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Mathematical Social Sciences, 94, 65-81, 2018



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Education choices, longevity and optimal policy in a Ben-Porath economy[☆]

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HIGHLIGHTS

- According to Ben-Porath (1967), a rise in longevity increases education investment.
- We use an OLG model to examine conditions under which a Ben-Porath effect exists.
- Under exogenous longevity, a Ben-Porath effect requires a change in old-age labor.
- Under endogenous longevity, a Ben-Porath effect exists even if labor time is fixed.
- The Ben-Porath effect may not be robust to variations in production factor prices.

ARTICLE INFO

Article history:

Received 28 March 2017

Received in revised form 26 September 2017

Accepted 13 October 2017

Available online 31 October 2017

ABSTRACT

We develop a 3-period overlapping generations (OLG) model where individuals borrow at the young age in order to finance their education. Education does not only increase future wages, but also raises the duration of life, which, in turn, can affect education, in line with Ben-Porath (1967). We examine the conditions under which the Ben-Porath effect prevails. Although the existence of a positive Ben-Porath effect requires, under exogenous longevity, a change in lifetime hours of work, we find, under endogenous longevity, that a positive Ben-Porath effect arises even when old-age labor is fixed. It is also shown that the Ben-Porath effect may not be robust to allowing for adjustments in production factor prices. On the policy side, we show that the social optimum can be decentralized provided the capital stock is set to the Modified Golden Rule level. Finally, we introduce intracohort heterogeneity in learning ability, and we show that, under asymmetric information, the second-best optimal non-linear tax scheme involves a downward distortion in the education of less able types, which reinforces the longevity gap in comparison with the first-best.

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1. Introduction

In a seminal contribution, Ben-Porath (1967) highlighted a major channel through which demography affects economic development. According to Ben-Porath, a rise in life expectancy increases

lifetime returns from educational investment without increasing the cost of education, and, hence, encourages investments in education. Improvements in survival conditions can thus increase education and favor human capital accumulation and growth. The so-called “Ben-Porath effect” has become increasingly studied by growth theorists, when considering the impact of exogenous changes in mortality on education and development (see Ehrlich and Lui, 1991, Boucekine et al., 2002, Ludwig and Vogel, 2010), or when examining, in models with endogenous mortality, how development and longevity reinforce each other (see Blackburn and Cipriani, 2002, Chakraborty, 2004, Cervellati and Sunde, 2005, Soares, 2005).

On the empirical side, several studies tested whether life expectancy improvements affect economic growth. This impact may

[☆] The authors would like to thank the Editor, Juan Moreno-Ternero, and two anonymous referees for their helpful comments and suggestions on this manuscript. Yukihiro Nishimura acknowledges the financial support from the Grants-in-Aid for Scientific Research (C) from the Ministry of Education, Culture, Sports, Science, and Technology (24530348, 15K03511) and Strategic Young Researcher Overseas Visits Program for Accelerating Brain Circulation from the Japan Society for the Promotion of Science (J2402).

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not be due to the Ben-Porath effect, since life expectancy may affect growth through other channels (e.g. saving). But the Ben-Porath effect could explain, in theory, a positive impact of life expectancy growth on economic growth. Bloom et al. (2004) show that a 5-year increase in life expectancy generates a 21% rise of the growth rate.¹ Acemoglu and Johnson (2007) found that, once adequate instruments are used to avoid endogeneity biases, life expectancy does not seem to affect economic growth. Hazan (2009) argued that the Ben-Porath effect can only arise provided additional life-years are years of occupation, which has not been observed. On the contrary, de la Croix et al. (2009) find, for Sweden, that the longevity increase accounts for 20% of the rise in education over the last two centuries. Bloom et al. (2014) criticized Acemoglu and Johnson's approach, on the ground that they neglect the impact of initial conditions. Cervellati and Sunde (2011) argued also against Acemoglu and Johnson (2007) that, once we take into account fertility differences between countries, life expectancy increases economic growth in countries having accomplished their demographic transition.

Those mixed empirical results suggest that, although the Ben-Porath effect is theoretically plausible, that effect is far from being universally observed. On the contrary, its size depends on various kinds of factors. In particular, Hazan's (2009) criticism suggests that the plausibility of the Ben-Porath effect should be assessed in a theoretical model where the retirement age is not exogenous, but rather where individuals decide when they retire.

The goal of this paper is precisely to reexamine the Ben-Porath effect in a dynamic economy where individuals choose not only their education, but, also, the age at which they retire, and where the duration of their life depends on the amount of education.²

In other words, this paper proposes to examine, by means of a dynamic general equilibrium OLG model where education, saving and retirement are chosen by individuals, the conditions under which a rise in lifetime horizon contributes to increase education investment. Following Boucekkine et al. (2002), who emphasized that the Ben-Porath effect depends on the depreciation of human capital, we assume that there exists some decay of human capital, which may affect education decisions. We use that OLG model to identify the various determinants of the Ben-Porath effect.

Moreover, besides this positive goal, we would like also to explore some policy implications of the Ben-Porath effect. Actually, whereas the Ben-Porath effect has been widely studied at the theoretical and empirical levels, little emphasis has been laid, so far, on its policy implications. It is widely acknowledged that education constitutes a major vector of inequalities, and that inequalities in life expectancy and in education are strongly related (see Elo and Preston, 1996, Deboosere et al., 2009, Zarulli et al., 2012). This connection between education and life expectancy is consistent with the Ben-Porath effect. But few attempts were made to derive the optimal public intervention in an economy where education and life expectancy are correlated. For that purpose, we use our model to study the design of the optimal public policy in a Ben-Porath economy. We first consider the decentralization of the social optimum in an economy composed of homogeneous agents. Then, we introduce heterogeneity in individual learning ability, and we study the optimal public intervention in a second-best setting where learning ability is not observed by the government.

Anticipating our results, we first identify conditions that guarantee the existence of a stationary equilibrium with perfect foresight, and we compare those conditions in the cases of exogenous

longevity (i.e. longevity is fixed to a constant) and endogenous longevity (i.e. longevity depends on education choices). Then, we reexamine the conditions under which the Ben-Porath effect prevails. We first follow the existing literature, which generally abstracts from changes in production factor prices. When examining the Ben-Porath effect in a small open economy (where the wage and the interest rate are fixed), we show that the distinction between cases where longevity is exogenous or endogenous matters, especially concerning the impact of old-age labor on the Ben-Porath effect. Under exogenous longevity, the existence of a positive Ben-Porath effect requires a change in lifetime hours of work, in line with Hazan (2009). However, under endogenous longevity, a positive Ben-Porath effect can be found even in the absence of old-age labor, and even when old-age labor is fixed. As such, this study completes the recent work by Cervellati and Sunde (2013) on the link between life expectancy and education. Turning then to the closed economy, we show that a rise in longevity does not necessarily increase steady-state education, because longevity variations affect wages and interest rates. This general-equilibrium effect has remained so far largely unnoticed in the literature.

On the policy side, we compare the *laissez-faire* with the social optimum, and show that the latter can be decentralized provided the *laissez-faire* capital stock corresponds to the one satisfying the Modified Golden Rule. In a second stage, introducing intracohort heterogeneity in the learning ability allows us to show that, under asymmetric information, the second-best optimal non-linear tax scheme involves a downward distortion in the level of education of less able types, which reinforces the longevity gap in comparison with the first-best.

The rest of the paper is organized as follows. Section 2 presents the model. The temporary equilibrium is characterized in Section 3. Section 4 studies the conditions under which a stationary equilibrium with perfect foresight exists. Section 5 examines, at the stationary equilibrium with perfect foresight, the determinants of the Ben-Porath effect. The social optimum is characterized in Section 6. Section 7 examines the second-best problem when the population is heterogeneous in terms of learning capacity. Section 8 concludes.

2. The model

Let us consider a three-period OLG model. Period 1 is childhood, during which children borrow in order to invest an amount e in their higher education. During period 2, individuals work, pay back the cost of their education, consume and save. Period 3 is the old age. Whereas the durations of periods 1 and 2 are normalized to unity, the duration of period 3 is equal to ℓ ($0 < \ell < 1$).³ During period 3, individuals work some fraction $z \leq \ell$ and consume their saving. We consider, as a baseline, a closed economy.⁴ Moreover, we consider, for the sake of simplicity, an economy composed of identical individuals.⁵

Demography. Fertility is exogenous, and equal to its replacement level. There is no risk about the duration of life. Survival curves are perfectly rectangular. However, the duration of the old age varies over time, as a function of the educational level enjoyed during childhood, according to the function:

$$\ell_{t+1} = \ell(e_{t-1}) \quad (1)$$

where $\ell(0) = \ell > 0$.

³ Alternatively, we could here consider survival probabilities dependent on education. This would imply introducing an annuity market. Further, it would face Bommi er's critique of risk neutrality with respect to the length of life (see Bommi er, 2007).

⁴ When examining the conditions under which the Ben-Porath effect prevails, Section 5 will also consider, in addition, the case of a small open economy (see Section 5.1).

⁵ The case of heterogeneous individuals is discussed in Section 7.

¹ Similar effects are found in Bloom and Sachs (1998), Bloom and Williamson (1998), and Gallup and Sachs (2001).

² Assuming that longevity depends on education is in line with the empirical literature on the increasing (but non-linear) relationship between education and longevity (see Everett et al., 2013).

In order to examine the robustness of the determinants of the Ben-Porath effect to the modeling of survival conditions, we will, in the following, examine two distinct scenarios: (1) longevity is exogenous, that is, $\ell_t = \bar{\ell}$ and $\ell'(e_{t-1}) = 0$; (2) longevity is endogenous, i.e. $\ell'(e_{t-1}) > 0$. The latter reflects the case where people develop through their education a cognitive capacity to choose better survival strategies. In the latter case we assume $\ell''(e_{t-1}) < 0$, $\lim_{e_{t-1} \rightarrow 0} \ell'(e_{t-1}) = +\infty$, $\lim_{e_{t-1} \rightarrow +\infty} \ell'(e_{t-1}) = 0$ and $\lim_{e_{t-1} \rightarrow \infty} \ell(e_{t-1}) = \bar{\ell} < 1$.

Production. Production takes place with physical capital and effective labor, according to a production function $F(\cdot)$ that exhibits constant returns to scale (CRS):

$$Y_t = F(K_t, L_t) \quad (2)$$

where K_t denotes the stock of capital at time t , while L_t is the total amount of effective labor. As usual, we assume that $F_K, F_L > 0$ and $F_{KK}, F_{LL} < 0$ on the non-negative orthant.

In our closed economy, the total amount of effective labor is equal to:

$$L_t = h_t N_t + z_t \alpha h_{t-1} N_{t-1} \quad (3)$$

where h_t denotes the stock of human capital for each young worker at time t , and N_t is the number of young workers at time t , so that $h_t N_t$ is the number of effective labor units from young workers at time t . Moreover, $z_t \alpha h_{t-1} N_{t-1}$ amounts to the number of effective labor units from old workers at time t . This number depends on the retirement age z_t , as well as on the human capital decay, which is captured by the parameter α . When there is no decay, we have $\alpha = 1$. On the contrary, when there is some strong decay, α is close to 0. As to the retirement age z_t , we later show that individuals choose z_t according to the duration of the old age, ℓ_t . In this way, the total labor supply of the economy depends on life expectancy.

Under CRS, and given $N_{t-1} = N_t = N$, the production process can be written as:

$$\tilde{y}_t = \frac{Y_t}{N} = F\left(\tilde{k}_t, h_t + z_t \alpha h_{t-1}\right) \quad (4)$$

where $\tilde{y}_t \equiv \frac{Y_t}{N}$ is the output per young worker and $\tilde{k}_t \equiv \frac{K_t}{N}$ is the capital per young worker.

We assume that capital fully depreciates after one period of use. Regarding the capital market equilibrium, physical capital and borrowing for children's education investment are financed on the basis of individual saving. Given that only the young save for their old days, the capital market clearing condition is:

$$K_{t+1} + N e_t = N s_t \quad (5)$$

where s_t denotes the saving of each young adult.

Let us define capital per effective working unit as $k_t \equiv \frac{K_t}{L_t}$. Forwarding this by one period and using the definition of the total effective labor, we obtain:

$$k_{t+1} = \frac{K_{t+1}}{N h_{t+1} + N z_{t+1} \alpha h_t}. \quad (6)$$

We can rewrite the capital accumulation equation in per effective working units terms as:

$$k_{t+1} = \frac{N(s_t - e_t)}{N h_{t+1} + N z_{t+1} \alpha h_t} = \frac{s_t - e_t}{h_{t+1} + z_{t+1} \alpha h_t}. \quad (7)$$

Regarding the accumulation of human capital, we assume that the level of human capital depends on the amount of education investment at the young age, according to the relation:

$$h_t = h(e_{t-1}) \quad (8)$$

with $h(0) = 1$, $h'(e_{t-1}) > 0$, $h''(e_{t-1}) < 0$, $\lim_{e_{t-1} \rightarrow 0} h'(e_{t-1}) = +\infty$ and $\lim_{e_{t-1} \rightarrow +\infty} h'(e_{t-1}) = 0$. An investment e_{t-1} in education

during childhood yields a human capital h_t equal to $h(e_{t-1})$ at the young age, and a human capital $\alpha h(e_{t-1})$ at the old age. We assume also that $\lim_{e_{t-1} \rightarrow 0} h'(e_{t-1}) = +\infty$ and that $\lim_{e_{t-1} \rightarrow +\infty} h'(e_{t-1}) = 0$.

The economy is perfectly competitive. Workers and capital holders take market prices as given. The wage rate w_t is equal to the marginal productivity of labor, and the interest factor R_t is equal to the marginal productivity of capital:

$$w_t = F_L(K_t, L_t) \quad (9)$$

$$R_t = F_K(K_t, L_t). \quad (10)$$

Given CRS, we have that $F(K_t, L_t) = L_t F(\frac{K_t}{L_t}, 1) = L_t f(k_t)$, so that:

$$w_t = F_L(K_t, L_t) = f(k_t) - k_t f'(k_t) \quad (11)$$

$$R_t = F_K(K_t, L_t) = f'(k_t). \quad (12)$$

We assume that $\lim_{k \rightarrow 0} f'(k) = +\infty$ and $\lim_{k \rightarrow +\infty} f'(k) = 0$.

Preferences. Individuals derive some welfare from consumption during periods 2 and 3. There is no direct utility of education, in the sense that the only disutility from education comes from the foregone consumption due to financing education at the young age. At the old age, there is some disutility of labor. For the sake of analytical convenience, individual lifetime welfare for a young adult at time t is represented by the following function:

$$U_t = u(c_t) + \ell_{t+1} u(d_{t+1}) \quad (13)$$

where c_t denotes consumption in period 2,⁶ and d_{t+1} denotes consumption at the old age. As usual, we assume $u'(\cdot) > 0$, $u''(\cdot) < 0$ on the interior domain (non-negative real numbers), with $\lim_{x \rightarrow 0} u'(x) = +\infty$ and $\lim_{x \rightarrow +\infty} u'(x) = 0$.

