Mixed QCD-EW corrections for Higgs boson production at e^+e^- colliders

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Since the discovery of the Higgs boson at the Large Hadron Collider, a future electron-position collider has been proposed for precisely studying its properties. We investigate the production of the Higgs boson at such an e^+e^- collider and calculate for the first time the mixed QCD-electroweak corrections to the total cross sections. We provide an approximate analytic formula for the cross section and show that it reproduces the exact numeric results rather well for collider energies up to 350 GeV. We also provide numeric results for $\sqrt{s} = 500$ GeV, where the approximate formula is no longer valid. We find that the $\mathcal{O}(\alpha\alpha_s)$ corrections amount to a 1.3% increase of the cross section for a center-of-mass energy around 250 GeV. This is significantly larger than the expected experimental accuracy and has to be included for extracting the properties of the Higgs boson from the measurements of the cross sections in the future.

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INTRODUCTION

The discovery of the Higgs boson [1, 2] by the ATLAS and CMS collaborations at the Large Hadron Collider (LHC) marks a major milestone of high energy physics. Since then, extracting the properties of the Higgs boson by measuring its productions and decays has become a top priority of both experimental and theoretical particle physics. Since the proton-proton collisions at the LHC always generate enormous amounts of complicated backgrounds, the expected precisions for measuring the various couplings will not be sufficient to arrive at a conclusion on the nature of the Higgs boson. For this reason, a next-generation high-luminosity electron-positron collider served as a Higgs factory is undergoing active discussions in the community. Notable candidates include the Circular Electron-Positron Collider (CEPC) [3], the International Linear Collider (ILC) [4] and the FCC-ee [5]. Thanks to the clean environment and the high luminosity, many important properties of the Higgs boson can be measured to extremely high precisions at these machines, often orders of magnitude better than the LHC. This will provide stringent tests of the Standard Model (SM) and has the potential to indirectly probe new physics beyond the SM which might exist at very high energy scales.

At an electron-positron collider with the center-of-mass energy $\sqrt{s} \sim 250$ GeV, the main production mechanism for the Higgs boson is the associated production of a Higgs boson and a Z boson. At the CEPC, the production cross section $\sigma(e^+e^- \to ZH)$ can be measured to an extremely high precision of 0.51% [3]. As a result, the coupling of the Higgs boson with the Z boson can be extracted with a precision of 0.25%. The FCC-ee has claimed an even higher accuracy for this cross section [6, 7]. The studies on the various properties of the Higgs boson rely crucially on detailed investigations of this production channel. Obviously, the theoretical prediction for this process within the SM has to be known with a similar or even higher precision than the experimental one in order for these investigations to be meaningful. The calculations of higher order radiative corrections are therefore indispensable for the Higgs factory projects.

The next-to-leading order (NLO) electroweak (EW) corrections to $\sigma(ZH)$ have been calculated in [8–10]. The purely weak corrections decrease the total cross section for $\sqrt{s} \sim 250$ GeV, whose size depends on the renormalization scheme and amounts to a few percent of the leading order (LO) cross section. The purely quantum electrodynamics (QED) corrections involve virtual photon exchanges and initial state radiations (ISRs). The QED corrections contain logarithmic enhancements and can be quite sizable. A recent study [11] shows that the ISR effects reduce the ZH cross section by about 10% at the CEPC. No improvements over the NLO calculations were attempted till now, while apparently higher order corrections beyond the NLO will be necessary in order to match the high precision of the experimental measurements at the Higgs factories. In [12], higher order corrections for a closely related process $H \to ZZ^* \to Zl^+l^-$ were estimated in the large m_t approximation. For $e^+e^- \to ZH$, one needs to go beyond the large m_t limit due to the higher energy scales involved. In this Letter, we take the first step forward by computing exactly the mixed QCD-EW $\mathcal{O}(\alpha\alpha_s)$ corrections to $\sigma(ZH)$, which consist of the main part of the next-to-next-to-leading order (NNLO) corrections for this observable. Our results provide the most precision theoretical predictions for Higgs boson production at Higgs factories, and must be used when extracting the properties of the Higgs boson from experimental measurements in the future.

