Alternatives to Polynomial Trend-Corrected Differences-In-Differences Models

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Abstract

A common problem with differences-in-differences (DD) estimates is the failure of the parallel-trend assumption. To cope with this, most authors include polynomial (linear, quadratic…) trends among the regressors, and estimate the treatment effect as a once-in-a-time trend shift. In practice that strategy does not work very well, because *inter alia* the estimation of the trend uses post-treatment data. An extreme case is when sample covers only one period before treatment and many after. Then the trend's estimate relies almost completely on post-treatment developments, and absorbs most of the treatment effect. What is needed is a method that *i)* uses pre-treatment observations to capture linear or non-linear trend differences, and *ii)* extrapolates these to compute the treatment effect. This paper shows how this can be achieved using a fully-flexible version of the canonical DD equation. It also contains an illustration using data on a 1994-2000 EU programme that was implemented in the Belgian province of Hainaut.

JEL Classification: C21, C4, C5

Keywords: Treatment-Effect Analysis, Differences-in-Differences Models, Correction for trend differences

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1. Introduction

When the parallel-trend assumption fails, most authors (e.g. Friedberg, 1999; Autor, 2003; Besley & Burgess, 2004) resort to a polynomial (linear,…) trend-augmented version of the canonical DD model (Angrist & Pischke, 2009).

$$Y_{it} = \alpha + \sum_{\tau=t2}^{T} \alpha^{\tau} I_{\tau,t} + \alpha^{D} D_i + \eta AFTER_t{*}D_i \; [+ \boldsymbol{\theta}\, \boldsymbol{t}{*}D_i] \qquad [1.]$$

with $I_{\tau,t}$=1 if $t=\tau$ and 0 otherwise, and where Y_{it} is entity *i*'s outcome in time *t*, *D* the treatment dummy, *AFTER* the after treatment dummy, and here *t* is a continuous variable.

