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Redistributing leisure and consumption

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À la mémoire de Gérard Dispa

General introduction

Les distinctions sociales ne peuvent être fondées que sur l'utilité commune.

Art. 1, Declaration of the Rights of Man and of the Citizen (1789)

Inequalities

HUMANS ARE UNEQUAL. They live unequal lives: some consume more, live longer, work more, earn more, are happier, or study more than others. Societies are made of unequal humans. Since the 1789 *Declaration of the Rights of Man and of the Citizen*, democratic societies are organized so as to eliminate inequalities that do not benefit the general good. The present topic precisely studies this: *what are the consequences of the elimination of some inequalities?* In particular, we will mostly deal with the consequences of canceling some inequalities in consumption and leisure. The thesis shall contain both normative and positive evaluations of these eliminations.

On the positive side, we shall link these eliminations with their consequences for some determinants of consumption and leisure, i.e. inequalities in wages, productivity, employment, participation, and hours worked. Such a positive analysis needs not be merely descriptive, it can also have explanatory powers. In the present thesis, both empirical and theoretical explanations shall be mobilized. In the Popperian tradition, theoretical explanations shall be falsifiable and falsified by the data whenever possible.

On the normative side, we shall mostly focus on the consequences for *welfare*, understood as the metric that guides the assessment of various situations i.e. *l'utilité commune* or the general good. A key feature of our analysis will be to uphold the assumption that individuals are unequal in their tastes for consumption and leisure. When aggregating¹ such tastes to build *l'utilité commune*, we will impose consistency with individual tastes: a change of situation desired by all individuals must improve collective welfare. Two caveats should be raised.

First, in no way will it be assumed that all inequalities should be canceled. Interesting research questions were raised by the study of the elimination of some, but not all inequalities. In other words, we will assume that part of the observed inequalities are fair, but part of them are unfair. Second, in no way should this been seen as an endorsement of the ethical view under study, and as Samuelson (1947, p.220) puts it: "*It is a legitimate exercise of economic analysis to examine the consequences of various value judgments, whether or not they are shared by the theorist*".

In order to study this question, one ought to consider an actor of societies that effectively performs this elimination of inequalities. It is worth pointing out that many actors achieve such role: a kindergarten teacher minimizes the cognitive inequalities for children who received unequal nurturing from their parents while a nurse provides equal care to individuals having unequal genes. A major tool of intervention to perform this elimination is the *legislation* understood the set of rules determined by the society that coerce its members in their lives. I will personalize this concept by referring interchangeably to the *government* or the *planner*.

Overall, the thesis explores only a very small subset of the large array of consequences that arise from the elimination of inequalities. I shall argue below that this subset is a meaningful share of the challenges of our time.

¹The reader interested in epistemological considerations will have understood that we intend to abide by methodological individualism as well as non-paternalism, which characterize most of economics and a large fraction of social sciences.

Leisure and Consumption

In order to understand their relevance for today's challenges, one can put these inequalities in historical perspective. I now describe the long-run evolution of consumption and leisure. As in most of the thesis, I shall mostly focus on high-income countries.

The first stylized fact about the long-term evolution of living standards is the formidable rise of average income over the past century. In Figure 1(a), I plot the average real post-tax income since 1913 which is a good proxy for the average standards of living. This measure has been multiplied by 6 in one century: the average post-tax income was around 12,500 USD in 1913 and exceeds 73,000 USD in 2018.

Figure 1(b) decomposes this national average in two subgroups: the average post-tax income for the top 10% of the income distribution and for the bottom 90%. It turns out that the bottom 90% experienced a similar growth in their post-tax income as the average: from 8,100 USD in 1913 to 50,000 USD in 2018, i.e. around a factor of 6.2. However, the average individual in bottom 90% earns in 2018 what the average individual in the top 10% was earning in 1913.

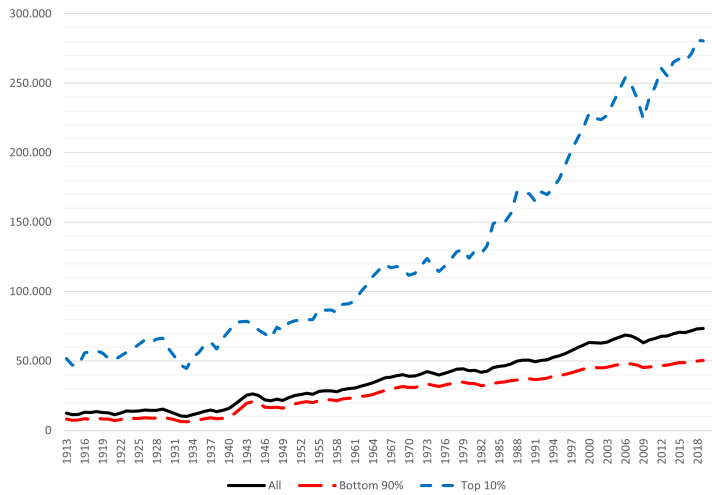
In this thesis, GDP per capita will not necessarily coincide with welfare. This merely come from the fact that individuals' tastes will trade-off leisure with consumption: an individual may become richer by working more but, when this comes at the cost of forgone leisure, it is not necessarily welfare-improving.

Because of this, it is interesting to put this income growth in perspective with the evolution of time devoted to labor. As is traditional, I shall differentiate the intensive and extensive margins of labor, i.e. hours worked spent in employment from the mere fact of being in employment.

Since the Industrial Revolution, hours worked per worker have steadily



(A) Average income per capita, post-tax national income, USA, in constant 2018 USD .
Data from Piketty, Saez, and Zucman (2018)



(B) Average income per capita, post-tax national income, USA, in constant 2018 USD, for the top 10% of the distribution (in blue) and the bottom 90% (in red). The black line coincides with panel (a). Data from Piketty, Saez, and Zucman (2018)

FIGURE 1: Income growth and distribution

decreased around high-income countries, as depicted in Figure 2.² In particular, France and Germany divided by two their annual hours worked per worker between 1900 and 2000: they went from 3000 annual hours to 1500 annual hours, respectively. It is worth pointing out that the United States is an exception in this graph as it does not feature a declining trend after 1945.

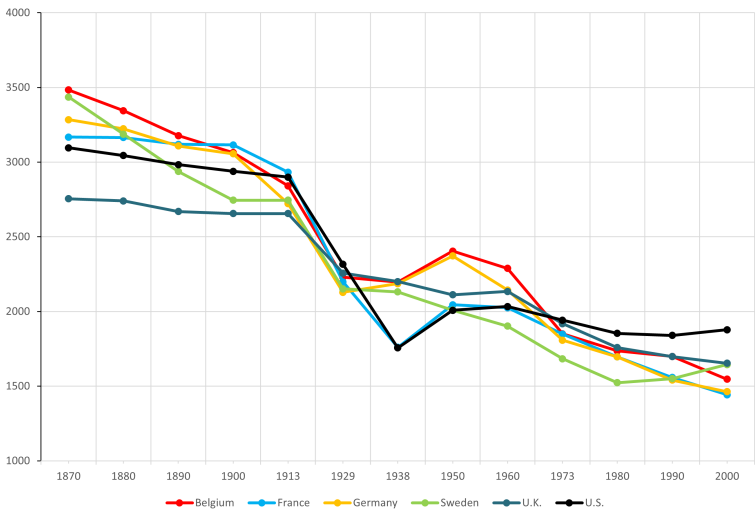


FIGURE 2: Annual hours worked per worker in selected countries retrieved from Boppart and Krusell (2020, Figure 1, p.119) and Huberman and Minns (2007).

As for the extensive margin, we traditionally compute employment rates as the fraction of individuals having a formal job over the total working age population. One of the most striking feature of employment rates since the Industrial Revolution is the spectacular rise of female labor force participation : in the United States, the share of female in formal

²Clark (1994) argues that in pre-industrial societies, workers were working less hours: they were paid per hour worked and had the freedom to choose their schedules while industrial workers externalize the scheduling to their managers. It is also believed that hunter-gatherers societies had more leisure than feudal societies. Hence, over human history, it seems that hours worked had an inverted U-shaped evolution.

employment went from 20% in 1900 to 70% in 2020.

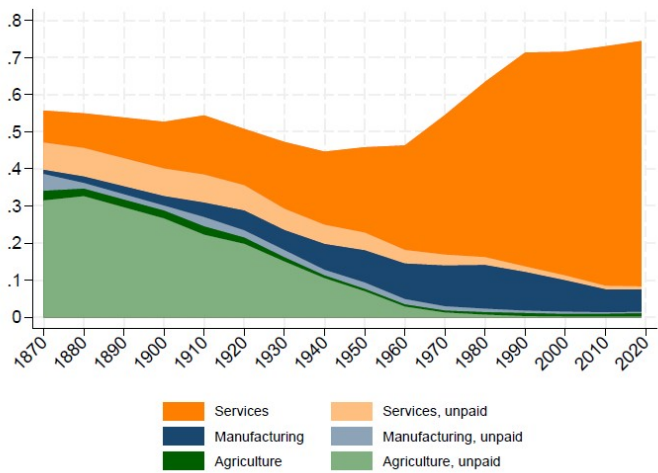
However, recent work by Ngai, Olivetti, and Petrongolo (2024) suggests that this conventional wisdom misses important facts about historical labor markets: many women were working in 1900 but a large majority of them were unpaid. They use time surveys and census data to reconstruct employment rates series accounting for unpaid employment. I report their findings in Figure 3. Strikingly, when accounting for unpaid employment, it seems that female labor force participation did not rise monotonically but evolved in a U-shaped fashion.

In this thesis, we shall bear in mind these facts when building models of the labor market. As a consequence, all models of the labor market in this thesis differentiate the unemployed from the non-participant. Moreover, Chapter 1 will explicitly study governmental intervention targeting inequalities in leisure and consumption among non-participants.

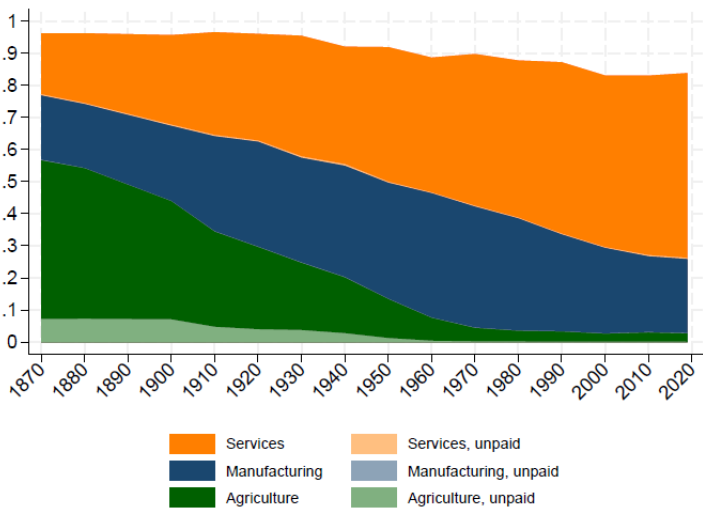
Redistribution

Governments are intervening in economies with a set of policies to perform the elimination of some inequalities. Rodrik and Stantcheva (2021) proposed a typology of public interventions along two dimensions: the income class that it targets (low income, middle class and top incomes) and its precedence in the income-generating process (pre-production, production, or post-production stage).

Although all these nine types of policies deserve to be studied, this thesis will only deal with two. In chapter 1, I study the implementation of a universal basic income and its interaction with traditional tax and transfer programs thereby contributing to the study of post-production stage policies targeting low-income agents. In chapter 2, I study the effect of working time reductions which can be seen as policies affecting the production stage of low-income agents. In chapter 3, I study an abstract post-production problem. Because of this, this thesis can be seen as contributing to the field of public economics which studies the intervention of government in societies.



(A) Female employment rate by industry adjusted for unpaid work, United States, individuals aged between 18 and 64 taken from Ngai, Olivetti, and Petrongolo (2024, p.8, Figure 2).



(B) Male employment rate by industry adjusted for unpaid work, United States, individuals aged between 18 and 64 taken from Ngai, Olivetti, and Petrongolo (2024, p.8, Figure 2).

FIGURE 3: Employment rates by sex accounting for unpaid employment.

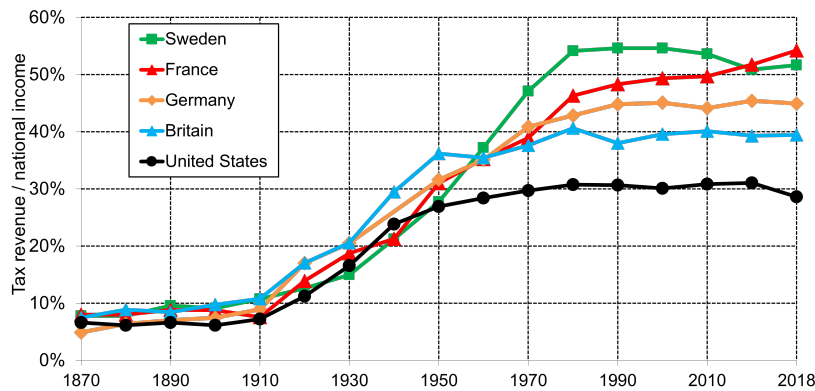
Income	Pre-production	Production	Post-production
Low	Early education	Minimum wage, labor laws	Tax-transfer system
Middle	Higher education, adult training	Industrial policies, collective bargaining	Unemployment insurance, pensions
Top	Inheritance, gift, estate taxes	R&D tax credits, antitrust policies	Top income taxes, wealth taxes, corporate taxes

TABLE 1: Typology of government intervention borrowed from the policy matrix of Rodrik and Stantcheva (2021, p.8).

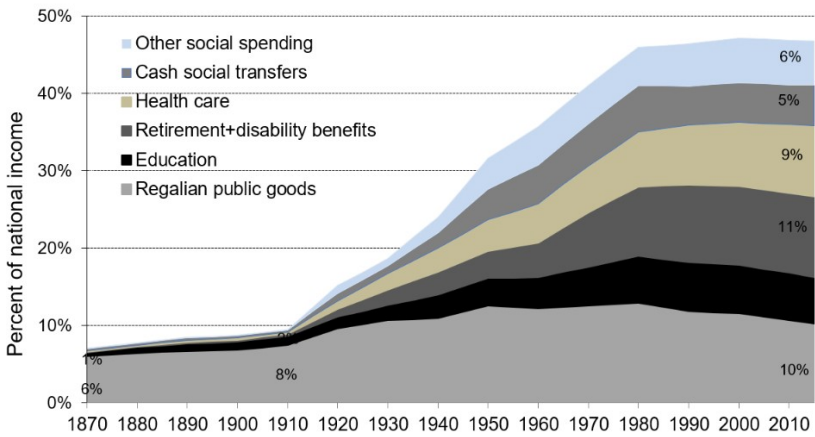
Size of governments

If one ignores the effect of government policies on pre-production stages, one can compute the size of the government by dividing all taxes collected, or equivalently all public expenditures, by the national income. Saez (2021) discussed the evolution of this government size from prehistoric hunter-gatherer societies to modern-day post-industrial societies. He argued that hunter-gatherer societies probably socialized around a half of their income while the first coercive states in sedentary societies collected much less taxes: around 10% of output often in a regressive fashion. As for industrial societies, I plot in Figure 4(a) the size of government for five high-income countries, what Piketty (2019) and Saez (2021) called the rise of the fiscal state: today, France and Sweden collect more than half of their national income in tax revenues.

Governments not only collect money but they also spend it, or redistribute it. In Figure 4(b), I show the evolution of public expenditures by broad spending categories, averaged over the same set of countries excluding the United States. Clearly, the rise of revenue has been spent mostly in health care, education and social spending.



(A) Share of government's revenue in national income over time taken from Piketty (2019, p.535, Figure 10.14).



(B) Share of government's expenditures in national income by categories over time taken from Piketty (2019, p.537, Figure 10.15). These numbers are averaged over Germany, France, the United Kingdom and Sweden.

FIGURE 4: The size of governments over time in terms of tax collection and public expenditures.

I wish now to argue that approximating the governments' sizes by the tax revenues over GDP is an underestimation of their actual importance. This thesis will study production-stage government intervention for which no money is collected, yet the legislation binds the production. In particular, I will study working time regulations in Chapter 2. In Table 2, I compare the historical evolution of the French and American legislation on working time.

France	United States
1841: 12h/day if age $\in [12, 16]$	1872: "8 hours" strike song
1848: 12h/day for all	1896: SCOTUS <i>blue laws</i>
1904: 10.5h/day for all	1933: Nat. Indus. Recovery Act
1919: 48h workweek, 8h/day	1935: SCOTUS cancels NIRA
1936: 40h week, 2 weeks PTO	1938: Fair Labor Standards Act
1956: 3 weeks of PTO	- 40h workweek
1968: 4 weeks of PTO	- <i>Time-and-a-half</i>
1982: 39h week, 5 weeks PTO	- Minimum wage
2000: 35h week	

TABLE 2: Political history of hours regulations: comparison between France and the USA

If one compares Table 2 with Figure 2, one can say that the decline in hours worked pre-dates working time reductions. This should be humbling for the public economists: hours worked are determined by much more than the sole intervention of the government. Table 2 also shows that the United States do not have any federal law mandating paid time off, while France has today five weeks of paid vacations.

In Figure 5, I show the state of working time regulations across countries today, both in terms of days worked in a year and of hours worked in a week. This heterogeneity alone is interesting for the public economist.

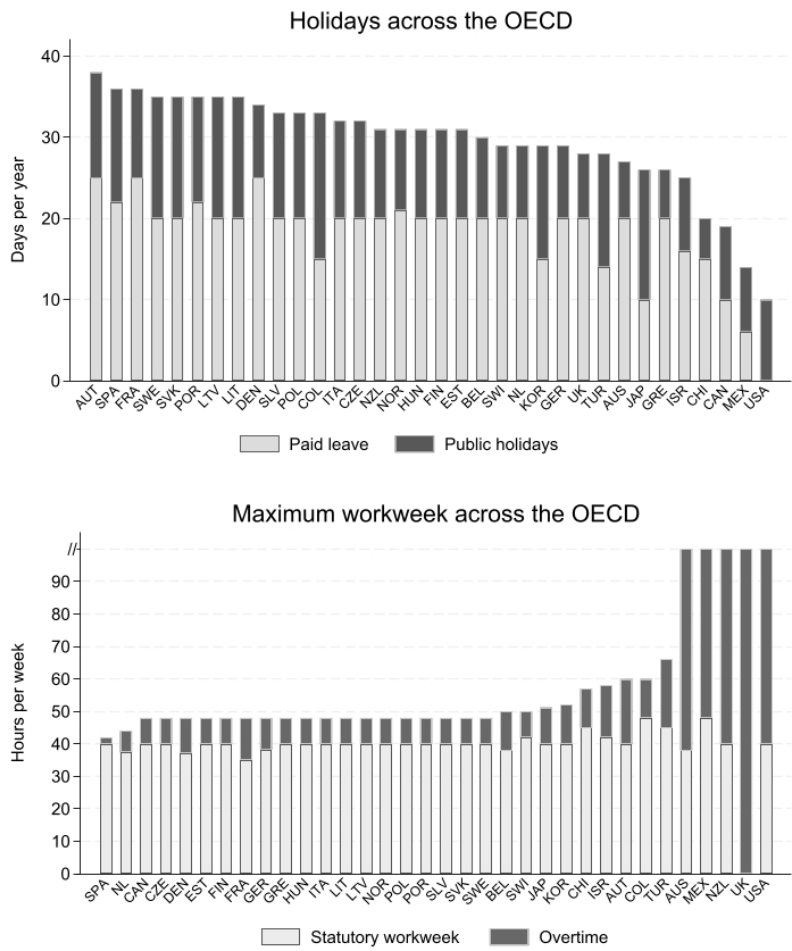


FIGURE 5: Working time regulations in the OECD. Source: OECD (2021).

Emergence of governments

Why did government emerge and reached such large sizes? The traditional answer of economists is that governments step up to repair market failures.³ Indeed, the second fundamental theorem of welfare economics states that any equilibrium on the Pareto frontier can be achieved by a market mechanism provided some individualized transfers are put in place.

While this was often used as a justification to promote market-based solutions, many economists now believe that such theorem is useful for what it does not say: if one assumption of the model is violated, the market fails and governmental intervention may be justified. This is why most textbook in microeconomics now devote a majority of their content to the study of market failures.

In this thesis, I shall mostly study two market failures. In Chapter 1, I will assume that the government is unable to observe the origins of inequality for each individuals. This private information à la Mirrlees (1971) leaves room for the introduction of a redistributive tax-transfer system. In Chapter 2, I will study the possibility of firms' labor market power: if firms are able to set the terms of the contract, the second fundamental theorem fails.

Saez (2021) argues that public economics should go beyond market failures and calls on studying the *social nature* of individuals⁴, as it is the only one capable of explaining the emergence and the size of large social states. By contrast, in this thesis I shall only focus on self-interested individuals who derive pleasure from consuming and distaste from working. I do believe that the present thesis, as a contribution to normative public economics, offers a way to think about governments and self-interested

³It is worth pointing out that recent empirical papers have tested the various theories of government's emergence. In particular, Allen, Bertazzini, and Heldring (2023) shows that its cooperative function was decisive in Mesopotamia.

⁴This concept must be understood as the fact that individuals care about beyond themselves, that is they care about groups.

individuals in a way that is consistent with societal concerns for inequalities. The two central features that allows to retrieve this consistency is (1) the fact that agents are often considered to be unequal in several dimensions in this thesis and (2) the fact that their aggregation as groups is cautiously examined through the lens of welfare economics.

Contributions by chapter

In chapter 1, I study a model of the labor market where there are three possible types of labor market statuses: employed, unemployed and non-participants. Inequalities in consumption and leisure are fully determined by three fundamentals inequalities: tastes, home production skills and wage rates. I consider a government that wishes to eliminate inequalities in consumption and leisure that originate from unequal home production skills and wage rates, but does not wish to eliminate inequalities inherited from tastes.

I derive three main results. First, I fully characterize the welfare metric that the government should use to assess tax-benefit reforms in this labor market. Second, I translate this welfare metric into sufficient statistics. Third, I apply this statistic to the introduction of a basic income in many OECD countries and show that it is unlikely to be desirable for a government with such views on inequalities.

In chapter 2, I study again a labor market with these three participation statuses. However, I now assume that workers do not control their schedules but firms set hours and workers only decide on participation. I explore how firms' labor market power influences this model and show that workers will tend to work long hours when the labor market deviates from perfect competition. I consider a different policy tool to address inequalities in leisure: working time reductions.

I derive three main results. First, the wage effect of a working time reduction are negative in monopsonistic labor market but positive in perfect competition. Second, I test these predictions by evaluating empirically the first-ever working time reduction in Belgium: the 9h maximum workday

in 1910 coal mines. Third, I bring these results in a directed search and matching model of the labor market and quantify that the welfare effects of this policy could be positive only if the value attached to leisure by workers is particularly large.

In chapter 3, I focus on inequalities in consumption and I delve into the issue of measuring consumers' welfare when all inequalities in consumption originate from tastes, incomes, or prices. I consider a government that wishes to eliminate inequalities in consumption due to unequal budgets, but that does not wish to eliminate inequalities spawned by different tastes. The main results show that the welfare metric that is endogenized by such an ethical view is a social welfare function that maximizes a money-metric utility function whose reference situation is given by the average purchasing power in the economy. Contrary to the two other chapters, the results of this chapter do not guide normatively policies, yet they are informative for policy-making. I show applications to the measurement of standards of living, costs of living and purchasing power parities.

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Chapter 1

Basic income versus fairness: redistribution with inactive agents

Some philosophers diverge on whether redistributive transfers to able-bodied inactives would be fair. This paper evaluates their claims. Labor markets feature multidimensional heterogeneity in leisure preferences, disutilities of participation, wages and home production. The social objective champions the ethics of equality of opportunity while upholding the Pareto principle. In the Mirrleesian second-best, it turns out that welfare analysis is reduced to a sufficient statistic. Its empirical application suggests that an inactivity benefit would not be welfare-improving in most high-income countries. Overall, the equity gains of introducing a basic income with respect to equality of opportunity are tenuous, whatever its efficiency costs.

1.1 Introduction

Across developed countries, a flagship feature of safety net programs is that able-bodied inactive agents are not eligible because transfers are conditioned on labor market participation¹. Typically, cash transfers are

¹This is the case for all social assistance programs for the 29 developed economies studied in this paper (MISSOC, 2021).

granted either to active job-seekers through social assistance and unemployment insurance, or to low-income earners via in-work benefits.

However, in recent years, this standard scheme has been criticized by those advocating for the introduction of a universal basic income, which is a social benefit granted on an individual basis without a means test nor any work requirements.² In the labor market, this tax-benefit reform would amount to grant some positive transfers to able-bodied inactives. Because of this equivalence, the paper uses the terms ‘basic income’ and ‘inactivity benefit’ interchangeably.

On the one hand, introducing a basic income may improve equity by reducing the welfare inequality between inactive and active agents. On the other hand, granting some subsidy to inactive individuals comes at the cost of disincentivizing job-seeking efforts of unemployed agents as well as work effort provided by employed individuals. This tension echoes a familiar discussion³ on the trade-off between the equity gains and the efficiency costs of welfare benefits. While the former have been scarcely explored, the latter are subject to considerable disagreement in the literature: labor supply elasticities and the associated efficiency costs of transfers are large in Conesa, Li, and Li (2023), Daruich and Fernández (2024), and Golosov et al. (2024) but small in Cesarini et al. (2017).

In this paper, I measure the equity gains of introducing a basic income *whatever the size of its efficiency costs*. In order to do so, I build a parsimonious model that rationalizes the choice of voluntarily inactive agents. The model displays multidimensional heterogeneity in both preferences and productive skills which allows to capture relevant features of the redistribution problem at hand like disutility of participation and home production.

²Academic supporters include notably Atkinson (1996) or Van Parijs and Vanderborght (2017) and the policy movement includes the Basic Income Earth Network (BIEN) (<https://basicincome.org/faqs/>).

³This tradeoff is the cornerstone of the theory of optimal taxation *à la* Mirrlees (1971).

Yet, as agents are arbitrarily heterogeneous along their ordinal preferences, there are multiple ways to aggregate (cardinally) these heterogeneous preferences in a single social objective. Therefore, the government must gauge the desirability of any tax-benefit system by taking an ethical stance on how individual welfare should be measured, compared interpersonally, and aggregated into a social objective.

I study the case of a government that seeks to equalize opportunities in the economy while respecting the celebrated Pareto principle. In particular, the social objective will be axiomatically constructed under the premise that inequalities in productive skills should be compensated for, whereas inequalities spawned by different preferences should be respected. This compensation (for one's skills)-responsibility (for one's preferences) approach has been pioneered by Fleurbaey and Maniquet (2006, 2007, 2011, 2018) for the standard income tax problem.

Assessing the desirability of basic income with respect to equality of opportunity social welfare is particularly relevant for several reasons⁴. First, recent surveys have suggested that equality of opportunity has received some public support (Saez & Stantcheva, 2016; Stantcheva, 2021; Weinzierl, 2017). Second, these axioms are rooted in a long tradition in political philosophy (Dworkin, 1981; Fleurbaey, 2008; Rawls, 1971; Van Parijs, 1995, 2021). Third and most importantly, philosophers sharing closely related ethical standpoints disagree on the policy recommendations that it entails⁵. Rawls (1988) argued that his difference principle does not justify that *Malibu surfers should be fed* as they enjoy so much leisure that they are not among the worst-off. Van Parijs (1991) countered that one should not take a stance on what a good life is, and granting a basic income would allow anyone to enjoy as much leisure as one wishes, including the worst-off, thereby providing Rawlsian justification to basic income.

⁴Let me note that I do not aim to endorse a single view of social welfare but rather to link a practical policy recommendation with transparent ethical underpinnings.

⁵Notoriously, neither Rawls nor Dworkin endorsed themselves the view that an inactivity benefit would be justified by their theories of justice. As Dworkin (2000) puts it "*forced transfers from the ant to the grasshopper are inherently unfair*" (p.329).

These fairness axioms endogenize the social objective. It is then transposed in a second-best Mirrlees (1971) environment where the government collects distortive (nonlinear) taxes on employed agents in order to finance transfers that may be decomposed into social assistance (e.g. TANF), in-work benefits (e.g. EITC), and an inactivity benefit. The exercise yields two analytical results which are the main contributions of this paper.

First, I characterize the optimal inactivity benefit. Despite multidimensional heterogeneity in the model, it follows a surprisingly simple additive formula from which one can derive qualitative properties and perform comparative statics. In particular, whenever the public finance constraint is marginally relaxed, the inactivity benefit covaries positively with the traditional social assistance, suggesting that basic income should supplement rather than crowd out existing transfers programs.

Second, I derive a sufficient statistic for the desirability of any tax-benefit reform, even in a suboptimal world⁶. Because the model does not impose any structural assumptions on primitive preferences and abilities, this sufficient statistic holds for any empirical correlations between them. The sufficient statistic guides the government on where it should spend any extra dollar of transfers to achieve a welfare improvement. In particular, it prescribes that this extra dollar should be spent in transfers to inactives (resp. actives) whenever inequality among them is larger than inequality among actives (resp. inactives). The inequality measure pinned down by the axioms is crudely the difference between the worst consumption and the average production for each sector.

Whether introducing a positive inactivity benefit in our economies is welfare-improving and/or optimal is ultimately an empirical question. While values for average production among actives are readily available statistics, estimating the inactives' average home production surplus is

⁶Second-best optimum might be unreachable for real-world tax-benefit systems because of political economy constraints (Bierbrauer, Boyer, & Peichl, 2021) or more generally because actual governments do not behave like the Mirrleesian planner (Stantcheva, 2016). Hence, policy recommendations of practical use are more likely to emerge from the study of welfare-improving reforms.

a challenging task. I elicit conservative bounds on home production by exploiting recent data from the *Global Survey on Working Arrangements* (G-SWA) on time savings when working from home (Aksoy et al., 2023). I combine these estimates with data on current tax-benefit systems on childless singles and lone parents (OECD, 2020) to compute the sufficient statistics for 29 developed economies.

Even under a series of conservative assumptions, the empirical application finds that the worst inactive is closer to the average home production than the worst active is to the average wage. As a result, all 29 governments should first increase transfers to actives before any dollar spent on inactivity benefit constitutes a welfare improvement. This is equivalent to say that introducing a basic income would not be welfare-improving in all those countries.

Next, I quantify lower bounds on these increases in transfers to inactives to justify any dollar of basic income. I find that their magnitude are almost always unrealistically large: on average, governments should at least triple the safety net before any dollar of basic income is welfare-improving. In sum, the inquiry shows that either the overall amount of social transfers is much too low in all developed economies, or granting an inactivity benefit cannot be welfare-improving.

Overall, this paper suggests that the equity gains of granting some benefits to voluntarily inactive agents are tenuous. This holds against a series of conservative assumptions: the social objective has most extreme inequality-averse assumptions and the government does not have a preference for labor market production over home production.

This demonstrates a normative tension between the allowance of basic income⁷ and equality of opportunity. Hence, the present analysis suggests that a government wishing to fight unequal opportunities outside of the

⁷I only study the desirability of the conditionality of social benefits to labor market participation, while basic income proposals additionally requires waiving conditionalities to means and to the household composition (Van Parijs, 1995). Arguably, the desirability of an inactivity benefit is a first step for the study of the desirability of a fully fledged basic income.

labor market should do so by providing better opportunities within the labor market when the latter is much more productive.

Literature

The study of conditionality of welfare benefits has a long history in economics⁸ and basic income has been recently studied by e.g. Banerjee, Niehaus, and Suri (2019), Conesa, Li, and Li (2023), Daruich and Fernández (2024), Ghatak and Maniquet (2019), Golosov et al. (2024), and Hoynes and Rothstein (2019). However, most papers consider utilitarian welfare or do not exploit heterogeneous tastes. The main contribution of the present paper is to derive a simple and estimable sufficient statistic to guide policy-making even under multidimensional heterogeneity in preferences, home production and wages. This is used to characterize the difference of transfers that should hold between unemployed and inactives.

The paper also contributes to optimal taxation theory. In the canonical Mirrlees (1971) model, agents may only react to tax-benefit reforms by decreasing their hours worked i.e. on the intensive margin. This class of models has been amended to allow for participation decisions in Choné and Laroque (2005, 2011), Diamond (1980), Hansen (2021), Jacquet, Lehmann, and Van der Linden (2013), and Saez (2001, 2002). However in these *pure* extensive margins models, there is no difference between an unemployed and an inactive. Several papers have then added search frictions to rationalize involuntary unemployment together with endogenous participation decisions (Hungerbühler & Lehmann, 2009; Hungerbühler et al., 2006; Jacquet, Lehmann, & Van der Linden, 2014; Lehmann, Parmentier, & Van der Linden, 2011). Yet, they do not allow the government to distinguish the transfers it gives to the inactive from the one to the unemployed.

⁸See Besley and Coate (1992b, 1995), Boadway and Cuff (2014), Boadway, Cuff, and Marceau (2003), and Boone and Bovenberg (2013) among many others.

