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Shopping Versus Shipping: A Spatial Competition Theory of E-commerce*

Toshitaka Gokan[†] Jacques-François Thisse[‡] Xiwei Zhu[§]

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Abstract

Online retailers compete not only on price but also on delivery performance, making logistics a central issue in the structure of the retail industry. Online retailers may choose either a uniform delivered price or location-specific delivered prices, whereas offline retailers rely on uniform mill pricing. Using a game-theoretic model that incorporates shopping costs, shipping costs, distaste costs, and consumer taste heterogeneity, we characterize the equilibrium structure of the retail industry. The existence of equilibrium critically depends on the degree of taste heterogeneity. An offline retailer chooses to move online when distaste or shipping costs are sufficiently low. However, firms become trapped in a Prisoner's Dilemma when both adopt uniform delivered pricing. When both firms operate as e-retailers, they engage in a Game of Chicken and soften price competition by adopting different pricing formats. We also describe the evolution of the retail industry from conventional retailing to personalized pricing as distaste and shipping costs decline.

Keywords — Offline retailing, online retailing, spatial price policies

JEL codes —L13; L81; R10.

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1 Introduction

In 2024, Amazon's revenue reached approximately \$638 billion, with shipping expenditures totaling \$95.8 billion—a substantial share of sales. For the fiscal year ending January 31, 2025, Walmart's total operating expenses reached approximately \$182 billion, which encompasses all fulfillment, labor, and logistics costs. By adopting automation technology, Walmart has cut its shipping costs by around 30%, underscoring how significant these costs are. On average, shipping costs account for about $10-15\%$ of online retailers' total cost (Gümüş et al., 2013; Gosling et al., 2025). Online firms compete not only on prices but also on delivery performance, making logistics a central strategic variable.

Unlike offline retail—where consumers incur travel costs to reach the store—online retailers deliver products directly to geographically dispersed consumers. This distinction underscores the need to explicitly model online retailers' shipping costs, an aspect largely overlooked in the literature (Ratchford et al., 2022). By shaping effective distance and delivery times, logistics influences the spatial structure of competition and, consequently, consumer choice. In this paper, we analyze a setting in which e-commerce gives rise to distinct costs arising from the shipment of goods to consumers.

Because e-commerce relies on physical delivery while conventional retail depends on consumers' travel, the analysis of competition across retail formats within a spatial competition framework, where firms adopt different pricing policies regarding the extent to which shipping costs are passed on to consumers (Forman et al., 2009; Bonfrer et al., 2022). While several studies have employed spatial models to examine e-retailing, the literature did not analyze price competition across different retail formats within a spatial framework that explicitly incorporates the logistical constraints faced by e-retailers.

To address this gap, we propose a spatial competition model in which industry equilibrium arises endogenously from firms' pricing strategies. Our unified game-theoretic framework integrates shopping and shipping costs, distaste costs, and consumer heterogeneity in preferences. We then characterize the resulting equilibrium and analyze how spatial pricing policies shape market structure and welfare outcomes. In particular, we identify the conditions under which retailers operate in markets offering both formats, as well as those under which all retailers choose to operate exclusively online, conditional on format-specific transportation costs. Our framework also allows us to evaluate which retail format is most desirable from both the consumer's and the firm's perspective, as well as the first-best optimum.

Consumers are heterogeneous in two dimensions. Beyond consumers' spatial dispersion, we also assume that consumers exhibit heterogeneous preferences, which we model by means of a discrete choice setting. This allows us to distinguish the effects of delivery/shopping frictions from the effects of product differentiation when comparing retail formats. Assuming taste heterogeneity is consistent with Hottman et al. (2016), who document that $50-70\%$ of the variance in firm size can be attributed to differences in firm appeal.

We consider three basic retail formats. An offline retailer charges a *uniform mill price* (M) to all customers, while each buyer faces a total price equal to the sum of the in-store price and her travel cost to the retailer (hereafter, the shopping cost). By contrast, firms offering home delivery face a wider array of potential pricing strategies, primarily related to how shipping costs are passed on to customers.

It is well established that free shipping and fast delivery enhance consumer satisfaction and positively influence purchase intention (Lewis et al., 2006; Gümüş et al., 2013). For example, more than 95% of a proprietary McKinsey survey of more than $1,000$ US consumers say they prefer free shipping with standard delivery versus paid shipping with expedited delivery (Gosling et al., 2025). Accordingly, we first assume that e-retailers adopt a *uniform delivered pricing* policy (U), in which a firm charges the same delivered price—inclusive of shipping and handling—regardless of the customer's location.

To underscore the empirical relevance of uniform delivered pricing, note that Amazon implements a uniform pricing strategy, ensuring that each product variety is available at identical prices within counties. Similar pricing practices have been documented for several other major retailers, such as BestBuy and Target (Cavallo, 2019). Consistent with these observations, Cavallo (2017), in a detailed study of online sellers in 10 countries, finds that "online prices are the same for all locations." Taken together, these findings suggest that uniform delivered pricing provides a relevant proxy for e-retailers' pricing policies (Cavallo, 2019). Despite its empirical importance, however, uniform delivered pricing has rarely been used to model e-retailers' pricing, possibly because the existence of an equilibrium is problematic (Kats and Thisse, 1993).

With firms gaining access to increasingly granular data on consumers' locations and purchasing behavior, the potential for more sophisticated differential pricing has expanded significantly (Chen et al., 2021; Elmachtoub et al., 2021; Aparicio et al., 2024; Rhodes and Zhou, 2024). Just as products can be tailored to individual consumers, customized pricing aims to match prices to consumers' willingness to pay. This practice constitutes third-degree price discrimination, in which firms set prices that vary across observable consumer characteristics.

For e-retailers, geographic location provides a natural basis for such discrimination, as firms can often observe consumers' precise locations or infer them from IP addresses or zipcodes. These considerations induce firms to adopt *personalized delivered pricing* (D), under which price schedules are defined over buyer locations and reflect location-specific demand conditions.[1] Because consumer preferences are frequently spatially correlated, the D-pricing framework is particularly well suited to capturing pricing behavior in online retail markets (Goldfarb and Tucker, 2019).

Considering retail expansion as a two-step process provides valuable insights into the evolution of digital markets. In the initial phase of online market development, e-retailers predominantly adopted uniform delivered pricing. A central question, therefore, is to identify the conditions under which a firm optimally chooses to operate as a U-retailer rather than being an offline retailer. The second phase emerged with the expansion of data availability and advances in delivery technologies. In this environment, e-retailers gained the ability to implement personalized delivered pricing strategies. Conditional on entering the online market, firms must then decide whether to maintain a uniform pricing regime or adopt D-pricing. The paper is organized accordingly in Sections 4 and 5.

Our main findings may be summarized as follows.

(i) A minimum degree of consumer heterogeneity and/or product differentiation across firms is necessary to ensure the existence of a price equilibrium when shipping costs associated with e-commerce are positive. Equally important, consumer heterogeneity plays a pivotal role in the emergence of distinct retail formats, even under otherwise identical market conditions. As usual, in all formats, prices increase when consumers become more heterogeneous.

(ii) As consumers are sensitive to shipping fees and delivery times, this has led online firms to implement sophisticated logistic strategies that allow them to serve at low cost geographically dispersed customers (Smith and Brynjolfsson, 2001). Doing this requires solving an NP-hard optimization problem which aims to determine the least-cost delivery routes. This problem, called the vehicle routing problem [VRP], has been extensively studied in operations research and management science (Laporte, 2009; Koç *et al.*, 2020). We determine the shipping cost function by solving the vehicle routing problem in the special case of a linear space.

(iii) Choosing a format amounts to choosing a spatial price policy. We show that such a transformation is profitable when distaste costs are sufficiently low. This result helps explain why high-priced luxury goods—characterized by strong idiosyncratic consumer preferences—remain predominantly distributed through physical stores. By contrast, standardized or easily digitized products (e.g., books, CDs, inexpensive apparel) are associated with low distaste costs and are

[1] We use D instead of P because P denotes delivered prices of online retailers.

more likely to be purchased online (Smith and Brynjolfsson 2001).

Improvements in logistics further increase the likelihood of adopting an online format by reducing shipping costs. Rising shopping costs have a similar effect. This point is worth emphasizing, as intra-city travel costs vary substantially depending on a city's shape, its transportation infrastructure, and its congestion policies. Finally, we show that e-retailing is detrimental to conventional retailers. Moreover, when online and offline formats coexist, sales of the online retailer are larger than those of the offline retailer.

(iv) When retailers can choose their format, three cases may arise. First, both firms optimally remain conventional retailers when the distaste cost is sufficiently high. Second, for intermediate values, the industry exhibits a mixed structure, with online and offline formats coexisting. Last, when distaste costs are sufficiently low, both firms adopt the e-retailing format. In this parameter region, the market involves a U-retailer and a D-retailer. In other words, firms avoid selecting the same channel to soften price rivalry.

Further, the price policy game between online firms is a game of chicken and features two Nash equilibria. Given that both firms operate in a perfectly symmetric environment, the existence of two asymmetric Nash equilibria is a priori unexpected.

(v) A central issue in the literature concerns the procompetitive nature of online retailing, driven by lower distribution and search costs (Brown and Goolsbee, 2002; Brynjolfsson et al., 2003; Lieber and Syverson, 2012). The standard view of e-commerce posits that it reduces market frictions, intensifies competition, and enhances consumer welfare.

In a duopoly involving a conventional retailer and an e-retailer, our analysis yields a more nuanced conclusion: consumers' preferred retail format depends on their location relative to retailers. Accordingly, it is difficult to derive general statements regarding the welfare implications of the online channel in a market involving offline and online retailers. By contrast, in a market comprising only e-retailers, the configuration formed by two D-retailers maximizes consumer surplus.

From the firms' perspective, the U-retailer earns higher profits than the M-retailer. Under competition between e-retailers, total industry profits are maximized at each Nash equilibrium of the price policy game. This is because competition between two D-retailers or two U-retailers is tougher than competition between a U-retailer and a D-retailer. In other words, price policy differentiation relaxes competition.

Related literature. Gauri et al. (2021) and Ratchford et al. (2022) are two detailed overviews of the now vast literature on competition between different retail formats. In what follows, we only discuss papers closely related to ours.

The idea that consumers are heterogeneous along multiple dimensions is not new; however, the existing literature typically focuses on conventional retailers. Balasubramanian (1998) and Bouckaert (2000) provide pioneering contributions by developing spatial competition models that incorporate both offline and online retailers. Several subsequent papers employ spatial competition frameworks to examine related issues, but they focus on different types of heterogeneity and disregard shipping costs (e.g., Guo and Lai, 2017; Lu and Matsushima, 2024; Li et al., 2024). A noticeable exception is Yoo and Lee (2011), who assume that spatially dispersed consumers differ in the disutility they incur from using an Internet channel. However, they disregard shipping costs in their analysis of uniform pricing. Colombo and Matsushima (2020) consider a two-dimensional approach of the demand side, but they focus on heterogeneous distaste costs. Rhodes and Zhou (2024) use a general discrete choice model to assess how varying degrees of market coverage affect consumer surplus and firm profits.

There is a growing body of literature on personalized pricing, where firms set delivered prices based on consumers' characteristics, such as their location (e.g., Elmachtoub et al., 2021; Rhodes and Zhou, 2024). However, these studies are not directly relevant to our analysis because they disregard transportation costs.

By facilitating consumers' search, e-commerce was expected to strongly reduce price dispersion. Brynjolfsson and Smith (2000) observe that price dispersion still prevails among e-retailers. In their empirical study, Smith and Brynjolfsson (2001) use the multinomial logit to rationalize customers' behavior who do not always buy the cheapest homogeneous good, such as books or CDs. Bernstein et al. (2008) also use the multinomial logit and consider different markets involving only offline retailers, offline retailers and retailers that combine off- and online sales, and retailers selling through both channels. They find that combining the two channels is a dominant strategy for all retailers. However, their setting includes no transportation costs.

