INVESTMENT UNDER DEMAND UNCERTAINTY AND CAPACITY CONSTRAINTS
An empirical application to Belgium

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ABSTRACT

This paper evaluates the relevance of q theories under stochastic demand and capacity constraints by estimating an investment function for the Belgian economy. Under these theoretical conditions, we find that the investment rate depends on average q and on the expectations about the degree of capacity utilisation. The dynamics of the empirical model is derived without using any "time to build" or "delivery lags" assumptions. Our estimation stresses how important are the expectations about the degree of capacity utilisation in explaining investment.

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INTRODUCTION

The major contribution of q investment theories inspired by Tobin (1969) is that all expectations about future revenues are contained in the q term. After Hayashi's (1982) seminal paper in which the conditions for the equality between marginal and average q were derived, various authors tried to test empirically the influence of the stock market index on the investment rate[1]. Average q turns out to be highly significant in explaining investment, even though the empirical estimations of q models are not totally satisfactory, mainly for two related reasons: (a) *lagged* values of q also have significant coefficients, and (b) the residual terms are autocorrelated. The existence of "time to build" or "delivery lags" in the investment process has been used as an informal justification for significant lagged q coefficients and autocorrelated residuals.

Malinvaud's (1987) paper suggests an alternative way of explaining Tobin's (1969) argument. The emphasis is shifted from adjustment costs to both demand uncertainty and capacity constraints. In Malinvaud's view, the firm is essentially concerned with forecasting future demand when it must decide about capacities. The fact that demand is not known with certainty implies that capacities can be underutilised. This leads the firm to take into account expected capacity utilisation when she must decide about investment. Empirical estimation of Malinvaud's q model was carried out by Lambert and Mulkay (1987).

Malinvaud's idea is related to previous papers in investment theory under fixed prices[2]. Precious (1987) shows that, if Hayashi's assumptions

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[1] The papers of Summers (1981) and Blanchard and Wyplosz (1983) are of particular relevance. Abel and Blanchard (1986) estimate an investment function depending on a calculated series for marginal q. They conclude that their estimation is very similar to those obtained using average q valuated on the stock market.

[2] See Malgrange and Villa (1984), Michel (1986), and Precious (1987).

on both the production function and the installation function hold, *marginal q is smaller than average q* when the firm expects to face demand constraints in future periods. The difference is explained by the marginal value of all future demand constraints. Empirical estimations have been carried out for this model by Chan-Lee and Torres (1987), using the degree of capacity utilisation (henceforth DUC) as a proxy for the marginal value of all future demand constraints. They conclude that DUC plays a significant role in explaining investment. This is also consistent with one of the main results of the European Unemployment Programme[3], according to which "profitability is mostly dependent upon the rate of capacity utilisation and capital accumulation should depend heavily upon the expected degree of utilisation of additional capacity."

The aim of this paper is to evaluate the relevance of q theories under stochastic demand constraints by estimating an investment function for Belgium. The theoretical background is taken from Licandro (1990a), and combines the adjustment cost hypothesis with demand uncertainty and capacity constraints. The first section is devoted to the theoretical model. The main result is that investment depends on expectations about DUC as well as on average q.

The empirical specification and the estimation results are presented in the second section. Our main concern is to show that the introduction of DUC as an explanatory variable significantly improves the investment rate equation and allows an explanation of the two mentioned empirical problems, i.e., (1) the autocorrelation of the residuals and (2) the role of the lagged values of q. The specification presented below does not require "time to build" or "delivery lags" assumptions in order to justify the introduction of some dynamics in the model.

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[3] The EUP has coordinated the estimation of the a non-Walrasian macroeconomic model across 10 countries. Some of the conclusions are presented by Drèze (1990).

THE MODEL

UNCERTAINTY AND CAPACITY CONSTRAINT

Following Malinvaud (1987) it is assumed that the representative firm faces demand uncertainty and capacity constraints when it must decide about investment. Uncertainty is modeled by a random demand shock and a productivity shock, which follow an i.i.d. bivariate lognormal distribution. This particular way of modeling uncertainty is taken from Sneessens (1987). On the other hand, the technology is Leontief[4] and investment becomes productive after one period. This assumption allows for the existence of a potential output, which plays the role of a capacity constraint.

As in Malinvaud (1987) the firm is assumed to be price-taker, with perfect foresight concerning prices[5]. Because capacities are determined before the stochastic terms become known, excess capacity and excess demand are possible ex-post. Production is defined by

$$Y(t) = \min\{YD(t), YP(t)\}, \tag{1}$$

where Y(t) is production, YD(t) is demand and YP(t) is potential output.

As long as the stochastic terms in YD(t) and YP(t) are jointly lognormally distributed, Lambert's (1988) theorem applies and expected

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[4] In a two-period model Malinvaud (1987) assumes that the production function is putty-clay. A dynamic version of the Malinvaud (1987) model is discussed by Artus (1990). The dynamic problem with a putty-clay technology, uncertainty and adjustment costs is a subject for future research.

production can be approximated by[6]

$$(2) \qquad E_t(Y(s)) = \left[E_t(YD(s))^{-\rho} + E_t(YP(s))^{-\rho}\right]^{-\frac{1}{\rho}} \quad \forall\, s>t,$$

where $E_t(.)$ represents expectations conditional to the information set at time t.

The parameter ρ is a function of the variance-covariance matrix. In particular, when ρ goes to infinity expected production is equal to the minimum of expected demand and expected potential output. In this case there is no uncertainty at all, and the model becomes equivalent to the investment models under fixed prices mentioned in the introduction.

An interesting characteristic of equation (2) is that the elasticity of expected production with respect to expected potential output, denoted Φ_p, is

$$(3) \qquad \Phi_p(s) = \left[E_t(DUC(s))\right]^{\rho}$$

$$\text{where} \quad E_t(DUC(s)) = \frac{E_t(Y(s))}{E_t(YP(s))}.$$

Lambert (1988) shows that Φ_p is the weighted probability for the firm of being capacity constrained. The variable $\Phi_p(s)$ is a function of expected values at times s, conditional to the information set at t. The index t is omitted for simplicity.

