GROWTH AND EQUALITY EFFECTS OF PENSION PLANS*

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Abstract

We investigate the balanced growth effects of pension plans on the rate of growth and on the equality in a closed economy where individual decisions about education are the engine of growth. We distinguish between two pension systems, a Beveridgean one and a Bismarckian one, where the latter may depend on one's entire earning history or not, and show that an economy with a pay-as-you-go Beveridgean system has a lower growth rate than with a Bismarckian one, yet is also "fairer" in terms of intergenerational equality. We also show that the rate of growth in a FF Bismarckian scheme may be increasing in the rate of contributions.

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Introduction[1]

There is a large body of research on the expenditure and provision of education. For one, countries spend a significant proportion of public expenditure on this item (more than 5.6% on average in OECD countries). Furthermore, education is also considered to be not only a major contributor to growth (see e.g. Barro & Sala-i-Martin (1995), Benhabib & Spiegel (1994) and Mankiw, Romer & Weil (1992) who all spell this more or less out), but also to changes in inequality. In fact, various studies have looked at the role of public expenditure on either one or in fact both of these issues (we mention only a few papers): Kaganovich & Zilcha (1999) show that there is a double benefit of public funding of education which consists in reducing income inequality and yielding a positive growth effect. Bearse, Glomm & Ravikumar (1998) investigate how the degree of centralisation of education funding influences income distribution. Fernandez & Rogerson (1999) examine the effect on the level and distribution of different education financing systems and show that there may be no simple trade-off between equity and resources to financing. Glomm & Ravikumar (1992) look at the effects of education on inequality and concentrate on comparing public vis-à-vis private education in terms of levels of wealth and income inequality.

This paper also investigates the effects of growth and equality where education is involved. We adopt an approach that is a little less straightforward, however at least as appealing and relevant: we investigate how growth and equality are affected by the choice of a public pension system.

Over the last years, research on public pension schemes has become a major player in the field of public economics as well as of late an important issue in actual public policy. The reasons for this are fairly straightforward: public pension plans not only represent a large proportion of total GDP (OECD countries already spend more than 10% of GDP on public pensions), more-

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over, given the demographic change that is taking place, this proportion is forecast to increase to over 16% in the next 30 years if no action is taken. Hence, it is of no surprise that numerous implications of pension policies have already been examined in the literature (e.g. induced distortions on the labour markets, effects on saving rates and capital accumulation, and consequences for the provision of public goods, see World Bank (1994)).

An important aspect of pensions is how they affect growth in the economy. Hence, from a normative point of view the implications of pension plans has found some attention. Saint-Paul (1992) investigates e.g. the case for public debt in an Arrow-Romer endogenous growth model (i.e. an 'AK' model) and shows that the introduction of an unfunded social security system reduces capital accumulation and hence the growth rate, such that it cannot be Pareto-improving. Marchand et al. (1996) show that the case for ascending net transfers (such as a PAYG pension plan) in such an endogenous growth setting is rather weak on the balanced growth path.

It seems surprising that there have been few attempts to combine both aspects, education and public pension schemes, in a single model and investigate the effect of one on the other. This is not to say that education and public pensions have not been combined. Turning to theoretical models first, in an endogenous growth framework Docquier & Michel (1999) examine a similar problem to Marchand et al. (1996), yet do so in a Lucas-Uzawa setting where education is the engine of growth. They reach quite a similar result to Marchand et al. (1996) regarding the weak case for ascending net transfers. Glomm & Kaganovich (1999) investigate the growth and inequality effects of increased spending on public provision of education that comes at a cost of decreased pension outlays and show that higher spending on public education leads to higher income inequality. Turning to empirical work, Zhang & Zhang (2000) perform an analysis of how social security affects growth determinants. They do so by allowing for a simultaneous interaction between growth and social security and find in a cross-country selection that the empirical evidence indicates that social security has a positive impact on growth in such that it stimulates investment in human capital.

However, to the best of our knowledge, there is no study examining in detail how *different* pension schemes will affect the investment in education and hence how growth and inequality changes resulting from differences in pension schemes arise. Although Zhang & Zhang (1995) look at effects of social security on the steady state growth path of per capita income where investment in education is motivated by parental altruism and fertility plays a role, their analysis of pension schemes is limited due to the two-period overlapping generation set-up with homogenous agents (see below for more details). Hence we investigate how growth and equality are affected by the choice of a public pension scheme. We contrast economies with different pension schemes and accordingly different incentives to invest in education. These pension schemes differ for one in the way that they are financed: they may be pay-as-you-go (PAYG) pension schemes where contributions are immediately paid out to retired agents, or they may be fully-funded (FF) pension schemes where contributions are invested on capital markets until the contributing generation is retired.[2] Furthermore, we deal in our model with heterogeneous agents and can thus add a dimension by looking at the way the pension is awarded: pensions may be Beveridgean, i.e. a lump-sum transfer that is identical to all agents, or pensions may be Bismarckian, i.e. earnings related. This too will of course be an important determinant in the decision of whether to invest in education or not.

Finally, as we are working in a three period overlapping generations model, we are able to add an interesting feature that has so far not been considered in the literature, yet is of real-world importance: Bismarckian pension benefits can either be based on one's entire earnings history or only on one's partial earnings history (as e.g. in Germany and in the US respectively).

We should point out that we only focus on the balanced growth path in our model. Although a model such as ours would necessarily exhibit rich dynamics in moving from one retirement system to another, we merely wish

[2] If pensions are paid as annuities in both a PAYG and a FF scheme, it is clear that uncertainty of life expectancy no longer matters when comparing schemes.

to compare schemes that are already in place. Hence, there are no explicit dynamics which must be taken into account.

The structure of the model is discussed in section 1. Section 2 analytically presents the comparison as far as possible. We present a numerical example in section 3 to study the long-run effects for schemes where no definite analytical conclusion is possible, compare growth and equality effects of a FF scheme as opposed to a PAYG scheme and give a brief summary of our results. Finally section 4 concludes.

