Should prevention campaigns disclose the transmission rate of HIV/AIDS? Theory and evidence frome Burundi

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Should prevention campaigns disclose the transmission rate of HIV/AIDS? Theory and evidence from Burundi.

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Abstract

Among non-specialists, the estimates of the HIV/AIDS transmission rate are generally upwardly biased. This overestimation may be perceived as a godsend, as it increases the incentives to have protected sexual relationships. However, a pernicious effect may counterbalance this positive effect. Combined with the overestimation of the transmission rate, an occasional unprotected sexual encounter may induce the feeling that "the die is cast", and hence lead to a permanent neglect of condom use. In this paper, I construct a model that reflects such insidious and unexpected behavior. I calculate that the optimal transmission rate to be disclosed for safer sexual practices ranges between 5% and 24.9%.

Keywords: HIV/AIDS, transmission rate, prevention, risk perception, condom, Burundi

1. Introduction

1.1. The question

When non-specialists are asked about the probability of getting infected with HIV during an unprotected sexual relationship with an infected individual, their answer is always sharply upward biased. In fact, HIV/AIDS is

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perceived as a very prolific disease whose transmission is easy, if not mechanical. Such a conception is probably due to both the high number of infected individuals and the large media coverage of AIDS prevention campaigns.

However, the transmission rate for normal heterosexual intercourse is always lower than 1% (Wawer et al., 2005; Boily et al., 2009). Intuitively, one may think that the overestimation of the transmission rate of HIV is favorable: higher risk estimation induces a more cautious behavior and consequently lowers the incidence of the disease. This is probably why prevention campaigns neglect to inform people about true infection rates.

However, a more subtle and pernicious factor may counterbalance the positive effect of overestimating the transmission rate. When deciding to have a sexual encounter, a couple decides to use or neglect protection, knowing that condoms are effective for preventing between 80 and 95 percent of all HIV transmissions through sexual intercourse (Pinkerton and Abramson, 1997; Weller and Davis, 2001; Hearst and Chen, 2004). However, although people are generally likely to use condoms during casual and commercial sexual encounters, they are rarely used in longer-term relationships in which there is a sense of commitment and trust (Meekers and Klein, 2002; Meekers et al., 2003; Flood, 2003). In fact, condom use is strongly associated with the notions of unfaithfulness, distrust, lack of love, and decrease of penile sensation (Leclerc-Madlala, 2002; Flood, 2003).In a qualitative study in Australia carried among young heterosexual men, Flood (2003) found that condom use mainly serves as a contraceptive method rather than as a prophylaxis. His study also emphasized that condoms interrupt the “heat of the moment”. Hence, couples often engage in unsafe practices, either because of confidence in the partner, for practical reasons or because of the impulsive character of sex (risky sex may also be due to involuntary condom failure).

If unsafe practices were occasional and exceptional, the disease would die out in a few years, considering its low transmission rate. However, couples who engage in risky sex once will generally repeat this practice over time (Flood, 2003; Meekers et al., 2003). This may be partly due to addiction to an already overridden rule. Another explanation, not covered in the HIV/AIDS literature, is the wrong estimation of the transmission rate. Let us imagine a young, irresponsible adolescent male engaging in unsafe sex because of instantaneous passion, and let us assume he trusts the contraceptive methods and the fidelity of his partner. If his estimation of the transmission rate is high, for instance 80%, he may not want to protect himself anymore with this partner after their first unprotected encounter. In fact, he may think "if the girl was HIV positive, I surely got infected and hence, we don't have to care anymore about protection ". Conversely, if his estimation is low, he should be more willing to protect himself in further sexual intercourses. Besides, if he thinks the probability of being infected is perceived as zero, he will probably continue to engage in unsafe practices.

Hence, before the first unprotected intercourse, I expect a positive correlation between the expected transmission rate and the use of condom. However, after a first neglect in protection, the rate of protected sex follows a non-linear function of the expected transmission rate. The aim of this paper is therefore to construct an empirically testable model of this U-shaped relation between the expected transmission rate and condom use. The simulations of the model answer three main questions: "Should prevention campaigns disclose the true transmission rate of the HIV/AIDS virus?", "What would be the optimal transmission rate to disclose?" and "What would happen if a big hearsay or an Internet buzz propagates the rumor that the transmission rate is very low?".

1.2. Literature review

The perception of the risk has barely been taken into account in empirical research on condom use. To the best of my knowledge, no theoretical and only a few empirical papers have included people's assessments of HIV transmission rate. To this regard, three studies are important to notice. First, Meekers and Klein (2002) found a positive correlation between risk perception and condom use for males with casual partners, but no significant correlation for men with regular partners [2]. For both men with regular and casual partners, later levels of condom use were significantly lower than earlier levels. This evidence backs my model: a high expected transmission rate implies low risk for first encounters (and hence for a casual encounter), but causes higher risk after a couple has failed to protect itself once.

Second, Maharaj and Cleland (2005), building on data from South African

[2] This analysis was not done for women because too few females in their sample reported having a casual partner in the year before the survey

couples, noticed that wives who perceived a risk of HIV infection from their partner were more or less four times more likely to report consistent or occasional condom use. Unfortunately, the authors have not reported the same statistics for husband, nor did they try to distinguish consistent and occasional condom use.

Third, Lammers et al. (2009) used data collected in several markets in Lagos (Nigeria) to show that the perceived level of risk is positively correlated with safer behavior for men only. Risk perception was measured on a 7-step scale ranging from "not at all risky" to "extremely risky". In this context, males who perceived unprotected sex as extremely risky appeared to be 50% more likely to have used a condom during the last intercourse than those who perceived it as not risky at all. Regarding women, there was a significant correlation between risk perception and condom use for single, but not for married women. The fact that culturally, a married woman cannot stop intentionally having children, together with their lack of bargaining power, could explain the irrelevance of the result for married women. The authors indeed showed that single women who know that condoms are effective in preventing HIV transmission used them significantly more often during the last intercourse than their married counterparts. This difference was also significant regarding males, which confirms both the importance differentiating between casual and regular relationships, and the crucial dimension of time in studying the effects of perceived risk. Unfortunately, their questionnaire did not ask with whom respondents had their last sexual intercourse, nor did the measure of perceived risk distinguished between prevalence and expected transmission rate. Moreover, risk perception enters only linearly in their probit model, without taking into account a possible quadratic relationship between perceived risk and condom use.

These three empirical studies provide crucial insights but lack an integrated theoretical background that would both explain the pernicious side effects of a misevaluated infection rate of the HIV virus, and evaluate the consequences of educational programs aiming to teach the true scale of the transmission rate. Despite the crucial importance of safe-sex practices, the complex links between AIDS education and the decision to use condoms during sexual encounters have rarely been examined through economic model. In this context, general approaches stand out and are worth exploring before presenting a new model. First, Philipson and Posner (1994) have confronted the private demand for information with the public provision of AIDS education. They noticed that the effect of public prevention is likely to be small. Indeed, in high-prevalence sub-populations, private incentives to get information are important enough to avoid public funding on AIDS information. Conversely, when the prevalence inside a sub-population is low, public provision of information tends to be useless since the risk is too low to deter unsafe practices. Assuming that accurate information enables people to differentiate between safe and risky behavior, Philipson and Posner (1994) subsequently discussed the hypothesis that providing truthful information may encourage dangerous behavior because people often exaggerate the infectivity of the AIDS virus. However, their model does not include the potential harmful repercussions of learning the true value of parameters such as the expected transmission rate or the prevalence of HIV/AIDS.

Second, much of the theoretical literature on HIV/AIDS is based on the SI (susceptible-infected) epidemiological model (Rottingen and Garnett, 2002; Viladent and van Ackere, 2007). In this type of model, healthy individuals (the susceptibles) match with infected people and become infected with a certain probability. Few differential equations determine how the populations of susceptible and infected people evolve across time. For instance, it allows the impact of different prevention strategies or different network organisations on the propagation of the virus to be evaluated. The propagation of the virus in the population of susceptibles is mainly measured by the basic reproductive rate R^0 which is above 1 if the disease will spread in the population (epidemic) and below 1 if the disease is expected to disappear. A recurring quest in this literature is the best specification for the transmission process of the virus (Rottingen and Garnett, 2002). Is the transmission rate of the virus a constant for each partner? Or for each encounter? Does the transmission process follow a more complex specification with heterogeneity of infectiousness across individuals or across the different stages of the disease? Although Kaplan (1990) and Rottingen and Garnett (2002) have shown that a constant transmission rate for each encounter is not empirically consistent, I assume people do not have a perfect knowledge of the different HIV/AIDS stages and they act having in mind a constant per-act probability of transmission denoted β. With that simple specification, the probability of being infected after n sexual encounters is given by the binomial model: $p[1-(1-\beta)^n]$ where p is the probability that the partner is infected. I will apply that specification in my model.

The SI model generally considers individual behavior as exogenous. However, some researchers such as Kremer (1994, 1996) and more recently Mechoulan (2004) introduced behavioral choices into the SI model. Kremer (1994, 1996) showed that prevention strategies that tend to decrease the number of partners may increase long-run prevalence. These two theoretical models conclude than condom promotion is more efficient than strategies based on the reduction of partners. In their framework, learning the true value of the parameters is not considered as an alternative prevention method. Mechoulan (2004) studied the impact of HIV/AIDS testing on sexual behavior theoretically. Even if selfish individuals may increase their number of partners after learning they are infected, Mechoulan (2004) showed that minimal altruism is sufficient for testing to become beneficial. Empirically, public health literature indicates that people are altruist enough when good counselling is provided after a positive test. This model focuses mainly on the behavior of HIV positive people. Again, the infection rate is considered to be known by individuals.

More recently, Tremblay and Ling (2005) set up a model of people's decisions to engage in sexual encounters and to protect themselves, in the light of their perceptions of the probability of HIV transmission. In their model education may favor best practice by promoting safer intercourse methods, by encouraging correct condom use (risk-altering strategy) or by increasing the perceived probability of infection. However, AIDS education may also encourage risky behavior if it conveys social approval of sexuality or if it provides information diminishing the perceived probability of infection. Empirically, Tremblay and Ling (2005) showed that AIDS education does not significantly discourage abstinence but does significantly encourage condom use. The concern about the possible dangerous consequences of social approval of intercourse does not seem consistent with US data. Regrettably, Tremblay and Ling (2005) did not take into account the fact that people misjudge the risk of infection. Neither does their model neither predict the potential negative consequences of an overestimated transmission rate.