Boadway and Cuff (2018) allow the government to differentiate the transfers to nonparticipants from the transfers to the (involuntary) unemployed. Nonetheless, their model assumes homogeneous preferences, no intensive margin and a piece-wise linear income tax schedule while the present paper relaxes all these three assumptions. Kroft et al. (2020) also operates this differentiation and do not have any of the aforementioned shortcomings. Let me pinpoint three main differences with that paper. First, they model wages as endogenously determined in general equilibrium while in the present paper, wages are left exogenous. Second, they have a (Bergson-Samuelson) weighted utilitarian social welfare function whereas I consider an Arrowian social ordering function that reflects the ethics of equality of opportunity. Third, they focus on the derivation of the optimal tax system but I will also look at welfare-improving reforms, even in a suboptimal world. As I model a home sector⁹ and a formal sector, the paper is related to the Mirrleesian optimal tax derivation of Rothschild and Scheuer (2013) in the multi-sector Roy (1951) model. The important difference is that they assumed that the tax schedule is uniform across sectors while I do not. In particular, it will be assumed that outcomes from the home sector are unobservable, and that the government can only give a lump-sum amount to all inactives. Beaudry, Blackorby, and Szalay (2009) also assume that home production is unobservable but hours worked are observed and they focus on the difference between social assistance and unemployment insurance.¹⁰ In contrast here, I focus on the difference between social assistance and inactivity and assume that gross income are observed along with the activity binary decision.

The observability of the binary activity decision supposes that there exists a screening mechanism that the government uses to differentiate inactives from actives. In the main body, this screening mechanism is costless and perfect but we discuss the relevance of the findings for the real world where that device is costly and fuzzy in sections 1.5.1 and 1.6.

⁹Gayle and Shephard (2019) introduced home production in an optimal income tax model. However, their focus is different as they estimate a large structural microeconomic model, with a marriage market and they focused on the jointness of spouses taxation.

¹⁰For a more recent treatment of the redistribution versus insurance problem in cash transfers and unemployment insurance, see Ferey (2022).

The question of the design of ordeals is left aside and has been treated by e.g. Kleven and Kopczuk (2011) and Rafkin, Solomon, and Soltas (2024).

I outline two additional differences with standard optimal tax papers. First, the literature since Saez (2001, 2002) typically studies small local tax reforms. By contrast, this paper derives a sufficient statistic that can assess any tax reform, including (potentially suboptimal) large and global ones such as the introduction of a basic income. Second, this paper does not need to impose any structural assumption on preferences¹¹ nor any correlation between heterogeneity dimensions, while optimal tax papers typically do so to solve the multidimensional screening problem. Both facts are possible thanks to the axiomatic derivation of a social objective as a transitive ordering between any two allocations i.e. any two tax-benefit systems. This endeavor has been inspired by the fair income tax literature (Fleurbaey & Maniquet, 2006, 2007, 2011, 2018). The present paper also contributes to the latter by including inactive agents as well as additional dimensions of heterogeneity and deriving the axiomatic characterization in this new environment.

In section 1.2, I formalize the environment. In section 1.3, I construct the social objective in the first-best. In section 1.4, I introduce the Mirrleesian second-best environment and derive the main theoretical results. In section 1.5, I present the empirical application. In section 1.6, I assume a stigma associated to benefits recipients while in section 1.7 I conclude.

1.2 Model

There is a finite set $\mathcal{I} = \{1, 2, \dots, I\}$ of agents whose generic element is denoted by i . There are only two goods, consumption and labor, forming a bundle denoted by $(c_i, l_i) \in X$. The homogeneous consumption good $c_i \in \mathbb{R}_+$ is produced either in the home sector or in the labor market.

¹¹Dworczak et al. (2023) mechanism design exercise finds a sufficient condition for the transfers to those meeting ordeals to be larger than transfers to those not meeting ordeals if preferences are quasilinear. In the present paper, there are no quasilinear restrictions. Moreover, the optimal transfers exercise precisely quantifies the optimal difference of transfers between inactives and unemployed.

The labor supply variable l_i is set to -1 when the agent stays at home, or takes value of hours worked in a normalized interval $[0, 1]$ when the agent is in the labor market. Hence, an inactive agent has $l_i = -1$, an unemployed agent has $l_i = 0$ and an employed agent has $l_i \in (0, 1]$.

Each agent is endowed with a convex preference ordering $\succsim_i$ that can be represented by a continuous ordinal utility function $u_i(c_i, l_i)$ which is strictly increasing in c_i and nonincreasing in l_i over $[0, 1]$. An agent has an idiosyncratic disutility of participation¹² to the labor market whenever $u_i(c_i, -1) - u_i(c_i, 0) \geq 0$ which captures the utility loss for an inactive that becomes unemployed while keeping the same level of consumption. Importantly, the disutility of participation must be distinguished from the willingness to work¹³. In this framework, disutility of participation embodies a preference to produce at home rather than in the labor market, while willingness to work reflects the substitutability between consumption and hours worked on the labor market.

In addition to their preferences, agents are also heterogeneous along their vector of innate productive abilities $(w_i, h_i) \in [\underline{w}, \bar{w}] \times [\underline{h}, \bar{h}] \subseteq \mathbb{R}_+^2$, where the first coordinate denotes the marginal productivity used on the labor market and the second coordinate captures the *surplus* of home production that inactivity allows for relative to activity, not its *level*.

In the labor market, I retain the standard assumption of a constant return to scale technology whose sole input is hours worked, and its marginal productivity is given by w_i . In the home sector, h_i captures the surpluses of production that active agents loose by joining the labor market. It is a reduced-form term for outcomes of activities such as gardening, child rearing or housekeeping. Obviously in reality all agents produce at home, be they inactive, unemployed or employed. Hence, it

¹²It is known that the disutility of participation displays substantial heterogeneity in the cross-section of households (Kaplan & Schulhofer-Wohl, 2018). Here, it may capture (but it is not restricted to) the stigma utility cost of welfare conditionality as in Friedrichsen, König, and Schmacker (2018) and Moffitt (1983), see Appendix C. .

¹³In this setup, the willingness to work may be approximated by the marginal rate of substitution over non-negative values for l . A low (resp. high) marginal rate of substitution in absolute value reflects a high (resp. low) willingness to work

is implicitly assumed here that all actives produce an identical level at home, normalized to 0 (i.e. the first-best Laissez-faire consumption of unemployed agents) and inactives produce h_i more than them¹⁴.

In the first-best, the second fundamental welfare theorem prescribes that efficient redistribution could be achieved through lump-sum transfers. Denoting these transfers by t_i , the first-best budgets, illustrated in Figure 1.1, are defined by :

$$B(t_i, w_i, h_i) = \left\{ (c_i, l_i) \in X : c_i \leq a(l_i)w_i l_i + (1 - a(l_i))h_i + t_i \right\}$$

where $a(\cdot)$ is an indicator function that takes value 1 if the agent is active (i.e. $l_i \in [0, 1]$) or 0 if the agent stays at home (i.e. $l_i = -1$)¹⁵.

Observe that when $a(\cdot) = 1$, this model nests the standard linear production model used e.g. in optimal taxation theory. Hence, agents can react to tax reforms along three margins in this model: (i) intensive (by decreasing hours worked when employed), (ii) standard-extensive (by switching from employed to unemployment), and (iii) participation-extensive (by switching from activity to inactivity). Four important remarks must be raised.

First, this model allows for both involuntary and voluntary unemployment. In the former case, the (primitive) state of the labor market nullifies the productivity of some agents (such that $\underline{w} = 0$) which can remain active with $l_i = 0$. By contrast, voluntary unemployment arises when $l_i = 0$ is the utility-maximizing choice of an agent endowed with some positive wage rate $w > 0$ ¹⁶.

¹⁴In the empirical section 5, I argue that, while absolute levels of home production are typically unobservable, reasonable bounds for home surpluses h_i may be found.

¹⁵Observe that the use of this indicator function renders moot the actual value of l_i when inactive as long as it is not in $[0, 1]$. The choice of -1 is arbitrary and harmless.

¹⁶In the first-best, unemployment (both voluntary and involuntary) only happens when $h = 0$. In the second-best, unemployment may happen when transfers to unemployed are larger than transfers to inactives, which is the empirically relevant case.

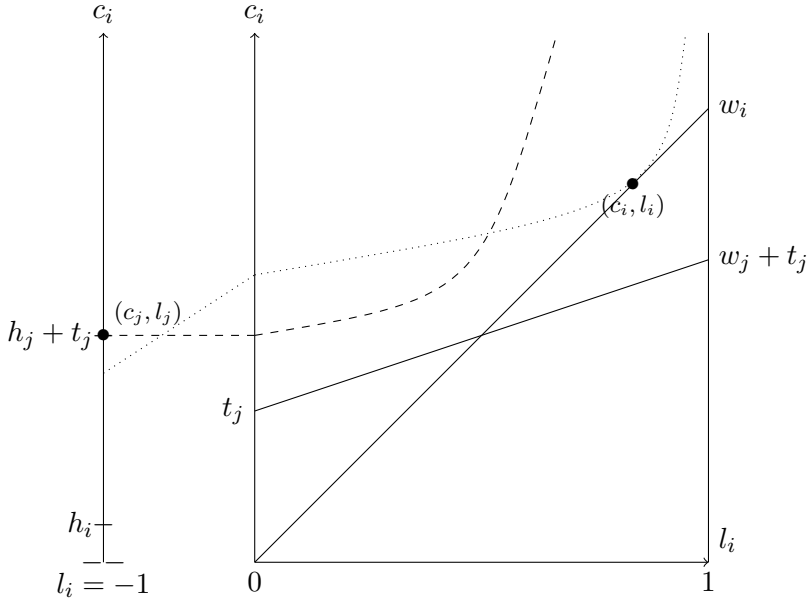


FIGURE 1.1: Illustration with two agents i, j such that $w_i > w_j$ but $h_j > h_i$. Dotted agent i receives no lump-sum transfer and has a positive disutility of participation, but still chooses optimally to be active. Contrarily, the dashed agent j receives $t_j > 0$ and does not display disutility of participation but chooses to be inactive.

Second, I assume that there is no intensive margin in the home sector¹⁷. Of course, in reality agents partition their time between leisure, paid work and home production. However, labor market inactivity is a binary status for any tax-benefit system, such that there is no such thing as a part-time inactive in the eyes of the fiscal authority : agents can either be full time in the home sector or not at all in this sector.

Third, inactive and unemployed agents both enjoy the full unit of

¹⁷Moreover, an intensive margin in the home sector would imply that the inactivity benefit distorts effort in the home production which would be unfavorable to the emergence of a basic income. Hence, this modelling assumption is conservative with respect to my result.

leisure. However, only the former are able to produce at home. This is equivalent to say that there is a fixed time cost spent looking for a job when unemployed that can be used productively when inactive. Hence, h_i must be understood (and measured) as the product of this time cost with the hourly idiosyncratic¹⁸ value of production in the home sector. For example, in a legal working week of, say, 40 hours, the inactive and the unemployed do not provide any hours worked and thus enjoy 40 hours of leisure. However, an unemployed must spend, say, 10 hours sending job applications during which the inactive takes care of his children. The inactive's h_i is then the product of $\frac{10}{40}$ with the shadow price of an hour of day care services in that economy. This estimation procedure for h_i is discussed at length in section 1.5.

Fourth, the model assumes away any positive externality of job search. This is again a conservative assumption, in the sense that such externality would lay the grounds for Pigouvian subsidies to the unemployed (or Pigouvian tax on inactivity) in the redistributive problem, which would be unfavorable to the emergence of a basic income. Despite that, I find below that basic income is not desirable such that this result would be even stronger with such an externality.

Note that throughout the paper, no parametric specification of utility functions is imposed. Moreover, I will not impose any correlation between primitives $\succsim_i$, w_i and h_i , neither at the individual level nor in the cross-section. As a consequence, the results are valid *for any* empirical moments observed in the data.

Finally, let me define an economy e by a list of endowments for each agent in each heterogeneity dimension $e = \{(\succsim_i, w_i, h_i)\}_{i \in \mathcal{I}}$. I denote the set of all such economies by E . An allocation is a bundle for each individual and is denoted $(c, l) = \{(c_i, l_i)_{i \in \mathcal{I}}\} \subseteq X^{\mathcal{I}}$.

¹⁸In a structural macro exercise with time use data, Boerma and Karabarbounis (2021) show that (1) inequalities in the home sector are quantitatively important and (2) inequalities in home production efficiency are needed to explain the variance of home inputs conditional on wages and preferences.

1.3 Fair social objective

1.3.1 Characterization

Each economy can yield many allocations which are associated with observable inequalities in consumption-labor outcomes (c_i, l_i) all originating from unobserved heterogeneity in primitives $(\succsim_i, w_i, h_i)$. The key question for a government is: when should an allocation (c, l) be socially preferred to an allocation (c', l') ?

There are infinitely many ways to answer this question. The standard approach used in economics in general and in optimal taxation theory in particular consists in using a weighted sum of utility functions to assess welfare improvements. However, when agents have heterogeneous preferences, such summation is sensitive to the cardinalization of preferences, i.e. to the choice of the particular $u_i(\cdot)$ function that represents $\succsim_i$.

There is no commonly admitted way to choose such a cardinalization. Yet, it implicitly embodies a way to compare welfare interpersonally, and as such it is ethically-loaded. The literature in normative economics has acknowledged that policy recommendations are contingent on an ethical view and as such it is better to visibilize it rather than leaving it to the researcher's choice. But how can we link ethics with a social welfare function?

A popular method in recent years has been to rely on Saez and Stantcheva (2016)'s generalized weighted sum of utilities where weights can be a function of $(\succsim_i, w_i, h_i)$ to reflect different fairness views. While this approach is flexible and tractable, it has a major drawback: it may fail transitivity in its policy recommendation (Sher, 2024).

In the present paper, I build a Social Ordering Function (SOF), i.e. a function that associates to each economy a transitive ordering of allocations.¹⁹ In particular, among the many possible SOF, I will select the

¹⁹This aggregation from individual preferences to social welfare follows the Arrow (1950) tradition and constructs the SOF axiomatically as in the pioneering work of Fleurbaey and Maniquet (2007).

one that respects three key fairness principles: (1) **Weak Pareto**, (2) **Responsibility** and (3) **Weak Transfer**.

Weak Pareto ensures that a Pareto-dominated allocation will never be chosen by the planner. Yet, there are typically many points on the Pareto frontier, and the next two axioms single out the one that reflects equality of opportunity. **Responsibility** requires that in the knife-edge case where all agents have the very same productive endowments w and h , then the *Laissez-faire* allocation (i.e. the absence of transfers) is the social optimum, even if unequal $\succsim_i$ still generates inequalities. As a result, **Responsibility** does not fight inequalities spawned by unequal preferences. Conversely, **Weak Transfer** requires that if two agents have identical preferences and identical labor supply decisions (i.e. identical l_i), a transfer from the richer to the poorer agents is a social improvement. Hence, **Weak transfer** fights inequalities spawned by unequal skills. As argued above, this ethical view may be relevant for the policy debate on basic income.

I now present the social objective function that pursue the ethics of equality of opportunity: it maximins $M_i(c_i, l_i)$. In short, $M_i(c_i, l_i)$ is a money-metric utility function that measures the well-being of agent i when she consumes the bundle (c_i, l_i) as the transfer that renders this agent indifferent between (c_i, l_i) and her preferred bundle in the budget determined by the average productive vector. Loosely speaking, the further away the individual sees herself from an average agent, the worst will be her well-being.

Definition 1.1. *The FSOF considers that (c, l) is socially strictly preferred to (c', l') whenever*

$$\min_{i \in \mathcal{I}} M_i(c_i, l_i) > \min_{i \in \mathcal{I}} M_i(c'_i, l'_i)$$

where $M_i(c_i, l_i)$ is a money-metric utility function defined as

$$M_i(c_i, l_i) = \min t \quad \text{such that } (c_i, l_i) \sim_i \max_{\tilde{\succsim}_i} B(t, \tilde{w}, \tilde{h})$$

$$\text{where } \tilde{w} = \frac{1}{I} \sum_i w_i \quad \tilde{h} = \frac{1}{I} \sum_i h_i \quad \text{and } t \in \mathbb{R}$$

The graphical construction of $M_i(c_i, l_i)$ is illustrated in Figure 1.2 for the case of a two-agent economy where agent i is inactive and agent j is employed.

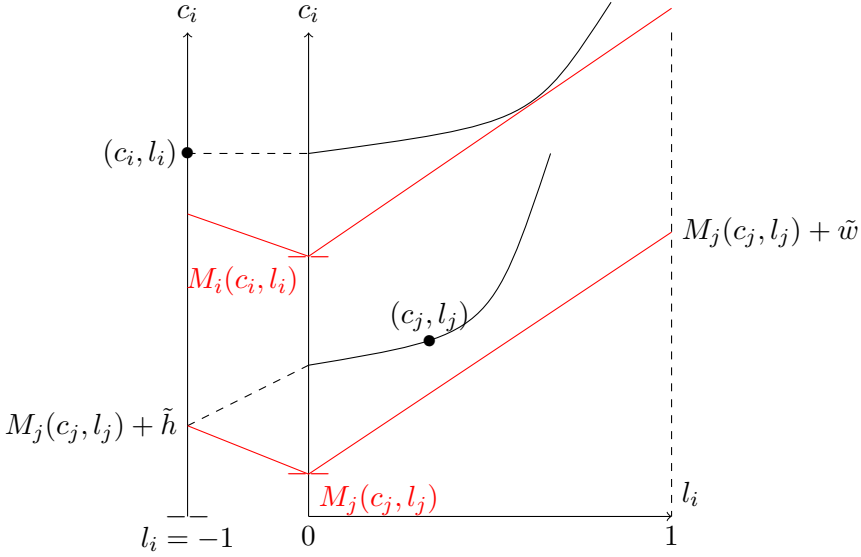


FIGURE 1.2: The construction of $M_i(c_i, l_i)$. The slope of the budget lines are given by arithmetic average $\tilde{h}$ and $\tilde{w}$ and define reference budgets $B(t, \tilde{w}, \tilde{h})$. The well-being at (c_i, l_i) is found by finding the transfer t such that the reference budget is tangent to the indifference curve of i at (c_i, l_i) . The transfer t is measured on the vertical axis at $l = 0$.

Definition 1.1 implies that the social welfare function will focus on the agent having the smallest $M_i(c_i, l_i)$, i.e. the agent with the lowest well-being. In other words, the social objective is of the *maximin* type. At this stage, it must be recalled that an infinite degree of aversion to inequality in a well-being measure does not necessarily lead to an infinite taste for redistribution (i.e. pure egalitarianism). For example, *maximinning* a money-metric utility function with individual-specific reference prices (w_i, h_i) imply that the absence of redistribution is optimal

(Fleurbaey & Maniquet, 2018). Hence, the key determinant of the following results lies more on the use of $M_i(c_i, l_i)$ than on its *maximin* nature.

The next theorem shows that the aforementioned three axioms single out the social welfare function in Definition 1.1 when they are combined with two technical axiom. The fourth axiom, **Mean-Preserving Separability** is reminiscent of the Separability axiom used in virtually all social welfare function since Fleming (1952): indifferent agents should not matter in the social decision.²⁰ The last axiom, **Hansson (1973) Independence**, allows the social welfare function to use indifference curves rather than mere utility levels to escape Arrow (1950)'s impossibility. All formal definitions of axioms are relegated to Appendix 1.A.1.

Theorem 1.1. *If a SOF satisfies Weak Pareto, Responsibility, Weak Transfer, Mean-Preserving Separability and Hansson Independence, then it is the FSOF in Definition 1.1.*

Proof. See Appendix 1.A.2. ■

Before providing intuition behind this result, it is useful to study its consequences for optimal policy. What would be an optimal schedule of lump-sum transfers in the first-best with respect to FSOF? Because of the maximin nature of the social objective, optimality in the first-best is achieved whenever all agents in the economy have an identical well-being measure, i.e. $M_i(c_i, l_i) = M_j(c_j, l_j)$ for all $i, j \in \mathcal{I}$. All agents then reach their indifference curve tangent to this reference budget set, thereby all enjoying the same level of well-being in the eyes of the planner. Crucially, observe that, given the heterogeneity in preferences, this does not mean that all agents consume the same bundle.²¹ I illustrate a first-best optimal allocation in Figure 1.3.

²⁰The axiom used in this paper is logically weaker than the standard Separability axiom. This weakening is required to avoid an impossibility, as is proven in the Appendix 1.A

²¹Also, note that such strongly egalitarian allocations may also be reached with a standard utilitarian setup in the first-best, as it is known since Edgeworth (1897).

new with **Theorem 1.1** is that the reference prices in the money-metric utility are pinned down by arithmetic averages $\tilde{h}$ and $\tilde{w}$. I now sketch an example to provide intuition for this result. It is illustrated in **Figure 1.4**.

Consider four agents: two agents with preferences $\succsim_a$ and two with preferences $\succsim_b$. For the purpose of this example, let us assume that these are purely inelastic preferences, i.e. the labor supply choice is independent of their endowments. We know that the social ordering should be *maximizing* a money-metric utility function and wonder which reference endowments are pinned down by the axioms.

When these four agents have identical skills, the economy is $\{(\succsim_a, h, w), (\succsim_a, h, w), (\succsim_b, h, w), (\succsim_b, h, w)\}$. In such case, **Responsibility** implies that the absence of redistribution is optimal and *Laissez-faire* allocation (c^*, l^*) is optimal. Because of the maximin nature of the social objective, we must have that the money-metric welfare of all agents is equal at the optimal allocation. Which reference endowments in the economy $\{(\succsim_a, h, w), (\succsim_a, h, w), (\succsim_b, h, w), (\succsim_b, h, w)\}$ ensures that the money-metric welfare of agents a and b is equal at (c^*, l^*) ? The answer is : all those that are such that

$$R(\{h, w\}, \{h, w\}, \{h, w\}, \{h, w\}) = \{h, w\}$$

where R is the reference operator. This equation tells us that $R(\cdot)$ is a mean. At this stage though, it could be many means: the quadratic mean, the cubic mean and all other Hölder means respect that equation.

Now imagine that there is a shock to the endowments of agents and the economy becomes $\{(\succsim_a, h + \delta, w + \delta), (\succsim_a, h - \delta, w - \delta), (\succsim_b, h + \delta, w + \delta), (\succsim_b, h - \delta, w - \delta)\}$ where $\delta > 0$. In other words, among each pair of agents with preferences $\succsim_a$ and $\succsim_b$, there is one winning (δ, δ) in her endowments and one losing (δ, δ) . In that case, **Weak Transfer** imposes that a within-pair transfer from the fortunate to the unfortunate is a social improvement. Iterating **Weak Transfer**, one finds that the social optimum in the new economy is also the allocation (c^*, l^*) .

We have shown that the social optimum is identical in two economies,

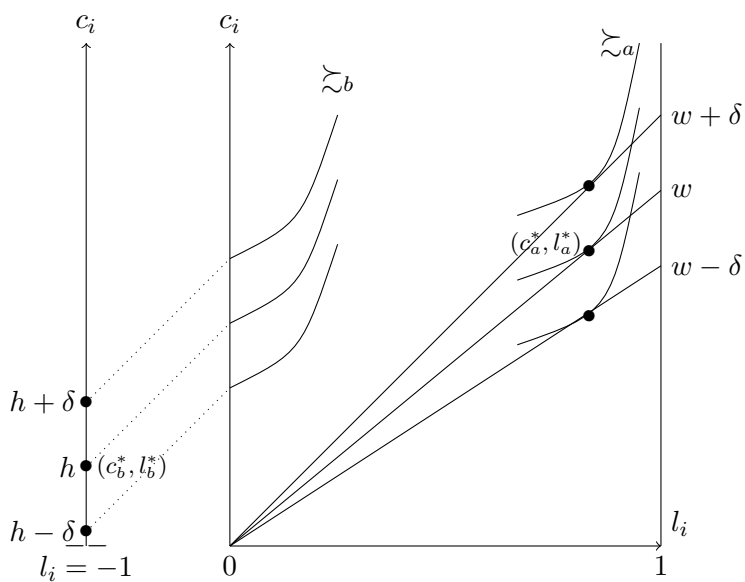


FIGURE 1.4: Why the average? By **Responsibility**, (c^*, l^*) is optimal if everyone has the same (w, h) endowment. By **Weak Transfer**, (c^*, l^*) is also optimal for arithmetic mean-preserving spreads from the equal endowment economy.

where one economy is constructed as an arithmetic mean-preserving spread from the other. This forces the reference price to be the same in both economies i.e.

$$\begin{aligned} R(\{h + \delta, w + \delta\}, \{h - \delta, w - \delta\}, \{h + \delta, w + \delta\}, \{h - \delta, w - \delta\}) \\ = R(\{h, w\}, \{h, w\}, \{h, w\}, \{h, w\}) \end{aligned}$$

There is only one operator $R(\cdot)$ that satisfies the last two equations : the arithmetic average. This shows why arithmetic averages as reference prices are compatible with the axioms of **Weak Transfer** and **Responsibility**.

The formal proofs relegated in the [Appendix 1.A.1](#) build on this intuition. In [Theorem 1.A.1](#) I prove that there an impossibility between **Weak Transfer**, **Responsibility** and **Separability**. Indeed, removing indifferent agents from the economy is not innocuous in our setting because they change the size of the pie, which influences the potential redistribution that the axioms of equality of opportunity will be able to achieve. In the proof of [Theorem 1.1](#), I show that **Mean-Preserving Separability** is enough to escape the impossibility, i.e. restricting the Separability axiom to case that do not affect the per-capita amount of resources in the economy.²⁴

1.4 Redistributive taxes and transfers

In this section I characterize the tax-benefit system pursuing equality of opportunity under incentive-compatibility constraints. As in Mirrlees (1971), these constraints arise because the government is unable to observe the endowment vector $(\tilde{z}_i, w_i, h_i)$ of each individual despite

²⁴Formally, this does not prove that arithmetic averages are the only reference prices compatible with **Weak Transfer** and **Responsibility**. Such a proof is beyond the scope of the present paper. Yet, it can be proven that popular alternatives to the arithmetic mean will violate some axioms, in particular the geometric and harmonic means, the min and the max operator. All results in [section 1.4](#) are valid for a general reference vector $(\tilde{h}, \tilde{w})$. In the empirical [section 1.5](#), I benefit from the fact that arithmetic averages for wages are readily available statistics in many countries.

knowing its joint distribution in the population.

Moreover, the government is unable to observe l_i and can only observe $y_i = a(l_i)w_i l_i$, the gross labor income reported in tax returns. I will also assume that, when $y_i = 0$, the government can perfectly distinguish the inactive from the unemployed agent. In other words, $a_i(\cdot)$ is observable even if l_i is not. This is a key assumption : it allows the government to differentiate the transfers it gives to unemployed agents from transfers to inactives. This is consistent with the observation that in most developed economies, there exists a screening mechanism enforcing the conditionality of welfare benefits to a job-seeking behavior.²⁵

For active agents with $a(l_i) = 1$, the government designs the tax schedule on the labor market through the nonlinear tax function $\tau(y)$ which is a subsidy whenever $\tau(y) < 0$ on some y . For agents out of the labor market with $a(l_i) = 0$, the government cannot observe home production yet these inactive agents may receive a subsidy $D \in \mathbb{R}$ which is a tax if $D < 0$ such that the second-best budgets are

$$B(\tau, D, w_i, h_i) = \left\{ (c_i, l_i) \in X \mid c_i = a(l_i)[a(l_i)w_i l_i - \tau(a(l_i)w_i l_i)] \right. \\ \left. + (1 - a(l_i))[h_i + D] \right\}$$

I call these variables: D the inactivity benefit, $-\tau(0)$ the social assistance and $\tau(y)$ the tax schedule. Agents choose l_i to maximizes $u_i(c_i, l_i)$ under this second-best budget constraint.

One may wonder about the relationship between the tax-benefit system (τ, D) studied here and the basic income proposals. Indeed, why wouldn't we give a basic income *universally* to both actives and inactives? Observe that the tax-benefit system (τ, D) studied here is completely equivalent to a universal basic income $(\tau' - D, D)$ for a τ' chosen such as $\tau' - D = \tau$, i.e. both systems decentralize the very same allocation. In other words, it is the (consequentialist) difference between active and

²⁵In practice, such screening mechanism may be imperfect and costly. I come back in the conclusion on the sensitivity of the results to this assumption.

inactive transfers that has welfare consequences, not the (deontological) means of transfers²⁶. For clarity, I retain the formulation with an inactivity benefit.

An incentive-compatible allocation (c, l) is such that :

$$\forall i, j \in \mathcal{I}, (c_i, l_i) \succsim_i (c_j, l_j) \text{ or } (c_j, l_j) \notin B(\tau, D, w_i, h_i)$$

Before turning to the main results, we need one assumption. It ensures that there exists at least one agent in the economy that has the worst endowments and *hardworking* preferences, i.e. no preference for leisure and a disutility of participation such that she is indifferent between social assistance and inactivity.

Assumption 1 (Hardworking Poor Existence). *For any incentive-compatible allocation (c, l) decentralized by (τ, D) and any economy $e \in E$, there exists $j \in \mathcal{I}$ such that $(w_j, h_j) = (\underline{w}, \underline{h})$ and $(\underline{h} + D, -1) \sim_j (-\tau(0), l_j)$ for all $l_j \in [0, 1]$.*

This assumption may seem strong for small economies but is arguably more reasonable when designing tax-benefit systems for large economies, as the present paper is concerned about. Moreover, since governments cannot observe indifference curves, it seems prudent to design policies as if the worst-off agents existed, should that indeed be the case in reality.

The next theorem translates the allocation ordering implied by the axioms from a function expressed in abstract well-being measures (in [Theorem 1.1](#)) to a function expressed in terms of policy tools and economy's parameters. It is stated and proven for the case where the smallest wage rate in the population is 0, i.e. $\underline{w} = 0$. It can be interpreted either as agents unable to work or as involuntary unemployed agents.²⁷ The case

²⁶In layman terms, whether a job-seeker receives 1000 USD as social assistance or 500 USD as basic income combined with 500 as social assistance yield the same bundle for that jobseeker.

²⁷Involuntary unemployment corresponds to the case in which the state of the labor market nullifies an agent's productivity.

$\underline{w} > 0$ can be derived in a similar way and is relegated to Appendix 1.B.1.

Theorem 1.2. *For $e \in E$ such that $\underline{w} = 0$, and any two incentive-compatible allocations (c, l) and (c', l') decentralized by (τ, D) and (τ', D') respectively, one has that (c, l) is socially preferred to (c', l') with respect to the FSOF whenever*

$$\min \left\{ \underline{h} + D - \tilde{h}; -\tau(0) - \tilde{w} \right\} > \min \left\{ \underline{h} + D' - \tilde{h}; -\tau'(0) - \tilde{w} \right\} \quad (1.1)$$

Proof. By Theorem 1.1, it suffices to show that for all $e \in E$ with $\underline{w} = 0$

$$\min_{i \in \mathcal{I}} M_i(c_i, l_i) = \min \left\{ \underline{h} + D - \tilde{h}; -\tau(0) - \tilde{w} \right\}$$

By Hardworking Poor Existence, for any (c, l) , $\exists j \in \mathcal{I}$ with $(w_j, h_j) = (\underline{w}, \underline{h})$ with preferences such that she is indifferent between all feasible bundles for her, i.e. $(\underline{h} + D, -1) \sim_j (-\tau(0), l_j)$ for all $l_j \in [0, 1]$. In other words, because $M_j(\cdot)$ represents $\succeq_j$ one has $M_j(c_j, l_j) = M_j(c'_j, l'_j)$ for any $(c_j, l_j), (c'_j, l'_j) \in B(\tau, D, 0, \underline{h})$. Using Definition 1.1, and the flat indifference curve of j over $l_j \in [0, 1]$ one has

$$M_j(c_j, l_j) = \min_{\hat{l}} t \text{ s.t. } \begin{cases} -\tau(0) = \tilde{w}\hat{l} + t & \text{if } \hat{l} \in [0, 1], \\ \underline{h} + D = \tilde{h} + t & \text{if } \hat{l} = -1 \end{cases}$$

This is equivalent to

$$M_j(c_j, l_j) = \left\{ \underline{h} + D - \tilde{h}; -\tau(0) - \tilde{w} \right\}$$

We are left to prove that j is the agent with the lowest well-being among all agents. By incentive-compatibility, all agents i with $(w_i, h_i) \geq (\underline{w}, \underline{h})$ must have $(c_i, l_i) \succeq_i (c_j, l_j)$. Because the M_i function is a money-metric utility function and represents preferences, it follows that $M_i(c_i, l_i) \geq M_j(c_j, l_j)$ for all $i \in \mathcal{I}$ which completes the proof. ■

Theorem 1.2 tells us that the worst-off in the economy is the agent who is consuming the bundle that is the furthest away from her sectoral average production. The link with Theorem 1.1 is apparent here : the smallest ‘distance to the average’ utility function $M_i(c_i, l_i)$ can be written in sufficient statistics in the second-best. In other words, if the distance to the average in the home sector ($\underline{h} + D - \tilde{h}$) is larger than in the labor market ($-\tau(0) - \tilde{w}$), the worst-off in the society are the agents consuming $(\underline{h} + D, -1)$. As a consequence, a welfare-improving reform would consist in raising D in that case. In other words, Theorem 1.2 implies that reducing within-sector inequality is welfare-improving in the sector that has the largest level of inequality. The sectoral inequality measure is simply the difference between the smallest consumption and the average productivity.