1 The Model

1.1 Individuals

Our model depicts a closed economy consisting of individuals, firms and the government. We use an overlapping generations structure and split each individual's life into three equally long periods, the length of which we normalise to one, and which we label youth, adulthood and retirement respectively. We understand the term 'young' as being over the age of mandatory schooling yet in an age where agents may decide to invest in higher education, be it by attending a university or similar. Thus, agents born at time t have the opportunity to allocate a proportion $\overline{e}$ of their youth to education ($0 \leq \overline{e} \leq 1$), the remaining time of this period is spent working. This proportion $\overline{e}$ of time is exogenously given and agents only face a "take-it-or-leave-it" decision: they decide to either devote $\overline{e}$ of their youth to education or not to invest in education at all. This is not an unreasonable assumption if one regards the time spent investing in human capital when young as time attending a university or taking part in a training course: the length of such a program is given and one 'cannot' decide to stop attending before it is over.[3]

The consequence of investing in the accumulation of human capital is that

[3]'Cannot' may seem a harsh notion, but one could capture the notion by assuming that without a certificate/diploma confirming successful completion of the programme, no productivity gains are realised. Hence, no rational individual would start investing and then decide to quit.

it will increase one's productivity when adult - a period that is spent in full-time work (labour is supplied inelastically). In the third period of life, agents are retired and do not work. The population grows at a constant rate n, hence denoting the size of generation t by N_t we have

$$N_t = N_{t-1}(1+n). \tag{1}$$

Agents differ in their individual ability, denoted by b, to transform education into productivity. Thus, for a given time spent investing in education, agents that have a higher ability to transform education into human capital will increase their human capital more than an agent with a lower ability. We capture this in the following equation which characterises the productivity of an adult generation t individuals with ability b

$$h_{b,t+1} = \tilde{h}_t\left[1 + b e_{b,t}^{\alpha}\right] \tag{2}$$

where $h_{b,t+1}$ denotes the level of human capital this agent born at time t has when he is adult (i.e. at time $t+1$) and $\tilde{h}_t$ denotes the level of human capital young agents are endowed with at time t; if the agents invests in human capital, $e_{b,t} = \overline{e}$; if he does not, $e_{b,t} = 0$. Hence, agents who do not invest in human capital witness no change in their productivity when adult.

Formally, we assume that abilities to educate, b, are uniformly distributed between an upper and a lower bound, which we normalise for convenience to 1 and 0 respectively. Denoting the distribution of abilities in the population by $f(b)$, the level of human capital generation t inherits is:

$$\tilde{h}_t = \int_0^1 h_{b,t} f(b)\, db. \tag{3}$$

We see that while agents are identical at birth in terms of their inherited human capital, due to differing abilities, individuals are heterogeneous in terms of their levels of human capital when adult.

We assume that the level of human capital each young generation starts off with, $\tilde{h}_t$, is the average level of human capital of its parent generation. This assumption is common in the literature (see e.g. Azariadiz & Drazen

(1990), Glomm & Ravikumar (1992) and Lucas (1988)) and is made in order to have education as the (endogenous) engine of growth. It can be justified by the fact that human capital accumulation profiles are very similar during the period of mandatory schooling - a period not considered in our model – and largely dependent on the knowledge of the adult *generation*.[4] An alternative setting would have been to study an endogenous growth model without human capital but where investment in physical capital is the engine of growth (an 'AK' model). However, examining inequality effects in this case would require to differentiate agents on the basis of another exogenous criterion (such as the discount rate). Also, in terms of incentives due to the social security system, the story would be somewhat different. A model with human capital accumulation says a similar story with a realistic heterogeneity criterion (the ability to educate) and endogenous differences (in wages).

The lifetime income of an agent of type b is the discounted sum of his net wage income earned during the time devoted to work during his youth, his net wage income earned during adulthood and his pension. As $h_{b,t+1}$ is also the productivity expressed in efficiency units of labour, lifetime income is given by:

$$W_{b,t} = \omega_t \widetilde{h}_t \left(1 - e_{b,t}\right) + \frac{\omega_{t+1} h_{b,t+1}}{R_{t+1}} + \frac{p_{b,t+2}}{R_{t+1} R_{t+2}} \tag{4}$$

where $\omega_t = (1 - \tau_t)\, w_t$ denotes the net wage per efficiency unit of labour, w_t the gross wage rate and τ_t the proportional rate of contributions to the retirement system at time t. $p_{b,t+2}$ denotes the pension individual b of generation t will receive, $R_t = 1 + r_t$ is one plus the interest rate paid on savings at time t. Note that in this descriptive exposition of our model we will for the time being not be more explicit about the pension as we come to this at a later point.

Each individual maximises an additively time separable utility function subject to his budget constraint. We denote youth consumption of generation

[4] The more educated the adult generation is, the higher the average productivity of their children will be.

t with c_t, middle age consumption with d_{t+1} and old age consumption with z_{t+2}. Furthermore, we assume for simplicity that utility takes a logarithmic form, hence we have individuals maximising

$$(5) \qquad \text{Max } U_{b,t} = Log\left(c_{b,t}\right) + \beta_1 Log\left(d_{b,t+1}\right) + \beta_2 Log\left(z_{b,t+2}\right)$$

subject to

$$(6) \qquad W_{b,t} = c_{b,t} + \frac{d_{b,t+1}}{R_{t+1}} + \frac{z_{b,t+2}}{R_{t+1}R_{t+2}}.$$

Agents are of course allowed to supplement their state pension with private savings. In other words, old age consumption is financed by the pension and private savings. We do not take liquidity constraints into account but assume for reasons of simplicity and tractability that individuals have perfect access to credit markets, therefore the agent's optimal solution to the above problem yields:

$$(7) \qquad c_{b,t} = \frac{1}{\beta_1}\frac{d_{b,t+1}}{R_{t+1}} = \frac{1}{\beta_2}\frac{z_{b,t+2}}{R_{t+1}R_{t+2}} = \frac{W_{b,t}}{1+\beta_1+\beta_2}.$$

1.1.1 Investment Decision

So far we have not dwelled on which individuals will decide to invest in education and which will not. Clearly, it is preferable to select $e_{b,t} = \overline{e}$ if the resulting lifetime income is higher than with $e_{b,t} = 0$. Agents therefore compare what their lifetime income would be if they decided not to invest in education to what they would enjoy if they decided to opt for the investment (they do not take account of general equilibrium effects as each agents considers his actions to be negligible upon the aggregate). As the abilities of agents are uniformly and continuously distributed, there will be an agent who is indifferent to investing in education or not: his cost of doing so, the foregone income when young, will just balance the present value of the additional gain of his higher productivity. We will label the ability this agent has "the critical value of ability", b_t^c. Agents with a lower ability will choose not to invest whilst agents with a higher ability will choose to do so. As perfect access to credit markets is given, b_t^c is obtained by comparing the income

Figure 1: Income distribution among adults

one would have with education to that without education. It is clear that the critical cut-off of abilities will depend on the wage, the rate of return on capital as well as on the pension scheme. As we have not yet discussed the latter, we give the form of b_t^c later. In any case, the distribution of labour efficiency amongst adults is shown in figure ??.

