Dispersion curve calculation using the Method of Moments: the impact of Macro Basis Functions

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***Abstract*—The Method of Moments (MoM) is a powerful tool for computing the dispersion curves of open and possibly lossy structures. However, the brute-force computation of the dispersion curves associated to a 2D periodic structure using the MoM can be computationally intensive. One way to accelerate the computation is to reduce the number of unknowns used to model the periodic structure using Macro Basis Functions (MBFs). However, using MBFs to compute dispersion curves may seem counter-intuitive, since MBFs are usually obtained from the response of the structure to various excitations, while the dispersion curves are the excitation-free homogeneous solution to Maxwell's equations. In this paper, we investigate the error introduced by the use of MBFs when computing the dispersion curves of printed periodic structure on a layered substrate. We study both the impact of the selected set of excitations used to build the MBFs and, for a given set of excitations, the impact of the number of MBFs used to model the printed patches. From the obtained results, practical rules are proposed for metasurface-type periodic array of printed patches.**

***Index Terms*—Computational electromagnetics, Method of Moments, dispersion curves, macro-basis functions.**

I. Introduction

For many years, researchers have been looking for concise models to describe fields in periodic structures. When looking at the propagation or dissipation of the electromagnetic fields along periodic structure, a good trade-off between simplicity and accuracy consists of providing the dispersion curves of the periodic structure, i.e. the combination of frequencies and phase shifts between consecutive unit cells for which a homogeneous solution to Maxwell's equations exists. The dispersion curves of a periodic structure provide interesting information [1], [2], [3], such as the phase and group velocities as a function of the frequency and angle of propagation, the presence of electromagnetic band gaps, or the asymptotic evolution of the fields generated by a current source located close to the periodic structure. Dispersion curves are also useful for the design of metasurfaces: under a local periodicity assumption, the equivalent surface impedance of metallic patches printed on a layered medium can be estimated and used for design purpose [4], [5].

To compute the dispersion curves of 2D-periodic printed patches using full-wave simulations, two different techniques can be used. The first one is based on the transfer-matrix (T-matrix) of the unit cell (see e.g. [6] and references therein). While being very efficient from a numerical point of view, this method cannot rigorously handle open periodic structures since the accurate modelling of the infinite volume of the unit cell would, strictly speaking, require an infinitely large T-matrix. The second one is based on integral-equation methods (see e.g. [7], [8], [10] and references therein). Instead of looking for the fields in the infinite volume, one looks for the currents on the finite patch, which can be expressed using a finite set of basis functions. On one hand, this technique enables the rigorous study of the periodic printed patches in possibly lossy environment [10], the radiation boundary condition being implicitly satisfied. On the other hand, the problem becomes mathematically more complicated and is generally formulated as a non-linear root or pole searching problem. One has to look for combinations of phase shifts and frequency for which a homogeneous solution exists, i.e. for which the impedance matrix describing the patch is rank-deficient. This search is generally time-consuming since a new periodic impedance matrix needs to be computed for each new combination of phase shifts and frequency. Recently, the authors have proposed an approach based on the Method of Moments (MoM) that can be used to rapidly compute the dispersion of periodic printed patches [10]. The method makes use of Macro Basis Functions (MBFs) to reduce the size of the periodic impedance matrices.

MBFs are specialized basis functions defined on the entire patch [11], [12], [13], [14]. The idea is that, among all the mathematical degrees of freedom offered by the mesh, only a few of them can be excited in practical scenarios. Thus, it is theoretically possible to reduce the number of degrees of freedom involved in the problem without impacting significantly the accuracy. The vectors spanning the reduced basis are called the MBFs. To build the MBFs, the patches are excited using a set of sources (e.g. incident plane waves) and the resulting current distributions on the patches are computed. The MBFs can be obtained from the resulting set of current distribution, usually through the use of a truncated Singular Value Decomposition (t-SVD).

