(Un)stable vertical collusive agreements

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Abstract. In this paper, we extend the concept of stability to vertical collusive agreements involving downstream and upstream firms, using a setup of successive Cournot oligopolies. We show that a stable vertical agreement, the unanimous vertical agreement involving all downstream and upstream firms, always exists. Thus, stable vertical collusive agreements exist even for market structures in which horizontal cartels would be unstable. We also show that there are economies for which the unanimous agreement is not the only stable one. Furthermore, the Stigler statement according to which the only ones who benefit from a collusive agreement are the outsiders need not be valid in vertical agreements.

Résumé. Accords collusifs verticaux stables et instables. Ce texte utilise une approche en termes d'oligopoles à la Cournot successifs pour développer le concept de stabilité et l'appliquer aux accords collusifs verticaux impliquant des firmes en amont et en aval. On montre qu'un accord vertical stable existe toujours : l'accord vertical unanime impliquant toutes les firmes en amont et en aval. Des accords collusifs verticaux existent même pour des structures de marché dans lesquels les cartels horizontaux seraient instables. On montre aussi qu'il existe des économies pour lesquelles l'accord unanime n'est pas la seule solution stable. De plus, l'assertion de Stigler à savoir que les seuls qui bénéficient d'un accord collusif sont les acteurs de l'extérieur de ces accords n'est pas nécessairement valide pour les accords verticaux.

JEL classification: D43, L13

1. Introduction

In the present paper, we study the existence of stable *vertical collusive agreements* in the context of successive oligopolies. Such collusive agreements simultaneously embody downstream and upstream firms. Collusion is represented as an agreement through which the insiders act in unison, reducing thereby the total number of decision units operating in the downstream and upstream markets and, thus, the corresponding number of oligopolists in each of them. Collusive

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outcomes are the Cournot equilibria corresponding to these reduced numbers of oligopolists, which are then compared with those arising when downstream and upstream firms act independently from each other in their respective markets.

This definition of stability is a direct extension of the definition of stability in d'Aspremont et al. (1983). It requires that no firm in the entity, whether an upstream or a downstream firm, would get more when leaving the entity than when staying inside (internal stability), taking into account the change in profits resulting from its move. Furthermore, it requires that no firm outside the entity would obtain more when entering the entity than staying outside, again taking into account the change in profits resulting from its move (external stability).

More than half a century ago, Stigler (1950) stressed the main difficulty encountered by a cartel promoter: "...the major difficulty in forming a merger is that it is more profitable to be outside a merger than to be a participant. The outsider sells at the same price but at the much larger output at which marginal cost equals price. Hence, the promoter of a merger is likely to receive much encouragement from each firm, almost every encouragement, in fact, except participation." This sentence clearly illustrates the need for careful analysis of the conditions under which a cartel is expected to resist the forces acting against its stability.

A definition of cartel stability relying on two natural requirements, namely, external stability and internal stability, was proposed by d'Aspremont et al. (1983) in the context of horizontal mergers. A cartel with K firms in an industry embodying n firms ($n \geqslant K$) is said to be *internally stable* when the profits realized by each firm that is a member of the cartel exceed the profits obtained when outside of it, *taking into account the change in the profits of an outsider resulting from this exit.* Similarly, a cartel with K members is *externally stable* when the profits of a firm member of a cartel of size $K+1$ are smaller than the profits realized by an outsider when the cartel is of size K. A cartel of size K is stable when it is both internally and externally stable. Formally, assuming that all firms are identical, and defining $\Pi^f(K)$ (resp. $\Pi^c(K)$), i.e. the payoff received by each outsider (resp. by each cartel member), a cartel of size K is stable if both the inequalities:

$$\Pi^c(K) \geqslant \Pi^f(K-1)$$

(internal stability) and

$$\Pi^c(K+1) \leqslant \Pi^f(K)$$

(external stability) hold simultaneously.

This definition of stability is rather abstract since it does not state how the profits of an insider or an outsider are defined or, to which market structure it corresponds. As a consequence, the above abstract definition can be applied to a wide variety of market situations and corresponding payoffs' structures. Nevertheless, the definition of stability assumes that each member of the agreement receives the same share of profits, $\Pi^c(K)$ and, similarly, that each outsider obtains an equal amount of profits, $\Pi^f(K)$. One way to rationalize this assumption consists in supposing that all firms in the industry are identical, with the sole exception that they either participate in the collusive agreement or decide to remain outsiders. This assumption then allows sharing of profits equally among members inside the entity through an argument of equal treatment. Furthermore, supposing an identical strategic behaviour for the outsiders allows us to state, by an argument of symmetry, that each one of them should also obtain an equal amount of profits at the solution. With these assumptions, given an entity of size K, profit sharing among firms is fully described by two numbers: the share of profits received by each participant, $\Pi^c(K)$, and the profits received by each outsider, $\Pi^f(K)$.

