Persistent Inequality through Schooling: the Role of Limited School Capacity.

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Abstract

Children of educated parents systematically perform better at school than children of uneducated parents. It is then natural, if the location of families in the city follows a socially stratified pattern, to observe differences in public school's performances even when they are identically financed. Free school choice is not enough to change this outcome. Capacity cannot be changed overnight and schools facing an excess demand may be forced to turn applications down. If the aim of the school is to maximize expected achievement of children, those from a less favored environment will be rejected first and segregation will increase. In order to overcome this negative result we propose a system of grants to disadvantaged children as a means to finance public schools.

Key words: Educational Finance.

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1 Introduction

Many are of the opinion that the task of public schools should be to provide equal educational opportunity to students of heterogeneous talent and background (Roemer [92]). In reality, schools systematically fail to attain this objective. The aim of this paper is to account for the role of limited school capacity in explaining the observed persistent inequality in public school's performances.

Because the students are different, average school performances are most likely to differ. Moreover, a small initial difference between schools' average performances can set off a series of reactions in demand and supply that, feeding back into each other, may result in maximum differentiation. In particular, demand will move attracted by the better performing school. In turn, schools confronted to an excess demand may be able to select the best students among their applicants. As a consequence, the difference between school's average performances will grow deeper and demand will increasingly address the better school, which will as a result be able to become more selective. We will study the limits to this chain reaction.

Previous literature has attributed the observed systematic differences in school's attainments to the fact that public education may be financed at the local level. Therefore, richer districts benefit from larger amounts of funds (Bénabou [93] and [94], Fernandez and Rogerson [96]). However, and although the subject remains controversial, no relation between attainment and per-pupil expenditure has been systematically found. [1]

Instead, the socioeconomic profile of the family and, above all, the cultural background have of late been increasingly recognized as the main determinants of achievement of children at school. A statistically significant advantage of 4 to 19 % of the grade has been found in Science and Mathematics exams of children whose fathers hold a university degree as compared to children whose fathers have minimum education in 18 OECD countries or regions.[2] If, as it has long been accepted, the location of families in the city follows a socially stratified pattern (Hartwick *et al.* [76]), differences in average school performances may be explained by the fact that public schools assemble

[1] See Vandenberghe (96) for an insightful discussion of this literature.

[2] TIMSS (1995), Third International Mathematics and Science Study& IEA, Amsterdam, the Netherlands. See also (Vandenberghe et al (2000)).

children living in the same neighborhood,[3] i.e. with similar socioeconomic profiles.

Suppose that we decide to facilitate the free choice of school across neighborhoods in order to change this pattern. All parents will prefer the better school. Since capacity cannot be changed overnight, schools facing an excess demand will be forced to turn some applications down. Schools can generally observe children characteristics and backgrounds upon application of their parents. If they aim at maximizing the average performance of their pupils, each school will reject the children with lower expected performance among those who applied. Therefore, the greater the demand for one specific school by highly educated parents (i.e., the larger the difference in performances), the lower the number of children of less educated parents that will effectively be able to access this school if its capacity is limited. Moreover, in this case, free school choice will provide the better of the schools with a greater power of selection: it will be *the school* and not the parents, who will exert the final choice.

In this paper, we consider two identical schools with limited capacity and a continuum of parents that can freely choose school for their children. Parents, who differ in skills (or education), are unevenly distributed in the city. Achievement of children at school is positively related to their parents' education level. For this reason, schools prefer to admit children of educated parents in order to maximize their average performance. In this framework, we show that higher mobility results in higher segregation when schools' capacities are limited. Then, we show that the government can induce a greater mixing of children types within schools by means of a grant to disadvantaged students directly provided to the school where they enroll. A high degree of mobility will be a necessary condition for the relative success of the policy. Finally, we point out that, if schools passively accept all applications (that is, with unlimited school capacities) supporting free school choice can be an equalizing measure, provided that all parent types react identically to the policy.

The paper is organized as follows: Section 2 presents the model and describes the interaction of the agents. Sections 3 and 4 analyze, respectively, the optimal choices

[3] Indeed, evidence for a certain degree of continuity in the concentration of (richer) highly educated families as we cross cities can be found everywhere. Roughly speaking, the concentration of higher income families is higher as we move towards the West of London, the South-West of Paris or the North of Madrid.

of schools and parents. Section 5 provides and discusses all possible equilibria under limited capacity, with and without specific grants to disadvantaged students. Section 6 shows how, if schools accept all the applications, the chance that a disadvantaged student may attend the better school can in fact increase, as this school becomes larger in size. Finally, section 7 concludes.

2 The Model

There is a continuum of $P = 1$ parents in this economy. One half of them are high skilled or educated ($j = H$) and the other half are low skilled ($j = L$). Each parent has one child that attends some school i. The achievement of this child, s_i^j, is a function of the inputs provided by both the school (investment per pupil) and the parent (home education). Children leave school i to pursue higher education or work. In any case the exact value of their achievement at school is not observed by future professors or employers. Instead, the latter will know the value of the average performance of children at the school they attended (S_i, $i = 0, 1$) and will use this information. In this sense, children can be assumed to leave school endowed with S_i. This assumption allows to account for the role of social networks, which through peer effects, role models, job contacts or norms of behavior will have an important effect on opportunities in adult life (Bénabou [96]).

Families are located along the line $0 - 1$ and distributed in a way such that, when moving towards the east, the proportion of highly educated parents increases at rate 1. Then, at location x, there are $x\,(\%)$ families of type H and $(1 - x)\,(\%)$ families of type L. Each family is thus characterized by $j = H, L$ and x.

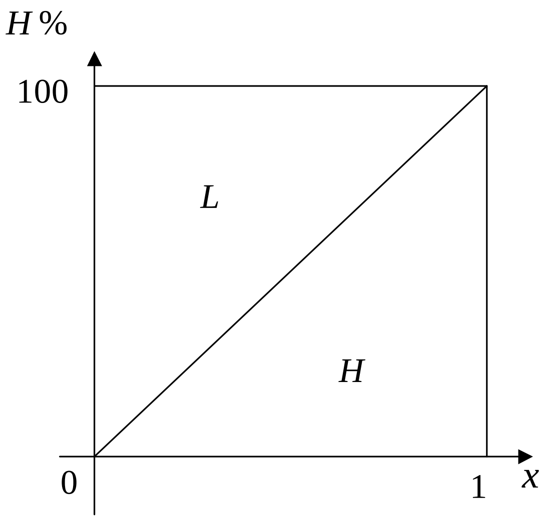

Figure 1: Distribution of Families.

We consider two schools $i = 0, 1$ placed at $x = 0$ and $x = 1$ respectively.[4] The performance of children of j-parents attending school i is

$$s_i^j = \gamma^j k_i \ \ (i = 0, 1 \ \ j = H, L)$$

where k_i is per-student investment at school i . In order to reflect the effect of home education or the family background on achievement at school we assume

$$\gamma^H > \gamma^L > 0$$

Let H_i and L_i be the number of children of H and L parents respectively enrolled at school i. The capacity of each school is identical and limited to one half of the market, so that $H_i + L_i = 1/2$ at both schools. Hence, the average productivity obtained by children at school i is

$$S_i = 2(H_i\gamma^H + L_i\gamma^L)k_i \tag{1}$$

Schools care for this average performance of their pupils, that provides them with recognition and prestige.

Parents choose school for their children. In order to do so, they anticipate and compare the average performances of pupils at each school, information that will provide their children with a signal in the market for higher education and/or labor.

[4] It can be shown that both letting schools be closer to each other and allowing for a greater segregation of families in the city will only exacerbate the results in section 5.

We make the simplifying assumption that children's innate abilities are identical. Thus the only difference in performances is due to the different social origin of parents and per-pupil investment by the school. Utility derived by a j-parent located at x when sending her child to school 0 and to school 1 is respectively:

$$v^j(S_0, x) = S_0 - c_j\, x \tag{2}$$

$$v^j(S_1, x) = S_1 - c_j(1 - x) \tag{3}$$

where c_j stands for mobility costs of j-parents.