1.2 Production and factor prices

There are a large number of firms, each producing an identical composite good, Y_t, using capital, K_t , and labour, L_t , measured in efficiency units . The production function is assumed, for simplicity, to be of the Cobb-Douglas type:

$$Y_t = AK_t^{\beta} L_t^{1-\beta} \tag{8}$$

where β is the share of capital in income and A is a scale parameter. As noted above, human capital is equivalent to productivity, which is expressed in efficiency units. Hence, labour supply during period t is given in efficiency units by

$$L_t = N_t \left(1 - \left(\overline{1} - b_t^c\right) \overline{e}\right) \widetilde{h}_t + N_{t-1} \widetilde{h}_t \tag{9}$$

where the first term denotes total labour supply of the young at time t and the second term labour supply of the adult at time t. Each firm acts competitively so that it equates the gross wage and the rate of return on capital with the respective marginal productivity. Denoting per capita efficient capital by k_t ($k_t = K_t/L_t$), we have

$$w_t = A\beta k_t^{\beta-1} \tag{10}$$

$$R_t = A\left(1-\beta\right)k_t^{\beta}. \tag{11}$$

Capital is assumed to depreciate completely each period.

1.3 Government's Role

The sole role of the government is to levy a proportional tax τ_t on wage income which is used to finance pensions. So far we have not been explicit about the pension $p_{b,t}$. Given the setup of our model, we have various different pension schemes that we can compare. For one, the pension may either be of the PAYG or of the FF type. Furthermore, as we are dealing with heterogeneous agents, the pension may either be a Beveridgean pension or a Bismarckian one. Finally, if we are considering a Bismarckian pension, our three period overlapping generations setup allows us to distinguish between Bismarckian pensions that depend on one's entire earnings/contributing history or on a partial earnings/contributing history. We can capture of all these features of the pension scheme if we use the following equation for pensions:

$$p_{b,t} = \rho_t\left[\theta\left(1-e_{b,t-2}\right)\omega_{t-2}\widetilde{h}_{t-2}R_{t-1} + \omega_{t-1}h_{b,t-1}\right] + B_t. \tag{12}$$

B_t denotes the Beveridgean pension and is hence independent of types. The bracketed term denotes the Bismarckian pension and ρ_t denotes the proportion of past wage income that the individuals receive (the so-called Bismarckian coefficient). If $\theta = 1$, the Bismarckian pension depends on one's entire earnings history, if $\theta = 0$, only ones adult income is taken into account when calculating pension benefits. This will have important implications on the incentive to invest in education.

The government's budget constraint in each respective pension scheme (PAYG or FF) is therefore

$$\begin{aligned} \text{PAYG} &: \quad t_t^1 N_t + t_t^2 N_{t-1} = N_{t-2}\int_0^1 p_{b,t} f(b)\, db \\ \text{FF} &: \quad t_t^1 R_{t+1} R_{t+2} + t_{t+1}^2 R_{t+2} = \int_0^1 p_{b,t+2} f(b)\, db \end{aligned}$$

where t_t^1 and t_{t+1}^2 refer to the contributions of the young and adult generation respectively ($t_t^1 = \tau_t w_t h_t \left[1 - \overline{e}(1-b_t^c)\right]$ and $t_{t+1}^2 = \tau_{t+1} w_{t+1} h_t \left[1 + \overline{e}^{\alpha} \frac{1-b_t^{c2}}{2}\right] = \tau_{t+1} w_{t+1} h_{t+1}$).

Finally, now that we have given the form for pensions, one can calculate the critical level of abilities. This is done by setting the lifetime income of an agent who does not invest in education (i.e. $e_{b,t} = 0$) equal to that of one who does, denoting his ability by b_t^c and solving for the critical level of abilities. Of course, as we have an upper bound on b (we assumed above that $b \in [0,1]$), we therefore obtain

$$b_t^c = Min\left[\frac{\omega_t}{\omega_{t+1}} \overline{e}^{1-a} R_{t+1}\left(\frac{R_{t+2} + \rho_{t+2}\theta}{R_{t+2} + \rho_{t+2}}\right), 1\right]. \tag{13}$$

We then have to close the model by introducing the capital stock dynamics. As the capital stock in period $t+1$, K_{t+1}, is given by the total savings of period t, we have accordingly :

$$\begin{aligned} \text{PAYG} &: \quad K_{t+1} = N_t \widetilde{s}_{t,1} + N_{t-1}\widetilde{s}_{t,2} \\ \text{FF} &: \quad K_{t+1} = N_t\left[\widetilde{s}_{t,1} + t_t^1\right] + N_{t-1}\left[\widetilde{s}_{t,1} + R_t t_{t-1}^1 + t_t^2\right] \end{aligned}$$

where $\widetilde{s}_{t,1} = \int_0^1 s_{t,1} f(b)\, db$ and $\widetilde{s}_{t,2} = \int_0^1 s_{t,2} f(b)\, db$ are the average savings of the young and of adults at time t.

