m/s.

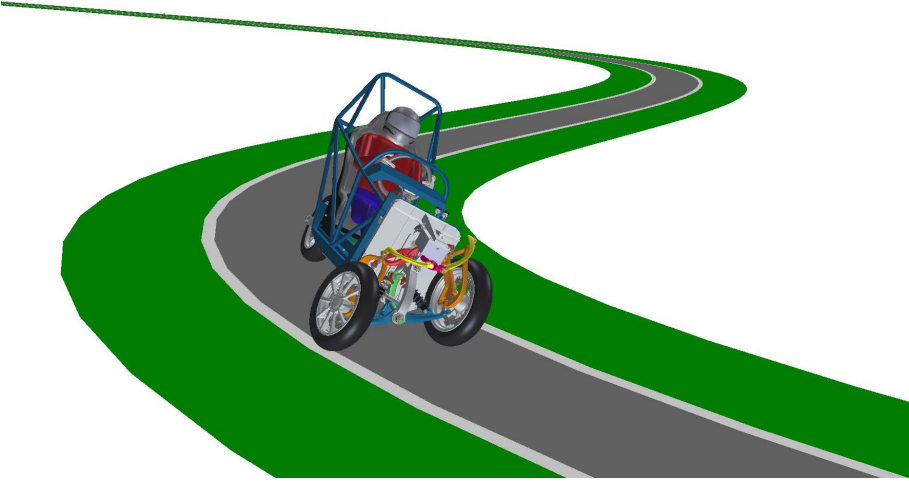


Figure 4.8: Three-wheeler on the S-shape reference trajectory.

It must be noticed that, thanks to the mathematical formulation of the equilibrium solving, the process finds the nearest equilibrium with respect to initial conditions, between all possible equilibria, included unstable ones. It allows to study the three-wheeler without tilting actuation and to find, in this particular case, unstable steady-state configurations if any. A special attention must be paid.

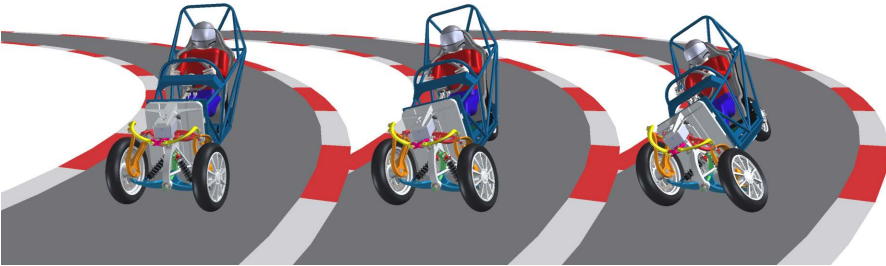


Figure 4.9: Three-wheeler steady-state equilibrium in curve with a radius of 10 m, for three different speeds: 2.5, 5.0 and 7.5 m/s.

4.4.4 Evaluator

Applied criteria

The generic criteria introduced in section 3.5.1 are particularized for the Wemoov optimization. Due to the three-wheeler configuration, some changes must be carried out.

The under-oversteering coefficient becomes:

$$E_{\alpha} = \left| \frac{\alpha_{lt} + \alpha_{rt}}{2} \right| - |\alpha_{rr}| \quad (4.8)$$

where α_{lt} , α_{rt} , α_{rr} are respectively the slip angles of the front left, front right and rear wheels.

The pseudo-Ackermann criterion remains identical:

$$E_{Ack} = \alpha_{lt} - \alpha_{rt} \quad (4.9)$$

As discussed earlier, we opt for a very simplified steering torque feed-back criterion:

$$E_{T_{sw}} = T_{sw} = \frac{Qq(25)r_{st}}{\eta_{st}} \quad (4.10)$$

with:

$Qq(25)$ the generalized force i.e. the torque in the steering fork joint,

r_{st} the ratio of the steering transmission [-],

η_{st} the efficiency of the steering transmission between the wheel and the fork [-].

Symb.	Description	Unit	Value
r_{st}	Steering transmission ratio	[-]	5:1
η_{st}	Steering transmission efficiency	[-]	0.5

Table 4.15: Steering transmission setup.

It must be noticed that the steering torque feed-back¹ is, over a more or less part of the driving range, such the steering angle is self-increased i.e. in case of a release of the wheel, the steering angle tends to increase by itself. This behavior is not surprising because of the intrinsic tilting dynamics of the Wemoov which is closer to that of a bike or a motorbike than that of a car. It must be checked if such a control is pleasant and, above all, safe given the use of a steering wheel instead of a handle bar. This point will be discussed in the section 5.2.2.

The track following error remains identical:

$$E_{\epsilon} = \epsilon \quad (4.11)$$

¹In order to remove a possible ambiguity, despite the denomination of steering torque "feed-back", the given values correspond to the torque of the driver to the steering wheel.

These generic criteria are completed by others that are very specific to the development of a tilting three-wheeler. They are listed below:

Unballasting of the front axle

The unballasting coefficient is computed by the ratio of the difference and the sum of the normal forces on the front tires:

$$E_{F_n} = \frac{F_{n\,lt} - F_{n\,rt}}{F_{n\,lt} + F_{n\,rt}} \quad (4.12)$$

where $F_{n\,lt}$ and $F_{n\,rt}$ are respectively the normal forces for left and right front wheels. This criterion is prominent in the study of the roll equilibrium. It measures the limit of roll equilibrium and in particular, the capabilities of tilting actuation. A right awareness of this criterion helps to prevent the roll-over of the vehicle and the related material and, much worse, human consequences. Due to the vital importance, this is the most serious criterion to consider.

Tilting actuation torque

The two last criteria refer to the DTC actuation. We introduce both the torque and the power because these criteria have different impacts on the resulting three-wheeler. Both the torque and the power are measured at the level of the T-pendulum revolute joint. Note that these criteria are taken into account only in the case of the transient analysis. In facts, for the steady-state analysis, the T-pendulum joint is free so the torque and the power are null, by definition of the problem.

So, the torque criterion is simply given by:

$$E_{T_{ta}} = T_{ta} = Qq(11) \quad (4.13)$$

The encountered maximum torque specifies the strength of the transmission and of all the mechanical elements involved in the kinematic chain. This has a large impact on the weight of the system, itself impacting the torque requirement. It is a cyclic problem, what reinforces the interest of the optimization.

In addition to the inertia characteristics, like the roll inertia, the body mass and the height of the center of mass which are very sensitive but not considered as parameters, the tilting torque mainly depends on the tilting controller parametrization. The geometry of the DTC mechanism inside the suspension has a second order effect. The potential gain of the optimization is all the same significant.

Tilting actuation power

The power of the tilting actuation, probed at the same point as the torque, is simply computed as:

$$E_{P_{ta}} = P_{ta} = T_{ta}\omega_{ta} = Qq(11)\dot{q}(11) \quad (4.14)$$

Obviously, the elements that affect the torque affect also the power. Furthermore, the stiffness and, even more, the damping rate of the shock through which the DTC effort path passes, influence this criterion.

Let us note that we consider, in the evaluator, only the criteria directly related to true performances of the vehicle. We do not take attention to intermediate characteristics that are supposed to provides properties or to impact the true performances. As example, we are not concerned by the evolution of the track and the wheelbase, or more generally, by the evolution of the footprint of the vehicle, in function of the tilting or the steering, because of the influence on the roll stability. However, the unballasting of the front axle that is selected as criterion, is a direct measure of the stability, allowing to reach, indirectly, a suitable front suspension kinematics.

Evaluator setup

As described in section 3.5, the evaluator is a complex processing of the data recorded along the hybrid simulator, in order to synthesize a unique objective function representative of the global vehicle quality. As a reminder, the process includes the criteria clustering processing and the mix law respectively described in the sections 3.5.2 and 3.5.3. The determination of the numerous coefficients of this algorithm is a very important task to properly express the requirements about the vehicle. However, despite the proposed method, it is still difficult to mathematize the specifications addressed to the Wemoov.

The table 4.16 contains the general evaluator setup i.e. the settings that do not relate directly to the criteria. Notice that the exponent of equilibrium success ratio has been chosen large enough to strongly promote the candidates that succeed in a maximum of the tested steady-states.

Symb.	Description	Unit	Value
n_{kref}	Reference number of the successful equilibria	[-]	159
z_{SS}	Exponent of equilibrium success ratio	[-]	8
t_{go}	Evaluator activation time	[s]	1.0
t_{ref}	Reference time for the test trajectory	[s]	30.0

Table 4.16: Evaluator setup.

The tables 4.17 and 4.18 give the satisfaction and acceptance thresholds and consequent weights, w , and exponents, z , for each meta-criterion, for the transient simulation and for the steady-state analysis respectively. Quite a few observations can be done.

All the meta-criteria are not exploited. For the transient simulation, the maximal value of the under-oversteering coefficient is not used because this value is not representative of the front axle quality, but rather of the intensity of the steering change which is not relevant for the vehicle quality evaluation. In a similar way, the mean of unballasting is not taken into account. Only the maximal value encountered in the demanding reference track is relevant. The

mean tilting actuation torque does not affect the vehicle property contrary to the maximum torque that determines the capacity of the actuation system. We agree that the justification of these choices is not perfect. One can argue to take these meta-criteria into account with some valid arguments. The deliberated exclusions mainly aim to avoid perturbing the objective function with meta-criteria for which we think they are lowly, or not at all, relevant and, in any case, difficult to set. In this regards, for the steady-state analysis, the track following error, the tilting actuation torque and power do not make sense and consequently are not taken into account.

The value of the satisfaction and acceptance levels whose depends the weight and exponents are left to the designer appreciation. The selection of levels are governed by some guiding principle. Obviously, the mean values are smaller than the maximum values. Broadly speaking, the levels are more stringent for steady-state analysis than for the transient simulation.

For the under-oversteering coefficient and the pseudo-Ackermann criterion, the base of thinking is the characteristics of the tire lateral force as a function of the slip angle (section 2.1.2). Indeed, the slip angle of tires is at the basis of the computation of these criteria. The levels are selected with respect to the limits between linearity, inflection, saturation of the curve (figure 2.8). These limits are approximate (almost subjective) as well as the subsequent satisfaction and acceptance levels.

The levels related to the steering torque feed-back stem from some elementary experiments. Physically, for a mean driver, the limit of steady-state torque is around 6 Nm. For a transient situation, the developed torque can easily rise up to 15 Nm in peaks. Similar values could be founded in the literature [67, 69]. Some lower values have been selected for comfort reasons.

We have decided to target low track following error to get excellent handling properties. Another goal of that criterion is avoiding the deterioration of the initial track following, allowing, in the way, the other criteria improvement.

For the unballasting of the front axle, the values for the levels corresponding to the state-state analysis are set very low. The existence of suspensions that do not generate unballasting convinces us that such levels are reachable. In fixing the satisfaction level at 20 % and the acceptance to 60 % for the maximum value in transient simulation, we clear a security margin with the risk of a wheel take-off. The dramatic consequences of such an event justify this choice.

The levels for the tilting actuation torque and power are selected to be challenging but reachable in comparison to the values observed along the preliminary analysis achieved on the unoptimized three-wheeler.

Except for the mean value of unballasting of the front axle, the exponent coefficients are often higher than 2, and up to 6.6. This implies a high sensitivity of the global objective function with respect to the criteria. So, the optimization is more severe with a vehicle which exhibit a downrightly poor criterion than for a vehicle moderately bad in each criterion. This is in line with our wish.

		Transient simulation (TS)								
		Mean				Max				
ID	Symb.	Name	$\bar{E}_{s\ i}$	$\bar{E}_{a\ i}$	$\bar{w}_i$	$\bar{z}_i$	$\hat{E}_{s\ i}$	$\hat{E}_{a\ i}$	$\hat{w}_i$	$\hat{z}_i$
1	E_α	Under-oversteering coefficient	3.162e-4	3.162e-3	1.000e+8	2.000				
2	E_{Ack}	Pseudo-Ackermann criterion	1.000e-3	8.000e-3	4.404e+7	2.215	5.000e-3	2.500e-2	3.838e+7	2.861
3	$E_{T_{sw}}$	Steering torque feed-back	5.000e-1	2.000e+0	1.000e+2	3.322	4.000e+0	8.000e+0	1.000e-3	6.644
4	E_ϵ	Track following error	1.000e-2	5.000e-2	5.281e+6	2.861	5.000e-2	2.000e-1	2.099e+5	3.322
5	E_{F_n}	Unballasting of the front axle					2.000e-1	6.000e-1	8.510e+3	4.192
6	$E_{T_{ta}}$	Tilting actuation torque					1.000e+1	1.000e+2	1.000e-1	2.000
7	$E_{P_{ta}}$	Tilting actuation power	1.000e+1	1.000e+2	1.000e-1	2.000	2.500e+1	5.000e+2	7.096e-2	1.537

Table 4.17: Evaluator setup: transient simulation.

		Steady-state analysis (SS)								
		Mean				Max				
ID	Symb.	Name	$\bar{E}_{s\ i}$	$\bar{E}_{a\ i}$	$\bar{w}_i$	$\bar{z}_i$	$\bar{E}_{s\ i}$	$\bar{E}_{a\ i}$	$\bar{w}_i$	$\bar{z}_i$
1	E_α	Under-oversteering coefficient	5.000e-4	5.000e-3	4.000e+7	2.000	5.000e-4	5.000e-3	4.000e+7	2.000
2	E_{Ack}	Pseudo-Ackermann criterion	5.000e-4	4.000e-3	2.044e+8	2.215	2.500e-3	1.250e-2	2.789e+8	2.861
3	$E_{T_{sw}}$	Steering torque feed-back	5.000e-1	2.000e+0	1.000e+2	3.322	4.000e+0	8.000e+0	1.000e-3	6.644
4	E_ϵ	Track following error								
5	E_{F_n}	Unballasting of the front axle	1.000e-8	1.000e-4	1.000e+5	0.500	2.500e-2	1.000e-1	2.099e+6	3.322
6	$E_{T_{ta}}$	Tilting actuation torque								
7	$E_{P_{ta}}$	Tilting actuation power								

Table 4.18: Evaluator setup: steady-state analysis.

4.4.5 CMA-ES parametrization

The most relevant parameters of CMA-ES, the optimization algorithm, are given in the table 4.19.

The number of particles is over the recommendation of the algorithm developers.

We have deliberately chosen to reduce the initial variance, aware to increase the convergence time but hoping to increase, in the same way, the robustness of the optimization. In retrospect, we are not sure of the interest of this decision. Indeed, this change could lead to an increased sensitivity to a local minimum. A deeper analysis of the effect of such settings could be very interesting if the performances of the optimization process become critical, what is not the case here.

Only the termination conditions mentioned in the table 4.19 were activated for the optimization runs. The maximum number of generations is fixed to 400. As illustrated in the section 4.5.2, it is largely sufficient. The termination conditions related to the change of objective function has been risen with respect to the default values. Despite this, such conditions were never satisfied in practice. Most of the time, the condition based on the step size was on the origin of the process termination. The desired precision is so high that it can not be reached in practice during the mechanical implementation. A so much demanding value makes only sense for the optimization process characterization.

CMA-ES parameter	Description	Default value	Run value
n_{core}	Number of cores		50
N (n_{part})	Number of particles		49
$Xsdev[i]$ ($\sigma_{0\ i}$)	Initial variance	0.3	0.01
stopMaxIter	Maximum number of iterations (generations)	∞	400
stopTolFun	Change of objective function (between two consecutive generations)	1e-12	1e-6
stopTolFunHist	Change of objective function (considering the historical best candidate)	0	1e-13
stopTolX	Step size in $\mathbf{x}$ -space	0	1e-5

Table 4.19: CMA-ES parametrization.

4.5 Optimization results

4.5.1 Reference selection

Before showing the result of the optimization, it is useful to select a reference which allows an objective comparison. The initial configuration, which usually plays this role, is not suitable, simply because it does not succeed the data conditioner. A relevant comparison is only possible if each vehicle fits the requirements of the assessment by the phase 3 of the evaluator (section 3.5.4). So, we want the reference vehicle passes the data conditioner, succeeds in 100

steady-states as a minimum and finishes the travel of the reference track without any problem.

In order to find an interesting reference, we have implemented a pure random "optimization" scheme. This generates lots of aleatory optimized parameter vectors and evaluates the corresponding vehicles via the trial. 200 candidates have been tested. 31 of them succeed in over than 100 steady-states. Only 5 three-wheelers could be assessed in phase 3. All in all, these 5 configurations are retained as references. They are labeled REF1, REF2, REF3, REF4 and REF5. Particularly representative, the third reference, REF3, is considered as the main one and is simply labeled REF in the subsequent sections.

4.5.2 Optimized parameter evolution

This section deals with the evolution of the parameter along the optimization. The table 4.21 gives the boundaries of ranges for the optimized parameters and their values and normalized values for the initial, reference, REF, and optimized, CF1, configurations. The table 4.22 gives the optimized parameter vectors for the 5 references. The color code is identical to those of the graphics of evolution of the parameters along the optimization leading to CF1 configuration (figures 4.10 and 4.11).

The figure 4.11 shows the filtered¹ evolution that can not be distinguished on the unfiltered version (figure 4.10). This last allows to reveal the high variance of the parameter evolution along the first part of the optimization.

Among the 12 parameters, 2 come down to their lower boundary, 4 cap to their top boundary and 6 find a "true" optimal value. The carrier_ft_lt_to_wishbone_ft_lt_up_z, p_{o5} , that reaches its lowest value, cannot be decreased without encounter any mechanical interaction with the tire. The carrier_ft_lt_to_wheel_ft_lt_z, p_{o6} , is minimal because the lower suspension ball joint cannot be nearer to the wheel axis, the space being filled by the hub. In contrast, the carrier_ft_lt_to_steering_rod_ft_lt_z, p_{o8} , is limited to do not obstruct the wheel arch too much. The steering_fork_to_steering_rod_lt_y, p_{o9} , is maximum to avoid a bifurcation of the steering mechanism. The R2_steering_fork, p_{o11} and R2_body_nose_up, p_{o12} are limited because the resulting wishbone pivot angle leads to a harmful coupling between the longitudinal and suspension dynamics, if too high. We notice that a moderate pivot angle is suitable to get anti-dive effect as already mentioned in the section 2.1.3.

One can observe some parameters shift very much what proves the optimization algorithm works well. Generally speaking, the parameters change freely each other but some local correlations can also be observed. The changes are more pronounced at the beginning of the optimization and the change rate diminishes quickly thereafter what indicates a suitable convergence. Most of changes occurs before the 100th generation while the termination condition is fulfilled at the 333th generation in 3 hours 25 minutes and 25 seconds on the Lemaitre3 cluster (table 3.3).

¹The Matlab second order Butterworth filter has been used with a low-pass coefficient of 1/5. In order to obtain better results, the filtering has been processed backwards, the signals being smoother at the end.

4.5.3 Fitness analysis

After the analysis of the parameter evolution, we propose to observe the resulting effect on the fitness. The table 4.20 gives the fitness for the transient simulation, the steady-state analysis and the sum of these two contributions, for each configuration. Due to the large non-linearity of the evaluator, the fitness spans over a large range: there are three orders of magnitude between the worst fitness and the best one.

In order to understand these results, the details of the objective function must be observed. The values of meta-criteria for each criterion are given in the next tables 4.23, 4.24, 4.25, 4.26, 4.27, 4.28 for REF1, REF2, REF3, REF4, REF5 and CF1 respectively. As a reminder, the values below 10 are satisfactory (green), the values below 1000 are acceptable (orange) and those over 1000 are unacceptable (red).

Generally, the satisfaction and acceptance levels of criteria relative to the transient simulation are more often reached. For the references, the number of successful steady-states is never equal to the best result. In fact, the conquest of the steady-state mapping is really an optimization goal per se.

One can remark that the optimization does not especially increase the number of satisfied meta-criteria. For example, REF1 and CF1 have both 5 satisfied criteria. However, the optimization drastically reduces the number of unacceptable criteria. Despite the action of the optimizer, 5 meta-criteria remain unacceptable according to the selected levels.

Some meta-criteria are satisfied for each configuration, as for examples the following track error and the mean track power. Unlikely, some meta-criteria are systematically highly unacceptable, such as the unballasting of the front axle (Steady-state analysis (SS)) or the tilting actuation torque (Transient simulation (TS)). We could question the right choice of the satisfaction and acceptance levels. Are the selected criteria levels too less or too much challenging? Or, are these criteria inherently easy or difficult to reach? Fortunately, some meta-criteria are sometimes unacceptable, sometimes acceptable for the reference configurations and satisfactory for the optimized one, CF1. This does not prove the levels are perfectly determined, but it attests the potential improvement of meta-criteria and the positive impact on the vitality of the optimization process.

Fitness		REF1	REF2	REF3	REF4	REF5	CF1
$F_{0\ TS}$	Transient simulation	9387.77	8215.55	132507.55	18714.81	14144.59	6642.22
$F_{0\ SS}$	Steady-state analysis	16080302.37	71424.67	16839646.39	189884.70	455521.62	27381.93
F_0	Total hybrid simulator	16089690.13	79640.22	16972153.94	208599.50	469666.21	34024.15

Table 4.20: Evaluator result overview.

Color	ID	Name	Type	Boundaries		Initial		REF		CF1	
				p_o	p_o	x_o	p_o	x_o	p_o	x_o	p_o
	1	body_to_steering_fork_x	PC	0.5000	0.6000	0.8000	0.5800	0.8848	0.5885	0.9978	0.5998
	2	body_to_steering_fork_z	PC	0.3500	0.4000	0.2000	0.3600	0.7383	0.3869	0.3539	0.3677
	3	body_to_body_nose_up_x	PC	0.4100	0.4500	0.2500	0.4200	0.4575	0.4283	0.5569	0.4323
	4	body_to_body_nose_up_z	PC	0.2800	0.3500	0.2857	0.3000	0.8603	0.3402	0.3444	0.3041
	5	carrier_ft_lt_to_wishbone_ft_lt_up_z	PC	0.3700	0.4000	0.0000	0.3700	0.2714	0.3781	0.0002	0.3700
	6	carrier_ft_lt_to_wheel_ft_lt_z	PC	0.0650	0.0800	0.1000	0.0665	0.7787	0.0767	0.0005	0.0650
	7	carrier_ft_lt_to_steering_rod_ft_lt_y	PC	0.0100	0.0700	0.5833	0.0450	0.7375	0.0542	0.7367	0.0542
	8	carrier_ft_lt_to_steering_rod_ft_lt_z	PC	0.2500	0.4200	0.8824	0.4000	0.8112	0.3879	1.0000	0.4200
	9	steering_fork_to_steering_rod_lt_y	PC	0.0250	0.0400	0.3333	0.0300	0.0635	0.0260	1.0000	0.0400
	10	R2_body_nose_dn	FJ	0.1000	0.2000	0.6000	0.1600	0.4147	0.1415	0.2329	0.1233
	11	R2_steering_fork	FJ	0.1000	0.1500	0.8000	0.1400	0.9502	0.1475	0.9997	0.1500
	12	R2_body_nose_up	FJ	0.1000	0.2000	0.9000	0.1900	0.3358	0.1336	0.9996	0.2000

Table 4.21: Optimized parameters: boundaries, initial and result.

Color	ID	Name	Type	REF1		REF2		REF3		REF4		REF5	
				x_o	p_o	x_o	p_o	x_o	p_o	x_o	p_o	x_o	p_o
	1	body_to_steering_fork_x	PC	0.1398	0.5140	0.0416	0.5042	0.8848	0.5885	0.6853	0.5685	0.4319	0.5432
	2	body_to_steering_fork_z	PC	0.4233	0.3712	0.1874	0.3594	0.7383	0.3869	0.4450	0.3722	0.9547	0.3977
	3	body_to_body_nose_up_x	PC	0.0912	0.4136	0.4913	0.4297	0.4575	0.4283	0.3171	0.4227	0.6101	0.4344
	4	body_to_body_nose_up_z	PC	0.7755	0.3343	0.1934	0.2935	0.8603	0.3402	0.4282	0.3100	0.8725	0.3411
	5	carrier_ft_lt_to_wishbone_ft_lt_up_z	PC	0.4273	0.3828	0.2339	0.3770	0.2714	0.3781	0.8693	0.3961	0.1994	0.3760
	6	carrier_ft_lt_to_wheel_ft_lt_z	PC	0.5513	0.0733	0.2085	0.0681	0.7787	0.0767	0.8534	0.0778	0.9754	0.0796
	7	carrier_ft_lt_to_steering_rod_ft_lt_y	PC	0.8629	0.0618	0.8859	0.0632	0.7375	0.0542	0.5875	0.0453	0.5798	0.0448
	8	carrier_ft_lt_to_steering_rod_ft_lt_z	PC	0.6129	0.3542	0.8798	0.3996	0.8112	0.3879	0.9990	0.4198	0.8617	0.3965
	9	steering_fork_to_steering_rod_lt_y	PC	0.6007	0.0340	0.8318	0.0375	0.0635	0.0260	0.4033	0.0310	0.5544	0.0333
	10	R2_body_nose_dn	FJ	0.5466	0.1547	0.6417	0.1642	0.4147	0.1415	0.3344	0.1334	0.8896	0.1890
	11	R2_steering_fork	FJ	0.6136	0.1307	0.3149	0.1157	0.9502	0.1475	0.4284	0.1214	0.0575	0.1029
	12	R2_body_nose_up	FJ	0.3012	0.1301	0.1826	0.1183	0.3358	0.1336	0.0254	0.1025	0.2925	0.1292

Table 4.22: Optimized parameters: 5 references.

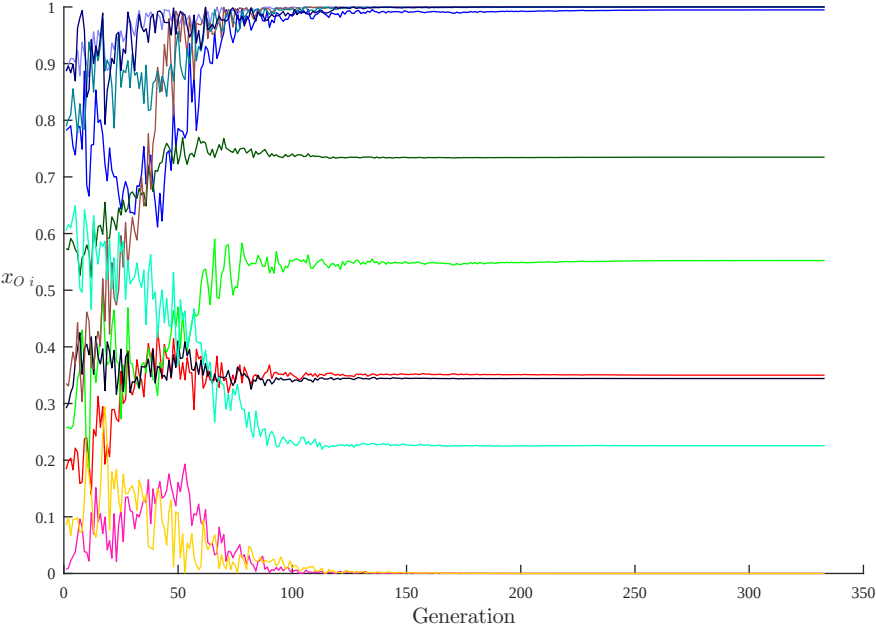


Figure 4.10: Unfiltered evolution of optimized parameters.

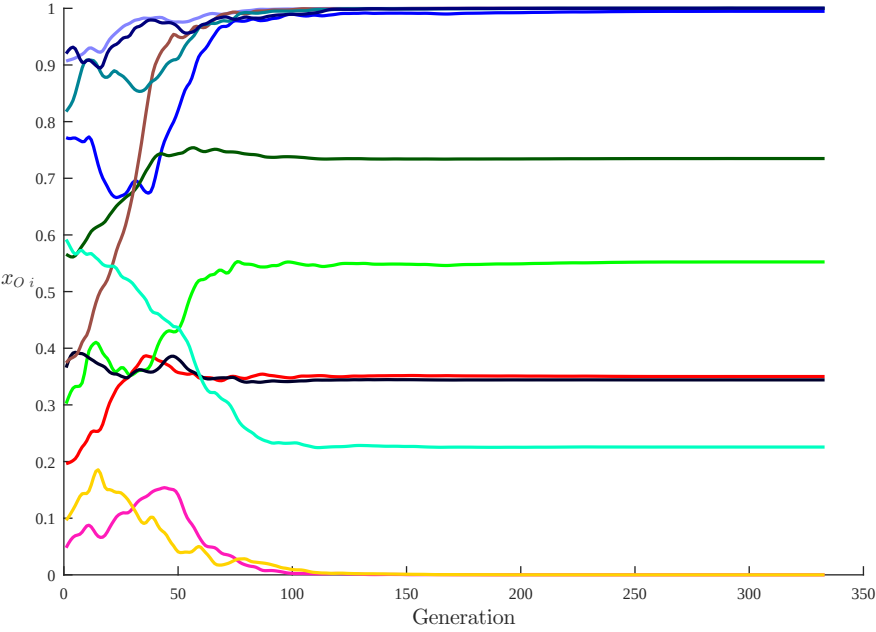


Figure 4.11: Filtered evolution of optimized parameters.

			Transient simulation		Steady-state analysis	
			Mean	Max	Mean	Max
ID	Symb.	Name	$\bar{E}_i$	$\hat{E}_i$	$\check{E}_i$	$\bar{E}_i$
1	E_α	Under-oversteering coefficient	125.99		1732.70	323891.76
2	E_{Ack}	Pseudo-Ackermann criterion	654.47	6339.55	162501.07	5642858.59
3	$E_{T_{sw}}$	Steering torque feed-back	16.40	0.17	21.76	5.04
4	E_ϵ	Track following error	1.76	3.33		
5	E_{F_n}	Unballasting of the front axle		82.86	16799.05	2119.26
6	$E_{T_{ta}}$	Tilting actuation torque		2127.22		
7	$E_{P_{ta}}$	Tilting actuation power	3.88	32.13		
$n_k/n_{k\ ref}$						141 / 159
$F_0\ TS\ F_0\ SS$		Total simulation		9387.77		16080302.37
F_0		Total hybrid simulator				16089690.13

Table 4.23: Evaluator results for the REF1 configuration.

			Transient simulation		Steady-state analysis	
			Mean	Max	Mean	Max
ID	Symb.	Name	$\bar{E}_i$	$\hat{E}_i$	$\check{E}_i$	$\bar{E}_i$
1	E_α	Under-oversteering coefficient	31.01		116.45	816.61
2	E_{Ack}	Pseudo-Ackermann criterion	223.68	1623.27	5636.58	28914.50
3	$E_{T_{sw}}$	Steering torque feed-back	86.57	1143.11	10.84	494.56
4	E_ϵ	Track following error	1.72	3.34		
5	E_{F_n}	Unballasting of the front axle		182.85	21598.53	3741.23
6	$E_{T_{ta}}$	Tilting actuation torque		4840.04		
7	$E_{P_{ta}}$	Tilting actuation power	5.84	74.13		
$n_k/n_{k\ ref}$						156 / 159
$F_0\ TS\ F_0\ SS$		Total simulation		8215.55		71424.67
F_0		Total hybrid simulator				79640.22

Table 4.24: Evaluator results for the REF2 configuration.

			Transient simulation		Steady-state analysis	
			Mean	Max	Mean	Max
ID	Symb.	Name	$\bar{E}_i$	$\hat{E}_i$	$\check{E}_i$	$\bar{E}_i$
1	E_α	Under-oversteering coefficient	810.76		994.80	416169.86
2	E_{Ack}	Pseudo-Ackermann criterion	3834.12	119809.64	149846.02	7468861.31
3	$E_{T_{sw}}$	Steering torque feed-back	41.15	46.45	13.26	12.42
4	E_ϵ	Track following error	2.03	2.69		
5	E_{F_n}	Unballasting of the front axle		379.76	17009.20	2732.04
6	$E_{T_{ta}}$	Tilting actuation torque		7445.02		
7	$E_{P_{ta}}$	Tilting actuation power	5.37	130.55		
$n_k/n_{k\ ref}$						145 / 159
$F_0\ TS\ F_0\ SS$		Total simulation		132507.55		16839646.39
F_0		Total hybrid simulator				16972153.94

Table 4.25: Evaluator results for the REF3 configuration.

			Transient simulation		Steady-state analysis	
			Mean	Max	Mean	Max
ID	Symb.	Name	$\bar{E}_i$	$\hat{E}_i$	$\check{E}_i$	$\bar{E}_i$
1	E_α	Under-oversteering coefficient	53.61		66.67	552.75
2	E_{Ack}	Pseudo-Ackermann criterion	760.89	6708.35	12316.62	87949.28
3	$E_{T_{sw}}$	Steering torque feed-back	160.97	4845.41	38.91	3177.47
4	E_ϵ	Track following error	1.71	3.27		
5	E_{F_n}	Unballasting of the front axle		252.68	22601.77	5749.50
6	$E_{T_{ta}}$	Tilting actuation torque		5834.73		
7	$E_{P_{ta}}$	Tilting actuation power	5.80	87.38		
$n_k/n_{k\ ref}$						152 / 159
$F_0\ TS\ F_0\ SS$		Total simulation		18714.81		189884.70
F_0		Total hybrid simulator				208599.50

Table 4.26: Evaluator results for the REF4 configuration.

			Transient simulation		Steady-state analysis	
			Mean	Max	Mean	Max
ID	Symb.	Name	$\bar{E}_i$	$\hat{E}_i$	$\check{E}_i$	$\bar{E}_i$
1	E_α	Under-oversteering coefficient	60.07		1.99	2591.58
2	E_{Ack}	Pseudo-Ackermann criterion	65.64	329.93	9433.39	247552.72
3	$E_{T_{sw}}$	Steering torque feed-back	486.70	5026.41	284.50	2513.01
4	E_ϵ	Track following error	1.88	3.12		
5	E_{F_n}	Unballasting of the front axle		498.07	19513.65	3909.05
6	$E_{T_{ta}}$	Tilting actuation torque		7562.02		
7	$E_{P_{ta}}$	Tilting actuation power	6.77	103.98		
$n_k/n_{k\ ref}$						150 / 159
$F_0\ TS\ F_0\ SS$		Total simulation		14144.59		455521.62
F_0		Total hybrid simulator				469666.21

Table 4.27: Evaluator results for the REF5 configuration.