METHODS

We consider corrections to the leading order process

$$e^-(k_1) + e^+(k_2) \to Z^*(q) \to Z(p_1) + H(p_2)\,, \quad (1)$$

and we define $s \equiv Q^2 \equiv q^2$. The leading order cross section can be written as

$$\sigma^{\rm LO} = \frac{2\pi\alpha^2|\vec{p}_1|/\sqrt{s}}{3\big(s-m_Z^2\big)^2 s_w^2 c_w^2}\big(E_Z^2+2m_Z^2\big)\big(v_e^2+a_e^2\big)\,, \quad (2)$$

where $E_Z = (Q^2+m_Z^2-m_H^2)/(2Q)$ is the energy of the on-shell Z boson in the rest frame of q^μ (i.e., the e^+e^- center-of-mass frame), $s_w = \sin\theta_w$ and $c_w = \cos\theta_w$ with θ_w the weak mixing angle, $v_e = (-1/4+s_w^2)/(s_w c_w)$ and $a_e = 1/(4 s_w c_w)$ coming from the vector and axial-vector couplings of the electron with the Z boson, and $|\vec{p}_1|^2 = E_Z^2 - m_Z^2$.

The $\mathcal{O}(\alpha\alpha_s)$ corrections to the cross section can be decomposed into 3 parts

$$\sigma^{\alpha\alpha_s} = \sigma_Z^{\alpha\alpha_s} + \sigma_\gamma^{\alpha\alpha_s} + \sigma_{eeZ}^{\alpha\alpha_s}\,, \quad (3)$$

where σ_Z consists of corrections to the HZZ form factors (including bubble insertions on the off-shell Z^* propagator), σ_γ contains contributions from the loop-induced $HZ\gamma^*$ form factors (including the Z^*-γ^* bubble diagrams), and σ_{eeZ} arises from the counter-term for the eeZ vertex. The HZV form factors are defined by

$$\mathcal{T}^{\mu\nu}_{HZV} = \frac{iem_Z}{s_w c_w}\bigg(p_1^\mu p_1^\nu T_{1,V} + q^\mu q^\nu T_{2,V} - p_1^\mu q^\nu T_{3,V} - \frac{q^\mu p_1^\nu}{QE_Z}T_{4,V} + g^{\mu\nu}T_{5,V} - \epsilon^{\mu\nu\rho\sigma}p_{1\rho}q_\sigma T_{6,V}\bigg)\,, \quad (4)$$

among which only $T_{4,V}$ and $T_{5,V}$ contribute to the cross section:

$$\sigma_V^{\alpha\alpha_s} = \frac{4\pi\alpha^2|\vec{p}_1|/\sqrt{s}}{3\big(s-m_Z^2\big)s_w^2c_w^2}C_V \times\Big[\big(E_Z^2+2m_Z^2\big)\,{\rm Re}T_{5,V}^{\alpha\alpha_s} - \big(E_Z^2-m_Z^2\big)\,{\rm Re}T_{4,V}^{\alpha\alpha_s}\Big]\,, \quad (5)$$

where $C_Z = (v_e^2+a_e^2)/(s-m_Z^2)$ and $C_\gamma = v_e/s$. The contribution from the counter-term for the eeZ vertex is given by

$$\sigma_{eeZ}^{\alpha\alpha_s} = \frac{4\pi\alpha^2|\vec{p}_1|/\sqrt{s}}{3\big(s-m_Z^2\big)^2 s_w^2c_w^2}\big(E_Z^2+2m_Z^2\big)\Bigg[\frac{\delta Z_{\gamma Z}^{\alpha\alpha_s}}{2}v_e + \bigg(\frac{\delta Z_{ZZ}^{\alpha\alpha_s}}{2}+\delta Z_e^{\alpha\alpha_s}\bigg)\big(v_e^2+a_e^2\big) + v_e\delta v_e^{\alpha\alpha_s} + a_e\delta a_e^{\alpha\alpha_s}\Bigg]\,, \quad (6)$$

where $\delta v_e = -\delta c_w^2(1+2s_w^2)/(8s_w^3c_w^3)$ and $\delta a_e = \delta c_w^2(1-2s_w^2)/(8s_w^3c_w^3)$.

For the calculation of $T_{4,V}$ and $T_{5,V}$, we generate the relevant Feynman diagrams using both FeynArts [13] and QGRAF [14]. The resulting amplitudes are further manipulated with FORM [15]. The 2-loop integrals arising from the triangle diagrams with a top quark loop are reduced to a minimal set of 41 master integrals using Reduze 2 [16] and FIRE [17], which implement the Laporta algorithm [18] for solving integrate by parts (IBP) identities. Part of these master integrals, together with those in the bubble diagrams and those in the calculation of renormalization constants, can be calculated analytically using the method of differential equations [19]. The remaining master integrals do not admit analytic solutions and we evaluate them with two different methods.