How can one relate this result to the axioms in section 1.3? Recall that **Weak Transfer** implied that among equal-preferences agents a transfer from the richer to the poorer was a social improvement *provided they have the same labor supply choice*. This is precisely the driver of the government’s inequality aversion within a sector. Now, even if perfect equality was achieved in each sector of the economy, there might still be inequalities between sector, say because $\tilde{h} < \tilde{w}$. But in that case, **Responsibility** would prevent the government from further reducing these inequalities, as agents choosing the home sector with a smaller consumption do so because of their tastes. As a result, the axioms treat each sector equally : one dollar produced in the labor market is not better than a dollar produced at home per se. The combination of these two arguments yields a social welfare function that fights within-sector inequality in the sector that has the largest level of inequality.

Theorem 1.2 is useful because it amounts to identifying a sufficient statistic for the evaluation of the desirability of *any* tax-benefit reform, even in a suboptimal world. It can be used to compare a reform scenario (τ, D) against the status quo tax-benefit system (τ', D') . I come back to the usefulness of this theorem in the empirical section 1.5.

Let me now turn to the characterization of the optimal tax benefit-system. It is enough to concentrate on D^* for our purposes.

Theorem 1.3. *For $e \in E$ with $\underline{w} = 0$, if there exists an incentive-compatible allocation decentralized by (τ^*, D^*) that is optimal with respect to the FSOF, then*

$$D^* = \tilde{h} - \underline{h} - \tau^*(0) - \tilde{w}$$

The proof is immediate from Theorem 1.2. The optimal inactivity benefit follows a similar formula for economies with $\underline{w} > 0$ ²⁸ and is relegated to Appendix 1.B.1.

This optimal inactivity benefit formula is interesting for several reasons. First, it gives us an idea of the consumption difference that should hold between actives and inactive. In other words, it pinpoints the premium that active agents *fairly* deserve because of their participation in the labor market, if any, the magnitude of which must be pinned down by empirical moments of the economy.

Second, this premium is increasing in the discrepancy between average productivity in the labor market and at home ($\tilde{w} - \tilde{h}$). This suggests that as the labor market becomes relatively more productive, the inactivity benefit should decrease for a given τ^* . This would come as no surprise if we had introduced a government budget constraint. What is peculiar, surprising and new here is that this holds only because of fairness considerations, whatever the size of efficiency costs.

Third, this theorem also informs us on the complementarity or substitutability of the inactivity benefit with the traditional safety net programs. In particular, basic income should supplement, rather than crowd out, the standard safety net as they covary in identical rather than opposite directions: an increase in one dollar of optimal social assistance $-\tau^*(0)$ should be accompanied with an increase of one dollar in optimal inactivity benefit D^* , *ceteris paribus*.

Finally, while Theorem 1.3 characterizes the optimal relationship between the safety net and the inactivity benefit, it is silent about the

²⁸The main difference is that the worst-off in the labor market may be different from the ones earnings $-\tau(0)$. For example, they may work full time at the minimum wage.

optimal joint level of (τ^*, D^*) . To sketch its design²⁹, consider a given economy e and the *Laissez-faire* policy $(0, 0)$. The government computes the well-being of the worst-off in the labor market and the worst-off in the home sector using Theorem 1.2. In most cases, the distance to the average is much greater in the former than in the latter sector. Hence, the government will start by directing some transfers to the worst-off in the labor market, up to the point where the distances to the averages are equalized across sectors, as embodied by Theorem 1.3. In turn, the Rawlsian government will pursue tax collection as much as efficiency permits. In other words, it will direct redistribution such that any dollar spent on D is matched with a dollar spent on the safety net, thereby setting their joint level as high as efficiency permits.

1.5 Empirical application

Whether introducing an inactivity benefit is a welfare-improving reform or an optimal policy is ultimately an empirical question, as one can notice from Theorem 1.2 and Theorem 1.3, respectively. Let me start with the analysis of welfare-improving reforms.³⁰ I therefore need to estimate parameters $-\tau(0)$, $\underline{h}$, $\tilde{w}$, $\tilde{h}$ from equation (1) in Theorem 1.2.

I set the right handside equation (1) with the status-quo tax-benefit system such that $(-\tau'(0), D') = (-\tau'(0), 0)$ for all countries as to date no OECD government has waived the conditionality of welfare benefits to labor market participation. Then, I calibrate the left handside with the reform that consists in giving a positive inactivity benefit, i.e. the reform $(-\tau(0), D)$ with $D > 0$. By Theorem 1.2, This reform is welfare-improving

²⁹It is easy to prove that τ^* would have the same qualitative properties as Fleurbaey and Maniquet (2007), i.e. setting $\underline{w} - \tau(\underline{w})$ as high as possible with negative marginal tax rates over $[0, \underline{w}]$.

³⁰Observed tax-benefit systems around the world are probably far away from the Mirrleesian optimum such that policy recommendations of practical use are more likely to emerge from the study welfare-improving reforms

if

$$\begin{aligned} -\tau(0) - \tilde{w} &\geq \underline{h} - \tilde{h} + D \\ -\tau(0) &\geq \underline{h} - \tilde{h} + D + \tilde{w} \end{aligned}$$

The right handside of this inequality reads as a sufficient statistic for the desirability of the reform of the inactivity benefit. In what follows, I consider a dollar increase of D (i.e. from 0 dollar to 1 dollar) and I compute the right handside. I obtain a sufficient statistic that answers the following question : *by how much should one increase the safety net in order for 1 dollar of inactivity benefit to be welfare-improving ?* Hence, this section performs a bounding exercise.

The advantage of the sufficient statistics (1) is that estimates for the current social assistance $-\tau'(0)$ as well as the average gross earnings $\tilde{w}$ are readily available statistics for many countries³¹. However, the difficulty of this exercise is that home production surpluses estimates for $\underline{h}$ and $\tilde{h}$ are not readily available. From section 1.2, we know that h_i must be measured as the product of the sunk time cost of participating in the labor market³² with the productivity in the home sector. Let me denote the former by F and the latter by γ_i :

$$\underbrace{h_i}_{\text{home surplus}} = \underbrace{\overbrace{F}^{\text{Time cost of participation}}}_{\text{Time cost of participation}} \times \underbrace{\gamma_i}_{\text{home hourly productivity}}$$

To estimate these two key unobservables, I will take a Beckerian view on home production and set average productivities to be identical across

³¹I recover $-\tau'(0)$ and $\tilde{w}$ from the OECD (2020) tax-benefit simulator for 29 developed economies. The results are differentiated for two different family compositions (i.e. two different status-quo τ') : childless singles and lone parents with two children. The reference year is set to 2020 for the case $\underline{w} = 0$ because there is little doubt that there has been involuntary unemployment.

³²Observe that in the real world, this sunk time cost can only be partly controlled by the government (through job-seeking ordeals). This paper focuses on the redistribution problem considering that these ordeals are set for other reasons (see e.g. Rafkin, Solomon, and Soltas (2024)). Accordingly, the measurement of F is set to match commuting time, which is largely independent of these ordeals.

the two sectors, i.e. $\tilde{\gamma} = \tilde{w}$ (Becker, 1965). Moreover, I impose that $\underline{\gamma} = 0$. Observe that I only impose two moments restrictions, i.e. on the minimum and the average, while staying completely agnostic about the shape of the γ_i distribution. Section 1.5.1 addresses robustness.

To get an estimate of F , I exploit the recent G-SWA survey on time savings when working from home (Aksoy et al., 2023). On average across countries, workers spent 72 minutes per day commuting which is taken to reflect the sunk cost of labor market participation³³. In turn, F is expressed as the fraction of this time cost over the statutory length of the working week³⁴ because l is normalized to 1. For example, the average American spends 55 minutes commuting per day over a 40 hours workweek, yielding a $F_{US} = 11.56\%$.

The results are reported in table 1.1.

Despite a series of conservative assumptions, I find that for all 29 countries, the safety net should be increased by very large amounts before any dollar spent on inactivity benefit constitutes a welfare improvement. The magnitude of estimates are in general smaller for lone parents than for childless singles, as most countries typically offer more generous coverage to the former than the latter. These results come from the fact that distances to the average are much larger in the labor market than in the home sector, or equivalently, that the well-being measurement pinned down by the axioms always identify the worst-off as being a job-seeker in these economies, but never an inactive. It is worth pointing out that, while results for the case $\underline{w} > 0$ are qualitatively similar, the magnitude is considerably smaller (see Appendix 1.B.2). Yet, the policy conclusion is unchanged.

This analysis of welfare-improving reforms suggests that before introducing a basic income, a prioritarian government should significantly

³³An alternative strategy would have been to use estimates of the time devoted to job search activities. However, Mukoyama, Patterson, and Şahin (2018) documented that unemployed Americans spend on average 31.1 minutes per day searching for a job, such that my choice is again conservative.

³⁴Additional details on the empirical application are relegated to Appendix D.

TABLE 1.1: Summary of results. Sufficient increase in $-\tau(0)$ for $D = 1$ to be a welfare-improving reform, in percentage of current $-\tau(0)$.

Country	Lone parents	Singles
Australia	365,99%	410,22%
Belgium	142,67%	248,18%
Bulgaria	352,10%	1109,41%
Canada	353,95%	600,39%
Czech Republic	303,34%	730,34%
Estonia	116,44%	657,54%
France	121,11%	371,55%
Greece	294,24%	589,93%
Germany	218,40%	744,30%
Croatia	325,47%	793,48%
Hungary	1490,03%	1490,03%
Israel	223,35%	549,81%
Ireland	186,29%	288,84%
Japan	68,66%	319,12%
Lithuania	224,54%	788,99%
Latvia	302,19%	1307,68%
Luxembourg	82,64%	189,04%
Malta	205,47%	235,06%
Netherlands	257,60%	257,60%
New Zealand	164,09%	295,15%
Poland	188,73%	994,14%
Portugal	244,71%	589,42%
Romania	1012,90%	2697,91%
Slovenia	35,97%	245,36%
Slovak Republic	511,34%	1365,17%
Spain	220,58%	374,47%
Turkey	3639,13%	∞
United Kingdom	226,69%	638,80%
United States	762,76%	2163,63%

(most of the time, unrealistically) increase the safety net coverage offered to the actives. This comes from the fact that the empirical difference between inactives consuming $\underline{h}$ and unemployed consuming $-\tau(0)$ is smaller than $\tilde{h} - \tilde{w}$.

Now what we have computed in Table 1.1 can also be read using Theorem 1.3. If one assumes that the observed social assistance programs are optimal ($-\tau'(0) = -\tau^*(0)$), then the optimal inactivity benefit D^* should be the opposite of the estimates of Table 1.1, i.e. the optimal inactivity benefit for lone parents in the Luxembourg should amount to -82.64% multiplied by the level of social assistance. In other words, all optimal inactivity benefits should be negative. This interpretation of the results follows the line of the inverse-optimum literature³⁵. Overall, this goes against the idea that introducing a basic income without modifying the safety net would be welfare-improving, let alone optimal.

1.5.1 Discussion

I made several assumptions that, if relaxed, would render the conflict between basic income and equality of opportunity even stronger. I discuss them in turn for the empirical application, the theoretical framework and the government's objective.

From the theorems of section 1.4, we know that a positive basic income is more likely to emerge if the gap $\tilde{w} - \tilde{h}$ is small. In order for my negative result on its desirability to be robust, I have made measurement assumptions that were favorable to the emergence of a basic income, i.e. conservative assumptions. First, the ratio of the average productivity in the home sector to the labor market $\frac{\hat{z}}{\tilde{w}}$ has been set to 1 while Bridgman, Duernecker, and Herrendorf (2018) found it closer to 0.3. Second, the choice of F could have been smaller if it had reflected time devoted to job search rather than commuting (see Mukoyama, Patterson, and Şahin (2018)). Third, I assumed that the average marginal product of labor $\tilde{w}$ is equal to the average gross earnings. However, as firms have monopsony

³⁵ See Bourguignon and Spadaro (2012) and Stantcheva (2016) for an introduction and a critique of the inverse-optimum approach, respectively.

power, the marginal product of labor is higher than the wage³⁶. All in all, these choices suggest that estimates in Table 1.1 are lower bounds.

Moreover, several features of the theoretical framework are also conservative with respect to this conflict. First, I considered that there is no negative externality of home production. However, if it includes black market activities, the government might wish to impose a Pigouvian tax on inactives and/or hold them responsible for their h_i , thereby decreasing even more the desirable level of D^* . Second, the model assumed away the existence of an intensive margin in the inactives' production function. As in Saez (2001, 2002) or Rothschild and Scheuer (2013), such an intensive margin would have driven an additional efficiency cost of raising D by disincentivizing effort in the home sector. Hence, these results would only be reinforced by including an intensive margin.

However, I have assumed that the government can perfectly distinguish a job-seeker from an inactive at a zero cost, which might seem a strong assumption. Yet, as long as the monitoring cost for the government has a lower dollar value than estimates from Table 1.1, the government should fully invest in it. Given the size of the estimates, the inclusion of costly monitoring is unlikely to overturn the policy recommendation. In the same vein, if the screening mechanism is imperfect and a basic income emerges as optimal policy, then this policy would be justified on third-best considerations but not on second-best arguments (see Ghatak and Maniquet (2019) for a discussion).

Finally, this result holds despite the fact that the social objective have not favored the production in the formal sector with respect to the one in the home sector³⁷ even if there may be good reasons to do so. Indeed, there has been recent calls in the public debate for an increase in the employment rate, notably in order to maintain the sustainability of public

³⁶Mas and Pallais (2019) review the literature and consider that the marginal product of labor may be 25% larger than the average wage.

³⁷This is especially relevant to the debate because proponents of basic income have argued that one should not be paternalistic about what a good life is (see in particular Van Parijs (1995)).

pension systems in aging economies.

However, this paper did not exclude the possibility that other social objectives may lead to different policy recommendations. In particular, among the family of social objectives pursuing equality of opportunity, the FSOE studied above may be criticized by some because it holds agents responsible for their disutilities of participation. In [Appendix 1.C](#), I study the case where disutilities of participation are driven by a welfare recipient stigma³⁸ that the government wishes to compensate for. It is shown that the estimates in [Table 1.1](#) can be then interpreted as lower bounds for the government's willingness to compensate for the stigma cost of conditionality.

1.6 Conclusion

This article has explored redistribution between active and inactive agents. The inquiry showed that abandoning the conditionality of social benefits to labor market participation is unlikely to be justified by the ethics of equality of opportunity in most developed economies.³⁹

The main explanation lies in the fact that as the value of home production is small, inactivity is driven by preferences for which agents are held responsible. Hence, an inequality-averse government can fight inequalities outside the labor force by providing better opportunities within

³⁸This *welfare stigma hypothesis* has received attention from the theoretical literature (Besley & Coate, 1992a; Hupkau & Maniquet, 2018; Lindbeck, Nyberg, & Weibull, 1999; Moffitt, 1983) and was recently backed by experimental evidence (Friedrichsen, König, & Schmacker, 2018).

³⁹A priori, this ethical standpoint could have justified both an anti- and a pro-basic income argument as could be attested by the debate between Rawls (1988) and Van Parijs (1991) on whether Malibu surfers should be fed. What the present paper has done is precisely to reconcile these two diametrically opposed interpretation of a single fairness viewpoint, by proving the conjecture outlined in Van Parijs and Vanderborght (2017, p.112) : "[Once leisure is included in the index], the optimal option, by the standard of the difference principle, will crucially depend on the relative weights the index places on income and leisure, [...] and on a great many contingent empirical facts". In this paper, the relative weights on these dimensions are those of the agents themselves because of the non-paternalistic nature of the social objective. Then I quantify where and when these empirical facts are such that a basic income may not be justified: in developed economies.

the labor market, which is desirable under the proviso that the aggregate technology in the formal sector is productive enough.

This paper assumed a perfect observability of the activity status a_i which might be noisy in practice.⁴⁰ This could justify the emergence of a basic income for third-best considerations while the present paper is confined to the analysis of the second-best. Similarly, this paper has left exogenous the definition of the eligibility requirements to be considered as an active job-seeker. However, it is well-known that there exists a heterogeneity in the stringency of these requirements across developed countries, whose positive and normative study is left for future research.

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⁴⁰Rafkin, Solomon, and Soltas (2024) study the correlations between eligibility and take-up for various US transfer programs. They find that the take-up is concentrated among eligible individuals with the worst consumption levels. Hence, the correlation is positive but below 1.

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1.A Axiomatic proofs

1.A.1 Axioms

In this section, I define formally each axiom used in the paper. Notation-wise, the social ordering function (SOF) $\succsim$ for an economy $e \in E$ is a transitive ordering such that for any two allocations $(c, l), (c', l') \in X^{\mathcal{I}}$, we write $(c, l) \succsim (c', l')$ whenever (c, l) is socially weakly preferred to (c', l') . The strict social preference and the social indifference are denoted by $\succ$ and $\sim$, respectively.

The first axiom is the celebrated Pareto principle that imposes that the planner agrees with unanimous decisions.

Axiom 1 : Weak Pareto

If $(c_i, l_i) \succ_i (c'_i, l'_i)$ for all $i \in \mathcal{I}$ then $(c, l) \succ (c', l')$.

The second axiom hold agents responsible for inequalities spawned by their preferences. It consists in saying that when all agents have identical endowments, reducing lump-sum transfers inequality is desirable. In the limit, Laissez-faire is optimal in this knife-hedge case where there is no inequality in endowments. It is illustrated in Figure 1.A.1.

Axiom 2: Responsibility

For all economy $e \in E$ and allocations with $(w_i, h_i) = (w_0, h_0) \forall i \in \mathcal{I}$,
If $\exists i, j \in \mathcal{I}$ and $(c, l), (c', l') \in X^{\mathcal{I}}$ with

$$\begin{aligned} (c_i, l_i) &\in \max_{\succsim_i} B(t_i, w_0, h_0) & (c'_i, l'_i) &\in \max_{\succsim_i} B(t'_i, w_0, h_0) \\ (c_j, l_j) &\in \max_{\succsim_j} B(t_j, w_0, h_0) & (c'_j, l'_j) &\in \max_{\succsim_j} B(t'_j, w_0, h_0) \end{aligned}$$

and $(c_k, l_k) = (c'_k, l'_k)$ for all $k \in \mathcal{I} \setminus \{i, j\}$ and $\exists \delta > 0$ such that $t'_i - \delta = t_i \geq t_j = t'_j + \delta$, then $(c, l) \succ (c', l')$.

The third axiom compensates agents for unequal skills (w_i, h_i) . It is illustrated in Figure 1.A.2.

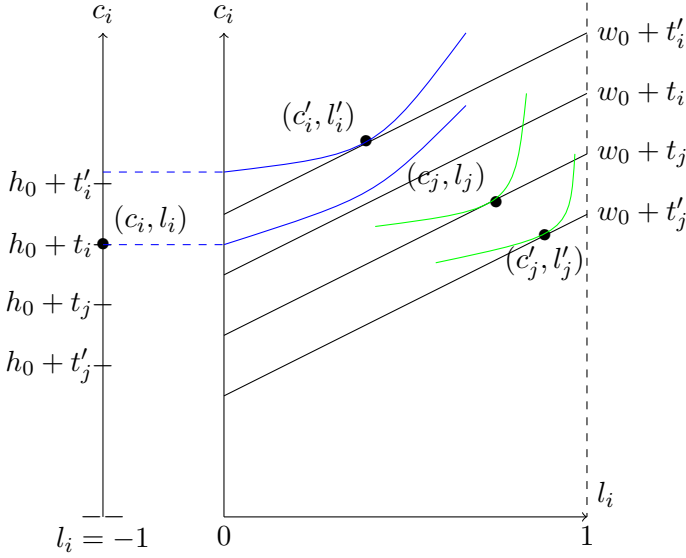


FIGURE 1.A.1: In this figure, all agents have the same endowments w_i and h_i . Yet, they have received different lump-sum transfers for some reasons. **Responsibility** imposes that $(c, l) \succ (c', l')$ because lump-sum transfer inequality is reduced in (c, l) .

Axiom 3 : Weak Transfer

For all economy $e \in E$, all allocations $(c, l), (c', l') \in X^{\mathcal{I}}$, if $\exists i, j \in \mathcal{I}$ two agents with $\succsim_i = \succsim_j$ such that

$$l_i = l_j = l'_i = l'_j$$

and for some $\delta > 0$

$$c'_i - \delta = c_i \geq c_j = c'_j + \delta$$

while $(c_k, l_k) = (c'_k, l'_k) \forall i \in \mathcal{I} \setminus \{i, j\}$;

Then, $(c, l) \succsim (c', l')$

Now, in order to build a SOF for all economies one needs consistency

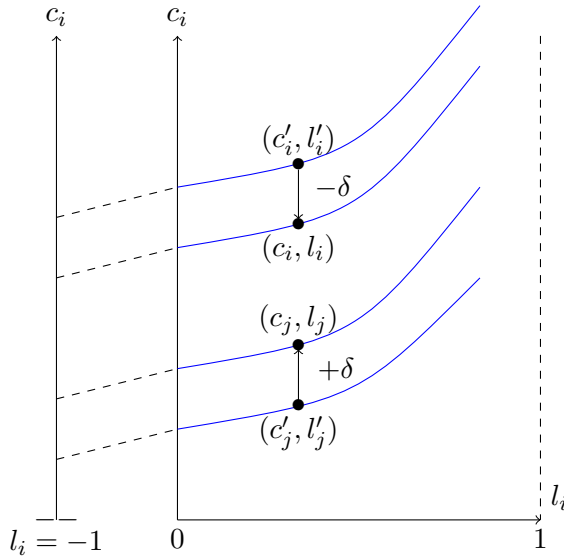


FIGURE 1.A.2: **Weak Transfer** imposes that $(c, l) \succsim (c', l')$

conditions i.e. invariance rules of the social evaluation when the economy changes. A popular choice in the literature is *Separability* which prescribes that adding or removing from the economy indifferent agents should not affect the ranking between two allocations.

Axiom : Separability

If there exists $(c, l), (c', l') \in X^I$ such that $(c_j, l_j) = (c'_j, l'_j)$ then

$$(c, l) \succsim (c', l') \iff (c, l)_{(I \setminus \{j\})} \succsim_{(I \setminus \{j\})} (c', l')_{(I \setminus \{j\})}$$

with $(c, l)_{(I \setminus \{j\})} = \left((c_1, l_1), \dots, (c_{j-1}, l_{j-1}), (c_{j+1}, l_{j+1}), \dots, (c_I, l_I) \right)$

and $(c', l')_{(I \setminus \{j\})} = \left((c'_1, l'_1), \dots, (c'_{j-1}, l'_{j-1}), (c'_{j+1}, l'_{j+1}), \dots, (c'_I, l'_I) \right)$ and $\succsim_{(I \setminus \{j\})}$ is the SOF when the set of agents is $I \setminus \{j\}$

We will also use a logical weakening of the latter, i.e. **Mean-Preserving Separability** which prescribes that removing indifferent agents from the economy does not affect the social ranking only if the absence of these agents do not change the per-capita amount of resources.

Axiom 4 : Mean-Preserving Separability

For all $\mathcal{S} \subset \mathcal{I}$ and for all economy $e, e_{(\mathcal{I} \setminus \mathcal{S})} \in E$ with $e_{(\mathcal{I} \setminus \mathcal{S})} = \left((w_i, h_i, \succsim_i)_{\forall i \in \mathcal{I} \setminus \mathcal{S}} \right)$, and for all $(c, l), (c', l') \in X^{\mathcal{I}}$ such that

- $(c_i, l_i) = (c'_i, l'_i)$ for all $i \in \mathcal{S}$ and
- $\frac{1}{I} \sum_{i=1}^I w_i = \frac{1}{S} \sum_{i=1}^S w_i$ and $\frac{1}{I} \sum_{i=1}^I h_i = \frac{1}{S} \sum_{i=1}^S h_i$

Then $(c, l) \succsim (c', l') \iff (c, l)_{(\mathcal{I} \setminus \mathcal{S})} \succsim_{(\mathcal{I} \setminus \mathcal{S})} (c', l')_{(\mathcal{I} \setminus \mathcal{S})}$
 where $(c, l)_{(\mathcal{I} \setminus \mathcal{S})}, (c', l')_{(\mathcal{I} \setminus \mathcal{S})} \in X^{\mathcal{I} \setminus \mathcal{S}}$ and $\succsim_{(\mathcal{I} \setminus \mathcal{S})}$ is the social ordering over the subpopulation $\mathcal{I} \setminus \mathcal{S}$.

Finally, the fifth axiom, **Hansson (1973) Independence**, deals with the informational structure of the SOF. It weakens the Arrovian binary independence in order to escape the impossibility of social choice. It imposes that when the indifference curves over two allocations are unchanged between two economies, then the social ordering over these allocations is unchanged as well.

Axiom 5 : Hansson (1973) independence

For all economy $e = \left((w_i, h_i, \succsim_i)_{\forall i \in I} \right)$ $e' = \left((w_i, h_i, \succsim'_i)_{\forall i \in I} \right) \in E$ with $(\succsim_i)_{i \in \mathcal{I}}$ and $(\succsim'_i)_{i \in \mathcal{I}}$ two profiles of preferences, let $(c, l), (c', l') \in X^{\mathcal{I}}$ be

two allocations,

$$\begin{aligned} \text{If } \forall q \in X \quad & \left[(c_i, l_i) \sim_i q \iff (c_i, l_i) \sim'_i q \right. \\ & \left. \text{and } (c'_i, l'_i) \sim_i q \iff (c'_i, l'_i) \sim'_i q \quad \forall i \in \mathcal{I} \right] \\ \text{Then, } [z \succsim z' & \iff z \succsim' z'] \end{aligned}$$

where $\succsim'$ is the SOF when the economy is e' .

1.A.2 Theorems and proofs

I now prove that the standard axiom of *Separability* clashes with **Responsibility** and **Weak Transfer**. To the best of my knowledge, this result is new. The key difference with previous papers is that the present paper has three dimensions of heterogeneity $(\succsim_i, w_i, h_i)$ while most previous papers only had two (Fleurbaey & Maniquet, 2018).

Theorem 1.A.2. *There is no SOF that satisfies **Responsibility**, **Weak Transfer** and **Separability** for all $e \in E$*

Proof. By contradiction, suppose the statement does not hold. Consider the economies $e_1 = \{(\succsim_i, \bar{h}, \underline{w}), (\succsim_j, \bar{h}, \underline{w})\}$, $e_2 = \{(\succsim_i, \underline{h}, \bar{w}), (\succsim_j, \underline{h}, \bar{w})\}$, $e_3 = \{(\succsim_i, \bar{h}, \underline{w}), (\succsim_j, \bar{h}, \underline{w}), (\succsim_i, \underline{h}, \bar{w}), (\succsim_j, \underline{h}, \bar{w})\}$, with $\bar{h} > \underline{h}$ and $\bar{w} > \underline{w}$, as well as their associated utility-maximizing bundles:

$$\begin{aligned} (c_1, l_1) &\in \max_{\succsim_i} B(0, \bar{h}, \underline{w}) & (c_2, l_2) &\in \max_{\succsim_i} B(0, \underline{h}, \bar{w}) \\ (c_3, l_3) &\in \max_{\succsim_j} B(0, \bar{h}, \underline{w}) & (c_4, l_4) &\in \max_{\succsim_j} B(0, \underline{h}, \bar{w}) \end{aligned}$$

Let me add the additional restrictions that $c_3 - c_4 = c_1 - c_2$, and $l_1 = l_2$ as well $l_3 = l_4$. I define the average bundles $(c_5, l_5) = \frac{(c_1, l_1) + (c_2, l_2)}{2}$ and $(c_6, l_6) = \frac{(c_3, l_3) + (c_4, l_4)}{2}$ and illustrate the setup in Figure 1.A.3..

By **Responsibility**, $\left((c_1, l_1), (c_3, l_3) \right) \succ_{(e_1)} \left((c_5, l_5), (c_6, l_6) \right)$,

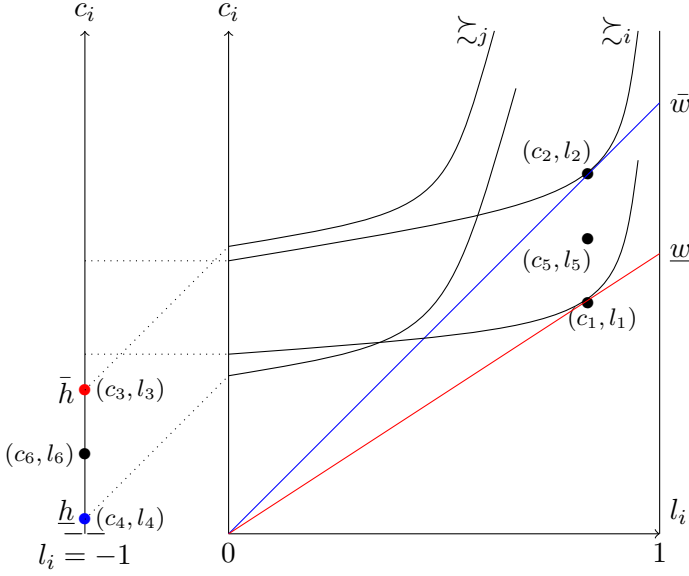


FIGURE 1.A.3: Illustration of the proof of Theorem 1.A.1.

By **Separability**,

$$\left((c_1, l_1), (c_3, l_3), (c_2, l_2), (c_4, l_4) \right) \succ_{(e_3)} \left((c_5, l_5), (c_6, l_6), (c_2, l_2), (c_4, l_4) \right),$$

By **Compensation**,

$$\left((c_5, l_5), (c_6, l_6), (c_5, l_5), (c_6, l_6) \right) \succsim_{(e_3)} \left((c_1, l_1), (c_3, l_3), (c_2, l_2), (c_4, l_4) \right),$$

By **Transitivity**,

$$\left((c_5, l_5), (c_6, l_6), (c_5, l_5), (c_6, l_6) \right) \succ_{(e_3)} \left((c_5, l_5), (c_6, l_6), (c_2, l_2), (c_4, l_4) \right),$$

By **Separability**,

$$\left((c_5, l_5), (c_6, l_6) \right) \succ_{(e_2)} \left((c_2, l_2), (c_4, l_4) \right),$$

But this contradicts **Responsibility**, proving the statement. ■

The main-text Theorem 1.1 shows that one can escape the impossibility of Theorem 1.A.1 by weakening **Separability** to **Mean-Preserving**

Separability. I prove Theorem 1.1 by using two lemmas. This proof is reminiscent to Fleurbaey and Maniquet (2006, 2011) and Valletta (2014)⁴¹. Lemma 1.1 implies the maximin⁴² nature of the social ordering while Lemma 1.2 characterizes the well-being measure.

Lemma 1.1. *If the SOF satisfies **Weak Pareto**, **Hansson independence** and **Weak Transfer**, then $\forall e \in E, (c, l), (c', l') \in X^I$ if there exists $\{i, j\} \in \mathcal{I}$ such that $\succsim_i = \succsim_j \equiv \succsim_0$ and*

$$(c'_i, l'_i) \succ_0 (c_i, l_i) \succ_0 (c_j, l_j) \succ_0 (c'_j, l'_j)$$

while $(c'_k, l'_k) = (c_k, l_k)$ for all $k \in \mathcal{I} \setminus \{i, j\}$, one has $(c, l) \succ (c', l')$.

Proof of Lemma 1.1. Follows mutatis mutandis the proof of Lemma 1.1 in Fleurbaey and Maniquet (2006). ■

Lemma 1.2. *If the SOF satisfies **Weak Pareto**, **Responsibility**, **Weak Transfer**, **Hansson Independence**, and **Mean-Preserving Separability**, and $\forall e \in E, \exists (c, l), (c', l') \in X^I$ such that*

$$M_i(c'_i, l'_i) > M_i(c_i, l_i) > M_j(c_j, l_j) > M_j(c'_j, l'_j)$$

and $(c_k, l_k) = (c'_k, l'_k) \forall k \in \mathcal{I} \setminus \{i, j\}$

Then,

$$(c, l) \succ (c', l')$$

Proof of Lemma 1.2. By contradiction, suppose that $(c', l') \succsim (c, l)$.

Let me introduce two new agents, a, b such that :

$$\bullet (w_a, h_a) = (w_b, h_b) = (\tilde{w}, \tilde{h})$$

⁴¹In comparison with Fleurbaey and Maniquet (2006), I have a stronger Responsibility requirement but a weaker Separability requirement and more dimensions of heterogeneity. With respect to Valletta (2014), I have weaker versions of Pareto and Transfer axiom. That paper dealt with the fair income tax if there are two consumption goods but only one productive skill. In the present paper, I have a homogeneous consumption good but productive skills in two sectors.