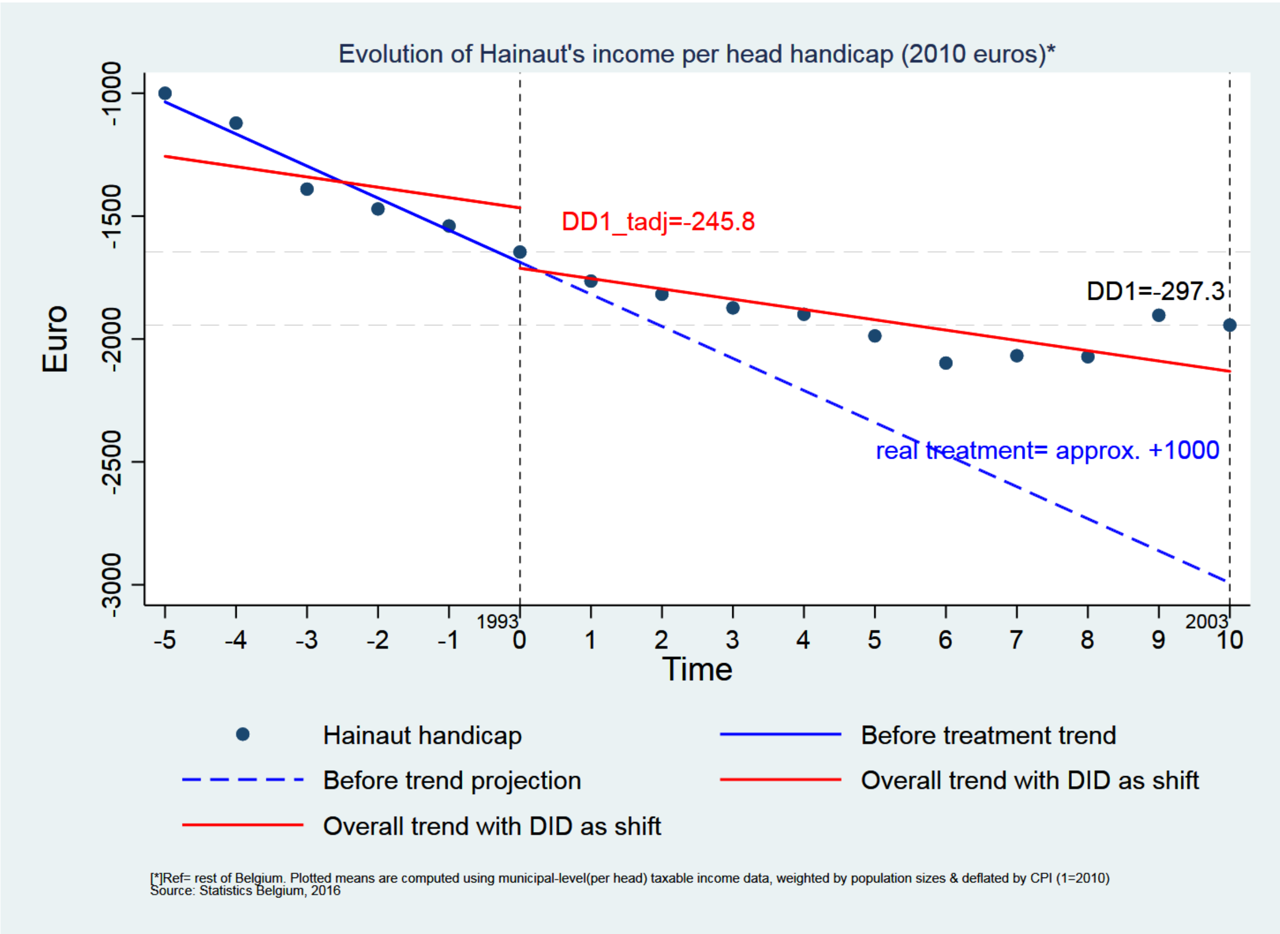
Coefficient $\boldsymbol{\theta}$ captures the linear trend characterizing the treated entities. And η - a trend shift around $t=0$ – measures the treatment effect. As suggested by Wolfers (2006), the problem with this strategy is that it uses post-treatment observations, and that the treatment outcome takes the form of a once-in-a-time trend shift. A case in point is visible on Figure 1. The latter describes the evolution of income per head in the Belgian province of Hainaut (in deviation to the rest of Belgium), before and after it benefited from EU money.[1] That treatment began in 1994 and lasted until 2000. The trend is clearly negative prior to treatment, and still so after. The estimation of η, using the canonical DD model 10 years after treatment, delivers a negative value, in the range of -300€. A 'placebo' estimation of that model evidently reveals that there was no parallelism before the treatment started. So, the - 300€ figure is not trustworthy. This justifies estimating the trend-augmented eq.[1]. The red line on Figure 1 depicts the result. After treatment, the income handicap tends to stabilize, and this explains the moderately negative estimated trend ($\boldsymbol{\theta} <0$). By construction, this trend applies to the pre-treatment period. Being negative, it delivers "corrected" DD estimates that are less negative than the traditional ones (-245.8€>-297.3€). Also, η corresponds to the trend shift just after $t=0$.[2] And as income handicap after treatment is larger, that shift is still negative; suggesting that the EU policy failed (it "caused" approx: - 245€ of additional income handicap). Yet, $\boldsymbol{\theta}$ underestimates the actual pre-treatment trend (in blue on Figure 1). Before treatment, the handicap was growing faster than after. Prolonging the initial trend up to *t*=10 suggests that, *ceteris paribus*, the income handicap might have reached -3,000€, while it ended being less than -2,000€. The tentative conclusion is that the real treatment outcome was positive (in

[1] See Vandenberghe (2016) for more details about EU-Objective 1-Hainaut.

[2] Defining *t=year-1993*

the range of +1,000€). What we propose hereafter is an alternative way of correcting DD estimation, that solely uses pre-treatment observations.

Figure 1 – The liminations of trend-augmented DD[$]



[$] Plotted values are (municipal)- population-weighted mean differences between Hainaut and rest-of-Belgium. These are used to estimate a linear trend-adjusted DD model.

2. Beyond polynomial trend-corrected DD

Mora & Reggio (2012) suggest that DD analysis can be done by estimating a generalize fully-flexible equation, where the right-hand part only consists of time, treatment and time*X*treatment dummies:

$$Y_{it} = \gamma + \sum_{\tau=t_2}^{T} \gamma_\tau I_{\tau,t} + \gamma^D D_i + \sum_{\tau=t_2}^{T} \gamma_\tau^D I_{\tau,t} D_i \qquad [2.]$$

with $t=t_1,\ldots T$ and $I_{\tau,t}=1$ if $t=\tau$ and 0 otherwise, covering before and after treatment periods.

