

Communication

On the Maximum Absorbed Power in Receiving Antenna Arrays

Ha Bui-Van^{1b}, Michel Arts, Christophe Craeye^{1b}, and Eloy de Lera Acedo^{1b}

Abstract—The loading condition to maximize the total received power in arbitrary antenna arrays is presented. It is found that the loads should equal the conjugate of the active impedance of the array in the transmitting case. A physical insight into the solution is provided. Numerical validation on a 2-D nonregular antenna array is provided.

Index Terms—Mutual coupling, optimum loads, received power, receiving antenna array.

I. INTRODUCTION

In receiving antenna arrays, the loads are often designed so as to maximize the power transferred to the loads from the incident wave. De Hoop [1] has shown that the optimum impedance matrix of the receiving network should be equal to the complex conjugate of the array impedance matrix in the transmit case. To design such a solution, one, however needs to realize a matrix of loads, where the loads are coupled from an electromagnetic point of view. In many systems, each antenna is connected to an isolated receiver. The antenna is thus connected directly to a single load rather than to a coupled receiving network. For such systems, a systematic approach to design/adjust the loads to maximize the received power is looked for.

de Lera Acedo *et al.* [2] have estimated that these optimal loads might link to the active impedance, as could be proven for the special case of infinite periodic arrays [3]. In this communication, we present the optimum condition for designing such loads. An analytical solution for isolated loads is derived from the case of coupled loads [1]; it requires the knowledge of the impedance matrix of the array and of the open-circuit voltages defined on receive. That solution can be applied to power transfer for both far-field and near-field sources. The link with the active impedance solution used in infinite periodic arrays is established and the steps need to put this solution into practice are described. Finally, the solution is validated by comparison with solutions from a global optimization on a 2-D array of complex antennas.

The remainder of this communication is organized as follows. In Section II, the theory on optimum loads is presented. The solution is derived and its physical meaning is discussed. Numerical validation is provided in Section III. Concluding remarks are made in Section IV.

II. THEORY ON MAXIMIZING RECEIVED POWER

Considering an array of N antennas, the array is characterized by the impedance matrix $\mathbf{Z}$. In the receiving case, the open-circuit

voltage at the port of antenna m for a given incident wave can be defined as follows [4, Sec. 4.2.1], [5, eqs. (49)–(51)]:

$$\mathbf{v}_m^{\text{o.c.}} = \frac{1}{I_m^T} \iint_{S_b} (\bar{\mathbf{E}}^I \times \bar{\mathbf{H}}_m^T - \bar{\mathbf{E}}_m^T \times \bar{\mathbf{H}}^I) \cdot \hat{n}_b dS \quad (1)$$

with S_b a surface enclosing the array, $\hat{n}_b$ is the (outward) unit vector normal to S_b , $(\bar{\mathbf{E}}^I, \bar{\mathbf{H}}^I)$ the electric and magnetic incident fields, and $(\bar{\mathbf{E}}_m^T, \bar{\mathbf{H}}_m^T)$ the electric and magnetic fields radiated by the array in the transmit case when the element m is excited by a current source I_m^T while the other elements are left open. Equation (1) is valid for both near- and far-field incident waves. For the far-field case, where the incident wave is a plane wave, the integration (1) becomes the scalar product of incident plane wave with the open-circuit pattern [1]. In this communication, we will derive the loading condition to maximize the received power, given the information of $\mathbf{v}^{\text{o.c.}}$, including the near-field case, i.e., when the fields incident on the array present a substantially curved phase front. The only condition for application to the near-field is that the physical presence of the receiving array does not significantly affect the incident fields themselves $(\bar{\mathbf{E}}^I, \bar{\mathbf{H}}^I)$, which means that currents on the source device are not altered (no significant multiple-scattering between source device and receiving array).