Empirically, numerous evaluations of the long-term impact of condom promotion programs yielded mixed results (Meekers et al., 2003; Hearst and Chen, 2004; Wong et al., 2005). However, when information is well provided, prevention may induce people to conduct safer sexual encounters, i.e. to use condoms, to avoid intergenerational sex, and to avoid multiple concomitant partners (Tremblay and Ling, 2005; Duflo et al., 2006; Dupas, 2009; Thornton, 2008).

As part of this review of the existing literature, I considered a few empirical papers that included risk perception in their analysis of condom use. Unfortunately, these papers were never based on a strong theoretical basis. Also, the existing theoretical literature about AIDS education does not include the possible pernicious effect of biased knowledge about specific parameters. The overestimation of the riskiness of unprotected sexual behavior is always seen as favorable and exceptional non-use of condoms is never dissociated from permanently risky behavior.

1.3. My contribution

In this paper, I will study both theoretically and empirically, the links between regular/casual condom use and the per-act expected transmission rate. I propose a behavioral model incorporating risk evaluation. I include the time dimension of the condom-use decision and distinguish the misevaluation of the prevalence and the overestimation of the expected transmission rate. The aim is to show that biased knowledge of the expected transmission rate may have a positive and a pernicious effect, yielding a U-shaped relationship between the risk and the expected transmission rate. By calibrating the model with existing data, I find it optimal to disclose that the transmission rate is between 5% and 24.9%.

In the second section, I present some stylised facts drawn from field data collected among students in Burundi. I show that their basic knowledge about the virus is good for broad questions such as the Demographic and Health Surveys (DHS), but sharply biased when a precise evaluation of some parameters is requested. In the third section, I develop a simple model of a risk-neutral agent who has to choose between safe and risky sex. I show that an upward-biased evaluation of the infection rate may be harmful when an unplanned risky sexual intercourse is quite probable. In the fourth part, I propose several numerical estimations of the model. Those different calibrations allow me to calculate the optimal disclosed transmission rate, i.e. the transmission rate that minimizes the risk taken by individuals. In the two last section of this paper, I discuss the validity of the conclusions and I give suggestions for further research to evaluate the impact of educational programs that provide correct information on the infection rate of HIV/AIDS.

2. Stylized facts

The few empirical studies linking risk evaluation and condom use suggest that the beliefs of people about the riskiness of unprotected sex are strongly biased. Using data from Nigerian markets in 2008, Lammers et al. (2009) found that 49.8% of interviewees evaluated unprotected sex as extremely risky. Only 7.5% of the unmarried indicated that unprotected sex is not risky at all. Men's and women's answers were not statistically different. Lammers et al. (2009) did not identify whether unprotected sex was seen as risky because of high perceived prevalence, because of high perceived infectivity, or both. Their questionnaire distinguished different degrees of risk but did not ask for a numerical equivalent (i.e. an explicit evaluation of the parameters). Similarly, Meekers and Klein (2002) found that in South African couples, 22% of husbands and 57% of wives perceived a risk of HIV infection from their partner.

To the best of my knowledge, there is no available data on people's numerical evaluations of very specific parameters such as the expected transmission rate of the HIV/AIDS virus, the expected prevalence or the expected length of life while infected. However, these parameters are the ones rational individuals use in deciding whether to protect themselves or not. Hence, I conducted a field survey in Burundi in May 2010 in order to assess the knowledge of individuals about HIV/AIDS and the impact on their behavior.

Some 509 volunteers from different courses at the University of Burundi in Bujumbura were interviewed in May 2010. In order to attract students, a team of workers distributed some flyers at the two main entrances of the University advising that volunteers could take part to a survey and earn 1,000 FBU in 10 minutes. Of course, this sampling method is not random at all and the sample will not reflect the behavior of the whole Burundian population. However, because classes are compulsory at the University of Burundi, the sample is fairly representative of the student population because the reward offered for participation was sufficiently high. With too low a reward, only poor people and individuals who care about HIV/AIDS would have answered the questionnaire. Their expectations and behavior might be very different from the rest of the students. In order to minimize this risk of possible bias, I gave a reward which was large enough to attract every student.

The expression of probabilities and the evaluation of numbers require very specific skills which are generally not mastered by the general population. The sample was hence limited to highly educated people. Of course, when taking a risky decision related to sex, everybody - educated and less educated - tries to assess the dangerousness of their behavior. However, expressing the complexity of the calculation may be very difficult. In order to minimize the bias due to the fact that less educated people have difficulty evaluating probabilities, I decided to focus on educated people, at the expense of a less representative sample. The goal was first to show that even educated people who have followed specific prevention programs have a really biased knowledge of transmission probabilities. Second, I wanted to draw up a list of different sexuality profiles and to compare them with the level of knowledge of the virus. These profiles will permit us to calibrate the simulations in order predict an optimal expected transmission rate level that would minimize the risk of transmission of the virus.

I used self-completion questionnaires for three reasons. First, not hiring interviewers was less time consuming and less expensive. Second, in order to assess the validity of the model, I did not need a complete measurement of all aspects of each individual in the sample. I only needed to evaluate their beliefs about HIV/AIDS and to collect data about their past sexual behavior. Because this information is very private and sensitive, I hoped to achieve more reliable information if respondents answered by themselves in an anonymous way. No name was requested on the questionnaire and the students deposited their completed questionnaire in a closed ballot box. Finally, because all the respondents were highly educated, they easily understood how to fill in the questionnaire.

The main descriptive statistics are presented by gender in table 1. The sample was 66% male. The average age was 25.2 years. The questionnaire was divided into 3 parts: a probability test, an HIV/AIDS knowledge test, and a part on past sexual behavior. First, I evaluated the ability of interviewees to assess probabilities in simple games (coins and dice). The results of this probability test were disappointing: only 5 students answered correctly to all 4 questions (less than 1% of the sample) and 27 answered correctly to 3 out of the four questions (about 5.3%). 388 respondents (about 76% of the students) gave no correct answers.

Second, I assessed the beliefs of students about the HIV/AIDS virus. The answers on HIV/AIDS confirmed my intuition: basics about the HIV/AIDS

Table 1: HIV beliefs and behavior

Variable	**Obs.**	**Total**	**Male**	**Female**	**p-value** (H0: M=F)
Age	508	25.2	25.5	24.7	0.001
Sex	506	66.8%	337	168	
Prob. (2 heads)	402	16.2%	18.0%	12.0%	0.111
Prob. (tails then heads)	403	16.4%	15.4%	19.1%	0.380
Prob. (dice: double 6)	390	14.6%	14.5%	15.0%	0.901
Prob. (dice: sum=3)	375	7.5%	7.6%	7.3%	0.911
HIV+ may look healthy	504	98.4%	99.1%	97.0%	0.141
Avoid sex diminish P(infected)	492	88.0%	91.1%	82.3%	0.010
Condom diminish P(infected)	460	90.4%	93.2%	84.5%	0.009
Expected transmission rate (B)	478	81.4%	81.6%	81.1%	0.820
Exp. trans. rate with condom (B)	474	17.2%	16.3%	19.0%	0.241
Had an HIV/AIDS test	506	65.0%	62.5%	69.5%	0.118
Prevalence in Burundi	440	27.9%	20.8%	42.1%	0.000
Life expectancy while infected	419	8.0	7.8	8.3	0.562
Heard about ARV therapies	504	94.0%	95.2%	92.2%	0.211
Participated in a prevention training	508	87.8%	86.9%	89.3%	0.438
Belong to an association	491	34.0%	33.4%	34.0%	0.907
Already had sexual relationship	505	48.3%	50.1%	43.7%	0.174
Age at the first encounter	241	18.1	17.2	20.1	0.000
Always uses a condom	240	43.33%	48.48%	30.56%	0.008
Condom at the first encounter	242	48.35%	50.30%	43.06%	0.305
Condom at the last encounter	239	46.44%	50.00%	38.57%	0.107
Guilt if no condom use	212	78.77%	81.38%	71.88%	0.148
Experienced a condom failure	211	24.64%	26.17%	22.03%	0.527
Know a place to buy condoms	455	85.71%	91.30%	74.51%	0.000
Feel guilty if ask for condom use	362	44.20%	47.22%	37.04%	0.072

virus are well-known, but specific information about the transmission are sharply biased. Indeed, 98.4% of the respondents knew that infected people look healthy, 88% that it is possible to reduce the risk of infection by avoiding any sexual relationships, and 90.4% that the probability of transmission of the HIV/AIDS virus could be reduced by using condoms during sexual encounters. These promising statistics are probably due to the fact that 87.8% of the respondents had had some prevention training. However, specific parameters about the transmission of the virus remained largely unknown. Although the transmission rate of the HIV/AIDS virus during any unprotected sexual encounter with an infected partner is almost always lower than 1% (Boily et al., 2009; Wawer et al., 2005), the average transmission rate estimated by the respondents was about 81% (Figure B.1). Similarly, the students estimated the average risk of transmission with condoms at 17.2%, which is much higher than the true transmission rate for unprotected encounters (Figure B.2). The prevalence of HIV-positive people in Burundi was largely unknown to the students. On average, they estimated that 27.9% of the population was infected by the virus, with a very large standard deviation of 21.4 (Figure B.3). In 2007, the UNAIDS and the WHO estimated that 2% of the adult population in Burundi was infected. The overestimation of the risk of transmission is a strong argument in favor of a theoretical study about the indirect consequences of such a dramatization of HIV/AIDS infectivity. It is worth noting that the students' estimates of the life expectancy of HIV-positive people with and without ARV therapies were quite accurate (Figures B.4 and B.5).

Finally, I asked about past sexual behavior in order to build different sexuality profiles. Surprisingly, 51.7% of the interviewees claimed they had never had any sexual relationships. The average age for the first sexual encounter of the others was 17.2 years for men and 20.1 years for women. Sadly, 28.6 % of the first sexual relationships were by children less than 16 years old, and 13.2% below 12 years old. Half of the sexually active males and 38.57% of females affirmed that they used a condom during their last intercourse. Under 14 years old, none of them used condoms, either because of the violent nature of the encounter (rape) or because they were too young to be aware of the riskiness of the behavior. 73.33% of the respondents declared that condom use brings less pleasure than risky sex (Figure B.6). Some 37.4% of students who were sexually active never used a condom with their last partner, 22% used often condoms, 11.5% had used condoms at some point in the past, and 29% of respondents always used a condom with their last partner.

