

OPTIMAL QUOTAS, PRICE COMPETITION AND PRODUCTS' ATTRIBUTES*

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We characterize the Nash equilibrium in the Hotelling model in the presence of an import quota. The optimal quota is identified and shown to be invariant to the mode of competition. We also prove that in the presence of a quota maximal differentiation is not achieved at equilibrium.

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1. Introduction

Despite the global trend towards freer trade, many governments are still trying to enforce protectionist measures in numerous industries. The “Market Access Sectoral and Trade Barriers” database maintained by the European Commission provides ample evidence on this point.¹ As recently argued by Maggi and Rodriguez-Clare (2000), if tariffs tend to disappear because of political pressures, they are replaced by quantitative restrictions in the form of quotas, voluntary export restraints (VERs), or anti-dumping regulations.² Still, quotas may have very specific implications under oligopolistic competition, and their full implications need to be understood better.

In a seminal contribution, Krishna (1989) shows that a quota imposed at the free-trade equilibrium level (hereafter FTE) is likely to be ineffective under quantity competition while it affects market outcomes under price competition. Her paper provides a striking illustration of the fragility of trade policy recommendations in oligopolistic industries. If the government lacks the relevant information about the nature of the strategic interaction, it may well implement an inappropriate policy. In this respect, trade policy instruments that are invariant to the mode of competition (as put forward for instance by Maggi, 1996) look particularly attractive. Moreover, because a quota affects equilibrium outcomes at the market stage, it will also affect the strategic behaviour of firms in those stages of the game where the technology or the products' attributes are selected. Taking Krishna's analysis as a starting point, we suspect that the mode of competition could play a key role in this respect.

There is already a vast literature studying the implications of trade quotas in oligopolistic market. However, Brander (1995) notes that very few theoretical papers have elaborated on Krishna's result, i.e. on the precise effect of a quota under price competition.

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¹ See <http://mkaccd.db.eu.int/mkdb/mkdb.pl>

² As argued recently by Veugelers and Vandenbussche (1999) and Vandenbussche and Wauthy (2001), anti-dumping actions may alter price competition in a way that facilitates collusive practices.

Instead, theoretical research about trade quotas focus mostly on quantity and/or quality-setting games. (See Herguera *et al.* (2000) for a very recent contribution in this area.) On the other hand, empirical work dealing with price setting and trade quotas assumes the existence of a price equilibrium in pure strategies.³ Yet, a key insight of Krishna (1989) is to show that quotas in the vicinity of free trade tend to prevent the existence of such pure strategy equilibria.

In this paper we test the relevance of Krishna's result by identifying the optimal value of the quota as a function of the mode of competition. Moreover, in order to assess the magnitude of the strategic effects at work in the presence of trade quotas, we take into account how quotas affect the choice of products' attributes and hence trade composition. To this end, we build on the Hotelling (1929) model of horizontal differentiation. This model has been recently used in the international trade literature (see e.g. Schmitt, 1990, 1995; Schachmurove and Spiegel, 1995), and it neatly captures the idea that intra-industry trade occurs because of the variety in consumers' tastes and products' characteristics, while allowing for strategic interactions between firms.

In addition to providing an original characterization of price equilibria in the Hotelling model for all values of the quotas (unlike the analysis of Krishna, which is confined to the vicinity of FTE), we establish two main results. First, in our Hotelling framework, the optimal level of the quota is invariant to the mode of competition. Second, focusing then on the effect of the quota on products' selection, we show that, for most relevant values of the quota, maximum differentiation is not achieved in a subgame-perfect equilibrium, contrary to what happens under free trade.⁴

The paper is organized as follows. In Section 2 we characterize Nash equilibrium in prices for all possible values of the quota. Section 3 is devoted to the analysis of the optimal quota level under Cournot–Bertrand competition. Then in Section 4 we deal with the choice of products' attributes. Section 5 concludes. Most proofs are only a matter of calculations and are displayed in the Appendix.

2. Price equilibrium in the presence of a quota

In this section, we characterize the Nash equilibrium in prices.⁵ We first analyse a simplified Hotelling model under free trade, because this provides a useful starting point for the (more complex) analysis of price competition with a quota.

2.1 The free trade benchmark

Consider a domestic market consisting of a street of unit length. A homogeneous good is sold at two shops. One sells a domestic product at a price p_d and the other sells a foreign product at a price p_f . They are located, respectively, at the left and right ends of the unit segment. Consumers are uniformly distributed in this segment. Each consumer is

³ See e.g. Goldberg (1995), Verboven (1996), Goldberg and Verboven (1998). Feenstra (1988) assumes that firms behave competitively.

⁴ Similar issues have been widely studied under Cournot competition and for vertical differentiation; Das and Donnenfeld (1989), Herguera *et al.* (2000) and Boccard and Wauthy (1998) are recent examples.