The array is then connected to a series of loads, described by the diagonal matrix $\mathbf{Z}^L$, and its Thevenin equivalent circuit is shown in Fig. 1(b) [1], [6]. Denoting $\mathbf{v}$ and $\mathbf{i}$ as the column vectors of voltages across the ports and of currents flowing through the ports, respectively, the following relations are found:

$$\mathbf{v}_m^{\text{o.c.}} = \mathbf{v}_m + \mathbf{u}_m^T \mathbf{Z}^T \mathbf{i} \quad (2)$$

where $\mathbf{u}_m$ is a $N \times 1$ vector of zeros, except for a unit value at the m th position, while $(\cdot)^T$ denotes the transpose operation. The total time-average power dissipated in the loads is obtained as

$$P^L = \frac{1}{2} \text{Re} \left(\sum_{m=1}^N \mathbf{v}_m \mathbf{i}_m^* \right) \quad (3)$$

where $(\cdot)^*$ denotes complex conjugation. Sections II-A and II-B derive the optimum values of the loads for maximum received power. It makes use of the result provided by De Hoop [1] regarding the optimal receiving network. It goes beyond that result in that, here, a series of isolated loads attached to the ports rather than a receiving network represented by an impedance matrix. This is made possible through the knowledge of the open-circuit voltages, whose general expression has been reminded in (1). Moreover, an interpretation is provided in terms of active impedances.

A. Existence of an Optimum

Following [1], to maximize the received power, the load matrix impedance of the receiving network is equal to $\mathbf{Z}^{T,*}$, i.e., the conjugate of the array impedance matrix in the transmit case. For this load matrix, the port voltages and currents are $\mathbf{v}$ and $\mathbf{i}$, respectively. We will now derive an equivalent series of loads $\mathbf{Z}_{\text{port}}^L$ attached to the ports that is able to produce the same vector current as the load

Manuscript received February 12, 2018; revised October 10, 2018; accepted December 2, 2018. Date of publication December 21, 2018; date of current version March 5, 2019. This work was supported in part by the European Union's Horizon 2020 Research and Innovation Programme under Grant Agreement No 736937, in part by the Science and Technology Facilities Council, U.K., (SKA) under Grant ST/M001393/1 and in part by the University of Cambridge, U.K. (Corresponding author: Ha Bui-Van.)

H. Bui-Van is with AGC Glass Europe, AGC Technovation Centre, 6041 Gosselies, Belgium (e-mail: buivanhabk@gmail.com).

M. Arts is with ASTRON, 7991 PD Dwingeloo, The Netherlands.

C. Craeye is with ICTEAM, Université catholique de Louvain, 1348 Ottignies-Louvain-la-Neuve, Belgium (e-mail: christophe.craeye@uclouvain.be).

E. de Lera Acedo is with the Cavendish Astrophysics Group, University of Cambridge, Cambridge CB2 1TN, U.K.

Color versions of one or more of the figures in this communication are available online at <http://ieeexplore.ieee.org>.

Digital Object Identifier 10.1109/TAP.2018.2889137

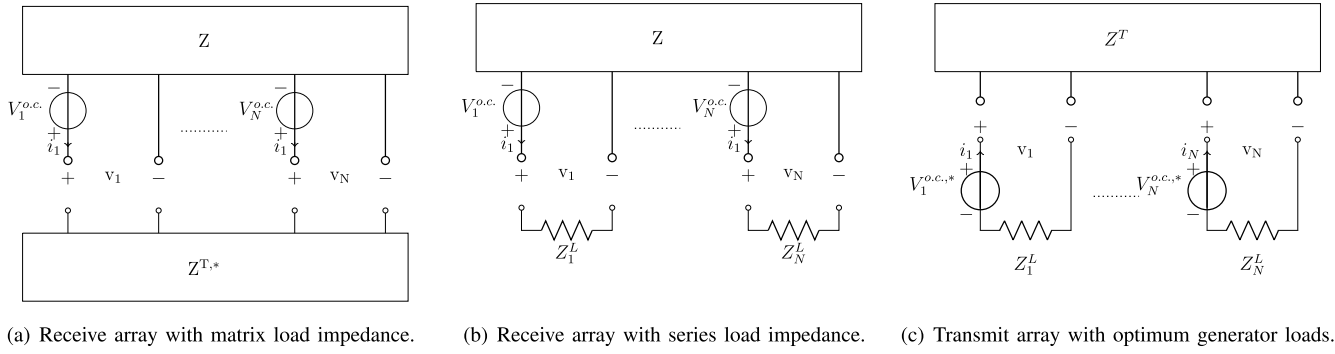


Fig. 1. Equivalent circuit of receiving and transmitting arrays. (a) Receive array with matrix load impedance. (b) Receive array with series load impedance. (c) Transmit array with optimum generator loads.