A good choice for the MBFs should significantly reduce the number of degrees of freedom without impacting too much the accuracy of the results. First, it requires a well chosen set of excitations. If the set is too small, the acccuracy of the final results will be degraded. However, if the set is too large, MBFs that are not physically relevant may be included in the set, increasing the number of unknowns without significantly improving the accuracy of the results. Second, it requires to carefully choose the threshold of the t-SVD. Generally, this threshold can be defined either as a value below which singular values are considered as being negligible, or as a number of modes that should be kept.

In this paper, we study the impact of the MBFs on the calculation of the dispersion curves. Using the method proposed in [10], we compute the dispersion curves of different types of metallic patches printed on a grounded dielectric substrate. Different sets of excitations are used to build the MBFs. For each set, the convergence with the threshold number of the t-SVD and with the number of MBFs is analyzed. Using these results, we propose empirical guidelines concerning the set of excitations that should be considered and the number of MBFs that should be used to model metasurface-type metallic printed patches.

The paper is organized as follows. First, Section II summarizes the method used to compute the dispersion curves. In Section III, the procedure used to build the MBFs is explained. In Section IV, the impact of the MBFs on the obtained dispersion curves is studied for two types of printed metallic patches: rectangular and coffee-bean patches [15], [5]. Based on the results obtained, guidelines in the choice of the set of excitations and the threshold of the t-SVD are proposed.

II. Computation of the dispersion curves

To compute the dispersion curves of the periodic printed metallic patches, the method described in [10] is used. In this section, we rapidly summarize the method for self-consistency. More details are available in the original paper.

First, a specialized interpolation model is built to accelerate the evaluation of the MoM periodic impedance matrix of the patches over a chosen range of frequencies and phase shifts (typically, the first Brillouin zone over an octave or more). To do so, the periodic impedance matrix of the patches is evaluated for a few frequencies f and phase shifts ϕ_x, ϕ_y along the two directions of periodicity, noted $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$. The method used to compute the periodic impedance matrices is described in details in [10, Section II.A]. From the few reference matrices, a specialized interpolation model is built. The matrices are separated in two terms: one that can be evaluated rapidly on-the-flight, and a remainder that is smooth versus frequency and phase shifts [9], [10]. The smooth term, is then interpolated using a polynomial model.

Using the interpolation model, the response of the structure to propagative and evanescent plane waves can be evaluated rapidly for several phase shifts and frequencies. From the currents distributions obtained, a set of MBFs is built and used to compress the matrix (see Section III). Then, a new interpolation model is built to directly interpolate the reduced impedance matrix itself. The interpolation method is then used to rapidly evaluate the reduced impedance matrix and map its determinant over a regular grid in the (f, ϕ_x, ϕ_y) space. The dispersion curve of the structure corresponds to the positions of the map at which the determinant vanishes. For simplicity, an automatic tracking procedure [10] is used to extract the dispersion curves from the determinant map and express it as a parameterized curve:

$$\rho(\psi) = a_0 + \sum_{n=1}^{N} \left(a_n \cos(2\pi n \psi) + b_n \sin(2\pi n \psi)\right) \quad (1)$$

where ρ and ψ are the polar coordinates of points on the curve.

Using this technique, the reduced impedance matrix can be interpolated with a relative precision better than 10^{-3}. Moreover, interpolating the reduced impedance matrix and evaluating its determinant takes a few milliseconds, so that dense maps can be obtained in a few minutes.

III. Building the MBFs

To build the MBFs, the periodic structure is excited using several propagative and evanescent plane waves. To the i^{th} plane wave can be associated a frequency $f^{(i)}$ and phase shifts $\phi_x^{(i)}$ and $\phi_y^{(i)}$ in the $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ directions. First, the periodic impedance matrix $\underline{\underline{Z}}(f^{(i)}, \phi_x^{(i)}, \phi_y^{(i)})$ is interpolated. Then, the plane wave is projected on the testing function of the patch to obtain the excitation vector $\mathbf{b}^{(i)}$. The corresponding current distribution is obtained by solving the system of equation:

$$\underline{\underline{Z}} \mathbf{x}^{(i)} = -\mathbf{b}^{(i)} \quad (2)$$

Once a sufficient number of excitations have been considered, the coefficients of the different current distributions are concatenated in a large matrix $\underline{\underline{X}}$:

$$\underline{\underline{X}} = \begin{bmatrix} \mathbf{x}^{(1)} & \mathbf{x}^{(2)} & \ldots & \mathbf{x}^{(N_e)} \end{bmatrix} \quad (3)$$

with N_e being the total number of excitations considered. Then, to obtain the MBFs, a singular value decomposition of the $\underline{\underline{X}}$ matrix is used:

$$\underline{\underline{U}} \cdot \underline{\underline{S}} \cdot \underline{\underline{V}} = \underline{\underline{X}} \quad (4)$$

with $\underline{\underline{U}}$ and $\underline{\underline{V}}$ being two unitary matrices and $\underline{\underline{S}}$ the diagonal matrix containing the singular values. For a given number of MBFs M, the best possible choice consists of taking the columns of $\underline{\underline{U}}$ associated to the M highest eigenvalues. Those MBFs are the ones that can reproduce the current excitations $\mathbf{x}^{(i)}$ with the lowest root mean square (RMS) error.

Once the MBFs have been chosen, they are concatenated into a matrix $\underline{\underline{M}}$. The reduced periodic impedance matrix $\tilde{\underline{\underline{Z}}}$ is then defined as

$$\underline{\underline{Z}}^r(f, \phi_x, \phi_y) = \underline{\underline{M}}^\dagger \cdot \underline{\underline{Z}}(f, \phi_x, \phi_y) \cdot \underline{\underline{M}} \quad (5)$$

with $\underline{\underline{M}}^\dagger$ the conjugate transpose of $\underline{\underline{M}}$. This reduced impedance matrix is then used to find the dispersion curves (cf. Section II).

IV. Numerical results

We studied three different geometries: rectangular patches with different orientations and coffee-bean patches [15], [10]. The rectangular patches are printed on a 1 mm thick grounded dielectric substrate of relative permittivity $\varepsilon_r = 9.8$. Two different orientations of the patches are considered, called hereafter the *aligned* and *diagonal* ones (see Fig. 1(a)). The

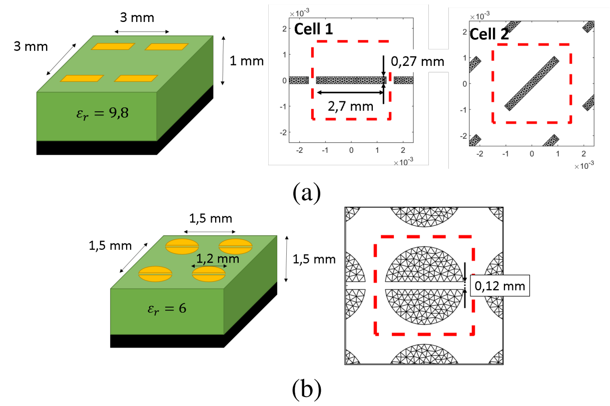


Fig. 1. Illustration of the printed structure whose isofrequency curves are computed. (a) The aligned (Cell 1) and diagonal (Cell 2) rectangular strips and (b) the coffee-bean patches.

coffee-bean patches are printed on a 1.5 mm thick grounded dielectric substrate of relative permittivity $\varepsilon_r = 6$ (see Fig. 1(b)). The dimensions and geometries are illustrated in Fig. 1.

Three different sets of excitations have been considered to build the MBFs. First, a large reference set of excitations (set S_1). TE and TM planes waves corresponding to a regular sampling of the first Brillouin zone and the 8 surrounding Brillouin zones are used to excite the structure at 5 regularly spaced frequencies ranging from 9 GHz to 19 GHz for the rectangular patches and from 10 GHz to 45 GHz for the coffee-bean patches. Second, we considered a smaller set in which only the first Brillouin zone is sampled (set S_2), thus eliminating the most evanescent plane waves. Last, we considered a set where the 9 first Brillouin zones are sampled, but only the lowest frequency of the frequency interval is considered: 9 GHz for the rectangular patches and 10 GHz for the coffee-bean patches (set S_3). The latter set is used to evaluate the dependence of the MBFs on frequency.