The first method involves an series expansion in $1/m_t$, which is expected to converge for $\sqrt{s} < 2m_t$. The leading contribution to the $\mathcal{O}(\alpha\alpha_s)$ correction is of order m_t^2, and we perform the expansion up to order m_t^{-4}. In this way we obtain an approximate analytic formula for the cross section. This approximate result is to be compared with the result from a purely numerical evaluation of the difficult master integrals using the method of sector decomposition [20, 21]. For this second method we have used a fast private code documented in [22], and cross-checked with SecDec [23] whenever possible.

We renormalize the fields and the masses in the on-shell scheme. The weak mixing angle is defined by the on-shell relation $c_w = m_W/m_Z$, whose renormalization constant is given by $\delta c_w^2/c_w^2 = \delta m_W^2/m_W^2 - \delta m_Z^2/m_Z^2$. The fine structure constant α was renormalized on-shell in the original NLO calculations [8–10]. In contrast, we choose to renormalize α in the $\overline{\rm MS}$ scheme for all contributions except the top quark loop, which is subtracted on-shell. In this scheme the final results are insensitive to non-perturbative effects and to the masses of light fermions such that we can safely take them to zero. Alternative renormalization (and expansion) schemes are also widely adopted in electroweak physics. For example, one may choose the Fermi constant G_F as an input parameter instead of m_W; or one may perform the perturbative expansion in terms of G_F instead of α. The impact of these choices at the NLO level was investigated in [9, 10]. The phenomenological consequences of these schemes for our mixed QCD-EW corrections will be studied in a forthcoming article [28].

As mentioned before, the benefit of performing the expansion in $1/m_t$ is that we obtain an approximate analytical formula for the cross section, which allows much faster numerical evaluations compared to the sector decomposition method. For our numerical results in the next section we have used the expansion up to order m_t^{-4}. Due to the limited space, we give below the analytic results up to order m_t^0, which will prove to be a sufficiently accurate approximation for $\sqrt{s} \sim 250$ GeV. We begin with the simpler ones:

$$T_{4,\gamma}^{\alpha\alpha_s} = \frac{\alpha}{4\pi}\frac{\alpha_s}{4\pi}C_F\frac{8QE_Zv_t}{m_Z^2} + \mathcal{O}(m_t^{-2})\,, \tag{7}$$

$$\begin{aligned} T_{5,\gamma}^{\alpha\alpha_s} = \frac{\alpha}{4\pi}\frac{\alpha_s}{4\pi}C_F\Bigg[&\frac{8QE_Zv_t}{m_Z^2} \\ &- \frac{(21-44s_w^2)Q^2}{3s_wc_w(Q^2-m_Z^2)}\left(\ln\frac{Q^2}{m_Z^2}+i\pi\right)\Bigg] + \mathcal{O}(m_t^{-2})\,, \end{aligned} \tag{8}$$

$$T_{4,Z}^{\alpha\alpha_s} = \frac{\alpha}{4\pi}\frac{\alpha_s}{4\pi}C_F\frac{QE_Z}{m_Z^2}\left(-12v_t^2+\frac{4}{3}a_t^2\right) + \mathcal{O}(m_t^{-2})\,, \tag{9}$$

where $v_t = (1/4 - 2s_w^2/3)/(s_wc_w)$ and $a_t = -1/(4s_wc_w)$ come from the vector and axial-vector couplings of the top quark with the Z boson. Note that all the above 3 form factors vanish at the leading order. The most complicated form factor is $T_{5,Z}$, which equals 1 at tree-level. It is given by

$$\begin{aligned} T_{5,Z}^{\alpha\alpha_s} = \frac{\alpha}{4\pi}\frac{\alpha_s}{4\pi}C_F\Bigg\{&\frac{m_t^2}{m_Z^2}a_t^2\left(30-12\pi^2-264L_t-144L_t^2\right) \\ &+ \frac{(45-84s_w^2+88s_w^4)}{6s_w^2c_w^2(Q^2-m_Z^2)}\left[m_Z^2+Q^2\left(\ln\frac{Q^2}{m_Z^2}+i\pi-1\right)\right] \\ &- 12(v_t^2+a_t^2)\frac{QE_Z}{m_Z^2} - \frac{4}{3}a_t^2\frac{m_H^2}{m_Z^2}\Bigg\} + \mathcal{O}(m_t^{-2}) \\ &+ \left[\delta Z_e + \delta Z_{ZZ} + \frac{1}{2}\delta Z_H + \frac{\delta m_Z^2}{2m_Z^2} + \frac{\delta c_w^2(c_w^2-s_w^2)}{2s_w^2c_w^2}\right]_{\text{finite}}^{\alpha\alpha_s}\,, \end{aligned} \tag{10}$$

where $L_t = \ln(\mu^2/m_t^2)$, and the subscript "finite" refers to the finite part of the various renormalization constants.