⁵ The analysis presented in this section is a direct application of the methodology laid out in Krishna (1989).

identified by its address $x \in [0; 1]$ and buys at most one unit of the good. The common reservation price is S . When buying one of the products, the consumer goes to a shop and bears a linear transportation cost which we normalize to \$1. The utility derived by a consumer located at x in the interval $[0; 1]$ is

$$\begin{cases} 0 & \text{if the consumer does not purchase} \\ S - (1 - x) - p_f & \text{if the product is bought at the foreign firm} \\ S - x - p_d & \text{if the product is bought at the domestic firm} \end{cases}$$

Assume that prices are such that all consumers buy a product (the market is covered) and that the market is shared by the two firms. By definition, the *marginal consumer* $\tilde{x}(p_d, p_f)$ satisfies $S - (1 - x) - p_f = S - x - p_d$, so that we obtain $\tilde{x}(p_d, p_f) = (1 - p_d + p_f)/2$. Demands are therefore $D_d(p_d, p_f) = \tilde{x}(p_d, p_f)$ and $D_f(p_d, p_f) = 1 - \tilde{x}(p_d, p_f)$. Interiority conditions are satisfied whenever $\tilde{x}(p_d, p_f) \in [0; 1]$ and $S - \tilde{x}(p_d, p_f) - p_d \geq 0$. The first condition requires $|p_d - p_f| \leq 1$, whereas the second is satisfied if $p_d + p_f \leq 2S - 1$. When the latter condition fails, the market is not covered and both firms act as local monopolists.

Relying on the symmetry of the firms' positions, we concentrate on the price constellations where $p_d \geq p_f$ (a symmetric analysis prevails in the other case) and define demand functions as follows:

- Whenever $p_d + p_f \leq 2S - 1$,

$$D_d(p_d, p_f) = \begin{cases} \tilde{x}(p_d, p_f) & \text{if } p_d \leq p_f + 1 \\ 0 & \text{if } p_d \geq p_f + 1 \end{cases} \quad (1)$$

$$D_f(p_d, p_f) = \begin{cases} 1 - \tilde{x}(p_d, p_f) & \text{if } p_d \leq p_f + 1 \\ \max\{1, S - p_f\} & \text{if } p_d \geq p_f + 1 \end{cases} \quad (2)$$

- Whenever $p_d + p_f \geq 2S - 1$,

$$D_d(p_d) = \begin{cases} S - p_d & \text{if } p_d \leq S \\ 0 & \text{if } p_d \geq S \end{cases} \quad (3)$$

$$D_f(p_f) = \begin{cases} S - p_f & \text{if } p_f \leq S \\ 0 & \text{if } p_f \geq S. \end{cases} \quad (4)$$

In the region where $p_d + p_f < 2S - 1$, each firm's best reply is defined by $\varphi_i(p_j) = \max\{p_j - 1, H_i(p_j)\}$ with $H_i(p_j) \equiv (1 + p_j)/2$. We characterize the Hotelling equilibrium in the following lemma.⁶

Lemma 1: *If $S \geq 3/2$, and if firms face no quantitative constraint, the unique Nash equilibrium of the pricing game is $(1, 1)$; no consumer refrains from buying in equilibrium.*

We shall assume hereafter that $S \geq 3/2$.

⁶ We do not prove this standard result formally. Interested readers are referred to e.g. Mas-Colell *et al.* (1995).

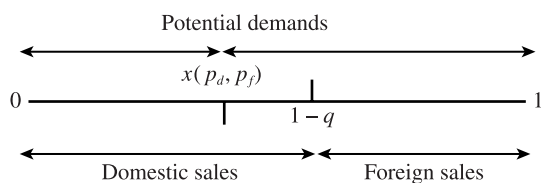


FIGURE 1. Distribution of sales in the presence of a quota

2.2 Import quotas

Consider two price-competing firms in a differentiated industry and assume that a unique Nash equilibrium (p_1^*, p_2^*) exists. Suppose now that in this market firm 2 is facing a quota q at the FTE level; i.e., $q = D_2(p_1^*, p_2^*)$. Is (p_1^*, p_2^*) still an equilibrium? Presumably not. Indeed, if firm 1 raises p_1 , given p_2^* , its demand should decrease, whereas the demand addressed to firm 2 should increase. However, firm 2 cannot meet demand since it exceeds the quota. Accordingly, rationing appears in the market. It is then sufficient that some rationed consumers turn back to firm 1 in order to make the upward deviation from p_1^* profitable. If this is so, (p_1^*, p_2^*) cannot define a Nash equilibrium.

In order to characterize firms' sales, we first specify a rationing rule. We assume, like Kreps and Scheinkman (1983), that the efficient rationing rule is at work in the market so that the closer a consumer is to the shop the earlier he is served. It follows that *rationed consumers are those who exhibit the lowest reservation prices for the foreign product*. We identify then the distributions of sales when the quota is binding. Consider the example depicted in Figure 1, where $p_d > p_f$, $p_d + p_f \leq 2S - 1$ and q is "small". The market is covered and some consumers willing to buy from the foreign firm are rationed. Under efficient rationing, they are located in the interval $[\tilde{x}(p_d, p_f); 1 - q]$. Thus, the effective demand of the domestic firm is $1 - q$, as long as p_d is less than $S - (1 - q)$.

2.3 The quota-constrained equilibrium

The analysis of the pricing game with a quota q proceeds as follows. Given a foreigner's price, the domestic producer may either name a high price (which will make the quota binding, thereby generating rationing and spillovers) or name a low price (in which case it enjoys its free trade demand). We first characterize the shape of demands corresponding to these strategic options and then compute the firms' best replies.

