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THE ROLE OF EDUCATION SUPPLY IN ECONOMIC GROWTH AND THE DYNAMICS OF SKILLS

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Abstract

This paper analyzes the role of the structure of skills in economic development through investment in human capital. With a lack of credit market for education and the presence of indivisibilities in investment in human capital as well as of congestion in the educational system, the initial distribution of skills affects aggregate output and investment both in the short and the long run, as there are multiple balanced growth paths. This paper provides an additional explanation for the persistent differences in per-capita output across countries. Moreover, it shows that cross-country differences in macroeconomic adjustment to educational policies can be attributed, among other factors, to differences in the distribution of skills.

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1 Introduction

This paper explores the theoretical linkage between social mobility and growth, through investment in human capital. Our main interest is how high skilled population evolves through time in light of educational policies and how it is related to growth. It is shown that the distribution of skills can significantly affect aggregate economic activity both in the short and in the long-run. Countries which have similar educational policies but different historically determined skills structures, follow different growth paths. Moreover, in the long-run, transitory changes in educational policies may have quite different effects according to the skills structure in the economy. Some countries may reach a new stage of development while others are trapped at a low development equilibrium. Hence the paper suggests an explanation for the differences in growth patterns between countries.

One of the major motivations to study the relationship between education and growth is the observed correlation between skills and economic performance. In developed countries, we observe two main levels of formal education. The first is the compulsory schooling which provides individuals with fundamental knowledge. The second includes subsequent trainings (college and university). For several decades, this structure evolves with an increasing proportion of students, even if this tendancy seems to slow down. For example, in the United States, the college population increased by 4.4% yearly on average from 1970 to 1980, and by 2.6% yearly on average since the beginning of the 1990's. On the contrary, many developing countries fail to promote university education and remain, at best, with a very large proportion of low skilled individuals and a very low growth rate. Finally, a set of countries, including "new dragons" succeeded in developing university and have experienced high growth rates.

Following empirical studies showing the importance of the workforce quality on the growth process (Denison, 1962; Jorgenson, 1984; Barro, 1991), some models of economic growth, such as Lucas (1988) or Azariadis and Drazen (1990), have emphasized investment in human capital as an important factor contributing to growth. These models generate persistent growth endogeneously. Azariadis and Drazen (1990) provide an explanation for economic development related to increasing returns to education. To the extent that a significant component of human capital is due to formal schooling, these models do not account for education supply (and its organization). Further, they are representative agent models and then can't stress any issues concerning the structure of skills an economy. This shortcoming applies to Tamura (1991) since, in his model, an initial heterogeneous population converges to an homogenous one.

Banerjee and Newman (1993) provide an explanation for different economic development paths based on wealth distribution, regardless of education. Galor and Zeira (1993) analyzed the linkage between wealth distribution, education choices and growth. But they ignored the efficiency of the educational system and the impact of educational policies on the growth process. The importance of education supply has been stressed in a general equilibrium model by Glomm and Ravikumar (1992) who show how educational incentives could be different according to the nature of the educational system and how this leads to different income distributions and growth paths. In another framework, Bénabou (1996) considers dynasties in which original parent spends time to educate his child and votes taxes to provide him with public education but he is not concerned with economic development.

This paper develops a general equilibrium model with overlapping generations and intergenerational altruism. It focuses on the educational organization with two levels of human capital. The basic level, called low skill, is reached by all agents in the economy and is entirely financed by taxes on wages. The second level, called high skill, is only partially financed by the government and implies an individual choice of paying or not the additional private cost, the tuition fee. Without credit markets for education, parents choose to pay or not the fee for their children, according to their preferences (which depend on the human capital level of their children) and to their income[1] (section 2). This contrasts with Gradstein and Justman (1996) who consider that additional private spendings are perfectly divisible and may complete public ones. Our formulation highlights the existence of fixed costs, such as fees, to access university (see Chiu, 1998). Eckstein and Zilcha (1994) formerly introduced public education which depends on educators in a non political context but they only focused on the effect of the introduction of compulsory schooling.

In each period, parents are divided into two categories, the high-skilled (HS) who have had the second level of formation and the low-skilled (LS) who have not had it. Each parent has one child, and he chooses whether or not to

[1] In most of the mentioned litterature, there are no credits markets for education. Education takes place because of parental altruism. A notable exception is Fehrstman and *al* (1996).

pay the fee for his child. There are then three possibilities for the proportion of HS in the new generation (section 3). No change arises when all HS choose to pay the fee and no LS pays it. An increase in the HS proportion arises when some of the LS choose to pay the fee. A decrease of the HS proportion arises when no LS and only part of the HS pays the fee. These three outcomes appear in the study of the short-term equilibrium. In the intertemporal equilibrium with perfect foresight (section 4), these possible changes in the proportion of HS combine with the accumulation of physical capital resulting from the savings decisions and with the accumulation of human capital: there is an external effect of the mean level of human capital from one generation to the next.

We show that the dynamics is monotonic for both the proportion of the HS agents and the ratio of physical capital to the mean human capital. In the long-run, there is an endogenous growth rate which depends on the long run proportion of HS agents. Three tendancies appear concerning the proportion of HS. First, there are intervals where this proportion is constant. Hence, identical schooling systems lead to different growth rates according to different proportions of HS agents. Second, this proportion decreases when the tax rate is too high and it may converges to a "low-skill trap", which can be viewed as an underdevelopment trap. Third, the proportion of HS increases when the tax rate is low enough and it converges to a higher growth path.

The paper is organized as follows. The following section presents the basic model. Section 3 provides an analysis of the short-term equilibrium. In section 4, we present the dynamics. The study of the effects of the proportion of high skilled is provided in section 5. In section 6, we study development paths and assess the effects of educational policies. The last section contains concluding remarks.

2 The model

We consider a three period lived overlapping generations model. Each period t, there is a continuum of agents born at t of weight 1 which are identical, but their parents are not.

Each agent born at t receives a basic education, and a proportion of them receives an additional education (say at university) during their first period of life. In period $t+1$, when adult, he works, consumes, saves and may pay the fee (finance the university) for his child. In his last period of life $t+2$, he is retired and consumes the gross return of his savings.

Parents in period t are either high skilled (HS) or low skilled (LS). The p_t HS workers are those who benefited the additional education in period $t-1$. Their human capital level is h_t^s and their wage income is $w_t h_t^s$, where w_t is the wage per unit of efficient labor. The human capital of the $1-p_t$ LS parents (those who only received the basic education in period $t-1$) is h_t^o and their wage income is $w_t h_t^o$. Parents choose to pay or not to pay the fee (cost of additional training) for their child.