Although the literature on digital platforms acknowledges that goods are delivered to consumers either by platforms or by third-party sellers, relatively few studies explicitly incorporate shipping costs into the analysis. Notable exceptions include Sun et al. (2022, 2025), who analyze membership-based free-shipping programs—such as Amazon Prime and Walmart+—as mechanisms to mitigate consumers' shipping cost burden.

The remainder of the paper is organized as follows. Section 2 introduces the model and discusses the roles of shipping and distaste costs. Section 3 characterizes the equilibrium in the benchmark case of two offline retailers. Section 4 analyzes a mixed duopoly with one offline and one online firm; we first characterize the pricing equilibrium and then derive the conditions under which a conventional retailer adopts the online format. Section 5 examines competition between e-retailers that may choose between U and D. In addition, we show that D-D is the retail system most preferred by all consumers, while the total industry profits are maximized at each Nash equilibrium of the price policy game. Section 7 presents an integrated synthesis that characterizes the equilibrium structure of the retail industry, wherein firms select a format from the strategy set $\{M, U, D\}$. Section 8 concludes with a brief discussion of possible extensions.

2 The model

In line with most of the literature, we consider a duopoly. Firms, which are denoted $i = 1, 2$, supply a horizontally differentiated good; both firms have access to the same delivery technology; marginal production costs are equal and normalized to zero. We make this assumption to uncover the intrinsic characteristics of each format. There is a unit-mass of consumers; each consumer buys a fixed amount of the differentiated good, which is normalized to one.

Consumers and firms are located in a linear city $[0, 1]$. When Firm i operates as an offline retailer, its location corresponds to its place of establishment; when it operates as an online retailer, its location refers to the fulfillment center that serves the residents of the city. Shopping centers accommodating conventional retailers and warehouses used by e-retailers are typically located at the city limits, where land prices are low. Hence, Firms 1 and 2 are set up at the endpoints $x = 0$ and $x = 1$ of the unit interval $[0, 1]$. Consumers are uniformly distributed over $[0, 1]$ with unit density.

Our focus on spatial pricing requires a clear distinction between *producer prices* and *consumer prices*. For a conventional retailer, the producer price is the mill price, while the consumer price equals the mill price plus a location-dependent shopping cost that increases with the consumer's distance from the retailer. As a result, producer prices are spatially uniform, whereas consumer prices vary across locations. By contrast, in the case of e-retailing, consumer prices are spatially uniform, as consumers pay the same delivered price regardless of location. Producer prices, however, are decreasing, reflecting the shipping costs borne by the firm.

2.1 Consumer demand

We now come to the second dimension of heterogeneity. Following the literature on product differentiation, we assume that the indirect utility of a consumer located at $x \in [0, 1]$ who chooses variety i is given by

$$V_i(x) = \nu - p_i(x) + \varepsilon_i,$$

where ν is the gross utility from consuming either variety, $p_i(x)$ the consumer price of variety i at x, and ε_i a random variable whose realization measures the quality of the match between individual preferences (e.g., her ideal product) and variety i. Since we consider a duopoly, we can adopt the canonical discrete choice model in which the random variable $\varepsilon \equiv \varepsilon_2 - \varepsilon_1$ is uniformly distributed over the compact interval $[-L, L]$ with density $1/(2L)$. In other words, consumer heterogeneity is represented through the *linear probability model* (Amemiya, 1981).

The parameter L serves as an index of heterogeneity in matching costs: when $L = 0$, consumers have identical tastes, whereas higher values of L correspond to greater dispersion in individual preferences. Alternatively, the parameter L can measure consumers' attitude toward diversity: as L becomes larger, consumers care more about diversity in consumption and less about prices (Sajeesh and Raju, 2010). In sum, each consumer is characterized by her geographical location $x \in [0, 1]$ and her preference address $z \in [-L, L]$. The location and matching distributions are independent.

The demand for variety i of a consumer located at x is defined by the Lebesgue measure of the set of realizations of ε such that $V_i(x) > V_j(x)$ (Anderson et al., 1992):

$$\begin{aligned}
q_1(p_1(x), p_2(x)) &= \begin{cases} 0 & \text{if } x \geq x_{\max} \\ \frac{L - p_1(x) + p_2(x)}{2L} & \text{if } x_{\min} < x < x_{\max} \\ 1 & \text{if } \quad x \leq x_{\min} \end{cases}, \\
q_2(p_1(x), p_2(x)) &= 1 - q_1(p_1(x), p_2(x)),
\end{aligned} \tag{1}$$

where $x_{\max}$ and $x_{\min}$ are, respectively, the locations of consumers indifferent between buying both varieties or only one of them, i.e., $q_1(p_{1m}(x_{\max}), p_{2m}(x_{\max})) = 0$ and $q_2(p_{1m}(x_{\min}), p_{2m}(x_{\min})) = 0$.

There is full market coverage when $x_{\min} = 0$ and $x_{\max} = 1$, and partial market coverage when $x_{\min} > 0$ and $x_{\max} < 1$. Consumers located in $(x_{\min}, x_{\max})$ buy from both stores, whereas those in $[0, x_{\min})$ and $(x_{\max}, 1]$ patronize a single store. To limit the number of possible cases, we focus on full market coverage.

As shown in Appendix A, the demand functions implied by the linear probability model coincide with those obtained in a Hotelling-like framework where consumers' ideal products are uniformly distributed over $[-L, L]$.

Using linear demand systems is common in industrial economics (Vives, 1999). Less common, our demand functions are symmetric in prices, like in Hotelling. However, our analysis can easily be extended to account for variety-specific characteristics that lead to weighted prices in (8). For example, asymmetry could be introduced by assuming that the random variable $\varepsilon = \varepsilon_2 - \varepsilon_1$ is uniformly distributed over the interval $[-L_1, L_2]$ with density $1/(L_1 + L_2)$. Likewise, quality difference between varieties may be taken into account by adding a positive constant to V_i, which measures the intrinsic quality of variety i. Our main results remain qualitatively unchanged in those extensions.

Competition between firms is modeled as a two-stage game. In the first stage, firms choose a retail format, either offline or online. In the second stage, they compete in prices, contingent on their chosen price policies. The market outcome is given by a subgame perfect Nash equilibrium. As usual, the game is solved by backward induction.

2.2 Distaste costs

Consumers who purchase online are deprived from the advantages associated with buying from a conventional retailer. This includes (i) the physical inspection of goods, (ii) the delay in receiving the product, (iii) the difficulty in returning products, (iv) personal and trusted advice, and (v) the after-sales services supplied by a conventional retailer. All of this generate a utility loss, which is expressed as a monetary loss that is called a *distaste cost* (Balasubramanian, 1998; Loginova, 2009). Smith and Brynjolfsson (2001) find that the distaste cost is positive even for homogeneous goods like books. Note that a high distaste cost may reflect the inefficiency of the logistics used by an e-retailer through a late delivery.

We capture this cost by assuming that consumers at x make purchase decisions based on the consumer prices $p_1(x) = p + tx$ for an offline firm and $p_2(x) = \theta P$ for an online firm where p and P are the producer prices and $t > 0$ shopping cost. The distaste cost $\theta \geq 1$ can be viewed as a statistic that subsumes the various costs and benefits of purchasing from e-retailing. This cost is often supposed to be additive, $P + \theta$ (Balasubramanian, 1998; Guo and Lai, 2017). We choose instead a multiplicative cost because the distaste cost is likely to be higher when consumers buy a luxury and expensive product, such as leather goods, jewelry, and watches, than when they acquire more standardized products like books or CDs.

Note that some consumers dislike shopping and therefore choose to buy online to avoid shopping costs. In this case, $\theta < 1$, and our results remain valid under this condition. Ideally, the model would allow consumers to have different attitudes toward different types of shopping, which would require introducing a third source of heterogeneity (Colombo and Matsushima, 2020). To keep the analysis simple, we assume that θ is the same across consumers.

2.3 Shipping costs

The extant literature pays little attention to the logistics problem faced by e-retailers. We propose a solution to the vehicle routing problem and then, determine the shipping cost function of an e-retailer.

Assume that Firm 2 is an e-retailer. Using (1), the demand for variety 2 of a consumer located at x is given by

$$Q(p_{mU}, P_{mU}; x) = \begin{cases} 1 & \text{if} \quad x \geq 1 \\ \frac{L+p_{mU}+tx-\theta P_{mU}}{2L} & \text{if} \quad 0 \leq x \leq 1 \\ 0 & \text{if} \quad x \leq 0 \end{cases}, \tag{2}$$

where lower case letters, (p_{mU}, t), are used for the offline retailer and capital letters, (P_{mU}, T), for the online retailer; T is the unit shipping cost. When a variable has two subscripts, the first one specifies Firm 1's pricing policy, while the second subscript shows the pricing policy of Firm 2. For example, p_{mU} is the price of Firm 1 when it is a M-retailer and Firm 2 a U-retailer.

The total demand for Firm 2's variety is given by

$$\bar{Q}_1 = \int_0^1 Q(p_{mU}, P_{mU}; x)dx = \frac{1}{2} + \frac{p_{mU} - \theta P_{mU}}{2L} + \frac{t}{4L}. \tag{3}$$

In what follows, we build on the linear space framework to develop a simplified version of the VRP in which the e-retailer uses a single truck and bears the cost of shipping Firm 2's variety to all consumers. To determine the corresponding cost, we consider n zones $[x_{i+1}, x_i]$ where $x_i = 1-(i-1)/n$, with $i = 1, 2, ..., n$. At x_{i+1}, the last-mile principle is applied to the consumers located between x_i and x_{i+1} (Boysen et al., 2021).

At $x_1 = 1$, a light commercial vehicle [LCV] has the capacity $\bar{Q}_1$ required to meet the total demand of consumers located between 0 and 1. The LCV distributes

$$Q_1 = \frac{2Ln + 2n(p_{mU} - \theta P_{mU}) + (2n-1)t}{4Ln^2}$$

units of Firm 2's variety to the consumers located in $[1 - 1/n, 1]$. When $x_2 = 1 - 1/n$ is reached, it remains to ship the quantity

$$\bar{Q}_2 = \bar{Q}_1 - Q_1 = (1 - 1/n)\,\frac{2L + 2(p_{mU} - \theta P_{mU}) + (1 - 1/n)t}{4L}.$$

Upon arrival at x_i, the LCV's remaining load is equal to

$$\bar{Q}_i = \int_0^{1-(i-1)/n} \frac{L + p_{mU} + tx - \theta P_{mU}}{2L}dx = \left(1 - \frac{i-1}{n}\right)\frac{2L + 2(p_{mU} - \theta P_{mU}) + (1 - (i-1)/n)t}{4L},$$

which is just sufficient to supply all the consumers located between x_i and 0.

Finally, when the location $x_n = 1/n$ is reached, the LCV distributes the remaining load $\bar{Q}_n$ among consumers located in the interval $[0, 1/n]$. Since $\bar{Q}_i > \bar{Q}_{i+1}$, the occupancy rate of the LCV decreases as it gets closer to its final destination.

We now assume that the shipping cost is proportional to the LCV's load and linear in distance. Total shipping costs borne by the e-retailer are thus equal to

$$TSC(p_{mU}, P_{mU}; n) = \frac{T}{n} \sum_{i=1}^{n} \bar{Q}_i = \left(1 + \frac{1}{n}\right) \frac{T}{2} \left(\frac{1}{2} + \frac{p_{mU} - \theta P_{mU}}{2L} + \frac{(1 + 1/2n)t}{6L}\right), \qquad (4)$$

which decreases with the number n of trucks. Thus, the shipping cost borne by the online firm is minimized when $n \to \infty$:

$$TSC(p_{mU}, P_{mU}) \equiv \lim_{n \to \infty} TSC(p_{mU}, P_{mU}; n) = \frac{T}{2} \left(\frac{1}{2} + \frac{p_{mU} - \theta P_{mU}}{2L} + \frac{t}{6L}\right). \qquad (5)$$

Alternatively, we could assume that the e-retailer uses a fleet of a given number $n > 1$ of vehicles, which range from bikes to small LCV, whose global capacity is $\bar{Q}_1$. At $x_1 = 1$, the vehicle whose capacity is $\bar{Q}_1$ supplies the customers located in $[1 - 1/n, 1)$ and remains at x_2. The other $n - 1$ vehicles depart from $x_2 = 1 - 1/n$ to reach x_3. At $x_n = 1/n$, a single vehicle departs from x_n and delivers the variety to the consumers located in $[0, 1/n)$.

