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ORIGINAL ARTICLE

Detecting communities with the multi-scale Louvain method: robustness test on the metropolitan area of Brussels

Arnaud Adam[1] · **Jean-Charles Delvenne**[2] · **Isabelle Thomas**[1]

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Abstract
Detecting communities in large networks has become a common practice in socio-spatial analyses and has led to the development of numerous dedicated mathematical algorithms. Nowadays, however, researchers face a deluge of data and algorithms, and great care must be taken regarding methodological questions such as the values of the parameters and the geographical characteristics of the data. We aim here at testing the sensitivity of multi-scale modularity optimized by the Louvain method to the value of the resolution parameter (introduced by Reichardt and Bornholdt (Phys Rev Lett 93(21):218701, 2004. https://doi.org/10.1103/PhysRevLett.93.218701) and controlling the size of the communities) and to a number of spatial issues such as the inclusion of internal loops and the delineation of the study area. We compare the community structures with those found by another well-known community detection algorithm (Infomap), and we further interpret the final results in terms of urban geography. Sensitivity analyses are conducted for commuting movements in and around Brussels. Results reveal slight effects of spatial issues (inclusion of the internal loops, definition of the study area) on the partition into job basins, while the resolution parameter plays a major role in the final results and their interpretation in terms of urban geography. Community detection methods seem to reveal a surprisingly strong spatial effect of commuting patterns: Similar partitions are obtained with different methods. This paper highlights the advantages and sensitivities of the multi-scale Louvain method and more particularly of defining communities of places. Despite these sensitivities, the method proves to be a valuable tool for geographers and planners.

Keywords Community detection · Sensitivity analyses · Urban planning · Commuting movements

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✉ Arnaud Adam
a.adam@uclouvain.be

Extended author information available on the last page of the article

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Abbreviations

Aud	Auderghem
And	Anderlecht
B	City of Brussels (municipality)
BCR	Brussels Capital Region
ET	Etterbeek
EVE	Evere
For	Forest
IX	Ixelles
Mol	Molenbeek-Saint-Jean
NMI	Normalized mutual information
Ottignies	Ottignies Louvain-la-Neuve
S	Schaerbeeck
SG	Saint-Gilles
SJ	Saint-Josse-ten-Noode
U	Uccle
VI	Variation of information
WL	Woluwe-Saint-Lambert
WP	Woluwe-Saint-Pierre
Z	Zaventem

1 Introduction

Urban areas concentrate a large number and a great diversity of people, buildings, activities, and functions. Consequently, interactions are numerous, intense, complex, and vary in nature (Scott and Storper 2015; Fujita and Thisse 2013; Anas et al. 1998). Quantitative geographers often measure urban realities as a set of characteristics X associated with places i and/or as a set of interactions W between places i and j. Both data types have led to the development of methods and theories for determining the delineation and the composition of urban areas. While urban factorial ecology was early and rapidly accepted as a standard approach for clustering places with similar characteristics X_i that fitted well with classical urban models (Hoyt, Burgess, see e.g., Kitchin and Tate 1999), there is less methodological consensus about the delineation of the *interaction fields* W_{ij} (see Fujita and Thisse 2013) required for defining urban boundaries or for identifying specific intra-urban basins such as job basins (Duranton 2015; Dujardin et al. 2007). A job basin pertains to only one facet of the interaction fields. It corresponds to a group of places having a strong economic relation or linked between by intense commuting (residence to place of work) and it is generally dominated by one or to multiple centers (Géoconfluences 2017). Job basins are generally delineated on the basis of origin–destination flow matrices (O'Kelly et al. 2012; Li and Farber 2016).

The delineation of *interaction fields* has been the subject of numerous publications, covering methods from the simple a priori defined threshold up to interaction models with sophisticated parameters (for a critical review; Haynes and Fotheringham 1984; Jacobs 1969; Dujardin et al. 2007). For example, the interactions between places can be classically clustered based on graph theory and hierarchical clustering; many methods, such as the Anabel method developed by INSEE in France (INSEE 2015) have been applied in regional research and in public administration. This method iteratively groups municipalities together based on the highest flow found in the transition matrix. At a larger scale, communication patterns between regions have been modeled by analyzing phone calls with techniques such as Intramax and IPFP approaches (Fischer et al. 1993).

Although these methods show interesting results in terms of spatial organization, the partitioning is deterministic and may be sensitive to the thresholds or the choices to be made during the implementation. Instead of producing dendrograms that visualize the *interaction patterns*, we aim to detect the organization of a network into groups of nodes (Fortunato 2009). Indeed, a network is a set of nodes linked together by edges that can be weighted (for instance by the number of commuters between places), and a community is a set of highly connected nodes that are more intensively linked together than with the rest of the network (Danon et al. 2005; Fortunato and Lancichinetti 2009). Such communities at different resolution levels can be identified by means of a new generation of algorithms. Because of their simplicity and the availability of new data sources (sensors, big data, ICT data), the interest in these methods has increased with the application in various research fields such as physics and mathematics (for a general review, please read Fortunato 2009), socioeconomics (Beguerisse-Díaz et al. 2014; Cerina et al. 2014; Crampton et al. 2013; Onnela et al. 2011; Papadopoulos et al. 2011), and spatial planning (Blondel et al. 2010; Chi et al. 2016; Montis et al. 2013; Ratti et al. 2010; Jones et al. 2016).

This paper relies on one of the most widely used community detection methods (the multi-scale Louvain method), and tests its usefulness for delineating interaction fields as well as regional science topic and, more precisely, job basins within the Brussels metropolitan area. The Louvain method does not require any information about job centers and allows researchers to analyze communities of commuting flows without relying on spatial information. No a priori center is here defined, and spatial information is not used as an input for these community detection methods (such as the localization of places).

Moreover, only the total flow between places is considered here, and not the direction of flows. Let us consider two locations, m and n: If the flow between m and n is equal to 50 and the flow between n and m corresponds to 90, we here consider only the total exchange (140). The contribution of this paper is threefold: to test the robustness of the Louvain method to some crucial inputs; to illustrate the potential of the method for urban delineation in the Brussels metropolitan area; and to, finally, compare the results with an alternative method, called Infomap. For this purpose, sensitivity analyses are performed with respect to the resolution parameter (Reichardt and Bornholdt 2004), the effect of the inclusion (or not) of internal loops in the data set, and extending of the study area. Computations are based on commuting flows in and around Brussels (Belgium), and results are further commented in terms of well-known Belgian geographical issues. We show that distance still matters (Ioannides 2012) and that nearby places are more strongly related than the distant places. This paper is organized as follows: Methodological issues and data choices are dealt with in Sect. 2; Sect. 3 presents and discusses the results, while Sect. 4 provides conclusions.

