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Boris Chafwehé

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Composition du Jury:

Rigas Oikonomou	(*UCLouvain*, Promoteur)
Michal Kobielarz	(*KULeuven*)
Ricardo Nunes	(*University of Surrey*)
Gonzague Vannoorenberghe	(*UCLouvain*)
Raf Wouters	(*National Bank of Belgium*)

Président du Jury:

Gonzague Vannoorenberghe (*UCLouvain*)

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Introduction

The financial crisis of 2008-2009 and the ensuing recession (the so-called *Global Financial Crisis* (GFC) and *Great Recession*), as well as the more recent Covid-19 pandemic, have brought up many challenges for macroeconomics. Following the GFC, economists who were engaged in the development of models aimed at analyzing business cycle fluctuations have soon realised that these models were ill-equipped to explain the type of disturbances which were at the root of the crisis, and, looking forward, to tackle the challenges imposed by extraordinary circumstances which ultimately became a "new normal".[1] Consequently, in the last decade a lot of research has been done to extend the tools at our disposal to better model and understand the GFC and this "new normal". To introduce the research contained in this thesis, some of these recent developments in the macroeconomics discipline are worth emphasizing.[2]

First, a growing number of studies have acknowledged the importance of accounting for heterogeneity between households to understand the drivers of economic fluctuations and the appropriate policy response to disturbances such as the ones that have lead to the aforementioned crises.[3,4] Households differ in terms of skills, wealth and many other dimensions that influence their exposure and response to changes in the macroeconomic environment. While this might seem obvious to many, the use of models featuring a 'representative agent' has been the norm rather than the exception in modern macroeconomics.[5] Abstracting from heterogeneity is sometimes inevitable: economists are con-

[1] See for instance Lindé, Smets, and Wouters (2016).

[2] For brevity, I only expose here the advances in the macroeconomics field that are relevant in the context of this thesis, thereby abstracting from other important developments. For instance, the importance of financial frictions and macro-financial linkages have received a lot of attention in the recent literature (see, for instance, Brunnermeier and Sannikov, 2014), but this topic is not treated here.

[3] In this introduction, and throughout the thesis, emphasis is given to heterogeneity between households. It should, however, be stressed that other strands of the literature have also focused on heterogeneity across other types of agents (see, for instance, Khan and Thomas (2013) for the case of heterogeneous firms).

[4] Examples of work on household heterogeneity with a focus of the 2008-09 financial crisis include Mian, Rao, and Sufi (2013), Krueger, Mitman, and Perri (2016a) and references therein. Kaplan and Violante (2018) provide a broad overview of the literature studying the implications of micro heterogeneity for the transmission of macroeconomic shocks.

[5] For instance, the Smets and Wouters (2007) model, the workhorse model of modern business cycle analysis, features a representative agent. The same holds true for Chapter 3 of the present thesis.

fronted with a trade-off between tractability and realism, and in many cases privileging tractability is necessary to make progress. Nonetheless, we also need frameworks that allow us to understand how micro heterogeneity shapes macroeconomic outcomes. This is a route that has been taken in the recent decade.

Second, the post-crisis macroeconomic environment has been characterized by low levels of interest rates. The long-run downward trend in global interest rates, together with the magnitude of the shocks implied by the recent financial and Covid crises, have pushed many economies in a liquidity trap, where the effective lower bound on nominal interest rates becomes binding.[6] This environment implies a considerable challenge for central banks, which have been forced to innovate and engage in unconventional policies. Understanding the implications of a binding lower bound constraint, and the effect of unconventional policies (such as 'forward guidance' or 'quantitative easing') for the macroeconomy, is the focus of many papers in the recent literature.[7]

Rising government debt levels over the last decade have led many economists to question the ability of fiscal authorities to finance their liabilities in scenarios where borrowing conditions for governments could worsen, for instance if interest rates were to rise. This, together with the quantitative easing programs run by central banks, has led many to speculate that central bank independence is compromised, and to study the boundaries between monetary and fiscal policies.

The first two chapters of this thesis develop models featuring heterogeneity between households, in line with the first set of developments in the macroeconomics literature mentioned above. More precisely, in Chapter 1 I develop a framework in which households are heterogeneous because they get hit by labor income shocks that cannot be fully warded off as financial markets are incomplete.[8] I use the model to study the dynamics of purchases and sales of durable goods, focusing on the equilibrium in the secondary market, where households can trade their used goods. The resale price of durables is determined in equilibrium by households' optimal decisions, along with the distribution of wealth. The model allows me to shed light on the forces behind the strong negative correlation between the resale price and aggregate unemployment risk, an empirical fact that is documented in the chapter. As is shown, the negative correlation can be explained through the increase in precautionary savings of households following a rise in unemploy-

[6]The downward trend in the so-called *natural* rate of interest has been documented by many, and is usually associated with the *secular stagnation hypothesis*. This phenomenon is usually attributed to factors such as demography, the rise in inequality, economic uncertainty, and the global savings glut that these elements imply. See, for instance, Summers (2014) and Eggertsson, Mehrotra, and Robbins (2019).

[7]See, for instance, Eggertsson et al. (2003), Chen, Cúrdia, and Ferrero (2012), and Del Negro, Giannoni, and Patterson (2012).

[8]This is the so-called Bewley-Aiyagari class of models (following Bewley, 1983 and Aiyagari, 1994), in which a continuum of households face idiosyncratic risk, and accumulate financial assets to partially insure against it.

ment risk, and the aggregation of heterogeneous households' decisions in the market for durables. The chapter therefore provides a concrete example of heterogeneity mattering for macroeconomic outcomes.

In Chapter 2 (co-authored with F. Courtoy), we analyze optimal fiscal policies in a model where households are heterogeneous in terms of labor productivity and holdings of financial assets. The model allows us to shed light on the properties of the optimal tax schedule when heterogeneity is accounted for. We investigate how the volatility of optimal labor taxes is impacted by the presence of heterogeneity and the availability of transfers as a fiscal instrument for the government. We study to which extent variations in transfers rather than distortionary taxes are used to finance deficits, when it comes at the cost of increasing inequality in consumption and hours worked between households. To assess whether heterogeneity matters for the design of the optimal policy, we contrast our results to the ones obtained in the representative agent framework typically used in the literature. In the representative model an optimizing government would always finance spending and debt in a lump sum fashion, whereas under heterogeneous agents this is not so. The government uses distortionary taxation when heterogeneity between households is sufficiently high and the transfer system has a redistributive component, thus allowing to partially reduce inequality between agents. Over the business cycle, the behavior of taxes and transfers is governed by two features which are central to our model: inequality and financial market incompleteness. We show that incomplete markets are much more important as a force driving fluctuations in the optimal tax system. We finally show that the presence of both heterogeneity and optimal transfers brings the cyclical behaviour of macroeconomic and fiscal variables in the optimal policy model closer to the actual policies observed in the United States.

The third chapter of this thesis (co-authored with R. Oikonomou, R. Priftis and L. Vogel) falls within the second body of research done after the financial crisis. In particular, it focuses on the interactions between monetary and fiscal policies. In this chapter we study optimal monetary policy when government debt sustainability is a constraint for the central bank. Monetary policy becomes subservient to fiscal policy, and partially gives up on the objective of stabilizing inflation and the output gap in order to make debt sustainable. Moreover, we show that in liquidity trap episodes, monetary policy is much less effective in stabilizing the macroeconomy, and unconventional policies in the form of 'forward guidance' (committing to keeping interest rates at zero for a long while) are not effective either.

Chapter 1

Durable Goods Markets in Heterogeneous Agents Economies

Abstract

I provide a theoretical framework of optimal purchases of new and used consumer durables in an economy with heterogeneous agents, idiosyncratic income risk and incomplete financial markets. Agents choose optimally between consuming nondurable and durable goods and accumulating a risk-free asset. The price of durable goods in the secondary market is determined endogenously, through market clearing. The model is used to study the impact of idiosyncratic unemployment risk and incomplete financial markets on market outcomes, and especially on the resale price of durables. I find that an unexpected shock to unemployment probabilities has the effect of lowering this price on impact, which is consistent with the available empirical evidence for the US economy. The mechanism behind this result is that, following the increase in risk, durable goods become less attractive to households, which rebalance their portfolio towards liquid asset holdings. This decreases the demand for durable goods and exerts a downward pressure on their price.

1.1 Introduction

Purchases of consumer durables constitute an important component of total consumption expenditures in developed economies.[1] The volatility and lumpiness of durable spending suggest that the dynamics of durable markets should be studied in isolation from those in the market for non-durable goods and services.[2] At the micro level, consumer durables represent a significant share of total asset holdings in the balance sheets of private households, especially for households at the bottom end of the income and wealth distributions.[3] Following a bad income shock (implied, for instance, by a job loss), consumption smoothing arguments might lead one to argue that selling durable goods to finance non-durable consumption can be optimal for a household that wishes to maintain a stable consumption stream; the existence of well-established secondary markets for durable goods allows households to do so.[4] However, agents are likely to incur a loss if the resale price for their good is low when they wish to sell it. If resale prices in durable markets are pro-cyclical (and therefore co-move negatively with measures of unemployment risk), this implies that durable goods constitute a poor hedge against labor income risks.

Given the significance of the secondary market for durable goods such as cars, understanding how the dynamics of this market are shaped by and influence households' decisions and their interaction with the macroeconomic environment, is of high relevance. However, little work has been done on this topic in the macroeconomic literature. This paper intents to fill this gap, by providing a framework in which the co-movement between unemployment risk and the properties of secondary markets for durables can be studied. The introduction of a secondary durable market in an otherwise standard Bewley-Aiyagari model, allows me to study how the reaction of individual households to an increase in unemployment risk affects optimal purchase and selling decisions in this market and, in turn, the equilibrium resale price. I show that the model generates a pro-cyclical resale price: the equilibrium price drops when unemployment (risk) increases, a feature which, as I show in the next section, also holds empirically.

A pro-cyclical price of used durable goods implies that these goods constitute a bad hedge

[1] According to NIPA Table 2.3.5, provided by the Bureau of Economic Analysis, in the US household expenditures in durable goods have accounted for more than 13% of personal consumption expenditures over the period 1960-2019.

[2] See Mankiw (1982) for an early contribution on this topic.

[3] For instance, about 65% of US car owners (households who own at least one car constitute themselves 85% of the total) report that the total value of their car holdings exceeds the amount of liquid assets they hold. For more details, see the empirical evidence in Section 1.2 of this paper).

[4] For goods suchs as cars, the size of the secondary market is significant. For instance, estimates show that in 2017 more than 39 million of vehicles were sold in the US, and the total value of these sales were approximately equal to 3.9% of GDP. The numbers on the used car markets come from the *Edmunds 2017 Used Car Report*, available on `http://edmunds.com`.

against labor income risks, which has important implications. As I show using both the model and household-level data, households that are the most affected by this feature are the ones which have low wealth and hold low levels of liquid assets. For these households, the welfare losses implied by a counter-cyclical price of used durables can be significant. This provides a rationale for policies that aim at providing income support for agents at the bottom of the income and wealth distribution in recessions, and to policies that help stabilizing purchases of durables in primary and secondary markets over the business cycle.

The model developed in this paper features heterogeneous agents which face labor income shocks that cannot be fully warded off because financial markets are incomplete. Agents can accumulate wealth investing in a risk-free asset, and they can run down their wealth endowments when labor income drops in unemployment. The main innovation lies in the modelling of the market for durable goods. Agents in the model can trade in durables through accessing the primary and secondary markets. They can sell used durable goods in the secondary market and buy new goods in the primary market. They can also sell used durables to acquire cash (liquid assets) when they become unemployed. However, these sales are costly and agents have to pay transaction costs. Moreover, since goods are sold at market-clearing prices, which in the presence of aggregate risk are time-varying, households may incur capital losses when they sell their goods. These modelling assumptions are in line with recent work on heterogeneous agents models with durable goods (see the literature review below). The key innovation here is that the resale price is endogenous, determined in equilibrium by households' optimal decisions, along with the distribution of wealth. This allows me to use my model in order to identify the forces behind the strong negative correlation between resale prices and aggregate unemployment risk, an empirical feature that I document in Section 1.2 of this paper.

Using the model, I investigate the impact of higher unemployment risk through studying the transition from a low unemployment steady-state, to a high unemployment steady-state. In the high unemployment scenario, the job finding probability drops sharply and the likelihood of a job loss is higher. The mechanisms through which unemployment risk affects the price of durables in the model are the following: First, an increase in risk makes durable holdings less attractive to households. A stronger 'precautionary savings' motive induces agents to accumulate more liquid assets, which enables households to more effectively smooth consumption if they become unemployed. As durable goods are less liquid (due to transaction costs), households rebalance their portfolio away from them and postpone their purchases. This decreases the demand for durable goods and exerts a downward pressure on the resale price. In addition, in the high unemployment case, more durable good owners that become unemployed desire to sell their goods to free up liquid resources. This increases the supply of used goods, adding another negative impact on the resale price.

Second, higher risk makes used goods more attractive than new ones, and wealthier households find it optimal to shift their portfolios towards used goods. This property also reflects stronger precautionary savings motives; the fact that households sell their used goods in the secondary market to buy new goods, effectively makes 'new' goods sales subject to the transaction costs and lower resale prices of used good sales. Because of this property, agents prefer to hold on to their used durables rather than to sell them and buy new goods. This decreases the supply in the secondary market and tends to increase the resale price.

The balance of the above forces determines the ultimate effect of high unemployment on the resale price. Using a carefully calibrated model I find that the unexpected shock to unemployment lowers the equilibrium resale price: the first channel dominates over the second channel in the micro-founded model.

Related literature A sizeable literature has studied the behaviour of household spending in durable markets. The well-known failures of the lifecycle-permanent income hypothesis to match the behaviour of durable spending (see for example Mankiw, 1982; Bernanke, 1984 and Caballero, 1990) has led to the development of (s,S)-type of models of durable stock adjustments (Grossman and Laroque, 1990; Caballero, 1993; Eberly, 1994). In these models, the presence of non-convex adjustment costs gives rise to 'inactivity regions' in which agents find it optimal not to adjust their stock of durables. The framework proposed in this paper can be seen as partly endogenizing these non-convexities; even without exogenous transaction costs, agents in my model would still avoid continuously adjusting their durable stock because selling an old good and replacing it with a new one entails a capital loss as the resale price is endogenously lower.[5] Moreover, since the resale price is endogenous and responds to unemployment risks, the model predicts that shocks to the economy lead to variations in the size of non-convexities. The types of shocks considered in this paper lead to countercyclical adjustment costs. This feature of my model can be seen as complementary to other sources of endonegous potentially time-varying adjustment costs, for example adverse selection (Hendel and Lizzeri,

[5] To see how nonconvexities appear from the price gap between new and used goods, consider the following. Assume for simplicity that durable goods never depreciate and that the only way for an agent to adjust her durable stock is to sell her current good on the secondary market at the price p_t^u, and buy new goods at the price p_t^n. Denote z_t the total market resources spent in period t on durable investment. The part of an agent's budget constraint related to durable spending can therefore be written as:

$$z_t = \begin{cases} p_t^n d_t - p_t^u d_{t-1} & \text{if the agent adjusts its stock of durables} \\ 0 & \text{otherwise} \end{cases}$$

This equation can be rewritten as $z_t = p_t^n \Delta d_t + (p_t^n - p_t^u) d_{t-1} \mathbf{1}_{(\Delta d_t \neq 0)}$. From this we see clearly that there is a discontinuity at $\Delta d_t = 0$, provided that $p_t^n - p_t^u \neq 0$.

1999; House and Leahy, 2004) and search frictions (Caplin and Leahy, 2011).

Several papers have used frameworks similar to the one employed in this paper, to study durable adjustments with heterogeneous agents and precautionary savings. An early reference is the work of Carroll and Dunn (1997), who show that an increase in unemployment risk extends the region where it is optimal for households not to adjust their durable stock. More recently, Berger and Vavra (2015) estimate a model where individual durable adjustments are subject to non-convexities and show that the response of durable spending to exogenous shocks is strongly procyclical. Harmenberg and Öberg (2020) use a similar framework to study the impact of time-varying labor income risk and show that their model can generate a volatility and procyclicality of durable spending similar to what is observed in the data. Gruber and Martin (2003) introduce durable goods to the Aiyagari (1994) model and look at the implications of the framework on the long-run distribution of wealth. Luengo-Prado (2006) uses a buffer-stock model with convex adjustment costs and down-payment requirements when borrowing to accumulate durables, in order to study how changes in credit market conditions influence the behaviour of non-durable consumption. Guerrieri and Lorenzoni (2017) look at the implications of a credit crunch in a general equilibrium model with nominal rigidities and look at the transition between two long-run steady states. All the above mentioned papers assume that the price of durables is exogenous and constant over time. In the framework proposed here, this assumption is relaxed, and the price of goods in the secondary market is determined endogenously through a market-clearing condition. This provides an additional channel of adjustment for durable spending, which is seen as complementary to the ones that have been emphasized in the previous literature.

The treatment of secondary markets in my model is similar to Stolyarov (2002) and Gavazza, Lizzeri, and Roketskiy (2014). These papers use models with different vintages of durable goods and impose market clearing in the market for each vintage. They show that the presence of transaction costs (and income inequality in Gavazza et al., 2014) in durable markets allows to explain the resale patterns of durable goods over their lifetime. However, while these papers have a thorough treatment of the durable market, they assign no role to precautionary savings and the interaction between liquid assets and durable goods. The model presented here extends these papers through introducing curved utility, jointly modeling durable and nondurable consumption, and allowing for portfolio decisions through the inclusion of a riskless asset to the model.

The recent work of Gavazza and Lanteri (2021, GL), written at the same time as the first version of this paper, is closely related and therefore deserves special mention in this literature review. These authors develop a framework similar to the one presented in this paper. Moreover, they also endogenize the price of consumer durables in secondary markets. However, while GL study the economy's response to credit shocks during the

Great Recession, I focus on the interaction between unemployment risk and the market for durables. Our two papers can therefore be seen as complementary.

The remainder of the paper is structured as follows. Section 1.2 documents empirical facts relating consumer durables and unemployment, obtained from aggregate and household-level data. Section 1.3 provides a description of the model economy and defines its competitive equilibrium. Section 2.5 presents the results obtained from numerical simulations of the model, and analyzes the mechanisms at work through the lens of households' policy functions. A final section concludes.

1.2 Consumer durables in US data

In this section, I present several facts on households' consumer durables stocks and purchases in the US. I use both aggregate and household-level data. I focus on data on motor vehicles such as cars and trucks to document the properties of durables. First, because these goods are an important component of households' balance sheets. Second, vehicles constitute a good proxy for the types of durable goods that are considered in the theoretical model presented in this paper: the market value of old cars and trucks depreciates over time, and there exists a well functioning secondary market for these types of goods.

1.2.1 Durable goods in households' balance sheets

In this section I use data from from the 2013 Survey of Consumer Finances (SCF), a survey of US households with detailed data on wealth and its components, to provide summary statistics on the importance of vehicles in households' balance sheets. Details on the data source and the definition of variables used in this section are provided in Appendix 1.6.2.

[Table 1.1 approximately here]

Table 1.1 provides summary statistics on vehicle holdings by US households. As can be seen from the table, about 88% of households own at least one vehicle. Comparing the total value of liquid assets to the value of owned vehicles for the sub-sample of vehicle owners, I find that more than 65% of households report that the value of their cars is greater than their liquid asset holdings, and across the whole sample, the average value of cars to total assets is equal to 22.81%. These statistics show that holdings of vehicles constitute a big component in the balance sheets of private households in the US. This might therefore leads us to believe that, following income losses (such as a job loss), sales of vehicles could constitute a margin of adjustment for households that need to free up resources in order to finance non-durable consumption.

[Figures 1.1 and 1.2 approximately here]

Figures 1.1 and 1.2 display related statistics across wealth and labor income deciles. The key takeaway from these figures are the following. First, car ownership evolves positively with wealth and income, as can be seen from panel (a) of each figure. Similarly, it can be observed from panels (c) that, as wealth and income grow, households tend to own newer vehicles. As shown later in this paper, these properties also arise endogenously from the theoretical model presented in the next section.

Finally, panels (b) and (d) show that, unsurprisingly, cars become less important in households' balance sheets at high levels of wealth and labor income: both the fraction of households with car holdings greater than liquid assets, and the average value of cars to total assets is a decreasing function of wealth/income. These stylized facts suggest that, to the extent that the 'durable adjustment margin' described in the previous paragraph exists, low-wealth/low-income households are the most likely to use it, as durable holdings occupy a bigger place in their balance sheet. This prediction also holds in the theoretical model presented below.

1.2.2 Cyclical properties of durable holdings and prices

I now present three new facts that constitute an important benchmark against which the predictions of the model developed in this paper will be tested. The first fact concerns the properties of the resale price of used cars and trucks along the business cycle:

Fact 1: *The resale price of vehicles is pro-cyclical.*

To document this I study the cyclical properties of aggregate time series on the resale price of vehicles. Figure 1.3 displays the cyclical components of the CPI for used cars and trucks, and aggregate unemployment. Both series are at the quarterly frequency.[6]

[Figure 1.3 approximately here]

As can be seen from the Figure, the two series are negatively correlated (the correlation coefficient is -0.56): when unemployment is high, the price of vehicles in the secondary

[6] In Appendix 1.6.2 I use regional data from principal census regions in the US, and show that the properties presented in this section also hold at this disaggregate level.

market tends to drop.[7]

To explain this pattern the theory presented in this paper will rely on the following mechanism. Households facing high unemployment risk desire to postpone durable purchases, and unemployed households are induced to sell their cars to acquire cash. At the macro level, this behavioural response implies a decrease in the demand for used goods which leads to a drop in the equilibrium price. The following two facts provide evidence in favor of this mechanism.

Fact 2: *Households are less likely to buy vehicles in times of high unemployment.*

Fact 3: *Low-income households are more likely than high-income households to sell a vehicle in times of high unemployment.*

Facts 2 and 3 derive from the Consumer Expenditure Survey (CEX) over the period 1994Q1–2018Q4. I study the vehicle purchase and selling decisions of households in the US and aggregate series across all households in my sample (details on the construction of the series that follow are provided in Appendix 1.6.2).

First, I find that the share of households that buy a vehicle (either new or used) is negatively correlated with unemployment. Figure 1.4 displays the cyclical components of the fraction of buyers (dashed red line) together with the detrended unemployment series (the solid black line). As can be seen from the Figure the two series are negatively correlated (the coefficient of correlation is -0.68).

[Figures 1.4 and 1.5 approximately here]

The observed negative correlation leads me to conclude that households postpone their purchases of vehicles in the fact of high unemployment risk. To shed more light on this, I display in Figure 1.5 the year-to-year change in the average age of vehicles owned by car owners in the CEX (dashed blue line), together with the unemployment series (solid black line).[8] The two series are positively correlated (the correlation coefficient is 0.47). Thus, when unemployment is above trend, households delay their durable purchases as displayed in Figure 1.4, and therefore the age of owned vehicles tends to rise.

[7] In the theoretical model presented below, price movements in the secondary market reflect relative price movements between used and new durables, rather than the absolute price of used goods as presented here. In Appendix 1.6.4, I provide the counterpart of Figure 1.3 using the ratio of the price index for used vs. new vehicles, and show that this relative price measure is also negatively correlated with unemployment (the correlation coefficient between the cyclical components of unemployment and the relative price of used goods is -0.34).

[8] I chose to compute the age of the newest vehicle that households own rather than the average age of the stock of vehicles.