The expectations about DUC, called $E_t(DUC)$[7], and the elasticity Φ_p

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[5] Malinvaud (1987) also analyses the behaviour of a monopolistic firm. Licandro (1990a) assumes that the firm is working in a monopolistically competitive market and fixes its price before knowing the stochastic demand term. Under these conditions the optimal investment path is the same as in the present paper, but the investment equation belongs to a two-equation model, where investment and prices are determined simultaneously.

[6] Alternatively we can assume that both YD(t) and YP(t) are stochastic with joint iid process. If the two random shocks at t are identically independently Weibull distributed we can follow Gourieroux and al. (1984) and show that equality (2) holds exactly.

[7] Strictly speaking E(DUC) is the ratio of expected production to expected capacity.

play an important role in the model. This will become clear below when we compute the optimal condition for investment and analyze the relation between marginal and average q.

THE FIRM'S BEHAVIOUR

Under the previous assumptions the optimisation problem for the firm can be written as[8]

$$(4) \qquad V_t(K(t-1)) = \underset{\{K(s)\}_{s=t}^{\infty}}{\text{Max}} \sum_{s=t}^{\infty} \left[a(s)\, E_t(Y(s)) - P(s)\, \Psi(I(s), K(s-1)) \right] (1+r)^{-(s-t)}$$

such that

$$a(s) = P(s) - MPC(s)$$

$$E_t(Y(s)) = \left[E_t(YD(s))^{-\rho} + E_t(YP(s))^{-\rho} \right]^{-\frac{1}{\rho}} \quad \forall\, s>t,$$

$$E_t(YP(s)) = B(s)\, K(s-1)$$

$$K(s) = K(s-1)(1-\delta) + I(s),$$

where P(s) represents the price level, I(s) investment, MPC(s) the marginal production cost, K(s) the capital stock, and B(s) the Leontief coefficient for capital. The discount rate r, the depreciation rate δ and the parameter ρ are assumed constant. The certainty path of the exogenous variables P(s), MPC(s) and B(s) are taken as known by the firm. The adjustment cost function $\Psi(I,K)$ will be characterised later.

The first-order condition for K(s-1), $\forall$ s>t, is

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[8] This is a discrete-time version of Licandro's (1990a) paper, which develops a similar model in continous time. The non-autocorrelation assumption on the stochastic terms allows for a non-stochastic path in the control variable, which implies that profits are a linear function of only one random variable, production. Under these conditions, expected profits can be written as a linear function of expected production.

$$(5) \qquad P(s-1)\,\Psi_I(s-1)\,(1+r) = a(s)\,\frac{\partial E_t[Y(s)]}{\partial K(s-1)} - P(s)\,\left[\Psi_I(s)\,(\delta-1) + \Psi_K(s)\right],$$

$$\text{where} \quad \Psi_I = \frac{\partial \Psi}{\partial I} \quad \text{and} \quad \Psi_K = \frac{\partial \Psi}{\partial K}.$$

Multiplying both sides by K(s-1), and taking into account the law of motion for the capital stock, the optimal condition becomes

$$P(s-1)\,K(s-1)\,\Psi_I(s-1) = P(s)\,K(s)\,\Psi_I(s)\,(1+r)^{-1} +$$
$$\left\{a(s)\,\Phi_p(s)\,E_t[Y(s)] - P(s)\,\left[I(s)\,\Psi_I(s) + K(s-1)\,\Psi_K(s)\right]\right\}(1+r)^{-1}.$$

Summing over s, the optimal condition for investment becomes

$$(6) \qquad P(t)\,K(t)\,\Psi_I[I(t),K(t-1)] =$$
$$\sum_{s=t+1}^{\infty}\left\{a(s)\,\Phi_p(s)\,E_t[Y(s)] - P(s)\,\left[I(s)\,\Psi_I(s) + K(s-1)\,\Psi_K(s)\right]\right\}(1+r)^{-(s-t)}.$$

Equation (6) states the well-known optimal condition that the marginal cost of investment must be equal to the marginal value of capital. The functional form of $\Psi(I,K)$ plays an important role in this relation. In the next section we impose a particular form in order to get a simple relation between marginal and average q.

The right-hand side of equation (6) is the nominal marginal value of the total capital stock. Marginal q is obtained by dividing the right-hand side of (6) by P(t) K(t).

AVERAGE Q AND MARGINAL Q

To derive more tractable implications in terms of marginal and average q, we need to add one last assumption concerning the adjustment cost function, namely,

$$(7) \qquad \Psi[I(s),K(s-1)] = \Omega[\alpha(s)]\,K(s-1)\,E_t[DUC(s)],$$

$$\text{where} \quad \alpha(s) = \frac{I(s)}{K(s-1)}.$$

α is the investment rate, i.e., the ratio of investment to the capital stock. Hayashi (1982) assumes that $\Psi(I,K)$ is linearly homogeneous in I and K, as sufficient conditions to show the equality between average and marginal q. Equation (7) differs from Hayashi's assumption in that $\Omega(\alpha)$ is weighted by the *expected utilised* capital and not with respect to the capital stock. This assumption implies that the more capacity is utilised, the higher are the adjustment costs. Additionally, if the firm expects full utilisation of capacity, both assumptions become equivalent.

Moreover, we add the following standard conditions for the installation function:

$$\Omega(0) = 0,$$

$$\Omega'(.) > 0 \quad \text{and} \quad \Omega''(.) > 0.$$

By assumption, equation (7) verifies

$$I(s)\,\Psi_I(s) + K(s\text{-}1)\,\Psi_K(s) = \Psi(s)\,\Phi_p(s),$$

in which case marginal q becomes

$$(8) \qquad q(t) = \frac{1}{P(t)\,K(t)} \sum_{s=t+1}^{\infty} \left\{ a(s)\,E_t[Y(s)] - P(s)\,\Psi(s) \right\} \Phi_p(s)\,(1+r)^{-(s-t)}.$$

Equation (8) can be rewritten as

$$(8') \qquad q(t) = Q(t)\,\Phi^*_p(t),$$

$$\text{where} \quad Q(t) = \frac{V_{t+1}[K(t)]}{P(t)K(t)}\,(1+r)^{-1};$$

$$\Phi^*_p(t) = \sum_{s=t+1}^{\infty} \Phi_p(s)\,f(s);$$

$$f(s) = \frac{\left[a(s)\,E_t[Y(s)] - P(s)\,\Psi(s)\right]\,(1+r)^{-(s-t)+1}}{V_{t+1}[K(t)]} \quad \text{s.t.} \quad \sum_{s=t+1}^{\infty} f(s) = 1.$$

The relation between marginal and average q is represented by equation (8'). The variable Q(t) denotes average q, while marginal q is denoted by q(t). They differ by a multiplicative factor, Φ^*_p.