matrix. The voltage at port m must be the same for coupled and isolated loads, it can be expressed as

$$v_m = \sum_{n=1}^N Z_{m,n}^* i_n = Z_{port,m}^L i_m. \quad (4)$$

Dividing both sides by i_m , the load at the port is then obtained as

$$Z_{port,m}^L = \frac{\sum_{n=1}^N Z_{m,n}^* i_n}{i_m} = \frac{\mathbf{u}_m^T \mathbf{Z}^* \mathbf{i}}{\mathbf{u}_m^T \mathbf{i}}. \quad (5)$$

The isolated loads in (5) depend on the knowledge of the currents in the optimal coupled receiving network, which can be traced back to array impedance matrix and open-circuit voltages. Indeed, based on the solution from [1] [shown in Fig. 1(a)], we also have

$$\mathbf{i} = -(\mathbf{Z}^T + \mathbf{Z}^*)^{-1} \mathbf{v}^{o.c.}. \quad (6)$$

Hence, given the knowledge of the source, it is possible to find a set of loads producing the same port voltages as in the presence of the fully coupled optimal receiving network. Since the boundary condition at the ports level is directly given in terms of equivalent isolated load impedances, it also ensures that the currents are the same as in the solution proposed in [1]. This will allow us to also write the port current as

$$\mathbf{i} = -(\mathbf{Z}^T + \mathbf{Z}^L)^{-1} \mathbf{v}^{o.c.}. \quad (7)$$

Therefore, the maximum power absorbed by the isolated loads must also correspond to the maximum power obtained with a coupled receiver.

B. Determination of the Series Load Impedance

Now, the existence of the optimal series of load impedance is ensured, we establish an explicit expression for calculating these loads. From (5) and (6), the load impedance is obtained as

$$Z_{port,m}^L = \frac{\mathbf{u}_m^T \mathbf{Z}^* \mathbf{i}}{\mathbf{u}_m^T \mathbf{i}} = \frac{\mathbf{u}_m^T \mathbf{Z}^* (\mathbf{Z}^T + \mathbf{Z}^*)^{-1} \mathbf{v}^{o.c.}}{\mathbf{u}_m^T (\mathbf{Z}^T + \mathbf{Z}^*)^{-1} \mathbf{v}^{o.c.}}. \quad (8)$$

From (8), the values of these loads are directly obtained as soon as the open-circuit voltage is determined.

C. Interpretation of the Solution

It is known that for infinitely periodic arrays, the optimum impedance for maximum power absorption corresponds to the conjugate of the active impedance. In this way, arrays able to absorb all the power incident from a plane wave have been designed. The active impedance corresponds to the ratio between voltage and current

in the infinite periodic array when, in a transmitting configuration, it is scanned in the direction of interest, which supposed that all elements are transmitting (from there the name). For the general array referred to earlier, one may also define active impedances. They are obtained through a reinterpretation of the equivalent circuit of the receiving array and its termination as that of a transmitting array, shown in Fig. 1(c). In this case, what used to correspond to open-circuit voltages of the receiving arrays now, after conjugation, corresponds to the open-circuit voltages of the different generators. The use of voltages equal to the conjugates of the open-circuit voltages on receive ensures scanning in the direction of the incident wave, or time reversal for the more general case of near-field sources (see the theory and implementation of focusing through time reversal [8], [9]). Moreover, the generator loads are designed as Z^H (with H being the Hermitian operator) as it would allow maximum power deliver to the antenna (see [1, Appendix]).

From Fig. 1(c), the active impedance with $\mathbf{v}^{o.c.,*}$ voltage source is written as

$$Z_m^{\text{act}} = \frac{\mathbf{u}_m^T \mathbf{Z} (\mathbf{Z} + \mathbf{Z}^H)^{-1} \mathbf{v}^{o.c.,*}}{\mathbf{u}_m^T (\mathbf{Z} + \mathbf{Z}^H)^{-1} \mathbf{v}^{o.c.,*}}. \quad (9)$$

From (8) and (9), it is possible to show that

$$Z_m^{L,o} = Z_m^{\text{act},*}. \quad (10)$$

This configuration is also consistent with the classical solution referred to above for infinite periodic arrays.