1.4 Growth

In our model, economic growth comes from the fact that the average stock of human capital of adults is transmitted to the young of the next generation. This gives the growth rate of human capital, given the uniform distribution of abilities, as

$$g_{t+1} = \frac{\widetilde{h}_{t+1} - \widetilde{h}_t}{\widetilde{h}_t} = \left(1 - b_t^{c^2}\right) \frac{\overline{e}^{\alpha}}{2}. \tag{14}$$

We see that the rate of growth is decreasing in the critical level of abilities: the lower the proportion of agents who decide to invest in education, the lower the level of growth as the transmitted average level of human capital is low:

$$\frac{dg_{t+1}}{db_t^c} = -\overline{e}^{\alpha} b_t^c < 0.$$

2 Comparisons

We want to compare various pension plans in terms of how they affect the rate of growth and inequality in the economy on the balanced growth path.[5] Hence, observing that one pension scheme may entail higher growth and/or lower inequality than another does not necessarily imply that the latter should be replaced by the former. This is due to the fact that any transition to a new steady state would hence entail dynamics that would have to be analysed to see whether after taking costs of transition into account, the new pension scheme would still be more desirable than the existing one. Investigating this is however beyond the scope of this paper; instead we wish to compare how otherwise identical economies will differ only due to their choice of the pension scheme. We should also point out that we will only

[5] Given the utility function we selected in (5), we saw above that an individual's consumption in all periods of his life is proportional to his lifetime income. As the average level of human capital grows over time at the rate g, so does wealth, hence does consumption. We therefore have to deflate the latter three and all other intensive variables (stock of capital, savings) by the average level of human capital to have the balanced growth path.

consider 'pure' pension plans. We will not consider a plan where pensions are *partly* PAYG and *partly* FF financed, nor when pensions are *partly* Beveridgean and Bismarckian. Considering only pure plans will allow us to focus solely on effects that arise due to the specific plan.

The way we compare in terms of steady state growth and inequality is by observing the effect of changing the rate of contributions to the pension scheme. The reason we proceed as such is that even though the setup of our model is fairly simple, it is impossible to find any closed form solution for our model (we show this the appendix 5.1). However, as we consider mandatory pension schemes, varying the contribution rate will allow us to see how growth and inequality move as the scope of the pension scheme is changed. As with a zero contribution rate there is no pension scheme that can affect the steady state level of growth and inequality, we can draw conclusions which scheme will result in higher growth/inequality.

Before comparing pension schemes in terms of inequality we first have to define a measure of inequality. No doubt, selecting a measure of inequality is not an easy task.[6] We choose here a simple measure defined by the difference in total lifetime income of the least and the most able agents. Although this is a simple measure and far from perfect, it allows us to make analytical conclusions on how inequality will change. The measure, denoted on the balanced growth path by η, is given by $\eta_t = \frac{W_{b=b_t^c,t}}{\widetilde{h}_t} - \frac{W_{b=0,t}}{\widetilde{h}_t}$ and can be simplified to be on the balanced growth path

$$\eta = w(1-\tau)\frac{\overline{e}^{\alpha}}{R}\left(1+\frac{\rho}{R}\right) - w(1-\tau)\overline{e}\left(1+\frac{\theta\rho}{R}\right). \tag{15}$$

Note that we assume the rate of contribution to be such that the most able agent ($b = 1$) always finds it worthwhile to invest in education ($e_1 = \overline{e}$). If this were not the case, all agents have the same lifetime income and we would simply have $\eta = 0$. Also, note that the fact that our measure is not scale-invariant does not pose any problems for our investigation: all we are interested in is how inequality changes as the scope of the pension scheme is

[6]See Sen (1973) for a discussion of positive and normative measures of inequality.

enlarged (i.e. the contribution rate is increased), and how the benefit schemes compare to each other in terms of inequality.

Rewriting the critical level of abilities (13) for the balanced growth path, we get

$$(16) \qquad b^c = Min\left[\overline{e}^{1-\alpha}R\frac{R+\theta\rho}{R+\rho}, 1\right].$$

We see in (15) and (16) that a Beveridgean transfer does not have a direct effect on growth or on inequality. However, the Bismarckian coefficient ρ is important. Using the social security budget constraint, one can show that the Bismarckian coefficient is related to the tax rate:

$$(17) \qquad \rho = \begin{cases} 0 & \text{if FF } or \text{ PAYG Beveridgean} \\ \frac{\tau}{1-\tau}R\frac{[1-\overline{e}(1-b_t^c)]R+\left[1+\frac{\overline{e}^\alpha}{2}\left(1-b_t^{c2}\right)\right]}{[1-\overline{e}(1-b_t^c)]\theta R+\left[1+\frac{\overline{e}^\alpha}{2}\left(1-b_t^{c2}\right)\right]} & \text{if FF Bismarckian} \\ \frac{\tau}{1-\tau}\frac{[2-\overline{e}(1-b_t^c)]\left[1+\frac{\overline{e}^\alpha}{2}\left(1-b_t^{c2}\right)\right]^2}{[1-\overline{e}(1-b_t^c)]\theta R+\left[1+\frac{\overline{e}^\alpha}{2}\left(1-b_t^{c2}\right)\right]} & \text{if PAYG Bismarckian} \end{cases}$$

We proceed in the remainder of this section as follows: we first of all only consider pension schemes that are financed under a FF principle before considering a PAYG scheme. In each case, we will compare the Beveridgean plan to the Bismarckian plan as well as distinguish in the latter between the full ($\theta = 1$) or partial ($\theta = 0$) earnings history.

2.1 Fully funded schemes

Let us recall a standard result of the literature: in a model with exogenous growth, perfect capital markets (i.e. no liquidity constraints), an exogenous labour supply (the pension regime does not affect the labour market) and an actuarially fair pension scheme (i.e. the present value of benefits equals the present value of taxes), a fully funded pension scheme does not modify the capital stock as private savings and savings of the social security system are perfect substitutes. Hence any decrease in the capital stock due to a decrease in savings is neutralised by an equal increase in the pension fund: the interest rate is not affected by a change in the contribution rate.

We shall see however that in our model as long as the FF scheme is actuarially fair for each generation as a whole (i.e. the discounted value of benefits allocated to the members of each generation equals the discounted value of the taxes paid), this is also true, even if we have redistribution *within* a generation and hence the system is not actuarial fair at the *individual* level. In this case there is no *direct* incentive to save less or to save more as a *generation*, which means that, for a given rate of growth and given factor prices, the rate of contributions does not influence the aggregated amount of saving in the economy.[7] However, this does *not* mean that capital accumulation and growth are unaffected by the choice of the pension scheme. Indeed, the pension scheme may modify the incentive to educate and hence may change the labour supply (given in equation (9)) and thus the wage and interest rates. These indirect effects on the labour supply can then be observed on capital accumulation.