A first example of market structure and associated payoffs for which stability has been analyzed can be found in the paper referred to above (d'Aspremont et al. 1983). The solution at which the profits are evaluated is that corresponding to the *price leadership* model introduced by Markham (1951). In this version, the cartel (collusive agreement) is assumed to be the price-leader (the dominant firm) maximizing its profits on the "residual demand function," while the outsiders are behaving competitively, taking the price set by the cartel as given. It is easily seen that, for this specific market situation and payoff's structure, any cartel is always internally unstable simply because, according to the argument put forward by Stigler (1950), the per-firm profits of a cartel member are smaller than the profits obtained by each outsider. However, this remark does not prevent the existence of at least one stable cartel, as demonstrated in d'Aspremont et al. (1983).

A second example of market structure in which a collusive agreement is considered corresponds to the version of the Cournot model proposed by Salant et al. (1983). Compared with the price-leadership model, this approach consists in assuming that a collusive agreement among K firms takes place, according to which these firms maximize per-firm profits against the output choice of the outsiders. This situation represents the Cournot equilibrium of the game consisting of the entity and $n-K$ outsiders. In this setup, stability has been analyzed by Shaffer (1995) and Belleflamme and Peitz (2010). Assuming linear inverse demand and constant marginal cost, Shaffer (1995) shows that, whenever $n \geqslant 3$, there exists no stable horizontal cartel in the Cournot game. In a recent paper, Zu et al. (2012), borrowing from Konishi and Lin (1999), study the size of horizontal stable cartels using general specifications for the output demand and cost function. These authors confirm that, even with a more general specification, the size of stable horizontal cartels remains quite reduced. The main reason why cartel stability fails in Cournot competition refers to the Stigler statement at the beginning of this paper—outsiders are always better off than insiders, thereby destroying internal stability in the Cournot game. Accordingly, contrary to the price leadership model in which there always exists a stable cartel, the Cournot game with linear output demand and constant marginal cost never has a stable cartel when the number of firms exceeds three.

The above studies all refer to collusive agreements embodying downstream firms only, excluding thereby more general forms of collusive agreements, like those arising when some downstream and upstream firms are allowed to combine together. Extending stability analysis to such collusive agreements is undoubtedly interesting and important. Real-life collusive entities often share the property that, among the participating firms, some of them operate in the upstream market(s) and produce the input(s) used by the downstream firms in the production of the final good. Such agreements have been studied by Salinger (1988), Gaudet and van Long (1996) and, more recently, by Gabszewicz and Zanaj (2011). However, to the best of our knowledge, the analysis of collusive stability in this context has not yet been pursued. Probably this is so because some assumptions used in the traditional approach seem inappropriate in this new setup. Among these assumptions is the one stating that each member of the agreement must receive the same share of the entity's profits. By the very nature of the problem, the firms participating in the entity belong to different types, some producing the final good (downstream firms) while the others produce the input (upstream firms). When downstream and upstream firms are allowed to combine together, with both upstream and downstream firms in the collusive agreement, there is no longer any reason to assume that both types of firms should get the same share of the entity's profits. Thus a conceptual problem arises: how should profits be shared among the members of the collusive entity, knowing that these members are not all identical but belong to two different types? We address this difficulty in the present paper by requiring that the redistribution of total profits of the entity among its members guarantees to each of them an amount of profits at least as high as the profits it would obtain when leaving the entity, taking into account the change in profits resulting from its move. In this paper, we extend the definition of stability from pure horizontal to vertical collusive agreements by requiring that internal and external stability hold for both types of firms in the agreement, namely downstream and upstream firms, simultaneously. In this paper, stability is analyzed for this definition, introduced formally below.

A *vertical* collusive agreement has three effects: (i) it softens double marginalization for the entity, boosting its profit; (ii) it reduces the number of active upstream firms in the upstream market leading to an upward shift of the input supply, and it reduces the number of downstream firms that buy the input in the input market, causing a downward shift of the input demand schedule. The balance of these two shifts can lead to an increase or a decrease of the equilibrium input price; and (iii) it creates asymmetries in the production costs of the downstream firms: the entity produces at the marginal cost while the downstream firms produce at the market price. Thus, the effect on the equilibrium output price is ambiguous. The equilibrium output price decreases if the output quantity increases due to the presence of the entity, or it may decrease, otherwise. The stability of the vertical entity depends on the balance of these three effects. It is clear that the combination of these effects leads to extremely complex consequences that are difficult to disentangle. However, we are able to show that, in the class of linear economies (linear demand function in the downstream market and constant returns to scale in production), the vertical agreement involving *all* firms, downstream and upstream, always constitutes a stable vertical agreement. We also provide examples of some economies, in which this agreement is not the only one to be stable: we provide an example in which both the unanimous agreement and another one, involving a strictly smaller number of firms, are simultaneously stable.