			Transient simulation		Steady-state analysis	
			Mean	Max	Mean	Max
ID	Symb.	Name	$\bar{E}_i$	$\hat{E}_i$	$\check{E}_i$	$\bar{E}_i$
1	E_α	Under-oversteering coefficient	0.34		36.43	455.39
2	E_{Ack}	Pseudo-Ackermann criterion	1.97	71.64	1802.26	2200.52
3	$E_{T_{sw}}$	Steering torque feed-back	63.21	221.64	101.94	32.18
4	E_ϵ	Track following error	1.70	3.31		
5	E_{F_n}	Unballasting of the front axle		289.24	20656.90	2096.31
6	$E_{T_{ta}}$	Tilting actuation torque		5887.21		
7	$E_{P_{ta}}$	Tilting actuation power	6.60	95.35		
$n_k/n_{k\ ref}$						159 / 159
$F_0\ TS\ F_0\ SS$		Total simulation		6642.22		27381.93
F_0		Total hybrid simulator				34024.15

Table 4.28: Evaluator results for the CF1 configuration.

The evolution of the global objective function with respect to the successive generation is shown in the figure 4.12. The values correspond to the objective function for the best candidate of each generation. After a quick improvement at the beginning of the optimization, the function converges asymptotically to its final value. Passed the 100th generation, the evolution can not be distinguished anymore.

A far more interesting point of view of the optimization evolution is obtained by the computation of the offset-free fitness, F'_o , as the difference between the current objective function and that of the best-ever candidate:

$$F'_o = F_o - F_{o\ best} \quad (4.15)$$

The logarithmic plot of such an offset free fitness (figure 4.13) highlights the remarkable regularity of the convergence which is virtually logarithmic as attested by the comparison with the curve fitting. The fit allows to measure the convergence rate of 2.132 orders of magnitude per 100 generations. This is an outstanding performance considering the complexity of the problem.

We notice the sudden improvement occurred during the last generations (>300) is artificial. This is due to the choice of the best ever candidate as the offset reference instead of the absolute optimum which is unknown. This does not adversely contradict the above observations about the quality of the optimization process.

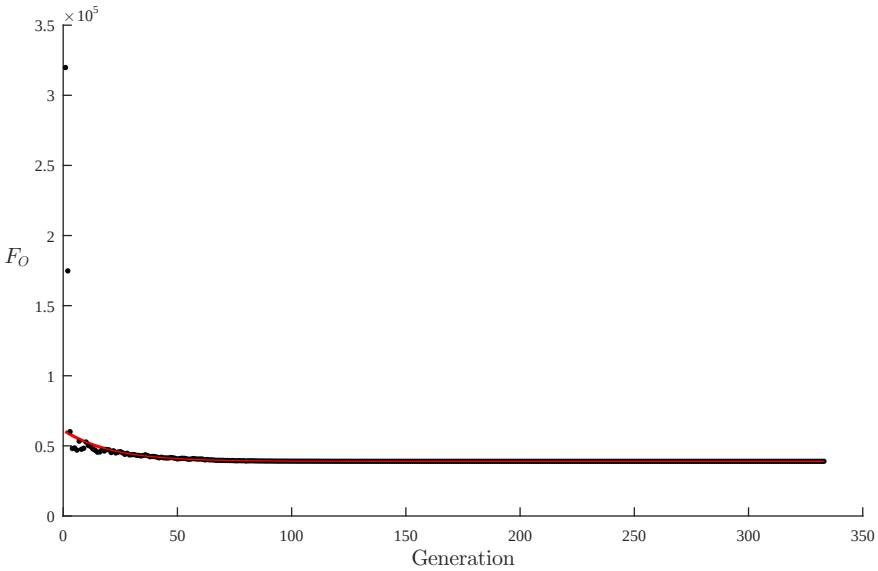


Figure 4.12: Convergence diagram of the objective function.

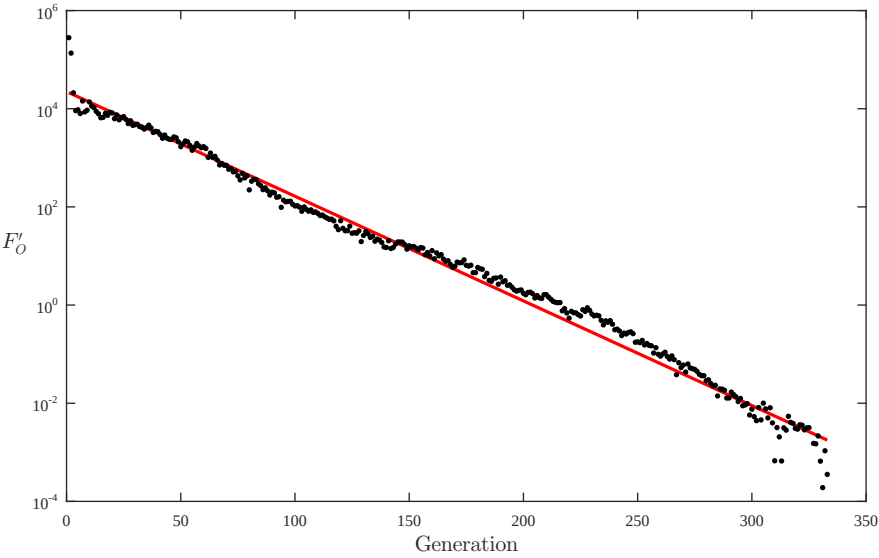


Figure 4.13: Convergence diagram of the offset-free objective function (logarithmic scale).

4.5.4 Criterion analysis

In this section, the impact of the optimization on the dynamic behavior of the vehicle is analyzed separately for each criterion. The color code used in the plots relative to the transient simulation is given in the table 4.29. The mean and maximum, satisfaction and acceptance levels color code is given in the table 4.30. In order to be exhaustive, these results are completed by the data for the steering wheel angle, the tilting angle, the attitude and the speed. These complementary results can be found in the appendix C.1.

Color	Configuration
	REF1
	REF2
	REF3
	REF4
	REF5
	CF1

Table 4.29: Configuration color code.

Color	Symbol	Level
	$\bar{E}_{s\ i}$	Mean criteria satisfaction level
	$\bar{E}_{a\ i}$	Mean criteria acceptance level
	$\hat{E}_{s\ i}$	Max criteria satisfaction level
	$\hat{E}_{a\ i}$	Max criteria acceptance level

Table 4.30: Satisfaction and acceptance level color code.

Under-oversteering coefficient

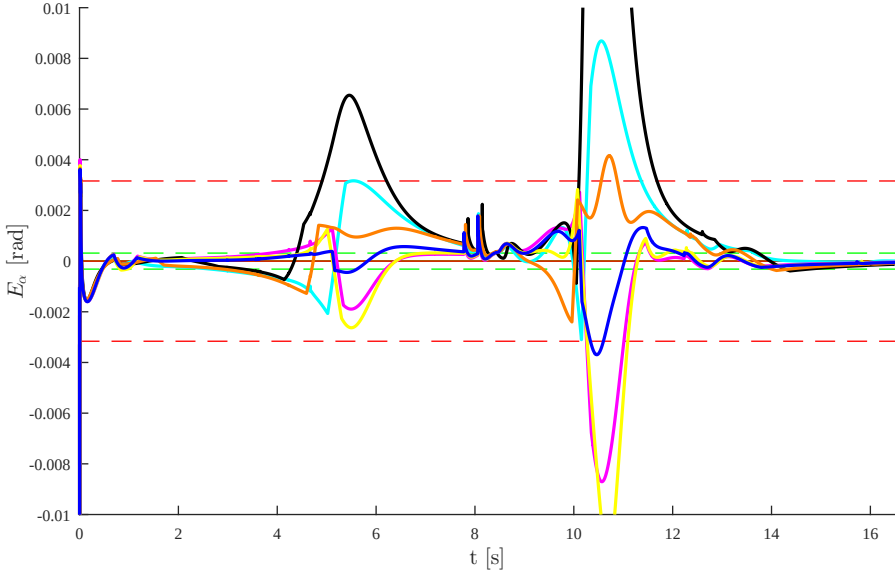


Figure 4.14: Transient simulation analysis: under-oversteering coefficient.

In the transient simulation (figure 4.14), the biggest peaks for the under-oversteering coefficient (equation 4.8) occur in the second turn, as one might expect. The curves have a similar shape but with different magnitude, including sign changes. For this criterion, the CF1 configuration is the best one, which is well better than REF3 and REF4, which show alarming extreme values.

The steady-state analysis results show how the parametrization can deeply change the dynamic behavior of the vehicle (figure 4.15). REF1 and REF3 that are similar, are first neutral, then rather oversteering before being completely and extremely understeering at high lateral acceleration. REF2 and REF4 as well as CF1 mapping exhibit a similar shape but in some very moderate amplitudes. The CF1 is the best candidate because the most neutral and regular over the SS mapping.

The REF5 is very specific. It is quite neutral over a large part of the mapping. At low radius, it is understeering. At high radius and high speed, the behavior switches abruptly from a large oversteering to an extreme understeering. The change that is too quick generates instability in operations. It is symptomatic of a vehicle very difficult to drive because of its insidious behavior. This abrupt changing behavior probably explains the oscillations which occur during the transient simulation only for this configuration.

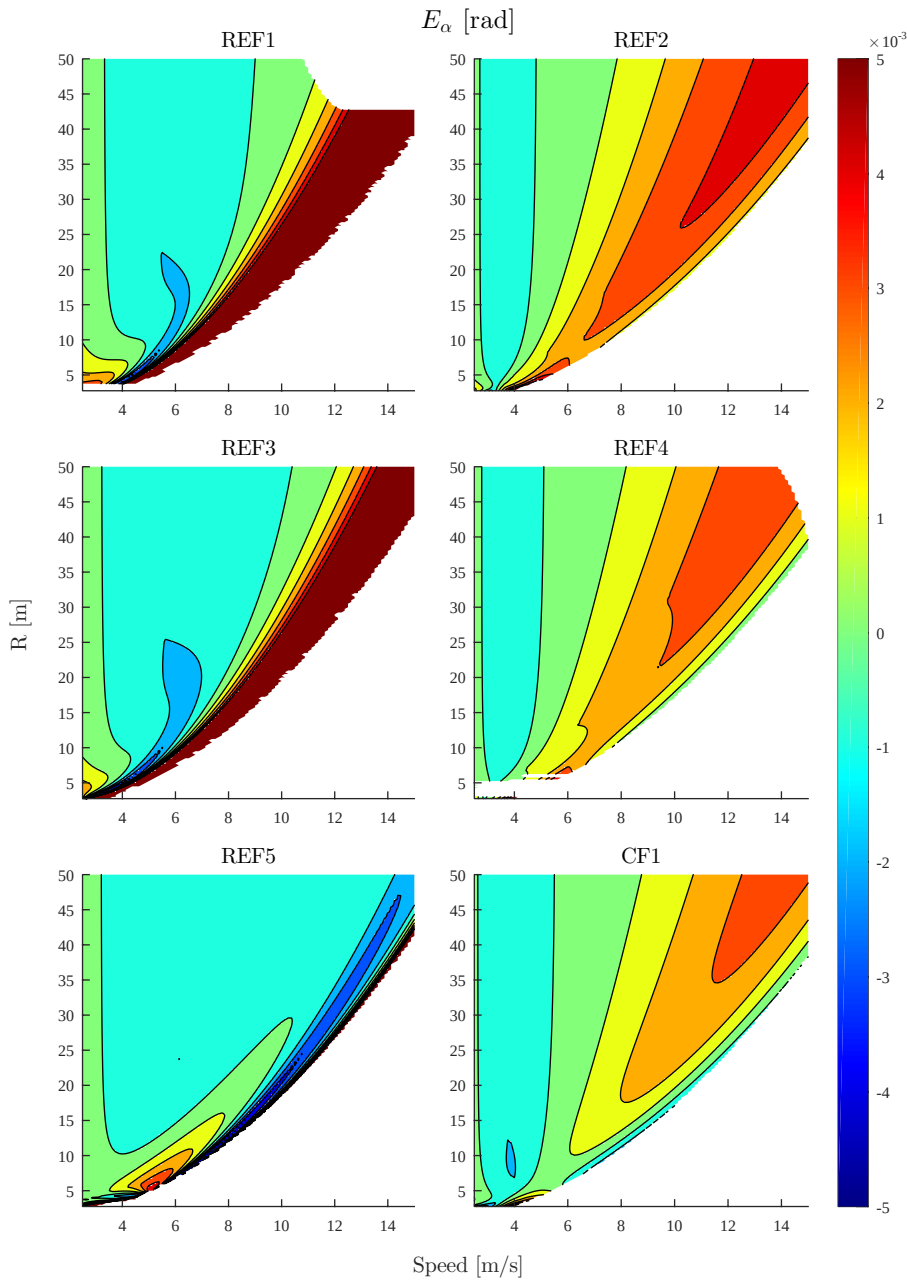


Figure 4.15: Steady-state analysis: under-oversteering coefficient.

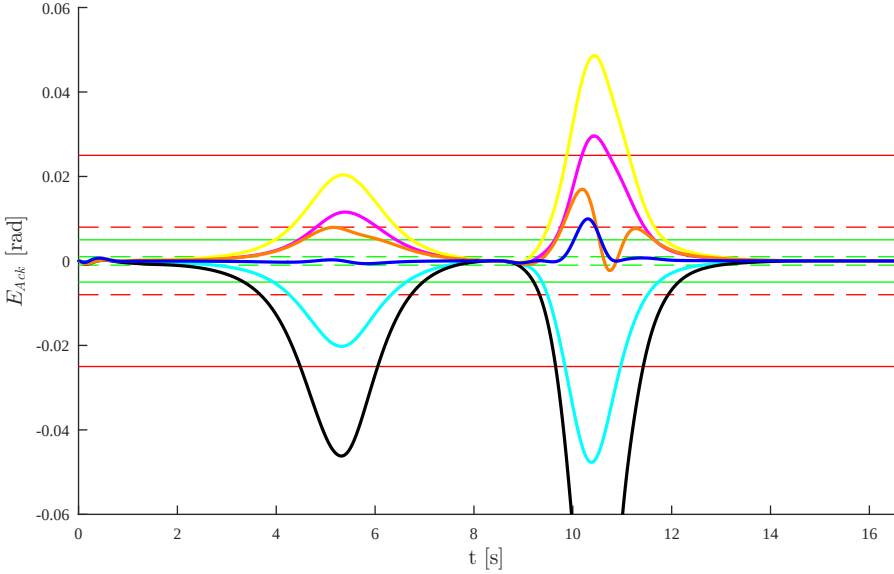
Pseudo-Ackermann criterion

Figure 4.16: Transient simulation: pseudo-Ackermann criterion.

For the pseudo-Ackermann criterion (equation 4.9), the remark can be similar as for the under-oversteering, what is quite logic because they are twins criteria. The performance in the under-oversteering coefficient is conditioned to the performance in this criterion. Due to its definition, the present criterion is more "geometric", or, at least, less "dynamic" than the under-oversteering, what would explain that the curves are more regular (figure 4.16). In any case, CF1 is the best configuration.

The steady-state analysis for the pseudo-Ackermann criterion (figure 4.17) explains also the result obtained for the under-oversteering. REF1 and REF3 are regular but some really bad values occur quickly. REF5 combines special pattern and bad results. CF1 is not especially regular but the criterion remains very low on the entire mapping.

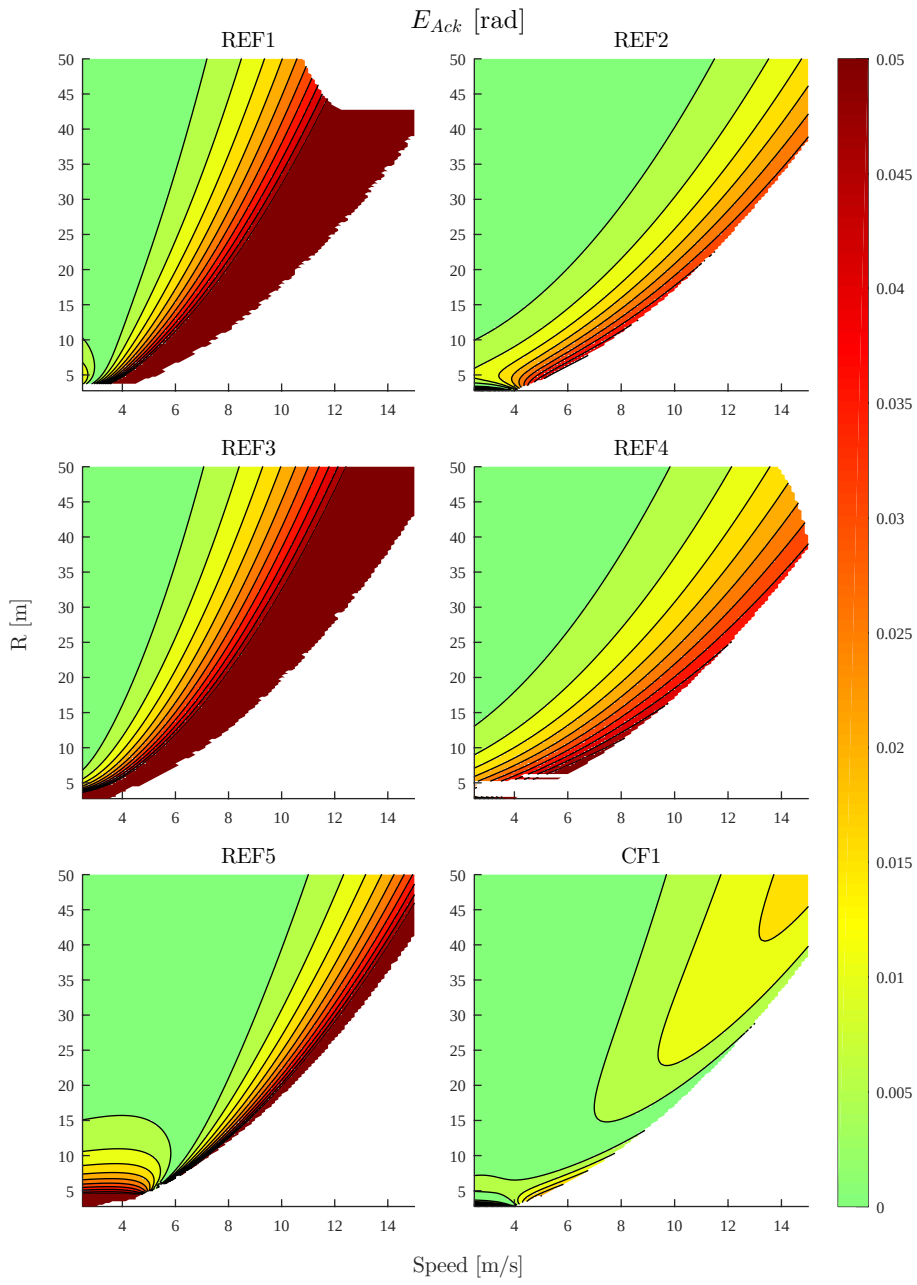


Figure 4.17: Steady-state analysis: pseudo-Ackermann criterion.

Steering torque feed-back

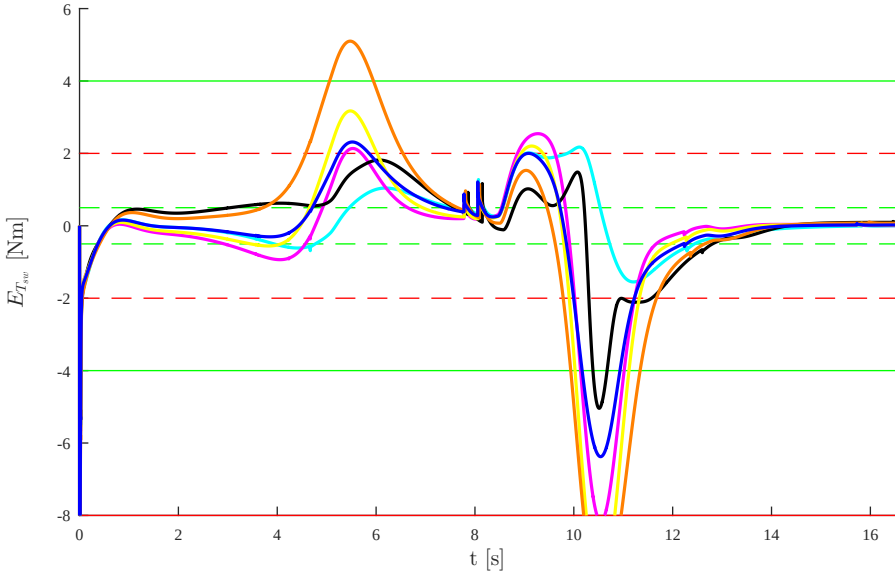


Figure 4.18: Transient simulation: steering torque feed-back.

Regarding the self-increasing behavior of the steering torque feed-back (section 4.4.4), we are more interested in assessing the amplitude and the regularity of the criterion than in its sign itself. By regularity, it must be understood, for the case of the transient simulation, we expect the absence of oscillation and, in a same curve phase, the absence of sign change. For the steady-state analysis, the regularity stands for, the absence of inflection in the characteristic surface.

In transient simulation, the steering torque feed-back (equation 4.10) of the CF1 is not the best one but a kind of trade-off between REF1 and REF3, more reduced, and the others that are less. Obviously, the optimized vehicle is not necessarily the best candidate in each criterion¹. The REF1 and REF3 feed-backs are less regular than for other configurations. So, such setups are not so enviable than the meta-criteria could suggest.

In steady-state analysis, REF1 and REF3 mapping are characterized by a banana shape patch for which the torque feed-back is negative, thus, apparently closer to the targeted handling feeling. Nonetheless, this behavior is not more suitable because irregular. Moreover, at the center of the patch, the torque is high. REF2 and REF4 involve also a negative patch but not at same area of the mapping and not so important in magnitude. Fortunately, CF1 mapping exhibits very regular and progressive behavior. These characteristics could provide the suitable handling. REF5 mapping contains higher values and at high lateral acceleration, the peak of torque is followed by a decrease.

¹In fact, it could be the best in no criterion but simply the best on the trade-off i.e. on the sum of them.

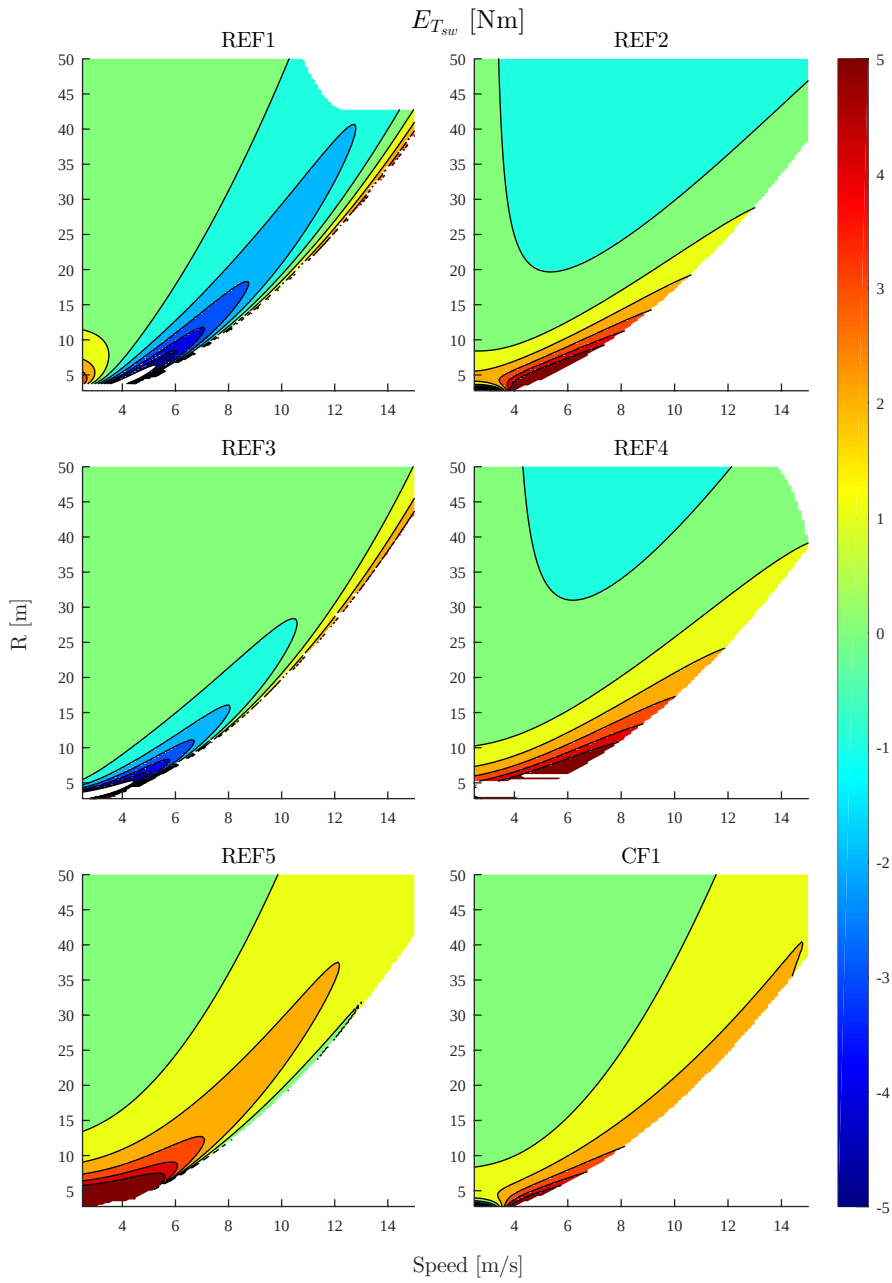


Figure 4.19: Steady-state analysis: steering torque feed-back.

Unballasting of the front axle

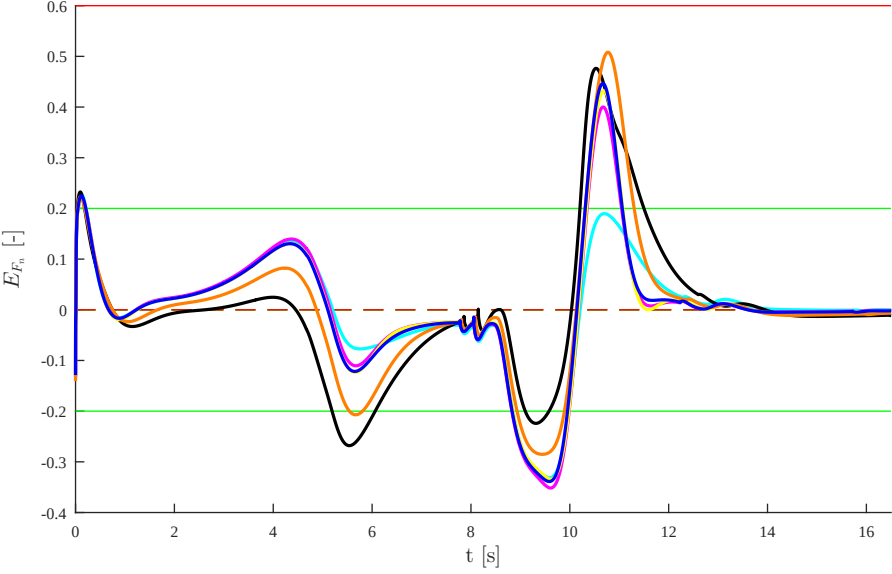


Figure 4.20: Transient simulation: unballasting of the front axle.

For this criterion, the transient simulation is very interesting (figure 4.20). One can remark the vehicles for which the unballasting (equation 4.12) is the lowest in the initiation of the turn are those for which the unballasting is the highest in the turn termination. From this point of view, the CF1 configuration offers a good trade-off. Despite the challenging track, the maximum unballasting is only around 50 %.

In the steady-state analysis, the REF1 and REF3 seem offer the best performances, REF2 and REF4 are the worst, while CF1 is intermediate. Once more, REF5 surprises by its particular profile. The slopes are steeper and a peak is located at very low radius.

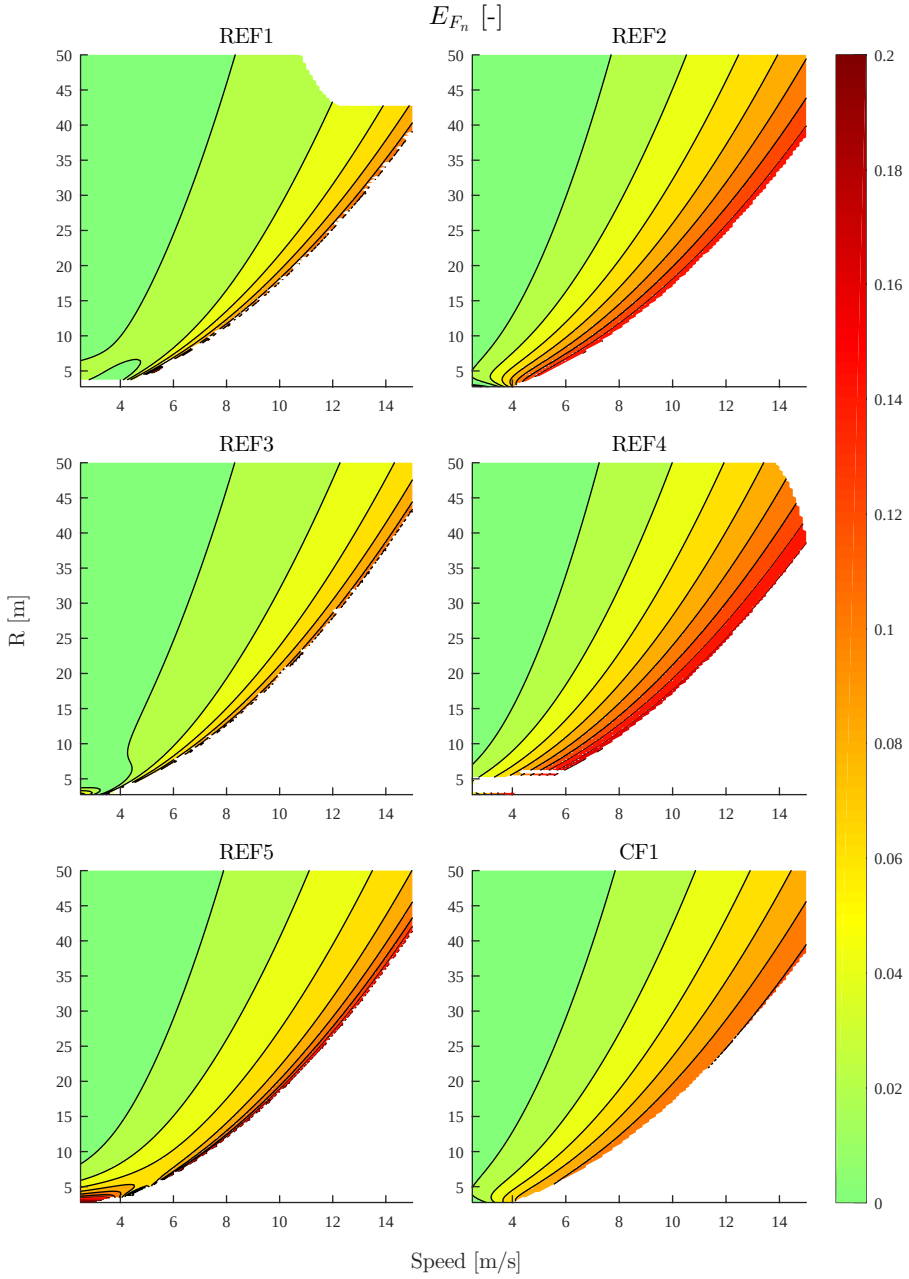


Figure 4.21: Steady-state analysis: unballasting of the front axle.

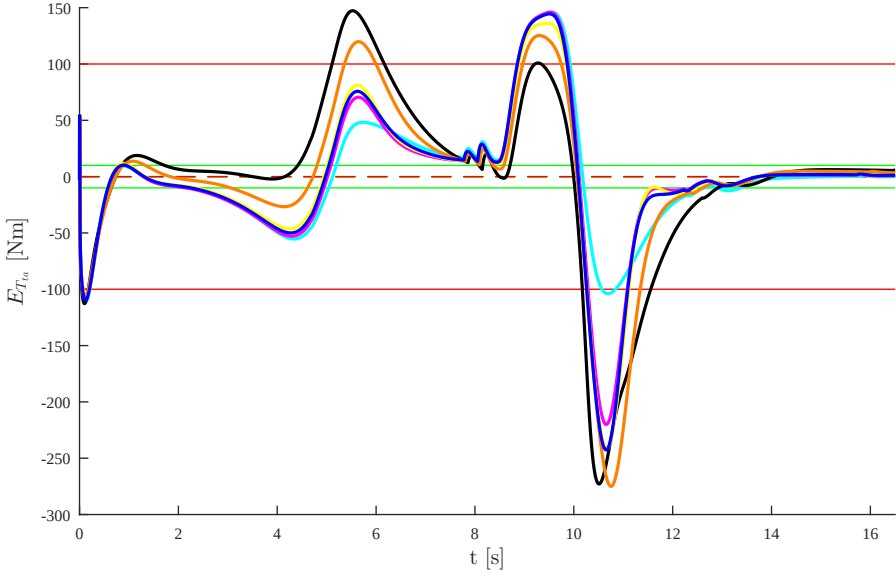
Tilting actuation torque

Figure 4.22: Transient simulation: tilting actuation torque.

Due to the strong link between both criteria, the results for the tilting actuation torque (equation 4.13)(figure 4.22) are alike those of the unballasting. REF1 stands out from the crowd. It is the only vehicle for which the torque is higher in the inclination phase than in the redressment phase. In addition, its reduction of the maximum torque is amazing. It proves that if this criterion should be more critic, a solution could be found, at the risk of degradation of the other criteria.