The dashed blue line of Figure 1.4 displays the cyclical component of the fraction of households selling a vehicle.[9] This series is uncorrelated with unemployment (the correlation coefficient is equal to -0.03). There are two forces determining the effect of aggregate macroeconomic conditions on the selling rate displayed in the figure. First, selling takes place when households desire to buy a new durable good. Given the strong negative correlation between the buying rate of vehicles and unemployment, shown previously, this force would imply a negative impact of unemployment on the share of households selling vehicles. However, second, selling decisions may originate from negative income shocks, when households become unemployed and sell their durable goods to obtain cash. Such behaviour would produce a positive correlation between the unemployment rate and the selling rate. Taking stock of this, the observed zero correlation between the two series in Figure 1.4 seems two suggest that the two forces balance each other out.

To shed further light on the two margins of adjustment just mentioned, I next exploit variation in individual income levels, and the presumption that the second force (selling for cash) is more pertinent for low-income households (as is known these types of households suffer from more cyclical unemployment and are more likely to be cash constrained) . Figure 1.6 shows the cyclical components of the share of households selling a vehicle for two population groups: households in the top 90% of the income distribution (dashed blue line), and in the bottom 10% (dashed red line). The figure for the top 90% has the same features as the aggregate series presented in Figure 1.4 and has a very small correlation with the unemployment series. The counterpart for the low-income households, however, has a positive correlation with unemployment. This suggests that the selling for cash margin following a rise in unemployment risk is indeed more present at the bottom of the income distribution.

[Figure 1.6 approximately here]

To take a deeper look at how the buying and selling decisions are influenced by households' characteristics I now run logit regressions using data from the CEX. I run separate models for buying and selling decisions. I use as explanatory variables, demographic controls, quarter and year fixed effects, income decile dummies, the aggregate unemployment rate as well as their interaction. The detailed specification of the models, as well as the full estimation results, are presented in the Appendix.

To study how changes in the unemployment rate affect households' adjustment decisions, in Figure 1.7 I show the percentage deviations from mean of the predicted probabilities of adjustment over sample period, obtained from the logit model, for three different income

[9] The series accounts both for goods sold in the secondary market, and vehicles that are traded-in. Details on the construction of the series are provided in the Appendix.

deciles. To produce the Figure, I use the observed unemployment rates in sample periods, set all the individual controls to their median value, and normalize fixed effects to zero .

[Figure 1.7 approximately here]

Two main fresults emerge from the figure: first, from the top panel, we see that the probabilities of buying do not vary much across income deciles and over time. It therefore seems that on average, individual income does not affect the response in buying probabilities to the aggregate unemployment rate. Second, when looking at the bottom panel, we observe that there is significant variation across income deciles in selling probabilities. As can be seen from the Figure, for lower deciles, there is a positive correlation between unemployment and the probability of selling, while for higher deciles, the correlation is negative. It therefore seems likely that for low-income households, selling decisions reflect the 'selling for cash' mechanism, as households time their sales in times of high unemployment. For median and high income households, the pattern of adjustment seems more compatible with selling in order to buy new vehicles. This produces a positive co-movement between the buying and selling rates.

1.3 The model economy

I construct a model in which agents face uninsurable income risks and choose optimally between consuming nondurable and durable goods and accumulating a risk-free asset. The model distinguishes between new and used durables. Agents can choose to buy new goods, or used goods through accessing the secondary market. The supply of new goods is perfectly elastic at an exogenously given price, whereas in the case of used goods, the price in the secondary market is determined endogenously, through market clearing. Finally, trades in the risk-free asset are modelled in the standard fashion of the heterogeneous agents literature: households face ad hoc constraint which limit the amount of borrowing. Markets are therefore incomplete.

1.3.1 The baseline model

Preferences There is a continuum of infinitely-lived agents that derive utility from nondurable consumption, and from the services provided by the stock of durable goods they hold. Let c_t denote the nondurable consumption level of a generic agent and d_t her durable stock. Preferences are time-additive and all agents discount the future at rate β.

Individual lifetime utility can be written as:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, d_t)$$

where $u(\cdot,\cdot)$ is the period utility function.

Individual income and aggregate shocks Individuals face uncertainty in income which is formalized as follows: First, each individual can be in any period t either employed or unemployed. If the agent is employed, she earns income $\overline{y}$, whereas in unemployment she earns 0. Second, transitions across employment and unemployment are governed by a first order Markov process with transition matrix Π whose elements vary depending on whether the economy is in recession or in expansion. More specifically, let $\epsilon_t \in \{0, 1\}$ denote the employment status, taking the value 1 if the agent is employed and 0 otherwise. Let $j \in \{G, B\}$ denote the aggregate state, with G representing 'good times'- i.e. expansions and B 'bad times' - recessions. Transition probabilities are given by:

$$\Pi^j(\epsilon_{t+1}|\epsilon_t) = \begin{pmatrix} \pi^j_{ee} & 1-\pi^j_{ee} \\ 1-\pi^j_{uu} & \pi^j_{uu} \end{pmatrix}$$

where $1-\pi^j_{ee}$ is the job separation rate in state j and $1-\pi^j_{uu}$ is the job finding probability. Naturally, it is easier to find a job during an expansion than it is in a recession and also agents experience transitions from employment to unemployment at higher rate in recessions. The elements of Π^j vary accordingly across states j.

The focus of this paper will be to study how changes in unemployment risks, that is changes in the aggregate state j, influence the equilibrium in durable good markets. For tractability I treat the two aggregate states as separate model versions (I denote them the *low-risk* and *high-risk* economies). In Section 2.5 I study the transition between the low-risk and the high-risk steady-states, following an unexpected switch to the model calibration from state G to state B.

Liquid assets Agents can save in a one-period, risk-free asset, subject to an ad-hoc borrowing limit. Variable a_t denotes the quantity of the asset accumulated in period t by a generic household. This asset is modelled as a discount bond: one unit of the bond sells at the price $q = \frac{1}{1+r}$ in period t and brings forward one unit of the consumption good in $t+1$. The real return on the asset r is exogenous; since the focus here is to study the equilibrium in the market of durable goods, this assumption can be seen as a simplification.

Finally, in every period t individual asset positions have to satisfy the following borrowing constraint:

$$a_t \geqslant 0$$

Ruling out negative savings (debt) is a standard friction in the heterogeneous agents literature.

Durable goods Durable goods enter in the model as a discrete choice. An agent can purchase/hold $\overline{d}$ units of a durable good or hold 0 units of the good and be a non-owner. As discussed previously, durable goods can be either old or new. New goods are in perfectly elastic supply: The price of a new durable always equals 1, and the quantity is demand-determined through the agents's optimal decisions which I describe below. The price of used goods, denoted p^u in a stationary equilibrium, is determined endogenously in a secondary market where households buy and sell their goods.

New durables are subject to a depreciation shock; I assume that with probability $1-\pi_1$, the good becomes *used* (old). Used goods differ from new goods in terms of the utility services that they provide to their owners. In particular, $\overline{d}$ units of a used good enter in the agent's utility function as $(1-\delta)\overline{d}$. Since u is monotonically increasing in its arguments, individuals derive lower utility from a used good than they do from a new good. Moreover, used goods are subject to a risk of becoming scrapped, in which case their utility services are zero. This occurs at rate $1-\pi_2$ in the model, and I assume for simplicity that when a good is scrapped, no compensation is made to its owner.

The above can be summarized as follows. Let the set $\{n, u, o\}$ denote the 'vintage' of a durable good (where n stands for a new good, u for a used one, and o designates the non-ownership status), and let Ω denote the matrix of transition probabilities across n, u and o. We have:

$$\Omega = \begin{pmatrix} \pi_1 & 1-\pi_1 & 0 \\ 0 & \pi_2 & 1-\pi_2 \\ 0 & 0 & 1 \end{pmatrix}$$

Ω is not the only object that influences the 'ownership status' of a household (whether it holds a new, a used, or no good) in a given period. Ownership is above all influenced by agents' choices. In particular, at any point in time, agents can choose (i) whether or not to hold a durable good, and (ii) if they decide to hold a good, the vintage of the good (new or used) they wish to hold. Agents can adjust their stock of durable holdings by selling them in the secondary market, for instance to sell a used good in order to buy a new one, or to free-up market resources and become a non-owner ('selling for cash').

I assume that new goods that are sold automatically depreciate and become used goods, thereby having to be sold on the secondary market at the equilibrium price p^u. Notice that basically this assumption aims at capturing the empirical fact that resale prices of durable goods drop significantly right after their purchase.[10] This also holds in the model: an agent can purchase a new good at price 1, and if they sell after one model period, the price drops discretely to p^u, the price of a used good in the secondary market.

Finally, it is assumed that agents face proportional transaction costs χ when they sell their goods; sellers receive $(1-\chi)p^u\overline{d}$ when they sell $\overline{d}$ units of a durable in the market. This assumption is standard in the literature on durable goods accumulation.

1.3.2 Recursive formulation

I now state the agent's program recursively. Let

$$(a_{t-1}, \epsilon_t, h_t) \subset (\mathbb{R}^+, \{0,1\}, \{n,u,o\})$$

denote the vector of individual state variables, where a_{t-1} is asset holdings at the beginning of t, ϵ_t is the employment status and h_t is the durable good 'ownership status' of the household. Based on these states variables (h_t and ϵ_t are revealed at the beginning of t through the realization of depreciation and labor income shocks) individuals will make optimal consumption and savings decisions. The timing of these decisions is represented in Figure 1.8: at the beginning of the period, agents draw ϵ_t and h_t. Subsequently, they have to decide their 'ownership' status. Owners of goods (new and old) decide whether to sell or not and conditional on selling, they receive the amount $(1-\chi)p^u\overline{d}$ and choose a new ownership status. They can buy a new good at price equal to 1, a used good in the secondary market, or no good at all. Agents who at the beginning of t are non-owners simply choose whether or not to be owners (and if they become owner, the type of good they wish to buy) in t. The optimal ownership decision is summarized by the policy function $\tilde{h}(a_{t-1}, \epsilon_t, h_t)$.

Finally, agents choose asset holdings a_t and nondurable consumption c_t. The consumption and asset policy function, conditional on the durable ownership choice $\tilde{h}_t$, are denoted $a_{\tilde{h}_t}(a_{t-1}, \epsilon_t, h_t)$ and $c_{\tilde{h}_t}(a_{t-1}, \epsilon_t, h_t)$ respectively.

[Figure 1.8 approximately here]

[10] According to Kelly Blue Books, a website that provides information on car prices in the U.S, the resale value of cars drops by about 20% one year after they are bought. Clearly, the price drop can be explained through many channels, including incomplete information about the quality of the good, but also preferences for new goods over used goods, which is what I assume here.

Formally, let $V^{\tilde{h}_t}$ denote the lifetime utility of an agent who chooses status $\tilde{h}_t \in \{n, u, o\}$ in period t. Then

$$V(a_{t-1}, \epsilon_t, h_t) = \max_{\tilde{h}_t \in \{n,u,o\}} V^{\tilde{h}_t}(a_{t-1}, \epsilon_t, h_t) \tag{1.1}$$

is the envelope which gives the lifetime utility at the beginning of t after observing $(a_{t-1}, \epsilon_t, h_t)$ and before the 'ownership status' decision $\tilde{h}_t$.

After the choice of $\tilde{h}_t$ the agent's program can be described through the following functional equation:

$$\begin{aligned} V^{\tilde{h}_t}(a_{t-1}, \epsilon_t, h_t) &= \max_{c_t, a_t} u(c_t, d^{\tilde{h}_t}) + \beta \sum_{h'} \sum_{\epsilon'} \Pi(\epsilon'|\epsilon_t)\ \Omega(h'|\tilde{h}_t) V(a_t, \epsilon', h') \\ \text{s.t} \quad & a_t = (1+r)\big(x(a_{t-1}, \epsilon_t, h_t, \tilde{h}_t) - c_t - p^{\tilde{h}_t} d^{\tilde{h}_t}\big) \\ & a_t \geqslant 0 \end{aligned} \tag{1.2}$$

for $\tilde{h}_t \in \{n, u, o\}$, and where x defines the available resources of the agent given her status h_t at the beginning of t and her choice $\tilde{h}_t$ in t:

$$x(a_{t-1}, \epsilon_t, h_t, \tilde{h}_t) = \begin{cases} a_{t-1} + \epsilon_t \bar{y} + \big(1 - \mathbf{1}[\tilde{h}_t = h_t]\big)(1-\chi) p^u \bar{d} & \text{if } h_t = n, u \\ a_{t-1} + \epsilon_t \bar{y} & \text{if } h_t = o \end{cases}$$

and

$$d_t^{\tilde{h}} = \begin{cases} \bar{d} & \text{if } \tilde{h} = n \\ (1-\delta)\bar{d} & \text{if } \tilde{h} = u \\ 0 & \text{if } \tilde{h} = o \end{cases} \qquad p_t^{\tilde{h}} = \begin{cases} 1 & \text{if } \tilde{h} = n \\ p^u & \text{if } \tilde{h} = u \end{cases}$$

1.3.3 The competitive equilibrium

This section defines the competitive equilibrium. As discussed previously, in the model the return on the liquid asset a and the price of new goods are exogenous. The competitive equilibrium therefore determines the market clearing price p^u as a function of the agents' optimal decisions and the distribution of agents across the state space.

Definition The competitive equilibrium is objects $V^{\tilde{h}_t}$ and V described above, optimal policies $\tilde{h}(a_{t-1}, \epsilon_t, h_t)$ (mapping state variables to 'ownership status'), $a_{\tilde{h}_t}(a_{t-1}, \epsilon_t, h_t)$ and $c_{\tilde{h}_t}(a_{t-1}, \epsilon_t, h_t)$ (mapping states to liquid asset holdings and consumption of non-durable goods), a resale price of durables p^u and a measure F of agents across the state space, that are such that:

1. Optimal policies $\tilde{h}(a_{t-1}, \epsilon_t, h_t)$, $a_{\tilde{h}}(a_{t-1}, \epsilon_t, h_t)$ and $c_{\tilde{h}}(a_{t-1}, \epsilon_t, h_t)$ derive from the solution to the functional equations (1.1) and (1.2).

2. The secondary market for durables clears:

$$\sum_{h_t=n,o} \sum_{\epsilon_t=0,1} \int_{a_{t-1}\in\mathbb{R}^+} \mathbf{1}\big(\tilde{h}(a_{t-1}, \epsilon_t, h_t) = u\big) dF(a_{t-1}, \epsilon_t, h_t)$$
$$= \sum_{h_t=n,u} \sum_{\epsilon_t=0,1} \int_{a_{t-1}\in\mathbb{R}^+} \big(1 - \mathbf{1}\big[\tilde{h}(a_{t-1}, \epsilon_t, h_t) = h_t\big]\big) dF(a_{t-1}, \epsilon_t, h_t)$$

   The dependence of policy functions $\tilde{h}(\cdot)$ on the equilibrium resale price of durables p^u is made implicit here.

3. The measure F is consistent. In particular, it obeys the following law of motion:

$$F^{(t)}(\mathcal{A}, \epsilon', h') = \int_{a\in\mathbb{R}^+} \sum_{\epsilon\in\{0,1\}} \sum_{h\in\{n,u,o\}} \Pi(\epsilon'|\epsilon)\Omega\big(h'|\tilde{h}(a,\epsilon,h)\big)\mathbf{1}\big[a_{\tilde{h}(a,\epsilon,h)}(a,\epsilon,h) \in \mathcal{A}\big] dF^{(t-1)}(a,\epsilon,h)$$

Conditions 1 & 3 are standard in the class of incomplete markets models such as the one considered in this paper: they ensure that households are optimizing along the equilibrium path, and describe the law of motion of the measure F of households across the state space, that we need to keep track of in order to impose equilibrium.

Condition 2 ensures that the secondary market for durable goods clears. The LHS of the expression denotes the demand for the good, which pertains to households that did not hold a used good at the beginning of the period (after uncertainty is resolved) and wish to acquire one. The RHS represents the supply side of the market, which is composed of households that held a new or used good before optimizing, and decide to change their ownership status, thereby selling their good in the resale market.

In the next section, I provide quantitative results obtained from numerical simulations of the model economy. I first look at households' optimal policies for given prices of durables, then I study the long-run equilibrium of the model, and the transition dynamics induced by changes in aggregate unemployment risk.

1.4 Quantitative Analysis

In this section, I use numerical simulations of the model to tease out the effect of unemployment shocks on the secondary market for durables. I first describe the properties of the optimal policies over durable goods and liquid assets for different values of the resale price of durables, in order to shed light on some of the key mechanisms at work in the model. Then I focus on an equilibrium where the resale price is equal to its steady-state value and the economy is permanently in the expansion phase. Finally, I consider a change in the aggregate state assuming that following an unexpected (essentially zero probability) shock, the economy permanently enters in recession. Looking at the transition path after this shock occurs, I can characterize the short-run effects of an increase in unemployment risks on durable goods market outcomes.

The non-convexities present in households' optimization programs make my model difficult to solve. To approximate households' optimal policies, I use the method proposed in Fella (2014). Details about the solution method are provided in Appendix 1.6.3.

1.4.1 Calibration

Preferences The model period is one quarter and the discount factor is set to $\beta = 0.99$. The period utility function is of the following form:

$$u(c_t, d_t) = \frac{\left(c_t^{\alpha}(\underline{d} + d_t)^{1-\alpha}\right)^{1-\gamma}}{1-\gamma} \quad \text{, where } d_t = \begin{cases} \overline{d} & \text{if the good is new} \\ (1-\delta)\overline{d} & \text{if the good is used} \\ 0 & \text{otherwise} \end{cases}$$

The coefficient of risk aversion, γ, is set to 2, a standard value in the macro literature. The parameter α, which represents the relative share of nondurable goods in utility is set to 0.78, which is the average expenditure share of nondurable goods observed in U.S data over the period 1992-2016.[11] This follows Guerrieri and Lorenzoni (2017).

The term $\underline{d}$ is introduced in utility to allow for the 'non-ownership' state. If $\underline{d} = 0$ then obviously, given the curvature of utility, no agent would be a non-owner. I set $\underline{d} = 3.875$ to match the non-ownership rate for cars in the 2013 Survey of Consumer Finances, which equals 15%.

Individual income process and employment status Conditional on employment ($\epsilon_t = 1$) individual income is constant and equal to $\bar{y}$. I normalize this value to 1. For the aggregate state G I choose the elements of Π so that in the long-run, the unemployment rate is equal to 4 percent and the average duration of an unemployment spell is 1.5 quarters. In aggregate state B the long-run unemployment rate is at 10 percent and the

[11] This share was computed using data from the U.S Bureau of Economic Analysis. The share is $\frac{ND}{ND+D}$, where ND stands for personal consumption expenditures on nondurable goods and services, except spending for health care and financial services expenses. D stands for expenditures on durable goods.

duration of unemployment 2.5 quarters. These values are taken from Krusell and Smith (1998). The transition probabilities are summarized in Table 1.2.

[Table 1.2 approximately here]

Liquid assets I set $r = 0.025/4$ which gives an annualized interest rate of 2.5%. Notice that it holds that $\beta(1+r) < 1$ which is a necessary condition under incomplete financial markets for assets not to diverge (see e.g. Deaton, 1991).

Durable goods The transition probabilities between vintages, $1-\pi_1$ and $1-\pi_2$, determine both the relative share of new and used goods in the long-run equilibrium, and the average lifetime of durable goods. I set the average lifetime to 12 years (or 48 quarters), which gives me the restriction $\frac{1}{1-\pi_1} + \frac{1}{1-\pi_2} = 48$. The steady-state ratio of used to new goods, $\frac{1-\pi_1}{\pi_2}$ in the model, is assumed to be equal to 3. I thus obtain $\pi_1 = 2/3$ and $\pi_2 = 35/36$ to satisfy both targets. With these numbers the expected duration of a new good is 3 years.

The depreciation rate δ is chosen in order to match the depreciation of an average used car. Since on average a good becomes used after 3 years and scrapped after 12, the average age of used goods is 7.5 years. I set $\delta = 7.5 \times \delta^A$, where δ_A is an annual depreciation rate for durable goods. Following Stolyarov (2002) I set $\delta_A = 0.085$. Then $\delta = 0.638$.

According to Kelly Blue Books, the average value of new cars bought in the U.S was around \$30,000 in 2015. This is roughly equivalent to 3/4 of average annual disposable income. To match this ratio, the size of goods $\overline{d}$ is set to 3. Finally, the proportional transaction cost χ incurred by agents who sell a used good is 0.05. This value is standard in the literature (see e.g. Eberly, 1994).

1.4.2 Inspecting individual policy functions: the role of prices

This section describes agents' policy functions. I investigate the impact of varying the price p^u in the secondary market and the impact of a change in unemployment risks. The analysis presented here is useful to understand how the resale price is determined in equilibrium, which I analyze in a subsequent section. Here the focus is on partial equilibrium; this enables me to identify separately the forces at work.

[Figure 1.9 approximately here]

In Figure 1.9, I consider the low unemployment risk economy (aggregate state G). The figure shows the optimal choices of new, used and no goods as functions of individual resources (variable z_t in the figure) for employed agent ($\epsilon_t = 1$). z_t is defined as follows:

$$z_t \equiv a_{t-1} + \epsilon_t \bar{y} \tag{1.3}$$

and so it represents 'cash-in-hand' (the sum of liquid wealth and current labor income) in the beginning of t.

The left panel of Figure 1.9 represents non-owners' policy functions, which is basically the demand side of the secondary market for durables. Black triangles and blue circles depict the regions in which households find it optimal to buy, respectively, used and new goods, and red dots represent the region where households prefer to be non-owners. The right panel displays the policies for used goods owners, the supply side of the market: black triangles populate the region where agents want to keep their used good, and blue circles the region in which they wish to sell their good in the secondary market.[12] This figure shows how reservation prices on the secondary market (defined as the highest price at which agents want to buy a used good - in the case of non owners -, and the lowest price at which they are willing to sell - in the case of owners) vary with individual wealth.

The first noteworthy feature is that poor agents never wish to purchase durable goods, no matter what the price is. These households prefer to accumulate a buffer stock of liquid assets in order to insure their nondurable consumption against idiosyncratic income shocks. Once this buffer stock is built, then households start buying durable goods. The counterpart of this property on the selling side of the market (panel b) is that very poor agents always sell their durable good to free up market resources and finance nondurable consumption or the accumulation of liquid assets.

The second key property shown in the figure is that, beyond a certain threshold, reservation prices drop with the wealth level. Richer agents prefer new durable goods since the utility services these yield is higher. Therefore, the richer the agent, the higher the relative price she accepts to pay for a new good, and since prices of new goods are normalized to unity, this translates into a lower reservation price for used goods. The same intuition applies to owners of used goods (panel b). Even at relatively low resale prices, agents with high wealth desire to sell their used goods in order to buy new ones, and thereby adjust their durable stock upwards.

[12] Notice that in Section 2 the value functions allowed owners of new goods to sell them in the secondary market. Clearly, agents would never wish to sell a new good and buy a used good and they would never sell to buy a new good (recall that new goods' prices depreciate when they are offered in the secondary market). However, they can sell a new good (as a used good) and become non-owners. Though this policy emerges from the value functions, because it is costly, it is not included in the part of the state space considered in the figure. Moreover, in equilibrium, the measure of agents in the region where this policy is active is basically zero.

[Figure 1.10 approximately here]

To study the impact of unemployment risk on reservation prices, in Figure 1.10 I plot households' polices under low (state G, black circles) and high (state B, red crosses) risk. The left panel depicts areas where non-owners buy used goods , and the right panel shows regions where used goods owners decide to keep their goods (these policies coincide with the ones depicted by the black triangles in Figure 1.9).