We call Φ^*_p the "mean potential output elasticity." The variable f(s) is the ratio of date-s cash flows to total firm value. Because f(s) has the characteristics of a probability function, Φ^*_p is a mean of all future $\Phi_p(t)$ elasticities. As shown by equation (3), the "mean potential output elasticity" depends on the expected DUC.

As in the fix-price investment models, marginal q is smaller than or equal to average q, which follows directly from equation (8') because $\Phi_p(t) \leq 1$ for all t. Marginal q and average q are equal only if the firm expects to fully employ its capacity during all future periods.

Finally, as long as DUC(t) is assumed to belong to the information set at time t, we have $E_t(DUC(t))=DUC(t)$. Thus equation (6) can be rewritten as

$$\Omega'\left(\frac{I(t)}{K(t-1)}\right) DUC(t) = Q(t)\,\Phi^*_p(t). \tag{9}$$

The left-hand side of equation (9) is the marginal cost of investment. It is equal to marginal q, which is average q times the "mean potential output elasticity."

In order to better understand this result, let us divide both sides of equation (9) by DUC(t) to obtain the following investment function:

$$\Omega'\left(\frac{I(t)}{K(t-1)}\right) = Q(t)\,\frac{\Phi^*_p(t)}{DUC(t)}. \tag{9'}$$

The investment rate function is described by equation (9') as depending on average q and on the expectations about DUC.

From equation (9') we can see that DUC has two opposite influences on investment:

(1) The restrictions imposed by equation (7) on the $\Psi(I,K)$ function imply that a rise in DUC(t) increases adjustment costs, with a negative effect on investment.

(2) The mean elasticity Φ^*_p is a positive function of the $\{E_t(DUC(s))\}$ sequence, implying that an increase in expected DUC has a positive effect on investment. Intuitively, if expected demand rises, the "mean potential output elasticity" increases and the firm hopes that its output will respond more to a rise in capacity; raising the investment rate will therefore be more profitable.

The total effect on investment depends on the relative weight of both terms. Leaving aside dynamic considerations, the total effect of DUC on investment depends on the parameter ρ: in the long run, DUC has a positive effect on investment if $\rho > 1$. For reasonable values of the covariance matrix of the stochastic shocks, it is indeed likely that $\rho > 1$ holds; this will be discussed in the next section. In this sense, Tobin's q model under demand uncertainty and capacity constraints gives a theoretical argument to an intuitive perception: the closer the firms are to the full utilisation of capacity, the more they invest.

EMPIRICAL ANALYSIS

The aim of the second part of this paper is to estimate on Belgian data the model developed above in order to evaluate the empirical relevance of the q theory under stochastic demand and capacity constraints.

EMPIRICAL SPECIFICATION

For empirical purposes we must give a particular functional form to $\Omega'(\alpha)$ and approximate the mean value $\Phi^*_p(t)$ of the sequence of elasticities $\{\Phi_p(s)\}$, with $s>t$. This will lead us to specify the investment rate

as a function of average q and the DUC variable.

Let the function $\Omega'(\alpha)$ take the particular form:

(10) $$\Omega'(\alpha) = \exp\{a + b\,\alpha\}\,.$$

This specific functional form verifies the conditions imposed on $\Omega(\alpha)$ when b>0. The optimality condition for investment then becomes

(11) $$\alpha(t) = a_0 + a_1 \ln[Q(t)] + a_1 \ln[\Phi^*_p(t)] - a_1 \ln[DUC(t)].$$

Combining equations (3) and (9) we see that the logarithm of Φ^*_p is a non-linear function of the $\{E_t(DUC(s))\}$ sequence, for s>t:

$$\Phi^*_p(t) = \sum_{s=t+1}^{\infty} [E_t[DUC(s)]]^{\rho} f(s).$$

We assume that each particular value of $E_t(DUC(s))$, s>t, can be seen as depending on all the past values (s≤t) of the {DUC(s)} sequence. We cannot in general solve for Φ^*_p as a loglinear function of the past of DUC. However, let us assume that the conditions for such a linearisation are verified and that Φ^*_p follows[9]

(12) $$\ln\Phi^*_p(t) = \beta_1 \rho \ln[DUC(t)] + \beta_2 \rho \ln[DUC(t-1)] + \beta_3 \ln\Phi^*_p(t-1) + \beta_4 \ln\Phi^*_p(t-2)$$

$$\text{with} \quad \beta_1 + \beta_2 + \beta_3 + \beta_4 = 1\,.$$

The condition imposed on the β's is necessary to retrieve equation (3) in the steady state. The parameter ρ is assumed constant over time. The particular assumption (12) allows to specify equation (11) in an estimable form:

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[9] If our model is static, then $\Phi^*_p(t) = \Phi_p(t)$ and only the expectation formation process of DUC matters. In this case, equation (12) can be interpreted as a specification of this process.

$$(13) \qquad \alpha(t) = a_0(1-\beta_3-\beta_4) + a_1[\ln(Q(t)) - \beta_3 \ln(Q(t-1)) - \beta_4 \ln(Q(t-2))]$$

$$+a_1(\beta_1 \rho-1) \ln(DUC(t)) + a_1(\beta_2 \rho+\beta_3) \ln(DUC(t-1)) + a_1\beta_4 \ln(DUC(t-2))$$

$$+\beta_3 \alpha(t-1) + \beta_4 \alpha(t-2) + \varepsilon(t).$$

where the stochastic term $\varepsilon(t)$ is assumed to be a white noise.

Equation (13) allows for lagged values of average q and for lagged endogenous terms, even though all expectations about future average profits are contained in the current average q. Informal arguments about plausible "time to build" or "delivery lags" phenomena are not necessary. The dynamics imposed on the Φ^*_p term is enough to generate the dynamics of the investment rate. As indicated before, the particular novelty of this type of model is the important role played by DUC in explaining investment. Equation (13) stresses this essential element.