Referring to Figure 2, we identify four critical regions in the price space. Suppose that $q > 1/2$ and consider prices such that the market is covered and the quota is not binding. This is the case for $p_d < p_f$ with $p_d + p_f \leq 2S - 1$, which corresponds to area A in Figure 2. For higher p_f , we leave area A to enter area D : the marginal consumer $\tilde{x}(p_d, p_f)$ stops purchasing the good and the market is not covered. Consider instead a higher p_d . Then $D_f(\cdot)$ increases whereas $D_d(\cdot)$ decreases until the quota becomes binding. This occurs when we reach area B . Note that in this area $p_d \leq S - 1 + q$; accordingly, the domestic firm recovers all rationed consumers and its sales are equal to $1 - q$. If p_d is above $S - 1 + q$, some consumers cease to buy. This is area C : the foreign producer is still constrained by the quota but the market is not covered. Formally, the equation of the A – B frontier is defined by the solution to $1 - \tilde{x}(p_d, p_f) = q$, i.e. $p_d = p_f + 1 - 2q$. The equation of the A – D frontier is $p_d = p_f + 1 - 2q$.

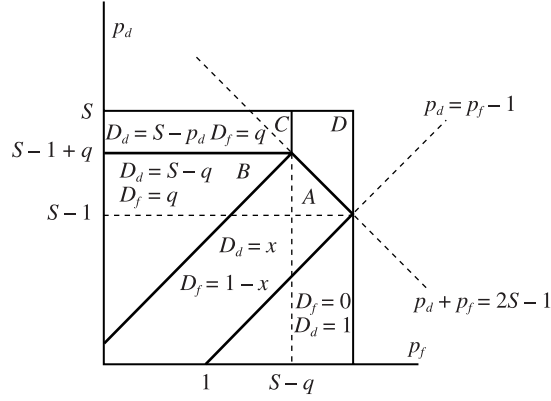


FIGURE 2. Partition of the price space according to the definition of demands

Thus, in the presence of a quota $q < 1$, demand functions may be defined as follows:

- Whenever $p_f \leq S - q$,

$$D_d(p_d, p_f) = \begin{cases} S - p_d & \text{if } p_d \geq S - 1 + q \\ 1 - q & \text{if } p_f + 1 - 2q \leq p_d \leq S - 1 + q \\ \max\{0, \tilde{x}(p_d, p_f)\} & \text{if } p_d \leq p_f + 1 - 2q \end{cases} \quad (5)$$

$$D_f(p_d, p_f) = \begin{cases} q & \text{if } p_f \leq p_d - 1 + 2q \\ \max\{0, 1 - \tilde{x}(p_d, p_f)\} & \text{if } p_f \geq p_d - 1 + 2q \end{cases} \quad (6)$$

- Whenever $p_f \geq S - q$, demand functions are defined as in equations (1)–(4).

Notice that (5) leads to a non-concave profit function as the domestic firm's payoff exhibits two local maxima: $S - 1 + q$, and $H_d(p_f) = (1 + p_f)/2$. We define by $\hat{p}_f$ the level of the foreign price such that the domestic firm's profit is equal at these two maxima. Since the associated profits are, respectively, $(1 + p_f)^2/8$ and $(S - 1 + q)(1 - q)$, solving for $(1 + p_f)^2/8 = (S - 1 + q)(1 - q)$ gives $\hat{p}_f \equiv \sqrt{[8(S - 1 + q)(1 - q)]} - 1$. We also identify the level of p_d , such that the foreign firm hits the quota when playing $H_f(p_d)$. Denoting this price by $\hat{p}_d$, and solving for $D_f(p_d, H_f(p_d)) = (1 + p_d)/4$, we obtain $\hat{p}_d = 4q - 1$. With these definitions in hand, we define the firms' best replies in the following lemma.

Lemma 2: Assume $q \geq 1 - S/2$, then the firms' best replies are given by

$$\psi_d(p_f) = \begin{cases} S - 1 + q & \text{if } p_f \leq \hat{p}_f \\ (1 + p_f)/2 & \text{if } \hat{p}_f \leq p_f \leq 3S/2 - 1 \\ S/2 & \text{if } 3S/2 - 1 \leq p_f \end{cases} \quad (7)$$

$$\psi_f(p_d) = \begin{cases} (1 + p_d)/2 & \text{if } p_d \leq \hat{p}_d \\ p_d + (1 - 2q) & \text{if } \hat{p}_d \leq p_d \leq S - 1 + q \\ S - q & \text{if } S - 1 + q \leq p_d \end{cases} \quad (8)$$

The proof of this lemma is detailed in the appendix. The key point to notice is that the best reply of the domestic firm is discontinuous, jumping “down” at $\hat{p}_f$, while the best reply of the foreign firm is continuous but exhibits a kink at $\hat{p}_d$. In the following proposition we characterize the Nash equilibrium for the complete range of quota levels under the assumption that $S < 3$.⁷

Proposition 1: *When $S < 3$ there exists a unique equilibrium of the pricing game. Its shape depends on the level of the quota.*

- (i) *If $q < 1 - S/2$, the domestic firm acts as a pure monopolist and the foreign firm sells its quota; the market is not covered.*
- (ii) *If $1 - S/2 \leq q < 1 - S/3$, the foreign firm sells its quota at price $S - q$, and the domestic firm covers the market with price $S - 1 + q$.*
- (iii) *If $1 - S/3 \leq q \leq \bar{q} = 1 - [S - \sqrt{(S^2 - 2)}]/2$, the equilibrium is in mixed strategies: the domestic firm randomizes between $S - 1 + q$ and $H_d(\hat{p}_f)$ while the foreign firm plays $\hat{p}_f$.*
- (iv) *If $q \geq \bar{q}$, firms play the Hotelling equilibrium prices $(1, 1)$.*

The proof essentially replicates the argument developed in Krishna (1989) to the particular case of the Hotelling model; it is provided in the appendix.