2 Methodology and data

2.1 Community detection

Many recent community detection algorithms are based on modularity maximization (Newman and Girvan 2004) and are applied to networks, allowing the detection of structures consisting of nodes linked together by (un)weighted edges. The higher the weight between two nodes, the more they are related and the more the modularity will increase by classifying these two nodes in the same community. A community is hence a group of tightly connected nodes. To quantify the quality of a partition into communities, modularity compares how densely the nodes are connected within the communities in comparison with a random null model. Indeed, modularity is the sum of the weighted edges inside each community minus the number of expected edges between the same nodes in a random null model with the same degree of distribution and the same number of edges (Newman and Girvan 2004). Modularity therefore quantifies how densely the communities are connected, with respect to the density of edges that would be expected of the same group of nodes in a null model where nodes are linked randomly while maintaining the same degree.

Using modularity as an objective function produces an unknown number of communities of homogeneous size where the size is the sum of all the weighted edges classified in the community and does not refer to the number of nodes (Fortunato and Barthélemy 2007; Delvenne et al. 2010). Delvenne et al. (2013) reinterpret modularity (Q) by writing simplified Eq. 1:

$$Q = Diversity - Cut, \tag{1}$$

$$Diversity = 1 - \sum_{c \in P} \left(\frac{in[c]}{2m} \right)^2, \tag{2}$$

$$Cut = 1 - \frac{in[c]}{2m}, \tag{3}$$

where P is a partition (a set of communities c), $in[c]$ is the sum of all the weighted edges totally included in community c, and m is the sum of all the weighted edges of the network. More precisely, determining the organization of the network into communities, modularity (Eq. 1) aims minimizing the *Cut* size (the total weighted edges between communities) while maximizing the *Diversity* index (the number of communities of homogeneous size detected) that are two opposed objectives (see Delvenne et al. 2013). Minimization of the *Cut* size would lead to one single community including all the nodes of the network, while the maximization of the *Diversity*

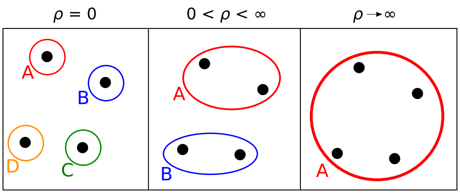


Fig. 1 Partitions detected using different values of ρ, in multi-scale modularity (A, B, C, D communities)

index provides a partition of nodes into one community. A main drawback of the modularity function is the *resolution limit* (Fortunato and Barthélemy 2007). This limit is due to the *trade-off* made between maximization of the *Diversity* and minimization of the *Cut*. In some cases, depending on the number of nodes and edges, some small communities are incorporated into larger communities although these are only linked by one edge.

The Louvain method is a greedy heuristic that searches for a maximum modularity partition of the nodes in successive steps, which are briefly described as follows (for more details, see Blondel et al. 2008). The method starts with an initial partition where each node is a community: The number of communities (Z) equals the number of nodes (N). Then, the method merges the nodes that are highly related in successive steps and creates communities that increase the value of the modularity. These merges are made until a local maximum of modularity is reached.

This paper proposes to deal with a modified version of the Louvain method[1] to evaluate the multi-scale component of the modularity. Reichardt and Bornholdt (2004) introduce a resolution parameter ρ in Eq. 4 to influence the maximization of modularity. This modification allows the detection of partitions composed of communities of different sizes. The modified modularity Q_ρ is given by

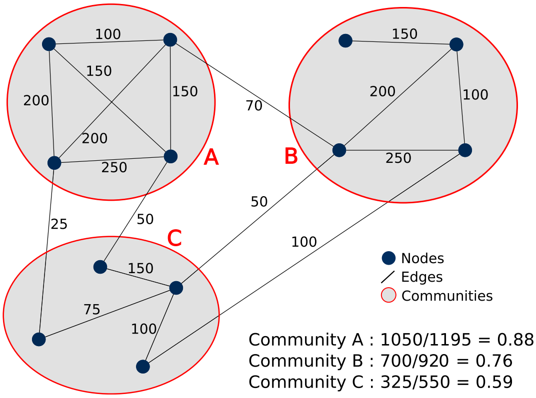
$$Q_\rho = Diversity - \rho\, Cut. \tag{4}$$

For Lambiotte (2010), the parameter ρ is seen as a value of time for modularity to group together the highly linked nodes. The effect of the use of different values of ρ is illustrated in Fig. 1. With a zero value, there is no gain of modularity and the communities detected are those of the initialization step ($Z = N$). With a high value of ρ, modularity always increases with each merge and the Louvain method detects one single large community including all nodes ($Z = 1$). When intermediate values of ρ are used, Z ranges between N and one. Let us note that when ρ is equal to one, the partition corresponds to the one detected endogenously by the original Louvain method.

Although the method can detect communities of different sizes, the problem of the resolution limit is still present and using modularity as a general indicator of the *quality of the partition* (Lambiotte et al. 2014; Steinhaeuser and Chawla 2010) only gives a value of quality for the entire partition and not for each community. To

[1] For the sake of clarity, the term Louvain method used in this contribution refers to the multi-scale Louvain method.

Fig. 2 Compactness of communities



describe each community independently, we suggest to calculate a measure evaluating the *Cut* (Eq. 4) which we here call *compactness*. The *compactness* F of a community c is defined as:

$$F(c) = \frac{in[c]}{all[c]}, \tag{5}$$

where $F(c)$ evaluates the ratio between the sum of the weighted edges included within a community c and the total weighted edges of this community c (Fig. 2). It ranges from zero to one, where a value close to zero expresses that a community has more edges going to other communities, and a value close to one reveals that the community is self-centered and densely connected. This indicator provides information about the connections of a community to the rest of the network.

Last but not least, let us point out three major characteristics of modularity. First, all edges have an influence on the detection of communities without taking into account the spatial location of the nodes; only the relational information is used. Second, the method detects endogenously the number of communities and the size of these structures tends to be homogeneous. Finally, the partitions obtained are sensitive to the order in which the nodes are considered by the method. Thus, if the order of the nodes is not changed, the Louvain method gives the same deterministic results; sensitivity to the change of order is hence a way of testing the robustness of the results. In order to detect robust communities, we run the method 1000 times, randomly shuffle the order of the nodes, and extract the dominant structure from the multiple partitions obtained. The applied methodology (see Appendix for more details) allows to calculate the instability of the classification of the nodes. The instability corresponds to the number of times that the nodes are not classified in their dominant community during the 1000 runs of the Louvain method. Instability moderates the classification of municipalities into communities. Some municipalities do not have a clear classification, indicating that a tendency in terms of commuting movements is fuzzier.

2.2 Data

Belgium is a small and densely inhabited European country (11 M inhabitants and 30,528 km^2), with a strong urban hierarchy (Van Hecke et al. 2009) dominated by Brussels. Belgium is made up of three administrative regions separated by a linguistic border: the Flemish (Dutch-speaking), the Walloon (French-speaking), and the Brussels Capital Region (bilingual). The Brussels Capital Region (BCR) contains 19 municipalities corresponding to the core of the city, and sprawls over a large territory. The BCR extends beyond its administrative border, leading to extensive debate about the delineation of the Brussels metropolitan area (Thomas et al. 2012). In order to avoid this discussion of the limits of the study area, this paper focuses on the former province of Brabant; that is to say, the 19 municipalities of the BCR and 92 municipalities located around the BCR, in Flanders (north of the linguistic border) and in Wallonia (south) (Fig. 3).

