

GENERAL DISCUSSION

As we have outlined in the Foreword, the starting point of the present thesis was the single-case study of an acalculic patient, CO, who became unable, after bilateral parietal damage, to process numbers according to their relative position in the sequence, whereas he was still able to process their quantity meaning. CO's peculiar pattern of performance drove us to the hypothesis that numerical quantity and numerical order (that is, the rank or relative position of a number within the counting sequence) could rely on potentially dissociable mental representations and/or entail (partially) distinct processing mechanisms. Besides, CO was also impaired in processing the relative position of non-numerical items (e.g. letters, days and months) in their corresponding sequence, which further suggested that order coding could rely on a more general mechanism, independent of stimulus material. An ERP study and two behavioural experiments were subsequently devised to address the issue of whether a similar dissociation between quantity and numerical order could be observed in healthy adult individuals. The question of whether processing the order of numbers and the order of non-numerical stimuli (e.g. letters) in their corresponding sequence involved similar cognitive mechanisms was also addressed.

We will first briefly summarize the data of each empirical study presented in the previous chapters. We will then discuss these empirical data and examine how each study helps to clarify the issue of a potential association or dissociation between numerical quantity and numerical order, both with respect to their mental representation(s) and their underlying processing mechanism(s). We will then integrate our empirical data into a tentative model.

1. Summary of empirical data

In our first, single-case, study we examined number processing abilities in a patient showing Gerstmann's syndrome following bilateral parietal damage. Overall, this study revealed an interesting dissociation between largely preserved quantity meaning of numbers in the face of impaired sequence processing skills. Preserved understanding and processing of numerical quantity was observed with symbolic and non-symbolic stimuli in various tasks (comparison, give a number smaller/larger than a target, cardinality tasks: 'How many' questions, inclusion tasks, Arabic-to-dot matching, and so on) and a standard distance effect was observed on RTs when comparing small Arabic numerals. The data suggest that CO had preserved access to a mental representation of numerical quantity (possibly taking the form of a left-to-right oriented continuum). With respect to sequence knowledge, preserved (up to a certain level) forwards and impaired backwards sequence recitation skills could explain CO's performance in some of the tasks (e.g. give the number coming just before/after a target), but couldn't account for the whole story, and crucially, it couldn't explain why the patient was unable to use the written ordered series to perform sequence tasks. What is more, CO was unable to perform sequence tasks (e.g. does 3 come before/after 5 in the counting sequence?) for which he had an almost perfect performance with the corresponding quantity instructions (e.g. is 3 smaller/larger than 5?). Taken together, these data suggest that the patient became unable, after parietal damage, to use (preserved) quantity meaning of numbers to process their corresponding position in the counting sequence. Besides, CO's impaired sequence knowledge was not restricted to numbers but extended to the non-numerical ordered series of letters, days and months, thus supporting the involvement of a more general sequence processing deficit. We will propose two alternative hypotheses to account for CO's pattern of performance in numerical tasks: numerical quantity and order information (i.e. knowledge of an item's relative position in its corresponding sequence) either address separate mental representations, or rely on distinct processing mechanisms. These two hypotheses will be developed below.

Our second, electrophysiological, study aimed at examining whether processing numerical quantity (judging a number as smaller or larger than 15) and numerical order (judging a number as coming before or after 15 in the counting sequence) entailed similar behavioural and, more crucially, electrophysiological markers, in healthy adult volunteers. An alphabetic order task (judging a letter as coming before or after M in the alphabet) was also included. Behavioural results showed a standard distance effect in all three tasks that was identical in the two numerical tasks. Notwithstanding the behavioural similarity, electrophysiological results showed that different spatio-temporal patterns were associated with the distance effect in each numerical task: quantity processing recruited an early left parietal network, whereas judging the relative position of a number in the sequence recruited a delayed bilateral parietal network. Bilateral parietal areas were also involved in alphabetic order judgments. Besides, prefrontal regions were recruited more when judging the relative order of numbers, rather than their associated quantity. Because material, methods (except for instructions), task difficulty, and subjects were identical in the two numerical tasks, their different spatio-temporal patterns could reflect the involvement of distinct operational mechanisms when processing quantity and numerical order. Yet, beyond their differential recruitment of parietal and prefrontal regions, the specific nature of these cognitive mechanisms could hardly be derived from the present experiment.