3. A theoretical model

3.1. Set-up

My aim was to construct a simple n-period model in discrete time that links condom use and the evaluation of specific parameters such as the expected transmission rate and the prevalence. In the simplest framework, a healthy and risk-neutral individual i begin his or her sexual life at time $t = 0$. He or she has n repeated sexual encounters with a partner j (one relation per period). Let us assume that the serostatus of j, denoted p_j, is fixed and unknown. Each period, the individual i chooses whether or not to use a condom. If protection is used at time t, $s_{it} = 0$, and the sexual relationship is assumed to be risk free. Otherwise, $s_{it} = 1$ and there is a constant positive probability β that the virus is transmitted if the partner is infected. In reality, the transmission rate β is not constant: the infectivity is higher in the first and the later stages of the disease (Wawer et al., 2005; Boily et al., 2009). However, I assume a constant expected transmission rate $E(\beta)$ because individuals are not well informed about the different stages of the AIDS disease and because it is very difficult to visually distinguish HIV positive people from uninfected individuals. While infected, an individual has a probability $\frac{1}{\lambda} > 1$ of dying in each period of time where λ is the life expectancy of a sick person.

The optimization program is:

$$MaxE[U_i(s_{it}; E(d_{it}))] = \sum_{t=0}^{n} \rho^t u_i(s_{it})(1 - E(d_{it})) \tag{1}$$

where $E(d_{it})$ is the expected probability of dying from AIDS, s_{it} is equal to 1 for risky sex and 0 if a condom is used, and ρ is the discount rate. We assume a separable linear utility function:

$$u_i(s_{it}) = \alpha + s_{it} + \theta(1 - s_{it}) \tag{2}$$

with $0 < \theta < 1$ because protected sex is less highly valued than risky sex (as mentioned above and observed during the survey). α represents all the other pleasures that an individual enjoys and is assumed to be constant over time. If α is high, people are less willing to exchange years of life against risky sex. If α is low, people prefer more the pleasure of risky practices even if it may shorten their life. In the simulations, I show that the results are insensitive to changes in α, meaning that the conclusions remain valid if the assumption of risk neutrality with respect to length of life is relaxed.

I assume that AIDS is the only cause of death. Therefore, the expected probability of dying depends only on self-protection choices during preceding sexual encounters. At time t, the individual i may be in four different states. He or she may be uninfected with an uninfected partner ($p_{it} = 0$ and $p_j = 0$), uninfected with an infected partner ($p_{it} = 0$ and $p_j = 1$), infected and alive ($d_{it} = 0$ and $p_{it} = 1$) or infected and dead ($d_{it} = 1$ and $p_{it} = 1$). I assume that people take rational decisions based on a binomial epidemiological model with a constant probability of transmission per coïtal act. As explained in the introduction, more complex epidemiological models are available (Rottingen and Garnett, 2002), but they seem to be less consistent with people's biased beliefs. If $E(p_{jt})$ is the unknown expected status of partner j, $E(\beta)$ the expected transmission rate, and $E(\lambda)$ the expected life expectancy of an HIV-positive individual, the subjective probabilities of being in one of these states at time t are:

$$E(p_{it} = 0 \text{ and } p_j = 0) = \underbrace{1 - E(p_j)}_{\text{j not infected}}, \tag{3}$$

$$E(p_{it} = 0 \text{ and } p_j = 1) = \underbrace{E(p_j)\prod_{k=0}^{t-1}(1 - E(\beta))^{s_k}}_{\text{j infected - no transmission}}, \tag{4}$$

$$E(d_{it} = 0 \text{ and } p_{it} = 1) = \tag{5}$$
$$\underbrace{E(p_j)}_{\text{j infected}} \sum_{k=0}^{t-1} \left(\underbrace{[1 - (1 - E(\beta))^{s_k}]}_{\text{transmission occurs at k}} \underbrace{(1 - \frac{1}{E(\lambda)})^{t-k}}_{\text{alive between k and t}} \underbrace{[\prod_{j=0}^{k-1}(1 - E(\beta))^{s_j}]}_{\text{no infection before k}} \right),$$

$$E(d_{it} = 1 \text{ and } p_{it} = 1) = \tag{6}$$
$$\underbrace{E(p_j)}_{\text{j infected}} \sum_{k=0}^{t-1} \left(\underbrace{[1 - (1 - E(\beta))^{s_k}]}_{\text{transmission occurs at k}} \underbrace{[1 - (1 - \frac{1}{E(\lambda)})^{t-k}]}_{\text{die between k and t}} \underbrace{[\prod_{j=0}^{k-1}(1 - E(\beta))^{s_j}]}_{\text{no infection before k}} \right).$$

3.2. Solution of the infinite period model

In this section, I will solve the model for an agent i with an infinite horizon. In order to clarify the formulas, I forget the expectations in the equations. However, I keep in mind that all variables are subjective for the individual i. By assuming a possible infinite life, an "end of life" effect in which the individual chooses risky behavior because he or she knows that the optimization problem will finish soon is avoided. In this section, I will show that i chooses either safe sex, or risky sex throughout his or her life, but never both. A higher expected transmission rate β implies lower risk as long as no unprotected encounters occur. However, when a sudden unprotected sexual relationship occurs, a higher expected transmission rate increases the chances of risky behavior.

The individual i has to choose s_{it} for all t in order to maximise:

$$MaxE[U_i(s_{it};E(d_{it}))] = \sum_{t=0}^{\infty} \rho^t [\alpha + s_{it} + \theta(1-s_{it})]$$
$$\left[1 - p_j \sum_{k=0}^{t-1} \left([1-(1-\beta)^{s_k}][1-(1-\frac{1}{\lambda})^{t-k}][\prod_{j=0}^{k-1}(1-\beta)^{s_j}] \right)\right]. \quad (7)$$

Proposition 3.1. *If a safe sexual encounter is chosen at the first period, then safe sex will be chosen for all periods.*

Proof If an individual chooses to engage in safe sex in the first period, it is straightforward to show that the optimization program remains the same in the second period. Indeed, because we assume that a safe sexual encounter has a zero expected risk of transmission, a safe encounter in t=0 has no influence on the expected probability of dying and hence on the maximisation program for the next period. □

Proposition 3.2. *When the individual chooses to engage in risky sex with partner j at time $t = 0$, protection will never be used.*

Proof Proof in appendix.

Propositions (3.1) and (3.2) imply that individual i will always engage either in safe sex or in risky sex.

Proposition 3.3. *The individual chooses safe sex if:*

$$E[U_i(s_{it})|s_{it} = 0, \forall t] \geq E[U_i(s_{it})|s_{it} = 1, \forall t].$$

This inequality is equivalent to:

$$\frac{\theta+\alpha}{1-\rho} \geq \frac{1+\alpha}{1-\rho} - \frac{(1+\alpha)\rho p_j \beta}{1-\rho(1-\beta)} \left[\frac{1}{1-\rho} - \frac{(1-\frac{1}{\lambda})}{1-\rho(1-\frac{1}{\lambda})}\right]. \quad (8)$$

Proof Proof in appendix.

Equation (8) implies that risky sex is avoided if the expected transmission rate β, the expected serostatus of partner p_j and the utility of safe sex θ are sufficiently high. Indeed, the fear of infection and death prevents the taking of excessive risks. This is the favorable direct effect of an upward-biased expected transmission rate. A higher life expectancy, λ, of HIV-positive individuals encourages risky behavior, suggesting the pernicious effect of antiretroviral therapies on the level of risk taken by individuals. I will now formally prove this statement for the expected transmission rate β.

Proposition 3.4. *When individuals know they are not infected, overestimating the transmission rate favors safe sex.*

Proof The derivative of the left-hand side of inequality (8) is zero and the derivative of the right-hand side is negative:

$$\frac{dE[U_i(s_{it})|s_{it}=1,\forall t]}{d\beta} = \\ -\frac{(1+\alpha)\rho p_j(1-\rho)}{[1-\rho(1-\beta)]^2}\left[\frac{1}{1-\rho}-\frac{(1-\frac{1}{\lambda})}{1-\rho(1-\frac{1}{\lambda})}\right] < 0. \tag{9}$$

Hence, a higher β deters risky sex because it raises the expected probability of becoming infected and to dying. □

3.3. Decision after an unplanned risky intercourse

Within the same framework, assume that an unprotected relationship occurs in any one time period, like an external shock. This unprotected relationship may be due to the impulsive nature of sex or to a condom failure. If all the parameters remain equal, the expected probability of being alive, $1-E(d_t)$, is reduced for all future periods of time. Without loss of generality, let us assume that risky intercourse occurs at $t=-1$.

If individual i would choose risky sex even without this unplanned unprotected intercourse, the optimisation program is not affected and the individual will continue to choose risky encounters. However, if i would choose safe sex, the optimisation program is affected and i's reaction depends on his or her assessment of the parameters. Indeed, the satisfaction from safe behavior decreases for two reasons. First, life expectancy is lower and hence i

will be more prone to enjoy certain present satisfactions. Second, if he or she survives across time, individual i learns that the unprotected encounter was not so dangerous and may be tempted to change his or her attitude toward risky sex. Proposition (3.1) and (3.2) have to be extended.

Proposition 3.5. *When the individual i chooses to engage in risky sex at time t, he or she will choose risky sex for all future periods of time, independently of the amount of risk taken before t. Similarly, if i chooses protection at time t, he or she will choose safe sex for all future periods of time, whatever the amount of risk taken before t.*

Proof Proof in appendix.

Proposition (3.5) shows that individual i will always engage either in safe sex or in risky sex after the unplanned risky encounter.

Proposition 3.6. *After the unplanned risky encounter in $t = -1$, the individual chooses safe sex if:*

$$E[U_i(s_{it})|s_{it} = 0, \forall t > 0] \geq E[U_i(s_{it})|s_{it} = 1, \forall t > 0].$$

This inequality is equivalent to:

$$\begin{aligned} & (\alpha+\theta)\left(\frac{1-p_j\beta}{1-\rho}+\frac{p_j\beta(1-\frac{1}{\lambda})}{1-\rho(1-\frac{1}{\lambda})}\right) \\ \geq\ & (1+\alpha)\left[\frac{1-p_j}{1-\rho}+\frac{\beta p_j(1-\frac{1}{\lambda})}{1-\rho(1-\frac{1}{\lambda})}\right. \\ & \left.+\frac{p_j[1-\beta]}{1-\rho[1-\beta]}+\frac{\rho p_j\beta[1-\beta](1-\frac{1}{\lambda})}{[1-\rho[1-\beta]][1-\rho(1-\frac{1}{\lambda})]}\right]. \end{aligned} \tag{10}$$

Proof Proof in appendix.

The presence of β on the left-hand side of the inequality 10 constitutes the crucial difference between inequality (8) and inequality (10). This new term sharply modifies the incentives for conducting risky sex. Indeed, a high expected transmission rate, and a low expected life expectancy for HIV-positive individuals raise the value of the left-hand side of the inequality 10 and increase the probability of engaging risky behavior after an unplanned risky encounter. Hence, an upward-biased expected transmission rate may induce less cautious behavior when unplanned risky behavior are likely to happen. This bias implies that a wise and careful individual may take greater risks because he or she wrongly thinks that he or she is necessarily infected if the partner was HIV positive. I will prove this statement formally below.