Two remarks are in order. Notice first that we can relate the size of the interval $[1 - S/3; \bar{q}]$ to S , the fundamental parameter of the model. As shown in the proof, $\partial \bar{q} / \partial S > 0$ so that, the larger the parameter S , the larger the interval that supports a mixed strategy equilibrium. The reason for this is quite intuitive: when S is large, the profit levels at the Hotelling equilibrium are well below the monopoly profit level since the Hotelling prices do not depend on S (see Lemma 1). As revealed by our previous analysis, the main implication of the quota is that the domestic firm can play on its monopolist profit curve by naming $S - 1 + q$ with positive probability. Therefore, a higher S brings about a greater incentive to use the quota strategically.

Second, Proposition 1 states that a pure strategy equilibrium exists under highly restrictive quota levels (case (ii)). This possibility was not considered by Krishna (1989) and indeed was not relevant in her setting. This result is very specific to the Hotelling model. In the equilibrium of type (ii), the domestic firm has lost *any* incentive to compete in price, and both firms name the highest price ensuring full market coverage, given the quota.

3. Optimal quota and the mode of competition

A quota affects domestic welfare in three different ways. First, there is a profit diversion effect; that is, a part of the total welfare is captured by the foreign producer. A quota decreases welfare if, other things being equal, it increases foreign profits. Second, there is a price differential effect: total welfare is maximized when prices are equal because this minimizes consumers’ utility losses. Therefore, a quota inducing a price difference negatively affects welfare. Third, a quota negatively affects welfare if it prevents full market coverage.

The domestic welfare W_d is computed from the total welfare function by subtracting the foreign profit. In the Hotelling model, total welfare, W , depends only on the position of the marginal consumer, as long as the market is covered. Formally, we define

⁷ This last assumption is made to keep the exposition simple. Readers are referred to Boccard and Wauthy (1997) for a treatment of equilibria for arbitrary S .

$$W(p_d, p_f) = \int_0^{\tilde{x}} (S - x) dx + \int_{\tilde{x}}^1 (S - 1 + x) dx = S - \frac{2\tilde{x}^2 - 2\tilde{x} + 1}{2}. \quad (9)$$

W is maximal when $\tilde{x}(p_d, p_f) = 1/2$, i.e. when prices are identical. Proposition 2 (whose proof is in the appendix) characterizes the optimal quota.

Proposition 2: *The domestic government implements the most restrictive quota that is compatible with market coverage.*

Notice that it is only if the reservation price is very low ($3/2 < S < 2$) that neither complete protectionism nor free trade is optimal for the government. The analysis is performed under zero production cost, but it is easy to see that, with symmetric constant marginal cost c , the relevant constellation would be $S - c \in]3/2; 2[$. Thus, the case for a restrictive quota depends in fact on the difference between the valuation of the product on the consumers' side in the domestic market and the marginal cost of production.

Is this result sensitive to the mode of competition? Rather surprisingly, we shall show that *in the present model, the level of the optimal quota is invariant to the mode of competition*. To this end, we first characterize a Cournot equilibrium in the Hotelling model.

Lemma 3 (Cournot Equilibrium): *There exists a continuum of Cournot equilibria (q_d, q_f) where the market is exactly covered ($q_d + q_f = 1$). The equilibrium market-clearing prices are $p_d = S - q_d$ and $p_f = S - q_f$.*

Proof: Under Cournot competition, firms supply quantities q_i and q_j in a competitive market. (i and j belong to $\{d, f\}$.) If the quantities q_i and q_j do not cover the market, prices are set to equalize supply and demand on each side of the market, i.e. to satisfy $q_i = S - p_i$ and $q_j = S - p_j$. If $S > 1$, this quantity pair cannot define an equilibrium because at least one firm has an incentive to increase its quantity above the complement of the other. If the proposed quantities q_i and q_j exceed the market size, excess supply prevails and at least one of the prices, say p_i , must be zero in order to clear the market. Such quantities cannot form an equilibrium either, since firm i prefers to deviate by offering a quantity slightly lower than $1 - q_j$, which is sold at positive price.

The remaining candidates for a Cournot equilibrium are $(q, 1 - q)$ with $q \leq 1/2$. The market-clearing prices are $S - q$ and $S - 1 + q$.⁸ Without loss of generality, firm i offers q , thus selling less than $1/2$ in equilibrium. Hence p_i cannot be zero, for it would attract at least one half of the consumers, thereby implying an excess demand. Firm j cannot profitably deviate to a quantity larger than $1 - q$ because it would face a zero price. (One price must be zero, and from the above argument, it must be the price of the large firm.)

Firm i may however profitably deviate to some $Q \in [q; 1/2]$. Since there is excess supply, p_j is nil; thus, firm j sells Q and the consumer located at $x = Q$ must be indifferent between the two products in equilibrium. Hence we must have $p_i = 1 - 2Q$. The associated profit $Q(1 - 2Q)$ reaches a maximum of $1/8$ when $Q = 1/4$, which is compared with $q(S - q)$. Since $q \leq 1/2$, the only relevant root is $q^* = S/2 - \sqrt{(2S^2 - 1)/2} > 0$. Therefore, a Cournot

⁸ Notice that for exact market coverage there is a continuum of prices that clear the market, but only the highest possible ones can appear in a subgame-perfect equilibrium.

equilibrium features exact market coverage $(q, 1 - q)$ for any $q > q^*$ if $S > 2$. For $S < 2$, the support of the continuum is possibly bounded by $S/2$. ■

The intuition underlying the lemma is straightforward. If quantities exceed the market size, at least one of the two prices must be equal to zero in order to clear the market. Therefore the firm, say firm i , selling at a zero price gains by reducing its quantity to $1 - q_j$ so as to ensure a positive market-clearing price. When quantities are small, the market-clearing prices are computed along each firm's local monopoly demand and each of them has an incentive to raise its quantity. Using Lemma 3, we establish the following proposition.