⁴²There has been recent axiomatizations of money-metric aggregator with finite inequality aversion which consists in weakening either **Weak Transfer** (Bosmans, Decancq, & Ooghe, 2018) or **Hansson Independence** (Piacquadio, 2017). However, I kept the present structure as the maximin has been crucial in the basic income debate.

- $\succsim_a = \succsim_i$ and $\succsim_b = \succsim_j$

I denote the relevant economies in the following way :

$$e_{(\{a,b\})} = \left((w_i, h_i, \succsim_i)_{\forall i \in \{a,b\}} \right) \quad e_{(\mathcal{I} \cup \{a,b\})} = \left((w_i, h_i, \succsim_i)_{\forall i \in \mathcal{I} \cup \{a,b\}} \right)$$

Let $\left((c_a, l_a), (c_b, l_b) \right), \left((c'_a, l'_a), (c'_b, l'_b) \right)$ be two allocations in $e^{\{a,b\}}$ such that

$$\begin{aligned} (c_a, l_a) &\in \max_{\succsim_a} B(t_a, \tilde{w}, \tilde{h}) & (c'_a, l'_a) &\in \max_{\succsim_a} B(t'_a, \tilde{w}, \tilde{h}) \\ (c_b, l_b) &\in \max_{\succsim_b} B(t_b, \tilde{w}, \tilde{h}) & (c'_b, l'_b) &\in \max_{\succsim_b} B(t'_b, \tilde{w}, \tilde{h}) \end{aligned}$$

with $t_a > t'_a > t'_b > t_b$

and

$$\begin{aligned} M_i(c'_i, l'_i) &> M_i(c_i, l_i) > M_a(c_a, l_a) > M_a(c'_a, l'_a) > \dots \\ &\dots M_b(c'_b, l'_b) > M_b(c_b, l_b) > M_j(c_j, l_j) > M_j(c'_j, l'_j) \end{aligned}$$

By Mean-Preserving Separability,

$$\left((c', l'), (c'_a, l'_a), (c'_b, l'_b) \right) \succsim_{(\mathcal{I} \cup \{a,b\})} \left((c, l), (c'_a, l'_a), (c'_b, l'_b) \right)$$

Recall that $(c_k, l_k) = (c'_k, l'_k)$ for all $k \in I \setminus \{i, j\}$. Moreover, because $M_i(\cdot)$ and $M_a(\cdot)$ represents the same preferences (resp. $M_j(\cdot)$ and $M_b(\cdot)$), we can apply Lemma 1 between agents i and a (resp. between agents j and b) to get

$$\left((c, l), (c_a, l_a), (c_b, l_b) \right) \succ_{(\mathcal{I} \cup \{a,b\})} \left((c', l'), (c'_a, l'_a), (c'_b, l'_b) \right)$$

By transitivity of the SOF,

$$\left((c, l), (c_a, l_a), (c_b, l_b) \right) \succ_{(\mathcal{I} \cup \{a,b\})} \left((c, l), (c'_a, l'_a), (c'_b, l'_b) \right)$$

By **Mean-Preserving Separability** one has,

$$\left((c'_a, l'_a), (c'_b, l'_b) \right) \succ_{(\{a,b\})} \left((c_a, l_a), (c_b, l_b) \right)$$

and this contradicts **Responsibility**, which completes the proof for lemma 1.2. ■

Proof of Theorem 1.1. The proof of theorem 1.1 is immediate from the combination of Lemma 1.1 and Lemma 1.2 with the characterization of the maximin ordering by Hammond (1976). □

1.B Voluntary unemployment

1.B.1 Theoretical results

All the results in the main body of the paper have been presented with a worst wage rate in the economy equal to zero $\underline{w} = 0$. In this section, I show that the results can be extended to the case where $\underline{w} > 0$. This subsection derives the equivalent results as those presented in section 1.4 in the main text. It will not be assumed here that *Hardworking Poor Existence* holds. Rather, I will impose *Minimality* which restricts the number of tax-benefit systems (τ, D) that decentralizes a particular incentive-compatible allocation (c, l) by focusing on those where no inconsequential tax cut are left. It formally requires that the second-best budget $B(\tau, D, \underline{w}, \underline{h})$ coincides with the envelope curve of agents' indifference curves at (c, l) . It is enough to restrict this assumption only for agents with the worst endowments.

Assumption 2 (Minimality). For $e \in E$ where there is at least one agent j with $(w_j, h_j) = (\underline{w}, \underline{h})$, a tax-benefit system (τ, D) that decentralizes the incentive-compatible (c, l) satisfies Minimality if

$$B(\tau, D, \underline{w}, \underline{h}) = cl \left\{ \bigcup_{i:(w_i, h_i):(\underline{w}, \underline{h})} Q_i(c_i, l_i) \right\}$$

where cl denotes the closure of a set, $Q_i(c_i, l_i)$ is the upper contour set of agent i at bundle (c_i, l_i) i.e. $Q_i(c_i, l_i) = \{(c'_i, l'_i) | (c'_i, l'_i) \succeq_i (c_i, l_i)\}$.

When the tax-benefit system is not minimal, one can devise tax cuts that do not affect any individual nor the budget constraint of the government. It is therefore a quite natural assumption. Figure 1.A.4 provides an example of a violation of Minimality. Although (τ, D) decentralizes (c, l) in this two-agent economy, one could find a tax cut such that no one is affected. It would amount to make the blue locus coincide with the envelope of agents' indifference curves, i.e. to annihilate the shaded area in Figure 1.A.4.

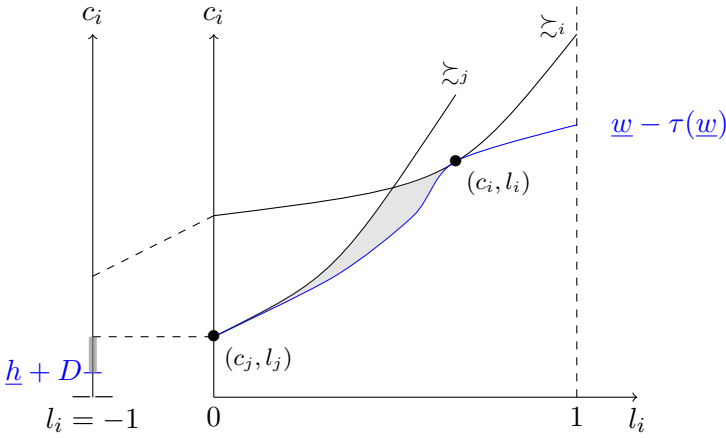


FIGURE 1.B.4: (τ, D) decentralize (c, l) but violates Minimality.

It is worth noting that when (τ, D) satisfies *Minimality*, $y - \tau(y)$ is non-decreasing in y because preferences are monotonic. As a consequence, *Minimality* forbids $\frac{\partial \tau(y)}{\partial y} > 1$ on some $y \in [0, \underline{w}]$, i.e. a confiscatory tax rate on some interval of low incomes. The next theorem is reminiscent to Theorem 1.2 in the main text.

Theorem 1.B.4. For $e \in E$ such that $\underline{w} > 0$, and any two incentive-compatible allocations (c, l) and (c', l') decentralized by minimal (τ, D) and (τ', D') respectively, one has that (c, l) is socially preferred to (c', l') with

respect to the FSOF whenever

$$\min \left\{ \underline{h} + D - \tilde{h}; \min_{0 \leq y \leq \underline{w}} \left(1 - \frac{\tilde{w}}{\underline{w}}\right)y - \tau(y) \right\} > \\ \min \left\{ \underline{h} + D' - \tilde{h}; \min_{0 \leq y \leq \underline{w}} \left(1 - \frac{\tilde{w}}{\underline{w}}\right)y - \tau'(y) \right\}$$

Proof. By Minimality, we know that there will be at least an agent with endowments $(\succsim_j, \underline{w}, \underline{h})$ such that she is indifferent between a bundle $(c_j, l_j) \in B(\tau, D, \underline{w}, \underline{h})$ and $\max_{\succsim_i} B(t, \tilde{w}, \tilde{h})$ for some t . One computes $M_j(c_j, l_j)$ as

$$M_j(c_j, l_j) = \min_i t \text{ s.t. } \begin{cases} \underline{w}\hat{l} - \tau(\underline{w}\hat{l}) = \tilde{w}\hat{l} + t & \text{if } \hat{l} \in [0, 1], \\ \underline{h} + D = \tilde{h} + t & \text{if } \hat{l} = -1 \end{cases}$$

This is equivalent to

$$M_j(c_j, l_j) = \min \left\{ \underline{h} + D - \tilde{h}; \min_{0 \leq y \leq \underline{w}} \left(1 - \frac{\tilde{w}}{\underline{w}}\right)y - \tau(y) \right\}$$

All agents $i \in \mathcal{I}$ with $(w_i, h_i) \geq (\underline{w}, \underline{h})$ must have $(c_i, l_i) \succsim_i (c_j, l_j)$. Because the M_i function is a money-metric utility function and represents preferences, it follows that $M_i(c_i, l_i) \geq M_j(c_j, l_j)$ for all $i \in \mathcal{I}$ which completes the proof. ■

The main difference with the main-text Theorem 1.2 is that now the worst-off in the labor market may be different from those earning the social assistance. In particular, they could be working with low pay, such as full time minimum wage earners. We conclude this subsection by deriving the optimal inactivity benefit with a formula reminiscent to Theorem 1.3.

Theorem 1.B.4. *For $e \in E$ with $\underline{w} > 0$, if there exists an incentive-compatible allocation decentralized by (τ^*, D^*) that is optimal with respect to the FSOF, then*

$$D^* = \tilde{h} - \underline{h} + \min_{0 \leq y \leq \underline{w}} \left(1 - \frac{\tilde{w}}{\underline{w}}\right)y - \tau^*(y)$$

To sketch the optimal tax system, consider again the Laissez-faire policies $(\tau, D) = (0, 0)$. The government computes the smallest well-being in each sector using Theorem 1.A.2. If it sets marginal tax rates such that $\frac{\partial \tau(y)}{\partial y} = 1 - \frac{\bar{w}}{w}$ for all $y \in (0, \underline{w})$, i.e. negative marginal tax rates on low incomes, then all low-incomes have identical well-being. Then, the government collects taxes as much as efficiency permits and sets the joint level of D^* and $-\tau^*(0)$ as in Theorem 1.A.3. Again, this is very close to the optimal tax schedule of Fleurbaey and Maniquet (2007), where the present paper adds the key difference between D^* and $-\tau^*(0)$.

1.B.2 Empirical results

This subsection extends the results of section 1.5 in the main text to the case where $\underline{w} > 0$. I have assumed that $\underline{w} = \arg \min_{0 \leq y \leq \underline{w}} (1 - \frac{\bar{w}}{w})y - \tau^*(y)$ for all countries. The simulation is run for 2019 with the exact same parameters as in the main text, except that $\underline{w} - \tau(\underline{w})$ is now interpret as the disposable income of a minimum wage earners working full time.

I conclude that the results are qualitatively similar even if quantitatively different. The policy conclusion is unchanged: governments should first increase support for low-income workers before any dollar of inactivity benefit is welfare-improving.

1.C Welfare recipient stigma

In this section, I consider the case of a government that does not wish to hold agents fully responsible for their preferences because their disutilities of participation have been partly driven by the *welfare recipient stigma*. It captures the idea that some agents remain inactive and do not take up conditional social benefits because enduring the screening device of the government entails a mental burden, rooted in the stigma that societies attach to benefits recipients.

Consider the following structure for the disutility of participation d_i :

$$d_i(c_i) \equiv u_i(c_i, -1) - u_i(c_i, 0) = s_i + \delta_i(c_i)$$

TABLE 1.B.4: Summary of results. Sufficient increase in $\underline{w} - \tau(\underline{w})$ for $D = 1$ to be a welfare-improving reform, in percentage of current $\underline{w} - \tau(\underline{w})$.

Country	Lone parents	Singles
Australia	102,75%	116,50%
Belgium	50,47%	112,84%
Bulgaria	74,41%	139,96%
Canada	118,73%	165,95%
Czech Republic	96,99%	147,41%
Estonia	53,97%	114,82%
France	46,91%	91,72%
Greece	158,22%	158,22%
Germany	132,58%	214,20%
Croatia	129,42%	129,42%
Hungary	137,39%	233,21%
Israel	52,77%	119,34%
Ireland	46,10%	111,43%
Japan	54,23%	179,36%
Lithuania	88,95%	157,29%
Latvia	43,39%	139,87%
Luxembourg	50,49%	110,96%
Malta	93,07%	138,38%
Netherlands	116,16%	142,99%
New Zealand	44,68%	59,68%
Poland	125,05%	165,15%
Portugal	107,27%	107,27%
Romania	185,09%	192,31%
Slovenia	11,61%	97,30%
Slovak Republic	77,42%	118,30%
Spain	89,53%	102,38%
Turkey	119,83%	126,20%
United Kingdom	83,42%	134,01%
United States	103,51%	289,04%

where $\delta_i \in [-s_i, +\infty)$ is the idiosyncratic taste parameter⁴³ and $s_i \geq 0$ is the value of stigma. . Observe that s_i has a money-metric interpretation: it is the maximal amount of consumption that an agent is willing to forgo in order to escape enduring the screening device of the government.

For clarity of the exposition, let me assume that there are only two draws of this stigma utility cost : $s_i \in \{S, 0\}$ with $S > 0$. Hence, there are only two different exposures to the stigma cost of conditionality in the population : those that do suffer from it and those that do not⁴⁴. The social planner wishes to compensate for s_i as well as w_i and h_i , while holding responsible for δ_i and their willingness to work.

Rather than deriving rigorously the full axiomatization that this compensation for s_i would entail, I sketch the result by an example. Consider the case of two fraternal twin sisters, agents k and j . They are identical in every respect but they differ in their exposure to the stigma utility cost: agent k suffers from it such that $s_k = S$ while agent j do not and $s_j = 0$.

Consider the bundle (c_j, l_j) and (c'_j, l'_j) that are such that agent j is indifferent between the two, and they are consumed when inactive and unemployed, respectively. I sketch this setup in Figure 1.A.5.

In red are drawn the indifference curves of agent k . Obviously, when she is unemployed and consumes (c'_j, l'_j) she suffers the stigma $s_k = S > 0$ associated to this labor market status. If one computes the $M_i(c_i, l_i)$ utility of these two agents when they consume (c'_j, l'_j) , one gets that $M_k(c'_j, l'_j) = M_j(c'_j, l'_j) - S$. In other words, *when unemployed*, the well-being measure derived in previous sections already accounts for the fact that those suffering from a larger disutility of participation (here, coming

⁴³The fact that δ_i can now have negative values reflect the possibility for some agents to derive non-pecuniary benefits from labor market participation (such as friendliness from colleagues for example) which could partly offset the burden S puts on them.

⁴⁴If this partitioning in two sets is degenerate, we are back to the analysis of the previous sections as there is no inequality to compensate for. In other words, if all agents experience the same stigma cost of conditionality, or if none of them does, the main formulas are unaffected.

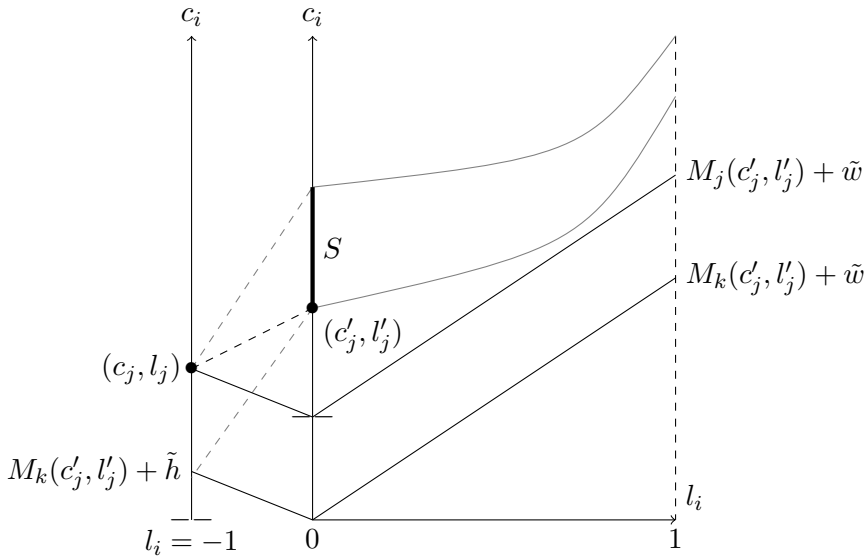


FIGURE 1.C.5: Agents k and j are identical in every primitive but one: the inactive k suffers a stigma utility cost of S while the unemployed j does not. In gray, the indifference curves of k . Agent j is indifferent between (c_j, l_j) and (c'_j, l'_j) . Observe that $M_k(c_j, l_j) + \tilde{w} = M_j(c_j, l_j) + \tilde{w} = M_j(c'_j, l'_j) + \tilde{w}$

from the stigma) have a lower well-being.

Now, the difference is when the agents are inactive and consume (c_j, l_j) . In this case, the indifference curve of k lies above the one of j , and they only coincide at (c_j, l_j) . If one applies the $M_i(c_i, l_i)$ utility to this situation, one gets that $M_k(c_j, l_j) = M_j(c_j, l_j)$, i.e. both agents have the same level of well-being, even if agent k would have suffered from the stigma had she joined the labor market.

A government that compensates for S should treat the well-being of k when inactive as if she had actually experienced this stigma cost. In other words, the well-being measure when inactive must be reduced by s_i with respect to the $M_i(c_i, l_i)$ utility. Hence, the new well-being measure

compensating for the stigma, is given by

$$W_i(c_j, l_j) = M_i(c_i, l_i) - (1 - a_i)s_i$$

In turn, all the analysis above can be repeated using $W_i(c_j, l_j)$ instead of $M_i(c_i, l_i)$. Obviously, the worst-off in the home sector will now have a well-being measure of $\underline{h} - S + D - \tilde{h}$. Hence, the optimal (τ^{**}, D^{**}) that maximins $W_i(c_j, l_j)$ follows:

$$D^{**} = S + \tilde{h} - \underline{h} + \min_{0 \leq w} \left(1 - \frac{\tilde{w}}{w}\right) y - \tau^{**}(y)$$

I conclude that S positively influences the optimal inactivity benefit in an additive fashion and thereby weakens the conflict between fairness and basic income outlined in previous sections. Because of the formalization, S has a money-metric interpretation and should be measured as an answer to this question : *how much would one be willing to pay (i.e. forgo consumption) to escape enduring the screening device of the government?*

To the best of my knowledge, such empirical estimates for S are not available in the literature. In order to justify that one dollar of inactivity benefit is welfare-improving, S must be at least larger than the estimates of Table 1.1. In many instances, it seems unrealistically large. Alternatively, S could also be interpreted as an ethical parameter. In that case, estimates from Table 1.1 would provide lower bounds on the government's willingness to pay to escape its own screening device.

1.D Details on the empirical application

I used the OECD tax-benefit simulator version 2.5.0. with the following parameters:

- Childless single 2020 : aged 40, unemployed for 6 months, with 216 months of social security contributions accumulated over the lifetime, earning social assistance, for the year 2020. Eligible to social assistance, net of income tax and social security contributions.

- Lone parents 2020 : aged 40, unemployed for 6 months, with 216 months of social security contributions accumulated over the lifetime, earning social assistance, for the year 2020. Children aged 4 and 6. Eligible to social assistance, lone parents support, net of income tax and social security contributions.
- Childless single 2019 : aged 40, unemployed for 6 months, with 216 months of social security contributions accumulated over the lifetime, earning statutory minimum wage. Eligible to social assistance, in-work benefits, net of income tax and social security contributions.
- Lone parents 2019 : aged 40, unemployed for 6 months, with 216 months of social security contributions accumulated over the lifetime, earning statutory minimum wage. Children aged 4 and 6. Eligible to social assistance, in-work benefits, lone parents support, net of income tax and social security contributions

The set of countries covered by the G-SWA survey (Aksoy et al., 2023) is a strict subset of the 29 countries I study. Whenever the estimate for F was not available, I kept the maximum of the series, i.e. 100 minutes per day.

The legal length of the working week is taken from the OECD (2021) (Annex Table 5.A.1.). The statutory length may differ from the negotiated length in some countries. When both are present, I took the maximum among the two. If both are absent, I set the length of the working week to 45 hours, corresponding to the maximum of the series. I considered that the working week lasts 5 days.

Chapter 2

Working time reductions and monopsony power

This paper studies the consequences of working time reductions when labor markets may be monopsonistic. A toy model shows that the marginal utility of a small working time reduction is negative when workers control schedules but positive when firms set hours. However, the policy increases wage rates in perfect competition but decreases monopsonistic wage rates. I test these predictions empirically by evaluating the first-ever working time reduction in Belgium: the maximum 9h work-day in 1910's coal mines. I find that the policy had sizable negative effects on profits, employment and earnings. To assess welfare, I build a directed search model with matching frictions where firms have heterogeneous productivity and post wages and hours. Utilitarian welfare is expressed in terms of a sufficient statistic whose application to the 1910 reform suggests that the value of leisure was particularly large.

2.1 Introduction

Since the Industrial Revolution, labor movements have fought for higher wages and shorter working time per day, per week and per year. Government-mandated working time reductions emerged at the beginning of the 20th century and remain highly heterogeneous across countries to

this date. For instance, in France minimal paid time off was set at two weeks in 1936 and amounts to five weeks today, while the United States do not have any nationwide mandate on vacations.

The present paper studies the consequences of working time reductions for wages and welfare. It shows that these effects depend on the timing (i.e. who chooses what) as well as the extent of imperfect competition of the labor market.

In the first part of the paper, I build a simple general equilibrium toy model of the labor market where production features decreasing returns to hours and workers face the canonical leisure-consumption tradeoff: an extra hour increases utility through consumption but decreases utility through leisure. This model brings two key findings.

First, the welfare effects depend on *who sets schedules*. If workers choose hours as in neoclassical labor supply theory, the marginal utility of an extra hour worked must be zero. Conversely, when firms choose hours, the labor contract will feature long hours and workers would welcome a working time reduction for a given wage. This suggests that workers are more likely to support government intervention on working time when they do not control schedules.

Second, the wage effects depend on *who sets wages*: a working time reduction decreases wage rates in an oligopsonistic labor market but the very same policy increases wage rates in perfect competition. Intuitively, in the pure monopsony case, labor supply increases with wages so that a working time reduction must be associated with a wage cut. Conversely, in a perfectly competitive labor market, workers' earnings are equal to their marginal product. Decreasing returns per hour implies that a working time reduction by one unit engenders a reduction of marginal product by less than one unit. Hence, earnings decrease proportionally less than hours worked, which implies that competitive wage rates have increased after the reform.

In order to evaluate these empirical predictions, the paper studies the first-ever working time reduction implemented in Belgium: the 9 hour

maximum workday in coal mines in 1910. It is an ideal setup for at least two reasons. First, as one of the first piece of labor legislation in the country, the law was not coupled with compensatory measures nor wage regulations such that we should be able to observe the pure effect of the policy.¹ Second, there was no downward nominal wage rigidity at the time which should allow to test for wage cuts.

I digitized and assembled administrative datasets from archival sources covering the near-universe of coal mines in the country from 1903 to 1913. The identification strategy follows an event study design and relies on comparing post-reform outcomes of firms with different scheduling practices before the reform. I find sizable, short-run, negative effects of the reform on profits: a one-hour reduction in miners' working day reduces profits by 93%. I also find negative yet imprecise effects on employment and wages: a reduction of working time by one hour decreases employment and wages by 6% on average over affected firms.

While leisure increased, wages, employment and profits fell. To assess whether this reform was welfare-improving, I embed the model and these results into a directed search model and competitive search equilibrium of the labor market à la Moen (1997) where firms are heterogeneous in productivities. Matching frictions ensure that all participating workers have some non-degenerate job-finding probability. Hence, when a government grants extra holidays, ex-post utilities increase conditional on employment but job-finding probabilities might decrease. In turn, this model aggregates individuals' micro leisure-consumption trade-offs in a macro arbitrage between wages, employment and hours worked.

As in Vergara (2023), this competitive search equilibrium is compatible with any degree of monopsony power. Indeed, when a firm increases wage rates, it will attract new applicants. Yet, because of matching frictions, the probability that these additional applications fill a vacancy is

¹For example, the French 35h workweek reform studied in Chemin and Wasmer (2009) was accompanied with payroll tax cuts. In the US, Roosevelt's working time reductions were accompanied with wage regulations (see Fishback, Vickers, and Ziebarth (2024) for the National Industrial Recovery Act of 1933 and Costa (2000) for the Fair Labor Standards Act of 1938).

less than one. Workers internalize these frictions such that only a finite number apply. As a result, the labor supply curve observed by the firm is not infinitely elastic and this creates some monopsony power in equilibrium. I derive two main findings.

First, I study how heterogeneous firm sort in the contract space. It is shown that firms with higher productivity offer contracts with higher wage rates, shorter hours, and a higher job quality but lower job-finding probability. On the workers' side, a local increase in the preference for leisure leads to a higher equilibrium wage rate and lower hours worked.

Second, I derive sufficient statistics and quantify welfare gains. Interestingly, welfare may be assessed without assumptions on production nor on the degree of monopsony power in labor markets, which is known to vary significantly across contexts and industries (Azar, Marinescu, & Steinbaum, 2022; Card, 2022). I estimate that the average utility gains in leisure must have been sizable for the Belgian reform to produce welfare gains.

Overall, the paper makes two kind of contributions. On the positive side, the paper derives empirical predictions for the wage effects of working time reductions and shows that they differ with the degree of firms' labor market power, i.e. monopsony power. These results can act both as a characterization of monopsony power as well as a detection test. On the normative side, the paper derives sufficient statistics for welfare evaluation of any working time regulation in a general equilibrium environment with rich heterogeneity, matching frictions and imperfect competition in the labor market. In terms of policy, the paper suggests that working time reductions are appealing for workers in monopsonistic markets but their equilibrium effects on wages and employment may offset the welfare gains.

Section 2.2 discusses the relationship with the literature. Section 2.3 presents the toy model which contrasts the effect of working time reductions on marginal utility and wage rates depending on who sets hours and wages. Section 2.4 presents a reduced-form empirical evaluation of the Belgian coal mines 1910 maximum workday and test for these

predictions. Section 2.5 generalizes the model with a competitive search equilibrium between firms with heterogeneous productivities and workers with heterogeneous leisure preferences and it studies the effects on welfare. Section 2.6 concludes.

2.2 Literature

Canonical labor models This paper merges two standard models in labor economics. On the one hand, leisure-consumption trade-offs determines labor market equilibrium like in Rosen (1974, 1986)’s hedonic theory of wages. On the other hand, the model features directed search and competitive search equilibrium². While matching frictions were added to hedonic wages by Hwang, Mortensen, and Reed (1998) and Lang and Majumdar (2004), they assumed random search rather than directed search. The present paper adds working time and hedonic wages to Vergara (2023) who studied minimum wages in a competitive search equilibrium. As a result, labor supply is not derived from utility-maximization but from profit maximization under a utility constraint.

Working time regulations Descriptive facts on vacations can be found in Altonji and Oldham (2003) and Altonji and Usui (2007). Marimon and Zilibotti (2000) and Rocheteau (2002)³ considered that a working time reduction is desirable if it reduces unemployment. In the present paper, social welfare encompasses the tradeoff between the terms of jobs and the number of jobs. Carry (2023) studied a unique minimum workweek policy in France and builds a model with quasilinear utilities and random search. All of these papers model wage-hour determination as the outcome of bargaining while the present model has contract posting which features two advantages. First, it allows me to escape the assumption that

²Key references include Acemoglu and Shimer (1999), Eeckhout and Kircher (2010a, 2018), Guerrieri, Shimer, and Wright (2010), Kircher (2009), Moen (1997), Vergara (2023), and Wright et al. (2021)

³Other related papers include Fagnart, Germain, and Van der Linden (2023), Lang and Majumdar (2004), Osuna and Rios-Rull (2003), and Willington and Navarro (2015). Chemin and Wasmer (2009) studied the French 35-hour workweek and Fishback, Vickers, and Ziebarth (2024) studied the Roosevelt working time reduction during the Great Depression.

bargaining power is invariant to policy changes, which seems implausible in the present context. Second, posting has been found more relevant than bargaining empirically for low-wage jobs (Caldwell & Harmon, 2019; Hall & Krueger, 2012; Lachowska et al., 2022).

Importantly, Fishback, Vickers, and Ziebarth (2024) studied the effect of the introduction of the maximum workweek in the US during the Great Depression which was accompanied with wage and earnings floors. There are several important differences with the current paper. First, the policy in the US was motivated by work-sharing arguments in a acute recession where unemployment was plaguing the economy. This is not the case in the Belgian 1910 context: the business cycle was neither booming nor recessionary. Second, they find massive positive effect on employment while we find the opposite effect. Third, they study a perfectly competitive labor markets with voluntary unemployment, while we allow for the possibility of monopsony power and involuntary unemployment.

Hours and employment Empirical studies found conflicting effects of wage and hours regulations. Some papers documented that increases in minimum wages lead to decreases in hours worked (Di Nola, Wang, & Haywood, 2023; Gandhi & Ruffini, 2022; Jardim et al., 2022; Kim, Lee, & Lee, 2023) but Vergara (2023) finds no effect. Most minimum wage studies finds no effect on employment (Cengiz et al., 2019; Manning, 2021). Gravoueille (2023) showed that an increase in low-earners wage subsidies led to an increase in hours worked but a decrease in wage rates. Carry (2023) finds that low-hours contract restrictions led to a decrease of low-hour jobs at the extensive margin and an increase in the intensive margin of full-time jobs. The present paper provides a unifying theory for such results. Its key elements are the output elasticities of hours versus hires as well as the degree of monopsony power.

Amenities The literature has focused on positive questions⁴ such as the relationship of amenities to earnings and productivity (Mas & Pallais, 2017, 2020; Ouimet & Tate, 2023; Sockin, 2022; Sorkin, 2018), monopsony power (Lamadon, Mogstad, & Setzler, 2022), job search (Hall &

⁴See Lavetti (2023) for a recent review.

Mueller, 2018), and minimum wages (Clemens, 2021; Clemens, Kahn, & Meer, 2018). However, the normative aspects of regulating amenities have not been addressed such that the present paper complements this literature. A notable exception is Nekoei (2023) who suggests that mandating amenities can improve efficiency if there is adverse selection à la Akerlof. Yet, in Nekoei (2023) amenities do not have productive value for the firm per se, contrary to hours worked in the present paper.

2.3 Toy model: who chooses what?

In this section, I contrast the theoretical effects of working time reductions on wages and utility depending on who sets hours and wages. I build a simple general equilibrium model of the codetermination of wages and hours.

A firm produces using only hours, denoted by l , through a production function $F(l)$ which is assumed to be increasing and concave. Hours are bounded above by a natural constraint, normalized to 1 such that the government must choose a legal maximum $\bar{l}$ in the interval $[0, 1]$. The labor contract specifies a quantity of hours worked l at a unit price w such that firm's profits $F(l) - wl$ must be nonnegative for it to participate.

Workers have a utility function $u(c, l)$ that is increasing in consumption c and decreasing in hours worked $l \in [0, 1]$. They have an outside option to the labor contract that gives a reservation utility u_r . When working, their consumption is equal to their earnings $c = wl$. Interestingly, the marginal utility of an extra hour $\frac{\partial u(\cdot)}{\partial l} = wu'_c + u'_l$ has an ambiguous sign: at low hours, workers would like to work more at the wage rate w because the positive consumption effect dominates the negative leisure effect while this is reverted for long hours. For regularity, I assume that the workers' preferred schedule is neither 0 nor 1⁵.

I have two main objects of interest. First, I will sign the marginal utility of an extra hour $\frac{\partial u(\cdot)}{\partial l}$ before the introduction the policy $\bar{l}$. When it is negative, workers would like to work less at the given wage rate

⁵Formally, I impose that for any w we have a $l^* \in (0, 1)$ such that $\frac{\partial u(wl^*, l^*)}{\partial l} = 0$.

such that they would welcome a working time reduction with rigid wages. Second, I study how the introduction of a small binding working time reduction $\bar{l} < l$ affect equilibrium wage rates.

The theoretical predictions depend on the definition of strategy sets: who chooses hours and who chooses wages? I will consider four different models. In model 1, workers choose hours to maximize utility and wage rates are competitive. In model 2, workers choose hours but the firm sets wage rates. In model 3, the firm sets hours to maximize profits but wage rates are competitive. In model 4, the firm sets both hours and wage rates. Models 1 and 2 use the neoclassical theory of labor supply while models 3 and 4 use the hedonic theory of labor supply. Models 1 and 3 correspond to perfectly competitive wages while models 2 and 4 correspond to monopsonistic wages. These models are illustrated in Figure 2.1.

