

Reconsidering the Market Size Effect in Innovation and Growth

Hélène Latzer^a

Kiminori Matsuyama^b

Mathieu Parenti^c

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Abstract

In the standard horizontal innovation model of endogenous growth, larger economies innovate more and grow faster. Due to the homotheticity of preferences, however, it does not matter whether the large market size comes from a large population or a high per capita expenditure. In this paper, we extend the standard model to allow for nonhomothetic preferences. Among others, we show that, holding the size fixed, economies with higher per capita expenditure and smaller populations innovate more and grow faster for the empirically relevant case of incomplete pass-through, strategic complementarity in pricing, and procompetitive entry.

Keywords: Endogenous growth, Balanced growth, Horizontal innovation, Nonhomothetic preferences, Directly explicitly additive (DEA) preferences, Demand composition, Incomplete pass-through, Strategic complementarity in pricing, Procompetitive entry, Competition and growth

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^aParis-I, CNRS, France & Université Saint-Louis, Belgium.

^bNorthwestern University, USA. Corresponding author, email: k-matsuyama@northwestern.edu.

^cUniversité Libre de Bruxelles, Belgium.

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1. Introduction

The standard horizontal innovation models of endogenous growth—say, Romer (1990), Grossman and Helpman (1993, ch.3), Gancia and Zilibotti (2005), and Acemoglu (2008; ch.13.4) just to name a few—, predict that countries with larger market sizes innovate more and hence grow faster. However, due to the assumption of consumer demand homotheticity, the composition of the aggregate demand has no effect. In particular, it does not matter whether the large market size comes from a larger population size or a higher per capita expenditure. Once the aggregate market size is controlled for, poorer countries with larger population sizes innovate as much as richer countries with smaller population sizes. Furthermore, without any effects of the demand composition, the (demand-side) *market size effect* becomes indistinguishable from the (supply-side) *scale effect*, for which there exists little supporting evidence.

Another well-known feature of the standard horizontal innovation models of endogenous growth is that the markup rate at which innovated products are sold is determined solely by an exogenous preference parameter. Thus, the markup rate can change only by changing the preferences, which makes it impossible to conduct any welfare analysis of markup rate changes. Furthermore, any (exogenous) increase in the markup rate leads to an increase in the innovation rate, contrary to some empirical evidence suggesting that competition fosters innovation and hence growth.¹ Another implication of this feature is that a change in the production cost of innovated products has no effect on the profit share earned by the innovators, and hence on the innovation and growth rates of the economy.

In this paper, we extend textbook horizontal innovation models of endogenous growth to allow for nonhomothetic preferences to examine the effect of the market size composition on innovation and growth. We consider an economy, populated by N identical agents, each endowed with h units of labor.² For the preferences, we follow the footsteps of Dixit and Stiglitz (1977). Virtually all the existing horizontal innovation models of endogenous growth build on their well-known model of monopolistic competition with homothetic CES preferences in Dixit-Stiglitz (1977; Section I). Instead, we build on their lesser-known model of monopolistic

¹ See, e.g., Blundell, Griffith, van Reenen (1995, 1999). Some economists view this as *prima facie* evidence against horizontal innovation models of endogenous growth, in favor of vertical innovation and creative destruction models of endogenous growth: see, Aghion et. al. (2005), Aghion, Akcigit, and Howitt (2015), and Acemoglu et. al. (2018).

² Thus, h measures the worker efficiency, and can be interpreted as the level of human capital or the quality of the labor force. The total labor endowment of the economy is hence equal to $L = hN$. With labor being the only factor of production, L is also the size of the economy.

competition with directly explicitly additive (DEA) nonhomothetic preferences in Dixit-Stiglitz (1977; Section II), which contains homothetic CES as a knife-edge case.³ A distinctive feature of monopolistic competition model with DEA is that the price elasticity of demand each firm faces is a function of per capita consumption of its product only. We use this class of nonhomothetic preferences for two reasons. First, it allows us to develop a horizontal innovation model of endogenous growth that preserves the balanced growth property in spite of nonhomotheticity. Second, it allows us to account for the following empirical observations:⁴

- i) *Incomplete (less than 100%) Pass-Through*; An increase in the production cost reduces the markup rate of monopolistically competitive firms.
- ii) *Strategic Complementarity in Pricing*; Each firm responds by increasing its price when competing firms increase their prices.
- iii) *Procompetitive Entry*; Other things being equal, the presence of more firms reduces the markup rates,

by departing from homothetic CES within this class to allow the elasticity of substitution between products to be smaller at higher indifference curves.⁵

Here are our main results. First, after controlling for the size $L = hN$, a richer country with a smaller population (a higher h with a smaller N) innovates more and hence grows faster. Hence, the composition of the aggregate demand matters, and the demand-side market size effect can be distinguished from the supply-side scale effect. Second, even though both the markup rate

³ See also Zhelobodko et al. (2012). Although Dixit and Stiglitz (1977) called Section II “Variable Elasticity Case”, the well-known Bergson’s Law states that, within this class of preferences, they are homothetic if and only if they are CES. In other words, any departure from CES within this class introduces nonhomotheticity. Hence, one could equally call Section II “Nonhomothetic Case.” Of course, nonhomotheticity and non-CES are generally distinct properties of preferences. Indeed, it is possible to have nonhomothetic CES as well as homothetic non-CES in a broader class of preferences. However, such preferences would be incompatible with the balanced growth property, which we would like to preserve in order to keep our departure from the standard model to the minimum.