The renormalization constants appearing in Eqs. (6) and (10) are calculated exactly with the help of differential equations. We have checked that our results agree with those in [24, 25].[1] Since we have used a different renormalization scheme for the electromagnetic coupling, we list its renormalization constant here:

$$\delta Z_e^{\alpha\alpha_s} = \frac{\alpha}{4\pi}\frac{\alpha_s}{4\pi}C_F\left(\frac{5}{\epsilon}+10+\frac{8}{3}L_t\right). \tag{11}$$

RESULTS

In this section we present the numerical predictions from our calculations. We choose the input parameters as $m_t = 173.3$ GeV, $m_H = 125.1$ GeV, $m_Z = 91.1876$ GeV, $m_W = 80.385$ GeV, $\alpha(m_Z) = 1/127.94$ and $\alpha_s(m_Z) = 0.118$ [27]. The default renormalization scale is chosen as $\mu_0 = \sqrt{s}/2$. The renormalization group evolutions of the coupling constants are performed at 4 loops for α and 2 loops for α_s.

$\sqrt{s}$ (GeV)	σ_{LO} (fb)	σ_{NLO} (fb)	σ_{NNLO} (fb)	$\sigma_{\text{NNLO}}^{\text{exp.}}$ (fb)
240	256.3(9)	228.0(1)	230.9(4)	230.9(4)
250	256.3(9)	227.3(1)	230.2(4)	230.2(4)
300	193.4(7)	170.2(1)	172.4(3)	172.4(3)
350	138.2(5)	122.1(1)	123.9(2)	123.6(2)
500	61.38(22)	53.86(2)	54.24(7)	54.64(10)

TABLE I. The NNLO predictions for the total cross sections at various collider energies.

In Table I we show the NNLO predictions with the LO and the NLO cross sections for center-of-mass energies $\sqrt{s} = 240$ GeV, 250 GeV, 300 GeV, 350 GeV and 500 GeV. The results from the $1/m_t$ expansion up to order m_t^{-4} are also shown in the 5th column in the table. We find that the $\mathcal{O}(\alpha\alpha_s)$ corrections increase the NLO cross section by about 1.3% for all 3 collider energies below the $t\bar{t}$ threshold of about 346 GeV. This effect is significantly larger than the expected experimental accuracies of Higgs factories. Our results are therefore crucial for extracting useful information from precision measurements at these future facilities. We also see that the $1/m_t$ expansion approximates the exact results remarkably well for these 3 energies. The digits in the parentheses reflect the variations of the cross sections with respect to the renormalization scale μ by a factor of 2 around the default scale $\mu_0 = \sqrt{s}/2$. We observe that the variations of the NLO cross sections are too small to cover the higher order corrections, which is common for electroweak observables. The mixed QCD-EW corrections introduce dependence on strong interactions for the first time in the perturbative series. As a result, the NNLO cross sections exhibit larger scale variations than the NLO ones. We use this to estimate that the size of even higher order corrections amounts to about 0.2%.

Once we go for higher energies above the $t\bar{t}$ threshold, the $1/m_t$ expansion is expected to break down. In this case one has to rely on the numerical methods. Nevertheless, we observe from Table I that for $\sqrt{s} = 350$ GeV, the $1/m_t$ expansion still does a reasonable job to describe the $\mathcal{O}(\alpha\alpha_s)$ correction. We also see that, due to the threshold enhancement, the NNLO correction can reach 1.5% of the NLO cross section. The energy $\sqrt{s} = 350$ GeV is just slightly above the $t\bar{t}$ threshold[2], and is a design energy of the ILC and the FCC-ee to study the properties of the

[1] There is a typo in [24] relevant for the $W^\pm$ boson self-energies, which was corrected in [26].

[2] This fact also makes the numerical evaluation of the master integrals for $\sqrt{s} = 350$ GeV rather difficult. For this reason, many optimizations over the original version of the program reported

$\sqrt{s}$ (GeV)	$\mathcal{O}(m_t^2)$	$\mathcal{O}(m_t^0)$	$\mathcal{O}(m_t^{-2})$	$\mathcal{O}(m_t^{-4})$
240	81.8%	16.2%	1.4%	0.4%
250	81.7%	16.1%	1.5%	0.5%
300	80.0%	15.2%	2.1%	1.1%
350	69.7%	12.6%	2.7%	2.1%
500	137%	18.6%	17.3%	31.1%

TABLE II. Convergence of the $1/m_t^2$ expansion for the mixed QCD-EW corrections.

top quark, which makes it particularly interesting. Our result provides the essential theoretical input to continue investigating the Higgs boson at this collider energy.