Since vehicle i delivers Firm 2's variety to the customers living in $[1 - i/n, 1 - (i-1)/n)$ and travels from $1 - (i-1)/n$ to $1 - i/n$, the total distance traveled by the i-th vehicle is i/n, while its load is given by

$$\int_{1-i/n}^{1-(i-1)/n} Q(p_{mU}, P_{mU}; x)dx = \frac{2L + 2(p_{mU} - \theta P_{mU}) + [2 + (1 - 2i)/n]\, t}{4nL},$$

which is smaller than the capacity Q_1 of the first vehicle. In line with the last-mile principle, the number of vehicles decreases as the distance to the last delivery point diminishes.

The total shipping cost borne by the e-retailer is then equal to

$$T \sum_{i=1}^{n} \frac{i}{n} \int_{1-i/n}^{1-(i-1)/n} Q(p_{mU}, P_{mU}; x)dx = \left(1 + \frac{1}{n}\right) \frac{T}{2} \left(\frac{1}{2} + \frac{p_{mU} - \theta P_{mU}}{2L} + \frac{(1 + 1/2n)t}{6L}\right),$$

which is equal to (4).

Which solution to choose will depend on the size of city streets and customer density because congestion and accessibility issues raise delivery costs. We treat the per-unit cost T exogenously and do not specify whether it represents a payment to a shipper or a cost borne by the e-retailer when arranging delivery itself.

First, we assume that $t > \theta T$. The value of travel time for a European car driver is estimated at €0.93 per kilometer (Wardman et al., 2016). If the consumer purchases a 5-kg package, the cost per kg·km is $t =$ €0.186. Using French data, Béziat et al. (2026) show that $T \ll t$. For a LCV with a capacity of 350kg, the driver's salary is €0.96 per km and an average load of 175kg, the cost per kg·km is equal to $T =$ €0.005. Therefore, even when consumers make multipurpose trips, we find it reasonable to assume that *the cost per kg·km for a LCV is significantly lower than that for a personal car*. This is consistent with Chintagunta et al. (2012) who find that buying online is cheaper than travelling to an offline retailer. Using the above values of t and T implies that $t > \theta T$ holds provided that $\theta < t/T = 37.2$, which is a loose constraint.

Note that the expression (5) has been derived for the case of a *M-U* duopoly. It can easily be extended to other spatial price policies by replacing p_{mU} or P_{mU} by the corresponding prices.

Second, as shown below by Proposition 1, studying the case of full market coverage implies $L > t$. Furthermore, we will see that L must exceed some threshold for a price equilibrium in pure strategies to exist in some retailing formats.

In sum, throughout the remainder of the paper, we assume the following inequalities:

$$L \geq t \geq \theta T. \tag{6}$$

All the propositions discussed below hold under (6). However, several results remain valid under weaker conditions. In the special case of goods that can be digitized, such as books, music, and softwares, we may consider that T is close to 0, while θ is close to 1.

3 Competition between offline retailers

Our benchmark setting features two conventional retailers. In this case, consumers incur travel costs to patronize conventional retailers. Accordingly, the consumer price is equal to the retailer's mill price plus the individual travel cost to the corresponding outlet: $p_1(x) = p_{1m} + tx$ and $p_2(x) = p_{2m} + t(1-x)$. Hence, the quantity that a consumer located at x buys from Firm 1 is equal to

$$q_1(p_{1m}, p_{2m}; x) = \begin{cases} 0 & \text{if} \quad x \geq 1 \\ \frac{L-(p_{1m}+tx)+p_{2m}+t(1-x)}{2L} & \text{if} \quad 0 < x < 1 \\ 1 & \text{if} \quad x \leq 0 \end{cases}, \tag{7}$$

while a symmetric expression holds for variety 2.

Integrating (7) over x, it is easy to show that firms' demands are linear and given by

$$q_1(p_{1m}, p_{2m}) = \frac{L - p_{1m} + p_{2m}}{2L}, \qquad q_2(p_{1m}, p_{2m}) = \frac{L + p_{1m} - p_{2m}}{2L}, \tag{8}$$

which shows that demands are less price-sensitive when L is larger.

Since firms' profits $\pi_1 = p_{1m}q_1(p_{1m}, p_{2m})$ and $\pi_2 = p_{2m}q_2(p_{1m}, p_{2m})$ are concave in their respective price, applying the first-order conditions yields the equilibrium prices:

$$p_{1m}^* = p_{2m}^* = L, \tag{9}$$

which increase with the degree of heterogeneity because firms' demands (8) become less elastic when L rises, thus allowing firms to charge higher prices. Since $q_1^*(1) = q_2^*(0) = (L - t)/2L \geq 0$, there is full market coverage at the equilibrium prices if $L \geq t$. In this case, no firm has an incentive to shift to partial market coverage by raising its price above L (see Appendix B).

Using (8) shows that firms' outputs are equal to 1/2, while the equilibrium profits are given by

$$\pi_m^* = \frac{L}{2}. \tag{10}$$

Summarizing, we have:

Proposition 1. *Assume a market with two conventional stores. If* $L \geq t$*, there exists a unique price equilibrium with full market coverage.*

4 Competition between offline and online retailers

In this section, we consider a mixed duopoly that involves a conventional retailer (Firm 1) and a U- or a D-retailer (Firm 2). We first determine the condition for a price equilibrium to exist and then, ask when Firm 2 finds it profitable to become an e-retailer, and which type of e-retailer.

Consumers bear location-specific shopping costs to Firm 1 whereas Firm 2 incurs location-specific delivery costs to consumers. Consumers reveal their true locations to the U-retailer as arbitrage between consumers is not desirable because they pay the same consumer price. A mill pricing firm encounters the following strategic trade-off: it is incentivized to lower its mill price to attract distant consumers, yet it has a preference for charging higher prices to the local customers. Conversely, the firm using U-pricing faces the opposite trade-off: to cover the transportation costs associated with serving remote consumers, it is inclined to set a higher uniform price; however, doing so limit its ability to capture a higher surplus from nearby customers.

4.1 *M-U* duopoly

For simplicity, we consider the case where total shipping costs are given by (5). We do not specify whether shipping costs represents a payment to an independent shipper or the cost incurred by the e-retailer when arranging delivery itself. Firms' profits are defined as follows:

$$\begin{aligned}
\Pi_{1mU}(p_{mU}, P_{mU}) &= p_{mU} \int_0^1 q(p_{mU}, P_{mU}; x)dx = p_{mU}\left(\frac{1}{2} + \frac{\theta P_{mU} - p_{mU}}{2L} - \frac{t}{4L}\right), \\
\Pi_{2mU}(p_{mU}, P_{mU}) &= P_{mU} \int_0^1 Q(p_{mU}, P_{mU}; x)dx - TSC(p_{mU}, P_{mU}) \\
&= \left(P_{mU} - \frac{T}{2}\right)\left(\frac{1}{2} + \frac{p_{mU} - \theta P_{mU}}{2L} + \frac{t}{4L}\right).
\end{aligned}$$

It follows from (3) that $\Pi_{2mU}(p_{mU}, P_{mU}) = (P_{mU} - T/2)\bar{Q}_1$. Therefore, $P_{mU} - T/2$ may be interpreted as a markup, reflecting a marginal cost that is lower than T.

As both profit functions are concave in their own price, applying the first-order conditions yield the candidate equilibrium prices of the offline and online firms:

$$p^*_{mU} = L - \frac{t - \theta T}{6}, \qquad P^*_{mU} = \frac{L}{\theta} + \frac{t + 2\theta T}{6\theta}, \tag{11}$$

which are positive because $L > t$. The first terms account for the degree of heterogeneity, while the second terms are equal to 0 in the absence of transportation costs.

As an e-retailer pays the shipping cost, it never chooses a delivered price smaller than T. Therefore, for the prices (11) to be a Nash equilibrium of the game between the offline and online retailers, the following two conditions must be satisfied:

$$p^*_{mU} > 0 \Leftrightarrow 6L > t - \theta T \Leftrightarrow \theta T > 0 > t - 6L,$$

and

$$P^*_{mU} > T \Leftrightarrow 6L > 4\theta T - t \Leftrightarrow \theta T < \frac{6L + t}{4}.$$

Furthermore, the condition for the two firms to serve the whole market at the prices (11) is given by

$$-L < -p^*_{mU} - tx + \theta P^*_{mU} < L.$$

Substituting $x = 0$ and $x = 1$ leads to the following inequalities:

$$-p^*_{mU} + \theta P^*_{mU} < L \Leftrightarrow 6L > 2t + \theta T \Leftrightarrow \theta T < 6L - 2t,$$

$$-L < -p^*_{mU} - t + \theta P^*_{mU} \Leftrightarrow 6L > 4t - \theta T \Leftrightarrow \theta T > 0 > 4t - 6L.$$

It follows from (6) that the above four conditions are always satisfied.[2] We prove in Appendix B that Firm 1 does not find it profitable to raise its price such that $x_{\max} < 1$.

[2] The most demanding condition is given by $\theta T < 6L - 2t$, which is sufficient for a price equilibrium to exist.

Summarizing, we have: *when* (6) *holds, there exists a unique price equilibrium in a M-U duopoly, which is given by given by* (11). Note that the condition $L > t/3 + \theta T/6$, which is less stringent than (6), is sufficient for a price equilibrium to exist because $p^*_{mU} > 0$.

Firm 2's producer price

$$P^*_{mU} - T(1-x) = \frac{L}{\theta} + \frac{t}{6\theta} - T\left(\frac{2}{3} - x\right)$$

increases over $[0,1]$ with the distance to the supplied consumer, but the pass-through is incomplete. As the distance from $x = 0$ increases, the consumer price of the offline retailer $(p^*_{mU} + tx)$ increases with x while that of the online retailer remains constant (θP^*_{mU}). The former is smaller than the latter if and only if $x < 1/3 + \theta T/6t < 1/2$. Beyond this location, Firm 2's consumer price is lower than that of its competitor.

At the equilibrium prices (11), the quantities sold at x are as follows:

$$q^*_{mU}(x) = \frac{1}{2} + \frac{\theta T + 2t(1-3x)}{12L}, \qquad Q^*_{mU}(x) = 1 - q^*_{mU}(x), \tag{12}$$

which shows that $q^*_{mU}(x)$ decreases with x while $Q^*_{mU}(x)$ increases with x. In other words, the consumption of a variety decreases as the distance to its supplier rises (Lieber and Syverson, 2012). On the other hand, the markup of the offline retailer is constant across space, whereas the markup of the online retailer $P^*_{mU} - T(1-x)$ decreases with the distance to the fulfillment center. Dolfen et al. (2023) observe that the main share of the gains generated by e-commerce in the U.S. stems from substituting to retailers available online but not locally. This concurs with (12) which shows that when consumers have better access to a conventional retailer ($t \downarrow$), they substitute away from the e-retailer. By contrast, a more efficient shipping strategy ($T \downarrow$) implies that consumers will buy more from the online channel.

Furthermore, since $t > \theta T$, the online firm's sales are larger than those of the offline firm:

$$0 < \int_0^1 q^*_{mU}(x)dx = \frac{1}{2} - \frac{t - \theta T}{12L} < \frac{1}{2} < \int_0^1 Q^*_{mU}(x)dx = \frac{1}{2} + \frac{t - \theta T}{12L},$$

which suggests that on average e-retailing is more competitive than conventional retailing.