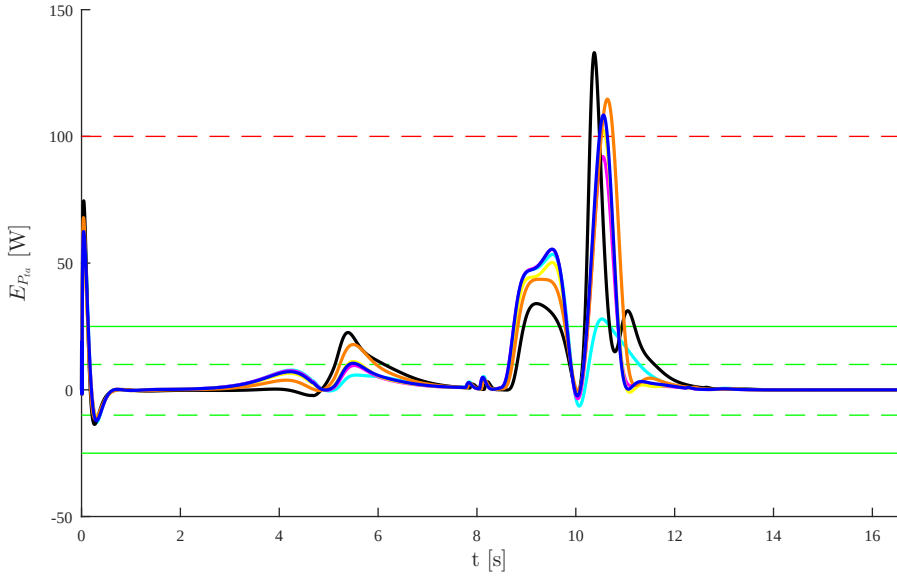
Tilting actuation power

Figure 4.23: Transient simulation: tilting actuation power.

Like for the tilting torque, the maximum tilting power (equation 4.14) is 50 % lower for REF1. For its part, REF3, besides involving the highest peak, is oscillating in the redressment phase, which is detrimental. The CF1 configuration appears as a trade-off offering, however, rather poor performances.

Despite the optimization, the tilting torque hits almost 250 Nm and the power overcomes 100 W. As a reminder, these values respectively correspond to the torque and power measured at the articulation of the T-pendulum on the front body nose. The values have to be inflated according to the efficiency of the electromechanical chain including a high ratio transmission and the electric motor (section 4.6.2). Thus, the consumed power is much higher. In addition, the reference track is challenging but not the most difficult one that the three-wheeler must travel and a reserve torque must always be available to ensure the controllability of the tilting DOF. Following these considerations, the specified maximum torque is fixed to 400 Nm at 1 rad/s to which a power of 400 W corresponds.

Track following error

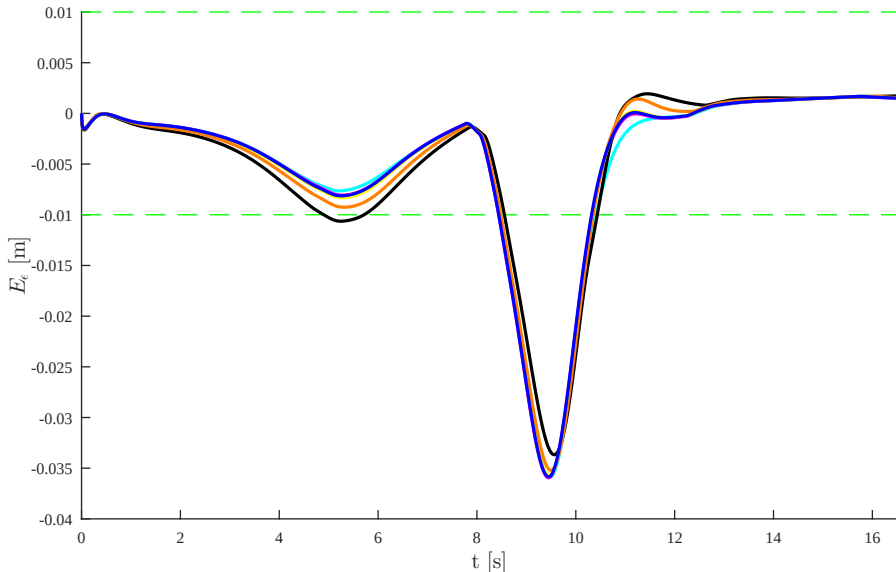


Figure 4.24: Transient simulation: track following error.

The track following error (equation 4.11) has been marginally affected by the optimization. The mission of this criterion, i.e. to maintain a correct track following under the pressure of the optimizer, is reached.

4.5.5 Time performances

The optimization process takes 333 generations and 3 hours 25 minutes and 25 seconds to converge¹. Here, we are interested by the time sharing between the different sub-processes of the optimization scheme. The table 4.31 gives the time performance² for the main elements of the trial: the data conditioner, the transient simulation and the steady-state analysis. The computation time for the other processes is negligible with respect to the 3 main ones.

The data conditioner is relatively fast to run: around 5 % of the full trial process. Given it is able to satisfy all the design constraints, it is relatively efficient.

The transient simulation is the biggest process. Despite a relatively simple model, two thirds of the CPU time are dedicated to its computation. The used integrator, a Runge-Kunta 4 algorithm with fixed step of 2.10^{-4} s is certainly neither the most precise nor the faster but it is robust and regular, two important features when used inside an optimization scheme. On the development

¹The optimization run has been achieved on the Lemaitre3 cluster (table 3.3).

²The laptop performances are measured on the basis of the mean of 5 runs of the trial process. The cluster performances correspond to the actual mean performances during the optimization leading to the CF1 configuration.

laptop, the reference simulation is performed in 14.1 s, what is better than real time simulation. The process is perfectible but pretty efficient, nevertheless.

Thanks to the steady-state analysis, for which a single equilibrium computation takes less than 10^{-2} s of time on the development laptop, the behavior of the vehicle is characterized over its entire range of use in 6.02 s. In the evaluation part of the process, the steady-state analysis, with 159 equilibria spread over the range of use, takes only approximately 43 % of the time needed to perform the simulation over 16.7 seconds of real time and 28 % of the total CPU cost. Because it supplies very instructive and relevant information, one can appreciate its great interest in the optimization process.

Process	Laptop time [s]	Cluster time [s]	Part [%]
Data conditioner	1.04	1.82	4.9
Transient simulation (TS)	14.10	24.66	66.7
Steady-state analysis (SS)	6.02	10.53	28.4
Total	21.16	37.01	100.0

Table 4.31: Computation time of the optimization sub-processes.

4.5.6 Conclusion about optimization

The optimization of the Wemoov leads to an optimized configuration, CF1, which has been compared to 5 references. The process shows its ability to improve the criteria related to the tire work as the under-oversteering and pseudo-Ackermann criteria. A regular steering torque feed-back was also obtained. However, the criteria corresponding to the roll dynamics and the DTC actuating do not find the same success. For them, the result is thinly better than a trade-off between the different references. That said let us recall that the 5 references are not just random candidates but a selection of the 5 best ones from 200 of them. The gains on the tilting actuation torque and power are moderate. This is a bit discouraging especially since the optimization of these particular criteria was the main motivation of the Wemoov optimization.

Globally, however, the optimization allows to find the vehicle with the best performances regarding the specifications, including the dynamics objectives and the design constraints. From this point of view, the proposed schemes are fully satisfactory.

The good performances are all the more significant as the efficiency is high. The large evolution of parameters mainly occurs in the first hour. After two hours, the precision on the parameters is over what can be reached in production. Despite the abusive termination conditions, the optimization finishes after three and half hours. Moreover, this fast and extremely regular convergence requires only the use of 50 CPU cores. Such an optimization process is thus completely affordable inside the general design process.

4.6 Mechanical implementation

Based on the solution of the above optimization process, we have designed a first demonstrator¹ of the three-wheeler, denoted D0. This demonstrator aims to verify dynamics performances of the vehicle in real conditions. The design is centered on the rolling functionalities. So, the superfluous elements are not implemented. Nevertheless, the security components are anyway incorporated for obvious reasons.

Due to the high compactness and the severe dynamic performance requirements, the mechanical design is rather unusual. The underlying techniques have been sometimes borrowed from car or motorbike designs or they are completely specific to the Wemoov application.

In the rest of this section, the mechanical design and the related CAD model are described. Besides giving ideas about the conception, the goal is also to highlight the conflicts between design and dynamics requirements, which are numerous and complex in the case of the three-wheeler. In most of the cases, the result is a trade-off between both points of view.

One must notice that only the final version of the design is presented, the entire progress through this long task including the trial of numerous alternatives and design reversals can not be detailed in such a work, despite their intellectual interest. We limit the descriptive approach to a functional description. The implied specific kinematic analyses and the strength and deformation analyses are not treated. In the same way, the detail design is substantively omitted for conciseness reasons.

Note also that the CAD model is designed with SolidWorks which allows a very accurate and complete representation. Nevertheless, we have chosen to represent the assembly as it should be built, but with the following notable exceptions.

Firstly, some components can be excluded from the model:

- the fasteners,
- the brake pipes,
- the electrical wiring,
- the electrical control elements:
 - the mother board,
 - the switches and other buttons,
 - the IMU,
 - the other sensors,
- the fluid:
 - the lubricants,
 - the brake fluid,
- the ball joint rubber protections.

¹The demonstrator appellation refers to the verification step of a sub-assembly of functionalities, generally, the most difficult to reach and the most innovative. Only the elements needed to realize the targeted tests are implemented in this case. This contrasts with the prototype which is as similar as possible to the series production, although being still potentially produced with some different methods.

Secondly, some details of represented parts are committed or simplified:

- the weld beads,
- the bore and screw threads,
- the gear teeth,
- the tire threads,
- the complex seat shell.

These elements are not represented either because their excessively burden the CAD model, their geometry is too complex to be drawn efficiently, their flexibility can not be represented in the software, or the information is not available at this step. Often, multiple reasons are combined to justified the simplifications or the omission. Despite all this, these component and features must be kept in mind in order to take them into account for the rest of the design.

4.6.1 General description

The final version of the Wemoov D0 demonstrator design is illustrated in the figure 4.25. The main characteristics are picked up in the table 4.32. The comparison with the preliminary design data can be done in reference to the table 4.2. Naturally, small changes have occurred.

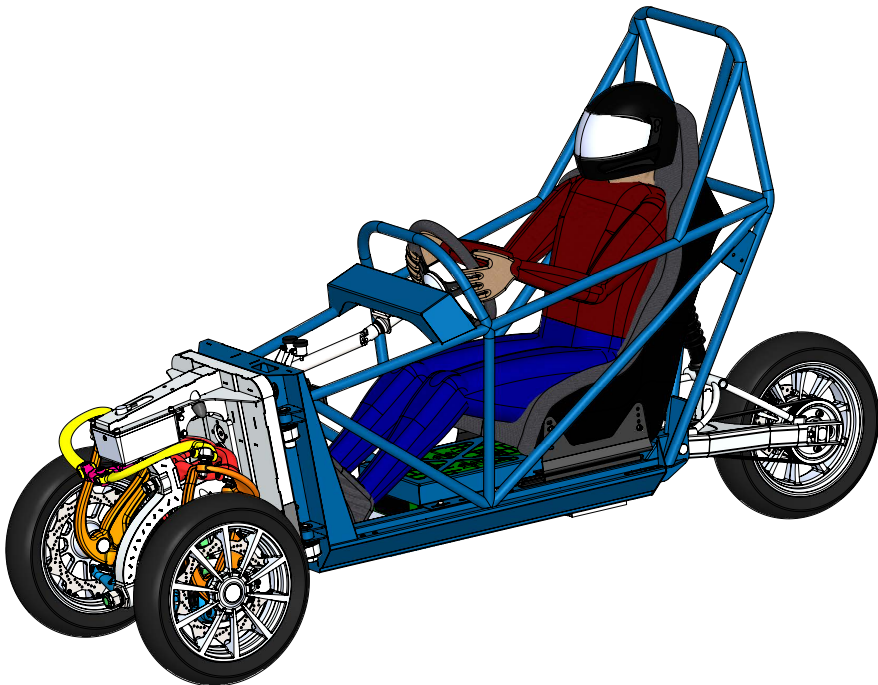


Figure 4.25: Wemoov mechanical design.

Wheelbase	2.11 m
Track	680 mm
Mass	348 kg
Mass distribution	57/43
Power	9.5 hp
Speed	74 km/h

Table 4.32: Final design characteristics.

The three-wheeler is divided into three main assemblies: the front suspension (1), the main body (2), the rear suspension (3). Each of them is subject to a dedicated section.

4.6.2 Front suspension

The front suspension is conform to the morphology developed in the section 4.2. The topology of the modified double wishbone suspension is not so unconventional. However, the geometrical parameters resulting from the optimization and the large targeted motion stroke make the kinematics really challenging to implement.

Narrow track

Wishbone and structure

The optimization, for which the suspension part geometry and their anchor point locations have been considered as parameters, shows that the best setup maximizes the length of wishbones and tends to merge their axes along the median plane of the vehicle.

According to this observation, we have decided to articulate the left and right wishbones on the same shafts. Both shafts, top (4) and bottom (5), are naturally located on the median plan. As a consequence of this, the wishbones are offset impaled on the shaft in the same manner as a hinge. So, the wishbones can not be symmetric what demands more design work and an increased production cost.

The shafts are highly loaded when the vehicle is rolling and in particular in case of unilateral impacts of the wheel. A worst case study has proven that a cantilever mounting of the wishbone shafts can not be envisaged. Due to the suspension kinematics, the structure, that supports the shaft at each end, must be reduced to a simple narrow panel centered on the median plane. The structure has a long protrusion to support the end of the bottom wishbone shaft, what justifies the *trunk* (6) appellation. The panel is formed by two aluminum sheets closed all around and reinforced by internal spacers. The welded assembly targets the best rigidity, despite its complex geometry and large lever. But, such structure does not provide a sufficient mechanical resistance and must be reinforced by two triangulation bars (7) on each side. The location of these

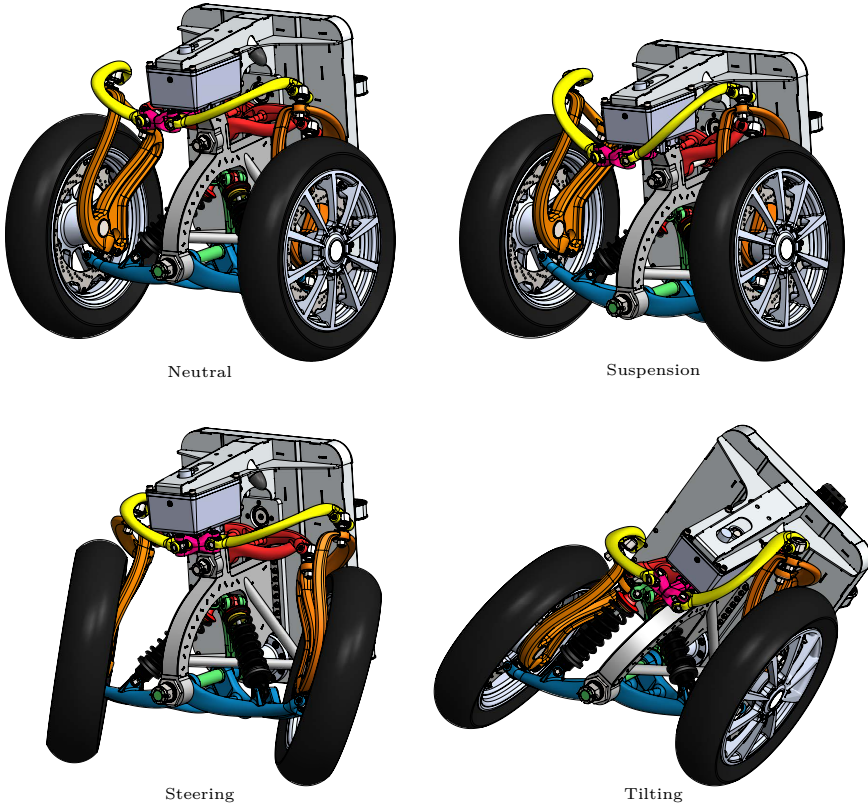


Figure 4.26: Three DOF of the front axle.

bars and the close moving part designs must be carefully analyzed to avoid any interactions with the suspension shocks (8) and the bottom wishbones (9). It partially explains the twisted shape of the bottom wishbones.

The wishbone shafts are positioned by the tilting system housing (10) (see below). On the other end, the shaft interfaces with the trunk through the reveal adjustment systems (11). Such system allows to get the suitable axial clearance for the wishbones, articulated on the shafts through some polymer flanged bushings (12).

Notice the bottom shaft is part of the T-pendulum (13) assembly and so is free to rotate while the up shaft is rigidly fixed. The trunk, including the triangulation reinforcement bars (7), is assembled with the tilting system housing (10) with screws, allowing the mounting of the whole.

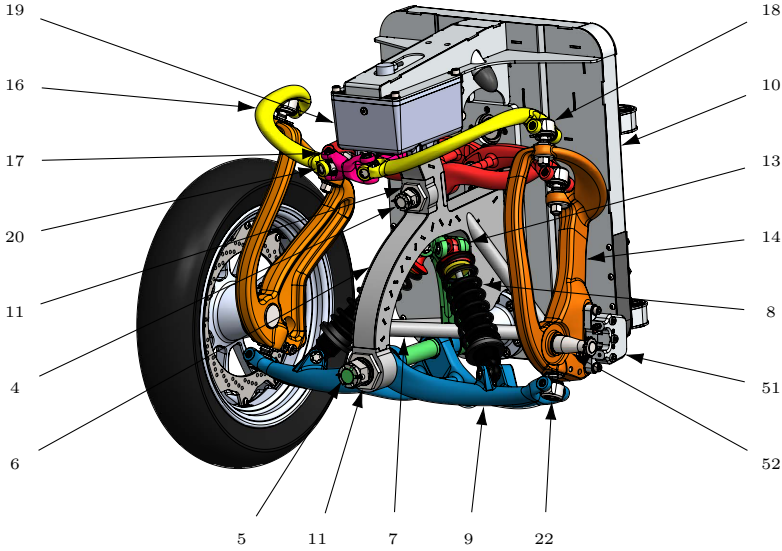


Figure 4.27: Front axle structure.

External steering system

Another consequence of the narrowness of the front axle and of the tilting actuation is that the space area delimited by the wishbones and the carriers (14) are so full that the steering system must be pushed up over the top wishbones. Once more, the compression in term of vehicle width fatally leads to an expansion in another direction. Furthermore, this involves a serious advantage: the inclination of the steering column (15) is least. Consequently, the steering action more regular.

In the same manner as for the wishbones, the optimization method tends to maximize as possible the steering rod (16) lengths promoting the steering box topology instead of the steering rack. This justifies the choice done in the morphological determination step. As a result, the steering fork (17) is as narrow as possible given the requirement of the articulation with the rods. At the other ends, the ball joints (18) between the steering rods and the wheel carriers (14) overhang the wheels.

Due to the tilting DOF, the motions of the steering rods (16) are so large that their shapes must be curved to avoid interactions with the box (19) itself. Two consequences of this particular shape must be noticed. First, the curve must be oriented in space in order to avoid the box. Then, the rotation around the main axis of the steering rods cannot be free as it is the case, in a given range, for traditional car steering rods. To face this requirement, the ball joint is replaced by a universal joint (20) at the inner end of rods. The deleted degree of freedom corresponds to the harmful rod rotation. Secondly, the curved shape of the rod makes difficult or impossible the elongation of its own body in order

to set the toe of front axle. The easiest way to implement the toe setting is to mount the ball joint pin with respect to the wheel carrier (14) via an eccentric mounting (21). A similar arrangement is used for ball joints (22) between carrier and wishbones in order to set the camber, the caster and the caster angle (combined settings).

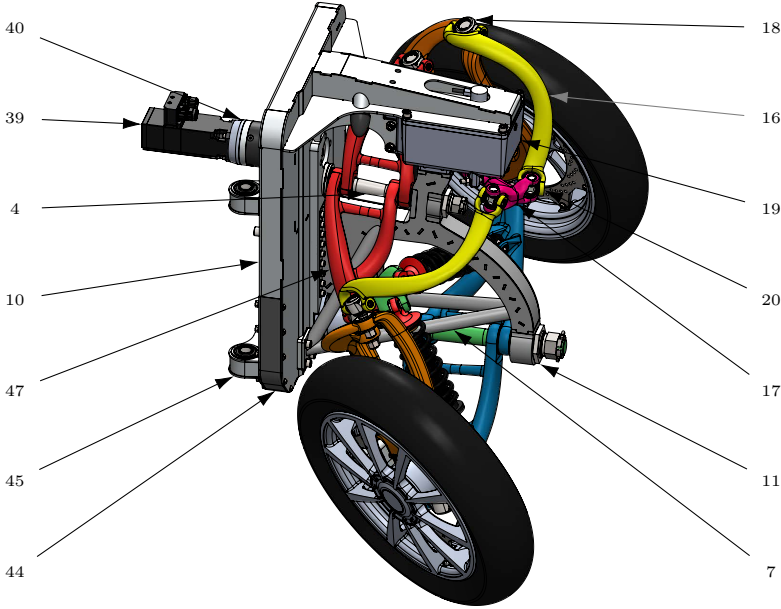


Figure 4.28: Front axle for combined steering and tilting: steering rod.

Steering box

The steering box (19) aims to transform the large motion of the steering wheel (23) to the reduced steering fork (17) amplitude, while the torque is increased. A 90° direction change also occurs in this way. The ratio is fixed to 5:1, such that the range of wheel rotation is $5/8$ of turns in each direction corresponds to a bilateral range of 45° at the steering fork. The two successive stages of gears that realize the transmission are enclosed in the box (figure 4.29).

The first stage (24) is made of a pair of spiro-bevel gear with ratio 2:1. The second stage (25) is a simple pair of parallel helicoidal gear for which the ratio is 2.5:1. Each shaft is mounted on a pair of sintered bronze flanged bushings (26). The mounting is allowed by two removable flange plates (27) and the main box cover (28) which respectively lock the input (29), output (30) and intermediate shafts (31). The lubrication of the whole is realized with grease.

The steering fork is directly fixed on the output shaft by clamping and double parallel key. The adjustment of the two end-of-stroke screws (32) allow to limit the steering motion. A precise sensor (33) is mounted on the output

shaft end to measure the steering fork angle, μ , the main state of the tilting torque controller.

It is possible to lock the steering DOF in its neutral position with a dedicated pin (34) inserting through the steering fork and the box. This is useful to perform the different settings of the front axle e.g. the toe.

The box (19) is fixed on a support welded on the tilting system housing (10). This excrescence is reinforced in each direction to face the huge counter effort applied on it. The box is also down linked to the trunk (6) to reinforce the whole.

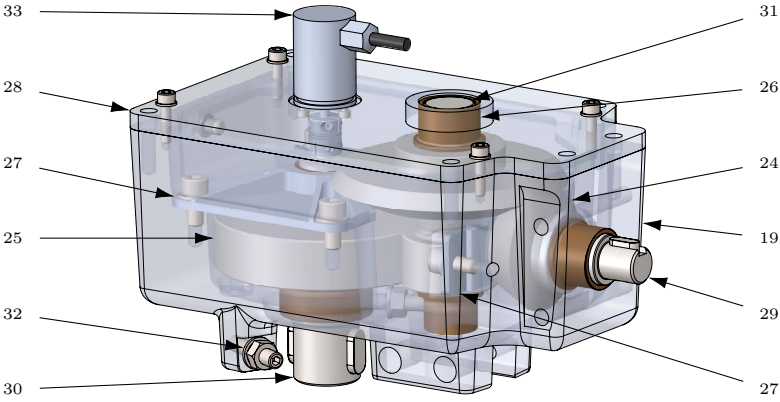


Figure 4.29: Steering box.

Tilting actuation

Tilting shaft assembly

The tilting actuation is performed thanks to the T-pendulum (13) to which the top connections of the shocks (8) are linked and which is articulated on the same shaft as the bottom wishbones (9). The design of this assembly is relatively complex due to the multitude of handled functionalities. The tilting shaft (5) is locked both radially and axially between the body nose chassis (35) and a tulip hub (36) which underpins the shaft, as close as possible to the T-pendulum. The bottom wishbones are stuck between the T-pendulum and the axial reveal adjustment system (11) located at the end of the trunk. Notice, the wishbones rotate on the tilting shaft thanks to polymer bushings because this side is lubricant free. In contrast, the interfaces with the chassis and the hub are done through some sintered bronze bushings (37) because working with the same lubricant as that of the tilting transmission: special gear grease. Both the T-pendulum and the gear sector (38) are locked in rotation with respect to the shaft by involute splines.

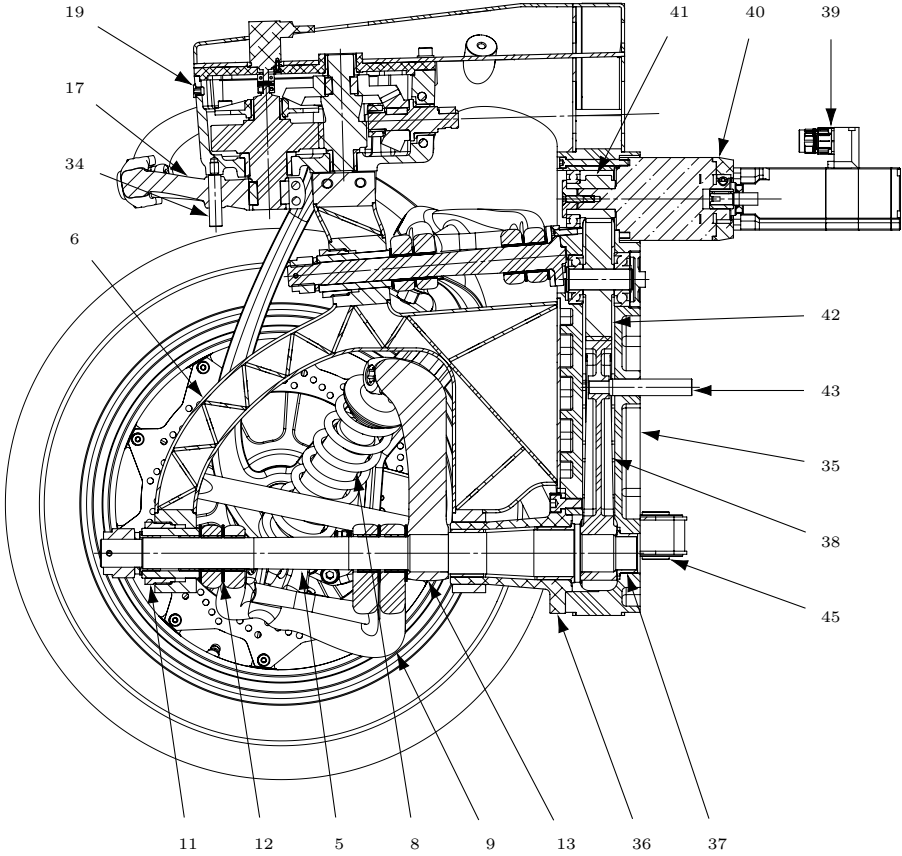


Figure 4.30: Front axle median cut.

Tilting actuation and transmission

The tilting actuator is a brushless electric motor (39) with a maximum continuous torque of 1.4 Nm. It is mounted in cantilever along the two stage planetary gear box (40) through a custom spacer. The transmission ratio of this device is 40:1. The gear box is flanged to the body nose chassis (35). The sprocket (41) is directly fitted on the output shaft of the gear box which is too weak to take up the transverse efforts due to gear teeth contacts. So, the extremity of the sprocket fit, as a shaft, in a ball bearing which float axially in its bore. This output mounting is unconventional and could be improved with more adapted components i.e. a customized planetary gear box, which is too expensive for a demonstrator.

The driving sprocket is not directly geared to the sector. The transmission is realized through an intermediate pinion (42) in order to pushed up the motor and the gear box which, otherwise, would be harmful for the pedal placement.

This pinion is free to rotate with respect to the body nose chassis thanks to a pair of angular contact ball bearings mounted in X. The system made of the sprocket, the intermediate pinion and the gear sector has a ratio 10:1 such that the total tilting system ratio is 400:1 while being reversible as required.

Tilting system housing

Due to the needed gearing precision, the three stages of transmission fit in the same monobloc part: the body nose chassis (35). This last supports also the top wishbone shaft (4) and the trunk reinforcement. As for the steering fork, the tilting gear sector can be locked by a specific timing pin (43) in order to perform the settings.

On both sides of the body nose chassis, the gear sector is protected from external contamination by a complete fairing. Each side, polymer spacers (44) close the openings provided for the introduction of the sector in the mounting task. The housing supports the steering box (19) through the dedicated ex-crescence and four elastomer bushes (45) which link the front axle (1) to the body (2). Two pockets are deepened in the fairing to enable, once more, the suitable pedal placement. They increase the rigidity of the whole at the same time.

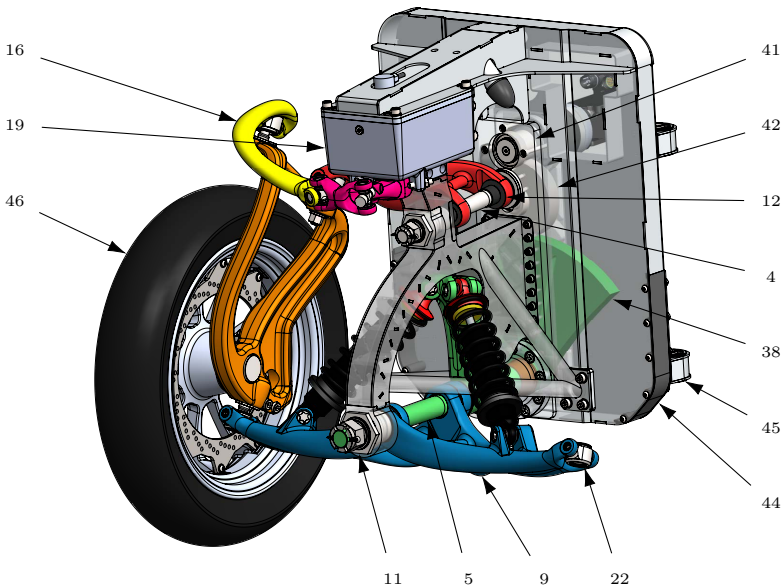


Figure 4.31: Tilting system housing.

Combination of tilting and steering motion

The major specificity of this vehicle is its tilting motion. As for a bike or a motorbike, the tilt angle is directly linked to the lateral acceleration and consequently to the curve radius and to the speed. In fact, the maximum tilt angle must be as large as possible in order to increase the dynamic capabilities of the vehicle. Nevertheless, the tilting angle involves large displacement of the front axle wheels (46). As already introduced in the morphological study, the maximum tilting angle is limited, rather by the mechanical implementation than by the morphology itself. So, the maximum tilt angle must be determined as a trade-off between the limits of dynamic capabilities and the mechanical complexity of the front suspensions. In this respect, we target to reach a maximum lateral acceleration 0.7 m/s^2 . In this way, the targeted maximum tilt angle is fixed to 35° . Due to urban targeted use, the vehicle must provide an agile handling, especially for maneuvers, for which the turn radius must be as low as possible. To reach a turn radius of 2.8 m, which is in the common range for this kind of vehicle, the targeted maximum steering angle of the front wheels must be pushed to 40° .

The combined tilting and steering angles for such amplitudes are outright not compatible with standard ball joints used as connections between the wishbones and the carriers, and at the end of the steering rods. Standard ball joint are built so that the main rotation is unlimited while the maximum amplitude in the two auxiliary axes extends on a range from $2 \times 10^\circ$ to $2 \times 30^\circ$ depending on the mechanical implementation. The upper limit of this range corresponding to weakest models, that are unsuitable for vehicle applications. As done in some off-road prototypes in order to increase the wheel travels or in drifting race cars to extend the steering angle, we need specific ball joints allowing large amplitude of motion in, at least, two directions.

The practical solution consists in a combination of a traditional ball joint with an additional pivot perpendicular to the main axis of the ball joint. Obviously, the arrangement proposes only 3 true degrees of freedom, the additional one is redundant but replaces one auxiliary ball joint degree of freedom by extension of its amplitude.

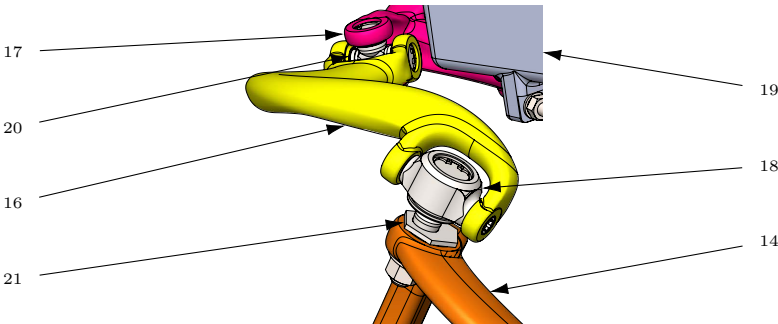


Figure 4.32: Specific ball joint.

This mechanical implementation is more complex than a standard ball joint but allows to reach the targeted steering and tilting amplitudes. However, its cumbersome design involves other drawbacks explained below.

The same customized ball joints (18) are used for bottom (9) and top wishbones (47) and for the steering rod (16) outside extremities. The forks are systematically implemented on the wishbones or the rods instead of the carrier (14) side. This facilitates the design and improves the motion strokes. The ball pivots fit to the carriers through an eccentric mounting (21) providing suspension toe, camber and caster settings as previously discussed. For the four top ball joints, the eccentric settings are locked by irreversible conical taper fittings where the ball pivot screw are tighten. For the connection between the bottom wishbone and the carrier, such tightening is not possible due to the proximity with the wheel axis. So, the setting is locked by clamping on the cylindrical pin. The orientation of the main ball pivot axis and the additional pivot axis are not defined by the dynamics optimization. In fact, they are simply not included in the MBS model. The orientations are set to optimize the steering and tilting strokes facing mechanical interactions.

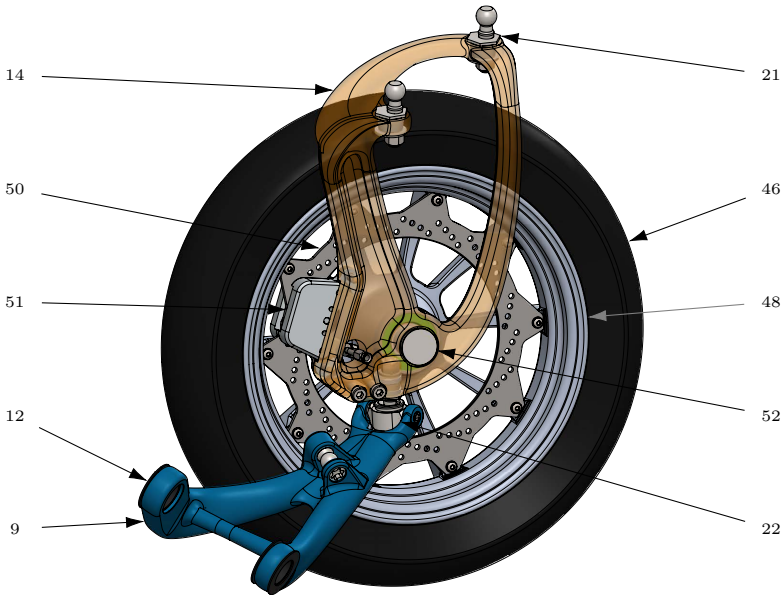


Figure 4.33: Bottom suspension ball joint and peripheral brake.