The following properties emerge from the figure: first, the response to unemployment risk is different at low and high levels of wealth. Agents with low wealth now prefer the outside option of non-ownership and as a result, their reservation prices drop when the unemployment risk is higher. On the other hand, agents with high wealth optimize at the margin new/used goods and as can be seen from the figure, the reservation price increases (in the sense that non-owners are willing to pay a higher price to buy them, and owners require a higher price before deciding to sell used goods). This can be explained by the fact that new goods are a worse hedge against unemployment risks. If agents become unemployed and are forced to sell, new good owners suffer a capital loss, since their new goods automatically become used. Hence, many agents prefer to hold used goods because they can be liquidated at a lower cost.

From the above findings we can identify the key forces that will influence the response of the equilibrium price of used goods to a change in unemployment risks. On the one hand, low-wealth agents are less willing to buy durables and prefer to hold no good, since focusing on the accumulation of liquid assets is better, in terms of hedging against the higher income risk. So the demand for used goods weakens. On the other hand, used goods become relatively more attractive to wealthier agents, and this will give a potent portfolio rebalancing effect which will increase demand. The balance of these forces determines the impact of unemployment on p^u. If there is a sufficiently high number of low-wealth (hand-to-mouth) households in the economy, it can be expected that the first effect dominates, and then p^u will be procyclical, like in the data. Otherwise, p^u will be counterfactually positively correlated with unemployment.

1.4.3 Long-run steady states

[Table 1.3 approximately here]

Before studying the transition following a shock to unemployment, I consider here the properties of the long-run equilibrium, comparing steady states with high and low unemployment risks. Table 1.3 presents the results for each of the models, which can be summarized as follows. First, in the high risk economy, liquid asset holdings are more than twice as high as in the low-risk economy. Panel b of Figure 1.11 plots the distribution of agents over liquid wealth. From the figure we can see that in the high risk case (the dashed red line) there is a shift to the right in the distribution of wealth as compared to the low-risk economy (solid blue line). This property may seem surprising, because aggregate income is lower in this case, as a larger fraction of the population is unemployed. However, this shift can be explained by standard precautionary savings arguments and the fact that the model features two types of assets- liquid and illiquid: notice that when individuals face an uncertain income stream they are willing to accumulate wealth to finance consumption in unemployment; moreover, as discussed previously, agents are now less willing to buy durable goods and thus rebalance their portfolios towards liquid wealth. As shown in the table, the non-ownership rate increases in the high-risk steady-state to 19 percent, compared to 15 percent in the low-risk case. To show how this leads to higher assets holdings in equilibrium, in panel a of Figure 1.11 I show the policy functions of liquid assets for each ownership status separately. From the figure it can clearly be seen that optimal asset holdings jump when a used good is bought, and also jump when the agent decides to hold no good at all. This is the portfolio rebalancing effect: as overall household wealth (including the value of durables) decreases households substitute towards liquid assets. A stronger precautionary savings motive in the high-risk economy is consistent with the graphs since the asset policy functions are shifted upwards in the high-risk scenario (dotted red lines v.s. blue solid lines). However this effect is weaker than the portfolio rebalancing one operating following the change in the ownership ratio implied by the higher labor income risk.

[Figure 1.11 approximately here]

Another noteworthy result is that in the long run equilibrium, durable good prices are not at all sensitive to the change in the unemployment risks. This is also shown in Table 1.3; the steady state price is basically the same across the two models. As discussed previously, higher unemployment risk makes less wealthy agents rebalance their portfolios towards liquid assets, but also wealthier agents value used durables more than new goods. In the long-run, these two forces therefore seem to balance each other. The next section will revisit this result, to show that changes in unemployment risks impart large effects on p^u in the transition between the two long-run equilibria.

1.4.4 Transitional dynamics

I now investigate the effects of changes in unemployment risk in the transition. The structure is the following. The economy has settled in the steady-state with low unemployment risk when an unanticipated (zero probability) shock is realized. Households perceive this shock as permanent and therefore anticipate that the economy will eventually reach a new steady-state with high unemployment. However, after 6 quarters, there is another unanticipated shock which changes the aggregate state back to G.[13]

Figure 1.12 presents the path followed by model variables from $t = 0$ to 50. In each panel, the red circle represents the initial steady-state value, and the black solid line shows the behaviour of the variables of interest under the baseline specification. Shaded areas represent periods in which the aggregate state is B.

[Figure 1.12 approximately here]

Panel a shows the response of the resale price of durables p_t^u. As can be seen, there is a huge drop in the price on impact: it drops to 0.434 in $t = 1$, when the first shock occurs, and so its value is 20% lower than in the initial steady state.

Recall that the model imparts two separate forces which determine the equilibrium outcome in the secondary market. On the one hand high unemployment induces agents to rebalance their portfolios towards liquid assets and away from used goods. On the other hand, wealthier households rebalance away from new durables and towards used goods. The behavior of the price reveals that the first force is the dominant one.

In order to show explicitly the above using individual policy functions, in Figure 1.13 I plot optimal policies under aggregate states G (regions denoted A_1, A_2, A_3) and B (regions denoted B_1, B_2, B_3) at the *initial* price level (i.e. the steady state price in the low-unemployment economy), as a function of liquid wealth z_t (defined in equation (1.3)). Panels a and b show the regions where owners of used goods optimally keep their goods and therefore do not participate in the market. The shaded areas in panel c represent the values of z_t at which non-owners are happy to buy a good in the secondary market. The solid blue line represents the distribution of agents over liquid wealth. The last panel shows the relative populations (densities) of the types of agents considered in panels a to c.[14]

[Figure 1.13 approximately here]

[13] Modelling unanticipated permanent shocks is very common in the literature. Here, I put one shock on top of the other to also consider the adjustment back to the original steady state, before the economy has time to settle to the high unemployment steady state. In other words, the economy will be on the 'saddle path' which leads to the high risk steady state after the first shock, but when the second shock arrives it jumps on the old saddle path which leads to the low risk steady state. Clearly, the behavior of model quantities during the first 6 periods (after the first shock occurs) is not impacted by the second shock, since this is not anticipated.

[14] Unemployed non-owners never want to buy used goods at the price p_0^u, both under low and high unemployment risk. For this reason their policy functions are not displayed in Figure 1.13.

At the initial steady-state, most of the goods sold in the secondary market are supplied by unemployed owners, as can be seen from panel b of Figure 1.13, where the lower end of the distribution of agents lies outside the region A_2. The richest employed owners also sell their used goods (the part of the distribution in panel a which is on the right hand side of region A_1). These are the agents that have accumulated enough liquid assets to find it optimal to upgrade their holdings of durable goods; they therefore choose to sell used goods in order to buy new ones. The durables sold on the market are all bought by employed non-owners. These agents find it optimal to buy used durable at levels of z_t in the region A_3; it can be observed from the distribution that this region is occupied by the richest agents among non-owners. Only a negligible share of agents are on the right of this region and buy new goods.

When the aggregate shock hits, at the prevailing price p_0^u, the inaction region of owners moves towards higher wealth levels (from A_1 to B_1 and A_2 to B_2). It becomes optimal for agents with low wealth to sell their goods, and used goods become more attractive than new goods at higher wealth levels. However, only the first effect matters in the determination of equilibrium prices, because the mass of agents in the right part of regions B_1 and B_2 is zero. At the initial price level, the supply of used goods is therefore higher under aggregate state B. Notice that under high unemployment risk most of the unemployed owners want to sell their goods: an important part of the line depicting the distribution of agents in panel b lies outside the region B_2.

On the demand side, panel c shows that higher unemployment risk moves the region where non owners want to buy used goods towards higher levels of wealth (from A_3 to B_3 for employed agents). It reflects the fact that the non-ownership option is more attractive when uncertainty increases. Because the mass of agents in the new region is zero, there is no demand for used goods at price p_0^u. As a result, there is excess supply in the secondary market. This calls from a reduction of the equilibrium p_1^u after the occurrence of the unemployment risk shock.

[Figure 1.14 approximately here]

Figure 1.14 compares policies at the initial price p_0^u (the regions B_1, B_2, B_3, also depicted in Figure 1.13) and at the *equilibirum* price in $t = 1$, p_1^u (regions C_1, C_2, C_3, C_4). On the supply side (where the change in price moves the inaction regions from B_1 and B_2 to C_1 and C_2), the price drop has the effect of extending the levels of z_t at which owners find it optimal to keep their used durables instead of selling them. This is especially relevant for employed agents (panel a), for which the inaction region C_1 includes levels such that there is almost no agent that wish to sell goods, as can be seen from the plotted density. Therefore, in the new equilibrium most of the goods in the market are sold by unemployed owners who a want to sell durables to finance nondurable consumption and accumulate liquid assets (there is still a significant share of agents at levels of z_t outside the region C_2 in panel b).

For employed non-owners, panel c of Figure 1.14 shows that the region where it is optimal to buy used durables also extends on both sides (from B_3 to C_3). The price decline makes agents in the right tail of the initial distribution buy used goods, which accommodates the supply coming from unemployed owners. Concerning unemployed non-owners (panel d), the price drop makes it optimal for them to buy used goods at high levels of z_t (the region C_4 is nonempty, in contrast with regions A_4 and B_4, which were not depicted in the previous figure). However, the mass of agents at such levels of wealth is zero; they therefore do not participate to the durable market, both in the initial steady-state and in the new equilibrium.

Notice that at the equilibrium price p_1^u, at all levels of cash-in-hand z_t depicted in Figure 1.14 agents prefer used goods to new ones. This can be observed from the fact that in the figure, above a certain threshold, regions C_1, C_2, C_3, C_4 cover all the levels of z_t included on the x-axis. Agents therefore never find it optimal to buy goods in the primary market and only decide to be either non-owner or to own used goods. As a result, the demand for new durables becomes zero in the first period of the transition, and the ratio of new-to-used goods in the economy decreases. Even though this model prediction is a bit extreme, this is well in line with the empirical facts presented in Section 1.2 (according to which, for instance, the average age of vehicles held in the economy increases with unemployment). The (partial) shutdown of primary durable markets in recessions is a well-known business cycle feature.

Let us now turn back to the analysis of Figure 1.12 and look at the response of other model variables to the recessionary shock. Panel b plots aggregate holdings of liquid assets during the transition. For reasons identical to the ones given in the steady-state analysis (see previous section), households start accumulating more liquid assets when the economy turns to its bad aggregate state. This is the result of higher precautionary savings, and the associated portfolio rebalancing from durables towards holdings of financial assets. The model predicts that asset holdings increase during recessions for all employed agents. The precautionary savings mechanism at work behind higher asset accumulation is much stronger than the negative effect implied by the higher share of agents actually getting unemployed and running down their buffer-stock of savings. More on this below.

Panel c presents the response of aggregate nondurable spending. A significant drop is observed on impact, as nondurable consumption drops by 17%. This outcome is also the result of higher precautionary savings. On impact, agents prefer to reduce consumption to accumulate assets faster and be able to cope with possible future unemployment spells, which are more likely and last longer in aggregate state B. Once their buffer-stock of savings becomes high enough, households start consuming more again.

Finally, panels d and e represent respectively the share of new-to-used good owners and the share of non-owners in the economy. It can be seen that the non-ownership ratio increases up to 10 percentage points above its steady-state level, reaching 0.26 in the last period of the recession. Then, when the economy turns back to aggregate state G, the ratio starts decreasing and goes as low as 0.09 in $t = 10$. Therefore, when the economy goes back to normal, many agents start accumulating durables and the ownership rate overshoots its steady state level. The same type of behaviour is observed for the ratio of new-to-used owners. On impact, the ratio falls as used goods start to become more attractive, both because they are a better hedge against labor income shocks, and from the fact that their relative price falls. When unemployment risk becomes low again, the ratio starts rising and reaches a level more than 50% above its long-run value. The main intuition behind the impact response of these quantities has already been discussed above. Higher unemployment risk makes households less willing to accumulate durables. The non-ownership option therefore becomes more attractive to households and, conditional on durable ownership, used goods become more attractive than new goods, for their better hedging properties against shock, and also because they become cheaper. Concerning the strong reversal in the behaviour of these variables after $t = 6$, the main explanation goes as follows. Having accumulated a lot of liquid assets, many agents (mostly employed households) have become wealthier in recessionary times. As a result, when the economy reverts back to the good aggregate state, many agents find it useless to keep high levels of assets, and durable ownership becomes more attractive to them. In other words, the portfolio rebalancing effect described above is reversed, and households start to increase the durable goods share of total assets in their balance sheet, as the lower unemployment risk decreases the relative attractiveness of liquid assets. As a result, the ownership ratio increases significantly. As there is not enough supply of used goods to accommodate the overall increase in durables' demand, many agents start buying goods on the primary market. The ratio of new-to-used goods in the economy therefore increases.

In order to distinguish the effect of the shock coming from the behavioural response of agents to increasing risk and the response coming mechanically from the higher share of agents getting unemployed, the transition path of the model was also simulated for the following cases: (i) with changes in 'perceived' unemployment risk only and (ii) with only 'realized' unemployment risk. More precisely, in case (i) during the recessionary period the model was run using the same households' policy functions as in the baseline case, but the true realizations of unemployment status continued to be set according to the transition matrix of the good state, Π^G. In case (ii) the realized labor market status are the same as in the baseline (i.e. from $t = 1$ to 6, the transition matrix of the bad state Π^B is used, and otherwise the matrix of Π^G), but the policy functions under the good aggregate state have been used throughout the entire transition period. Results are also presented in Figure 1.12. The blue dashed lines presents model outcomes from exercise (i), and the cyan dashed-dotted lines results from case (ii). It can be observed that under case (ii) there is almost no price change in the durable market. Therefore, most of the price response comes from the precautionary behaviour and the portfolio rebalancing towards liquid assets described above. For all the other variables present in the figure, it is also the case that most of the variation comes from changes in households' policies, and not from the mechanical response due to the higher share of unemployed agents in the economy. Overall, the two mechanisms have an opposite effect, but the response under case (i) is always stronger in absolute value. Therefore, changes in actual unemployment realizations only contribute marginally to the model outcomes implied by the optimal response of agents to higher unemployment risk.

1.4.5 Robustness to separable preferences

As a robustness check, the model was solved with an alternative specification for households' preferences, with separability between nondurable and durable consumption. The following utility function was used:

$$u(c_t, d_t) = \frac{c_t^{1-\gamma_c}}{1-\gamma_c} + \theta \frac{(\underline{d} + d_t)^{1-\gamma_d}}{1-\gamma_d}$$

It was assumed that $\gamma_c = \gamma_d \equiv \gamma$. The parameter γ was set to 2, as in the baseline case with non-separable preferences. The parameter θ was set to $\theta = \frac{1-\alpha}{\alpha}$, where α is the non-durable utility share in the non-separable specification. This implies that if γ is equal to 1 (log-utility), the separable and non-separable cases are equivalent. The parameter $\underline{d}$ was also changed in order to keep the non-ownership ratio at 0.15 in the steady-state of the low unemployment economy. This implied $\underline{d} = 1.306$.

The steady state quantities and the figure depicting the transitional path of model variables obtained under this model specification are provided in Appendix 1.6.4. The only changes in model outcomes when using separable preferences are level changes for the equilibrium resale price and the aggregate stock of assets. In the initial equilibrium, the resale price of durables is equal to 0.65, as compared to 0.55 in the non-separable case. The aggregate stock of liquid assets is 2.89 instead of 2.66. The behaviour of variables following the recessionary shock, as well as the steady-state differences between the model versions with low and high unemployment risk, are similar under the two specifications, and the analysis provided above therefore also applies to the case of non-separable preferences.

1.5 Conclusions

The impact of idiosyncratic unemployment risk and incomplete financial markets on the resale price of durable goods is investigated in this paper. I develop a macroeconomic framework where individual households face uninsurable income risks and choose optimally between consuming nondurable and durable goods and accumulating a risk-free asset. The model distinguishes between new and used durables, and the price in the secondary market is determined endogenously, through market clearing.

The mechanisms through which unemployment risk affects the price of durables are the following. First, an increase in risk makes durable goods less attractive to households. Therefore, they rebalance their portfolio from durables to liquid asset holdings. This decreases the demand for used durable goods and exerts a downward pressure on their price. Second, increasing risk makes used durable goods more attractive than new goods, because new goods cannot be sold at their true value (reflecting the empirical observation that the resale value of new durables drops discretely after purchase). Therefore, agents prefer to keep their used durables rather than selling them to buy new goods, which decreases the supply in the secondary market and impacts the price positively. The main finding is that, starting from a steady-state with relatively low risk, an unexpected shock to unemployment probabilities has the effect of lowering the resale price of durables on impact, giving more weight to the first channel.

1.6 Appendix

1.6.1 Tables and Figures

Table 1.1: **Vehicle ownership in the US**

Description	**Value**
Share of vehicle owners	88.6%
Average number of vehicles	1.65
Average age of newest vehicle (in years)*	7.56
Fraction with cars > liquid assets*	65.96%
Average value of vehicle holdings (in USD)*	18,090
Average cars-to-assets ratio	22.81%
Average cars-to-net-worth ratio	84.71%

Notes: Summary statistics from the 2013 Survey of Consumer Finances. Details on the data source and the construction of variables is provided in Appendix 1.6.2.
* Computed excluding non-owners of vehicles.

Table 1.2: **Baseline calibration**

Parameter	Description	Target	Value
β	discount factor	standard	0.99
γ	risk aversion	standard	2
α	non durable share in utility	nondurable-to-durable spending	0.78
$\underline{d}$	minimum durable services	share of non owners = 0.15	3.875
r	interest rate on asset holdings	annual rate = 2.5%	0.025/4
$\overline{d}$	size of a durable unit	av. value of new cars = 2/3 yearly income	3
$1-\pi_1$	transition prob new→used	share of new-to-used goods = 1/3	0.917
$1-\pi_2$	transition prob used→scrap	average lifetime of durables = 12 years	0.972
δ	depreciation rate	av. annualized rate = 0.085	0.638
χ	transaction cost	standard	0.05
$\bar{y}$	income of employed	normalization	1
$1-\pi^g_{ee}$	job separation rate, normal times	steady-state unemployment = 0.04	0.028
$1-\pi^g_{uu}$	job finding rate, normal times	av. unemployment spell of 1.5 quarters	0.67
$1-\pi^b_{ee}$	job separation rate, recession	steady-state unemployment = 0.10	0.044
$1-\pi^b_{uu}$	job finding rate, recession	av. unemployment spell of 2.5 quarters	0.40

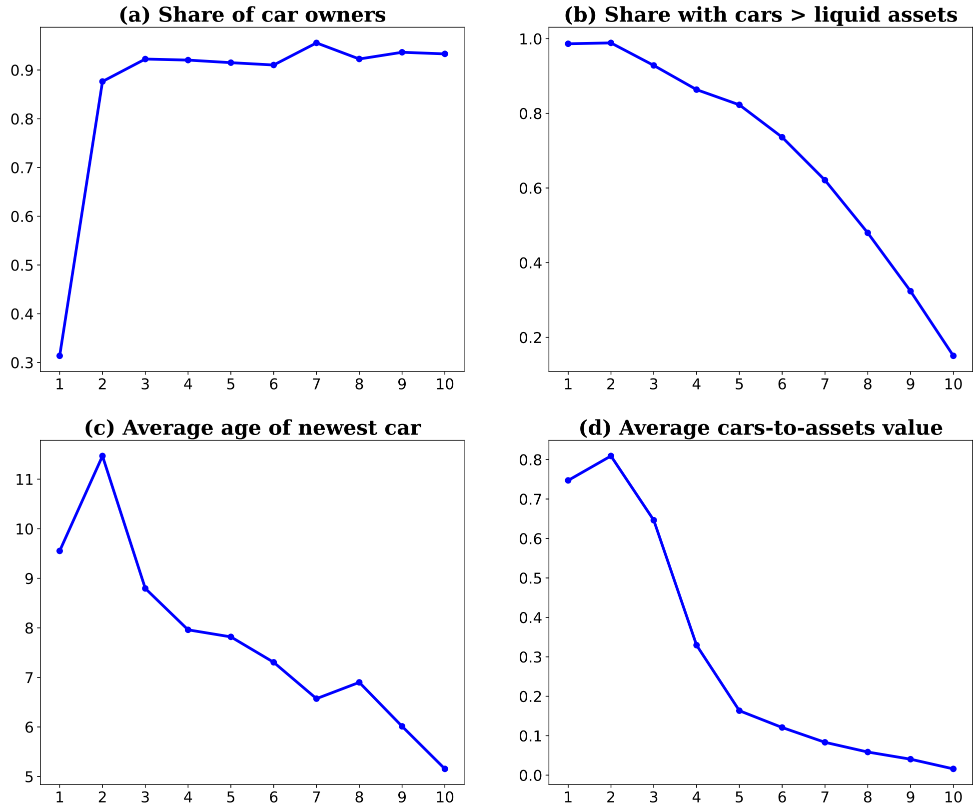
Notes: The model period is one quarter. Parameters $\alpha, \underline{d}, \overline{d}, \pi_1, \pi_2, \delta$ are the ones linked to durable goods. They are set in order to match non durable-to-durables sending, the average value of new cars as a ratio of yearly income, the non-ownership ratio observed in the U.S, and a share of new-to-used goods of 1/3, an average lifetime of 12 years for durables, and an average annual depreciation of 0.085 for used goods.

Table 1.3: **Steady state results**

	Low unemployment risk	**High unemployment risk**
Price of used durables	0.549	0.549
Owners of new goods	0.20	0.19
Owners of used goods	0.65	0.62
Non-owners	0.15	0.19
Nondurable consumption	0.92	0.89
Liquid asset holdings	2.66	6.30

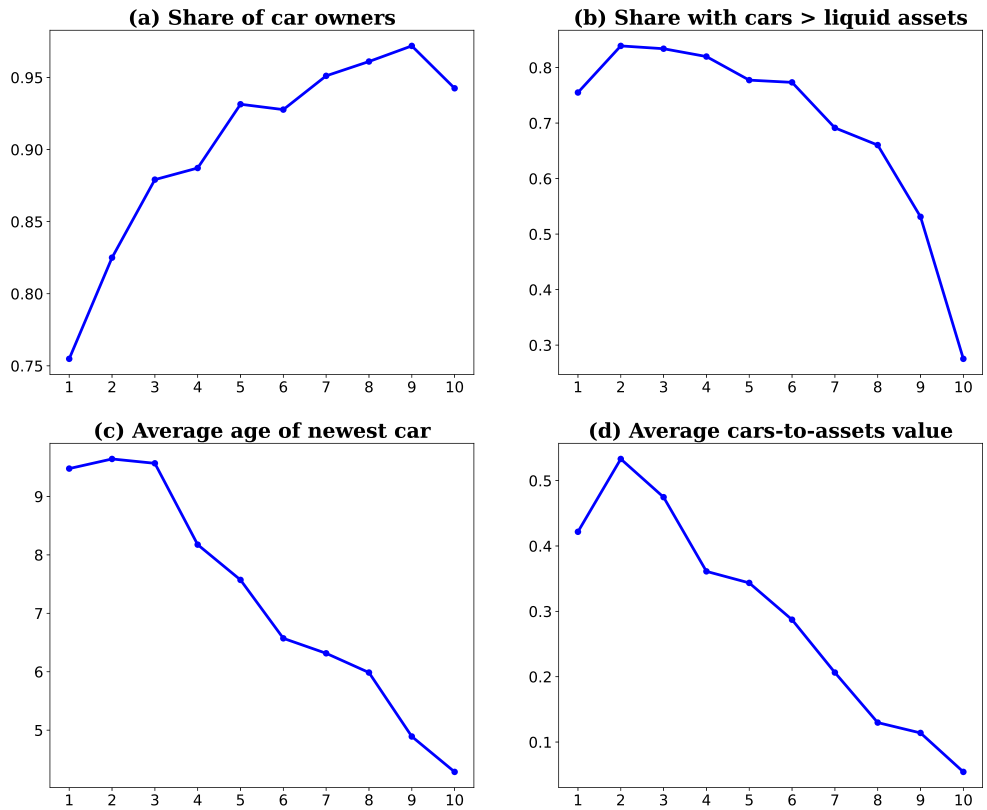
Notes: Steady-state results. The first column shows results for the economy that is permanently in state G (the low-risk economy). The second column is the equivalent for aggregate state B (the high-risk, recessionary economy).