In model 1, workers choose hours to maximize utility such that one must have that $l = \arg \max u(wl, l)$ for any given w and $l \in [0, \bar{l}]$. Hence, it must be that the marginal utility of working an extra hour is nonnegative $\frac{\partial u(\cdot)}{\partial l} \geq 0$ in equilibrium and equal to 0 in interior solutions. Because wage rates are competitive, workers are paid at their marginal product and $w = F'(l)$. Concavity in production function implies that the marginal product is decreasing in hours such that a binding working time reduction must increase $F'(l)$ and hence equilibrium wage rates $dw \geq 0$.

In model 2, workers set again hours such that $\frac{\partial u(\cdot)}{\partial l} \geq 0$. However, firm sets monopsonistic wage rates to maximize profits internalizing the neoclassical labor supply. This gives rises to the standard Robinson (1933) monopsony equation $F'(l) - w = \frac{l}{l'(w)}$ where $l'(w)$ is the derivative of the labor supply function. When $l(w)$ is increasing⁶, a working time reduction must necessarily be associated with a decrease in equilibrium wage rates

⁶The fact that $l(w)$ is increasing implies that the substitution effect dominates the income effect in the Slutsky equation of workers. This guarantees that workers are paid below their marginal product $w < F'(l)$ which is a widely-documented empirical fact by the monopsony literature (see Card (2022) e.g.).

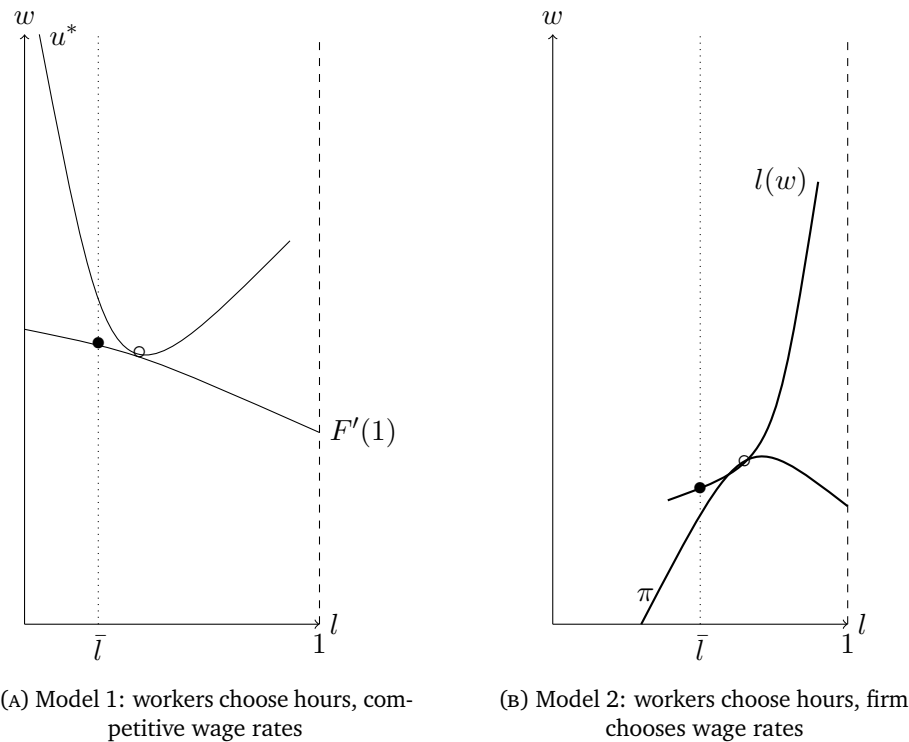


FIGURE 2.1: Equilibrium before (○) and after (●) a small binding working time reduction $\bar{l} < l$.

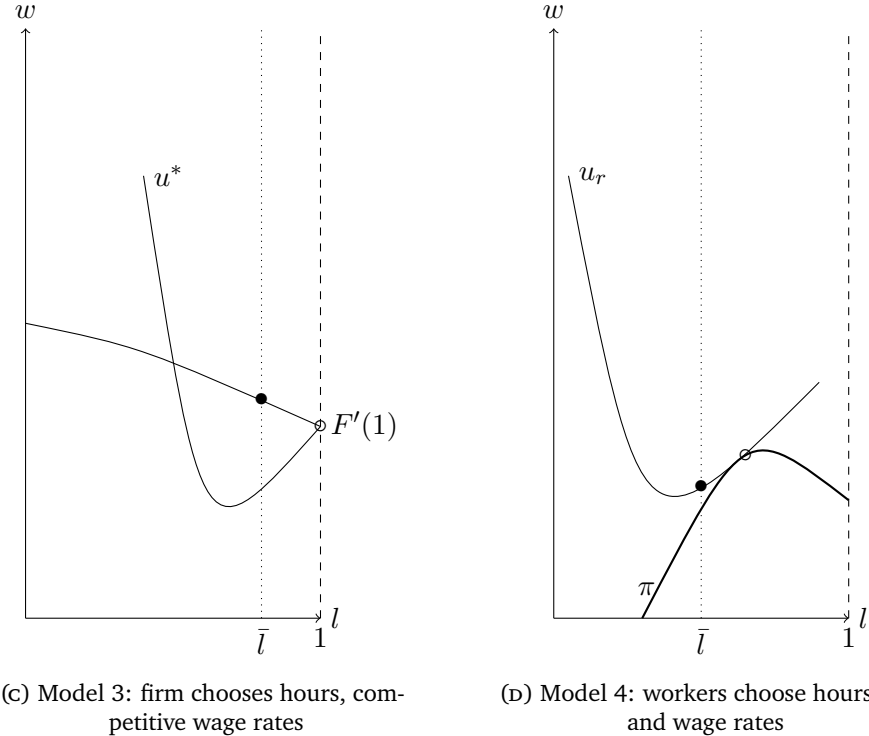


FIGURE 2.1: Equilibrium before (○) and after (●) a small binding working time reduction $\bar{l} < l$.

$$dw \leq 0.$$

In model 3, the firm chooses hours to maximize profits and wages are competitive $w = F'(l)$ such that the optimal schedule solves $l = \arg \max F(l) - lF'(l)$. Because $F(\cdot)$ is concave, this objective increases in l such that the solution is a corner solution: hours will be set at their largest admissible value, that is $l = \bar{l} \leq 1$. When workers have no leisure and $l = 1$, they will welcome a working time reduction such that $\frac{\partial u(\cdot)}{\partial l} \leq 0$. Here again, because $F'(l)$ is decreasing in hours, a working time reduction increases wage rates in equilibrium $dw \geq 0$.

Finally, in model 4, firms choose both wage rates and hours to maximize profits. Equivalently, the firm chooses the profit-maximizing contract on the workers' reservation indifference curve u_r . Necessarily, this is a contract with long hours and low wage rates such that the marginal utility of an extra hour at that wage rate is negative $\frac{\partial u(\cdot)}{\partial l} \leq 0$ and workers would welcome a working time reduction. However, in equilibrium they must remain on the same indifference curve u_r , such that the utility gains generated by the policy on leisure will be fully compensated by a loss in wage rates, such that $dw < 0$.

In Table 2.1, I summarize the theoretical predictions on $\frac{\partial u(\cdot)}{\partial l}$ and dw for the four models. The main results of this section can be summarized as follows. First, if wages are rigid, workers are more likely to be favorable to government-mandated working time reductions when firms set hours. Second, the very same working time reduction increases competitive wage rates but decreases monopsonistic wage rates⁷.

All these results are derived with only one labor input in the production function. In Appendix 2.7, I consider a production function $F(N, l)$ that may have an imperfect sustainability between jobs N and hours worked l . I show that the main results carry over provided that the output elasticity of hours exceeds the output elasticity of jobs $lF'_l > NF'_N$. In section 2.5,

⁷Interestingly, the Roosevelt administration introduced the minimum wage alongside a working time reduction, see Fishback, Vickers, and Ziebarth (2024).

Who sets l ?	Utility-maximizing l		Profit-maximizing l	
Who sets w ?	Competitive <i>Model 1</i>	Monopsony <i>Model 2</i>	Competitive <i>Model 3</i>	Monopsony <i>Model 4</i>
Before $\bar{l} < l$	$\frac{\partial u(\cdot)}{\partial l} \geq 0$	$\frac{\partial u(\cdot)}{\partial l} \geq 0$	$\frac{\partial u(\cdot)}{\partial l} \leq 0$	$\frac{\partial u(\cdot)}{\partial l} \leq 0$
After $\bar{l} < l$	$dw \geq 0$	$dw \leq 0$	$dw \geq 0$	$dw \leq 0$

TABLE 2.1: Summary of theoretical predictions in the four different models.

I build a search and matching model of the labor market with production function $F(N, l)$ and study the welfare effects of the policy.⁸

2.4 Reduced-form policy evaluation

2.4.1 Institutional context

At the time of the reform, Belgium's coal industry was mature and the coal-producing provinces were among the highest GDP per capita regions in Europe (Rosés & Wolf, 2021). As the first country to industrialize on the European continent, Belgium relied heavily on coal extraction to provide manufacturing industries with cheap energy (Philips & Buyst, 2020).

The policy under study is the first major piece of labor regulation affecting prime-age males in the country.⁹ Meanwhile, neighboring countries already had some form of working time regulations in the mining industry: for example, France had a maximum workday of 10 hours

⁸It would be easy to show that models 1 and 2 imply that working time reductions are inefficient while models 3 and 4 give a redistributive role to working time reductions. Welfare when production includes employment is less obvious (and therefore more interesting) because working time reductions affect not only the quality but also the quantity of jobs.

⁹Earlier regulations were mostly targeted to female and child labor. In the mining industry, underground labor was prohibited for women younger than 21 and male children below 12 from 1892 onward (Annales des Mines, 1907). The 1910 reform was among the first policies to break the *Laissez-faire* tradition on which Belgium was built, which consisted in low tariffs and few regulations (Abbeloos, 2008).

in 1900 and 8 hours in 1905.¹⁰ Compared their competitors, Belgian coal mines were relying more on manual labor and less on mechanized extraction (Denoël, 1909). They also exhibited a smaller mortality risk (Leboutte, 1991) despite longer workdays (Cousot, 1908).

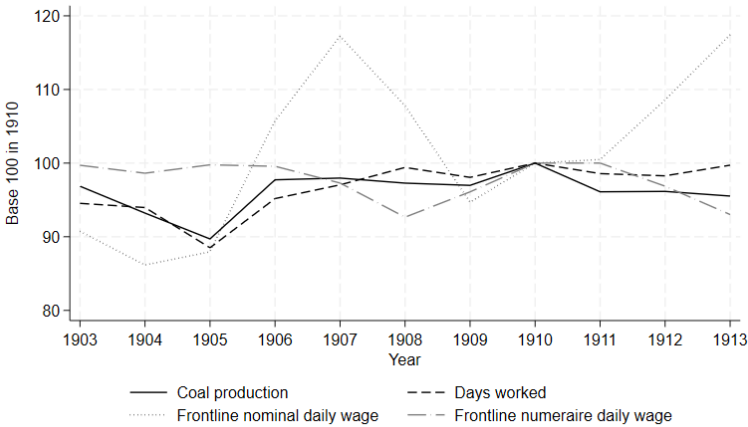
The policy was passed on December 31, 1909 and imposed that a workday for underground workers could not exceed 9 hours 30 minutes from January 1, 1911 and 9 hours from January 1, 1912.¹¹ Before this, there was no regulations on daily schedules. Exceptions were granted for some specific underground occupations such as horseman or cagers, but these exceptions may not exceed one hour per day. Violations were subject to civil fines or criminal charges.

There are two main reasons why studying this reform is interesting with respect to our research question. First, there is no confounding policy at the same time: there was no payroll tax and barely any other labor regulations. This contrasts with modern-day reforms on working time where these policies are typically coupled with support measures such as payroll tax reductions, rendering identification of the pure impact of working time reductions tedious.

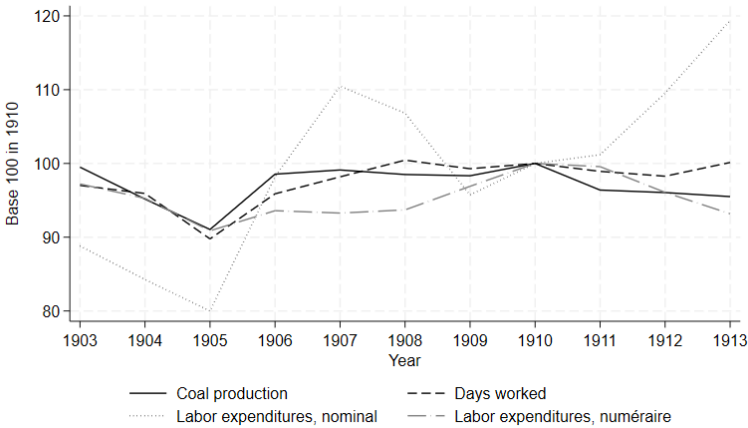
Second, there was no downward wage rigidity at the time. In Figure 2.2, we show that wages display a striking cyclical both in nominal and real terms. This holds both for the average firm-level wage in panel (a) and for the aggregate variables in panel (b). It also holds in terms of average wage per worker as well as firm-level labor expenditures. In principle, if working time reductions were inducing equilibrium wage cuts $dw \leq 0$, our setting should allow us to observe them.

¹⁰In the Netherlands, the maximum workday was 8.5 hours in 1908. In Prussia, the maximum workday was 8 hours in 1905 and even limited to 6 hours in mines where temperature exceeds 28 degrees Celsius (Cousot, 1908).

¹¹Working time must be understood as *from bank to bank*, i.e. the time from the surface at the beginning of the working day to the time at the surface at the end of the working day. Hence, it includes the time in the lift as well as time needed to walk from the lift to the work station.



(A) Evolution of the cross-sectional average for selected outcomes.



(B) Evolution of aggregates over firms for selected outcomes.

FIGURE 2.2: Descriptive statistics over the business cycle of Belgian coal mines, 1903-1913. Coal production is measured in tons of coal while total days worked include all occupations within the firm. Monetary units are shown in current Belgian francs and then converted to the numéraire by dividing by the price of output. Base year is set to 1910.

2.4.2 Data

I use administrative data retrieved from archival sources. The coal mining industry was closely scrutinized by the government for several reasons. First, coal mines were subject to a tax (composed of a fee and a linear rate), although it only raised a modest amount: 1.5 million BEF in 1903 i.e. 0.3% of the state's revenue (Chamber of Representatives, 1903). Second, coal mines were important to the state for industrial, political, social, and economic factors. More than 100,000 workers were directly employed in coal mining while 37% of GDP was produced by the manufacturing sector (Buyst, Smits, & Van Zanden, 1995). Third, as all mineral resources belonged to the state but were leased for private exploitation¹², the government was keen on monitoring production.

As a result, the Mining Administration kept a consistent record of data of remarkable quality for the time. Each year in August, state officials¹³ were sent to each mine to collect statistics on production, prices, employment and wages. These reports were then collected by the administration to establish a firm-level panel dataset. In Figure 2.C.1, I provide examples of a report in (a) and of the panel dataset in (b).

I digitized the panel dataset for the province of Hainaut while Delabastita and Rubens (2025) digitized the provinces of Liège and Namur. In 1910, the province of Hainaut accounted for 71% of national coal production and 67% of labor expenses. The combination of these efforts yields a dataset covering the universe of mines in the country from 1903 to 1913.

This dataset contains firm-level information on annual production, costs, employment and wages but not on daily working time. Yet, as

¹²This tradition was inherited from the French domination rather than the Dutch domination. The Imperial Law of April 21, 1810 promulgated by Napoleon set the basis for such leasing on minerals and served as backbone of Belgian legislation on mining. By contrast, in 1900 the Dutch government still had a monopoly on two third of national production.

¹³These officials were public servants with a high level of education such as mine engineers. Besides collecting statistics, these officials also had a role of policing, advising and studying the mines.

one of the first piece of labor legislation, this reform was highly controversial in parliament¹⁴ which initiated a parliamentary commission. This commission produced over 3000 pages of documents and requested information on daily hours worked to the Mining Administration. The latter collected firm-level hours worked in August 1900 and published the data in parliamentary proceedings in 1907 while noting that "*the situation has barely changed since then*" (Annales des Mines, 1907, p.556). Table 2.2 reports summary statistics for this cross-sectional data on hours worked by geological region.

Hours	Mons	Centre	Charleroi	Namur	Liège	Total
(7.5, 9]	2	2	0	5	30	39
(9, 9.5]	2	0	4	1	9	16
(9.5, 10)	1	0	0	0	2	3
[10, 12]	13	8	31	5	4	61
Total	18	10	35	11	40	114

TABLE 2.2: Firm-level average daily hours worked for frontline workers before the reform, by geological region.

Before the reform, more than 70% of the mines were above the 9 hour threshold. While geology seems to play a role as workdays are longer in Mons than in Liège geological regions (Vandervelde, 1911), there also exists within-region variation.

Overall, the cross-section of hours worked contains 114 firms which is a fraction of the universe of firms contained in the panel. All estimations below are computed on a dataset matching this panel with the cross-section of hours worked. As reported in appendix Table 2.C.2, this

¹⁴The bill was introduced by socialist M.P. Destrée in 1903. From 1884 and until World War I and despite electoral reforms toward universal suffrage, the Catholic party had an absolute majority and initially opposed the maximum workday in the coal mines, as did King Leopold II. In 1907, some Catholic MP flipped their vote and created a political crisis which culminated in the resignation of the prime minister and mine owner Count de Smet de Nayer (Neuville, 1981). The new government initiated a Parliamentary Commission. Documents contain interviews of workers and mines owners but also technical reports from academics in economics and engineering.

matched dataset covers more than 96% of production and employment. The unbalanced panel of firms contains entities that merged, were acquired or exited the market. In the remainder, I consider that a merged entity is a new firm while an acquired firm espouses the identity of its acquirer. For details on the construction of variables, I refer the reader to Appendix 2.C.2.

Besides datasets, these archival sources also provide qualitative information on the context of the reform. In particular, the parliamentary commission interviewed mine owners and workers in various locations and asked their opinion on the reform. These interviews are helpful to understand the labor conflict : all owners opposed the reform and threatened to compensate its effect by cutting wages. On the workers' side, all were in favor of a reform cutting hours holding wages constant. Some were in favor of the reform even if wages were cut while others opposed wage cuts (Parliamentary Commission, 1909). This qualitative evidence is consistent with model 4 in section 2.3 where firms set both hours and wages and the marginal utility of an extra hour is negative $\frac{\partial u(\cdot)}{\partial l} \leq 0$.

2.4.3 Policy evaluation

The empirical strategy uses pre-reform hours worked as a measure of exposure to the reform. I construct the continuous variable Exposure_j as the difference between the hours worked by frontline workers at firm j and the maximum workday of 9 hours. It is set to 0 for a firm which was already below the 9h threshold before the reform.

$$\text{Exposure}_j = \max\{\text{Frontline_Hours}_j - 9; 0\}$$

The average Exposure_j among firms with positive exposure is 1.03 hours with a standard deviation of 0.43. The main specification follows the following regression equation:

$$y_{j,t} = \beta_0 + \sum_{k \neq 1909} \beta_k \times \text{Exposure}_j \times \mathbf{1}_{t=k} + \mu_j + \nu_{r,t} + \beta X_{j,t} + \epsilon_{j,t}$$

where $y_{j,t}$ denotes the outcome for firm j in year t , μ_j and $\nu_{r,t}$ are firm and region-by-year fixed effects respectively, $X_{j,t}$ are a set of controls

and standard errors $\epsilon_{j,t}$ are clustered at the firm level. The coefficients β_k are the objects of interest. Because of the construction of $\text{Exposure}_{j,t}$, β_k can conveniently be interpreted as the average effect in year k across treated firms of a mandated one-hour reduction. The omitted year is set to 1909 which is the last year before the law is passed. As compliance is unobserved, this effect should be interpreted as an intention to treat.

This strategy is standard in the policy evaluation literature (Carry, 2023; Harasztosi & Lindner, 2019; Saez, Schoefer, & Seim, 2019). The identifying assumption is that firms with different hours worked before the reform would have had parallel evolution in their outcomes if the reform had not happen. This assumption could be rejected if β_k significantly differs from 0 in years prior to the reform.

Profits. I find a large and negative effect of the reform on profits: a reduction of one hour per day reduces profits by 7962 and 6009 tons of coal in 1911 and 1912 respectively as reported in Figure 2.3. These effects are economically significant as the average profit for firms above the 9h threshold before 1910 was 8473 tons of coal. This confirms anecdotal evidence that the reform was not anticipated and had largely adverse effects on mine owners. I now decompose the results into firms' reactions along the extensive and the intensive margins.

Extensive margin Given adverse effects on profits, firms may react to the policy along the extensive margin by exiting the market. In Table 2.C.3, I report the count of firms' exit per year. While only one firm exited the market in the period 1903-1909, 6 firms shut down operations after 1909.

Intensive margin: employment The intensive margin reactions are computed on the subsample of firms that never shut down operations.¹⁵ In Figure 2.4, I report the values of β_k for regressions with $y_{j,t}$ being the number of frontline workers in panel (a) and the log of that number in panel (b). Both effects are somewhat noisily estimated and should be

¹⁵This is motivated by the fact that intensive margins responses are comparable when one uses log transformations for which 0 values create indeterminacy. See Chen and Roth (2024) for a characterization of the problem and motivations for splitting results along the intensive and extensive margins.

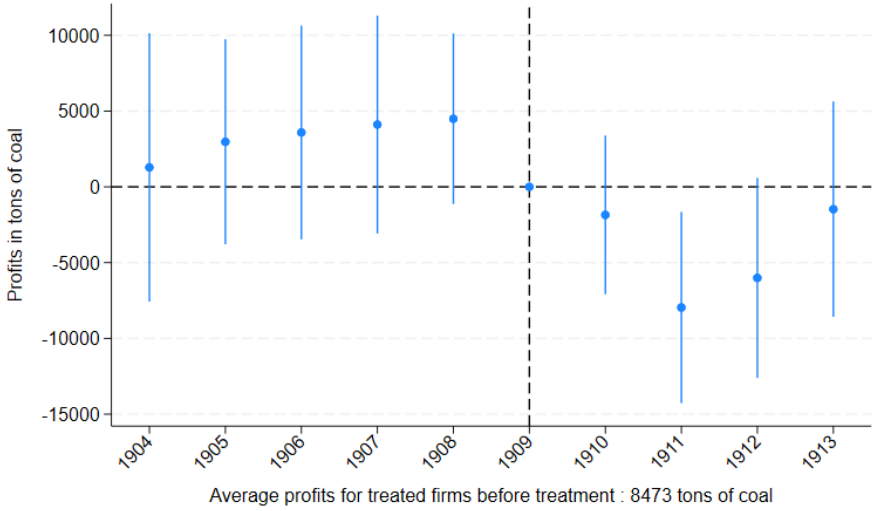


FIGURE 2.3: β_k for each year if $y_{j,t}$ are firms' profits in units of the numéraire. Controls $X_{j,t}$ are size of exploitation at the surface in squared meters, TFP, average power of coal veins, share of frontline workers, share of underground workers and tons of coal of auto-consumption. The p-value for testing $H_0 : \beta_k = 0 \forall k < 1909$ is 0.64. The within R-squared is 0.3207.

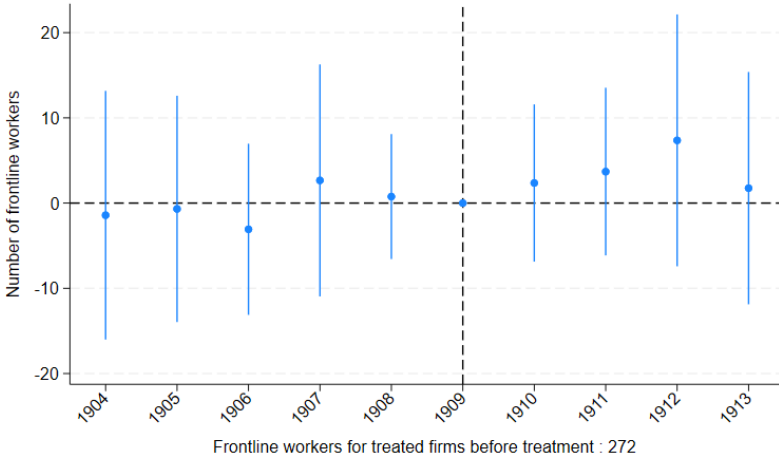
interpreted with caution: panel (a) suggests that a mandated reduction of working time by one hour does not affect differentially firms with unequal Exposure_j in absolute terms but panel (b) suggests a negative effect in relative terms. In Table 2.C.3, I show that firms with positive Exposure_j (i.e. targeted by the reform) were on average twice larger than firms with zero Exposure_j (i.e. non-targeted) before the reform. Hence, these results are consistent with a story where the reform increases the number of frontline workers by a similar magnitude in both targeted and non-targeted firms, which implies that the percentage change is larger in the latter than the former.

Intensive margin: earnings In Figure 2.5, I show β_k for each year if the dependent variable is either the log of frontline workers' earnings in panel (a) or the log of all underground workers' earnings in panel (b). Both panels suggests that a working time reduction has a non-positive effect on earnings although here again the estimation is noisy: a mandated working time reduction of an hour reduces average earnings for underground worker by 6.18% in targeted firms related to non-targeted firms in 1912.

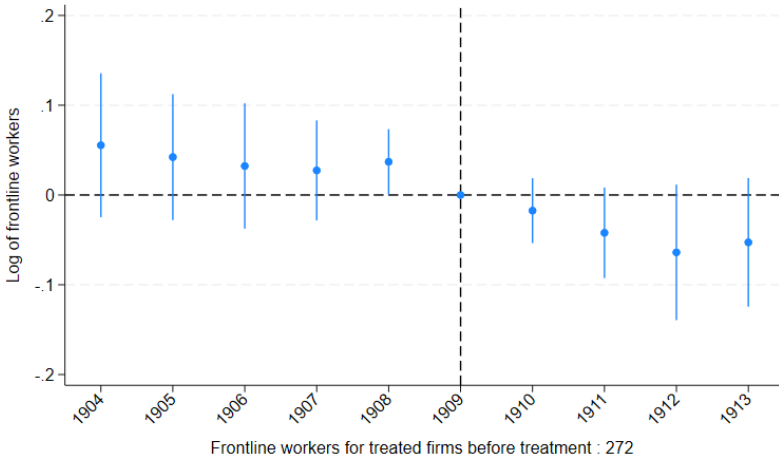
In Table 2.C.4, I show that the results obtained here with event-study regressions are broadly consistent with those obtained with standard difference-in-differences strategy, whether the treatment is binary as in the second column or continuous as in the third column. Overall, hours worked l decreased by 7.6% on average while earnings wl decreased by a smaller proportion which implies that average wage rates must have increased and $dw \geq 0$ as in Model 3 of section 2.3. Importantly, these average effect may hide significant heterogeneity across firms which is studied in the next section.

2.5 General model

In this section, I embed models 3 and 4 of section 2.3 where firms set hours into a larger model where firms have heterogeneous productivities. This is consistent with the observation that coal mines workers were not free to choose their hours. The labor market features directed search and

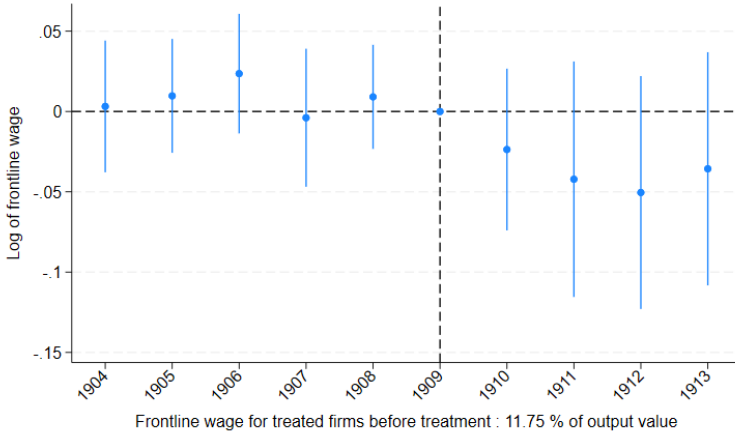


(A) Number of frontline workers. P-value for H_0 is 0.73. Within R-squared is 0.55.

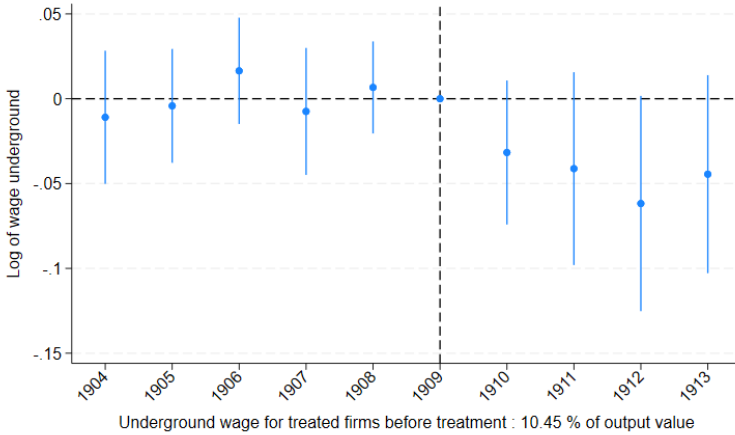


(B) Log of number for frontline workers. P-value for H_0 is 0.34. Within R-squared is 0.45.

FIGURE 2.4: β_k for each year. Standard errors are reported at 95% confidence level. I report the p-value of the joint statistical test with $H_0 : \beta_k = 0$ for all $k < 1909$. Controls $X_{j,t}$ are size of exploitation at the surface in squared meters, TFP, average power of coal veins, share of frontline workers, share of underground workers and the output price in current BEF.



(A) Log of earnings of frontline workers. P-value for H_0 is 0.16. Within R^2 is 0.66.



(B) Log of earnings for underground workers. P-value for H_0 is 0.10. Within R^2 is 0.72.

FIGURE 2.5: β_k for each year. Standard errors are reported at 95% confidence level. I report the p-value of the joint statistical test with $H_0 : \beta_k = 0$ for all $k < 1909$. Controls $X_{j,t}$ are size of exploitation at the surface in squared meters, TFP, average power of coal veins, share of frontline workers, share of underground workers and the output price in current BEF.

contract posting¹⁶ as in the seminal paper by Moen (1997) but it is kept static as in Vergara (2023).¹⁷

2.5.1 Directed search model

Labor market Consider a set of measure L of workers who are each endowed with a unit of time.¹⁸ Firms are posting vacancies with a pair $m = (w_m, l_m)$ of wage rate and hours worked. All vacancies v_m at a given pair m form a submarket and there may be potentially many submarkets for each type. Workers are applying to one vacancy among the various submarkets m and the number of applicants for a given submarket is denoted by a_m .

Matching There is a technology $\mathcal{M}(a_m, v_m)$ matching applicants and vacancies in a submarket. I assume that it is constant returns to scale.¹⁹ As a result, one can compute the job-finding probability as

$$p_m = \frac{\mathcal{M}(a_m, v_m)}{a_m} = \mathcal{M}(1, \theta_m) = p(\theta_m)$$

where $\theta_m = \frac{v_m}{a_m}$ is the submarket tightness. Similarly, the job-filling probability is

$$q_m = \frac{\mathcal{M}(a_m, v_m)}{v_m} = \mathcal{M}\left(\frac{1}{\theta_m}, 1\right) = q(\theta_m)$$

¹⁶There is a long literature exploring the relevance of posting in labor market. For example, Eeckhout and Kircher (2010b) showed that price posting (sorting workers types ex-ante) emerges as an equilibrium trading mechanism rather than auctions (screening workers types ex-post) when the meeting technology is sufficiently rival. A review can be found in Wright et al. (2021).

¹⁷The key difference with Vergara (2023) is the presence of leisure.

¹⁸All the analysis in this section could be carried out in a model where workers have heterogeneous types as long as firms observe it and the labor market is segmented.

¹⁹See Hall and Schulhofer-Wohl (2018) and Petrongolo and Pissarides (2001) for empirical evidence on the matching function.

and $q_m = p(\theta_m) \times \frac{1}{\theta_m}$. It is further assumed that the matching technology is twice continuously differentiable, increasing and concave. Hence,

$$\frac{\partial p(\theta_m)}{\partial \theta_m} > 0 \quad \frac{\partial q(\theta_m)}{\partial \theta_m} < 0$$

In other words, the tighter submarket, the higher will be the job-finding probability and the lower will be the job-filling rate.

Workers Each worker decides whether or not to enter the labor market. Workers are only heterogeneous in their disutility of participation, denoted by d and drawn from a cdf $G(\cdot)$. They have preferences over consumption and leisure represented by an ordinal utility function $u(c, l)$. The government grants some benefits B to all nonemployed agents, be they inactive or unemployed.²⁰ I assume that each worker may only apply to one submarket, such that the expected utility of participating to the labor market for a worker of disutility of participation d reads

$$\max_m \left\{ p_m u(w_m l_m, l_m) + (1 - p_m) u(B, 0) \right\} - d$$

An individual worker applies by taking p_m as given but the aggregate behavior of all workers will pin down p_m . Hence, in equilibrium, it must be that all agents have the same expected utility²¹ (net of their d) that I will denote by $\bar{U}$.²² However, the fact that all workers of the same type enjoy the same expected utility ex-ante does not imply that they all enjoy the same ex-post utility.