Proposition 3.7. *An upward-biased expected transmission rate favors risky sex after an unplanned risky encounter if $\beta \geq \beta^*$ with:*

$$\beta^* = \frac{-(\alpha+\theta)(1-\rho)\rho \pm \sqrt{(1+\alpha)(\alpha+\theta)(1-\rho)\rho^2}}{(\alpha+\theta)\rho^2}. \tag{11}$$

Proof If the derivative of the inequality (10) with respect to β is positive, then increasing β encourages safe sex. Conversely, if this derivative is negative, increasing β encourages risky practices. The derivative of inequality (10) with respect to β is given by:

$$p\left(\frac{1-(1-\alpha+2\alpha\beta)\rho-\alpha(1-\beta)^2\rho^2-\theta[1-\rho(1-\beta)]^2}{(1-\rho)[\rho+l(1-\rho)][1-\rho(1-\beta)]^2}\right).$$

This derivative has one positive and one negative root:

$$\frac{-(\alpha+\theta)(1-\rho)\rho \pm \sqrt{(1+\alpha)(\alpha+\theta)(1-\rho)\rho^2}}{(\alpha+\theta)\rho^2}$$

Between these two roots, the derivative is positive, i.e. raising the expected transmission rate β encourages safer sex. The optimal (i.e. safer) expected transmission rate is given by the positive root β^* of the derivative of inequality (10) with respect to β. □

4. Numerical estimation

4.1. Estimation of β^* for long-lasting and careless relationships

In this subsection, I will derive different estimates of β^*, i.e. the expected transmission rate that maximizes condom use. I will focus on a specific population with a high probability of unprotected encounters, where relations between partners are long-lasting. First, I suggest some realistic values for the parameters and I evaluate β^*. Then, I will assess the robustness of the estimate to changes in the parameters.

The parameters of the model are:

- α, which represents the utility gained from other sources of pleasure than sex. I will take 10 as a benchmark value and I will check for changes in this parameter between 0 and $1,000$.

- ρ, that is the discount factor from one period to the other. In the model, one period of time corresponds to one sexual act. I will take 0.999 as a benchmark value (it is the weekly discount factor corresponding to a yearly discount factor of 0.95) and will check for changes in this parameter between 0 and 1.

- θ, which represents the relative utility of safe sex compared to unprotected sex. I will take 0.5 as a benchmark value and I will check for changes in this parameter from 0 to 1.

- $E(p_j)$, which represents the expected probability that the partner is infected. I will take 0.05 as a benchmark value and I will check for changes in this parameter between 0 and 1.

- λ, that is the life expectancy of an infected person (expressed as the remaining number of sexual encounters). Assuming two sexual encounters per week and knowing that a seropositive person has an average life expectancy of 10 years, the benchmark value for λ is $1,000$. I will test the robustness of the results for variations of λ between 0 and 10.000.

- $E(\beta)$, that is the expected transmission rate. Because it is a probability, it ranges between 0 and 1. I will not fix a benchmark value for β because it is the main variable of interest.

Inserting the benchmark values in equation (11) gives an optimal transmission rate that prevention campaigns should disclose of $\beta^* = 3.14\%$. The optimal disclosed transmission rate is the expected transmission rate that minimizes the risks taken after an unprotected encounter. The mean expected transmission rate in the Burundian sample is $81,4\%$. Hence, the optimal disclosed transmission rate is much lower than the rate most people expect. This first result suggests that the overestimation of the transmission rate is not optimal. The next step is to assess the robustness of the results to parameter changes.

Firstly, figure B.7 assesses the robustess of the estimate against changes in the discount rate ρ, and changes in the relative utility of other pleasures than sex, α. It appears that β^* is not significantly affected by changes in α. On the contrary, changes in ρ affect a lot the optimal disclosed transmission rate. β^* is strictly decreasing in ρ. Between the benchmark values for ρ, β^*d ranges from $\beta^* = 24.9\%$ for $\rho = 0.9$ to $\beta^* = 0.32\%$ for $\rho = 0.99999$.

Secondly, figure B.8 assesses the robustess of the estimate of β^* against changes in the discount rate ρ, and changes in the relative utility of safe sex θ. β^* is not affected by changes in θ. Again, ρ has a big impact on the optimal disclosed transmission rate. β^* is a strictly decreasing function of ρ.

Finally, figure B.9 considers changes in α and θ and has a similar interpretation: the relative utility of sex compared to other pleasures and the relative utility of safe sex compared to risky sex do not matter in the estimation of the optimal disclosed transmission rate.

When unexpected risky encounters are likely, I conclude that the optimal disclosed transmission rate ranges between 3% and 24.9%. An upward-biased expected transmission rate, as in the sample of Burundian students, is then probably dangerous. The estimation of the optimal disclosed transmission rate is very sensitive to slight changes in the discount rate ρ, especially between $\rho = 0.9$ and $\rho = 1$. A discount rate per sexual relationship (i.e. a for few days) of 0.9 seems very low, but sexuality may be prone to instinctive or short-termist behavior, justifying the"small" discount rate chosen the upper bound of the range. More empirical research on sexuality and implicit discount rates is needed to strengthen the estimate.

4.2. Estimation of β^* for different sexuality profiles

Not all relationships are as long-lasting as suggested in the last subsection. Some sexual encounters are one-off events, some are repeated a few times, and others are long-lasting. For any individual aiming to have only one-off sexual encounters, the analysis after an unplanned encounter is of course not meaningful. This may be the case in specific sexual networks. On the contrary, in long-lasting relationships, especially when condoms are not easily available or not always used properly, numerous sexual encounters can occur after an unprotected encounter, endorsing the simulation of the optimal disclosed transmission rate. Of course, it would be very difficult to reveal separately different transmission rates to subpopulations that have different sexuality profiles. Hence, it is crucial to assess globally the relative importance of casual and regular encounters.

With the data from the survey among students in Burundi, I studied the distribution of the length of relationships and the number of sexual encounters. Focusing on the lastest sexual partner, less than 0.8% of the sexual encounters were with a casual partner, 0.4% were a single encounter within a long-lasting relationship, and 7.9% were in a relationship where sex is not common (less than one encounter per month). By contrast, 90.7% of sexual acts took place within long-lasting relationships where sex was regular (one or more relation per month). 34% of the interviewees admited irregular condom use, and 24.6% had already experienced a condom failure. Thus, a large proportion of sexual encounters occured in long-lasting relationships where condom use was not ensured. These stylized facts justify the pertinence of the model and the simulations, at least among educated Burundian students.

However, casual encounters and relationships where condoms are perfectly used are not marginal. The next step is then to find an optimal disclosed transmission rate β^* lying between the optimal β for casual encounters and for relationships where a condom is used perfectly (i.e. $\beta = 1$), and the optimal β for long-lasting relationships where condoms are not perfectly used (i.e. $\beta = \beta^*$). Let us simulate inequality (8) that determine condom use before any unplanned risky intercourse. The aim is to assess the impact of β on the risk taken during casual encounters.

Figures B.10 to B.14 are simulations of the inequality (8). The vertical axis measures the utility of risky sex divided by the utility of safe sex. Hence, when the outcome is greater than one, individual i chooses risky sex. Otherwise, i chooses to use condoms. The figures show that β has no influence on the decision to use condoms, except for very small values of β ($\beta < 0.05$). A higher expected prevalence $E(p_j)$ implies safer behavior because people want to avoid a dangerous disease (Figure B.10). Other pleasures than sex, α, also imply more cautious behavior (Figure B.11). A high discount rate implies condom use because future periods are as important as the present (Figure B.12). If safe sex gives as much pleasure as risky sex, people will choose to protect themselves (Figure B.13). Finally, if the life expectancy of infected people is very long, people do not fear being sick and engage in risky behavior (Figure B.14).

I conclude that the expected transmission rate $E(\beta)$ has almost no impact on the decision to use condoms for $0.05 < E(\beta) < 1$. For $E(\beta) < 0.05$, that is if people think that the transmission rate is very low, the utility of risky sex increases sharply. Hence, disclosing too low a transmission rate during prevention campaigns may therefore be dangerous for people engaging in casual sex. Above 5%, the risk of becoming infected is sufficient to prevent all dangerous behavior. I conclude that the optimal disclosed transmission rate lies between 5 and 24.9%. More research on the implicit discount rate during sexual encounters is needed in order to narrow this estimate.

5. Discussion

5.1. Implementation and external validity

The model and simulations disscussed here confirm the idea of a U-shaped relation between risk perception and condom use. If unexpected encounters without condoms are quite common, the calibrations of the model suggest that it would be optimal to tell the population that the transmission rate ranges between 3 and 24.9%. This range is too large in populations where some sexual encounters occur in one-night stands. Indeed, an expected transmission rate below 5% encourages risky behavior because of the marginal of being infected. In a general population with different sexuality profiles, the optimal disclosed transmission rate ranges between 5 and 24.9%

It is morally difficult to promote prevention campaigns which lie to people by advertising a transmission rate of 5, 10 or 20%. Moreover, the sustainability of such policies is questionnable because well-informed people would react against the misleading campaign. Some alternatives are however morally more defensible. First, it would be possible to disclose the per-relationship transmission rate which is about 20% (Kaplan, 1990). Another possibility would be to announce an upper bound for the transmission rate. By saying that the infectivity is never higher than 20% and by insisting on the fact that protection is crucial even if an unprotected encounter has occurred, people may be encouraged to engage in safer behavior. Otherwise, it is possible to reveal imprecise information, insisting on the importance of condom use after an unplanned risky encounter because the virus is not always transmitted. Finally, it would be possible to disclose the true transmission rate of the virus. Given these imperfect strategies, empirical testing of the model and of those proposals is essential before engaging in any prevention campaign aiming to disclose the transmission rate.

5.2. Empirical test

Random experiments are probably the least criticized and most promising tool to evaluate development programs. However, they often raise ethical concerns because some people are randomly assigned to a program that has unknown consequences. Conversely, the control group is deliberately removed from a potentially beneficial treatment. For this topic (i.e. the disclosure of an optimal transmission rate), random experiments also raise ethical concerns because it is not known how people will react to the information. It would be morally untenable to engage some people in a prevention campaign that may be harmful for them.