2.1 The educational system

The basic education in period t applies to all agents and it is financed by the government. It leads to a human capital level

$$h_{t+1}^o = \Phi\left(\alpha_t^s p_t, \alpha_t^o\left(1-p_t\right)\right)\bar{h}_t \tag{1}$$

which depends on the number $\alpha_t^s p_t$ of HS teachers (α_t^s is the proportion of the p_t HS workers who are teachers in the basic school), on the number $\alpha_t^o\left(1-p_t\right)$ of LS teachers and on the mean human capital level (of adults) in the society at date t:

$$\bar{h}_t = p_t h_t^s + \left(1-p_t\right)h_t^o \tag{2}$$

Φ is an increasing function of its arguments: $\Phi_1 > 0$, $\Phi_2 > 0$.

The cost of the basic education is the total wage cost of teachers[2]

$\alpha_t^s p_t w_t h_t^s + \alpha_t^o\left(1-p_t\right)w_t h_t^o$

This cost will be paid by the government through taxes on wage income.

The additional training in period t only applies to p_{t+1} children, those for which parents accept to pay for it. The fee is proportional to the current HS wage

[2] Workers and teachers wages are the same for an equal qualification (non arbitrage property). A simple interpretation of the two types of teachers is the following. To teach the basic education it is sufficient to have had it, i.e. both LS and HS can do it. But HS are more efficient and there is some complementarity.

$$e_t^s = \gamma_t w_t h_t^s \tag{3}$$

The production of h^s depends on three arguments: the number of university professors the government finances $(\beta_t p_t)$, the fee, γ_t, and the number of students p_{t+1}. Specifically:

$$h_{t+1}^s = \Psi\left(\beta_t p_t, \gamma_t, p_{t+1}\right) h_{t+1}^o \tag{4}$$

Ψ is an increasing function of $\beta_t p_t$ and γ_t; it is a decreasing function of the number of students p_{t+1} (dilution effect).

The university education increases the human capital level which results from the basic education. Without spendings $(\beta_t = \gamma_t = 0)$ there is no supplementary human capital accumulation: $\Psi(0, 0, p) = 1$ for all $p \geq 0$.

The total financing by the government (including the basic education) is

$$g_t = \left(\alpha_t^s + \beta_t\right) p_t w_t h_t^s + \alpha_t^o \left(1 - p_t\right) w_t h_t^o \tag{5}$$

We shall assume positive proportion of all teachers and a positive fee $\alpha_t^s > 0$, $\alpha_t^o > 0$, $\beta_t > 0$, $\gamma_t > 0$, $\alpha_t^s + \beta_t < 1$, $\alpha_t^o < 1$.

2.2 The consumers

An agent born in $t-1$ works in period t. His human capital h_t is either h_t^s or h_t^o according to he has received or not the university education.

With a tax rate τ_t on his wage income, the after-tax income of an agent with a human capital h_t is $(1-\tau_t) w_t h_t$. He consumes c_t, saves s_t, and spends e_t for the university education of his child. He chooses e_t either equal to e_t^s and the child receives the university education, or he chooses $e_t = 0$ and his child does not receive this education. Consequently:

$h_{t+1} = h_{t+1}^s$ if $e_t = e_t^s$ and $h_{t+1} = h_{t+1}^o$ if $e_t = 0$

He maximizes his life cycle utility which depends on the human capital level of his child[3]:

[3] The utility may depend on the net income of the child $(1-\tau_{t+1}) w_{t+1} h_{t+1}$: this would not modify the decisions since the utility function is assumed to be log-linear.

$$U_{t-1} = (1-a)\ln c_t + a\ln d_{t+1} + b\ln h_{t+1} \tag{6}$$

subject to the budget constraints:

$$(1-\tau_t)\, w_t h_t = c_t + s_t + e_t \text{ and } d_{t+1} = (1+r_{t+1})\, s_t \tag{7}$$

a and b verify: $0 < a < 1$ and $b > 0$; r_{t+1} is the interest rate on savings and d_{t+1} the consumption in period $t+1$.

Whatever the choice of e_t, the optimal decisions of consumption and savings are:

$$c_t = (1-a)\left(\Omega_t - e_t\right), s_t = a\left(\Omega_t - e_t\right), d_{t+1} = (1+r_{t+1})\, s_t \tag{8}$$

where $\Omega_t = (1-\tau_t)\, w_t h_t$ is the after-tax income.

The difference of the two life-cycle utilities U^s_{t-1} and U^o_{t-1} corresponding the choice of $e_t = e^s_t$ and $e_t = 0$ is

$$\begin{aligned} U^s_{t-1} - U^o_{t-1} &= \ln\left(\Omega_t - e^s_t\right) + b\ln h^s_{t+1} - \left(\ln\Omega_t + b\ln h^o_{t+1}\right) \\ &= \ln\left(\tfrac{(\Omega_t - e^s_t)\Psi^b_{t+1}}{\Omega_t}\right) \end{aligned}$$

where $\Psi_{t+1} = h^s_{t+1}/h^o_{t+1} = \Psi\left(\beta_t p_t, \gamma_t, p_{t+1}\right)$ is the ratio of the two possible human capital levels for children.

As a consequence, at date t, the optimal choice of an agent with an after-tax income Ω_t is

$$e_t = e^s_t \text{ if } \Omega_t > \frac{e^s_t}{1-\Psi^{-b}_{t+1}}$$

$$e_t = 0 \text{ if } \Omega_t < \frac{e^s_t}{1-\Psi^{-b}_{t+1}} \tag{9}$$

He is indifferent between the two choices when $\Omega_t = \frac{e^s_t}{1-\Psi^{-b}_{t+1}}$ and may choose $e_t = e^s_t$ or $e_t = 0$.