If $\hat{x}_j$ is the location of the j-parent who is indifferent among schools, then

$$\hat{x}_H = \frac{1}{2} + \frac{S_0 - S_1}{2c_H} \tag{4}$$

$$\hat{x}_L = \frac{1}{2} + \frac{S_0 - S_1}{2c_L} \tag{5}$$

and the number of applicants of each type at each of the schools is represented in figure 2 and given by

$$D_0^H = \frac{\hat{x}_H^2}{2} \tag{6}$$

$$D_1^H = \frac{1}{2} - \frac{\hat{x}_H^2}{2} \tag{7}$$

$$D_0^L = \frac{1}{2} - \frac{(1 - \hat{x}_L)^2}{2} \tag{8}$$

$$D_1^L = \frac{(1 - \hat{x}_L)^2}{2} \tag{9}$$

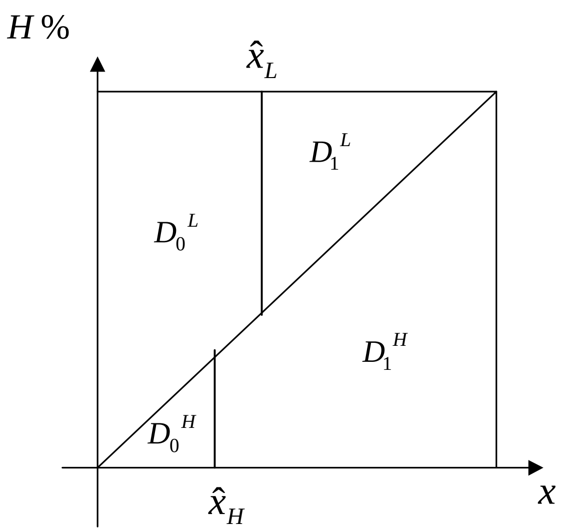

Figure 2: Demand by H and L parents.

Total demand for School 0 is then:

$$D_0 = \frac{1}{2} - \frac{(1-\hat{x}_L)^2}{2} + \frac{(\hat{x}_H)^2}{2} \tag{10}$$

and total demand for school 1

$$D_1 = 1 - D_0 = \frac{1}{2} + \frac{(1-\hat{x}_L)^2}{2} - \frac{(\hat{x}_H)^2}{2} \tag{11}$$

Then, since $\hat{x}_H, \hat{x}_L \in [0,1]$ we can write

$$D_1 \geq D_0 \Longleftrightarrow (1-\hat{x}_L) \geq \hat{x}_H \Longleftrightarrow \hat{x}_H + \hat{x}_L \leq 1$$

The government provides the funds in the form of a per-student voucher that, given that school capacity is given, is equivalent to a lump-sum amount F. As there are disadvantages associated to the education of children of L-parents, an additional grant ϕ may be provided per L-child enrolled in order to induce schools to admit these children and support their education. Then, per-student investment at school i is

$$k_i = \frac{F + \phi L_i}{H_i + L_i} = 2(F + \phi L_i) \tag{12}$$

since $H_i + L_i = 1/2$.

Each period the interaction between schools and parents takes place in two stages. First, parents apply to schools. Since capacity is limited, the school facing the largest demand selects (among applicants) the children it wants to keep. The other school is forced to take the remaining children. Schools observe the type of parents, information that they use to decide on admissions.

Before analyzing in detail the choices of schools and parents, let us define two limiting patterns of children repartition among schools that will be used as points of reference in the rest of the paper.

No-choice benchmark: If parents cannot choose school for their children, but are assigned to the school closest to their homes (i.e. $\hat{x} = 1/2$) it is straightforward to show that the distribution of children types within schools will be such that $L_0 = 3/8$ and $L_1 = 1/8$. School 1 will have a higher average performance than school 0. As discussed in the introduction, this is the type of situation that motivated the first voucher proposals.

Egalitarian benchmark: If children are equally distributed among schools, $L_0 = L_1 = 1/4$, average school performances will be identical. Then $S_0 = S_1$. In our setting, this is the ultimate objective of the government.

3 Choice of Student Mix by Schools

In this model, due to the fixed capacity assumption, the school facing the larger overall demand determines through its choice of H or L the mix of students in both schools. We arbitrarily choose L_i as the variable of reference. If $D_0 = D_1 = 1/2$, demand equals capacity in both schools and no choice is available to any school. Otherwise, the school with largest demand, henceforth the leading school, chooses who to keep and who to reject up to the point in which full capacity is met.[5] The other school has to accommodate, taking the rest of the children in the market.

At this stage demand D_i is given. If school i is the leading school ($D_i > 1/2$) it chooses L_i that maximizes

$$S_i = 4\left(\left(\frac{1}{2} - L_i\right)\gamma^H + L_i\gamma^L\right)(F + \phi L_i) \tag{13}$$

The choice of L_i is constrained by the type of demand addressed to the leading school.

Consider first the situation where school 0 is the leading school. Admissions of H-children are limited by D_0^H. From (6)[6]

$$H_0^* \leq \frac{(\hat{x}_H)^2}{2}$$

which, since $H_i + L_i = 1/2$, can be written

$$L_0^* \geq \frac{1}{2} - \frac{(\hat{x}_H)^2}{2} \tag{14}$$

[5] As a result, some families may end up suffering mobility costs that are higher than what they would be willing to pay. This families would in this case prefer not to send their children to school, but will be forced to by law.

[6] Throughout the paper, * stands for optimal choice.

On the other hand, admission of L-children are bounded above by L-parents' demand for school 0, D_0^L. From (8)

$$L_0^* \leq \frac{1}{2} - \frac{(1-\hat{x}_L)^2}{2} \tag{15}$$

Likewise, if school 1 is the leading school, we have from (7)

$$H_1^* \leq \frac{1}{2} - \frac{(\hat{x}_H)^2}{2}$$

that we can write

$$L_1^* \geq \frac{(\hat{x}_H)^2}{2} \tag{16}$$

and, from (9)

$$L_1^* \leq \frac{(1-\hat{x}_L)^2}{2} \tag{17}$$

From the maximization of (13) subject to (14) and (15) or (16) and (17) respectively we obtain:

$$\begin{array}{rcl}
(\gamma^L - \gamma^H)(F + \phi L_i) + (H_i\gamma^H + L_i\gamma^L)\phi & < & 0 \text{ if lower limit on } L_i \text{ binding} \\
(\gamma^L - \gamma^H)(F + \phi L_i) + (H_i\gamma^H + L_i\gamma^L)\phi & > & 0 \text{ if upper limit on } L_i \text{ binding} \\
(\gamma^L - \gamma^H)(F + \phi L_i) + (H_i\gamma^H + L_i\gamma^L)\phi & = & 0 \text{ at the interior solution}
\end{array}$$

If the *lower* limit on L_i is binding, school i will decide to keep the fewest possible L-children. That is to say, the admission of one more child of type L always decreases the payoff of the leading school. If existing, per L student allocations ϕ are insufficient to compensate the differences in performance of the children due to the dissimilar familiar backgrounds. Therefore, when ϕ is small relative to $(\gamma^H - \gamma^L)$, schools will accept all the applications by H-parents and use admissions of L-children only to fill capacity. In other words, *the school facing an excess demand will reject the applications of as many disadvantaged children as possible, unless it is sufficiently compensated for not doing so.*

If the *upper* limit on L_i is binding, L_i will be set at the maximum level available to the leading school. In other words, the admission of one more L-child always pays off better. This is due to the fact that ϕ is large as compared to the difference in performances of L with respect to H children. Then, the school with more L-children will enjoy larger funds and will be able to perform better in spite of its *lower quality inputs.*

Finally, at the interior solution L_i^* will be optimally chosen by the leading school at

$$L_i^* = \frac{2(\gamma^L - \gamma^H)F + \gamma^H \phi}{4(\gamma^H - \gamma^L)\phi} \tag{18}$$

In general, the admission of one additional L-child reduces the average performance of the school due to her own lower performance. However, the "production" possibilities of the school increase due to the higher funds associated to the admission of L through ϕ. At the interior solution these two effects must counteract each other.

Note that since the objective function is the same, the optimal admissions level is identical in both schools. However, due to the uneven distribution of families in the city, *the constraints are not* (see (14) to (17)) and for this reason the maximization programs are not the same. In other words, although optimality is achieved at the same point by both schools, the conditions under which each of the schools will be able to attain the maximum will differ.

Consider now a marginal increase in the grant to disadvantaged students ϕ. In order for the planner's budget for schools $B = 2F + (L_0 + L_1)\phi$ to remain balanced (i.e. $dB = 0$) we need marginal increases in ϕ, $d\phi$, to be accompanied by

$$dF = -\frac{1}{4}d\phi$$

Budget balanced rises in ϕ then have the following effect on admissions of L-children by schools:

$$\left(\frac{dL_i^*}{d\phi} - \frac{1}{4}\frac{dL_i^*}{dF}\right)d\phi = \frac{F}{2\phi^2} + \frac{1}{8\phi} = \frac{4F+\phi}{8\phi^2} > 0$$

We can thus conclude that L_i^* is monotonically increasing in budget-balanced increases of ϕ. When ϕ small, the lower limit on L_i is binding and school i maximizes its payoff at the lower possible admissions of L. When ϕ is large, the upper limit on L_i is binding and school i maximizes its payoff at the highest possible admissions of L.