Let us consider commuting movements in 2011 (Census 2011 2016). Due to data limitation, there is no information on commuting flows from locations outside of Belgium. The nodes represent the 589 municipalities of the country, and the edges are weighted by the number of commuters between pairs of nodes. The BCR offers job opportunities attracting commuters from all over the country (Riguelle et al. 2007). More specifically, many workers living in the province of Brabant commute to the BCR for work: 32 percent of workers living in Flemish Brabant work in the BCR (more than 0.32 M), 35 percent of workers living in the Walloon Brabant

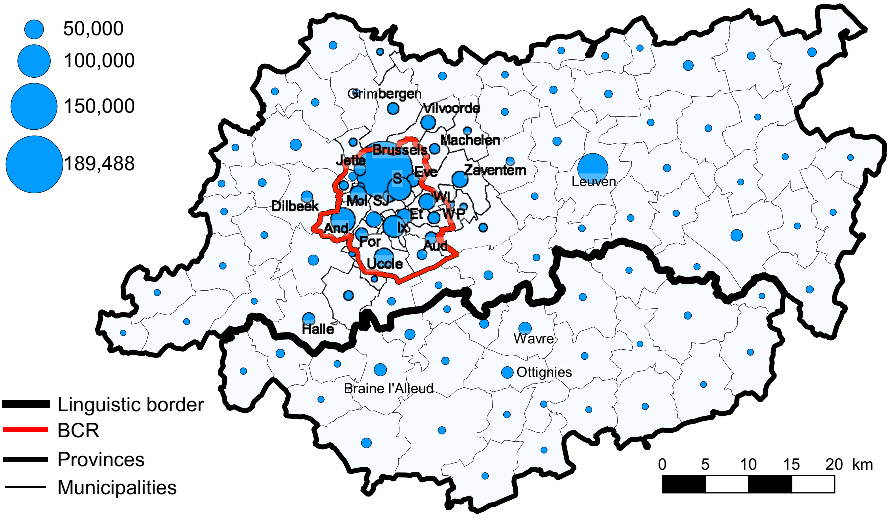


Fig. 3 Weighted degree of the municipalities within the former province of Brabant (including self-loops)

Table 1 Weighted degree size-ranked municipalities within the former province of Brabant

Municipality	With self-loops		Without self-loops	
Bruxelles	189488	(1)	145910	(1)
Leuven	91837	(2)	49931	(2)
Anderlecht	69127	(3)	49267	(3)
Schaerbeek	65718	(4)	48310	(4)
Ixelles	59274	(5)	43166	(5)
Uccle	50859	(6)	32569	(6)
Molenbeek-Saint-Jean	43228	(7)	32320	(7)
Zaventem	38405	(8)	31557	(8)
Woluwe-Saint-Lambert	37867	(9)	28625	(9)
Saint-Gilles	35559	(10)	26899	(11)
Etterbeek	34517	(11)	27229	(10)

commute to the BCR (more than 0.13 M), and finally, 87.4 percent of workers living in the BCR work in the region (more than 0.32 M) (Census 2011 2016).

By analyzing the communities of commuters in the province of Brabant, we aim to delineate job basins in and around Brussels. As explained in Introduction, a job basin is a group of municipalities linked together by strong commuting (residence–place of work) (Géoconfluences 2017). Figure 4 illustrates for two municipalities (one of the 19 within the BCR, the city of Brussels, and the municipality of Leuven) the proportion of workers who commute from other municipalities to reach these specific places for work. On the one hand, a large proportion of workers in the city of Brussels live in the numerous municipalities located within or close to the BCR, as well as in the Flemish and the Walloon municipalities. On the other hand, workers in the municipality of Leuven have a residence close to this center in the eastern part of the former province of Brabant. Figure 4 illustrates two distinct spatial residential choices based on the place of work.

As already explained above, the Louvain method delineates communities without a priori definition of a center. To quantify the centrality of places, the weighted

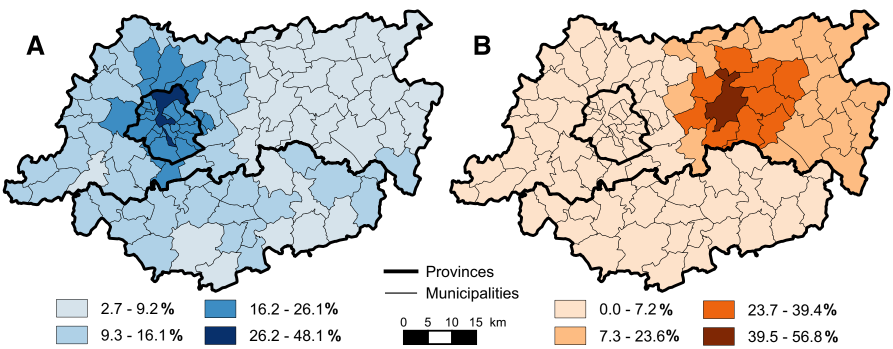


Fig. 4 Proportion of in-commuters working in the municipality of Brussels (**A**) and in the municipality of Leuven (**B**)

degree of each node is calculated. The weighted degree of a municipality i is the sum of all the weighted edges of this node (Freeman 1978). In other words, the weighted degree represents the total number of commuters arriving at and leaving from a municipality as well as the intra-municipal workers. These intra-municipal workers (internal loops) correspond to what is called here “self-loops”. As expected, Fig. 3 shows that the highest weighted degree values are located within the BCR as well as in the secondary centers (Leuven, Zaventem, Wavre) in accordance with the urban sprawl mechanism observed in that area (Riguelle et al. 2007). To evaluate the importance of the self-loops in this commuting network, we calculate the weighted degree of the municipalities when all the self-loops are erased and we notice that the spatial distribution of the weighted degree is similar to the distribution obtained with the inclusion of self-loops. Table 1 shows that the municipalities located in or close to the BCR (Bruxelles and Zaventem) have a smaller loss of weighted degree (23.0% and 17.8%) than the municipalities further away such as Leuven (46.6%). Indeed, municipalities located close to or within the BCR are less influenced by intra-municipal workers because many workers commute from places outside of the BCR (Cheshire 2010; Verhetsel et al. 2010; Thisse and Thomas 2010).

To categorize the municipalities in two groups, the Jenks classification is applied on the weighted degree distribution. The classes are defined according to the distribution of the data values, and the limits of the classes are based on break points of the distribution; it minimizes the internal variance of classes and maximizes the variance between classes (Longley et al. 2010). The municipalities characterized by the highest values of weighted degree are represented on the figures with black dots located at the center of these places. In Fig. 3, the highest values of weighted degree (Jenks classification) are highlighted by their names. They correspond to employment centers and edge cities (Riguelle et al. 2007), but not all small urban centers appear (Tienen, Nivelles, Asse, etc.).