2.3 The firms

There is in each period one representative firm producing one good with two inputs, physical capital K_t and human capital (or efficient labor) H_t. The production function $Y_t = F(K_t, H_t)$ is homogenous of degree one, increasing and strictly concave with respect to each of the two production factors. The firm maximizes its profits: $F(K_t, H_t) - w_t H_t - (\delta + r_t) K_t$. w_t is the wage, δ the depreciation rate of capital and r_t the interest rate. At equilibrium the wage is equal to the marginal productivity of labor:

$$w_t = F_L'(K_t, H_t) = F_L'(k_t, 1) \tag{10}$$

where $k_t = K_t/H_t$ is the ratio of the capital stock and the human capital level used in production. The interest rate is equal to the marginal productivity of capital net of depreciation:

$$r_t = F_K'(K_t, H_t) - \delta = F_K'(k_t, 1) - \delta \tag{11}$$

3 Short-term equilibrium

At date t, the capital stock K_t is given and results from the savings decisions made in period $t-1$. The proportions p_t of HS and $1 - p_t$ of LS workers are given and result from the education decisions taken by parents in period $t-1$. The corresponding human capital levels h_t^s and h_t^o are known.

3.1 The tax rate

Given educational policies $\alpha_t^o, \alpha_t^s, \beta_t$, the government spendings are given by the relation (5). The tax rate τ_t is assumed to be chosen in order to balance the government constraint. Since all workers (including teachers) pay the same tax rate, the tax income $\tau_t (p_t w_t h_t^s + (1 - p_t) w_t h_t^o)$ should be equal to the government spendings g_t. Using the ratio $\Psi_t = h_t^s / h_t^o$, we obtain:

$$\tau_t = \frac{(\alpha_t^s + \beta_t) p_t \Psi_t + \alpha_t^o (1 - p_t)}{1 - p_t + p_t \Psi_t} \tag{12}$$

Note that $\tau_t \leq \max\{\alpha_t^s + \beta_t, \alpha_t^o\}$ which implies $\tau_t < 1$.

3.2 The number of students at equilibrium

The efficiency of the university education $\Psi(\beta_t p_t, \gamma_t, p_{t+1})$ depends on the number p_{t+1} of students. Since the after-tax income of HS workers is larger than the LS one, there are three possibilities:

a) all HS pay the university cost for their child, and no LS does it.
b) at least some LS pay it, and then all HS pay it.
c) at leat some HS do not pay it, and then no LS pay it.

Let us define, for fixed values of $\beta_t \geq 0$, p_t, $0 \leq p_t \leq 1$ and $\gamma_t > 0$,

$$\rho_t(p_{t+1}) = \frac{\gamma_t}{1 - \Psi(\beta_t p_t, \gamma_t, p_{t+1})^{-b}} \tag{13}$$

$\rho_t(p_{t+1}) w_t h_t^s$ is the minimum after-tax income for which a parent accepts to pay the university cost for his child; $\rho_t(p_{t+1})$ is a measure of the ratio of costs and benefits of the additional education. The function $\rho_t(p_{t+1})$ is well defined and increasing for all $p_{t+1} \geq 0$, since the denominator doen't vanish, $\Psi(\beta_t p_t, \gamma_t, p_{t+1}) \geq \Psi(0, \gamma_t, 1) > 1$.

We first assume $0 < p_t < 1$. The three possibilities are the following

Case a. The students are the children of the HS workers, and $p_{t+1} = p_t$. For this to be an equilibrium with optimal choice of education by both types of parents, the following inequalities should be satisfed with $p_{t+1} = p_t$:

$\Omega_t^s = (1 - \tau_t) w_t h_t^s \geq \rho_t(p_{t+1}) w_t h_t^s$ and $\Omega_t^o = \frac{1}{\Psi_t} \Omega_t^s \leq \rho_t(p_{t+1}) w_t h_t^s$
or equivalently, the tax rate should satisfy:

$$\rho_t(p_t) \leq 1 - \tau_t \leq \Psi_t \rho_t(p_t) \text{ with } \Psi_t = h_t^s / h_t^o \tag{14}$$

Case b. At least some of the LS workers pay the university fee and $p_{t+1} > p_t$. There are two subcases:

- when all LS workers pay the cost, $p_{t+1} = 1$, their income Ω_t^o should at least be $\rho_t(1) w_t h_t^s$, i.e.

$$1 - \tau_t \geq \Psi_t \rho_t(1) \tag{15}$$

and then of course the HS workers pay it: $1 - \tau_t > \rho_t(1)$

- When some LS-workers pay the cost, but not all, $p_{t+1} < 1$, then these workers are indifferent between paying or not paying it and

$$1 - \tau_t = \Psi_t \rho_t(p_{t+1}) \text{ with } p_t < p_{t+1} < 1 \tag{16}$$

Then of course all HS workers pay the fee: $1 - \tau_t > \rho_t(p_{t+1})$

Case c. At least some HS workers do not pay the cost and thus $p_{t+1} < p_t$.

- When no HS worker pay the cost, $p_{t+1} = 0$, then

$$\rho_t(0) \geq 1 - \tau_t \tag{17}$$

- When some but not all HS workers pay the cost, then

$$\rho_t(p_{t+1}) = 1 - \tau_t \text{ with } 0 < p_{t+1} < p_t \tag{18}$$

In both subcases, LS workers choose optimally not to pay the cost: $1-\tau_t < \Psi_t \rho_t(p_{t+1})$

Remark: We have assumed $0 < p_t < 1$. In the special case where $p_t = 0$, there are no HS workers in period t, and thus no university teachers. Then the fee does not exist (it can be assumed arbitrarely large) and necessarely $p_{t+1} = 0$. In the other special case, when all workers are HS, $p_t = 1$, then case b is excluded and there are the three possibilities:

if $\rho_t(1) \leq 1 - \tau_t$ (case a), then $p_{t+1} = 1$

if $\rho_t(0) \geq 1 - \tau_t$, then $p_{t+1} = 0$

if $\rho_t(0) < 1 - \tau_t < \rho_t(1)$, then there exists p_{t+1}, $0 < p_{t+1} < 1$, such that $\rho_t(p_{t+1}) = 1 - \tau_t$.