Going further up to higher energies, the main task of the colliders becomes producing new particles below the TeV scale rather than precisely measuring standard model processes, and the ZH cross section is not as important as in previous cases. Nevertheless, we give the results for $\sqrt{s} = 500$ GeV in Table I for demonstration purposes. It is clear that the asymptotic expansion completely fails here: the $1/m_t$ expansion up to order m_t^{-4} overestimates the size of the NNLO correction by a factor of 2. To further assess the behavior of the $1/m_t$ expansion, we show in Table II the fractions of different orders of the expansion in the full $\mathcal{O}(\alpha\alpha_s)$ corrections at the default scale $\mu = \sqrt{s}/2$. Again we show the result for 5 different center-of-mass energies. The most important one is $\sqrt{s} = 240$ GeV, which exhibits the largest production cross section and also very high luminosity can be achieved experimentally, and therefore is the design energy of Higgs factories. At this energy, we see that the leading $\mathcal{O}(m_t^2)$ term accounts for about 82% of the total corrections, while the subleading $\mathcal{O}(m_t^0)$ term accounts for another 16%. The even higher power contributions are negligible here. These demonstrate the good convergence of the $1/m_t$ expansion and the usefulness of our approximate analytical formula, which evaluates much faster than the sector decomposition method. It provides an efficient and reliable way to perform high precision physics analyses for Higgs factories.

As we increase the center-of-mass energy, it can be seen that the size of the power corrections starts to grow gradually. The $1/m_t$ expansion still provides very good approximations to the full results as long as the energies are below or even slightly above the $t\bar{t}$ threshold. For $\sqrt{s} = 500$ GeV which is far beyond the threshold, the power series tends to diverge as expected.

in [22] are implemented to further improve the efficiency. We are not able to cross-check this result using the current public version of SecDec (3.0.9) with the computation resouce attainable to us.

SUMMARY AND OUTLOOK

In this Letter, we calculated the mixed QCD-electroweak corrections to the associated production of a Higgs boson and a Z boson at future electro-position colliders. We found that the $\mathcal{O}(\alpha\alpha_s)$ corrections increase the cross sections by about 1.3%, which is significantly larger than the expected experimental accuracies of the Higgs factories. Our results must be included when extracting the properties of the Higgs boson from future precision measurements of the ZH production cross section.

We have shown that for center-of-mass energies below the $t\bar{t}$ threshold, the approximate analytic formula obtained in the $1/m_t$ expansion agrees remarkably well with the exact numeric results. This is especially important for the design energy of the Higgs factories $\sqrt{s} \sim 250$ GeV, as it provides a fast and reliable method to perform physics analyses with high precisions. We have also shown that even for $\sqrt{s} = 350$ GeV, which is slightly above the $t\bar{t}$ threshold and is a design energy of the ILC and the FCC-ee to study the top quarks, the approximate formula is still valid with good precisions. For higher collider energies which can be achieved at the ILC and the FCC-ee, such an expansion breaks down for $\sqrt{s}$ much larger than $2m_t$, and we have explicitly demonstrated that for $\sqrt{s} = 500$ GeV.

While we only presented our predictions for the total cross sections in this Letter, the same formula can be used to study the kinematic distributions as well as polarized scatterings. These investigations will be presented in a forthcoming article [28], together with many details of our calculations.

The mixed QCD-EW corrections calculated in this Letter brings the accuracy of the theoretical prediction for the ZH cross section to a regime comparable to the expected experimental accuracy at Higgs factories. At this point, one should further consider several other important effects. First of all, as we stated in the Introduction, one should add the QED corrections upon our results. While the NLO QED corrections are known, higher order contributions could be important due to logarithmic enhancements arising from ISR effects. It is possible to resum such effects to all orders in perturbation theory, which will be a subject of future investigations. Secondly, in addition to the production process, one should combine the NNLO corrections to the decay of the on-shell Z boson to leptons and also consider the corrections from the finite width of the Z boson. Finally, it is interesting to know the size of the two-loop weak corrections. Such a calculation is unlikely to be done with analytic methods. However, with the recent developments in the numerical evaluation of loop integrals, a purely numerical estimation of the remaining NNLO effects should be possible in the near future.

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Note added after submission: while we are finishing this work, we are aware of an independent calculation [29]. We have checked that our results are compatible with theirs.

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