⁴ See Berman et al (2012) as well as the review provided by De Loecker and Goldberg (2014) for incomplete pass-through; Fontagné et al (2018) and Amiti et al (2019) for strategic complementarity in pricing; Campbell and Hopenhayn (2005) and Feenstra and Weinstein (2017) for procompetitive entry. The Dixit-Stiglitz (1977, Section I) model with homothetic CES has been widely criticized because they are at odds with these empirical observations; the markup rate never changes in response to a change in the production cost, or in the market environment (i.e., pricing and entry of other firms).

⁵ Zhelobodko et.al. (2012) called it the case of “increasing relative love for variety (RLV).” It turns out that this is equivalent to assuming that demand for each product is more price elastic at a higher price (Marshall’s second law of demand) within DEA. Of course, homothetic CES is a knife-edge case within DEA, so that, if we departed in the opposite direction of decreasing RLV (i.e., the elasticity of substitution being larger at higher indifference curves), we would have obtained the opposite results of more than 100% pass-through, strategic substitutes in pricing, and anti-competitive effects of entry. We view such a departure as empirically irrelevant.

and the innovation (and growth) rate stay constant over time along the balanced growth path, they are both endogenous and hence they could either be positively or negatively correlated across the balanced growth paths, depending on the sources of underlying variations. We show that the markup and innovation rates move in the same direction if changes are caused by exogenous variations in production cost or per capita expenditure, while they move in the opposite directions if changes are caused by variations in the discount rate, the innovation cost or the population size. This implies, in particular, that the measure of competitiveness and the growth rate are positively correlated in cross-sections of countries, if countries differ mostly in the innovation (or firm entry) cost.

It should be noted that the (demand-side) *market size effect* on innovation and growth we study is conceptually distinct from the (supply-side) *scale effect*, whose empirical validity has been questioned by Jones (1995) and many others. Nonhomotheticity provides one natural way of distinguishing these two effects.⁶ Indeed, our results suggest that the difference in per capita income across countries could potentially be one reason why there is little supporting evidence for the scale effect. We also show that correlations between competition and growth across countries depend on the sources of variations across countries. As such, our results suggest that horizontal innovation models of growth can also contribute to the debate regarding competition and growth.

Boucekkine et. al. (2017) also investigated the implications of departing from CES preferences on the markup and growth rates in a balanced growth model of horizontal innovation. However, their framework is too restrictive to study the effect of the demand composition. Moreover, under the class of preferences they consider, the markup rule of the firms is independent of the market environment (i.e., pricing and entry of other firms). Hence, it can account neither for procompetitive effects of entry nor for strategic complementarity in pricing. In contrast, our framework is consistent with these two empirical findings, in addition to incomplete pass-through.

2. Back to the Basics: Innovation and Growth under CES

⁶ An alternative approach to distinguish the two has been pursued in the directed technological change literature, using multi-sector, multi-factor extensions of endogenous growth models, see, e.g., Acemoglu (2008, ch.15), Aghion and Howitt (2008, ch.8), and Gancia and Zilibotti (2009).

We start with a benchmark balanced growth model with homothetic CES, which encompasses two versions of the textbook models (Grossman and Helpman, 1993; Gancia and Zilibotti, 2005) in order to facilitate comparisons with our model with DEA preferences.

2.1. Intratemporal problem

Labor is the only factor of production. We consider an economy populated by N identical agents, each supplying inelastically h units of labor measured in efficiency units. Hence, the total labor supply measured in efficiency units is given by $L = hN$, which is also the size of the economy.

Time is continuous and extends from $t = 0$ to infinity. Intertemporal preferences of each agent take the following form:

$$u_0 = \int_0^\infty \log(U(\mathbf{x}_t)) e^{-\rho} dt,$$

where $U(\mathbf{x}_t)$ is the intratemporal utility, $\mathbf{x}_t = \{x_t(\omega); \omega \in [0, V_t]\}$ is the consumption profile, with V_t being the range of the products that have been innovated by time t , and $x_t(\omega)$ denoting the consumption of product $\omega \in [0, V_t]$.

At time t , each agent earns a wage income equal to $w_t h$, and spends E_t , and chooses $\mathbf{x}_t$ to maximize $U(\mathbf{x}_t)$ subject to the intratemporal budget constraint,

$$\int_0^{V_t} p_t(\omega) x_t(\omega) d\omega = E_t, \quad (1)$$

where $p_t(\omega)$ denotes the price of product $\omega \in [0, V_t]$. When $U(\mathbf{x}_t)$ is a CES with the elasticity of substitution $\sigma > 1$:

$$U(\mathbf{x}_t) = \int_0^{V_t} (x_t(\omega))^{1-\frac{1}{\sigma}} d\omega, \quad (2)$$

this intratemporal maximization problem yields the per capita demand curve for each product:

$$x_t(\omega) = \frac{[p_t(\omega)]^{-\sigma}}{(P_t)^{1-\sigma}} E_t, \quad (3)$$

with the price elasticity being constant and equal to $\sigma > 1$, where

$$(P_t)^{1-\sigma} \equiv \int_0^{V_t} [p_t(\omega')]^{1-\sigma} d\omega'.$$

The total demand for product ω is simply given by $q_t(\omega) = N x_t(\omega)$.