For all three sets, different samplings of the Brillouin zone have been considered: 1×1, 3×3 and 5×5 regular samplings centered inside the Brillouin zone [13]. For compactness, we will note S_i^j the $j\times j$ sampling version of set S_i. For example, the 1×1 sampling version of the second set (S_2^1) corresponds to normally incident TE and TM plane waves at 5 different frequencies, for a total of 10 excitations. Similarly, the most abundant set of excitations (S_1^5) is made of TE and TM plane waves of phase shifts $\phi_x, \phi_y \in \{n2\pi/5 \mid -7 \leq n \leq 7\}$ and frequencies $f \in \{9, 11.5, 14, 16.5, 19\}$ GHz for the rectangular patches and $f \in \{10, 18.75, 27.5, 36.25, 45\}$ for coffee-bean patches, for a total of 2250 excitations.

Using each of these sets, we studied the convergence of the dispersion curve position. The error e_i^j associated with a given set S_i^j is defined as the RMS distance between the obtained curve and the reference curve from [10]:

$$e = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\tilde{\rho}(\psi_i) - \rho(\psi_i)\right)^2} \tag{6}$$

where ψ_i are the directions in which the two curves are sampled and ρ and $\tilde{\rho}$ are the radial positions of the reference and computed curves, respectively. The azimuthal coordinate of the sampling points are given by $\psi_i = 2\pi i/N$ with $N = 100$.

In fig. 2, the results are displayed for the rectangular patches at 16.5 GHz (fig. 2(a)) and 36.25 GHz for the rectangular and coffee-bean patches (fig. 2(b)). First, it can be noticed that the number of MBFs required to reach a given accuracy strongly depends on the set of excitations. However, nearly all the sets of excitation show similar convergence with respect to the threshold of the t-SVD. Thus, imposing a threshold instead of a given number of MBFs seems to be a safer choice.