The third, behavioural, study aimed at further investigating the hypothesis (and the nature) of different underlying mechanisms when processing numerical quantity and numbers' relative position in the counting sequence, in healthy adult individuals. More specifically, we sought whether relative-order (and not quantity) judgments on numbers entailed similar (specific) behavioural effects (e.g. reverse distance and pair-order effects) as those reported in order judgments with non-numerical stimuli (e.g. letters of the alphabet). Results for the numerical quantity task (2 > 3, which is smaller/larger?) showed the expected standard distance effect (i.e. slower processing of close than far numbers), which was also found in relative-order judgments on descending number pairs (e.g. 3 > 2, is the pair in the conventional sequence order?). Whereas judging the relative order of

ascending number pairs revealed the opposite pattern that is, faster judgments of adjacent (2 3) than non-adjacent pairs (2 4). Moreover, order-judgments on numbers and letters, but not quantity judgments, were affected by pair-order (i.e. showed faster processing of ascending relative to descending pairs). Altogether these data were taken as evidence for the involvement of different cognitive mechanisms in numerical quantity and relative-order judgments: the standard distance effect was consistent with the recruitment of a magnitude comparison process, whereas the reverse distance effect could be explained by the contribution of a serial search (i.e. recitation) mechanism. Yet, because of their familiarity, we may not preclude that adjacent conventionally-ordered pairs were processed faster because of the involvement of a special encoding mechanism of direct order recognition (instead of serial search). Another interesting finding was that numerical quantity and numerical order judgments were equally affected by the physical size of digits which points to some common underlying representation of quantity and relative order (see below).

2. What do our empirical data tell us about the relationship between numerical quantity and numerical order?

We have argued, in the introductory chapter of this dissertation, that numbers may refer with equal efficiency to different properties of objects (the cardinality of sets, or the relative position of an element in an ordered set) and convey different meanings (respectively numerical quantity and numerical rank¹) according to their use in cardinal or ordinal assignments (e.g. Wiese, 2003). The relationship between numerical quantity and numerical order is a debated issue to which, we hope, our thesis will provide some clarifying contribution. The evidence gathered thus far in the literature appears inconclusive. On the one hand, some behavioural, neuroimaging and neuropsychological data suggest (with mostly indirect evidence) that they might rely on a common underlying representation and/or involve some

¹ We have also referred to that concept as ‘numerical order’.

common processing mechanism, as they entail similar distance and SNARC effects, recruit similar parietal regions and can be conjointly impaired or spared after cerebral lesion. On the other hand, however, evidence from developmental, neuropsychological and behavioural studies suggest that sequence knowledge and quantity number meaning might rely on at least partially distinct mechanisms (and/or representations). A developmental distinction was reported, in fact, between children's (early) acquisition of the number sequence and their later knowledge of the numbers' corresponding cardinal meaning. Besides, a brain lesioned patient was described to have preserved sequence knowledge in the face of impaired (automatic access to) cardinal number meaning. Finally, processing the relative order of items in a sequence was shown to entail specific behavioural markers (namely a reverse distance effect and a pair-order effect) that were not, or less consistently, reported in numerical quantity judgments.

Altogether, the empirical data collected and discussed in the present thesis will lend some crucial support to the second line of evidence. That is, they suggest that processing numbers according to their quantity meaning or to their relative position in the sequence might rely on separate mental representations and/or on distinct processing mechanisms. Yet, one crucial question is, indeed, whether there are separate mental representations for quantity and (numerical) order, or whether the distinction we have outlined purely refers to the involvement of distinct mechanisms, that apply to a common representation coding for quantity and relative positions in the sequence?

We will now discuss each hypothesis in turn (separate mental representations and separate processing mechanisms), and see whether and how our data, together with what we learn from the literature, can favour one or the other interpretation. First, we will present the likelihood that quantity information and relative positions in an ordering could be coded on distinct mental representations, as our patient's pattern of performance might suggest. We will then argue that an integrated representation of quantity and order information cannot be excluded, and provide some support for this latter, more parsimonious, hypothesis. Second, we will ask whether and how quantity and the relative position of numbers in the sequence could rely on

distinct processing mechanisms and discuss the evidence gathered in our ERP and behavioural studies. Each mechanism will then be described in turn. Finally, we will propose our own view, and a tentative model, of the relationship between numerical quantity and numerical order and their processing mechanisms.

2.1. Separate or integrated mental representation(s) for quantity and order information?

** Some evidence for separate representations*

We have mentioned above that there might be (at least) two possible ways we could account for our patient's pattern of performance (i.e. spared quantity processing in the face of impaired sequence knowledge). One of these is to make the tentative hypothesis that quantity and order information could be coded on separate mental representations. Hence, this is one way we might account for the fact that CO's preserved knowledge of numerical quantity didn't help him retrieving the relative position of numbers in the sequence (e.g. he was unable to derive that 3 comes before 5 in the counting sequence from his preserved knowledge that 3 is smaller than 5). What could the separate representations of quantity and relative order each look like?

We may speculate that the representation of numerical quantity would be based on the inclusion property of cardinality (see Figure 1, below), and that it would not easily allow to derive information about sequence order. On the contrary, relative positions in the sequence would be coded on a different medium, possibly an oriented line, upon which proximity relations between numbers would not simultaneously code for their respective quantity (see Figure 2, below). Hence, this latter representation might be similar to a mental 'number line' that would be devoid of any numerosity meaning, though. This representation might not be restricted to the coding of numerical order, but could be shared by non-numerical series which items have a well-defined order. Some evidence for such common representation comes from CO's extended impairment about sequence knowledge to the non-numerical ordered series of letters, days and months. In a related vein,

2-year-old children at early stages of the number sequence acquisition were reported to sometimes use the letters of the alphabet instead of counting words in counting situations (Fuson, 1982; Gelman & Gallistel, 1978). These intrusion errors, together with those reported in aphasic patients (e.g. producing “March” instead of “Wednesday”; Deloche & Seron, 1984) might also argue for some common mental representation of ordered series held in LTM. Presence of a SNARC effect on numerical (Dehaene & al., 1993) and non-numerical ordered series might also fit along these lines (Gevers & al., 2003).