2.2. Firms' intratemporal problem

Each product $\omega \in [0, V_t]$ is produced and sold exclusively by a single firm, which is also indexed by $\omega \in [0, V_t]$. Producing one unit of each product requires ψ_t efficiency units of labor. Each firm chooses its price, $p_t(\omega)$ or the quantity, $q_t(\omega) = Nx_t(\omega)$, to maximize the profit,

$$\pi_t(\omega) \equiv (p_t(\omega) - w_t\psi_t)q_t(\omega) = (p_t(\omega) - w_t\psi_t)Nx_t(\omega)$$

subject to eq.(3) taking w_t, ψ_t, P_t , and E_t as given. This profit maximization problem has a unique solution, and hence all firms adopt the same pricing rule:

$$p_t(\omega) \left(1 - \frac{1}{\sigma}\right) = w_t\psi_t \Leftrightarrow p_t(\omega) = Mw_t\psi_t \equiv p_t,$$

where $M \equiv \sigma/(\sigma - 1)$ is the markup rate, which is exogenously constant under CES.

Because all firms set the same price, firm symmetry entails that all products are produced and consumed by the same amount, and all firms earn the same level of profits:

$$q_t(\omega) = Nx_t(\omega) = Nx_t = q_t; \pi_t(\omega) = \pi_t$$

and the intratemporal budget constraint, eq.(1), becomes simplified to:

$$p_t x_t V_t = E_t.$$

The above mark-up rule also implies that the share of the aggregate expenditure that goes to the firms' profits is also exogenously constant and given by:

$$\frac{\pi_t V_t}{NE_t} = \frac{(p_t - w_t\psi_t)q_t V_t}{p_t q_t V_t} = \frac{p_t - w_t\psi_t}{p_t} = \frac{1}{\sigma} \quad (4)$$

Likewise, the share of the aggregate expenditure that goes to the wage payment in the production sector is also exogenously constant, and given by:

$$\frac{w_t L_{Xt}}{NE_t} = 1 - \frac{1}{\sigma} = \frac{1}{M} \quad (5)$$

where $L_{Xt} = \psi_t q_t V_t = \psi_t Nx_t V_t$ denotes the total number of efficiency units of labor employed in the production of the existing products.

2.3. R&D and resource constraints

Because firms are symmetric, the market value of each firm is the same and equal to $B_t \equiv \int_t^\infty \pi_s e^{-(R_s - R_t)} ds$, where $R_s \equiv \int_0^s r_\tau d\tau$ is the cumulative interest rate and r_t is the instantaneous one. Log-differentiating this expression of B_t with respect to t , we obtain:

$$\frac{\dot{B}_t + \pi_t}{B_t} = r_t. \quad (6)$$

Innovating per unit of new products requires F_t efficiency units of labor:

$$F_t \dot{V}_t = L_{Rt},$$

where L_{Rt} is the number of units of labor being employed in the R&D sector at time t . There is free entry in the R&D sector. Hence, whenever the R&D sector is active, net returns from R&D, $B_t \dot{V}_t - w_t L_{Rt} = (B_t - w_t F_t) \dot{V}_t$, are equal to zero, which means that the cost of creating a product (the R&D cost) and the value of creating a product (the value of a firm) are equalized:

$$B_t = w_t F_t. \quad (7)$$

Finally, the labor resource constraint, or the labor market equilibrium condition, is given by:

$$hN = L = L_{Rt} + L_{Xt} = F_t \dot{V}_t + \psi_t N x_t V_t. \quad (8)$$

2.4. Intertemporal problem

To describe the intertemporal maximization problem of the agent, we first derive the intertemporal budget constraint. Each agent holds $1/N$ fraction of the ownership shares of the profit-making firms, hence their asset holding is $a_t = B_t V_t / N$. At time t , an agent earns the wage income $w_t h$ and the profit income $\pi_t V_t / N$, spends $E_t = p_t x_t V_t$ and purchases assets (the ownership shares of the new profit-making firms) by $B_t \dot{V}_t / N$. The flow budget constraint is hence:

$$B_t \dot{V}_t / N + E_t = w_t h + \pi_t V_t / N$$

By adding the capital gains $\dot{B}_t V_t / N$ on both sides, and using eq.(6) and the fact that $a_t = B_t V_t / N$, the above expression can be written as:

$$\dot{a}_t + E_t = w_t h + r_t a_t$$

By integrating this expression from $t = 0$ to infinity, we obtain the intertemporal budget constraint:

$$\int_0^\infty E_t e^{-Rt} dt \leq a_0 + \int_0^\infty w_t h e^{-Rt} dt, \quad (9)$$

with the no-Ponzi scheme condition, $\lim_{t \rightarrow \infty} a_t e^{-Rt} \geq 0$. Subject to this intertemporal budget constraint, eq.(9), agents choose an expenditure path, $\{E_t\}_{t=0}^\infty$, so as to maximize:

$$\mathcal{U}_0 = \int_0^\infty \log(U(\mathbf{x}_t)) e^{-\rho t} dt = \int_0^\infty \log \left(V_t (x_t)^{1-\frac{1}{\sigma}} \right) e^{-\rho t} dt = \int_0^\infty \log \left(V_t \left(\frac{E_t}{p_t V_t} \right)^{1-\frac{1}{\sigma}} \right) e^{-\rho t} dt.$$

The first-order condition is given by

$$\frac{1}{E_t} e^{-\rho t} = \lambda_0 e^{-Rt},$$

where λ_0 is the Lagrange multiplier associated with eq.(9). Log-differentiating this first-order condition with respect to t leads to the familiar Euler equation:

$$\frac{\dot{E}_t}{E_t} = r_t - \rho. \quad (10)$$

2.5. The Balanced Growth Path

The balanced growth path (BGP) is defined as an equilibrium path satisfying the following three conditions:

- i) The growth rate of the range of products $g_t \equiv \dot{V}_t/V_t$ is constant and positive.
- ii) The allocation of labor between the production and R&D sectors is constant: $L_{Xt} = L_X^*$ and $L_{Rt} = L_R^*$.
- iii) The markup rate, $M \equiv \sigma/(\sigma - 1)$, is constant, satisfied automatically under CES.