2.3 Sensitivity analyses

First, we try to understand and measure how far ρ affects the partitions. Second, we evaluate the impact of the inclusion (or not) of the self-loops (intra-municipal commuters) in the data on the partitions and, third, we estimate the effect of isolating the province of Brabant from the rest of Belgium (that is to say, considering 111 municipalities instead of 589). Each analysis is run 1000 times, and the order of nodes is randomly shuffled for each iteration. These two points guarantee the robustness of the structure and allow the detection of the unstable municipalities (places that are not always classified in the same community through the 1000 runs). Each municipality is hence characterized by a percentage, representing the relative number of times it is classified in its dominant community. A value of 100 percent corresponds to a fully stable structure. We here decide that the municipalities classified less than 95 percent with their dominant community are represented on the maps with white hatches to allow comparison over the various maps.

To quantify the comparison between two partitions, we use the normalized mutual information criterion (*NMI*, Strehl and Ghosh 2002), defined as

$$NMI(\omega, \upsilon) = \frac{I(\omega, \upsilon)}{(H(\omega) + H(\upsilon))/2}, \tag{6}$$

where ω and υ are two partitions, $I(\omega, \upsilon) = H(\omega) + H(\upsilon) - H(\omega, \upsilon)$, with $H(\omega)$ and $H(\upsilon)$ representing the Shannon entropy of the partitions ω and υ, respectively, while $H(\omega, \upsilon)$ is the Shannon entropy of the joint probability. *NMI* returns a value of similarity that ranges between zero and one. A value close to one expresses a very good similarity between the two partitions.

3 Results

The sensitivity of the Louvain method is first presented for the variation of ρ, secondly for the inclusion or not of self-loops, and finally for using two different study areas. For the sake of clarity, we only compare partitions with a same number of communities in Sects. 3.2 and 3.3.

3.1 Effect of parameter ρ

We apply the linearized stability procedure developed by Delmotte et al. (2011) to select a set of values of ρ used for the sensitivity analysis. This method tests the robustness of the communities found for various values of ρ (range from 0.1 to 10.0 with a step of 0.1) by applying the Louvain method several times. The stability method thus provides multiple partitions for each ρ. Delvenne et al. (2013) explain that a robust community structure is persistent through the multiple partitions found for each ρ and can be observed (a) when the *variation of information* (*VI*) is minimal between the partitions made with a specific value of ρ or, (b) when the curve of the number of communities detected has a plateau shape. *VI* between two distinct partitions ω and υ is defined as:

$$VI(\omega, \upsilon) = H(\omega, \upsilon) - I(\omega, \upsilon). \tag{7}$$

Hence, a low value of *VI* indicates that two partitions are very close to each other and the structure of the communities to be robust to varying ρ. Let us note that *NMI* and *VI* (Eqs. 6 and 7) both aim to test the mutual information shared by two partitions (minimal for the *VI* and close to one for the *NMI*).

Figure 5 illustrates the use of the stability method for the former province of Brabant including the intra-municipal movements (self-loops). In this case, we propose to select values of ρ that provide different characteristics: two values with the smallest *VI* (0.7 and 1.5), two values on a constant slope (0.5 and 0.9), and two breaks of the slope (1.0 and 2.0). For each of these values, communities are detected, analyzed, and mapped (Fig. 6). The color used to discriminate the communities are chosen arbitrarily, and the municipalities distinguished by white hatches correspond to places that are classified as less than 95 percent of their dominant community.

As mentioned above, we run 1000 times the Louvain method for the selected values of ρ to obtain 1000 partitions. We find that the number of communities detected

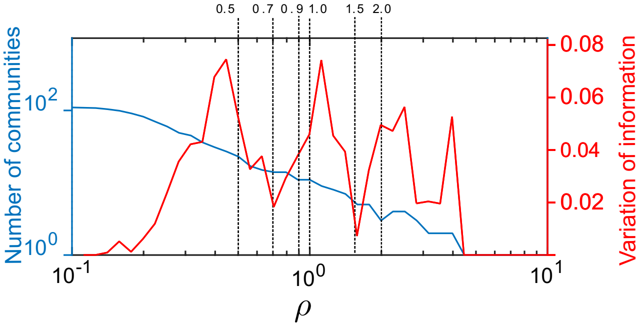


Fig. 5 Stability method: variation of information and number of communities detected by varying ρ

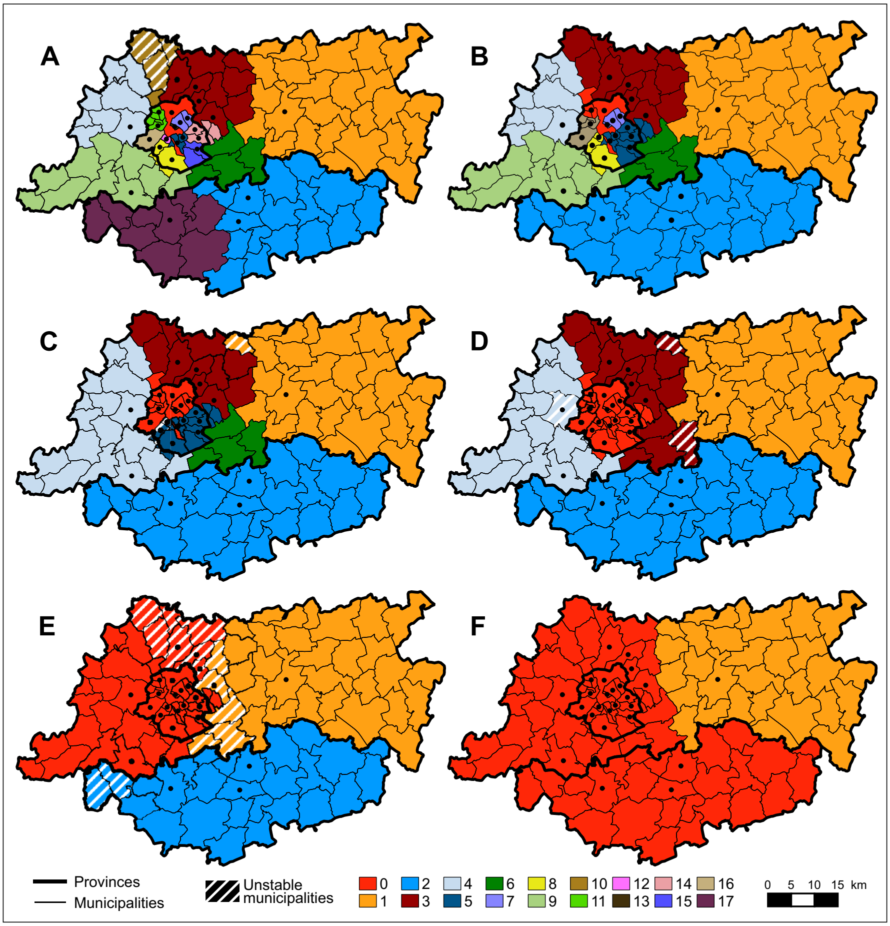
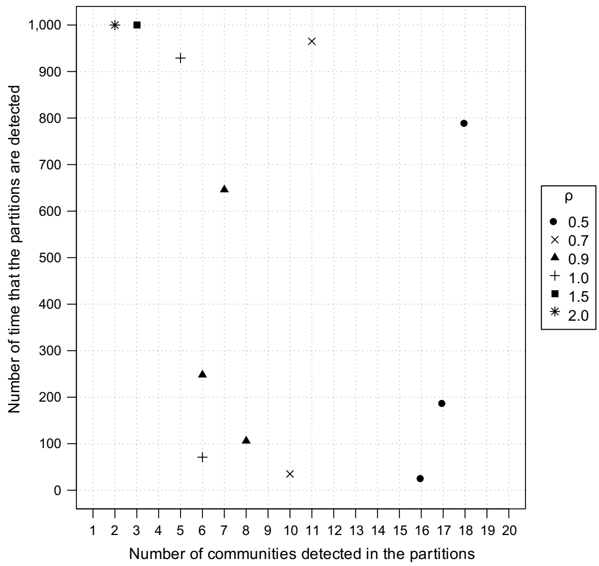