We therefore now turn to each of the different possible pension benefit rules: a Bismarckian benefit based on a full earnings history ($\theta = 1$), a Bismarckian benefit based on a partial earnings history ($\theta = 0$) and a Beveridgean benefit.

In the case of a **Beveridgean benefit rule**, inequality and the critical level of abilities are

$$\eta = w(1-\tau)\left[\frac{\overline{e}^{\alpha}}{R} - \overline{e}\right] \tag{18}$$

$$b^{c} = Min\left[\overline{e}^{1-\alpha}R, 1\right]. \tag{19}$$

We see that there is no incentive to change one's education investment decision as the contribution rate changes ($\partial b^c/\partial\tau = 0$). Hence the labour supply is unaffected. It follows that the capital per efficiency unit of labour does not change, so factor prices and the growth rate stay constant. Since the wage and the interest rates are constant, the inequality measure only evolves with

[7] This is shown in the appendix: the capital stock is independent of contributions paid to a fully funded system.

the tax rate: the higher the rate of contributions, the more inequality we have.

Conclusion 1: *With a Beveridgean FF system, one obtains* $dG/d\tau = 0$ *and* $d\eta/d\tau < 0$. *Redistribution lowers inequality but does not modify the incentive to educate compared to a laisser-faire solution (i.e. no pension scheme). Since capital accumulation is unchanged, a Beveridgean FF system has no macroeconomic implications.*

When benefits are however **Bismarckian** and are based on a **full earnings history** ($\theta = 1$), inequality and the critical level of abilities are

$$\eta = w\,(1-\tau)\left[\frac{\overline{e}^{\alpha}}{R} - \overline{e}\right]\left[1 + \frac{\rho}{R}\right] \tag{20}$$

$$b^c = Min\left[\overline{e}^{1-\alpha}R, 1\right]. \tag{21}$$

Here again, there is no incentive to change one's education investment decision as the contribution rate changes ($\partial b^c/\partial\tau = 0$), so that the labour supply is again unaffected. Again, the capital per efficiency unit of labour does not change, so neither do factor prices or the growth rate. Using (17) to derive the equilibrium value of the Bismarckian coefficient we get

$$\rho = \frac{\tau}{1-\tau}R$$

and hence the inequality measure is given by

$$\eta = w\left[\frac{\overline{e}^{\alpha}}{R} - \overline{e}\right] \tag{22}$$

which is completely independent of the pension scheme in place and is not affected by the contribution rate. We therefore use this benefit rule as a benchmark case.

Conclusion 2: *A Bismarckian FF system where benefits are linked to full earnings is completely neutral: one obtains* $dG/d\tau = 0$ *and* $d\eta/d\tau = 0$.

There is no intragenerational redistribution so that the system is actuarially fair at the individual level. Hence, there is no effect on human capital accumulation, physical capital accumulation or on inequality of changing the contribution rate. This is our benchmark case.

As however noted above, given our three period overlapping generations structure, we can also consider a **Bismarckian benefit rule** where pensions are based only on a **partial earnings** history $(\theta = 0)$. In this case inequality and the critical level of abilities are given by

$$\eta = w(1-\tau)\left[\frac{\overline{e}^{\alpha}}{R} - \overline{e}\right] + w(1-\tau)\frac{\overline{e}^{\alpha}\rho}{R} \tag{23}$$

$$b^c = Min\left[\overline{e}^{1-\alpha}R\frac{R}{R+\rho}, 1\right]. \tag{24}$$

In such a case the pension scheme is still actuarially fair at the level of the generation but of course ceases to be actuarially fair at the individual's level (as in the Beveridgean system). In this case we are unable to claim that $\frac{dR}{d\tau} = 0$. In fact, the uneducated are penalised for being so not only by earning lower wages and having a lower lifetime income, but also due to the fact that their contributions made to the retirement system when young are not taken into account when calculating the pension benefit. Hence, there is a clear incentive to invest in human capital as the Bismarckian pension based on partial earnings $(\theta = 0)$ 'subsidises' the cost of education. Therefore, as the rate of contributions increases, investment in human capital becomes more and more attractive even to low-ability agents and the level of growth in the economy increases. Formally, the Bismarckian coefficient is

$$\rho = \frac{\tau R}{1-\tau}\left(1 + \frac{1-(1-b_t^c)\,\overline{e}}{1+\frac{\overline{e}^{\alpha}}{2}(1-b_t^{c2})}R\right). \tag{25}$$

The partial derivative of this term with respect to τ is clearly positive: the higher the tax rate, the higher the Bismarckian parameter. Therefore we will observe that there is a direct incentive to invest more in education since the critical ability decreases with the contribution rate $(\partial b^c/\partial\tau < 0)$. The growth rate thus rises and the number of workers decreases. In the long run however the effect on the labour supply in efficiency units is ambiguous: the number of workers decreases yet their efficiency increases. This change in L affects factor prices and general equilibrium effects come into play: the wage and interest rates move, hence aggregate savings and physical capital accumulation change too. Unfortunately, the global effect cannot be studied analytically. For plausible values of the parameters, we use a numerical example to later reveal that the capital stock per efficiency unit of labour falls: the interest rate thus rises and the wage rate decreases. This general equilibrium effect on R reinforces the stimulating effect of the pension plan on growth: the growth rate is highly increasing with τ.

Substituting ρ in the inequality measure, we get

$$\eta = w\frac{\overline{e}^{\alpha}}{R}\left[1+\frac{\left[1-\overline{e}(1-b_t^c)\right]\tau R}{1+\frac{\overline{e}^{\alpha}}{2}\left(1-b_t^{c2}\right)}\right]-w(1-\tau)\overline{e} \tag{26}$$

We have a positive partial derivative with respect to τ: the direct effect of the pension plan on inequality is positive. Incorporating general equilibrium effect alters the size of the direct effect. If R rises and w decreases, this lowers the direct effect on inequality. A numerical example reveals that the income gap rises rather linearly with the payroll tax.

