Modal analysis of thermal noise from lossy dielectric medium

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***Abstract*—The modal characterization of the thermal noise generated by lossy dielectric media can be used to improve the design of antenna arrays and improve the overall system performance. In this paper we study the thermal noise generated above a circular ground plane by a lossy semi-infinite medium. To do so, we consider an array of infinitesimal dipoles above the ground plane. The fields generated by the dipoles in presence of the finite ground plane are computed using the Method of Moments and used to compute the power dissipated by the dielectric medium. Combining the fluctuation-dissipation theorem to Lorentz reciprocity, the noise correlation matrix at the location of the dipoles is obtained. Using this matrix, we apply the eigenmode decomposition to obtain a modal representation of the noise. From there, we study the behaviour of the different modes and the dependence on the size of the ground plane. We show that increasing the size of the ground plane decreases the noise power and increases its coherence, potentially facilitating its filtering.**

***Index Terms*—Thermal noise, Method of Moments, Finite ground plane, Modal analysis.**

I. Introduction

The estimation of noise is of great importance in numerous applications such as in direction finding algorithms [1], MIMO systems [2] and radioastronomy [3]. Let us consider a system composed of a single antenna and a lossy object, both at the same temperature. Using the Lorentz reciprocity, the authors of [4] showed that the noise power at the antenna port due to the lossy object is directly proportional to the amount of power radiated by the antenna into the lossy object. An important extension of that result to multiple ports is provided in [5], used in the framework of the detection of radiometric signals. It is also shown in [5] that the noise power at the output of the beamformer can be written as a function of the noise correlation matrix between ports. A given entry of the noise correlation matrix can be calculated by evaluating the fields radiated into the lossy object when each of the corresponding ports to the entry are activated.

Techniques using the Method of Moments (MoM) can be used to analyse the noise due to the lossy antenna itself, including losses due to feed lines and front-end generators [6], [7]. The spatial correlation of thermal noise radiated by lossy antennas is studied in [7]. It is shown that the effect of the noise correlation on system noise temperature depends on scan angle. Figures of merit of arrays involving the noise have been developed over the years, such as efficiency metrics and noise system temperature. The first definitions were convenient for the single antenna case. The authors of [8] gave new unified definitions for arrays. An alternative representation is also provided in [9] for phased array feeds, which makes use of the Lange parameters [10].

In radioastronomy, antenna arrays are often place in remote locations to avoid unwanted radio interference. Moreover, a finite ground plane is usually installed to reflect back-radiation and to provide protection against the thermal noise radiated by the soil. The size of the ground plane is generally the result of a trade-off between the cost and screening efficiency that is required. Thus, to guide the design of a given antenna array, it is important to be able to accurately characterize the noise that can leak around a given ground plane, both in terms of intensity and spatial correlation.

In this paper, we address the modal representation of thermal noise generated on top of a circular finite ground plane by a semi-infinite lossy medium. To do so, we start by considering an array of infinitesimal dipoles located at a given height above the ground plane. For each dipole, the electric field radiated on the ground plane is calculated and the MoM is solved using the method described in [11]. The presence of the dielectric medium is accounted for by using the appropriate Green's functions. From the fields radiated by each dipole inside the dielectric medium, we can compute the mixed losses of the dipole array [12], [13], i.e. the matrix describing both the coupling of each dipole with the dielectric medium and the interference effects that appear when a given combination of dipoles is active. Using the reciprocity derived in [12], [13], the noise correlation matrix is directly obtained from the mixed losses. The noise modes are obtained from the eigenmode decomposition of the correlation matrix, which is Hermitian [14]. The amplitude of the modes is related to the noise intensity, while the number of modes is directly related to the spatial correlation of the noise. Since highly correlated noise is easier to filter out, a good ground plane should lower as much as possible the amplitude and number of modes. In this study, we show that the noisy modes can be roughly grouped into two families depending on the orientation of their electric field. One family decreases much faster with the size of the ground plane.

It should be noted that ideal dipoles of currents are used in this study in order to isolate the behaviour of the ground plane. When considering a realistic antenna array, diffraction by the antenna should be taken into account, which will be the subject of a future work.

The paper is structured as follows. Section II contains the mathematical details regarding the computation of the fields and the use of the MoM. In Section III, the thermal noise generated by the dielectric medium on top of circular ground planes of various sizes is presented and analyzed. Section IV draws the conclusion.