As shown by (9) and (11), we also have $p^*_{mU} < p^*_m$. In other words, Firm 1 must decrease its mill price when Firms 2 becomes an e-retailer. Since its output also declines, *Firm* 2*'s shift to e-retailing proves highly detrimental to Firm* 1. In other words, by shifting from M- to U-pricing, Firm 2 adopts a more aggressive stance. On the other hand, profits made by Firm 2 when it becomes an e-retailer need not be larger than the profits earned when it is a conventional retailer. We will see below when Firm 2 finds it profitable to adopt the online channel.

In addition, the price gap $p_m^* - p_{mU}^* > 0$ widens as shopping costs increase or shipping costs decrease. For example, a higher density of population is likely to permit better access to conventional retailers in larger cities than in rural areas. This in turn should foster the gradual disappearance of conventional retailing in small and medium-size cities. This concurs with Fan et al. (2018) who find that the average welfare gains from e-commerce are 1.11% for Chinese cities in the largest population quintile but 2.01% for those in the lowest quintile.

As shown by (11), higher shopping costs ($t \uparrow$) lead the conventional retailer to decrease its price because its demand is reduced. By contrast, higher shipping costs ($T \uparrow$) lead the e-retailer to raise its delivered price because its transportation cost rises. Shopping and shipping costs thus have different impacts on market prices, confirming once more that the two retail formats differ in nature. However, both equilibrium prices increase when consumers have more heterogeneous tastes.

Note also that a higher distaste cost θ allows the conventional retailer to raise its price because the e-retailer becomes less competitive. Consequently, the e-retailer must lower its price (P_{mU}^* decreases with θ). Therefore, *the costs T and θ have different impacts on the online firm's price*, highlighting the need to differentiate between them.

Substituting the equilibrium prices (11) into firms' profits yields

$$\pi_{mU}^* = \frac{1}{72L}(6L + \theta T - t)^2 > 0, \qquad \Pi_{mU}^* = \frac{1}{72\theta L}(6L + t - \theta T)^2 + \frac{tT}{24L} > 0. \tag{13}$$

Observe that the offline retailer's profits decrease with shopping costs ($t \uparrow$), but increase with shipping costs ($T \uparrow$), whereas the opposite holds for the e-retailer. Next, a higher distaste cost ($\theta \uparrow$) is always detrimental to the online retailer but is beneficial to the offline retailer. In sum, by relaxing competition, a retailer's higher transaction cost is always detrimental to this retailer, which agrees with Chintagunta et al. (2012). Moreover, when consumers are more heterogeneous, both types of firms charge higher prices because firms have more market power.

Last, the e-retailer does not drive its rival out of business because products are sufficiently differentiated for $q_{mU}^*(0) > 0$, even when $T \approx 0$. In other words, *product differentiation protects the conventional retailer*. However, the e-retailer earns higher profits than the conventional one.

4.2 Does e-commerce make consumers better-off?

Are consumers better-off when Firm 2 adopts U-retailing, or do they prefer to keep conventional retailers? Using Theorems 3.3 and 3.4 of Anderson et al. (1992) implies that the linear probability model describes a population of heterogeneous consumers who can be represented by a single consumer whose indirect utility is given by

$$V(Y, \mathbf{p}(x)) = \begin{cases} Y + \frac{-2L(p_1(x)+p_2(x))+(p_1(x)-p_2(x))^2}{4L} & \text{if } q_1(\mathbf{p}(x)) + q_2(\mathbf{p}(x)) = 1 \\ -\infty & \text{otherwise} \end{cases} . \tag{14}$$

Since individual preferences are quasi-linear, the expression

$$S = \int_0^1 V(Y, \mathbf{p}(x))dx$$

measures the consumer surplus.

(i) In the M-M duopoly, the individual utility of a consumer located at x is given by

$$V_m(x) = Y + \frac{t^2 (1-2x)^2 - 2L(2L+t)}{4L},$$

which is convex and symmetric around $x = 1/2$. Hence, $V_m(x)$ decreases over $[0, 1/2)$ and increases over $(1/2, 1]$.

(ii) In the M-U duopoly, we have

$$V_{mU}(x) = Y - L - \frac{2tx + \theta T}{4} + \frac{[t(2-6x) + \theta T]^2}{144L},$$

which decreases over $[0, 1]$ because the consumer price P^*_{2mU} is constant while the consumer price $p^*_m + tx$ keeps increasing with x.

It remains to determine the sign of the gap $V_m(x) - V_{mU}(x)$. Setting $X \equiv \theta T$, we have:

$$V_m(x) - V_{mU}(x) = \frac{108t^2x^2 - 120t^2x + 12txX - 72Lt(1-x) + 32t^2 + 36LX - 4tX - X^2}{144L},$$

which is convex in x, negative at $x = 0$ and positive at $x = 1$. It is readily verified that $V_m(x) - V_{mU}(x)$ has unique root in $(0, 1)$ given by

$$\bar{x} = \frac{1}{18t} \left(2\sqrt{24L(t-X) + 9L^2 + (t-X)^2} - (6L - 10t + X) \right).$$

Therefore, $V_m(x) - V_{mU}(x)$ is negative for $x < \bar{x}$ and positive otherwise. This finding shows that *the more competitive retail format depends on the location of consumers relative to firms*, thus confirming the idea that spatial proximity still matters in consumers' welfare. As a result, no single retail format is universally preferred across the entire population. This divergence is intuitive—consumers' preferred format largely depends on their geographic proximity to retailers, consistent with the empirical findings of Forman et al. (2009). For example, residents of rural areas who have no digital barriers are likely to benefit significantly from e-commerce (Couture et al., 2021)

Let us now compare the consumer surpluses given by

$$S_m = Y - L - \frac{t}{2} + \frac{t^2}{12L}, \qquad S_{mU} = Y - L + \frac{t + \theta T}{4} - \frac{3t^2 + (t - \theta T)^2}{144L},$$

which are obtained by integrating $V_m(x)$ and $V_{mU}(x)$. Setting $X = \theta T$ yields

$$S_m - S_{mU} \equiv I(X) = \frac{-X^2 - 36Lt + 8t^2 + 36LX + 2tX}{144L},$$

which is concave and maximized at $\bar{X} = 18L + t > t$. Since $I(0) < 0$ and $I(t) > 0$, $I(X) = 0$ at

$$\tilde{X} = 18L + t - 3\sqrt{36L^2 + t^2} < t.$$

Consequently, $S_m < S_{mU}$ when $\theta T < \tilde{X}$, whereas $S_m > S_{mU}$ when $\theta T > \tilde{X}$. Stated differently, consumers as a whole prefer *M-D* to *M-M* when the distaste cost and/or the shipping cost are sufficiently small for $\theta T < \tilde{X}$ to hold. Otherwise, the consumer surplus is higher at *M-M*.

(iii) Since both firms charge the same delivered price in the *U-U* duopoly, the individual utility of a consumer located at x is constant and equal to the consumer surplus:

$$S_U = V_U(x) = Y + \frac{-2L(\theta P_{1U} + \theta P_{2U}) + (\theta P_{1U} - \theta P_{2U})^2}{4L} = Y - L - \frac{\theta T}{2}. \tag{15}$$

Therefore,

$$S_m - S_U = \frac{t^2 - 6L(t - \theta T)}{12L}.$$

Clearly, consumer surplus is higher in an online duopoly than in an offline duopoly if and only if $\theta T < t - t^2/6L$, which holds when consumers are very heterogeneous.

Since

$$S_{mU} - S_U \equiv J(X) = \frac{X^2 + 2(18L - t)X - 4t(9L - t)}{144L}$$

is increasing, negative at $X = 0$ and positive at $X = t$, there exists a unique value

$$\hat{X} = t - 18L + \sqrt{324L^2 - 3t^2} \in (0, t)$$

such that $J(\hat{X}) = 0$. Accordingly, $S_{mU} < S_U$ if and only if $\theta T < \hat{X}$, while $S_{mU} > S_U$ otherwise.

To sum up, *there is no retail format that is most preferred by consumers as a whole.* However, when the distaste and/or shipping costs are sufficiently high, *M-M* is both the market equilibrium and the most preferred format pattern. This explains the limited development of mail-order selling prior to the emergence of the new ICTs.

4.3 When does an offline retailer become a U-retailer?

Does profit maximization incentivize Firm 2 to shift from M- to U-pricing, and if so under which conditions? Using (10) and (13), we have:

$$\Pi^*_{mU} - \pi^*_m = F(\theta, T) \equiv \frac{\theta^2 T^2 - 12L\left(3(\theta - 1)L - (t - \theta T)\right) + t^2 + \theta T t}{72\theta L}.$$

Observe that $F(1,T)>0$. Differentiating $F(\theta,T)$ with respect to θ shows that $F(\theta,T)$ decreases with θ over the interval $(1,\theta_{\min})$ where $\theta_{\min}=(6L+t)/T>t/T>1$ is the minimizer of convex function $F(\theta,T)$. Since $F(\theta_{\min},T)<0$ because $L\geq t$, there exists a unique value $\theta_U\in(1,\theta_{\min})$ such that $F(\theta_U,T)=0$, $F(\theta,T)>0$ for $\theta\in(1,\theta_U)$, and $F(\theta,T)<0$ for $\theta>\theta_U$.

To sum up, we have:

Proposition 2. *Assume that Firm* 1 *is a M-retailer while Firm* 2 *chooses between M and U. When* (6) *holds, there exists a unique value* $\theta_U>1$ *such that Firm* 2 *chooses to become a U-retailer if and only if* $\theta<\min\{\theta_U,t/T\}$.

Note that the above proposition can be restated as follows. Since there exists a unique positive value T_U that solves $F(\theta,T)=0$, Proposition 2 also holds when shipping costs are sufficiently low, that is, $T<T_U$. Specifically, lower shipping costs, driven by a more efficient delivery system, make the emergence of a U-retailer more likely.

Proposition 2 has noteworthy implications. First, this proposition may explain why luxury and expensive goods, such as fashion clothes, jewelry, and watches, for which consumers have strong idiosyncratic preferences are still provided by conventional shops. By contrast, more standardized goods, such as books, CDs, appliances or inexpensive clothes, involve low distaste costs. It is no surprise, therefore, that Amazon started by selling such products. Furthermore, the adoption of new information technologies that allow for a better perception of goods by consumers favors e-retailing at the expense of conventional retailers.

Second, F tends to infinity when $L\to 0$. As F decreases with L and tends to $-\infty$ when $L\to\infty$, $\hat{L}>0$ exists such that $\Pi^*_{mU}-\pi^*_m>0$ if and only if $L<\hat{L}$. In other words, if the product is very differentiated, no retailer becomes a U-retailer. Luxury goods, for instance, are typically highly differentiated, so offline firms selling such products have less incentive to change their format because they can already charge high prices (9). Likewise, since larger cities tend to host more heterogeneous populations, *an offline retailer is more likely to remain in business in larger cities than in smaller ones*, consistent with empirical evidence (see, e.g. Delage et al., 2020).

Last, an offline retailer does not change its format when distaste costs are sufficiently high. Hence, we have:

Proposition 3. *The retail industry involves two conventional retailers when* $\theta>\min\{\theta_U,t/T\}$.

In other words, conventional retailers are more likely to remain in business when distaste or shipping costs are high.

The following question suggests itself: what do Propositions 2 and 3 become when D-pricing is added to Firm 2's strategy set. Following the same approach as in the above, we show in Proposition

A in Appendix C that there exists a threshold $\theta_D > \theta_U$ such that both propositions continue to hold when θ_U is replaced by θ_D. In other words, the above analysis remains valid over a larger interval of distaste costs.

4.4 *U-U* duopoly

When one firm chooses to become a U-retailer, it is natural to anticipate the analysis in the next section by asking whether the other firm is incentivized to make the same choice. Let P_{iU} be the producer price selected by Firm $i = 1, 2$ and θP_{iU} the corresponding consumer price.