Old-age consumption is defined as:

$$d_{t+1} = \frac{\tilde{d}_{t+1} - v(z_{t+1}, \ell_{t+1})}{\ell_{t+1}} \quad (14)$$

where $\tilde{d}_{t+1}$ denotes the material resources consumed in period 3, whereas $v(z_{t+1}, \ell_{t+1})$ denotes the disutility of old-age labor in monetary terms. The numerator of (14) is expressed in a quasi-linear form with respect to consumption good. This assumption is merely for simplification of the analytical features of the first-order conditions (e.g., (22) and (28) below). The expression of d_{t+1} is per unit time. Implicitly, there is smoothing of consumption and labor across ℓ_{t+1} years.

It is assumed that the monetary disutility of old-age labor is increasing in the retirement age z_{t+1} , and decreasing with the duration of the old age ℓ_{t+1} . The underlying intuition is that the shorter the old age, the larger is the disutility from old-age labor. On the contrary, the longer the old age, the lower is the disutility of old-age labor. We thus have: $v_z(z_{t+1}, \ell_{t+1}) > 0$, $v_{zz}(z_{t+1}, \ell_{t+1}) > 0$, and $v_\ell(z_{t+1}, \ell_{t+1}) < 0$. For the sake of simplicity, we will use the following quadratic function for the disutility of old-age labor:

$$v(z_{t+1}, \ell_{t+1}) = \frac{(z_{t+1})^2}{2\gamma \ell_{t+1}} \quad (15)$$

⁶ Labor at young adulthood (i.e. period 2) is exogenous, but it is not the focus of this paper. The current model can be readily extended to include (i) a labor-leisure choice in period 2 and (ii) a trade-off between education and labor supply in period 2. One could write the second-period utility as $u(c_t - V(e_t, a_t))$, where second-period disutility $V(\cdot)$ would depend on education e_t (which would then takes place at the same period as labor, unlike in our model) and on second-period labor time a_t .

where the parameter $\gamma > 0$ affects the marginal disutility of old-age labor. We have $v_\ell(z_{t+1}, \ell_{t+1}) = \frac{-(z_{t+1})^2}{2\gamma(\ell_{t+1})^2}$ and $v_{\ell\ell}(z_{t+1}, \ell_{t+1}) = \frac{(z_{t+1})^2}{\gamma(\ell_{t+1})^3} > 0$.

Resource constraints. Young individuals need to borrow to finance their education, and must pay this back when they start to work. Denoting the interest factor by R_t , the budget constraint at the young age can be written as:

$$c_t = w_t h(e_{t-1}) - e_{t-1} R_t - s_t \quad (16)$$

whereas the budget constraint at the old age is:

$$\tilde{d}_{t+1} = z_{t+1} \alpha w_{t+1} h(e_{t-1}) + R_{t+1} s_t \quad (17)$$

From (14) we have:

$$d_{t+1} = \frac{z_{t+1} \alpha w_{t+1} h(e_{t-1}) + R_{t+1} s_t - v(z_{t+1}, \ell_{t+1})}{\ell_{t+1}}. \quad (18)$$

3. Temporary equilibrium

This section characterizes the temporary equilibrium of our economy. For that purpose, we will proceed in two stages. We will first consider the case where longevity is exogenous, and, then, the case where longevity is endogenous.

In each case, individuals choose the education investment, their saving as well as their retirement age, conditionally on their resource constraints, and conditionally on some beliefs regarding future factor prices (wages and interest rates), with superscript e .

3.1. Exogenous longevity

In the case where longevity is exogenous, i.e. $\ell_t = \bar{\ell}$, the problem of agents can be written as:

$$\max_{e_{t-1}, s_t, z_{t+1}} u[w_t h(e_{t-1}) - e_{t-1} R_t - s_t] + \bar{\ell} u \left[\frac{z_{t+1} \alpha w_{t+1}^e h(e_{t-1}) - v(z_{t+1}, \bar{\ell}) + R_{t+1}^e s_t}{\bar{\ell}} \right].$$

The first-order conditions (FOCs) are:

$$u'(c_t) = R_{t+1}^e u'(d_{t+1}) \quad (19)$$

$$\alpha w_{t+1}^e h(e_{t-1}) = v_z(z_{t+1}, \bar{\ell}) \quad (20)$$

$$u'(d_{t+1}) [z_{t+1} \alpha w_{t+1}^e h'(e_{t-1})] = u'(c_t) [R_t - w_t h'(e_{t-1})]. \quad (21)$$

(20) and (21) become:

$$z_{t+1} = \alpha w_{t+1}^e h(e_{t-1}) \gamma \bar{\ell} \quad (22)$$

$$h'(e_{t-1}) [\alpha^2 (w_{t+1}^e)^2 h(e_{t-1}) \gamma \bar{\ell} + R_{t+1}^e w_t] = R_{t+1}^e R_t. \quad (23)$$

The last condition equalizes the marginal welfare gains from extending education investment with the marginal cost of education. The marginal welfare gain from increasing education concerns both periods 2 and 3, and goes through two channels. On the one hand, a higher education level increases the level of hourly labor earnings in periods 2 and 3, for a given retirement age in period 3, and, hence, expands consumption possibilities and welfare. Note that, in period 3, that effect is mitigated by the decay of human capital. On the other hand, a higher education level tends to increase

the retirement age in period 3, which also expands consumption possibilities and welfare (net, of course, of the additional disutility of old-age labor).

Given the anticipated levels of future factor prices w_{t+1}^e and R_{t+1}^e , and given the saving of the previous cohort s_{t-1} , the temporary equilibrium under exogenous longevity is a vector $(e_{t-1}, c_t, s_t, d_{t+1}, z_{t+1}, w_t, R_t, K_t, L_t)$ satisfying (3), (9), (10), (16), (18), (19), (22), (23) and

$$K_t + N e_{t-1} = N s_{t-1}. \quad (24)$$

The last condition is a slightly modified version of (5) above.

The following proposition states conditions that guarantee that the optimal education level is interior and is unique under exogenous longevity.

Proposition 1. Given w_t, R_t, w_{t+1}^e and R_{t+1}^e , if $\lim_{e_{t-1} \rightarrow 0} h'(e_{t-1}) = +\infty$ and $\lim_{e_{t-1} \rightarrow +\infty} h'(e_{t-1}) = 0$, there exists an interior optimal level of e_{t-1} .

If $h(e_{t-1})$ satisfies $\frac{|h''(e_{t-1})|}{h'(e_{t-1})} > \frac{h'(e_{t-1})}{h(e_{t-1})}$ for all e_{t-1} , then that interior optimal level for e_{t-1} is unique.

Proof. See Proposition 2 and its proof (for the case where $\ell'(e_{t-1}) = 0$). ■

Note that, whereas the interiority conditions constitute some form of classical Inada conditions, the uniqueness condition is far stronger. To illustrate this, let us take the case of an isoelastic function $h(e_{t-1})$, satisfying $h(0) = 1$, $h'(\cdot) > 0$ and $h''(\cdot) < 0$. We have:

$$h(e_{t-1}) = \frac{e_{t-1}^{1-\phi}}{1-\phi} + 1$$

with $0 < \phi < 1$. The derivatives yield:

$$h'(e_{t-1}) = e_{t-1}^{-\phi} > 0$$

$$h''(e_{t-1}) = -\phi e_{t-1}^{-\phi-1} < 0.$$

Hence the uniqueness condition requires:

$$\phi e_{t-1}^{-\phi-1} \left(\frac{e_{t-1}^{1-\phi}}{1-\phi} + 1 \right) > e_{t-1}^{-2\phi}.$$

This is equivalent to:

$$e_{t-1}^{\phi-1} > \frac{1-2\phi}{\phi(1-\phi)},$$

which is true for all levels of $e_{t-1}^{\phi-1}$ when $\phi > 1/2$. However, if $\phi \leq 1/2$, then the RHS exceeds the LHS for e_{t-1} sufficiently large.

Hence, the condition on $h(e_{t-1})$ guaranteeing the uniqueness of the optimal education level is not trivial. Whereas this result may seem surprising given the simplicity of the assumption, remind that the FOC for optimal education is:

$$h'(e_{t-1}) [\alpha^2 (w_{t+1}^e)^2 h(e_{t-1}) \gamma \bar{\ell} + R_{t+1}^e w_t] = R_{t+1}^e R_t.$$

Given the product $h'(e_{t-1}) [\alpha^2 (w_{t+1}^e)^2 h(e_{t-1}) \gamma \bar{\ell} + R_{t+1}^e w_t]$ on the LHS of that condition, it is not surprising that, for a given RHS, various levels of education e_{t-1} can satisfy that condition. When e_{t-1} increases, $h'(e_{t-1})$ decreases, but $h(e_{t-1})$ increases. Hence, without imposing conditions on $h(e_{t-1})$, it is difficult to guarantee the uniqueness of the optimal education level. The condition $|h''(e_{t-1})| h(e_{t-1}) > [h'(e_{t-1})]^2$ guarantees that the LHS of the FOC is strictly decreasing in e_{t-1} for all levels of e_{t-1} , and, hence, guarantees the uniqueness of the level of e_{t-1} for which the LHS equals the RHS of the FOC.

⁷ We assume here that individuals choose education for themselves. Alternative modeling would include parents choosing education for their children (see de la Croix and Licandro, 2013) or parents and children bargaining about education (see Leker and Ponthiere, 2015).

Finally, note that, when the condition stated in Proposition 1 is not satisfied, it does not necessarily imply that uniqueness does not hold. Actually, that condition is a *sufficient* condition for uniqueness, but not a necessary condition. However, when that condition does not hold, one can have, under some parametrizations, several education levels satisfying (23), each of them being associated with a retirement age that is increasing with the education level. In that case, it may be equivalent, in welfare terms, either to invest less in education and retire earlier, or to invest more in education and retire later on.

3.2. Endogenous longevity

The problem faced by the individual is now:

$$\max_{e_{t-1}, s_t, z_{t+1}} \left\{ u[w_t h(e_{t-1}) - e_{t-1} R_t - s_t] + \ell(e_{t-1}) u \left[\frac{z_{t+1} \alpha w_{t+1}^e h(e_{t-1}) - v(z_{t+1}, \ell(e_{t-1})) + R_{t+1}^e s_t}{\ell(e_{t-1})} \right] \right\}$$

FOCs are:

$$u'(c_t) = R_{t+1}^e u'(d_{t+1}) \quad (25)$$

$$\alpha w_{t+1}^e h(e_{t-1}) = v(z_{t+1}, \ell(e_{t-1})) \quad (26)$$

$$\begin{aligned} u'(c_t) [R_t - w_t h'(e_{t-1})] &= \ell'(e_{t-1}) u(d_{t+1}) \\ &\quad + u'(d_{t+1}) \\ &\quad \times \left[\frac{z_{t+1} \alpha w_{t+1}^e h'(e_{t-1})}{-v_\ell(z_{t+1}, \ell(e_{t-1})) \ell'(e_{t-1})} \right] \\ &\quad - u'(d_{t+1}) \frac{\ell'(e_{t-1})}{\ell(e_{t-1})} \\ &\quad \times \left[\frac{z_{t+1} \alpha w_{t+1}^e h(e_{t-1})}{-v(z_{t+1}, \ell(e_{t-1})) + R_{t+1}^e s_t} \right] \end{aligned} \quad (27)$$

(26) is rearranged to yield:

$$z_{t+1} = \alpha w_{t+1}^e h(e_{t-1}) \gamma \ell(e_{t-1}). \quad (28)$$

The FOC of education becomes:

$$\begin{aligned} u'(c_t) [R_t - w_t h'(e_{t-1})] &= u'(d_{t+1}) z_{t+1} \alpha w_{t+1}^e h'(e_{t-1}) \\ &\quad + \ell'(e_{t-1}) \\ &\quad \times \left[\frac{u(d_{t+1}) - u'(d_{t+1}) d_{t+1}}{-u'(d_{t+1}) v_\ell(z_{t+1}, \ell(e_{t-1}))} \right]. \end{aligned} \quad (29)$$

The FOC for education can be further rewritten as:

$$\begin{aligned} h'(e_{t-1}) \left[\alpha^2 (w_{t+1}^e)^2 h(e_{t-1}) \gamma \ell(e_{t-1}) + R_{t+1}^e w_t \right] \\ + \ell'(e_{t-1}) \left[d_{t+1} \left(\frac{1}{F^R} - 1 \right) - v_\ell(z_{t+1}, \ell(e_{t-1})) \right] \\ = R_{t+1}^e R_t \end{aligned} \quad (30)$$

where $F^R \equiv \frac{u'(d) d}{u(d)}$. In the literature on attitudes with respect to longevity risk (see [Eeckhoudt and Pestieau, 2008](#)), F^R is called the fear of ruin, and is a measure of risk aversion. In the present context, where there is no risk about the duration of life, it is more appropriate to interpret F^R as the elasticity of instantaneous utility with respect to its scalar input. As such, F^R informs us about the change in the degree of substitutability of utility across lifetime periods with respect to variations in the level of its input. In the following, we assume that F^R is a positive constant lower than unity ($0 < F^R < 1$).

In (30), in comparison with the case where longevity is exogenous, we have two additional terms on the LHS. The first additional term captures the pure effect of education on longevity, which is positive from $0 < F^R < 1$. The second additional term captures the effect of education on the disutility of old-age labor. Note that,

since $v_\ell(z_{t+1}, \ell(e_{t-1})) < 0$, that effect is also positive: a higher investment in education tends, by reducing the disutility of old-age labor, to raise lifetime welfare.

Given the anticipated levels of future factor prices w_{t+1}^e and R_{t+1}^e , and given the saving of the previous cohort s_{t-1} , the temporary equilibrium under endogenous longevity is a vector $(e_{t-1}, c_t, s_t, d_{t+1}, z_{t+1}, w_t, R_t, K_t, L_t)$ satisfying (3), (9), (10), (16), (18), (24), (25), (28), and (30).

Proposition 2 studies conditions that guarantee that the optimal education level is interior and is unique under endogenous longevity.

Proposition 2. Given w_t, R_t, w_{t+1}^e and R_{t+1}^e , if $\lim_{e_{t-1} \rightarrow 0} h'(e_{t-1}) = +\infty$, $\lim_{e_{t-1} \rightarrow +\infty} h'(e_{t-1}) = 0$, $\lim_{e_{t-1} \rightarrow 0} \ell'(e_{t-1}) = +\infty$ and $\lim_{e_{t-1} \rightarrow +\infty} \ell'(e_{t-1}) = 0$, there exists an interior optimal level of e_{t-1} .

If $0 < F^R < 1$, and if $h(\cdot)$ and $\ell(\cdot)$ satisfy: $|h''(e_{t-1})| h(e_{t-1}) \ell(e_{t-1}) > (h'(e_{t-1}))^2 \ell(e_{t-1}) + 2h'(e_{t-1}) h(e_{t-1}) \ell'(e_{t-1})$, then the interior optimal level of e_{t-1} is unique.