II. Noise modal analysis

Let us consider an infinitesimal dipole $\vec{\jmath}_n(\vec{r}\,')$ with a given orientation located above a finite ground plane at position $\vec{r}\,' = (x', y', z')$. The ground plane is lying on a semi-infinite dielectric medium (typically, the soil). In order to obtain the fields radiated by the dipole in the presence of the structure, we begin by calculating the electric fields produced by the dipole on the ground plane in the presence of the dielectric medium. After that, the fields are projected on the ground plane testing functions and the MoM is solved. This yields the equivalent currents on the ground plane and, from there, the fields radiated by the overall structure can be inferred.

The electric field radiated by the dipole on the ground plane can be obtained as follows [11]:

$$\vec{E}(x,y,z) = -\frac{\mathrm{j}k_0\eta_0}{(2\pi)^2} \sum_{p\in\{TE,TM\}} \iint_{-\infty}^{\infty} \hat{\mathbf{e}}_p \times F_p(k_x,k_y)\ \tilde{G}_p(k_x,k_y)\ \mathrm{e}^{-\mathrm{j}(k_x x + k_y y - k_z z)}\,\mathrm{d}k_x\,\mathrm{d}k_y \quad (1)$$

where

$$F_p(k_x,k_y) = \hat{\mathbf{e}}_p \cdot \vec{\jmath}_n(\vec{r}\,')\ \mathrm{e}^{\mathrm{j}(k_x x' + k_y y' - k_z z')} \quad (2)$$

is the radiation pattern of the dipole. The polarization p is either TE or TM. We consider the wave vector $\vec{k}_0 = (k_x, k_y, k_z)$ satisfying $k_x^2 + k_y^2 + k_z^2 = k_0^2 = \omega^2\mu\epsilon$, with k_0 the wavenumber. μ and ϵ are the free-space permeability and permittivity, respectively. $\tilde{G}_p(k_x,k_y)$ is the Green's function with polarization p, which accounts for the presence of the semi-infinite medium. Note that, when considering infinitesimal dipoles, the double integral in Equation (1) can be reduced to a single Sommerfeld integral [15]. The electric field radiated by the dipole is projected onto the ground plane testing functions to obtain the vector $\mathbf{v}$. In this case, the ground plane is discretized with RWG basis functions [16] and a Galerkin testing procedure is used. Following the field projection, the MoM system of equations is solved to obtain the equivalent currents $\mathbf{x}$:

$$\mathbf{Z}_G\ \mathbf{x} = -\mathbf{v} \quad (3)$$

with $\mathbf{Z}_G$ the MoM impedance matrix of the ground plane. Memory and time efficient techniques exploiting the regular structure of the ground plane can be used when solving (3). The array scanning method can be efficiently applied to circular ground planes [11]. When considering rectangular ground planes, techniques exploiting the properties of block-Toeplitz matrices can be used [17].
Let us now consider an array of N dipoles uniformly distributed inside a circular domain and above a finite ground plane, as depicted in Fig. 1. For each position, we consider three dipoles having each a different orientation: along x, y or z. Considering N dipoles, we obtain N current distributions on the ground plane by solving (3) N times. After that, we calculate the fields due to each pair of dipoles inside the semi-infinite medium. From there, we can obtain the entries of the noise correlation matrix $\mathbf{R_e}$ [5], [12], [13], which is an Hermitian matrix. The eigenvalues and eigenvectors of $\mathbf{R_e}$ are found using the eigenmode decomposition [18]. The eigenvectors of matrix $\mathbf{R_e}$ correspond to modes of the thermal noise due to the semi-infinite medium on the finite ground plane. Each mode corresponds to a thermal field distribution that is fully deterministic within a constant phase factor. Different modes are superimposed incoherently, leading to the partially correlated noise described by matrix $\mathbf{R_e}$ [14].

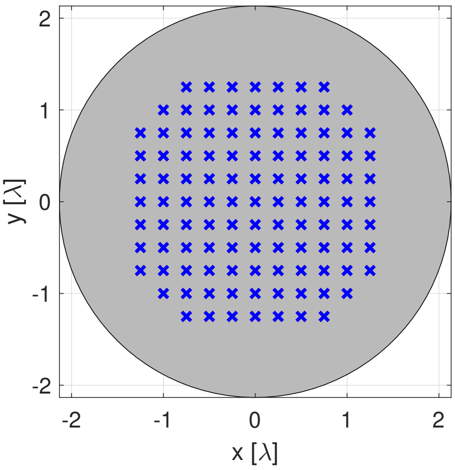


Fig. 1. Array of infinitesimal dipoles above a 4 m radius ground plane. The dimensions are given in free-space wavelength.