To conclude, while the literature on the stability of vertical collusive agreements is not developed yet, there is a related literature to our paper on the sustainability of a cartel in vertically related industries. Two important contributions in this literature are Nocke and White (2007) and Piccolo and Miklos-Thal (2012). Nocke and White (2007) analyze the collusive agreement among upstream firms, some of which might be vertically integrated with downstream firms. In their paper, vertical mergers have a positive impact on the upstream collusion. Piccolo and Miklos-Thal (2012) study collusive agreements among downstream firms that collude on their input supply contracts. They show that an implicit agreement on input supply contracts with above-cost wholesale prices and slotting fees facilitates collusion in the downstream market.

2. Stability of vertical agreements and successive oligopolies

2.1. The definition of stability

We call *vertical* agreement any collusive entity involving both downstream and upstream firms simultaneously. In order to examine the question of stability in the case of vertical agreements, we need a framework in which these are analyzed and firms' payoffs defined. This framework is provided in Salinger (1988) and, more recently, by the authors in Gabszewicz and Zanaj (2011), in which they propose a definition of *successive oligopolies* that allows a precise concept of vertical agreement. The nature of the agreement concerns the payoff division among downstream and upstream participants and the behaviour of the insider upstream firms with respect to the input market. As for the payoff division, the profit-sharing rule consists of any rule guaranteeing to each member of the entity at least as much as it would receive when being outside of it, taking into account the resulting change in the profits. More specifically, we assume that each participant receives the profit of an outsider firm plus a profit "bonus" that should in principle guarantee the interest to participate in the agreement. In this paper, we assume that upstream participants in the vertical agreement do not participate directly in the upstream market but instead foreclose the outsider downstream firms.

To define precisely the notion of stability of vertical agreements, consider two successive markets embodying n identical downstream firms and m identical upstream firms, $n, m \geqslant 2$. Assume that $K, K \leqslant n$, downstream firms and $H, H \leqslant m$, upstream firms decide to collude. For further use, we define the specific vertical agreement in which $K = n$ and $H = m$ as the *unanimous vertical agreement*: this vertical agreement involves all firms operating in either of the successive markets. Notice that this entity now involves *two* types of agents, all identical in each type.

An entity of size $K+H$ is stable when it is both internally and externally stable, for each type of agents. Define formally $\Pi^f(K,H)$ (resp. $\Pi^c(K,H)$) as the payoff received by each downstream outsider (resp. by each participant), and define $\Gamma^f(K,H)$ (resp. $\Gamma^c(K,H)$) as the payoff received by each upstream outsider (resp. by each participant). Then:

DEFINITION 1. *A vertical entity of size $K+H$ is stable if both the sets of inequalities*

$$\Pi^c(K,H) \geqslant \Pi^f(K-1,H) \text{ and } \Gamma^c(K,H) \geqslant \Gamma^f(K,H-1) \tag{1}$$

(internal stability) and

$$\Pi^c(K+1,H) \leqslant \Pi^f(K,H) \text{ and } \Gamma^c(K,H+1) \leqslant \Gamma^f(K,H) \tag{2}$$

(external stability) hold simultaneously.

This definition directly extends the definition of stability provided by d'Aspremont et al. (1983) to agreements that include two types of firms. More precisely, a vertical collusive agreement embodying $K+H$ is said *internally stable* when the profits realized by each type of (downstream and upstream) firm member of the entity exceed the profits obtained when being outside of it, taking into account the change in the profits of an outsider resulting from this exit. Similarly, a vertical entity with $K+H$ members is *externally stable* when the profits of a (downstream or upstream) firm member of an entity of size $K+1$ and/or $H+1$ are smaller than the profits realized by a (downstream or upstream) outsider when the entity is of size $K+H$. This definition of stability then requires four conditions to be satisfied.

2.2. The linear model

Now let us apply this definition of stability to the well-known case of linear output demand and constant returns to scale in successive Cournot oligopolies. This is the most common setup in the existing literature that studies vertical mergers using successive oligopolies. The same assumptions are also often used in the literature of horizontal mergers. Hence, using this model will allow us to make easy comparisons between stability in horizontal and vertical mergers. Let the demand function for some output in the downstream market be given by $p(Q)=1-Q$, where Q denotes aggregate supply. Consider n *downstream* firms producing the output *via* a constant returns technology $f(z)=\alpha z, \alpha>0$, as well as m *upstream firms* initially supplying the market for the input z at a constant marginal cost equal to $\beta, \beta>0$. Now assume that H upstream firms, $h=1,2..H$, form a vertical collusive agreement with K downstream firms, $k=1,2,\ldots K$, and maximize joint profits together. After this agreement, the downstream and upstream markets move from an initial situation with n active downstream firms and m active upstream firms to a market structure with $n-K+1$ active firms in the downstream market and $m-H$ in the upstream one.[1]

1 An example of this type of vertical collusive agreement is the agreement taking place in a market among one or several wholesalers with one or several retailers that fix the price at which the