To guarantee the existence of such a BGP, we follow Grossman and Helpman (1993) and Gancia and Zilibotti (2005) and many others by assuming that knowledge spillovers from past R&D experiences reduce the cost of R&D as follows:

$$F_t = \frac{F}{V_t}, \quad (11)$$

which implies that $L_{Rt} = F_t \dot{V}_t = F g_t$.

Regarding the production cost, ψ_t , Grossman and Helpman (1993) assume $\psi_t = \psi$ so that knowledge spillovers are limited to R&D. In contrast, Gancia and Zilibotti (2005) assume $\psi_t = \psi/V_t$ so that they benefit both R&D and production equally. As will become clear below, however, neither of these assumptions play any role in ensuring the existence of a BGP under CES, so we intentionally leave ψ_t unspecified in this section.

We now derive the law of motion for this economy under the assumption that the R&D sector is active: $L_{Rt} = F_t \dot{V}_t = F g_t > 0$. By inserting eq.(6) into the Euler equation, eq.(10), we obtain

$$\frac{\dot{E}_t}{E_t} = \frac{\dot{B}_t + \pi_t}{B_t} - \rho.$$

Using eqs. (4), (7) and (11), this can be written as

$$\frac{\dot{E}_t}{E_t} = \frac{\dot{w}_t}{w_t} - g_t + \frac{N}{\sigma F} \frac{E_t}{w_t} - \rho.$$

By defining $\mathcal{E}_t \equiv NE_t/w_t$, this can be simplified to:

$$\frac{\dot{\mathcal{E}}_t}{\mathcal{E}_t} = \frac{\mathcal{E}_t}{\sigma F} - g_t - \rho, \quad (12)$$

while the labor market equilibrium condition eq.(8) becomes, using eq.(5):

$$L = L_{Rt} + L_{Xt} = Fg_t + \left(1 - \frac{1}{\sigma}\right) \mathcal{E}_t. \quad (13)$$

By combining eq.(12) and eq.(13), we obtain the law of motion for $\mathcal{E}_t$:

$$\frac{\dot{\mathcal{E}}_t}{\mathcal{E}_t} = \frac{\mathcal{E}_t - \mathcal{E}^*}{F}, \text{ where } \mathcal{E}^* \equiv L + \rho F. \quad (14)$$

Since $\dot{\mathcal{E}}_t > 0$ for $\mathcal{E}_t > \mathcal{E}^*$ and $\dot{\mathcal{E}}_t < 0$ for $\mathcal{E}_t < \mathcal{E}^*$, eq.(14) would imply divergence, leading to a violation of the equilibrium conditions, unless the economy jumps immediately to $\mathcal{E}_0 = \mathcal{E}^*$ and stays at $\mathcal{E}_t = \mathcal{E}^*$. This in turns implies $L_{Xt} = (1 - 1/\sigma)\mathcal{E}^* \equiv L_X^*$ and $L_{Rt} = Fg_t = L - L_X^*$ are all constant along the only equilibrium path and the economy stays on the balanced growth path, as long as the parameters are such that the R&D sector is active: $L_R^* = L - L_X^* = L - (1 - 1/\sigma)\mathcal{E}^* > 0 \Leftrightarrow M \equiv \sigma/(\sigma - 1) > 1 + \rho F/L$. Hence, we have:

Proposition 1A: Balanced Growth Path under CES

Suppose $M \equiv \sigma/(\sigma - 1) > 1 + \rho F/L$. Then, the economy jumps immediately to the balanced growth path along which

$$\begin{aligned} L_{Xt} &= L_X^* = \left(1 - \frac{1}{\sigma}\right) (L + \rho F) = \frac{L + \rho F}{M} < L; \\ L_{Rt} &= L_R^* = \frac{L}{\sigma} - \left(1 - \frac{1}{\sigma}\right) \rho F = \left(1 - \frac{1}{M}\right) L - \frac{\rho F}{M} > 0; \\ g_t &= g^* = \frac{L}{\sigma F} - \left(1 - \frac{1}{\sigma}\right) \rho = \left(1 - \frac{1}{M}\right) \frac{L}{F} - \frac{\rho}{M} > 0. \end{aligned}$$

From Proposition 1A, one could immediately show

Proposition 1B: Comparative statics under CES

In the benchmark CES case,

- i) Both an increase in the discount rate ρ and in the R&D cost F leave the markup rate M unchanged, increase L_X^* , decrease L_R^* , and decrease the growth rate g^* ;
- ii) An increase in the total labor supply $L = hN$ leaves the markup rate M unchanged, and increases L_X^* , L_R^* , and g^* ;
- iii) An increase in the elasticity of substitution σ decreases the markup rate M , increases L_X^* , and decreases L_R^* , and g^* .

Table 1 summarizes Proposition 1B.