Proposition 1 *For any values of parameters ($\beta_t \geq 0$, $0 \leq p_t \leq 1$, $\gamma_t > 0$, $0 < \tau_t < 1$), there exists a unique number of students p_{t+1}, $0 \leq p_{t+1} \leq 1$, corresponding to the optimal choice to pay the fee or not by parents, i.e. one and only one of the five conditions ((14), (15), (16), (17), (18)) is verified. When $0 < p_t < 1$,*
case a ($p_{t+1} = p_t$) occurs if and only if $\rho_t(p_t) \leq 1 - \tau_t \leq \Psi_t \rho_t(p_t)$
case b ($p_{t+1} > p_t$) occurs iff $1 - \tau_t > \Psi_t \rho_t(p_t)$
case c ($p_{t+1} < p_t$) occurs iff $1 - \tau_t < \rho_t(p_t)$

We have seen in the remark that the special cases $p_t = 0$ and $p_t = 1$ lead to a unique value of p_{t+1}. Consider now the case $0 < p_t < 1$. When (14) is satisfied, then we also have for all $p_{t+1} > p_t$, $1 - \tau_t < \Psi_t \rho_t(p_{t+1})$ since ρ_t is increasing and case b is excluded; for $p_{t+1} < p_t$, we have $1 - \tau_t > \rho_t(p_{t+1})$ and case c is excluded. Thus when (14) is satisfied, $p_{t+1} = p_t$ is the unique solution.

Now assume (14) is not satisfied and first that $\Psi_t \rho_t(p_t) < 1 - \tau_t$. Then either (15) is satisfied and $p_{t+1} = 1$ is the unique solution, or (15) is not satisfied: $\Psi_t \rho_t(p_t) < 1 - \tau_t < \Psi_t \rho_t(1)$ and then there exists a unique value of p_{t+1}, $p_t < p_{t+1} < 1$ which satisfies (16).

Similarly the last possibility $\rho_t(p_t) > 1 - \tau_t$ implies either (17) and $p_{t+1} = 0$, or (18) defining a unique value of p_{t+1}, $0 < p_{t+1} < p_t$.

3.3 Equilibrium prices

The number of LS workers who are not teachers in the basic education system is $(1 - \alpha_t^o)(1 - p_t)$ which is non negative if $\alpha_t^o \leq 1$; but the number of the HS teachers (in the two types of training) $(\alpha_t^s + \beta_t) p_t + \gamma_t p_{t+1}$ should be less than the number p_t of HS agents. When this is the case, the total efficient labor supply made to the firm is

$$H_t = (1 - \alpha_t^o)(1 - p_t) h_t^o + [(1 - \alpha_t^s - \beta_t) p_t - \gamma_t p_{t+1}] h_t^s \qquad (19)$$

Then the labor market equilibrium determines the current wage and the current interest rate (since K_t is given) following (10) et (11).

In the case $p_t = 0$, the economy works without university education and $p_{t+1} = 0$. In the case $p_t > 0$, there is a restriction on the fee and the number of students. For an optimal choice of $p_{t+1} \leq p_t$ (cases a and c), it is sufficient that $\gamma_t < 1-\alpha_t^s-\beta_t$; but in the case b, the condition $(1-\alpha_t^s-\beta_t)\,p_t > \gamma_t p_{t+1}$ for existence of an equilibrim is more restrictive. It is certainly verified when $\gamma_t < (1-\alpha_t^s-\beta_t)\,p_t$.

Proposition 2 *Given positive parameters α_t^s, α_t^o, β_t and γ_t which verify: $\alpha_t^s+\beta_t<1$ and $\alpha_t^o<1$, and given the past variables $K_t>0$, $0\leq p_t\leq 1$ and $\Psi_t = h_t^s/h_t^o > 1$, there exists a unique short-term equilibrium at period t if either $p_t=0$ (and then $p_{t+1}=0$) or if $p_t>0$ and the unique solution p_{t+1} defined in proposition 1 verifies*

$$(1-\alpha_t^s-\beta_t)\,p_t \geq \gamma_t p_{t+1} \tag{20}$$

Given the parameters and the past variables, the equilibrium tax rate τ_t defined by (12) verifies $0<\tau_t<1$. There exists then a unique optimal number of students p_{t+1} (proposition 1); when $p_t=0$ or condition (20) is satified, the labor supply to the firm H_t is determined by relation (19) and the equilibrium prices are given by (10) et (11).

Finally, all agents save optimally a proportion a of their income net of tax and of fee. The total net income of the parents is

$p_t\Omega_t^s+(1-p_t)\Omega_t^o-p_{t+1}e_t^s = w_t\left[p_t\left(1-\tau_t\right)h_t^s+\left(1-p_t\right)\left(1-\tau_t\right)h_t^o-p_{t+1}\gamma_t h_t^s\right]$

The total net income is necessarily positive since agents do not pay the fee when it is larger than his income net of tax. Thus

$$K_{t+1} = a\left[\left(1-\tau_t\right)\bar{h}_t - p_t\gamma_t h_t^s\right]w_t \tag{21}$$

With $\Psi_{t+1} = h_{t+1}^s/h_{t+1}^o = (\beta_t p_t, \gamma_t, p_{t+1})$ all equilibrium variables obtain.

4 Dynamics

In order to maintain things simple, we consider constant positive values for the parameters α^s, α^o, β and γ of the education policies, and we assume: $\alpha^s+\beta=\alpha^o$; the government finances the same proportion of teachers in each type of agents HS and LS. With this assumption, the tax rate is constant $\tau=\alpha^o$. In this case, the total tax paid by the LS agents, $(1-p_t)\,\tau w_t h_t^o$ is just the cost of the LS teachers in the basic formation $(1-p_t)\,\alpha^o w_t h_t^o$. The total tax paid by the HS agents $p_t \tau w_t h_t^s$ is the cost of the HS teachers in both formations which is paid by the government $p_t\left(\alpha^s+\beta\right) w_t\, h_t^s$.

We also assume that the fee is compatible with at least one constant proportion p of HS-workers.

Assumption: the parameters α^s, α^o, β and γ are positive and verify: $\alpha^s+\beta=\alpha^o$ and $0<\alpha^s+\beta+\gamma<1$.

This implies $\tau=\alpha^o$ and $0<\tau<1$.

4.1 Dynamics with a constant proportion of HS

The number p of students (and of HS agents) is constant: all HS agents pay the fee and no LS pay it (case a). The condition (14) for this case is

$$\frac{\gamma}{1-\Psi^{-b}} \leq 1-\tau \leq \frac{\gamma\Psi}{1-\Psi^{-b}} \tag{22}$$

where $\Psi=\Psi\left(\beta p, \gamma, p\right)>1$. The condition for existence of the short term equilibrium when $p>0$, $\gamma<1-\alpha^s-\beta=1-\tau$ is then necessarely satisfied.