Consider first how the profit functions are written in the downstream market after the collusive agreement. To this end, denote by I the entity resulting from the agreement. The profits Π_I of the entity I to which K downstream firms participate are given by:

$$\Pi_I(q_I,q_{-I}) = \Big(1 - q_I - \sum_{i\neq I} q_i\Big) q_I - \beta \frac{q_I}{\alpha}, \tag{3}$$

where q_I (resp. $\Sigma_{i\neq I} q_i$) denotes the supply of the entity (resp. firms not in the agreement) in the downstream market and $\frac{\beta}{\alpha}$ is its unit production cost. As for the downstream firms that do not participate in the agreement, each of them obtains a payoff Π_i defined by:

$$\Pi_i(q_i, q_k) = (1 - q_I - q_i - \sum_{k\neq i} q_k) q_i - \omega\Big(\frac{q_i}{\alpha}\Big), \tag{4}$$

with $i, i \neq I$, and ω denoting the unit price in the input market.[2] Notice that from the comparison between (3) and (4), it immediately becomes apparent that, while the collusive members in the downstream market pay their input at marginal cost β, the rivals pay the input price ω. Since Π_I is concave in q_I, we may use the first order condition to get the best reply function q_I of the entity in the downstream market game as:

$$q_I(q_{i\neq I}) = \frac{1 - \frac{\beta}{\alpha} - \sum_{i\neq I} q_i}{2}.$$

As for an outsider downstream firm i, its best reply q_i in the downstream market is conditional on the input price ω realized in the upstream market, namely:

$$q_i(q_I, q_k, \omega) = \frac{1 - \frac{\omega}{\alpha} - \big(q_I + \sum_{k\neq i, k\neq I} q_k\big)}{2}.$$

Assuming a symmetric equilibrium *among the colluded downstream firms*, we get the resulting Cournot equilibrium in the downstream market, namely, the optimal supply coming from the entity q_I^* and from each of the rivals q_i^* that do not belong to the cartel, namely:

$$q_I^*(K,\omega) = \frac{\alpha - \beta + (n-K)(\omega - \beta)}{\alpha(n-K+2)} \tag{5}$$

and

$$q_i^*(K,\omega) = \frac{\alpha + \beta - 2\omega}{\alpha(n-K+2)}. \tag{6}$$

It is worth noting that the equilibrium in the downstream market depends on the input price obtained in the upstream market as an immediate consequence

market product is sold to the final consumers. For example, the French Competition Council in 2008 sanctioned five toy manufacturers and three distributors on the grounds of collusion during the Christmas period between 2001 and 2004 (La Revue 2008).

2 Notice that the set $\{k : k \neq i\}$ includes the index I.

of supply and demand for the input. Taking into account (6) and the fact that $q = f(z) = \alpha z$, it is easy to derive the input demand resulting from the $n-K$ outsider firms in the downstream market, i.e., $\Sigma_{i \neq I} z_i(\omega) = (n-K)\left(\frac{\beta+\alpha-2\omega}{\alpha^2(n-K+2)}\right)$. As for the input supply, it comes from the strategies $s_j, j \neq I$, selected by the outsider upstream firms in the input market. Consider the j_{th} upstream firm not participating in the entity. Its profits Γ_j at the vector of strategies (s_j, s_{-j}) are written as:

$$\Gamma_j(s_j, S_{-j}) = \omega(s_j, S_{-j})s_j - \beta s_j, \tag{7}$$

with $S_{-j} = \Sigma_{-j \neq I} s_{-j}$. Taking into account that $\omega(s_j, s_{-j})$ has to make demand equal to supply in the upstream market, namely, $\Sigma_{j \neq I} s_j = \Sigma_{i \neq I} z_i(\omega)$, we obtain:

$$\omega(s_j, s_{-j}) = \frac{(\alpha+\beta)(n-K) - \alpha^2(n-K+2)\sum_{j \neq I} s_j}{2(n-K)}, \tag{8}$$

where $\Sigma_{j \neq I} s_j = S_{-j} + s_j$. Accordingly, the payoff of the j-th upstream firm is written as:

$$\Gamma_j(s_j, s_{-j}) = \left(\frac{(\alpha+\beta)(n-K) - \alpha^2(n-K+2)\sum_{j \neq I} s_j}{2(n-K)}\right) s_j - \beta s_j.$$

It is immediately possible to derive from the above the best reply function, $s_j = s_j(S_{-j})$. Using the symmetry condition $S_{-j} = (m-H-1)s_{-j}$, we derive the optimal input supply s_j^* coming from the j-th outsider firm, namely:

$$s_j^*(K,H) = \frac{(\alpha-\beta)(n-K)}{\alpha^2(n-K+2)(m-H+1)}.$$

Substituting the expression of s_j^* in (8), we get the equilibrium input price:

$$\omega^*(H) = \frac{\alpha+\beta+2\beta(m-H)}{2(m-H+1)}. \tag{9}$$

Substituting (9) in (5) and (6), we get the output supply of each outsider downstream firm q_i^* and that of the cartel q_I^*, respectively:

$$q_i^*(K,H) = \frac{(m-H)(\alpha-\beta)}{\alpha(n-K+2)(m-H+1)} \tag{10}$$

and

$$q_I^*(K,H) = \frac{(\alpha-\beta)(n-K+2(m-H+1)}{2\alpha(n-K+2)(m-H+1)}.$$

It follows immediately that profits at equilibrium of the entity Π_I^* and of the outsider firms $\Pi_i^*(K,H)$ in the downstream market are written as:

$$\Pi_I^*(K,H) = \frac{(\alpha-\beta)^2}{4\alpha^2}\frac{(2(m-H)+n-K+2)^2}{(n-K+2)^2(m-H+1)^2} \tag{11}$$

and

$$\Pi_i^*(K,H)=\frac{(\alpha-\beta)^2}{\alpha^2}\frac{(m-H)^2}{(n-K+2)^2(m-H+1)^2}, \tag{12}$$

respectively. The profit of an outsider upstream firm is:

$$\Gamma_j^*(K,H)=\frac{(\alpha-\beta)^2}{2\alpha^2}\frac{(n-K)}{(n-K+2)(m-H+1)^2}. \tag{13}$$

Notice for later use that the profits of an upstream and a downstream firm when the entity I is the empty set (namely, $H=K=0$) are given by (for details, see the appendix):

$$\Gamma^*=\frac{(\alpha-\beta)^2}{\alpha^2}\frac{n}{(m+1)^2(n+1)} \text{ and } \Pi^*=\frac{(\alpha-\beta)^2}{\alpha^2}\frac{m^2}{(m+1)^2(n+1)^2}. \tag{14}$$

2.3. Stability in the linear model

Now we are in a position to examine stability properties for vertical agreements in the model we have just reviewed. In the next proposition, we show that the unanimous vertical agreement plays a crucial role in the analysis.

Remember that the profit-sharing rule consists of any rule guaranteeing to each member of the entity at least as much as it would receive when being outside of the entity, taking into account the resulting change in the profits. Given the total profits of the entity $\Pi_I^*(K,H)$, such a rule is equivalent to distributing at least an amount of profits $K*\Pi_i^*(K-1,H)$ to the insiders downstream firms and an amount of profits at least equal to $H*\Gamma_j^*(K,H-1)$ to insiders upstream firms. Translated into the linear model described above, the condition for internal stability is thus rewritten as:

$$\Pi_i^*(K-1,H)+\frac{\Pi_I(K,H)-[K*\Pi_i^*(K-1,H)+H*\Gamma_j^*(K,H-1)]}{K+H} \geqslant \Pi_i^*(K-1,H) \tag{15}$$

for the downstream participants and

$$\Gamma_j^*(K,H-1)+\frac{\Pi_I(K,H)-[K*\Pi_i^*(K-1,H)+H*\Gamma_j^*(K,H-1)]}{K+H} \geqslant \Gamma_j^*(K,H-1) \tag{16}$$

for the upstream participants. The second term in the left-hand side of both inequalities simply tells us that all participant firms, i.e, $K+H$, share equally the remaining part of the entity's profit once each downstream firm has received $\Pi_i^*(K-1,H)$ and each upstream firm has received $\Gamma_j^*(K,H-1)$.

Similarly, for the external stability, the conditions are given by:

$$\Pi_i^*(K,H)+\frac{\Pi_I(K+1,H)-[(K+1)*\Pi_i^*(K,H)+H*\Gamma_j^*(K+1,H-1)]}{K+1+H} \leqslant \Pi_i^*(K,H) \tag{17}$$

for each outsider downstream firm i and

$$\Gamma_j^*(K,H)+\frac{\Pi_I(K,H+1)-[K*\Pi_i^*(K-1,H+1)+(H+1)*\Gamma_j^*(K,H)]}{K+H+1} \leqslant \Gamma_j^*(K,H) \tag{18}$$

for each outsider upstream firm j.

The natural question one may ask at this point of the analysis is whether some stable vertical agreement always exists. Using the definition of stability, the payoffs in (11), (12), (13), (14) and the profit sharing rule, we can state the following:

Proposition 1 *For all m and n, the corresponding unanimous vertical agreement is stable.*

Proof. Notice that the unanimous vertical agreement is always externally stable. Indeed, it requires the inequalities (17) and (18) to be satisfied at $K+1$ and $H+1$. However, $K=n$ and $H=m$ at the unanimous vertical agreement. Therefore these two inequalities are redundant and accordingly should not be taken into account when checking for stability of the unanimous vertical agreement. Accordingly, the check for stability is complete when the conditions for internal stability are satisfied. These conditions are immediately obtained from (15) and (16) by letting $K=n$ and $H=m$ in these expressions. It is easy to see that both these conditions are satisfied if and only if:

$$\frac{\Pi_I(K,H)-[K*\Pi_i^*(K-1,H)+H*\Gamma_j^*(K,H-1)]}{K+H} \geqslant 0.$$

This last condition for $K=n$ and $H=m$ boils down to:

$$\frac{\Pi_I(K,H)-[K*\Pi_i^*(K-1,H)+H*\Gamma_j^*(K,H-1)]}{K+H} = \frac{1}{4(m+n)},$$

which is clearly positive. ■

This proposition should be contrasted with the result provided by Shaffer (1995) for the case of pure horizontal agreements. In Shaffer's framework, the corresponding unanimous agreement is never stable for n exceeding 3. Thus, allowing the upstream firms to also participate in the collusive agreement considerably enhances the interest of the participants to agglomerate all firms in a single entity.