Table 1: Comparative Statics under CES (Proposition 1B)

	M	L_X^*	L_R^*	$g^* = L_R^*/F$
$\rho \uparrow$	0	+	—	—
$F \uparrow$	0	+	—	—
$L = hN \uparrow$	0	+	+	+
$\sigma \uparrow$	—	+	—	—

Three features of these results under CES deserve special emphasis. First, the per capita labor endowment h and the population size N enter in the law of motion for $\mathcal{E}_t$, eq.(14), as well as the expressions for $L_R^* = L - L_X^*$ and g^* only through their product $L = hN$. In short, what matters is the aggregate market size, not its composition. Once the country size, $L = hN$, is controlled for, a richer country with a higher h and a lower N innovates as much as a poorer country with a lower h and a higher N . This also implies that the (demand-side) market size effect is indistinguishable from the (supply-side) scale effect. This property is due to the homotheticity of preferences and does not hold under nonhomothetic preferences. Second, the production cost, ψ_t , has no effect on the aggregate dynamics. This neutrality of ψ_t is due to the exogeneity of the markup rate, which depends solely on the preference parameter σ . Thus, the share of the aggregate expenditure accruing to the firms' profits never change when the production cost changes. Hence, the production cost never affects incentive to innovate, and hence it has no impact on the labor allocation between the production and R&D sectors. This is the reason why we left it unspecified in this section.⁷ Again, this feature will disappear as we depart from CES. Third, the markup rate changes only with a variation in σ , which leads to a positive correlation between the markup and growth rates. This result is at odds with some empirical evidence suggesting that competition fosters innovation and growth, if we use the markup rate as an inverse measure of competition, as commonly done.

3. Innovation and Growth under Directly Explicitly Additive (DEA) Preferences

⁷Note however that the time path of ψ_t affects that of x_t (and hence the welfare), because $L_X^* = \psi_t N x_t V_t$. For example, if we assume $\psi_t = \psi$, as in Grossman and Helpman (1993), x_t must shrink at the rate equal to g^* , so that $x_t V_t$ stays constant. Instead, if we assume $\psi_t = \psi/V_t$, as in Gancia and Zilibotti (2005), x_t stays constant.

We now depart from CES preferences and consider a broader class of DEA preferences, which admits CES as a limit case.

3.1. Intratemporal problem

The intratemporal preferences are called *directly explicitly additive* (DEA) if the *direct* utility function is *explicitly additive*:

$$U(\mathbf{x}_t) \equiv \int_0^{V_t} u(x_t(\omega)) d\omega,$$

where the sub-utility function, $u(\cdot)$, satisfies $u(0) = 0$, $u'(x) > 0$, and $u''(x) < 0$. For a technical reason, it is assumed to be thrice-differentiable. The agents maximize this intratemporal utility subject to their intratemporal budget constraint, eq.(1), which yields the inverse demand curve for each product;

$$p_t(\omega) = \frac{u'(x_t(\omega))E_t}{\Delta_t}, \quad (15)$$

where

$$\Delta_t \equiv \int_0^{V_t} u'(x_t(\omega')) x_t(\omega') d\omega'$$

captures the effects of the competing firms in the market.

The firms choose $p_t(\omega)$ or $q_t(\omega) = Nx_t(\omega)$ to maximize the profit $\pi_t(\omega) = (p_t(\omega) - w_t\psi_t)Nx_t(\omega)$ subject to eq.(15), taking w_t, ψ_t, Δ_t , and E_t as given, which yields the pricing rule:

$$\frac{u'(x_t(\omega))E_t}{\Delta_t} \left(1 - \frac{1}{\sigma(x_t(\omega))}\right) = p_t(\omega) \left(1 - \frac{1}{\sigma(x_t(\omega))}\right) = w_t\psi_t,$$

or

$$\frac{u'(x_t(\omega))E_t}{M(x_t(\omega))\Delta_t} = w_t\psi_t, \quad (16)$$

where

$$\sigma(x) \equiv -\frac{u'(x)}{xu''(x)} > 1$$

is the price elasticity, and $M(x) \equiv \sigma(x)/[\sigma(x) - 1] > 1$ is the markup rate.⁸

⁸Without the condition, $\sigma(x) > 1$, the firm's profit-maximization problem, its pricing rule, eq.(16), and its markup rate, $M(x) > 1$ would not be well-defined. It is equivalent to assuming that the firm's revenue, $p_t(\omega)q_t(\omega) = u'(x_t(\omega))x_t(\omega)(NE_t/\Delta_t)$ is increasing in $x_t(\omega)$, and hence its marginal revenue, the LHS of eq.(16), is positive.

A distinctive feature of monopolistic competition under DEA is that the price elasticity of demand each firm faces, $\sigma(x)$, is a function of per capita consumption of its product and nothing else. Notice that, for any differentiable price elasticity function $\sigma(\cdot) > 1$, one could define the sub-utility function as $u(x) = \int_0^x \exp \left[- \int_{y_0}^y \frac{ds}{s\sigma(s)} \right] dy$, which satisfies $u(0) = 0$; $u'(x) > 0$, $u''(x) < 0$, and is thrice-differentiable. Hence, one could also use the price elasticity function $\sigma(\cdot)$ as the primitive of the DEA preferences. Clearly, CES is a special case, where $\sigma(x) = \sigma > 1 \Leftrightarrow u(x) = A(x)^{1-\frac{1}{\sigma}}$, with A being a positive constant.