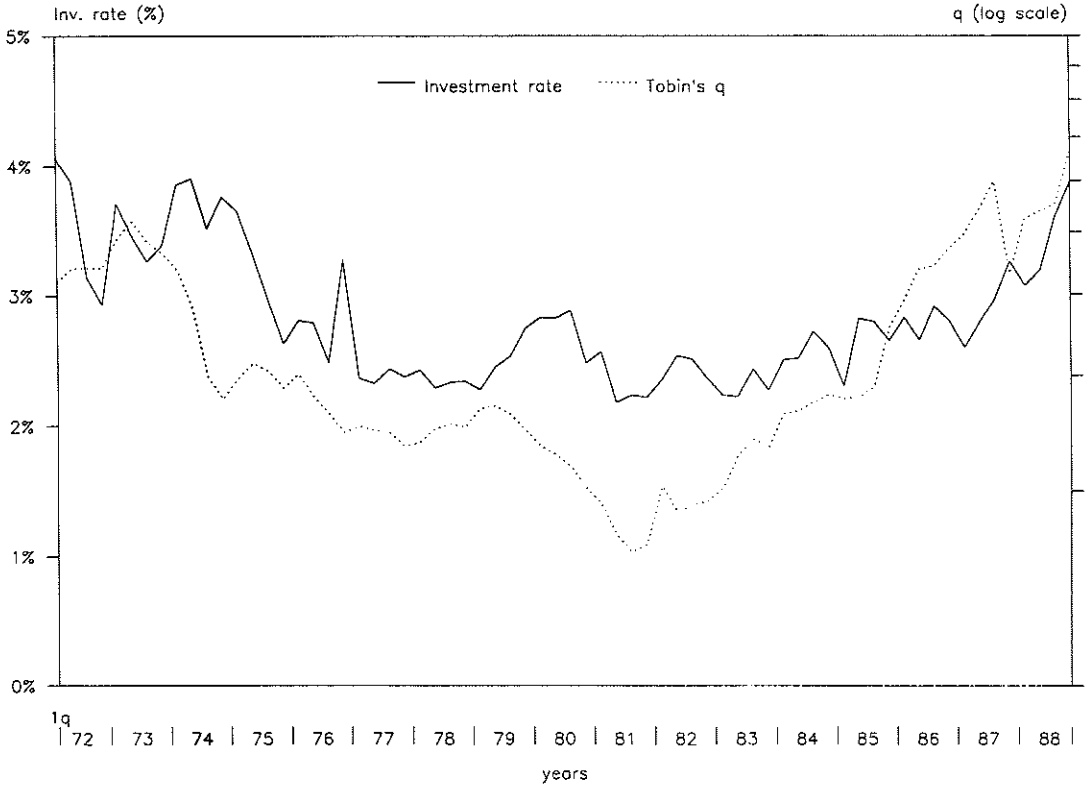
DESCRIPTIVE STATISTICS FOR BELGIUM

Before discussing the estimation results, we need to say something about the data. Data are quarterly and extend from 1972 to 1988. Figure 1 shows the evolution of the rate of industrial investment compared to the logarithm of average q[10]. The investment rate is defined as I(t)/K(t-1), where the capital stock has been computed from the investment series with a depreciation rate of 2% per quarter. Average q is measured by the stock market index of industrial values divided by an index of the output price of the investment goods sector. The figure shows that the general trend of investment seems to be well captured by the average q series; however, the short-term fluctuations of investment do not seem to be captured by the q variable.

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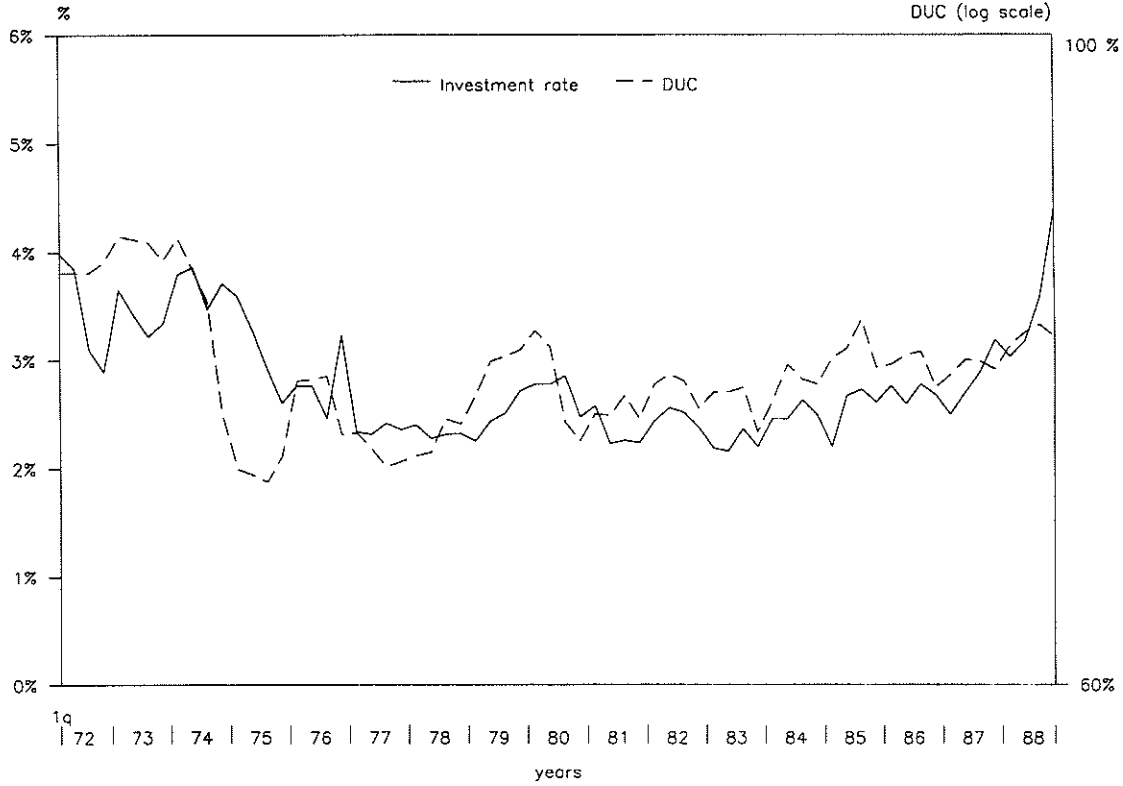
10 The series have been seasonally adjusted by a moving average in order to make the graphical presentation more readable. In the estimation, we use the raw series and the seasonal adjustment is performed with dummy variables.