Fig. 6 Partitions detected within the former province of Brabant for different values of ρ: **A** = 0.5; **B** = 0.7; **C** = 0.9; **D** = 1; **E** = 1.5; **F** = 2

Fig. 7 Number of time that each partition is detected for variation of ρ



for each ρ-value is not always the same for each run of the Louvain method. For each value of ρ, Fig. 7 shows (a) the number of communities detected and (b) the number of times that these communities are found. When $\rho = 0.5$, the Louvain method detects 789 partitions composed of 18 communities, 186 partitions of 17 communities, and 25 partitions of 16 communities. Hence, for $\rho = 0.5$, the Louvain method detects 18 communities that we consider to be the dominant community structure. Figure 7 shows that each ρ-value leads to a very dominant community structure (with an occurrence of 80 percent or more) except for $\rho = 0.9$ where the occurrence of the most represented community is only 65 percent.

Next, for each of the 1000 partitions detected with the selected values of ρ, we renumber the ID of the communities (for details see Appendix) and analyze the dominant community structure. Figure 7 shows that the number of communities detected by the Louvain algorithm is equal to 18, 11, 7, 5, 3, and 2 with $\rho = 0.5$, 0.7, 0.9, 1.0, 1.5, and 2.0, respectively. As described in Sect. 2.1, when ρ increases, the partitions change from a large number of small communities to a small number of large ones.

Moreover, we find that each value of ρ leads to a delineation of communities that is not hierarchical. Indeed, the allocation of municipalities to communities is carried out under the objective function given by Eq. 4. Each partition is thus independent of the others detected with different values of ρ. Table 2 represents the similarities between pairs of partitions and shows that the partitions obtained with values of ρ higher than one are less similar between them. We explain these differences by the gap between the values chosen for ρ. The gaps are smaller for values below one and, thus, the objective function of the maximization modularity is slightly modified.

As expected, the size of the communities (corresponding to the sum of all the weighted edges) tends to be homogeneous across the partitions. This homogeneity

Table 2 *NMI* values between partitions resulting from different ρ-values

ρ	0.5	0.7	0.9	1.0	1.5	2
0.5	1	0.90	0.84	0.79	0.63	0.43
0.7		1	0.90	0.87	0.66	0.45
0.9			1	0.90	0.72	0.50
1.0				1	0.74	0.52
1.5					1	0.58
2.0						1

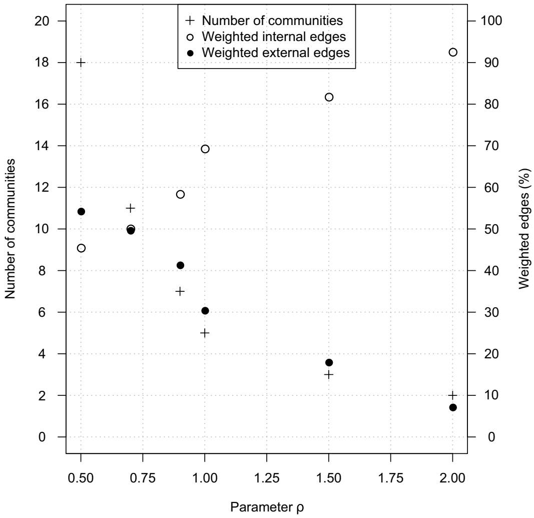


Fig. 8 Number of communities detected, internal and external weighted edges for each partition

can lead to the problem of *resolution limit*. To test the veracity of the organization of the communities, we calculate the total weight of edges located within (internal weighted edges) and between (external weighted communities) communities for each partition (Fig. 8). Let us remind that the modularity finds the best partition under the *Cut* and *Diversity* criterion. The *Cut* (external edges) has to be minimized and the *Diversity* (number of equal size communities) maximized (Delvenne et al. 2013). Here, we look at how the internal and external weighted edges evolve with the increasing ρ. With the rise in ρ, the proportion of internal weighted edges increases and when $\rho = 2$, 92 percent of the weighted edges are found within only two communities. When $\rho = 0.5$, we observe more external weighted edges located between communities than inside the structure and, for $\rho = 0.7$, we observe that 50 percent of the weighted edges are homogeneously distributed in and between the communities. Hence, it is hard to consider the partition obtained with $\rho = 0.5$ as a "good" community structure due to the large proportion of external weighted edges.

Next, let us examine the spatial structures detected. Space is often polarized and jobs (commuters) concentrated in certain places. In the case of Brabant, these concentrations correspond to the highest values of weighted degree (Fig. 3). When ρ is equal to 0.5, for example, the ten municipalities with the highest values of weighted degree are all classified in different communities that may be considered as “job basins” organized around some central places with high value of weighted degree.

The underlying geographical structure may play an important role in data analysis and, surprisingly enough, the observed communities are always sets of contiguous municipalities, although the Louvain method does not take into account the location of the nodes. This confirms the strong effect of distance in commuting in Brabant. Commuters seem to work and live in the same area (Riguelle et al. 2007; Verhetsel et al. 2010).

Within the province of Brabant, we detect two large communities that are slightly affected by variations in ρ. On the one hand, the community centered on Leuven (say, Community 1) is characterized by a *compactness* value close to 0.7 in all cases, meaning that this community is self-centered (Table 3). This corroborates the findings of Thomas et al. (2012). On the other hand, although the delineation of the southern community (Walloon Brabant) is little modified by an increasing ρ, the *compactness* values are close to 0.59. Walloon Brabant has many internal commuting flows, and 40 percent of the flows are linked with municipalities located outside of the community. When $\rho = 2$ (Fig. 6F), the community is merged with Brussels (Community 0), confirming the importance of the commuting flows between these two areas (Thisse and Thomas 2010).