Proposition 3 *Consider fixed values of the parameters: $\tau=\alpha^o=\alpha^s+\beta$, with $0<\tau<1$, $\alpha^s\geq 0$ and $\beta>0$. Then for any value of p_0, $0<p_0<1$, there exists an interval of values for the fee $\gamma>0$ and an interval of values for the proportion p which contains p_0 such that the short-term equilibrium leads to a constant p.*

The set of values of (p,γ) which verify the strict inequalities

$$\frac{\gamma}{1-\Psi^{-b}} < 1-\tau < \frac{\gamma\Psi}{1-\Psi^{-b}} \tag{23}$$

is an open set, by continuity of Ψ. It is enough to show that (p_0,γ_0) belongs to this set for some $\gamma_0>0$. The function of γ:

$\theta(\gamma)=\frac{\gamma\Psi(\beta p_0,\gamma,p_0)}{1-\Psi(\beta p_0,\gamma,p_0)^{-b}}$

is continuous on $[0,1]$ since the denominator does not vanish (for $\gamma=0$, $\Psi\left(\beta p_0,0,p_0\right)>1$ since $\beta>0$). It verifies: $\theta(0)=0$ and $\theta(1-\tau)>1-\tau$ since $\Psi\left(\beta p_0,1-\tau,p_0\right)>1$. Thus there exists a largest value γ_1 of γ, $\gamma<1-\tau$ such that $\theta(\gamma_1)=1-\tau$ For $\gamma>\gamma_1$, we have $\theta(\gamma)>1-\tau$. But we also have at γ_1:

$\frac{\gamma_1}{1-\Psi(\beta p_0,\gamma_1,p_0)^{-b}} < \theta(\gamma_1) = 1-\tau$

Thus for $\gamma_0 > \gamma_1$, and γ_0 near γ_1, this inequality also holds by continuity. This shows that (p_0, γ_0) does belong to the open set.

With such values of the parameters, all short-term equilibria verify that the proportion p of HS is constant, and the two types of human capital accumulation verify for all t:

$h^o_{t+1} = \Phi\left(\alpha^s p, \alpha^o (1-p)\right) \bar{h}_t$ and $h^s_{t+1} = \Psi\left(\beta p, \gamma, p\right) h^o_{t+1}$

Thus the mean human capital level grows at a constant rate

$\bar{h}_{t+1} = G\bar{h}_t$ with $G = \Phi\left(\alpha^s p, \alpha^o (1-p)\right)\left[1 - p + p\Psi\left(\beta p, \gamma, p\right)\right]$.

The human capital devoted to production is with this assumption: $H_t = \left(1 - \tau - \frac{\gamma p \Psi}{1-\Psi^{-b}}\right)\bar{h}_t$. And the physical capital accumulation is:

$K_{t+1} = aH_t w_t$

Since H_t grows at the same rate as $\bar{h}_t$, we have

$k_{t+1} = K_{t+1}/H_{t+1} = K_{t+1}/GH_t = aw_t/G$

and the dynamics of k_t are

$$k_{t+1} = \frac{a}{G} F'_L\left(k_t, 1\right) \tag{24}$$

These dynamics are monotonic (since $dk_{t+1}/dk_t > 0$) and thus k_t converges to some stable steady state k^* which is decreasing with respect to G. But there may be several steady states (see Galor and Ryder, 1989). Then the limit k^* depend on the initial value of k_0. Moreover 0 could be stable and the unique steady state. This is excluded with the assumption

$$\lim_{k \to 0} \frac{F'_L\left(k, 1\right)}{k} = +\infty \tag{25}$$

Note that all educational parameters α^0, α^s, β, γ have a positive effect on the growth rate. But in the short run, when the tax rate τ increases, then the income of all consumers diminishes. The effect of an increase in γ only reduces the net income of the HS who pay the fee. Note also that an increase in G has a negative effect on the dynamics of k_t (equation 24).

4.2 Dynamics with a decreasing proportion of HS

Let us define the cost-benefits ratio:

$B(p,q) = \frac{\gamma}{1-\Psi(\beta p,\gamma,q)^{-b}}$

This function is decreasing with respect to p and increasing with respect to q. Given the initial value p_0, $0 < p_0 \leq 1$, the number of HS decreases iff $B(p_0,p_0) > 1-\tau$.

Proposition 4 *Consider p_0, $0 < p_0 \leq 1$ such that $B(p_0,p_0) > 1-\tau$. Then the sequence of HS at equilibrium p_t is decreasing and*

either there exists some p, $0 < p < p_0$, verifying $B(p,p) \leq 1-\tau$ and in this case the sequence of p_t converges to a steady state value $\check{p}$ where $B(\check{p},\check{p}) = 1-\tau$.

or for all p, $0 < p < p_0$, we have $B(p,p) > 1-\tau$, and in this case the sequence p_t converges to 0; and it may reach 0 after a finite number of periods.

In the case where $B(p,p) \leq 1-\tau$ for some p, $0 < p < p_0$, we have: $B(p_0,p) < B(p,p) \leq 1-\tau < B(p_0,p_0)$. Then there exists a unique value p_1, $p < p_1 < p_0$ such that $B(p_0,p_1) = 1-\tau$; and p_1 is the unique optimal parents choice leading to an equilibrium in period 1 (propositions 1 and 2). Indeed condition (20) is verified: $p_1\gamma < p_0\gamma < (1-\tau)p_0$. Moreover, $B(p_1,p_1) > B(p_0,p_1) = 1-\tau$ and $p_1 > p$. Thus the same argument shows that there exists a unique equilibrium value in period 2, p_2 which verifies $p < p_2 < p_1$. By induction the sequence of p_t is decreasing and verifies $p_t > p$ for all t. Thus the sequence of p_t admits a limit $\check{p} \geq p$ and we have: $B(\check{p},\check{p}) = \lim B(p_t,p_{t+1}) = 1-\tau$.

In the other case, when $B(p,p) > 1-\tau$ for all p, $0 < p < p_0$, we know from proposition 1 that there exists a unique p_1, $0 \leq p_1 < p_0$, which corresponds to the optimal choice of parents an which is an equilibrium by proposition 2 (verifying $p_1\gamma < p_0\gamma < (1-\tau)p_0$). When $p_1 = 0$, then all future periods equilibria verify $p_t = 0$ (there is no more university education). When $p_1 > 0$, we have $B(p_1,p_1) > 1-\tau$ and the same argument as for p_0 applies for p_1. It may need an infinite number of periods for p_t to converge, but the limit of p_t cannot be positive. A positive limit would verify: $B(\check{p},\check{p}) = \lim B(p_t,p_{t+1}) = 1-\tau$ which is excluded.