Figure 1: Investment rate and Tobin's q



Note: Investment is seasonally adjusted

Figure 2: Investment rate and DUC



Note: Investment is seasonally adjusted

For the estimation of equation (13), we assume that the average q computed with stock market values is a good representation of the actual average q. This assumption implies that: (a) the stock market value of the firm is the best estimate of future average profits and, (b) the firm's managers look at the stock market to evaluate the average value of the firm when they must decide about investment.

Figure 2 compares the investment rate to the logarithm of the current DUC, as reported by the Belgian National Bank in its business surveys. Following equation (12), this variable will be used to compute the "mean potential output elasticity". The evolution of DUC is not far from that of the investment rate.

CAUSALITY AND COINTEGRATION TESTS

Prior to the estimation of the model, we check the existence and the direction of causality (in Granger's or Sims' sense) between our three variables. We estimate the equations of Table 1 where the length of the lagged endogenous variable is fixed at eight quarters and the possible Granger-causing variables are lagged four quarters. Table 1 presents the specification of the equations and the corresponding value of the logarithm of the likelihood function. Z represent the deterministic part of each equation (constant and seasonal dummies).

From equations (a) and (b) of Table 1, we see that the restrictions $\kappa_{2j}=0$, $\forall$ j=1to 4, are not accepted by the likelihood ratio test. Therefore, the introduction of ln(DUC) improves the predictions about the investment rate in a significant way so that ln(DUC) Granger-causes α. The comparison of the likelihood value of equations (a) and (c) leads to the same conclusions concerning average q: ln(Q) Granger-causes α.

With equations (d)-(e) and (f)-(g), we show that the introduction of the investment rate improves neither the predictions about ln(Q) nor the predictions about ln(DUC). The hypothesis $\kappa_{5j}=0$ and $\kappa_{7j}=0$, $\forall$ j=1 to 4, cannot be rejected by the likelihood ratio test at the significance level of 5%.

Therefore, the data indicate unidirectional causality from ln(DUC) and ln(Q) to the investment rate.

TABLE 1: CAUSALITY TESTS

likelihood		equation
274.4	(a)	$\alpha(t) = Z + \sum_{i=1}^{8} \gamma_{1i}\, \alpha(t-i)$
286.4	(b)	$\alpha(t) = Z + \sum_{i=1}^{8} \gamma_{2i}\, \alpha(t-i) + \sum_{j=1}^{4} \kappa_{2j} \ln[DUC(t-j)]$
281.9	(c)	$\alpha(t) = Z + \sum_{i=1}^{8} \gamma_{3i}\, \alpha(t-i) + \sum_{j=1}^{4} \kappa_{3j} \ln[Q(t-j)]$
69.2	(d)	$\ln[Q(t)] = Z + \sum_{i=1}^{8} \gamma_{4i}\, \ln[Q(t-i)]$
69.7	(e)	$\ln[Q(t)] = Z + \sum_{i=1}^{8} \gamma_{5i}\, \ln[Q(t-i)] + \sum_{j=1}^{4} \kappa_{5j}\, \alpha(t-j)$
174.5	(f)	$\ln[DUC(t)] = Z + \sum_{i=1}^{8} \gamma_{6i}\, \ln[DUC(t-i)]$
175.8	(g)	$\ln[DUC(t)] = Z + \sum_{i=1}^{8} \gamma_{7i} \ln[DUC(t-i)] + \sum_{j=1}^{4} \kappa_{7j} \alpha(t-j)$

In order to be non-explosive in the long run, equation (13) requires that the investment rate, the log of average q and the log of DUC be cointegrated[11]. It was first verified that these three variables were integrated of order 1. Regressing the change in each variable on their past level and two past changes (plus seasonal dummies) gave t-statistics on the past level of 0.1, 0.26 and -0.33 respectively. This shows that these variables are not stationary in levels. Running the same model with second differences on lagged first differences and two lags of second

[11] We use the methodology presented in Engle and Granger (1987) and Engle and Yoo (1987).

differences (plus seasonal dummies), the t-statistics on the lagged first difference were -3.68, -3.02 and -4.11. This indicates that the first differences of our three variables are stationary.

The cointegration regression of α on ln(Q), ln(DUC), a constant and seasonal dummies gives a Durbin-Watson test of 0.76, indicating that by the CRDW test cointegration is not rejected at the 5 per cent level. Regressing the change in the residuals on past levels and four lagged changes, the t statistic on the level is -2.32 which is below the critical value for the 5% augmented Dickey-Fuller test. Because the lags are not significant, the Dickey-Fuller regression was run, giving a t-stat of -3.48 which is significant. Therefore, we accept that α, ln(Q) and ln(DUC) are cointegrated.

ESTIMATION

Equation (13) is estimated with non-linear least squares for the period 1972:1-1988:4. In order to take into account the seasonal fluctuations of the investment series, we have added three seasonal dummies (d1, d2 and d3). The result is shown in Table 2. We have tested and accepted the restrictions on the parameters imposed by our equation (13) against the unconstrained model, which includes two lags for each variable[12] .

In order to well understand the relevance of this estimation, we will analyse the effects of the average q and DUC variables on the investment rate.

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[12] The unconstrained model reads:

$$\alpha(t) = 3\,\text{seasonal dummies} + \gamma_0 + \gamma_1 \ln(Q(t)) + \gamma_2 \ln(Q(t-1)) + \gamma_3 \ln(Q(t-2))$$
$$+ \gamma_4 \ln(DUC(t)) + \gamma_5 \ln(DUC(t-1)) + \gamma_6 \ln(DUC(t-2)) + \gamma_7 \alpha(t-1) + \gamma_8 \alpha(t-2)$$

The main statistics of this estimation are : Log of likelihood function = 300.479 , Adjusted R-Squared = 0.818 , Durbin T-stat for AR(1) = -0.1

Equation (13) imposes 3 restrictions on the parameters. The value of the likelihood ratio test is -2 (298.18 - 300.479) = 4.6 to be compared with the critical value of the Chi-squared distribution with 3 degrees of freedom, which is 7.815. The restrictions are therefore accepted.