When both firms adopt U-pricing, their profit functions are as follows:

$$\begin{aligned}\Pi_{1U} &= \int_0^1 P_{1U} \frac{L + \theta P_{2U} - \theta P_{1U}}{2L} dx - TSC(P_{1U}, P_{2U}) = \left(P_{1U} - \frac{T}{2}\right) \frac{L + \theta P_{2U} - \theta P_{1U}}{2L}, \\ \Pi_{2U} &= \left(P_{2U} - \frac{T}{2}\right) \frac{L + \theta P_{1U} - \theta P_{2U}}{2L}.\end{aligned}$$

Since each profit function is concave in its own price, the first-order conditions yield the common candidate equilibrium price:

$$P^*_{1U} = P^*_{2U} = \frac{L}{\theta} + \frac{T}{2}. \tag{16}$$

Furthermore, (6) implies that P^*_{iU} is larger than T while Firm 1 does not find it profitable to supply a submarket by setting a price lower than T (see Appendix C). Hence, we have: *in a U-U duopoly, there exists a unique price equilibrium given by* (16).

Observe that L must be large enough for a price equilibrium to exist under full market coverage in an *U-U* duopoly ($L > \theta T/2$). This should not come as a surprise as U-pricing has the nature of an aggressive pricing strategy that leads to the non-existence of an equilibrium in pure strategies when consumers are differentiated along the sole spatial dimension (Kats and Thisse, 1993).

The pass-through of shipping cost is positive but smaller than 1. The equilibrium uniform delivered price increases with shipping costs, but decreases with the distaste cost. This confirms that shipping and distaste costs play different roles.

Finally, Firm 2 must decrease its delivered price when Firm 1 shifts to the U-channel because the market becomes more competitive ($P^*_{2U} < P^*_{mU}$).

Firms' equilibrium profits are equal and given by

$$\Pi^*_U = \frac{L}{2\theta}, \tag{17}$$

which decreases with the distaste cost.

We now come to our main question. Firm 1 prefers U-pricing to M-pricing when

$$\pi^*_{mU} = \frac{1}{72L} (6L + \theta T - t)^2 < \Pi^*_U = \frac{L}{2\theta}.$$

which holds if and only if

$$T < \frac{t - 6(1 - 1/\sqrt{\theta})L}{\theta}. \tag{18}$$

Comparing (10) and (17) shows that conventional stores always earn higher profits ($L/2$) than the U-retailers ($L/2\theta$). In other words, Firms 1 and 2 face a Prisoner's Dilemma. This result is in line with Thisse and Vives (1988) and Bernstein et al. (2008), but price policies differ.

Summarizing leads to the following proposition.

Proposition 4. *Assume that both firms choose between M and U. There exists a subgame perfect Nash equilibrium in which Firms* 1 *and* 2 *choose to be U-retailers if and only if* (18) *holds. Furthermore, firms face a Prisoner's Dilemma.*

Alternatively, since π^*_{mU} increases with θ whereas Π^*_U decreases, the two curves intersects once because $\pi^*_{mU} < \Pi^*_U$ at $\theta = 1$ and $\pi^*_{mU} > \Pi^*_U$ at $\theta = t/T$. Therefore, $\theta_{UU} > 1$ exists such that $\Pi^*_U > \pi^*_{mU}$ if and only if $\theta < \theta_{UU}$.

Since the condition (18) always holds in the case of digitized goods ($T = 0$ and $\theta \simeq 1$), Proposition 4 helps explain why bookstores have nearly disappeared from many cities when T is sufficiently small.

Equally important, when product varieties are sufficiently differentiated, condition (18) no longer holds. Consequently, when the degree of product differentiation is high, a M-retailer competing with a U-retailer remains in business. This is likely to be so in a larger city where consumers are often more heterogeneous than in a smaller one.

5 Competition between e-retailers

Assume that the distaste and shipping cost are sufficiently small for both firms to be e-retailers. Competition between e-retailers is described by a two-stage game in which firms first choose between U- and D-pricing and then select their corresponding prices. As usual, the game is solved by backward induction.

When a variable has three subscripts, the first identifies the firm, the second specifies Firm 1's pricing policy, and the third indicates Firm 2's pricing policy. For example, P_{1UD} is the price of Firm 1 when it is a U-retailer and Firm 2 a D-retailer.

5.1 *D-D* duopoly

Consider a duopoly in which both firms choose simultaneously their price schedules $P_{1D}(x)$ and $P_{2D}(x)$. Firm 1's profit function is defined as follows:

$$\Pi_{1D} = \int_0^1 (P_{1D}(x) - Tx)\frac{L - \theta P_{1D}(x) + \theta P_{2D}(x)}{2L}dx,$$

while a similar expression holds for Firm 2. Profit maximization yields the equilibrium price schedules:

$$P_{1D}^*(x) = \frac{L}{\theta} + \frac{T(1+x)}{3}, \qquad P_{2D}^*(x) = \frac{L}{\theta} + \frac{T(2-x)}{3}, \tag{19}$$

which are symmetric around $x = 1/2$. Furthermore, we have

$$P_{1D}^*(x) - Tx = \frac{L}{\theta} + \frac{1}{3}T(1-2x) > 0, \qquad P_{2D}^*(x) - T(1-x) > 0,$$

which hold because of (6). More specifically, an equilibrium exists if and only if $L > \theta T/3$.

The consumer price $P_{iD}^*(x)$ increases with the distance from the firm, whereas it decreases when the good is homogeneous (Hoover, 1937). This is because product differentiation relaxes price competition and allows Firm i to charge higher prices to more distant consumers. By contrast, Firm i's producer price decreases with the distance from the firm as it absorbs 2/3 of the shipping costs. Note also that there is no arbitrage among customers residing in different locations because $|P_{1D}^*(x) - P_{1D}^*(y)| = T|x-y|/3$ is smaller than the cost of shipping the good between x and y.

We may conclude as follows: *in a D-D duopoly, there exists a unique price equilibrium given by* (19).

Comparing (16) and (19) shows that

$$P_{1D}^*(x) - P_{1U}^* = \frac{T}{6}(2x-1) < 0 \quad \text{iff } x < 1/2.$$

Hence, consumers located on the left of the market center pay a higher price to Firm 1 under *U-U* than under *D-D*, whereas those located on the right pay a lower price. Thus, *adopting personalized rather than uniform pricing does not necessarily result in an overall reduction in prices*, thus confirming Ali et al. (2023).

Substituting $P_{1D}^*(x)$ and $P_{2D}^*(x)$ into Π_{1D} and Π_{2D}, it is readily verified that the profits made by Firm 1 and Firm 2 are equal and given by

$$\Pi_D^* = \frac{L}{2\theta} + \frac{\theta T^2}{54L}. \tag{20}$$

Furthermore, it follows from (17) and (20) that *both firms earn higher profits under D-D than under U-U* ($\Pi_D^* > \Pi_U^*$). While *D-D* is usually regarded as a competitive format, *U-U* leads to an even more competitive outcome for the two retailers. This is because they compete for dispersed consumers through a single delivered price rather than through pointwise price schedules. Reducing shipping costs leads to lower profits for the D-retailers because price competition is tougher at each location.

Let us take a pause and consider the case of an homogeneous good ($L = 0$). It is well known that there exists a unique price equilibrium in which each firm sets a delivered price at x equal to its competitor's delivery cost, and thus each firm supplies only half of the market (Hoover, 1937; Lederer and Hurter, 1986). In contrast, when varieties are differentiated and consumers value diversity, firms do not engage in price wars at each location, which allows them to charge prices that rises with distance. It is, therefore, natural to ask how to reconcile these different results.

In Hoover (1937), only one half of the market is supplied by each firm. In other words, partial market coverage prevails. Consequently, we must determine the equilibrium prices in a *D-D* duopoly and partial coverage to make a comparison. We have seen that the condition $L > \theta T/3$ is necessary and sufficient for full market converge. Therefore, for partial market coverage to arise, it must be that $L < \theta T/3$, which covers the case of homogeneous consumers ($L = 0$).

Assume that Firm 1 supplies the segment $[0, z)$ where $z < 1$ is such that $P_{1D}(z) = Tz$, for otherwise Firm 1 would ship its output beyond z or would stop delivering before z. Hence,

$$z = \frac{3L}{2\theta T} + \frac{1}{2},$$

which is equal to 1 when $L = \theta T/3$ and to 1/2 when $L = 0$. For the same reason, we have $P_{2D}(y) = T(1 - y)$ whose solution is

$$y = \frac{1}{2} - \frac{3L}{2\theta T}.$$

Consequently, the equilibrium price schedules are still given by (19) over (y, z), whereas $Q_1^*(x) = 1$ over $(0, y)$ and $Q_1^*(x) = 0$ over $(z, 1]$. As L decreases below $\theta T/3$, the area of contention (y, z) shrinks and vanishes when $L = 0$. Over the remaining two intervals, firms compete under different costs, so that each submarket is supplied by the lower-cost firm at a price equal to the delivery cost of its competitor, $P_{1D}^*(x) = \max\{Tx, T(1-x)\}$ and $P_{2D}^*(x) = \max\{Tx, T(1-x)\}$, which decrease with the distance to the provider.

5.2 *U-D* duopoly

Consider now a duopoly in which Firm 1 sets a uniform delivered price P_{1UD} while Firm 2 chooses a price schedule $P_{2UD}(x)$ defined over $[0, 1]$. Then, Firm 1's profits at x are given by

$$\Pi_{1UD}(P_{1UD}, P_{2UD}(x)) = \frac{L - \theta P_{1UD} + \theta P_{2UD}(x)}{2L}(P_{1UD} - Tx).$$

Setting

$$\bar{P}_{2UD} = \int_0^1 P_{2UD}(x)dx, \qquad \tilde{P}_{2UD} = \int_0^1 xP_{2UD}(x)dx,$$

we obtain:

$$\begin{aligned}\Pi_{1UD} &= \int_0^1 \Pi_{1UD}(P_{1UD}, P_{2UD}(x))dx \\ &= \frac{L - \theta P_{1UD}}{2L}\left(P_{1UD} - \frac{T}{2}\right) + \frac{\theta}{2L}\left(P_{1UD}\bar{P}_{2UD} - T\tilde{P}_{2UD}\right),\end{aligned}$$

which is concave in P_{1UD}.

Firm 2's profits at x are given by

$$\Pi_{2UD}(P_{1UD}; x) = \frac{L - \theta P_{2UD}(x) + \theta P_{1UD}}{2L}(P_{2UD}(x) - T(1-x)),$$

which is concave in $P_{2UD}(x)$.

Differentiating Π_{1UD} with respect to P_{1UD} yields Firm 1's best reply:

$$P^*_{1UD}(\bar{P}_{2UD}) = \frac{2L + \theta T + 2\theta \bar{P}_{2UD}}{4\theta}, \tag{21}$$

while differentiating $\Pi_{2UD}(P_{1UD}, P_{2UD}(x))$ with respect to $P_{2UD}(x)$ yields Firm 2's best reply at x:

$$P^*_{2UD}(P_{1UD}; x) = \frac{L + \theta T(1-x) + \theta P_{1UD}}{2\theta}. \tag{22}$$

Plugging

$$\bar{P}_{2UD} = \int_0^1 P^*_{2UD}(P_{1UD}; x)dx = \frac{2L + \theta T + 2\theta P_{1UD}}{4\theta},$$

in (21) and solving for P_{1UD} yields the equilibrium price of the U-retailer:

$$P^*_{1UD} = \frac{L}{\theta} + \frac{T}{2}. \tag{23}$$

Substituting P^*_{1UD} in (22), we obtain the equilibrium price at x of the D-retailer:

$$P^*_{2UD}(x) = \frac{L}{\theta} + \frac{(3-2x)T}{4}, \tag{24}$$

which increases with the distance to Firm 2 with a slope $T/2$. In other words, Firm 2 absorbs half of the additional transportation cost it bears to supply a more distant consumer, which yields a positive pass-on.

It remains to check that there is full market coverage. First, prices must be larger than shipping costs for both firms. It is readily verified that both $P^*_{1UD} - Tx > 0$ and $P^*_{2UD}(x) - T(1-x) > 0$ hold for all x.