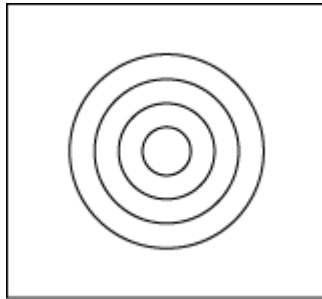


Figure 1. Tentative representation of (numerical) quantity without concomitantly coding for relative positions in a sequence.

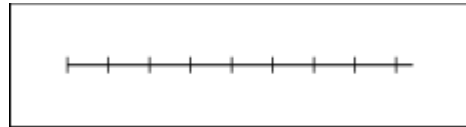


Figure 2. Tentative representation of relative positions in a sequence, without any associated quantities.

Some support for the ‘separate-representations hypothesis’ comes from studies with preverbal infants and young children. Infants were found to have a preverbal grasp of quantities, as demonstrated by their abilities to discriminate sets of either small (up to 3 or 4 items) or large numerosities (provided that their ratio was large enough; see Feigenson, & al., 2004, for review), and thus to possess some kind of mental representation of numerical quantity (see the discussion about “object-files” and analogue magnitudes in the introductory chapter) before they were able to recite the sequence of number words. Moreover, once they start learning the number sequence, developmental data suggest that children don’t immediately map their recitation skills onto these preverbal numerosity representations. In fact, it will take the child a long time to understand that the position of a number in the counting sequence is directly related to its cardinality (see Fuson, 1988, and Wynn, 1992).

Beyond this early distinction however, Fuson (1988) proposes that sequence knowledge and cardinal number meaning become progressively integrated during development, through the use of number words in various contexts, and end up into a full-blown number concept. Little is known, however, as to when and how this integration takes place (see Piaget, 1941, for a similar position about integrated development of ‘cardinal and ordinal’ aspects of number). The question of whether initially separate representations of (small exact and large approximate) numerosities and sequence knowledge become progressively integrated into a common mental representation (see our proposal of an integrated representation in Figure 3), or whether they remain separate up to adulthood has received, so far, only little consideration (but see Verguts & al.’s model, 2005, below). This lack of attention can be partly explained by the fact that mechanisms underlying (number) sequence processing have been only rarely studied. Moreover, it is usually taken for granted that numerical quantity and numbers’ relative position in the sequence are two indivisible aspects of numbers and should thus be coded on the same mental representation, which is consistent with adults’ ability to easily shift from one number meaning to another. Hence, there probably were no strong reasons to propose separate representations for numerical quantity and numerical order until the description a patient like CO, who provides a more complicated picture. However, the hypothesis that quantity and sequence knowledge might address separate mental representations is only tentative (but would deserve further consideration) since other data, both from our empirical studies and from the literature, point, instead, to a common integrated representation.

* Integrated Representation of quantity and order

The more parsimonious hypothesis of a common representation coding for both numbers’ quantity and their relative-order in the sequence has received some support from our behavioural study. In Experiment 1, we found a similar size congruity effect during quantity and relative-order judgments about numbers; thus, because interference between numerical quantity and physical size has been shown to arise at a central (semantic)

stage in number comparison (Schwarz & Heinze, 1998; but see Otten, Sudevan, Logan, & Coles, 1996 for a later interference locus), occurrence of a similar size congruity effect in the two numerical tasks might be taken as evidence for a common underlying representation of quantity and relative positions in the number sequence.

Because numbers were usually studied with respect to their quantity meaning, we know little about how their relative position in the sequence would be coded on a mental representation. The default conception is that of a unique semantic representation of numbers, which is magnitude-based¹. This semantic representation is thought to take the form of a left-to-right oriented continuum, coding for proximity relations among numbers and upon which information about both quantity and numbers' relative position could be derived (but see our discussion about 'number line' models in the introductory chapter and their limits in coding for sequential order). Hence, a representation initially devoted to approximate analogue magnitudes, as that found in preverbal infants (e.g. Feigenson, & al., 2004), would be progressively adapted, possibly through the acquisition of language (see Spelke and Tsivkin, 2001) and of the number sequence acquisition, to enable the coding of pure sequential order as well (see Nieder, 2005, who further suggests that numerical quantity and 'rank order' information probably share the same neural system). The use of fingers in counting might also play a role in this mapping between sequence order and quantity representation (e.g. through the concomitant activation of the number sequence recitation, the raising of fingers in a certain order while counting objects and the cardinality of the finger set/object set; see Fayol & Seron, 2004).