In what follows, we consider the subclass of DEA preferences that satisfy the following two assumptions:

Assumption (D1):

$$\frac{1}{\sigma(x)} + \frac{xM'(x)}{M(x)} > 0$$

This inequality is equivalent to assuming that $u'(x)/M(x)$ is decreasing in x . In words, the firm's marginal revenue, the LHS of eq.(16), is decreasing in $x_t(\omega)$. Hence, eq.(16) has a unique solution, $x_t(\omega) = x_t$, which is decreasing in $w_t\psi_t\Delta_t/E_t$. This implies the symmetry of equilibrium across firms and products, $p_t(\omega) = p_t$, $q_t(\omega) = q_t$, and $\pi_t(\omega) = \pi_t$. Furthermore, (D1) ensures that the balanced growth path is the only equilibrium path of the economy: see Proposition 2A.

Assumption (D2):

$$\sigma'(x) < 0 \Leftrightarrow M'(x) > 0$$

In words, the demand for each product becomes more price elastic as one moves up along the demand curve, eq.(15), (i.e., for a higher price/a lower quantity), which is sometimes called “Marshall's Second Law of Demand”. (D2) is also equivalent to assuming that the elasticity of substitution between any two products, ω_1 and ω_2 evaluated at $x(\omega_1) = x(\omega_2) = x$ is decreasing in x .⁹ Obviously, CES is a borderline case of the subclass of DEA satisfying (D2). Note also that (D2) implies (D1).

⁹ Zhelobodkho et.al. (2012) studied what they called “relative love for variety (RLV),” which is nothing but the *inverse* of $\sigma(\cdot)$. Thus, (D2) is equivalent to assuming that RLV is increasing.

(D2) plays a crucial role in the comparative static results in Proposition 2B. It is thus important to understand its empirical implications. Since (D2) implies (D1), the unique solution of eq.(16), $x_t(\omega)$, is strictly decreasing in $\psi_t \Delta_t$. Hence, we can show:

- i) *Imperfect Pass-Through*: A higher production cost, ψ_t , reduces $x_t(\omega)$. This leads to a lower markup rate, $M(x_t(\omega))$ under (D2);
- ii) *Strategic Complementarity in Pricing*: If the competitors reduced their prices and increased their sales, Δ_t would go up, which would reduce $x_t(\omega)$. This would lead to a lower markup rate, $M(x_t(\omega))$, and a lower price, $p_t(\omega)$ under (D2);
- iii) *Procompetitive Entry*: The presence of more firms, an exogenous increase in V_t would lead to a higher Δ_t , which would reduce $x_t(\omega)$. This would lead to a lower markup, $M(x_t(\omega))$ under (D2),

for which there exists ample empirical evidence, as already discussed in the introduction. To further underscore the importance of (D2), consider the implications of maintaining (D1) but assuming the opposite of (D2), so that $\frac{1}{\sigma(x)} > -\frac{xM'(x)}{M(x)} > 0$. Then, a decline in $x_t(\omega)$, caused by an increase in ψ_t or Δ_t , would lead to an increase in $M(x_t(\omega))$ with $M'(x) < 0$. Thus, it would imply more than 100% pass-through, strategic substitutes in pricing, and anti-competitive entry, contrary to the empirical evidence. This is the reason why we depart from CES in the direction of (D2). It is worth noting that, in other classes of non-CES preferences, assuming Marshall's Second Law of Demand, i.e., demand for each product is more price elastic at a higher price, may not imply strategic complementarity in pricing, and procompetitive entry.¹⁰ This is another reason why we use DEA.

Because (D1) ensures the symmetry of equilibrium, the pricing rule now becomes

$$p_t \left(1 - \frac{1}{\sigma(x_t)}\right) = w_t \psi_t \Leftrightarrow p_t = M(x_t) w_t \psi_t. \quad (17)$$

Using this expression, and following the same steps as in the CES case, the profit share in the aggregate expenditure, eq.(4) now becomes

$$\frac{\pi_t V_t}{NE_t} = \frac{1}{\sigma(x_t)} \equiv 1 - \frac{1}{M(x_t)}, \quad (18)$$

¹⁰ For example, in a class of preferences used in Boucekine et.al. (2017), the pricing rule of each firm is *independent* of the pricing behaviors of other firms or the number of firms competing.

while the share of the wage payment to the production sector in the aggregate expenditure, eq.(5) becomes

$$\frac{w_t L_{xt}}{NE_t} = 1 - \frac{1}{\sigma(x_t)} \equiv \frac{1}{M(x_t)}. \quad (19)$$

Note that departing from CES to DEA does not change the relations between these shares and the markup rate. However, these shares are endogenous under DEA, because the markup rate is a function of per capita consumption of each product, which is increasing under (D2).

3.2. Intertemporal problem

Under DEA, the intertemporal utility is now given by

$$u_0 = \int_0^\infty \log \left(V_t u \left(\frac{E_t}{p_t V_t} \right) \right) e^{-\rho t} dt,$$

which agents maximize subject to the intertemporal budget constraint eq.(9). This leads to the first-order condition given by

$$\frac{\zeta(x_t)}{E_t} e^{-\rho t} = \lambda_0 e^{-Rt},$$

where $\zeta(x) \equiv u'(x)x/u(x) > 0$, while λ_0 is again the Lagrange multiplier associated with eq.(9). Log-differentiating this first-order condition with respect to t yields an augmented Euler equation,

$$\frac{\dot{E}_t}{E_t} = r_t - \rho + \frac{\dot{\zeta}(x_t)}{\zeta(x_t)} = r_t - \rho + \left[1 - \frac{1}{\sigma(x_t)} - \zeta(x_t) \right] \frac{\dot{x}_t}{x_t}, \quad (20)$$

which features an additional term, which is absent in the original Euler equation, eq.(10).¹¹

3.3. Balanced Growth Path

Following the same definition for the BGP as before, a constant markup rate requires that x_t must be constant. Furthermore, from $L_{xt} = \psi_t N x_t V_t$, a constant x_t and a constant L_{xt} requires that $\psi_t V_t$, must be constant. Thus, in order to ensure the existence of a BGP, it is now necessary to assume along Gancia and Zilibotti (2005) that knowledge spillovers improve productivity not only in R&D but also in production, as $\psi_t = \psi/V_t$.¹²

¹¹ For CES, $\sigma(x_t) = \sigma$ and $\zeta(x_t) = 1 - 1/\sigma$, so that the last term disappears.

