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SUBSCRIPTION AS A PRICE DISCRIMINATION DEVICE

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Abstract

We propose a model representing a newspaper producer supplying a product which can be acquired by the readers either every day per one unit at a time, or by subscription. The population of potential buyers is differentiated according to the frequency at which they want to read the newspaper. First we identify the optimal pricing policy, namely, the optimal price for issues sold per unit at the newspaper-stall, as well as the optimal subscription fee. Then we show that it is always more profitable to supply the market with both possibilities, — free sale purchase and subscription —, than to concentrate the sales on either of these alternatives.

Keywords: Subscription, price discrimination, press industry.

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1 Introduction

There are several goods and services for which sellers offer to the consumers the possibility of either buying one unit at a time, or buying a subscription giving them the right of consuming these units repeatedly over the year, on a regular basis (like a newspaper) or irregular (like entry tickets to a swimming pool). Other examples would include magazines, theater tickets, entry tickets to recreation parks, airplane or train tickets. On the other hand, there are goods and services for which this opportunity does not arise either because consumers can buy them only one unit at a time, or, on the contrary, only by subscription. Most of feeding goods,- like bread, meat, fish,- fall in the first category ; in the second, one finds, for instance, scientific journals, which can be bought only by subscribing for at least one year (*Econometrica, the Journal of Political Economy,...*), or access to TV networks.

The practice of sorting consumers into different classes, those who buy a good or a service once at a time, and those who prefer to subscribe, raises the question of why this practice applies to some goods, and not to others, which are as well repeatedly consumed over time. Certainly there are several reasons why it is so. The first deals with the profitability of *price discrimination.* When a product is offered simultaneously for purchase once at a time and by subscription, it is as if the seller would supply two different goods, which are of course substitutes. Undoubtedly, it allows the seller to discriminate among the buyers, by selling the units of the good at a different price, according to whether the buyer chooses a one-unit-purchase, or buys by subscription. However, it forces the seller "to compete with himself' by offering two substitute products. It is unclear whether the advantage to be gained from price discrimination compensates the loss in surplus resulting from selling simultaneously two products which are close substitutes. An example where this compensation does not occur is provided by Gabszewicz (1983). Accordingly, it may be that, for some types of goods, the price discrimination effect is advantageous while it is not for others.

Another reason why the subscription opportunity is offered for some goods with repeated purchase and not for others, deals probably with *random demand fluctuations.* Consider the example of a swimming pool. According to whether or not the sun is shining , the number of people willing to buy an entry ticket may vary considerably. When the sun shines, customers consist simultaneously of the regular swimmers who would be willing to swim whatever the type of weather, and of the occasional swimmers who are willing

to swim only if the weather is fine. If the weather is not fine, only the first category of customers would like to buy an entry ticket. By offering to the customers the possibility of buying a subscription, the owner of the swimming pool "smooths" random demand fluctuations, since all the "unconditional" customers who buy a subscription constitute all together a block of demand which is no longer fluctuating over time. The reduction of randomness accordingly obtained is particularly profitable when the cost of serving the marginal user is low, which is of course the case of a swimming pool when the capacity is not yet reached. As a consequence, it must be expected that in industries selling goods with repeated purchase and randomly fluctuating demand, there exists a strong incentive for the sellers to offer both per unit-purchases and subscriptions. This incentive is probably enhanced when the marginal cost of providing the service is low, compared with the amount of overhead costs.

Finally, there could be some reasons which are *industry-specific* for privileging the purchase by subscriptions. A good example is provided by the press industry, for which an interesting feature is that it sells, in fact, two products to two different categories of consumers : the newspaper to readers, and an advertising support to firms which advertise their products. In this situation, there is a double advantage in reducing the randomness of demand by offering to customers the possibility to subscribe. Not only this benefits directly to the producer of the newspaper because it allows a reduction in the number of unsold copies, but also it benefits indirectly to the firms which use the newspaper as a support for advertising: subscribers are exposed to advertising each time they get their copy, which of course reinforces the impact of the advertising message.