Proposition 3: *The optimal quota is invariant to the mode of competition.*

Proof: Under quantity competition, the presence of a quota reduces the range of the equilibria continua. The strategy set for the foreign firm is defined as $q_f \in [0; q]$ where q is the quota set by the domestic government. Total welfare is $W(p_d, p_f)$ as defined in (9), but evaluated at Cournot prices. Letting $q_d = 1 - q_f$, we obtain a total welfare $W(S, q_d) = S - q_d^2 + q_d - 1/2$. The foreigner's profit is $\Pi_f = (1 - q_d)(S - 1 + q_d)$. Thus, the domestic welfare is $W_d(S, q_d) = q_d(S - 1) + 1/2$. It is increasing in q_d since $S > 3/2$. Therefore, the government chooses the most restrictive quota compatible with market coverage. If $S > 2$, this corresponds to complete protectionism ($q = 0$), whereas if $S < 2$, the government chooses a restrictive quota allowing the foreign producer to sell the complement of the domestic monopoly equilibrium ($q = 1 - S/2$). In both cases, the continuum of Cournot equilibria is reduced to a single point: the one obtained under price competition. ■

4. Import quota and location choice

As shown in Section 2, a quota deeply alters price competition. Accordingly, it could also influence the choice of products' attributes. Given the existence problems in the Hotelling model for *linear* transportation costs,⁹ we modify our basic model by considering *quadratic* transportation costs. It is well known that under quadratic transportation costs firms maximize differentiation. In this section we show that, in the presence of the quota, maximal differentiation cannot be sustained at the equilibrium.

Regarding the price competition stage, the only changes we consider are the locations $0 \leq x_f < x_d \leq 1$ of the domestic and foreign firms and the assumption of quadratic transportation cost for consumers. The utility derived by a consumer located at x in the interval $[0; 1]$ is defined as follows:

$$\begin{cases} 0 & \text{if no purchase is made} \\ S - (x_d - x)^2 - p_d & \text{if the product is bought at the domestic firm} \\ S - (x_f - x)^2 - p_f & \text{if the product is bought at the foreign firm} \end{cases}$$

The following lemma (whose proof can be found in the appendix) characterizes the subgame-perfect equilibrium in the location-price Hotelling game under free trade.

⁹ It is well known however that, under linear transportation costs, firms want to move towards the centre, but also that, once they are located too close to the centre, i.e. within $[1/4; 3/4]$, no pure strategy equilibrium exists.

Lemma 4 (The maximum differentiation principle): *If $S > 5/4$, the unique Nash equilibrium of the pricing game is $(1, 1)$ and the market is covered. The location equilibrium is characterized by maximal differentiation.*

Consider then the presence of an import quota q . We assume again that rationing is efficient. Defining $\bar{x} \equiv (x_f + x_d)/2$, we solve equation $D_f(p_d, p_f) = q$ to define $p_f = p_d - 2(q - \bar{x})(x_d - x_f)$. For a low price p_d , the foreign firm's best reply is $H_f(p_d) = p_d/2 + (x_d - x_f)\bar{x}$ whenever $\bar{x}(p_d, H_f(p_d)) \leq q$. Solving for prices, we obtain $p_d \leq \hat{p}_d = 2(x_d - x_f)(2q - \bar{x})$. If $p_d \geq \hat{p}_d$, the optimal strategy for the foreign firm is to sell the quota at the maximum price $p_f = p_d - 2(q - \bar{x})(x_d - x_f)$. Thus, the best reply of the foreign firm is defined as

$$\psi_f(p_d) = \begin{cases} \frac{p_d}{2} + (x_d - x_f)\bar{x} & \text{if } p_d \leq \hat{p}_d \\ p_d - 2(q - \bar{x})(x_d - x_f) & \text{if } p_d \geq \hat{p}_d \end{cases} \quad (10)$$

The domestic firm can either play $H_d(p_f) = p_f/2 + (x_d - x_f)(1 - \bar{x})$ or name $p_d^s = S - (x_d - q)^2$, in which case it sells $1 - q$. The latter option is optimal. The profits associated with each strategy are

$$\frac{\left(\frac{p_f}{2} + (x_d - x_f)(1 - \bar{x})\right)^2}{8(x_d x_f)} \quad \text{and} \quad (1 - q)[S - (x_d - q)^2].$$

They are equal for

$$p_f = \hat{p}_f \equiv 4\sqrt{[2(x_d - x_f)(1 - q)(S - (x_d - q)^2)] - 2(x_d - x_f)(1 - \bar{x})}.$$

The discontinuous best-reply function of the domestic firm is therefore defined as

$$\psi_d(p_f) = \begin{cases} S - (x_d - q)^2 & \text{if } p_f \leq \hat{p}_f \\ H_d(p_f) & \text{if } p_f \geq \hat{p}_f \end{cases} \quad (11)$$

Finally, defining $q(S, x_d, x_f)$ as the critical level of the quota such that $\hat{p}_f < p_f^*$, (with p_f^* denoting the free-trade equilibrium price) we can see that the Hotelling equilibrium for locations (x_d, x_f) does not exist in the quota game for $q < q(S, x_d, x_f)$. Characterizing Nash equilibrium as in Proposition 1, we may go backward and show that there exists a threshold level for the quota below which maximum differentiation cannot be sustained in a subgame-perfect equilibrium.