Our problem may seem trivial at this point: given demand and the budget, it suffices to choose F and ϕ in a way such that, from (18), the Egalitarian Benchmark is attained. However, we need to verify that such a distribution of pupils among schools is compatible with their demand. Next section studies the choice of schools by parents. It will make clear that selective admission policies by schools may not be the only responsible for segregation.

4 Choice of School by Parents

At stage 1 demand results from the anticipation of school performances by parents. If demand exactly equals capacity in both schools the mix of students will be determined by parental choice. Otherwise, the leading school will reject some of the applicants and become the ultimate responsible for the mix of children types within schools.

Recall that

$$D_1 = 1 - D_0 = \frac{1}{2} + \frac{(1-\hat{x}_L)^2}{2} - \frac{(\hat{x}_H)^2}{2}$$

where $\hat{x}_H$, $\hat{x}_L$ are defined by (4) and (5).

We assume that $c_H = c_L = c$. Therefore, $\hat{x}_H = \hat{x}_L = \hat{x}$. Without changing the main results, this assumption will simplify the exposition and, above all, allow to make comparisons among equilibria with and without the grant to disadvantaged students. Since

$$S_0 - S_1 = \left(2F(\gamma^H - \gamma^L) - \phi\gamma^L\right) 2(L_1 - L_0)$$

then,

$$\hat{x} = \frac{1}{2} + \frac{S_0 - S_1}{2c} = \frac{1}{2} + \frac{\left(2F(\gamma^H - \gamma^L) - \phi\gamma^L\right)}{c}(L_1 - L_0) \tag{19}$$

It is now straightforward to verify that if $L_0^* = L_1^* = 1/4$, demand is such that $\hat{x} = 1/2$ and, hence, schools do not have any choice power. Children are distributed as in the *No-Choice Benchmark* ($L_0 = 3/8$ and $L_1 = 1/8$). Therefore, the *Egalitarian Benchmark* can never be attained. Moreover, the distribution of children according to the *No-Choice Benchmark* generates differences in average attainment that, provided that the choice of school is free, will imply differences in demand. Therefore, unless mobility costs are large enough, this benchmark will not be an equilibrium solution. Let us, however, conclude the analysis of demand before deriving the equilibria of this game between schools and parents.

The factor $\big(2F(\gamma^H - \gamma^L) - \phi\gamma^L\big)$ in (19) *transforms* differences in children mix within schools $(L_1 - L_0)$ into differences in school performances $S_0 - S_1$. The sign of this transformation critically depends on the size of the grant ϕ. If $2F(\gamma^H - \gamma^L) > \phi\gamma^L$, the school with a higher proportion of H-type children always performs better (since $L_1 - L_0 = H_0 - H_1$ due to limited school capacity). Then, parents, in their search for school quality, will try to avoid schools with higher proportions of L children. If $2F(\gamma^H - \gamma^L) < \phi\gamma^L$ the specific grant ϕ is large enough to more than compensate the disadvantages associated to the lower cultural background of L-type children in the average performance of the school. In this case, the better performing school will be that with more disadvantaged children (and more funds). Consequently, parents will try to avoid schools with higher proportions of H children.

Most evidence on educational production functions tends however to undermine the later case. Indeed, educational attainment seems essentially correlated to each child's socio-economic background. As it was explained in the introduction, although the subject is still controversial, no systematic relation between attainment and per-pupil expenditure has been found. For the sake of generality, we will refer this analysis to all possible values of ϕ. Still, the reader shall keep in mind the existing evidence and take with caution the results concerning "high" levels of ϕ which imply that disadvantaged students are preferred by schools.

On behalf of tractability, let us define

$$C \equiv \frac{c}{2F(\gamma^H - \gamma^L) - \phi\gamma^L} \tag{20}$$

A lower $|C|$ implies a higher responsiveness of parents to differences in the mix of children types within schools (or anticipated difference in performances). Hence, C represents some sort of *weighted mobility cost* taking account for both unit transport costs c and school differences in average performance due to the socioeconomic profile of the pupils. We can then write

$$\hat{x} = \frac{1}{2} + \frac{S_0 - S_1}{2c} = \frac{1}{2} + \frac{(L_1 - L_0)}{C} \tag{21}$$

Note that, for an interior $\hat{x}$ it must be the case that, if $C > 0$,

$$\frac{1}{2} + \frac{(L_1 - L_0)}{C} \geq 0 \Leftrightarrow \frac{C}{4} + L_1 \geq \frac{1}{4} \Leftrightarrow L_1 \geq \frac{1}{4} - \frac{C}{4} \tag{22}$$

$$\frac{1}{2} + \frac{(L_1 - L_0)}{C} \leq 1 \Leftrightarrow L_1 - \frac{1}{4} \leq \frac{C}{4} \Leftrightarrow L_1 \leq \frac{1}{4} + \frac{C}{4} \tag{23}$$

and, if $C < 0$,

$$\frac{1}{2} + \frac{(L_1 - L_0)}{C} \geq 0 \Leftrightarrow \frac{C}{4} + L_1 \leq \frac{1}{4} \Leftrightarrow L_1 \leq \frac{1}{4} - \frac{C}{4} \tag{24}$$

$$\frac{1}{2} + \frac{(L_1 - L_0)}{C} \leq 1 \Leftrightarrow L_1 - \frac{1}{4} \geq \frac{C}{4} \Leftrightarrow L_1 \geq \frac{1}{4} + \frac{C}{4} \tag{25}$$

If the difference in L enrollments at each school is "too large" ((22) and (23) or (24) and (25) not satisfied respectively) demand will react "shooting" to a corner solution (zero or one). In other words, the large difference in student mix within the two schools will make it worthwhile for all parents to incur the mobility cost associated to taking their children to the school with the larger average performance. Thus, *all* demand will turn to this one school.

We can then define, for $C > 0$

$$\hat{x} = \begin{cases} 1 & \text{if} \quad L_1 > \frac{1}{4} + \frac{C}{4} \\ \frac{1}{2} + \frac{L_1 - L_0}{C} & \text{if} \quad \frac{1}{4} - \frac{C}{4} \leq L_1 \leq \frac{1}{4} + \frac{C}{4} \\ 0 & \text{if} \quad L_1 < \frac{1}{4} - \frac{C}{4} \end{cases} \tag{26}$$

and, for $C < 0$

$$\hat{x} = \begin{cases} 1 & \text{if} \quad L_1 < \frac{1}{4} - \frac{C}{4} \\ \frac{1}{2} + \frac{L_1 - L_0}{C} & \text{if} \quad \frac{1}{4} - \frac{C}{4} \geq L_1 \geq \frac{1}{4} + \frac{C}{4} \\ 0 & \text{if} \quad L_1 > \frac{1}{4} - \frac{C}{4} \end{cases} \tag{27}$$

5 Equilibrium

At equilibrium, optimal admissions must be compatible with effective demands. Firstly we study the two limiting cases in which either the lower or the upper limit on L are binding in the leading school (it prefers only H-type or only L-type children as pupils). Then we will discuss the interior solution. In order to determine the equilibrium mix of children within schools in each case we proceed as follows: first we look for a fixed point such that the choice of schools corresponds to available demand and verify whether the solution is interior; then, we check for stability. Corner solutions are independently considered.

5.1 Equilibrium without the Specific Grant

A particular and interesting case in which the lower limit on L_i is always binding is that in which schools are financed by means of a lump-sum transfer alone.[7] We then consider in this subsection equilibrium without specific grants.

When the lower limit on L is binding, schools admit a minimum of children of L-type parents. Then, since H-parents are able to register their children at the school of their choice, their demand alone is responsible for the equilibrium mix of children types.

From (14), (16) and (21), given that $L_0 + L_1 = 1/2$, we can write the following equilibrium conditions for the case in which the lower limit on L_i is binding

$$L_0 \geq \frac{1}{2} - \frac{1}{2}\left(\frac{1}{2} + \frac{1}{2C} - \frac{2L_0}{C}\right)^2 \tag{28}$$

$$L_1 \geq \frac{1}{2}\left(\frac{1}{2} + \frac{2L_1}{C} - \frac{1}{2C}\right)^2 \tag{29}$$

where, since $\phi = 0$, $C = c/2F(\gamma^H - \gamma^L) > 0$.