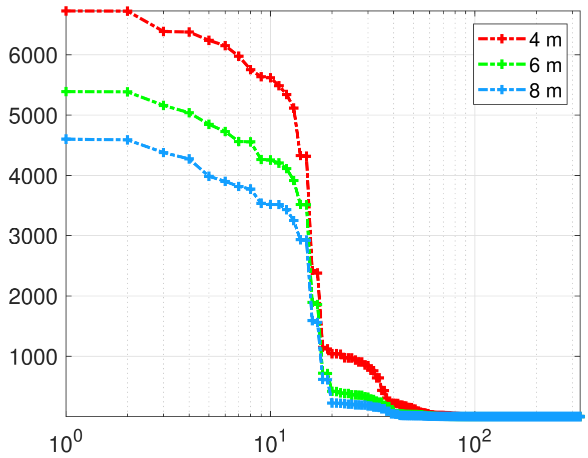
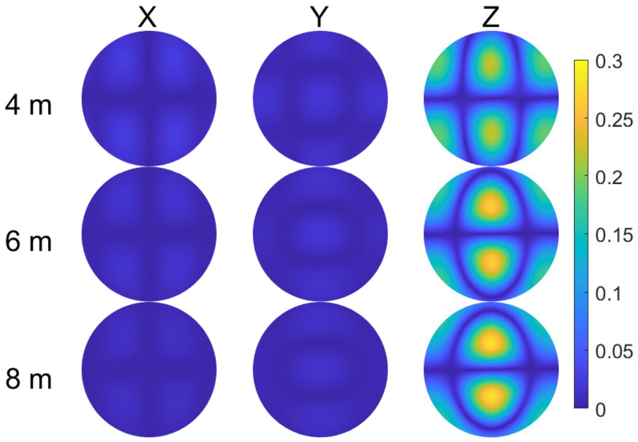
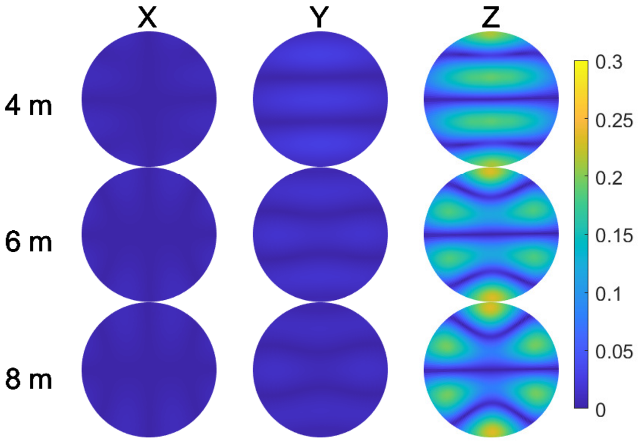


Fig. 2. Eigenvalues absolute values of the noise correlation matrix considering three ground planes of radii 4 m , 6 m and 8 m.

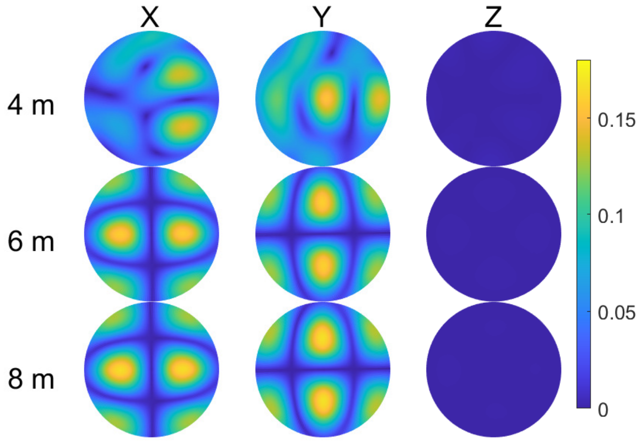


(a)