The complex geometry of the suspension components aims to avoid the mechanical interactions in the entire range of tilting and steering strokes. The bottom wishbones (9) avoid the trunk reinforcement bars (7) and the inside face of the wheel rims (48). The top wishbones (47) avoid the trunk panel, the steering box (19) and the T-pendulum (13). The carriers (14), in addition to mold as closely as possible to the wheel, are shaped like a harp to envelop the shocks

without any contact. The steering rods (16) circumvent the box. Moreover, these parts avoid each other. Note that the resulting complex geometry combined with the large mechanical loadings encountered oblige to produce these parts from blanks obtained by steel foundry, possibly combined with precision forging. The production techniques are not affordable for a demonstrator: this is the reason why the metal additive manufacturing techniques were chosen for the demonstrator.

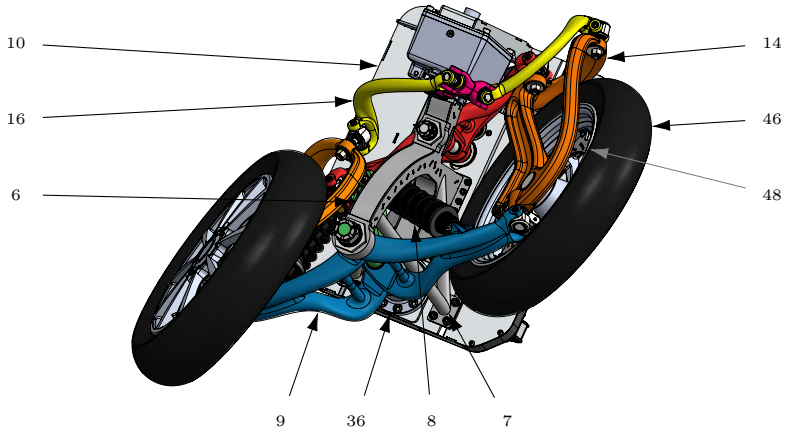


Figure 4.34: Front axle for combined steering and tilting: wishbone.

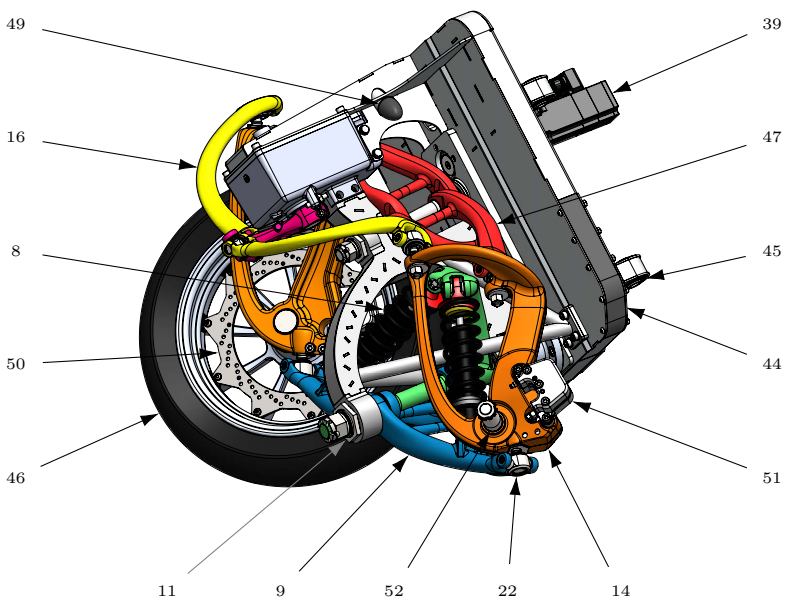


Figure 4.35: Front axle for combined steering and tilting: shock.

The extreme motion due to the tilting DOF leads also to a very specific placement for the shocks (8). The top connections are as close as possible. The top of main axes are unusually inclined inward and rearward. The shocks are connected with specific spherical bearings at their two ends.

In order to avoid any contact between the shocks and the carrier at high tilt, a pair of elastomer bump stop (49) limit the tilting stroke by contact with the top wishbones (47).

Location of the bottom ball joint of the wheel carrier

Another troublesome result for the optimization process refers to the location of the ball joint (22) between the bottom wishbone (9) and the wheel carrier (14) which is very close to the wheel axis, as close as possible in fact. Also, it must be deeply embedded as much possible inside the rim (48). The point location is all the more critical because, in the three-wheeler, no steering assistance is present to correct an inadequate steering torque that could be caused by a wrong caster and a wrong scrub radius. This problem is again reinforced by the cumbersome design of a specific ball joint described above.

A first problem is the interaction with the front suspension brake system. The figure 4.36 (right) illustrates the traditional arrangement for wheel, spindle and brake system. The optimized position for the bottom wishbone ball joint is located and its clutter zone is also represented by a dotted circle. The unwelcome interaction between the ball joint and the brake disc (50) is well visible. This is counteracted by an uncommon mechanical arrangement for the front suspension brake system: a peripheral brake is used instead of a traditional one (figure 4.36 left) to free up space around the optimal ball joint position.

There are several drawbacks due to the peripheral brake. The major one appears in case of disassembling the wheel. It imposes to disassemble the brake caliper (51) from the carrier (14), to allow disengaging the wheel out of the spindle (52). On the contrary, because the wheel hub does not support the brake disc anymore, it can be "melted" with the rim. It compensates the weight gain due to rim reinforcement and the related increase of the unsprung masses with the well-known repercussions on the dynamic behavior. Such reinforcement is required to ensure in-load good positioning of the disc (50) with respect to the brake caliper despite high efforts from tire-ground contact.

The front rims (48) are very specific. The bead tire profile is that of a motorbike type tire but, on the contrary, the rim flange are lateral, as for a car, and not central as for a motorbike. At the center, the rim is assembled on the spindle by a pair of angular contact bearing mounted following the O configuration. As usual, a seal protects them against external pollution. This direct mounting, i.e. hubless, refers rather to a motorbike type mounting. The rim is thus so specific that a dedicated design has been required, despite the serious difficulties to manufacture demonstrator of such parts. In the same manner, the brake discs which must fit perfectly with the rims have been specifically designed. Fortunately, it has been possible to adapt some existing brake calipers (51). This selected model was developed for the Brudelli

Leanster [107], another tilting reverse three-wheeler endowed with a peripheral brake system. Some light modifications have been applied to allow a tightening through the rim and the use of a brake disc for which the inner diameter is a bit smaller.

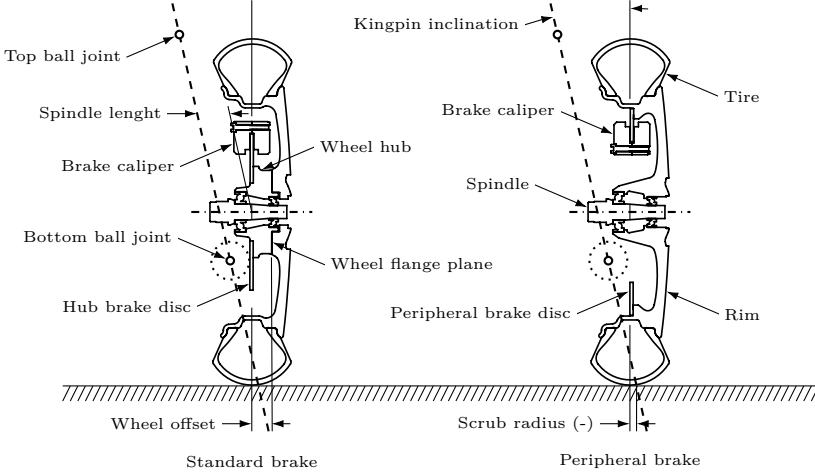


Figure 4.36: Front brake: alternative designs.

Another consequence of the ball joint location is related to the motorization. The wheel hub size is limited, leading to difficulties or the impossibility to insert an electric actuator to form an in-wheel-motor. Anyway, this technical option was previously rejected for its negative influence on unsprung masses. This consideration definitively closes the question about the front wheel motorization; a transmission via a universal joint, or other similar systems, having been excluded from the morphological determination step because totally incompatible with the front suspension kinematics.

Note that the design difficulties encountered in respecting the best position of the bottom ball joint are common in vehicle suspension design. In the double wishbone suspension, the bottom ball joint is often very close to the brake disc. It is very easy to observe for a quad-bike. The particularity of the Wemoov design is the determination to implement the result of the optimization process giving, as expected, the best dynamics performances.

4.6.3 Main body

The main body (2) has three principal functions:

- to support the front (1) and the rear axles (3),
- to host the driver and its control interfaces,
- to support the motorization (53).

The version of the design that is presented here is particularly dedicated to the D0 demonstrator and fundamentally different from a series model. The D0 version is essentially realized on the basis of steel sheets and tubes whereas the serie one will be made of some composite material panels fixed on a light aluminum structure. This D0 design is not equipped with elements like the fairing, the lightning, the signalization, the windscreen and the related wiper blade system, the doors or other entrance mean, the cabin interior garnishments, the instrument panel, the optional multimedia system and other multiple devices which will equip the series vehicles. The D0 body is very strong and heavy despite the simplifications. In fact, the structure has been largely reinforced, on the one hand, to increase the security during the test phase in particular the setting of the tilting controller and on the other hand, to overburden the body in order the get equivalent inertia properties between the series and the demonstrator versions.

A main concern in the design of the cabin is ensuring a correct driving position. Contrary to what could be expected, the driver wellness is very sensitive with respect to the position and the orientation of the interfaces: heel plate, thrust and brake pedals, steering wheel and seat. In order to get a position suitable for a large range of drivers, we have tested several setups via the Human In The Loop (HITL) simulator (appendix D). Indeed, the modular structure, on which the driving interfaces are fixed, allows to envisage each possible adjustment to reach the satisfying position.

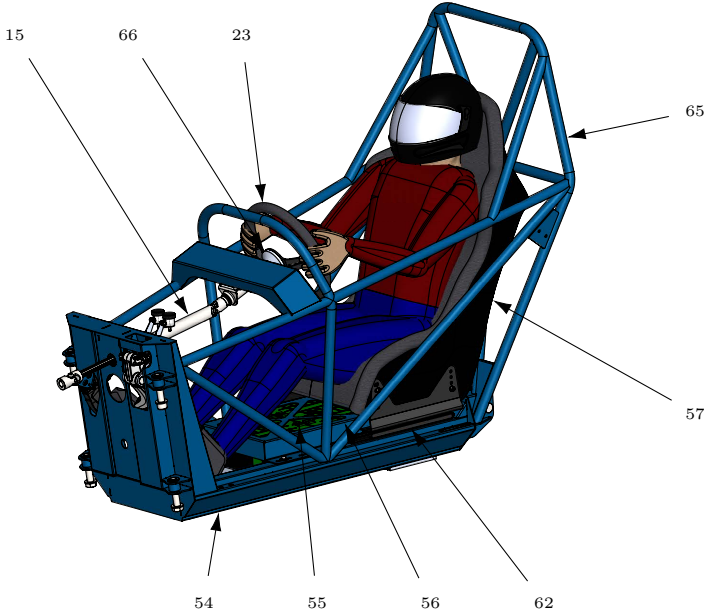


Figure 4.37: Main body.

Body structure

The body is a welded assembly made of steel sheets and tubes. The structure is based on two longerons (54) made of folded sheets and linked together through a flat bottom. This forms a receptacle for the battery pack (55) which is maintained in place with a removable closing frame (56). The battery pack is thus partially located under the seat (57), as low as possible. On each side from the batteries, the structure is reinforced, forming two closed boxes that hold electronics. The main controller, the tilting actuator driver and the Battery Management System (BMS) (58) are stored in the front box. The top of the box coincides with the foot support. The propulsion motor driver (59) is isolated in the rear box. This strategic position between batteries and the motor (60) shortens the high power electric cables. The cooling of this device, which is a very important feature, is realized thanks to a high capacity radiator (61) glued on the bottom face of the body. The racing seat is fixed to the main longerons through a pair of slide runners (62) allowing a leg length setting. The four point harness is not represented on the CAD model. The external bottom edge of longerons are bevelled to allow tilting.

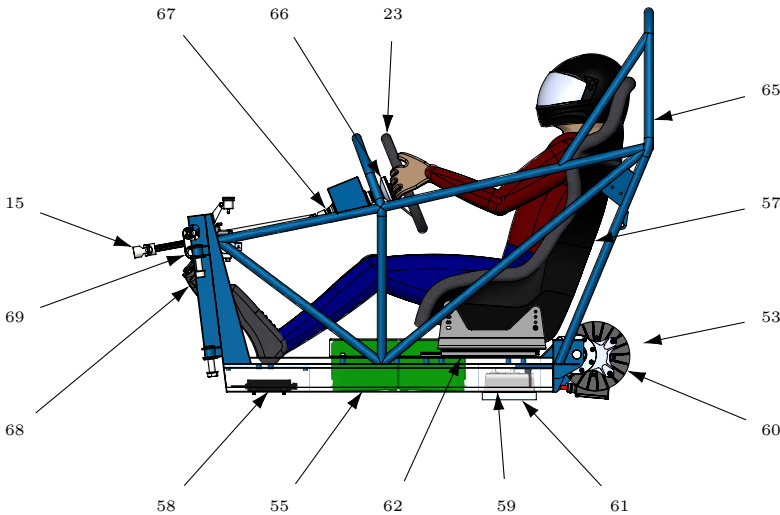


Figure 4.38: Battery pack location.

At the front side, a complex assembly of folded sheets forms a bulkhead (63) supporting the front suspension (1) through the four elastomer bushes (45). The design aims to get a stiffness as high as possible and a large strength in case of crash. In particular, the bulkhead, like a shield, prevents the penetration of suspension parts in the cabin which, could cause dramatic injuries. From this point of view, the tilting motor (39) and the associated gear box constitute a security weakness. They penetrate the cabin through a hole in the bulkhead.

The hole is over-sized to take the relative motion of the front axle into account. This arrangement is not optimal, but unfortunately, it is the only possible one for such a configuration. The bulkhead supports also the pedals for which the mechanisms are partially ahead of the bulkhead main sheet. As mentioned before, some pockets are created in the tilting system housing for that purpose. The front axle and the main body intertwine of such manner to reduce as much as possible the wheelbase of the vehicle. The way to reach this objective must be revised for the series model design.

At the rear side, a panel closes the longeron assembly. The motorization (53) and the rear swingarm (64) articulate with this panel through several supports and reinforcements.

All the media between the front and rear axle connections are built with sheet assemblies what is more easy to position the elements with the required precision. The L chassis is topped by a tube-based roll bar frame (65). Two arcades, one just ahead the steering wheel and one just back the seat, are completed by longitudinal stringers, including the link to the front bulkhead. Several struts triangulate the structure to stiffen as much as possible the whole, making the cell safe even in case of rollover. The rear shock top mounting is connected to the frame.

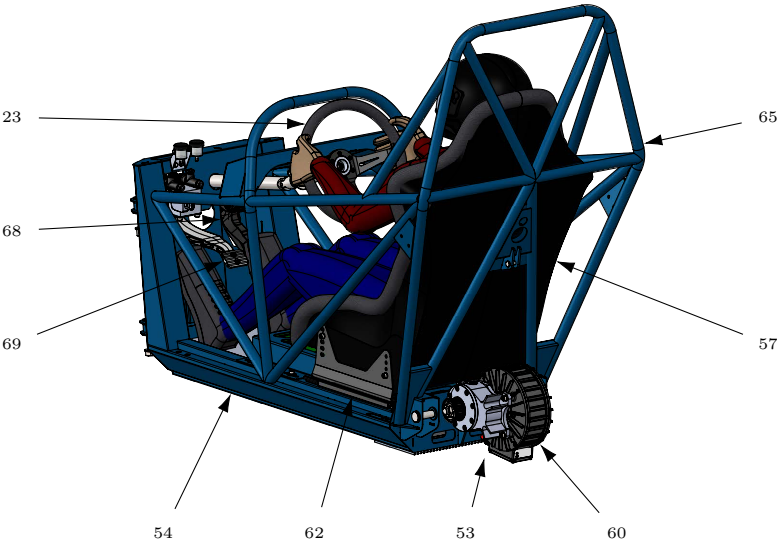


Figure 4.39: Rear axle interface.

Notice the structure of the body was explained before the elements integrated on the main body. However, from the point of view of the design method, such elements are placed in the space after which the body structure is post-designed aiming to support them as determined.

Steering

The steering wheel (23), mounted on the hub (66), is tightened to a short shaft (67). This last is guided by the steering sleeve mounted on a crossbeam of the chassis through a spacer. The thickness of the spacer enables a steering height setting while the slotted mounting holes provide the depth adjustment, in this demonstrator version. A telescopic double universal joint (15) connects the steering wheel shaft and the steering box (19) input shaft. Some holes allow to cross the body bulkhead and the tilting system housing (10). This system avoids the risk of kickback of the steering wheel in case of possible front crash.

Pedals

The thrust pedal (68) is based on a Hall effect sensor and communicates its angular position to the main controller. The whole set is sold by a car Original Equipment Manufacturer (OEM) supplier.

Contrariwise, the brake pedal (69) is completely designed in house. A S-shaped pedal enables right foot braking¹ despite the mechanism is left-sided because it is the only available space.

The optimal brake distribution ratio has been computed in some preliminary analyses [32] but must be refined experimentally, the imprecisions of the MBS model being incompatible with this sensitive adjustment. This unusual setting is obtained by an arrangement coming from racing cars. The pedal is in rocker arm configuration and acts on the pair of hydraulic master cylinders (70) through an adjustable spreader bar (71). The adjustment is obtained by rotation of the bar leading to the displacement of the ball joint connection with the pedal bore thanks to left and right hand threaded connections between the bar and the trunnions (72). Indeed, this operation change the lever ratio and, finally, the distribution of braking between front and rear hydraulic circuits. Finally, the forked rods (73) transmit the motion from the trunnions to the master cylinders².

A rotational hall effect sensor (74) measures the pedal position thanks to an arm (75) and a precision rod (76) connecting the pedal, forming a four-bar mechanism. The neutral position of the pedal and brake clearance are adjustable. This generated signal is important for the implementation of the envisaged regenerative braking feature.

Obviously, a kinematic study has been realized to ensure the strokes of all motions are coherent.

¹This particular specification came from the ordering consortium. From our point of view, it is more suitable to brake left with a straight pedal. This control is less intuitive for car drivers but much more efficient regarding the response time.

²The so-implemented system acts in "push" configuration. Kinematically, this arrangement suffers a parasitic unstable DOF: the cant of the spreader bar. The problem is roughly solved by low clearance between forks, washers and pedal flanges. Opting for a "pull" configuration for which the kinematics is reversed, makes the system unwelcome DOF stable, but is much more complex and expensive.

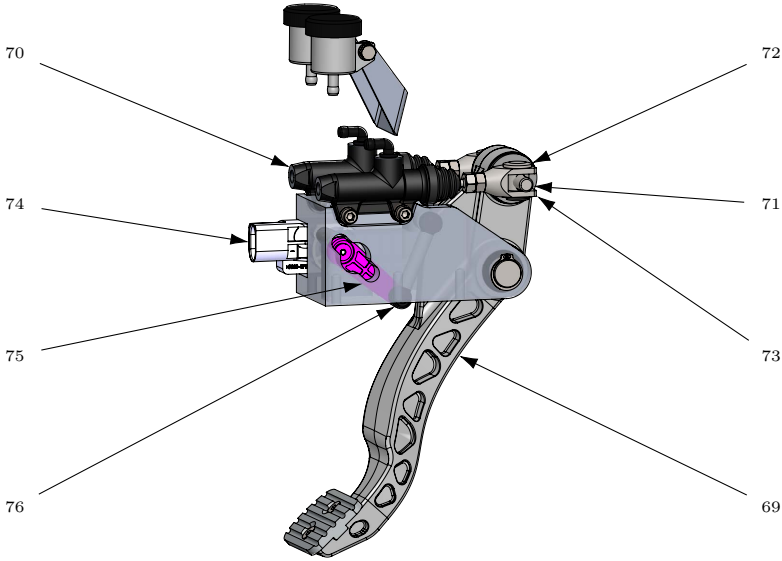


Figure 4.40: Brake pedal assembly.

Motorization group

Unfortunately, it was not possible to find an electric motor matching with the ideal specifications of the Wemoov: a power around 10 hp and a maximum operational speed which fit the suitable speed resulting from the rolling maximum speed, the wheel diameter and the possible transmission belt ratio range. It must be noted that we have targeted a high power density motor but only the air-cooled models in order to avoid the implementation of a complex and expensive water cooling system. The CE marking imperative reinforces again the difficulty to find a motor produced "for stock". However, a specific design motor which could be ideal was judged inaccessible for our demonstrator.

The problem has found a solution by the addition of a geared reductor (77) mounted on the motor output shaft. It enables to exploit the motor in its optimal range of speed. The selected propulsion motor (60) is a synchronized brushless electric motor which develops 7 kW (9.5 hp) at 4500 rpm. The speed of 74 km/h is obtained with a total transmission ratio of 6:1 divided in 2 stages: the geared reductor and the belt-pulley system exhibit respectively a ratio 2:1 and 3:1. This setup is sufficient to provide an acceleration of about 2 m/s^2 which is comparable to that of a common car.

To ensure the best interface with the motor, the geared reductor has been specifically designed. The mono-stage reductor is based on a helical gear. The pinion (78) is directly mounted on the output shaft of the motor. The output geared shaft (79) is guided by a pair of angular contact ball bearings mounted in X. The axial pre-constraint is given by the main cover (80) tightening. The output shaft supports and drives the in-pulley (81) of the belt (82) system.

The motorization group and its mounting on the chassis are such that the input pulley axis coincides with those of swingarm (64), so that the rear suspension stroke does not affect the belt tension. An aluminum casing handles structure features as well as the function of heat radiator. Two seals allow a lubrication by high viscosity oil, ensuring a good efficiency.

Despite the combination of the motor and reductor allow to exploit the first in its best operational area, some drawbacks of the complete solution must be mentioned. The system is more heavy, more bulky, more complex and more expensive, in addition to exhibiting more internal friction, more inertia than a specifically developed motor used without gearbox. Retrospectively, we doubt actually about the long term efficiency of such design strategic choice.

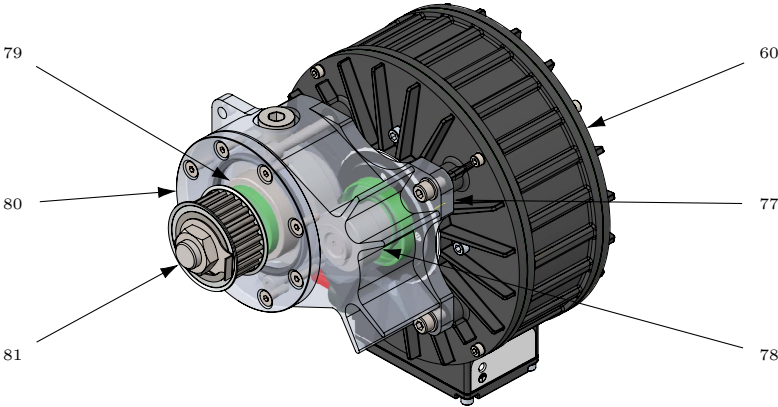


Figure 4.41: Motorization group.

Anti-tip

The tuning of the DTC system on true vehicle could be extremely dangerous. Indeed, during this phase, a rollover must be envisaged. The consequence of such events have to be minimized as possible. The robust security frame (65) of the chassis combined with racing seat and four point harness are already very secure. However, a reduction of the impact is preferable: it is why, in the setting phase, the demonstrator is equipped with an anti-tip (83) system. The principle is inspired from the bike stabilizers for children training. On each side of the main body (2), a triangle (84) is articulated along the longitudinal axis and ends by a small wheel (85). The inclination of the triangle is determined by a rods (86) articulated at both ends by ball joint. The length of the rods can be discretely fixed in order to set the allowed tilting stroke. A progressive opening should reduce the risks during test phases.

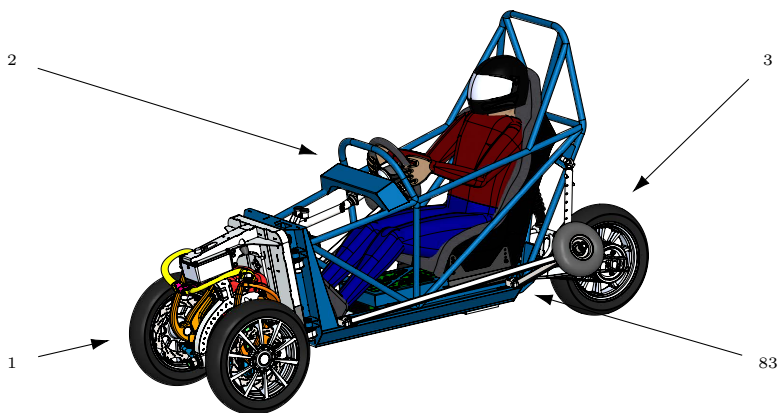


Figure 4.42: Anti-tip.

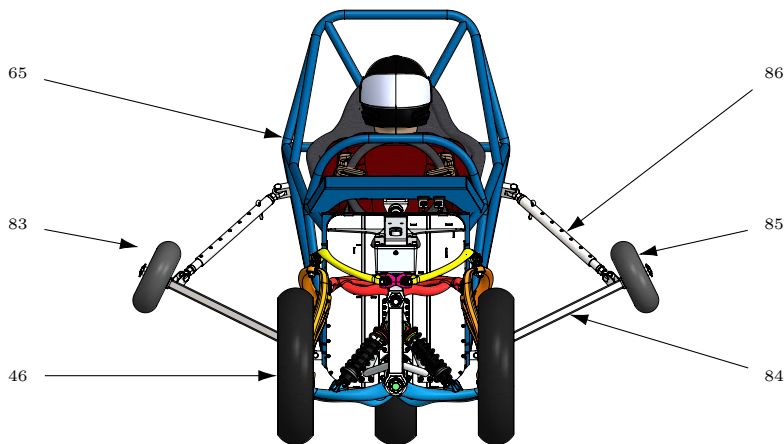


Figure 4.43: Anti-tip allowed angle.

4.6.4 Rear suspension

The rear suspension (3) is based on a swingarm (64) like for most the of motor-bikes. The particularities come from the regenerative braking which strongly suggests the use of a synchronized belt (82) instead of a classical chain in order to minimize the backlash due to torque inversion occurred between acceleration and braking phases. Indeed, the belt, due to its construction, is closed and can not be opened as a chain. Thus, the swingarm must be designed to allow the assembly despite this obstacle. The swingarm main part is a welded assembly of several folded aluminum sheets which forms a H-shaped plane structure. At the center, a pillar is erected to link the bottom rear shock (87) connection. Each side, a removable V-shape reinforcement (88) based on tubes is tightened on the H structure to form a very stiff tridimensional structure, while allowing the belt mounting.

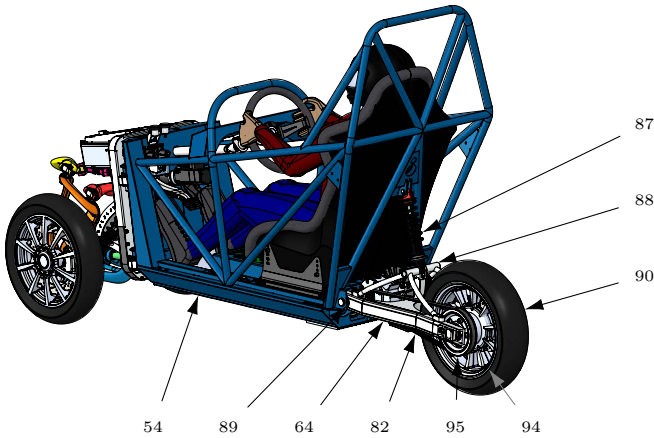


Figure 4.44: Rear axle: belt transmission.

The articulation of the rear axle with the body is achieved by two free pivots for which the bores are the inner ring of the elastomer bushes (89). This mount reduces the transmission of vibrations and solves the problem of alignment between the two ends of the fork. The space between both connections is free to host the motorization group (53).

In order to ensure the belt tensioning, the entire wheel (90) shaft (91) is mounted on sliding supports (92). The setting of the tension is achieved with a pair of screws (93) which push the shaft supports backward. The shaft end nut performs both the complete locking of the sliding supports on the fork ends and the pre-loading of the pair of angular contact ball bearings mounted in O configuration. On the belt side, an additional ball bearing reinforces the mount to face the large belt tensioning loading applied on the wheel main bearings. Its outside race is caught between the rim (94) and a complementary hub that extents the rim to support the propulsion pulley. The inner race is free to

slide on the shaft. Note the mounting is radially over-constrained, which is compensated by a high precision. This exotic bearing mount is protected by seals.

The brake is implemented at the opposite side of the belt. The disc (96) is directly held by the rim while the caliper (97) is fixed on a support (98) centered on the shaft, in order to ensure the brake coaxiality, independently of the belt tensioning. A sliding pin prevents any support rotation.

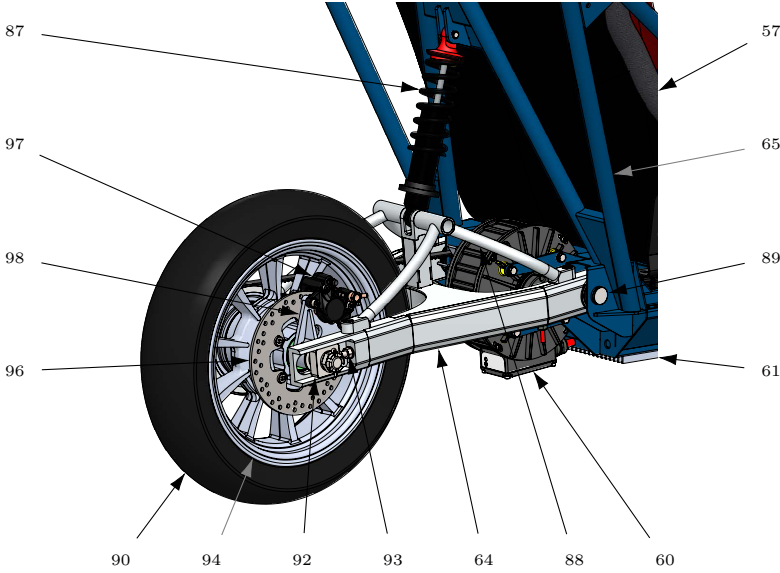


Figure 4.45: Rear axle: brake.

Finally, the rear axle looks like a standard motorbike rear axle. It is simply a bit more massive due to the chassis width and weight.

4.6.5 Conclusion about the mechanical design

In this section, we have briefly presented the mechanical design based on the optimization results. This important step required about two years of work. The result is a complex and coherent set of numerous components that satisfy together the entire rolling requirements. Moreover, the CAD model, presented under the functional point of view, the drawing plotting and the associated detail design have been accurately conducted, leading to complete manufacturing files for each component category. These results are not presented in this manuscript because they are a notch out of scope of the main thesis research issues. The manufacturers and suppliers have been identified. Their feed-backs, remarks and advice suggested multiple corrections, whose main ones have already been applied to this design. These contacts aimed also to make a quote for the complete demonstrator manufacturing.

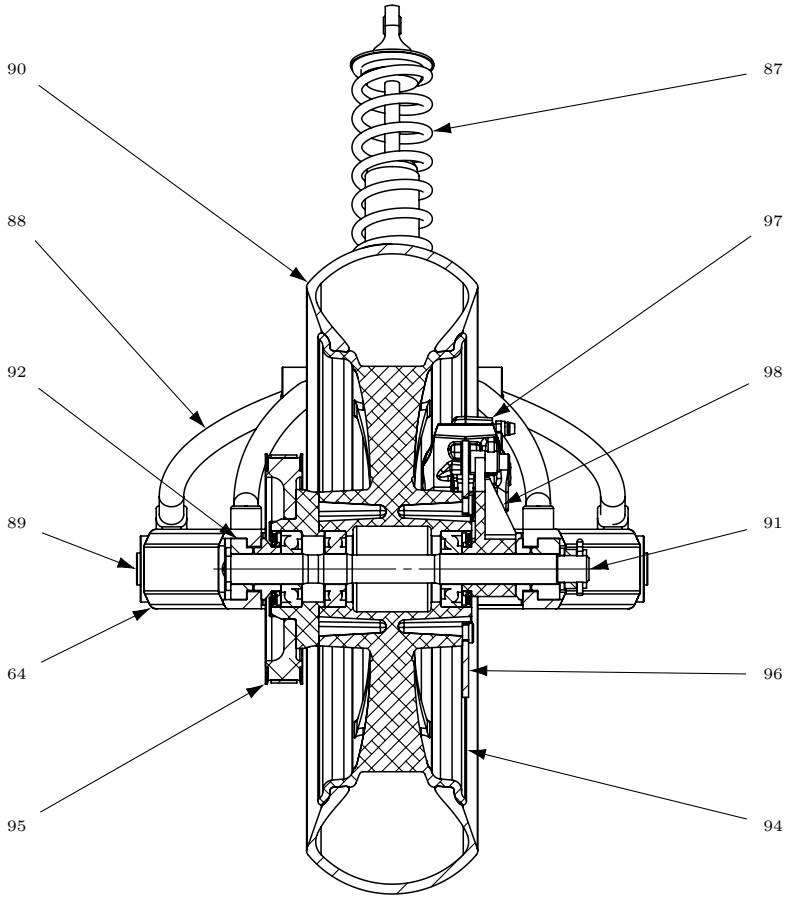


Figure 4.46: Rear wheel shaft.

The cost of such first true vehicle largely exceeds the initial budget, outright too tight, in all likelihood. Unfortunately, for this reason between several others, the consortium decided to abort the Wemoov project before the demonstrator manufacturing. At this time, the demonstrator can not be built and can not be tested to prove or to refute the dynamic quality of the three-wheeler, estimated and obtained thanks to the optimization process.

4.7 Semi-recovery

As expected, the mechanical design, which has been presented in the previous section, has given rise to some changes in the model. Some data are also known with much more precision. So, the impact of the CAD development on the model have to be precise and the effect on the dynamics has to be measured. It is important to verify if the good performances obtained thanks to the optimization are affected by these design modifications and, if so, how do they evolve ?