The practice of subscription has similitudes and differences with alternative contexts in which price discrimination is observed, like commodity bundling (Adams and Yellen (1977)), quality pricing (Mussa and Rosen (1979)) or multiple-part tariffs (Oi (1971)). Subscription can be viewed as a kind of mixed commodity bundling since it consists in selling "en bloc" a given number of units of the same good, which can also be bought separately. However, the notion of bundling generally applies to situations where the seller proposes units of different, and generally complementary, goods. Also the property of repeated purchase seems essential in the case of subscription, while it does not appear as a typical characteristic of commodity bundling. Quality pricing and multipart tariffs apply to situations in which consumers are sorted by proposing different variants of the same good (a luxury and a standard variant),

or by offering different classes of services related to the same good (the Disneyland dilemma). Offering both per unit purchase and subscription is a way of differentiating a product according to the preferences of consumers for the frequencies at which they want to consume. Those with a high frequency may prefer to buy a subscription when the subscription fee is low enough, compared with the price to be paid when the product is bought "à la pièce". On the contrary, consumers with a low consumption frequency may prefer to buy once at a time, simply because the price advantage of the subscription does not justify the increase in consumption entailed by the subscription practice. Again the idea of repeated purchase seems inseparable of this practice.

We concentrate in this paper on the price discrimination aspect of the subscription practice. To illustrate this aspect, we propose a model representing a newspaper producer supplying a product which can be acquired by the readers either every day, or by subscription. The population of potential buyers is differentiated according to the frequency at which they want to read the newspaper. First, we identify the optimal pricing policy, namely, the optimal price for issues sold at the newspaper-stall, as well as the optimal subscription fee. Second, we show that it is always more profitable to supply the market with both possibilities, than to concentrate the sales on either of these alternatives. In other words, we show that a pure price discrimination argument is sufficient to explane why, in most of the press industry, the practice of subscription is observed. We plan to pursue our analysis in another paper, and to determine for the press industry which part of this practice can be imputed to random fluctuations of demand and to the benefit which can be expected from advertising.

In Section 2 we analyse the optimal pricing policy for the monopolist while in Section 3 we characterize the optimal product-mix. Production costs are introduced in Section 4. Finally, in the conclusion, we discuss the assumptions introduced in Section 2, and propose some possible research extensions.

2 The optimal pricing policy

We consider a monopolist selling, to a population of potential buyers, a newspaper (or a weekly or monthly magazine) which they can buy either at the newspaper-stall at unit price p, or by subscription covering the whole period of reference, typically one year, at a total price r.

Each consumer selecting the option of buying the newspaper on free sale

is characterised by a *reading frequency* expressing the number of times over the year this consumer buys the newspaper divided by the number of issues over the year. We assume that the reading frequency of each consumer is given ex-ante and does not depend on the prices p and r. Rather it depends on the reading habits of the consumer, which depend themselves on features like whether the consumer is living close to, or far away from, a newspaper-stall, or whether he goes often to holidays, or business trips, etc. We also suppose the distribution of the consumers' reading frequencies to be known by the monopolist. To express the disparities in the reading frequency across the population while keeping the analysis tractable, we assume that the members of this population are uniformly distributed over an interval $[m, v]$, $[m, v]$ included in $[0, 1]$, an element t in $[m, v]$ representing a particular reading frequency of a consumer buying the newspaper on free sale. Thus an individual of type t is assumed to buy and read on average over the year a number of issues equal to tn, where n denotes the number of issues sold by the monopolist during the year (12 for a monthly magazine, 52 for a weekly periodical, about 300 for a newspaper, ...). Without loss of generality, we assume the number of potential readers with reading frequency t be equal to one. Furthermore, we assume that consumers buying the newspaper at the shop incur at each purchase a *moving cost* equal to c, $c > 0$. Finally, their *reservation price* for a particular issue of the newspaper is equal to w, $w > c$. We shall assume that

$$\frac{w}{w+c} > \frac{m}{v}. \tag{1}$$

This assumption says that total utility derived by the consumer with the highest reading frequency when he subscribes, exceeds the total utility derived from reading by the consumer with the lowest reading frequency when he buys from the stall. This condition bears on the domain of reading frequencies which are present in the population. It is trivially satisfied when $m = 0$.