In order to test the model, it is therefore necessary to find two very similar groups of people who differ only in the fact that they have different perception of the transmission rate. Random samples of the population do not fit this characterization because nearly all individuals expect the transmission rate to be very high. Hence, no comparison group is available. However, a field experiment among students of health might be feasible. By comparing the behavior of students who had already followed a course disclosing the true transmission rate with that of other students, it should be possible to measure the impact of the disclosure of the transmission rate on behavior. Nevertheless, although this identification strategy seems unbiased, its external validity is questionable.

5.3. Possible extensions

This simple model is a first step in the understanding of the different behavioral consequences of the disclosure of the rate of HIV/AIDS infectivity. This paper may be extended in many directions to achieve a more realistic model and more precise estimates of the optimal disclosed transmission rate. In this section, I will discuss some possible extensions that are beyond the scope of this paper. First, a more complex model could include all other factors that may influence condom use. For instance, the possibility of HIV/AIDS testing, the bargaining power of the two individuals, the contraception decision and the desire to procreate, all partly explain condom use. As long as seropositivity is a private information, the conclusions of this model are valid because unprotected sex remains risky. For people using condoms as a contraceptive, the expected transmission rate has no influence on their decision to protect themselves. This model is not meaningful for them, but its conclusions hold for other people. The bargaining power of partners probably has no influence on the results if women and men have similar expectations about HIV infectivity.

Secondly, in this simple specification, the only cause of death is the AIDS virus, and HIV does not enter directly into the utility function, except through the probability of death. In addition, antiretroviral therapies (ARV) are not considered and it is assumed that condoms are perfectly safe. More complex specifications of the risk should not significantly alter the positive and the negative effects of a biased expected transmission rate. For instance, the availability of ARV would raise the life expectancy while infected. As shown in the simulation, it has no impact on the optimal disclosed transmission rate. Also, adding HIV disutility or condom failure would modify the utility of safe and risky sex, but would not remove the pernicious effect of an over-estimated transmission rate.

Finally, allowing for multiple partners would be realistic. In this model, changing partner is never optimal for i if the pleasure of sex remains unchanged throughout the relationship and if $E(p_j)$ is equal among all possible partners. The optimality of partner change would require either diminishing the pleasure of sex during the relationship, the existence of safer partners, or the inclusion of other factors than sex in the utility function. The conclusions should not be affected by such modifications.

6. Conclusion

The estimation of the infectivity of the HIV/AIDS virus is generally upwardly biased. In this paper, a behavioral model was constructed in order to assess the positive and negative impacts of such biased knowledge. On the one hand, a high expected transmission rate implies safer behavior because people want to protect themselves against a disease that is seen as very infectious. On the other hand, individuals expecting a very high transmission rate may cease to protect themselves after an unprotected encounter because they think that they will have already been infected if the partner was HIV positive. Simulations indicate that the optimal transmission rate to disclose during prevention campaigns lies between 5% and 24.9%.

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Appendix A. Proofs

Proposition 3.2. When the individual chooses to engage in risky sex with partner j at time $t = 0$, then protection will never be used.

Proof Because of the last proposition, we know that an individual who chooses unprotected sex at least once will not use protection in the first period. Indeed, from proposition (3.1), we know that if i chooses to use protection in the first period, then he or she will always opt for protected encounters. Hence, we have to prove that the choice $s_{il-1} = 1$ implies the choice $s_{il} = 1$ $\forall l > 1$.

Assuming i is alive at time $t = l - 1$, $s_{il-1} = 1$ is optimal if $\forall s_{il}, s_{il+1}, ...$:

$$\frac{\Delta U}{\Delta s_{il-1}}|_{d_{il-1}=0} = U(1, ..., 1, s_{il}, ...)|_{d_{il-1}=0} - U(1, ..., 0, s_{il}, ...)|_{d_{il-1}=0} > 0. \quad (A.1)$$

The fact that i had unprotected encounters and is still alive at $t = l - 1$ gives him or her information about the true parameters. Assuming rational behavior, he or she will correct the optimization program, knowing him- or herself to be in one of the three states described by the equations (3) to (5) (or alternatively, knowing him- or herself not to be in the fourth state described by equation (6) i.e. not dead). Hence, if i is alive at $t = l - 1$ and had unprotected sexual relationships in the past, the probabilities of being in each of the 3 possible states are:

$$E(p_{il-1} = 0 \text{ and } p_j = 0) = m_{l-1}(1 - p_j); \quad (A.2)$$

$$E(p_{il-1} = 0 \text{ and } p_j = 1) = m_{l-1}p_j(1 - \beta)^{l-1}; \quad (A.3)$$

$$E(d_{il-1} = 0 \text{ and } p_{il-1} = 1) = m_{l-1}p_j \sum_{k=0}^{l-2} \beta(1 - \frac{1}{\lambda})^{l-k-1}(1 - \beta)^k; \quad (A.4)$$

with $m > 1$ fixed such that the sum of the probabilities of these three states is equal to one:

$$m_{l-1} = \frac{1}{1 - p_j\beta[1 - (1 - \frac{1}{\lambda})^{l-1} - \sum_{k=1}^{l-2}(1 - 1 - \frac{1}{\lambda})^{l-k-1}(1 - \beta)^k]}. \quad (A.5)$$

This factor m_{l-1} may be seen has a correction for "trust". The fact that i is still alive indicates that the unprotected encounters are safer than expected. We have $\frac{dm_t}{dt} > 0$ because the probability of being dead because of having unprotected sexual relationships rises over time.

The probability of dying at time $t > l-1$, knowing that i is alive at time $l-1$ is given by:

$$E(d_{it}=1)|_{d_{il-1}=0} = \underbrace{m_{l-1}p_j \sum_{k=0}^{l-2} \beta(1-\frac{1}{\lambda})^{l-k-1}(1-\beta)^k}_{\text{infected alive in l-1}} \underbrace{[1-(1-\frac{1}{\lambda})]^{t-l+1}}_{\text{die between l-1 and t}}$$
$$+ \underbrace{m_{l-1}p_j(1-\beta)^{l-1}}_{j \text{ infected, i uninfected at l-1}} \sum_{k=l-1}^{t-1} \underbrace{[1-(1-\beta)^{s_k}]}_{\text{transmission at k}} \underbrace{[1-(1-\frac{1}{\lambda})^{t-k}]}_{\text{die between k and t}} \underbrace{[\prod_{j=l-1}^{k-1}(1-\beta)^{s_j}]}_{\text{no infection before k}}. \tag{A.6}$$

Let us now compute in detail each of the terms of equation (A.1):

$$U(1,...,1,s_{il},...)|_{d_{il-1}=0} = \underbrace{\sum_{t=0}^{l-1}\rho^t(1+\alpha)}_{\text{risky sex till l-1}} + \sum_{t=l}^{\infty}\rho^t[\alpha+s_{it}+\theta(1-s_{it})] \tag{A.7}$$

$$\Big[1 - \underbrace{m_{l-1}p_j \sum_{k=0}^{l-2}\beta(1-\frac{1}{\lambda})^{l-k-1}(1-\beta)^k[1-(1-\frac{1}{\lambda})]^{t-l+1}}_{\text{P(dying due to risk taken before l-1)}}$$
$$\underbrace{-m_{l-1}p_j[1-\beta]^{l-1}\beta[1-(1-\frac{1}{\lambda})^{t-l+1}]}_{\text{P(dying due to risk taken in l-1)}}$$
$$\underbrace{-m_{l-1}p_j(1-\beta)(1-\beta)^{l-1}\sum_{k=l}^{t-1}[1-(1-\beta)^{s_k}][1-(1-\frac{1}{\lambda})^{t-k}][\prod_{j=l}^{k-1}(1-\beta)^{s_j}]}_{\text{P(dying due to risk taken after l-1)}}\Big];$$

$$U(1,...,0,s_{it},...)|_{d_{il-1}=0} = \tag{A.8}$$

$$\sum_{t=0}^{l-2}\rho^t(1+\alpha) + \underbrace{\rho^{l-1}(\theta+\alpha)}_{\text{safe sex in l-1}} + \sum_{t=l}^{\infty}\rho^t[\alpha + s_{it} + \theta(1-s_{it})]$$

$$\Bigg[1 - \underbrace{m_{l-1}p_j\sum_{k=0}^{l-2}\beta(1-\frac{1}{\lambda})^{l-k-1}(1-\beta)^k[1-(1-\frac{1}{\lambda})]^{t-l+1}}_{\text{P(dying due to risk taken before l-1)}}$$

$$\underbrace{-m_{l-1}p_j(1-\beta)^{l-1}\sum_{k=l}^{t-1}[1-(1-\beta)^{s_k}][1-(1-\frac{1}{\lambda})^{t-k}][\prod_{j=l}^{k-1}(1-\beta)^{s_j}]\Bigg]}_{\text{P(dying due to risk taken after l-1)}}.$$

The difference between these two terms is:

$$\frac{\Delta U}{\Delta s_{il-1}}|_{d_{il-1}=0} = \rho^{l-1}(1-\theta) - \sum_{t=l}^{\infty}\rho^t[\alpha + s_{it} + \theta(1-s_{it})] \tag{A.9}$$

$$\Bigg[\underbrace{m_{l-1}p_j[1-\beta]^{l-1}\beta[1-(1-\frac{1}{\lambda})^{t-l+1}]}_{\text{P(dying due to risk taken in l-1)}}$$

$$\underbrace{-m_{l-1}p_j\beta(1-\beta)^{l-1}\sum_{k=l}^{t-1}[1-(1-\beta)^{s_k}][1-(1-\frac{1}{\lambda})^{t-k}][\prod_{j=l}^{k-1}(1-\beta)^{s_j}]\Bigg]}_{\Delta\text{ P(dying due to risk taken in l-1)}}.$$

The first term of the subtraction represents the utility gain from the unprotected relationship at time $t = l - 1$ while the second term accounts for the utility loss due to a higher probability of dying in the future. If $\frac{\Delta U}{\Delta s_{il-1}}|_{d_{il-1}=0}$ is positive, i chooses to engage in unprotected sex.