Conclusion 3: *A FF Bismarckian system based on partial earning ($\theta = 0$) redistributes income from the unskilled to the skilled by subsidising the return on education (it increases the return on human capital investment while not its cost) under the assumption that general equilibrium effects do not dominate the direct effects. This stimulates growth but also increases inequality; we have $dG/d\tau > 0$ and $d\eta/d\tau > 0$.*

2.2 Pay-as-you-go schemes

A PAYG system differs from a FF system in two important aspects: it encompasses intergenerational transfers and modifies aggregated savings and capital accumulation. Since contributions are not invested, a PAYG system reduces capital accumulation and pushes the interest rate upward and the wage rate downward.[8] We should note that the direct effect on saving does not depend on the way benefits are allocated. Rather, the analytical study of growth and inequality consequences in a PAYG scheme are strongly affected by general equilibrium reactions of factor prices. Hence, we must base our analysis on the intuitive conjecture that the higher the contribution rate, the less agents save (as they receive a higher pension and hence the need for private savings is lower). As a result, we assume that $\frac{\partial R}{\partial \tau} > 0$ and $\frac{\partial w}{\partial \tau} < 0$.

In a PAYG Beveridgean scheme the incentive to invest is the same as in the case of the FF Beveridgean system. As a higher level of contributions reduces the need for private savings, this increases the cost of investing in human capital by lowering the discounted value of future income. Hence it is straightforward that a higher contribution rate will imply a lower growth rate. In fact, in the case of a **Beveridgean benefit rule** we also have the same equation for inequality as in the FF case:

$$\eta = w(1-\tau)\left[\frac{\overline{e}^{\alpha}}{R} - \overline{e}\right] \tag{27}$$

$$b^{c} = Min\left[\overline{e}^{1-\alpha}R, 1\right]. \tag{28}$$

A higher payroll tax also reduces the inequality measure. This direct effect ($\partial\eta/\partial\tau < 0$) is reinforced by general equilibrium effects through w ($\partial\eta/\partial w > 0$)and R ($\partial\eta/\partial R < 0$). Therefore a Beveridgean PAYG system reduces inequality more strongly than a FF Beveridgean system.

Conclusion 4: *With a Beveridgean PAYG system one obtains $dG/d\tau < 0$ and $d\eta/d\tau < 0$. A PAYG system discourages human capital investment through its effects on the interest rate. Redistribution and factor price movements lower inequality.*

When benefits are **Bismarckian**, based on one's **full earnings history**

[8] This is shown in the appendix.

$(\theta = 1)$, we get

$$\eta = w(1-\tau)\left[\frac{\overline{e}^{\alpha}}{R} - \overline{e}\right]\left[1 + \frac{\rho}{R}\right] \tag{29}$$

$$b^{c} = Min\left[\overline{e}^{1-\alpha}R, 1\right]. \tag{30}$$

We would have assumed that in this case investment in human capital would be more attractive than under a Beveridgean benefit rule as agents profit from their investment when adult as well as when retired. However, we see that the incentive to invest is no different from the incentive under the Beveridgean PAYG benefit rule (equation 28)). Therefore the incidence on the growth rate and factor prices is also identical. The reason for this is that as one's entire earnings history is taken into account, an investment in human capital implies higher adult earnings which increases one's pension claims, yet on the other hand lowers youth income and hence also lowers one's pension claims. These two effects cancel and we observe that the growth effects of altering the rate of contributions are identical to the growth effects of the Beveridgean scheme.

Turning to inequality, although we have the same condition as in the respective FF case (condition (23)), inequality will actually be different as the Bismarckian factor is not identical. Using the equilibrium value of $\rho = \frac{\tau}{1-\tau}\Psi$, where $\Psi = \frac{(2-\overline{e}(1-b_t^c))\left(1+\frac{\overline{e}^{\alpha}}{2}\left(1-b_t^{c2}\right)\right)^2}{(1-\overline{e}(1-b_t^c))R+\left(1+\frac{\overline{e}^{\alpha}}{2}\left(1-b_t^{c2}\right)\right)} > 0.$ we have

$$\eta = w\left(1-\tau\right)\left[\frac{\overline{e}^{\alpha}}{R} - \overline{e}\right]\left[1 + \frac{\tau}{1-\tau}\frac{\Psi}{R}\right] \tag{31}$$

This is clearly larger than the one obtained in a Beveridgean system. The multiplicative term $(1-\tau)$ decreases with τ and the term $\left[1 + \frac{\tau\Psi}{(1-\tau)R}\right]$ is likely to increase with τ. The global result on inequality seems ambiguous and we use a numerical example in section 3 to illustrate the sign of the effects.

Conclusion 5: *Under a PAYG Bismarckian benefit rule based on full earnings $(\theta = 1)$, the effect on growth and other macroeconomic implications are* *similar to those obtained under a Beveridgean benefit rule. A PAYG system discourages human capital investment through its effects on the interest rate, a priori the effect on inequality is ambiguous.*

Finally, we turn to a **Bismarckian** scheme where the pension only depends on one's **partial earnings history** (more specifically, on one's adult income, i.e. $\theta = 0$). In this case inequality and the critical level of abilities are given by

$$\eta = w(1-\tau)\left[\frac{\overline{e}^{\alpha}}{R} - \overline{e}\right] + w(1-\tau)\frac{\overline{e}^{\alpha}\rho}{R} \tag{32}$$

$$b^c = Min\left[\overline{e}^{1-\alpha}R\frac{R}{R+\rho}, 1\right]. \tag{33}$$

There will clearly be again a larger incentive to invest in human capital compared to the case where pensions depend on one's complete earnings history as contributions made during one's youth are not taken into account when calculating pensions, hence increasing the cost of being uneducated. We will therefore observe a substitution towards education. Here the Bismarckian parameter is

$$\rho = \frac{\tau}{1-\tau}\left[2 - \overline{e}(1-b_t^c)\right]\left[1 + \frac{\overline{e}^{\alpha}}{2}\left(1 - b_t^{c2}\right)\right]. \tag{34}$$

The partial derivative of this with respect to τ is clearly positive: the higher the rate of contributions, the higher the Bismarckian coefficient. As a result, compared to the two previous regimes, there is a direct incentive to invest more in education since the critical ability decreases ($\partial b^c/\partial\rho < 0$). However, this decreases the number of workers, hence the total effect on labour supply in efficiency units is again ambiguous. Accordingly, we cannot analytically deduce what the effect on aggregated savings will be and hence on factor prices, therefore the total effect on growth is ambiguous too. Substituting ρ in the inequality measure, one gets

$$\eta = w(1-\tau)\left[\frac{\overline{e}^{\alpha}}{R} - \overline{e}\right] + w\tau\left[1 - \overline{e}(1-b_t^c)\right]\left[1 + \overline{e}^{\alpha}\frac{1-b_t^{c2}}{2}\right] \tag{35}$$

The tax rate induces two effects on the inequality measure: it reduces inequality via a decrease in net wages, but also increases inequality via the stimulating effect on growth (the return on education is stimulated). The overall effect is ambiguous and again, we will turn to a numerical example to illustrate the sign of the effects on inequality.