The key mechanism of the model is already visible here: a worker prefers submarkets that pays higher income $w_m l_m$ for lower hours worked

²⁰This formulation supposes that inactives are entitled to the same benefit coverage B than unemployed. This is made solely for analytical tractability, as it is typically not the case in actual economies. Germain (2023) studies this mismatch in depth. For the value of nonemployment versus unemployment, see Jäger et al. (2020).

²¹Proof: assume it is not the case such that m' yields higher expected utility than m . Then some agents will move towards m' , which decreases $\theta_{m'}$ and decreases $p_{m'}$ in turn. This marginally reduces $U(m')$. The process continues until $U(m) = U(m')$.

²²This is called the *market utility* by (Wright et al., 2021).

l_m but their tightness θ_m will be lower. In other words, as elsewhere in competitive search models, there is a tradeoff between favorable terms of trade and probability of trade. Observe that this equation also defines the level of tightness θ_m on the equilibrium path. In particular, we have equilibrium tightness as an implicit function of three variables $\theta(w_m, l_m, \bar{U})$.

A worker of type d participates if and only if $\bar{U} \geq d + u(B, 0)$ such that the total number of participants is given by $G(\bar{U} - u(B, 0)) \times L$.²³

Firms All firms face perfect competition in the single output market whose homogeneous good is set as the numéraire. Indeed, Delabastita and Rubens (2025) documented that Belgian coal mines had no price-setting power in their output market. Firms are only heterogeneous in their total factor of productivity ψ_j .²⁴ They share the same production technology $F(N, l)$ whose inputs are jobs $N = qv$ and hours worked l . Firms internalize workers' decisions such that their job-filling probabilities $\tilde{q}_m$ are implicitly defined by equation (3) with $\tilde{q}_m = q(\theta(w_m, l_m, \bar{U}))$. Conditional on entering the labor market, they maximize expected profit by choosing the number of vacancies v_m to post in each submarket m along with the associated wage-hours (w_m, l_m) :

$$\pi(\psi_j) = \max_{\forall m: w_m, l_m, v_m} \int_m \psi_j F(\tilde{q}_m v_m, l_m) - \tilde{q}_m v_m w_m l_m - k(v_m) dm$$

where $k(\cdot)$ is the increasing and convex cost of vacancy posting. Because firms face a cost of creating vacancies independently of the hours worked in that vacancy, it is *ex-post* more costly to hire two workers each working

²³We can mention current endeavors in the literature to estimate $G_i(\cdot)$, that is to compute workers' outside options (Caldwell & Danieli, 2024; Caldwell & Harmon, 2019; Caldwell & Oehlsen, 2018; Jäger, Schoefer, & Zweimüller, 2023; Jäger et al., 2020, 2022).

²⁴The analysis in this section would be unchanged if ψ_j was assumed to be the productivity of the match between firms and workers, both of them having heterogeneous skills, as long as the skill-matching function exhibits supermodularity. See Eeckhout and Kircher (2010a, 2018) for a rigorous treatment.

half-time rather than one full-time worker, *ceteris paribus*. However, *ex-ante* the probability that vacancies will be filled might differ between half-time and full-time contracts.²⁵

Firms entry A set of measure K contains capitalists who are heterogeneous in productivity ψ_j drawn from a compact set $[\underline{\psi}, \bar{\psi}]$. A capitalist enters the labor market if and only if their expected profit is greater than a fixed cost denoted by x . Because the profit function is monotonically increasing in ψ_j , there exists a decisive ψ^* such that $\pi(\psi^*) - x = 0$. All capitalists with $\psi_j < \psi^*$ abstain from entering the labor market and remain inactive while all with $\psi_j \geq \psi^*$ participate.

Economy, allocation and equilibrium The directed search economy e is a set of vNM agents of measure L with leisure preferences $\succsim$ and disutility of participation d drawn from a cdf G , as well as a set K of firms with productivity ψ_j , the vacancy posting cost function $k(\cdot)$, the entry cost x and the production function $F(\cdot)$ and the matching function $\mathcal{M}$.

$$e = \left\{ L, \succsim, G, \{\psi_j\}_{\forall j \in K}, k, x, F(\cdot), \mathcal{M} \right\}$$

Definition 2.1. An allocation is a **competitive search equilibrium** for e if it is characterized by the market utility $\bar{U}$, the zero-profit firm ψ^* , applications a_m , vacancies v_m in each submarket $m = (w_m, l_m)$ as well as a mapping $P(\cdot)$ from productivity to submarkets

1. **Firms are expected profit-maximizers:**

The tuples (v_m, w_m, l_m) solve the FOC of firms of type $\psi_j = P^{-1}(m)$

²⁵See Carry (2023) for empirical evidence of such imperfect substitutability the French case.

for $m \in [P(\psi^*), P(\bar{\psi})]$ taking ψ^* and $\bar{U}$ as given

$$v_m : \tilde{q}_m^i(\psi_j F'_N - w_m l_m) \leq k'(v_m) \text{ with equality if } v_m > 0 \quad (2.1)$$

$$w_m : v_m \tilde{q}'_{m,w}(\psi_j F'_N - w_m l_m) \leq q_m v_m l_m \text{ with equality if } w_m > 0 \quad (2.2)$$

$$l_m : \tilde{q}'_{m,l}(\psi_j F'_N - w_m l_m) + \psi F'_l \geq \tilde{q}_m w_m \text{ with equality if } l_m < \bar{l} \quad (2.3)$$

where $\bar{l} \leq 1$ the legal maximum workweek, and the partial variation of tightness with respect to wages and hours worked are denoted by $\tilde{q}'_{m,w} = \frac{\partial \tilde{q}_m}{\partial w}$ and $\tilde{q}'_{m,l} = \frac{\partial \tilde{q}_m}{\partial l}$.

2. Firm's entry constraints :

$$\psi^* \text{ solves } \pi(\psi^*) = x \quad \text{taking } \bar{U} \text{ given} \quad (2.4)$$

3. Across-submarket equilibrium condition :

Applications ensures that all submarkets yields market utility

$$a_m \text{ solves } \bar{U} = p_m u(w_m l_m, l_m) + (1 - p_m) u(B, 0) \quad (2.5)$$

with $p_m = p(\frac{v_m}{a_m})$, taking ψ^*, v_m, l_m, w_m as given for all m

4. Workers' participation constraints :

$$\bar{U} \text{ solves } \int_{P(\psi^*)}^{P(\bar{\psi})} a_m dm = G(\bar{U} - u(B, 0)) \times L \quad (2.6)$$

taking ψ^*, u and a_m as given.

2.5.2 Properties

I now turn to the properties of the competitive search equilibrium just defined.

Firm and submarket sizes In a single submarket may be found vacancies from several firms. Say that we find two firms $\psi_j > \psi_k$ in some given

submarket (w, l) . By equations 2.1 and 2.3 respectively, it must be that

$$\begin{aligned}\psi_j F'_N(v_j q, l) &= \psi_k F'_N(v_k q, l) \\ \frac{1}{v_j} \psi_j F'_l(v_j q, l) &= \frac{1}{v_k} \psi_k F'_l(v_k q, l)\end{aligned}$$

In other words, the marginal product of jobs must be equal for both firms, and their relative number of posted vacancies must be proportional to their relative marginal product of hours. As a consequence, their marginal rate of technical substitution must be equal.

Conversely, can a single firm post vacancies in several submarkets in equilibrium? Observe that production is separable in submarkets, such that the model assumes away potential complementarities between submarkets in production. However, in general firms will post vacancies in several submarkets, i.e. equation 2.1 may be saturated for many m . This is so because the presence of risk and non-degenerate job-filling probabilities induces a portfolio choice for the firm, whose optimal strategy may consist in diversification to hedge against risk. To see why firms are not risk-neutral, observe that (i) the vacancy posting cost $k(v)$ is paid almost surely and (ii) concave production may imply risk aversion. Hence, consistent with the empirical setup studied in section 2.4, this model produces large firms in the sense of Eeckhout and Kircher (2018).

Monopsony power Although search is competitive, the labor market remuneration might not be. Indeed, this model produces an earnings markdown and workers are paid below their marginal product. If the firm-specific wage elasticity of labor supply is $\epsilon_m^w = \frac{\partial \hat{q}_m v_m}{\partial w_m} \frac{w_m}{\hat{q}_m v_m}$, then equation 2.2 reads

$$\frac{\psi_j F'_N - w_m l_m}{w_m l_m} = \frac{1}{\epsilon_m^w}$$

which is the original Robinson (1933) markdown equation: the earnings markdown is inversely proportional to the firm-specific elasticity of labor

supply. It is noteworthy that the present model has two sources of monopsony power.²⁶ As models 2 and 4 in section 2.3, there is monopsony power because agents' utility function implies that the wage elasticity of labor supply is finite. Yet, contrary to section 2.3, there is monopsony power because of matching frictions. Indeed, observe that when matching frictions increase, ϵ_m^w decrease and the earnings markdown increase.

Given this proximity to standard monopsony equations, it can be shown that a working time reduction may also have negative equilibrium effects on wage rates $dw \leq 0$ provided the production function satisfies $lF'_l \geq NF'_N$, i.e. that the marginal product of an hour exceeds the marginal product of a job. In that case, workers have a negative marginal utility of hours worked and would like to work less conditional on wage rates. As a result, the analysis of the wage effect of working time regulations follow the same line as model 4 in section 2.3 and appendix 2.A.2.

Contract dispersion and sorting The model produces wage dispersion among (observably) identical workers²⁷ which is a long-standing finding of empirical labor studies (see Card et al. (2018)) because there is a continuum of submarkets in equilibrium. It is naturally interesting to study how heterogeneous firms are sorting in the contract space. Taking the total derivative of equation 2.1 in a particular submarket away from corner solutions one gets

$$\underbrace{dv (\tilde{q}^2 \psi F''_N - k''(v))}_{<0 \text{ if } F''_N \leq 0} + \underbrace{dw (\tilde{q}'_w (\psi F'_N - wl) - \tilde{q}l + \tilde{q}v \tilde{q}'_w \psi F''_N)}_{<0 \text{ by equation 2.2}} + dl \left(\gamma_l \right) + d\psi \underbrace{(\tilde{q} F'_N)}_{>0} = 0$$

where indices are omitted for brevity. Hence, a firm with higher productivity ψ_j will post more vacancies and higher wages. With respect

²⁶Berger et al. (2024) quantifies the empirical importance of several determinants of monopsony power including preferences and search frictions.

²⁷It escapes the Diamond (1971) paradox of homogeneous contract in search models by allowing heterogeneous firms' productivities.

to hours worked, the coefficient γ_l multiplying dl is difficult to sign in the general case. However, if one assumes that labor production follows $F(N, l) = N^\alpha l^\beta$ with $\alpha < \beta < 1$, one gets that $\gamma_l > 0$, such that higher productivity firms post contracts with shorter hours. The proof is relegated to appendix 2.B.1.

Under these assumptions, firms with higher productivities offer higher wage rates, and lower hours worked, hence a higher ex-post utility to their workers. However, ex-ante utilities are equal for all workers. This implies that job-finding probabilities are smaller in firms with higher productivities. In other words, the larger number of vacancies in these firms is dominated by the larger number of applications such that equilibrium tightness $\theta_m = \frac{v_m}{a_m}$ decreases with firms' productivity.

We can also study comparative statics when the preferences for leisure of agents locally increase: the slope of its indifference curve gets steeper. This does not affect labor supply in level, but it affects labor supply elasticities. As a result, the firm reacts by modifying w_m and l_m according to equations 2.2 and 2.3. In particular, observe that a simple rearrangement of these equations for interior solutions yields

$$-\frac{\tilde{q}'_l l}{\tilde{q}'_w w} = \frac{\psi F'_l}{\tilde{q}vw}$$

where the left handside coincides with the marginal rate of substitution derived from $u(\cdot)$ at the particular contract. Hence, when the indifference curve gets steeper, the left handside decreases. The equilibrium conditions imposes that on the right handside firms must marginally increase w or decrease l . As a consequence, a marginal increase in the preference for leisure locally increases wage rates and decreases hours worked.

2.5.3 Welfare analysis

The competitive search equilibrium features monopsony power, large firms, and sorting patterns that are consistent with the empirical context of section 2.4. Moreover, the welfare effects of the policy is ambiguous here because the negative effects on wages and employment may be offset

by positive effects on leisure. In this section, I quantify welfare gains and losses of the Belgian 1910 coal mine reform using the directed search model.

As discussed above, the market utility $\bar{U}$ summarizes workers' ex-ante welfare. Interestingly, it features a trade-off between jobs quantity and job quality. I now express it in terms of sufficient statistics as in Vergara (2023). Recall that for each submarket m we have

$$\begin{aligned}\bar{U} &= p_m u(w_m l_m, l_m) + (1 - p_m) u(B, 0) \\ a_m [\bar{U} - u(B, 0)] &= N_m [u(w_m l_m, l_m) - u(B, 0)]\end{aligned}$$

where N_m is the number of workers employed in submarket m . Integrating over submarkets yields

$$\begin{aligned}[\bar{U} - u(B, 0)] \int_m a_m dm &= \int_m N_m [u(w_m l_m, l_m) - u(B, 0)] dm \\ \bar{U} - u(B, 0) &= \frac{\int_m N_m [u(w_m l_m, l_m) - u(B, 0)] dm}{G(\bar{U} - u(B, 0)) \times L}\end{aligned}$$

where the last equation is obtained using equation 2.6. The denominator is simply the total number of workers participating to the labor market and the numerator is a weighted sum of utilities across submarkets whose weights are the submarkets' employment size N_m . This ratio is simply giving the average utility among active workers. To see this, observe that the last equation can be written as

$$\begin{aligned}\bar{U} - u(B, 0) &= \frac{\int_m N_m dm}{G(\bar{U} - u(B, 0)) \times L} \frac{\int_m N_m [u(w_m l_m, l_m) - u(B, 0)] dm}{\int_m N_m dm} \\ &= \mu \mathbb{E}_m [u(w_m l_m, l_m) - u(B, 0)]\end{aligned}$$

where μ is the employment rate and $\mathbb{E}_m [u(w_m l_m, l_m)]$ is the average utility of a job among workers. It is straightforward to show the following result.

Proposition 2.1. *A small working time reduction²⁸ $d\bar{l}$ has a positive impact on on ex-ante welfare $\frac{d\bar{U}}{d\bar{l}} > 0$ if*

$$\eta^N + \eta^u > 0$$

where η^N is the percentage change in employment rate due to the reform and η^u is the percentage change in workers' average (ex-post) utility.

Taking the total derivative of $u(c, l)$ with respect to $\bar{l}$ we can write the percentage change in utility η^u as a function of the percentage change in consumption and leisure:

$$\begin{aligned} \frac{du}{d\bar{l}} &= u'_c \frac{dc}{d\bar{l}} + u'_l \frac{dl}{d\bar{l}} \\ \eta^u &= \frac{\bar{l}}{u} \frac{du}{d\bar{l}} = \frac{cu'_c}{u} \frac{\bar{l}}{c} \frac{dc}{d\bar{l}} + \frac{lu'_l}{u} \frac{\bar{l}}{l} \frac{dl}{d\bar{l}} = \zeta_c^u \eta^c + \zeta_l^u \eta^l \end{aligned}$$

where η^c and η^l are the percentage change in consumption and hours worked while ζ_c^u and ζ_l^u are the elasticity of the utility function to consumption and hours respectively.

These sufficient statistics are strikingly simple. One can assess welfare effects of the reform simply by observing the average treatment effect of the reform on employment, earnings and leisure and postulating a cardinal utility function. In particular, it does not require to take any stance on the production side nor the extent of imperfect competition in the labor market, which is known to vary significantly across contexts and industries (Azar, Marinescu, & Steinbaum, 2022; Card et al., 2018). I note that these elasticities must be understood as macro-elasticities in the sense of Landais, Michaillat, and Saez (2018), i.e. incorporating all general equilibrium effects. The estimates derived in section 2.4 may

²⁸In order to enforce such reforms, governments must observe hours and hence contract (w, l) which contrasts with the typical assumption in public finance since Mirrlees (1971) that only income is observed. If governments observe (w, l) , why don't they use lump-sum transfers to decentralize any first-best allocation rather than using the maximum workweek? In this model, we may reconcile the second-best environment of Mirrlees (1971) with an information set that contains w and l by assuming that the government does not observe the identity of firms in a particular submarket, hence the second fundamental theorem of welfare economics may not be used.

be used to compute the welfare effects using these sufficient statistics.²⁹ However, two caveats must be raised.

First, I must assume a cardinal utility function to derive ζ_c^u and ζ_l^u . Obviously, welfare assessment will be sensitive to this choice.³⁰ Second, the estimates of section 2.4 are average treatment effects on the treated. As a result, I apply the welfare analysis only on workers from treated firms i.e. workers whose hours worked were above the maximum workweek prior to the reform. I also assume that the number of workers sending job applications to these firms is unaffected by the reform, which is a conservative assumption for my results.³¹

The point estimates from section 2.4 give us $\hat{\eta}^N = -6$ and $\hat{\eta}^c = -6$ while $\hat{\eta}^l = -7.9\%$. If preferences can be represented by a Cobb-Douglas utility, $\zeta_c^u = \delta = 1 - \zeta_l^u$, the reform is increasing treated workers' ex-welfare if

$$\begin{aligned} -6 + \delta(-6) - (1 - \delta)(-7.9) &> 0 \\ 1 - \delta &> 0.86 \end{aligned}$$

In others words, workers must spend at least 86% of their income buying leisure for this reform to be welfare-improving in the Cobb-Douglas case. In future iteration of the paper, I intend to use the equilibrium conditions in equations 2.2 and 2.3 to pin down values of ζ_c^u and ζ_l^u and then

²⁹The estimates of section 2.4 are expressed as the effect of a working time reduction by one hour, while these statistics are the effect of the overall reform. This does not change the analysis because multiplying $\eta^N + \eta^u$ by a scalar does not modify the sign. Moreover, the mean working time reduction was 1.03 hour such that our estimates are quantitatively close to the average impact of the reform.

³⁰One could have picked Boppart and Krusell (2020) utility function who show in a structural macro exercise that the functional form consistent with labor supply dynamics over the past century take the form of $u(w, l) = w l \phi(l c^{\frac{\nu}{1-\nu}})$ with ϕ a decreasing function, where $\nu \in (0, 1)$ is such that if productivity grows by g , then hours decrease at a rate g^ν and consumption increase at a rate $c^{1-\nu}$. However, 1910 mine workers were unlikely to use the preferences that lead to the decline in hours worked over the century that succeeds them.

³¹To see this, observe that $\eta^N = \frac{d\mu}{dt} \frac{\bar{l}}{\mu} = \frac{dN}{dt} \frac{\bar{l}}{N} - \frac{dA}{dt} \frac{\bar{l}}{A}$ where A is the number of applicants. Assuming the latter term to be equal to 0 pushes η^N upwards.

estimating these moments with my data to provide a nonparametric welfare estimation.

2.6 Conclusion

The paper has studied theoretically and empirically the effects of working time reductions on wages, employment, profits and welfare. On the normative side, we have shown that the empirical effect of working time reductions on employment, wages and profits are sufficient for welfare evaluation. The empirical application uncovered large and negative effects on profits, wages and employment in the first-ever working time reduction in Belgium such that the overall welfare effects may only be positive if workers had a large value attached to leisure.

There are several ways in which this line of research can be extended. First, the interaction with progressive taxes is of obvious interest for public economists. Second, the characterization of optimal policy and efficiency has been left aside, while heterogeneous preferences for leisure would allow us to discuss the desirability of gendered holidays or retirement policies. Third, the (counter)cyclicality of working time regulations could be interesting to relate to the literature on short-time work (Giupponi & Landais, 2023).

2.7 Bibliography

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2.A Frictionless hedonic model with employment

This section presents a general equilibrium model that unbundles hours and jobs but does not feature search and matching frictions. The determination of hours worked is hedonic, in the sense that hours are set by firms contrary to the neoclassical labor supply theory where hours are utility-maximizing. First, I present a game-theoretic microfoundation for the model. Second, I explain the sufficient conditions for the results of the toy model in section 2.3 to be preserved.

2.A.1 A game-theoretic microfoundation

I build a game-theoretic model where firms set schedules and which is compatible with any degree of monopsony power and nest the case of perfect competition.³² It generalizes models 3 and 4 of section 2.3.

Labor supply There is a mass of workers with a utility function $u(c, l)$ which is increasing and concave in consumption c and decreasing in hours worked l . Workers receive a wage rate w for each hour worked such that $c = wl$ in equilibrium. Workers are only heterogeneous in an outside option to the labor market which gives utility $d \geq 0$ distributed by a strictly increasing and concave cumulative distribution function $G(d)$. As a result, labor supply N_s is given by $N_s(w, l) = G(u(wl, l))$ and it follows from workers' preferences that $N_s(w, l)$ is increasing in wage rates w and inversely U-shaped (hence concave) in hours worked l .

Labor demand Consider J identical firms whose profits are defined by the difference between their production $F(N_j, l_j)$ and their total wage bill $N_j w_j l_j$ where N_j is the number of workers employed at firms j while w_j and l_j define the wage rate and hours worked. Firms choose which

³²Interestingly, this game-theoretic model can be seen as the labor market equivalent of the Cournot-Bertrand games of d'Aspremont and Dos Santos Ferreira (2021) who study imperfect competition on output markets (à la Dixit and Stiglitz (1977)) when firms choose both prices and quantities, thereby competing both for the market size and the market share. Here, the product market is competitive but the labor market is oligopsonistic.

contract (w_j, l_j) to offer, understanding that all workers accept the utility-maximizing contracts. Firms participate to the market only if profits are nonnegative. As a result, the problem for the firm j reads:

$$\max_{w_j, l_j, N_j} F(N_j, l_j) - N_j w_j l_j$$

$$N_j \leq \begin{cases} 0 & \text{if } u(w_j l_j, l_j) < \max_{-j \in J \setminus \{j\}} u(w_{-j} l_{-j}, l_{-j}) \\ \frac{N_s(w, l)}{J} & \text{if } u(w_j l_j, l_j) = u(w_{-j} l_{-j}, l_{-j}) \quad \forall -j \in J \setminus \{j\} \\ N_s(w_j, l_j) & \text{if } u(w_j l_j, l_j) > \max_{-j \in J \setminus \{j\}} u(w_{-j} l_{-j}, l_{-j}) \end{cases}$$

Given that this maximization program holds for all (w_{-j}, l_{-j}) , it also completely describes the best-response function of firm j . The following proposition will be useful.

Theorem 2.A.1. *If production is concave in N , any Nash equilibrium of this game with positive production and participation is symmetric.*

Proof. I prove the statement for the duopsony case where $J = 2$.

By contradiction, consider $(w_1^*, l_1^*) \neq (w_2^*, l_2^*)$ but $\{(w_1^*, l_1^*), (w_2^*, l_2^*)\}$ is a Nash equilibrium of this game.

Step 1 : in a Nash equilibrium we must have $u(w_1^*, l_1^*) = u(w_2^*, l_2^*)$.

By contradiction, imagine that $u(w_1^*, l_1^*) < u(w_2^*, l_2^*)$ such that $\pi_1^* = 0$ and $\pi_2^* = F(N_s(w_2^*, l_2^*), l_2^*) - N_s(w_2^*, l_2^*) w_2^* l_2^*$.

If $\pi_2^* < 0$ then it violates the participation constraint of firm 2. If $\pi_2^* = 0$ then earnings are equal to average product. Yet, first order conditions indicate that $F'_N > w_2^* l_2^*$ and concavity implies that average product is larger than marginal product. As a result, it must be that $\pi_2^* > 0$.

If firm 2 has positive profits, observe that agent 1 can simply deviate by

replicating agent 2's strategy. Agent 1's profits by deviating would be

$$\begin{aligned}
 \pi_1^d &= F\left(\frac{N_s(w_2^*, l_2^*)}{2}, l_2^*\right) - \frac{N_s(w_2^*, l_2^*)}{2} w_2^* l_2^* \\
 &= F\left(\frac{N_s(w_2^*, l_2^*)}{2}, l_2^*\right) - \frac{N_s(w_2^*, l_2^*)}{2} \left(\frac{F(N_s(w_2^*, l_2^*), l_2^*)}{N_s(w_2^*, l_2^*)} - \frac{\pi_2^*}{N_s(w_2^*, l_2^*)} \right) \\
 &= F\left(\frac{N_s(w_2^*, l_2^*)}{2}, l_2^*\right) - \frac{F(N_s(w_2^*, l_2^*), l_2^*)}{2} + \frac{\pi_2^*}{2}
 \end{aligned}$$

Because of Jensen's inequality, the last line is positive which contradicts that (w_1^*, l_1^*) is a best-response to (w_2^*, l_2^*) . The argument proves that it cannot be the case that $u(w_1^*, l_1^*) < u(w_2^*, l_2^*)$ in equilibrium. A symmetric argument can be made to prove the converse strict inequality. Hence, it must be that $u(w_1^*, l_1^*) = u(w_2^*, l_2^*)$ and all firms offer contracts on the same indifference curve and hire the same number of employees $\frac{N_s}{2}$.

Step 2 : In a Nash equilibrium, we must have $\pi_1^* = \pi_2^*$.

If it were not the case, say $\pi_1^* < \pi_2^*$, we would have a profitable deviation for firm 1 : mimicking 2's strategy does not affect the number of workers employed in firm 1 and yet increases profits. As a consequence, firms lie on the same iso-profit curve.

Step 3 : In a Nash equilibrium, we must have $(w_1^*, l_1^*) = (w_2^*, l_2^*)$

By contradiction, assume $l_1^* > l_2^*$. By step 1, equal utility imposes that $w_1^* l_1^* > w_2^* l_2^*$ as utility function is strictly decreasing in l . Hence we have

$$\begin{aligned}
 \pi_1^* &= F\left(\frac{N_s}{2}, l_1^*\right) - \frac{N_s}{2} w_1^* l_1^* < F\left(\frac{N_s}{2}, l_1^*\right) - \frac{N_s}{2} w_2^* l_2^* \\
 &\leq F\left(\frac{N_s}{2}, l_2^*\right) - \frac{N_s}{2} w_2^* l_2^* = \pi_2^*
 \end{aligned}$$

which contradicts step 2. A symmetric argument can be made for the case $l_1^* < l_2^*$. Hence, it must be that $l_1^* = l_2^*$. Given that utility is strictly increasing in consumption, we must have $w_1^* = w_2^*$. This completes the proof. ■

A consequence of this proposition is that each firm faces that same program that can be written

$$\begin{aligned} \max_{w,l,N} & F(N, l) - Nwl \\ \text{s.t. } & N \leq \frac{N_s(w, l)}{J} \end{aligned}$$

where j subscripts may be omitted for brevity. The associated Lagrangian for this problem reads

$$\mathcal{L} = F(N, l) - Nwl + \lambda \left[\frac{N_s(w, l)}{J} - N \right]$$

where $\lambda \geq 0$ is the Lagrangian multiplier. The remaining KKT conditions are

$$F'_N - wl = \lambda \quad (2.7)$$

$$F'_l - wN + \lambda \frac{1}{J} \frac{\partial N_s}{\partial l} = 0 \quad (2.8)$$

$$lN = \lambda \frac{1}{J} \frac{\partial N_s}{\partial w} \quad (2.9)$$

$$\lambda \left[\frac{N_s(w, l)}{J} - N \right] = 0$$

When $J = 1$, these KKT conditions are describing a pure monopsony problem where the Robinson wage markdown is obtained by substituting out λ using equation 2.9 in equation 2.7. Moreover, equation 2.9 implies that $\lambda > 0$ whenever there is positive production. Hence, the inequality constraint is binding in equilibrium and all participating workers are employed in the monopsony equilibrium : there is no involuntary unemployment.

Now in perfect competition, firms lose their wage-setting power and competitive wage rates adjust so that each worker gets paid at its marginal

productivity, i.e. $F'_N = wl$. The problem for the firm reads:

$$\begin{aligned} & \max_{l, N} F(N, l) - Nwl \\ & \text{s.t. } N \leq \frac{N_s(w, l)}{J} \text{ and } F'_N = wl \end{aligned}$$

The associated Lagrangian for this problem reads

$$\mathcal{L} = F(N, l) - Nwl + \lambda \left[\frac{N_s(w, l)}{J} - N \right] + \mu [F'_N - wl]$$

where $\lambda \geq 0$ and $\mu \in \mathbb{R}$ are the Lagrangian multipliers. The remaining KKT conditions are

$$F'_N - wl - \lambda + \mu F''_N = 0 \quad (2.10)$$

$$F'_l - Nw + \lambda \frac{1}{J} \frac{\partial N_s}{\partial l} + \mu (F''_{N,l} - w) = 0 \quad (2.11)$$

$$\lambda \left[\frac{N_s(w, l)}{J} - N \right] = 0 \quad F'_N = wl \quad (2.12)$$

To prove again that the inequality constraint must be binding in equilibrium, observe that if it were not the case, by equation 2.10 we should have $\lambda = 0 = \mu$ such that equation 2.11 implies that $F'_l - Nw = 0 = F'_N - wl$. This can only hold whenever the marginal rate of technical substitution is 1, that is when hours and jobs are perfect substitutes. In the general case, we have $N_j = \frac{N_s(w, l)}{J}$ and there is no involuntary unemployment. This contrasts with the search and matching model where matching frictions engender involuntary unemployment.

Observe that combining the equality constraints with equations 2.10 and 2.11 implies that competitive hours are set by firms such that

$$F'_l - Nw + \lambda \left(\frac{1}{J} \frac{\partial N_s}{\partial l} + \frac{F''_{N,l}}{F''_N} - \frac{F'_N}{lF''_N} \right) = 0 \quad (2.13)$$

2.A.2 Wage-hour covariance and imperfect competition

In this section, I derive a set of sufficient conditions for the results of the toy model with a general production function $F(N, l)$ increasing in its arguments. I assume $F(\cdot, 0) = F(0, \cdot) = 0$.

Perfect competition

The competitive equilibrium wage is characterized by $F'_N - wl = 0$ and $N_j = \frac{N_s(w, l)}{J}$. Taking the total derivative of the former, one gets

$$dw \left(\frac{1}{J} \frac{\partial N_s}{\partial w} F''_N - l \right) + dl \left(\frac{1}{J} \frac{\partial N_s}{\partial l} F''_N + F''_{N,l} - w \right) = 0 \quad (2.14)$$

The coefficient multiplying dw is negative if $F''_N \leq 0$.

With respect to the coefficient multiplying dl , observe that

$$\begin{aligned} \text{sign} \left(\frac{\partial N_s}{\partial l} F''_N + F''_{N,l} - w \right) &= -\text{sign} \left(\frac{\partial N_s}{\partial l} + \frac{F''_{N,l}}{F''_N} - \frac{F'_N}{l F''_N} \right) \\ &= \text{sign} \left(F'_l - wN \right) \end{aligned}$$

where the last equality is obtained by using equation 2.13. Hence, a sufficient condition for the coefficient multiplying dl to be positive is that

$$F'_l - wN > F'_N - wl = \lambda \geq 0$$

which is true whenever

$$lF'_l > NF'_N$$

To sum up, sufficient conditions for a negative equilibrium covariance between hours worked and wage rates in perfect competition are the concavity of production in jobs and the assumption that the output elasticity of hours exceeds the output elasticity of jobs.

$$\begin{aligned} F''_N &\leq 0 \\ lF'_l &> NF'_N \end{aligned}$$

The latter can be justified by experience effect: the marginal hours produced by existing workers should yield more output than those done by the marginal hire.

Pure monopsony

In the case where $J = 1$, equations 2.7 and 2.9 imply that the monopsony equilibrium wage is characterized by $\frac{\partial N_s}{\partial w}(F'_N - wl) = N_s l$. The total derivative reads

$$\begin{aligned} dw \left(\frac{\partial^2 N}{\partial^2 w} (F'_N - wl) + \left(\frac{\partial N}{\partial w} \right)^2 F''_N - 2 \frac{\partial N}{\partial w} l \right) + dl \left(\frac{\partial^2 N}{\partial w \partial l} (F'_N - wl) \right. \\ \left. + \frac{\partial N}{\partial w} \left(\frac{\partial N}{\partial l} F''_N + F''_{N,l} - \frac{F'_N}{l} \right) - \frac{\partial N}{\partial l} l \right) \end{aligned}$$

and must be equal to 0 in equilibrium. Observe that the coefficient multiplying dw is negative whenever $\frac{\partial^2 N}{\partial^2 w} \leq 0$ and $F''_N \leq 0$. Note that the latter was already required by the perfect competition equilibrium. As to the former, observe that $N_s(w, l) = G(u(wl, l))$ implies that

$$\frac{\partial^2 N_s}{\partial^2 w} = (lu'_c)^2 \frac{\partial g(u(wl, l))}{\partial d} + g(u(wl, l)) l^2 u''_c$$

Hence, the concavity of the labor supply N_s with respect to w is an immediate consequence of the concavity of the cdf $G(d)$ and the concavity of utility in consumption.