Now, let us undertake the same reasoning for i alive at time $t = l$:

$$\frac{\Delta U}{\Delta s_{il}}|_{d_{il}=0} = \rho^l(1-\theta) - \sum_{t=l+1}^{\infty} \rho^t[\alpha + s_{it} + \theta(1-s_{it})] \tag{A.10}$$

$$\Bigg[\underbrace{m_l p_j [1-\beta]^l \beta [1-(1-\frac{1}{\lambda})^{t-l}]}_{\text{P(dying due to risk taken in l)}}$$

$$\underbrace{-m_l p_j \beta(1-\beta)^l \sum_{k=l+1}^{t-1} \left([1-(1-\beta)^{s_k}][1-(1-\frac{1}{\lambda})^{t-k}][\prod_{j=l+1}^{k-1}(1-\beta)^{s_j}] \right)}_{\Delta \text{ P(dying due to risk taken in l)}}\Bigg].$$

To complete the proof of proposition (3.2), we need to show that if risky sex is preferred at time $t = l - 1$ (equation (A.9) greater than 0), then risky sex is also preferred at time $t = l$ (equation (A.10) is also greater than 0). This is, of course, the case: the opportunity cost of an unprotected sexual encounter is lower in the future because remaining life is shorter. Formally, after dividing equation (A.10) by ρ, we have to compare the factor m_{l-1} that enters negatively into equation (A.9) and the factor $m_l(1-\beta)$ that enters negatively into equation (A.10). This completes the proof because:

$$m_{l-1} > m_l(1-\beta). \tag{A.11}$$

□

Proposition 3.3. The individual chooses safe sex if:

$$E[U_i(s_{it})|s_{it} = 0, \forall t] \geq E[U_i(s_{it})|s_{it} = 1, \forall t].$$

This inequality is equivalent to:

$$\frac{\theta+\alpha}{1-\rho} \geq \frac{1+\alpha}{1-\rho} - \frac{(1+\alpha)\rho p_j \beta}{1-\rho(1-\beta)} \left[\frac{1}{1-\rho} - \frac{(1-\frac{1}{\lambda})}{1-\rho(1-\frac{1}{\lambda})}\right]. \tag{A.12}$$

Proof Because the probability of dying for i is zero if protection is always used, the total utility of i's infinite life is:

$$E[U_i(s_{it})|s_{it} = 0, \forall t] = \sum_{t=0}^{\infty} \rho^t(\theta+\alpha) = \frac{\theta+\alpha}{1-\rho}. \tag{A.13}$$

Conversely, if individual i only practices risky sex, his or her expected total utility is:

$$E[U_i(s_{it})|s_{it}=1,\forall t] \tag{A.14}$$
$$=\sum_{t=0}^{\infty}\rho^t(1+\alpha)\left[1-p_j\beta\left([1-(1-\frac{1}{\lambda})^t]+\sum_{k=1}^{t-1}[1-(1-\frac{1}{\lambda})^{t-k}](1-\beta)^k\right)\right].$$

This infinite sum can be decomposed for each period of time:

$$t=0\Rightarrow 1+\alpha;$$
$$t=1\Rightarrow \rho^1(1+\alpha)\left[1-p_j\beta([1-(1-\frac{1}{\lambda})](1-\beta))\right];$$
$$t=2\Rightarrow \rho^2(1+\alpha)\left[1-p_j\beta([1-(1-\frac{1}{\lambda})^2](1-\beta)+[1-(1-\frac{1}{\lambda})](1-\beta))\right];$$
$$t=3\Rightarrow \rho^3(1+\alpha)\left[1-p_j\beta([1-(1-\frac{1}{\lambda})^3](1-\beta)+[1-(1-\frac{1}{\lambda})^2]...)\right].$$

Equation (A.14) can be rewritten by grouping in one infinite sum the first vertical column, and in a suite of infinite sums the remaining diagonal terms:

$$\sum_{t=0}^{\infty}\rho^t(1+\alpha)-(1+\alpha)p_j\beta\Bigg[\rho[1-(1-\frac{1}{\lambda})]\sum_{t=0}^{\infty}\rho^t(1-\beta)^t$$
$$+\rho^2[1-(1-\frac{1}{\lambda})^2]\sum_{t=0}^{\infty}\rho^t(1-\beta)^t+...\Bigg]. \tag{A.15}$$

Using the properties of infinite sums, we obtain:

$$\frac{1+\alpha}{1-\rho}-\frac{(1+\alpha)p_j\beta}{1-\rho(1-\beta)}\left[\rho[1-(1-\frac{1}{\lambda})]+\rho^2[1-(1-\frac{1}{\lambda})^2]+...\right]. \tag{A.16}$$

Again, we form an infinite sum with the right-hand term:

$$\frac{1+\alpha}{1-\rho}-\frac{(1+\alpha)p_j\beta}{1-\rho(1-\beta)}\left[\rho\sum_{t=0}^{\infty}\rho^t-\rho(1-\frac{1}{\lambda})\sum_{t=0}^{\infty}[\rho(1-\frac{1}{\lambda})]^t\right], \tag{A.17}$$

which simplifies to:

$$\frac{1+\alpha}{1-\rho} - \frac{(1+\alpha)\rho p_j\beta}{1-\rho(1-\beta)}\left[\frac{1}{1-\rho} - \frac{(1-\frac{1}{\lambda})}{1-\rho(1-\frac{1}{\lambda})}\right]. \tag{A.18}$$

$\square$

Proposition 3.5. When the individual i chooses to engage in risky sex at time t, he or she will choose risky sex for all future periods of time, independently of the amount of risk taken before t. Similarly, if i chooses protection at time t, he or she will choose safe sex for all future periods of time, whatever the amount of risk taken before t.

Proof Following the same lines as the proof of proposition 3.2, let us assume that risky sex is optimal until time $t = l - 1$; we will show that it implies that risky sex is optimal at time $t = l$. The same applies to safe sex. The exponents are slightly different from those for proposition 3.2 because we now assume that an unprotected encounter occurs at time $t = -1$.

Equation (A.7) shows that the utility of i when taking a decision at time $t = l - 1$ is equal to:

$$U(...s_{il-1}, ...)|_{d_{il-1}=0} = \text{past utility} + \underbrace{\rho^{l-1}[\alpha + s_{il-1} + \theta(1 - s_{il-1})]}_{\text{utility in l-1}} \tag{A.19}$$

$$+\sum_{t=l}^{\infty}\rho^t[\alpha + s_{it} + \theta(1 - s_{it})]\Big[1 - \text{P(dying due to risk before l-1)}$$

$$- \text{P(dying due to risk taken in l-1)} - \text{P(dying due to risk after l-1)}\Big].$$

Again, the choice of risk at time $t = l - 1$ depends on the sign of the difference ΔU:

$$\frac{\Delta U}{\Delta s_{il-1}}|_{d_{il-1}=0} = \underbrace{\rho^{l-1}(1-\theta)}_{\Delta\text{ utility in l-1}} - \sum_{t=l}^{\infty}\rho^t[\alpha + s_{it} + \theta(1 - s_{it})] \tag{A.20}$$

$$\Big[\text{P(dying due to infection in l-1)} - \Delta\text{P(dying due to risk taken after l-1)}\Big].$$

Similarly, the choice of risk at time $t = l$ depends on the sign of the difference ΔU:

$$\frac{\Delta U}{\Delta s_{il}}|_{d_{il}=0} = \underbrace{\rho^l(1-\theta)}_{\Delta \text{ utility in l}} - \sum_{t=l+1}^{\infty} \rho^t[\alpha + s_{it} + \theta(1-s_{it})] \tag{A.21}$$

$$\Bigg[\text{P(dying due to infection in l)} - \Delta\text{P(dying due to risk taken after l)}\Bigg].$$

Let us analyze in detail the term on the second line of these equations for each kind of past behavior. This term is positive and represents the change in the probability of dying in the future given today's behavior.

Let us begin with the proof of the first statement of proposition (3.5). If risky sex is optimal for all $t \leq l-1$, the second line of equation (A.20) is equal to:

$$\underbrace{m_l p_j[1-\beta]^l}_{\text{not infected at } t=l-1} \underbrace{\beta}_{\text{infected at l-1}} \underbrace{[1-(1-\frac{1}{\lambda})^{t-l+1}]}_{\text{dead in } t} \tag{A.22}$$

$$\underbrace{-\, m_l p_j[1-\beta]^l \beta \sum_{k=l}^{t-1}[1-(1-\beta)^{s_k}][1-(1-\frac{1}{\lambda})^{t-k}][\prod_{j=l}^{k-1}(1-\beta)^{s_j}]}_{\Delta \text{ P(dying due to future behavior)}}.$$

Assuming that individual i prefers risky sex at time $t = l-1$, we know that this term will be lower when i has to take the decision at time $t = l$. The probability of not being infected will be lower. Formally, it is clear that $m_l p_j[1-\beta]^l$ is decreasing with l. Because this term affects negatively equation (A.20), we conclude that risky sex will be chosen for all future periods of time if risky sex is chosen once.

Similarly, we prove the second statement of proposition (3.5). If safe sex is optimal for all $t \leq l-1$, the second line of equation (A.20) is equal to:

$$\underbrace{m^*_{l-1}p_j[1-\beta]}_{\text{not infected at } l-1}\underbrace{\beta}_{\text{infected in l-1}}\underbrace{[1-(1-\frac{1}{\lambda})^{t-l+1}]}_{\text{dead in } t} \tag{A.23}$$
$$\underbrace{-m^*_{l-1}p_j[1-\beta]\beta\sum_{k=l}^{t-1}[1-(1-\beta)^{s_k}][1-(1-\frac{1}{\lambda})^{t-k}][\prod_{j=l}^{k-1}(1-\beta)^{s_j}],}_{\Delta\text{P(dying due to future behavior)}}$$

with:

$$m^*_{l-1} = \frac{1}{1-p_j\beta(1-(1-\frac{1}{\lambda})^l)}. \tag{A.24}$$

Because m^*_{l-1} is increasing with time, we see that this coefficient will be higher from one period to the next. Again, because this term enters negatively into equation (A.20), we conclude that safe sex will be chosen for all future periods of time if safe sex is chosen once. □

Proposition 10. After the unplanned risky encounter in $t = -1$, the individual chooses safe sex if:

$$E[U_i(s_{it})|s_{it} = 0, \forall t > 0] \geq E[U_i(s_{it})|s_{it} = 1, \forall t > 0].$$

This inequality is equivalent to:

$$\begin{aligned}
& (\alpha+\theta)\left(\frac{1-p_j\beta}{1-\rho} + \frac{p_j\beta(1-\frac{1}{\lambda})}{1-\rho(1-\frac{1}{\lambda})}\right) \\
\geq \; & (1+\alpha)\Bigg[\frac{1-p_j}{1-\rho} + \frac{\beta p_j(1-\frac{1}{\lambda})}{1-\rho(1-\frac{1}{\lambda})} \\
& + \frac{p_j[1-\beta]}{1-\rho[1-\beta]} + \frac{\rho p_j\beta[1-\beta](1-\frac{1}{\lambda})}{[1-\rho[1-\beta]][1-\rho(1-\frac{1}{\lambda})]}\Bigg].
\end{aligned} \tag{A.25}$$

Proof The proof is similar to that of proposition (3.3), but with a higher probability of dying because of the unprotected encounter at time $t = -1$.