¹² This assumption has been made in other horizontal innovation models of endogenous growth, such as Foellmi and Zweimuller (2006). Note also that it is isomorphic to assuming that $F_t = F$, $\psi_t = \psi$, and $h_t = hV_t$.

Again, we derive the law of motion for the economy. Following the same steps as in the CES case, but noticing that the augmented Euler equation, eq.(20) now has the additional term, eq.(12) is now modified to:

$$\frac{\dot{\mathcal{E}}_t}{\mathcal{E}_t} = \frac{\mathcal{E}_t}{\sigma(x_t)F} - g_t - \rho + \left[1 - \frac{1}{\sigma(x_t)} - \zeta(x_t)\right] \frac{\dot{x}_t}{x_t},$$

while eq.(13) is modified to

$$L = L_{Rt} + L_{Xt} = Fg_t + \psi Nx_t.$$

Combining these two equations, and using eq.(18), $\mathcal{E}_t \equiv NE_t/w_t = N\psi x_t M(x_t)$, we obtain the following law of motion for x_t :

$$\left[\zeta(x_t) + \frac{1}{\sigma(x_t)} + \frac{x_t M'(x_t)}{M(x_t)} \right] \frac{\dot{x}_t}{x_t} = \frac{N\psi x_t M(x_t) - (L + \rho F)}{F}. \quad (21)$$

(D1) implies that the bracket term in front of $\dot{x}_t/x_t$ on the LHS of eq.(21) is positive. (D1) also implies that $x_t M(x_t)$ is increasing in x_t . Thus, $\dot{x}_t > 0$ for $x_t > x^*$ and $\dot{x}_t < 0$ for $x_t < x^*$, where x^* is defined implicitly by

$$x^* M(x^*) = \frac{L + \rho F}{N\psi} = \frac{h + \rho F/N}{\psi}. \quad (22)$$

Thus, eq.(21) would imply divergence, leading to a violation of the equilibrium conditions, unless the economy jumps immediately to $x_0 = x^*$ and stays at $x_t = x^*$. This in turns implies $L_{Xt} = \psi Nx^*$ and $L_{Rt} = Fg_t = L - L_{Xt} = L - \psi Nx^*$ are all constant along the only equilibrium path and the economy stays on the balanced growth path, as long as the parameters are such that the R&D sector is active: $L_{Rt} = Fg_t = L - \psi Nx^* = N(h - \psi x^*) > 0$. Since (D1) implies that the LHS of eq.(22) is increasing in x^* , this condition can be written as $M(h/\psi) > 1 + \rho F/L$.

Hence, we have:

Proposition 2A: Balanced Growth Path under DEA with (D1)

Suppose $M(h/\psi) > 1 + \rho F/L$. Then, the economy jumps immediately to the balanced growth path along which

$$\begin{aligned} L_{Xt} &= L_X^* = \left(1 - \frac{1}{\sigma(x^*)}\right) (L + \rho F) = \frac{L + \rho F}{M(x^*)} < L; \\ L_{Rt} &= L_R^* = \frac{L}{\sigma(x^*)} - \left(1 - \frac{1}{\sigma(x^*)}\right) \rho F = \left(1 - \frac{1}{M(x^*)}\right) L - \frac{\rho F}{M(x^*)} > 0; \\ g_t &= g^* = \frac{L}{\sigma(x^*)F} - \left(1 - \frac{1}{\sigma(x^*)}\right) \rho = \left(1 - \frac{1}{M(x^*)}\right) \frac{L}{F} - \frac{\rho}{M(x^*)} > 0, \end{aligned}$$

where x^* is defined implicitly by eq.(22).

Note that this proposition requires (D1), but not (D2). Note also that, by comparing Proposition 1A and Proposition 2A, departing from CES to DEA does not alter the functional relations between L_X^* , L_R^* , g^* on one hand and $\sigma(x^*)$ and $M(x^*)$ on the other hand. The *only but significant* difference is that, under DEA, the parameters, ρ, F, h, N , and ψ affect $\sigma(x^*)$ and $M(x^*)$ through eq.(22).

From Proposition 2A, it is straightforward to show:

Proposition 2B: Comparative Statics under DEA with (D2)

Under DEA preferences with (D2),

- i) Both an increase in the discount rate ρ and in the R&D cost F increase per capita per product consumption x^* and the markup rate $M(x^*)$, increase L_X^* , and decrease L_R^* and the growth rate g^* ;
- ii) An increase in the population size N decreases x^* and $M(x^*)$, increases L_X^* , L_R^* , and g^* ;
- iii) An increase in per capita labor endowment h increases x^* and $M(x^*)$, increases L_X^* , L_R^* , and g^* ;
- iv) Both an increase in N (a decrease in h) for a fixed $hN = L$, and an increase in ψ decrease x^* and $M(x^*)$, increase L_X^* , and decrease L_R^* and g^* .