Proof. See [Appendix](#). ■

Note that, in comparison with [Proposition 1](#), the uniqueness of an interior optimal level of education depends not only on the properties of the education return function $h(\cdot)$, but also on the shape of the longevity function $\ell(\cdot)$. In the exogenous case, we had $\ell'(e_{t-1}) = 0$, so that the above condition collapses to

$$\frac{|h''(e_{t-1})|}{h'(e_{t-1})} > \frac{h'(e_{t-1})}{h(e_{t-1})} \quad (31)$$

as in [Proposition 1](#). However, in the endogenous case, $\ell'(e_{t-1}) > 0$ and thus the condition becomes also dependent on the shape of $\ell'(e_{t-1})$.

4. Existence of a stationary equilibrium

This section examines the conditions under which there exists a stationary equilibrium in our economy. For that purpose, we will, as above, distinguish between the two cases, where longevity is either exogenous or endogenous.

4.1. Exogenous longevity

To examine the existence of a stationary optimum under exogenous longevity, we need to start from the FOCs of the individual problem. From expression (19), we can rewrite optimal saving per young adult as a function of present and future factor prices, as well as of the education level:

$$s_t \equiv s(w_t, R_t, w_{t+1}^e, R_{t+1}^e, e_{t-1}). \quad (32)$$

Indeed, given some values for parameters $\bar{\ell}, \alpha, \gamma$ and some functional forms for $u(\cdot)$ and $h(\cdot)$, saving depends only on education and present and expected future factor prices.

Regarding the education choice, we can, under the conditions for existence and uniqueness provided in [Proposition 1](#), and using expression (23), rewrite the optimal education as a function of present and future factor prices:

$$e_{t-1} \equiv E(w_t, R_t, w_{t+1}^e, R_{t+1}^e). \quad (33)$$

Indeed, given some values for parameters $\bar{\ell}, \alpha, \gamma$ and some functional forms for $u(\cdot)$ and $h(\cdot)$, the chosen level of education depends only on present and expected future factor prices. We also know from our conditions that this optimal education level is unique.

Hence, given that the optimal education depends only on present and future factor prices, we can deduce that saving per young adult can be rewritten as:

$$\begin{aligned} s_t &\equiv s(w_t, R_t, w_{t+1}^e, R_{t+1}^e, E(w_t, R_t, w_{t+1}^e, R_{t+1}^e)) \\ &\equiv S(w_t, R_t, w_{t+1}^e, R_{t+1}^e). \end{aligned} \quad (34)$$

Given that factor prices obviously depend on the level of k_t , since $w_t = f(k_t) - k_t f'(k_t)$ and $R_t = f'(k_t)$, the capital accumulation equation can be written in intensive terms (i.e. per effective unit of labor) as:

$$\begin{aligned} k_{t+1} &= \frac{S(w(k_t), R(k_t), w(k_{t+1}^e), R(k_{t+1}^e)) - e_t}{h(e_t) + z_{t+1}\alpha h(e_{t-1})} \\ &= \frac{S(w(k_t), R(k_t), w(k_{t+1}^e), R(k_{t+1}^e)) - e_t}{h(e_t) + \alpha^2 w(k_{t+1}^e)[h(e_{t-1})]^2 \gamma \bar{\ell}}, \end{aligned} \quad (35)$$

where we made use of $z_{t+1} = \alpha w(k_{t+1}^e)h(e_{t-1})\gamma \bar{\ell}$.

At the same time, education depends on present and future expected factor prices:

$$e_{t-1} \equiv E(w(k_t), R(k_t), w(k_{t+1}^e), R(k_{t+1}^e)).$$

We are thus in presence of a two-dimensional system with three time lags.

Let us explore the conditions under which a stationary equilibrium with perfect foresight can exist. Under such an equilibrium, we have $k_{t+1}^e = k_{t+1} = k_t = k$, and $e_t = e_{t-1} = e_{t-2} = e$, so that the pair (e, k) must satisfy:

$$k = \frac{S(w(k), R(k), w(k), R(k)) - e}{h(e) + \alpha^2 w(k)[h(e)]^2 \gamma \bar{\ell}} \equiv \frac{\tilde{S}(k) - e}{h(e) + \alpha^2 w(k)[h(e)]^2 \gamma \bar{\ell}} \quad (36)$$

$$e \equiv E(w(k), R(k), w(k), R(k)) \equiv \bar{e}(k). \quad (37)$$

To examine the existence of such a pair, let us study some properties of those two relations. The following proposition summarizes our results.

Proposition 3. Suppose that the existence and uniqueness conditions of Proposition 1 hold. Denote the level of e satisfying $h(e)[1 + \alpha^2 w(k)[h(e)]^2 \gamma \bar{\ell}] + \frac{e}{k} = \frac{\tilde{S}(k)}{k}$ for a given k by $\check{e}(k)$. Suppose that the level of $k > 0$ such that $\check{e}(k) = 0$ is unique. Suppose also that $\bar{e}(0) = 0$. Suppose further that $\check{e}(k)$ is continuous in k , with $\check{e}'(0) > \bar{e}'(0)$ and $\check{e}(0) \geq 0$. Then a stationary equilibrium (k^E, e^E) that satisfies (36) and (37) with $k^E > 0$ exists.

Proof. See Appendix. ■

Proposition 3 states conditions that are sufficient for existence of a stationary equilibrium with perfect foresight. It should be stressed that Proposition 3 relies on the conditions for a unique interior optimal education level stated in Proposition 1. Those conditions allow us to write education as a function $\bar{e}(k)$, which assigns to each level of k a unique level for educational investment. In Appendix we show that $\bar{e}(k) > 0$. For $\check{e}(k)$, see Fig. 1. Fig. 1a illustrates the LHS of (36) (a 45° line) and the RHS of (36) with different levels of e . The dotted curve denotes the case when $e = 0$ ($h(e) = 1$). At $k = k^a > 0$, the capital market clears. In the figure there is another intersection at $k = 0$: this corresponds to an unstable steady-state in the OLG model. With higher levels of e , the curve that represents the RHS of (36) shifts down since the RHS of (36) is decreasing in e (higher e decreases the numerator⁸

⁸ In Appendix we show that s_t is decreasing in e_{t-1} under exogenous longevity (Eq. (67) in the proof of Proposition 2 when $\ell'(e) = 0$). This also reinforces the capital market tightening.

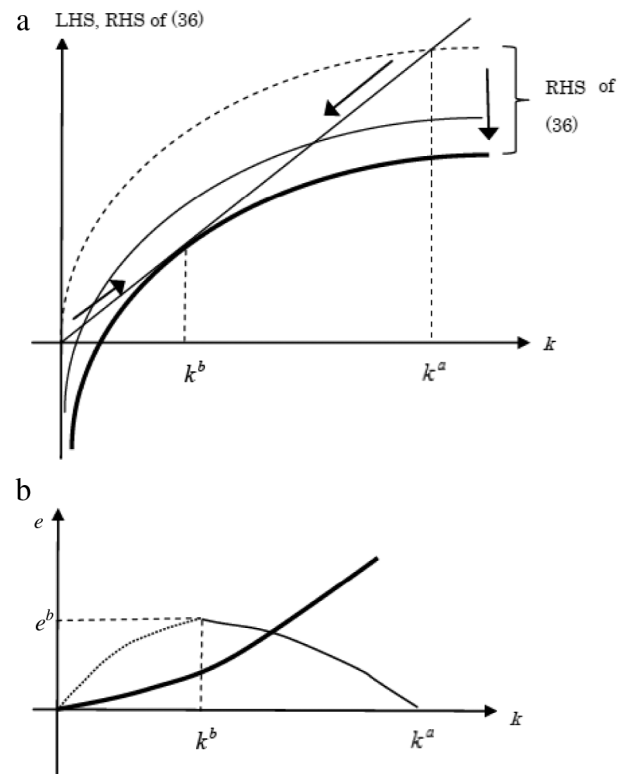


Fig. 1. Existence of a stationary equilibrium.

and increases the denominator). Accordingly, the stable solution, starting from $k = k^a$, goes to the left; on the other hand, the unstable solution, starting from $k = 0$, goes to the right.⁹ The curve in bold in Fig. 1a corresponds to the critical value of $e \equiv e^b$, above which the RHS does not have an intersection with the 45° line. In Fig. 1b, the solid downward-sloping curve is the stable solutions, and the dotted upward-sloping curve is the unstable solutions. They meet at $k = k^b \equiv \check{e}^{-1}(e^b)$.¹⁰ The curve in bold in Fig. 1b is $\bar{e}(k)$ in Eq. (37). With the assumptions of the proposition the intersection of $\bar{e}(k)$ and $\check{e}(k)$ with $k > 0$ exists.

Several remarks are in order. First, the intersection in Fig. 1b may be at the downward-sloping curve (stable solutions) or at the dotted upward-sloping curve (unstable solutions). Intuitively, if the “education demand” along $\bar{e}(k)$ cannot be afforded in the capital market along $k \in [k^b, k^a]$, then the intersection may be at the dotted curve. Second, note that Proposition 3 only informs us about the existence of a stationary equilibrium with perfect foresight, but does not inform us about the uniqueness. Uniqueness can only be examined provided particular functional forms are imposed for production, utility and education returns. The next subsection examines existence of a stationary equilibrium with perfect foresight once education increases longevity.

⁹ This multiplicity of the capital market equilibrium (with given e) is normal, and it is indeed very similar to Phelps and Shell (1969, Figure 2). The education investment e here corresponds to the government’s debt in Phelps and Shell (1969), and their “classical” and “anticlassical” ranges correspond to our stable and unstable solutions, respectively.

¹⁰ In Fig. 1b, it is reasonable to assume continuity of the solid downward-sloping curve since all relevant functions are continuous and differentiable. Same for the dotted upward-sloping curve, and these two curves meet at $(k, e) = (k^b, e^b)$ at which the continuity is assured.

4.2. Endogenous longevity

To examine the conditions under which a stationary equilibrium with perfect foresight exists, we start from the FOCs of the agent's problem in the endogenous longevity case. From the FOC for saving (expression (25)), we can, as above, rewrite optimal saving per young adult as a function of present and future factor prices, as well as the education level:

$$s_t \equiv \hat{s}(w_t, R_t, w_{t+1}^e, R_{t+1}^e, e_{t-1}).$$

Indeed, given some values for parameters α, γ and some functional forms for $u(\cdot), h(\cdot)$ and $\ell(\cdot)$, saving depends only on education and present and expected future factor prices.

Regarding the education choice, we can, under the conditions for existence and uniqueness provided in Proposition 2, and using expression (30), rewrite the optimal education as a function of present and future factor prices, as well as saving:

$$e_{t-1} \equiv \hat{e}(w_t, R_t, w_{t+1}^e, R_{t+1}^e, s_t).$$

Indeed, given some values for parameters α, γ and some functional forms for $u(\cdot), h(\cdot)$ and $\ell(\cdot)$, the chosen level of education depends only on present and expected future factor prices, and on the saving level. We also know from our conditions that this optimal education level is unique. Given that $s_t \equiv \hat{s}(w_t, R_t, w_{t+1}^e, R_{t+1}^e, e_{t-1})$, we can rewrite the education relation as:

$$e_{t-1} \equiv \hat{e}(w_t, R_t, w_{t+1}^e, R_{t+1}^e, \hat{s}(w_t, R_t, w_{t+1}^e, R_{t+1}^e, e_{t-1})).$$

The capital accumulation equation can be written in intensive terms as:

$$\begin{aligned} k_{t+1} &= \frac{\hat{s}(w_t, R_t, w_{t+1}^e, R_{t+1}^e, e_{t-1}) - e_t}{h(e_t) + z_{t+1}\alpha h(e_{t-1})} \\ &= \frac{\hat{s}(w_t, R_t, w_{t+1}^e, R_{t+1}^e, e_{t-1}) - e_t}{h(e_t) + \alpha^2 w_{t+1}^e [h(e_{t-1})]^2 \gamma \ell(e_{t-1})}, \end{aligned} \quad (38)$$

where we made use of $z_{t+1} = \alpha w_{t+1}^e h(e_{t-1}) \gamma \ell(e_{t-1})$.

Let us explore the conditions under which a stationary equilibrium with perfect foresight can exist. Under such an equilibrium, we have $k_{t+1}^e = k_{t+1} = k_t = k$, $e_t = e_{t-1} = e_{t-2} = e$, $w = f(k) - kf'(k)$ and $R = f'(k)$, so that:

$$\begin{aligned} k &= \frac{\hat{s}(w(k), R(k), w(k), R(k), e) - e}{h(e) + \alpha^2 w(k)[h(e)]^2 \gamma \ell(e)} \\ &= \frac{\tilde{s}(k, e) - e}{h(e) + \alpha^2 w(k)[h(e)]^2 \gamma \ell(e)} \end{aligned} \quad (39)$$

$$e = \hat{e}(w(k), R(k), w(k), R(k), \tilde{s}(k, e)) = \tilde{e}(k, \tilde{s}(k, e)) \quad (40)$$

where $\tilde{s}(k, e) \equiv \hat{s}(w(k), R(k), w(k), R(k), e)$.

A stationary equilibrium with perfect foresight is a pair (k, e) satisfying those two conditions. The following proposition summarizes our results.

Proposition 4. Suppose that the existence and uniqueness conditions of Proposition 2 hold. Denote the level of e satisfying $e = \hat{e}(k, \tilde{s}(k, e))$ for a given k by $e = \tilde{e}(k)$. Suppose that $\tilde{e}(0) = 0$ and $\tilde{e}'(k) > 0$. Denote the level of e satisfying $h(e)[1 + \alpha^2 w(k)h(e)\gamma\ell(e)] + \frac{e}{k} = \frac{\tilde{s}(k, e)}{k}$ for a given k by $e = \check{e}(k)$. Suppose that the level of $k > 0$ such that $\check{e}(k) = 0$ is unique. Suppose also that $\check{e}(k)$ is continuous in k , with $\check{e}'(0) > \tilde{e}'(0)$ and $\check{e}(0) \geq 0$. Then a stationary equilibrium (k^E, e^E) that satisfies (39) and (40) with $k^E > 0$ exists.

Proof. See Appendix. ■

Here again, it should be stressed, as in the case of Proposition 3, that the above conditions only guarantee that there exists at least

one stationary equilibrium with perfect foresight, but do not exclude the possibility of several stationary equilibria with perfect foresight. In order to study the uniqueness of that equilibrium, we would have to impose particular functional forms on the utility functions, the production functions, and on relations $h(e)$ and $\ell(e)$. Given that the emphasis of this paper lies on the Ben-Porath effect, we will not carry out the discussion on the uniqueness of the stationary equilibrium here.

5. The Ben-Porath effect reexamined

Having shown that, under mild conditions, a stationary equilibrium with perfect foresight exists in our economy, we can now turn back to the study of the Ben-Porath effect. This section reexamines the conditions under which a rise in the lifetime horizon affects the education investment. For that purpose, we consider an economy at a stationary equilibrium with perfect foresight.

Note that, within our dynamic general equilibrium model, a change in longevity can affect the education decision either directly, or indirectly through changes in production factor prices w and R . The latter indirect effects being quite complex, this section will first reexamine the Ben-Porath effect within a small open economy (where production factor prices are fixed). Then, we will consider the more complex case of a closed economy.