As an alternative to free sale purchase, individuals are offered the option of taking a *subscription* according to which they receive all issues of the newspaper in their mailing box. In this case, they do not incur any moving cost and have only to pay the subscription fee r. For simplicity, we assume that when buying a subscription, a consumer reads with the same frequency the issues he gets over the year than he would do if he would buy them at the store. Then, whether buying from the newspaper-stall or buying the subscription, the utility derived from reading can be measured by the number of issues effectively read over the year. Accordingly, a consumer with reading frequency t will buy

from the newspaper-stall rather than taking the subscription, if, and only if,

$$(p+c)tn \leq r.$$

Denote by $t(p, r)$ the consumer who is just indifferent between the two options when the monopolist quotes the price p at the newspaper-stall and the fee r for the subscription. We obtain

$$t(p, r) = \frac{r}{(p+c)n}: \tag{2}$$

at the pair of prices (p, r), all consumers with reading frequency smaller than $t(p, r)$ buy from the newspaper-stall while those with reading frequency higher than $t(p, r)$ prefer to buy a subscription. Accordingly, the receipt $R(p, r)$ of the monopolist is given by

$$R(p, r) = p \int_m^{\frac{r}{(p+c)n}} ntdt + r \int_{\frac{r}{(p+c)n}}^v dt$$

or

$$R(p, r) = p\left[\frac{1}{2n}\frac{r^2}{(p+c)^2} - \frac{1}{2}nm^2\right] + r\left[v - \frac{r}{[(p+c)n]}\right]. \tag{3}$$

For any price p, the optimal subscription fee r^* must satisfy the first-order condition $\frac{dR}{dr} = 0$, i.e.

$$\frac{dR}{dr} = \frac{p}{n}\frac{r}{(p+c)^2} + v - 2\frac{r}{[(p+c)n]} = 0,$$

which gives

$$r^* = vn\frac{(p+c)^2}{(p+2c)}.$$

Furthermore, we notice that $\frac{d^2R}{dr^2}$ is negative, so that r^* is a maximizer of $R(p, r)$ for all values of p. Substituting the value of r^* into $t(p, r)$ as given by (2), we obtain

$$t(p, r^*) = v\frac{(p+c)}{(p+2c)}.$$

Now consider the function $R(p, r^*)$ obtained by substituting r^* to r in (3), i.e.

$$R(p, r^*) = \frac{nv^2(p+c)^2}{2(p+2c)} - \frac{nm^2p}{2}. \tag{4}$$

It is shown in the appendix (Lemma 1) that this function reaches its maximum in the interval $(0, w-c)$ at the upper boundary $w-c$. Thus, the optimal price p^* of an issue sold at the newspaper-stall obtains as $p^* = w - c$, the reservation price of the consumer minus the moving cost. The corresponding profit $R(p^*, r^*)$ obtains as :

$$R(p^*, r^*) = \frac{n(m^2c^2 + v^2w^2 - m^2w^2)}{2(w+c)}.$$

Accordingly, we have the following:

Proposition 1 *When the monopolist decides to sell the newspaper both at the newspaper-stall and by subscription, there exists a unique optimal pricing policy for the monopolist, which is given by*

$$p^* = w - c; \quad r^* = \frac{vnw^2}{w+c}. \tag{5}$$

First we notice that both optimal prices, p^* and r^*, increase with w and decrease with c; furthermore, the subscription fee increases with the number of issues.

Second, the pricing policy identified in Proposition 1 is meaningful only if some consistency conditions are satisfied. The first condition required is that the marginal consumer who is indifferent between the alternative of buying at the newspaper-stall and taking a subscription, i.e., $t(p^*, r^*)$, has to lie in the interval $[m, v]$. Substituting the optimal price $p^* = w - c$ in the expression for $t(p, r^*)$, we obtain

$$t(p^*, r^*) = \frac{vw}{w+c}, \tag{6}$$

so that the inequality $t(p^*, r^*) \leq v$ is always satisfied. But is is also necessary that $t(p^*, r^*) \geq m$, or

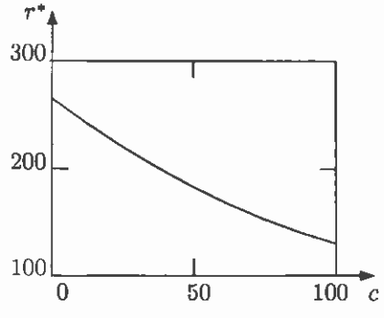
$$\frac{w}{w+c} \geq \frac{m}{v}.$$

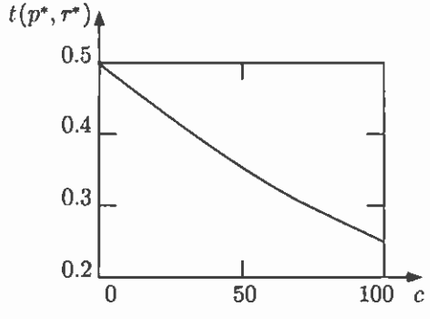
This condition holds by assumption (see condition (1)).