FIGURE 1.1: **SCF Data: Summary statistics per net worth decile**



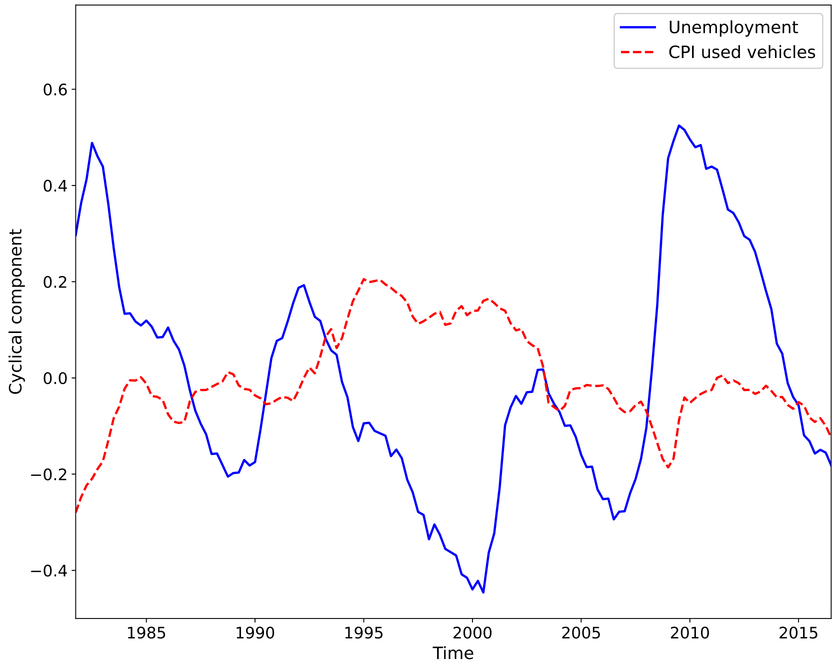
Notes: The Figure provides descriptive statistics about vehicle holdings across net worth deciles in the US, using data from the 2013 Survey of Consumer Finances. Panel (a) provides the share of car owners for each net worth decile. Panel (b) depicts the share of households with a reported value of vehicle holdings greater than the value of their liquid assets. Panel (c) presents, for car owners, the average age of the newest car held across net worth deciles. Panel (d) displays the average value of the ratio of vehicle holdings to total assets. Details on the data source and the construction of variables are provided in Appendix 1.6.2.

FIGURE 1.2: **SCF Data: Summary statistics per income decile**



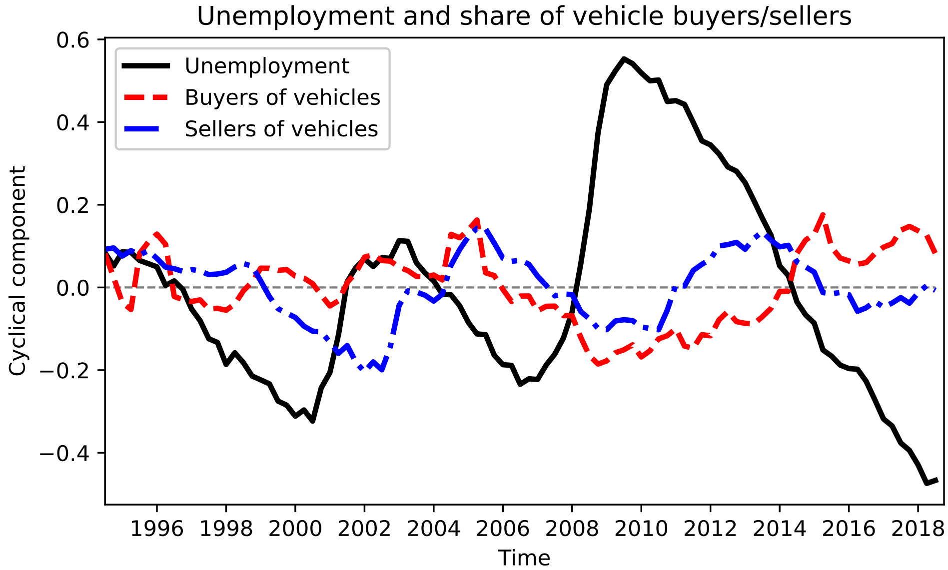
Notes: The Figure provides descriptive statistics about vehicle holdings across labor income deciles in the US, for households with a positive reported value of labor income. These figures are constructed using data from the 2013 Survey of Consumer Finances. Panel (a) provides the share of car owners for each income decile. Panel (b) depicts the share of households with a reported value of vehicle holdings greater than the value of their liquid assets. Panel (c) presents, for car owners, the average age of the newest car held across income deciles. Panel (d) displays the average value of the ratio of vehicle holdings to total assets. Details on the data source and the construction of variables are provided in Appendix 1.6.2.

FIGURE 1.3: **Unemployment rate and resale price of durables in U.S data**



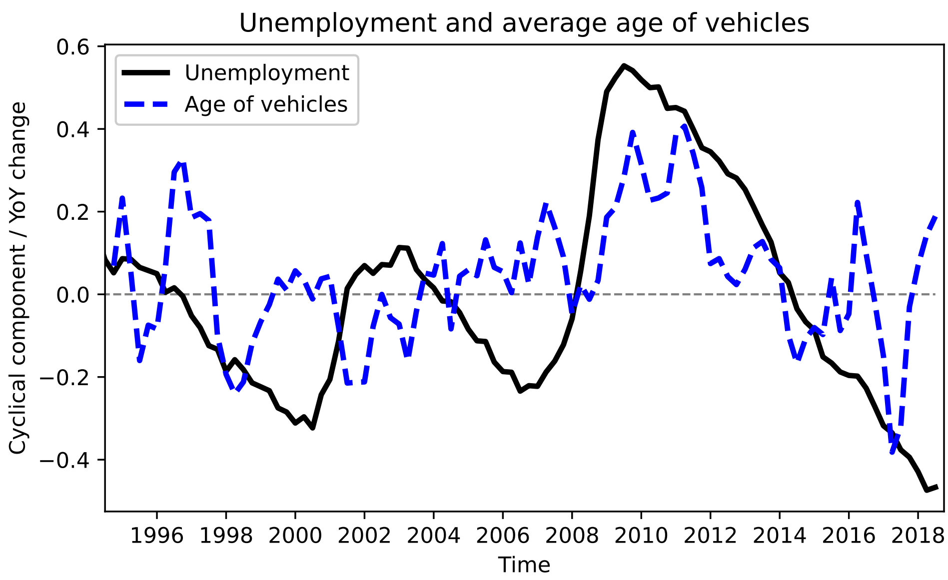
Notes: Cyclical components of quarterly U.S data. Solid blue line: Civilian Unemployment Rate. Dashed red line: Consumer Price Index for All Urban Consumers: Used cars and trucks. Both series are seasonally-adjusted. The series are logged and detrended using a linear filter. The correlation coefficient of the two series is -0.56.

FIGURE 1.4: **Unemployment rate and shares of buyers/sellers of vehicles**



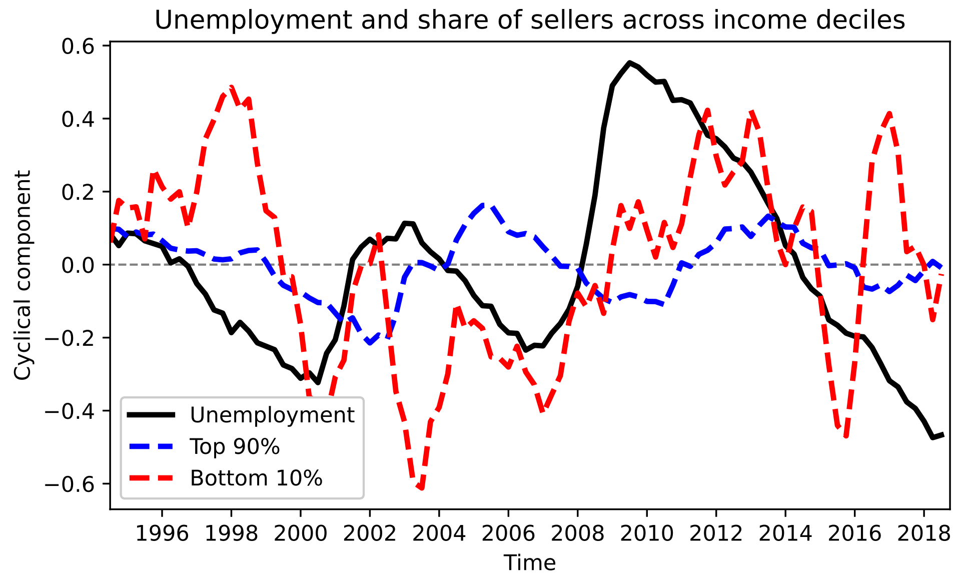
Notes: The solid black line depicts the cyclical component of aggregate US unemployment (as displayed in Figure 1.3). The dashed red line displays the cyclical component of the (log-)share of households who reported to have bought a new or used vehicle in the CEX data. The dashed-dotted blue line presents the counterpart for the vehicle selling decision. The correlation coefficient of the buyers series with unemployment is -0.68. The correlation between the sellers series and unemployment is -0.03.

FIGURE 1.5: **Unemployment rate and changes in the average age of owned vehicles**



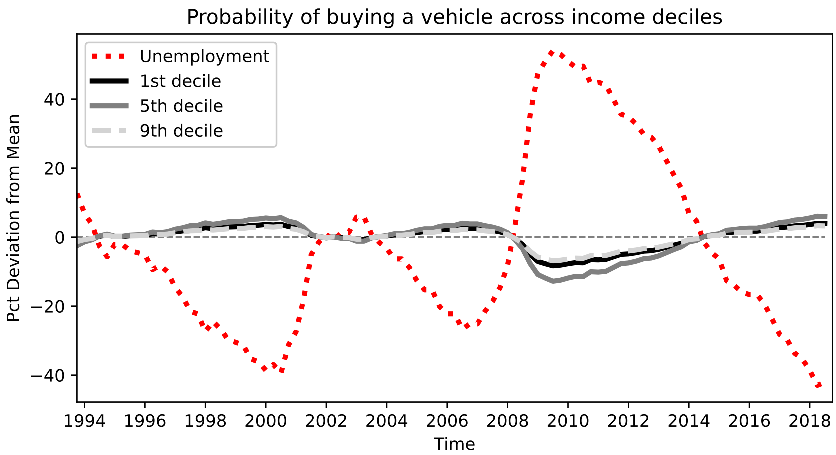
Notes: The solid black line depicts the cyclical component of aggregate US unemployment (as displayed in Figure 1.3). The dashed blue line displays the year-on-year change in the average age of the newest vehicle owned by car owners in the CEX data. The correlation coefficient between the two series is 0.47.

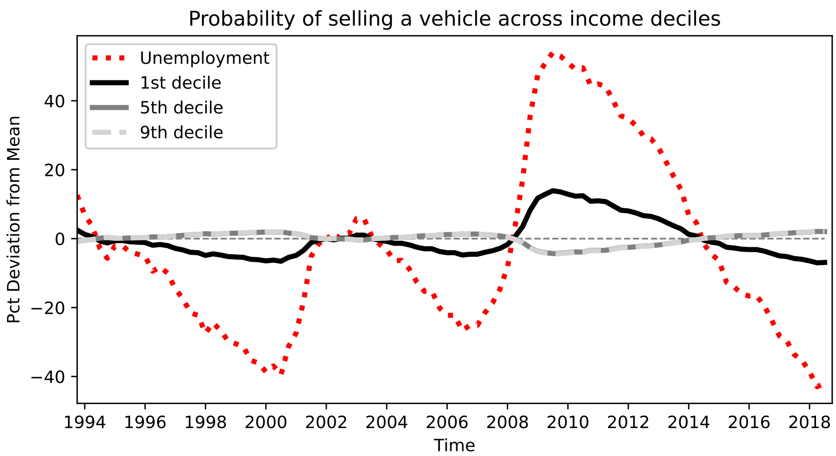
FIGURE 1.6: **Unemployment rate and shares of sellers of vehicles across income deciles**



Notes: The solid black line depicts the cyclical component of aggregate US unemployment (as displayed in Figure 1.3). The dashed blue line displays the share of vehicle sellers among households in the top 90% of the income distribution (correlation with unemployment: -0.05). The dashed red line presents the counterpart for households in the first decile (correlation with unemployment: 0.25).

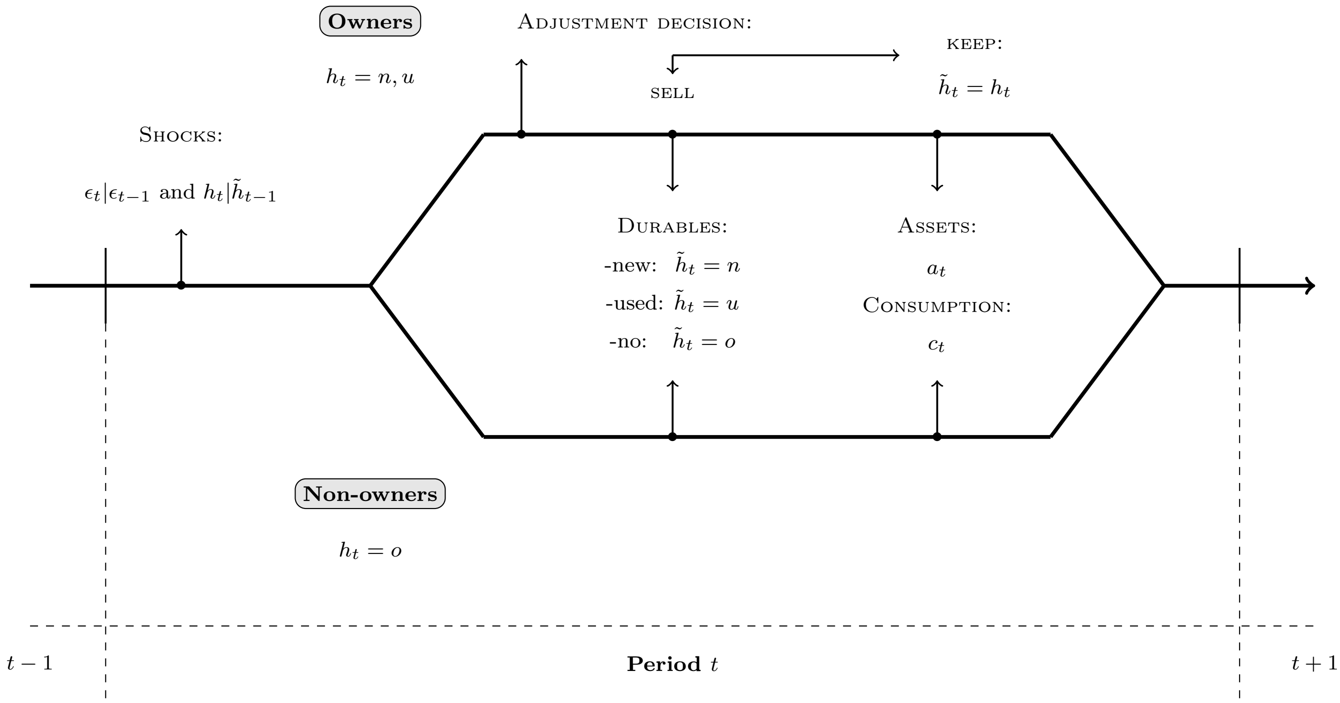
FIGURE 1.7: **Fitted probabilities of buying/selling a vehicle**





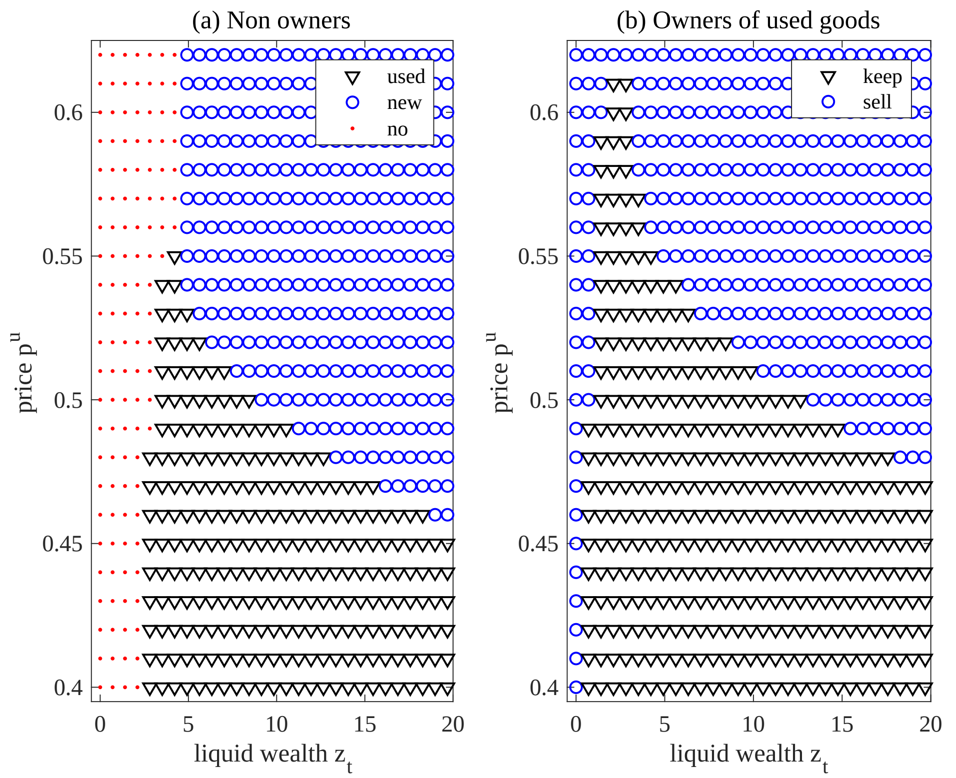
Notes: The figure presents the fitted probability of adjusting car holdings obtained from the Logit model described in Appendix 1.6.2. The top panel depicts the probability of buying a (new or used) vehicle for households in the 1st, 5th and 9th decile of the income distribution. The bottom panel depicts the probability of selling a vehicle for the same groups of households. In both panels, the dotted red lines displays the cyclical component of unemployment at the US level, as displayed in Figure 1.3.

FIGURE 1.8: **Timing of events in the model**



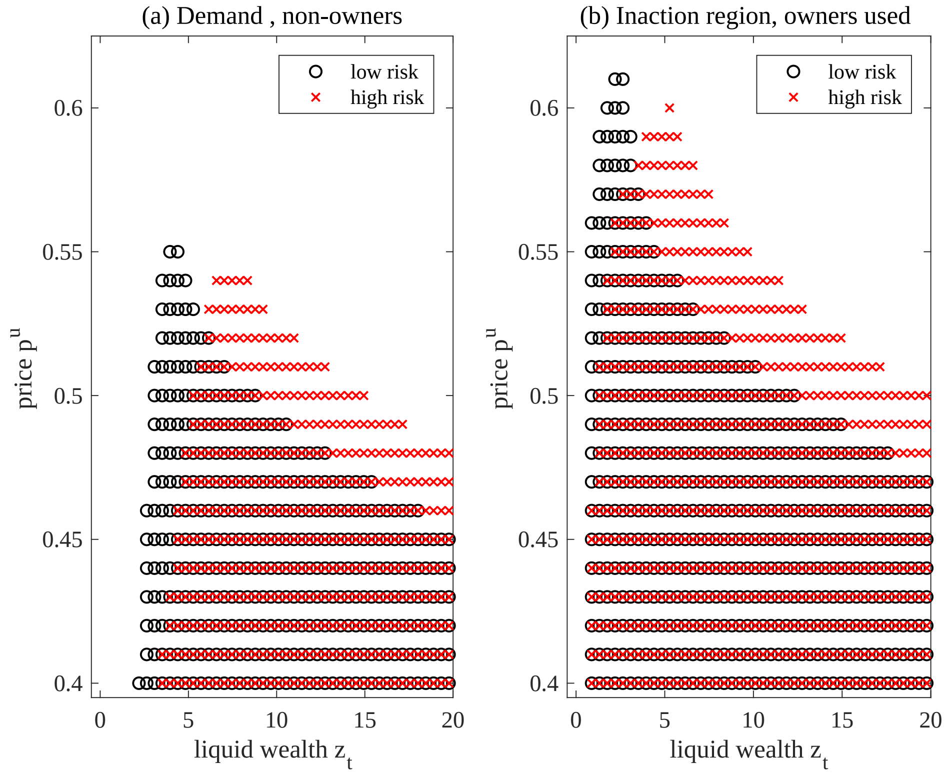
Notes: Timing convention adopted in the model: at the beginning of period t, durable and employment status are updated. Then, durable market adjustment takes place. Owners ($h_t \in \{n, u\}$) choose whether to sell their good or not. If yes, they join the pool of non-owners ($h_t = o$) and freely choose their new status $\tilde{h}_t$. Finally, holdings of liquid assets a_t and nondurable consumption levels c_t are chosen.

FIGURE 1.9: **Optimal durable policies in the low unemployment economy**



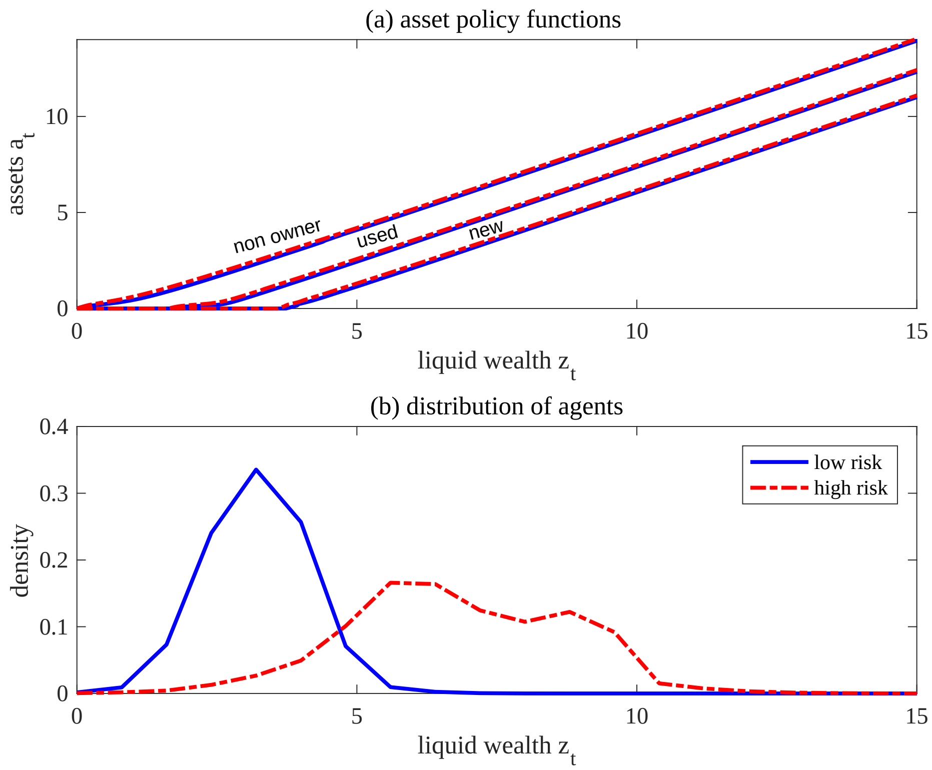
Notes: Optimal durable policies $\tilde{h}(a_{t-1}, \epsilon_t, h_t)$ over different values of the resale price of durables p^u, and aggregate state G. Optimal policies for employed agents ($\epsilon_t = 1$). Panel a: policies of non-owners ($h_t = o$). Panel b: policies of owners of used goods ($h_t = u$). For the latter, only the decision between adjusting ($\tilde{h}(a_{t-1}, \epsilon_t, h_t) \neq h_t$, the 'sell' region) or not ($\tilde{h}(a_{t-1}, \epsilon_t, h_t) = h_t$, the 'keep' region) is displayed. The x-axis denotes liquid wealth $z_t = a_{t-1} + \bar{y}$, and the y-axis the resale price p^u.