4.2.1 Interpretation

By assumption, when $B(p_0,p_0) > 1-\tau$, the initial cost is too large when all HS workers would pay the fee. Thus, only a fraction of them will pay it, and the number of HS decreases. But this will reduce the number of HS teachers in the following period who are paid by the government (the tax rate τ is the same but the tax basis is reduced). Then the decreasing sequence p_t necessarely converges to a limit which is

either equal to 0 and the university education disappears in the long run or a positive level $\check{p}$ which is a steady state of the proportion of HS.

4.2.2 Dynamics

Since p_t converges to 0 or $\check{p} > 0$, the sequence of capital-efficient labor ratio also converges and its limits is a steady state of

$k_{t+1} = \frac{a}{G_\infty} F'_L(k_t, 1)$

where G_∞ is the limit growth rate of the mean human capital stock.

In the case where p_t converges to $\check{p} > 0$,

$G_\infty = \Phi(\alpha^s \check{p}, \alpha^o (1-\check{p})) [1 - \check{p} + \check{p}\Psi(\beta\check{p}, \gamma, \check{p})]$

In the case where p_t converges to $\check{p} > 0$,

$G_\infty = \Phi(0, \alpha^o)$

4.2.3 The cases of monotonicity of function $B(p, p)$

1) The case in which $B(p, p)$ is increasing for all p, $0 < p < 1$.

This is the case in which $\Psi(\beta p, \gamma, p)$ is decreasing with respect to p, i.e. the dilution effect Ψ_3 dominates $\beta\Psi_1$ the effect of the university financing by the government. Assume $B(\check{p}, \check{p}) = 1 - \tau$ for some $\check{p}$, $0 < \check{p} < 1$[4]. Then the condition $B(p_0, p_0) > 1 - \tau$ is equivalent to $p_0 > \check{p}$. For any $p_0 > \check{p}$ the sequence p_t decreases and converges to $\check{p}$. In this case, $\check{p}$ is the largest possible proportion of HS in the long run.

2) The case in which $B(p, p)$ is decreasing for all p, $0 < p < 1$.

This case corresponds to $\beta\Psi_1 > -\Psi_3$: the financing by the government dominates the dilution effect.

When there exists $\check{p}$ such that $B(\check{p}, \check{p}) = 1 - \tau$, the condition $B(p_0, p_0) > 1 - \tau$ is now equivalent to $p_0 < \check{p}$; and for $p_0 < \check{p}$, the sequence p_t is decreasing and converges to 0. In this case, $\check{p}$ is the lowest positive proportion of HS for

[4] When $B(p, p) > 1 - \tau$ for all p, $0 < p < 1$, then for all p_0 the sequence p_t decreases and converges to 0. When $B(p, p) < 1 - \tau$ for all p, $0 < p < 1$, then whatever p_0, we have $p_1 \geq p_0$ (since $p_1 < p_0$ is excluded) and the sequence of p_t is non-decreasing).

the university education to exist in the long term: if at any date t, p_t comes below $\check{p}$, then the sequence p_t converges to 0.

The ratio Ψ of h^s on h^o is equal to the ratio of individual net incomes of HS and LS. The larger Ψ, the widest is the inequality between the two types of individuals.

4.3 Dynamics with an increasing proportion of HS

Given p_0 and $\Psi_0 = h_0^s/h_0^o$, we have $p_1 > p_0$ (case b) iff

$$\Psi_0 B\left(p_0, p_0\right) < 1 - \tau$$

and there exists a unique $p_1 > p_0$ which is either 1 when $\Psi_0 B\left(p_0, 1\right) \leq 1 - \tau$, or the unique value p_1, $p_0 < p_1 < 1$ which verifies $\Psi_0 B\left(p_0, p_1\right) = 1 - \tau$ (proposition 1). In order to determine a short-term equilibrium in period $t = 1$, the following condition should be verified (condition (20) in proposition 2):

$$\gamma p_1 < \left(1 - \tau\right) p_0$$

For this to be verified, it is sufficient that $\gamma < (1 - \tau)p_0$.

Proposition 5 *Assume $p_0 > 0$ and $\Psi_0 = h_0^s/h_0^o$ such that $\gamma < (1 - \tau)p_0$ and $\Psi_0 B\left(p_0, p_0\right) < 1 - \tau$.*

Then the intertemporal equilibrium starting with p_0 exists and the sequence p_t is non-decreasing.

The assumptions imply: $p_1 > p_0$ and for all t, the sequence p_t is uniquely defined. Consider any date t such that $p_t \geq p_{t-1} > 0$.

If $p_t = p_{t-1}$, then case a applies in period $t - 1$, and we have (condition (14)) $B\left(p_t, p_t\right) = B\left(p_{t-1}, p_{t-1}\right) \leq 1 - \tau$and this property excludes $p_{t+1} < p_t$.

If $p_t > p_{t-1}$, then case b applies in period $t - 1$, and we have (15) or (16), and $B\left(p_{t-1}, p_t\right) < \Psi_{t-1} B\left(p_{t-1}, p_t\right) \leq 1 - \tau$and since $B_1 < 0$: $B\left(p_t, p_t\right) < 1 - \tau$, which also excludes $p_{t+1} < p_t$.

Thus in all cases we have $p_{t+1} \geq p_t$.

Remark: In the increasing case, the dynamic equation is (relation (16)):

$$\Psi\left(\beta p_{t-1},\gamma,p_t\right)B\left(p_t,p_{t+1}\right)=1-\tau,$$

which is a second order difference equation in p_t. But we have shown that, when this equation applies, the dynamics of p_t is monotonic non-decreasing.

The non-decreasing sequence p_t necessarely converges to a limit $\bar{p}\leq 1$ and in the long term the growth rate is

$$G_\infty=\Phi\left(\alpha^s\bar{p},\alpha^o\left(1-\bar{p}\right)\right)\left[1-\bar{p}+\bar{p}\Psi\left(\beta\bar{p},\gamma,\bar{p}\right)\right]$$

Thus the ratio k_t of the K_t and H_t converges to a stable steady state of the dynamics $k_{t+1}=\frac{a}{G_\infty}F_L'\left(k_t,1\right)$.