4.7.1 Impact of the mechanical design

The first impact of the mechanical design is the inertia properties of the bodies which are well better estimated after the CAD modeling. In order to compare the evolution of these properties before and after the mechanical design, the tables 4.33 and 4.34 gives the mass, the center of mass and the inertia matrix components for each body, respectively for the REF and CF1, and for CF2 and CF3 configurations. Besides the corrections – sometimes significant – of the previous values, numerous null values, i.e. neglected properties, are now affected with a suitable estimation.

ID	Name	m	cm _x	cm _y	cm _z	I _{xx}	I _{xy}	I _{xz}	I _{yy}	I _{yz}	I _{zz}
1	body	235.0	-0.570	0	0.330	19.000	0	-7.000	52.000	0	41.000
2	fork	10.0	-0.200	0	0	0	0	0	0	0	0
3	wheel_rr	14.0	0	0	0	0.200	0	0	0.350	0	0.200
4	body_nose_dn	15.0	0	0	0	0	0	0	0	0	0
5	pendulum	5.0	0	0	0.100	0.100	0	0	0	0	0
6	wishbone_ft_rt_dn	0	0	0	0	0	0	0	0	0	0
7	carrier_ft_rt	10.0	0	0	0	0	0	0	0	0	0
8	wheel_ft_rt	9.5	0	0	0	0.150	0	0	0.270	0	0.150
9	wishbone_ft_lt_dn	0	0	0	0	0	0	0	0	0	0
10	carrier_ft_lt	10.0	0	0	0	0	0	0	0	0	0
11	wheel_ft_lt	9.5	0	0	0	0.150	0	0	0.270	0	0.150
12	steering_fork	0	0	0	0	0	0	0	0	0	0
13	body_nose_up	0	0	0	0	0	0	0	0	0	0
14	wishbone_ft_rt_up	0	0	0	0	0	0	0	0	0	0
15	wishbone_ft_lt_up	0	0	0	0	0	0	0	0	0	0

Table 4.33: Wemoov body masses for REF and CF1 configurations.

ID	Name	m	cm _x	cm _y	cm _z	I _{xx}	I _{xy}	I _{xz}	I _{yy}	I _{yz}	I _{zz}
1	body	227.9	-0.613	0	0.331	20.475	0	-7.673	52.534	0	40.989
2	fork	7.5	-0.315	-0.013	0.020	0.139	0	0	0.326	0	0.438
3	wheel_rr	12.0	0	0	0	0.170	0	0	0.286	0	0.170
4	body_nose_dn	32.9	-0.202	0	0.277	1.031	0	0	1.577	0	0.960
5	pendulum	9.0	-0.197	0	0.059	0.064	0	0	0.223	0	0.197
6	wishbone_ft_rt_dn	6.5	-0.007	-0.139	-0.030	0.059	0	0	0.019	0	0.071
7	carrier_ft_rt	10.6	0.012	-0.021	0.186	0.154	0	0	0.192	0	0.052
8	wheel_ft_rt	8.5	0	0	0	0.140	0	0	0.270	0	0.140
9	wishbone_ft_lt_dn	6.5	-0.007	0.139	-0.030	0.059	0	0	0.019	0	0.071
10	carrier_ft_lt	10.6	0.012	0.021	0.186	0.154	0	0	0.192	0	0.052
11	wheel_ft_lt	8.5	0	0	0	0.140	0	0	0.270	0	0.140
12	steering_fork	0	0	0	0	0	0	0	0	0	0
13	body_nose_up	0	0	0	0	0	0	0	0	0	0
14	wishbone_ft_rt_up	3.7	-0.029	-0.112	0.012	0.020	0	0	0.007	0	0.026
15	wishbone_ft_lt_up	3.7	-0.029	0.112	0.012	0.020	0	0	0.007	0	0.026

Table 4.34: Wemoov body masses for CF2 and CF3 configurations.

The second impact relates to the shock anchor points. Those have been moved to allow their placement and to prevent any mechanical interaction. The table 4.35 lists the modified fixed parameters, whereas the unique geometrically constrained parameter that has been modified is given in the table 4.36.

ID	Name	Nature	CF1	CF2
3	body_to_shock_rr_x	PC	-1.0900	-1.1800
10	pendulum_to_shock_ft_lt_x	PC	-0.1700	-0.1100
11	pendulum_to_shock_ft_lt_y	PC	0.0200	0.0250
12	pendulum_to_shock_ft_lt_z	PC	0.1600	0.1700
13	wishbone_ft_lt_dn_to_shock_ft_lt_x	PC	-0.0300	0.0050
14	wishbone_ft_lt_dn_to_shock_ft_lt_z	PC	0.0100	0.0050

Table 4.35: Modified fixed parameters.

ID	Name	Nature	CF1	CF2
1	body_to_shock_rr_z	PC	0.5048	0.5423

Table 4.36: Modified geometrically constrained parameter.

The whole of changes impact the optimized configuration, CF1, what constitutes the corrected configuration CF2. Despite these modifications are as restricted as possible, the three-wheeler is anymore neither optimal from the point of view of the objectives, nor constraint compliant. A new optimization cannot be envisaged because the geometrical parameters are definitively fixed by the finished mechanical design. However, it is still possible to run again a part of the data conditioner. Indeed, the shock parameters are still free at this point and can be set once again to satisfy specific constraints.

In this *semi-recovery* phase, only the geometrical constraints 9 and 10, which correspond to chassis pith and height settings (table 4.9), all the modal constraints (table 4.11) and all the algebraic constraints (table 4.8) are activated. The result of such partial process is denoted as the configuration CF3. The next sections aim to characterize and to compare the different configurations. Among the five references, the third one is considered as the most representative and is maintained in the analysis.

4.7.2 Setup analysis

The shock setup is given for the different configurations in the table 4.37. The semi-recovery acts moderately on the shock characteristics. The rear shock is more affected than the front one but the maximum change remains below 17 %.

	Symb.	Description	Unit	REF	CF1	CF2	CF3
Front	$K_{sh\ ft}$	Spring rate	[N/m]	41139	38640	38640	36502
	$D_{sh\ ft}$	Damping rate	[N.s/m]	3502	3277	3277	3154
	$L_{0\ sh\ ft}$	Neutral length	[m]	0.3170	0.3186	0.3186	0.3209
Rear	$K_{sh\ rr}$	Spring rate	[N/m]	66036	66011	66011	76713
	$D_{sh\ rr}$	Damping rate	[N.s/m]	4204	4202	4202	4626
	$L_{0\ sh\ rr}$	Neutral length	[m]	0.3535	0.3536	0.3536	0.3489

Table 4.37: Wemoov shock setup.

The table 4.38 summarizes the front axle setup. One can remark, from the CF2 configuration, the constraints are not satisfied anymore. However, the imperfection about constraints are very small, in the order of the tenth of millimeter, for the linear dimensions or of hundredth of degree for the angles.

It is very interesting to notice that the correction of the main body pitch and height rectifies also partially the other constraints. This result is not surprising but fully appreciable. So, for CF3, the gap between actual and targeted constrained values is really low: negligible for the trail and the scrub radius, overshadowed by some other effects for the camber and the toe.

Symb.	Name	Unit	REF	CF1	CF2	CF3
D_{caster}	Caster	[°]	18.6046	17.2115	17.3180	17.2145
D_{trail}	Trail	[mm]	50.0000	50.0000	50.4123	49.9206
$D_{kingpin}$	Kingpin	[°]	12.8195	12.6848	12.6457	12.6756
D_{scrub}	Scrub	[mm]	-10.0000	-10.0000	-9.8984	-9.9215
D_{camber}	Camber	[°]	-0.0000	-0.0000	-0.0420	-0.0086
D_{toe}	Toe	[°]	0.0000	0.0000	-0.0140	-0.0028

Table 4.38: Wemoov front axle setup.

4.7.3 Fitness analysis

The fitness analysis is summarized in the table 4.39 and detailed, for each configuration, in the tables 4.40, 4.41, 4.42 and 4.43.

One can remark the corrections, leading to CF2, improve the results in the transient simulation and degrade the steady-state one. It is particularly surprising that this remark is true for nearly all the meta-criteria. In total, the fitness is worse. However, given the observed small change combined to the highly non-linear evaluator function, the impact of the correction on the dynamics performances should be very restricted.

The evolution to CF3 are completely opposed. This time, the transient simulation results regress but are still better than CF1. The steady-state performances are improved without recovering the CF1 level. In addition, the total fitness nearly regains the same level as that for CF1.

Fitness		REF	CF1	CF2	CF3
$F_{0\ TS}$	Transient simulation	132507.55	6642.22	5849.04	6003.52
$F_{0\ SS}$	Steady-state analysis	16839646.39	27381.93	29029.02	28103.27
F_0	Total hybrid simulator	16972153.94	34024.15	34878.06	34106.79

Table 4.39: Evaluator result overview.

			Transient simulation		Steady-state analysis	
			Mean	Max	Mean	Max
ID	Symb.	Name	$\bar{E}_i$	$\bar{E}_i$	$\bar{E}_i$	$\bar{E}_i$
1	E_α	Under-oversteering coefficient	810.76		994.80	416169.86
2	E_{Ack}	Pseudo-Ackermann criterion	3834.12	119809.64	149846.02	7468861.31
3	$E_{T_{sw}}$	Steering torque feed-back	41.15	46.45	13.26	12.42
4	E_e	Track following error	2.03	2.69		
5	E_{F_n}	Unballasting of the front axle		379.76	17009.20	2732.04
6	$E_{T_{ta}}$	Tilting actuation torque		7445.02		
7	$E_{P_{ta}}$	Tilting actuation power	5.37	130.55		
$n_k/n_{k\ ref}$					145 / 159	
$F_0\ TS\ \ F_0\ SS$		Total simulation	132507.55		16839646.39	
F_0		Total hybrid simulator			16972153.94	

Table 4.40: Evaluator results for the REF configuration.

			Transient simulation		Steady-state analysis	
			Mean	Max	Mean	Max
ID	Symb.	Name	$\bar{E}_i$	$\hat{E}_i$	$\check{E}_i$	$\bar{E}_i$
1	E_α	Under-oversteering coefficient	0.34		36.43	455.39
2	E_{Ack}	Pseudo-Ackermann criterion	1.97	71.64	1802.26	2200.52
3	$E_{T_{sw}}$	Steering torque feed-back	63.21	221.64	101.94	32.18
4	E_ϵ	Track following error	1.70	3.31		
5	E_{F_n}	Unballasting of the front axle		289.24	20656.90	2096.31
6	$E_{T_{ta}}$	Tilting actuation torque		5887.21		
7	$E_{P_{ta}}$	Tilting actuation power	6.60	95.35		
$n_k/n_{k\ ref}$						159 / 159
$F_0\ TS\ F_0\ SS$		Total simulation		6642.22		27381.93
F_0		Total hybrid simulator				34024.15

Table 4.41: Evaluator results for the CF1 configuration.

			Transient simulation		Steady-state analysis	
			Mean	Max	Mean	Max
ID	Symb.	Name	$\bar{E}_i$	$\hat{E}_i$	$\check{E}_i$	$\bar{E}_i$
1	E_α	Under-oversteering coefficient	0.09		37.33	460.72
2	E_{Ack}	Pseudo-Ackermann criterion	4.18	42.44	2148.09	2739.28
3	$E_{T_{sw}}$	Steering torque feed-back	92.69	286.80	165.52	45.57
4	E_ϵ	Track following error	1.70	3.30		
5	E_{F_n}	Unballasting of the front axle		157.12	21037.81	2394.70
6	$E_{T_{ta}}$	Tilting actuation torque		5194.45		
7	$E_{P_{ta}}$	Tilting actuation power	5.49	60.78		
$n_k/n_{k\ ref}$						159 / 159
$F_0\ TS\ F_0\ SS$		Total simulation		5849.04		29029.02
F_0		Total hybrid simulator				34878.06

Table 4.42: Evaluator results for the CF2 configuration.

			Transient simulation		Steady-state analysis	
			Mean	Max	Mean	Max
ID	Symb.	Name	$\bar{E}_i$	$\hat{E}_i$	$\check{E}_i$	$\bar{E}_i$
1	E_α	Under-oversteering coefficient	0.28		38.68	466.71
2	E_{Ack}	Pseudo-Ackermann criterion	1.99	57.30	1899.18	2357.34
3	$E_{T_{sw}}$	Steering torque feed-back	90.80	298.04	157.73	51.29
4	E_ϵ	Track following error	1.70	3.31		
5	E_{F_n}	Unballasting of the front axle		166.37	20878.40	2253.94
6	$E_{T_{ta}}$	Tilting actuation torque		5311.71		
7	$E_{P_{ta}}$	Tilting actuation power	5.67	66.34		
$n_k/n_{k\ ref}$						159 / 159
$F_0\ TS\ F_0\ SS$		Total simulation		6003.52		28103.27
F_0		Total hybrid simulator				34106.79

Table 4.43: Evaluator results for the CF3 configuration.

4.7.4 Criterion analysis

In order to better understand the results of the fitness analysis, the evolution of each criterion through the different configurations, in both the transient simulation and the steady-state analysis, is examined. To do so, the results of the transient simulation are drawn according to the color code defined in the table 4.44. As for the optimization step, these results are completed by the data for the steering wheel angle, the tilting angle, the attitude and the speed. These complementary results can be found in the appendix C.2.

Color	Configuration
	REF
	CF1
	CF2
	CF3

Table 4.44: Configuration color code.

Under-oversteering coefficient

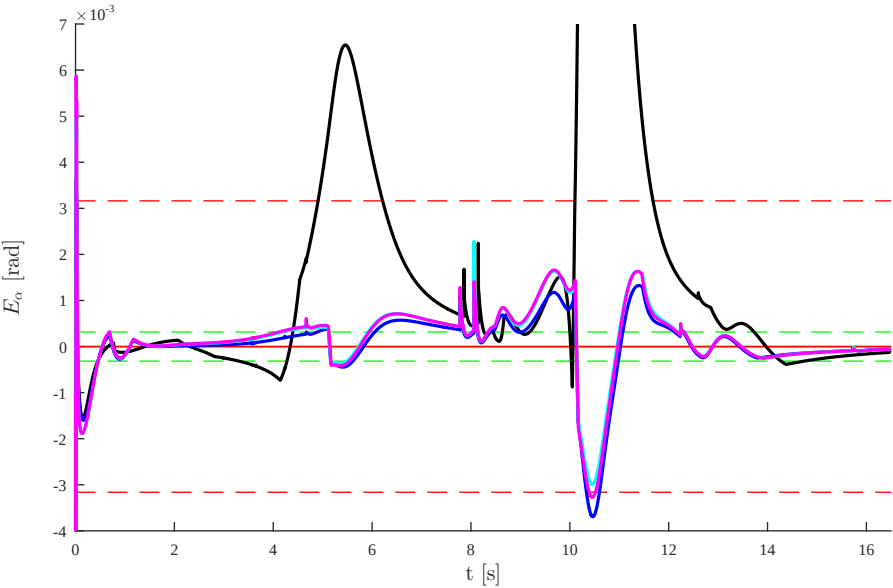


Figure 4.47: Transient simulation: under-oversteering coefficient.

In the transient simulation, the under-oversteering coefficient is relatively unaffected. For CF2, the maximal value is a bit improved with respect to CF1 but at low under-oversteering, the behavior is degraded. CF3 is rather intermediate between CF1 and CF2.

The changes occurred in the steady-state performances are imperceptible.

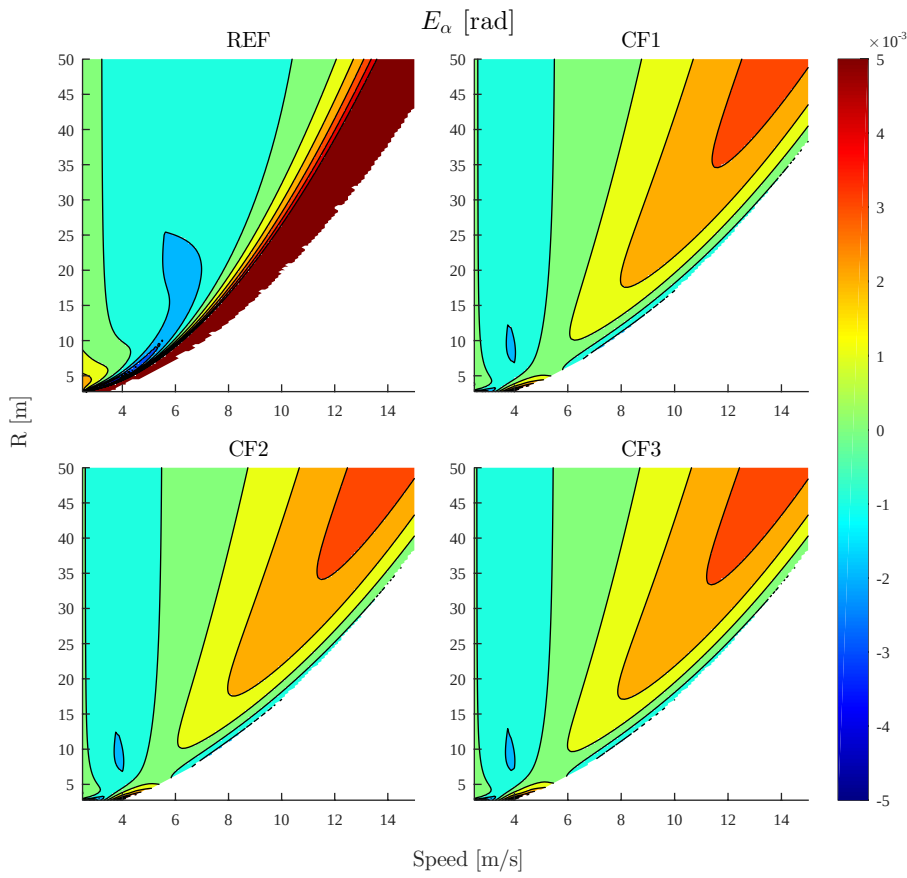


Figure 4.48: Steady-state analysis: under-oversteering coefficient.

Pseudo-Ackermann criterion

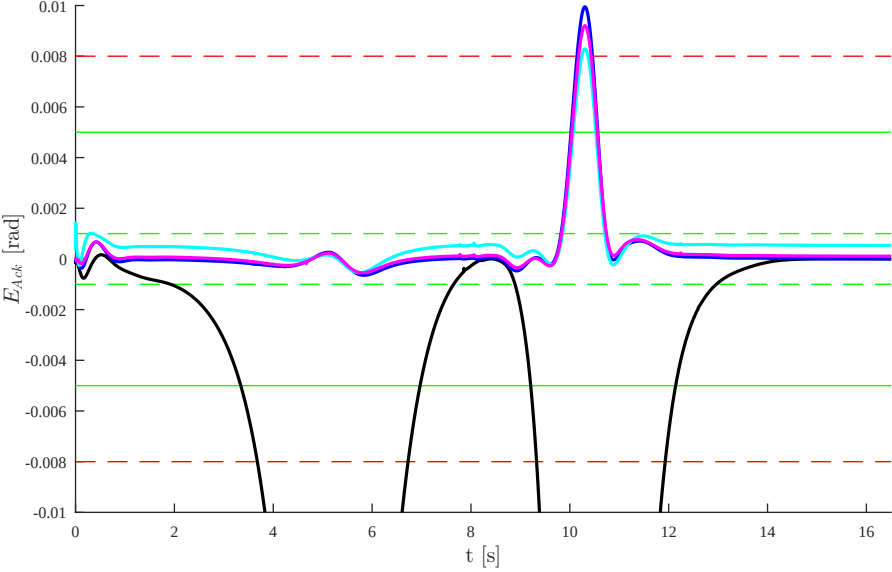


Figure 4.49: Transient simulation: pseudo-Ackermann criterion.

For the pseudo-Ackermann criterion, the difference between changes at low and high radius is very marked. At low radius, the criterion is degraded due to the violation of toe constraints. At high radius, the problem disappears and is even replaced by an improvement for the highest peak which correspond to the second turn. In this case, the initial toe corrects the imperfections of the steering mechanism. With the semi-recovery, CF3 comes back very close to CF1. These observations explain also most of the behavior changes highlighted for the under-oversteering coefficient.

On the steady-state mappings, a degradation of CF2 with respect to CF1 can be observed as well as the nearly perfect recovery for the CF3 behavior.

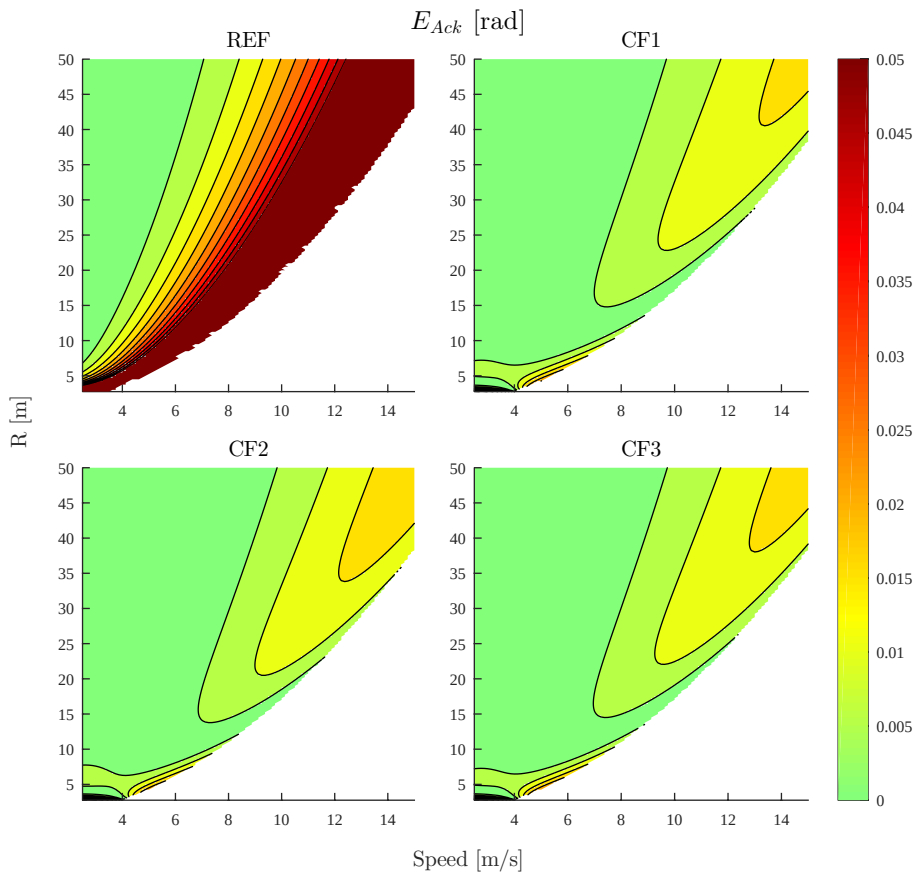


Figure 4.50: Steady-state analysis: pseudo-Ackermann criterion.

Steering torque feed-back

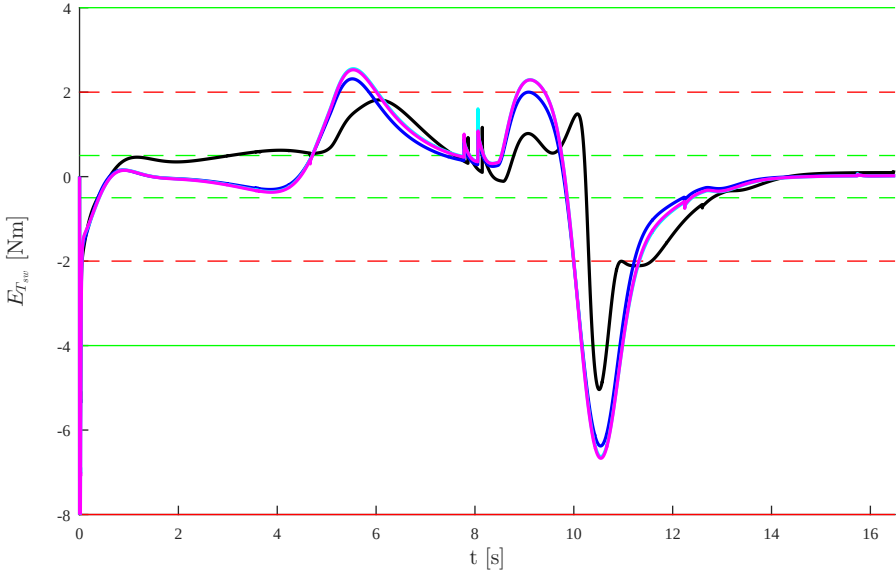


Figure 4.51: Transient simulation: steering torque feed-back.

The torque feed-back is lightly increased due to the correction. However, the effect of the recovery is practically invisible. It is rather normal, since these last modifications are somewhat "orthogonal" to the steering mechanism.

For the steady-state evaluation, the same remarks can be addressed: the corrections lead to a light degrade and the semi-recovery is without any real effect.

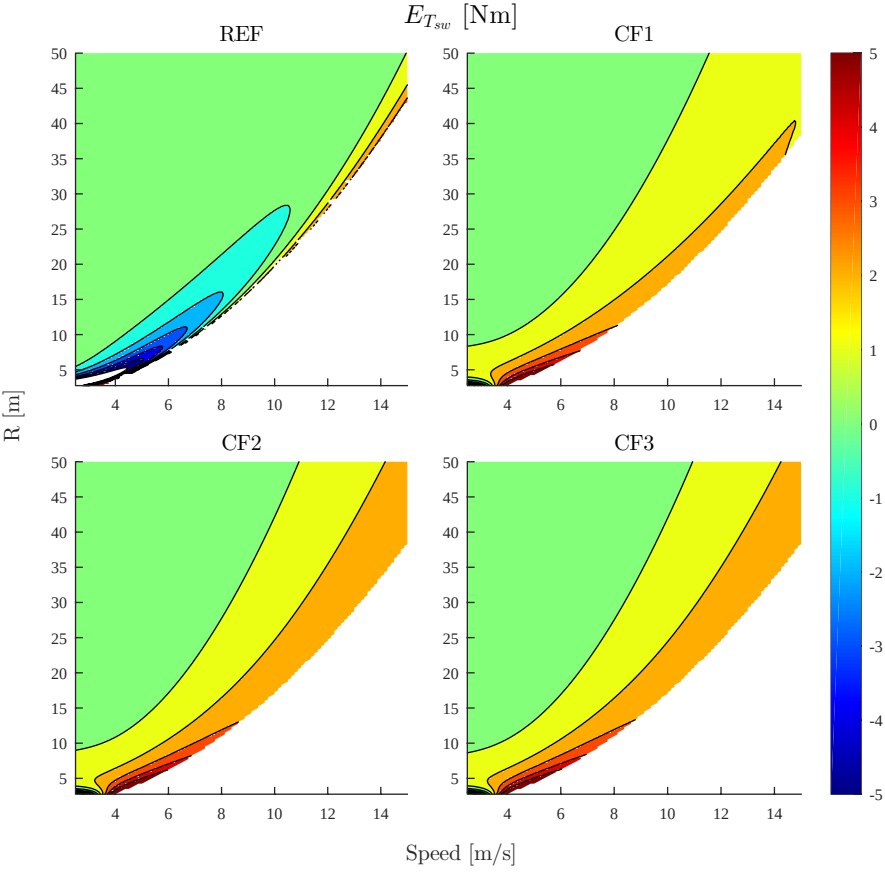


Figure 4.52: Steady-state analysis: steering torque feed-back.

Unballasting of the front axle

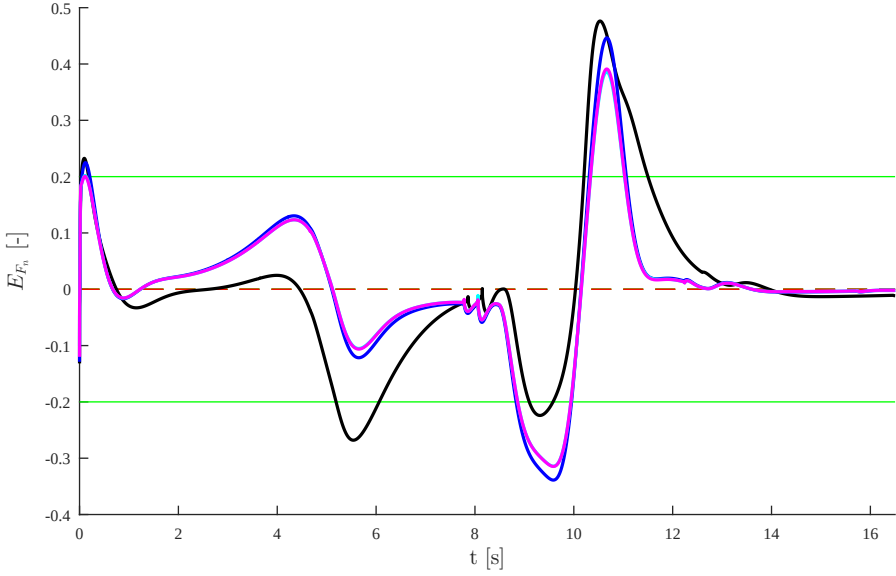


Figure 4.53: Transient simulation: unballasting of the front axle.

Contrary to the steering torque feed-back criterion, the unballasting of the front axle is a bit improved. Once more, the recovery is effectless.

The steady-state analysis allows to better measure the effect of each phase. However, as for all the criteria, the change due to correction and recovery are very small.

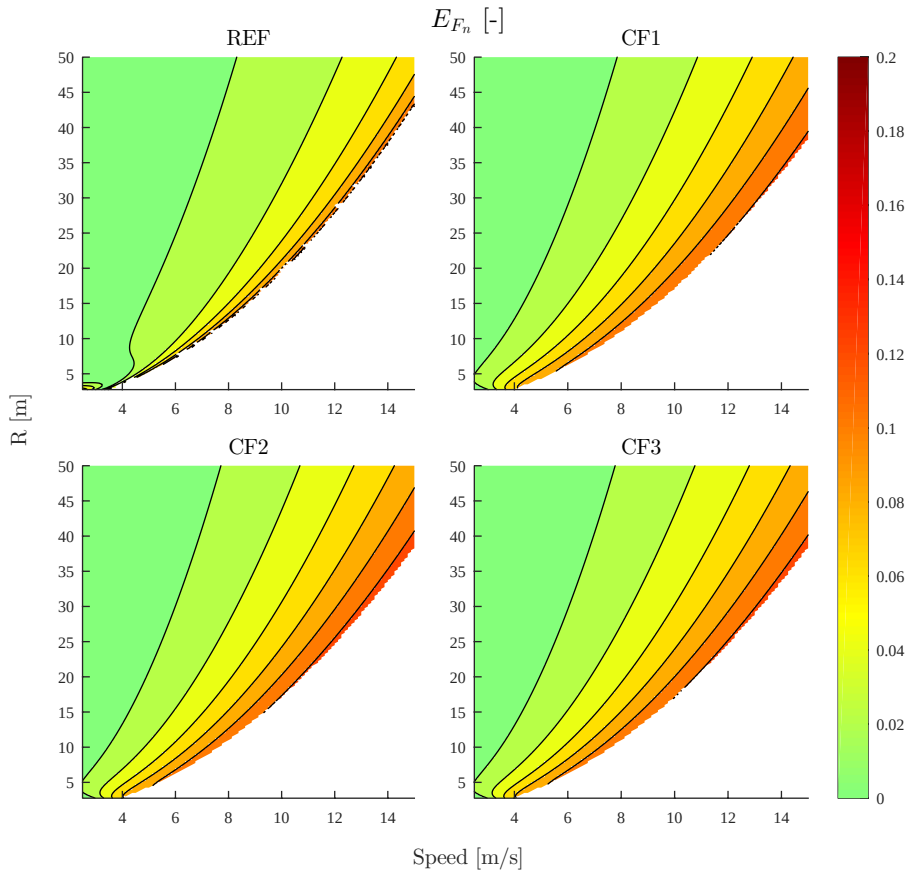


Figure 4.54: Steady-state analysis: unballasting of the front axle.

Tilting actuation torque

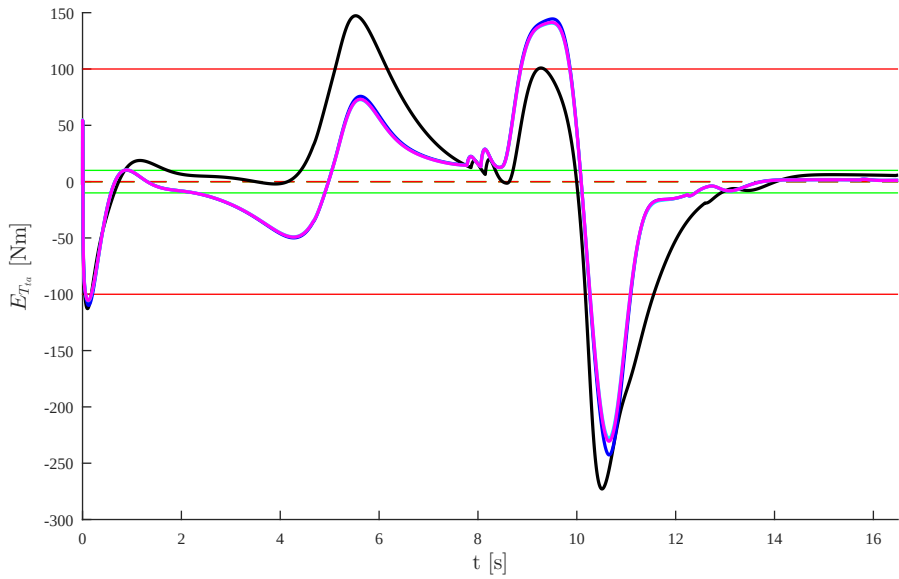


Figure 4.55: Transient simulation: tilting actuation torque.

The reduction of the tilting torque is minor as well.