Once we account for the fact that $L_0 = 1/2 - L_1$ it is clear that (28) and (29) are the same condition: independently of which school has the leading position, the equilibrium condition is the same. We refer the analysis to school 1. Since demand for school 1 will be larger when it enrolls less L-children, $L_1 < 1/4$ will guarantee

[7] At the end of this section it will appear clear to the reader that $\phi < 2(\gamma^H - \gamma^L)F/\gamma^H$ is enough to make this constraint binding.

that school 1 is the *leading* school.

We find two values of $L_1 \in [0, \frac{1}{2}]$ that satisfy (29) with equality:

$$L_1^A = \frac{1}{4}\left(1 + C^2 - C - C\sqrt{2 - 2C + C^2}\right) \tag{30}$$

$$L_1^B = \frac{1}{4}\left(1 + C^2 - C + C\sqrt{2 - 2C + C^2}\right) \tag{31}$$

Figure 3 below depicts the roots of (29) together with the limiting values of L_1 that, for a given C, support an interior $\hat{x}$ according to (26). We use this figure to explain what will be shown formally in propositions 1 and 2.

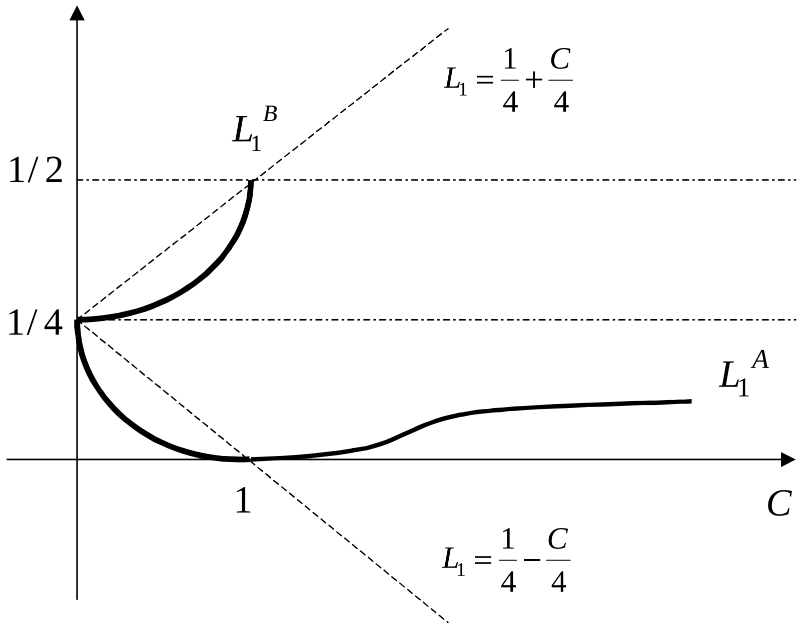

Figure 3

Note first that L_1^A does not satisfy the conditions for interior demand (22) when $c \leq 2F(\gamma^H - \gamma^L)$ (or $C < 1$): the differences in admissions and hence in performances are too large relative to mobility costs to sustain a distribution of demand among the two schools that allows for this equilibrium mix. If L_1^A is chosen by school 1 when $C < 1$ it will attract all demand. Then, since the lower limit on L_1 is binding, admissions of L_1 will be minimum, i.e. zero (and not L_1^A).

We then investigate whether corner solutions for demand $\hat{x} = 0$ ($L_1 = 0$) and $\hat{x} = 1$ ($L_1 = 1/2$) may take place at equilibrium. Proposition 1 shows that this is indeed the case.

By way of contrast, L_1^B corresponds to differences in admissions by the schools that are small relative to the weighted mobility costs $C < 1$. Hence, the better performing school (0 in this case) will not attract all demand but only that which will allow the admission of L_1^B students, thus yielding an equilibrium. However, this equilibrium is not robust to group-deviations (or is not stable): if a small set of children are moved to the other school, their parents will not want to put them back to their original school. In fact the existence of this equilibrium relies entirely on the fact that, since there is a continuum of families, each of them is atomless, i.e. unable to change the outcome by deviating. If allocated to schools according to L_1^B no single parent will have an incentive to deviate. And yet, from the moment that *a few families* realize that they can benefit from turning to the other school, this equilibrium will fall apart: once *all* H-children are put into the same school none of their parents will have an incentive to move them when $C < 1$.

Finally, proposition 2 shows that, when $C \geq 1$, the equilibrium is unique and described by L_1^A.

Proposition 1 *Assume that $\phi = 0$. Then, if $C < 1$, there exist three equilibria. Two of them are stable and involve total segregation of children types.*

Proof. The proof has several parts. We first show that L_1^A is not an equilibrium when $C < 1$. Then we show that L_1^B is an equilibrium and refer to the Appendix for a proof of its instability. Finally, we show that both $L_1 = 0$ and $L_1 = 1/2$ are equilibrium distributions of children among schools when $C \leq 1$.

Part 1. $L_1^A < 1/4 - C/4$ implying, from (26), that $\hat{x} = 0$. Then, $L_1^* = 0 \neq L_1^A$.

Part 2. Consider L_1^B when $C < 1$. Families of type-H living at

$$\hat{x}(L_1^B) = \frac{1}{2} + \frac{2L_1^B - \frac{1}{2}}{C} = \frac{C + \sqrt{2 - 2C + C^2}}{2}$$

derive an identical utility level from sending their children to school 0 or 1

$$v^H(S_0(L_1^B), \hat{x}(L_1^B)) = v^H(S_1(L_1^B), \hat{x}(L_1^B)) = F(\gamma^H + \gamma^L) - \frac{c}{2}$$

All H-families living at $x > \hat{x}(L_1^B)$ prefer school 1: although they derive the same benefit from schooling of their children at school 1 than the family who is indifferent among schools, their incurred costs are lower, since they live closer to school 1. For the same reason, all H-families living at $x < \hat{x}(L_1^B)$ prefer school 0. Also, $\hat{x}(L_1^B) \in (0, 1)$

when $C < 1$.

As it is shown in Appendix A, this equilibrium is not robust to group deviations.

Part 3. If $L_1 = 0$, the utility to the H-families living furthest from school 1 ($x = 0$) is

$$v_1 = S_1(L_1 = 0) - c = \; 2\gamma^H F - c$$

If one of these families decided to move her child to the closest school 0, she would then benefit from utility

$$v_0 = S_0 \left(L_0 = \frac{1}{2} \right) = \; 2\gamma^L F$$

Since $c < 2(\gamma^H - \gamma^L)F$ this family does not have an interest to change school.

A similar argument shows that if all H-children attend school 0 no single H-family will have an incentive to change school when $C < 1$. Then, $L_1 = 1/2$ is also an equilibrium.

Both $L_1 = 0$ and $L_1 = 1/2$ are robust to ε-deviations (see Appendix B). ■

Therefore, when schools are financed by lump-sum transfers and $C < 1$, stable equilibria involve total segregation: either all H attend school 1 and all L school 0 or vice versa. *The difference in school's average performances is extreme.*

Proposition 2 *Assume that $\phi = 0$. Then, if $C \geq 1$, there exists a single stable equilibrium. This equilibrium is such that school 1 is the leader and involves more segregation than the no-choice benchmark, although it approaches this point of reference as $C \to \infty$.*

Proof. $L_1^B > 1/2$ when $C \geq 1$, hence it cannot take place. Neither $L_1 = 0$ nor $L_1 = 1/2$ are equilibrium outcomes when $C \geq 1$. To see this just note that when $C \geq 1$

$$\hat{x}_H = \frac{1}{2} + \frac{L_1 - L_0}{C} \begin{cases} < 1 & \text{if} \quad L_1 = 1/2 \\ > 0 & \text{if} \quad L_1 = 0 \end{cases}$$

hence $L_1^* = \dfrac{\hat{x}_H^2}{2} > 0$ and $L_1 = \dfrac{1}{2} - L_0^* = \dfrac{\hat{x}_H^2}{2} < \dfrac{1}{2}$.

The equilibrium, determined by $L_1^A \in \left(\frac{1}{4} - \frac{C}{4}, \frac{1}{4} + \frac{C}{4}\right)$, is unique and stable. Also,

$$lim_{C \to \infty} \; L_1^A \; = \frac{1}{8}$$

■

Therefore, at the unique and stable equilibrium corresponding to any $C \geq 1$ (or $c \geq 2F(\gamma^H - \gamma^L)$) the leading school 1 will admit only $L_1^* < 1/8$, thus increasing segregation with respect to the no-choice benchmark. We can now depict (stable) equilibria as a function of weighted mobility costs of H-families when schools are equally financed and parents are free to choose school for their children. The line $L_1 = 1/4$ (implying $L_0 = 1/4$) represents the Egalitarian Benchmark (schools equally share children types). Conversely, the line $L_1 = 1/8$ ($L_0 = 3/4$) represents the No-Choice Benchmark.