Although the communities located in Brabant Wallon and in the BCR are merged together when $\rho = 2$, the linguistic border between the Flemish and the Walloon Regions is accurately followed by the delineation of the communities obtained with ρ equal to 0.5, 0.7, 0.9, 1.0, and 1.5. The linguistic border appears thus to be a barrier in commuting within the province of Brabant. Nevertheless, municipalities located within the BCR are connected with some Flemish places. For small values of ρ, three Flemish municipalities are included in the BCR (Linkebeek, Kraainem, and Wezembeck Oppem) and their number increases with ρ. This can easily be linked with the sprawl of the city (see Thomas et al. 2012). When $\rho = 1.5$, the Western part of the area is totally merged with Brussels, confirming the history of commuting (see Verhetsel et al. 2010) and when $\rho = 2$ the Brussels community crosses the linguistic border. This Brussels community presents a high value of compactness (0.98), revealing that the majority of the weighted edges are within the community. Unexpectedly, we notice that some municipalities squeezed between Brussels (Community 0) and Leuven (Community 1) have an unstable classification when $\rho = 1$ (Fig. 6C). Both cities attract commuters, and the Louvain method is unable to provide a robust solution. This is confirmed by Fig. 6E, F. When $\rho = 1.5$, these municipalities belong to the Leuven community, while when $\rho = 2$, they are incorporated in the community around the BCR. This area is polarized by both cities in terms of residence and employment (see Vandermotten et al. 2009).

The advantage of introducing ρ in the Louvain method is to modify the objective function that optimizes modularity and to detect partitions with communities of different sizes. ρ allows us to detect small communities within the BCR and to understand how the municipalities interact. Within the BCR, the number of communities detected is equal to nine when $\rho = 0.5$, two when $\rho = 0.9$ and one when $\rho = 1.0$. Each of these partitions can be easily linked with the literature on Brussels.

Table 3 Compactness of the communities detected in the province of Brabant for different values of ρ

Community		ρ					
ID	Name	0.5	0.7	0.9	1.0	1.5	2.0
0	Bruxelles	0.48	0.50	0.67	0.88	0.93	0.98
1	Leuven	0.71	0.71	0.71	0.71	0.70	0.70
2	Walloon Brabant (East)	0.56	0.59	0.59	0.59	0.59	
3	Vilvoorde	0.48	0.50	0.50	0.49		
4	Halle	0.42	0.42	0.45	0.45		
5	Ixelles	0.35	0.47	0.54			
6	Overijse	0.33	0.33	0.33			
7	Schaerbeek	0.29	0.33				
8	Uccle	0.37	0.40				
9	Dilbeek	0.38	0.38				
10	Meise	0.35					
11	Ganshoren	0.30					
12	Saint-Gilles	0.29					
13	Saint-Josse-ten-Noode	0.28					
14	Woluwe-Saint-Lambert	0.35					
15	Auderghem	0.30					
16	Anderlecht	0.32					
17	Walloon Brabant (West)	0.47					

The communities detected with $\rho = 0.5$ and 0.7 (Fig. 6A, B) are in line with the historical suburban organization of Brussels (see Eggerickx 2013) where each municipality presents its own socioeconomic characteristics resulting from its particular history. Moreover, when $\rho = 0.9$ (Fig. 6C), the BCR is split in two parts: the northwestern and the southeastern area. The population in the southeastern area has a higher average income. The patterns detected by the Louvain method perfectly fit with the socioeconomic reality as depicted by Vandermotten et al. (2009). The small central communities detected with ρ equal to 0.5 and 0.7 are characterized by rather small *compactness* indices (0.25–0.48), meaning that these communities are more connected with the rest of the study area. Consequently, when $\rho = 0.9$, these many small structures merge into two larger communities (northwest and southeast) with *compactness* values equal to 0.67 and 0.54, respectively (Fig. 6C). When $\rho = 1$, we observe only one central community that almost perfectly corresponds to the BCR (Fig. 6D). The *compactness* value of this community is 0.88. The small communities located within the BCR are hence more affected by the variations of ρ than the communities having larger values of *compactness*. As a general overview, the steps illustrated in Fig. 6A–F corroborate the history of urban sprawl around Brussels as well as the history of commuting and transport infrastructure, all of them constrained by administrative realities (see Dujardin et al. 2007; Poelmans and Van Rompaey 2009).

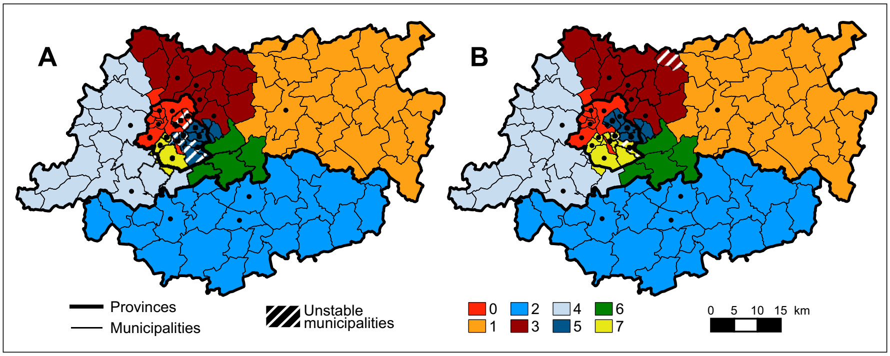


Fig. 9 Partitions detected in the former province of Brabant **A** with and **B** without self-loops (eight communities)

3.2 Effect of self-loops

The effect of the diagonal in origin–destination matrices (self-loops) has already widely been discussed in geography (Grasland 1999). One can show (see Delvenne et al. 2013; Lambiotte et al. 2014) that adding self-loops to each node of a weight proportional to the degree of the node is strictly equivalent to changing the resolution parameter, when identifying communities. Indeed, the self-loops change the Cut term of Q_ρ by scaling the total weight $2m$, but leave other terms unchanged. This is congruent with a similar observation of Arenas et al. (2008) who deal with constant-size self-loops instead. A second set of sensitivity analyses is thus performed in order to quantify the influence of self-loops on the results. For the sake of clarity, we present only comparisons between partitions with the same number of communities (11, 8, and 4). Figure 9 illustrates the case of eight communities. To obtain eight communities by including or not self-loops, the Louvain method was applied with $\rho = 0.9$ for the network including self-loops and with $\rho = 0.7$ when not. As expected, smaller self-loops tend to produce larger communities and we were thus led to decrease the value of ρ to detect eight communities.

Results show that the assumption that self-loops are sufficiently congruent in size with the total degree of communities to leave partitions unchanged. Nevertheless, to obtain the same number of communities, the resolution parameter has to be rescaled. Deleting/including self-loops has little effect on the composition and the delineation of the communities evidenced by the *NMI* values that range between 0.98 and 0.93. Figure 9 shows that the few differences are mainly located within or close to the BCR (Communities 0, 4, 5, and 7); the delineation and the composition of these communities are slightly modified by the removal of the self-loops. Let us note that although Community 5 has the same number of municipalities (Table 4), its composition and delineation are different. The *compactness* slightly decreases for Communities 0 and 5 when deleting self-loops and slightly increases for Communities 4 and 7 (Table 4). These variations are small and do not lead to important changes in the connections of these communities with the others. Let us here recall that 1000 runs were computed for each use of

Table 4 *Compactness* values of the communities detected with the inclusion of or without self-loops (the number of municipalities within communities is given in brackets)

Community ID	With self-loops		Without self-loops	
0	0.670	(11)	0.603	(8))
1	0.711	(26)	0.711	(26)
2	0.587	(27)	0.587	(27)
3	0.501	(12)	0.501	(12)
4	0.446	(18)	0.449	(19)
5	0.466	(8)	0.407	(8)
6	0.328	(4)	0.328	(4)
7	0.402	(5)	0.495	(7)

the Louvain method and that the municipalities that are not always classified in their dominant structure are hatched in Fig. 6. Hatched municipalities also correspond to those that are affected by the self-loops; internal links slightly affect the results of the Louvain method.