5.1. The small open economy

Focusing on a small open economy allows us to study the Ben-Porath effect as if we were in a partial equilibrium setting, in line with the existing literature on the Ben-Porath effect, where variations in production factor prices are generally neglected. In a small open economy, given the wage rate w , the labor market is not bounded by condition (3). Moreover, education e_t and saving s_t are not bound to condition (5), and the difference $K_{t+1} - N(s_t - e_t)$ can be borrowed from or loaned to the rest of the world.

Assuming a perfect international capital mobility is a common assumption in macroeconomic models for small open economies. As to the labor mobility, assuming perfect international mobility constitutes a stronger assumption. Note, however, that this is actually the case, for example, in the European Union, where shortages on the domestic market in certain sectors of the economy (e.g. construction) in old European Union member states are mitigated by an influx of labor from new member states.

Given that the Ben-Porath effect takes different forms when longevity is exogenous or when longevity depends on the education level, we will, here again, proceed in two cases, and consider first the case where longevity is exogenous, and, then, the case where longevity depends positively on education investment.

5.1.1. Exogenous longevity

In the case where longevity is exogenous, i.e. $\ell_t = \bar{\ell}$, the FOCs of the individual problem are (19) and (23) of the previous section. At a stationary equilibrium, abstracting here from time indices, we have:

$$u'(c) = Ru'(d) \quad (41)$$

$$wh'(e)[R + \alpha^2 wh(e)\gamma\bar{\ell}] = R^2. \quad (42)$$

As discussed above, in a small open economy, education e and saving s are not bound to $K = N(s - e)$, since the difference $K - N(s - e)$ can be borrowed from or loaned to the rest of the world. Likewise, given the wage rate w , the labor market is not bounded by $L = h(e)N + \alpha h(e)N$. In the light of this, we can, for the case

of a small open economy, characterize the stationary equilibrium with perfect foresight under exogenous longevity as follows.¹¹

Proposition 5. *Given w and R , the stationary equilibrium in a small open economy under exogenous longevity is a vector (e, c, s, d, z) satisfying the conditions:*

$$\begin{aligned} c &= wh(e) - eR - s \\ d &= \frac{z\alpha wh(e) + Rs - v(z, \bar{\ell})}{\bar{\ell}} \\ u'(c) &= Ru'(d) \\ z &= \alpha wh(e)\gamma \bar{\ell} \\ h'(e)[\alpha^2(w)^2 h(e)\gamma \bar{\ell} + Rw] &= R^2. \end{aligned}$$

Proof. The proof follows immediately from the FOCs of the agent. ■

Let us use (42) to study the conditions under which the Ben-Porath effect prevails in our economy. Define:

$$wh'(e)[R + \alpha^2 wh(e)\gamma \bar{\ell}] - R^2 \equiv \Delta. \quad (43)$$

In order to identify the effect of the life horizon on the education investment, we can compute:

$$\frac{de}{d\bar{\ell}} = \frac{\Delta_{\bar{\ell}}}{-\Delta_e} = \frac{h'(e)\alpha^2 w^2 h(e)\gamma}{-\left[\alpha^2 w^2 \gamma \bar{\ell}\right] \left[h''(e)h(e) + [h'(e)]^2\right] - h''(e)Rw} \quad (44)$$

where $\Delta_e < 0$ because of the second-order condition.

In line with the literature on the Ben-Porath effect, one usually expects that a longer life horizon leads to a larger investment in education. Note, however, that, in our model, we have $\frac{de}{d\bar{\ell}} = 0$ when $\alpha = 0$, that is, when there is a complete decay of human capital. Indeed, in that particular case, a rise in the duration of life does not favor more education, since at the old age the return of those educational investments are low because of the complete decay. Moreover, when $\gamma \rightarrow 0$, the marginal disutility of old-age labor tends to be infinite, which implies that individuals retire at the beginning of the old age. Therefore, when $\gamma \rightarrow 0$, a rise in the time horizon has no impact on the optimal education, since those additional years will not be worked. The same result would hold in case of mandatory retirement at the beginning of the old age, implying $z = 0$.

Hence our calculations seem to qualify the Ben-Porath effect: the size of that effect depends on how large the decay of human capital is, and on how large the marginal disutility of old-age labor is. Proposition 6 summarizes our results.

Proposition 6. *Consider a small open economy with exogenous longevity at a stationary equilibrium with perfect foresight. The impact of life horizon on the education is given by:*

$$\frac{de}{d\bar{\ell}} = \frac{h'(e)\alpha^2 w^2 h(e)\gamma}{-\left[\alpha^2 w^2 \gamma \bar{\ell}\right] \left[h''(e)h(e) + [h'(e)]^2\right] - h''(e)Rw}.$$

The impact of life horizon on education is:

- increasing with the wage rate w
- decreasing with the human capital decay $1/\alpha$
- decreasing with the strength of the marginal disutility of labor $1/\gamma$.

Proof. See above. ■

¹¹ Note that the existence of a stationary equilibrium in the open economy model poses no problem, as wages and interest rates are determined on the international market, and domestic excess demand or supply of labor or capital is resolved on the international market.

The above proposition shows that the size of the Ben-Porath effect depends, in our economy, on several forces, which affect the Ben-Porath effect in various ways. Some forces at work are related to preferences, such as the parameter γ capturing the disutility of old-age labor. Another important determinant, which is often neglected, consists of the extent of human capital decay α . This affects the size of the Ben-Porath effect significantly. Indeed, the impact of a marginal change in $\bar{\ell}$ on education is proportional to the square of α , which shows the significance of that parameter in the determination of the Ben-Porath effect. A simple corollary of the role of the decay is that, in societies with quicker technological progress, human capital's decay is stronger, implying, *ceteris paribus*, a reduction of the size of the Ben-Porath effect.

Finally, under exogenous longevity, we find that old-age labor is necessary to have the Ben-Porath effect, that is, to have a positive impact of a rise in longevity on education. To see this, note that, in case of full decay of human skills (i.e. $\alpha = 0$), individuals choose not to work at the old age (i.e. $z = 0$), and in that case longevity has no effect on education choices, that is, $\frac{de}{d\bar{\ell}} = 0$. Thus old-age labor is necessary, under exogenous longevity, to have the Ben-Porath effect. Moreover, our results are here in line with Hazan (2009), who argued that a rise in the retirement age is a necessary condition for the presence of a Ben-Porath effect. We find the same result here. To see this, note that, if z was an exogenous parameter $\bar{z}$ (thus not varying with $\bar{\ell}$), then we would have $\frac{de}{d\bar{\ell}} = 0$, that is, longevity would have no impact on education choices.

5.1.2. Endogenous longevity

Having studied the determinants of the Ben-Porath effect in the case of exogenous longevity, let us now consider the case where longevity is endogenous, and increasing in the amount of education investment. Under endogenous longevity, and focusing on a stationary equilibrium with perfect foresight, the FOCs of the individual problem can be rewritten as:

$$u'(c) = Ru'(d) \quad (45)$$

$$R^2 = wh'(e)[R + \alpha z] + \ell'(e) \left[d \left(\frac{1}{F^R} - 1 \right) - v_\ell(z, \ell(e)) \right] \quad (46)$$

where $z = \alpha wh(e)\gamma \ell(e)$ derived from (28), and $F^R \equiv \frac{u'(d)d}{u(d)} < 1$. Focusing, here again, on a small open economy, the stationary equilibrium with perfect foresight can be characterized as follows.

Proposition 7. *Given w and R , the stationary equilibrium in a small open economy under endogenous longevity is a vector (e, c, s, d, z) satisfying the conditions:*

$$\begin{aligned} c &= wh(e) - eR - s \\ d &= \frac{z\alpha wh(e) + Rs - v(z, \ell(e))}{\ell(e)} \\ u'(c) &= Ru'(d) \\ z &= \alpha wh(e)\gamma \ell(e) \end{aligned}$$

$$Rwh'(e) + z\alpha wh'(e) + \frac{\ell'(e)}{u'(d)} \left(\frac{u(d) - u'(d)d}{-u'(d)v_\ell(z, \ell(e))} \right) = R^2.$$

Proof. The proof follows immediately from the FOCs of the agent. ■

Alternatively, (46) can be written as:

$$\begin{aligned} R &= wh'(e) \left[1 + \frac{\alpha z}{R} \right] + \frac{\ell'(e)}{R} \left(\frac{1}{F^R} - 1 \right) \\ &\quad - \frac{\ell'(e)v_\ell(z, \ell(e))}{R}. \end{aligned} \quad (47)$$

Hence, at the equilibrium, the marginal cost of education R must be equal to the marginal benefits from education, which are

composed of three terms: (1) its pure return on labor earnings; (2) its effect on longevity; (3) its positive effect on the disutility of old-age labor.

As we did in the case of exogenous longevity, we can now consider whether the Ben-Porath effect remains true in our framework. For that purpose, we consider now a displacement of the function $\ell(e)$. Let us now define $\tilde{\ell}(e) = \lambda\ell(e)$ and $\tilde{z} = \lambda z$ for $\lambda > 0$. As in (43),

$$wh'(e)[R + \alpha\lambda z] + \lambda\ell'(e) \left[d \left(\frac{1}{FR} - 1 \right) - v_{\bar{\ell}}(\lambda z, \lambda\ell(e)) \right] \equiv \Delta.$$

We want to know the value of

$$\frac{de}{d\lambda} = \frac{\Delta_{\lambda}}{-\Delta_e},$$

where $\Delta_e < 0$ from the second-order condition. In Appendix we show Eq. (48) which is given in Box I.

That formula is more complex than in the case of exogenous longevity. However, it is easy to see that, when $\ell'(e)$ tends towards 0, that expression reduces to the one prevailing in the case of exogenous longevity. In comparison with that formula, two additional terms are present.

The term $-\ell'(e)v_{\bar{\ell}}(\lambda z, \lambda\ell(e))$ reflects the fact that an improvement of longevity, by reducing the extent of disutility of old-age labor, tends to make a larger investment in education more desirable. Hence, *ceteris paribus*, this effect pushes towards a higher investment in education. The extent of that effect depends on the marginal effect of a rise in education on longevity, as well as on the form of the disutility of old-age labor function $v(\lambda z, \lambda\ell(e))$.

The term $\ell'(e) \left(\frac{1}{FR} - 1 \right) (d + \lambda \frac{\partial d}{\partial \lambda})$ reflects the effect of a favorable displacement of the function $\ell(e)$ on longevity. Note that there is an effect of increasing λ on the second-period consumption, $\frac{\partial d}{\partial \lambda}$. In Appendix we show that $\frac{\partial d}{\partial \lambda} < 0$. Intuitively, an increase in longevity has a depressive effect on the flow of consumption in the second period. This can be called a dilution effect. However, in Appendix we show that $d + \lambda \frac{\partial d}{\partial \lambda} > 0$: the overall effect is unambiguously positive, so that this term also makes a larger investment in education more desirable. Note also that this additional effect depends on the degree of fear of ruin F^R , which can be interpreted, in the present context, as the elasticity of instantaneous utility with respect to its scalar input. The higher F^R is, the lower this additional effect is.

It is worthy to note that, even when there is complete decay of human capital (i.e. $\alpha = 0$), or even when $z = 0$, an increase in λ tends unambiguously to boost education. Clearly, even if we abstract from the first term of (48), and from the second term (related to old-age labor disutility), we see that the numerator of (48) still includes a term which captures the positive impact of longevity on education through the larger lifetime utility gains induced by education when longevity is larger. Proposition 8 summarizes our results.

Proposition 8. Consider an economy with endogenous longevity at its stationary equilibrium with perfect foresight. Let us define $\tilde{\ell}(e) = \lambda\ell(e)$ and $\tilde{z} = \lambda z$, and consider the impact of a displacement of the function $\tilde{\ell}(e)$ by a change of λ . The impact of that displacement on the education is given bysee the equation in Box II.

Proof. See Appendix. ■

The determinants of the size of the Ben-Porath effect are, in the case of endogenous longevity, more numerous than in the case of exogenous longevity. Quite importantly, due to the presence of the three additional effects mentioned above, the size of the Ben-Porath effect depends now on the extent to which better

survival conditions reinforce the marginal welfare gains, in terms of a longer life, from education investment, as well as on the extent to which education will contribute to allow individuals to work longer. The size of those three effects depends crucially on the shape of individual preferences, in particular the degree of fear of ruin (or elasticity of instantaneous utility with respect to its scalar input), a factor that was absent in the case of exogenous longevity.

In contrast with the case of exogenous longevity, an important difference concerns the role of old-age labor in the existence of the Ben-Porath effect. As stressed above, a rise in longevity can, unlike in the exogenous case, have here a positive impact on education even in the absence of old-age labor. To see this, note that, even under $\alpha = 0$ (implying $z = 0$), we can still have $\frac{de}{d\lambda} > 0$. More importantly, we have here the Ben-Porath effect even when the retirement age does not vary with the rise in longevity. Indeed, even if z was assumed to be a constant $\tilde{z}$, we would still have $\frac{de}{d\lambda} > 0$, that is, a positive impact of longevity growth on education. Our results in the endogenous case are thus quite different from the ones derived in Hazan (2009), in the sense that we find that old-age labor is not a necessary condition for the existence of the Ben-Porath effect.¹²

5.2. The closed economy

Let us now consider the case of a closed economy, where the economy cannot borrow from or rent to the rest of the world, and where the wage w and the interest rate R may adjust to variations in longevity, and, as a consequence, may also affect the education decision. For that purpose, we will consider only the case of exogenous longevity changes, that is, a change in longevity $\bar{\ell}$.¹³

In a closed economy, there is no possibility to borrow from the rest of the world, so that the variables e and s are bound to $K = N(s - e)$, i.e. the saving equation. Moreover, the labor supply must satisfy $L = h(e)N + \alpha h(e)N$. The following proposition defines the stationary equilibrium with perfect foresight under exogenous longevity in a closed economy.

Proposition 9. The stationary equilibrium in a closed economy under exogenous longevity is a vector $(e, c, s, d, z, w, R, L, K)$ satisfying the conditions:

$$\begin{aligned} c &= wh(e) - eR - s \\ d &= \frac{z\alpha wh(e) + Rs - v(z, \bar{\ell})}{\bar{\ell}} \\ u'(c) &= Ru'(d) \\ z &= \alpha wh(e)\gamma\bar{\ell} \\ h'(e) [\alpha^2 w^2 h(e)\gamma\bar{\ell} + Rw] &= R^2 \\ L &= h(e)N + \alpha h(e)N \\ K &= N(s - e) \\ w &= F_L(K, L) \\ R &= F_K(K, L). \end{aligned}$$

Proof. The proof follows immediately from the FOCs of the agent and the FOCs of the competitive firms. ■

¹² As such, our theoretical predictions better fit the recent work by Cervellati and Sunde (2013) showing a positive link between life expectancy and education independently from an adjustment of old-age labor.