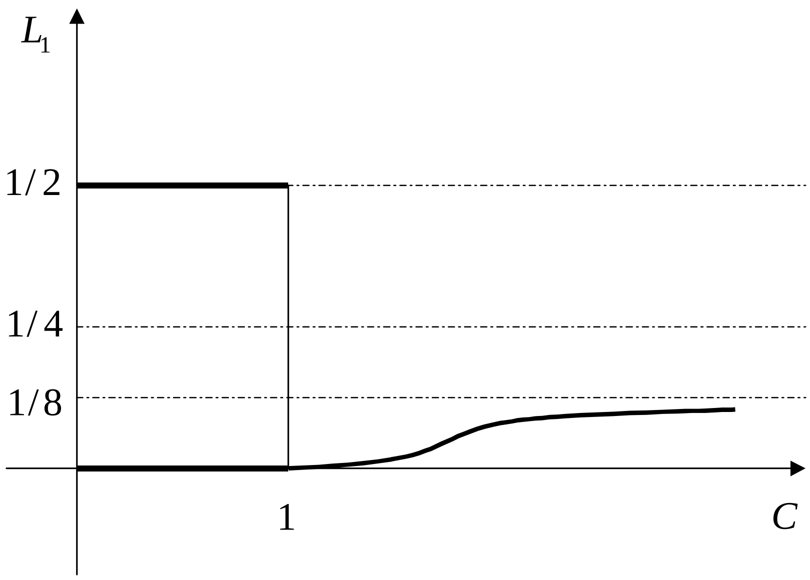

Figure 4: Equilibria when $\phi = 0$.

Since the equilibrium outcome tends to the No-Choice Benchmark as C tends to infinity, we can conclude that free school choice will only change the no-choice distribution of children in schools if mobility costs are low relative to the differences in performances of children types. But if weighted mobility costs are lower, then free choice of schools will result in *greater* segregation and discrepancy of average performances when schools have limited capacity and are lump-sum financed.

Note that, in our model, different parent types agree on what is good for their children and suffer identical (unit) mobility costs. Moreover, differences in income do not play a role in demand for schooling. Total segregation and the maximum difference in school average performances result from the fact that *schools* of limited capacity can choose their pupils among all applicants. As we discussed in the introduction, there are a few reasons why we could be interested in changing this.

5.2 Equilibrium with a Large Specific Grant

Consider now the other extreme case. Not only we allow for a grant ϕ but also it is sufficiently large relatively to lump-sum transfers to make the upper limit on L_i binding for the school enjoying the leading role. From the conditions for the maximization of (13) given in section 4.3,[8]

$$\phi > \frac{2(\gamma^H - \gamma^L)F}{\gamma^H - 4(\gamma^H - \gamma^L)L_i}$$

Then, the benefits in terms of larger funds associated to the admission of L children outweight the effects of their lower marginal achievement. The leading school would like to admit at least as many L students as have applied. From (15), (17) and (21), we can write

$$L_0 \leq \frac{1}{2} - \frac{1}{2}\left(\frac{1}{2} - \frac{1}{2C} + \frac{2L_0}{C}\right)^2 \tag{32}$$

or

$$L_1 \leq \frac{1}{2}\left(\frac{1}{2} - \frac{2L_1}{C} + \frac{1}{2C}\right)^2 \tag{33}$$

where the denominator of C, $2F(\gamma^H - \gamma^L) - \phi\gamma^L$ can now be positive or negative. In other words, parents may now prefer the school with more H-children as before, or they may prefer the school where there are more L. It will certainly depend upon whether the better school has more students of one or the other type. In turn, this will depend upon the size of ϕ and the ability of this variable to completely overcome the negative effect of disadvantaged cultural backgrounds on average performances

[8] Some caution is required when, like in this case, limiting levels of ϕ are expressed as a function of L_i, which depends itself on ϕ. However, this way of proceeding greatly simplifies the exposition. We then stick to it while carefully verifying ex-post the validity of the inequality.

of the schools. We have already discussed the controversial nature of this possibility. The case in which schools may end up preferring L children as a result of a large ϕ ($C < 0$) shall then be considered with caution.

5.2.1 *H* students are preferred by schools

Let us first consider $C > 0$ or, equivalently

$$\phi < \frac{2F(\gamma^H - \gamma^L)}{\gamma^L}$$

As before, (32) and (33) are equivalent equilibrium conditions. When the upper limit on L_1 is binding, we find two roots of $L_1 \in [0, \frac{1}{2}]$ that satisfy (33) with equality:

$$L_1^C = \frac{1}{4}\left(1 + C + C^2 - C\sqrt{2 + 2C + C^2}\right) \tag{34}$$

$$L_1^D = \frac{1}{4}\left(1 + C + C^2 + C\sqrt{2 + 2C + C^2}\right) \tag{35}$$

Figure 5 below depicts these roots of (33).

Clearly, from the figure, the only interior equilibrium is given by L_1^C (L_1^D does not satisfy (23)). Proposition 3 shows that L_1^C is in fact the only equilibrium when the upper limit of L_i is binding.

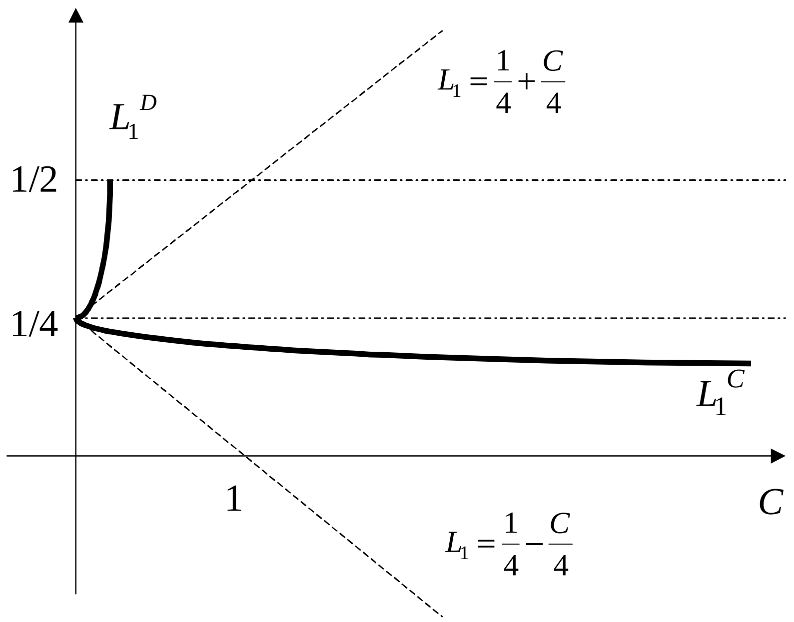

Figure 5

Proposition 3 *Assume that*

$$\frac{2F(\gamma^H - \gamma^L)}{\gamma^L} > \phi > \frac{2F(\gamma^H - \gamma^L)}{\gamma^H - 4(\gamma^H - \gamma^L)L_1^C}$$

Then, the equilibrium is unique and stable and involves more mixing of children types than those resulting at the No-Choice Benchmark. However, the equilibrium mix approaches this point of reference as $C \to \infty$.

Proof. The locus of admissions of L compatible with demand levels is represented by either L_1^C or L_1^D. In fact, since L_1^D does not satisfy (23), it cannot be an interior equilibrium. In contrast with the case when the lower limit for L_1 was binding, the corner solution $L_1 = 1/2$ is no longer an equilibrium. The reason is that, if school 1 admits all L, its average performance will be lower than that of school 0 (ϕ is not large enough to make the best school choose $L_i^* > 1/4$). Then, demand will turn to school 0, that will become the leader. If, on the other hand, school 0 is the leader, it will not choose $L_0 = 0$ (which would imply that $L_1 = 1/2$) since ϕ is large. A similar argument shows that $L_1 = 0$ ($L_0 = 1/2$) is not an equilibrium when the upper limit on L_1 is binding.