Another consistency condition requires that the unit price of an issue, p^*, augmented of the moving cost, c, be higher than the equilibrium subscription fee, r^*, divided by the number of issues provided over the year. Otherwise, it would be always advantageous to buy from the newspaper-stall, whatever the reading frequency of the cousumer. This must necessarily hold, since for all t in the interval $(t(p^*, r^*), v]$, we have $nt(p^* + c) > r^*$, which implies the

inequality $p^* + c > \frac{r^*}{n}$, since $t < 1$. Finally, we must check that the receipt of the monopolist is positive when he uses his (her) optimal policy. This follows from the fact that both prices are positive, as well as market shares, when $t(p^*, r^*)$ lies in the interval $[m, v]$. We have just seen that this is the case when condition (1) holds.

We have computed hereafter the optimal policy of the monopolist for specific values of the parameters. The example is defined by the parametric values: $m = 0$, $n = 52$, $v = .5$, $c = 50$ and $w = 100$. Then we find $p^* = 50$; $r^* = 1733,33$; $t(p^*, r^*) = 0,33$. The following figures plot the values of r^* and $t(p^*, r^*)$ as a function of c.





It is interesting to examine the behaviour of the market solution as a function of the moving cost c. First, it must be noticed that when c is equal to 0, the model is degenerate. Indeed, performing the above analysis with $c = 0$ leads to an optimal value r^* equal to vnw, so that $t(p, r^*)$ is then equal to v for all values of p. Accordingly, with zero moving costs, the subscription fee is always chosen in a way such as the demand for subscription vanishes. A contrario, when the moving cost is strictly positive, it is worthy for the monopolist to offer to consumers with higher reading frequencies the possibility of avoiding the repetition of moving costs by taking a subscription. To realise this program, the monopolist starts to decrease the subscription fee when the moving cost increases $\left(\frac{dr^*}{dc} < 0\right)$; as a consequence, the number of consumers who decide to take the subscription increases with c $\left(\frac{dt(p^*, r^*)}{dc} < 0\right)$. However this is not to say that, as soon as the moving cost is strictly positive, it is always advantageous to offer the possibility of buying a subscription. Indeed, the optimal policy derived above was analysed under the presumption that both alternatives, free sale purchase and subscription, are offered to the

consumers. But the monopolist has always the possibility of either supplying the market with free sale purchase only, or supplying the newspaper only by selling subscriptions, avoiding thereby cannibalization by selling two substitute products. We now examine which of these possibilities is the most advantageous.

3 The optimal product mix

To this end, we must first compare the profit $R(p^*, r^*)$ realised at equilibrium when both products are sold, with the profit obtained when selling the newspaper at the store only. Then the optimal price is $w - c$, so that the profit is equal to $\frac{n(v^2 - m^2)(w-c)}{2}$. Comparing this profit with the profit realised at equilibrium with two products, we see that the second exceeds the first, whenever

$$G(c) = \frac{n(m^2c^2 + v^2w^2 - m^2w^2)}{2(w+c)} - \frac{n(v^2 - m^2)(w-c)}{2} > 0.$$

We notice that $\frac{dG}{dc} > 0$ and that $G(c) = 0$, if and only if $c = 0$. Accordingly, the above inequality is always positive, and it is never more advantageous to sell the newspaper at the store only, rather than selling both at the store and by subscription.

On the other hand, it is proved in the appendix (see Lemma 2) that it could never be more profitable to sell by subscription only, compared with the profit realised by selling simultaneously at the store and by subscription. Accordingly, we have

Proposition 2 *The optimal product selection for the monopolist consists in selling the newspaper both on free sale and by subscription.*

4 Production costs

So far, we have performed the analysis by assuming zero production costs. However, the model can easily be accommodated so as to account for linear production costs. Let d denote the unit production cost of an issue. In view of simplifying notation, we shall henceforth assume that $m = 0$. Under this assumption, the monopolist's profits $R(p, r)$ write as

$$R(p,r) = \frac{1}{2}\frac{p}{n}\frac{r^2}{(p+c)^2} + r\left[v - \frac{r}{[(p+c)n]}\right] - \frac{1}{2}\frac{d}{n}\frac{r^2}{(p+c)^2} - dn\left[v - \frac{r}{[(p+c)n]}\right].$$