The precise way this adaptation process takes place is still largely unknown, but crucial for a more comprehensive understanding of how the number concept develops and should deserve better attention in future studies. One attempt towards a better understanding of how a system initially dedicated to the representation of approximate magnitudes would end up in coding for exact positions along the number line has yet been recently

¹ We should probably say that the hypothesis of separate representations for distinct number meanings (e.g. cardinal, ordinal) has never been seriously considered in the literature thus far.

proposed by Verguts and Fias (2004).

We might imagine that coding for relative positions requires a much finer representation, coding for the exact location of items in the sequence, whereas representation of numerosities could be approximate. Yet, the way exact positions could be derived from an initially approximate magnitude coding system can be understood with the recent neural model of Verguts and Fias (2004). The authors propose that the number line might first develop to represent non-symbolic numerosities, and could be later adapted to code for symbolic numbers. This was shown in a simulation study: a number line was found to develop spontaneously under unsupervised learning conditions when a neural network was presented with numbers as non-symbolic numerosities (collections of dots or objects); the network was then presented with symbolic input (verbal numeral labels) in conjunction with non-symbolic numerosities and the same network nodes that represented a given numerosity also learned to represent the value of the corresponding symbol. Hence, the meaning of numbers in a symbolic format was represented on the same number line that initially reacted to non-symbolic stimuli. Yet, the properties of this unique number line were different depending on whether symbolic or non-symbolic inputs were presented. For instance, the tuning curves were more peaked for symbolic material, which leads to a more precise representation. We make the hypothesis that such precise representation might then enable for the coding of the exact position of numbers along the sequence.

Hence, quantity and sequence meanings would initially develop in parallel and then be progressively integrated on a common representation. On the one hand, the initial grasp of cardinalities would lead to the construction of an analogue magnitude representation (the ‘number line’) that becomes increasingly precise with the acquisition of symbolic systems; on the other hand, the preverbal grasp of serial order and the elaboration of sequence recitation would allow sequential order to be progressively mapped onto the representation initially devoted to cardinalities. The ‘number line’ would then be suitable for the coding of both quantity and sequential order. Nonetheless, we make the hypothesis that each meaning would still remain potentially distinct from the other on this mental representation, and that

each meaning could be preferentially processed through a specific cognitive mechanism. As we will see in the next section, we make the hypothesis that quantity would be more easily derived using a comparison strategy, whereas order would be more directly processed through a serial search (i.e., sequence recitation) process.

2.2. Distinct mechanisms underlying the processing of quantity and order information

The second hypothesis we have outlined above to account for our patient's pattern of performance, is that of the involvement of separate mechanisms when processing numerical quantity and sequence relations. Along these lines, CO would have preserved mechanisms to access quantity information in the face of impaired sequence processing mechanisms. Yet, the patient's pattern of performance in sequence tasks (and in particular better accuracy in answering 'what comes next' than 'what comes before' questions) might suggest that sequence processing would rely on some kind of (sequence) recitation process (that was better preserved forwards than backwards in our patient) that could be common to most ordered series held in LTM. With respect to quantity processing, the presence of a standard distance effect in number comparison has usually been taken as evidence for the involvement of a magnitude comparison mechanism (see below). Let us note that the hypothesis of the use of different strategies according to task instructions (numerical quantity or sequence order) was already outlined by Fuson (1988) more than 15 years ago; she proposed that "children might address sequence order questions by the use of an overt or covert run-through procedure (...) but approach cardinal word questions by the use of a magnitude comparison process" (Fuson & Hall, 1983, p. 97).

The results from both our electrophysiological and behavioural studies lend further support to the involvement of separate mechanisms when processing numerical quantity and the relative order of numbers. In the ERP study, the distance effect had a different spatio-temporal course in parietal and prefrontal regions whether numbers had to be judged as smaller or larger than a standard (quantity instructions) or as coming before or after that

standard (numerical order instructions). This was taken as evidence for the recruitment of slightly different strategies in each task. Hence, similar strategy modulation by instructions was reported in an fMRI study by Fink and colleagues (Fink, Marshall, Weiss, Toni, & Zilles, 2002) in a perceptual variant of a line bisection task (the Landmark test). Normal volunteers were shown lines with a bisection mark and had to judge whether the mark was on the middle of the line. Two different instructions were proposed: subjects had to judge (1) whether line segments on either side of the bisection mark were of equal length, and (2) whether the bisection mark was on the centre of the line. Different instructions were associated with specific cerebral activations (as well as common activation) that were taken as evidence for the involvement of different cognitive strategies. Our ERP study suggests similar modulation of strategy use, and of concomitant cerebral activations, by instructions. Hence, when instructions emphasized on the quantity meaning of numbers, or on their relative position in the sequence, different cognitive strategies were elicited that recruited parietal and prefrontal regions with a specific time course. The study tells us little however with respect to the nature of the strategies used by the subjects. In both numerical tasks and in an alphabetic order judgment, a standard behavioural distance effect was observed. This has usually been taken as evidence for the involvement of a comparison process. Yet, in what ways did this process differ according to instructions (quantity and order on numbers) and material (numbers and letters) is still unclear.