Equilibrium is then represented by $L_1^C(C) < 1/4$, that satisfies (22) and (23) and is robust to deviations, with

$$\begin{aligned} lim_{C\to 0}\, L_1^C &= 1/4 \\ lim_{C\to \infty}\, L_1^C &= 1/8 \end{aligned}$$

Note that

$$\begin{aligned} C &\to 0 \Leftrightarrow c/\left(2F(\gamma^H-\gamma^L)-\phi\gamma^L\right)\to 0 \\ C &\to \infty \Leftrightarrow c/\left(2F(\gamma^H-\gamma^L)-\phi\gamma^L\right)\to \infty \end{aligned}$$

On the other hand, there exists a range of ϕ that supports this equilibrium, since for that range to exist we need

$$\frac{1}{2}\gamma^L < \frac{1}{2}\gamma^H - 2(\gamma^H-\gamma^L)L_1^C \Leftrightarrow 2(\gamma^H-\gamma^L)L_1^C < \frac{1}{2}\left(\gamma^H-\gamma^L\right) \Leftrightarrow L_1^C < \frac{1}{4}$$

which is indeed the case. Hence, the domain of ϕ in this proposition is non empty. ■

Equilibrium when the upper limit on L is binding with $C > 0$ (H students are preferred by schools or $\phi < 2F(\gamma^H-\gamma^L)/\gamma^L$) is depicted in figure 6. It approaches the Egalitarian Benchmark as C tends to zero and the No-Choice Benchmark as C tends to infinity.

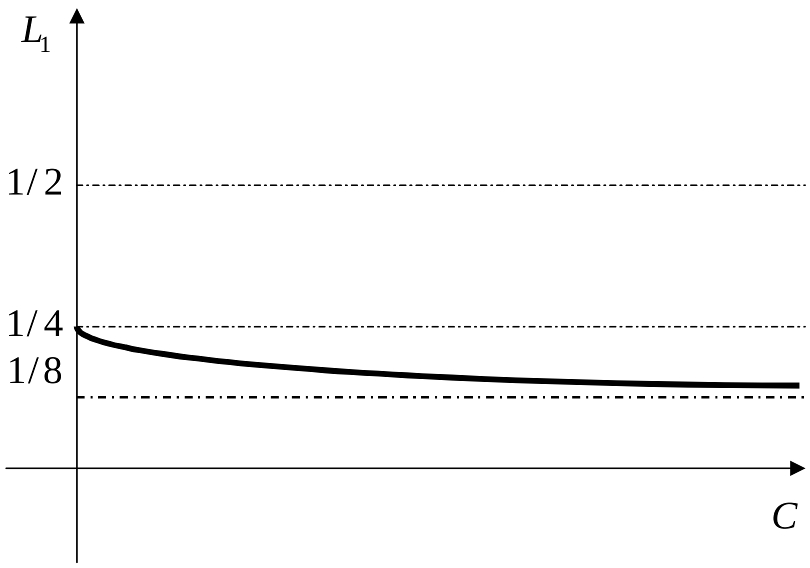

Figure 6: Equilibrium when ϕ "large".

By comparing figures 4 and 6 we can see that, in terms of reducing the differences in student mix within schools (and hence in average performances), i.e. approaching the benchmark $L_0 = L_1 = 1/4$, a substantial improvement can take place for low, positive, levels of C. This suggests that a level of ϕ sufficiently large to make the upper limit on L_i binding combined with the subsidization of transport costs (∇c) may prove efficient in reducing differences in schools' performances. A "too large" grant ϕ, able to make schools prefer L students, could however yield perverse effects, as it will shortly be clarified.

5.2.2 L students are preferred by schools

Consider now the case $C < 0$ or

$$\phi > \frac{2F(\gamma^H - \gamma^L)}{\gamma^L}$$

The grant associated to disadvantaged students is so large that the best school will be that which manages to enroll the larger number of L. In this case, not only schools try to admit as many L as possible (the upper limit on L is binding) but also, parent's demands turn to the more performing school, that is now attended by a higher proportion of L relative to H-children. Therefore, in contrast with the rest of the paper, $L_i > 1/4$ corresponds to optimal choices of school i.

In fact, this problem is symmetric to that described by propositions 1 and 2 when schools admit as many H-children as possible and parents prefer schools where there are more H. In that case, admissions of $H_i > 1/4$ were optimal choices of school i. By writing H instead of L we can directly obtain the equilibria corresponding to $\phi > 2F(\gamma^H - \gamma^L)/\gamma^L$.

Proposition 4 *Assume that $\phi > 2F(\gamma^H - \gamma^L)/\gamma^L$. Then there exist three equilibria when $-1 < C < 0$. Two of them are stable and involve total segregation. When $C < -1$ the equilibrium is unique and such that school 0 is the leader. It involves more segregation than the No-Choice Benchmark, although it approaches this point of reference as $C \to -\infty$.*

Proof. Take (33) and replace C by $\widehat{C} = -C > 0$. The condition is now (29) in terms of $\widehat{C}$. Therefore, the derivation of the equilibria follows the steps of propositions 1 and 2. ■

5.3 The Interior Solution

The aim of this section is to determine the domain of all possible equilibrium allocations of children to schools. Note first that interior equilibria can only take place if $C > 0$. When $C < 0$ the upper limit on L_1 is always binding.

We know from (29) and (33) that an interior solution for L_1 when $\hat{x} \in (0, 1)$ must be such that

$$\begin{aligned} L_1 &\geq \frac{1}{2}\left(\frac{1}{2} + \frac{2L_1}{C} - \frac{1}{2C}\right)^2 \\ L_1 &\leq \frac{1}{2}\left(\frac{1}{2} - \frac{2L_1}{C} + \frac{1}{2C}\right)^2 \end{aligned}$$

These inequalities are satisfied, respectively if

$$L_1^A \leq L_1^* \leq L_1^B$$

and

$$L_1^C \geq L_1^* \geq L_1^D$$

This limits the domain of interior solutions to the surface between L_1^A and L_1^C.

If, on the other hand, $\hat{x}$ is constrained at 0 or 1, interior solutions of L_i $(i = 0, 1)$ can take place provided that $L_1 < 1/4 - C/4$:[9] the admission of a sufficiently low number of L by the school preferred by all parents will not reduce its demand.

This fully determines the areas in figure 7.

[9] To better understand figure 12 note that $L_0 \leq 1/4 - C/4$ is equivalent to $L_1 > 1/4 + C/4$.

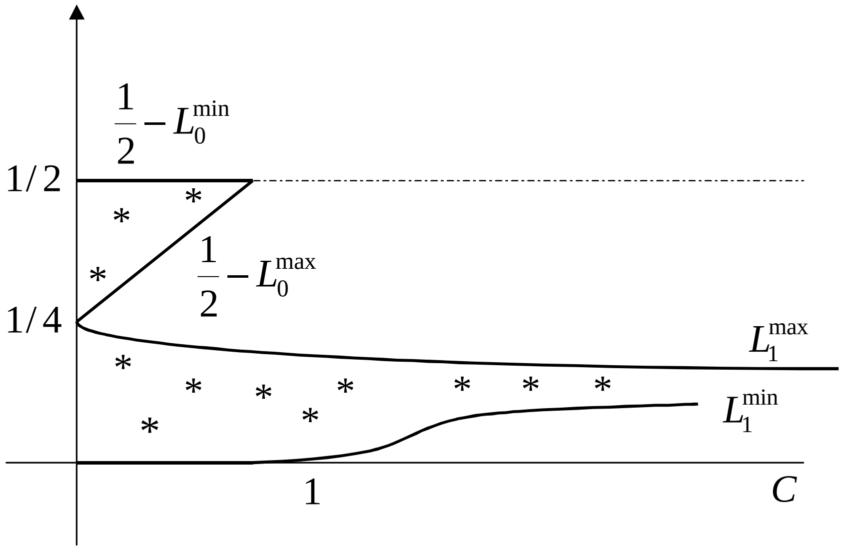

Figure 7

We can now determine the limiting levels of ϕ, $\underline{\phi}$ and $\overline{\phi}$ that support an interior equilibrium when school 1 is preferred by all parents. In this case minimum admissions of L at school 1 are given by L_1^{min} and maximum admissions of L are given by L_1^{max} in figure 8. From the first order conditions for maximization when neither constraint is binding at $L_1^* = L_1^{min}$, $L_1^* = L_1^{max}$ respectively,

$$\underline{\phi} = \frac{2(\gamma^H - \gamma^L)F}{\gamma^H - 4(\gamma^H - \gamma^L)L_1^{min}} \text{ and } \overline{\phi} = \frac{2(\gamma^H - \gamma^L)F}{\gamma^H - 4(\gamma^H - \gamma^L)L_1^{max}}$$

When school 0 is leading $L_0^{min} = 0$, $L_0^{max} = 1/4 - C/4$. Then, the first order conditions give us the limiting levels of ϕ, $\underline{\underline{\phi}}$ and $\overline{\overline{\phi}}$, that support an interior equilibrium when school 0 is the leader

$$\underline{\underline{\phi}} = \frac{2(\gamma^H - \gamma^L)F}{\gamma^H} \text{ and } \overline{\overline{\phi}} = \frac{2(\gamma^H - \gamma^L)F}{2\gamma^H - 4(\gamma^H - \gamma^L)(\frac{1}{4} - \frac{C}{4})}$$

Note that $\underline{\phi} \geq \underline{\underline{\phi}}$ since $L_1^{min} \geq L_0^{min}$. However, if $\underline{\phi} > \underline{\underline{\phi}}$ ($L_1^{min} > 0$), no equilibrium in which school 0 is the leading school can take place (see figure 7). On the other hand, $L_1^{max} > (1/4 - C/4)$. Hence, $\overline{\phi} > \overline{\overline{\phi}}$.