The first-order necessary condition with respect to the variable r implies that

$$r^* = (p+c)n\frac{vp + vc + d}{p + 2c + d}.$$

It is easily checked that $\frac{d^2R}{dr^2} < 0$, so that r^* is a maximizer of $R(p, r)$ for all values of p. Substituting the value of r^* into $t(p, r)$, we now obtain

$$t(p, r^*) = \frac{vp + vc + d}{p + 2c + d}. \tag{7}$$

Similarly, we get

$$R(p, r^*) = \frac{1}{2}\frac{pn(vp + vc + d)^2}{(p + 2c + d)^2} + \frac{(p+c)n(vp + vc + d)}{p + 2c + d}\left(v - \frac{vp + vc + d}{p + 2c + d}\right) - \\ -\frac{1}{2}dn\frac{(vp + vc + d)^2}{(p + 2c + d)^2} - dn\left(v - \frac{vp + vc + d}{p + 2c + d}\right). \tag{8}$$

Tedious calculations (see Lemma 3 in the appendix) show that $\frac{dR(p, r^*)}{dp}$ is strictly positive so that the optimal price p^* of the issue sold at the newspaper-stall is equal to the highest price at which consumers are still willing to buy, i.e. $w - c$. Substituting p^* into r^* and $t(p, r^*)$, we obtain

$$r^* = \frac{wr(vw + d)}{w + c + d} \tag{9}$$

and

$$t(p^*, r^*) = \frac{vw + d}{w + c + d}. \tag{10}$$

When we compare the equilibrium value of r^*, without or with production costs, as provided by Proposition 1 and by (9), we obtain that the latter exceeds the former when $vw \leq w + c$, a condition which is always satisfied, since $v < 1$.

On the other hand, comparing $t(p^*, r^*)$ without or with production costs, as provided by (6) and (10), we conclude that the marginal consumer indifferent between buying the two options, has a lower reading frequency in the first alternative than in the second one. Accordingly, there are less consumers who subscribe when there are production costs than in the opposite case. This is a direct consequence of the increase in the subscription fee resulting from the introduction of productive costs.

5 Conclusion

The analysis above shows that the argument based on price discrimination is sufficient for explaning the rise of the subscription practice in the press industry. Unfortunately, this analysis has been conducted at the cost of severe assumptions. First we have assumed that the reading frequency of a consumer does not depend on the prices p and r. Most probably however, this frequency is influenced by these prices, and should be determined as the solution of an optimization problem balancing an increase in frequency against the increase in expenses resulting from this supplementary purchase of information. Furthermore, one must not expect the reading frequency to be, as implicitely assumed, independent of the events happening over the year. On the contrary, it should vary with the nature and the importance of these events. When there is a presidential election, or when the death of a celebrated actress is announced, peaks in the individual reading frequencies are observed. Our analysis neglects such random elements which influence the reading frequency of the consumers. Finally, the assumption that the reading frequency remains the same, whether the consumer buys at the newspaper-stall or by subscription, is not satisfactory either. Many consumers change their reading habits when they take a subscription. Some of them even buy a subscription in order to constrain themselves to read everyday the newspaper, a practice they would not be able to obey, should they be deprived from receiving every day the newspaper in their mailing box... Nevertheless, the above simplifying assumptions have brought some tractability in the treatment of a problem which seemed at first sight extremely difficult to tackle. But this is clearly a first step, and the analysis should be deepened.

In our opinion, the first extension of the analysis should deal with the problem of randomly fluctuating demand. In other words the assumption according to which reading frequencies are independent from the events happening over the year, should be dropped and replaced by an assumption tolerating some random fluctuations of individual frequencies related to the various possible states of nature which can appear during the period of reference. The profitability of the subscription strategy resulting from smoothing the random fluctuations of demand, could then be identified, and contrasted with the profitability resulting from price discrimination, as analysed in the present paper.

Furhermore, it would be interesting to extend the analysis in view of taking care of the incidence of advertising on the subscription strategies of editorial firms. This approach would be more "industry-specific", but would certainly

shed some light on the functioning of a fascinating industry.