With respect to the coefficient multiplying dl , it is positive if

$$\frac{\partial N_s}{\partial l} \leq 0 \quad (2.15)$$

$$\frac{\partial^2 N_s}{\partial w \partial l} \geq 0 \quad (2.16)$$

$$\frac{\partial N_s}{\partial l} F''_N + F''_{N,l} - \frac{F'_N}{l} \geq 0 \quad (2.17)$$

With respect to equation 2.15, observe that substituting out λ in the first-order condition 2.8 using 2.7 yields

$$\begin{aligned}\frac{\partial N_s}{\partial l}(F'_N - wl) + F'_l - wN &= 0 \\ \frac{\partial N_s}{\partial l}(F'_N - wl) + \frac{N}{l}(\frac{l}{N}F'_l - wl) &= 0\end{aligned}$$

Hence, sufficient condition 2.15 is verified whenever

$$\begin{aligned}\frac{l}{N}F'_l - wl &\geq F'_N - wl = \lambda > 0 \\ lF'_l &\geq NF'_N\end{aligned}$$

This is exactly the same condition as in the perfect competition case : the marginal product of an hour must be larger than the marginal product of a job. Under this assumption, we obtain equation 2.15 which says that the marginal utility of an extra hour worked is negative in the monopsony equilibrium : workers would like to work less at the monopsony wage rate.

With respect to sufficient condition 2.16, it consists in saying that labor supply should be supermodular: in other words, the wage elasticity of labor supply increases when schedules get longer. I now provide sufficient conditions on the utility functions for it to be the case.

$$\frac{\partial^2 N_s}{\partial w \partial l} = \frac{\partial g(u(wl, l))}{\partial d} (u'_c w + u'_l) u'_c l + g(u(wl, l)) (u'_c + w l u''_c + l u''_{c,l})$$

The first term must be positive because $G(\cdot)$ is assumed to be concave³³ and equation 2.15 implies that the marginal utility of hours $(u'_c w + u'_l)$ is negative. As a result, a sufficient condition for 2.16 to be true is that the second term is positive, i.e.

$$\frac{w l u''_c + l u''_{c,l}}{u'_c} \geq -1 \quad (2.18)$$

³³Concavity of the cdf of outside option means that there are less and less individuals with higher and higher outside options. For example, Pareto distributions have this property.

An example of utility functions satisfying 2.18 are separable utility functions $u(c, l) = v(c) - \phi(l)$ with an Arrow-Pratt coefficient of relative risk aversion $-\frac{cv''_c}{v'_c} \leq 1$. This includes, among others, $v(c) = \log c$.

To sum up, the set of sufficient conditions for obtaining a positive equilibrium covariance between wage rates and hours worked in monopsony are

$$\begin{aligned} lF'_l &\geq NF'_N \\ \frac{wlu''_c + lu''_{c,l}}{u'_c} &\geq -1 \\ \frac{\partial N_s}{\partial l} F''_N + F''_{N,l} - \frac{F'_N}{l} &\geq 0 \end{aligned}$$

2.B Competitive search model : additional results

2.B.1 Sorting

We prove here that $\gamma_l > 0$ in interior solutions whenever production is $F(N, l) = N^\alpha l^\beta$ with $\alpha < \beta < 1$.

$$\gamma_l = \tilde{q}'_l(\psi F'_N - wl + \tilde{q}vF''_N) + \psi \tilde{q}F''_{N,l} - \tilde{q}w$$

Substituting equation (2.5) for the last term yields

$$\gamma_l = \tilde{q}'_l(\tilde{q}vF''_N) + \psi \tilde{q}F''_{N,l} - \psi F'_l \frac{1}{v}$$

Multiplying by v preserves the sign

$$v\gamma_l = v\tilde{q}'_l(\tilde{q}vF''_N) + \psi \tilde{q}vF''_{N,l} - \psi F'_l$$

Developing using the functional form and using the identity $N = \tilde{q}l$, one gets

$$\begin{aligned} v\gamma_l &= v\tilde{q}'_l(\psi\alpha(\alpha-1)N^{\alpha-1}l^\beta) + \psi\alpha\beta N^\alpha l^{\beta-1} - \psi\beta N^\alpha l^{\beta-1} \\ &= (\alpha-1)\psi N^\alpha l^{\beta-1} \left(\beta + \alpha l \frac{\tilde{q}'_l}{\tilde{q}} \right) \end{aligned}$$

We are left with signing the last bracketed factor. Observe that equation (2.5) can be written as

$$v\tilde{q}'_l = - \frac{\psi F'_l - \tilde{q}vw}{\psi F'_N - wl}$$

Hence we get

$$\begin{aligned} l \frac{\tilde{q}'_l}{\tilde{q}} &= - \frac{\psi\beta N^{\alpha-1}l^\beta - wl}{\psi\alpha N^{\alpha-1}l^\beta - wl} \\ \beta + \alpha l \frac{\tilde{q}'_l}{\tilde{q}} &= (\alpha - \beta) \frac{wl}{\psi\beta N^{\alpha-1}l^\beta - wl} \end{aligned}$$

This prove that the bracketed factor is negative whenever $\alpha < \beta$. Hence, it must be that $\gamma_l > 0$, proving that firms with higher productivity ψ_j offer contracts with shorter hours worked.

2.C Empirical appendix

2.C.1 Data collection

2.C.2 Construction of variables

Profits

I construct profits as the difference between total revenue and total expenses. Total revenue is computed as the sum of sales and the change in stock of coal evaluated at the firm's market price. The variable is expressed in real terms by deflating with the price of output, such that profits are measured in tons of coal.

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(A) A report for the mine *Carabinier-Pont du Loup* in 1903.

[illegible]

(B) A slice of the panel dataset for the province of Hainaut in 1913.

FIGURE 2.C.1: Pictures of the data collection

Earnings

Earnings are measured as the fraction of the monetary value of production that is paid to the worker.

$$y_{j,t} = \frac{\text{Daily wage in current BEF for occupation}_{j,t}}{\text{Output price, current BEF}_{j,t} * \text{Daily product per frontline worker}_{j,t}}$$

Daily production per frontline worker is obtained by dividing the annual production by the average days worked in the firm (which is total days worked in the firm divided by the total number of all workers). I obtained annual production per worker by dividing the annual quantity produced by the firm by the number of frontline workers.

TFP

TFP is used as control variable in various regressions. To measure it, I assume a Cobb-Douglas production function and take the log to obtain a regression equation of the form

$$\ln q_{j,t} = \beta_1 \ln l_{j,t} + \beta_2 \ln m_{j,t} + \ln \psi_{j,t} \quad (2.19)$$

where $q_{j,t}$ is the output in tons of coal and $l_{j,t}$ and $m_{j,t}$ are labor and non-labor expenditures, respectively and $\psi_{j,t}$ is unobserved TFP. Estimating this equation with OLS leads to bias estimates because unobserved productivity shocks are positively correlated with observed input choices, the well known simultaneity bias.

The typical way to overcome this identification challenge is to rely on the control function approach since Levinsohn and Petrin (2003) and Olley and Pakes (1996). Delabastita and Rubens (2025) estimate production functions of Belgian coal mines in that spirit. I pinpoint two differences with their estimation. First, the present paper has a much smaller set of years but a larger number of firms. Second and related, the present paper does not have information on capital. As a result, these methods may not be used.

I assume that $\ln \psi_{j,t}$ is a Markov process of order 1 with productivity shocks $\epsilon_{j,t}$. I consider the following moment restriction:

$$E[\epsilon_{j,t} | \ln l_{j,t-1}, \ln m_{j,t-1}] = 0$$

This moment restriction allows to retrieve β_1 and β_2 by GMM estimation. The results are reported below.

Outcome	$\ln \text{production}_{j,t}$	$\ln \text{production}_{j,t}$	$\ln \text{production}_{j,t}$
β_1	0.745 (0.026)	0.654 (0.052)	0.661 (0.041)
β_2	-0.070 (0.021)	0.315 (0.050)	0.237 (0.080)
$\ln \text{capital}$	n/a	n/a	0.102 (0.088)
Serial corr. $\ln \psi_{j,t}$	0.967 (0.008)	0.879 (0.018)	0.853 (0.157)
Years covered	1903-1913	1903-1913	1845-1913
Firms covered	Near universe	Near universe	Liège, Namur
Method	OLS	GMM (own)	Delabastita and Rubens (2025)

TABLE 2.C.1: Production function estimation

Despite differences in sample and in methodology, the estimates obtained by the GMM approach in the present paper are remarkably similar to those of Delabastita and Rubens (2025) both in their magnitudes and in their precision. The present paper computes $\psi_{j,t}$ for each firm-year cell by predicting equation 2.19 using the GMM estimated $\hat{\beta}_1, \hat{\beta}_2$. The use of lags implies that TFP cannot be estimated for 1903 such that the main results only show 1904-1913. For firms with positive production in year $t > 1903$ but zero production in $t - 1$, I set the value of $\psi_{j,t}$ equal to the one of $\psi_{j,t+1}$.

2.C.3 Additional empirical results

Observe that there are 7 events of mergers and acquisitions in Table 2.C.3 while there are only 6 firms less in 1913 than in 1903 in Table 2.C.2. This comes from the fact that one firm, *Charbonnages de Haine Saint Pierre, La Hestre et Houssu*, was divided in 2 parts merged with two different firms in 1912: the *Houssu* merged with *Ressaix* while the *Haine-Saint-Piere et La Hestre* merged with *Mariemont*.

Year	Matched	Population	Share of Covered (%)	
			Output	Days Worked
1903	114	124	99.51	99.33
1904	113	122	99.24	98.98
1905	112	121	99.26	98.74
1906	111	122	98.70	98.53
1907	111	125	98.37	98.11
1908	111	126	98.30	98.23
1909	111	125	98.16	98.03
1910	110	129	97.73	97.47
1911	110	132	97.44	97.12
1912	109	132	96.95	96.57
1913	108	132	96.86	96.17

TABLE 2.C.2: Number of firms for which there is information on hours worked (*Matched*) versus the total number of firms in the country (*Population*) by year.

Year	Mergers or Acquisitions	Firm exit
1903		
1904	1	1
1905	1	
1906	1	
1907		
1908		
1909		1
1910	1	
1911		2
1912	2	
1913	1	

TABLE 2.C.3: Count of mergers & acquisitions and firm exits per year. A firm exit is defined as a zero coal production in the calendar year.

	Non-targeted	Targeted
<i>TFP</i>	1.043 (0.234)	1.045 (0.153)
<i>Coal autoconsumption in tons</i>	8,883.33 (11,618.93)	24,280.00 (20,329.29)
<i>Surface land size in squared meters</i>	143,886.06 (177,947.45)	282,538.33 (189,259.33)
<i>Share of workers underground</i>	0.769 (0.056)	0.715 (0.055)
<i>Share of frontline workers among underground</i>	0.266 (0.143)	0.243 (0.045)
<i>Average coal vein power</i>	0.692 (0.196)	0.676 (0.153)
<i>Sales price in current BEF</i>	14.434 (2.375)	14.544 (2.712)
Profits in tons of coal	4,018.93 (14,260.24)	-904.49 (34,466.69)
Frontline employment	124.06 (169.36)	261.76 (192.36)
Frontline earnings	0.134 (0.073)	0.114 (0.033)
Average product per frontline worker	1,042.84 (484.67)	1,004.75 (305.31)
Observations	33	78

TABLE 2.C.4: Summary statistics of various variables in levels for the baseline year 1909. Means are reported while standard deviations are between brackets. Control (outcome) variables are in italics (bold).

Outcome	$Treated_j^* Post$	$Exposure_j^* Post$	Sample
Profits in levels	-8432.417** (3232.001)	-7076.853** (2742.410)	Full
ln frontline employment	-0.081 (0.049)	-0.076* (0.040)	Non-closing firms
ln frontline earnings	-0.047* (0.028)	-0.045 (0.030)	Non-closing firms
ln frontline hours worked	-0.076*** (0.008)	-0.079*** (0.003)	Non-closing firms

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

TABLE 2.C.5: Difference-in-differences (DiD) estimates. The second column reports the DiD results for various outcomes whenever the $Treated_j$ dummy takes value 1 if $Exposure_j > 0$ while the second column uses $Exposure_j$ as continuous treatment variable. Regressions for outcome $y_{j,t}$ use the same controls as their event-study counterpart in the main text. Frontline hours are computed assuming perfect compliance to the reform.

Chapter 3

Consumer welfare beyond GDP

In a general consumption space where agents may differ in their preferences, endowments and prices, this paper builds a theory of consumer welfare on axioms reflecting the ethics of equality of opportunity: unequal budgets create welfare-relevant inequalities but heterogeneous preferences do not. When combined with an appropriate cross-economy robustness condition, these axioms single out a consumer welfare measure. Like aggregate consumption in national accounts, this measure sums up individual expenditure functions. Unlike national accounts, these functions are evaluated at a common price vector for all individuals. I show that standard measures of cost of living, standards of living, and purchasing power parity can be modified to reflect welfare as equality of opportunity without additional data.

3.1 Introduction

When consumers have heterogeneous preferences and budgets, measuring individual and collective consumer welfare is riddled with obstacles. Practitioners are routinely facing such dilemmas and have to make choices notably on how to (i) compare consumers' with heterogeneous preferences interpersonally, (ii) deflate their consumption spawned by different prices and (iii) assess collective welfare changes.

For example, the Penn World Table¹ deflates each country's consumption by some "world" prices pinning down purchasing power parities. Then, it compares real income across countries which is assimilated to welfare. Deaton and Heston (2010) criticized this measure of income of nations because (i) it neglects international taste dispersion, (ii) the deflation is plutocratic as world prices are influenced more by richer countries and (iii) it does not account for inequalities.²

The present paper proposes an integrated theory that compares the welfare of consumers both collectively and interpersonally while informing on deflation practices. I build axiomatically a measure of consumer welfare in a general consumption space where agents have heterogeneous preferences while they receive heterogeneous endowments and face heterogeneous prices.

Axioms pinning down welfare shall reflect the ethics of equality of opportunity: on the one hand, inequalities in consumption due to unequal budgets should be reduced but on the other hand, inequalities in consumption spawned by heterogeneous preferences are unproblematic. Such an approach resonates with the debate on building economic indicators *beyond GDP* (Fleurbaey, 2009; Fleurbaey & Blanchet, 2013).

Comparing agents with heterogeneous preferences is notoriously difficult since Arrow (1950)'s celebrated impossibility theorem. Indeed, there is no obvious way of summing up utilities when utility functions are differing across individuals. The present paper escapes Arrow's impossibility by using the social choice theory of fairness initiated by Fleurbaey and Maniquet (2006, 2011, 2018). It consists in using information on indifference curves³ while Arrow's axioms only relied on utility levels. Yet,

¹See Feenstra, Inklaar, and Timmer (2015) and Summers and Heston (1991) for the construction of the Penn World Table.

²Another popular criticism is that many dimensions of well-being are not accounted for by GDP (Fleurbaey, 2009; Jones & Klenow, 2016). Although the present paper will only focus on the consumption dimension of overall welfare, its results could be used for more dimensions given the generality of the consumption space.

³This allows to characterize welfare measures in the realm of *equivalent income*, such as willingness to pay, expenditure functions or money-metric utility functions. All of these are amounts of money that renders the individual indifferent between her situation and a

this fairness theory typically considers parallel budget sets as it assumes homogeneous prices for all agents. However, it is well-known that the law of one price fails in many datasets.⁴

I show that the key axioms of this fairness theory leads to an impossibility in the general environment with multi-dimensional heterogeneity in prices, endowments and preferences. More precisely, I prove that the axioms of equality of opportunity conflict with the standard cross-economy robustness condition of Fleming (1952)⁵ whereby indifferent agents do not matter in the welfare assessment. In some sense, this result highlights an ethical tension between a democratic principle of neutrality of indifferent agents and equality of opportunity.

Next, I argue that one should escape this impossibility by weakening Fleming (1952) Separability rather than the axioms of equality of opportunity. Intuitively, when welfare is based on equality of opportunity, indifferent agents should matter in welfare assessments because, conditional on their consumption, they are endowed with resources that influence the overall amount of redistribution that can be done in the economy. The weakening strategy consists in applying Separability only when there is no aggregate growth of resources.

The main result of the paper is that this weaker Separability axiom allows to pin down a social welfare function that can be used to compare the welfare of consumers interpersonally and collectively. On the one hand, the interpersonal consumer welfare measure should be a money-metric utility function⁶ whose reference situation is the economy's average purchasing power. More precisely, the reference price for good l is the ratio between aggregate income and the total quantity of good l that

reference situation. For example, the Hicksian compensating and equivalent variations belongs to this class.

⁴For example, practitioners building purchasing power parities across countries have to face the Balassa-Samuelson effect whereby non-tradeable goods are more expensive in richer countries.

⁵Fleming (1952) proved that this Separability axiom combined by Strong Pareto leads to a utilitarian social welfare function.

⁶A money-metric utility is the amount of money that renders an individual indifferent between her situation and a reference scenario (McKenzie, 1957).

could ever be consumed in that economy.

On the other hand, the measure of collective consumer welfare aggregates these money-metric utility functions using a standard summation. An interesting parallel can be drawn with GDP-like measures here. The standard aggregate consumption in national accounts sums up individuals' expenditures using the prices they actually faced, i.e. $\sum_i e_i(p_i, z_i)$ where p_i and z_i are individual i 's price and consumption vectors and $e_i(\cdot)$ is the expenditure function. By contrast, the main results of this paper suggest that a consumer welfare measure that reflects the ethics of equality of opportunity is $\sum_i e_i(\tilde{p}, z_i)$, where $\tilde{p}$ is the price reflecting the average purchasing power in the economy.

This result also provides a normative basis for deflation techniques from which I derive applications to applied welfare problems. First, in the context of measuring real income across countries, I show that this paper's consumer welfare consists in changing the purchasing power parity from what is typically done in the Penn World Table. In particular, world prices for a good should be based on each countries' income and prices but not on countries' consumption of that good. Fortunately, this does not require more data than standard measures. Second, I show that this consumer welfare measure allows to reconcile inflation and growth measures with welfare provided that the base period on which inflation and growth indices are computed is carefully chosen. In particular, the price of good l in the base period should be the ratio between the sum over time of income and the sum over time of the total quantity of good l that could have ever been consumed.

Literature

First, the paper contributes to the literature on social choice and fairness. The fact that money-metric utility function is a useful tool to measure welfare interpersonally and collectively comes at no surprise in this literature. Bosmans, Decancq, and Ooghe (2018), Fleurbaey and Maniquet (2011, 2017), and Piacquadio (2017) all characterized similar social welfare

functions. A lasting⁷ critique of money-metric utility⁸ function is that they depend on a reference scenario that is left at the researcher's discretion. This motivated the study of reference-independent welfare assessment (Blackorby, Laisney, & Schmachtenberg, 1993; Eden & Freitas, 2024; Roberts, 1980). The contribution of the present paper with respect to that literature is that the reference situation is endogenized by the equality of opportunity axioms. More precisely, I show that the axioms used in that literature lead to an impossibility if there is price dispersion and that one way out of the impossibility implies a precise reference situation.

Second, the paper contributes to the index number theory as initiated by Fisher (1922). The index number problem consists in building price and quantity indices based on dataset of observed prices and expenditures for each consumer. It is customary to divide the field into two approaches (van Veelen & van der Weide, 2008). The axiomatic approach imposes series of desirable properties on the indices but is typically not linked with classical demand theory (Samuelson & Swamy, 1974; Van Veelen, 2002). The economic approach rationalizes observed differences from a representative consumer demand system (Diewert, 1976; Neary, 2004). The present paper rejects a representative consumer unlike the latter but has links with classical demand theory unlike the former. Moreover, the paper does not impose domain restrictions on individual preferences so that its results are valid whether preferences are homothetic (Diewert & Nakamura, 1993) or not (Jaravel & Lashkari, 2024). Finally, the indices constructed in this paper reflect ethical views whose foundations are transparently carried over by the axioms.

The paper is structured as follows. In section 3.2, I present the consumption space with multi-dimensional heterogeneity in prices, endowments and preferences. In section 3.3, I present the key axioms and prove

⁷A classical critique by Blackorby and Donaldson (1988) was that money-metric utility function may lead to inequalitarian assessment. However, Fleurbaey and Maniquet (2011) and Schlee and Khan (2022, 2023) that the Blackorby and Donaldson (1988) showed that this can be escaped rather easily.

⁸The fact that no theory guides the choice of the reference price is well-known and discussed recently in Bosmans, Decancq, and Ooghe (2018), Capéau, Decoster, and De Sadeleer (2023), and Fleurbaey and Blanchet (2013).

their incompatibilities. In section 3.4, I show that weakening Separability allows possibilities and characterize the maximin and the summation of the money-metric utility with average purchasing power reference. In section 3.5, I derive implications for various applications. In section 3.6, I conclude.

3.2 Model

A society $\mathcal{N} = \{1, \dots, N\}$ is composed of a finite number $N \geq 5$ of agents. There are $L \geq 2$ goods and an agent i 's bundle is denoted by $z_i \in \mathbb{R}_+^L$. Agents are heterogeneous in their preferences $\succsim_i$ that are complete, transitive, locally non-satiated and convex.

Agents receive some endowments in goods⁹ $\omega_i \in \mathbb{R}_+^L$ and face a vector personalized prices $p_i \in \mathbb{R}_+^L$. Personalized prices may include the standard case of no price dispersion in a Walrasian equilibrium, as well as price dispersion due to e.g. frictions, transport costs, discrimination, heterogeneous needs, comparative advantages in production, or intra-household bargaining *à la* Chiappori (1988, 1992). For brevity, I will sometimes refer to the scalar product $p_i \omega_i$ as income and denoted it by $y_i = p_i \omega_i \geq 0$.

An allocation z is a list of bundles for each agent $z = (z_1, z_2, \dots, z_N) \in \mathbb{R}_+^{LN}$ and it may or may not be rationalizable by agents' constrained utility maximization. An economy e is composed of N agents which are heterogeneous in preferences, endowments and prices $e = \{\mathcal{N}, (\succsim_i, \omega_i, p_i)_{i \in \mathcal{N}}\}$. The set of all economies is denoted by $\mathcal{E}$.

The main object of interest will be the ethical observer's ordering function that associates to each economy e a complete and transitive ordering of allocations. I denote this social preference relation by $\mathbf{R}(e)$, $\mathbf{P}(e)$, $\mathbf{I}(e)$ for weak preference, strict preference and indifference, respectively.

⁹Observe that this model nests a model where one good is set as the numéraire and endowments are only in that good, that is a monetary endowment.

I now define two useful objects. First, an allocation $\hat{z}$ is feasible by lump-sum transfers from e if there exists a vector of individual lump-sum transfers $(t_1, t_2, \dots, t_N) \in \mathbb{R}^N$ with $\sum_{i \in \mathcal{N}} t_i \leq 0$ such that bundles $\hat{z}_i$ are part of individuals' budgets $B(t_i, y_i, p_i)$:

$$\hat{z}_i \in B(t_i, \omega_i, p_i) = \{z_i \in \mathbb{R}_+^L : p_i z_i \leq p_i \omega_i + t_i\}$$

Second, the money-metric welfare of agent i consuming the bundle z_i and reference situation (ω_R, p_R) is the transfer that renders the agent indifferent between her bundle z_i and the bundle she would have chosen in the budget determined by that reference situation and the transfer. Formally, we define money-metric welfare as $W_i(z_i, p_R)$ with

$$W_i(z_i, \omega_R, p_R) = \min t \text{ s.t. } z_i \succsim z_0 = \arg \max_{\tilde{z}_i} B(t, \omega_R, p_R)$$

3.3 Impossibilities

The axiomatic construction of the social ordering $\mathbf{R}(\mathbf{e})$ consists in imposing desirable properties, i.e. axioms, restricting the scope of possibilities among the universe of transitive and complete $\mathbf{R}(\mathbf{e})$. This section introduces the key axioms of equality of opportunity and shows that they lead to an impossibility.

The first one, **Laissez-faire**, embodies the view that inequalities spawned by heterogeneous preferences should not be reduced by the ethical observer: agents should be held responsible for their preferences and the planner should not discriminate along different tastes.¹⁰ Formally, it requires that when there is no cross-sectional heterogeneity in prices and endowments, the Laissez-faire allocation is strictly preferred to any other allocations feasible by lump-sum transfers from the economy.

¹⁰Political philosophers such as Arneson (1990), Cohen (1989), Dworkin (1981), and Rawls (1971) all defended variants of this principle, as discussed in Fleurbaey (2008) and Fleurbaey and Maniquet (2006, 2011).

Laissez-faire: $\forall e \in \mathcal{E}$ such that $(\omega_i, p_i) = (\omega, p)$ for all $i \in \mathcal{N}$ and z feasible by lump-sum transfer for e ,

$$z^* \mathbf{P}(\mathbf{e}) z$$

where z^* is such that $z_i^* \in \max_{\succsim_i} B(0, \omega, p)$ for all $i \in \mathcal{N}$.

Conversely, the second axiom, **Compensation**, reflects the principle that inequalities spawned by heterogeneous prices and income should be fought against. In particular, the axiom imposes that when a pair of agents have identical preferences, a permutation of their bundle should be socially indifferent.¹¹ In other words, the social ranking is anonymous among agents with identical preferences.

Compensation: $\forall e \in \mathcal{E}$ where agents i, j are such that $\succsim_i = \succsim_j$ then

$$(z_i, z_j, z) \mathbf{I}(\mathbf{e}) (z_j, z_i, z).$$

Taken together, these two axioms reflect the ethics of equality of opportunity: heterogeneous tastes should not be materially relevant for welfare inequalities but unequal budgets are creating welfare-relevant inequalities in the eyes of the planner. Observe that both axioms only apply under some specific premises, i.e. for some but not all e in $\mathcal{E}$. In order to obtain a complete ordering $\mathbf{R}(\mathbf{e})$ for all economies, one must use a cross-economy robustness condition.

Since the characterization of utilitarianism by Fleming (1952), the standard cross-economy condition is Separability which requires that indifferent agents should not have a say in the social evaluation between two alternatives. This is sometimes defended on democratic grounds: only those effectively affected by the alternatives should be heard. More formally, **Fleming (1952) Separability** requires that if some agents are receiving the same bundle in two allocations, then the social ordering

¹¹This type of axioms also received support in political philosophy among liberal egalitarians (Dworkin, 1981). It is reminiscent to Suppes (1966)'s grading principle. See Hammond (1976) for an early treatment and Maniquet (2004) for its relationship to equality of opportunity.

between these two allocations is unaffected if these agents change their preferences, endowments, prices, or receive a different bundle.

Fleming (1952) Separability: $\forall e, e' \in \mathcal{E}$ and the partitions $\mathcal{K} + \mathcal{M} = \mathcal{N}$ with $e_{\mathcal{K}} = \{\mathcal{K}, (\underline{z}_i, \omega_i, p_i)_{i \in \mathcal{K}}\}$, $e'_{\mathcal{K}} = \{\mathcal{K}, (\underline{z}'_i, \omega'_i, p'_i)_{i \in \mathcal{K}}\}$, $e_{\mathcal{M}} = \{\mathcal{M}, (\underline{z}_i, \omega_i, p_i)_{i \in \mathcal{M}}\}$, and z, z' then,

$$(z_{\mathcal{K}}, z_{\mathcal{M}})\mathbf{R}(e_{\mathcal{K}}, e_{\mathcal{M}})(z_{\mathcal{K}}, z'_{\mathcal{M}}) \iff (z'_{\mathcal{K}}, z_{\mathcal{M}})\mathbf{R}(e'_{\mathcal{K}}, e_{\mathcal{M}})(z'_{\mathcal{K}}, z'_{\mathcal{M}})$$

I note that the version presented here is logically weaker than the axioms of separability in subpopulations used in d'Aspremont and Gevers (1977), Fleurbaey (2003), and Maniquet (2004). The next theorem shows that the three axioms outlined above lead to an impossibility when combined together.

Theorem 3.1. *There does not exist any $\mathbf{R}(e)$ that satisfies *Laissez-faire*, *Compensation* and *Fleming (1952) Separability**

Proof. By contradiction, assume that the statement does not hold. Consider the following four-agent economies constructed with two preference orderings and two budgets

$$\begin{aligned} e_1 &= \left\{ (\underline{z}_i, \bar{\omega}, \bar{p}), (\underline{z}_i, \bar{\omega}, \bar{p}), (\underline{z}_j, \bar{\omega}, \bar{p}), (\underline{z}_j, \bar{\omega}, \bar{p}) \right\} \\ e_2 &= \left\{ (\underline{z}_i, \underline{\omega}, \underline{p}), (\underline{z}_i, \underline{\omega}, \underline{p}), (\underline{z}_j, \underline{\omega}, \underline{p}), (\underline{z}_j, \underline{\omega}, \underline{p}) \right\} \\ e_3 &= \left\{ (\underline{z}_i, \bar{\omega}, \bar{p}), (\underline{z}_i, \underline{\omega}, \underline{p}), (\underline{z}_j, \underline{\omega}, \underline{p}), (\underline{z}_j, \bar{\omega}, \bar{p}) \right\} \end{aligned}$$

There are four bundles of interest that maximize either $\underline{z}_i$ or $\underline{z}_j$ over the budgets $B(0, \bar{\omega}, \bar{p})$ and $B(0, \underline{\omega}, \underline{p})$ and we denote them by z_1, z_2, z_3, z_4 respectively. Moreover, the values of $\bar{\omega}, \underline{\omega}, \bar{p}, \underline{p}$ are chosen such that z_1, z_2, z_3, z_4 are the vertices of a parallelogram. Because the diagonals of a parallelogram bisect each other, we know that (z_1, z_1, z_3, z_3) is feasible as lump-sum transfers from e_2 and (z_2, z_2, z_4, z_4) is feasible as lump-sum transfers from e_1 . The situation is depicted in Figure 3.1.

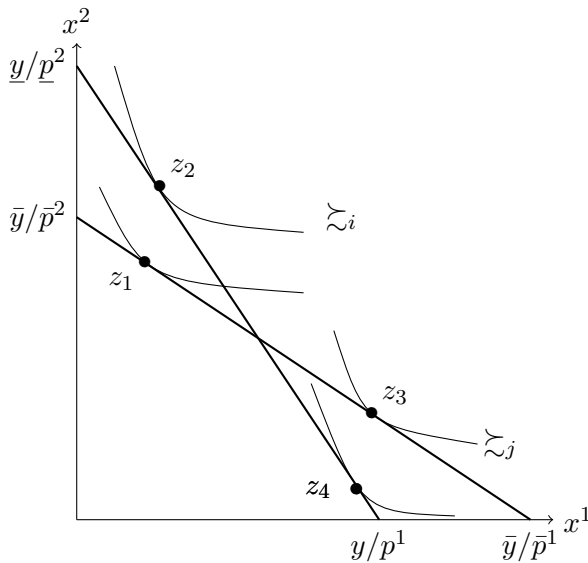


FIGURE 3.1: Illustration of the proof of Theorem 3.1 where $\bar{y} = \bar{p}\bar{\omega}$ and $\underline{y} = \underline{p}\underline{\omega}$.

By **Laisser-faire**,

$$(z_1, z_1, z_3, z_3)\mathbf{P}(\mathbf{e}_1)(z_2, z_1, z_3, z_4)$$

By **Fleming (1952) Separability**,

$$(z_1, z_2, z_4, z_3)\mathbf{P}(\mathbf{e}_3)(z_2, z_2, z_4, z_4)$$

By **Compensation**,

$$(z_2, z_1, z_3, z_4)\mathbf{I}(\mathbf{e}_3)(z_1, z_2, z_4, z_3)$$

By **Transitivity**,

$$(z_2, z_1, z_3, z_4)\mathbf{P}(\mathbf{e}_3)(z_2, z_2, z_4, z_4)$$

By **Fleming (1952) Separability**,

$$(z_2, z_1, z_3, z_4)P(e_2)(z_2, z_2, z_4, z_4)$$

But this contradicts **Laissez-faire**, proving the statement for $N = 4$. The statement for $N \geq 5$ follows by duplicating this reasoning. ■

A natural attempt to escape the impossibility would consist in weakening **Laissez-faire** to a weak social preference for the Laissez-faire allocation rather than a strict preference. This is what the next axiom, **Laissez-faire^W**, achieves.

Laissez-faire^W: $\forall e \in \mathcal{E}$ such that $(\omega_i, p_i) = (\omega, p)$ for all $i \in \mathcal{N}$ and z feasible by lump-sum transfers from e , then $z^*R(e)z$

Unfortunately, the next theorem shows that the impossibility persists if one looks for orderings that satisfy the widely-used **Weak Pareto** principle.¹²

Theorem 3.2. *There does not exist any $R(e)$ that satisfies **Weak Pareto**, **Laissez-Faire^W**, **Compensation** and **Fleming Separability***

Proof. I proceed again by contradiction. Consider the same economies as in the proof of Theorem 3.1 and z_5, z_6 such that (z_5, z_5, z_6, z_6) is feasible as lump-sum transfers from e_2 while $z_5 \succ_i z_1$ while $z_6 \succ_j z_3$.