¹³ Note, however, that a similar exercise could be made when considering the case where longevity is endogenous (that is, a variation in λ).

$$\frac{de}{d\lambda} = \frac{h'(e)w^2\alpha^2h(e)\gamma\ell(e) - \ell'(e)v_{\bar{\ell}}(\lambda z, \lambda\ell(e)) + \ell'(e)\left(\frac{1}{F^R} - 1\right)(d + \lambda\frac{\partial d}{\partial\lambda})}{-\Delta_e} \quad (48)$$

Box I.

$$\frac{de}{d\lambda} = \frac{h'(e)w^2\alpha^2h(e)\gamma\ell(e) - \ell'(e)v_{\bar{\ell}}(\lambda z, \lambda\ell(e)) + \ell'(e)\left(\frac{1}{F^R} - 1\right)(d + \lambda\frac{\partial d}{\partial\lambda})}{-\Delta_e} > 0.$$

Box II.

The stationary equilibrium of a closed economy is characterized by the intersection of education and saving equations:

$$wh'(e)[R + \alpha^2wh(e)\gamma\bar{\ell}] - R^2 \equiv \Delta = 0$$

$$k - \frac{\tilde{S}(k) - e}{h(e) + \alpha^2w(k)[h(e)]^2\gamma\bar{\ell}} \equiv \Gamma = 0.$$

Total differentiation of the former equation with respect to e and $\bar{\ell}$ yields Proposition 6. We obtain that, for a given level of k , a higher level of longevity $\bar{\ell}$ tends unambiguously to raise the optimal education level.

However, once we take the endogeneity of k into account, several difficulties arise, and it is not necessarily the case that a rise in $\bar{\ell}$ will raise the equilibrium education level. Indeed, in that case, the total effect of variations in $\bar{\ell}$ on the equilibrium level of education depends on how it affects the education equation and the saving equation.

To see this, note first that the saving equation depends on $\bar{\ell}$. Total differentiation of the saving equation (while taking education as fixed) yields the equation in Box III.

The sign of $\frac{dk}{d\bar{\ell}}$ depends on two factors. First, the denominator Γ_k is positive at stable solutions (the solid downward-sloping curve in Fig. 1b) but negative at unstable solutions (the dotted upward-sloping curve in Fig. 1b). Second, the numerator $\Gamma_{\bar{\ell}}$ depends on $\frac{\partial\tilde{S}(k)}{\partial\bar{\ell}}$ (the increase of saving due to longer period after retirement) and on $\bar{\ell}$ (the increase of the denominator), which are offsetting. Thus the downward-sloping and the upward-sloping segments of $\check{e}(k)$ can move either to the left or to the right.¹⁴

But even if one assumes, for simplicity, that the saving equation is invariant to increasing longevity (i.e. $\frac{dk}{d\bar{\ell}} = 0$), we still have that the sign of the Ben-Porath effect is indeterminate, because of the following reason. Since the saving equation is a hump-shaped curve, the rise in the education equation may lead to a higher or a lower equilibrium education level. Fig. 2a and b illustrate those two possibilities, i.e., equilibrium education may go up or go down.

Thus, once we take into account the endogeneity of k prevailing at the stationary equilibrium, the sign of the Ben-Porath effect becomes ambiguous, unlike in the case of a small open economy, where e and s were not bound to satisfy the saving equation. Note that the same kind of result would be obtained in the more complex case where longevity is endogenous.

Our analysis in this subsection reveals an important result concerning the conditions under which longevity raises education. In the existing literature, the Ben-Porath effect is generally presented without considering changes in production factor prices (as in the last subsection). Typically in existing models, education is increasing with life expectancy at the temporary equilibrium, and a rise

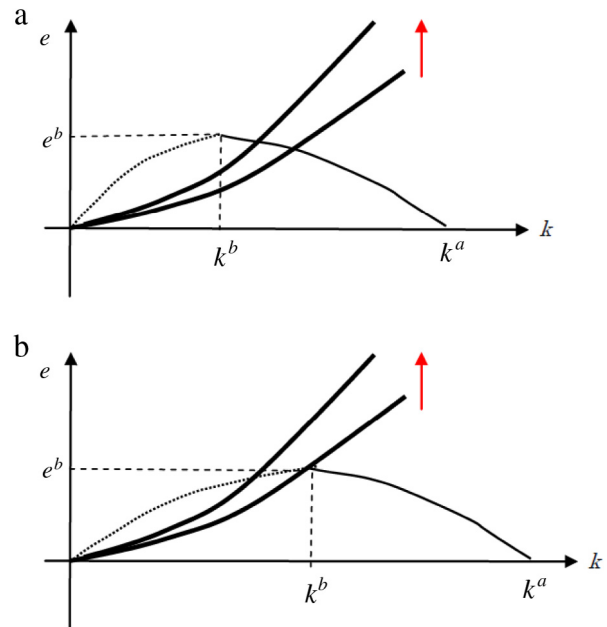


Fig. 2. Impact of a change in $\bar{\ell}$ on the equilibrium education level.

in education contributes positively to the accumulation of human capital and to long-run growth.¹⁵ However, if we allow for changes in prices (as in this subsection), the connexion between the temporary equilibrium and the long-run equilibrium is more complex. Including the fact that education may tighten the domestic capital market,¹⁶ the general-equilibrium effects studied in this section are quite intuitive and would be applicable generally, but have remained so far unnoticed in the literature. Our results contribute to cast some light on the quite mitigated results concerning the empirical tests of the Ben-Porath effect.

6. The social optimum

So far, we only considered the Ben-Porath effect from a purely positive perspective. Our explorations allowed us to identify the

¹⁵ The previous subsection confirmed the Ben-Porath effect in this setting, even when the retirement age does not vary with the rise in longevity (Proposition 8). It is worth noting that Propositions 6 and 8 hold not only at the stationary equilibrium but also at any temporary equilibrium.

¹⁶ In addition, a later retirement period decreases per-worker capital, and also, saving is substitutable with old-age labor income. Both of these factors affect (36) of the model.

¹⁴ This ambiguous effect would also hold under endogenous longevity.

$$\frac{dk}{d\bar{\ell}} = \frac{\Gamma_{\bar{\ell}}}{-\Gamma_k} = \frac{\frac{\partial \tilde{S}(k)}{\partial \bar{\ell}} [h(e) + \alpha^2 w(k)[h(e)]^2 \gamma \bar{\ell}] - (\tilde{S}(k) - e) \alpha^2 w(k)[h(e)]^2 \gamma}{\left[[h(e) + \alpha^2 w(k)[h(e)]^2 \gamma \bar{\ell}]^2 - \tilde{S}'(k) [h(e) + \alpha^2 w(k)[h(e)]^2 \gamma \bar{\ell}] \right] + (\tilde{S}(k) - e) [\alpha^2 w'(k)[h(e)]^2 \gamma \bar{\ell}]}$$

Box III.

conditions under which the Ben-Porath effect is more or less sizeable, in terms of the structural parameters of the economy. Besides those positive issues, another important question concerns the definition of the optimal public policy in an economy characterized by some form of Ben-Porath effect. To answer that question, a necessary stage consists in characterizing the social optimum.

Let us consider the following social planning problem. The social planner chooses consumptions, education, retirement age and physical capital in such a way as to maximize the sum of generational lifetime welfare levels, subject to the resource constraint of the economy.¹⁷ That problem can be written as:

$$\max_{c_t, d_t, e_t, z_t, k_t} \sum \left\{ \beta^t [u(c_t) + \ell_{t+1} (e_{t-1}) u(d_{t+1})] - \mu_t \left[c_t + \ell_t (e_{t-2}) d_t + e_t + \tilde{k}_{t+1} + v(z_t, \ell_t (e_{t-2})) - F(\tilde{k}_t, h(e_{t-1}) + \alpha z_t h(e_{t-2})) \right] \right\}$$

where β is the factor of social time preferences and μ_t is the Lagrange multiplier associated with the resource constraint at time t .

The FOCs of that social planning problem are:

$$c_t : \beta^t u'(c_t) = \mu_t \quad (49)$$

$$d_t : \beta^{t-1} u'(d_t) = \mu_t \quad (50)$$

$$k_t : \mu_t F_{\tilde{k}_t}(\cdot) = \mu_{t-1} \quad (51)$$

$$z_t : v_{z_t}(z_t, \ell(e_{t-2})) = F_{L_t}(\cdot) \alpha h(e_{t-2}) \quad (52)$$

$$e_t : \beta^{t+1} \frac{\partial \ell_{t+2}}{\partial e_t} u(d_{t+2}) - \mu_{t+2} \frac{\partial \ell_{t+2}}{\partial e_t} d_{t+2} - \mu_t - \mu_{t+2} v_{\ell_{t+2}}(\cdot) \frac{\partial \ell_{t+2}}{\partial e_t} + \mu_{t+1} F_{L_{t+1}}(\cdot) h'(e_t) + \mu_{t+2} F_{L_{t+2}}(\cdot) \alpha z_{t+2} h'(e_t) = 0. \quad (53)$$

Focusing on a stationary state in which $\mu_t = \beta \mu_{t-1}$, we have:

$$\beta^{-1} = F_L(\cdot) h'(e)(1 + \alpha \beta) + \ell'(e) \beta \left[d \left(\frac{1}{F^R} - 1 \right) - v_{\ell}(z, \ell(e)) \right]. \quad (54)$$

Let $F_L(\cdot) = w$ and $\beta^{-1} = F_{\tilde{k}}(\cdot) = R$. We then obtain:

$$R^2 = w h'(e)(R + \alpha \beta) + \ell'(e) \left[d \left(\frac{1}{F^R} - 1 \right) - v_{\ell}(z, \ell(e)) \right]. \quad (55)$$

This expression coincides with the FOC for optimal education at the laissez-faire in (46). Therefore, the social optimum can be decentralized provided the laissez-faire capital stock corresponds to the one satisfying the Modified Golden Rule:

$$F_{\tilde{k}}(\cdot) = \frac{\partial F(\tilde{k}, (1 + \alpha \beta) h(e))}{\partial \tilde{k}} = \frac{1}{\beta}. \quad (56)$$

¹⁷ In so doing, we deliberately neglect the objection that the utilitarian approach with variable longevity implies a bias towards long-lived individuals (see Pestieau and Ponthiere, 2016).

Hence, the social optimum can be decentralized by merely decentralizing the capital to labor ratio k satisfying the Modified Golden Rule, with $R = \beta^{-1}$ and $w = F_L(\tilde{k}, (1 + \alpha \beta) h(e))$, where e , z , and $\tilde{k}$ are fixed at the social optimum level.¹⁸ As soon as k takes that level, then all other variables, such as consumptions, education and retirement age, take their socially optimal levels. Proposition 10 summarizes our results.

Proposition 10. For any $\beta > 0$, the long-run social optimum is a vector (c, d, z, e, k) satisfying the conditions:

$$\begin{aligned} u'(c) &= u'(d) \beta^{-1} = u'(d) F_{\tilde{k}}(\cdot) \\ v_z(z, \ell(e)) &= F_L(\cdot) \alpha h(e) \\ \beta^{-2} &= F_L(\cdot) h'(e)(\beta^{-1} + \alpha \beta) \\ &\quad + \frac{\partial \ell}{\partial e} \left[d \left(\frac{1}{F^R} - 1 \right) - v_{\ell}(z, \ell(e)) \right] \end{aligned}$$

$$c + \ell(e) d + e + \tilde{k} + v(z, \ell(e)) = F(\tilde{k}, h(e) + \alpha z h(e)).$$

Provided the capital stock level in the laissez-faire is the one satisfying the Modified Golden Rule, the long-run social optimum can be decentralized by a laissez-faire allocation.

Proof. See the Appendix. ■

Proposition 10 corresponds to the Second Theorem of Welfare Economics, which characterizes the property for all β . A stationary solution to maximize the discounted sum of lifetime utilities leads to the Modified Golden Rule of capital accumulation. Proposition 10 contains the laissez-faire case, where the weight β is $\beta^* \equiv R^{-1}$ of the laissez-faire allocation in Section 4. Provided the Modified Golden Rule level of k can be maintained in a stationary equilibrium, such an equilibrium attains the first-best social optimum. Accordingly, if the policy-maker exhibits $\beta > \beta^*$ (namely, agents are more myopic than the social planner), the stationary equilibrium level of capital would be too low (under-accumulation), and vice versa for $\beta < \beta^*$. However, apart from $\beta = \beta^*$, Proposition 10 is silent on how the Modified Golden Rule k level can be obtained. This point is similar to the argument that the Second Theorem of Welfare Economics is silent on how to implement lump-sum transfers associated with any Pareto efficient allocation. As in the standard model, the laissez-faire level of k depends on the initial endowment and the individual preferences.

7. Optimal second-best policy

Up to now, we considered an economy composed of identical individuals. Let us now consider an economy where individuals differ regarding their ability to transform educational investment

¹⁸ See footnote 23 in Appendix that shows the capital-market clearing. Regarding the labor market, (52) is consistent with (26) and (54) is consistent with (46), and in footnote 23 we showed the household optimization condition (25) is consistent with $R = \beta^{-1}$ and w being the market-clearing price of $w = F_L(\tilde{k}, L/N)$ and $L = (1 + \alpha \beta) h(e) N$.

into human capital. That source of heterogeneity can be regarded as individual differences in their *learning ability*. Type-1 agents exhibit a lower ability to learn than type-2 agents:

$$h_2(e) > h_1(e) \text{ and } h'_2(e) > h'_1(e) \text{ for all } e > 0$$

with $\lim_{e \rightarrow 0} h'_i(e) = +\infty$ and $\lim_{e \rightarrow +\infty} h'_i(e) = 0$ ($i = 1, 2$). We consider a setting à la [Mirrlees \(1971\)](#) and [Stiglitz \(1982\)](#) in which learning abilities are private information. Our focus is how individual decisions for education and saving would get distorted at the social optimum when abilities are not observed by the government. For that purpose, we now abstract from the retirement decision, and suppose that the hourly wage and the interest factor are both equal to 1: $w = R = 1$.¹⁹

Let us first look at the *laissez-faire* outcome. Type- i 's utility maximization problem is as follows:

$$U_i \equiv \max_{e_i, s_i} u[h_i(e_i) - e_i - s_i] + \ell(e_i) u\left(\frac{s_i}{\ell(e_i)}\right).$$

The FOCs are:

$$s_i : -u'(h_i(e_i) - e_i - s_i) + u'\left(\frac{s_i}{\ell(e_i)}\right) = 0 \quad (57)$$

$$e_i : u'(c_i)[h'_i(e_i) - 1] + \ell'(e_i)(u(d_i) - d_i u'(d_i)) = 0. \quad (58)$$

In the *laissez-faire* solution, we have $e_2 > e_1$ and $s_2 \leq s_1$.²⁰ Also, since type 2 can receive higher incomes from the same unit of education investment, type 2 ends up with higher utility than type 1: that is, $U_2 > U_1$ at the *laissez-faire* solution. As in the conventional scenario, the social planner aims at redistributing income from type 2 to type 1.

In the rest of this section, we first characterize the social optimum, under the assumption that the learning ability of individuals cannot be easily observed by the social planner. Given the asymmetric information on the learning ability, we consider now a second-best social optimum. Then, we will derive the associated second-best optimal non-linear taxation scheme.