Finally, our behavioural study lends further support to the hypothesis that different mechanisms might underlie the processing of numerical quantity and relative order about numbers. The most striking result was that of a reverse distance effect when judging the order (but not the quantity) of ascending number pairs that was also found for conventionally ordered letter pairs from the beginning of the alphabet. Quantity judgments elicited, instead, a standard distance effect that was also found when judging the order of descending pairs of numbers and letters. The pair-order effect was also restricted to order judgments in our study. Hence, relative-order judgments entailed “order-specific” behavioural markers which may shed some light on the nature of the underlying mechanisms (as opposed to the

mechanisms involved in quantity judgments). Overall, we interpreted the standard distance effect as the result of a comparison process, and the reverse distance effect as originating from a serial search mechanism (see Jou & Aldridge, 1999; Leth-Steensen & Marley, 2000, for reviews). We thought this was the most direct way to account for our behavioural data (and it corroborated other authors' interpretation of similar results). Nonetheless, we may not preclude that the reverse distance effect in processing adjacent ascending pairs might entail some special processing mechanism (such as direct order recognition), because of their familiarity. Yet, in the absence of more convincing evidence (e.g. how do we tell apart a familiar from a non-familiar number pair?), we will consider the more widely accepted view of the involvement of a serial search process as underlying the reverse distance effect. We will now describe the comparison and the serial search mechanisms in turn.

* The comparison process

The way numerical information is coded and processed during comparative judgments is still controversial. Several models of number comparison have been proposed since the seventies, and beyond their peculiar differences, all were devised to account first and foremost for the behavioural (standard) distance effect. This was first reported by Moyer and Landauer (1967) in a number comparison task and explained by the conversion of symbolic numbers into analogue magnitudes that are then compared in much the same way as physical (perceptual) continua (e.g. locations along a linear ordering or 'number line'; Restle, 1970). In analog models (as opposed, for instance, to Semantic-coding models, Banks, 1977; Banks, Fujii, & Kayra-Stuart, 1976), the distance effect is explained by slower decision times with decreased discriminability that is, stimuli with positions that are close together in an ordering are more confusable. Hence, analog models usually propose that numbers are analogically encoded on a mental map or 'number line' (e.g. Buckley & Gillman, 1974; Jamieson & Petrusic, 1975; see the introductory chapter for the most recent developments about number line models). Little is known, however, as to the

exact nature of the comparison process itself (e.g., once positions are activated on the number line, how does a subject know that the number on the right is larger than the number on the left? Is this information retrieved from LTM?). One way this process has been described is as an iterative comparison process (or random walk; Birnbaum & Jou, 1990; Buckley & Gillman, 1974; Jou & Aldridge, 1999) that is, by sampling information iteratively from memory for each of the two items, until the cumulated (information) difference reaches the response threshold. Distance effects arise because the further apart two items are in the ordering, the more different their semantic features are, the fewer samplings needed, and the faster the response criterion is reached. Using a comparison process is not restricted to numerical tasks, but extends to comparative judgments of non-numerical stimuli as well (e.g. letters, Jou, 1997), as suggested by the presence of a standard distance effect.

* The serial search process

With respect to the serial search process, it is thought to proceed step-by-step along a memorized sequence to locate the position of an item in its relation to another item. This process is analogous to a chain association in which one item elicits the next and produces a positive correlation between the distance searched and RT (i.e., a reverse distance effect; Jou, 1997; Parkman, 1971; Tzelgov, Yehene, Kotler, & Alon, 2000; Woocher, Glass & Holyoak, 1978). This process acts much like a run-through (or recitation) mechanism: position of an item along the sequence (or with respect to another item) is identified by running-through the sequence (e.g. from the beginning). Among variations of this model, the ends-inward serial search model (e.g. Potts, 1974) proposes that subjects identify the target by serially searching from one end towards the interior of a series. Along these models, participants only need to locate one of the two items in a relative-order judgment tasks to make a decision. In fact, when presented, for instance, with a pair of items and asked to identify the one 'coming after/after' in the sequence, the search may start from the beginning of the sequence and as soon as one item is encountered, a decision can be made since the other item

is the expected response (i.e., the item ‘coming after’; Holyoak & Patterson, 1981; Polich & Potts, 1977). Yet in overlearned series like the number sequence the search can start from any interior element of the sequence (and not necessarily from the beginning). This might not be true, however, for less automatized sequences like the alphabet, which recitation cannot start from any given entry point (or letter, e.g. Lovelace & al., 1973; see the concept of ‘memory accessibility’, Jou, 2003).