Proposition 4: *Whenever the quota is set at a level lower than $\bar{q}(S, 0, 1) \simeq 0.9 - S/2 + 1.8\sqrt{S}$, firms are always located in the interior of the $[0, 1]$ interval in a subgame-perfect equilibrium.*

The argument underlying the proof (to be found in the appendix) is straightforward. As shown in Proposition 1, the presence of the quota allows the domestic firm to benefit from a local monopoly in the market area that cannot be served by the quota-constrained foreign firm. In order to maximize profits over this area, the domestic firm prefers to settle in the middle of the area, in order to minimize consumers' transportation costs and thereby increase their reservation prices. This is sufficient for us to conclude that maximal differentiation cannot be part of a subgame-perfect equilibrium.

5. Final remarks

After the seminal paper of Krishna (1989), very few papers investigated the effects of quotas under price competition. In this paper we have studied the implications of import restraints in the Hotelling model of horizontal differentiation. We have shown that the optimal policy consists in implementing a restrictive quota which guarantees full market coverage. Moreover, the *optimal* quota is invariant to the mode of competition. We have also shown that the presence of the quota induces a tendency towards less product differentiation, as compared with free trade.

Our results have been established under restrictive assumptions. We do not expect our invariance result to generalize to all models of horizontal differentiation. On the other hand, it is sensible to argue that the presence of a trade quota induces less differentiation in any model where equilibrium differentiation is used to avoid price competition. In Boccard and Wauthy (1998) we studied a similar problem under vertical differentiation and again concluded that the quota lowers the degree of differentiation.

Appendix

Proof of Lemma 2

In order to derive best replies, we consider in turn the optimal responses in each of the four areas of Figure 2 and then compare resulting payoffs to derive the effective best replies.

The foreign firm best reply

Assume first that the foreign firm is not constrained given p_d (area A). The best reply is $H_f(p_d) = (1 + p_d)/2$ and demand is $D_f(p_d, H_f(p_d)) = (1 + p_d)/4$. $D_f(p_d, H_f(p_d)) = q$ when $\hat{p}_d = 4q - 1$. If $p_d \geq \hat{p}_d$, the foreign firm's best reply is to sell the quota at price $p_d + 1 - 2q$ (the frontier between A and B). In areas B and C, $D_f(\cdot) = q$ and the profit is increasing in p_d . Thus, the optimal price is defined by the frontier between areas A and D. In region D there is a monopoly demand $S - p_d$. We shall assume in what follows that $p^m = S/2 < S - q$, so that the monopoly profit function is decreasing everywhere in D and the optimal choice is $S - q$.¹⁰ Note then that the optimal price in $B \cup C$ is dominated by that of $A \cup D$; therefore the best reply of the foreign firm is the continuous function:

$$\psi_f(p_d) = \begin{cases} \frac{1 + p_d}{2} & \text{if } p_d \leq \hat{p}_d \\ p_d + (1 - 2q) & \text{if } \hat{p}_d \leq p_d \leq S - 1 + q \\ S - q & \text{if } S - 1 + q \leq p_d \end{cases}$$

The domestic firm best reply

In area $C \cup D$ the monopolistic price $S/2$ is the overall maximizer of the profit. If $S/2 \geq S - 1 + q$, i.e. $q < 1 - S/2$, the monopoly price is a dominant strategy for the domestic

¹⁰ Otherwise when $q > 3/4$ as $S \geq 3/2$, the best reply of the domestic firm would thus create competition and drive us back to area A, which means that $S/2$ does not appear in equilibrium.

firm. Otherwise, the optimal price is $S - 1 + q$. In area $A \cup B$ there are two best reply candidates. The first one is the free-trade best reply, $(1 + p_f)/2$. The second one is $S - 1 + q$. The associated profits are, respectively, $[(1 + p_f)^2]/8$ and $(S - 1 + q)(1 - q)$. Equalizing these two expressions and solving for p_f gives $\hat{p}_f \equiv \sqrt{[8(S - 1 + q)(1 - q)]} - 1$. If $q \geq 1 - S/2$, we obtain the discontinuous best reply correspondence:

$$\psi_d(p_f) = \begin{cases} S - 1 + q & \text{if } p_f \leq \hat{p}_f \\ \frac{1 + p_f}{2} & \text{if } \hat{p}_f \leq p_f \leq \frac{3S}{2} - 1 \\ \frac{S}{2} & \text{if } \frac{3S}{2} - 1 \leq p_f \end{cases}$$

while if $q < 1 - S/2$, we have $\psi_d(p_f) = S/2$. ■

Proof of Proposition 1

Observe first that for any p_d , $D_f(p_d, \cdot)$ is constant and then linearly decreasing, so that the profit function $\Pi_f(p_d, \cdot)$ is concave for any p_d . Thus, whatever mixed strategy $F_d(p_d)$ the domestic firm might play, $\Pi_f(F_d, \cdot) = \int \Pi_f(p_d, \cdot) d(F_d(p_d))$ is concave and has a unique maximizer. This implies that in a Nash equilibrium the foreign firm plays a pure strategy.