TABLE 2: ESTIMATION

Parameter	Estimate	Standard Error	T-Statistic
a_0	0.038	0.0055	6.8
a_1	0.0069	0.0023	2.2
ρ	7.87	4.36	1.8
β_2	1.02	0.52	2.0
β_3	0.31	0.14	2.2
β_4	0.16	0.11	1.4
d1	-0.0059	0.0017	-3.5
d2	-0.0011	0.0016	-0.7
d3	-0.0021	0.0008	-2.3

Log of likelihood function = 298.18
Number of observations = 68
Standard error of the regression = 0.00323
Adjusted R-squared = .816
Durbin t-test = 1.23[13]
Chow test = 1.77[14]

Average q and investment

The coefficient a_1 associated with average q is significantly different from zero. By construction, the restrictions on the parameters of

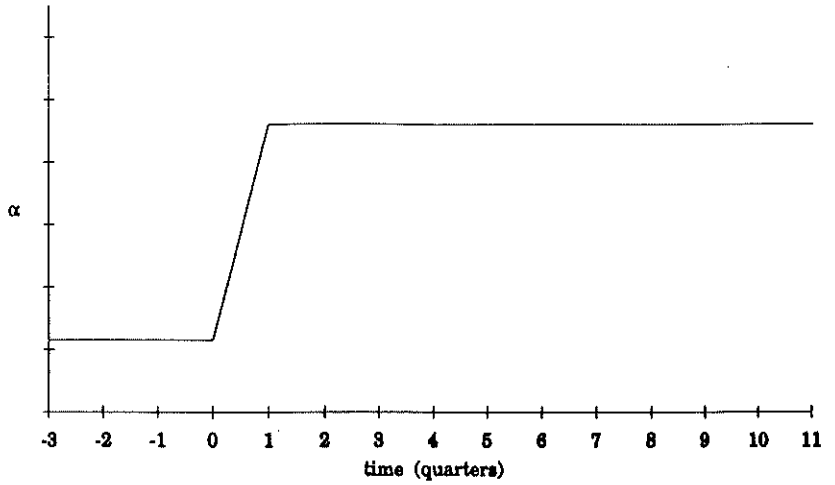
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13 In order to see if the residuals are time-independent, we have computed the Durbin (AR1) test by regressing the error term of the above equation on the lagged error term and all exogenous variables. The t-statistic of the lagged error term is 1.2, which allows us to reject the hypothesis that the residuals follow an AR(1) process. On the other hand, the error term is not explained by any of the exogenous variables; this is a sign that the restrictions we have imposed on the parameters (in equation (12)) are not too strong.

14 We have tested for structural instability by estimating the equation over two subperiods (71:1 - 79:4 and 80:1 - 88:4) and by computing the F-test for the constraint that the two sets of coefficients are equal (Chow test). We obtain a value of 1.77 which is smaller than the critical value 2.78 (F(9,50)). Therefore, this constraint is accepted.

equation (13) do not allow for any dynamic effects of average q on the investment rate. This is consistent with the assumptions that average q includes all available information about the future revenues of the firm. The dynamics of investment in the face of a variation in average q is illustrated by the Figure 3[15].

Figure 3: Reaction of the investment rate to an increase in average q at given DUC



DUC and investment

DUC influences the rate of investment through various parameters: the parameter ρ, which relates DUC with Φ_p and the parameters β, which are related with the dynamics of Φ^*_p.

First, what can we learn from the estimated ρ? As equation (3) states, ρ relates expectations about DUC with the elasticity Φ_p. In the

[15] In interpreting Figure 3 we must pay attention to the fact that average q cannot diverge from the steady state value for a long time if we take into account the full model and not only equation (13).

same way as in Lambert (1988), let us interpret Φ_p as the weighted probability for the firm of being capacity constrained. The particular case when the weighted probability of a demand constraint is equal to the weighted probability of a capacity constraint,i.e., Φ_p=0.5, corresponds to an expected DUC equal to 0.916. As we see in the appendix, the DUC series is always below 0.916. This implies that the weighted probability of being demand-constrained is larger than the weighted probability of being capacity-contrained. The observed importance of the expected demand constraints is linked with a small "mean potential output elasticity" Φ^*_p, and it is at the root of the bad performance of Belgian investment in the last two decades[16].

Finally, the dynamic effect of DUC on investment depends on Φ^*_p and on the specification of the adjustment cost function. The effect of an exogenous permanent increase in DUC on the investment rate is twofold (see Figure 4):

(1) *Adjustment cost effect.* As commented before, the restrictions imposed on the adjustment cost function imply that DUC has a direct negative effect on investment.

(2) *Demand effect.* By equation (12) an increase in DUC leads the firm to increase Φ^*_p, This implies a positive net effect on investment. In fact, the dynamic pattern described by the β coefficients tells us that the effect is negative in the very short run, but the total effect becomes positive after one period.

Because ρ is larger than 1, the second effect dominates. After two years (8 quarters), the rise in DUC is fully passed on to expectations. A permanent increase in DUC has a small permanent negative effect through the increase of adjustment costs and a large positive effect through Φ^*_p.

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16 The importance of demand constraints in the Belgian business cycle has already been stressed by Sneessens and Drèze (1986). Moreover, Mehta and Sneessens (1990) and Drèze (1990) emphasise the role of DUC in explaining investment behaviour in Belgium.