Tilting actuation power

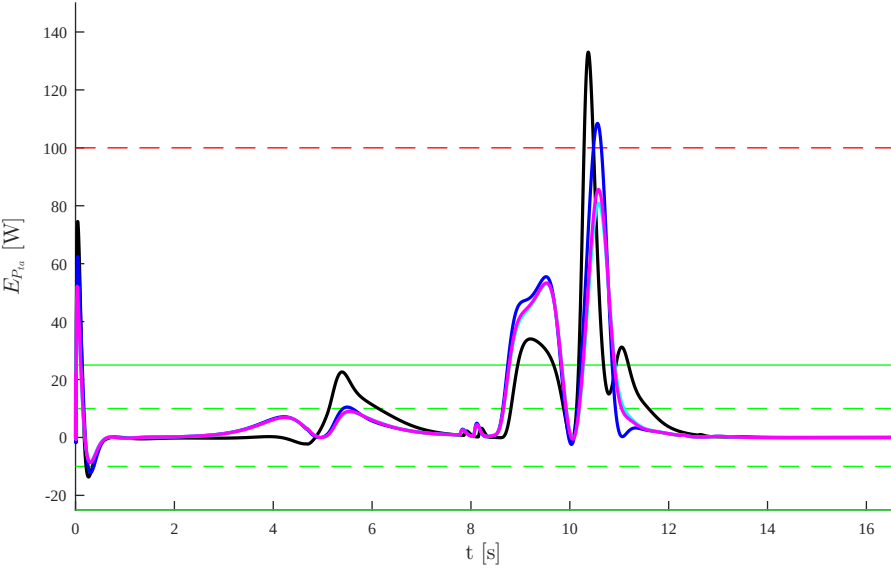


Figure 4.56: Transient simulation: tilting actuation power.

For that actuation torque, the gain is a bit larger. So, we can conclude the roll speed is reduced, during entering turn entry and exit, with respect to CF1. Moreover, the curve is more smooth for the CF2 and CF3 configurations. The mass gain and the related increase of inertia between the models are certainly responsible of these changes.

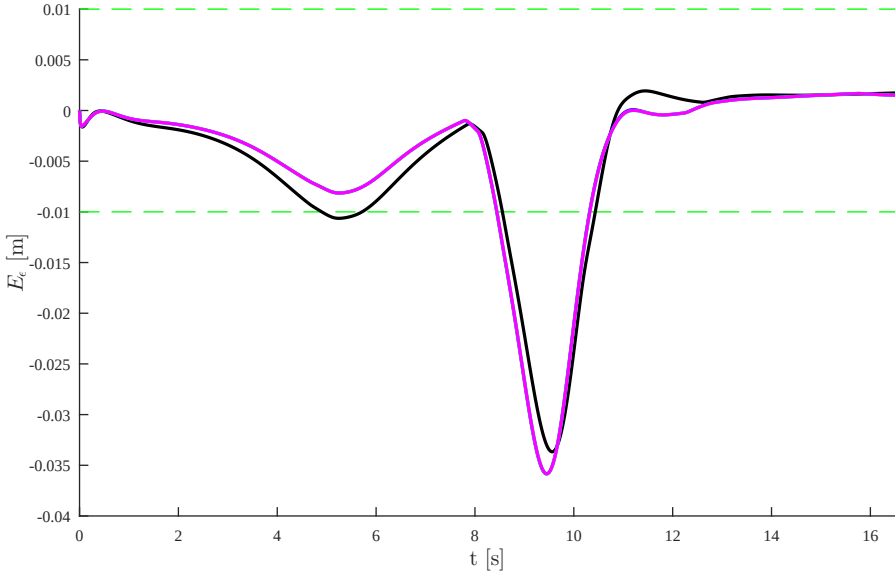
Track following error

Figure 4.57: Transient simulation: track following error.

There is no change at all for this criterion.

4.7.5 Conclusion about semi-recovery

The corrections based on the mechanical design have been applied on the optimized configuration. Thanks to a precise and conservative estimation of the inertia properties of bodies, the impact is as limited as possible. In the same way, the anchor point shifts occurred only on the shocks mounting anchors. So, the impact on the suspension kinematics is also moderate. As a consequence of these modifications, one observes some deviations with respect to design constraints and some criteria performance changes, which are globally rather negative. Fortunately, the observed dynamic behavior changes are minimal.

The semi-recovery which consists in applying a part of the data conditioner on the model allows to satisfy again the related constraints and to reduce the deviation observed for the others. The modal constraints which are the most impacted by the correction, can be completely satisfied again. From the point of view of the criteria, this operation allows to virtually restore the same level of performance as for the optimized configuration.

Finally, we consider the semi-recovered configuration, CF3, neither constraint compliant nor optimal but really close to such an – unknown – configuration. This result relativizes the interest of iterations on the entire design process including dynamics optimization and mechanical design, what was imagined at the genesis of this thesis. It is thanks to the high realization quality of some key steps such as the model synthesis and the first estimation of the inertia properties that this sophisticated iterative task can be avoided without waiving substantial potential gain.

4.8 Double lane change assessment

In the previous sections, we have highlighted the improvements of the Wemoov allowed by the optimization process. In the same way, we have observed that the correction and the semi-recovery steps lightly impact the performances, the last allowing to recover practically the same level than for the optimized configuration regarding the global objective function. The evaluation along this whole process is based on the hybrid simulator. Contrary to the steady-state analysis, which is very generic, the transient simulation relies on a particular reference trajectory. The s-shaped reference track, which is described in the section 4.4.3, is well adapted to the evaluation of the three-wheeler because relatively exciting for the tilting vehicles. However, this implementation could be lead to the improvement of the dynamics behavior for the special case of the reference track but, at the same time, to a deterioration for some other practical situations. As already discussed, it is not possible to assess the vehicle for all transient situations. Nonetheless, we have decided to select a second transient test and to measure the handling capabilities of the different configurations of the Wemoov. This way, we could check if the optimization leads to a generic improvement as expected.

For this second test track, we have decided to opt for a normalized test in order to have some performance references, by comparison to "standard" vehicles. We have selected the widespread Double Lane Change (DLC) maneuver. In the section 4.8.1, the normalized test is described and its relevance is discussed for the Wemoov. The section 4.8.2 describes the required changes of the model in order to simulate the DLC. The section 4.8.3 gives the main performance, i.e. the maximum travel speed, on the DLC track for the successive configurations of the Wemoov. Then, in the section 4.8.4, a criterion analysis is achieved for the final configuration, CF3, with the same criteria as for the evaluator. Finally, the main outcomes of this second assessment are summarized in the section 4.8.5.

4.8.1 Description of the double lane change maneuver

The double lane change, also called the *Moose test*¹ is a very common maneuver used for vehicle evaluation purposes. It is an evasive maneuver which consists in quickly swerve into the oncoming lane, in order to avoid an obstacle, before immediately come back on the original lane, to avoid oncoming traffic. The test is standardized in the ISO 3888-2 [50]. The geometry of the track is given in the figure 4.58. The track is divided in five sectors. The sectors 1 and 5 correspond to original lane. The sector 3 corresponds to oncoming lane. The sectors 2 and 4 aim towards the lane changes. The length of each of them is fixed independently of the considered vehicle. Alike, the lateral shift between the sectors 1 and 3 is fixed to 1 m and the width of sector 5 equals 3 m. In contrast, the widths of sectors 1 and 3, a and b , are computed according to

¹The Moose test or the Elk test appellations come from the frequent meeting between mooses, big animals from the deer family, and car drivers occurred in the North America and North Eurasia. Indeed, avoiding the fatal impact requires to perform a double lane change maneuver similar to the normalized test.

equations 4.16 and 4.17 respectively, with the vehicle width as variable. For the case of the Wemoov, the vehicle width is fixed to 0.8 m. In general, the traffic cones are used to delimit the boundaries of the maneuver track.

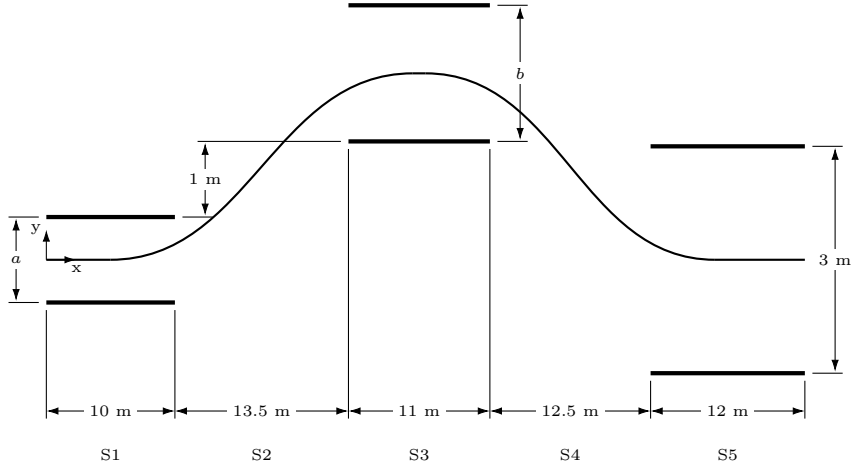


Figure 4.58: ISO 3888-2 double lane change maneuver¹.

$$a = 1.1 \text{ vehicle width} + 0.25 \text{ m} \quad (4.16)$$

$$b = \text{vehicle width} + 1.0 \text{ m} \quad (4.17)$$

The test is achieved with the throttle and brakes released from the origin point. The test is repeated, each time with an increased initial speed, until the vehicle loses grip (skids or spins around) or crosses a track boundary (knocks down cones). The test is successful if the maximum speed is at least 60 km/h. According to Teknikens Värld² [129], a reference automotive magazine in Sweden, the "good" cars perform the test with speed over 70 km/h and the "really good" car, with speed over 80 km/h³.

The main drawback of the DLC test is its large dependency with respect to the driver skills. As shown in the figure 4.59, the track can be negotiated with different trajectories, with significant impact on the maximal travel speed. For the purpose of the evaluation of the Wemoov, we have selected a "driver" trajectory at midway in the range between a pure "naive" trajectory and a fully optimized "pilot" trajectory as a trained pilot can do. These different trajectories and the footprint of the Wemoov in case of the driver trajectory are illustrated in the figure 4.59. We select the "driver" trajectory because it

¹For illustrative purpose, the scales in x and y directions are not the same. This representation could be misleading.

²The Moose test used by Teknikens Värld differs a bit from the ISO 3888-2. The sector 1, 3, 5 have each a width of 3 m. The sector 4 has the same length than the sector 2: 13.5 m. The length of the sector 1 and 5 is reduced to 6 m but this does not really influence the test. These changes have a moderate impact on the maximum travel speed. Indeed, the changes compensate each others such that a comparison remains relevant.

³The maximal successful speed record is surprisingly held by the Citroën Xantia Activa V6 with 85 km/h.

is more relevant for a common use vehicle evaluation, on the contrary of the optimized trajectory, unreachable in practice, especially in case of a sudden obstacle apparition.

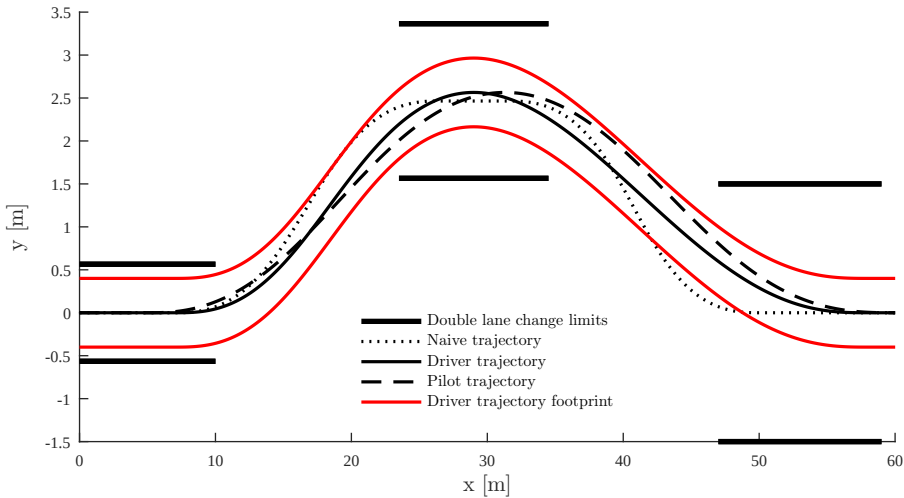


Figure 4.59: Double lane change: trajectories.

A priori, the ISO 3888-2 is applicable to passenger cars as defined in ISO 3833 [49] and light commercial vehicles up to a gross vehicle mass of 3.5 t. So, it is not specifically adapted to vehicles like the Wemoov. However, we find this test is relevant because very challenging but representative of realistic and common situation, also for urban tilting three-wheelers. Moreover, the standard implies dimensional adaptations with respect to the vehicle width. So, the test remains consistent for the narrow vehicles. Though, it must be kept in mind that the Wemoov is a tilting vehicle what implies a reduced capacity in term of lateral acceleration rate due to the second order control dynamics.

4.8.2 Steering controller adaptation

The double lane change maneuver is very different from the reference trajectory used for the optimization simulator. For several reasons, it is required to adapt the steering controller previously detailed in the section 3.4.1.

Firstly, the description of a suitable trajectory curve is not possible with the original track description (equations 3.10 and 3.11). For the DLC, it is required to impose the position of some check points but also the orientation i.e. the tangent at these points. Thus, the order of spline polynomials has been risen from 3 to 5, allowing more control on the shape.

Secondly, the condition of the test is different. On the one hand, the maximum curvatures is different: if, on the reference trajectory, the minimum radius is 8 m, for the DLC, it is 36 m. On the other hand, the speed is also very different: 5 m/s on the reference trajectory, twice or thrice more for the DLC. In

that respect, the distance between the nearest and the target points, L_{target} , is set to zero. The justification comes from the steering oscillations encountered at high speed for such different treatment of curvature and following error. Consequently, the PD part of the controller for the orientation becomes redundant with the PID for track following error. $K_{p\sigma}$ and $K_{d\sigma}$ have thus been also set to zero. To guarantee the stability, despite the high speed and the natural delay of the steering control chain, the reference point for the computation of the track following error has been pushed ahead 3 m with respect to the body reference origin. This point remains rigidly fix with respect to the main body.

This whole of changes allows to get a sufficiently reactive controller to perform a DLC maneuver while being stable and generating not too much oscillations.

4.8.3 Wemoov performances

Each Wemoov configuration has been tested in the DLC according to the ISO 3888-2. The initial speed has been risen incrementally by steps of 0.1 m/s, up to failure. The table 4.45 sums up the results of these tests.

Configuration	Maximum speed		Cause of failure
	[m/s]	[km/h]	
REF1	13.1	47.2	Front suspension reversing
REF2	14.9	53.6	Rollover
REF3	9.2	33.1	Front suspension reversing
REF4	15.1	54.4	Rollover
REF5	9.0	32.4	Front suspension reversing
CF1	16.7	60.1	Front suspension reversing
CF2	17.2	61.9	Front suspension reversing
CF3	17.2	61.9	Front suspension reversing

Table 4.45: Double lane change: maximum speed for Wemoov configurations.

Two cases of failure have been encountered. Firstly, the rollover occurs when the vehicle does not succeed to follow the track and ends by fall in the direction opposite to the curve due to an excessive steering. Secondly, the front suspension reversing happens while the tilting system is not able to redress the vehicle at the end of the curve negotiation and the roll angle surpasses the capacity of the front suspension. Because the wishbone stroke limit bump stops are not taken into account in the MBS model, the front suspension ends up reversing what indicates the vehicle is falling in the direction of the curve.

A first observation is that the maximum travel speeds are spread for the reference configurations: from only 33.1 km/h up to 54.4 km/h. The performance improvement consequent to the optimization is significant. Between the optimized configuration and the best random reference, a gain of 5.7 km/h is observed, that is 10 %. Fortunately, the correction allows to rise enough the speed by 1.8 km/h. Regarding the semi-recovery, the operated change is below

the increment threshold, so we do not observe any change with respect to the maximum travel speed.

For the final configuration, CF3, the maximum travel speed is 61.9 km/h i.e. a bit above the successful threshold. Considering the tilting dynamics nature of the three-wheeler and the conservative selected trajectory, we are very satisfied of this good result.

4.8.4 Criterion analysis

In order to pushed ahead the investigation about the performances of the We-moov passing the DLC, we propose a parametric study of the final configuration i.e. the semi-recovery configuration, CF3, with the speed as parameter. For this analysis, we focus on the criteria taken into account in the evaluator, in addition to the steering angle, the tilting inclination and the evolution of the speed along the simulation.

The results of the transient simulation are drawn according to the color code defined in the table 4.46. As usual, the mean and maximum, satisfaction and acceptance level code color can be found in the table 4.30.

Color	Speed	
	[m/s]	[km/h]
	12.0	43.2
	13.0	46.8
	14.0	50.4
	15.0	54.0
	16.0	57.6
	17.0	61.2
	17.5	63.0

Table 4.46: Speed color code.

Steering wheel angle

The steering wheel angle, Ω_{sw} , is computed on the basis of the steering fork angle and the steering transmission characteristics following:

$$\Omega_{sw} = \frac{\mu}{r_{st}} \quad (4.18)$$

$$= \frac{q(25)}{r_{st}} \quad (4.19)$$

with:

- μ the steering fork angle [rad],
- $q(25)$ the generalized coordinate i.e. the angle of the steering fork joint [rad],
- r_{st} the ratio of the steering transmission [-](given in the table 4.15).

The first observation is that the magnitude of the steering angle for the double lane change (figure 4.60), an emergency maneuver, is very low. Indeed, we observe a maximum steering about 12° , whereas, for a car in this situation, the angle would be over 90° . The steering seems too much sensitive due to a not sufficiently low steering ratio. The ratio, though, is fixed by the maximum steering wheel angle at very low radius curve. This highlights an interest for a variable ratio transmission (nonlinear) which is possible using very specific gears but which is expensive and reserved for large volume productions.

At a speed of 12 m/s, the steering angle is smooth. Above, some oscillations appear. This phenomenon increases with the speed due to saturation of the tilting controller as explained below. Apart from these oscillations, the graph change with respect to the speed is minor.

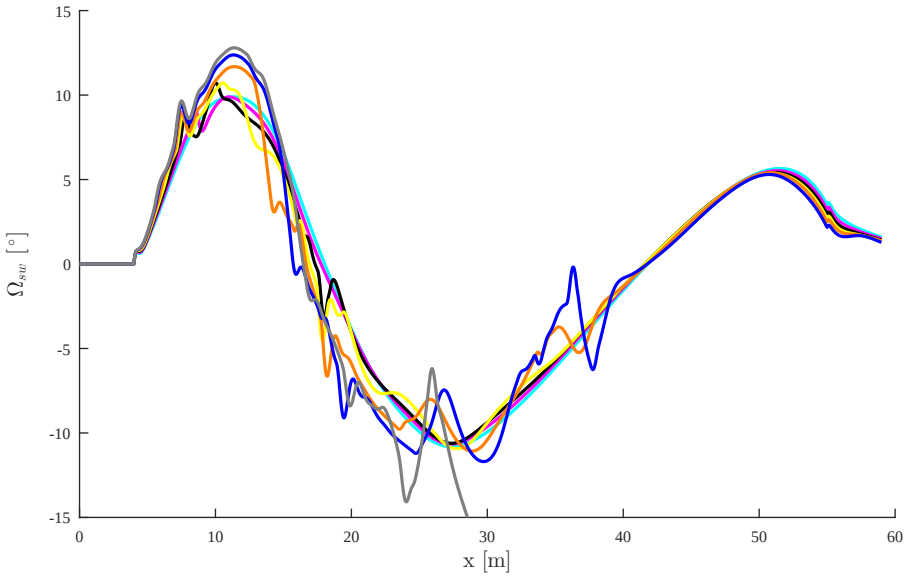


Figure 4.60: Double lane change: steering wheel angle.

Steering torque feed-back

The steering torque feed-back is plotted in the figure 4.61. The raise of torque is faster than the speed one. The unexpected allure of the graphs can be explained by the superposition of two effects: the transient component which is opposite to the motion and the steady-state effect which follows the motion, the delays applied on both being different. Naturally, the oscillations of the steering wheel angle affect the steering torque feed-back. Fortunately, the magnitude of the torque is fully compatible with the human actuation capabilities.

Track following error

The track following error, depicted in the figure 4.62, is very low. Obviously, the steering oscillations also impact the error. Probably, the simple steering controller is not fully adapted to such excited dynamics and react too nervously in order to keep the error as low as possible. Some developments could be beneficial to get a more natural behavior.

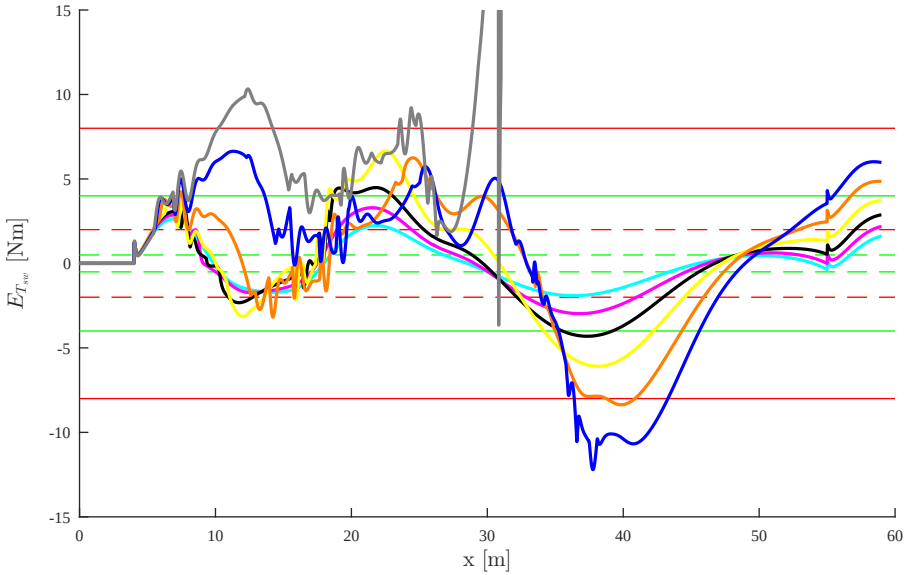


Figure 4.61: Double lane change: steering torque feed-back.

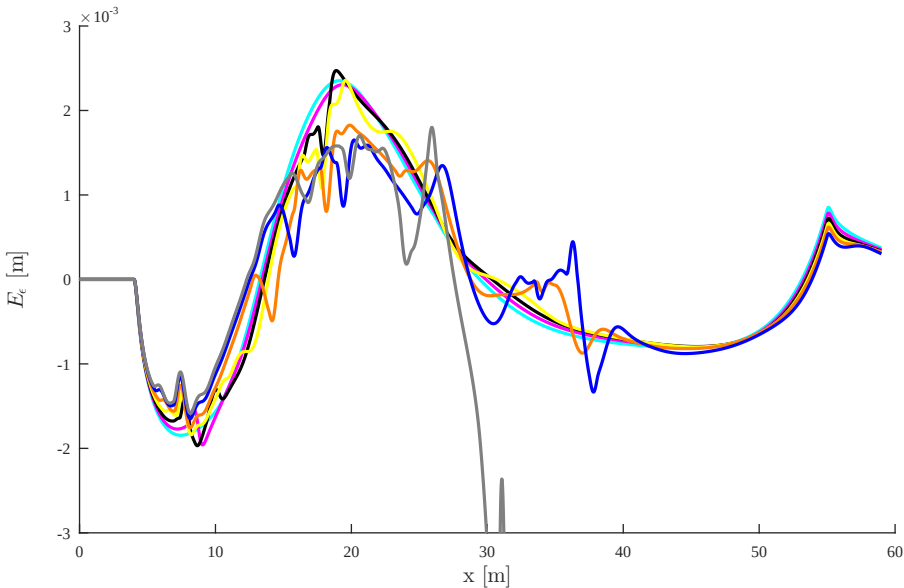


Figure 4.62: Double lane change: track following error.

Tilting angle

As expected, the tilting angle rises as the square of the speed as shown in the figure 4.63. It is also proportional to the curvature. Up to 13 m/s, the curves are very smooth. Above this threshold, the tilting system is saturated and some perturbations occur. Except for the last speed for which the three-wheeler fails, the maximum tilting angle, encountered in the sector 3, just exceeds 30° i.e. remains significantly below the technical maximum angle of 38° .

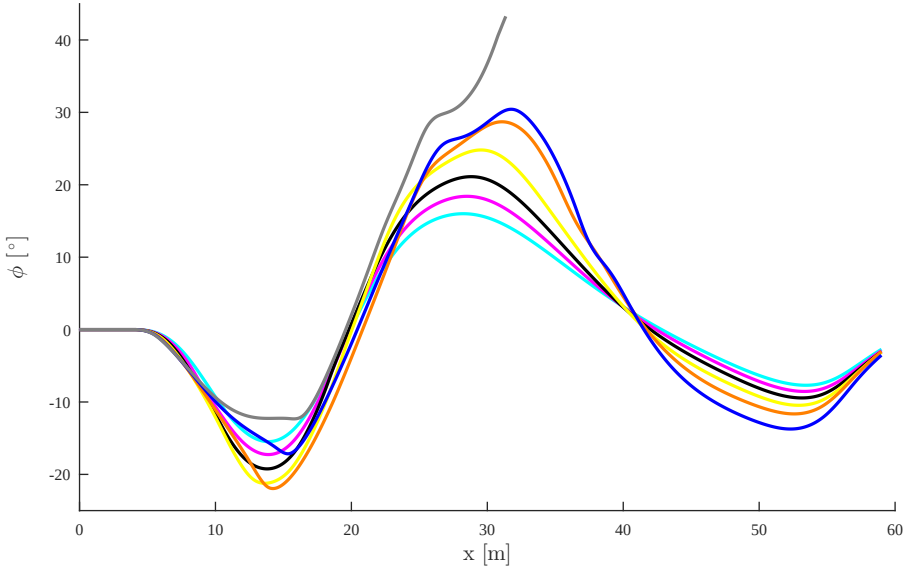


Figure 4.63: Double lane change: tilting angle.

Tilting actuation torque

The tilting actuation torque is given in the figure 4.64. We can observe that, over 14 m/s, the tilting torque reaches its maximum limit which has been fixed to 400 Nm in order to prevent to unballast up to take off one of the front wheels. Such discontinuities impact the tilting angle. At 17.5 m/s, the tilting system is so much saturated that it does not reach to clear the delay. Finally, the system does not succeed to redress to vehicle leading to the failure in the sector 3.

Tilting actuation power

The tilting actuation power ensues logically from the two last criteria and is illustrated in the figure 4.65. The maximum power is encountered at the curvature change throughout the first lane change. At these peaks, the power exceeds 2.5 kW what is really huge for a vehicle like the Wemoov. In fact, this represents more than a third of the motorization nominal power. The required power remains one of the main drawback of the DTC technology.

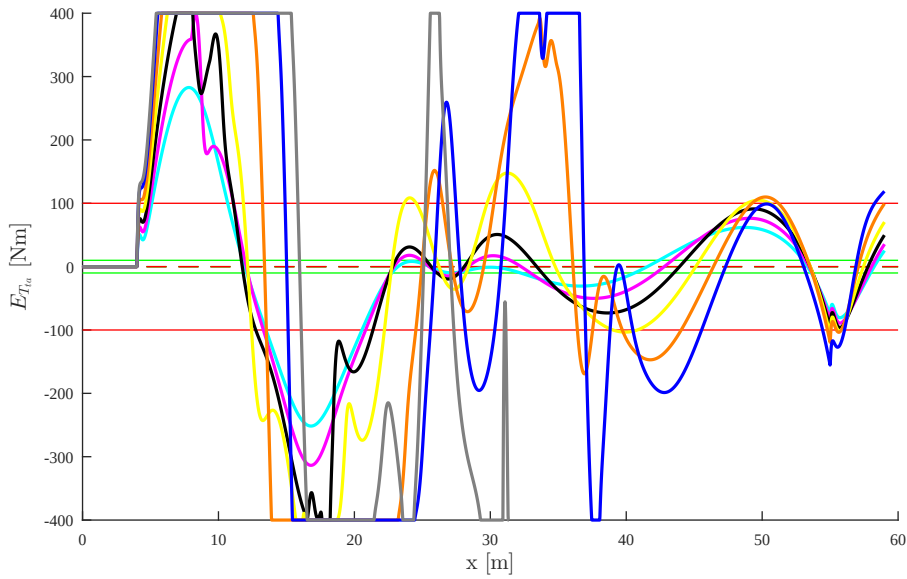


Figure 4.64: Double lane change: tilting actuation torque.

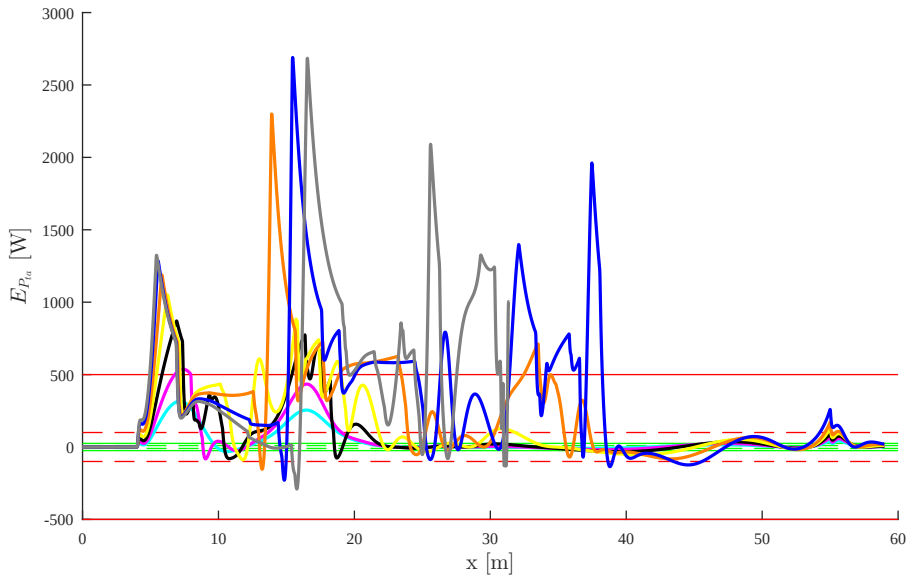


Figure 4.65: Double lane change: tilting actuation power.

Unballasting of the front axle

In the figure 4.66, one can observe the effect of the tilting torque on the unballasting of the front axle. Despite the imposed clipping of the torque that should maintain the unballasting below about 0.8, the transient dynamics leads to the full unballasting. For higher speeds, the wheels of the front axle take lightly off alternately. Thus, the double lane change pushes the Wemoov to its hard limits. It remains an emergency maneuver.

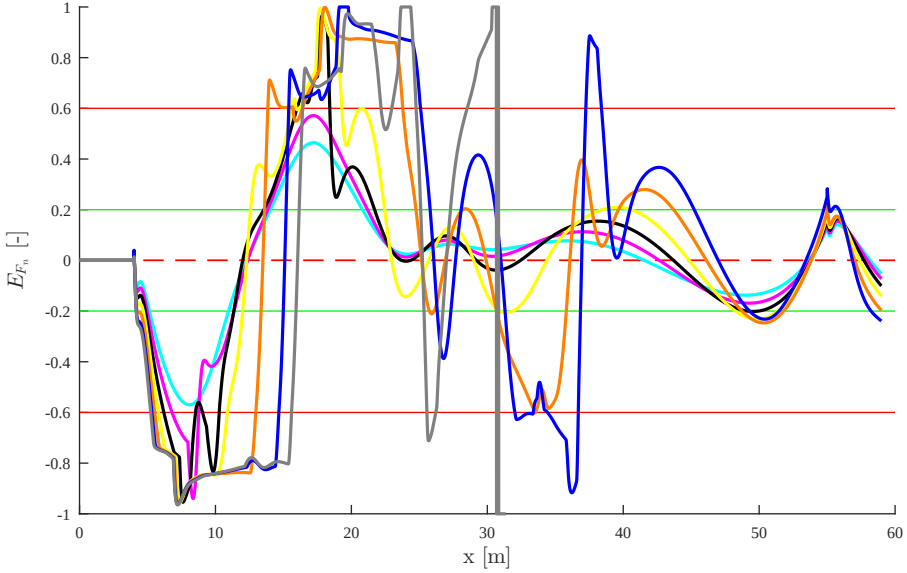


Figure 4.66: Double lane change: unballasting of the front axle.

Pseudo-Ackermann criterion

The pseudo-Ackermann coefficient, which increases naturally with the speed, keeps acceptable values given the nature of the test, as shown in the figure 4.67. Obviously, the wheel take-offs lead to some perturbations.

Under-oversteering coefficient

In comparison with the pseudo-Ackermann criterion, the under-oversteering coefficient, depicted in the figure 4.68, is much more disturbed. Indeed, the signals are really chaotic. This is logical because it is influenced by the steering and the tilting dynamics, themselves also very perturbed. The magnitude remains acceptable but such a disorderly behavior could be confusing for the driver.

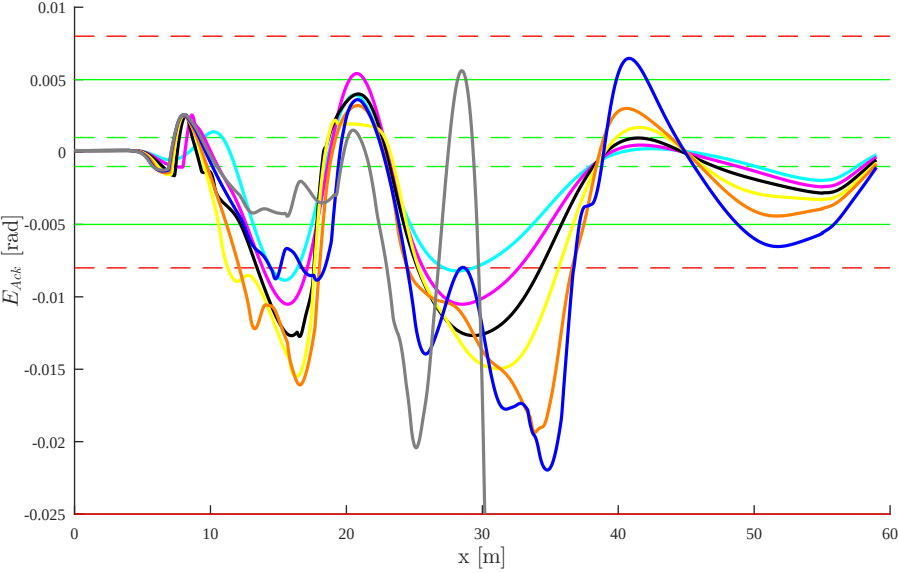


Figure 4.67: Double lane change: pseudo-Ackermann criterion.