Table 2 summarizes Proposition 2B. The signs in the shaded boxes are the consequences of (D2): $\sigma'(x) < 0$ ($M'(x) > 0$). If we had instead assumed $\sigma'(x) > 0$ ($M'(x) < 0$), there would be the opposite signs in the shaded boxes. Under CES, they would be “0”.

Table 2: Comparative Statics under DEA with (D2): Proposition 2B

	x^*	$M(x^*)$	$L_X^* = \psi N x^*$	L_R^*	$g^* = L_R^*/F$
$\rho \uparrow$	+	+	+	—	—
$F \uparrow$	+	+	+	—	—
$N \uparrow$	—	—	+	+	+
$h \uparrow$	+	+	+	+	+
$N = L/h \uparrow$, fixed L	—	—	+	—	—
$\psi \uparrow$	—	—	+	—	—

An immediate corollary of Proposition 2B is:

Corollary: Correlations between the Markup and Growth Rates under DEA with (D2)

Under DEA preferences with (D2),

- v) A change in the discount rate ρ , the R&D cost F , or the population size N causes the markup rate $M(x^*)$ and the growth rate g^* to move in the opposite direction.
- vi) A change in per capita labor endowment h or the production cost ψ causes the markup rate $M(x^*)$ and the growth rate g^* to move in the same direction.

We now discuss the implications of departing from CES within DEA in the direction of (D2), by comparing Proposition 1B with Proposition 2B and its corollary.

Just as in CES, both an increase in ρ and in F discourage R&D, which causes the reallocation of labor from the R&D sector to the production sector. This causes an increase in x^* , per capita consumption of each product. Unlike in CES, this increases the markup rate $M(x^*)$ under (D2). This secondary effect mitigates the impact on labor reallocation, but not enough to overturn it. However, this causes the negative correlations between the markup rate $M(x^*)$ and the innovation and growth rate, g^* . In particular, this implies that the measure of competitiveness and the growth rate are positively correlated in cross-section of countries, if countries differ mostly in the innovation (or firm entry) cost.

Just as in CES, both an increase in N and in h , by increasing the total labor supply, lead to an increase in the labor supply to both the production and the R&D sectors. The latter leads to an increase in the growth rate, due to the familiar scale effect. However, as seen in eq.(22), they have opposite impacts on x^* and hence on $M(x^*)$ under (D2). A higher N leads to a lower x^* , which in turn leads to a lower $M(x^*)$ under (D2), mitigating the effect on the growth rate as well as generating the negative correlations between $M(x^*)$ and g^* . In contrast, a higher h leads to a higher x^* , which in turn leads to a higher $M(x^*)$ under (D2), amplifying the effect on the growth rate as well as generating the positive correlations.

To see such differential effects of changes in h and in N more clearly, consider the effect of increasing N and decreasing h simultaneously to keep the total labor supply $L = hN$ unchanged. This removes the scale effect. Without a change in L , an increase in N would necessitate a decline in per capita consumption of each product. This would have no impact on the markup rate and the allocation of labor between the production and R&D sectors under CES. However, under (D2), this causes the markup rate to decline, reducing the incentive to innovate, which causes the reallocation of labor from the R&D sector to the production sector, and a decline in the innovation and growth rates. This result suggests that, once the aggregate market

size is controlled for, richer countries with smaller population sizes innovate more and hence grow faster than poorer countries with larger population sizes.

Indeed, this effect of an increase in N without an increase in L is completely isomorphic to the effect of an increase in ψ , as clearly seen from eq.(22). Without a change in L , a higher ψ , just like a higher N , necessitates per capita consumption of each product to decline. This change would be neutral under CES. Under (D2), however, it leads to a decline in the markup rate through imperfect pass-through, which discourages R&D, causing labor to reallocate to the production sector and the growth rate to decline.

It is worth pointing out that, though Proposition 2A does not require (D2), the comparative statics results reported in Proposition 2B and its corollary depend on (D2). If we had departed from CES in the opposite direction, many of these results would be overturned, as indicated by the shaded area in Table 2. That is why we have disciplined our departure from homotheticity by the empirical evidence of imperfect pass-through, strategic complementarity in pricing and procompetitive effects of entry.

4. Concluding Remarks

In the standard horizontal innovation model of endogenous growth, larger economies innovate more and hence grow faster. Due to the homotheticity of consumer demands, however, it does not matter whether the large market size comes from a large population or a high per capita expenditure. In this paper, we extended the standard textbook model by building on the Dixit-Stiglitz (1977; Section II) model of monopolistic competition with directly explicitly additive (DEA) nonhomothetic preferences. In spite of nonhomotheticity, the model preserves the balanced growth property of the standard model. Furthermore, we discipline our departure from homotheticity by the empirical evidence of incomplete pass-through, strategic complementarity in pricing, and procompetitive entry. Among others, it has been shown that, once the aggregate market size is controlled for, richer countries with smaller populations innovate more and grow faster, which suggests that the difference in per capita income across countries could potentially be one reason why there is little supporting evidence for the scale effect. It has also been shown that the correlations between the markup and growth rates across the balanced growth paths could be either positive or negative, depending on the source of variations. In particular, the measure of competitiveness and the growth rate are positively

correlated in cross-sections of countries, if countries differ mostly in the innovation (or firm entry) cost.

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