The advantages to this equation are manyfold. First, conditional on the availability of many pre-treatement periods in the data, the OLS-estimated coefficients can be used to compute a whole family of difference-in-difference estimators $DD_{[p]}$, where $p=1, 2...q$ is the degree of parallelism underpinning identification. The canonical DD model is noted $DD_{[1]}$, and rests on parallelism of degree 1 ($Parallel_{[1]}$ hereafter).[3] Without $Parallel_{[1]}$, one should estimate $DD_{[2]}$ that rests on $Parallel_{[2]}$, i.e. outcome growth rate parallelism.[4] If $Parallel_{[2]}$ fails, one should turn to $DD_{[3]}$ with requires $Parallel_{[3]}$ or outcome acceleration[5] parallelism…and so on up to degree $p=q$, if data permit. Second, eq. [2], unlike eq.[1] can capture dynamic (ie. lagged) responses to treatment.[6] Third, – and this is something we particularly stress in the contex to this paper as it brings a solution to Wolfer's trend & shift problem – corrections for the violation of $Parallel_{[p]}$ rests solely on pre-treatment observations.

Consider the canonical $DD_{[1]}/Parallel_{[1]}$ estimator, with just before-and-after observations t^* and t^*+1. Treatment effect writes[7],[8]

$$DD_{[p=1]}^{t^*+1;t^*}=(\gamma^D_{t^*+1}+\gamma^D)-(\gamma^D_{t^*}+\gamma^D)=\gamma^D_{t^*+1}-\gamma^D_{t^*}. \qquad [3.]$$

Also, Eq. [2] can be used to assess $Parallel_{[1]}$ prior to treatment. Using pre-treatment periods t^*-2, t^*-1, one can compute 'placebo' $DD_{[1]}$ capturing the deviation from $Parallel_{[1]}$ prior to treatment. For instance, $DD_{[1]}^{t^*;t^*-1}=\gamma^D_{t^*}-\gamma^D_{t^*-1}$. should not be statistically different from zero. It not, then treated and control trends diverge before treatment (as illustrated on Figure 1 or its stylised equivalent Figure 2). And identification should rests on $Parallel_{[2]}$. The point is this can be easly achieved by computing

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3 If outcome level change by unit of time (i.e 1st derivate) is "speed", then $Parallel_{[1]}$ means stable level differences due to identifical speeds.

4 If outcome growth rate change by unit of time (2nd derivative) is "acceleration" , then $Parallel_{[2]}$ means stable growth rate differences due to same accelerations.

5 If outcome acceleration change by unit of time (3rd derivative) is "surge", then $Parallel_{[3]}$ corresponds to a situation where acceleration differences remain stable due to identical surges.

6 The pattern of lagged effects is usually of substantive interest, e.g. if treatment effect should grow or fade as time passes.

7 When estimating eq. [2] with only 2 periods, $\gamma^D_{t^*}$ is subsumed into the constant $\gamma^D$and $DD_{[1]}$ is directly captured by the timeXtreatment coefficient.