Finally, we have considered that the monopolist proposes to the buyers the alternative: "subscribe for one year, or buy at the newspaper-stall". In most cases however, the alternative of subscription is refined since consumers are allowed to subscribe on a three-months, or six-months, or one year-basis, thereby reinforcing the possibility of price discrimination by the monopolist. It should not be difficult to extend the present analysis to such more elaborate pricing schemes.

References

[1] Adams, W.J. and J.L. Yellen (1976). Commodity bundling and the burden of monopoly. *Quarterly Journal of Economics*, 90, 475–498.

[2] Gabszewicz, J.J. (1983). Blue and red cars, or blue cars only? A note on product variety. *Economica*, 50, 203–206.

[3] Mussa, M. and S. Rosen (1978). Product quality and monopoly. *Journal of Economic Theory*, 19, 301–318.

[4] Oi, W. (1971). A Disneyland dilemma: Two-part tariffs for a Mickey Mouse monopoly. *The Quarterly Journal of Economics*, 85, 77–96.

Appendix

Lemma 1 *The maximizer of $R(p, r^*)$, as defined by (4), is $w-c$ in the domain $[0, w-c]$.*

Proof. The first derivative of $R(p, r^*)$ w.r.t. p obtains as

$$\frac{dR(p,r^*)}{dp} = \frac{n(-m^2p^2 - 4m^2pc + v^2p^2 + 4v^2pc - 4m^2c^2 + 3v^2c^2)}{2(p+2c)^2}.$$

The sign of $\frac{dR(p,r^*)}{dp}$ depends on the sign of its numerator, which is certainly positive when $\frac{\sqrt{3}}{2} > \frac{m}{v}$ and $p > 0$. Accordingly, when this condition holds, the first derivative of $R(p, r^*)$ is monotone increasing in $(0, w-c)$ and its maximizer is $w - c$. Now assume that the above condition does not hold, i.e. $\frac{\sqrt{3}}{2} < \frac{m}{v}$, then the solution to the equation $\frac{dR(p,r^*)}{dp} = 0$ are

$$\rho_1 = -2c + \frac{vc\sqrt{v^2 - m^2}}{v^2 - m^2};$$

$$\rho_2 = -2c - \frac{vc\sqrt{v^2 - m^2}}{v^2 - m^2}.$$

Both the roots ρ_1 and ρ_2 are negative when $\frac{\sqrt{3}}{2} > \frac{m}{v}$, while ρ_1 is positive when the reverse inequality holds. Taking the second derivative $\frac{d^2R}{dp^2}$, we get

$$\frac{d^2R}{dp^2} = nv^2\frac{c^2}{(p+2c)^3}.$$

Substituting ρ_1 in $\frac{d^2R}{dp^2}$, we obtain

$$\left.\frac{d^2R}{dp^2}\right|_{p=\rho_1} = \frac{n(v^2-m^2)^2}{vc\sqrt{v^2-m^2}} > 0,$$

so that $R(p, r^*)$ reaches a minimum value at ρ_1, and thus the maximizer of $R(p, r^*)$ in $[0, w-c]$ is either at zero, or at $w - c$. The value of $R(p, r^*)$ at zero is given by $\frac{nv^2c}{4}$, and at $w-c$, by $\frac{n(-m^2w^2 + m^2c^2 + v^2w^2)}{2(w+c)}$, so that the difference Δ between the two values obtains as

$$\Delta = \frac{n(-m^2w^2 + m^2c^2 + v^2w^2)}{2(w+c)} - \frac{nv^2c}{4}$$

or

$$\frac{\Delta}{n} = \frac{2[-m^2w^2 + m^2c^2 + v^2w^2]}{w+c} - v^2c.$$

Define $\Delta_1 = (w+c)\frac{\Delta}{n}$, $\Delta_2 = \frac{\Delta_1}{v^2}$ and $m_1 = \frac{m}{v}$. The sign of $\frac{\Delta}{n}$ is the same as the sign of Δ_2. We obtain

$$\frac{d\Delta_2}{dm_1} = -4w^2m_1 + 4c^2m_1 < 0,$$

where the last inequality follows from $c < w$. Now, evaluating Δ_2 at $m_1 = \frac{w}{w+c}$, we get

$$\begin{aligned}
\Delta_2\Big|_{m_1=\frac{w}{w+c}} &= -2w^2\left(\frac{w}{w+c}\right)^2 + 2c^2\left(\frac{w}{w+c}\right)^2 + 2w^2 - wc - c^2; \\
&= -2\left(\frac{w}{w+c}\right)^2(w-c)(w+c) + w(w-c) + (w-c)(w+c); \\
&= -\frac{2w^2}{w+c}(w-c) + w(w-c) + (w-c)(w+c).
\end{aligned}$$