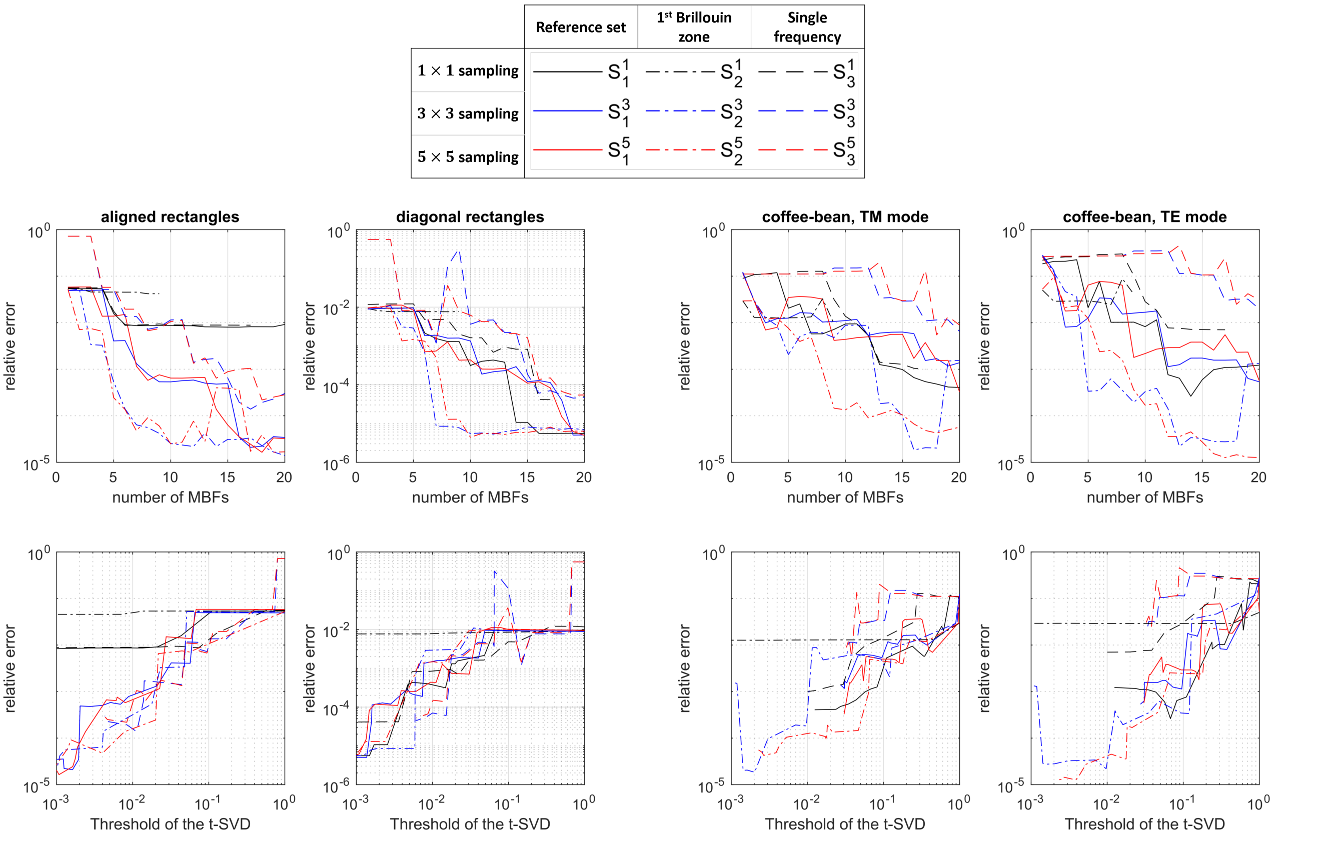
Second, it can be noticed that some sets of excitations tend to converge much more slowly than the others, and even saturate. Those correspond to the sets obtained using a single frequency (S_3^j, dashed curves) or using a single sampling point in the Brillouin zones (S_i^1, black curves). It means that the homogeneous solution at a given frequency cannot be accurately extrapolated from the response of the patches at much lower frequencies (quasi-static approximation) or their response to normally incident plane waves.

Last, to reach a given accuracy, the reference set that includes very evanescent plane waves located outside of the first Brillouin zone (S_1^j, plain curves) require more MBFs than the smaller set obtained by considering only plane waves inside the first Brillouin zone (S_2^j, dotted curve). Indeed, current distributions corresponding to highly evanescent plane waves are not excited in practice and thus do not significantly improve the accuracy of the solution.

As a rule of thumb, to obtain a complete yet small set of MBFs, a coarse sampling of the first Brillouin zone (typically 3×3 or 4×4) close to the frequency of interest is required. Additional modes only increase the number of unknowns without significantly improving the accuracy. Last, a threshold of 10^{-2} for the t-SVD, is sufficient for most applications (error below 1%).

V. Conclusion

The dispersion curves of open periodic structures, potentially in the presence of conduction or radiation losses, can be efficiently computed using the MoM. In [10], the MBFs have been used to drastically reduced the computation time. In this paper, we have investigated the impact of the chosen MBFs on the accuracy with which the dispersion curve of subwavelength printed periodic structures can be computed. We observed that the procedure used to generate the MBFs can have a strong impact on the convergence of the results with the number of MBFs. In the situation considered, an 'optimal' set of currents was obtained by exciting the structure at several frequencies with propagating and slightly evanescent plane waves spreading over the first Brillouin zone. When extracting the MBFs from the set of currents, we observed much less variability in the convergence when imposing a threshold value for the t-SVD, which is thus a safer criterion.



(a) (b)

Fig. 2. Convergence of the dispersion curves for (a) the rectangular patches at 16.5 GHz and (b) the coffee-bean patches at 36.25 GHz. In each case, the top graphs show the convergence with the number of MBFs, and the bottom graphs the convergence with the threshold of the t-SVD. Note that two modes are supported by the coffee-bean patches at 36.25 GHz. The error is displayed for the fundamental mode (left) and the high-order one (right).

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