One can also replace the coupled generator loads by isolated loads using the expression as follows:

$$Z_m^{\text{trans}} = \frac{\mathbf{u}_m^T \mathbf{Z}^H \mathbf{I}}{\mathbf{u}_m^T \mathbf{I}} = \frac{\mathbf{u}_m^T \mathbf{Z}^H (\mathbf{Z} + \mathbf{Z}^H)^{-1} \mathbf{v}^{o.c.,*}}{\mathbf{u}_m^T (\mathbf{Z} + \mathbf{Z}^H)^{-1} \mathbf{v}^{o.c.,*}}. \quad (11)$$

D. Implementation

In practice, to make this solution possible, one needs a method for determining the open-circuit voltages and a means to tune the impedance of each independent receiver. The former may be achieved through numerical simulation, while in a narrowband context, a different matching circuit may need to be introduced between the antennas and each receiver. The design of such a circuit is outside the scope of this communication.

III. VALIDATION

The analytical solution proposed in Section II is now applied to maximize the power received of an array of 16 Square Kilometer Array (SKA) log-periodic antennas (SKALAs), which have been proposed for the SKA low-frequency project [10]. In the following

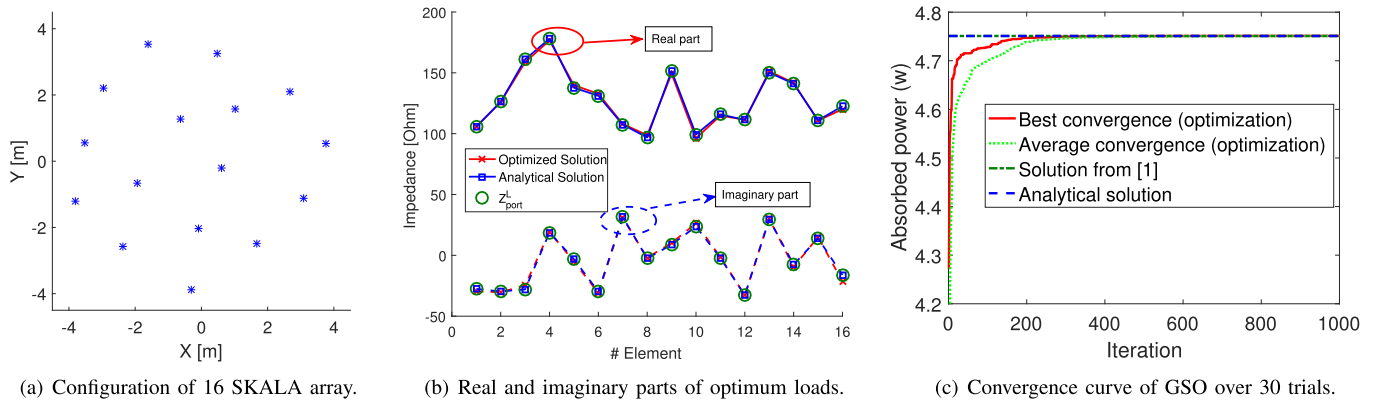


Fig. 2. Optimization of loads to maximize the received power of a 16-element array for the incident plane wave from direction $(\theta_0, \phi_0) = (90^\circ, 30^\circ)$. (a) Configuration of 16 SKALA array. (b) Real and imaginary parts of optimum loads. (c) Convergence curve of GSO over 30 trials.

example, one of the two polarizations (along x) of each antenna is excited. The incident plane wave is assumed to be coming from the $(\theta_0, \phi_0) = (90^\circ, 30^\circ)$ direction and to have the same polarization as the copolarization of the array (θ and ϕ are elevation and azimuth angles, respectively). To validate the result described in Section II-B, a global optimization is applied to optimize the loads. For the optimization, the genetical swarm optimization (GSO) technique [11] is implemented. The fitness function maximizes the power defined in (3) by controlling the imaginary and real parts of the loads. As for GSO's parameters, a population of 100 is set for the optimization, which is shared equally for genetic algorithm and particle swarm optimization kernels in the GSO. In this problem with infinite plane wave excitation, another validation considers the solution of [1] using a load matrix. For the analytical approach, series loads are directly obtained from (8) for given incident wave. To fully account for mutual coupling, the simulation technique HARP proposed in [12] is exploited to simulate the array and calculate all the open-circuit patterns. The array is analyzed at 110 MHz.