The sign of $\Delta_2\Big|_{m_1=\frac{w}{w+c}}$ is the same as the sign of $-\frac{2w^2}{w+c} + 2w + c$, which is itself of the same sign as $\Delta_2\Big|_{m_1=\frac{w}{w+c}}(w+c) = 3wc + c^2 > 0$. Accordingly,

$$\Delta_2\Big|_{\frac{m}{v}=\frac{w}{w+c}} > 0.$$

Since we have shown above that $\frac{d\Delta_2}{d(m/v)} < 0$, we must have $\Delta_2 > 0$ for all values of $\frac{m}{v} \le \frac{w}{w+c}$ and $\Delta > 0$, for all values $\frac{m}{v} < \frac{w}{w+c}$, which constitute the admissible domain of value $\frac{m}{v}$ (see condition (1)). Accordingly, we have $R(p, r^*)\Big|_{p=0} < R(p, r^*)\Big|_{p=w-c}$, and $p = w - c$ is the maximizer of $R(p, r^*)$ in the domain $(0, w-c)$.

■

Lemma 2 *The profit realized when selling only by subscription is smaller than the profit obtained with selling both at the newspaper-stall and by subscription.*

Proof. We start by identifying the optimal subscription fee when the monopoly sells only by subscription. A consumer with reading frequency t will take a subscription at a price r if and only if

$$wtn \geq r.$$

Accordingly, the set of consumers buying at price r is the interval $[\frac{r}{nw}, v]$, so that the receipt R writes as

$$\begin{aligned} R(r) &= r \int_{r/nw}^{v} dt \\ &= r\left(v - \tfrac{r}{nw}\right), \end{aligned}$$

when $m < \frac{r}{nw} \leq v$, and

$$\begin{aligned} R(r) &= r \int_{m}^{v} dt \\ &= r(v-m), \end{aligned}$$

when $\frac{r}{nw} \leq m$. Consequently, at an interior solution, the optimal fee r^* must satisfy the first order condition

$$\frac{dR}{dr} = v - \frac{2r}{nw} = 0,$$

so that $r^* = \frac{vnw}{2}$ and $t(r^*) = \frac{v}{2}$. In order for $t(r^*)$ to exceed m, it must be that $v \geq 2m$. Accordingly, when $v \geq 2m$, the optimal fee r^* obtains as $r^* = \frac{vnw}{2}$ and $R(r^*) = \frac{v^2nw}{4}$. Now assume that $v < 2m$. Then the optimal price r^* is the highest price at which the market is covered, i.e., r^* solves $m = \frac{r}{nw}$, or

$$r^* = mnw,$$

with $t(r^*) = m$ and $R(r^*) = mnw(v-m)$.

In order to prove the lemma, we shall successively consider the two cases: $v \geq 2m$ and $v < 2m$. In the first case, the difference D between the profit when selling the two products — selling at the store and by subscription —, or selling by subscription only, writes as

$$D = \frac{n(-m^2w^2 + m^2c^2 + v^2w^2)}{2(w+c)} - \frac{nv^2w}{4}.$$

It is easily checked that $D = 0 \Leftrightarrow c = c_1 = w$, or $c = c_2 = \frac{w(v^2 - 2m^2)}{2m^2}$. Since $v \geq 2m$, it follows that $c_2 \geq c_1$. On the other hand, we get

$$\frac{dD}{dc} = \frac{nm^2c}{w+c} - \frac{n(-m^2w^2 + m^2c^2 + v^2w^2)}{2(w+c)^2}$$

so that $\frac{dD}{dc} = 0 \Leftrightarrow c = c_1' = -\frac{w}{m}(v-m)$, or $c = c_2' = \frac{w}{m}(v-m) > w$, where the last inequality follows from $v \geq 2m$. Furthermore

$$\frac{d^2D}{dc^2} = \frac{nv^2w^2}{(w+c)^3}.$$

Substituting c_2' into $\frac{d^2D}{dc^2}$, we obtain

$$\left.\frac{d^2D}{dc^2}\right|_{c=c_2'} = \frac{nw^3}{vw} > 0.$$

Accordingly, the difference D, as a function of c, reaches a minimum at c_2', which exceeds w; it follows that D is positive for all values of c in the domain $(0, w)$. We conclude that, when $v \geq 2m$, it is more profitable to sell both at the store and by subscription.