Let y_i be type- i 's labor income and $E_i(y_i)$ be type i 's education investment required to achieve the level of y_i : that is, $E_i^{-1}(e) \equiv h_i(e) = y_i$. Also, let $\ell(E_i(y_i)) \equiv \hat{\ell}_i(y_i)$. The second-best social planning problem consists in deriving, for each types $i \in \{1, 2\}$, baskets $\{x_i, y_i, s_i\}$ that maximize social welfare, subject to the resource constraint of the economy, and subject to the incentive constraint.²¹ That planning problem can be written by means of the following Lagrangian:

$$\begin{aligned} \max_{x_i, y_i, s_i} \mathcal{L} = & \sum_{i=1,2} \left\{ \delta_i \left(u(x_i - E_i(y_i) - s_i) + \hat{\ell}_i(y_i) u\left(\frac{s_i}{\hat{\ell}_i(y_i)}\right) \right) \right. \\ & \left. + \mu (y_i - x_i) \right\} \\ & + \lambda \left[\begin{aligned} & u(x_2 - E_2(y_2) - s_2) + \hat{\ell}_2(y_2) u\left(\frac{s_2}{\hat{\ell}_2(y_2)}\right) \\ & - u(x_1 - E_2(y_1) - s_1) - \hat{\ell}_2(y_1) u\left(\frac{s_1}{\hat{\ell}_2(y_1)}\right) \end{aligned} \right] \end{aligned}$$

¹⁹ It is well-known that endogeneity of w and R generates distortionary second-best policies from the production side (violation of [Diamond and Mirrlees \(1971a, b\)](#) production efficiency) only if skilled and non-skilled labor are imperfect substitutes (see [Salanie, 2003](#), chapter 6). But we ruled out such effects in (3), by assuming perfect substitutability of labor. To the extent that production efficiency holds at the second-best optimum, linearity in the production sector is a common assumption in the optimal taxation models. Regarding retirement, the pattern of distortion of the retirement decision would be similar to the one in [Cremer et al. \(2004\)](#), so this section ignores deliberately the retirement decision.

²⁰ See [Appendix](#).

²¹ The variable $x_i \equiv c_i + e_i + s_i$ denotes the first-period total income, which is divided into consumption, education reimbursement and saving. We assume that s_i is observable and can be subject to a non-linear tax scheme.

where δ_i is the weight in the social welfare function ($\delta_1 \geq \delta_2 > 0$) that assures redistribution from type 2 to type 1,²² and μ is the Lagrange multiplier associated to the resource constraint for each type, while λ is the Lagrange multiplier associated with the incentive compatibility constraint. That constraint guarantees that individuals with a high learning ability will not be interested in pretending to have a low learning ability.

The FOCs for optimal x_1 and x_2 are:

$$x_1 : \delta_1 u'(c_1) = \mu + \lambda u'(\tilde{c}_2) \quad (59)$$

$$x_2 : (\delta_2 + \lambda) u'(c_2) = \mu \quad (60)$$

where $\tilde{c}_2 \equiv x_1 - E_2(y_1) - s_1$.

The FOCs for optimal y_1 and y_2 are:

$$\begin{aligned} -\delta_1 \left[u'(c_1) E'_1(y_1) - \hat{\ell}'_1(y_1) \Omega(d_1) \right] + \mu \\ + \lambda \left[u'(\tilde{c}_2) E'_2(y_1) - \hat{\ell}'_2(y_1) \Omega(\tilde{d}_2) \right] = 0 \end{aligned} \quad (61)$$

$$-(\delta_2 + \lambda) \left[u'(c_2) E'_2(y_2) - \hat{\ell}'_2(y_2) \Omega(d_2) \right] + \mu = 0 \quad (62)$$

where $\Omega(d_i) \equiv u(d_i) - d_i u'(d_i)$ and $\tilde{d}_2 \equiv \frac{s_1}{\hat{\ell}_2(y_1)}$.

Given $(\delta_2 + \lambda) u'(c_2) = \mu$, $\hat{\ell}'_2(y_2) = \ell'(e_2) E'_2(y_2)$ and $E'_2(y_2) = 1/h'_2(e_2)$, the FOC for y_2 can be written as:

$$\left[1 - \ell'_2(e_2) \frac{\Omega(d_2)}{u'(c_2)} \right] = h'_2(e_2). \quad (63)$$

Moreover, using the FOC for y_1 , it can be shown that:

$$\left[1 - \frac{\ell'(e_1) \Omega(d_1)}{u'(c_1)} \right] < h'_1(e_1). \quad (64)$$

Hence it follows that

$$h'_1(e_1) + \frac{\ell'(e_1) \Omega(d_1)}{u'(c_1)} = 1 + \theta \quad (65)$$

where $\theta > 0$ (see [Appendix](#)). We thus have, at the second-best, that the marginal rate of substitution between total income and labor income of the low ability type is lower than that obtained in the first-best. Namely, the non-linear tax scheme implies a downward distortion on the level of education of individuals with low learning ability. In other words, individuals with low learning ability are, at the second-best, subject to a tax on education investment.

The intuition behind this second-best argument for taxing the education of individuals with low learning ability goes as follows. Given the asymmetry of information, it is tempting for high learning ability types to pretend to have a low learning ability, in such a way as to benefit from redistribution. The introduction of an incentive compatibility constraint allows the government to avoid mimicking. However, avoiding mimicking has a cost. The only way to discourage the mimicking of the high learning ability individuals consists in taxing the education of those who claim to have low learning ability. By doing so, the government annihilates the incentives of high learning ability individuals to pretend to be a low learning ability type. Indeed, high learning ability individuals are those who most value education investment. Taxing the good they prefer most suffices thus to provide them the incentives to claim to have high learning ability, because they would not gain, under that tax, to pretend from having a low learning ability.

This downward distortion on the education of the low ability type leads to increase the longevity gap between the two types.

²² As $h_2(e_2) > h_1(e_1)$ and $s_2 \geq s_1$ at the *laissez-faire* solution, we cannot rank consumptions across agents, and have thus $c_2 \geq c_1$. However, as $U_2 > U_1$ at the *laissez-faire* solution, by assuming that δ_1 is sufficiently larger than δ_2 , we exclude a scenario where income is redistributed from type 1 (the worse-off) to type 2 (the better-off).

That result comes from the fact that the education of the high ability type is not distorted, whereas the education of the low ability type is distorted downwards, implying a lower education investment. Given that longevity depends only on education investment, the longevity gap between the two types is then increased. [Proposition 11](#) summarizes our results.

Proposition 11. *Consider an economy with two types of individuals, differing in their learning ability.*

Under asymmetric information, the optimal non-linear tax scheme implies a downward distortion on the level of education of the less able type.

This downward distortion contributes to increase the gaps on longevity between the two types.

Proof. See [Appendix](#). ■

The second part of [Proposition 11](#) appears counterintuitive: the second-best optimal policy involves to raise the longevity gap between the two types of agents, which seems paradoxical. However, this follows from the postulated relation between education and longevity. Given that relation, it is impossible to distort the education of the low-ability type downwards without also reducing their longevity, which contributes to raise longevity inequalities between types.

We finally comment on another type of heterogeneity that could be interesting to investigate. Suppose that education translates differently into a higher lifetime span: namely, $\ell_2(e) > \ell_1(e)$ and $h_2(e) = h_1(e)$ for all e . In this case, we can show that the second-best optimum involves a downward distortion on the saving of type-1 individuals (see [Appendix](#)). The intuition behind the saving taxation is simple. Given that mimickers live longer than true type 1, taxing their second-period consumption (again, the good preferred by the mimicker) discourages their incentives to claim to have low learning ability.

8. Concluding remarks

In the recent years, a large attention was paid to the study of the Ben-Porath effect, according to which a rise in life expectancy contributes, by increasing the lifetime return of education, to encourage education. The present paper proposed to reexamine the Ben-Porath effect in an OLG economy where individuals choose their education, their saving and their retirement age, and where education investments do not only raise future wages – possibly with some decay – but contribute also to raise longevity.

Our main results are the following. First, when analyzing the Ben-Porath effect in a small open economy (to abstract from variations in production factor prices), we emphasized that the Ben-Porath effect varies strongly depending on whether we consider an economy with exogenous or with endogenous longevity. In both cases, the strength of the Ben-Porath effect depends on future wages, the marginal return of education, the strength of the decay in human capital, as well as on the marginal disutility of old-age labor. However, once the feedback effect is introduced (i.e. endogenous longevity), the size of the Ben-Porath effect depends on a particular aspect of preferences – the fear of ruin (or the elasticity of instantaneous utility with respect to its scalar input) – which does not enter the Ben-Porath effect under exogenous longevity. Moreover, an additional effect is present under endogenous longevity: the fact that a rise in life expectancy reduces the disutility of old-age labor, which, in turn, increases even more the rise in education returns obtained through the increase in life expectancy.

The distinction between the cases of exogenous and endogenous longevity matters also regarding the (widely debated) impact of old-age labor on the Ben-Porath effect. Our analysis reveals that,

under exogenous longevity, the existence of a positive Ben-Porath effect requires a change in lifetime hours of work, in line with [Hazan \(2009\)](#). However, once longevity is endogenous, a positive Ben-Porath effect can arise even in the absence of old-age labor, and even if old-age labor is fixed, in line with [Cervellati and Sunde \(2013\)](#).

Our analysis of the Ben-Porath effect in a closed economy (where wages and interest rates can adjust) reveals another important result. Actually, we showed that the existence of a positive Ben-Porath effect may not be robust to allowing for adjustments in production factor prices. Once we allow for variations in wages and interest rates, a rise in longevity does not necessarily increase the level of education prevailing at the stationary equilibrium. The role of factor prices adjustments, which has been so far unnoticed in the literature, contributes to cast some light on the quite mitigated results concerning the empirical tests of the Ben-Porath effect.

On the normative side, our analyses highlighted that the long-run social optimum can be decentralized, within our economy, by merely decentralizing the Modified Golden Rule capital level. Such a decentralization, by reducing the interest rate, will also reduce the cost of education funding, and, hence, will induce the socially optimal level of education. We also considered a framework with heterogeneity on learning ability, and we showed that it is optimal, under asymmetric information on learning ability, to distort downwards the education of the low-ability type, which deters high-ability types from mimicking low-ability type, but at the cost of raising longevity inequalities.

All in all, this paper casts some light on the determinants of the Ben-Porath effect, and on some economic implications of that horizon effect. It should be stressed, however, that our analysis relied on several simplifying hypotheses. First, when studying the Ben-Porath effect, we considered the case of a small open economy (with perfect international mobility of both capital and labor) and the case of a closed economy (without mobility of capital and labor), whereas it may also be worth considering the (more realistic) intermediate case where there is only perfect international mobility of capital, but no international mobility of labor. Second, our model ignored fertility choices, and their interactions with education choices ([Hazan and Zoabi, 2006](#)). Changes in the life horizon may affect fertility decisions, and variations in fertility may strengthen or weaken the Ben-Porath effect. Another limitation is that our model presupposes that individuals choose education for themselves, whereas one could consider alternative frameworks where education is chosen by parents (as in [de la Croix and Licanthro, 2013](#)), or is subject to a bargaining between the child and his parent (as in [Leker and Ponthiere, 2015](#)). As shown in the latter article, whose model encompasses the choice by the child or by the parent, alternative decision structures do not annihilate the existence of a horizon effect concerning education choices, but affect the determinants and size of the Ben-Porath effect.

Appendix

A.1. Proof of [Proposition 2](#)

Suppose $e_{t-1} = 0$. Then the marginal welfare gain from extending education is, under $\lim_{e_{t-1} \rightarrow 0} h'(e_{t-1}) = +\infty$ and $\lim_{e_{t-1} \rightarrow 0} \ell'(e_{t-1}) = +\infty$,

$$\begin{aligned} \lim_{e_{t-1} \rightarrow 0} h'(e_{t-1}) & \left[\alpha^2 (w_{t+1}^e)^2 \gamma \ell + R_{t+1}^e w_t \right] \\ & + \frac{\ell'(e_{t-1})}{u'(d_{t+1})} \left[u(d_{t+1}) - u'(d_{t+1}) d_{t+1} \right. \\ & \quad \left. - u'(d_{t+1}) v_\ell(z_{t+1}, \ell) \right] \\ & = +\infty > R_{t+1}^e R_t \end{aligned}$$

so that the marginal welfare gain from increasing education exceeds the marginal welfare loss from increasing education, implying that $e_{t-1} = 0$ cannot be optimal.

Suppose $e_{t-1} = +\infty$. Then the marginal welfare gain from extending education is, under $\lim_{e_{t-1} \rightarrow +\infty} h'(e_{t-1}) = 0$ and $\lim_{e_{t-1} \rightarrow +\infty} \ell'(e_{t-1}) = 0$,

$$\begin{aligned} \lim_{e_{t-1} \rightarrow +\infty} & h'(e_{t-1}) \left[\alpha^2 (w_{t+1}^e)^2 h(e_{t-1}) \gamma \bar{\ell} + R_{t+1}^e w_t \right] \\ & + \frac{\ell'(e_{t-1})}{u'(d_{t+1})} \left[u(d_{t+1}) - u'(d_{t+1}) d_{t+1} \right] \\ & = 0 < R_{t+1}^e R_t \end{aligned}$$

so that the marginal welfare gain from increasing education is inferior to the marginal welfare loss from increasing education, implying that $e_{t-1} = +\infty$ cannot be optimal.

Given that the marginal welfare gain from increasing education exceeds the marginal welfare loss from increasing education at $e_{t-1} = 0$, and given that the marginal welfare gain from increasing education is inferior to the marginal welfare loss from increasing education when e_{t-1} tends to $+\infty$, it follows, by continuity, that there must exist an interior optimal education level.

Regarding the uniqueness of that optimal education level, note that the derivative of the LHS of the FOC w.r.t e_{t-1} is:

$$\begin{aligned} & h''(e_{t-1}) \left[\alpha^2 (w_{t+1}^e)^2 h(e_{t-1}) \gamma \ell(e_{t-1}) + R_{t+1}^e w_t \right] \\ & + h'(e_{t-1}) \left[\alpha^2 (w_{t+1}^e)^2 h'(e_{t-1}) \gamma \ell(e_{t-1}) \right. \\ & \quad \left. + \alpha^2 (w_{t+1}^e)^2 h(e_{t-1}) \gamma \ell'(e_{t-1}) \right] \\ & + \ell''(e_{t-1}) \left[d_{t+1} \left(\frac{1}{F^R} - 1 \right) - v_\ell(z_{t+1}, \ell(e_{t-1})) \right] \\ & + \ell'(e_{t-1}) \left[-v_{\ell\ell}(z_{t+1}, \ell(e_{t-1})) \ell'(e_{t-1}) \right. \\ & \quad \left. - v_{\ell z}(z_{t+1}, \ell(e_{t-1})) z_{t+1} \left(\frac{h'(e_{t-1})}{h(e_{t-1})} + \frac{\ell'(e_{t-1})}{\ell(e_{t-1})} \right) \right] \\ & + \ell'(e_{t-1}) \left(\frac{1}{F^R} - 1 \right) \frac{\partial d_{t+1}}{\partial e_{t-1}}. \end{aligned} \quad (66)$$

For the fourth term of (66), since $v_\ell(\cdot) = -\frac{z^2}{2\gamma\ell^2}$, $-v_{\ell\ell}(\cdot) - v_{\ell z}(\cdot) \cdot \frac{z}{\ell} = -\frac{z^2}{\gamma\ell^3} + \frac{z}{\gamma\ell^2} \cdot \frac{z}{\ell} = 0$. Taking account of $z_{t+1} = \alpha w_{t+1}^e h(e_{t-1}) \gamma \ell(e_{t-1})$, the fourth term is equal to $-\ell'(e_{t-1}) v_{\ell z}(z_{t+1}, \ell(e_{t-1})) z_{t+1} \frac{h'(e_{t-1})}{h(e_{t-1})} = \alpha^2 (w_{t+1}^e)^2 h(e_{t-1}) h'(e_{t-1}) \gamma \ell'(e_{t-1})$.