Fig. 2(a) shows the configuration of the array, while the optimized loads are plotted in Fig. 2(b). The loads optimized using GSO almost overlap those from the analytical solution. A maximum difference of 3Ω is observed, which leads to a slightly lower absorbed power using the solution of GSO, i.e., a relative deviation in terms of power smaller than 0.001 with respect to the analytical solution. Due to the stochastic nature of the GSO, in different trials, GSO has converged to different values, which are extremely close. These values are smaller than the one obtained using analytical solution. Fig. 2(c) plots the convergence of GSO over 30 independent trials, where the GSO converged to the optimum value of the analytical solution, which is the same as the case of load matrix [(see Fig. 2(c))]. This example validates the solution proposed in this communication.

IV. CONCLUSION

We have presented the optimum condition for the loads in receiving arrays. It corresponds to the conjugate of the active impedance of the array scanned in the direction of incidence. This generalizes the solution proposed in [3] to nonregular (and actually arbitrary) arrays. Moreover, we also linked the proposed solution to the one given in [1] for a fully coupled receiver. We showed that those two optimum conditions coincide in terms of absorbed power, with the essential

difference that the solution proposed for the isolated loads depends on the incident wave, through the open-circuit voltages. The obtained condition is validated on a 2-D array of complex antennas, where the solution of a global optimization converges to the proposed one.

ACKNOWLEDGMENT

The authors would like to thank Dr. M. Mussetta at the Politecnico di Milan, Milan, Italy, for helping with the GSO implementation.

REFERENCES

- [1] A. T. De Hoop, "The N -port receiving antenna and its equivalent electrical network," *Philips Res. Rep.*, vol. 30, pp. 302–315, 1975.
- [2] E. de Lera Acedo, M. Arts, C. Craeye, and H. Bui-Van, "On the effects of mutual coupling in the active reflection coefficient of wide-field scanning electrically large random phased arrays," in *Proc. Int. Conf. Electromagn. Adv. Appl. (ICEAA)*, Cairns, QLD, Australia, Sep. 2016, pp. 683–685.
- [3] C. Craeye and M. Arts, "On the receiving cross section of an antenna in infinite linear and planar arrays," *Radio Sci.*, vol. 39, no. 2, pp. 1–8, 2004.
- [4] G. Dolmans, "Effect of indoor fading on the performance of an adaptive antenna system," Ph.D. dissertation, Dept. Elect. Eng., Eindhoven Univ. Technol., Eindhoven, The Netherlands, 1997.
- [5] A. T. De Hoop and G. De Jong, "Power reciprocity in antenna theory," *Proc. Inst. Elect. Eng.*, vol. 121, no. 10, pp. 1051–1056, Oct. 1974.
- [6] J. W. Wallace and R. Mehmood, "On the accuracy of equivalent circuit models for multi-antenna systems," *IEEE Trans. Antennas Propag.*, vol. 60, no. 2, pp. 540–547, Feb. 2012.
- [7] C. Craeye, B. Parvais, and X. Dardenne, "MoM simulation of signal-to-noise patterns in infinite and finite receiving antenna arrays," *IEEE Trans. Antennas Propag.*, vol. 52, no. 12, pp. 3245–3256, Dec. 2004.
- [8] M. Fink and C. Prada, "Ultrasonic focusing with time reversal mirrors," in *Advances in Acoustic Microscopy*, A. Briggs and W. Arnold, Eds. Boston, MA, USA: Springer, 1996.
- [9] M. Ettore, W. A. Alomar, and A. Grbic, "Radiative wireless power-transfer system using wideband, wide-angle slot arrays," *IEEE Trans. Antennas Propag.*, vol. 65, no. 6, pp. 2975–2982, Jun. 2017.
- [10] E. de Lera Acedo, N. Razavi-Ghods, N. Troop, N. Drought, and A. J. Faulkner, "SKALA, a log-periodic array antenna for the SKA-low instrument: Design, simulations, tests and system considerations," *Exp. Astron.*, vol. 39, no. 3, pp. 567–594, 2015.
- [11] F. Grimaccia, M. Mussetta, and R. E. Zich, "Genetical swarm optimization: Self-adaptive hybrid evolutionary algorithm for electromagnetics," *IEEE Trans. Antennas Propag.*, vol. 55, no. 3, pp. 781–785, Mar. 2007.
- [12] H. Bui-Van *et al.*, "Fast and accurate simulation technique for large irregular arrays," *IEEE Trans. Antennas Propag.*, vol. 66, no. 4, pp. 1805–1817, Apr. 2018.