Now consider the case $v < 2m$. In this case, the difference D' between the profits writes as

$$D' = \frac{n(-m^2w^2 + m^2c^2 + v^2w^2)}{2(w+c)} - (v-m)nmw.$$

It is easily checked that $D' = 0 \Leftrightarrow c = c'' = \frac{w}{m}(v-m)$. On the other hand, we get

$$\frac{dD'}{dc} = \frac{n(2m^2wc + m^2c^2 + m^2w^2 - v^2w^2)}{(w+c)^2}$$

so that $\frac{dD'}{dc} = 0 \Leftrightarrow c = c'' < w$, when the last inequality follows from $v < 2m$. Furthermore,

$$\frac{d^2D'}{dc^2} = \frac{nvw^2}{(w+c)^3}.$$

Substituting c'' into $\frac{d^2D'}{dc^2}$, we obtain

$$\left.\frac{d^2D'}{dc^2}\right|_{c=c''} = \frac{nm^3}{vw} > 0.$$

Notice that D' is equal to 0 for the same value of c at which $\frac{dD'}{dc}$ vanishes, namely $c = c''$. Furthermore, $\left.\frac{d^2D'}{dc^2}\right|_{c=c''}$ is positive, so that D' reaches a minimum value at c'', which is smaller than w. It follows from the above that D is positive and decreasing in the interval $(0, c'')$ while it is positive and increasing in the interval (c'', w). In any case, D' is positive for all values of c in the domain $(0, w)$; which completes the proof of the lemma. ∎

Lemma 3 *The maximizer of $R(p, r^*)$ as defined by (8) is $w - c$.*

Proof. The first derivative of $\frac{1}{n}R(p, r^*)$ w.r.t. p obtains as:

$$\begin{aligned}
\tfrac{1}{n}\tfrac{dR(p,r^*)}{dp} &= \tfrac{1}{2}t^2 + \frac{pvt}{p+2c+d} - \frac{pt^2}{p+2c+d} + t(v-t) + \\
&+ (p+c)(v-t)\frac{v}{p+2c+d} - (v-t)(p+c)\frac{t}{p+2c+d} - \\
&- \frac{t(p+c)(v-t)}{p+2c+d} - \frac{dtv}{p+2c+d} + d\frac{t^2}{p+2c+d} + \\
&+ (v-t)\frac{d}{p+2c+d}
\end{aligned}$$

with $t = \frac{vp+vc+d}{p+2c+d}$ (see (10)).

The above expression can be rewritten as

$$\begin{aligned}
\tfrac{1}{n}\tfrac{dR(p,r^*)}{dp} &= \tfrac{1}{2}t^2 + \frac{pt(v-t)}{p+2c+d} + t(v-t) + \frac{(p+c)(v-t)^2}{p+2c+d} - \\
&- \frac{t(p+c)(v-t)}{p+2c+d} - \frac{dt(v-t)}{p+2c+d} + \frac{d(v-t)}{p+2c+d} \\
&= \tfrac{1}{2}t^2 + \frac{pt(v-t)}{p+2c+d} + \frac{(p+c)(v-t)^2}{p+2c+d} + \\
&+ \frac{d(v-t)}{p+2c+d} + t(v-t)\left[1 - \frac{p+c}{p+2c+d} - \frac{d}{p+2c+d}\right]
\end{aligned}$$

or

$$\begin{aligned}
\tfrac{1}{n}\tfrac{dR(p,r^*)}{dp} &= \tfrac{1}{2}t^2 + \frac{pt(v-t)}{p+2c+d} + \frac{(p+c)(v-t)^2}{p+2c+d} + \\
&+ \frac{d(v-t)}{p+2c+d} + t(v-t)\left(\frac{c}{p+2c+d}\right) \\
&= \tfrac{1}{2}t^2 + \frac{v-t}{p+2c+d}[pt + (p+c)(v-t) + d + tc] \\
&= \tfrac{1}{2}t^2 + (v-t)\left[\frac{vp+vc+d}{p+2c+d}\right] \\
&= t(v - \tfrac{1}{2}t) > 0,
\end{aligned}$$

where the last inequality follows from $t = t(p, r^*) < v$. The proof of the lemma follows. ∎

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