Therefore, (66) can be rewritten as:

$$\begin{aligned} & \alpha^2 (w_{t+1}^e)^2 \gamma \left[h''(e_{t-1}) h(e_{t-1}) \ell(e_{t-1}) + (h'(e_{t-1}))^2 \ell(e_{t-1}) \right. \\ & \quad \left. + 2h'(e_{t-1}) h(e_{t-1}) \ell'(e_{t-1}) \right] \\ & + h''(e_{t-1}) R_{t+1}^e w_t \\ & + \ell''(e_{t-1}) \left[d_{t+1} \left(\frac{1}{F^R} - 1 \right) - v_\ell(z_{t+1}, \ell(e_{t-1})) \right] \\ & + \ell'(e_{t-1}) \left(\frac{1}{F^R} - 1 \right) \frac{\partial d_{t+1}}{\partial e_{t-1}}. \end{aligned}$$

The second term is negative. The third term is negative when $\frac{1}{F^R} > 1$. For the fourth term, $d_{t+1} = \frac{z_{t+1} \alpha w_{t+1}^e h(e_{t-1}) - v(z_{t+1}, \ell(e_{t-1})) + R_{t+1}^e s_t}{\ell(e_{t-1})} = \frac{\alpha^2 (w_{t+1}^e)^2 (h(e_{t-1}))^2 \gamma}{2} + \frac{R_{t+1}^e s_t}{\ell(e_{t-1})}$. At this point we need to take account that e_{t-1} changes s_t through $-u'(w_t h(e_{t-1}) - e_{t-1} R_t - s_t) + R_{t+1}^e u' \left(\frac{\alpha^2 (w_{t+1}^e)^2 (h(e_{t-1}))^2 \gamma}{2} + \frac{R_{t+1}^e s_t}{\ell(e_{t-1})} \right) = 0$. Making use

of $R_t - w_t h'(e_{t-1}) = \frac{1}{R_{t+1}^e} [\alpha^2 (w_{t+1}^e)^2 h'(e_{t-1}) h(e_{t-1}) \gamma \ell(e_{t-1}) + \ell'(e_{t-1}) d_{t+1} \left(\frac{1}{F^R} - 1 \right) - \ell'(e_{t-1}) v_\ell(z, \ell(e_{t-1}))]$, we have:

$$\begin{aligned} \frac{\partial s_t}{\partial e_{t-1}} &= -\frac{\ell(e_{t-1})}{R_{t+1}^e} \alpha^2 (w_{t+1}^e)^2 h'(e_{t-1}) h(e_{t-1}) \gamma \\ &+ \frac{\ell'(e_{t-1})}{-u''(c_t) - \frac{(R_{t+1}^e)^2}{\ell(e_{t-1})} u''(d_{t+1})} \\ &\times \left[\frac{u''(c_t)}{R_{t+1}^e} \left(d_{t+1} \left(\frac{1}{F^R} - 1 \right) - v_\ell(z, \ell(e_{t-1})) \right) \right. \\ &\quad \left. - \frac{(R_{t+1}^e)^2 s_t u''(d_{t+1})}{(\ell(e_{t-1}))^2} \right], \\ \frac{\partial d_{t+1}}{\partial e_{t-1}} &= \alpha^2 (w_{t+1}^e)^2 h'(e_{t-1}) h(e_{t-1}) \gamma \\ &+ \frac{R_{t+1}^e}{\ell(e_{t-1})} \left(\frac{\partial s_t}{\partial e_{t-1}} - \frac{\ell'(e_{t-1})}{\ell(e_{t-1})} s_t \right) \\ &= \frac{\ell'(e_{t-1})}{-u''(c_t) - \frac{(R_{t+1}^e)^2}{\ell(e_{t-1})} u''(d_{t+1})} \\ &\times \left[\frac{u''(c_t)}{\ell(e_{t-1})} \left(d_{t+1} \left(\frac{1}{F^R} - 1 \right) - v_\ell(z, \ell(e_{t-1})) \right) \right. \\ &\quad \left. + \frac{R_{t+1}^e u''(c_t) s_t}{(\ell(e_{t-1}))^2} \right] \\ &< 0. \end{aligned} \quad (67)$$

Hence, a sufficient condition for the uniqueness of the optimal education level is:

$$\begin{aligned} & h''(e_{t-1}) h(e_{t-1}) \ell(e_{t-1}) + (h'(e_{t-1}))^2 \ell(e_{t-1}) \\ & + 2h'(e_{t-1}) h(e_{t-1}) \ell'(e_{t-1}) < 0. \end{aligned}$$

That condition, which guarantees the monotonicity of the LHS, can be written as:

$$\begin{aligned} & |h''(e_{t-1})| h(e_{t-1}) \ell(e_{t-1}) > (h'(e_{t-1}))^2 \ell(e_{t-1}) \\ & + 2h'(e_{t-1}) h(e_{t-1}) \ell'(e_{t-1}). \end{aligned}$$

A.2. Proof of Proposition 3

Under a stationary equilibrium with perfect foresight, we have $k_{t+1}^e = k_{t+1} = k_t = k$, and $e_t = e_{t-1} = e_{t-2} = e$, so that we have:

$$\begin{aligned} k &= \frac{S(w(k), R(k), w(k), R(k)) - e}{h(e) + \alpha^2 w(k) [h(e)]^2 \gamma \bar{\ell}} \\ &\equiv \frac{\tilde{S}(k) - e}{h(e) + \alpha^2 w(k) [h(e)]^2 \gamma \bar{\ell}} \end{aligned} \quad (69)$$

$$e \equiv E(w(k), R(k), w(k), R(k)) \equiv \bar{e}(k). \quad (70)$$

A stationary equilibrium with perfect foresight is a pair (e, k) satisfying those two conditions. To examine the existence of such a pair, let us study some properties of those two relations.

Take the first one. This equality can be rewritten as:

$$\begin{aligned} k &= \frac{\tilde{S}(k) - e}{h(e) + \alpha^2 w(k) [h(e)]^2 \gamma \bar{\ell}} \\ \iff k [h(e) + \alpha^2 w(k) [h(e)]^2 \gamma \bar{\ell}] &= \tilde{S}(k) - e \\ \iff h(e) [1 + \alpha^2 w(k) [h(e)] \gamma \bar{\ell}] &= \frac{\tilde{S}(k) - e}{k} \end{aligned}$$

$\frac{\tilde{S}(k)-e}{k}$ is a ratio: saving per young worker net of borrowing for education investment, divided by capital per effective labor unit. Denote the solution of $h(e) [1 + \alpha^2 w(k) [h(e)] \gamma \bar{\ell}] = \frac{\tilde{S}(k)-e}{k}$ as $e =$

$\check{e}(k)$, that is, the level of e such that $h(e)[1 + \alpha^2 w(k)h(e)\gamma\bar{\ell}] = \frac{\tilde{s}(k,e)-e}{k}$ holds for a given k . Suppose that the level of $k > 0$ such that $\check{e}(k) = 0$ is unique.

Take now (70). We first show that $\check{e}'(k) > 0$. Evaluated at the steady-state, Eq. (23) is:

$$wh'(e)[R + \alpha^2 wh(e)\gamma\bar{\ell}] - R^2 \equiv \Delta = 0. \quad (71)$$

We can compute:

$$\begin{aligned} \frac{d\bar{e}}{dk} &= \frac{\Delta_k}{-\Delta_e} \\ &= \frac{h'(e)(\alpha^2 2w(k)w'(k)h(e)\gamma\bar{\ell} + R w'(k) + f''(k)w(k)) - 2f''(k)R}{-\Delta_e} \end{aligned} \quad (72)$$

$\Delta_e < 0$ because of the second-order condition, so that the denominator of the above formula is positive. As for the numerator, $w'(k) = -kf''(k) > 0$ from (11), and $-f''(k) > 0$. Also, $h'(e)f''(k)w(k) - f''(k)R = f''(k)[h'(e)w(k) - R]$. From (71), $wh'(e) - R = -w^2 h'(e)\alpha^2 h(e)\gamma\bar{\ell}/R < 0$, so that $h'(e)f''(k)w(k) - f''(k)R > 0$. We then conclude that $d\bar{e}/dk > 0$.

Suppose that $\bar{e}(0) = 0$. Let $k^a > 0$ be the value such that $\check{e}(k^a) = 0$. Given $k^a > 0$ and $\check{e}'(k) > 0$, it follows that $\check{e}(k^a) < \bar{e}(k^a)$. On the other hand, by $\check{e}'(0) > \bar{e}'(0)$ and $\check{e}(0) \geq 0$, there must exist $\bar{k}$ such that $\check{e}(\bar{k}) > \bar{e}(\bar{k})$. Hence, by continuity, there must exist an intersection of $e = \bar{e}(k)$ and $e = \check{e}(k)$, namely, a pair (k, e) satisfying both conditions. Hence we proved existence of a stationary equilibrium with perfect foresight.

A.3. Proof of Proposition 4

Under a stationary equilibrium with perfect foresight, we have $k_{t+1} = k_{t+1} = k_t = k$, and $e_t = e_{t-1} = e_{t-2} = e$, so that we have:

$$\begin{aligned} k &= \frac{\hat{s}(w(k), R(k), w(k), R(k), e) - e}{h(e) + \alpha^2 w(k)[h(e)]^2 \gamma \bar{\ell}(e)} \\ &= \frac{\tilde{s}(k, e) - e}{h(e) + \alpha^2 w(k)[h(e)]^2 \gamma \bar{\ell}(e)} \end{aligned} \quad (73)$$

$$e = \hat{e}(w(k), R(k), w(k), R(k), \tilde{s}(k, e)) = \bar{e}(k, \tilde{s}(k, e)) \quad (74)$$

where $\tilde{s}(k, e) \equiv \hat{s}(w(k), R(k), w(k), R(k), e)$.

To examine the existence of such a pair, let us study some properties of those two relations. Take the first one. This equality can be rewritten as:

$$\begin{aligned} k &= \frac{\tilde{s}(k, e) - e}{h(e) + \alpha^2 w(k)[h(e)]^2 \gamma \bar{\ell}(e)} \\ \iff k[h(e) + \alpha^2 w(k)[h(e)]^2 \gamma \bar{\ell}(e)] &= \tilde{s}(k, e) - e \\ \iff h(e)[1 + \alpha^2 w(k)h(e)\gamma\bar{\ell}(e)] &= \frac{\tilde{s}(k, e) - e}{k}. \end{aligned}$$

Let us denote the solution of $h(e)[1 + \alpha^2 w(k)h(e)\gamma\bar{\ell}(e)] = \frac{\tilde{s}(k,e)-e}{k}$ as $e = \check{e}(k)$, that is, the level of e such that $h(e)[1 + \alpha^2 w(k)h(e)\gamma\bar{\ell}(e)] = \frac{\tilde{s}(k,e)-e}{k}$ holds for a given k .

Take the second relation. That equality can be rewritten as:

$$e = \bar{e}(k, \tilde{s}(k, e)).$$

Denote by $e = \bar{e}(k)$ the solution of $e = \bar{e}(k, \tilde{s}(k, e))$, that is, the level of e such that $e = \bar{e}(k, \tilde{s}(k, e))$ holds for a given k . There are good reasons to believe that $\bar{e}(k)$ is increasing in k . Clearly, a higher k implies a higher wage, and thus higher marginal returns from education, as well as a lower interest factor, implying a lower marginal cost of education. Assuming that $\check{e}'(k) > 0$ and $\bar{e}'(0) = 0$, $\bar{e}(k)$ is an increasing relationship passing through $(k, e) = (0, 0)$.

Let $k^a > 0$ be the value such that $\check{e}(k^a) = 0$. Given $k^a > 0$ and $\check{e}'(k) > 0$, it follows that $\check{e}(k^a) < \bar{e}(k^a)$. On the other hand, by

$\check{e}'(0) > \bar{e}'(0)$ and $\check{e}(0) \geq 0$, there must exist $\bar{k}$ such that $\check{e}(\bar{k}) > \bar{e}(\bar{k})$. Hence, by continuity, there must exist an intersection of $e = \bar{e}(k)$ and $e = \check{e}(k)$, namely, a pair (k, e) satisfying both conditions. Hence we proved existence of a stationary equilibrium with perfect foresight.

A.4. Proof of Proposition 8

The formula of Δ_e is (66) in the proof of Proposition 2, evaluated at the stationary equilibrium. When $\ell'(e) = 0$, the value of $-\Delta_e$ is equivalent to the denominator of (44).

Recall that $\lambda z = \alpha w h(e)\gamma\lambda\bar{\ell}(e)$, and also our assumption that F^R is constant. We have:

$$\begin{aligned} \Delta_\lambda &= h'(e)w^2\alpha^2 h(e)\gamma\bar{\ell}(e) + \ell'(e)\left(\frac{1}{F^R} - 1\right)\left(d + \lambda\frac{\partial d}{\partial \lambda}\right) \\ &\quad - \ell'(e)v_{\bar{\ell}}(\lambda z, \lambda\bar{\ell}(e)) \\ &\quad - \ell'(e)\left[\lambda\bar{\ell}(e)v_{\bar{\ell}\bar{\ell}}(\lambda z, \lambda\bar{\ell}(e)) + \lambda z v_{\bar{\ell}z}(\lambda z, \lambda\bar{\ell}(e))\right]. \end{aligned}$$

As for the last term in the above equation, since $v_{\bar{\ell}}(\cdot) = -\frac{z^2}{2\gamma\bar{\ell}^2}$, $v_{\bar{\ell}\bar{\ell}}(\cdot) \cdot \bar{\ell} = \frac{z^2}{\gamma\bar{\ell}^2}$ and $v_{\bar{\ell}z}(\cdot) \cdot \bar{z} = -\frac{z^2}{\gamma\bar{\ell}^2}$, so that $\lambda\bar{\ell}(e)v_{\bar{\ell}\bar{\ell}}(\lambda z, \lambda\bar{\ell}(e)) + \lambda z v_{\bar{\ell}z}(\lambda z, \lambda\bar{\ell}(e)) = 0$. Therefore, the above equation is identical to the numerator of (48).