Notice that the main difficulty of proving stability for vertical agreements different from the unanimous one comes from establishing the inequality (18). Indeed, when the vertical agreement comprises $H<m$ and $K<n$ firms, then the outsider upstream firms are always willing to enter into the entity because the inequality (18) is generally not satisfied (for instance, this is always the case for the entity composed of one downstream and one upstream firm, as in Gaudet and Van Long (1996). Thus, it appears that the entity exerts a strong attraction force on the outsider upstream firms. In fact, proposition 1 holds only because

there remains no outsider upstream firm available when all of them are already members of the entity.

A related result to proposition 1 concerns the effect of the simultaneous entry of a downstream and an upstream firm in the economy on the stability of the unanimous agreement. We show that:

COROLLARY 2 *The entry of a new pair of a downstream firm and an upstream firm destroys the stability of the unanimous agreement with n downstream firms and m upstream firms.*

Proof. The condition for internal stability of a vertical agreement including $n-1$ downstream and $m-1$ upstream firms requires that $m < (359-27n)/32$. Similarly, the condition for downstream firms for external stability of a vertical agreement including $n-1$ downstream and $m-1$ upstream firms requires that $n > 9$, whereas the condition for upstream firms for external stability requires that $m > 6$. Consequently, there exists no value of n and m for which these conditions can be simultaneously satisfied, implying that a vertical agreement with $n-1$ downstream and $m-1$ upstream firms can never be stable. ■

Of course, after the entry of a new pair of firms, the stability is restored for the unanimous agreement involving now $n+1$ downstream and $m+1$ upstream firms.

It would be interesting to know whether the unanimous vertical agreement is the only stable one for *any m* and *n*. The proof of this uniqueness property should require that the two following inequalities be satisfied:

$$\frac{\Pi_I(K,H)-[K*\Pi_i^*(K-1,H)+H*\Gamma_j^*(K,H-1)]}{K+H}>0$$

and

$$\frac{\Pi_I(K,H+1)-[K*\Pi_i^*(K-1,H+1)+(H+1)*\Gamma_j^*(K,H)]}{K+H+1} > \frac{\Pi_I(K,H)-[K*\Pi_i^*(K-1,H)+H*\Gamma_j^*(K,H-1)]}{K+H}.$$

The first inequality guarantees that the entity involving K downstream firms and H downstream firms is internally stable, whereas the second inequality implies that, whenever an entity involving K downstream firms and H downstream firms is internally stable, then the same entity is never externally stable for the upstream firms. These two conditions clearly imply that the entity involving $K, K < n$, downstream firms and $H, H < m$ downstream firms is unstable. The full-fledged analysis of these two inequalities turns out to be rather algebraically intricate due to the complexity of the functions and the number of variables. However, we were able to show that:

PROPOSITION 3 *There are values for K, H, m and n for which there exists a stable agreement that differs from the unanimous one.*

Proof. Internal stability of an entity including $K = n-1$ and $H = m-2$ requires that $m < (518-72n)/27$. External stability of a downstream firm requires that $n > 5$. External stability for an upstream firm in turn is satisfied if and only if $m > (110-27n)/8$. It turns out that all these inequalities are simultaneously compatible. For instance they are all simultaneously satisfied for $K = 5, H = 1, m = 3, n = 6$. ■

Hence, in an economy with three firms in the input market and six firms in the output market, not only is the unanimous agreement is stable but also a vertical agreement involving one upstream firm and five downstream firms is also stable. It can easily be checked numerically that these two vertical agreements are the only two stable agreements in this economy. Why is this the case? Why is only that particular "interior" vertical agreements stable? The intuition of the existence of the unanimous stable agreement is, as we explained above, that the upstream external stability is satisfied when no other firm is active in the market but all firms are inside the entity. The stability of the interior agreement is obtained by the balance of the three effects of a collusive agreement: the balance among the profit generated by the entity shared *per capita* to the downstream and upstream firms in the agreement, the profit of the downstream firm as an outsider and the profit of an upstream firm as an outsider. This balance is not a monotonic function of n, m, H or K. Consequently, a general rule on n, m, H and K that satisfies conditions (15), (16), (17) and (18) for the linear model is not feasible.