The expected total utility of safe sex is given by:

$$
\begin{aligned}
E[U_i(s_{it})|s_{it} = 0, \forall t > 0] &= \sum_{t=0}^{\infty} \rho^t m_1(\alpha + \theta)(1 - p_j\beta(1 - (1 - \frac{1}{\lambda})^{t+1}) \\
&= m_1(\alpha+\theta)\left(\frac{1 - p_j\beta}{1-\rho} + \frac{p_j\beta(1-\frac{1}{\lambda})}{1-\rho(1-\frac{1}{\lambda})}\right) \\
&= \frac{(\alpha+\theta)}{1-\frac{p_j\beta}{\lambda}}\left(\frac{1 - p_j\beta}{1-\rho} + \frac{p_j\beta(1-\frac{1}{\lambda})}{1-\rho(1-\frac{1}{\lambda})}\right) \\
&= \frac{\alpha+\theta}{1-\rho} - \frac{(\alpha+\theta)\rho p_j\beta(1-\frac{1}{\lambda})}{(\lambda - p_j\beta)(1-\rho)(1-\rho+\frac{\rho}{\lambda})}. \qquad \text{(A.26)}
\end{aligned}
$$

The expected total utility of risky sex is given by:

$$
\begin{aligned}
E[U_i(s_{it})|s_{it} &= 1, \forall t] \qquad \text{(A.27)} \\
&= \sum_{t=0}^{\infty} \rho^t m_1(1+\alpha)\Bigg[\underbrace{1-p_j}_{\text{j not infected}} + \underbrace{\beta p_j(1-\frac{1}{\lambda})^{t+1}}_{\text{i infected at -1 but alive}} \\
&\quad + \underbrace{p_j[1-\beta]^{t+1}}_{\text{i never infected}} + \underbrace{p_j\beta[1-\beta]\sum_{k=0}^{t-1}(1-\frac{1}{\lambda})^{t-k}[1-\beta]^k}_{\text{i infected at k but alive at t}}\Bigg] \\
&= m_1(1+\alpha)\Bigg[\frac{(1-p_j)}{1-\rho} + \frac{\beta p_j(1-\frac{1}{\lambda})}{1-\rho(1-\frac{1}{\lambda})} \\
&\quad + \frac{p_j[1-\beta]}{1-\rho[1-\beta]} + p_j\beta[1-\beta]\sum_{t=0}^{\infty}\rho^t\sum_{k=0}^{t-1}(1-\frac{1}{\lambda})^{t-k}[1-\beta]^k\Bigg].
\end{aligned}
$$

We develop the double infinite sum for each period of time:

$$
\begin{aligned}
t = 0 &\;\Rightarrow\; \rho^0 0, \qquad \text{(A.28)} \\
t = 1 &\;\Rightarrow\; \rho^1\left[(1-\frac{1}{\lambda})^1(1-\beta)^0\right], \\
t = 2 &\;\Rightarrow\; \rho^2\left[(1-\frac{1}{\lambda})^2(1-\beta)^0 + (1-\frac{1}{\lambda})^1(1-\beta)^1\right], \\
t = 3 &\;\Rightarrow\; \rho^3\left[(1-\frac{1}{\lambda})^3(1-\beta)^0 + (1-\frac{1}{\lambda})^2(1-\beta)^1 + (1-\frac{1}{\lambda})^1(1-\beta)^2\right].
\end{aligned}
$$

We then compute the infinite sums by grouping diagonal terms. By using the properties of infinite sums, we obtain:

$$\sum_{t=0}^{\infty} \rho^{t+1}(1-\beta)^t(1-\frac{1}{\lambda})^1 + \sum_{t=0}^{\infty} \rho^{t+2}(1-\beta)^t(1-\frac{1}{\lambda})^2 ...$$
$$= \sum_{t=0}^{\infty} \frac{\rho^{t+1}(1-\frac{1}{\lambda})^{t+1}}{1-\rho[1-\beta]} = \frac{\rho(1-\frac{1}{\lambda})}{[1-\rho[1-\beta]][1-\rho(1-\frac{1}{\lambda})]}. \tag{A.29}$$

Introducing this into equation (A.27) and simplifying, we finally have:

$$E[U_i(s_{it})|s_{it}=1,\forall t] = m_1(1+\alpha)\Bigg[\frac{1-p_j}{1-\rho} + \frac{\beta p_j(1-\frac{1}{\lambda})}{1-\rho(1-\frac{1}{\lambda})}$$
$$+ \frac{p_j[1-\beta]}{1-\rho[1-\beta]} + \frac{\rho p_j \beta[1-\beta](1-\frac{1}{\lambda})}{[1-\rho[1-\beta]][1-\rho(1-\frac{1}{\lambda})]}\Bigg]. \tag{A.30}$$