We have $d = \frac{\lambda z \alpha w h(e) - v(\lambda z, \lambda\bar{\ell}(e)) + Rs}{\lambda\bar{\ell}(e)} = \frac{\alpha^2 w^2 (h(e))^2 \gamma}{2} + \frac{Rs}{\lambda\bar{\ell}(e)}$. Taking account that λ changes s through $-u'(wh(e) - eR - s) + Ru'(\frac{\alpha^2 w^2 (h(e))^2 \gamma}{2} + \frac{Rs}{\lambda\bar{\ell}(e)}) = 0$,

$$\frac{\partial s}{\partial \lambda} = \frac{\frac{R^2}{\lambda\bar{\ell}(e)} u''(d)}{u''(c) + \frac{(R)^2}{\lambda\bar{\ell}(e)} u''(d)} \frac{s}{\lambda},$$

and thus,

$$\frac{\partial d}{\partial \lambda} = \frac{R}{\lambda\bar{\ell}(e)} \left(\frac{\partial s}{\partial \lambda} - \frac{s}{\lambda} \right) = \frac{-u''(c)}{u''(c) + \frac{(R)^2}{\lambda\bar{\ell}(e)} u''(d)} \frac{Rs}{\lambda^2 \bar{\ell}(e)} < 0.$$

Therefore,

$$d + \lambda \frac{\partial d}{\partial \lambda} \geq \frac{Rs}{\lambda\bar{\ell}(e)} + \lambda \frac{\partial d}{\partial \lambda} = \frac{Rs}{\lambda\bar{\ell}(e)} \frac{R^2}{u''(c) + \frac{(R)^2}{\lambda\bar{\ell}(e)} u''(d)} > 0.$$

Hence we conclude that all the terms in the numerator of (48) are positive.

A.5. Proof of Proposition 10

We have shown in the text that the optimum education level coincides with the laissez-faire, provided the level of the capital stock corresponds to the one satisfying the Modified Golden Rule ((55) and (56)). There remains to show how to assign the individual budget constraint consistently with $u'(c) = Ru'(d)$.

Evaluated at the long-run social optimum, let us define:

$$\bar{s}_1 \equiv wh(e) - eR - c, \quad R\bar{s}_2 \equiv d\bar{\ell}(e) + v(z, \bar{\ell}(e)) - w\alpha h(e)z,$$

which correspond to (16) and (18), respectively. Notice also that, at the temporary equilibrium with $R = \beta^{-1}$ and $w = F_L(\cdot)$, (25) corresponds to the social optimum in (49) and (50) without taxes or transfers from the young to the old.²³ Therefore, provided that

²³ Given CRS, $F(\bar{k}, h(e) + \alpha zh(e)) = R\bar{k} + w(h(e) + \alpha zh(e))$ for $R = F_k(\cdot)$ and $w = F_L(\cdot)$. From the resource constraint,

$$\begin{aligned} c + \bar{\ell}(e)d + e + \bar{k} + v(z, \bar{\ell}(e)) &= F(\bar{k}, h(e) + \alpha zh(e)) \\ &= R\bar{k} + w(h(e) + \alpha zh(e)) \iff \\ -eR - \bar{s}_1 + e + \bar{k} &= R(\bar{k} - \bar{s}_2) \\ \iff \bar{k} + e - \bar{s}_1 &= R(\bar{k} + e - \bar{s}_2). \end{aligned}$$

the level of the capital stock corresponds to the one satisfying the Modified Golden Rule, the social optimum corresponds to the laissez-faire allocation.

A.6. Proof that $e_2 > e_1$ at the laissez-faire

FOCs for saving and education are, respectively

$$\begin{aligned} s_i &: -u'(h_i(e_i) - e_i - s_i) + u'\left(\frac{s_i}{\ell(e_i)}\right) = 0 \\ e_i &: u'(c_i)[h'_i(e_i) - 1] + \ell'(e_i)(u(d_i) - d_i u'(d_i)) = 0 \\ &\iff (h'_i(e_i) - 1) + \ell'(e_i) \frac{s_i}{\ell(e_i)} \left(\frac{1}{F^R} - 1\right) = 0 \end{aligned}$$

where $\frac{1}{F^R} = \frac{u'(d_i)}{u'(c_i)}$. From (57), taking account that $u''(c_i) = u''(d_i)$, we have:

$$\begin{aligned} &\left\{ u''(\cdot) \left(-(h'_i(e_i) - 1) - \frac{s_i \ell'(e_i)}{(\ell(e_i))^2} \right) \right\} de_i \\ &+ \left\{ u''(\cdot) \left(1 + \frac{1}{\ell(e_i)} \right) \right\} ds_i = 0. \end{aligned}$$

Plugging in $-(h'_i(e_i) - 1) = \ell'(e_i) \frac{s_i}{\ell(e_i)} \left(\frac{1}{F^R} - 1\right)$, we have, by eliminating $u''(\cdot)$:

$$\begin{aligned} &\left\{ \left(\ell'(e_i) \frac{s_i}{\ell(e_i)} \left(\frac{1}{F^R} - 1\right) - \frac{s_i \ell'(e_i)}{(\ell(e_i))^2} \right) \right\} de_i \\ &+ \left\{ \left(1 + \frac{1}{\ell(e_i)} \right) \right\} ds_i = 0 \\ &- \frac{s_i \ell'(e_i)}{\ell(e_i)} \left\{ -\frac{1}{F^R} + 1 + \frac{1}{\ell(e_i)} \right\} de_i + \left\{ \left(1 + \frac{1}{\ell(e_i)} \right) \right\} ds_i = 0. \end{aligned}$$

Hence

$$\frac{ds_i}{de_i} \Big|_{(57)} = \frac{s_i \ell'(e_i)}{\ell(e_i)} \left(1 - \frac{1/F^R}{\left(1 + \frac{1}{\ell(e_i)}\right)} \right) \geq 0.$$

Since there are no other specifications other than $1/F^R > 1$ and $1 + \frac{1}{\ell(e_i)} > 1$, the sign of $\frac{ds_i}{de_i}$ based on (57) is ambiguous.

From (58), we have:

$$\begin{aligned} &\left\{ h''_i(e_i) + \left(\frac{\ell''(e_i)}{\ell(e_i)} - \frac{(\ell'(e_i))^2}{(\ell(e_i))^2} \right) s_i \left(\frac{1}{F^R} - 1 \right) \right\} de_i \\ &+ \frac{\ell'(e_i)}{\ell(e_i)} \left(\frac{1}{F^R} - 1 \right) ds_i = 0 \end{aligned}$$

$$\frac{ds_i}{de_i} \Big|_{(58)} = \frac{h''_i(e_i)}{-\frac{\ell'(e_i)}{\ell(e_i)} \left(\frac{1}{F^R} - 1\right)} - \left(\frac{\ell''(e_i)}{\ell'(e_i)} - \frac{\ell'(e_i)}{\ell(e_i)} \right) s_i > 0,$$

and also,

$$\begin{aligned} \frac{ds_i}{de_i} \Big|_{(58)} - \frac{ds_i}{de_i} \Big|_{(57)} &= \frac{h''_i(e_i)}{-\frac{\ell'(e_i)}{\ell(e_i)} \left(\frac{1}{F^R} - 1\right)} - \left(\frac{\ell''(e_i)}{\ell'(e_i)} \right) s_i - \frac{s_i \ell'(e_i)}{\ell(e_i)} \\ &\times \left(-\frac{1/F^R}{\left(1 + \frac{1}{\ell(e_i)}\right)} \right) > 0 \end{aligned}$$

so that $\frac{ds_i}{de_i} \Big|_{(58)} > \frac{ds_i}{de_i} \Big|_{(57)}$.

Therefore, along the resource constraint, the above equation can only be satisfied for $R = 1$ by letting $\bar{s}_1 = \bar{s}_2$, and in the case of $R \neq 1$ by $\bar{s}_1 = \bar{s}_2 = \bar{k} + e$. So (49) and (50) can be satisfied only at $\bar{s}_1 = \bar{s}_2 \equiv s$, with $u'(wh(e) - eR - s) = u'((\omega h(e)z + Rs - v(z, \ell(e)))/\ell(e))\beta^{-1}$.

The rest of the proof goes as follows. Let us denote by $s_i(e)$ the optimal level of saving for type $i = 1, 2$ based on FOC (57). Let us denote by $e_i(s)$ the optimal level of education for type $i = 1, 2$ based on FOC (58). The positions of the pairs (e_1, s_1) and (e_2, s_2) can be compared by drawing the relations $s_1(e)$, $s_2(e)$, $e_1(s)$ and $e_2(s)$ in the (e, s) space.

Let the coordinate $(e_1(\bar{s}), \bar{s})$ be the intersection of $e_1(s)$ and $s_2(e)$. Since $s_2(e) > s_1(e)$ for all e , $\frac{ds}{de} \Big|_{e_1(s)} > 0$ and $\frac{ds}{de} \Big|_{e_1(s)} > \frac{ds}{de} \Big|_{s_1(e)}$, then $(e_1(s_1), s_1) < (e_1(\bar{s}), \bar{s})$. Then, since $e_2(s) > e_1(s)$ for all s , $\frac{ds}{de} \Big|_{e_2(s)} > 0$ and $\frac{ds}{de} \Big|_{e_2(s)} > \frac{ds}{de} \Big|_{s_2(e)}$, we have $e_1(\bar{s}) \leq e_2$. Therefore $e_1 < e_2$. Thus two cases can arise. We have either $e_2 > e_1$ and $s_2 > s_1$ or $e_2 > e_1$ and $s_2 < s_1$. If $s_i(e)$ are non-decreasing for both $i = 1$ and 2 , then we can additionally conclude that $s_2 > s_1$.

A.7. Proof of Proposition 11

Given that $\delta_1 u'(c_1) = \mu + \lambda u'(\tilde{c}_2)$, $\hat{\ell}'_1(y_1) = \ell'(e_1)E'_1(y_1)$ and $\hat{\ell}'_2(y_1) = \ell'(\tilde{e}_2)E'_2(y_1)$ for $\tilde{e}_2 \equiv E_2(y_1)$, the FOCs for optimal y_1 can be written as:

$$\begin{aligned} &\left[-1 + \ell'_1(e_1) \frac{\Omega(d_1)}{u'(c_1)} \right] E'_1(y_1) \left[1 + \frac{\lambda}{\mu} u'(\tilde{c}_2) \right] \\ &+ 1 + \frac{\lambda}{\mu} u'(\tilde{c}_2) \left[1 - \ell'_2(\tilde{e}_2) \frac{\Omega(\tilde{d}_2)}{u'(\tilde{c}_2)} \right] E'_2(y_1) = 0. \end{aligned} \quad (75)$$

To show that the non-linear tax scheme implies a downward distortion on the level of education of the less able type, and, henceforth, increases the longevity gap between the two types, let us denote the marginal rate of substitution MRS for type 1 and type 2 (the mimicker) by, respectively A_1 and $\tilde{A}_2$, where:

$$\begin{aligned} A_1 &\equiv \left[1 - \frac{\ell'_1(e_1) \Omega(d_1)}{u'(c_1)} \right] E'_1(y_1) \\ \tilde{A}_2 &\equiv \left[1 - \frac{\ell'_2(\tilde{e}_2) \Omega(\tilde{d}_2)}{u'(\tilde{c}_2)} \right] E'_2(y_1). \end{aligned}$$

By assuming that the single crossing property holds, it must be the case that $\tilde{A}_2 < A_1$.

Using the FOCs, we obtain:

$$1 - A_1 = \frac{\lambda}{\mu} u'(\tilde{c}_2) [A_1 - \tilde{A}_2]. \quad (76)$$

It follows that $A_1 < 1$. Since $E'_1(y_1) = 1/h'_1(e_1)$, this implies (64) and (65) of the text. Thus the second-best optimum involves a downward distortion on the education of type-1 individuals, that is, of individuals with the low learning ability.

The FOCs for optimal s_1 and s_2 , taking account of (59), are:

$$\begin{aligned} s_1 &: -\delta_1 [u'(c_1) - u'(d_1)] + \lambda [u'(\tilde{c}_2) - u'(\tilde{d}_2)] = 0 \\ &\implies \delta_1 u'(d_1) = \mu + \lambda u'(\tilde{d}_2) \end{aligned} \quad (77)$$

$$s_2 : (\delta_2 + \lambda) [-u'(c_2) + u'(d_2)] = 0 \implies u'(d_2) = u'(c_2). \quad (78)$$

Eq. (78) shows that the Euler equation of intertemporal consumption holds for type 2.

On the other hand, for type 1's intertemporal consumption, $c_1 < \tilde{c}_2$ since $E_1(y_1) > E_2(y_1)$ and $d_1 = \frac{s_1}{\ell_1(y_1)} < \frac{s_1}{\ell_2(y_1)} = \tilde{d}_2$. Since there are no other specifications implied by (59) and (77), the direction of the distortion of type-1's saving ($u'(c_1) \geq u'(d_1)$) is ambiguous.

Here we consider an alternative scenario where education translates differently into a higher lifetime span instead of ability: namely, $h_2(e) = h_1(e)$ and $\ell_2(e) > \ell_1(e)$ for all e . As was mentioned in the text, we now show that the saving of type 1 is taxed. $c_1 =$

$x_1 - E_2(y_1) - s_1 = \tilde{c}_2$ and (59) together imply $(\delta_1 - \lambda)u'(c_1) = \mu$. Also, for $E_1(y_1) = E_2(y_1)$, we have $\hat{\ell}_1(y_1) = \ell_1(E_1(y_1)) < \ell_2(E_2(y_1)) = \hat{\ell}_2(y_1)$, so $d_1 = \frac{s_1}{\hat{\ell}_1(y_1)} > \frac{s_1}{\hat{\ell}_2(y_1)} = \tilde{d}_2$. So (77) implies $(\delta_1 - \lambda)u'(d_1) > \mu$. Therefore:

$$u'(c_1) < u'(d_1). \quad (79)$$

Hence we have:

$$(1 + \eta)u'(c_1) = u'(d_1) \quad (80)$$

where $\eta > 0$. Thus the second-best optimum involves a downward distortion on the saving of type-1 individuals, that is, of individuals with the low learning ability.

In the light of this, the decentralization of the second-best social optimum requires a tax T_a on the effective unit of education effort (y_1) and a tax T_b on saving of the individuals with low learning ability. Under such taxes, the problem for type-1 individuals becomes:

$$\max_{y_1, s_1} u[y_1 - E_1(y_1) - T_a(y_1) - s_1 - T_b(s_1)] \\ + \ell(E_1(y_1)) u\left(\frac{s_1}{\ell(E_1(y_1))}\right).$$

The FOCs are:

$$s_1 : (1 + T'_b(s_1))u'(c_1) = u'(d_1) \quad (81)$$

$$y_1 : u'(c_1)[1 - E'_1(y_1) - T'_a(y_1)] \\ + E'_1(y_1)\ell'(e_1)(u(d_1) - d_1 u'(d_1)) = 0. \quad (82)$$

From the last FOC, we obtain:

$$\frac{u'(d_1)}{u'(c_1)} = 1 + T'_b(s_1), \quad h'_1(e_1) + \ell'(e_1) \frac{\Omega(d_1)}{u'(c_1)} \\ = 1 + T'_a(y_1) \cdot h'_1(e_1) \quad (83)$$

which, by setting $T'_a(y_1) = \theta/h'_1(e_1)$ and $T'_b(s_1) = \eta$, coincides with the condition of the optimal level of s_1 and e_1 at the second-best.

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