Substituting P^*_{1UD} and $P^*_{2UD}(x)$ into market demand functions yields

$$Q^*_{1UD}(x) = \frac{L}{2} + \frac{\theta T(1-2x)}{8L} > 0, \qquad Q^*_{2UD}(x) = \frac{1}{2} - \frac{\theta T(1-2x)}{8L} > 0,$$

which are both positive at each location x if and only if $L > \theta T/4$. Hence, at P^*_{1UD} and $P^*_{2UD}(x)$, there is full market coverage if and only if $L > \theta T/4$.

Furthermore, comparing $P^*_{1D}(x)$ and P^*_{1UD} yields

$$P^*_{1D}(x) - P^*_{1UD} = \frac{T\,(2x-1)}{6}.$$

Since $P^*_{1D}(x)$ increases with x, the higher proximity to consumers in $[0, 1/2)$ makes D-pricing more competitive than U-pricing. However, over $(1/2, 1)$, as x increases Firm 1 charges an increasing price $P^*_{1D}(x)$ that exceeds P^*_{1UD} when $x > 1/2$.

Moreover, as

$$P^*_{2D}(x) - P^*_{2UD}(x) = \frac{T\,(2x-1)}{12},$$

under *D-D* Firm 2 charges each consumer a personalized price lower than the price it would set under *U-D* if and only if $x < 1/2$.

In the special case where $T = 0$, $P^*_{1D}(x)$ and P^*_{1UD} are equal to L/θ for all x while $P^*_{2D}(x) = P^*_{2UD}(x)$ for all x. That is, the difference between U- and D-pricing becomes immaterial when shipping costs are not taken into account.

To sum up, under *D-D* both firms set lower prices than under *U-D* over the interval $[0, 1/2)$. By contrast, both firms choose higher prices over $(1/2, 1]$ under *D-D*. Hence, *D-D* need not be more competitive than *U-D*.

Substituting P^*_{1UD} and $P^*_{2UD}(x)$ into $\Pi_{1UD}(x)$ and $\Pi_{2UD}(x)$ and integrating the resulting expressions over x yields the equilibrium profits

$$\Pi^*_{1UD} = \frac{L}{2\theta} + \frac{\theta T^2}{48L} > \Pi^*_{2UD} = \frac{L}{2\theta} + \frac{\theta T^2}{96L}, \tag{25}$$

which both increase with the shipping cost T and with L because price competition is relaxed.

5.3 The strategic choice of a digital format

The payoff matrix of the first-stage game is given by

$$\begin{array}{c|cc} & D & U \\ \hline D & (\Pi_D^*, \Pi_D^*) & (\Pi_{1DU}^*, \Pi_{2DU}^*) \\ U & (\Pi_{1UD}^*, \Pi_{2UD}^*) & (\Pi_U^*, \Pi_U^*) \end{array} \equiv \begin{array}{c|cc} & D & U \\ \hline D & (\gamma, \gamma) & (\beta, \delta) \\ U & (\delta, \beta) & (\alpha, \alpha) \end{array}.$$

By symmetry, we have $\Pi_{1DU}^* = \Pi_{2UD}^*$ and $\Pi_{1UD}^* = \Pi_{2DU}^*$.

Using (17), (20) and (25) shows that

$$\Pi_U^* < \Pi_{2UD}^* < \Pi_D^* < \Pi_{1UD}^*, \tag{26}$$

i.e., $\alpha < \beta < \gamma < \delta$. Hence, both firms earn higher profits under personalized delivered prices than under uniform delivered prices ($\Pi_D^* > \Pi_U^*$). However, this comparison, which occupies center-stage in the literature on e-retailing, disregards the possibility for different firms to adopt different spatial price policies, such as *U-D*.

It follows from (26) that the price policy game has *two asymmetric pure strategy equilibria given by U-D and D-U*. Firms do not earn the same profits at a Nash equilibrium: the firm choosing U makes a higher profit (δ) than the firm choosing D (β). The price policy game thus has the structure of the *game of chicken*: if one firm has the opportunity of moving before the other, it can secure the highest profit δ by choosing U, whereas its rival is worse-off if it deviates from D since $\beta > \alpha$. Each firm is incentivized to play U, but if both do so, they end up with the worst outcome (α).

To be complete, we consider the Nash equilibrium in mixed strategies. We assign Firm 1 the probability $\mathbf{p}$ of playing D-pricing and assign Firm 2 the probability $\mathbf{q}$ of playing the same strategy. The mixed strategy equilibrium is given by

$$\mathbf{p} = \mathbf{q} = \frac{\beta - \alpha}{\delta + \beta - \alpha - \gamma},$$

so that the expected profits are equal to

$$E(\text{profit}) = \frac{\beta\delta - \alpha\gamma}{\delta + \beta - \gamma - \alpha}.$$

Since $\alpha < \beta < E(\text{profit}) < \gamma < \delta$, the expected profits are between the equilibrium profits of the U-retailer and those made by the D-retailer at a Nash equilibrium in pure strategies. However, this equilibrium is a knife-edge outcome as adopting a maximin behavior implies that each retailer chooses D and receives $\gamma > E(\text{profit})$.

The following proposition comprises a summary.

Proposition 5. *The price policy game between e-retailers has two pure strategy subgame perfect Nash equilibria in which one retailer chooses U whereas its competitor chooses D. Furthermore, the U-firm earns higher profits than the D-firm.*

Thus, even when both firms adopt e-retailing, they strategically avoid selecting the same channel. That is, firms soften intra-channel competition by differentiating their spatial price policies. In doing so, they maximize joint profits.

Proposition 5 contrasts with the analysis of Thisse and Vives (1988) in which D is a dominant strategy. The discrepancy in results stems from the following difference: here, the price subgame is simultaneous, while in Thisse and Vives (1988), this subgame is sequential and the leader selects a uniform mill price. Equally important, consumers are heterogeneous here rather than homogeneous in those papers.

In the *U-D* duopoly, U-pricing implies a form of commitment, as the U-retailer must charge the same delivered price regardless of consumers' locations. By contrast, the D-retailer retains pointwise flexibility through its choice of a specific price at each location. This asymmetry has led several authors to assume that the U-retailer acts as the leader and the D-retailer as the follower of a two-stage price subgame, which implies that market competition is now described by a three-stage game (Thisse and Vives, 1988; Choe et al., 2018; Julien et al., 2023). Clearly, the equilibrium profits Π^*_{1UD} and Π^*_{2UD} change. This gives rise to a new payoff matrix for the price policy game, which now takes the form of a Hero Game, in which both retailers are better off at each of the two Nash equilibria (Rapoport, 1967). The equilibrium of the price policy game is the same as in the case where retailers move simultaneously in the price subgame.

To sum up, *even though products are differentiated, one retailer always chooses uniform delivered pricing to relax competition when its rival is a D-retailer.*[3]

5.4 Which pattern of e-retailers is preferred by consumers?

In what follows, we first determine consumer welfare under the following configurations of formats: *D-D*, *U-D* and *U-U*. We then consider the corresponding consumer surplus.

(i) Using (14), in the *D-D* duopoly the utility of a consumer located at x is given by

$$V_D(x) = Y - \frac{\theta P_{1D}(x) + \theta P_{2D}(x)}{2} + \frac{[\theta P_{1D}(x) - \theta P_{2D}(x)]^2}{4L}.$$

Substituting $P^*_{1D}(x)$ and $P^*_{2D}(x)$ given by (19) into this expression, we obtain:

$$V^*_D(x) = Y - L - \frac{\theta T}{2} + \frac{(1-2x)^2\,\theta^2 T^2}{36L}, \tag{27}$$

[3]The proof may be obtained from the authors upon request.

which decreases with x over $(0, 1/2)$, increases over $(1/2, 1)$ and is symmetric around $x = 1/2$. Hence, the worst-off consumers are located at the market center, whereas the consumers at the market endpoints reach the highest utility.

(ii) In the *U-D* duopoly, individual utility at x is

$$V_{UD}(x) = Y - \frac{\theta P_{1UD} + \theta P_{2UD}(x)}{2} + \frac{(\theta P_{1UD} - \theta P_{2UD}(x))^2}{4L}.$$

Substituting P^*_{1UD} and $P^*_{2UD}(x)$ given by (23) and (24) into $V_{UD}(x)$ yields

$$V^*_{UD}(x) = Y - L + \frac{T^2\theta^2 (2x-1)^2 + 8LT\theta (2x-5)}{64L}. \tag{28}$$

Since $V^*_{UD}(x)$ is convex and minimized at $x = -(4L - \theta T)/2\theta T < 0$, the utility level increases with the distance to Firm 1. In other words, consumers get better-off as they get closer to Firm 2. This is because consumers pay the same price to Firm 1 and a lower price to Firm 2 as x increases.

Using (15), (27) and (28) shows that

$$\begin{aligned} V^*_D(x) &> V^*_U > V^*_{UD}(x) \Leftrightarrow x < 1/2, \\ V^*_{UD}(x) &> V^*_D(x) > V^*_U \Leftrightarrow x > 1/2. \end{aligned}$$

In other words, consumers close to Firm 1 prefer *D-D* whereas the equilibrium *U-D* is their worst outcome. By contrast, consumers close to Firm 2 prefer the equilibrium *U-D* to the other two pricing formats. This is because both firms set lower prices under *D-D* (resp., *U-D*) over the interval $[0, 1/2)$ (resp., $(1/2, 1]$). Thus, as observed by Forman et al. (2009), the benefit of a retailing format system depends on where consumers live. Note, however, that $V^*_D(x) > V^*_U$ for all x because $P^*_{1D}(x) - P^*_U < 0$ over $[0, 1/2)$ and $P^*_{2D}(x) - P^*_U < 0$ over $(1/2, 1]$. Since the negative price gap matters more to consumers in $[0, 1/2)$ because they buy more from Firm 1 than from Firm 2, it must be that $V^*_D(x) \geq V^*_U$ for $x \in [0, 1/2)$. A similar argument applies to consumers located in $(1/2, 1]$.

(iii) Consider now the consumer surplus. In the *D-D* duopoly, we have

$$S_D = \int_0^1 V^*_D(x)dx = Y - L - \frac{\theta T}{2} + \frac{\theta^2 T^2}{108L},$$

while the consumer surplus in the *U-D* duopoly is equal to

$$S_{UD} = \int_0^1 V^*_{UD}(x)dx = Y - L - \frac{\theta T}{2} + \frac{\theta^2 T^2}{192L}.$$

Using (15), S_D and S_{UD} show that $S_D > S_{UD} > S_U$. Hence, we have the following proposition.

Proposition 6. *Consumers as a whole prefer D-D to U-D and U-D to U-U.*

Therefore, the market equilibrium is not the consumers' most preferred combination of formats. That total profits are maximized at any Nash equilibrium aligns with the findings of Dubé and Misra (2023) and Buchholz et al. (2025), who show that personalized pricing increases firms' profits relative to uniform pricing. In contrast, in our setting the consumer surplus is the highest in the *D*-*D* duopoly, which appears to conflict with the same authors who report a modest decline in consumer surplus under personalized pricing. These comparisons, however, should be interpreted cautiously, because it is not clear how these studies account for the spatial dimension of pricing—a factor that is central to our analysis.

Note also that personalized prices are often associated with anti-competitive behavior because firms would be able to capture a larger share of consumer surplus. Proposition 6 concurs with Thisse and Vives (1988) by showing that the toughness of competition between D-retailers may lead to the opposite result.

5.5 *Z*one pricing

The strategies U and D can be viewed as polar cases, involving either a single delivered price or a continuum of delivered prices. To bridge these two extremes, we introduce *zone delivered pricing* (**Z**) as an intermediate regime (Adams and Williams, 2019). Under this strategy, each firm sets the same delivered price within each zone (intra-zonal pricing) while allowing prices to differ across zones (inter-zonal pricing), and they endogenously determine the size and boundaries of the zones. We examine the simple case in which the market is divided into m zones, each of size $1/m$. When the number of zones equals one, zone pricing collapses to U-pricing, whereas D-pricing emerges when zones shrink to tiny areas inhabited by only a few consumers—formally, a single point.