FIGURE 1.10: **Optimal policies under low and high unemployment risk**



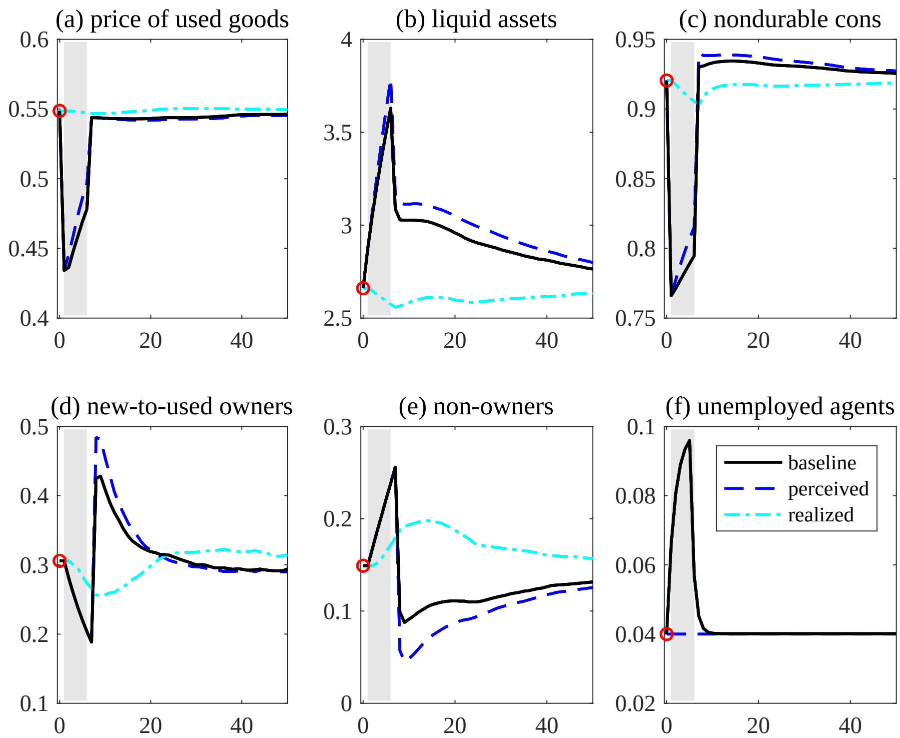
Notes: Optimal durable policies $\tilde{h}(a_{t-1}, \epsilon_t, h_t)$ as a function of liquid wealth z_t and resale price p^u, for employed agents ($\epsilon_t = 1$) under low and high unemployment risk (respectively aggregate state G, black circles - and B, red crosses). The left panel represents the region where non-owners ($h_t = o$) find it optimal to buy a used good ($\tilde{h}(a_{t-1}, \epsilon_t = 1, h_t = o) = u$). The right panel represents the region where owners of used goods ($h_t = u$) choose to be inactive in the resale market and keep their good.

FIGURE 1.11: **Liquid asset policy functions over durable ownership status**



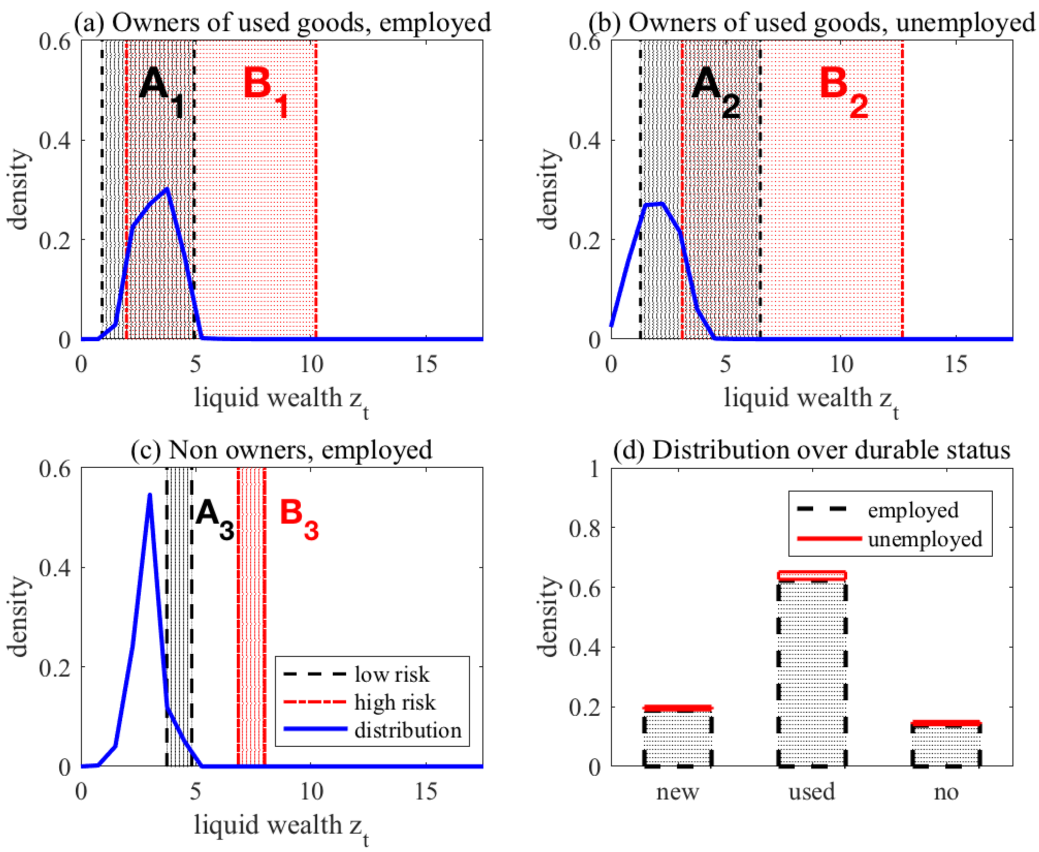
Notes: Solid blue lines are associated to quantities in the low-risk economy (aggregate state G). Dotted red lines concern aggregate state B. Panel a represents asset policies $a(a_{t-1}, \epsilon_t, h_t)$ as a function of liquid wealth z_t, conditional on the current choice $\tilde{h}_t$. The policies depicted are for non-owners ($h_t = o$) that are currently employed ($\epsilon_t = 1$). The text above policies described current durable policy $\tilde{h}_t$. Panel b depicts the distribution of agents over liquid wealth holdings z_t.

FIGURE 1.12: **Transition dynamics**



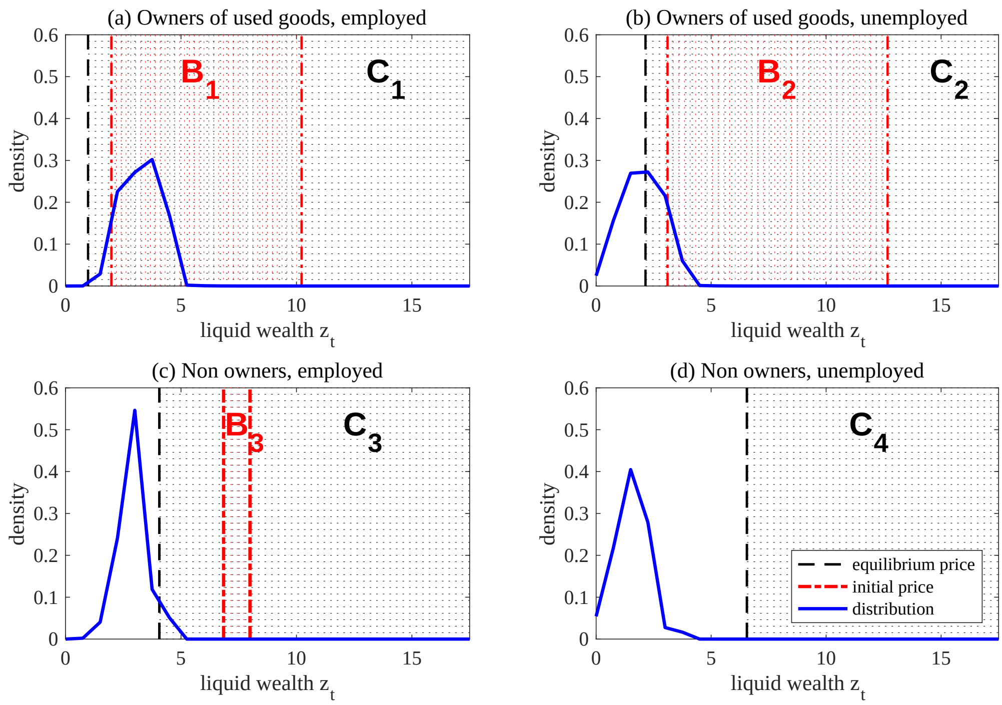
Notes: Path of model variables following a transitory shock to unemployment probabilities. In $t = 0$, the economy is assumed to be in the long-run equilibrium under aggregate state G. Then, from $t = 1$ to 6 (the shaded areas) the aggregate state switches to B, before returning indefinitely to G. Solid black lines represent the behaviour of variables in the baseline specification. The dashed blue lines depict the response of variables when only the policy functions switch to the bad aggregate state (and not the realized unemployment status). The dash-dotted cyan lines represent the response when only the realized unemployment status switches to state B.

FIGURE 1.13: **Durable policies at initial equilibrium price**



Notes: The three first panels represent optimal durable policies $\tilde{h}(a_{t-1}, \epsilon_t, h_t)$ as a function of liquid wealth z_t, at the initial equilibrium price p_0^u. Regions A_1, A_2, A_3 are associated to policies under aggregate state G, whereas the regions B_1, B_2, B_3 are associated to state B. Panels a and b represent the levels of liquid wealth z_t at which it is optimal for owners of used goods ($h_t = u$) to keep their good for employed agents ($\epsilon_t = 1$, panel a) and unemployed agents ($\epsilon_t = 0$, panel b). Panel c represents the regions where employed non-owners ($h_t = o$, $\epsilon_t = 1$) decide to buy a good on the secondary market. In all panels, the solid blue lines represent the conditional distributions of agents over levels of z_t. Finally, panel d depicts the distribution of agents across durable holdings and employment status.

FIGURE 1.14: **Durable policies at prices in $t = 0$ and $t = 1$**



Notes: Durable policies $\tilde{h}(a_{t-1}, \epsilon_t, h_t)$ as a function of liquid wealth z_t, at the equilibrium prices p_0^u (regions B_1, B_2, B_3) and p_1^u (regions C_1, C_2, C_3, C_4). Panels a and b represent the levels of z_t for which it is optimal for used good owners ($h_t = u$) to be inactive in the durable market, for respectively employed ($\epsilon_t = 1$) and unemployed agents ($\epsilon_t = 0$). Panels c and d represent the regions where it is optimal for non-owners to buy a used good. The solid blue lines represent the conditional distributions of agents over levels of z_t.

1.6.2 Data appendix

Macro data

The quarterly U.S macro data series used in the paper come from the Bureau of Labor Statistics (BLS), and consists of the *unemployment rate* series, and the *CPI for used cars and trucks*. The data series were extracted from the FRED database.

[Figure 1.16 approximately here]

For robustness I present here the counterpart of Figure 1.3 using the data series at the US regional level, thereby obtaining figures for the North East (NE), Midwest (MW), South (SO) and (WE) census regions. Results are provided in Figure 1.16. We can see from this figure that the negative correlation between unemployment and the price of used vehicles documented in the text also holds at this level of aggregation.

Household-level data

Survey of Consumer Finances (SCF) The data used to construct the summary statistics described in Section 1.2.1 come from the 2013 Survey of Consumer Finances (SCF), a survey of US households with detailed data on wealth and its components, conducted by the Federal Reserve Board every three years.

Liquid assets are defined as the sum of checking account and savings account balances, government mutual fund holdings, government bonds, mutual funds and other mutual fund holdings, stocks, combined funds, mortgage backed bonds, foreign bonds, other bonds and remaining funds, brokerage accounts and trusts.

Total assets are the sum of liquid assets and illiquid assets. Illiquid assets are obtained by taking the sum of IRA and Keogh accounts, life insurance accounts, savings bonds, certificate of deposits, annuities, housing, businesses and vehicles.

Net worth is computed as total assets minus total liabilities. Total liabilities are obtained as the sum of housing debt, vehicle debt, business debt, consumption debt, credit lines and margin loans, plus “other debt” as provided in the dataset. Households with a negative net worth are removed from the sample.

To report statistics across labor income deciles, I remove households with a self-reported labor income equal to zero.

Consumer Expenditure Survey (CEX) The data source for the household-level data used in Section 1.2.2 is the Consumer Expenditure Survey (CEX), a quarterly cross-section survey of US households conducted by the Bureau of Labor Statistics. The data on vehicle holdings used in the analysis is only available starting from 1994, so I focus on the period 1994Q1-2018Q4. The main dataset (the FMLI files) provides a broad set of variables on income expenses, income, and demographic characteristics.

To classify households along income deciles I use the *INC_RANK* variable present in the FMLI files, which provides a cumulative ranking of households in each quarter based on the value they reported as their current income before taxes.

To analyse car purchases, I make use of the OVB expenditure files, which provides data on purchases of new and used cars. To analyse car sales I make use of the OVC files, in which information on vehicle disposals is available. I consider a used car to be sold by a household if the latter explicitly mention a sale, or if the vehicle was traded-in.

Logit regressions The Logit regression results reported in Section 1.2 of the paper were obtained using two Logit models, one for the buying and one for the selling vehicle decisions. In the two models I make use of the following set of dependent variables: family size, age of the reference person, education of the reference person, number of vehicles, the aggregate unemployment rate, on top of income decile fixed effects, and the interaction between unemployment rate and income decile dummies. Full estimation results are reported in Table **??**.

1.6.3 Solution method

The household's problem

The household's problem, as described by equations (1.1) and (1.2), has been solved using the algorithm developed in Fella (2014). The method deals with the non-convexities present in the model efficiently, combining the endogenous gridpoint method (EGM) of Carroll (2006), and value function iteration in the region where the value function is identified as non-concave.

The EGM step makes use of the first order conditions of the household's problem. To derive them, let us rewrite the intensive margin problem of households for given choice in the durable market $\tilde{h}_t$, stated in equation (1.2), as follows:

$$\begin{aligned} V^{\tilde{h}_t}(a_{t-1}, \epsilon_t, h_t) &= \max_{c_t, a_t} u(c_t, d^{\tilde{h}_t}) + \beta \tilde{V}(a_t, \epsilon_t, \tilde{h}_t) \\ \text{s.t} \quad & a_t = (1+r)\big(\hat{x}(a_{t-1}, \epsilon_t, h_t, \tilde{h}_t) - c_t\big) \\ & a_t \geqslant 0 \end{aligned}$$

where the objects $\tilde{V}$ and $\hat{x}$ are defined as follows:

$$\tilde{V}(a_t, \epsilon_t, \tilde{h}_t) \equiv \sum_{\epsilon} \sum_{h'} \Pi(\epsilon'|\epsilon_t) \Omega(h'|\tilde{h}_t) V(a_t, \epsilon_{t+1}, h_{t+1}) \tag{A.1}$$

and

$$\hat{x}(a_{t-1}, \epsilon_t, h_t, \tilde{h}_t) \equiv x(a_{t-1}, \epsilon_t, h_t, \tilde{h}_t) - p_t^{\tilde{h}_t} d^{\tilde{h}_t} \tag{A.2}$$

The first order condition with respect to a_t, for given $\hat{x}_t$, can be written as:

$$u_{c,t}\Big(\hat{x}_t - \frac{a_t}{1+r},\ d^{\tilde{h}_t}\Big) \geqslant \beta(1+r)\tilde{V}_a(a_t, \epsilon_t, \tilde{h}_t)$$

with strict inequality if the borrowing constraint is not binding, and where

$$u_{c,t}(c_t, d_t) \equiv \frac{\partial}{\partial c_t} u(c_t, d_t)$$

and

$$\tilde{V}_a(a_t, \epsilon_t, \tilde{h}_t) \equiv \frac{\partial}{\partial a_t} \tilde{V}(a_t, \epsilon_t, \tilde{h}_t) = \frac{\partial}{\partial a_t} \sum_{\epsilon} \sum_{h'} \Pi(\epsilon'|\epsilon_t) \Omega(h'|\tilde{h}_t) V^{\tilde{h}(a_t, \epsilon', h')}(a_t, \epsilon', h') \tag{A.3}$$

where the last equality makes use (1.1), optimal extensive marginal policies $\tilde{h}(a_t, \epsilon_{t+1}, h_{t+1})$, and the fact that at the optimum the extensive margin value function $V(a_{t-1}, \epsilon_t, h_t)$ is never at a kink and is therefore differentiable (see Clausen and Strub, 2012). Defining $u_{c,t}^{-1}$ as the inverse of $u_{c,t}$ with respect to its first argument, we have:

$$\hat{x}_t = u_{c,t}^{-1}\Big[\beta(1+r)\tilde{V}_a(a_t, \epsilon_t, \tilde{h}_t)\ , d^{\tilde{h}_t}\Big] + \frac{a_t}{1+r} \tag{A.4}$$

Equation (A.4) allows us to get the current level of $\hat{x}_t$, for a given value of next period's assets a_t. This constitutes the EGM step of the solution method: a grid for next period's assets is defined, and for each gridpoint the above equation is used to know what is the current value of $\hat{x}_t$ associated with it. In what follows, I describe the entire algorithm used to solve the optimization problem faced by individual households in the model.

Algorithm I: Approximating households' policy functions

Preliminaries:

- Create a grid of size n_a for asset levels a_{t-1} and a_t: $\mathcal{A} = \{a^i\}_{i=1}^{n_a}$.
- Provide an initial guess for intensive margin value functions $V^{\tilde{h}_t}(a^j, \epsilon_t, h_t)$ and associated durable policies $\tilde{h}(a^j, \epsilon_t, h_t)$, for all $a^j \in \mathcal{A}$

Algorithm:

1. Compute $\tilde{V}(a_t, \epsilon_t, \tilde{h}_t)$ and its derivative $\tilde{V}_a(a_t, \epsilon_t, \tilde{h}_t)$ given previous guesses of and $V^{\tilde{h}_t}(a_{t-1}, \epsilon_t, h_t)$, using (A.1) and (A.3). A finite difference method is used to compute the derivatives of $V^{\tilde{h}_t}(a_t, \epsilon_{t+1}, h_{t+1})$ w.r.t a_t.
2. For every $(\epsilon_t, \tilde{h}_t)$ pair:
   (a) Find the region of the state space where the value function is non-concave. For details on how it is done, see Fella (2014, sec 3.2).
   (b) For each level of next period assets on the grid $\mathcal{A}$, use (A.4) to get the associated value of $\hat{x}_t$, that I denote $\hat{x}_t^{end,i}(\epsilon_t, \tilde{h}_t)$. Complement this with value function iteration in the non-concave region to discard the local maxima that are not global maxima.
   (c) Deal with the borrowing constraint in the case where the pair $(\hat{x}^{end,1}, a^1)$ is discarded, i.e. find the value of $\hat{x}_t$ below which the household is credit constrained. A root-finding method is used to do it.
   (d) Store the pairs $\left(\hat{x}_t^{end,i}(\epsilon_t, \tilde{h}_t), a^i\right)_{i=1}^{n_a}$, and the associated value for the intensive margin value function $V^{int}(\hat{x}_t^{endo,i}, \epsilon_t, \tilde{h}_t)$.
3. For every (a^j, ϵ_t, h_t) (where the gridpoints $a^j \in \mathcal{A}$ are now associated to a_{t-1} levels)
   (a) Compute $\hat{x}_t = \hat{x}(a^j, \epsilon_t, h_t, \tilde{h}_t)$ for each $\tilde{h}_t$ and use interpolation schemes on the objects obtained in the previous step to recover $V^{\tilde{h}_t}(a^j, \epsilon_t, h_t)$
   (b) Use the extensive margin value function (1.1) to choose the optimal $\tilde{h}_t$ and set $V^{new}(a^j, \epsilon_t, h_t) = \max_{\tilde{h}_t}\{V^{\tilde{h}_t}(a^j, \epsilon_t, h_t)\}$
4. Compute the convergence criterion $crit = \max\left(\left|\frac{V^{new}(a^j,\epsilon_t,h_t) - V(a^j,\epsilon_t,h_t)}{V(a^j,\epsilon_t,h_t)}\right|\right)$
   (a) If $crit < tol$, stop
   (b) Otherwise, set $V = V^{new}$ and go back to step 1

Distribution of agents and market clearing

To get the steady-state distribution of agents over values of $(a_{t-1}, \epsilon_t, h_t)$, I follow the method of Young (2010), which makes use of non-stochastic simulations. The algorithm is described in what follows.