Similarly, we extended the analysis to the entire country (see also Sect. 3.3). Figure 10A refers to the partition of the 589 municipalities into ten communities including self-loops, while Fig. 10B excludes self-loops. Both partitions are once again very similar visually, which is also evidenced in terms of *NMI* (0.916). The main difference between the two partitions is the extension of the community centered on Brussels, which is much larger in Fig. 10A than in Fig. 10B. When excluding self-loops (Fig. 10B), the eastern part of Brabant Wallon is not included anymore in the community centered on Brussels. Interestingly enough, this classification is unstable as illustrated by the hatching. The Louvain method is not very sensitive to the self-loop at any size of the study area for commuting movements. However, it would be interesting to test the sensitivity on another individual data set as well. Indeed, we used the aggregated commuting flows between municipalities and the importance of self-loops is here related to the values of weighted degree. From a geographical point of view, it is expected that a

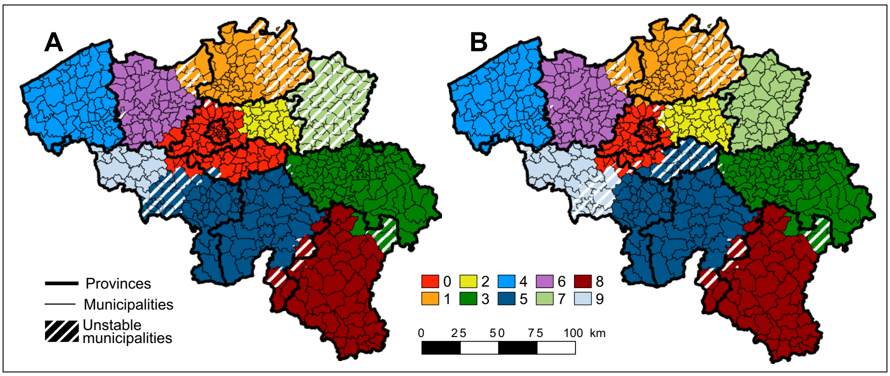


Fig. 10 Partitions detected in Belgium **A** with and **B** without self-loops (10 communities)

municipality that concentrates a lot of commuters (and thus can be called a *job center*) also has a large self-loop too. In consequence, the small sensitivity of the Louvain method to self-loops is clearly not due to the method itself, but to the strong structure of the data.

3.3 Delineation of the study area

The last set of sensitivity analyses deals with the extension of the study area. If we limit the analysis to the origin–destination matrix of flows between the 111 municipalities of the province of Brabant (Fig. 11B), do we get the same results as if we consider all the flows between the 589 municipalities of Belgium and zoom in on our study area (Fig. 11A)? If we ignore the relationship between the municipalities of Brabant and the rest of the country, do we get the same partitioning? Are the other cities exerting an influence? Geographers are familiar with this problem for several kinds of quantitative models (Horner and Murray 2002; Jones et al. 2016) even if often ignored (Jones et al. 2017). In this study more than others, the urban network is tight with a high density of closely located regional centers.

For the sake of clarity, we limit ourselves to two illustrations and for the same number of communities within the province of Brabant (Fig. 11). The *NMI* between these two maps takes the value of 0.86, hence confirming the visual observation. Both maps look alike without being perfectly identical. The same partition is observed with the exception of (a) the western peri-urban area where the urban network is tight (high density of closely located regional centers: high values of weighted degree), tending to a strong historical commuting movement (Verhetsel et al. 2010) and (b) with the exception of a small community within the BCR that appears in Fig. 11A and not in Fig. 11B. These exceptions correspond to the municipalities of Saint-Josse-ten-Noode, Evere and Schaerbeek which tend in practice to federate several public services within the urban agglomeration (a unique Police Zone 5344). Interestingly, in the partitions illustrated in Fig. 11A there are more unstable classifications. When eliminating the flows with all other Belgian

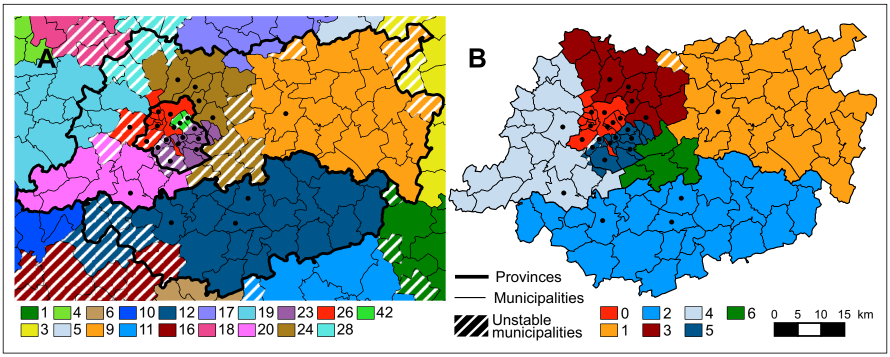


Fig. 11 Communities detected in Belgium (**A**) and in the former province of Brabant (**B**)

municipalities, the network is smaller and simpler and could lead to more robust results since the Louvain method has fewer partitions to explore than in the larger network (Fig. 11B).

3.4 Infomap

This section aims to compare the partitions detected by the Louvain method with those obtained by the Infomap algorithm. We propose a slight improvement and refer to Rosvall and Bergstrom (2008) for further methodological development. Infomap is an algorithm based on the idea that a random walker, following each weighted edges, should spend most of its time inside communities and rarely travel across communities. To compute the steps of this random walker, the authors represent the movements at two levels. The first level encodes the different communities, and the second level the individual nodes. Thus, they look for a partition of community c made of N nodes that minimize the expected description length of a random walk.

The Infomap version used here is the standard version. It is hence impossible to detect communities at different levels of resolution.