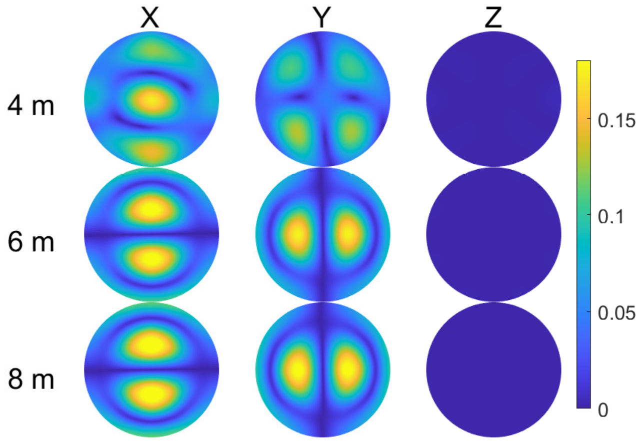


(b)

Fig. 3. (a) First noise mode and (b) third noise mode, both in absolute value and shown on a circular domain of 1.5λ radius. The mode is first obtained on the dipole positions and interpolated afterwards. Each line correspond to a given ground plane radius and each column correspond to the direction of the noise mode component.



(a)



(b)

Fig. 4. (a) 20^{th} noise mode and (b) 21^{st} noise mode shown on a circular domain of 1.5λ radius. The mode is first obtained at the dipole positions and interpolated afterwards. Each row corresponds to a given ground plane radius and each column corresponds to the direction of the noise mode component.

III. Numerical results

Let us first consider the array of 327 infinitesimal dipoles (3 dipoles with x, y and z orientations at each of the 109 positions) given in Fig. 1. The dipoles are uniformly distributed inside a 1.5 λ radius circular domain, with λ the free-space wavelength. The height of the dipoles is set to 1 m. A frequency of 160 MHz ($\lambda = 1.875$ m) is considered. We study the noise modes for three ground planes of radii 4 m, 6 m and 8 m, while we keep the same dipole array configuration. The ground plane is lying on a semi-infinite dielectric medium with relative permittivity $\epsilon_r = 3.7 - \text{j}\ 0.202$. This relative permittivity has been measured for the Murchison Radioastronomy Observatory (MRO) soil in dry conditions [19]. The eigenmode decomposition of the noise correlation matrix $\mathbf{R_e}$ provides the modes of the thermal noise on the finite ground plane. The magnitudes of the eigenvalues are given for the three ground planes in Fig. 2, while the normalized electric field distributions associated to some representative modes are provided in Figs. 3 and 4. Looking at the amplitude of the eigenvalues, we observe that the intensity of modes decreases with increasing size of the ground plane. According to the rate at which they decrease, the modes can be classified into two families. The amplitudes of the modes associated to the 19 highest eigenvalues decrease relatively slowly with the size of the ground plane. Comparatively, the amplitudes of the other modes decrease much more rapidly.

We now look at the shape of some representative modes of both families: modes 1 and 3 in Fig. 3(a) and (b) for the first family, and modes 20 and 21 in Fig. 4(a) and (b) for the second family. For each mode, the normalized electric field distribution is computed at the location of the dipoles and interpolated afterwards.

It can be observed that for the first mode (Fig. 3(a)), the z component is dominant compared to its x and y components. This can be explained by the stronger radiation of z oriented dipoles at grazing angles. The first mode and the second mode are degenerated. The third mode is shown in Fig. 3 (b), where we observe again the dominance of the z component.

We now look at modes of the second family (20^{th} and 21^{st}) in Fig. 4. For those modes, the intensities of the x and y components of the electric field are significantly larger than the one of the z component. The 20^{th} mode and 21^{st} are not the same when considering a 4 m radius ground plane compared to a 6 m or 8 m radius ground plane. This shows that the ranking of the modes can change at some point when increasing the size of the ground plane.

IV. Conclusion

In this paper, we performed a modal analysis of the thermal noise generated by a lossy semi-infinite dielectric medium on top of a finite ground plane. The analysis has been carried out by considering an array of infinitesimal dipoles above a circular ground plane. For each dipole, the electric field incident to the ground plane is calculated and projected on the ground plane testing functions. Then, a MoM system of equations is solved to find the equivalent currents on the ground plane. From the equivalent currents, the fields in the dielectric medium can be calculated, which enables the determination of the noise correlation matrix. The behaviour of the modes has been observed considering different sizes of the ground plane. It has been observed that the dominant modes have a stronger z component. Such modal analysis could prove useful to understand the behaviour of the noise on top of finite ground plane and guide the design of antenna arrays. Similar analysis in the presence of the antennas may be used to filter out the noise due to thermal radiation of dielectric media located below a finite ground plane. More precisely, they may allow the determination of beamforming weights able to suppress a large part of the noise originating from the lossy ground.

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