□

Appendix B. Figures

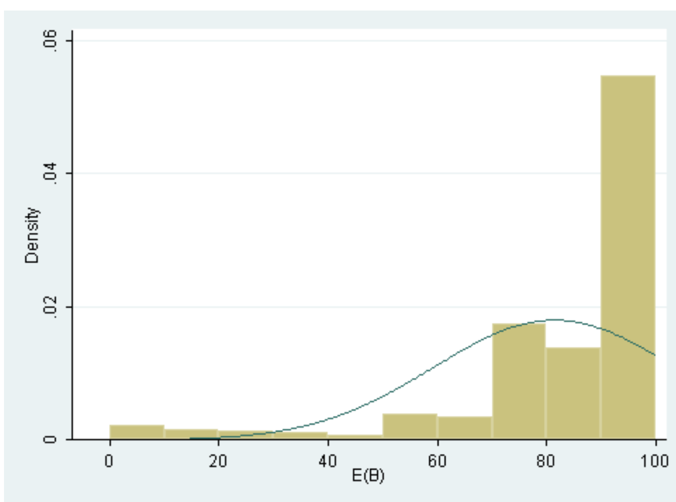

Figure B.1: Expected transmission rate without condom

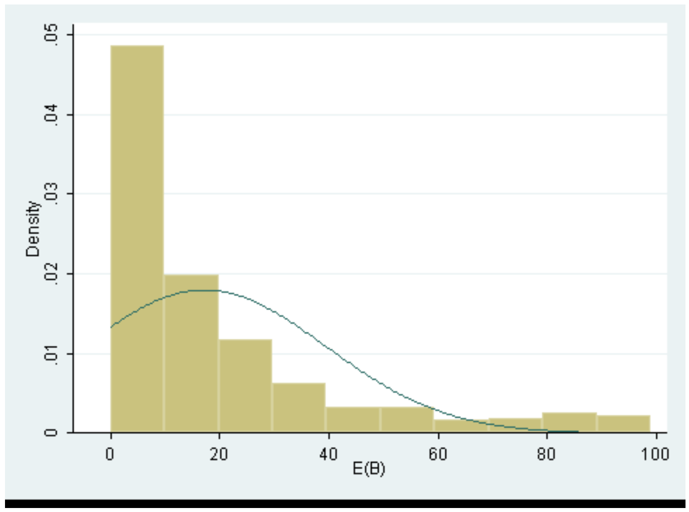

Figure B.2: Expected transmission rate with condom

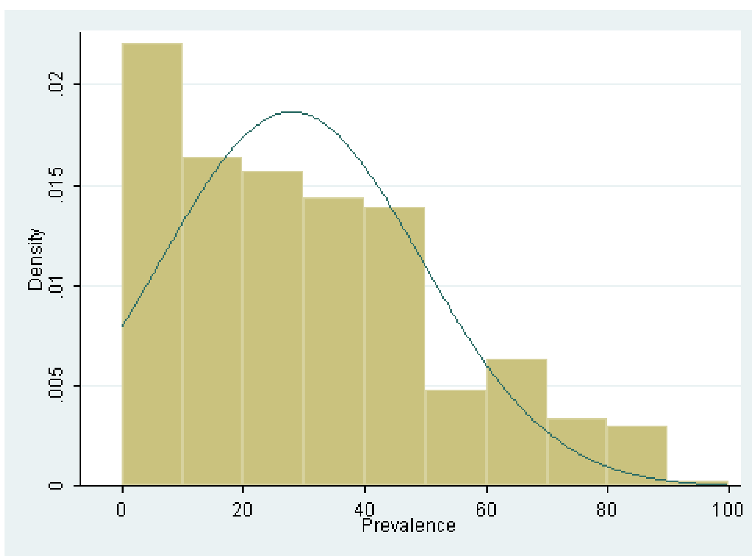

Figure B.3: Expected prevalence in Burundi

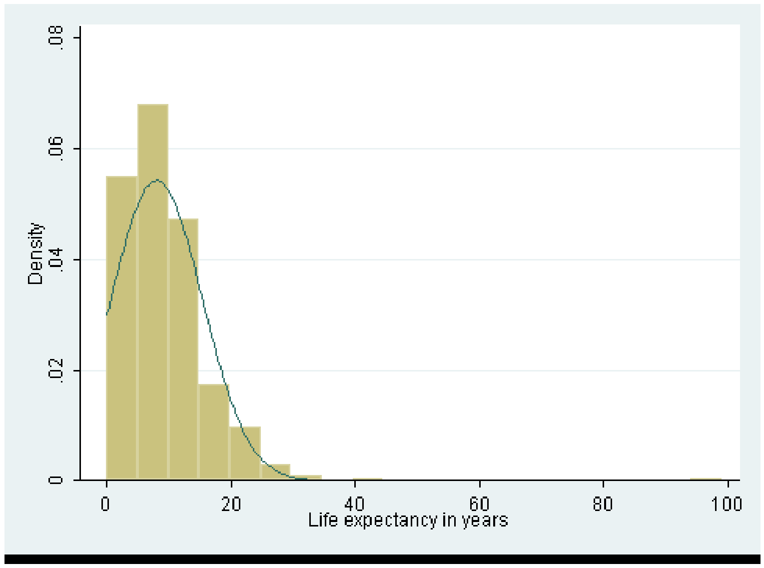

Figure B.4: Life expectancy of HIV-positive individuals

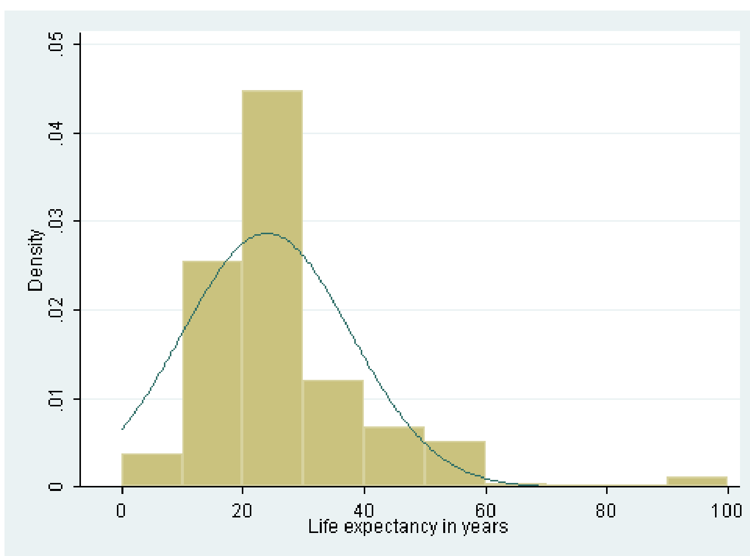

Figure B.5: Life expectancy of HIV-positive individuals with ARV treatment

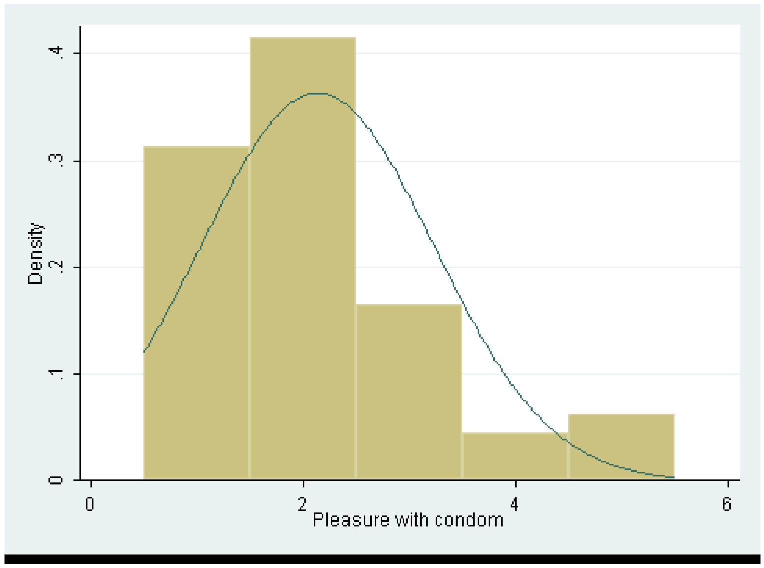

Figure B.6: Pleasure with condom

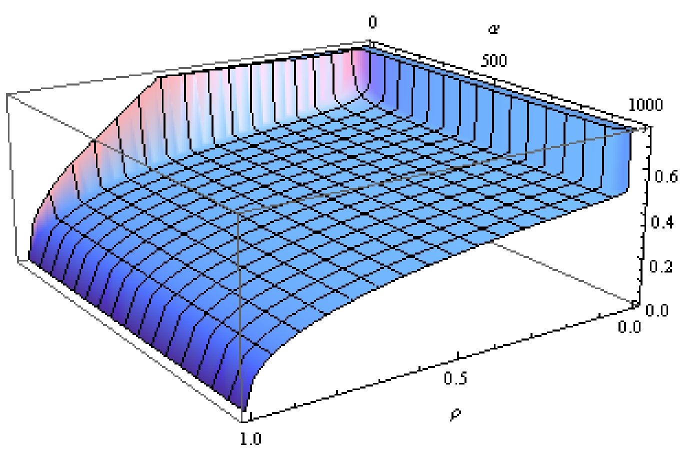

Figure B.7: The optimal disclosed transmission rate β^* as a function of ρ and α

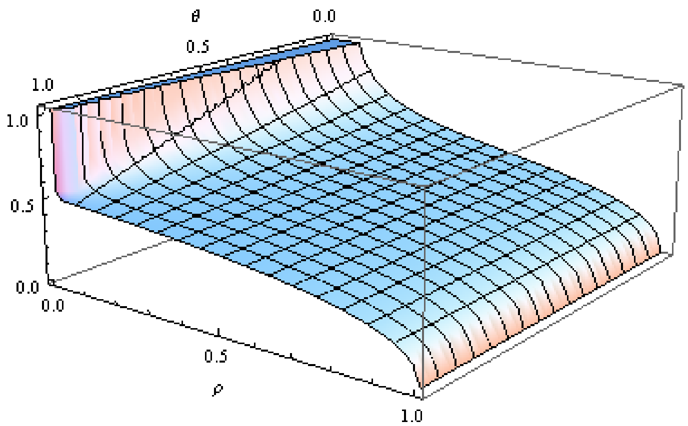

Figure B.8: The optimal disclosed transmission rate β^* as a function of ρ and θ

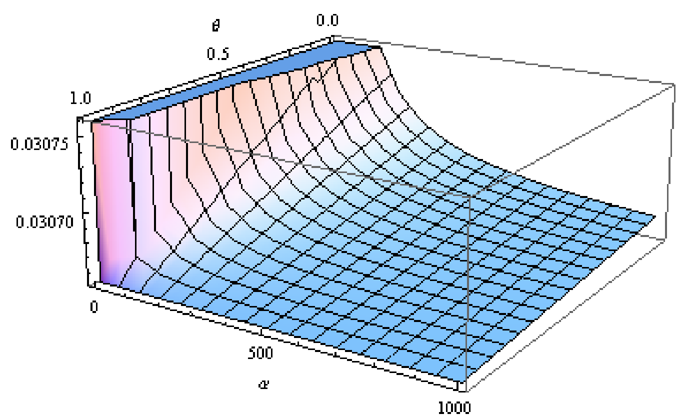

Figure B.9: The optimal disclosed transmission rate β^* as a function of θ and α

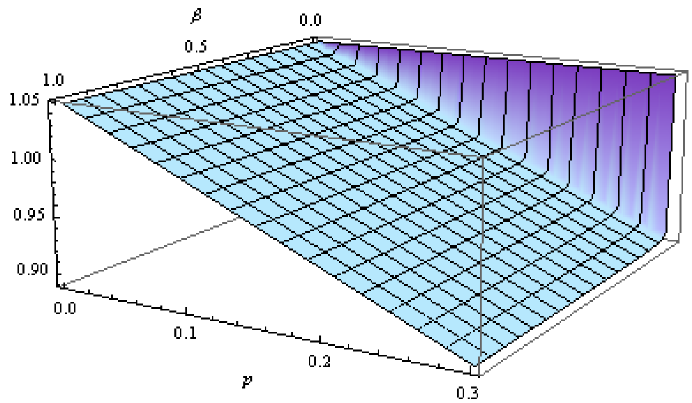

Figure B.10: Influence of $E(\beta)$ and $E(p_j)$ on condom use.

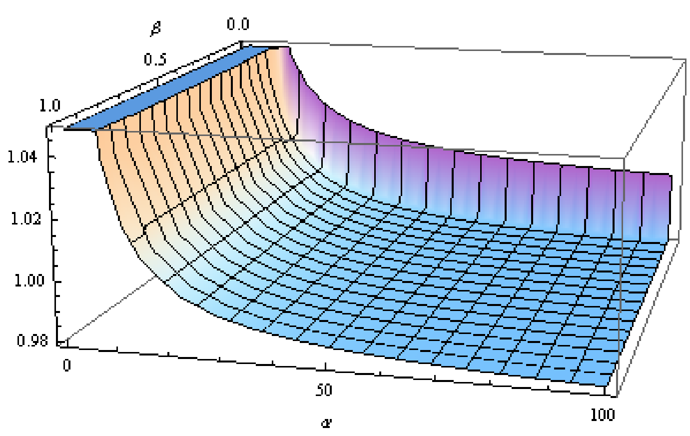

Figure B.11: Influence of $E(\beta)$ and α on condom use.

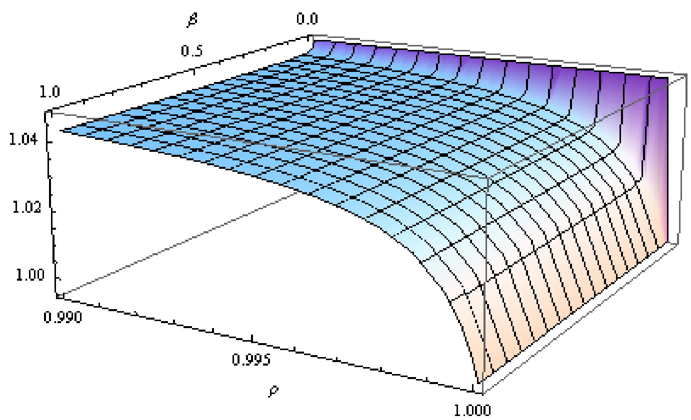

Figure B.12: Influence of $E(\beta)$ and ρ on condom use.

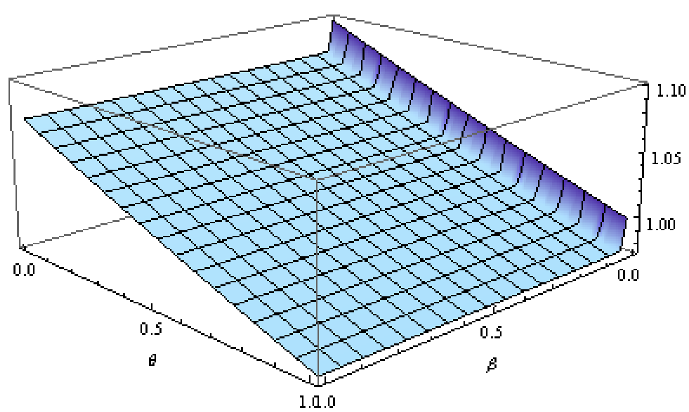

Figure B.13: Influence of $E(\beta)$ and θ on condom use.

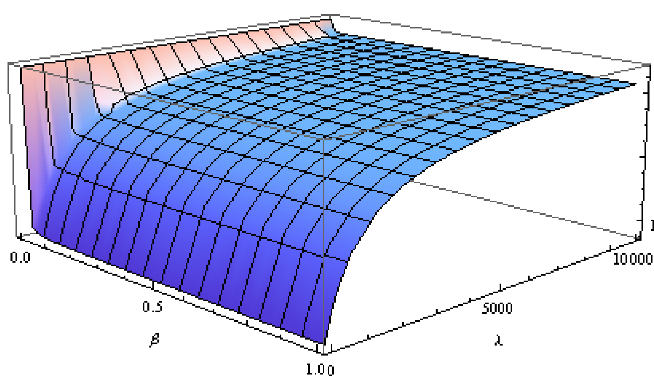

Figure B.14: Influence of $E(\beta)$ and $E(\lambda)$ on condom use.

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