Another interesting issue that concerns vertical agreements is whether Stilger's statement on vertical agreements is correct. Do the outsiders always win when a vertical collusive agreement is settled, whatever their type? To answer this question, consider the entity composed by one downstream and one upstream firm. Assume also that the number of downstream and upstream firms coincide. It is then easy to show that this entity is stable and that the outsider firms gain less then the participating firms. Hence, we can state the following:

Proposition 4 *Consider an entity involving one downstream and one upstream firm and $m = n$. The profit corresponding to each of the firms in the entity exceeds the profit of their equivalent outside the entity that operates in the downstream and in the upstream markets.*

Proof. Apply the inequalities (15), (16), (17) and (18) for $H = 1, K = 1, m = n$. ■

Hence, Stigler's statement according to which the only ones that benefit from a collusive agreement are the outsiders need not be true in vertical agreements. It is difficult to identify the precise threshold of n and m for which the result in proposition 4 is valid for any entity involving any number of downstream and/or upstream firms. Nonetheless, the result in proposition 4 shows that, in vertical agreements, the presence of upstream firms creates benefits for the insiders, as explained in the effects in (i) and (iii), in the introduction.

A final remark is in order. The analysis of stable vertical agreements developed above required a precise profit-sharing rule to specify the payoff for participants in the entity. The rule we put forward is not the only rule that can be used in vertical agreements. For instance, we could imagine that downstream firms share equally the profit of the entity net of the profit attributed to upstream firms that receive the same level of profits as the outsider upstream firms. It turns out that using this sharing rule, no stable cartel exists. The condition that guarantees the external stability of the upstream firms with respect to entry (or exit) of a downstream firm fails to hold. In fact, the profit of outsider upstream firms (which is also the payoff that they receive in the entity) is a decreasing function of K. Hence, the condition $\Gamma^f(K+1,H)=\Gamma^c(K+1,H)>\Gamma^c(K,H)$ is always violated.

This shows that the assumption about profit sharing is crucial for the analysis of stability, but it also reveals that our definition of stability is strong. Our notion of stability corresponds to a Nash equilibrium of the game with $n+m$ players and two (pure) strategies: enter the entity or remain outside the entity.

3. Conclusion

In this paper, we tackle the stability problem of collusive agreements not only involving some downstream firms but also embodying some upstream firms, providing the final market with a specific input. In other words, we extend the stability analysis from pure horizontal collusive agreements to entities involving some degree of vertical agreements. This endeavour is made possible due to the flexibility of the stability concept introduced above and to the framework of *successive oligopolies* introduced elsewhere by the authors (see Gabszewicz and Zanaj 2011). This extension is important because many real-life collusive agreements embody both upstream and downstream firms, thereby influencing the outcomes obtained in both the upstream and downstream markets. While the stability of horizontal collusive agreements has been extensively analyzed, the stability of vertical agreements has been neglected. Our paper is also useful because it tackles the analysis of the profit sharing rules when firms participating in the agreement are not of the same type. Furthermore, this paper completes and enriches the theory of successive oligopolies already introduced by the authors in Gabszewicz and Zanaj (2011).

The objective of this paper is not to endorse anti-trust policies, but it can be used to identify vertical agreements with stability properties, bearing in mind that such agreements can be anticompetitive and detrimental for consumers. Such stability properties of cartels are relevant to the extent that policies against consumers' detrimental cartels are meaningful only if such cartels are stable through time. Furthermore, our analysis is relevant for policy recommendation concerning not only the effects of vertical agreements themselves but also the sustainability of horizontal cartels in successive oligopolies, as analyzed in Nocke and White (2007) and Piccolo and Miklos-Thal (2012).

Our analysis reveals that, in the linear model, the unanimous vertical agreement is always stable: this proposition holds in full generality and should be contrasted with the proposition by Shaffer (1995), according to which no stable cartel exists in the case of horizontal cartel agreements when $n > 3$. Thus, stable vertical entities exist for market structures in which pure horizontal cartels would all be unstable. It also reveals that the introduction of vertical agreements weakens the Stigler statement according to which "...the major difficulty in forming a cartel is that it is more profitable to be outside a cartel than to be a participant...." In the framework of successive oligopolies, the marginal cost of downstream firms is no longer exogenous, as in the case of horizontal agreements. When the entity also comprises upstream firms, it reduces the number of input suppliers in the upstream market, thereby restricting competition in this market, leading in turn to an increase in the input price. Furthermore, the presence of the upstream firms in the agreement increases the differences in production costs between the insider and outsider downstream firms. Therefore, the entity exerts a strong attraction force on the outsiders' upstream firms, in contrast to what is argued by Stigler (1950), according to whom the only firms that gain from a cartel are the outsiders. In the case of vertical agreements, outsiders may be willing to become insiders. This attraction force explains why external stability is so difficult to obtain in the framework of vertical agreements.