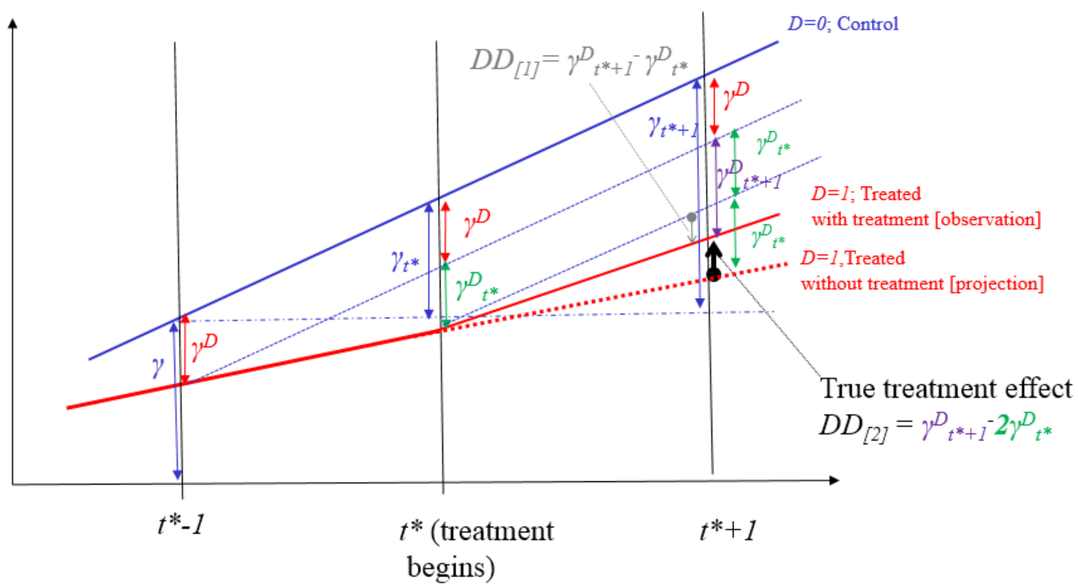
8 Treatment effect' standard error must account for the fact that it consists of a linear combination of estimated coefficients, and thus of the covariance between variables. That is automatically done by STATA `test` or `lincom` commands used hereafter, that exploit the variance-covariance matrix of the estimated coefficients.

$$DD_{[p=2]}^{t^*+1;\, t^*-1} = DD_{[1]}^{t^*+1;\, t^*} - DD_{[1]}^{t^*;t^*-1} = (\gamma^D_{t^*+1} - \gamma^D_{t^*}) - (\gamma^D_{t^*} - \gamma^D_{t^*-1}) = \gamma^D_{t^*+1} - 2\gamma^D_{t^*} + \gamma^D_{t^*-1} \quad [4.]$$

which is the difference between the observed t^*+1 outcome level handicap[9] $\gamma^D_{t^*+1}$ and its prediction $\gamma^D_{t^*} + DD_{[1]}^{t^*;t^*-1}$ given the handicap in t^* and its expected rise due to growth-rate difference between t^* and t^*-1. This prediction uses only regression coefficients driven by pre-treatment observations; a major difference with the trend-augmented method of eq.[1]. Note finally that the above logic can be generalized in many ways: to the case of lagged/dynamic treatment effects, or to $DD_{[p=q]}/Parallel_{[p=q]}$.where $q>2$ (Vandenberghe, 2016).

Figure 2 – How $DD_{[2]}$ copes with failure of $Parallel_{[1]}$

$$Y_{i,t} = \gamma + \gamma_t I_{t^*} + \gamma_{t+1} I_{t^*+1} + \gamma^D D_i + \gamma^D_{t^*} D_i . I_{t^*} + \gamma^D_{t^*+1} D_i . I_{t^*+1}$$



$ On this figure, t^*-1 is considered to be the first period observed in the data. Hence, $\gamma^D_{t^*-1}$ is subsumed into γ^D and, in contraxt with eq;[6], $DD_{[2]}$ is computed using only 2 coefficients.

3. Application to Hainaut data

To illustrate the properties of the eq. [2]-based generalized fully-flexible DD estimator, we use municipal data on taxable income per head. Time series are available for each of Belgium’s 589 municipalities, from 1988 to 2003, covering years before 1994 (start of EU policy) and after 2000 (end of the policy). The treated entities are the 69 municipalities of Hainaut (Table 1 & Figure 3).

9 Net of the initial handicap in t^*-1 : γ^D

The 520 other ones form the control group. All reported estimates are obtained using data that are weighted by municipal population sizes & deflated by 2010 consumer-price index.