Case (i): If $q < 1 - S/2$, the domestic firm uses its dominant strategy $S/2$ and the best reply of the foreign firm is then $S - q$; those prices form the unique Nash equilibrium. $D_d = S/2 < 1 - q$ and $D_f = q$, and the market is not covered.

Case (ii): Consider the equilibrium candidate $(p_d = S - 1 + q, p_f = S - q)$. The market is exactly covered; i.e., the consumer located at q is indifferent between buying domestic, foreign, or not buying. This behaviour is optimal for the domestic firm if $S - q < \hat{p}_f$, which leads to $q < 1 - S/3$. Note that $S - q$ remains optimal for the foreign firm.

Case (iii): We have $1 < \hat{p}_f < S - q$ if $q \in [1 - S/3, \bar{q}]$. Best reply curves do not intersect in this case, hence there exists no pure strategy equilibrium. Still, the foreign firm plays the pure strategy $\hat{p}_f$ at equilibrium,¹¹ while the domestic firm randomizes over $S - 1 + q$ and $(1 + \hat{p}_f)/2$. Let μ define the weight put by the domestic firm strategy F_d on $S - 1 + q$. The foreign profit against F_d is

$$\Pi_f(F_d, p_f) = p_f [\mu q + (1 - \mu) \tilde{x} \left(\frac{1 + \hat{p}_f}{2} \right), \hat{p}_f].$$

Solving for $[\partial \Pi_f(F_d, p_f)] / \partial p_f = 0$, we obtain the argmax of $\Pi_f(F_d, p_f)$ as a function $P(\cdot)$ of μ . We then solve $P(\mu) = \hat{p}_f$ to obtain $\bar{\mu} = 4q / (4q - 3 + 3\hat{p}_f)$. By construction, $\hat{p}_f$ is a best reply for the foreign firm against F_d .

¹¹ The technical condition is $\hat{p}_f < (1 + \hat{p}_f)/2 \Leftrightarrow (S - 1 + q)(1 - q) < 2$, for otherwise $D_d(\hat{p}_f, (1 + \hat{p}_f)/2) = 0$ implying that Π_f is locally increasing. $S < 3$ is sufficient to guarantee this condition.

Case (iv): Assume $q = 1$. The Hotelling equilibrium $(1, 1)$ remains an equilibrium. More generally, we may derive the conditions under which $\hat{p}_f < 1$ and $\hat{p}_d > 1$. From the first inequality, we obtain $(S - 1 + q)(1 - q) < 1/2$, which implies $q > \bar{q} = 1 - (S - \sqrt{S^2 - 2})/2$. (The other root is negative.) From the second inequality, we obtain $q > 1/2$, which is satisfied by $\bar{q}$ as $S > 3/2$. ■

Proof of Proposition 2

We use (9) to compute and compare welfare in the four equilibrium configurations defined in Proposition 1.

Case (i): For $q \in [0, 1 - S/2]$, the total welfare is given by

$$W^i(q) = \int_0^{\frac{S}{2}} (S - x) dx + \int_{1-q}^1 (S - 1 + x) dx = \frac{3S^2}{8} + q(S - \frac{q}{2}). \quad (12)$$

Since $\Pi_f(q) = q(S - q)$, we have $W_d^i(q) = 3S^2/8 + q^2/2$ and $\partial W_d^i(q)/\partial q > 0$. Thus, the optimal quota in this region is $1 - S/2$.

Case (ii): For $q \in [1 - S/2, 1 - S/3]$, the marginal consumer is located at q and (9) is given by

$$W^{ii}(q) = S - \frac{1}{2}[2(1 - q)^2 - 2(1 - q) + 1] = S - \frac{1}{2}(2q^2 - 2q) + 1. \quad (13)$$

Since $\Pi_f(q) = q(S - q)$, we have $W_d^{ii}(q) = q(1 - S) + S - 1/2$ and $\partial W_d^{ii}(q)/\partial q < 0$. Hence the optimal quota is $1 - S/2$,

Case (iii): For $q \in [1 - S/3, \bar{q}]$, the domestic firm mixes between $S - 1 + q$ and $(1 + \hat{p}_f)/2$ with probabilities $\bar{\mu}$ and $1 - \bar{\mu}$; the foreign firm quotes $\hat{p}_f$. Total welfare is defined as

$$\begin{aligned} W^{iii}(q) &= \bar{\mu}W(S - 1 + q, \hat{p}_f) + (1 - \bar{\mu})W\left(\frac{1 + \hat{p}_f}{2}, \hat{p}_f\right) \\ &= \frac{2(6S - 3 + 7q - 6q^2)\hat{p}_f - (4q - 3)(4S + 3q - 2) - 9q\hat{p}_f^2}{4(4q - 3 + 3\hat{p}_f)}. \end{aligned} \quad (14)$$

A numerical analysis¹² indicates that domestic welfare is strictly convex in the relevant domain; therefore it cannot be optimal for the government to set a quota in the interior of $[1 - S/3, \bar{q}]$.

Case (iv): For $q > \bar{q}$, domestic welfare is constant over $[\bar{q}, 1]$ and equal to $W_d^{iv} = S - 3/4$.