5 Effects of the proportion of high skilled agents

As shown by proposition 3, there is a continuum of different values of the proportion of HS leading (in general) to different growth rates in economies having the same educational parameters. Indeed, given an educational policy $(\alpha^s,\alpha^o,\beta,\gamma)$, the balanced growth rate is given by $G=\Phi\left(\alpha^s p,\alpha^o\left(1-p\right)\right)$ $\left[1-p+p\Psi\left(\beta p,\gamma,p\right)\right]$. There are three effects of the proportion of HS on growth. First, this proportion acts on the basic education through Φ. Second, it influences the rate of human capital accumulation in the university, Ψ. Finally, it has an effect on the mean level of skill. There are p HS having the university education, that is a higher human capital level ($\Psi>1$) than the $1-p$ LS. Let us study more precisely these three effects.

The structure of skills in the economy depends on the proportion of HS agents. Given the efficiency of both formations, an increase in the share of HS raises the (stationary) population undertaking the university formation. Some LS parents finance the university formation to their child. Next, their heir will also be HS. Since the human capital level of HS is always higher than the LS one ($\Psi>1$), an increase in the proportion of HS induces a structural effect improving the average human capital level in the economy. Nonetheless, a modification in the proportion of the HS population also changes the efficiency in both type of training. Given educational policies, the proportion of HS determines the number of teachers/professors but also the number of students in the university. This in turn alterates the efficiency in the basic formation as well as in the university. Moreover, the efficiency in the basic formation is shaped partly by the efficiency of HS teachers.

The university education is optional. Hence, the size of the student population corresponds to the proportion of the young whose parents finance university fees. Moreover, α^s HS adults are teaching in the university. Consequently, in the long-run, an increase in the number of teachers has two effects that are opposite in sign. On one hand, it worsens the congestion effect in the educational system and then reduces its efficiency. Given the number of professors, there are more university students. The quantity and/or the quality of knowledge professors can transmit to their students falls. On the other hand, this increased proportion of HS raises the number of professors. Given the number of students, this improves the efficiency in the university education (for the opposite reason than previously). In the particular case where the quality of education only depends on the teacher to pupil ratio, there is no long-run effect of a change in the skills structure. Indeed, this ratio remains unchanged since the number of professors as well as students grow at the same rate. The two effects neutralize each other so that the efficiency in the university education does not depend on the proportion of HS. This efficiency only depends on the proportion of HS financed publicly (and on fees). Given the government budget constraint, this acts upon the ability of financing HS teachers in the basic education.

The basic education is compulsory. Since there is no population growth, the number of students is constant. On the contrary, the number of teachers of each type changes with the proportion of HS. The number of LS teachers cuts back with this proportion whereas the number of HS ones increases. Hence, the overall effect is ambiguous and depends on the relative efficiency of each type of teachers. When the allocation of teachers is efficient, the marginal productivities are proportional to wages: $\Phi_1/\Phi_2 = w^s/w^o = \Psi$ and the derivative $\alpha^s\Phi_1 - \alpha^o\Phi_2$ has the sign of $\alpha^s\Psi - \alpha^o$. Under the assumption that the government finances the same proportion of teachers of each type ($\alpha^o = \alpha^s + \beta$), the proportion of LS teachers will always be higher than the HS one. The sign of the derivative is then unclear. In particular, a relatively high proportion of HS teachers implies a relatively low public financing effort in the university education. Indeed, an increase in the number of HS teachers in the basic formation significatively raises the number of HS teachers who might have relatively few additional knowledge. Then, a very high proportion of HS doesn't guarantee the efficiency in the basic formation. Even if the efficiency of university teaching is large when the proportion of HS teachers is relatively low in the basic education, the effect of an increasing proportion of HS is negative on Φ. The intuition of this result is very simple. If the basic education is principally made by LS teachers and if their proportion in the LS workers is constant, a decrease in the number $1-p$ of LS workers cuts back the number of teachers and, consequently, a decrease in the basic human capital level.

Proposition 6 The global effect of the proportion of high skilled on the growth rate results from three partial effects. First, a mean human capital effect which is positive. Second, the efficiency of the university formation, which is positive when the effect of the number of teachers dominates the congestion effect (there is no effect when this efficiency only depends on the teacher to pupil ratio). Third, the effect on the basic formation which depends on the proportion of the two types of teachers. It is positive iff $\alpha^s\Psi - \alpha^o > 0$.

We have $\frac{\partial G}{\partial p} = (\alpha^s\Phi_1 - \alpha^o\Phi_2)(1-p+p\Psi) + \Phi[\Psi - 1 + p\beta\Psi_1 + p\Psi_3]$. When the efficiency of a level of training only depends on the TPR, the human capital function in the university becomes $\Psi = \Psi\left(\frac{\gamma p_{t+1}+\beta p_t}{p_{t+1}}\right)$. In the long-run equilibrium, $p_t = p$. Then $\Psi = \Psi(\gamma+\beta)$ is independant from p. Since $\Psi > 1$, a sufficient condition to have $\frac{\partial G}{\partial p} > 0$ requires $\alpha^s\Phi_1 - \alpha^o\Phi_2 > 0$, that is $\alpha^s\Psi - \alpha^o > 0$ when the allocation of teachers in the basic formation is efficient.

The effect of the proportion of HS on the growth rate combines two positive effects and an ambiguous one. We shall assume in the next section that the overall effect is positive.

6 An heuristic illustration: educational policies and development

Let us consider two particular forms of the cost-benefits functions $B(p,p) - (1-\tau)$ and $\Psi(\beta p, \gamma, p)\, B(p,p) - (1-\tau)$ depicted respectively by the curves (C) and (D) in figure 1. They allow to describe the dynamics of the choices of HS and LS workers to pay the fee. The horizontal axis describes the dynamics of p which is contingent to its initial value, p_0.