Then, since the payoffs of schools 1 and 2 are concave and, hence, the interior solutions are unique, we can write the following proposition.

Proposition 5 *For all $\underline{\phi} < \phi < \overline{\phi}$, there exists an interior equilibrium given by*

$$L_1^* = \frac{2(\gamma^L - \gamma^H)F + \gamma^H \phi}{4(\gamma^H - \gamma^L)\phi}$$

For $\underline{\underline{\phi}} < \phi < \overline{\overline{\phi}}$ there is a second interior equilibrium given by

$$L_0^* = \frac{2(\gamma^L - \gamma^H)F + \gamma^H \phi}{4(\gamma^H - \gamma^L)\phi}$$

We have seen that balanced budget increases of ϕ can induce schools to admit children from disadvantaged environments. However, the grant also affects the range of possible interior solutions, i.e. the minimum and maximum values of available L. Indeed, as ϕ increases, demands tend to equalize. This makes the range of available choices of L lie closer to the *No-Choice Benchmark*.

Indeed, once $\phi > \overline{\phi}$, the upper limit of L remains binding at L_1^{max} and further increases in the per L-child allocation ϕ only affect the size of C. The point $\phi = 2F(\gamma^H - \gamma^L)/\gamma^L > \overline{\phi}$ embeds an important change of regime. It implies that the advantages related to the enrollment of better achieving H-type children are lost in favor of L-types. The grant ϕ then has the perverse effect of replicating the equilibria corresponding to free choice of school when schools are financed by lump-sum transfers alone.

The following table summarizes the optimal feasible choices of L by school 1 for given ϕ. The limiting optimal admissions of L children, L_1^{min}, L_1^{max}, L_0^{min} and L_0^{max} are those depicted in figure 7.

$\phi < \frac{2F(\gamma^H-\gamma^L)}{2\gamma^H-4(\gamma^H-\gamma^L)L_1^{min}}$	$L_1^* = L_1^{min}$ $\lvert S_0 - S_1\rvert$ maximal
$\frac{2F(\gamma^H-\gamma^L)}{2\gamma^H-4(\gamma^H-\gamma^L)L_1^{min}} < \phi < \frac{2F(\gamma^H-\gamma^L)}{2\gamma^H-4(\gamma^H-\gamma^L)L_1^{max}}$	L_1^* interior
$\frac{2F(\gamma^H-\gamma^L)}{\gamma^H-4(\gamma^H-\gamma^L)L_1^{max}} < \phi < \frac{2F(\gamma^H-\gamma^L)}{\gamma^L}$	$L_1^* = L_1^{max}$ $\lvert S_0 - S_1\rvert$ minimal
$\frac{2F(\gamma^H-\gamma^L)}{\gamma^L} < \phi$	$L_1^* = 1/2 - L_0^{min}$ $\lvert S_0 - S_1\rvert$ maximal

Similarly, the optimal feasible choices of L for school 0 given ϕ are

$\phi < \frac{2F(\gamma^H - \gamma^L)}{\gamma^H}$	$L_0^* = L_0^{min}$ $\|S_0 - S_1\|$ maximal
$\frac{2F(\gamma^H - \gamma^L)}{\gamma^H - 4(\gamma^H - \gamma^L)L_0^{min}} < \phi < \frac{2F(\gamma^H - \gamma^L)}{\gamma^H - 4(\gamma^H - \gamma^L)L_0^{max}}$	L_0^* interior
$\frac{2F(\gamma^H - \gamma^L)}{\gamma^H - 4(\gamma^H - \gamma^L)L_1^{max}} < \phi < \frac{2F(\gamma^H - \gamma^L)}{\gamma^L}$	no equilibrium
$\frac{2F(\gamma^H - \gamma^L)}{\gamma^L} < \phi$	$L_0^* = 1/2 - L_1^{min}$ $\|S_0 - S_1\|$ maximal

6 Passive Schools

In order to underline the importance of limited school capacity, consider, to conclude, the case in which schools passively accept all applications. In this case, parents cannot simply compare the absolute number of H in both schools, as the school with more H may also be the one with more L. Suppose that they consider instead the difference between the number of H and L at each school. Demand will then be determined by

$$x^* = \frac{1}{2} + \frac{H_0 - L_0 - H_1 + L_1}{2c} = \frac{1}{2} + \frac{L_1 - H_1}{c} \tag{36}$$

since $H_0 + H_1 = 1/2$ and $L_0 + L_1 = 1/2$.

Schools of unlimited capacity enroll now every applicant. Thus

$$L_1 = \frac{(1 - x^*)^2}{2} \text{ and } H_1 = \frac{1}{2} - \frac{x^{*2}}{2} \tag{37}$$

At equilibrium it must be the case that demand allows for such allocation of children to the schools:

$$x^* = \frac{1}{2} + \frac{1}{c}\left(\frac{(1 - x^*)^2}{2} - (\frac{1}{2} - \frac{x^{*2}}{2})\right)$$

This equilibrium condition has two solutions:

$$\begin{aligned} x^a &= \frac{1}{2}\left(c + 1 - \sqrt{c^2 + 1}\right) \\ x^b &= \frac{1}{2}\left(c + 1 + \sqrt{c^2 + 1}\right) \end{aligned}$$

x^b is larger than 1 for all c and therefore non-feasible. Demand cannot remain constrained at $x = 1$ since, if all the children were attending the same school, some of the H parents would prefer to send their children alone to the other one.

Therefore, the only equilibrium is that described by x^a. The corresponding distribution of children among schools is, from (37)

$$\begin{aligned} H_1 &= \frac{1}{2} - \frac{1}{8}\left(c+1-\sqrt{c^2+1}\right)^2 \\ L_1 &= \frac{1}{8}\left(1-c+\sqrt{c^2+1}\right)^2 \end{aligned}$$

depicted in figure 8

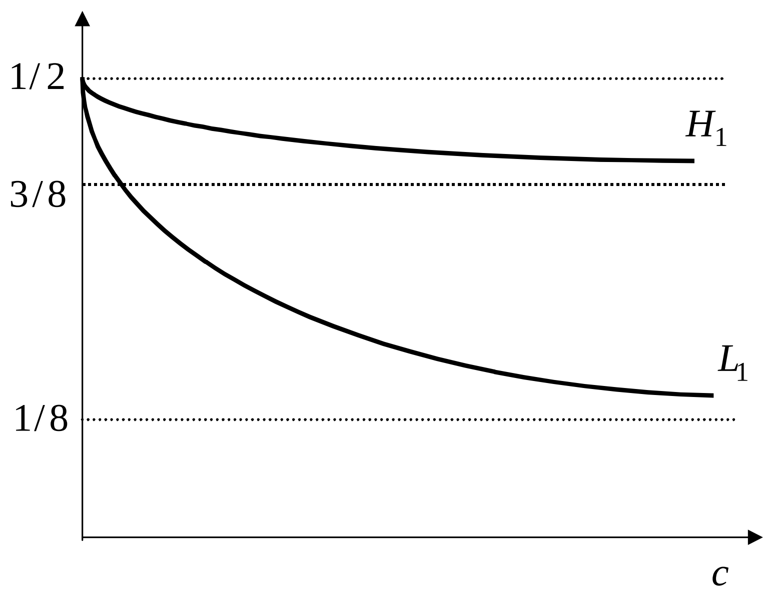

Figure 8

Note that school 0 tends to disappear as mobility costs fall, since the global amount of pupils in school 1 tends to 1(all the market) when mobility costs tend to zero. This is what has traditionally been claimed by free school choice advocates. Moreover, given the assumed distribution of families in the city, as the size of the better school increases from 1/2, the number of L children that get to attend this school is larger than the number of new H children ($L_1 - 1/8 > H_1 - 3/8$). Hence, if schools were able *and willing* to passively accept all applications (and all families were equally mobile) it could be the case that free school choice provided in fact disadvantaged children with better opportunities to attend the better school.