Algorithm II: Steady state distribution

Preliminaries:

- Create an equidistant grid for assets of size n_a^d (with $n_a^d > n_a$): $\mathcal{A}^d = \{a^i\}_{i=1}^{n_a^d}$
- Recover policy functions $a(a^i, \epsilon_t, h_t)$, $\tilde{h}(a^i, \epsilon_t, h_t)$ for all $a^i \in \mathcal{A}^d$ on the new grid from the ones obtained in Algorithm I.
- Set $k = 0$ and initialize the distribution: $F^{(0)}(a^i, \epsilon_t, h_t)$

Algorithm:

1. Set $F^{(k+1)}(a^i, \epsilon_t, h_t) = 0$ for all (a^i, ϵ_t, h_t) triples
2. For every (a^i, ϵ_t, h_t) combination:
   (a) Get $a' = a(a^i, \epsilon_t, h_t)$ and $\tilde{h}' = \tilde{h}(a^i, \epsilon_t, h_t)$
   (b) Find j such that $a^j \leqslant a' \leqslant a^{j+1}$ and compute $\omega = 1 - \frac{a'-a^j}{a^{j+1}-a^j}$
   (c) For every $(\epsilon_{t+1}, h_{t+1})$ pair, store:

$$\begin{aligned} F^{(k+1)}(a^j, \epsilon_{t+1}, h_{t+1}) &= F^{(k+1)}(a^j, \epsilon_{t+1}, h_{t+1}) \quad + \omega\ \Pi(\epsilon_{t+1}|\epsilon_t)\Omega(h_{t+1}|\tilde{h}')F^{(k)}(a^i, \epsilon_t, h_t) \\ F^{(k+1)}(a^{j+1}, \epsilon_{t+1}, h_{t+1}) &= F^{(k+1)}(a^{j+1}, \epsilon_{t+1}, h_{t+1}) + (1-\omega)\ \Pi(\epsilon_{t+1}|\epsilon_t)\Omega(h_{t+1}|\tilde{h}')F^{(k)}(a^i, \epsilon_t, h_t) \end{aligned}$$

3. Compute $conv = \max\left(\left|F^{(k+1)}(a^i, \epsilon_t, h_t) - F^{(k)}(a^i, \epsilon_t, h_t)\right|\right)$
   (a) if $conv < tol$, stop and store $F(a^i, \epsilon_t, h_t) = F^{(k+1)}(a^i, \epsilon_t, h_t)$ as the steady state distribution
   (b) Otherwise, set $k = k + 1$ and go back to step 1

Algorithm III: Equilibrium price

To compute the equilibrium steady-state resale price of durables p^u, I use a bisection search algorithm to find the price at which the excess demand for used durables is zero:

Preliminaries:

- Initialize P_l and P_r to loose enough levels in order to make sure that $p^u \in [P_l, P_r]$
- Set $k = 0$ and initialize $P^{(0)} = \frac{P_l+P_r}{2}$

Algorithm:

1. Use Algorithm I and Algorithm II to get household policy functions and steady-state distribution of agents given $p^u = P^{(k)}$
2. Compute excess demand for durables, denoted Z:

$$Z = \sum_{i=1}^{n_a^d} \sum_{\epsilon_t=0,1} \sum_{h_t=n,o} \mathbf{1}\big(\tilde{h}(a^i, \epsilon_t, h_t) = u\big) F(a^i, \epsilon_t, h_t) - \sum_{i=1}^{n_a^d} \sum_{\epsilon_t=0,1} \sum_{h_t=n,u} \mathbf{1}\big(\tilde{h}(a^i, \epsilon_t, h_t) \neq h_t\big) F(a^i, \epsilon_t, h_t)$$

3. If $Z < 0$, set $P_r = P^{(k)}$. Otherwise if $Z > 0$, set $P_l = P^{(k)}$
4. Set $P^{(k+1)} = \frac{P_l+P_r}{2}$
   - If $|Z| < tol_z$ or $|P^{k+1} - P^k| < tol_p$, stop and store $p^u = P^k$
   - Otherwise, go back to step 1

Algorithm IV: Transition path

To get the transition path of equilibrium prices and quantities (which is analyzed in section 1.4.4), the following algorithm is used:

Preliminaries:

- Set $t = 1$
- Initialize the distribution at the one obtained for the steady-state in Algorithm II: $F^{(0)}(a^i, \epsilon_t, h_t) = F(a^i, \epsilon_t, h_t)$
- Initialize p_0^u to the steady-state equilibrium price p^u
- Guess initial P_l, P_r, and p_t^u for all t (similarly to Algorithm III).

Algorithm:

1. Compute households' policies using Algorithm I
2. Use steps 1 to 2 of Algorithm II (replacing $k+1$ by t) to update the distribution of agents
3. Use steps 2 to 5 of Algorithm III to update the price p_t^u. If convergence, store the distribution $F^{(t)}(a^i, \epsilon_t, h_t)$, and set $t = t+1$. Otherwise, keep the same t and iterate until the excess demand Z_t is close to zero.

1.6.4 Additional tables and figures

Table 1.4: **Steady state results under separable preferences**

	Low unemployment risk	High unemployment risk
Price of used durables	0.656	0.655
Owners of new goods	0.20	0.18
Owners of used goods	0.65	0.59
Non-owners	0.15	0.23
Nondurable consumption	0.92	0.89
Liquid asset holdings	2.89	6.71

Notes: Steady-state results in the case of separable preferences between nondurable and durable consumption. The first column shows results for the economy that is permanently in state G (the low-risk economy). The second column is the equivalent for aggregate state B (the high-risk, recessionary economy).

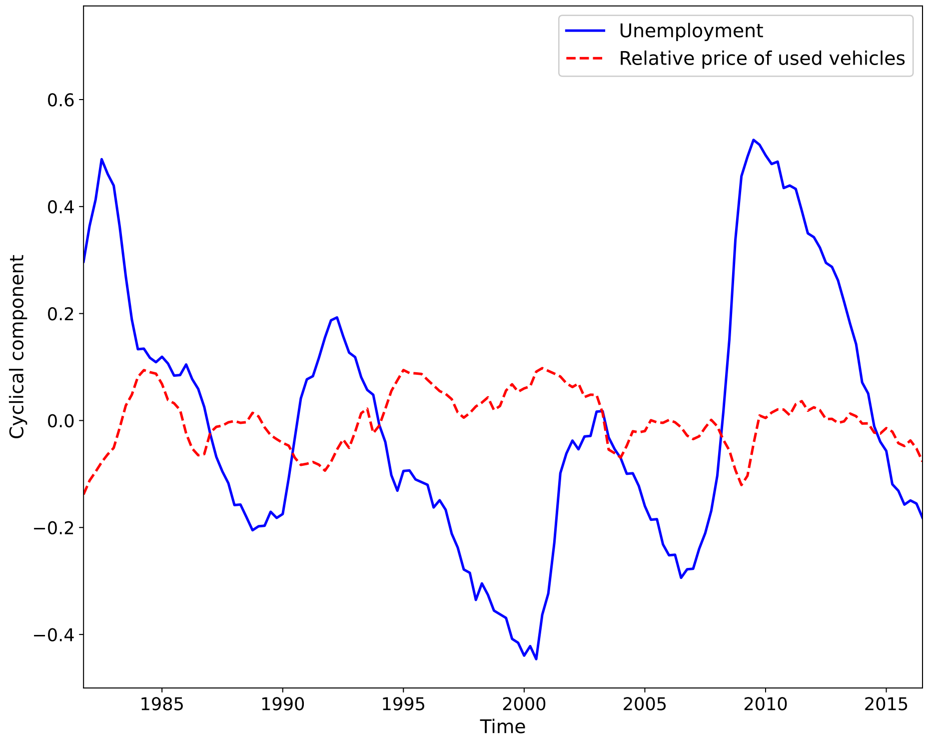
Table 1.5: **Output from Logit regressions**

	Model	
	Buy a vehicle	Sell a vehicle
Intercept	−4.0935*** (0.0021)	−4.0855*** (0.0016)
DECILE02	0.1367*** (0.0016)	0.3708*** (0.0012)
DECILE03	0.4222*** (0.0015)	0.7444*** (0.0012)
DECILE04	0.4844*** (0.0015)	0.8129*** (0.0011)
DECILE05	0.5088*** (0.0015)	0.8521*** (0.0011)
DECILE06	0.6323*** (0.0014)	0.986*** (0.0011)
DECILE07	0.5711*** (0.0014)	0.9296*** (0.0011)
DECILE08	0.2855*** (0.0014)	0.8108*** (0.0011)
DECILE09	0.5419*** (0.0014)	1.0533*** (0.0011)
DECILE10	0.3591*** (0.0014)	0.9162*** (0.0011)
UNRATE×DECILE02	0.014*** (0.0003)	−0.0252*** (0.0002)
UNRATE×DECILE03	−0.0067*** (0.0003)	−0.0528*** (0.0002)
UNRATE×DECILE04	0.0062*** (0.0002)	−0.0401*** (0.0002)
UNRATE×DECILE05	−0.0108*** (0.0002)	−0.0451*** (0.0002)
UNRATE×DECILE06	−0.0223*** (0.0002)	−0.0538*** (0.0002)
UNRATE×DECILE07	0.0015*** (0.0002)	−0.0327*** (0.0002)
UNRATE×DECILE08	0.0446*** (0.0002)	−0.0039*** (0.0002)
UNRATE×DECILE09	0.0034*** (0.0002)	−0.0454*** (0.0002)
UNRATE×DECILE10	0.0329*** (0.0002)	−0.0275*** (0.0002)
UNRATE	−0.0201*** (0.0003)	0.0344*** (0.0003)
FAM SIZE	0.0864*** (0.0)	0.033*** (0.0)
AGE REF	−0.0168*** (0.0)	−0.0091*** (0.0)
EDUC REF	−0.045*** (0.0)	−0.0497*** (0.0)
NUM AUTO	0.603*** (0.0001)	0.6002*** (0.0)
Quarter fixed effects	✓	✓
Year fixed effects	✓	✓
Income decile fixed effects	✓	✓
Observations	1016115	1016115

Notes: Estimation output from the Logit regressions with vehicle buying (first column) and selling (second column) decisions as dependent variables. The reported coefficient are the ones of the Logit regression and therefore do not have a direct interpretation. Standard errors are in parenthesis.

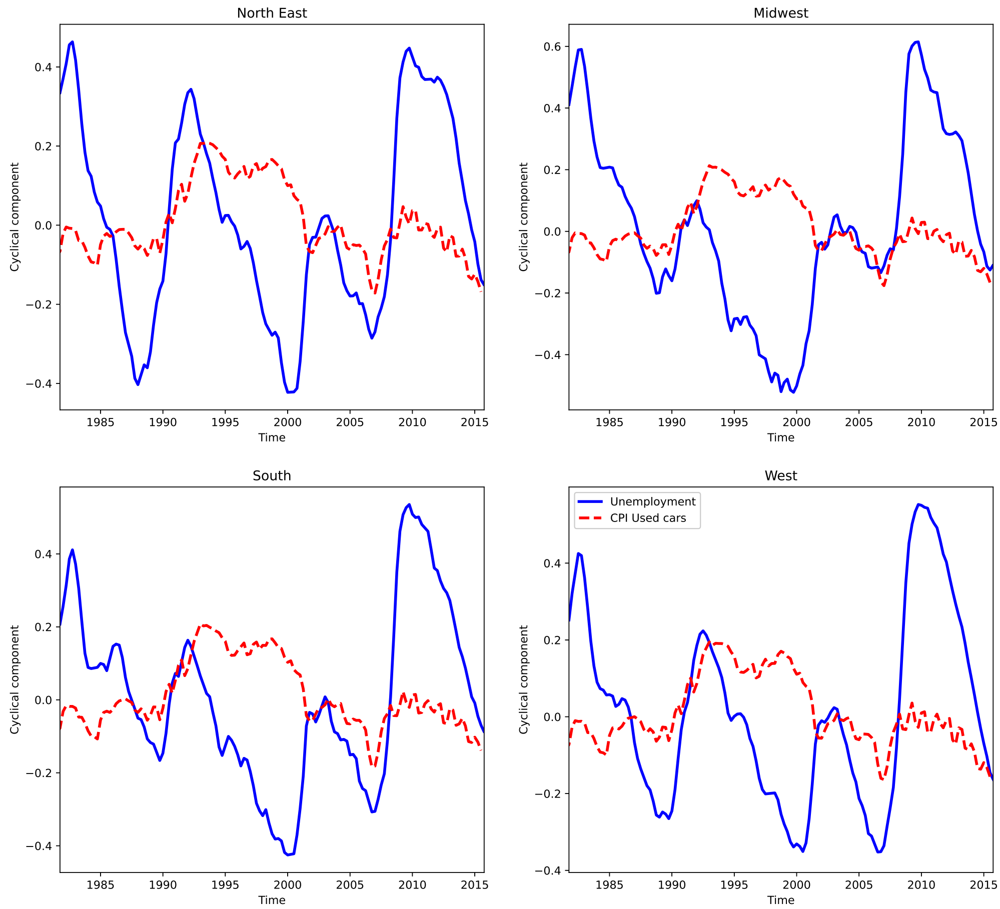
*10%, **5%, ***1%.

FIGURE 1.15: **Unemployment rate and relative price of used-to-new durables in U.S data**



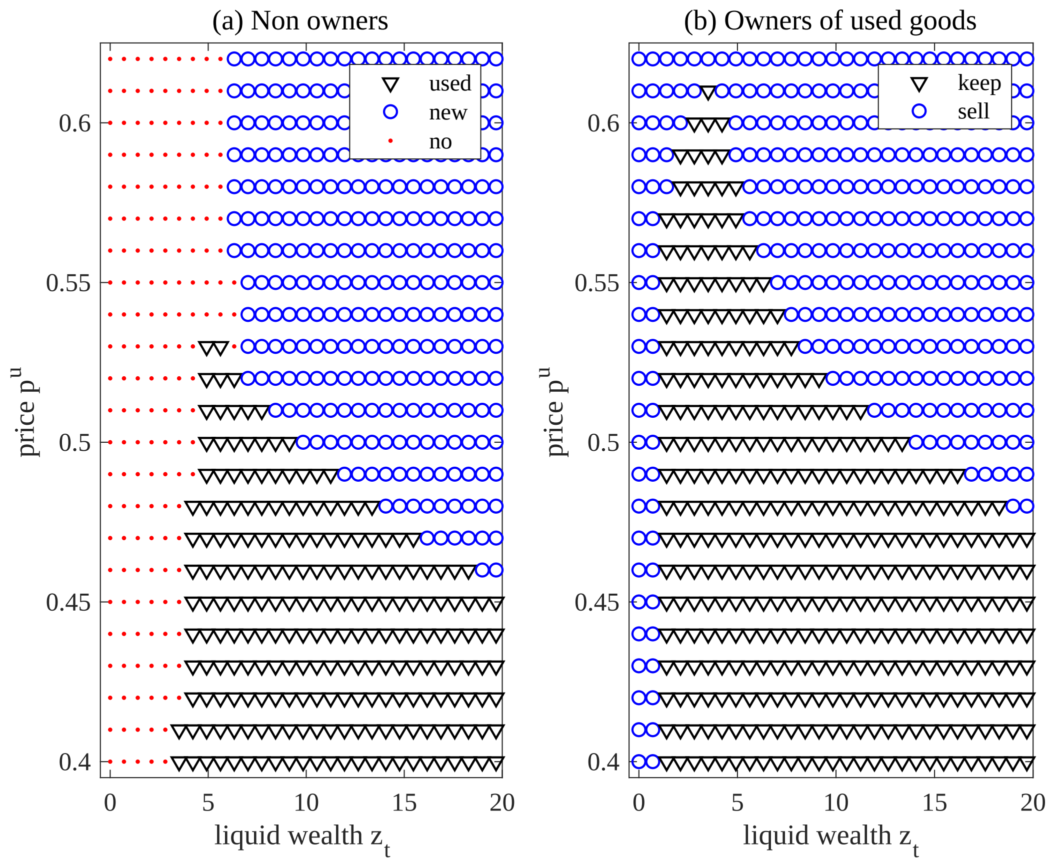
Notes: Cyclical components of quarterly U.S data. Solid blue line: Civilian Unemployment Rate. Dashed red line: Ratio of the Price Index for All Urban Consumers: Used cars and trucks to the Price Index for All Urban Consumers: New Vehicles. Both series are seasonally-adjusted. The series are logged and detrended using a linear filter. The correlation coefficient of the two series is -0.34.

FIGURE 1.16: **Unemployment and resale price of vehicle across US regions**



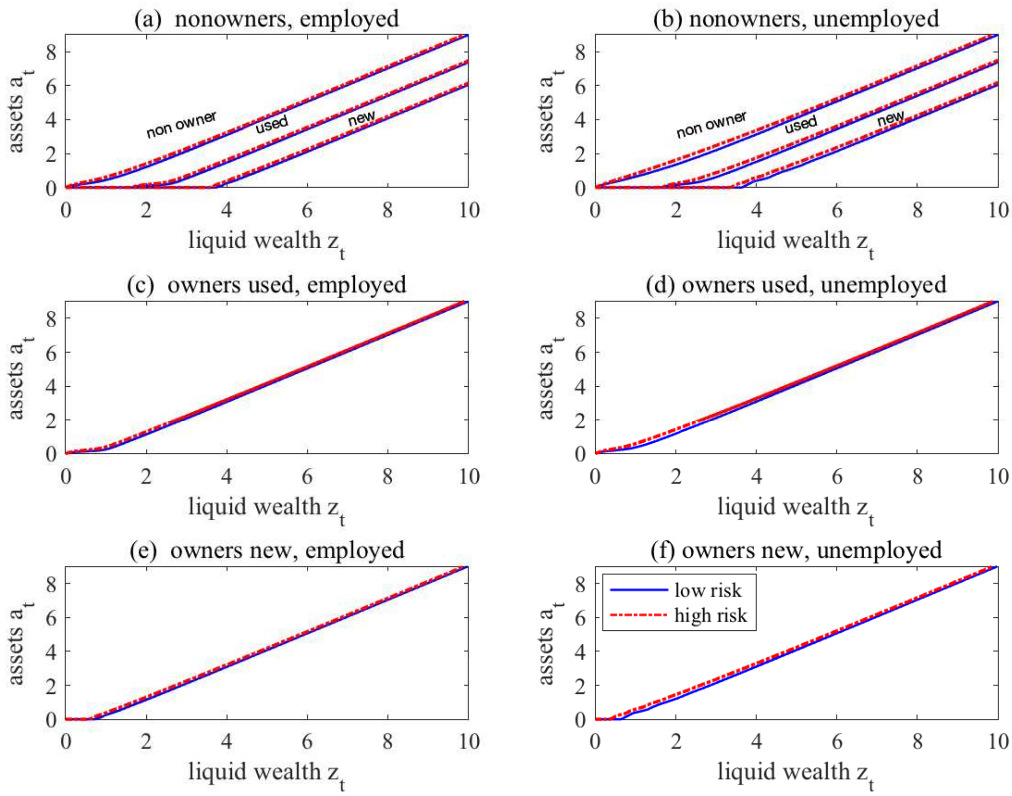
Notes: The figure presents the cyclical components of (log) unemployment (solid blue lines) and the (log) price index for used cars and trucks (dashed red lines), for each of the fours US census region. The figure is the counterpart of Figure 1.3 at the regional level.

FIGURE 1.17: **Optimal durable policies – low risk economy, unemployed agents**



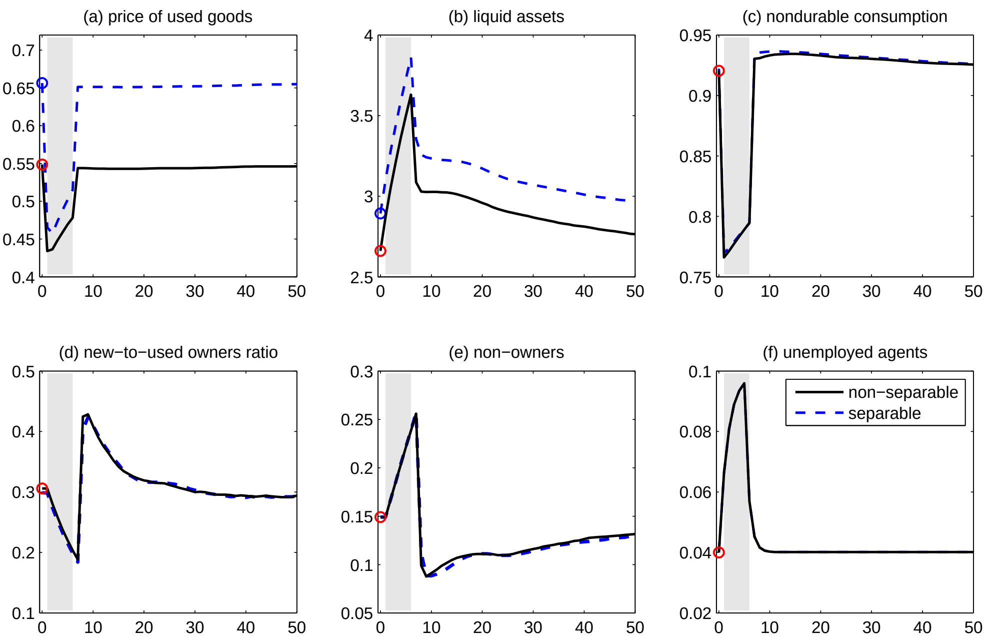
Notes: Optimal durable policies $\tilde{h}(a_{t-1}, \epsilon_t, h_t)$ over different values of the resale price of durables p^u, and aggregate state G. Optimal policies for **unemployed** agents ($\epsilon_t = 1$). Panel a: policies of non-owners ($h_t = o$). Panel b: policies of owners of used goods ($h_t = u$). For the latter, only the decision between adjusting ($\tilde{h}(a_{t-1}, \epsilon_t, h_t) \neq h_t$, the 'sell' region) or not ($\tilde{h}(a_{t-1}, \epsilon_t, h_t) = h_t$, the 'keep' region) is displayed. The x-axis denotes liquid wealth $z_t = a_{t-1} + \bar{y}$, and the y-axis the resale price p^u.

FIGURE 1.18: **Asset policies**



Notes: Asset policies $a(a_{t-1}, \epsilon_t, h_t)$. Solid blue lines are associated to quantities in the low-risk economy (aggregate state G). Dotted red lines concern aggregate state B. Panel a: $h_t = o$, $\tilde{h}_t = n, u, o$, $\epsilon_t = 1$; panel b: $h_t = o$, $\tilde{h}_t = n, u, o$, $\epsilon_t = 0$; panel c: $h_t = \tilde{h}_t = u$, $\epsilon_t = 1$; panel d: $h_t = \tilde{h}_t = u$, $\epsilon_t = 0$; panel e: $h_t = \tilde{h}_t = n$, $\epsilon_t = 1$; panel f: $h_t = \tilde{h}_t = n$, $\epsilon_t = 0$

FIGURE 1.19: **Transition path of model variables: non-separable vs. separable preferences**



Notes: Path of model variables following a transitory shock to unemployment probabilities. In $t = 0$, the economy is assumed to be in the long-run equilibrium under aggregate state G. Then, from $t = 1$ to 6 (the shaded areas) the aggregate state switches to B, before returning indefinitely to G. Solid black lines represent the behaviour of variables in the baseline specification with non-separable preferences between nondurable and durable consumption. The dashed blue lines are the counterpart for the case of separable preferences.

Chapter 2

Optimal Taxes and Transfers with Household Heterogeneity

Co-authored with François Courtoy (UCLouvain).

Abstract

We investigate the properties of optimal fiscal policy in a framework where household heterogeneity is accounted for. The Ramsey planner chooses (distortionary) labor taxes and transfers to maximize aggregate welfare in a two-agent economy. We contrast the properties of optimal labor taxes in our model to the ones obtained in the representative agent counterpart. We first show that the presence of household heterogeneity introduces an additional source of fluctuations in the optimal tax rate, as varying taxes allows the planner to use transfers for redistributive purposes. We then show that, depending on the assumptions that are made on how transfer receipts are distributed among households, and the type of shocks hitting the economy, the structure of government bond markets becomes more or less important in shaping the dynamics of the Ramsey allocation. In some cases, the presence of transfers brings the incomplete markets allocation close to the one in which the planner has access to state-contingent claims. We finally show that the presence of heterogeneity and optimal transfers helps bring the behaviour of fiscal variables in the Ramsey model closer to their counterpart in US data.

2.1 Introduction

The importance of heterogeneity at the household level in shaping economic fluctuations is the focus of a flourishing literature in macroeconomics. While many authors have studied the positive aspects of heterogeneity to analyze its impact on the cyclical properties of key macroeconomic aggregates, little work has been done that combines the cycle with the normative aspects of policy. This paper intents to fill this gap, by providing an analysis of optimal fiscal policies in models where households are heterogeneous in terms of labor productivity and holdings of financial assets.

The fiscal tools available to governments can broadly be seen as serving two main goals. First, deficit financing can be used along the business cycle to stabilize macroeconomic fluctuations; second, governments can use fiscal instruments to redistribute resources between economic agents and provide social insurance. While many papers in the optimal fiscal policy (OFP) literature have studied these two components in isolation, few authors have considered them at the same time. Indeed, broadly speaking, the OFP literature can be divided in two parts. On the one hand, several papers develop representative agent models in which fiscal instruments are used to mitigate the impact of aggregate shocks on macroeconomic variables[1]. These papers are therefore mostly concerned with the stabilization role of fiscal policy. On the other hand, numerous works have developed frameworks featuring household heterogeneity and in which the fiscal authority sets policy variables so as to redistribute resources between agents[2]. This line of work is therefore mostly concerned with the redistribution component of policy, and it usually relies on models where aggregate shocks are absent. In this paper, we consider a framework that weds these two approaches. We introduce a business cycle model which relies on a mild degree of heterogeneity, and consider fiscal instruments that allow the planner to redistribute resources across agents.[3] This implies a potential conflict between the stabilization of aggregate variables and the redistribution of market resources, and we are interested in looking at the trade-offs facing the fiscal authority.