Figure 4: Reaction of investment to an increase in the DUC at given q

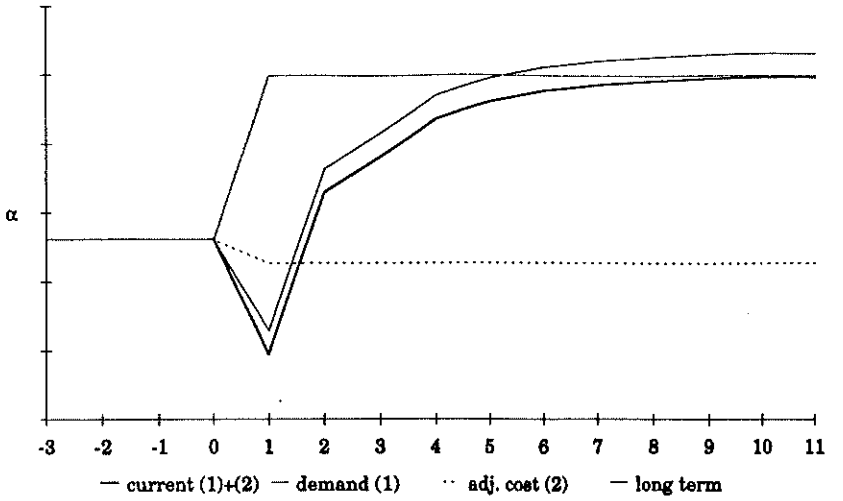
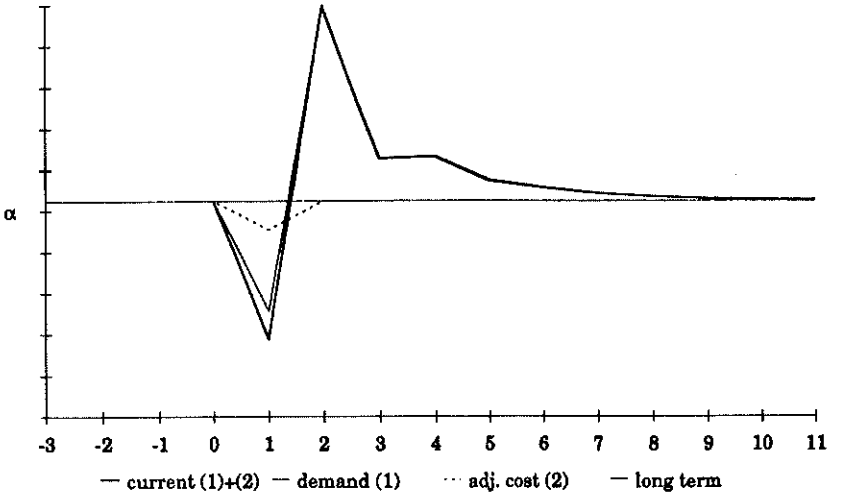


Figure 5: Reaction of investment to a transitory increase in DUC



If the shock on DUC is only transitory, the long-run value of the investment rate does not change. However, the short-run investment rate decreases at the time of the shock; one quarter later, we observe an overshooting process where the investment rate increases more than necessary to converge to the long-run level. Figure 5 shows the effect of a transitory variation of DUC at time 1 on the investment rate.

ALTERNATIVE SPECIFICATIONS

The dynamic structure of the investment equation we have estimated in the previous section comes from the expectation process about the DUC variable. We now try out four alternative specifications based on more usual assumptions in investment theory[17].

(a) THE PURE CLASSICAL MODEL: the present value of q contains all information about *marginal* profit.

$$\alpha(t) = a_0 + a_1 \ln[Q(t)] \tag{14}$$

The results are presented in Table 3. We see that q alone explains 66 percent of the variations in the investment rate, but the residuals are highly autocorrelated.

(b) THE PURE CLASSICAL MODEL WITH DELIVERY LAGS: There are some "delivery lags" for the investment goods or, alternatively, investment takes "time to build." Under these conditions the investment rate does not adjust instantaneously to its optimal level, which is defined by equation (14). Let us apply an error correction mechanism to explain and to test the dynamics of the equation (14):

$$\Delta\alpha(t) = \lambda_1 \left[a_1 \Delta \ln(Q(t))\right] + \lambda_2 \left[a_0 + a_1 \ln(Q(t-1)) - \alpha(t-1)\right] \tag{15}$$

As shown in Table 3, the error correction mechanism improves both the

---

[17] Note that the three seasonal dummies are always present in the following equations, without being mentioned.

likelihood and the autocorrelation of the residuals because it introduces some dynamics in the model. However, the parameter which measures the adjustment to objective speed is very low (0.06). This is partially compensated by a high value of the error correction parameter. The likelihood ratio test between equations (15) and the general form in footnote 12 allows us to reject the restrictions of equation (15).

(c) INTRODUCTION OF DUC: we assume that the DUC reported by the National Bank is a good proxy for Φ^*_p, so that $\ln(\Phi^*_p(t)) = \rho \ln(DUC(t))$. Under this assumption, equation (11) becomes

$$\alpha(t) = a_0 + a_1 \ln(Q(t)) + a_2 \ln(DUC(t)) \tag{16}$$

$$\text{where} \quad a_2 = a_1(\rho - 1).$$

The introduction of the DUC variable improves the log of likelihood significantly[18] but the autocorrelation of the residuals remains unchanged.

(d) INTRODUCTION OF DUC WITH DELIVERY LAGS: we apply an error correction mechanism on equation (16) to take into account "delivery lags" or "time-to-build" assumptions. The restrictions imposed here are of course different from the ones we have imposed in the model of the previous section.

$$\Delta\alpha(t) = \lambda_1 \left[a_1 \Delta\ln(Q(t)) + a_2 \Delta\ln(DUC(t))\right] \tag{17}$$

$$+\lambda_2 \left[a_0 + a_1 \ln(Q(t-1)) + a_2 \ln(DUC(t-1)) - \alpha(t-1)\right]$$

Contrary to equation (15) the likelihood ratio test between equation (17) and the general form does not allow us to reject the restrictions of equation (17) Thus, the restriction that removes DUC from equation (17) is not accepted by the data.

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[18] The likelihood ratio test between (14) and (16) allows us to reject the restriction that DUC is not important in explaining investment (a_2=0 in equation (16)).

However, the adjustment speed parameter is negative, which is very difficult to justify in economic terms.