Conclusion 6: *With a Bismarckian PAYG system based on partial earnings* $(\theta = 0)$*, the effect on growth and inequality is unfortunately ambiguous.*

3 A Numerical Example

We have seen that the effects of changing the contribution rate on inequality in a PAYG Bismarckian scheme under a full earnings rule and the effects on inequality and growth in a partial earnings rule are ambiguous as one is unable to find a closed form for our model. Hence, we use a numerical example of our model in order to deduce in which direction growth and inequality will move. Before simulating the model, we have to calibrate several parameters of our model, i.e. capital's share in output, β, the scaling parameter in the production function, A, the share of education investment in human capital accumulation, α, the individual's valuation of adult and old-age consumption, β_1 and β_2 respectively, and the proportion of youth to be invested in human capital, $\overline{e}$. We assigned following values: $\beta = .27$, $A = 6$, $\alpha = .4$, $\beta_1 = .8$, $\beta_2 = .64$ and $\overline{e} = .3$. Although these values may be debatable, the qualitative results are not very sensitive to changes in the values.

Our example gives us figures 2 and 3, showing the steady state rate of inequality and growth respectively, according to the benefit rule selected. Our numerical example hence shows that for the values chosen a PAYG Bismarckian system based on full earnings $(\theta = 1)$ reduces inequality. However, the inequality measure decreases slightly less than compared to a PAYG Beveridgean scheme. This is due to the absence of clear redistribution in the former, which naturally reduces the effect on inequality less.

Figure 2: Inequality

Figure 3: One plus the growth rate

Our numerical example also suggests that under a PAYG Bismarckian system based on partial earnings ($\theta = 0$) the interest rate rises more strongly than under the other two PAYG regimes as there is a larger decrease in the labour supply in efficiency units.[9] This general equilibrium effect reduces the direct growth rise. Thus, although the growth rate is stimulated by the pension rule, the discouraging effect of the interest rate dominates and leads to a decrease in the growth rate.

Finally, using the same parameters for simulations of a fully funded scheme, we see in figures 2 and 3 that under a Bismarckian benefit rule based on partial earnings ($\theta = 0$) the general equilibrium effects on the interest rate are sufficiently strong to increase the inequality as well as the growth rate. Note that in figure 2 we only see 5 curves and in figure 3 only 4. This is due to the fact that the FF and PAYG Beveridgean are, in terms of inequality, almost identical. In terms of growth the FF Beveridgean and FF Bismarckian ($\theta = 1$) are in fact identical as are the PAYG Beveridgean and PAYG Bismarckian ($\theta = 1$). Hence these respective curves overlap.

3.1 Summary

Table 1 summarises our main results. We have seen that growth and inequality in an economy where time-investment in education is the engine of growth can vary significantly, according to which pension scheme is selected. The reason is straightforward: the choice of one pension scheme rather than another will create a different incentive to invest in productivity increasing measures. This can have significant macroeconomic effects and affect the well-being of all generations and individuals.

[9] Graph omitted here.

4 Conclusion

A number of recent papers have investigated the growth effects of taxes in the context of neoclassical growth models. Our study also considers growth effects, however, we adopt an approach that is different from the literature in such that we focus on the implications of social security plans. Due to the set-up of our model, we are able to differentiate amongst a variety of these plans: we consider the way they are financed as well as the benefit rule. Since the benefit rule may induce intragenerational and/or intergenerational transfers, the macroeconomics of social security raises distributional issues. This is the reason why our study examines how pension plans affect both growth and inequality measures.

Our model depicts an endogenous growth economy where human capital accumulation is the engine of growth. We distinguish pay-as-you-go and fully funded pension plans and, by introducing heterogeneous agents, are also able to contrast a Beveridgean and a Bismarckian benefit rule. Finally, due to the particular three period overlapping generations structure of our model, we can distinguish two sub-cases of the Bismarckian benefit rule: benefits can either be based on a partial earnings history, meaning that not all contributions agents have made are taken into account when calculating their pension benefits, or benefits can be based on a full earnings history, i.e. all contributions are taken into account.

We find that whereas all pay-as-you-go regimes are detrimental for growth but beneficial for inequality, this need not be the case for fully funded schemes. Although the latter are usually neutral in terms of total capital accumulation in the economy (i.e. any decrease in individuals' savings resulting from an increase in the contribution rate to the social security system is offset by the increase in savings of the social security system) even if they are not actuarially fair at the individual level (intragenerational redistribution can be involved), we show that the growth rate of the economy may well be affected by the contribution rate as the benefit rule will affect the incentive to invest in human capital. We find that only under a Bismarckian fully funded rule where benefits are based on partial earnings will the growth rate be increasing in the contribution rate: by ignoring contributions made early in life when calculating the pension benefit, the cost of not investing in human capital is two-fold: not only does one have a low wage, rather more, one contributes longer, yet is not 'rewarded' for this. The inequality implications of fully funded systems also depend on the benefit rule: inequality decreases under a Beveridgean plan, stays constant under a Bismarckian plan based on full earnings and increases under a Bismarckian plan based on partial earnings history.

Finally, as in the literature on tax reforms, we should note that our analysis is purely descriptive and does not involve any normative judgement. Our only purpose is to show how the choice of a particular pension plan matters in terms of the long-run growth rate and income disparities. Given that an economy already has a pension scheme, our analysis does not allow us to recommend that a different scheme should be implemented: the choice of the best pension plan would require (i) solving a long-run efficiency-equity trade-off and (ii) comparing transitional costs and gains. This could be done by selecting a complex social welfare function taking into consideration intragenerational and intergenerational disparities. However, this is a line for future research.