In order to identify the optimal quota, we compare the free trade solution ($q = 1$) to either the “market-complement” solution ($q = 1 - S/2$) or the complete protectionism solution ($q = 0$). When $S > 2$, the complete protectionism is optimal because $W_d^{ii}(0) = S - 1/2$. However, when, $S < 2$, $W_d^i(1 - S/2) = (1 - S + S^2)/2 > S - 3/4 = W_d^{iv}$. Thus, it is optimal to let the foreign firm cover the part of the market that is not served by the domestic monopolist. ■

¹² The details are available upon request.

Proof of Lemma 4

Assume that S is large enough to ensure market coverage at the equilibrium. Solving $S - (x_d - x)^2 - p_d = S - (x_f - x)^2 - p_f$ for x , we obtain $\tilde{x}(p_d, p_f) = (p_d - p_f)/2(x_d - x_f) + \bar{x}$, where $\bar{x} \equiv (x_d + x_f)/2$. We have $D_f(\cdot) = \tilde{x}(p_d, p_f)$ and $D_d(\cdot) = 1 - \tilde{x}(p_d, p_f)$.

Direct computations yield the following best replies: $H_d(p_f) = p_f/2 + (x_d - x_f)(1 - \bar{x})$ and $H_f(p_d) = p_d/2 + (x_d - x_f)\bar{x}$. The Nash equilibrium is therefore $p_f^* = (2/3)(x_d - x_f)(1 + \bar{x})$, $p_d^* = 2/3(x_d - x_f)(2 - \bar{x})$. Note that $\tilde{x}(p_d^*, p_f^*) = (1 + \bar{x})/3$, the net price paid by the indifferent consumer is $(x_d - (1 + \bar{x})/3)^2 + p_d^*$, while the consumer located at 1 obtains $(1 - x_d)^2 + p_d^*$ and the consumer located at 0 obtains $x_f^2 + p_f^*$. Those expressions are maximal and are equal to $5/4$, 1 and 1 respectively when differentiation is maximized. Thus, $S > 5/4$ is a sufficient condition to guarantee market coverage at the Hotelling equilibrium for all possible pairs of locations.¹³

Plugging (p_f^*, p_d^*) in the profit functions, we obtain the reduced-form payoffs at the location stage:

$$\Pi_d = \frac{(4 - x_d - x_f)[(x_d(4 - x_d) - x_f(4 - x_f))]}{18}$$

$$\Pi_f = \frac{(2 + x_d + x_f)[x_d(2 + x_d) - x_f(2 + x_f)]}{18}$$

Direct computations yield the best replies in the location game: $L_d = (4 + x_f)/3$ and $L_f = (x_d - 2)/3$. Optimal locations candidates are $-1/4$ and $5/4$. The equilibrium locations are therefore the boundaries of the market segment. Firms' profits in the subgame-perfect equilibrium are equal to $1/2$. ■

Proof of Proposition 4

Suppose the quota is loose: the classical Hotelling pair (p_d^*, p_f^*) is the equilibrium. Applying the same methodology as in the linear case, we solve $\hat{p}_f < p_f^*$ and $\hat{p}_d > p_d^*$ for q . From the first condition, we obtain $q > \bar{q}(S, x_d, x_f)$.¹⁴ $\bar{q}(\cdot)$ is decreasing in the degree of differentiation. From the second inequality, we obtain $q > (1 + \bar{x})/3$, which is satisfied whenever $\bar{q}(S, x_d, x_f) > (1 + \bar{x})/3$. This last inequality is always satisfied.

When $q < \bar{q}(S, x_d, x_f)$, the foreign firm plays $\hat{p}_f$ while the domestic randomizes between p_d^s and $H_d(\cdot)$.

For all $q < \bar{q}(S, 0, 1)$, we have $q < \bar{q}(S, x_d, 1)$ and $q < \bar{q}(S, 0, x_f)$. Thus, when a firm moves towards the centre, the equilibrium is in mixed strategies. Using $\Pi_d(x_d) = (1 - q)[S - (x_d - q)^2]$, we derive the best reply as $x_d^* = 1 - (1 - q)/2 = (1 + q)/2$. Using $\Pi_f(p_f) = p_f(1 - \mu)q + \mu[1 - \tilde{x}(H_d(\hat{p}_f), \hat{p}_f)]$ and solving $\partial \Pi_f(p_f)/\partial p_f = 0$ at $p_f = \hat{p}_f$, we derive $\hat{\mu}$ to finally obtain

$$\Pi_f(x_f) = \frac{2q\hat{p}_f}{3\hat{p}_f - (x_d - x_f)(2 - 4q + x_d + x_f)}.$$

¹³ If firms play prices such that the market is not covered, then each one is a local monopoly. When $S > 5/4$, the pair of monopoly prices defines market segments that overlap, so that the solution is not feasible.

¹⁴ The explicit formula is available from the authors.

Assuming a covered market, the location best reply to $x_d = (1 + q)/2$ can be studied. A numerical computation shows that the optimal location is $x_f^* \approx 2q - 1$ for $q > 55\%$ (for S in the range $[3/2; 3]$) and zero otherwise. Since the domain of existence of the mixed strategy equilibrium is for intermediate quotas,¹⁵ we may conclude that a quota just below $\bar{q}(S, 0, 1)$ (which ranges from 65% to 85% for S in the range $[3/2; 3]$) would lead both firms to move towards the centre of the market at $x_d^* = (1 + q)/2$ and $x_f^* = 2q - 1$. Accordingly, maximal differentiation cannot prevail in a subgame-perfect equilibrium. ■

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¹⁵ Recall that a constrained pure strategy equilibrium, similar to case (ii) of Proposition 1, holds otherwise.