Thus, the nature of the series (e.g. numbers vs. letters) and their respective mastery by human adults will affect the underlying processing mechanisms and the behavioural effects, some of which will be specific to alphabetic order processing (e.g. serial position effect), while others will reflect more general order-processing mechanisms (like serial search). The use of a serial search process to code for the order of items in a sequence might thus not depend on the nature of the sequence (quantitative vs. non-quantitative) but on task demands and will be facilitated by certain conditions: proximity of the items, congruency with the direction of the sequence (ascending vs. descending order), serial position/accessibility of items in the sequence (Jou, 2003).

The series we have worked with in the present thesis (mostly numbers and letters) were all learned during childhood and held in LTM. The mechanisms we use to retrieve information about these series from LTM might thus not be the same as the mechanisms underlying the retrieving and processing of newly learned linear orders. Little is known however in this respect since we know of no study that has compared, to date, processing of overlearned series with that of new linear orders.

The way a linear order is constructed and retrieved from LTM has attracted cognitive psychologists for several decades (e.g. Sternberg, 1969). Different mechanisms have been proposed for the encoding, representation and retrieval of (new) linear order information in memory, such as inter-item associations (items are coded with respect to other items: “A precedes B, B precedes C”, etc.; e.g., Henson, 1999) and direct coding (i.e. explicit association of temporal or ordinate position to each item: A-first, B-second, C-third, etc.; Ebenholtz, 1963). Yet because we focused, in the present dissertation, on (over)learned series held in LTM, and more specifically on

the number sequence, and because our main objective was to better understand the relationship between numerical quantity and order *on numbers*, the debate of how a new serial order is learned, stored and retrieved from memory is beyond the scope of the present thesis. Yet the question of whether impaired processing of (overlearned) sequences would extend to newly learned arbitrary orders deserves further consideration.

3. A tentative model of representation

Altogether, the studies described in the present thesis lend support to the hypothesis that a number's quantity and its relative position in the sequence, though being integrated into a common representation, remain potentially disjoined: they can be selectively spared or impaired after cerebral damage and rely on different cognitive mechanisms. We made the hypothesis that each mechanism (generally speaking, comparison and serial search) would preferentially code for one kind of information (quantity and relative positions, respectively) from the activation of the corresponding information (i.e. quantity or relative positions) on the mental representation. We will see how existing models of the number line (don't fully) account for our data, and then briefly outline a very tentative model based on our empirical data and suggest that numerical quantity and numbers' relative position could be coded on an integrated representation, but remaining potentially disjoined and thus allowing for specific processing mechanisms.

The question of whether existing number line models could account for our empirical data cannot be answered directly since none of these models provide a clear explanation for relative order coding. Thus, we can only make assumptions as to whether and how they *would* code for the order of numbers. We already addressed this question in the introductory chapter and proposed that: (1) in place-coding models (e.g. Verguts, & al., 2005), the order relation between numbers might not be easily derived beyond adjacent numbers; (2) in magnitude-coding models (e.g. Zorzi & Butterworth, 1999), order must be derived from quantity information. What we could further add

is that, in place-coding models, the quantity a number refers to must be derived from its place along the line. It is unclear then whether and how a quantity could be processed independently of its corresponding place (or relative position). Hence, the problem with existing number line models is that they don't appear to (or it is unclear if they do) allow for the separate representation of quantity and relative positions (or order). In place-coding number line models, knowing that a number is larger than another should be derived from their relative positions along the line (e.g. knowing that 5 is larger than 3 because it is located on the right side of 3). On the contrary, magnitude-coding models would require the opposite inference: knowing the relative position of a number from its corresponding cardinality (e.g. knowing that 5 comes after 3 because cardinal representation of 5 contains the numerosity 3). Overall, it is unclear then how place-coding and magnitude coding models would account for CO's pattern of performance. Moreover, because quantity and the relative order of numbers appear to be tightly linked in these number line representations, whether and how each information could be activated through specific cognitive mechanisms is a challenging question.

Taking existing models of the number line as a starting point, we would like to propose a new (speculative) way about how the quantity and the relative position of numbers in the sequence could be coded on the same representation, though still remaining potentially dissociable. This 'model' should be taken, however, more as a thoughts exercise than as our definite conception of how numbers are represented in the brain. The aim is to see whether it is possible to represent quantity and order as integrated but still separable dimensions, and whether this kind of representation could account for our empirical data.

Just like number line models do, we would propose that the underlying 'integrated' representation of numbers takes the form of a left to right oriented continuum. However, contrary to other number line models we suggest that this representation is made up by two initially distinct representations (one for quantities, the other for serial order, see Figure 1 and 2 above) that are ultimately integrated into a single representation. Yet, this integration takes place "up to a certain point", by which we mean that

each underlying representation (the one for quantities and the one for relative positions) could still be accessed independently from the other.