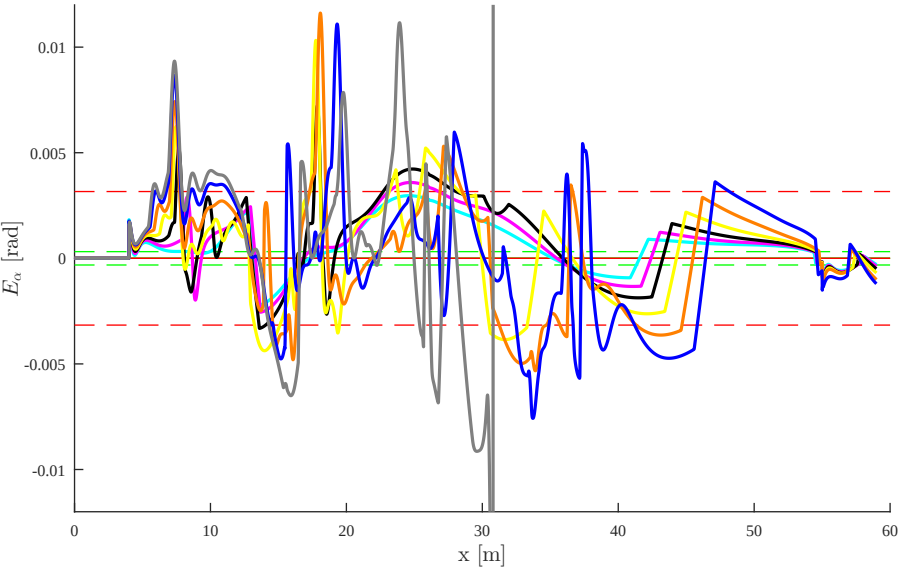


Figure 4.68: Double lane change: under-oversteering coefficient.

Speed

As regards the evolution of the speed (figure 4.69), one can observe the speed loss of the vehicles is relatively low. This indicates that the tire losses are low and so that the tire slips are moderate. This confirms the previously highlighted diagnostic: for the case of the double lane change, the maximum travel speed limit comes from the tilting system saturation rather than from a worst tire grip.

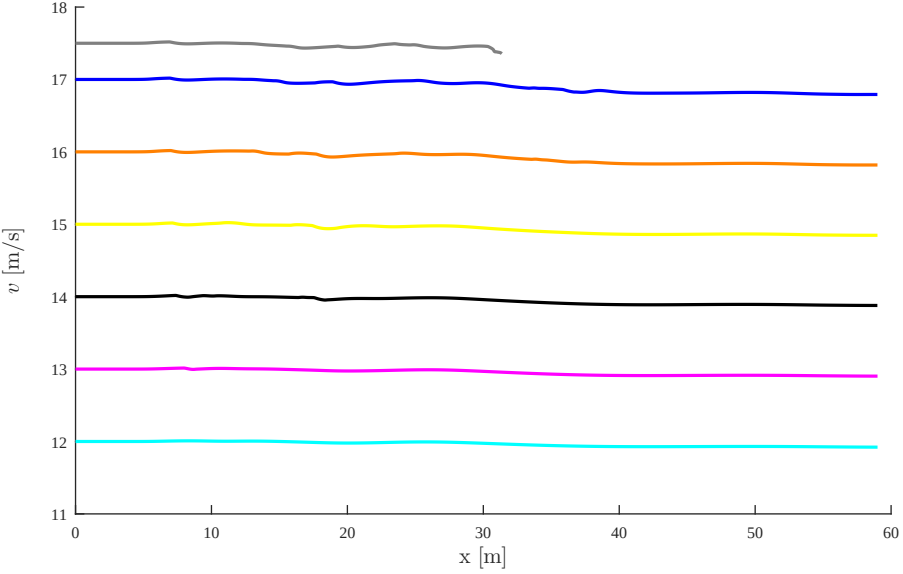


Figure 4.69: Double lane change: speed.

4.8.5 Conclusion about double lane change assessment

At the end of this second assessment of the Wemoov, achieved through the double lane change maneuver, three main results stand out.

First, it seems that the global optimization process leads to a generic improvement of the vehicle performances. Indeed, an improvement of 10 % of the maximum travel speed, a criterion unused until now, is outstanding given that this last test relies on a new type of trajectory, with very different curvature characteristics, traveled at very different speed ranges. This constitutes an important validation of our optimization methodology.

Secondly, the test demonstrates the Wemoov enables to successfully pass through the DLC up to 61.9 km/h what is also a good result given the tilting dynamics of the three-wheeler.

Thirdly, the assessment testifies than, from the dynamics point of view, the weakest point of the vehicle remains the tilting control. The DTC system does not entirely fulfill our challenging expectations.

Chapter 5

Conclusion

5.1 Contributions

5.1.1 Dynamics optimization methodology

This work mainly aims to develop an optimization methodology allowing to improve the performances of the devices for which the dynamics takes a prime importance. Road vehicles are especially concerned by this targeting. The dynamics optimization takes place in a more general design approach involving the morphological determination, the dynamics optimization itself and the mechanical implementation of a given vehicle. It is therefore relevant to take the mechanical design constraints into account as early as the dynamics optimization step.

Once the vehicle is modeled via the multi-body formalism, which plays a central role in our approach, the optimization problem must be formulated correctly. This means to distinguish the considerations treated as constraints to satisfy from the others considered as objectives to minimize. The formulation depends not only on the vehicle under study but also on the specifications addressed and on the context of development and utilization, so it is not unique but must be explicit.

The constraints are used either for themselves or to obtain some characteristics leading indirectly to desired dynamics qualities. In this second way, these performances must neither be evaluated through the costly simulations nor obtained thanks to the labor of the optimizer. Several characteristics of the front axle endow stability, while several vibration mode properties fix roughly the comfort level. In sum, the use of constraints on those features leads to suitable dynamics performances. Such provisions simplify the general dynamics optimization problem and allow to save a huge amount of computational time.

In this study, we are interested in several generic criteria: the understeering, the pseudo Ackermann, the steering torque feed-back. Obviously, some additional and specific criteria can enrich the evaluation. The pseudo-Ackermann criterion computed as the difference of the slip angles between two tires of a same axle is, if not original, under-employed in the literature. From

our point of view, the combination of this criterion with the under-oversteering one offers a good mean to improve the tire work through the suspension kinematics.

The proposed method relies on the parameter partitioning in order to organize the problem solving in a structured manner. Once identified, the parameters are classified in categories according to their incidence on the model. We differentiate null, fixed, constrained and optimized parameters.

The numerical process is based on the evaluation of candidates characterized by the optimized parameter vector generated by the optimizer.

As a first step, the data conditioner deals with the constrained parameters to solve the design constraints. The data conditioner is made of three embedded solving loops. The inner one solves the algebraic constraints. The intermediate loop solves the geometrical constraint and computes the neutral position of the vehicle thanks to the extended equilibrium equations for which the variable exchange method is used to directly target the desired solution. The outer one is used to solve the modal constraints, smartly extracted from a modal analysis. Such an arrangement for the solving of each type of constraints is at our knowledge an original contribution. Moreover, to be very efficient from the computational point of view, the data conditioner satisfies the constraints that have not to be handled any more by the optimizer. Consequently, this latter can be generic and deals with an unconstrained problem, well much easier to optimize. Another very interesting advantage of the data conditioner is the following simulations are performed for viable vehicles in the sense that they satisfy all the constraints. Given the computational cost of the simulations, this is much more efficient than other approaches based on the penalty method that imposes to simulate uselessly unrealistic vehicles that do not satisfy the constraints.

Once all the parameters are determined by the data conditioner, a hybrid simulator, made of both a transient simulation and a steady-state analysis, measures the dynamic behavior of the vehicle in action. The interest of the combination of these two different numerical processes is their complementary. The steady-state analysis, which is performed on a range of radii and speeds, characterizes, by the way, the vehicle on the entire range of use with a very low computational cost. On the contrary, the transient simulation enriches the analysis capturing the transient behavior of the vehicle that is completely omitted by the steady-state analysis. Despite the higher cost of such simulations, the transient effects are significant. Definitively, the hybrid simulation provides the required information with a good trade-off between cost and exhaustiveness.

The computation of the steady-state included in the simulator required the particular applications of the exchange variable technique on the quasi-static equilibrium equations. In this way, the exact calculation of the interest variables as the steering wheel orientation and the propulsion torque can be computed straightforwardly as a function of the imposed radius and speed. This formulation is more advantageous for both the performance optimization and the vehicle characterization purposes. Furthermore, we have proposed an adaptive tire model linearization in order to overcome the convergence prob-

lems occurred in the quasi-static equilibrium solving, allowing to get exact states. Moreover, the states very close to the tire saturation can be reached. These improvements brought to the vehicle steady-state computation constitute an additional major original contribution. This is more general because not specifically related to the optimization.

As the last step of the vehicle assessment, the evaluator computes the scalar objective function on the basis of the information recorded along the simulations, allowing the optimizer to proceed the vehicle improvement through the evolution of the parameters.

The evaluator is divided in two main steps. The clustering processing which agglomerates the discrete and continuous recordings from the simulations in some meta-criteria is rather classical. In contrast, the proposed mix law combines the meta-criteria into a unique non linear scalar global objectives function. The use of exponent coefficients in addition to the classical weights enables to easily take the diverse objectives exhibiting some disparate nature and impact into account. Besides the mix law itself, the method determining the coefficients on the basis of an acceptance and a satisfaction levels is, despite its clever simplicity, very efficient. Such contribution appears as a simple method that could be applied for other kids of multi-objective optimization problems.

Even more than these particular contributions, the interest of this work is the optimization process seen as a whole. Thanks to its coherence and its efficiency, the general process can handle the complex problem of dynamics optimization including the considerations related to the mechanical design, for systems as complex as road vehicles. Indeed, the method has proven its ability to robustly manage a large number of diversified and conflicting constraints and objectives, dealing with a multitude of parameters from different natures, the features that particularly characterize the vehicle dynamics challenges.

5.1.2 Wemoov three-wheeler

In addition to the conception of the dynamics optimization methodology, this work aims also to develop the Wemoov concept. This innovative vehicle, characterized as a narrow electric unipersonal tilting three-wheeler, intends to be an alternative to solve current mobility problems: the road network congestion, the energy sourcing and the environmental pollution.

Targeting a high nimbleness, the DTC has been quickly identified as the only system enable, at short term, to control the tilting motion, justified by the narrowness and the desired handling.

The suspension kinematics is rather conventional. The double front suspension is fitted with an additional articulated part, the T-pendulum, linked to shocks and conferring the tilting degree of freedom. A simple swingarm suspension is implemented for the rear axle. In contrast, the tilting actuation that is performed by an electric motor through an – offset – geared sector based transmission is an original contribution.

The complexity and the nature of the problem make this vehicle a perfect case study for the optimization methodology developed. Indeed, it allows to convince of its ability to improve the dynamics performances of some very

general vehicles for which the dynamics prevails.

In addition to the generic criteria, some specific ones have specially been developed for the three-wheeler: the unballasting especially refers to tilting vehicles and aims to prevent from the rollover, while the minimization of the tilting actuation torque and power improves the DTC capabilities.

The constraints have been applied to satisfy several requirements that are specific to vehicle dynamics issues about the stability, the comfort or to conform to some imperatives of the mechanical design.

The application of the optimization methodology has enabled a large improvement of the objectives. The dynamics performances of the optimized vehicle appear much better than those exhibited by the best candidates selected among hundreds of possible vehicles generated randomly. The trade-off drawn from this optimization seems to be a relevant and an interesting result.

The mechanical design aimed to build a first demonstrator of the Wemoov that involves all the features needed for the rolling tests in order to attest the performances promised by the dynamics optimization. Some specific mechanical arrangements have been designed to conform as much as possible to the optimal kinematics issuing from the optimization. Among others, the switch to a front peripheral brake system respects the best placement of the bottom wishbone end articulations. In the same way, the suspension and the steering ball joint are equipped with an additional pivot allowing the large required tilting and steering strokes. The implementation of the tilting actuation system has also been particularly challenging.

Beyond the mechanical particularities, it is mainly the design in its whole that can be brought to the fore. The front axle is extremely compact. From a design point of view, it could appear as over-constrained but each element finds its right place. The smart positioning of the elements combined with weird shapes of the suspension components allow to push the maximum tilting and steering strokes to 38° and 45° respectively, what is rather exceptional for a narrow actuated front axle.

Despite all attempts to conserve the exact optimal kinematics, the mechanical design has involved some local geometrical modifications. However, neither these infringements of the optimal kinematics nor the improved precision about the inertia properties of the bodies seem to induce significant degradation of the dynamics performances. The optimization results appear to be sufficiently robust with respect to the occurred changes.

The semi-recovery step, which consists in applying again a sub-part of the data conditioner, has allowed to counteract almost all the degradation noticed previously, simply by action on the limited set of parameters not locked by the mechanical design. Such a result makes irrelevant the iteration on both the optimization and the design phase, which was imagined at the beginning of this project.

Finally, a second assessment, achieved post optimization, according to the standardized double lane change test has been conducted under very different conditions than those used in the optimization: a different range of speeds and different trajectory type. The test has proved the final version of the Wemoov passes this test, attesting of the dynamic capabilities of the three-wheeler but

also that the improvements obtained through the optimization are generic.

The output of the Wemoov development is the complete mechanical design of the first demonstrator that is ready to build.



Figure 5.1: Wemoov design.

5.2 Prospects

5.2.1 Dynamics optimization methodology

Our solution for the dynamics optimization methodology has implied numerous choices and trade-off that could be called into question. In the same manner, there exist multiple possibilities to improve the method with new features in order to get more precision or better properties. The most relevant prospects addressed to the optimization methodology are briefly invoked here below.

The process works according to numerous parameters for which the effects were not sufficiently analyzed in this work. In particular, a better tuning of the CMA-ES parameters should allow to improve the convergence time.

The use of some other types of optimizer should be tested for a confrontation with the CMA-ES performances. Also, some hybrid schemes i.e. several methods used sequentially along the convergence, seem particularly promising.

Obviously, each improvement of the multi-body model leads to a better precision and a higher fidelity. In the context of vehicle dynamics, the deformations of the body and the suspension components under loading significantly impact the tire kinematics and thus the overall dynamics. The under-oversteering and the pseudo-Ackermann criteria are particularly affected. The introduction of flexible dynamics in the multi-body model refines the result sensitively at the expense of additional computational cost. In this regards, parsimony and common sense must apply.

Some developments of the tire kinematics computation algorithm are required to simulate a path on uneven and non horizontal ground. This could give the opportunity to test more realistic rolling cases encountered in practice.

The way the tires are modeled could be improved. On the one hand, a better model and the associated specific parametrization should improve the accuracy of the simulation. On the other hand, the tire model could be more integrated to the optimization. Indeed, because the tire customization is out of range for most of vehicle development projects, the tire model parameters are not considered eligible as constrained or optimized parameters. However, it is still possible to change the type, the size, or the load and speed indexes with obvious impacts on the whole vehicle dynamics. Offering this choice to the optimization process is not straightforward. From our point of view, we believe that the optimization should be divided in two phases. In the first one, the optimization would work on a continuous tire range model for which the parameters have to be optimized. In the second phase, a particular tire would be selected, for each axle, as close as possible to the optimal solution and, a second optimization would take place without any tire model degree of freedom. The major difficulty of such an approach is the creation of the continuous parametrized interpolation between the entire range of possible tires. The complexity of the tire models being very high, this task is clearly delicate to carry out.

The performance criteria addressed to the road vehicles are numerous and could not be tested exhaustively in a simple transient simulation as implemented in the proposed method. This weakness could be solved by a multi-scenario joined to additional criteria. The acceleration or braking possibly combined with diverse types of turns and direction changes could favorably enrich the simulator and, in the same way, the vehicle assessment.

Another way of simulator improvement is the use of some modal analyses computed around the steady-states. The stability of those modes can be measured and improved by the optimization.

Our optimization scheme allows to reach more than noteworthy performances for the selected criteria. Nevertheless, the robustness of these good results against the perturbations of any kind is not yet guaranteed. The introduction of robustness criteria alongside the base criteria themselves could boost the trust in the resulting optimized solution.

All the possibility offered by the symbolic multi-body approach are not fully

exploited in this work. Firstly, it could replace some algebraic constraints, in particular, those imposing the symmetry of the chassis. Some already available improvements that increase the acceleration computation can also be used. Mainly, the symbolic computation of gradients [73] offers interesting perspectives. The first possible application is the computation of some partial derivatives of the states involved in the controllers instead of very unstable finite differentiations. The second one is the computation of system sensitivities for establishing the robustness criteria highlighted just above. The third is the determination of the Jacobian matrix of criteria with respect to the optimized parameters, giving consequently the possibility to use some gradient based optimizers in some well targeted phases of the convergence. The potential gain of such improvement remains to be evaluated.

As already discussed in this work, the general procedure does not enable to reach the exact optimal solution because the dynamics optimization and the mechanical design interfere each other. If such exact optimal solution would be target, a costly iteration is the straight way. Presently, the automatization of the design loop process stays difficult. First, the creation of CAD model sufficiently robust to envisage the self incorporation of changes from the dynamics would require much more skills and time. Moreover, the rebuild process of CAD model is not sufficiently reliable and its functioning is completely closed. Finally, the use of the API functionalities of SolidWorks in order to interface the CAD model and the optimization would need deep developments.

In some extreme perspectives, some topological optimizations could be imagined. Although for the most simple systems and formulations, such operations have already been tackled [20], today, the dynamics topological optimization for systems as complex as complete road vehicle remains, at our best knowledge, out of range. Without proposing any particular solution to this end, we simply highlight here the absolute ability of such tool that could be so helpful and powerful for the vehicles dynamics purposes.

We stress that the possible improvements described have, in general, an operational cost: increase of development time and means, complexity of the formulation, increase of the optimization run time, emergence of convergence problems, difficulty to interpret the results, etc. In any case, a insightful trade-off must be found between the objectives of the optimization and the resources mobilized ensuring them.

The proposed dynamics optimization methodology has been successfully applied to the Wemoov project as explained in the chapter 4. Nevertheless, we claim that this method can be applied to any road vehicles under some small adaptations. As one can notice, the most of the involved elements are valuable for the purpose of the dynamics optimization of devices other than road vehicles. Two masters thesis projects under our supervision have already used the methodology with some success. The goal of the first one was the parametrical study of a hybrid electric vehicle performances according to its use [60]. The second project aimed at recovering the dynamics behavior of armored limousines after their transformation [55].

We are convinced the application of the method on other well calibrated

projects related to road vehicle optimization could also bring their lot of perspectives of developments.

5.2.2 Wemoov three-wheeler

The perspectives of the Wemoov project are conditional upon the actual road performances. Several auxiliary simulations must be conducted to increase the level of trust in the three-wheeler abilities.

First of all, the question about the drivability of the Wemoov regarding the unconventional torque steering feed-back must be clarified. It must be determined if the motorbike-like behavior is compatible with the steering wheel driving or if, on the contrary, a car-like feed-back must be imposed. Probably that some simulations achieved on the basis of a suitable HITL simulator could answer this unknown. If required, the car-like feed-back could be obtained thanks to a new optimization that takes additional considerations into account. In this case, some losses of performances for the other criteria remain possible. An alternative solution consist in adding a passive or active device inside the steering column to correct the intrinsic behavior. For that purpose, a simple rotational counter spring could perhaps be sufficient. In this context, some additional investigations are required.

Despite encountered difficulties, the success to obtain the model parametrization of the selected tires could be a real advantage in that way. The use of the well fitted relaxation model could also help to determine more accurately the effects of a particular tire setup.

Further analyses are required to examine the Wemoov stability on the entire range of use of the vehicle. As already discussed, additional modal analyses have to be performed for that purposes.

In the same manner, the simulations must be improved in order to determined the impact of uneven and sloping roads on the general behavior and, in particular, on the stability. Due to it narrowness, the Wemoov is particularly subjected to such perturbations.

These numerical validations, however important they might be, do not replace true experimental sessions that only authorize to validate a suitable and safe road handling. In particular, it is presently the only mean to consider the driver feeling, including the inertia feed-back. Due to the tilting motion, such a feeling is very difficult to anticipate. It could be very positive or, on the contrary, outright prohibitive.

The last major unknown concern the reaction in case of aerodynamics perturbations. Due to its large wind surface area in comparison with its lightness and its small support base, the Wemoov could be laterally destabilized. A dedicated analysis must also be conducted in that respect.

Thanks to the complete design of the first demonstrator, the accomplishment of some experiments is well pre-advanced. However, the demonstrator manufacturing could only be envisaged with a new research and development project, more ambitious than the previous one, but, above all, more realistic and more robust. The required means and, in particular, the financial cost of such project must be carefully estimated.

In order to improve the Wemoov concept, some advances for the tilting control could be implemented. The introduction of a STC, forming a HTC by the combination with the DTC is probably an interesting opportunity in order to improve the dynamics response, while cutting down the required capacity of the actuator and the energy consumption [85, 52].

From the market point of view, the lack of a passenger seat remains the major drawback of such unipersonal concept. This point was lengthly addressed in this project. Unfortunately, there is no possible trade-off from the design side. This must be done at the utilization side. Indeed, it is always possible to design a tandem two-seater, tilting three-wheeler but the second seat has a large negative impact on the vehicle characteristics, in particular, on the wheel base and the nimbleness. Thus, mono and bi-seater are clearly two different vehicles, exhibiting different specifications and thus different properties. The performances of the unipersonal three-wheeler can not be preserved in case of a bi-personal one.

Otherwise, on the contrary of some additional features aiming to improve the vehicles performances, a general design reflection centered on the simplification of the implementation should be fruitful. A deep costing analysis about the production of the Wemoov [98] highlights the strict need of such a reflection in order to propose the Wemoov at an acceptable prize.

The concept of narrow vehicle suffers of an inherent inconsistency. On the one hand, its major interest comes from the possibility to drive between stalled traffic lanes. This is valid only for a very small penetration ratio. Beyond a given level, this extra capacity of the road net is also saturated and the legislator must prohibit such driving tricks. On the other hand, the penetration must be as deep as possible to sufficiently increase the production rate in order to reach low costs. This is fundamentally antagonist. If the market exists, the latitude is apparently narrow.

Radical enhancements of the Wemoov implies a reconsideration of the basic specifications. Why a three-wheeler? How much narrow the vehicle must be? Is it really useful to tilt in curve? As one can notice, the tilting whose practical interest is moderate considering the width reduction of tens centimeters only, has required a very complex arrangement to incorporate this tilting DOF and to actuate it. A general hindsight is required. What really want the customers? An approach more functional than characterizing must be conducted. In the same vein, the frenzy for vehicle electrification has completely subrogated a true deep motorization analysis as for many other projects.

It is probable that the Wemoov concept and the other narrow tilting vehicles do not constitute the eagerly awaited revolutionary solution to face the mobility issues. They will be, at best, a piece of the puzzle.

The customers being always more and more interested by new experiences, the Wemoov has perhaps some perspectives as fun vehicle. In this context, the important cost associated to the inside technology is probably less disabling for such a market niche.

5.3 General conclusion

As the main achievement of this work, we have developed a complete methodology to jointly optimize the dynamics performances and the mechanical design of complex systems. Precisely, the method is especially dedicated to the synthesis of road vehicles suspensions and the optimization of their kinematics in order to reach the best road handling despite the constraints arising mainly from the mechanical design.

Based on a smart arrangement of several multi-body numerical processes, the optimization scheme solves the design constraints, simulates and evaluates the vehicle in view of its improvement according to the targeted objectives. This method enables clever trade-offs between the exhaustiveness of the involved road simulations and associated criteria, and the overall computational cost. Thanks to its consistency and efficiency, the hard problem of the road vehicle design can be handled in its wholeness.

The methodology was successful applied on the Wemoov concept: a innovative electric unipersonnal narrow tilting three-wheeler dedicated to urban areas. The dynamics optimization has effectively enabled to satisfy the constraints and to reach the objectives assigned to this vehicle. The upcoming ready-to-build demonstrator will allow to asses the actual road handing of this concept.

Thanks to the proposed optimization strategy, we hope help in the design of innovative vehicles, able to reach the today and future challenges addressed to the mobility.

“Die Liebe zum Erfinden höret nimmer auf.”

“The love of invention never ceases.”

Carl Benz

Appendices

A Other common car suspension arrangements

McPherson suspension

The McPherson suspension consist in fusing wheel carrier and shocks parts giving the *McPherson strut*. The bottom part of the shock cartridge fitted rigidly to the carrier knuckle. At the bottom, the strut-carrier is linked to the chassis by a wishbone, via a ball joint, or by several arms¹ (figure 2). At the top of strut, a compliant thrust bearing realizes the connection to the chassis. In this way, the carrier disposes of the suspension and steering DOF. The greatest advantage of this suspension is the occupied space, particularly easy to plug in a car design. Car-makers also appreciate the lightness, simplicity and cheapness. The longitudinal guidance can come in soft. Due to kinematics, the shocks piston rod is loaded in transverse direction that is avoided in other suspension categories. The shear encountered can cause tightness problem. Moreover, because the strut must avoid the inner side of wheel, the steering axis can't be positioned such a suitable scrub radius (see below) can't be reached. In final, the most troublesome effect is a too strong torque feed back in steering wheel when the engine load is high. It is critic for sportive, high powered, traction car. Despite their worrying drawbacks, the McPherson suspension is extensively used in micro, mini, small, and medium cars, often on the front axle.

Facing the problem of incorrect scrub radius, an successful improvement consists in uncoupling the suspension and steering motion by adding a pivot internally to the carrier, between the strut and the spindle knuckle. In this way, the structure becomes serial. It allows to position the strut axis and the steering axis independently. The handling performances are improved but the unsprung mass is heavier and the suspension is more complex and expensive. It is a very good example of trade-off between performances, design and cost considerations, that must be adopted to follow overall requirements. This arrangement is called McPherson with independent pivot. It is used in sport series of small and medium traction cars. The figure 3 compares the standard and the independent pivot configurations.

¹Properly speaking, in the original McPherson suspension, a simple arm was used. The use of a wishbone instead of the simple arm allows a better longitudinal wheel guidance. This improvement, actually systematically implemented, is called *pseudo McPherson* suspension.

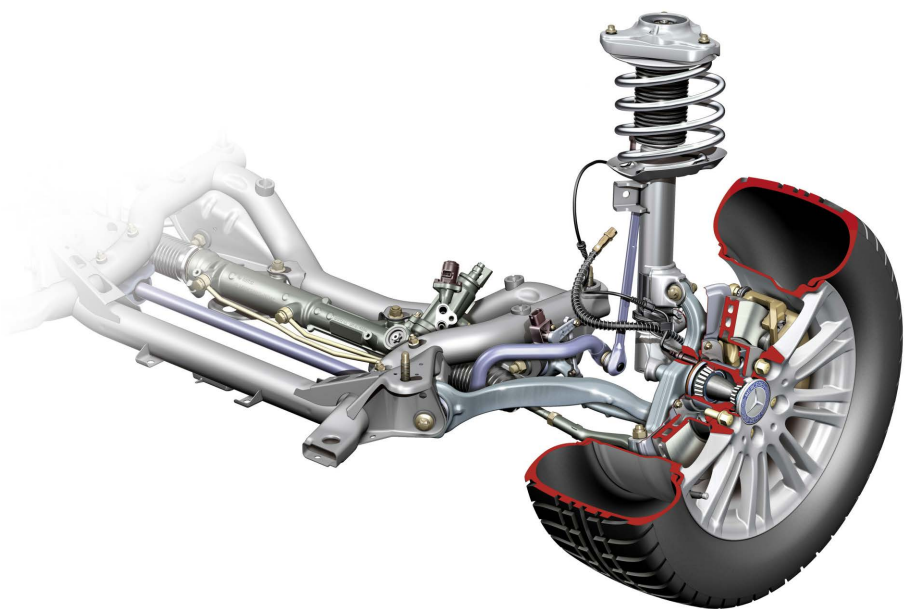


Figure 2: Mercedes-Benz E-class: front McPherson suspension [152].

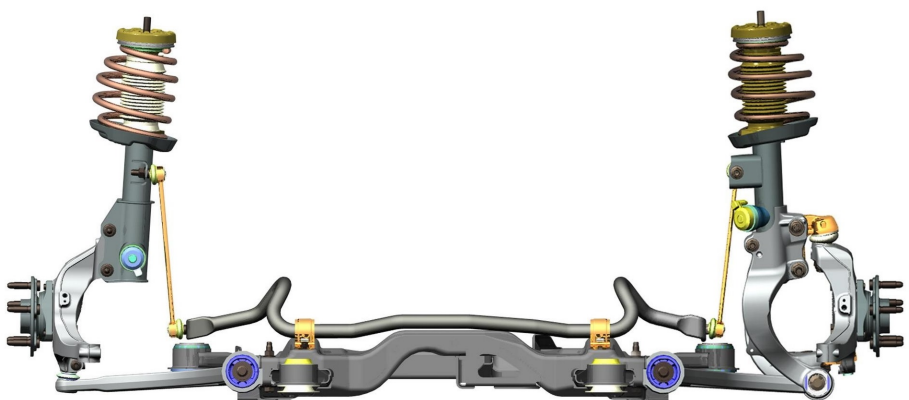


Figure 3: Opel Insignia: standard and independent pivot McPherson [163].

Multi-link suspension

Multi-link suspension is a wide generic name. The principle is simple. The carrier which have six DOF when free, is linked to the chassis with bi-articulated links which constrain the carrier. The number of required links depends on the target remaining DOF for carrier, one for the suspension, plus eventual one for the steering and from the nature of the link connections. A ball-ball, ball-pivot, pivot-pivot link suppress respectively one, two, three DOF¹. Lots of particular kinematic morphologies can be imagined. The already presented double wishbone is a particular case of these categories. An other commonly encountered suspension is the *five arm* suspension as depicted in the figure 4. In this case, there are five double ball articulated arms, including an eventual steering rod. It is the suspension that propose the larger number of design parameters and thus the more points of optimization and setting in order to obtain the suitable kinematic characteristics. With respect to double wishbone suspension, the five arm is potentially more efficient and compact but undoubtedly more complex and expensive. The five arm suspension is plugged in front or rear axle of premium car, sedan, limousine, essentially for road applications.

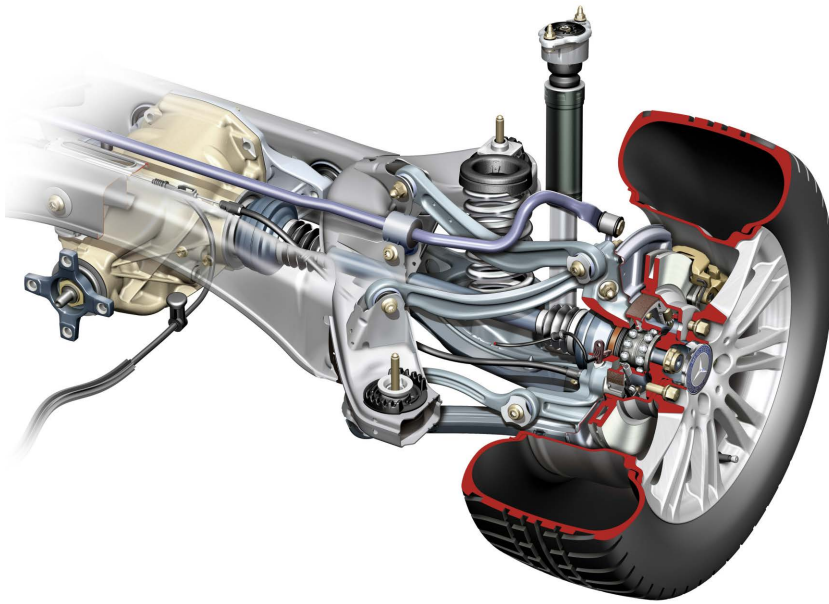


Figure 4: Mercedes-Benz E-class: rear multi-link suspension [151].

¹Constraint redundancy can arise such the number of lost DOF is reduced.

Semi-rigid axle

The last suspension presented here is the *semi-rigid axle*, also called the *torsion beam* suspension. As illustrated in the figure 5, each spindle is rigidly fixed on a trailing arm, themselves linked to the chassis through elastomer bushing. The two arms are bonded by a transverse beam. The torsion of the central beam and the deformation of bushing allow the wheel to travel. The central beam endorses thus structural and anti-roll features. The wheel can have different travel but their motion are coupled. In particular, parasitic motions are generated during cornering. This suspension is qualified as dependent. The dynamics performances is low but this suspension is particularly light, enduring, simple and cheap. The ratio price - performance is excellent so this arrangements is used in many micro to medium range traction cars, at rear, often in combination with McPherson for the front axle.



Figure 5: Semi-rigid axle [133].

B Wemoov optimization formulation

B.1 Fixed parameters

ID	Name	Nature	CF1	CF2
1	body_to_fork_x	PC	-1.0300	-1.0300
2	body_to_fork_z	PC	0.1100	0.1100
3	body_to_shock_rr_x	PC	-1.0900	-1.1800
4	body_to_body_nose_dn_z	PC	0.0100	0.0100
5	fork_to_wheel_rr_x	PC	-0.5250	-0.5250
6	fork_to_shock_rr_x	PC	-0.2100	-0.2100
7	fork_to_shock_rr_z	PC	0.1500	0.1500
8	body_nose_dn_to_wishbone_ft_lt_dn_y	PC	0.0000	0.0000
9	body_nose_dn_to_wishbone_ft_lt_dn_z	PC	0.0000	0.0000
10	pendulum_to_shock_ft_lt_x	PC	-0.1700	-0.1100
11	pendulum_to_shock_ft_lt_y	PC	0.0200	0.0250
12	pendulum_to_shock_ft_lt_z	PC	0.1600	0.1700
13	wishbone_ft_lt_dn_to_shock_ft_lt_x	PC	-0.0300	0.0050
14	wishbone_ft_lt_dn_to_shock_ft_lt_z	PC	0.0100	0.0050
15	carrier_ft_lt_to_wheel_ft_lt_y	PC	0.0450	0.0450
16	carrier_ft_lt_to_steering_rod_ft_lt_x	PC	0.1200	0.1200
17	steering_fork_to_steering_rod_lt_x	PC	0.1200	0.1200
18	body_nose_up_to_wishbone_ft_lt_up_y	PC	0.0000	0.0000
19	wishbone_ft_lt_up_to_carrier_ft_lt_x	PC	0.0000	0.0000
20	wishbone_ft_lt_up_to_carrier_ft_lt_z	PC	0.0000	0.0000
21	R3_wishbone_ft_lt_up	FJ	0.0000	0.0000
22	wheel_ft/K_tire	UP	200000	200000
23	wheel_ft/D_tire	UP	10000	10000
24	wheel_ft/r_n_tire	UP	0.2615	0.2615
25	wheel_ft/r_t_tire	UP	0.0600	0.0600
26	wheel_rr/K_tire	UP	200000	200000
27	wheel_rr/D_tire	UP	10000	10000
28	wheel_rr/r_n_tire	UP	0.2615	0.2615
29	wheel_rr/r_t_tire	UP	0.0700	0.0700
30	shock_ft/Z_min	UP	0.2350	0.2350
31	shock_ft/Z_max	UP	0.3040	0.3040
32	shock_ft/K_stop	UP	1000000	1000000
33	shock_rr/Z_min	UP	0.2650	0.2650
34	shock_rr/Z_max	UP	0.3640	0.3640
35	shock_rr/K_stop	UP	1000000	1000000
36	tilting_ctrl/Kp	UP	3000	3000
37	tilting_ctrl/Ki	UP	800	800
38	tilting_ctrl/Kd	UP	200	200

Table 1: Fixed parameters.