The set of questions we want to address in this paper are the following. First, we intend to understand the properties of the optimal tax schedule when heterogeneity is accounted for. In particular, we want to investigate how the volatility of optimal labor taxes is impacted by the presence of household heterogeneity and the availability of transfers

[1] Two seminal papers in this literature are the work of Lucas and Stokey (1983) and Aiyagari, Marcet, Sargent, and Seppälä (2002).

[2] See, for instance, Conesa and Krueger (2006); Conesa, Kitao, and Krueger (2009) and Heathcote, Storesletten, and Violante (2017).

[3] In this way our paper relates to the consequent literature using the so-called TANK model to study the impact of household heteorogeneity on the macroeconomy (following the work of Galí, López-Salido, and Vallés, 2007 and Bilbiie, 2008). Note, however, that most of the work that has been done using this type of models has focused on monetary policy, while we are interested in the role of fiscal policy.

as a fiscal instrument for the Ramsey planner. We study to which extent variations in transfers rather than distortionary taxes are used to finance deficits, when it comes at the cost of increasing inequality in consumption and hours worked between households. In order to assess whether heterogeneity matters for the design of the optimal policy, we contrast our results to the ones obtained in the representative agent framework typically used in the literature. Later in the paper, we investigate whether our simple model can reproduce key stylized facts related to the conduct of fiscal policy in the US. In particular, we assess whether the cyclical properties of output, deficits, transfers, and government debt, are better matched by our two-agent framework than by its representative agent counterpart, when we calibrate the model parameters plausibly.

In our model, *savers*, or *Ricardian* households, have access to financial markets and save by accumulating government bonds. *Non-savers*, or *hand-to-mouth* households, do not have access to financial assets and consume their entire disposable income in every period. Households also differ in terms of their labor productivity, and we allow these differences to move over the business cycle (in other words, we allow for non-trivial interactions between aggregate and 'household-specific' shocks). Using this framework, we study the properties of optimal fiscal policy under commitment, where a benevolent government sets distortionary labor taxes and issues debt to finance government spending and transfers, so as to maximize aggregate welfare in the economy. Transfers are restricted to follow an empirically-motivated targeting rule which specifies their repartition between agents, and implies a co-movement between the transfers received by each household. The level of transfers is also set optimally by the planner, and we do not impose restrictions on their value. Therefore a government that prioritizes to smooth the burden from distortionary taxation can set transfers negative, thus relying on lump-sum taxes to finance deficits.

We first analyze the steady-state properties of the model, in which the planner chooses labor taxes and transfers to maximize aggregate welfare, for given levels of government expenditures and government debt. We provide a condition for the optimal long-run tax rate on labor to be positive, and show that it is not always optimal to set the labor tax rate to zero and finance the entire deficit in lump-sum. The optimal labor tax is positive when heterogeneity between households is sufficiently high, and when the design of transfers allows for sufficient redistribution between agents. In this case, increasing the tax rate and letting transfers be positive, allows the planner to redistribute across agents.

We then study the properties of optimal taxes and transfers in the presence of shocks, and look at the cyclical properties of the Ramsey allocation. We thus extend the analysis of Lucas and Stokey (1983) and Aiyagari et al. (2002, AMSS) to a two-agent setting. We derive an analytical expression for the optimal labor tax schedule, showing that tax volatility can be attributed to two components/sources. The first is related to market incompleteness, since as in AMSS we assume that debt is not state contingent. We show that transfers enable the planner to reduce tax volatility through (for example) lowering transfers when the deficit is high. This tends to bring the economy closer to a complete market outcome (e.g. Lucas and Stokey, 1983). The second component of tax volatility derives from heterogeneity in consumption and hours worked between households. Tax volatility is shown to be a function of the cyclical behavior of variables that summarize consumption and hours inequality. Using our simplistic model we investigate the relative importance of these two sources of volatility. We show that the quantitative impact of heterogeneity is much weaker than the impact arising from incomplete markets.

In the last part of the paper, we investigate the potential of an optimal fiscal policy model with heterogeneity to match the US data. We do so mainly to gain insights on the relative importance of heterogeneity and incomplete markets in shaping the behavior of fiscal variables and on the necessity of explictly modeling heterogeneity in models of optimal fiscal policy. We use a simulated method of moments (SMM) estimator to choose the parameter values that minimize the distance between moments generated by the model and those observed in the data. We find that the estimated model does a good job in matching the empirical properties of fiscal variables and for plausible parameter values. Our estimates suggest that a key driver of fluctuations in model variables are the shocks affecting the relative labor productivity of hand-to-mouth agents. In response to these shocks, the planner implements a policy which implies co-movement between transfers, deficits and GDP that is very similar to the analogous moments in the data. At the estimated parameters values, we find that the optimal allocation features less tax volatility than in a representative agent, incomplete markets model.

Related Literature Our paper contributes to the literature in several ways. First, we consider a model where household heterogeneity is accounted for using a two-agent setup, along the lines of Campbell and Mankiw (1989), Galí et al. (2007) and Bilbiie (2008).[4] This class of models has widely been used to look at the positive aspects of monetary and fiscal policies;[5] we take a normative stance and study the properties of optimal fiscal policy implemented by a planner that operates under full commitment and has access to two types of instruments: distortionary labor taxes and transfers.

Our model also builds on the seminal contributions of Lucas and Stokey (1983) and

[4] A growing literature studies the impact of household heterogeneity on macroeconomic outcomes in models where agents face idiosyncratic risks. The seminal contribution of Kaplan, Moll, and Violante (2018) studies monetary policy in their so-called HANK model. Other examples of papers this literature include Krueger, Mitman, and Perri (2016b), Auclert (2019), McKay and Reis (2016b), Werning (2015). Debortoli and Galí (2017) provide a useful comparison between the predictions obtained from TANK versus HANK models.

[5] Bilbiie and Ragot (2017),Challe et al. (2017) and Bilbiie (2018) use this framework to study optimal monetary policies.

Aiyagari et al. (2002). These two papers study the cyclical properties of Ramsey policies when, respectively, the planner has access to a full set of contingent securities (financial markets are complete), and when the planner can only issue one-period risk free bonds, such that markets are incomplete.[6] Scott (2007) and Marcet and Scott (2009) show that the co-movement between deficits and government debt is negative under complete markets, while it is positive under incomplete markets. Moreover, they show that Ramsey models with complete markets imply no persistence in government debt while incomplete markets models imply very persistent levels.[7] Empirically, the behaviour of fiscal variables seems to favor incomplete markets, as the observed correlation between deficits and the market value of debt is positive. However, the persistence of government debt under incomplete markets that is derived in these models overshoots the data counterpart (see Marcet and Scott (2009)). In this paper, we show that, when household heterogeneity is accounted for, and transfers are introduced, the obtained persistence of government debt gets closer to what is observed in the data.

While models featuring household heterogeneity have been widely used to study the long-run properties of the optimal tax schedule (see Aiyagari (1995), Conesa et al. (2009), Heathcote et al. (2017) and references therein), little work has been done on Ramsey policies in frameworks that integrate both heterogeneous agents and aggregate shocks. Notable exceptions are Werning (2007), Bassetto (2014), Bhandari, Evans, Golosov, and Sargent (2017b, 2018), and Le Grand and Ragot (2017). Le Grand and Ragot (2017) and Bhandari et al. (2018) develop numerical algorithms that approximate Ramsey allocations in economies with heterogeneous agents, uninsurable idiosyncratic risks, and aggregate shocks. They apply their novel methods to study, respectively, optimal capital taxation, and jointly optimal monetary and fiscal policies. Compared to these papers, ours adopts a simpler 2-agent structure, and focuses on the optimal use of labor taxes and transfers under complete and incomplete government bond markets, in a model without capital and nominal rigidities. Our simpler setup enables us to derive analytical formulae that shed light on the trade-offs facing the government. Bassetto (2014), Werning (2007) and Bhandari et al. (2017b) study optimal policies in settings closely related to ours. The novelty in our paper lies in its focus on the role that transfers and market (in)completeness play for the conduct of optimal fiscal policy.

Finally, given the emphasis we give to transfers and their role for social insurance along the business cycle, our paper is also related to Oh and Reis (2012), who study the stabilizing

[6]The implications of the maturity of the government debt, i.e. the existence of both short and/or long term bonds, on the properties of optimal fiscal policy are well understood. See, for instance, Angeletos (2002), Buera, Nicolini et al. (2004), and Faraglia, Marcet, Oikonomou, and Scott (2019a,b).

[7]The literature has also explored the role of monetary policy in stabilizing government debt. It has been shown that, in the presence of nominal rigidities, inflation volatility has too big welfare costs to become an appropriate margin to stabilize debt. See e.g. Schmitt-Grohé and Uribe (2004) and Faraglia, Marcet, Oikonomou, and Scott (2013).

role of transfers in the Great Recession in a model with heterogeneous households and idiosyncratic risk; McKay and Reis (2016a,b) use a similar framework to look at the role of automatic stabilizers in the business cycle. Transfers in our model can also be used by the planner as a social insurance tool which has a role in mitigating the impact of aggregate shocks. However, contrarily to these papers, we focus on the joint use of transfers and other fiscal instruments such as debt and labor taxes and the implied trade-offs between providing insurance and minimizing labor tax distortions.

The remainder of the paper is organized as follows. In Section 2.2, we provide key stylized facts regarding the behaviour of fiscal variables in the US, using both macro and micro data sources. Section 2.3 describes the building blocks of the model that we use in the subsequent analysis. Section 2.4 and 2.5 contain the results obtained from the model, with a focus on the steady-state properties and the dynamics implied by the Ramsey allocation. Section 2.6 concludes.

2.2 Fiscal policy in the United States

In this section we present facts on the cyclical and distributional properties of fiscal variables in the US, focusing on the behaviour of transfers. The objective is twofold. First, we want to provide a clear definition of the notion of transfers that we will use throughout the paper. Second, we want to shed light on the data properties of transfers that we will use to justify the way we design transfers in the theoretical model presented in the next section. We first use aggregate data to describe the main cyclical properties of transfers. Then, we make use of household-level data to analyze the redistributive aspects of policy.

We use data from the NIPA tables to look at the aggregate properties of transfers. In this paper, we will focus on the components of transfers which are targeted towards households. We therefore construct our aggregate series accordingly, and define transfers as the sum of unemployment insurance payments, and other social benefits such as food stamps and income assistance programs. A complete description of our data series is provided in Appendix 2.7.3.[8] We now study the cyclical properties of our transfer series, and its relation with other components of the US federal budget. We summarize the observed properties with the following :

Fact 1: *Transfers are counter-cyclical, and are strongly correlated with primary deficits.*

[8] Oh and Reis (2012) show that the fiscal expansion during the Great Recession was also mostly driven by increases in social transfers. While unemployment benefits and income assistance programs explain a large share of the increase in transfers, social security and Medicare expenditures also increased significantly during the period 2007-2009.

In Table 2.1 we provide the correlation matrix of the cyclical components of transfers, primary deficits, and GDP. From this table we can observe that transfers and GDP have a negative correlation (-0.45). This observed counter-cyclicality of transfers can be attributed to the fact that, in recessions, governments use social insurance schemes to alleviate the drop in income that households suffer, and therefore (partially) insure them against fluctuations. We will further investigate this insurance component while analysing transfers using micro data sources.

[Table 2.1 approximately here]

Another noteworthy feature is the strong positive correlation between transfers and deficits observed from the table. It suggests that transfers are an important component of deficits, or at least that the government is not reducing transfers when deficits are high.

Figure 2.1 illustrates the negative correlation between GDP and transfers and the positive correlation between transfers and deficits. It shows that transfers are not used by the government to stabilize deficits. On the contrary, they are one of the most important components enabling public finances to fulfil their automatic stabilization role.

[Figure 2.1 approximately here]

We now provide empirical evidence on the behaviour of transfers in the cross-section. We use data from the Consumer Expenditure Survey (CEX), a quarterly survey of consumption expenditure by US households conducted by the Bureau of Labor Statistics. Besides detailed consumption data, the survey provides information on labor earnings and transfers received by households that are interviewed. We make use of these records to construct data series that summarize the behaviour of these variables across different household groups. Details on the construction of our series are provided in Appendix 2.7.3.

To perform our analysis, we proceed as follows. We first classify households according to the value of their yearly earnings, in order to divide them in two groups. The first group contains households at the bottom 30% of the income distribution, while the other group contains the remaining 70%.[9] Then, we aggregate data series across households in each

[9] In the theoretical framework developed in the next section, we distinguish between *hand-to-mouth* and *Ricardian households*. The first group is defined as households which do not have access to a savings technology and therefore consume their entire disposable income in every period. As the data we use do not contain information on wealth and asset holdings, we use labor earnings as a proxy for these variables, and assign households to subgroups according to this criterion.

group to look at how the empirical properties of income and transfers vary across income groups.[10]

We define individual transfers in a similar way as for the aggregate series described above. The aggregate transfer series obtained from the micro database shares the same cyclical properties as the macro data series used previously.

Let us now turn to the second stylized fact:

Fact 2: *Transfers are unevenly targeted towards low-income households.*

In Figures 2.2 and 2.3 we plot, respectively, the evolution over time of the value of per capita transfers for each of the two household groups defined above, and the total share of transfers received by households in the low-income group.[11] We can observe from these figures that on average, the value of transfers directed towards households at the bottom 30% of the income distribution is greater than the one received by households in the high income group. Indeed, as depicted in Figure 2.3, while hand-to-mouth agents represents 30% of the population, they receive about two third of total transfers: the average value for this share is 67% for our sample period. This confirms that transfers have a redistributive aspect, and that low-income individuals are more likely to benefit from them.

[Figures 2.2 and 2.3 approximately here]

Fact 3: *Transfers directed towards low and high income groups are strongly correlated.*
To finish this section, we want to emphasize the strong correlation between the cyclical component of transfers across groups. The correlation coefficient between the two series plotted in Figure 2.2 is equal to 0.79, implying a huge co-movement of the transfer series across the two considered income groups. This observation will be used in the next section to justify the assumption made in our theoretical model of a unique transfer that is split between households, rather than separate transfer processes targeting individual households independently.

[10]While CEX data are collected at a quarterly frequency, households only report the values of earnings and transfers received over the preceding 12 months. Hence, the quarterly series we get for these variables have some inertia, and are not as volatile as they should be, which does not allow us to study the dynamics of earnings and income inequality in greater detail. However, it is sufficient to get a broad overview of the dynamics of transfers at the micro level.

[11]In Appendix 2.7.3, we provide the transfer series by income quintile and show that the relation between income and transfer receipts described here is similar using this level of aggregation.

2.3 A simple two-agent model

In this section, we consider a simple two-agent model to outline the key features of optimal fiscal policy when household heterogeneity is accounted for. We first lay out a model where the government can only issue one period risk-free debt, and markets are incomplete. Then, we set up a model in which the planner issues state-contingent claims and markets are complete. Contrasting these two cases will allow us to study the implications of the market structure on the optimal behaviour of taxes and transfers, and characterize its interplay with heterogeneity.

Two types of households populate the economy: a fraction $1 - \lambda$ of agents are Ricardian, or *savers*, and have access to financial markets: they can accumulate government bonds to smooth consumption over time. The remaining fraction λ of agents are hand-to-mouth, or *non-savers*; they do not have access to the savings technology and consume their entire disposable income every period. Labor supply is chosen optimally by each household. There is no capital, and production is linear in total labor effort. We consider uncertainty deriving from three sources: shocks to government spending, to aggregate productivity, and individual productivity shocks relevant only for hand-to-mouth agents.

In what follows, we describe in detail the main building blocks of the model.

2.3.1 Model description

Government: The government has to finance an exogenous stream of spending denoted g_t, and transfers T_t, which are chosen optimally. Transfers are allowed to be negative, in which case they are essentially lump-sum taxes. To generate revenues, the government also taxes labor at the rate $0 \leqslant \tau_t \leqslant 1$. The latter is also chosen optimally by the Ramsey planner. Finally, the government issues debt, which takes the form of one-period discount government bonds. Current bond issuances are denoted b_t, and their price is q_t. The intertemporal government budget constraint is written as:

$$b_{t-1} = \tau_t a_t n_t - g_t - T_t + q_t b_t \tag{2.1}$$

where a_t and n_t denote respectively aggregate productivity and total labor supply. These objects are described below.

Modelling transfers: One of the goals of this paper is to explore the effect of transfers and their design on the properties of optimal fiscal policy. As we will show, the assumptions we make on the transfer schedule have important implications for the outcomes of the Ramsey allocation. It is therefore crucial to model transfers in an appropriate and empirically-relevant way. For instance, we could allow for household-specific transfers, that could be denoted T_t^i, for agent $i = h, s$, where h and s denote respectively hand-to-mouth and Ricardian households. The government would choose their optimal value and total transfers would simply equal their weighted sum: $T_t = \lambda T_t^h + (1-\lambda)T_t^s$. However, such an assumption is unappealing for two reasons. First, letting the planner choose household-specific transfer levels would allow her to reach the first-best allocation, thereby leading to trivial results. In such allocation, labor taxes would be set to zero and, as is shown below, the planner would choose transfers to equate marginal utilities of consumption across households.

Second, household-specific transfers, which imply that individual transfers can potentially evolve very differently across households, are not a good approximation of the behaviour of transfers observed in the data. This would violate the last stylized fact of the previous section, according to which individual transfers are strongly correlated across households.

For the above reasons, we choose to model transfers as a unique process T_t. However, to make it consistent with our second stylized fact (according to which transfers are unevenly targeted towards a subset of households), we introduce a targeting rule, which specifies the share of transfers going towards each of the two households populating the economy. More precisely, we assume that a share $\omega \in [0,1]$ of total tranfers is targeted towards hand-to-mouth agents. The per capita transfers to hand-to-mouth households and savers are therefore respectively given by $T_t^h = \frac{\omega}{\lambda}T_t$ and $T_t^s = \frac{1-\omega}{1-\lambda}T_t$. Therefore, when $\omega = \lambda$, transfers are equally distributed among the population ($T_t^h = T_t^s = T_t$), and whenever $\omega > \lambda$ transfers are targeted towards hand-to-mouth agents ($T_t^h > T_t^s$).[12] Although our targeting rule is simple and abstracts from many elements characterizing the design of transfers in the US, we believe that our assumptions enable us to develop a workable approximation of the actual transfer schedule.

Ricardian agents: Ricardian households choose consumption, labor supply, and holdings of government bonds to maximize their expected lifetime utility, which is expressed as:

$$E_0 \sum_{t=0}^{\infty} \beta^t \Big(u(c_t^s) - v(n_t^s) \Big) \tag{2.2}$$

where c_t^s and n_t^s denote respectively the consumption and the labor effort exerted by the agent at time t. Period utility is assumed to be separable between consumption and labor. We make common assumptions on the functions $u(\cdot)$ and $v(\cdot)$, namely $u' > 0$,

[12] Throughout the paper, we assume that ω is exogenous, and constant over time. While we believe that allowing the planner to optimally choose ω is an interesting exercise, we abstract from this in our model.

$u'' < 0$, and $v' > 0$, $v'' > 0$.

The budget constraint of savers, which has to be satisfied in every time period t, can be expressed as:

$$c_t^s + q_t b_t^s = (1 - \tau_t) a_t n_t^s + b_{t-1}^s + \frac{1-\omega}{1-\lambda} T_t \tag{2.3}$$

This constraint states that to finance consumption and the accumulation of newly issued government bonds (b_t^s denotes the quantity of bonds held by savers), the household uses its labor income net of taxes, transfers, and the payoff on government bonds issued in $t-1$. Note that the coefficient $\frac{1-\omega}{1-\lambda}$ appearing in front of aggregate transfers T_t is consistent with our targeting rule described above.

First order conditions for hand-to-mouth agents give:

$$\frac{v_{n,t}^s}{u_{c,t}^s} = a_t(1 - \tau_t) \tag{2.4}$$

$$q_t = \beta \frac{E_t u_{c,t+1}^s}{u_{c,t}^s} \tag{2.5}$$

where $u_c \equiv \frac{\partial u}{\partial c}$ and $v_n \equiv \frac{\partial v}{\partial n}$ denote, respectively, the marginal utility of consumption, and the marginal disutility of work. The first equation is a standard labor supply condition, equating the marginal rate of substitution between leisure and consumption to (net) labor income. The second equation is the usual Euler equation, which also constitutes the pricing condition for government bonds and has to be accounted for by the social planner.

Hand-to-mouth agents: Non-savers do not have access to financial markets and are therefore constrained in each period to consume their entire disposable income. They choose labor supply optimally to maximize their period utility $u(c_t^h) - v(n_t^h)$, subject to the budget constraint:

$$c_t^h = (1 - \tau_t)\theta_t^h a_t n_t^h + \frac{\omega}{\lambda} T_t \tag{2.6}$$

Variable θ_t^h denotes an exogenous shock to the relative productivity of hand-to-mouth agents. If $\theta_t^h < 1$, hand-to-mouth agents are less productive than Ricardian Households; this might provide incentives to the planner to tilt transfers towards hand-to-mouth agents.

The introduction of shocks to the relative productivity of hand-to-mouth households is useful to capture the following properties. First, on average the revenue of hand-to-mouth households is lower than the revenue of savers in the data. Second, the cyclical properties of their income are also likely to differ from the ones of savers. In the Appendix, we show that in our data, the earnings of households at the bottom of the distribution are more volatile than the ones of richer agents.[13]

Utility maximization by hand-to-mouth agents gives the labor supply condition:

$$\frac{v^h_{n,t}}{u^h_{c,t}} = \theta^h_t a_t (1 - \tau_t) \tag{2.7}$$

Equilibrium We define aggregate consumption and hours worked, respectively, as follows:

$$\begin{aligned} c_t &= \lambda c^h_t + (1-\lambda) c^s_t \\ n_t &= \lambda \theta^h_t n^h_t + (1-\lambda) n^s_t \end{aligned}$$

Equilibrium in the market for government debt implies $b^s_t = \frac{1}{1-\lambda} b_t$.

We can use the government budget constraint, and the individual budget constraints of both households to obtain the economy-wide resource constraint:

$$c_t + g_t = a_t n_t \tag{2.8}$$

2.3.2 Optimal policy

The Ramsey problem The Ramsey planner maximizes aggregate welfare, which we define as the weighted sum of individual expected lifetime utility functions:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \Big[\lambda \Big(u(c^h_t) - v(n^h_t) \Big) + (1-\lambda) \Big(u(c^s_t) - v(n^s_t) \Big) \Big] \tag{2.9}$$

The Ramsey planner maximizes (2.9) subject to her inter-temporal budget constraint (2.1), households optimality conditions (2.4), (2.5) and (2.7), the resource constraint (2.8), and the budget constraint of hand-to-mouth agents (2.6).

[13]We also show in the Appendix that the correlation between the revenues of low-income households and GDP is higher than the one obtained for high-income households: in other words, the revenues of low-income agents is more pro-cyclical. While we believe that the implications of this property for optimal fiscal policy is an interesting issue, we abstract from this in the current version of the paper.