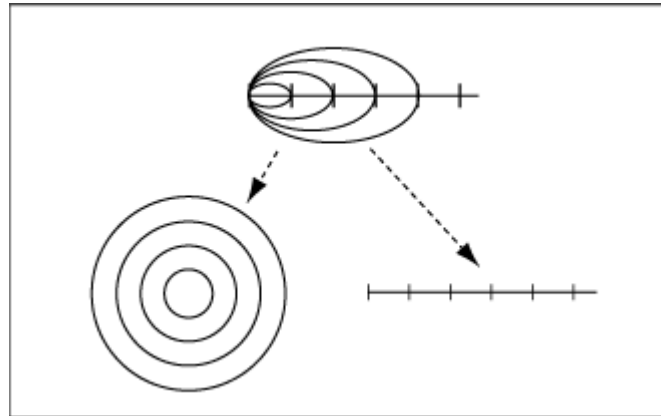


Figure 3. Tentative integrated representation of quantity and sequence order where each dimension remains nonetheless potentially disjointed from the other (i.e. can be processed through different, specific, mechanisms).

We propose in Figure 3 such a tentative integrated representation of numbers. This representation would equally code for quantities and the inclusion property of cardinality (represented by concentric ellipses), and for the relative position of numbers in the sequence (represented by tick marks on a horizontal line)². Yet, albeit being integrated, this representation would nonetheless allow for the processing of quantity information (possibly in a much perceptual, psychophysical manner), without the need to concomitantly activate or process the corresponding position of numbers in the sequence; and, conversely, relative positions could be derived independently from the number's corresponding magnitude.

Besides, we make the hypothesis that each dimension (quantity or relative position) would be preferentially accessed and processed through a specific cognitive mechanism: possibly comparison and serial search. In the comparison process, our representation would act much like the 'number line

² Let us note that, re-reading Piaget's (1941) conception of number we found that he proposed quite a similar view, cardinal numbers being conceptualized as seriated (according to an asymmetrical relation) and included within each other (as a class).

field'³ proposed by Verguts and colleagues (2005) in their place-coding model. In line with their model, we would propose that comparing numbers in a pair (i.e., telling which is larger or smaller) might require the simultaneous activation of each number's corresponding quantity representation (see Figure 4). Thus, when presented with two numbers in a comparison task, each number would activate its corresponding quantity on a distinct mental representation (i.e., we would have separate representations for each number), and the comparison process would take place on these quantity representations. This process might act like a psychophysical comparison mechanism and thus elicit a standard distance effect.

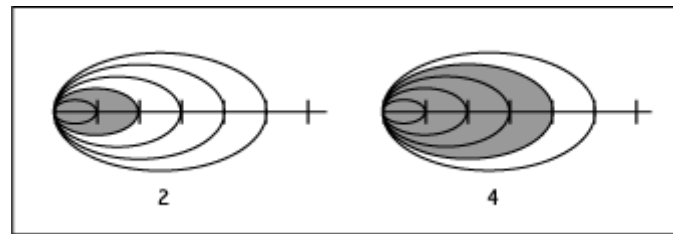


Figure 4. Schematic representation of numbers' 2 and 4 corresponding quantities, as activated in a number comparison task.

With respect to relative order judgments and the serial search process, we make the hypothesis, instead, that to better identify whether one number comes before or after another in the sequence, the corresponding position of each number should be coded on the same mental representation. Hence, when asked to judge the relative position of 2 numbers, the first number of the pair might activate its corresponding position on the mental representation, and the position of the second number would be derived through forwards or backwards serial search along the mental continuum (see Figure 5). Relative-order judgments would thus call for a scanning process, forth and back from one of the numbers, chosen as a reference point (e.g. the number on the left of the pair); this mechanism would be faster for

³ In line with their model we would also suggest that our proposed mental representation preserves the equidistant property of the number system (i.e. the distance between 2 and 3 is the same as the distance between 8 and 9).

adjacent compared to non-adjacent items and elicit a reverse distance effect, and this might be even more true for items in the conventional sequence order, as a forward scanning might be faster than backwards scanning. We won't go deeper into speculation, but the idea is that relative positions might be coded with respect to a reference point using a scanning (or recitation-like) process; moreover, deriving the relative position of numbers in the sequence might not require the additional activation of the corresponding quantities (which doesn't mean, yet, that these quantities could not be automatically activated in healthy adults).

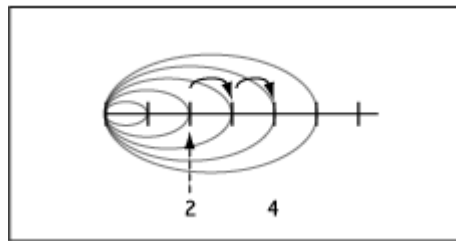


Figure 5. Schematic representation of how the relative position of number 4 could be derived from the position of number 2 along a tentative number sequence representation (without necessarily activating the corresponding quantities).

With respect to the relationship between quantity and a number's position in the sequence, we have argued that processing each aspect of numbers (e.g. quantities) might not require the additional processing (or activation on the representation) of the other (e.g. relative positions). Each numerical information might thus be processed independently of the other. Independent processing, however, doesn't mean that activating one dimension (e.g. the relative position of a number) prevents from the automatic activation of the other (quantity) on the mental representation. Such simultaneous intentional and automatic processing of quantity and relative positions is supported in our behavioural data of (quantity) number comparison by the influence of pair-order (i.e. through the automatic activation of relative positions in the sequence) on the quantity distance effect. Hence, quantity and relative positions might be independently and automatically activated from a common ('integrated') underlying representation.