We prove in Appendix D.1 that the following result holds.

Proposition 7. *Assume m pricing zones of size $1/m$. Then, there exists a single Nash equilibrium in the Z-Z duopoly given by* (D.1).

Furthermore, combining Appendices D.2 and D.3 yields the following proposition.

Proposition 8. *A Z-retailer prefers personalized pricing when its rival chooses U-pricing; it prefers U-pricing when its rival adopts D-pricing.*

Hence, the Nash equilibria of the game of chicken are robust against any unilateral deviation involving a finite number of pricing zones and chooses to increase m to ∞ (resp., to decrease m to 1) when its competitor adopts U (resp., D).

6 The equilibrium of the retail industry

As discussed in the introduction, we start with two conventional retailers. We know from Section 4 that one firm (say Firm 2) chooses D-pricing when the distaste—or shipping cost—is sufficiently small, i.e., $\theta < \theta_D$, where θ_D is the unique solution to $\Pi^*_{2mD} - \pi^*_m = 0$. How does Firm 1 react?

Since *D*-*D* is never an equilibrium, Firm 1 will consider U as an alternative by comparing Π^*_{1UD} and Π^*_{1mD}.

Differentiating

$$G(\theta, T) \equiv \Pi^*_{1UD} - \pi^*_{mD} = \frac{L}{2\theta} + \frac{\theta T^2}{48L} - \frac{(6L - t + \theta T)^2}{72L}$$

with respect to θ shows that $G(\theta, T)$ is decreasing in θ. Furthermore, $G(1, T) > 0$ because $G(1, T)$ increases with T while $G(t/T, T) < 0$ if and only if

$$T < \frac{24L^2}{24L^2 + t^2} t < t,$$

which is a mild requirement when consumers are heterogeneous. Therefore, when Firm 2 adopts D-pricing, $\theta_1 > 1$ exists such that Firm 1 prefer U to M if and only if $\theta < \theta_1$.

But when Firm 1 chooses U instead of M, does Firm 2 still choose D? Since *U*-*U* is never an equilibrium when the strategy D is available, Firm 2 does not choose U. Firm 2 then chooses D rather than M if and only if

$$H(\theta, T) \equiv \Pi^*_{2UD} - \pi^*_{Um} = \frac{L}{2\theta} + \frac{\theta T^2}{96L} - \frac{(6L + \theta T - t)^2}{72L} > 0.$$

Differentiating $H(\theta, T)$ with respect to θ shows that $H(\theta, T)$ is decreasing. Moreover, $H(1, T) > 0$ and $H(t/T, T) < 0$ if and only if

$$T < \frac{48L^2}{48L^2 + t^2} t < t.$$

Consequently, $\theta_2 > 1$ exists such that Firm 2 prefer D to M if and only if $\theta < \theta_2$.

It is readily verified that $H(\theta, T) < G(\theta, T)$ always holds. Therefore, as both functions are decreasing, it must be that $\theta_1 > \theta_2$. Consequently, when $\theta < \min\{\theta_D, \theta_2\}$, the retail industry involves a U-retailer and a D-retailer. By contrast, when $\theta > \min\{\theta_D, \theta_1\}$, the equilibrium involves two M-retailers because no firm wants to deviate.

When the above two inequalities do not hold, two cases may arise. If $\theta_D < \theta_2$, the interval $(\min\{\theta_D, \theta_2\}, \theta_D)$ is empty. If $\theta_D > \theta_2$, this interval has a positive measure. As a result, one firm chooses to be a D-retailer because $\theta < \theta_D$, while its rival prefers M to U because $\theta > \theta_2$.

Put differently, when the distaste cost takes intermediate values—$\theta_2 < \theta < \theta_D$—the equilibrium involves an offline retailer and an online retailer.

The following proposition comprises a summary.

Proposition 9. *If $\theta < \min\{\theta_D, \theta_2\}$, then the retail industry features a U-retailer and a D-retailer. If $\theta \in [\min\{\theta_D, \theta_2\}, \theta_D]$, one firm chooses to be a D-retailer while its rival remains a conventional retailer. Last, if $\theta > \theta_D$, the retail industry features two conventional retailers.*

Note also that $H(\theta, T)$ is shifted upward when T decreases, implying that θ_2 increases. It then follows from Proposition 9 that *the likelihood of a retail industry involving two e-retailers increases with falling shipping costs.* Thus, allowing retailers to adopt M-pricing leads to the emergence of upper bounds on distaste (or shipping) costs under which different combinations of retail formats can arise.

7 Concluding remarks

We developed a spatial discrete-choice model to examine competition between offline and online firms. The analysis demonstrates that the relative performance of these retail formats is critically contingent upon the spatial distribution of consumers and retailers. At first glance, this finding may appear counterintuitive, as e-commerce is often portrayed as a retail format in which spatial frictions are negligible. We argue, however, that this interpretation is overly reductive for at least two reasons. First, the geographical location of conventional retailers continues to exert a substantial influence on consumers' shopping behavior. For example, because shipping costs are positive, some consumers prefer an offline duopoly to a market with online retailers. Second, the spatial dispersion of consumers necessitates the design of transportation and logistics strategies whose efficiency directly depends on the underlying geography of demand. Together, these factors underscore the enduring relevance of spatial considerations in shaping competitive behaviors in the digital economy.

Our paper also highlights the central role of consumer two-dimensional heterogeneity in determining the market outcomes. While we have relied on a linear probability model—consistent with linear demand in a Hotelling-type duopoly—future research should explore more general frameworks. In particular, multinomial logit models represent a natural alternative, though their analytical tractability poses challenges. Recent advances in discrete choice modeling (Chambers et al., 2025) may help address these difficulties and open promising avenues for further work. Moreover, our analysis has taken firms' locations as exogenous. Understanding how firms operating under different formats endogenously choose their locations remains an important topic for future research.

Finally, a natural extension would be to examine the connections with models of platforms, which frequently abstract from the shipping dimension. Another one would be to assume that the online firm is multiproduct, while the offline retailer is a shopping street formed by independent retailers.

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Appendix A

Consider a one-dimensional Lancasterian characteristics space $[0, 1]$. Firms sell differentiated varieties defined by $z_1 = 0$ and $z_2 = 1 > 0$ at prices p_1 and p_2. There is a unit continuum of consumers whose ideal product z varies within $[0, 1]$. Each consumer buys one unit of the variety that maximizes her indirect utility. The reservation price r is high enough for each consumer to buy at the equilibrium. A consumer whose ideal product is z has the following indirect utility:

$$V_i(z) = \nu - p_i - \tau(z - z_i)^2, \qquad i = 1, 2. \tag{A.1}$$

In (A.1), the quadratic term stands for the utility loss incurred by an individual for consuming away from her ideal product, while $\tau > 0$ measures the intensity of this loss. In line with the literature, the utility loss is proportional to the square of the Euclidean distance between the consumer's ideal product z and variety z_i provided by the market.

The interval $[0, 1]$ is divided in two disjoint intervals formed by the consumers who buy variety 1 ($z < z_m$), and those who buy variety 2 ($z > z_m$) where the marginal consumer indifferent between the two varieties is set up at

$$z_m = \frac{\tau - p_1 + p_2}{2\tau}.$$

When consumers' ideal products are distributed according to the density $f(z) = 1$ over the interval $[0,1]$ and zero otherwise, the share of consumers who buy variety 1 is equal to

$$q_1(p_i,p_j) = \begin{cases} 0 & \text{if } p_1 - p_2 \geq \tau \\ \frac{\tau - p_1 + p_2}{2\tau}, & \text{if } -\tau < p_1 - p_2 < \tau \\ 1 & \text{if } \ p_1 - p_2 \leq -\tau \end{cases}, \tag{A.2}$$
$$q_2(p_i,p_j) = 1 - q_1(p_i,p_j).$$

Comparing (8) and (A.2) shows that τ and L play the same role in demand functions.

Appendix B

1. M-M duopoly. If Firm 1 increases its price above L for $x_{\max} < 1$ to hold when $p_2 = L$, Firm 1's demand is equal to

$$q_1(p_1, L) = \int_0^{\frac{2L+t-p_1}{2t}} \frac{-p_1 - tx + L + t(1-x) + L}{2L} dx = \frac{(2L + t - p_1)^2}{8tL}. \tag{B.1}$$

Differentiating $p_1 q_1(p_1, L)$ w.r.t. p_1 and solving the FOC for p_1 yields the following solutions: $\bar{p}_1 = (2L + t)/3 < L$ and $\hat{p}_1 = 2L + t > L$. Clearly, we have $q_1(\hat{p}_1, L) = 0$, and thus $\Pi_{1m}(\hat{p}_1, L) = 0 < \Pi_{1m}(L, L)$.

It remains to check that $x_{\min} = 0$ for (B.1) to be Firm 1's demand at $\hat{p}_1$. Plugging $\hat{p}_1$ in $x_{\min} = (-p_1 + t)/2t$ yields $-L/t < 0$. Therefore, $x_{\min} = \max\{-L/t, 0\} = 0$.

By symmetry, the same arguments apply to Firm 2.

2. M-D duopoly. Assume that Firm 1 raises its price for $x_{\max} < 1$ to hold when Firm 2 charges the uniform delivered price P^*_{mU}. Then, we have:

$$q_1(p_{mU}, P^*_{mU}) = \int_0^{\frac{-p+\theta P^* + L}{t}} \frac{L - p_{mU} - tx + \theta P^*_{mU}}{2L} dx = \frac{(12L - 6p_{mU} + t + 2\theta T)^2}{4tL}.$$

Differentiating $p_{mU} q_1(p_{mU}, P^*_{mU})$ w.r.t. p_{mU} and solving the FOC for p_{mU} yields the following solutions: $\bar{p} = (12L + t + 2\theta T)/18$ and $\hat{p} = (12L + t + 2\theta T)/6$. First, we have $\Pi_{1m}(\hat{p}, P^*_{mU}) = 0$ because of $x_{\max} = 0$ for $\hat{p}$. Second, plugging $\bar{p}$ in $x_{\max} = (-p + \theta P^*_{mU} + L)/t$ and using $L \geq t$ yields $x_{\max} = 1$, which contradicts $q_1(p, P^*_{mU})$.

3. D-D duopoly. Assume that Firm 1 decreases its price P_1 below T. In this case, the e-retailer does supplies the market until $x = P_1/T$, so that its profits are given by

$$\Pi_{1D}(P_{1D}, P^*_{2D}) = \int_0^{P_1/T} (P_{1D} - Tx) \frac{L - \theta P_{1D} + \theta P^*_{2D}}{2L} dx = P_1^2 \frac{L - \theta P_{1D} + L + \theta T/2}{4LT}.$$

Differentiating $\Pi_{1D}(P_{1D}, P^*_{2D})$ w.r.t. P_1 yields $\bar{P}_1 = 0$ and $\hat{P}_1 = (4L + T\theta)/3\theta > T$, which contradicts the definition of $\Pi_{1D}(P_{1D}, P^*_{2D})$.

Appendix C

Appendix C.1 Consider a duopoly in which Firm 1 sets a mill price p_{1mD} while Firm 2 chooses a price schedule $P_{2mD}(x)$ defined over $[0, 1]$.

Profits of Firm 1 (Π_{1mD}) and Firm 2 (Π_{2mD}) are defined as follows:

$$\Pi_{1mD} = p_{mD} \int_0^1 \frac{L - p_{mD} - tx + \theta P_{mD}(x)}{2L} dx, \qquad \Pi_{2mD}(p_{mD}) = \int_0^1 \Pi_{2mD}(p_{mD}, P_{mD}(x); x) dx,$$

with

$$\Pi_{2mD}(p_{1mD}, P_{2mD}(x); x) \equiv \frac{L + p_{mD} + tx - \theta P_{mD}(x)}{2L}(P_{mD}(x) - T(1 - x)), \tag{C.1}$$

where p_{mD} is the mill price chosen by Firm 1 and $P_{mD}(x)$ is Firm 2's price schedule.