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5 Appendix

5.1 Closed form solution

Given the setup of our model, total savings of the young and adult respectively at time t are given by

$$\begin{aligned}
\widetilde{s}_{t,1} &= \int_0^1 s_{t,1} f(b)\, db = \frac{((\beta_1+\beta_2)\omega_t h_t (1-\overline{e}(1-b_t^c))) - \frac{\omega_{t+1} h_t}{R_{t+1}} \left(1+\frac{\overline{e}^\alpha}{2}\left(1-b_t^{c^2}\right)\right) - \frac{\overline{p}_{t+2}}{R_{t+1} R_{t+2}}}{1+\beta_1+\beta_2} \\
\widetilde{s}_{t,2} &= \int_0^1 s_{t,2} f(b)\, db = \frac{\left(\beta_2 \omega_{t-1} h_{t-1} R_t \left(1-\overline{e}\left(1-b_{t-1}^c\right)\right)\right) + \beta_2 \omega_t h_{t-1} \left(1+\frac{\overline{e}^\alpha}{2}\left(1-b_{t-1}^{c^2}\right)\right) - \frac{(1+\beta_1)\overline{p}_{t+1}}{R_t R_{t+1}}}{1+\beta_1+\beta_2}
\end{aligned}$$

where $w_t = A\beta k_t^{\beta-1}$, $R_t = A(1-\beta) k_t^\beta$, $\overline{p}_t$ is given in (12), b_t^c depends on the respective pension scheme and labour supply is given in (9). To find a closed form we have so know solve

$$\begin{aligned}
\text{PAYG} &: \quad K_{t+1} = \widetilde{s}_{t,1} + \widetilde{s}_{t,2} \\
\text{FF} &: \quad K_{t+1} = \left[\widetilde{s}_{t,1} + t_t^1\right] + \left[\widetilde{s}_{t,1} + R_t t_{t-1}^1 + t_t^2\right]
\end{aligned}$$

with $t_t^1 = \tau_t w_t h_t \left[1 - \overline{e}(1-b_t^c)\right]$ and $t_{t+1}^2 = \tau_{t+1} w_{t+1} h_t \left[1 + \overline{e}^\alpha \frac{1-b_t^{c2}}{2}\right] = \tau_{t+1} w_{t+1} h_{t+1}$. We see that in the steady state we will receive an equation highly non-linear in k, which cannot be solved analytically.

5.2 Effect of taxes on capital

We derive here the partial derivative of the stock of capital with respect to payroll taxes. We examine this problem at the balanced growth equilibrium where the wage and interest rates are constant.

5.2.1 The fully funded system

With a fully funded system, the capital stock is given by

$$K_{t+1} = \left[\widetilde{s}_t^1 + t_t^1\right] + \left[\widetilde{s}_t^2 + R_t t_{t-1}^1 + t_t^2\right]$$

with $\widetilde{s}_{t,1} = \int_0^1 s_{t,1} f(b)\,db$ and $\widetilde{s}_{t,2} = \int_0^1 s_{t,2} f(b)\,db$ and

$$
\begin{aligned}
s_t^1 &= \frac{(\beta_1+\beta_2)(wh_t\left[1-\overline{e}(1-b^c)\right]-t_t^1) - \frac{wh_{t+1}-t_{t+1}^2}{R} - \frac{t_t^1 R^2 + t_{t+1}^2 R}{R^2}}{1+\beta_1+\beta_2} \\
s_t^2 &= \frac{\beta_2(wh_{t-1}\left[1-\overline{e}(1-b^c)\right]-t_{t-1}^1)R + \beta_2(wh_t - t_t^2) - \frac{(1+\beta_1)(t_{t-1}^1 R^2 + t_t^2 R)}{R}}{1+\beta_1+\beta_2}
\end{aligned}
$$

On the balanced growth path, we can write $t_t^1 = \tau w h_t\left[1-\overline{e}(1-b^c)\right] = t^1 h_t$ and $t_{t+1}^2 = \tau_{t+1} w_{t+1} h_{t+1} = t^2 h_{t+1}$. One thus obtains $\frac{\partial K_{t+1}}{\partial t^1} = \frac{\partial K_{t+1}}{\partial t_t^2} = 0$. The pension regime does not alter capital accumulation, whatever the form of redistribution within each generation. This is due to the fact that aggregated saving of a given generation only depends on the average benefit it receives and not on its distribution.

5.2.2 The PAYG system

With a PAYG system, the capital stock is given by

$$
K_{t+1} = \left(\widetilde{s}_t^1 + t_t^1\right) + \left(\widetilde{s}_t^2 + R_t t_{t-1}^1 + t_t^2\right)
$$

with $\widetilde{s}_{t,1} = \int_0^1 s_{t,1} f(b)\,db$ and $\widetilde{s}_{t,2} = \int_0^1 s_{t,2} f(b)\,db$ and

$$
\begin{aligned}
s_t^1 &= \frac{(\beta_1+\beta_2)(wh_t\left[1-\overline{e}(1-b^c)\right]-t_t^1) - \frac{wh_{t+1}-t_{t+1}^2}{R} - \frac{t_{t+2}^1 + t_{t+2}^2}{R^2}}{1+\beta_1+\beta_2} \\
s_t^2 &= \frac{\beta_2(wh_{t-1} - t_{t-1}^1)R + \beta_2(wh_t - t_t^2) - \frac{t_{t+2}^1 + t_{t+2}^2}{R}}{1+\beta_1+\beta_2}
\end{aligned}
$$

On the balanced growth path with $t_t^1 = t^1 h_t$ and $t_{t+1}^2 = t^2 h_{t+1}$, we obtain the following partial derivatives

$$
\begin{aligned}
\frac{\partial K_{t+1}}{\partial t^1} &= \frac{-h_t}{1+\beta_1+\beta_2}\left[(\beta_1+\beta_2) + \frac{\beta_2 G}{R} + \frac{G^2}{R^2} + \frac{G}{R}\right] < 0 \\
\frac{\partial K_{t+1}}{\partial t^2} &= \frac{-h_t}{1+\beta_1+\beta_2}\left[\beta_2 + \frac{G^2}{R^2}\right] < 0
\end{aligned}
$$

where G is one plus the growth rate of human capital. The effect on capital accumulation is obviously negative.