C Wemoov analysis complementary results

C.1 Optimization complementary results

The color code used in the plots relative to the transient simulation for the optimization step is given in the table 4.29. The mean and maximum, satisfaction and acceptance levels color code can be found in the table 4.30.

Steering wheel angle

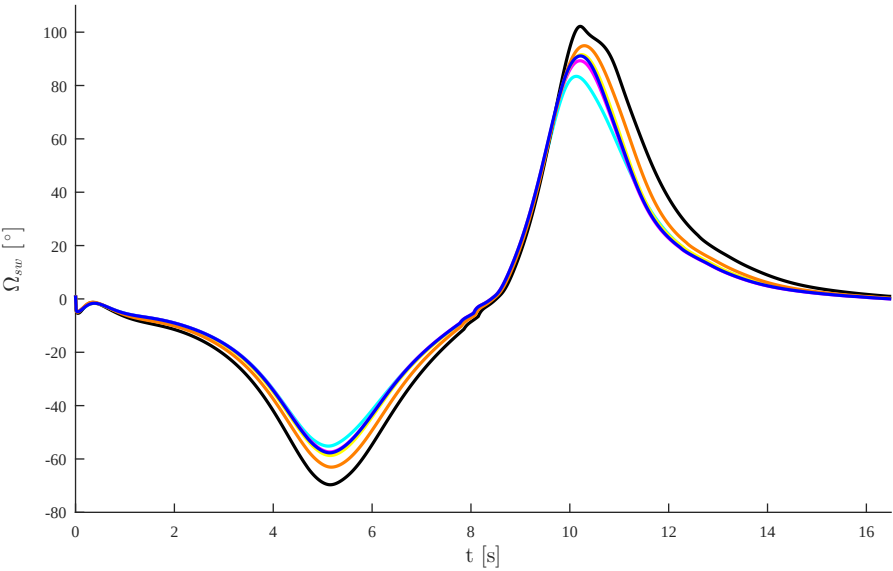


Figure 6: Transient simulation: steering wheel angle.

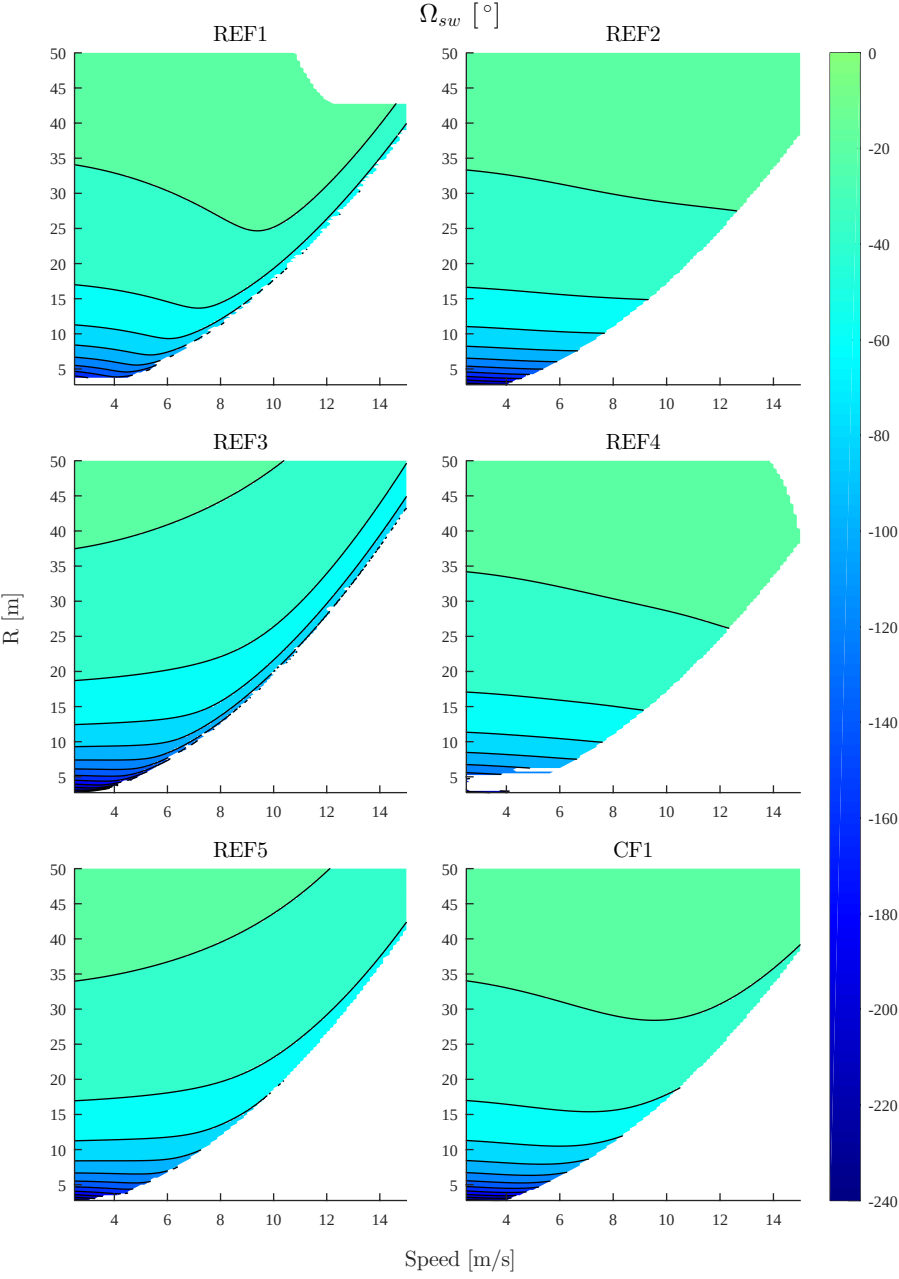


Figure 7: Steady-state analysis: steering wheel angle.

Tilting angle

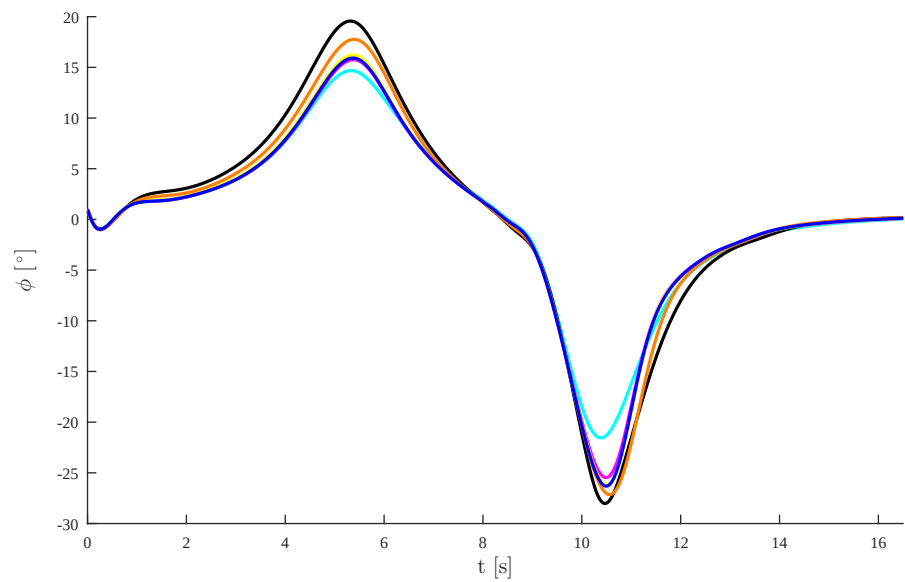


Figure 8: Transient simulation: tilting angle.

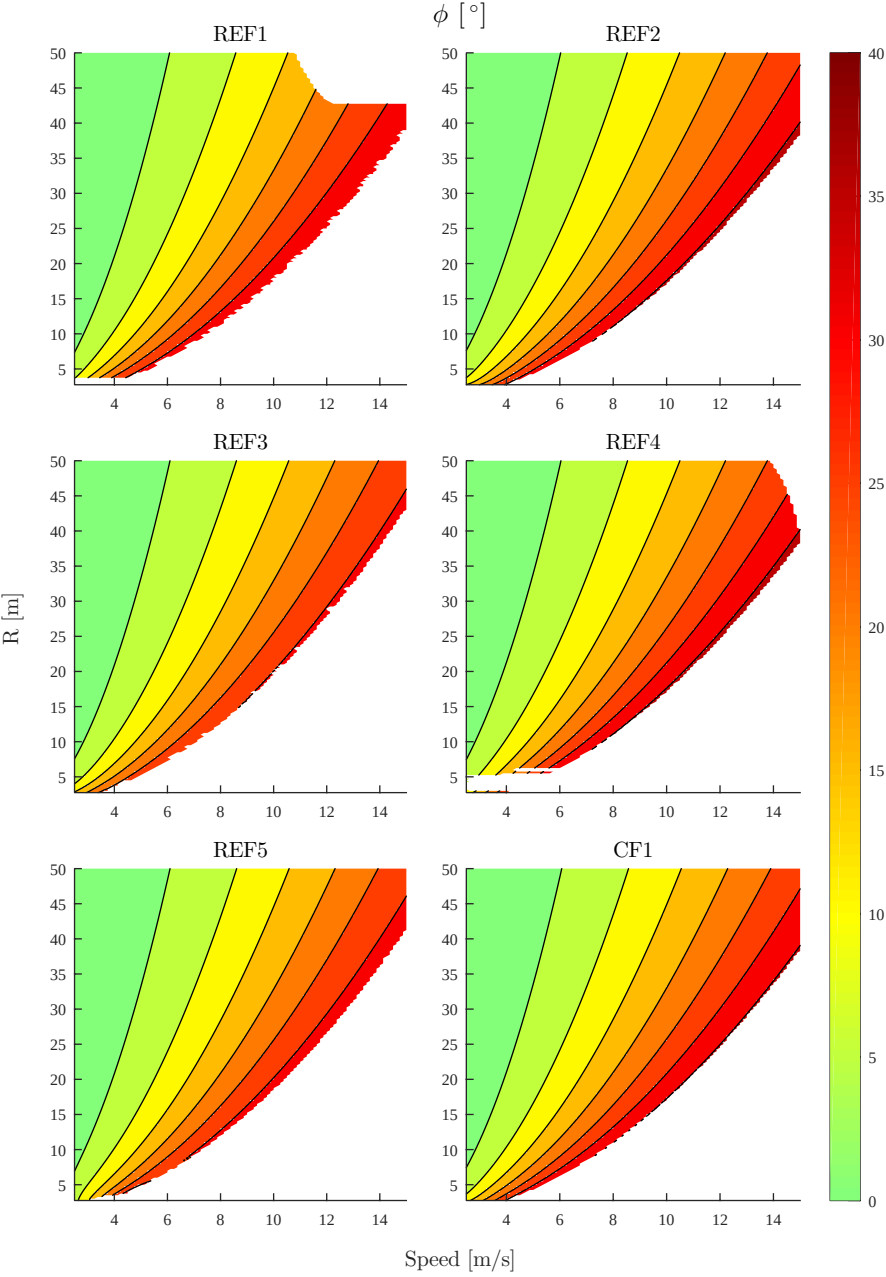


Figure 9: Steady-state analysis: tilting angle.

Attitude

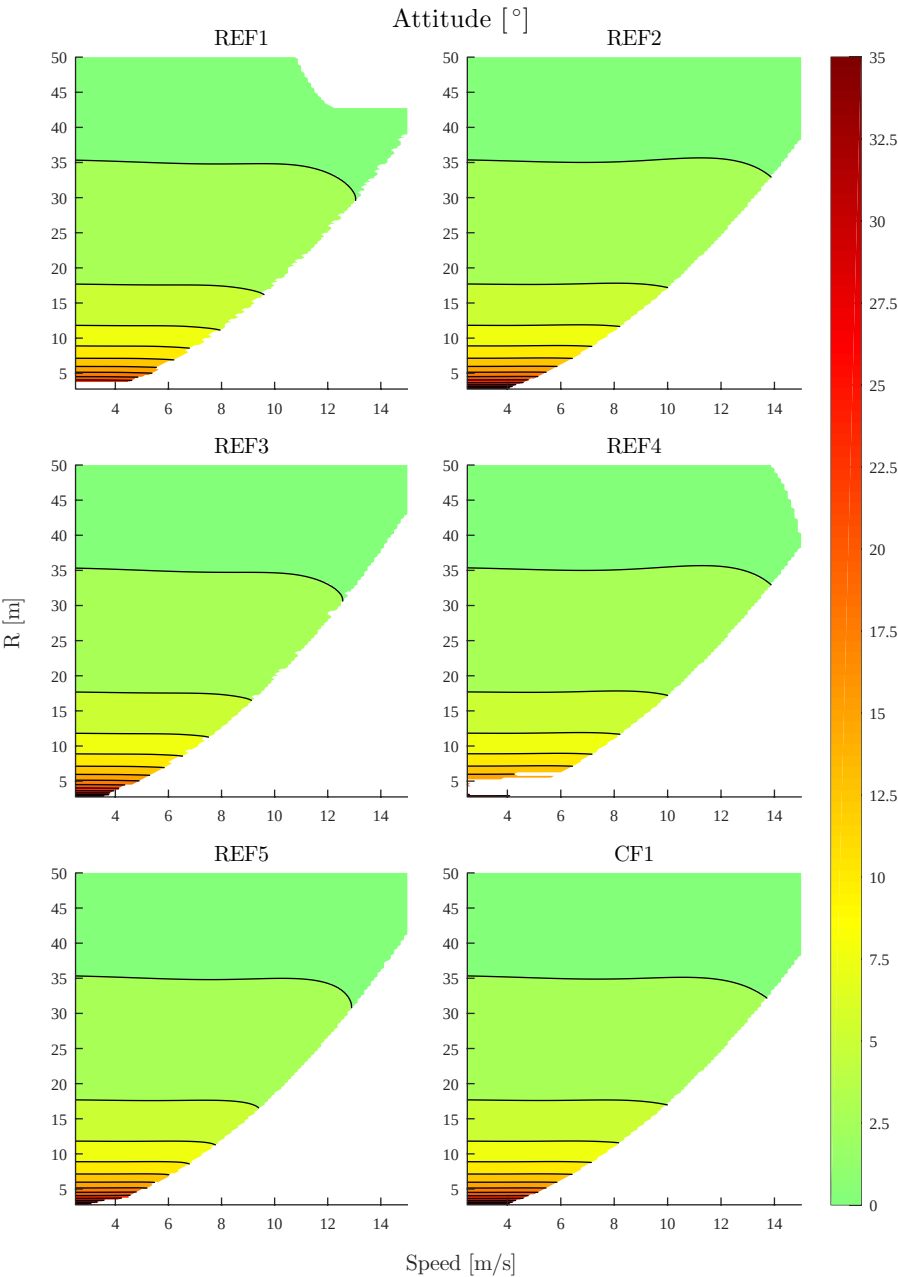


Figure 10: Steady-state analysis: attitude.

Speed

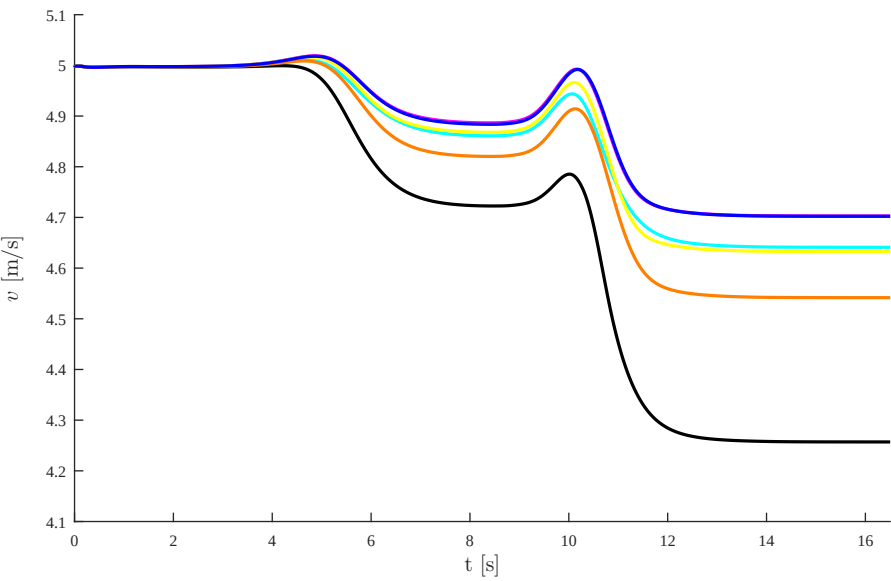


Figure 11: Transient simulation: speed.

C.2 Semi-recovery complementary results

The color code used in the plots relative to the transient simulation for the semi-recovery step is given in the table 4.44. The mean and maximum, satisfaction and acceptance levels code color can be found in the table 4.30.

Steering wheel angle

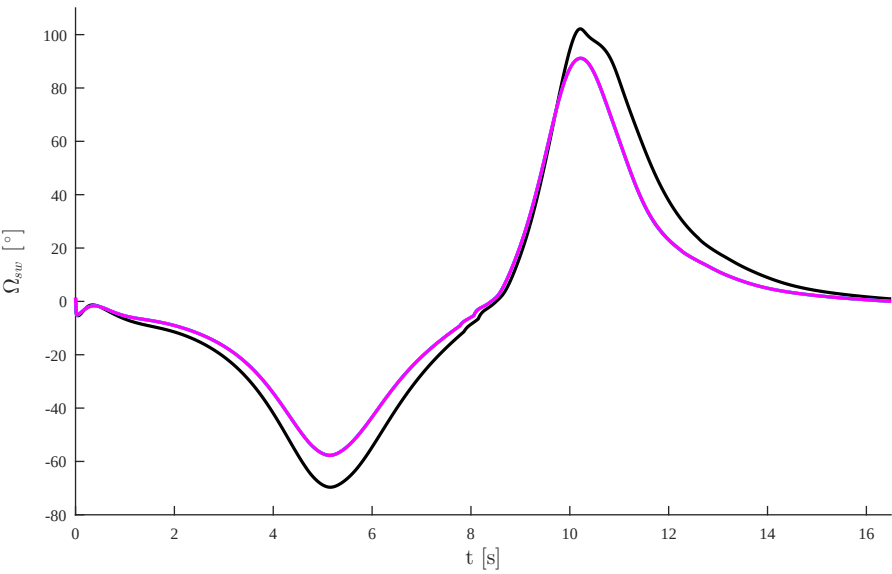


Figure 12: Transient simulation: steering wheel angle.

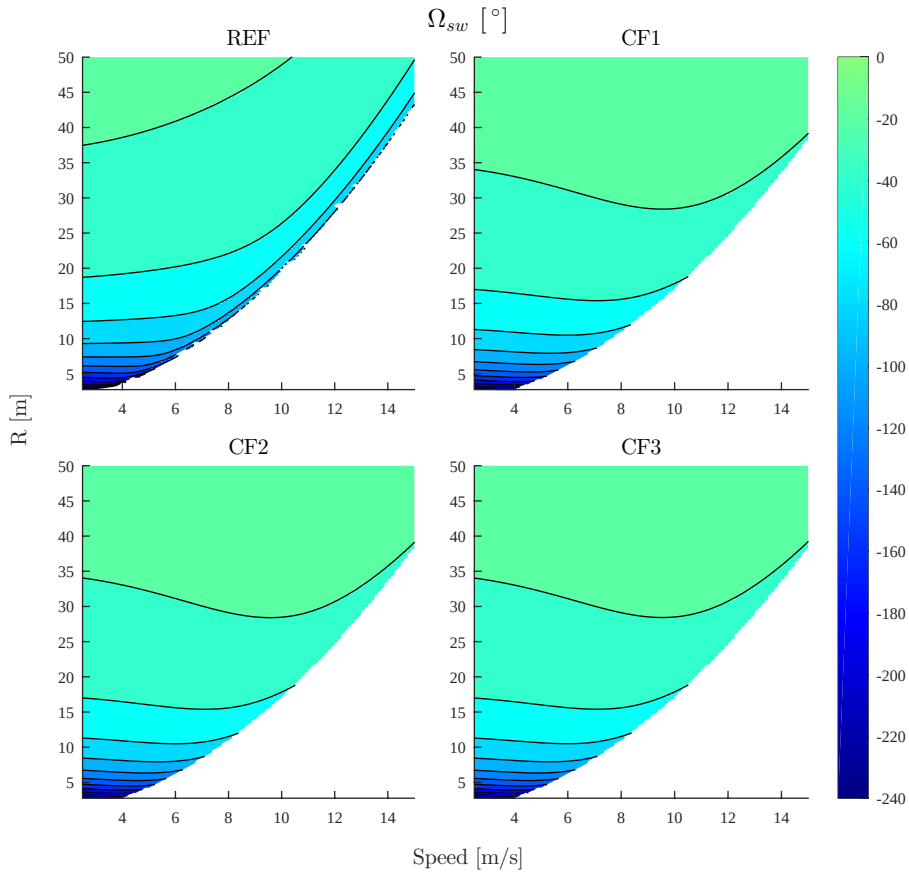


Figure 13: Steady-state analysis: steering wheel angle.

Tilting angle

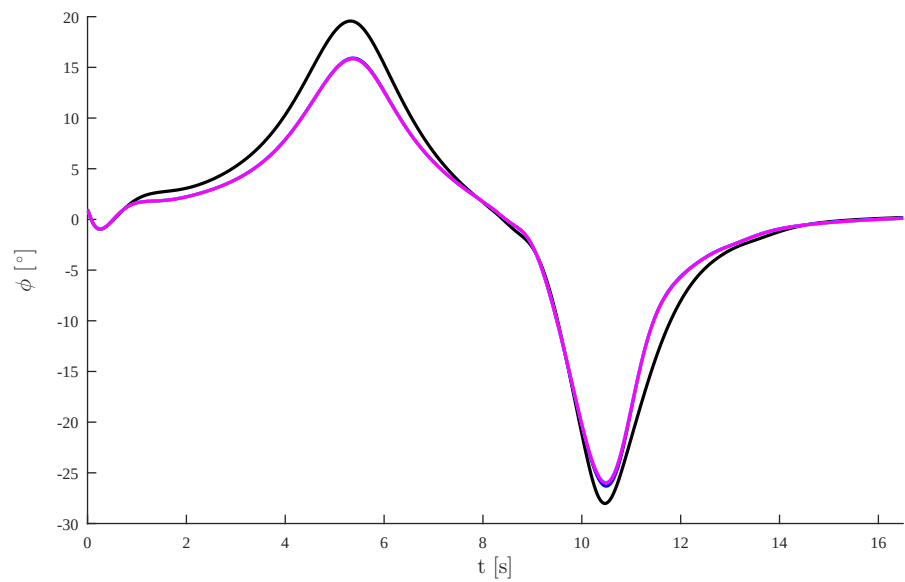


Figure 14: Transient simulation: tilting angle.

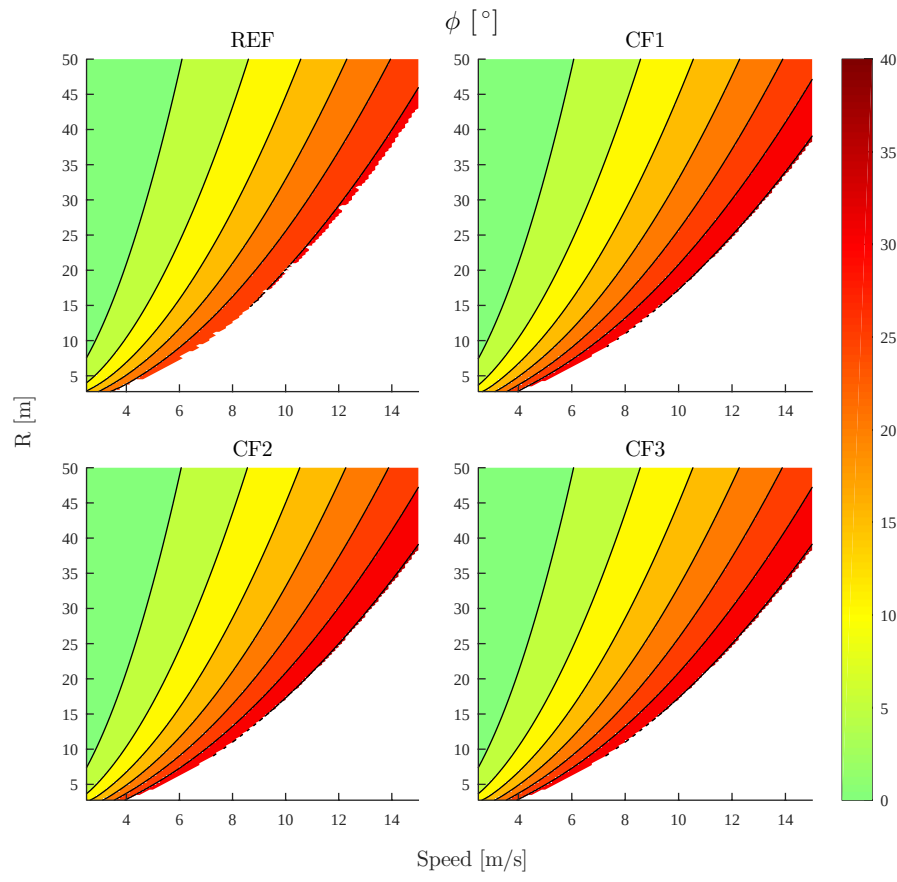


Figure 15: Steady-state analysis: tilting angle.

Attitude

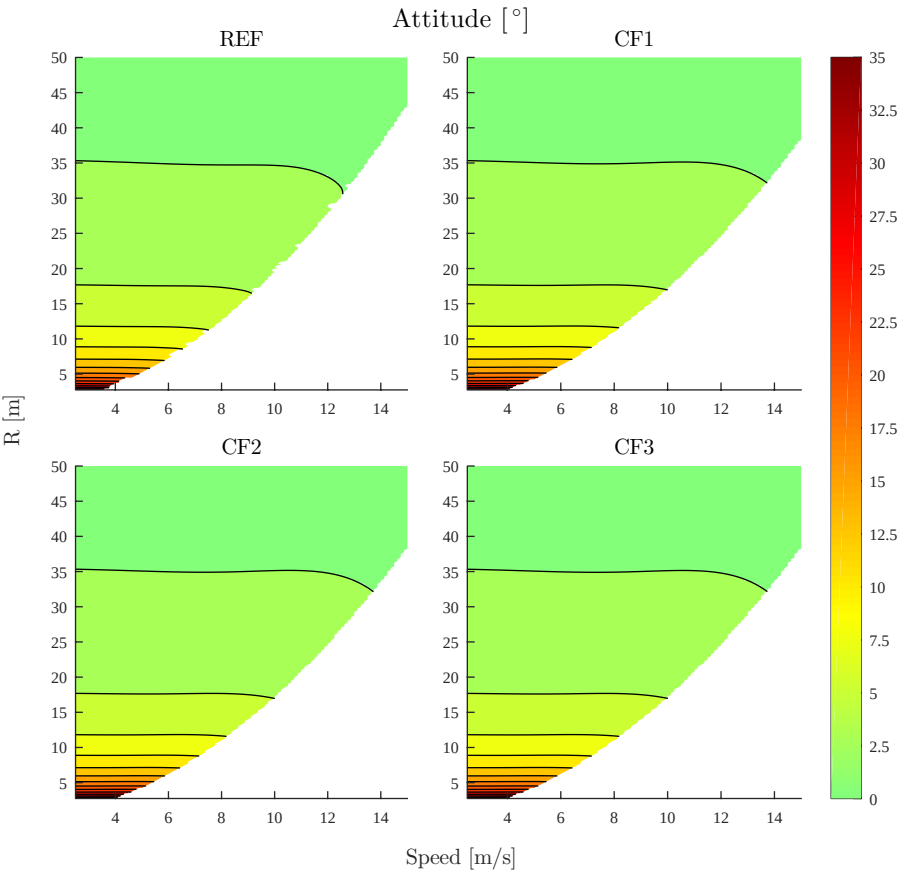


Figure 16: Steady-state analysis: attitude.

Speed

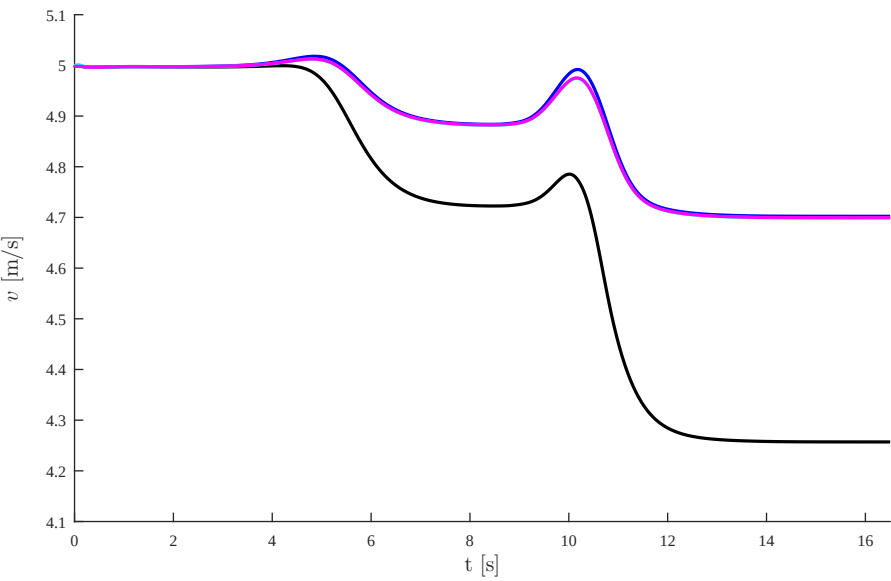


Figure 17: Transient simulation: speed.

D HITL simulation

As a complement to the present work, we want to highlight our development performed in the HITL simulation field. Rather than implementing a steering and speed controller, the action of a true human driver, digitalized through adequate medias, controls the simulation coordinate corresponding to the steering, as well as the motorization and braking torques. Factitious steering wheel and pedals are used as interfaces between the driver and the sensors that transmit the information to the simulation. In this case, the driver receives multiple feed-backs. Firstly, a visual feed-back is given by a screen displaying a driver view of the scene that represent the vehicle rolling on the scenario trajectory. Secondly, a steering torque feed-back is return to the driver through the wheel. Thirdly, a emulation of the inertial feed-back can be provided by a more or less complex actuation of the cabin support. The presence and the quality of such the feed-backs affect largely the human driver behavior ans so, the capability of the HITL simulation to represent the reality.

In our study we develop two different setups:

The first and simplest one is simply formed by a laptop and a commercial gaming wheel¹. The interface between the gaming device and the simulation code is realized via an USB port, from the point of view of the hardware, and via the Simple Directmedia Layer (SDL) library, from the software point of view. This library provides several functions to update the needed information from the wheel and the associated pedals, or from all other kind of computer interface devices. This setup allows to simulate the rolling of the vehicle with the same level of rendition as a 90'S arcade video game. The result is unsatisfactory foremost because the torque feed-back is poor due to the shabby manufacture that characterizes such the devices.

The second implementation², that aims to face the drawbacks of the first one, consists of a wheel mounted on a large synchronous electric motor by a simple hub, without any other transmission. In comparison with the other types of interfaces, this direct transmission arrangement provides the best performances. It is lag-free, virtually friction-free and the inertia is reduced at the most. In return, the device is cumbersome, heavy, expensive and difficult to control. The motor coder acts as the main sensor. The whole³ is managed by the motor driver, themself under the control of an interface board⁴ that links the host computer. In order to reach the suitable level of performances, a specific code architecture has been devised. Multi-tread is used to permit a fast communication up to 500 Hz.

¹The Thrustmaster T 100 Force feed-back was used in combination with a Dell Precision M6700 laptop.

²This setup has been developed by a large team. The mechanical part has been created by Nicola De Lellis and Quentin Materne in the context of their master thesis [24]. The software bonding between the controller and the simulation has been coded by Romain Poletti [72], here again, in the context of its master thesis. Nicolas Docquier and Martin Vanderwegen have also largely contributed to this work.

³The main electric motor is the Scheinder BMH140P12A2A with SinCos Hyperface coder and controlled by the driver Scheinder Lexium LXM32ADM2.

⁴The interface card is a Texas Instrument Industrial Communication Engine AM3359.



Figure 18: HITL simulator.

The bench has been able to characterize the behavior of a serie of drivers during a multi-scenario challenge. We select the kart as the target vehicle because, due to the lack of assistance, it offers a pure steering feed-back. This study proves the importance of the steering torque feed-back in the driver handling. As a consequence, we learn that the synthesize of the STC torque feed-back is very important despite the difficulties of realization.

The HITL simulator are regularly used in vehicle development either as validation or as setting purposes. Despite they never replace real testing, the analyses based on such tools represent a good numerical complement to the optimization process as developed in this work. Ones more, the quality of the device influences the nature of the mission which they can successfully handled. The development of an efficient HITL simulator requires huge technical and financial means, in particular to render the inertial feed-back.

The home-made HITL simulation tool has not been used in the context of the Wemoov project because as for motorbike and other tilting vehicles, the large roll dynamics enforces the need of inertial feed-back, a technique out the mean at our disposal. However, this tool could be used for further development concerning the synthesis of a suitable torque feed-back as discussed in the section 5.2.2.

Beyond the attempts of HITL simulation development, this parenthesis has allowed to improve the simulation time efficiency what benefit the optimization efficiency too. Succeeding to simulate a vehicle in real time with a very good precision proves the high quality of the transient simulator used in the optimization scheme.

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