This result requires a certain number of qualifications. In the first place, it is worth noting that not even under the most favorable assumptions is free school choice a Pareto improving policy. There are always winners and losers that are difficult to weight against each other. This is the reason why neither analytical nor empirical work seem conclusive on the desirability of this measure. On the other hand, there is some evidence that mobility could be more costly for disadvantaged families. If this is the case, then choice may only be effectively available for better endowed families.

Finally, we have shown that limited school capacity intensifies segregation of children types within schools, preventing all possible desirable effects of free school choice from taking place. Of course, schools could also chose to increase capacity. Although this issue deserves further research, we do not find this alternative very likely a priori. Limited capacity provides the leading school with the power to select the best inputs to the production of teaching. We find quite unlikely that this school shall be willing to give up the power to select the best performing students in order to be able to accept more applications of disadvantaged children.

7 Concluding Remarks

In our model, the Egalitarian Benchmark will under no circumstances result at equilibrium. This is due to the fact that any attempt to equalize schools will make parents indifferent among them. Then, children will attend the school that is closest to their homes. But since families, that differ in education level, are unevenly distributed in the city, schools will have uneven allocations of children types. And since children's achievements differ according to their background, average performances of the schools will differ and parents will no longer be indifferent among them. As we have seen, parent's reaction to these differences provides the better school with a power to choose among children that exacerbates segregation if schools are lump-sum financed.

In contrast, an adequate level of grants to disadvantaged pupils directly provided to schools and eventually combined with transport subsidies may take us arbitrarily close to the Egalitarian Benchmark: by modifying the school's incentives to accept applications of disadvantaged children we can induce the equalization of average school's performances.

It has been our goal to underline the role of school's choices as opposed to differences on the demand side (like differences in mobility costs or preferences) in explaining the persistent inequality of public school performances. In order to do so, we have constructed an very simple model of public schooling provision. Some of our assumptions, although strong, are unessential to obtain our results: it can be shown, for instance, that changes in the location of the schools and the families do not change the nature of the resulting equilibria (although they affect the ranges of ϕ for which each equilibrium results). Other features of our model, like the technology of education production or the partial and static nature of the analysis deserve to be challenged by future research. Additional feasible extensions to this piece of work include the endogeneization of the choice of residence, the introduction of parental and/or pupils's effort in the education technology and the endogeneization of school's capacity choice.

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A Instability of L_1^B when $C < 1$

When School 1 admits L_1^B children the location of the families that are indifferent among schools is:

$$\hat{x}(L_1^B) = \frac{1}{2} + \frac{2L_1^B - \frac{1}{2}}{C} = \frac{C + \sqrt{2 - 2C + C^2}}{2}$$

The utility they derive from sending their children to School 1 is the same as if they sent them to School 0. From substitution of L_1^B, $\hat{x}(L_1^B)$ and $C = \dfrac{c}{2F(\gamma^H - \gamma^L)}$ into (2) and (3) we obtain :

$$v^H(S_0(L_1^B), \hat{x}_H(L_1^B)) = v^H(S_1(L_1^B), \hat{x}_H(L_1^B)) = F(\gamma^H + \gamma^L) - \frac{c}{2}$$

Every $H-$family leaving at $x > \hat{x}(L_1^B)$ prefers School 1 for her child and every H-family leaving at $x < \hat{x}(L_1^B)$ prefers School 0.

Consider now a mass ε of H-families living to the right of $\hat{x}(L_1^B)$. Their utility from sending their children to School 1 is

$$v^H(S_1(L_1^B), \hat{x}(L_1^B) + \varepsilon) = F(\gamma^H + \gamma^L) - \frac{c}{2} + c\,\varepsilon \tag{38}$$

Suppose now that these families decide to send their children to School 0. They will be automatically admitted there, since schools prefer H-children. The minimum L that School 0 will admit is then

$$L_0^{dev} = \frac{1}{2} - \frac{\left(\hat{x}(L_1^B) + \varepsilon\right)^2}{2} < \frac{1}{2} - L_1^B = \frac{1}{2} - \frac{\left(\hat{x}(L_1^B)\right)^2}{2}$$

By changing school they improve the average performance of school 0. The utility of these families is then $V^H = v^H(S_0(L_1^B), \hat{x}(L_1^B) + \varepsilon)$

$$\begin{aligned} V^H &= F(\gamma^H + \gamma^L) - \frac{c}{2} + \varepsilon^2 2F(\gamma^H - \gamma^L) + \\ & \quad \varepsilon F(\gamma^H - \gamma^L)\sqrt{\frac{c^2 + 4F(\gamma^L - \gamma^H)c + 8F^2(\gamma^L - \gamma^H)^2}{F^2(\gamma^L - \gamma^H)^2}} \end{aligned} \tag{39}$$

Therefore, if we compare (38) and (39) we can see that it suffices that

$$c\,\varepsilon < \varepsilon F(\gamma^H - \gamma^L)\sqrt{\frac{c^2 + 4F(\gamma^L - \gamma^H)c + 8F^2(\gamma^L - \gamma^H)^2}{F^2(\gamma^L - \gamma^H)^2}} + 2\varepsilon^2 F(\gamma^H - \gamma^L)$$

for these families to benefit from deviation. Some manipulation transforms this condition into

$$\varepsilon > C - \sqrt{2 - 2C + C^2}$$

Since $C - \sqrt{2 - 2C + C^2} < 0$ for all $C < 1$, any $\varepsilon > 0$ makes the deviation profitable. Note that, as ε tends to zero the utility obtained when deviating converges towards the utility obtained when not deviating.

B Robustness of $L_1 = 0$, $L_1 = 1/2$ when $C < 1$.

If $L_1 = 0$ the utility derived from sending their children to school 1 by the ε H-families living furthest from this school ($x = \varepsilon$)

$$
\begin{aligned}
v_1 &= S_1(L_1) - (1-\varepsilon)c = \; 2\,\gamma^H F - (1-\varepsilon)c \\
&= 2(\gamma^H - \gamma^L)F\left(\frac{2\gamma^H F - 2\gamma^L F + 2\gamma^L F - (1-\varepsilon)c}{2(\gamma^H - \gamma^L)F}\right) \\
&= 2(\gamma^H - \gamma^L)F\left(\frac{2\gamma^L F}{2(\gamma^H - \gamma^L)F} + 1 - \frac{(1-\varepsilon)c}{2(\gamma^H - \gamma^L)F}\right)
\end{aligned}
$$

Suppose that they change school, then

$$
\begin{aligned}
v_0 &= S_0(L_0) - \varepsilon\, c = 2\gamma^H \varepsilon^2 F + 2\gamma^L(1-\varepsilon^2)F - \varepsilon\, c \\
&= 2\gamma^L F + 2(\gamma^H - \gamma^L)F\varepsilon^2 - \varepsilon\, c \\
&= 2(\gamma^H - \gamma^L)F\left(\frac{2\gamma^L F}{2(\gamma^H - \gamma^L)F} + \varepsilon^2 - \frac{\varepsilon\, c}{2(\gamma^H - \gamma^L)F}\right)
\end{aligned}
$$

The change will only be profitable if

$$
\begin{aligned}
\varepsilon^2 - \varepsilon C &> 1 - (1-\varepsilon)C \\
\varepsilon(\varepsilon - 2C) &> 1 - C
\end{aligned}
$$

which is not satisfied for small deviations ($\varepsilon \to 0$).

Consider now $L_1 = 1/2$, H-families living furthest from school 0 ($x = 1-\varepsilon$) enjoy utility

$$
v_0 = S_0(L_0) - (1-\varepsilon)\,c = 2\gamma^H F - (1-\varepsilon)c
$$

if they change their children to school 1

$$
\begin{aligned}
v_1 &= S_1(L_1) - \varepsilon c = 2\gamma^H(1-(1-\varepsilon)^2)F + 2\gamma^L(1-\varepsilon)^2 F - \varepsilon c \\
&= 2\gamma^H F + 2(\gamma^L - \gamma^H)(1-\varepsilon)^2 F - \varepsilon c
\end{aligned}
$$

The change will be profitable to these ε families if

$$
\begin{aligned}
2(\gamma^H - \gamma^L)(1-\varepsilon)^2 F + \varepsilon c &< (1-\varepsilon)c \\
(1-\varepsilon)^2 + \varepsilon C &< (1-\varepsilon)C \\
(1-\varepsilon)^2 &< C(1-2\varepsilon)
\end{aligned}
$$

which is not satisfied for small deviations ($\varepsilon \to 0$).