Using the exact same steps as in Theorem 3.1 but with a weak preference relation rather than a strict one, we get that by **Laissez-Faire^W**, **Compensation**, Transitivity and **Fleming (1952) Separability**,

$$(z_2, z_1, z_3, z_4)R(e_2)(z_2, z_2, z_4, z_4) \quad (3.1)$$

By **Fleming (1952) Separability**,

$$(z_1, z_1, z_3, z_3)R(e_2)(z_1, z_2, z_4, z_3)$$

By **Compensation** and Transitivity,

$$(z_1, z_1, z_3, z_3)R(e_2)(z_2, z_1, z_3, z_4)$$

¹²Weak Pareto imposes that $\forall e \in \mathcal{E}$ and z, z' such that $z_i \succ z'_i$ for all $i \in \mathcal{N}$, then $zP(e)z'$.

By Transitivity with equation 3.1,

$$(z_1, z_1, z_3, z_3)\mathbf{R}(\mathbf{e}_2)(z_2, z_2, z_4, z_4)$$

By **Laissez-Faire**^W,

$$(z_2, z_2, z_4, z_4)\mathbf{R}(\mathbf{e}_2)(z_5, z_5, z_6, z_6)$$

By Transitivity,

$$(z_1, z_1, z_3, z_3)\mathbf{R}(\mathbf{e}_2)(z_5, z_5, z_6, z_6)$$

which contradicts Weak Pareto and completes the proof. ■

These impossibilities are surprising¹³ because Bosmans, Decancq, and Ooghe (2018), Fleurbaey and Maniquet (2011, 2017), and Piacquadio (2017) all characterized possibilities with similar axioms.¹⁴ The present paper only differs by allowing price dispersion in the cross-section and this engenders impossibilities.

Impossibilities are sometimes discouraging for the unfamiliar reader. However, they show the way forward to the welfare economist: one needs to weaken one of these axioms to obtain a possibility. Moreover, these impossibilities have normative content: they suggest that the ethics of equality of opportunity are incompatible with the democratic ethics of **Fleming (1952) Separability**. In the next section, I will argue that the culprit is not to be found in the axioms of equality of opportunity but rather in **Fleming (1952) Separability**.

¹³I note that Bosmans and Öztürk (2022) studied the impossibility between a laissez-faire principle and the Pareto principle. Yet, their laissez-faire principle is much stronger than ours as it holds under any initial distribution of endowments while ours restrict it to economies with $(\omega_i, p_i) = (\omega, p)$ for all $i \in \mathcal{N}$. As such, the ethical view reflecting in their paper is closer to libertarianism while the present paper builds on the ethics of equality of opportunity.

¹⁴I note that the impossibility holds for a version of Compensation that use anonymity rather than transfers as in Bosmans, Decancq, and Ooghe (2018). It is not difficult to see from the proof of Theorem 3.1 that the impossibilities would persist with the transfer-based axiom of Compensation considered in their paper.

3.4 Possibilities

In the previous section, impossibilities arised because repeated applications of **Fleming (1952) Separability** allows us to move from e_1 to e_2 and compare an allocation derived from the Laissez-faire in e_1 with the Laissez-faire allocation in e_2 . More precisely, economies e_1 and e_2 have a different amount of aggregate income as $4\bar{y} > 4\underline{y}$. Yet, prices are such that the Laissez-faire allocation of economy e_1 is feasible as lump-sum transfers from economy e_2 and *vice versa*. To see this, observe that because the bundles in Figure 3.1 are vertices of a parallelogram the following identity holds

$$z_1 + z_1 + z_3 + z_3 = \tilde{Z} = z_2 + z_2 + z_4 + z_4$$

I depict aggregate budgets for e_1, e_2 and e_3 in Figure 3.2.

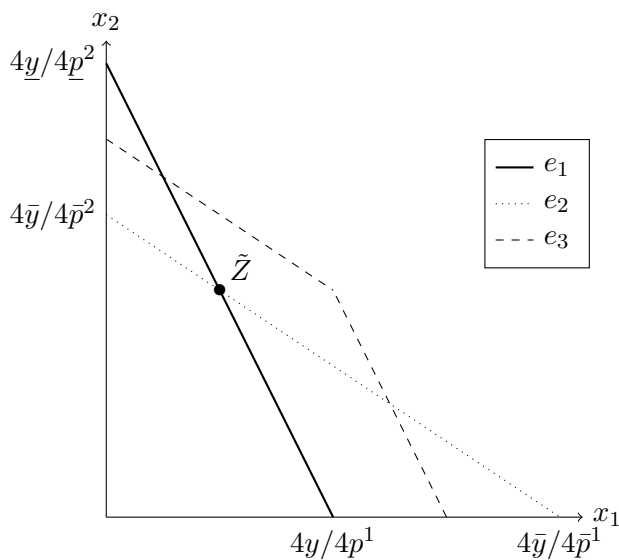


FIGURE 3.2: Aggregate budgets in the proof of Theorem 3.1.

In other words, the cross-economy robustness condition does not take

into account the fact that the aggregate trade-off between goods is completely different in e_1 and e_2 : aggregate budgets are crossing at $\tilde{Z}$. I argue that this shows that Separability should be weakened: when studying equality of opportunity, indifferent agents matter for the social evaluation not because of their bundles but rather because of their endowments and prices.

In order to escape the impossibility, we should make sure to apply Separability on a partition of economies whose elements do not contain two intersecting simplices as depicted in Figure 3.2. The most natural way is to define a partition by its intersection with the axis. The next axiom, **No-Growth Separability**, applies **Fleming (1952) Separability** only between economies where the maximal amount of each good that could be consumed is identical. This avoids the comparisons of non-nested aggregate budgets depicted in Figure 3.2.

No-Growth Separability: $\forall e, e' \in \mathcal{E}$ and the partition $\mathcal{K} + \mathcal{M} = \mathcal{N}$ with $e_{\mathcal{K}} = \{\mathcal{K}, (\tilde{z}_i, \omega_i, p_i)_{i \in \mathcal{K}}\}$, $e'_{\mathcal{K}} = \{\mathcal{K}, (\tilde{z}'_i, \omega'_i, p'_i)_{i \in \mathcal{K}}\}$, $e_{\mathcal{M}} = \{\mathcal{M}, (\tilde{z}_i, \omega_i, p_i)_{i \in \mathcal{M}}\}$ such that

$$\forall l = \{1, \dots, L\} : \sum_{i \in \mathcal{K}} \frac{y_i}{p_i^l} = \sum_{i \in \mathcal{K}} \frac{y'_i}{p_i'^l}$$

and for z, z' , then,

$$(z_{\mathcal{K}}, z_{\mathcal{M}}) \mathbf{R}(e_{\mathcal{K}}, e_{\mathcal{M}})(z_{\mathcal{K}}, z'_{\mathcal{M}}) \implies (z'_{\mathcal{K}}, z_{\mathcal{M}}) \mathbf{R}(e'_{\mathcal{K}}, e_{\mathcal{M}})(z'_{\mathcal{K}}, z'_{\mathcal{M}})$$

Why does this 'no-growth' requirement pop up? Intuitively, starting from any arbitrary economy e , we should only be able to compare it through the separability axiom with one and only one equal-price economy. When the maximal total amount of each good is identical in both economies, there is no way one can construct a parallelogram as in Theorem 3.1, thereby escaping the impossibility.

The two main theorems of this paper will make use of **No-Growth Separability** as well as **Compensation**. However, they will feature two stronger versions of the *laissez-faire* principle which I now define in turn.

First, **Egalitarian Responsibility** imposes that when all agents have equal endowments and prices, any reduction in lump-sum transfers inequality is a social improvement. Of course, the *Laissez-faire* allocation z^* , where no lump-sum transfers inequality remains, is still the social optimum. Second, by contrast, **Utilitarian Responsibility** imposes under the same premise that there should no be lump-sum transfers waste: all resources at the planner's disposal should be used.

More formally, $\forall e \in \mathcal{E}$ such that $(\omega_i, p_i) = (\omega, p) \forall i \in \mathcal{N}$, and z, z' such that $z_k = z'_k \forall k \in \mathcal{N} \setminus \{i, j\}$, and

$$\begin{array}{ll} z_i \in \max_{\tilde{z}_i} B(t_i, \omega, p) & z'_i \in \max_{\tilde{z}_i} B(t'_i, \omega, p) \\ z_j \in \max_{\tilde{z}_j} B(t_j, \omega, p) & z'_j \in \max_{\tilde{z}_j} B(t_j, \omega, p) \end{array}$$

If $\mathbf{R}(\mathbf{e})$ satisfies **Egalitarian Responsibility**, then $|t'_i - t'_j| \geq |t_i - t_j|$ implies that $z\mathbf{R}(\mathbf{e})z'$

If $\mathbf{R}(\mathbf{e})$ satisfies **Utilitarian Responsibility**, then $t_i + t_j \geq t'_i + t'_j$ implies that $z\mathbf{R}(\mathbf{e})z'$.

It is useful to observe that both **Egalitarian Responsibility** and **Utilitarian Responsibility** imply **Laissez-faire**^W. Indeed, the *laissez-faire* allocation is such that there is no lump-sum transfer inequality among agents, which **Egalitarian Responsibility** finds most desirable. However, the *laissez-faire* allocation is also the one where the sum of lump-sum transfers is 0, which is preferred to all other feasible allocations (i.e. $\sum t_i \leq 0$) by **Utilitarian Responsibility**. Both axioms confirm the supremacy of *laissez-faire* allocation in equal-budget economies, but for different reasons. They also have bite beyond that as any reform toward the *laissez-faire* allocation is desirable, thus being logically stronger than **Laissez-faire**^W.

I now present the main theorems of the paper. They characterize precisely the social welfare function that abides by the axioms.

Theorem 3.3 shows that **Egalitarian Responsibility**, combined with **Compensation** and **No-Growth Separability** leads to a social welfare function that has an infinite inequality aversion in a particular money-metric utility function whose reference situation is the cross-sectional average of purchasing powers.

Theorem 3.3. *$R(e)$ satisfies No-Growth Separability, Compensation and Egalitarian Responsibility if and only if for z, z' and e such that*

$$W_i(z'_i, \tilde{\omega}, \tilde{p}) \leq W_i(z_i, \tilde{\omega}, \tilde{p}) \leq W_j(z_j, \tilde{\omega}, \tilde{p}) \leq W_j(z'_j, \tilde{\omega}, \tilde{p})$$

with

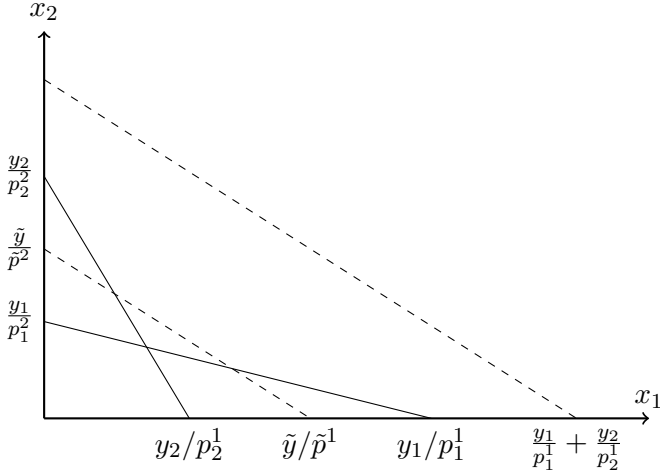
$$\begin{aligned} \tilde{p}\tilde{\omega} &= \tilde{y} = \frac{1}{N} \sum_{i \in \mathcal{N}} y_i = \frac{1}{N} \sum_{i \in \mathcal{N}} p_i \omega_i \\ \tilde{p}^l &= \frac{\sum_{i \in \mathcal{N}} y_i}{\sum_{i \in \mathcal{N}} \frac{y_i}{p_i^l}} \quad \text{for } l = \{1, \dots, L\} \end{aligned}$$

while $z'_k = z_k$ for all $k \in \mathcal{N} \setminus \{i, j\}$,

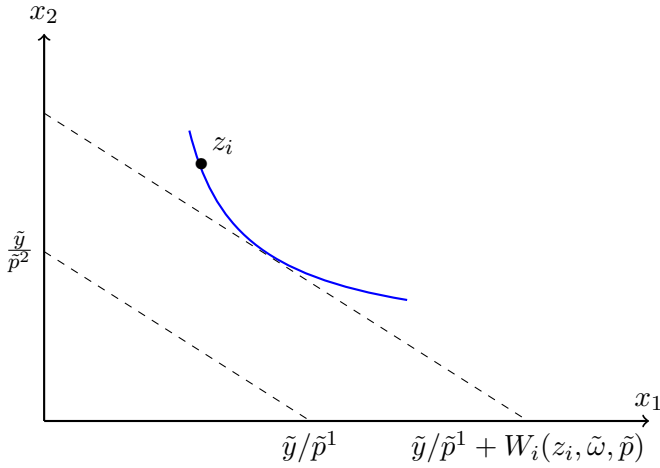
Then, ${}_z R(e) z'$

The proof is relegated to the appendix. I provide a graphical illustration of the construction of $W_i(z_i, \tilde{\omega}, \tilde{p})$ for a two-agent case with two goods in Figure 3.3. In panel (a), I show that $\tilde{y}$ and $\tilde{p}^l$ pin down the budget of the average purchasing power of buying l in the economy : the ratio $\frac{\tilde{y}}{\tilde{p}^l}$ gives the per-capita quantity of good l that could be bought if everyone was spending one's income in buying good l . In panel (b), I show that the welfare of agent i consuming z_i is the transfer that renders that agent i indifferent between z_i and the reference budget $B(0, \tilde{\omega}, \tilde{p})$.

The previous theorem has an infinite aversion to inequality in the particular money-metric utility $W_i(z_i, \tilde{\omega}, \tilde{p})$. However, observe that not all consumption inequalities are deemed unfair. Indeed, unequal preferences imply that agents consume different bundles in the Laissez-faire allocation even if they have the same budgets. Yet, this allocation is the optimal one with respect to Theorem 3.3. This is why these axioms reflect the ethics of equality of *opportunity* and not of *outcomes* : budget sets should be



(A) Illustration of the construction of $\tilde{y} = \tilde{p}\tilde{\omega}$ and $\tilde{p}$.



(B) Illustration of the construction of $W_i(z_i, \tilde{\omega}, \tilde{p})$.

FIGURE 3.3: Illustrations of the construction of the inter-personal consumer welfare measure.

equalized but not consumption vectors.

The next theorem shows the consequences of replacing **Egalitarian Responsibility** by **Utilitarian Responsibility**. Its proof is relegated to the appendix as well.

Theorem 3.4. *$R(e)$ satisfies No-Growth Separability, Compensation and Utilitarian Responsibility if and only if for z, z' and e such that*

$$W_i(z_i, \tilde{\omega}, \tilde{p}) + W_j(z_j, \tilde{\omega}, \tilde{p}) \geq W_i(z'_i, \tilde{\omega}, \tilde{p}) + W_j(z'_j, \tilde{\omega}, \tilde{p})$$

while $z_k = z'_k \ \forall k \in \mathcal{N} \setminus \{i, j\}$.

Then, $zR(e)z'$

These results deserve some comments. First, these two theorems characterize the social welfare function that are endogenized by the axioms. They differ in the aggregator used to compute social welfare (the *maximin* and the *sum*, respectively), but they use the very same measure of interpersonal welfare, i.e. $W_i(z_i, \tilde{\omega}, \tilde{p})$. It suggests that this money-metric utility function should be used to compare the welfare of heterogeneous consumers if one abides by **Compensation, Laissez-faire^W** and **No-Growth Separability**, while the choice of the aggregator depends on which logical strengthening the ethical observer prefers between **Utilitarian Responsibility** and **Egalitarian Responsibility**.

Second, this welfare measure only uses ordinal and non-comparable elements of preferences to build a welfare measure. Indeed, the only component of preferences needed to compute $W_i(z_i, \tilde{y}, \tilde{p})$ is the indifference curve at z_i , not the utility level. Despite that, this measure allows interpersonal comparisons of welfare between individuals. Thus, this measure is in line with the long-standing tradition since Robbins (1938) and Samuelson (1947) of not comparing cardinal utility levels across individuals.¹⁵ Besides, both measures also satisfy the Pareto principle as is widely accepted by the literature.

¹⁵This is not a new result in itself as it is characteristic of the fairness literature in social choice. See Bosmans, Decancq, and Ooghe (2018), Fleurbaey (2003), Fleurbaey and Maniquet (2011), and Piacquadio (2017) among others.

Third, it is interesting to contrast the result of Theorem 3.4 with the standard aggregate consumption computed in national accounts. Observe that $W_i(z_i, \tilde{\omega}, \tilde{p})$ is cardinally equivalent¹⁶ to the expenditure function at prices $\tilde{p}$, defined as

$$e(\tilde{p}, u_i(z_i)) = \min\{\tilde{p}z_0 : z_i \succsim_i z_0\}$$

Theorem 3.4 suggests that the social welfare function should be a sum of the form $\sum_i e(\tilde{p}, u_i(z_i))$. Interestingly, aggregate consumption in national accounts simply sums up individuals' expenditures using individuals' prices, i.e. $\sum_i e(p_i, u_i(z_i))$. This suggests that measuring consumer welfare with the ethics of equality of opportunity simply consists in changing the measuring rod, not the method. It also shows the importance of price normalization when building indicators beyond GDP. We come back on this point in the next section.

Fourth, that the sum of expenditure functions is an attractive way of measuring welfare has a long history in welfare economics marked with controversies over the choice of the reference price vector. First, the Hicksian compensating and equivalent variations are expenditure functions using post- and pre-reform prices as reference situation respectively¹⁷, but a social welfare function using their sum may lead to intransitive assessment as is known since Boadway (1974) and Scitovsky (1941).¹⁸ Second, money-metric utility functions have been criticized by Blackorby and Donaldson (1988) because they may yield anti-redistributive consequences for some reference prices¹⁹ even when the social welfare function is egalitarian. Third, Blackorby, Laisney, and Schmachtenberg (1993) and Roberts (1980) underlined the arbitrariness of the choice of the reference price and studied price-independent welfare assessment. What the present paper offers are social welfare functions based on money-metric utility that overcome each of these critiques: they are transitive,

¹⁶Indeed, one has that $W_i(z_i, \tilde{\omega}, \tilde{p}) = e(\tilde{p}, u_i(z_i)) - \tilde{p}\tilde{\omega}$ which is an affine transformation preserving cardinality.

¹⁷The widely-used consumer surplus is a Hicksian variation when preferences are quasilinear. See Hicks (1939, 1941, 1942, 1943, 1945) for the original contributions.

¹⁸See Blackorby and Donaldson (1990) for a review.

¹⁹Bosmans, Decancq, and Ooghe (2018), Fleurbaey and Maniquet (2011), and Schlee and Khan (2022, 2023) discussed several ways to escape this critique.

inequality-averse, and their reference situation is endogenized by the axioms. Moreover, $\tilde{p}$ is independent of the normalization of the numéraire of the economy. I now present some consequences for applied welfare analysis.

3.5 Applications

The following applications all use the interpersonal consumer welfare measure $W_i(\cdot)$ and are valid whether Theorems 3.3 or 3.4 are used, that is whether the aggregator of social welfare is the minimum or the sum. The fact that the axioms endogenize the same reference price $\tilde{p}$ allows to contribute to various applied welfare problems without taking a stance on which aggregator should be used.

3.5.1 Wealth of nations

In the index number problem²⁰ initiated by Fisher (1922), the economist must compare the *real income* of a set of countries index by i based on a dataset of observed prices p_i and quantities consumed z_i and decompose this real income into a price index and a quantity index. Our analysis of welfare above can readily be applied to this problem if each country is considered as an agent and *real income* is considered as $e(\tilde{p}, u_i(z_i))$. As is standard in index number theory, real income can be decomposed into a price index P_i and a quantity index $Q_{i,j}$ as follows

$$\begin{aligned}\tilde{P}_i &= \frac{e(p_i, u_i(z_i))}{e(\tilde{p}, u_i(z_i))} \\ \tilde{Q}_{i,j} &= \frac{e(\tilde{p}, u_i(z_i))}{e(\tilde{p}, u_j(z_j))}\end{aligned}$$

where the price index P_i informs on the deflation needed to compare the prices faced by country i and $Q_{i,j}$ informs on the real income difference

²⁰That expenditure functions and welfare economics are useful for index number theory is known at least since the classical contributions of Deaton (1979) and Samuelson and Swamy (1974).

between countries i and j .²¹

The key innovations with respect to the index number theory are that the present paper (i) allows for heterogeneous preferences across countries, (ii) does not restrict the set of admissible preferences and (iii) derives the indices from ethical judgments on equality of opportunity. These differences could have lead to completely new indices. Surprisingly, the indices defined here are close to those used by practitioners. Most notably, the Geary (1958) method that underpins the Penn World Table computes a world price $\bar{p}^l$ for good l , is such that

$$\bar{p}^l = \frac{\sum_i p_i^l z_i^l \epsilon_i}{\sum_i z_i^l}$$

where ϵ_i is a normalization of countries' income such that $\epsilon_i = \frac{\sum_l \bar{p}^l z_i^l}{\sum_l p_i^l z_i^l}$. The real income of country i is then computed as the cost of that country's bundle z_i at the world prices $\bar{p}$. While this approach ignores preferences, Neary (2004) shows that consistency with a representative agent preferences can be obtained by setting ϵ_i as the ratio of expenditure functions at these prices.

Deaton and Heston (2010) criticized²² these Geary and Neary measures because countries with larger consumption weight more in the world price, which renders it plutocratic. This is completely true, but the present paper suggests that it does not conflict with social welfare understood as equality of opportunity insofar as actual consumption z_i^l in the denominator is replaced by consumption possibility y_i/p_i^l of good l . In other words, a ranking of countries real income based on $e(\tilde{p}, u_i(z_i))$ takes into account the fact that poor countries have low income and faced lower prices and one wishes to cancel the between-country inequalities

²¹In the literature, the former is a true Kónus price index while the latter is an Allen true quantity index.

²²Deaton and Heston (2010) consider that the main advantage of the approach is the preservation of integrability. Indeed, because it uses a single world price for each good, a summation across goods does not perturb the ranking. This paper's measure escapes the Van Veelen (2002) impossibility by using third-country information when computing the real income of a pair of countries.

in budgets. The fact that the reference price should not depend on actual quantities consumed z_i^l but on consumption possibilities y_i/p_i^l comes from the axioms that imply that only budgets are creating welfare-relevant inequalities, not necessarily consumption vectors.

I note that this modification of the purchasing power parity does not require more data than what is commonly used. Moreover, because $\tilde{p}$ depends only on observed quantities and not on preferences, this paper's index is less data-intensive than Neary (2004)'s.

3.5.2 Lifetime inflation and growth

Consider the case where each good l is a time period. For clarity, let me substitute the superscript l by $t = \{1, \dots, T\}$. The problem for the growth economist consists in measuring inflation and real income growth over the past T periods for a set of countries based on a dataset of country i 's prices p_i^t and consumption z_i^t . Preferences are defined over the T periods, i.e. over the stream of consumption z_i , and must be understood as *lifetime* welfare.

The present paper suggests that the welfare-relevant inflation and growth indices between countries i and j over the T periods are

$$\Pi_i = \frac{e(\tilde{p}, u_i(z_i))}{e(p_i, u_i(z_i))}$$

$$Q_{i,j} = \frac{e(\tilde{p}, u_i(z_i))}{e(\tilde{p}, u_j(z_j))}$$

Interestingly, in such case the reference price vector $\tilde{p}$ becomes close to the GDP deflator i.e. $\tilde{p}^t = \frac{\sum_i y_i}{\sum_i y_i/p_i^t}$.²³ Unlike the Consumer Price Index (CPI), the GDP deflator does not depend on the weights of each consumption good in the bundle consumed, but rather only on income and prices. Again, this can be directly traced back to the axioms: only

²³In practice, the GDP deflator is computed on more goods than mere private consumption goods and include the price of investments as well as government expenditures.

budgets are creating welfare-relevant inequalities, not preferences.

Importantly, this does not mean that preferences are ignored. Indeed, Π_i and $Q_{i,j}$ both depend on preferences as they use expenditure functions as arguments. However, the measuring rod does not: $\tilde{p}$ only depends on consumption possibilities.

3.5.3 Year-by-year inflation and growth

Consider the case where each time period the preferences of the agent under study may vary. In such case, we can consider that the index i used above may be substituted by t . We can then see our contribution as a way to measure year-by-year inflation and welfare growth for a single country when prices, endowments, and preferences may change from one year to the next. The relevant inflation and growth indices are

$$\Pi_t = \frac{e(\tilde{p}, u_t(z_t))}{e(p_t, u_t(z_t))}$$

$$Q_{t,t-1} = \frac{e(\tilde{p}, u_t(z_t))}{e(\tilde{p}, u_{t-1}(z_{t-1}))}$$

In this setup, $\tilde{p}^l$, the reference price for a good l ; becomes the ratio between the sum of expenditures over time over the the sum of maximal quantity of good l that could have been consumed.

$$\tilde{p}^l = \frac{\sum_t y_t}{\sum_t y_t / p_t^l}$$

While traditional inflation and growth measures typically ignore preferences or assumed homothetic preferences, a recent contribution by Jaravel and Lashkari (2024) provides an algorithm to measure inflation and growth for non-homothetic and general preferences. Interestingly, they also rely on expenditure functions with a common, time-invariant reference price vector which they call *base period*.

However, neither their theory nor their algorithm provides guidance on which base period to choose. While their estimates significantly differ from

standard estimates derived under homothetic preferences, the magnitude of these differences depends on the *base period*. What the present paper gives is a normative justification for the base period that connects the measure with consumer welfare.

3.6 Conclusion

The present paper has built a measure of consumer welfare that is transitive and applicable to a wide set of problems. In essence, it builds interpersonal comparison between heterogeneous individuals, and as such may inform several other problems not mentioned so far, such as indexing the preference-based poverty line (Decerf, 2023), solving for optimal consumption taxes with preference heterogeneity (Ferey, Lockwood, & Taubinsky, 2024; Golosov et al., 2013), or the welfare costs of inflation (Adam & Weber, 2023; Blanco, 2021; Burstein & Hellwig, 2008; Craig & Rocheteau, 2008).

Moreover, by no means this paper has solved all issues of comparing heterogeneous consumers. There many practical issues that should be accounted for, for example, the incorporation of quality (Errico & Lashkari, *Forthcoming*) but also incomplete price information (Atkin et al., 2024) or individual information (Baqaee, Burstein, & Koike-Mori, 2024).

Moreover, individuals derive welfare beyond their mere consumption, notably from health and social relations. The inclusion of such well-being dimensions in the framework of equal opportunity is beyond the scope of the present paper.

3.7 Bibliography

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3.A Proofs

Proof of Theorem 3.3. The proof of the "if" proceeds by construction. Take a generic economy $e \in \mathcal{E}$ and two allocations z, z' such that the premise holds. Without loss of generality, set $i = 3$ and $j = 4$.

By **Egalitarian Responsibility**, we have

$$z \mathbf{R} \left((\succsim_i, \tilde{\omega}, \tilde{p})_{i \in \{1, \dots, N\}} \right) (z_1, z_2, z'_3, z'_4, \dots, z_N)$$

By **No-Growth Separability**, we can change the preferences of agents 1 and 2

$$z \mathbf{R} \left((\succsim_3, \tilde{\omega}, \tilde{p}) (\succsim_4, \tilde{\omega}, \tilde{p}) (\succsim_i, \tilde{\omega}, \tilde{p})_{i \in \{3, \dots, N\}} \right) (z_1, z_2, z'_3, z'_4, \dots, z_N)$$

By **Compensation** applied twice,

$$(z'_3, z'_4, z_1, z_2, \dots, z_N) \mathbf{I} \left((\succsim_3, \tilde{\omega}, \tilde{p}) (\succsim_4, \tilde{\omega}, \tilde{p}) (\succsim_i, \tilde{\omega}, \tilde{p})_{i \in \{3, \dots, N\}} \right) \\ (z_1, z_2, z'_3, z'_4, \dots, z_N)$$

By transitivity,

$$z \mathbf{R} \left((\succsim_3, \tilde{\omega}, \tilde{p}) (\succsim_4, \tilde{\omega}, \tilde{p}) (\succsim_i, \tilde{\omega}, \tilde{p})_{i \in \{3, \dots, N\}} \right) (z'_3, z'_4, z_1, z_2, \dots, z_N)$$

By **Compensation** applied twice,

$$z \mathbf{I} \left((\succsim_3, \tilde{\omega}, \tilde{p}) (\succsim_4, \tilde{\omega}, \tilde{p}) (\succsim_i, \tilde{\omega}, \tilde{p})_{i \in \{3, \dots, N\}} \right) (z_3, z_4, z_1, z_2, \dots, z_N)$$

By transitivity,

$$(z_3, z_4, z_1, z_2, \dots, z_N) \mathbf{R} \left((\succsim_3, \tilde{\omega}, \tilde{p}) (\succsim_4, \tilde{\omega}, \tilde{p}) (\succsim_i, \tilde{\omega}, \tilde{p})_{i \in \{3, \dots, N\}} \right) \\ (z'_3, z'_4, z_1, z_2, \dots, z_N)$$

By **No-Growth Separability**,

$$(z_3, z_4, z_1, z_2, \dots, z_N) \mathbf{R} \left((\tilde{z}_3, \tilde{\omega}, \tilde{p})(\tilde{z}_4, \tilde{\omega}, \tilde{p})(\tilde{z}_k, y_k, p_k)_{k \in \{3,4\}} \right. \\ \left. (\tilde{z}_5, \hat{\omega}_5, \hat{p})(\tilde{z}_i, \tilde{\omega}, \tilde{p})_{i \in \{6, \dots, N\}} \right) (z'_3, z'_4, z_1, z_2, \dots, z_N)$$

only if we can find $\hat{\omega}_5$ and $\hat{p}$ such that

$$\text{for each } l = \{1, \dots, L\} : 3 \frac{\tilde{y}}{\tilde{p}^l} = \frac{y_3}{p_3^l} + \frac{y_4}{p_4^l} + \frac{\hat{p}\hat{\omega}_5}{\hat{p}^l} \quad (3.2)$$

By **Compensation** applied four times and transitivity,

$$z \mathbf{R} \left((\tilde{z}_3, \tilde{\omega}, \tilde{p})(\tilde{z}_4, \tilde{\omega}, \tilde{p})(\tilde{z}_k, \omega_k, p_k)_{k \in \{3,4\}} \right. \\ \left. (\tilde{z}_5, \hat{\omega}_5, \hat{p})(\tilde{z}_i, \tilde{\omega}, \tilde{p})_{i \in \{6, \dots, N\}} \right) (z_1, z_2, z'_3, z'_4, \dots, z_N)$$

By **No-Growth Separability**,

$$z \mathbf{R}(\mathbf{e})(z_1, z_2, z'_3, z'_4, \dots, z_N)$$

if and only if

$$\text{for each } l = \{1, \dots, L\} : (N-3) \frac{\tilde{y}}{\tilde{p}^l} + \frac{\hat{p}\hat{\omega}_5}{\hat{p}^l} = \sum_{i \in \mathcal{N} \setminus \{3,4\}} \frac{y_i}{p_i^l} \quad (3.3)$$

Summing over equations (3.2) and (3.3), we get

$$\text{for each } l = \{1, \dots, L\} : N \frac{\tilde{y}}{\tilde{p}^l} + \frac{\hat{p}\hat{\omega}_5}{\hat{p}^l} = \sum_{i \in \mathcal{N}} \frac{y_i}{p_i^l} + \frac{\hat{p}\hat{\omega}_5}{\hat{p}^l}$$

which holds by construction of $\tilde{\omega}$ and $\tilde{p}$. The result is obtained by observing that $(z_1, z_2, z'_3, z'_4, \dots, z_N)$ is the same allocation as z' , proving $z \mathbf{R}(\mathbf{e}) z'$, the required result.

To prove the "only if", observe that $W_i(z_i, \tilde{\omega}, \tilde{p})$ does not depend on the individual's endowments (ω_i, p_i) such that a permutation between

equal-preference individuals leads the ordering unchanged, proving that **R(e)** satisfies **Compensation**.

As for **No-Growth Separability**, observe that any change of endowments such that $\sum_i \frac{y_i}{p_i}$ is unchanged for all l does not modify $\tilde{\omega}$ nor $\tilde{p}$. The proof for **Egalitarian Responsibility** is trivial and left to the reader. ■

Proof of Theorem 4. Without loss of generality, consider that agents 3 and 4 are i and j . Then, by **Utilitarian Responsibility**, we have

$$z \mathbf{R} \left((\tilde{\succ}_i, \tilde{\omega}, \tilde{p})_{i \in \{1, \dots, N\}} \right) (z_1, z_2, z'_3, z'_4, \dots, z_N)$$

The remaining steps of the proof follow exactly the proof of the previous theorem. ■