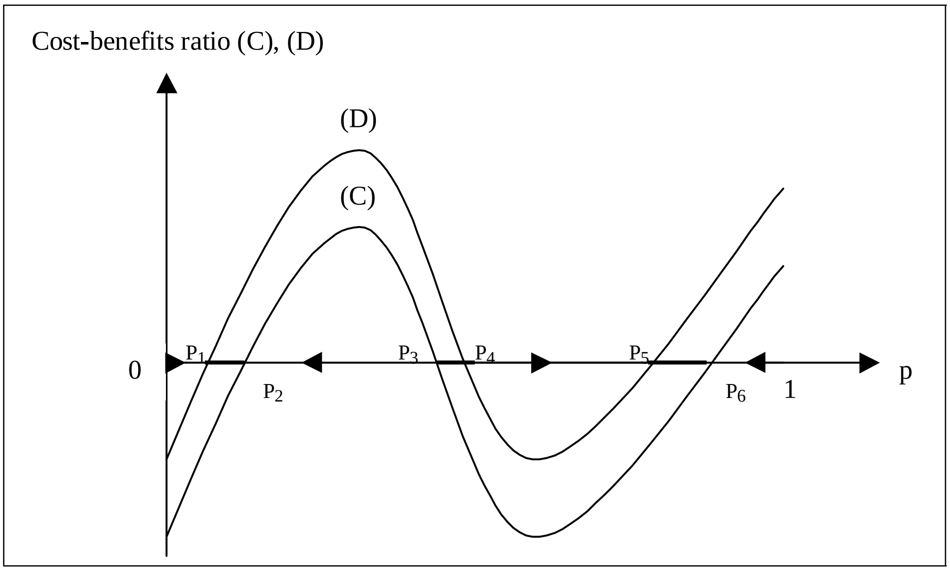

Figure 1: dynamics of p

We can observe the three types of dynamics analysed in the preceding sections.

The sets of p where $(C) \leq 0$ and $(D) \geq 0$, i.e. $[p_1, p_2] \cup [p_3, p_4] \cup [p_5, p_6]$, correspond to an invariant structure of skills in the economy. At each period $t \geq 0$ given the educational parameters, HS workers pay the fee and LS ones do not.

If p_0 lies in an interval where $(C) > 0$, i.e. $]p_2, p_3[\cup]p_6, 1]$, the sequence of p_t decreases. In the other sets $[0, p_1[\cup]p_4, p_5[$ where $(C) < 0$, the sequence of p_t increases.

Note that every interior point in any interval where the proportion of HS workers is constant has a property of stability according to Lyapounov definition. Consequently, there is no divergence following a local perturbation around such a point. Nonetheless, there exist two types of intervals differing in their boundary properties. "Stable extremities" intervals have the properties of a sink: if a shock initially moves the proportion of HS workers in the economy away from its boundary value (the proportions p_1, p_2, p_5 and p_6 in figure 1), this proportion will converge to this bound. Thus, the shock has no permanent, long-run growth effect. "Unstable extremities" intervals are such that any shock leading the proportion of HS workers to leave the interval has long run growth effects. Indeed, this proportion converges to a stable extremity of the closest "constant proportion" interval.

Let us consider an economy with a time-invariant structure of skill (no social mobility). In accordance on the discussion above, we can describe the consequences of a transitory educational policy. In particular, we consider pure redistributive policies which leave the tax rate unchanged ($\tau = \alpha^o = \alpha^s + \beta$). For example, an increase in the proportion of HS workers financed publicly (β) implies a decrease in the proportion of HS workers employed in the basic formation (α^s). If the economy lies in an "unstable extremities" interval[5], such a policy may allow to leave a trap with low growth and to reach a higher growth path. On the contrary, a policy promoting the basic formation leads to a lower growth path. If the economy lies in a "stable extremities" interval, these transitory educational policies won't have permanent growth effects. As in Azariadis-Drazen (1990), we show that little differences in human capital endowments, *i.e* here in the initial proportion of HS individuals, may lead to persistent differences in growth rates across countries. Consequently, some countries may initiate a higher growth path while others could stagnate. Contrary to Azariadis-Drazen, this result of multiple equilibria arises without increasing returns to education due to non convexities in the human capital technology; it is due to the existence of indivisibilities in the financing of the additional formation. Anyway, we extend this result in two ways. First, we exhibit intervals with no social mobility. Countries may initially lie in a state of social immobility (an interval with constant p). Families with HS parents remain HS permanently while parents of LS families never pay university to their children. As a consequence, countries with identical educational policies but different initial proportions of HS have different growth rates. Second, transitory educational policies may have quite different long-run effects according to the importance of the HS population. Despite transitory educational policies promoting university education, some countries can't jump from lower stages of the developement path to more rapid ones. They are trapped in a low development stage. For example, a temporary decrease in the fee or an increase in the proportion of teachers in the university training[6], which increases the return to the

[5] At the bounds of such intervals, the cost-benefits curves are decreasing.

[6] It is important to notice that these incentives have to be sufficient to leave an interval (which is characterized by no social mobility). If so, some LS parents will pay the additional formation to their children and there is an initial increase in p. Consequently, the

additional formation, incites some LS parents to finance university to their children. But, in subsequent periods, the university becomes too costly and some HS parents won't pay it to their children. In the long-run the proportion of HS workers converges to the upper bound of the initial interval. At the individual level, some families could benefit from the educational policy while others could suffer from it. But at an aggregate level, there is no social mobility since the proportion of HS remains constant (at least when the initial proportion of HS was at the upper bound of the interval).

On the contrary, temporary educational policies may help countries in reaching a new stage of development and in initiating a higher growth path. The initial incentives lead some LS parents to finance university to their children. In subsequent periods, the "return" to education is sufficiently high for new LS parents to invest. This is the case till the economy reaches the lower bound of the next interval, characterized by a higher growth rate. Hence, the cross-country differences in macroeconomic adjustements to educational policies can be attributed, among other factors, to differences in the proportion of HS workers. From this point of view, the model provides an explanation for the stages of development observed by Rostow (1960).

Note that extreme dynamics can arise. In the case where, for all $0 < p < p_0$, $B(p,p) - (1-\tau) > 0$, the economy ends up in a "low-skill trap". In the long run, the economy keeps running with LS workers only and the university education disappears.

7 Conclusion

This paper analyzes the effect of the distribution of skills on economic development through investment in human capital. The study demonstrates, that in the absence of a credit market for education, the distribution of skills significantly affects aggregate economic activity. Furthermore, in the presence of indivisibilities in investment in human capital, these effects are carried to the long run as well. Hence, growth is affected by the initial distribution of skills, or more specifically by the percentage of individuals who have college education level at the beginning. A major consequence due to the congestion effect of the educational system lies in the potential existence of many constant growth paths with no social mobility. Moreover, similar educational

magnitude of the educational policy has to be all the more important as the proportion of HS is low (in the interval).

policies have quite different long run effects according to the proportion of high skilled. Indeed, the model provides an explanation for quite different development paths. On one hand, economic development of many countries such as "dragons" which succeeded in promoting university and initiated a higher growth path. On the other hand, developing countries who failed to promote university and remain with a very large proportion of low skilled individuals. Such countries are trapped in a low development stage.

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