TABLE 3: ALTERNATIVE SPECIFICATIONS

Equation	a_0	a_1	a_2	λ_1	λ_2	$\bar{R}^2$	L.F.	D.W.
(14)	.024	.01				66%	275.5	.71
	(29)	(6.7)						
(15)	.023	.013		.06	.45	80%	294.3	1.9
	(16)	(4.7)		(.17)	(5)			
(16)	.032	.0078	.027			68%	277.7	.76
	(7.9)	(4.2)	(2.04)					
(17)	.036	.0081	.044	-.33	.55	81%	297.2	1.8
	(6.8)	(3.1)	(2.5)	(-.8)	(5.7)			

Comparing the error correction model (17) with the model of the previous section, we see that the negativity of the adjustment speed comes from the DUC variable, which seems to be negatively correlated with the contemporaneous investment rate. As the error correction mechanism imposes the *same* dynamic scheme for each of the explanatory variables, it imposes a negative short-term elasticity between average q and the investment rate if the influence of DUC is dominating. This problem is avoided in the model of the previous section since there it was assumed that the dynamics comes only from Φ^*_p. In that case, the impact of a change in average q does not lead to lagged effects.

CONCLUSION

This paper evaluates the relevance of q theories under stochastic demand and capacity constraints by estimating an investment function for Belgium.

In the first part, we developed a theoretical model adapted from Licandro (1990a). The main consequence of demand uncertainty is that capacities can be underutilised, implying that the elasticity of expected production to expected capacities is below unity. For this reason, when we combine the adjustment cost hypothesis with demand uncertainty and capacity constraints, marginal q differs from average q by the mean value of all future elasticities of expected production to expected capacities, called "mean potential output elasticity". Since this elasticity is a positive function of the expected degree of capacity utilisation (DUC), the investment rate depends on average q and on the expectations about DUC. In particular, it implies that expectations about DUC have a positive effect on investment. When the firm expects to make more intensive use of its capacity ,i.e., when DUC is rising, investment becomes more profitable.

In the second part, we give an empirical form to the investment rate equation. As in the neoclassical q models, average q contains all the information about expected average profits. However, as stated above, marginal q differs from average q by the "mean potential output elasticity." The expectations about the "mean potential output elasticity" depend on present and past values of DUC, allowing for a dynamic specification of the investment equation. After checking for causality and cointegration between investment rate, average q and DUC, we estimated an investment equation on quarterly data for Belgian manufacturing. The parameter estimates stress the importance of demand constraints in explaining the bad performance of Belgian investment during the last two decades. The estimation of alternative specifications show that the

dynamic scheme based on expectations about DUC works better than the one based on more usual delivery lags assumptions which imposes the identical dynamics to both q and DUC.

As a consequence, demand uncertainty and capacity constraints allow for the introduction of DUC as a new explicative variable in investment theory. They also provide a theoretical justification of the inclusion of lagged average q and autocorrelated residuals in existing empirical investment equations.

However, if we want to fully take into account the effect of q and DUC on investment, we have to endogenise these variables, as they could depend on each other. Licandro (1990b) stresses the influence of forecasts about the real economy on the equilibrium price of financial assets, through which average q and DUC are related. For this reason we believe that a general equilibrium approach to investment, under demand uncertainy and capacity constraints, could be fruitful in the future.

APPENDIX

date	α	Q	DUC	date	α	Q	DUC
1971:2	0.04283	2.16201	0.832	1980:1	0.02375	1.17180	0.793
1971:3	0.04732	2.15221	0.826	1980:2	0.02713	1.13687	0.783
1971:4	0.05922	1.94770	0.817	1980:3	0.02607	1.09037	0.737
1972:1	0.03411	2.09100	0.830	1980:4	0.03232	1.01213	0.726
1972:2	0.03754	2.19416	0.830	1981:1	0.02199	0.95591	0.742
1972:3	0.02826	2.19642	0.830	1981:2	0.02167	0.85665	0.741
1972:4	0.03790	2.19697	0.837	1981:3	0.02049	0.80531	0.753
1973:1	0.03124	2.43455	0.854	1981:4	0.02919	0.82576	0.739
1973:2	0.03344	2.59582	0.852	1982:1	0.02072	1.01758	0.760
1973:3	0.02938	2.41907	0.850	1982:2	0.02493	0.93025	0.766
1973:4	0.04387	2.31936	0.838	1982:3	0.02288	0.95271	0.762
1974:1	0.03247	2.18557	0.853	1982:4	0.03102	0.96658	0.745
1974:2	0.03771	1.91584	0.832	1983:1	0.01861	1.01121	0.755
1974:3	0.03171	1.49738	0.811	1983:2	0.02099	1.13446	0.755
1974:4	0.04875	1.38400	0.741	1983:3	0.02145	1.19981	0.758
1975:1	0.03076	1.48867	0.711	1983:4	0.02870	1.16820	0.731
1975:2	0.03208	1.56872	0.708	1984:1	0.02094	1.31054	0.750
1975:3	0.02669	1.52482	0.704	1984:2	0.02386	1.32532	0.772
1975:4	0.03408	1.43764	0.718	1984:3	0.02396	1.36812	0.763
1976:1	0.02363	1.50926	0.762	1984:4	0.03258	1.40275	0.760
1976:2	0.02696	1.39621	0.763	1985:1	0.01871	1.38507	0.776
1976:3	0.02238	1.31253	0.765	1985:2	0.02603	1.39299	0.782
1976:4	0.04247	1.22947	0.730	1985:3	0.02489	1.43988	0.800
1977:1	0.01991	1.25526	0.731	1985:4	0.03416	1.77545	0.770
1977:2	0.02248	1.23739	0.722	1986:1	0.02362	1.96300	0.772
1977:3	0.02196	1.22864	0.712	1986:2	0.02526	2.18927	0.778
1977:4	0.03079	1.17226	0.715	1986:3	0.02535	2.21329	0.780
1978:1	0.02044	1.18836	0.718	1986:4	0.03508	2.36097	0.758
1978:2	0.02212	1.24403	0.720	1987:1	0.02129	2.47548	0.766
1978:3	0.02102	1.26556	0.739	1987:2	0.02637	2.70891	0.775
1978:4	0.03034	1.25107	0.736	1987:3	0.02642	2.99407	0.774
1979:1	0.01917	1.33680	0.753	1987:4	0.04191	2.15024	0.769
1979:2	0.02369	1.34920	0.774	1988:1	0.02596	2.61647	0.784
1979:3	0.02283	1.30850	0.777	1988:2	0.03108	2.69201	0.792
1979:4	0.03561	1.23659	0.781	1988:3	0.03274	2.75729	0.797
				1988:4	0.05771	3.33470	0.789

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