Let us compare the partitions obtained by both methods (Louvain and Infomap) for a given number of communities. For Belgian commuting movements, Infomap detects 14 homogeneous communities. Based on the methodology described in Sect. 3.1, the value of $\rho = 0.6$ allows Louvain to detect 14 communities as well. The two partitions of Belgium into 14 communities are similar ($NMI = 0.82$). Results are mapped and analyzed at the scale of the former province of Brabant (Fig. 12). The former province of Brabant is dominated by three major communities by the Louvain method but only two by applying Infomap. The community centered on the area of Leuven (Community 2) is detected by both algorithms, and the composition and delineation tend to be similar (30 municipalities by using the Louvain method and 27 municipalities by using Infomap). The community located in the area of the BCR (Community 1) is detected by both algorithms but with two

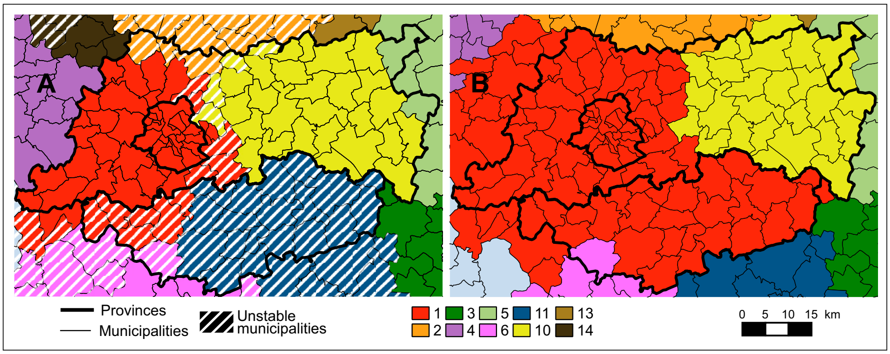


Fig. 12 Communities detected with the use of the Louvain method (**A**) and the Infomap algorithm (**B**)

major differences. On the one hand, this community is much larger in the Infomap partition (101 municipalities instead of 57 with the Louvain method), and, on the other hand, the majority of the municipalities of Walloon Brabant are classified to the BCR by applying Infomap, although these are allocated to another group in the case of the Louvain method (Community 9: Namur-Wavre). An interesting point is the instability in the classification of some municipalities with the Louvain method (Fig. 12), such as the municipalities located within the Province of Walloon Brabant and the one squeezed between one and two (BCR and Leuven).

4 Conclusions

Optimizing modularity using the Louvain method is a quantitative approach that enables to analyze the organization of a network that can be applied in the field of geography. The method partitions space into communities of highly connected places in terms of commuting flows. We analyzed the robustness of the results with respect to three modifications. Changing the parameter ρ has the most important impact, while the inclusion or not of self-loops (intra-communal commuters) and the definition of the study area have, respectively, no or little impact. The choice of ρ is essential, but there is no unique methodological guideline for choosing it, and each value of ρ leads to a different partition. This is found to be a major shortcoming of the approach. We used the method developed by Delmotte et al. (2011) and implemented in Delvenne et al. (2013) that quantifies the stability of partitions depending on ρ to detect the level of organization of commuting movements as well as the robustness of the partitions detected at different levels. Moreover, we examined the partitions in detail and tested how the communities are structured in terms of internal and external flows to characterize each group. Further research should be directed toward analyzing to find whether the most stable partitions are also the best partitioning of space or not. Interestingly enough, the impact of the delineation of the study area corresponds to a problem familiar to geographers. By modifying the size of the study area, we ignore a certain amount of information that may originate outside of the delineation and characterize different realities. The small differences observed with/without self-loops correspond to these places that are not classified unequivocally in the 1000 runs. Such unstable places are mainly located at the outskirts of the communities.

The Louvain method is an interesting method for extracting geographical information from relational data. The method delineates communities within networks, and we approximate their centers by calculating the weighted degree of each municipality. In our case, job basins are spatially continuous but without relying on locational information. Space matters and geographical maps can easily be interpreted in terms of the socioeconomic geography of Belgium. Not surprisingly, in the former province of Brabant, the linguistic and regional borders are not permeable (with some few exceptions located close to Brussels). Moreover, two communities are particularly little affected by the sensitivity analyses: Leuven and the Walloon Brabant. We also show that unstable municipalities are squeezed between Brussels and smaller secondary cities, especially in the eastern and western areas.

This strong effect of space is also present in case of the second community detection algorithm used for comparative reasons (Infomap). Partitions are similar to those obtained with the Louvain method but with some few exceptions. Some municipalities located close to the borders of the communities (job basins) show instability features. Space and the networks of cities within their environments have a strong effect on commuting patterns. Nearby places are more intensively linked than distant places. However, urban realities are by nature complex, and our study is limited only to one aspect (commuting, job basins). An exciting scientific challenge would be to compare results with other kind of flows and link them to the socioeconomic, environmental or other characteristics of the nodes. Future research in this direction might help planners to improve their perspective on the future of cities.

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Appendix

Extraction of the dominant community

With the multiple runs of the Louvain method, the nodes are characterized by a list of 1000 identification numbers (id) corresponding to the community number. It is important to underline that the allocation of nodes to communities is specific to each run of the Louvain method, implying that the ID of the communities is renumbered at each run. Hence, a community numbered "one" in the first run is not automatically labeled "one" in the other simulations; so we have to make the ID of the communities comparable through the runs of the Louvain method. The methodology is illustrated in Fig. 13.

First, each node is characterized by a list of 1000 elements corresponding to the communities where the nodes are allocated. We call this list the *signature of the node* and calculate the number of occurrences of each signature in the results. Second, the list of signatures is ordered by descending order of occurrence and we attribute to each specific id that we call the *new community number*. Third, the highly represented signature is used to renumber the communities detected. The renumbering starts with the nodes having the highest represented signature. The initial community numbers are modified with the *new community number*. Moreover, this renumbering is applied for the entire run and so, if the initial community number is still present, it is renumbered with the *new community number* too. This special case is illustrated in Fig. 13. In run 5, the nodes with the highest represented signature are nodes 1, 2, and 3 which belong to Community 1. In this run, we observe that a number of other nodes are initially classified within this community 1 (nodes 4 and 5). So, all these nodes (1, 2, 3, 4, and 5) are renumbered with the *new community number* 100. Once all the initial IDs of the communities have been modified by the *new community number*, the same procedure is applied to the second most frequent signature. This renumbering is applied until all the communities are renumbered.

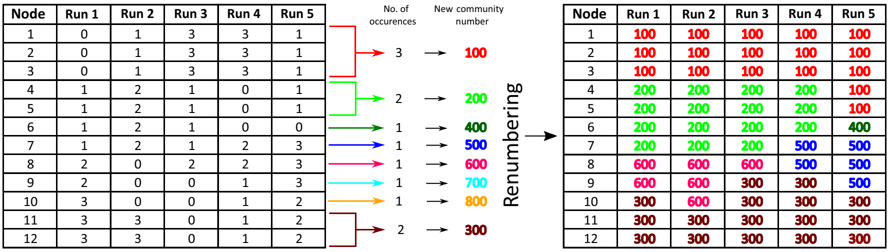

Fig. 13 Renumbering of the communities for five runs of the Louvain method

Finally, the dominant community (most frequent community) is extracted for each node and the stability of the nodes is calculated.

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Affiliations

Arnaud Adam[1] · Jean-Charles Delvenne[2] · Isabelle Thomas[1]

Jean-Charles Delvenne
jean-charles.delvenne@uclouvain.be

Isabelle Thomas
isabelle.thomas@uclouvain.be

[1] Center for Operations Research and Econometrics (CORE), Voie du Roman Pays, 34 bte L1.03.01, 1348 Louvain-la-Neuve, Belgium

[2] Institute of Information and Communication Technologies, Electronics and Applied Mathematics (ICTEAM), Avenue Georges Lemaître 4-6 bte L4.05.01 à, 1348 Louvain-la-Neuve, Belgium