Figure 3 – Hainaut vs rest of Belgium/

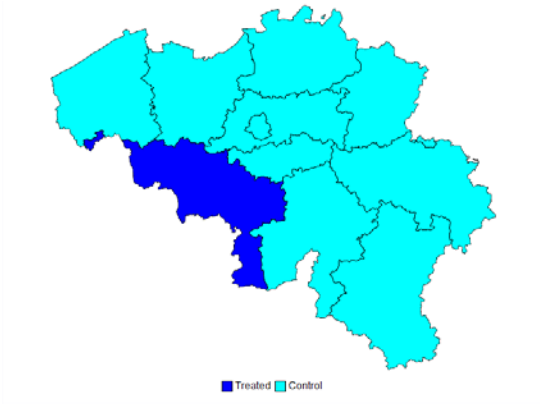


Table 1– Municipality count

Rest of Belgium	520
Hainaut	**69**
Total	589

Table 2 displays the results for the canonical $DD_{[1]}$/ $Parallel_{[1]}$. Year t^*=1993 is the most immediate year before the treatment, and t^*+s=2003 the moment the treatment is evaluated. Results confirm what was already visible on Figure 1. Compare to the rest of Belgium, the income handicap grew larger between 1993 and 2003 (-329.8€). But placebo $DD_{[1]}$ point at a rising income handicap prior to treatment. Thus $Parallel_{[1]}$ does no hold.

Table 2: $DD_{[1]}$ estimation + $DD_{[1]}$ placebo estimations

	$\boldsymbol{DD_{[1]}}$	**Placebo $\boldsymbol{DD_{[1]}}$**				
DD	-329.88*	-121.29***	-268.83***	-80.24***	-68.37*	-106.81***
prob DD=0$^{\$}$	*0.017*	*0.000*	*0.000*	*0.000*	*0.035*	*0.000*
Post-treat. year	2003	1989	1990	1991	1992	1993
Pre-treat. year	1993	1988	1989	1990	1991	1992
Nobs	21,832	21,832	21,832	21,832	21,832	21,832
R^2	0.92	0.92	0.92	0.92	0.92	0.92

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Thus, it is necessary to go beyond $Parallel_{[1]}$ to capture EU-Objective 1's true impact. Interestingly, as we possess many pre-treatment periods, we can implement both the traditional trend-corrected DD method and $DD_{[2]}$/ $Parallel_{[2]}$. Results are reported in Table 3. The last two columns correspond to year 2003 (t^*+10). As anticipated, the two estimators deliver treatment effect estimates that significantly diverge. Whereas the traditional linear-trend corrected method concludes to a negative impact (i.e. the income handicap rose by -238.5€), our preferred fully-flexible $DD_{[2]}$/ $Parallel_{[2]}$ method diplays a gain of 916.2€. This illustrates the striking differences induced by a method that only uses pre-treatment observations to account for trend differences, and also lift the constraint of outcome as one-in-a-time trend shift.

Table 3 - Linear trend-corrected DD vs $DD_{[2]}$

	2000		2003	
	Corrected $DD_{[1]}$	***$DD_{[2]}$***	**Corrected $DD_{[1]}$**	***$DD_{[2]}$***
DD	76.55	255.56***	-238.52***	961.19***
prob DD=0$	0.154	0.000	0.001	0.000
Post-treat. year	1997	1997	2003	2003
Pre-treat. year 1	1993	1993	1993	1993
Pre-treat. year 2	1988	1988	1988	1988
Nobs	5,890	21,832	9,421	21,832
R^2	0.96	0.92	0.95	0.92

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Finally, we assess the legitimacy of $Parallel_{[2]}$ by estimating placebo $DD_{[2]}$ using 3 pre-treatment years. Results (Table 4) are all supportive of $DD_{[2]}=0$, suggesting that $Parallel_{[2]}$ was a realistic description of the relative dynamics of Hainaut's income per head prior to EU-Objective 1, and that the positive $DD_{[2]}$ values in Table 3 properly identify the programme's causal impact.

Table 4- $DD_{[2]}$ 'placebo' estimation

	***$DD_{[2]}$* 1990**	***$DD_{[2]}$* 1991**	***$DD_{[2]}$* 1992**	***$DD_{[2]}$* 1993**
DD	-147.54	188.59	11.87	-38.43
prob DD=0$	0.593	0.448	0.959	0.843
Post-treat. year	1990	1991	1992	1993
Pre-treat. year 1	1989	1990	1991	1992
Pre-treat. year 2	1988	1989	1990	1991
Nobs	21,832	21,832	21,832	21,832
R^2	0.92	0.92	0.92	0.92

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

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