With respect to the time course of each processing mechanism (comparison and serial search), the spatio-temporal correlates of the distance effect in the ERP study (i.e. earlier distance effect for quantity than numerical order judgments) suggest that quantity would be derived faster than the relative order of numbers (or comparison executed faster than serial search). Faster comparison than serial search mechanism might explain why a reverse distance effect (depending on slower serial search) was only rarely reported in numerical tasks, probably being often overruled by a standard distance effect (i.e. by a faster comparison mechanism). Consequently, because numbers have an integrated representation of the quantity they convey and their relative position in the sequence, number processing tasks might more often activate fast quantity representations, unless specific emphasis is made on sequence order relations. Our data show however, that beyond fast quantity processing mechanisms, under specific paradigms and instructions, order-specific effects could still be observed. Thus, resorting to a comparison or a serial search mechanism in number processing tasks would be probabilistically determined on each trial according to task demands and specific characteristics of the stimulus (e.g. Jou, 1997), with possibly more frequent (since faster) activation of numbers' quantity meaning. Finally, let us note that several reason might explain, though none in a definitive way, why quantity-related effects have been more often reported than order specific-effects in number processing tasks; among these reasons, we can point out: that quantity processing was more often investigated than numerical order processing, that quantity is an evolutionarily more ancient number meaning (both phylogenetically and ontogenetically), that quantity might be processed faster than numerical order.

4. Conclusion

In the present thesis, we have emphasized that, beyond their quantity meaning, numbers can be used equally efficiently to refer to relative positions within a sequence or ordering. Sequential order is a crucial property of numbers, but yet a much neglected one.

Based on the neuropsychological investigation of a single case acalculic patient (CO), we proposed, as a working hypothesis, that the quantity meaning of numbers and their relative position in the sequence could rely on separate mental representations and/or on distinct processing mechanisms. As astonishing as it could appear, in fact, CO was unable to process sequence order relations about numbers (and other ordered series), whereas he could almost perfectly process their quantity meaning. Our hypothesis, from initially anecdotic turned out to be fascinating and well-grounded since the evidence for (at least partially) distinct processing mechanisms was collected in an electrophysiological study and in two behavioural experiments. A much tentative model was then proposed, and should be taken as a speculative (and only one) way we could integrate our empirical data into a more coherent story: we proposed a hypothetical number representation upon which quantity and relative positions would remain partially disjoined and would call for distinct processing mechanisms.

Overall, the present empirical work suggests that sequence knowledge about numbers deserves better consideration and should be better and more systematically investigated in both brain-damaged patients (especially patients experiencing difficulties in number processing), and in children during their acquisition of the number concept. Much work remains to be done to gain a better understanding of how numerical order (and sequential order in general) is processed and neurally implemented. The aim of the present thesis was not to provide any ultimate answers to long-lasting scientific debates, but, instead, to open the way towards a more comprehensive approach of numbers and to their fascinating peculiarities.

Some debated issues and the need for future research

One question that remains open and deserves further consideration is whether those features that were usually attributed to the quantity meaning of numbers, should be (re)considered, instead, as depending from their sequential order characteristic. We think, for instance, about the relationship between numbers and space. The SNARC effect, which is one of the most prominent substantiations of this relationship, has usually been taken as evidence for the activation of a magnitude representation, whenever numbers are processed. However, because a SNARC effect was recently reported with non-quantitative ordered series, its interpretation should deserve new analysis. It might be, in fact, that the sequential order of numbers, more than their quantity meaning, is the underlying feature that clarifies the relationship between numbers and space. Our patient's impairment in processing both sequence relations and spatial locations might provide some support in this direction. Yet, testing sequence knowledge in children with visuospatial disabilities might be an interesting way to better investigate the relationship between numerical order processing and space: how would these children, who lack a spatial representation of numbers (see the recent study by Bachot, Gevers, Fias, & Roeyers, 2005) behave in sequence processing tasks is a theoretically challenging question.

Besides, the question of whether order is biologically relevant, as much as numerical quantity is thought to be, should also deserve better attention. Some evidence in the animal literature suggests that monkeys are able to judge the relative social rank of other monkeys in their living groups, and thus that they can represent the ordinal position of other monkeys (Terrace & al., 2003). Yet, whether preverbal infants have a similar representation of relative (ordinal) positions is actually largely unknown and should be the object of future research.

Finally, order is not just (one of) the fundamental property of numbers, it is also highly important in the way it structures our lives. In fact, most of the things we do everyday have to be done *in a certain order*: from the way we prepare coffee in the morning, to the itinerary we follow to go to work, and the way we organize each hour of the day, most of our actions cannot be done randomly, otherwise causing a chaotic day...

5. References

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