Since $\Pi_{2mD}(p_{mD}, P_{mD}(x); x)$ is concave in $P_{mD}(x)$, solving the first order condition for profit maximization yields

$$P^*_{mD}(x; p_{mD}) = \frac{L + p_{mD} + \theta T + (t - \theta T)\, x}{2\theta}.$$

Similarly, differentiating Π_{1mD} with respect to p_{mD} yields

$$p^*_{mD}(\bar{P}) = \frac{1}{2}L - \frac{1}{4}t + \frac{1}{2}\theta\bar{P}.$$

Since

$$\bar{P} = \int_0^1 P^*_{mD}(x) dx = \frac{2L + t + T\theta + 2p_{mD}}{4\theta},$$

we obtain

$$p^*_{mD} = L - \frac{1}{6}(t - \theta T). \tag{C.2}$$

Plugging this expression in $P^*_{mD}(x; p_{mD})$, we obtain:

$$P^*_{mD}(x) = \frac{L}{\theta} + \frac{T}{2} + \frac{(t - T\theta)(6x - 1)}{12\theta}. \tag{C.3}$$

Therefore, the equilibrium profits are as follows:

$$\Pi^*_{1mD} = \frac{(6L - t + \theta T)^2}{72L}, \quad \Pi^*_{2mD} = \frac{3L + t - \theta T}{6\theta} + \frac{(t^2 + T^2\theta^2)}{48L\theta} + \frac{(t - \theta T)^2}{288L\theta}. \tag{C.4}$$

Since

$$P^*_{mD}(x) - T(1 - x) = \frac{12L - t - 5T\theta + 6(t + \theta T)x}{12\theta} > 0, \quad \text{for all } x$$

we have: *in a M-D duopoly, there exists a unique Nash equilibrium, which is given by* (C.2) *and* (C.3).

Appendix C.2 We are now equipped to determine when Firm 2 shifts to D-retailing. Comparing Π^*_{2mD} and Π^*_{2mU} shows that Firm 2's profits are higher under D-pricing than under U-pricing, i.e., $\Pi^*_{2mD} > \Pi^*_{2mU}$. Since Π^*_{2mD} and Π^*_{2mU} decrease over $[1, t/T]$, the equation $\Pi^*_{2mi} - \pi^*_m = 0$ has a unique solution θ_i for $i = D, U$. Since Π^*_{2mD} and Π^*_{2mU} decrease over $[1, t/T]$, the inequality $\Pi^*_{2mD} - \pi^*_m > \Pi^*_{2mU} - \pi^*_m$ implies that $\theta_U < \theta_D$.

Using the argument of Proposition 2 then yields:

Proposition A *Assume that Firm* 1 *is a M-retailer while Firm* 2 *chooses between M, U and D. When* (6) *holds, there exists a unique value* $\theta_D > \theta_U > 1$ *such that Firm* 2 *chooses D-pricing if and only if* $\theta < \min\{\theta_D, t/T\}$.

Appendix D

Appendix D.1. Firm i's profits are given by

$$\Pi_{1Z}(m) = \sum_{k=1}^{m} \int_{(k-1)/m}^{k/m} (P_{1k} - Tx) \frac{L - \theta P_{1k} + \theta P_{2k}}{2L} dx,$$

where P_{1k} is the delivered price charged by Firm 1 to the consumers residing in zone k; a similar expression holds for Firm 2. Solving the first-order conditions for profit maximization yields the equilibrium prices in zone $k = 1, .., m$:

$$P^*_{1k}(m) = \frac{L}{\theta} + \frac{T}{3} + \frac{2k-1}{2m}\frac{T}{3}, \qquad P^*_{2k}(m) = \frac{L}{\theta} + \frac{2T}{3} - \frac{2k-1}{2m}\frac{T}{3}. \tag{D.1}$$

Thus, $P^*_{1k}(m)$ increases while $P^*_{2k}(m)$ decreases with k. Furthermore, (D.1) is equal to (16) when $m = 1$. Further, as $m \to \infty$, the zone $((k-1)/m, k/m)$ shrinks and converges to the middle point $x_{kc}(m) = (2k-1)/2m$. Since $P^*_{iD}(x) = P^*_{ik}(m)$ if and only if $x_{kc}(m) = (2k-1)/2m$, we may conclude that $P^*_{ik}(m) \to P^*_{iD}(x)$ when $m \to \infty$. Since $P^*_{1D}(x)$ increases with x over $[0, 1]$ while $P^*_{1k}(m)$ is flat within the k-zone, the k-consumers located on the left of $x_{kc}(m)$ pay a lower price to Firm 1 in the *D-D* duopoly than in a *Z-Z* duopoly, whereas those located on the right pay a higher price. The opposite holds for the product bought from Firm 2.

Furthermore, we have

$$\begin{aligned} P^*_{1k}(m) - Tx &> 0 \text{ and } P^*_{2k}(m) - T(1-x) > 0 \text{ for } \forall x \in [0,1] \\ &\Leftrightarrow \quad L > \frac{\theta T}{3} + \frac{\theta T}{6m} > \theta T/3. \end{aligned}$$

Since $Tk/m > T(k-1)/m$, the demanding condition becomes

$$P^*_{1k}(m) - T\frac{k}{m} = \frac{L}{\theta} - \frac{1}{6}T\frac{4k - 2m + 1}{m},$$

which decrease with k, which implies

$$L > \frac{2m+1}{6m}\theta T.$$

Similarly, $T(1-k/m) > T[1-(k-1)/m]$ implies the same inequality.

In other words, the full market coverage needs a sufficiently high degree of product differentiation. When the number of zone tends to ∞, the above inequality becomes the same as the necessary and sufficient condition for existence of an equilibrium under D-D. When the number of zone tends to 1, the above inequality becomes the same as under U-U.

Plugging the equilibrium prices (D.1) in the above profit function, we obtain

$$\Pi_{1Z}^*(m) = \Pi_{2Z}^*(m) = \frac{L}{2\theta} + \frac{\theta T^2}{54L} - \frac{\theta T^2}{54Lm^2},$$

which increases with m. In other words, the profit earned by each firm increases with the number of zones because it permits a finer pricing segmentation of the market. When $m \to \infty$, $\Pi_{iZ}^*(m)$ converges to (19). In sum, if the number of zones increases, the market outcome converges to D-pricing (Lu and Matsushima, 2024; Rhodes and Zhou, 2024). On the other hand, when firms adopt U-pricing, there is one zone and $\Pi_i^* = L/2\theta$, which is equal to (17).

Appendix D.2. Under which condition does a firm choose Z-pricing when its competitor chooses U-pricing? Like in Section 5, we assume that Firm 1 chooses a uniform delivered price P while Firm 2 chooses a price vector $(P_1, ..., P_m)$. In this case, profit functions are such that

$$\begin{aligned}
\Pi_{1UZ} &= \sum_{k=1}^{m} \int_{(k-1)/m}^{k/m} (P - Tx)\frac{L - \theta P + \theta P_k}{2L} dx, \\
\Pi_{2UZ} &= \sum_{k=1}^{m} \int_{(k-1)/m}^{k/m} (P_k - T(1-x))\frac{L - \theta P_k + \theta P}{2L} dx.
\end{aligned}$$

Differentiating Π_{1UZ} with respect to P yields:

$$P^*(P_1, ..., P_m) = \frac{L}{2\theta} + \frac{\sum_{k=1}^{m} P_k}{2m} + \frac{T}{4}.$$

Differentiating Π_{2UZ} with respect to P_k yields

$$P_k^*(P) = \frac{L}{2\theta} + \frac{P}{2} + \frac{2m-2k+1}{4m}T.$$

Substituting $P_k^*(P)$ in $P^*(P_1, ..., P_m)$, we obtain

$$P^* = \frac{L}{\theta} + \frac{T}{2},$$

which is equal to (23).

Plugging P^* in $P_k^*(P)$ yields the equilibrium price set by Firm 2 in the k-th zone:

$$P_k^*(m) = \frac{L}{\theta} + \frac{3m - 2k + 1}{4m} T.$$

Furthermore, since $(k-1)/m < x < k/m$, full market coverage implies

$$P^* - Tx > 0, \qquad P_k^* - T(1-x) > 0.$$

This holds when the degree of heterogeneity is sufficiently large:

$$L > \max\left\{ \frac{\theta T}{2}, \frac{\theta T}{4m}(3m-1) \right\}$$

which is satisfied under (6).

It is then readily verified that the equilibrium profits are as follows:

$$\Pi_{1UZ}^* = \frac{L}{2\theta} + \frac{\theta T^2}{48L} - \frac{\theta T^2}{48Lm^2}, \qquad \Pi_{2UZ}^* = \frac{L}{2\theta} + \frac{\theta T^2}{96L} - \frac{\theta T^2}{96Lm^2},$$

which both increase with the total number of zones. In other words, both the Z- and U-firms are better-off when the number of zones increases, which implies that Firm 2 adopts D-pricing.

Appendix D.3. If Firm 2 adopts D-pricing, profits are given by

$$\begin{aligned}
\Pi_{1ZD} &= \sum_{k=1}^{m} \int_{(k-1)/m}^{k/m} (P_k - Tx) \frac{L - \theta P_k + \theta P_{2D}(x)}{2L} dx, \\
\Pi_{2ZD} &= \sum_{k=1}^{m} \int_{(k-1)/m}^{k/m} (P_{2D}(x) - T(1-x)) \frac{L - \theta P_{2D}(x) + \theta P_k}{2L} dx.
\end{aligned}$$

Differentiating $\Pi_{1ZD}(x)$ with respect to P_k, we obtain

$$P_k^*(P_{2D}(\cdot)) = \frac{L}{2\theta} + \frac{m}{2} \int_{(k-1)/m}^{k/m} P_{2D}(x) dx + \frac{T}{4} \frac{2k-1}{m}.$$

while differentiating $\Pi_{2ZD}(x)$ with respect to $P_{2D}(x)$ leads to

$$P_{2D}^*(x; P_k) = \frac{L + \theta P_k + \theta T(1-x)}{2\theta}$$

for $x \in ((k-1)/m, k/m)$. Plugging $P_{2D}^*(x; P_k)$ into $P_k^*(P_{2D}(x))$ yields Firm 1's equilibrium price in the k-th zone:

$$P_k^* = \frac{L}{\theta} + \frac{T}{3} + \frac{2k-1}{6m} T. \tag{D.2}$$

Substituting P_k^* into $P_{2D}^*(x; P_{1k})$ yields the equilibrium price schedule:

$$P_{2D}^*(x; k) = \frac{L}{\theta} + \frac{4-3x}{6} T + \frac{2k-1}{12m} T. \tag{D.3}$$

When $m \to 1$, $P_k^*(m)$ converges to the equilibrium price (23) set by the U-retailer, while the limit of $P_{2D}^*(x;k)$ is equal to the price schedule chosen by the D-retailer (24).

Plugging (D.2) and (D.3) into the functions Π_{1ZD} and Π_{2ZD}, we obtain the following equilibrium profits:

$$\Pi_{1ZD}^*(m) = \frac{L}{2\theta} + \frac{\theta T^2}{54L} + \frac{\theta T^2}{432L}\frac{1}{m^2}, \quad \Pi_{2ZD}^*(m) = \frac{L}{2\theta} + \frac{\theta T^2}{54L} - \frac{7\theta T^2}{864L}\frac{1}{m^2}.$$

Clearly, $\Pi_{1ZD}^*(m)$ decreases with m and is equal to Π_{1UD}^* given by (23) when $m = 1$ and Firm 2 chooses D-pricing. Similarly, $\Pi_{2ZD}^*(m)$ increases with m and converges to $\Pi_{2UD}^*(x)$ given by (24) when Firm 1 chooses U-pricing. Therefore, the D-retailer chooses a pointwise price schedule defined over $[0, 1]$.