Our paper has only scratched the surface of what looks like a promising area for further research. Many questions remain after our analysis. First, a natural question consists in evaluating the welfare effects of vertical agreements. Two forces operate in opposite directions: on the one hand, the entry of *upstream* firms should increase welfare to the extent that it dampens the negative effects of double marginalization; but on the other hand, the higher the number of *downstream* firms entering the agreement, the more concentrated the power in the downstream market.

Second, how robust are the conclusions of this paper? It is clear that, like most of the previous research in this field, the conclusions hold in the framework of the linear model. It would be interesting to examine more efficient types of pricing such as two part-tariffs. In that case, the first effect of the vertical collusive agreement, namely the elimination of double marginalization, would disappear. The second effect would remain, under the assumption that downstream firms do not buy from suppliers that are not part of the collusive agreement (see, for instance, Nocke and White 2007, on the *"outlet effect"*). Clearly, the profitability of the agreement in that setup would be different so different would be the type of stable equilibria that would emerge. Nevertheless, the main message of our paper would not change.

Finally, the institutional forms of collusive agreements observed in real life are by far more complex than those evoked in this paper, where the agreement reduces simply to accepting to belong to the entity or not. In particular, merging existing firms often takes the form of acquisition of one firm by another. Such acquisitions reveal the existence of a market where firms are exchanged among firms, opening the door to a whole range of potential arrangements among firms. The many potential forms of vertical agreements raise the interesting question of what could be the optimal structure of the market. This interesting issue is still to be explored in depth in the future.

Appendix

Here, we briefly summarize the equilibrium market solution of successive oligopolies when no vertical agreement takes place. Reconsider the same economy as in subsection 2.2 in the absence of any agreement. Then, the profit $\Pi_i(q_i, q_{-i})$ of a downstream firm is $\Pi_i(q_i, q_{-i}) = (1 - q_i - \Sigma_{j \neq i} q_j) q_i - \beta \frac{q_i}{\alpha}$. Assuming a symmetric equilibrium in the downstream market, we obtain the optimal output quantity as a function of the input price ω, namely $nq_i = \frac{n(\alpha - \omega)}{(n+1)\alpha}$. The market clearing condition in the input market, $nq_i = {}_{k=1}^{m} s_k$, gives the inverse input demand function $\omega({}_{k=1}^{m} s_k) = \alpha - \alpha^2 \frac{n+1}{n} {}_{k=1}^{m} s_k$. Then, the profit function of the j-th upstream firm is given by $\Gamma_j(s_j, s_{-j}) = (\omega({}_{j=1}^{m} s_j) - \beta) s_j$. At the symmetric equilibrium, we obtain $s^*(n, m) = \frac{n(\alpha - \beta)}{\alpha^2 (n+1)(m+1)}$ and $\omega^* = \frac{\alpha + m\beta}{m+1}$, $q^* = \frac{m(\alpha - \beta)}{\alpha(n+1)(m+1)}$. This equilibrium solution yields the level of profits in (14).

References

Belleflamme P., and M. Peitz (2010) *Industrial Organization: Markets and Strategies*. New York: Cambridge University Press

d'Aspremont C., J. Gabszewicz, and J. Weymark (1983) "On the stability of collusive price leadership," *Canadian Journal of Economics* 16(1) 17–25

Gabszewicz J., and S. Zanaj (2011) "Free entry in successive oligopolies," *International Journal of Economic Theory* 7(2), 179–88

Gaudet G., and N. van Long (1996) "Vertical integration, foreclosure, and profits in the presence of double marginalization," *Journal of Economics & Management Strategy* 5(3), 409–32

Konishi, H., and P. Lin (1999) "Stable cartels with a Cournot fringe in a symmetric oligopoly," *KEIO Economic Studies* 36(2), 1–10

La Revue (July 23, 2008) "Christmas price-fixing in the French toys market," available at larevue.ssd.com/Christmas-price-fixing-in-the-French-toys-market_a1043.html, accessed July 23, 2008

Markham, J. W. (1951) "The nature and significance of price leadership," *The American Economic Review* 41(5), 891–905

Nocke V., and L. White (2007) "Do vertical mergers facilitate upstream collusion?," *The American Economic Review* 97(4), 1321–39

Piccolo, S., and J. Miklós-Thal (2012) "Colluding through suppliers," *The RAND Journal of Economics* 43(3) 492–513

Salant, S. W., S. Schwitzer, and R. J. Reynolds (1983) "Losses from horizontal merger: The effects of an exogenous change in industry structure on Cournot-Nash equilibrium," *The Quarterly Journal of Economics* 98(2), 185–99

Salinger, M. A. (1988) "Vertical mergers and market foreclosure," *The Quarterly Journal of Economics* 103(2), 345–56

Shaffer, S. (1995) "Stable cartels with a Cournot fringe," *Southern Economic Journal* 61, 744–54

Stigler G. J. (1950) "The development of utility theory. II," *Journal of Political Economy* 58(5) 373–96
Zu L., J. Zhang, and S. Wang (2012) "The size of stable cartels: An analytical approach," *International Journal of Industrial Organization* 30(2), 217–22