The Lagrangian associated to the planner's optimization program is:

$$\begin{aligned}
\mathcal{L} \;=\; & E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ \lambda \Big[u(c_t^h) - v(n_t^h) \Big] + (1-\lambda) \Big[u(c_t^s) - v(n_t^s) \Big] \qquad (2.10) \\
& + \Psi_t^{IM} \Big[- u_{c,t}^s b_{t-1} + (u_{c,t}^s a_t - v_{n,t}^s)(\lambda \theta_t^h n_t^h + (1-\lambda) n_t^s) - u_{c,t}^s (g_t + T_t) + \beta u_{c,t+1}^s b_t \Big] \\
& + \Psi_t^1 \Big[a_t \lambda \theta_t^h n_t^h + (1-\lambda) n_t^s a_t - \lambda c_t^h - (1-\lambda) c_t^s - g_t \Big] + \Psi_t^2 \Big[\frac{v_{n,t}^s}{u_{c,t}^s} - \frac{1}{\theta^h} \frac{v_{n,t}^h}{u_{c,t}^h} \Big] \\
& + \Psi_t^3 \Big[- u_{c,t}^s c_t^h + v_{n,t}^s \theta_t^h n_t^h + \frac{\omega}{\lambda} u_{c,t}^s T_t \Big]
\end{aligned}$$

where we made use of Ricardian agents' Euler equation and labor supply condition to substitute away the bond price q_t and the labor tax rate τ_t, thereby making use of the primal approach to solve for optimal policies (see Lucas and Stokey, 1983). The optimality conditions associated to the planner's problem are the following:

$$\begin{aligned}
c_t^h \;&:\; \lambda u_{c,t}^h - \lambda \Psi_t^1 + \frac{\Psi_t^2}{\theta_t^h} \frac{v_{n,t}^h}{(u_{c,t}^h)^2} u_{cc,t}^h - \Psi_t^3 u_{c,t}^s = 0 \\
c_t^s \;&:\; (1-\lambda) u_{c,t}^s + \Psi_t^{IM} \Big[u_{cc,t}^s a_t n_t - u_{cc,t}^s (g_t + T_t) \Big] + (\Psi_{t-1}^{IM} - \Psi_t^{IM}) u_{cc,t}^s b_{t-1} \\
&\quad - \Psi_t^1 (1-\lambda) - \Psi_t^2 \frac{v_{n,t}^s}{(u_{c,t}^s)^2} u_{cc,t}^s + \Psi_t^3 (-u_{cc,t}^s c_t^h + \frac{\omega}{\lambda} T_t u_{cc,t}^s) = 0 \\
n_t^h \;&:\; -\lambda v_{n,t}^h + \Psi_t^{IM} (u_{c,t}^s a_t - v_{n,t}^s) \lambda \theta_t^h + \Psi_t^1 a_t \lambda \theta_t^h - \frac{\Psi_t^2}{\theta_t^h} \frac{v_{nn,t}^h}{u_{c,t}^h} + \Psi_t^3 v_{n,t}^s \theta_t^h = 0 \\
n_t^s \;&:\; -(1-\lambda) v_{n,t}^s + \Psi_t^{IM} \Big[(1-\lambda)(u_{c,t}^s a_t - v_{n,t}^s) - v_{nn,t}^s n_t \Big] + \Psi_t^1 (1-\lambda) a_t + \Psi_t^2 \frac{v_{nn,t}^s}{u_{c,t}^s} + \Psi_t^3 v_{nn,t}^s \theta_t^h n_t^h = 0 \\
T_t \;&:\; -\Psi_t^{IM} u_{c,t}^s + \Psi_t^3 \frac{\omega}{\lambda} u_{c,t}^s = 0 \\
b_t \;&:\; -E_t \Psi_{t+1}^{IM} u_{c,t+1}^s + \Psi_t^{IM} E_t u_{c,t+1}^s = 0
\end{aligned}$$

The first-best allocation as a useful benchmark In Appendix 2.7.2 we show that the first-best allocation in our two-agent model can be characterized by the following conditions:

$$\frac{v_{n,t}^h}{\theta_t^h u_{c,t}^h} = \frac{v_{n,t}^s}{u_{c,t}^s} = a_t$$
$$u_{c,t}^h = u_{c,t}^s$$

The first equalities provide a well-known optimality condition which can also be derived from representative agent models: it states that the marginal rate of substitution between consumption and leisure must be equal to the marginal product of labor, which is equal to a_t for Ricardian households, and $\theta_t^h a_t$ for hand-to-mouth households. This allocation can be attained by the planner if it is possible for her to set $\tau_t = 0$ for all t (which is the case if, for instance, she only makes use of variations in transfers to finance fiscal deficits), and therefore eliminate tax distortions.

The second condition equates marginal utilities of consumption across agents. It therefore states that the first-best allocation features complete consumption equality, given the assumptions made on the function $u(\cdot)$. In the framework considered here, it is in some cases possible for the planner to make this condition hold by using transfers to redistribute resources across agents. However, as such a policy often comes at the cost of increasing the level and volatility of labor taxes, this in turn prevents the planner from getting close to the first set of conditions mentioned above. This therefore creates a trade-off between efficiency and redistribution; one of the goals of this paper is to investigate the nature and the resolution of such a trade-off by the Ramsey planner.

2.3.3 Complete vs. incomplete markets

In this section, we briefly outline how the complete markets version of the model looks like. The Ramsey problem under complete markets is similar to the one which has been described above for the incomplete market case; the main difference lies in the government budget constraint.

In the model presented in the previous section, the state of the economy at time t can be represented by the object $s^t = \{s_0, s_1, ..., s_t\} = \{s^{t-1}, s_t\}$, where s_t denotes the state vector at time t. We assume that shocks are Markovian, implying that the evolution of exogenous processes can be represented by a density function of the form $f(s^t|s_{t-1})$.[14]

For the sake of exposition, in this section we rewrite model variables as $x_t = x(s^t)$ for any variable x. This way, we explicitly acknowledge the mapping between the state object s^t and the value taken by any variable x at time t. Using this notation, we can rewrite the government budget constraint under incomplete markets, specified in (2.1), as:

$$b(s^{t-1}) = \overline{S}(s^t) + q(s^t)b(s^t) \tag{2.11}$$

where $\overline{S}_t \equiv \tau_t a_t n_t - g_t - T_t$ is the primary surplus at time t. Using Ricardian agents' Euler equation (2.5) and iterating equation (2.11) forward, we can express the inter-temporal budget constraint as follows:

$$u_c^s(s^t)b(s^{t-1}) = E_0 \sum_{j=0}^{\infty} \beta^j \overline{S}(s^{t+j}) u_c^s(s^{t+j}) \tag{2.12}$$

[14]As stated above, we consider three types of shocks in this model version: shocks to government spending, to aggregate productivity, and to the relative productivity of hand-to-mouth households.

This equation has to hold for any t (acknowledging that $b(s^{-1}) = b_{-1}$: the initial value of government debt is taken as given by the government); this expression is very standard: it states that the inherited value of government debt (in units of marginal utility) has to be equal to the discounted sum of future primary surpluses.

When financial markets are complete, the government issues in each period t, a portfolio of state-contingent bonds $b^{CM}(s^{t+1})$ at prices $q^{CM}(s^{t+1})$. The dependence of b^{CM} on the state vector in $t+1$ makes it clear that the payoff on government debt depends on the state of the economy when bond payments are due. In this scenario, the government budget constraint can be written as follows:

$$b^{CM}(s^t) = \overline{S}^{CM}(s^t) + \int_{s^{t+1}} q^{CM}(s^{t+1}) b^{CM}(s^{t+1})\, ds^{t+1} \tag{2.13}$$

From savers' first order conditions (see Appendix 2.7.2), we obtain the pricing equation:

$$u_c^{s,CM}(s^t) q^{CM}(s^{t+1}) = \beta f(s^{t+1}|s_t) u_c^{s,CM}(s^{t+1}) \tag{2.14}$$

This equation has to hold for any s^{t+1}. Combining equations (2.13) and (2.14), and iterating forward, we get:

$$u_c^{s,CM}(s^t) b(s^t) = E_t \sum_{j=0}^{\infty} \beta^j \overline{S}^{CM}(s^{t+j}) u_c^{s,CM}(s^{t+j}) \quad \text{for } t \geqslant 1 \tag{2.15}$$

$$u_{c,0}^{s,CM} b_{-1} = E_0 \sum_{t=0}^{\infty} \beta^t \overline{S}^{CM}(s^t) u_c^{s,CM}(s^t) \quad \text{for } t = 0 \tag{2.16}$$

where $E_t x(s^{t+j}) \equiv \int_{s^{t+j}} x(s^{t+j}) f(s^{t+j}|s_t)\, ds^{t+j}$ is the conditional expectation of variable x at time $t+j$ based on the information set at time t. As mentioned earlier, in any period $t \geqslant 1$, the state-contingent nature of government debt makes debt payments b^{CM} a function of the current state vector s^t. It follows that the constraint (2.15) is slack (and therefore irrelevant for the Ramsey planner) for $t \geqslant 1$, as the government can freely choose the value of $b(s^t)$ which makes the constraint non-binding. In period 0, however, the inherited the level of government debt is b_{-1} is given; the planner has no influence on this value. This makes equation (2.16) the only binding constraint that has to be part of the planner's constraint set.

It follows from the above discussion that when markets are complete, only the intertemporal budget constraint in the initial period $t = 0$ needs to hold; this is a standard property of optimal fiscal policy models with complete financial markets (see Lucas and Stokey, 1983). The full set of constraints that have to hold in this model version is therefore constituted of equations (2.4), (2.6), (2.7), (2.8) and (2.16). A full description of the Ramsey program under complete markets and the associated first order conditions are provided in Appendix 2.7.2.

2.3.4 Calibration and functional forms

We describe here the parameter values and functional forms that will be used in the subsequent analysis and numerical simulations. The period utility function is separable between consumption and leisure, and takes the form:

$$u(c) - v(n) = \log(c) - \frac{n^{1+\phi}}{1+\phi} \tag{2.17}$$

The three shock processes are all assumed to follow AR(1) process in logs:

$$\log\left(\frac{x_t}{x}\right) = \rho_x \log\left(\frac{x_{t-1}}{x}\right) + \epsilon_{x,t} \tag{2.18}$$

for $x = g, a, \theta^h$. Shocks $\epsilon_{x,t}$ are iid with mean zero and standard deviations σ_x.

[Table 2.2 approximately here]

The parameter values are summarized in Table 2.2. The model horizon is annual, and we set the discount factor to $\beta = 0.95$. The inverse Frisch elasticity ϕ is set to unity. The share of hand-to-mouth agents is $\lambda = 0.3$, a value which is standard in the literature.[15]

We assume a steady-state debt-to-gdp ratio of 60%, and set the steady-state value of government expenditures to 15% of output.

Given the crucial role played by the individual productivity of constrained agents θ^h and the share of transfers going to these agents ω, we will allow for different values of these parameters in the next section. This will allow us to study how these parameters affect the properties of the Ramsey allocation. However, when not stated otherwise, we consider the benchmark case of $\theta^h = 1$, i.e. the scenario under which the two agents are equally productive in the steady-state. As for ω, there are two cases that we focus on: the case where $\omega = \lambda$ (transfers are evenly distributed among households, and can therefore be considered as purely lump-sum); and the case where $\omega = 1$ (all the transfers are targeted towards hand-to-mouth agents). We will present results for these two cases throughout the rest of the paper.

[15] This is also consistent with the empirical evidence provided in Kaplan and Violante (2014), according to which around 30% of US households can be considered as being hand-to-mouth.

2.4 Steady-state analysis

In this section we study the steady-state properties of the model. We first analyze the set of equilibrium outcomes that are attainable to the planner for the combinations of taxes and transfers satisfying the budget constraint; this gives us a sense of the objective of the planner. We then solve for the optimal long-run allocation, and study how taxes and transfers vary as a function of key model parameters.

2.4.1 Long-run tradeoffs

We assume that the government inherits a long-run level of government debt b_{-1}; the steady-state level of government spending is given and equal to g. In the steady-state, the government budget constraint is:[16]

$$(1-\beta)b_{-1} = \tau n - g - T \tag{2.19}$$

where dropping time subscripts from the variables denotes the steady-state. When choosing taxes and transfers, the planner faces a trade-off between efficiency and redistribution. On the increasing side of the Laffer curve, a higher labor tax rate τ increases tax revenues, thereby allowing the government to raise transfers T, which potentially has a redistributive role. However, this comes at the cost of distorting labor supply, and therefore reducing aggregate output. In standard models featuring a representative agent, the planner will always choose to set the distortionary tax on labor to zero, and to finance the entire deficit through lump-sum taxes (negative transfers). We study here whether the same property holds when agents are heterogeneous.

Figure 2.4 displays steady-state consumption inequality, measured as the ratio of hand-to-mouth to Ricardian households' consumption ($\frac{c^h}{c^s}$), as a function of the labor tax rate (τ). The first best allocation (which, as mentioned before, is the allocation which sets the labor tax rate to 0 and the consumption ratio to 1), is depicted in the figure by a red cross.

[Figure 2.4 approximately here]

We can see from the left panel of the figure (which assumes $\theta^h = 1$) that increases in the labor tax rate have almost no effect on consumption inequality if $\omega = \lambda$. In

[16]Note that the steady-state allocations under complete and incomplete markets, for a given level of inherited government debt, are the same. It implies, among others, that the steady-state multipliers on the government budget constraint are equal ($\psi^{IM} = \psi^{CM} = \psi$), and so are the optimal labor tax rate and transfers.

such a scenario transfers are evenly targeted between households, and as a result, a tax-financed increase in transfers does not allow for redistribution. However, when $\omega = 1$, an increase in transfers financed by an increase in the labor tax rate has a strong impact on the consumption ratio $\frac{c^h}{c^s}$, and such a policy allows the planner to reduce consumption inequality. Note that, as there is little ex-ante heterogeneity between households (the two agents are equally productive), the first-best is almost attainable by the planner when $\omega = \lambda$. Moreover, in such a case, increases in the tax rate *increase* consumption inequality, so there is no trade-off between efficiency and redistribution and under this parameterization (as will be shown below), the optimal allocation features a zero labor tax rate.

When hand-to-mouth households are less productive than Ricardian households (as in the right panel of Figure 2.4, for which we assume $\theta^h = 0.7$), a similar picture arises: increases in the labor tax rate affect consumption inequality only if transfers are unevenly distributed among households (which is the case when $\omega = 1$). However, the first best solution is now unreachable at any value of ω. Under an evenly distributed transfer rule, the planner never achieves to equalize consumption between households, while it requires a large labor tax rate to ensure consumption equality between households when transfers are targeted towards hand-to-mouth households.

Figure 2.4 shows that in some cases, increases in transfers (financed by positive labor taxes) allow the planner to reduce the consumption gap between households. Nevertheless, because such a policy implies efficiency losses (as higher labor taxes distort labor supply), we might wonder whether such a policy can be welfare improving. In the following section, we show under which conditions it is indeed optimal for the planner to set a positive labor tax rate.

2.4.2 Optimal taxes and transfers

The following proposition provides the condition under which it is optimal for the planner to set a positive labor tax.

Proposition 1: Given an inherited level of government debt b_{-1} and in the absence of stochastic shocks, the optimal long-run tax rate of labor is strictly positive if:

$$\frac{c^*}{c^{s*}} + \omega\Big(c^* - (1-\beta)b_{-1}\Big)\Big(\frac{1}{c^{h*}} - \frac{1}{c^{s*}}\frac{\theta^h(1+\phi\frac{c^{h*}}{n^{h*}})}{\theta^h + \phi\frac{c^{h*}}{n^{h*}}}\Big) > 1 \tag{2.20}$$

where variables with a "$*$" are evaluated at the allocation implied by setting $\tau = 0$: for any x, $x^* = x|_{\tau=0}$.

Proof: The above proposition essentially provides the condition under which, in the steady-state allocation, aggregate welfare is increasing in τ, when $\tau = 0$. More formally, it provides an expression for $\frac{dU}{d\tau}|_{\tau=0} > 0$, which is obtained by totally differentiating the system of steady-state model equations.

In the steady-state allocation, the competitive equilibrium can be represented by:

$$\begin{aligned}(1-\beta)b_{-1} &= \tau(\lambda n^h + (1-\lambda)n^s) - g - T \\ c^h &= (1-\tau)\theta^h n^h + \frac{\omega}{\lambda}T \\ c^s &= (1-\beta)b_{-1} + (1-\tau)n^s + \frac{1-\omega}{1-\lambda}T \\ (n^h)^\phi c^h &= \theta^h(1-\tau) \\ (n^s)^\phi c^s &= 1-\tau\end{aligned}$$

This gives us a system of 5 equations in $(c^h, c^s, n^h, n^s, T, \tau)$. Totally differentiating the equations, we obtain a system in $(dc^h, dc^s, dn^h, dn^s, dT, d\tau)$, that we can rearrange to get:

$$\begin{aligned}dn^h &= \left(\tau\lambda\theta^h + \phi\frac{\lambda}{\omega}\frac{c^h}{n^h} + \frac{\lambda}{\omega}(1-\tau)\theta^h - \tau\frac{\lambda\theta^h(1+\frac{c^h}{n^h}\phi)}{1+\phi\frac{c^s}{n^s}}\right)^{-1}\left(\frac{\lambda}{\omega}\theta^h n^h - n + \frac{\tau}{1-\tau}\frac{c}{1+\phi\frac{c^s}{n^s}} - \frac{\lambda}{\omega}\frac{c^h}{1-\lambda}\right)d\tau \\ dn^s &= \left((1-\lambda)(1+\phi\frac{c^s}{n^s})\right)^{-1}\left(-\lambda\theta^h(1+\phi\frac{c^h}{n^h})dn^h - \frac{c}{1-\tau}d\tau\right) \\ dc^h &= -\frac{c^h}{1-\tau}d\tau - \phi\frac{c^h}{n^h}dn^h \\ dc^s &= -\frac{c^s}{1-\tau}d\tau - \phi\frac{c^s}{n^s}dn^s\end{aligned} \tag{2.21}$$

Totally differentiating aggregate welfare gives:

$$dU = \lambda(u_c^h\, dc^h - v_n^h\, dn^h) + (1-\lambda)(u_c^s\, dc^s - v_n^s\, dn^s) \tag{2.22}$$

Making use of the system (2.21) in (2.22) , setting $\tau = 0$ and rearranging, we get the result stated in expression (2.20). ■

Proposition 1 gives us the condition under which setting positive labor taxes is welfare improving. In the next paragraphs, we describe three specific cases which help interpreting this result. Then, we study the effect of two specific parameters, the individual productivity of constrained agents (θ^h) and the share of transfers going to these agents (ω), on the optimal tax rate.

Representative agent: The first special case we consider is the representative agent version of the model, under which both distortionary taxes and lump-sum transfers (or taxes) are available. In such a setting, it is well-known that Ricardian equivalence holds and the planner chooses to entirely finance deficits at any time using lump-sum taxes, therefore setting $\tau = 0$. This can be seen by setting, $c^{s*} = c^{h*} = c^*$, and $\theta^h = 1$ in the above expression. In this case, the left-hand size of the condition is equal to one, and it is therefore sub-optimal to set $\tau > 0$.

Two-agent, evenly distributed transfers: Turning to the two-agent case, we assume first that $\omega = \lambda$, $\theta^h = 1$, $b_{-1} = 0$ and $g > 0$. In such a case, the two households have the same amount of resources available in the steady-state, as (i) they are equally productive, (ii) no agent holds assets, and (iii) transfers affect each agent in the same way. The assumption of positive government spending implies a need for the government to generate resources by taxation to satisfy its budget constraint. In such a scenario, we can show that, as in the representative agent case, the LHS of the expression stated in Proposition is equal to 1, and therefore the optimal labor tax is zero. It reflects the fact that, when the two households have similar characteristics, it is optimal to finance fiscal deficits with lump-sum taxes, if such an instrument is available to the planner. Because the two agents have similar characteristics, the optimal allocation is the one that maximizes aggregate consumption and output; this allocation is obtained by setting $\tau = 0$ and financing the entire deficit lump-sum.

Two-agent, transfers directed towards HTM households: As a third example, we still assume that $\theta^h = 1$, $b_{-1} = 0$ and $g > 0$, but we now set $\omega = 1$, such that transfers are fully targeted towards hand-to-mouth households. In this case, the LHS of the above condition boils down to $\frac{c^*}{c^{h*}}$. We can easily show from the steady-state model equations that, in such a scenario, $c^{s*} > c^{h*}$, and then $\frac{c^*}{c^{h*}} > 1$. Then, according to Proposition 1 the optimal long-run tax is positive. This is the case because, when the labor tax rate is zero, government expenditures are entirely financed by a lump-sum tax on hand-to-mouth agents, which reduces their consumption level, while Ricardian households are not impacted by such a tax. The planner thus finds it optimal to spread the tax burden across households by setting a positive labor tax rate. However, the optimal allocation does not equalize consumption/hours and welfare levels across household types: the efficiency costs associated with higher distortionary taxes imply that it is not optimal for the planner to fully wipe out any level of cross-sectional inequality.

[Figure 2.5 approximately here]

In Figure 2.5, we display aggregate steady-state welfare as a function of the labor tax, for the parametrization just described. We can see from the Figure that the tax rate which maximizes welfare (as shown by the red circle) is approximately 15%. At this value, the transfer is still negative, but is such that the costs associated to government spending are more equally shared between households.

Effect of key parameters We now solve for the steady-state of the Ramsey allocation, and look at the properties of the optimal tax schedule. We are primarily interested in identifying the effect of key model parameters on the behaviour of fiscal variables. We set the parameters to their baseline values, as described in Section 2.3 and summarized in Table 2.2, and then look at the impact of changing one parameter at a time on the behaviour of the steady-state labor tax. The results of this exercise are displayed in Figure 2.6.

[Figure 2.6 approximately here]

Panel (a) shows the effect of ω, the parameter targeting the share of transfers going to hand-to-mouth households, on the long-run optimal tax rate. As we can see from the Figure, the effect of ω on the optimal tax rate is non-monotonic: at low values of ω (when most of the transfer is directed towards Ricardian agents), the optimal tax rate is decreasing with ω, and reaches zero for values which are close to λ. Then, as ω starts increasing again, the tax rate becomes higher. This result emerges because, when transfers are evenly distributed among households (ω is close to λ), increasing them do not allow for much redistribution across households. Therefore, it is optimal to finance the deficit by decreasing transfers, which allows the planner to reduce the labor tax rate and thereby minimize its distortionary effects.[17]

In Panel (b) of the figure we look at the effects of the relative productivity of constrained agents (θ^h). The solid blue line provides results for the case where $\omega = \lambda$ (evenly distributed transfers), while the dashed black line is for the case where $\omega = 1$ (transfers targeted towards HTM agents). In this case too, the optimal labor tax is non-monotonic. At low values of θ^h, the optimal tax rate is positive: it is optimal for the planner to increase labor taxes to finance an increase in transfers and bring the consumption of hand-to-mouth agents closer to the one of Ricardian households, even if it comes with an efficiency cost. At high levels of θ^h, the optimal labor tax is increasing when $\omega = \lambda$, while it stays at zero when $\omega = 1$. In the former case, the relative productivity of hand-to-mouth agents becomes so high that it is optimal to redistribute resources towards Ricardian households by increasing labor taxes and transfers. In the latter case, redistribution towards savers is impossible, hence there is no tax response when $\omega = 1$ (dashed line).

[17] As it has been showed formally above, when the transfer is entirely lump-sum ($\omega = \lambda$), lump-sum taxes finance spending needs and the optimal labor tax is zero.

Panels (c) and (d) display the results of changes in steady-state debt levels and government spending. Unsurprisingly, we observe that, when spending needs increase (higher values of b and g), the optimal labor tax increases.

We have seen throughout this section that, in the long-run allocation, when heterogeneity is sufficiently high, it is optimal to redistribute across households through setting $\tau > 0$, even though this comes at the cost of higher tax distortions. In the next section we introduce dynamics and study the properties of the Ramsey allocation out of the steady-state.

2.5 Optimal responses to shocks

In the previous section, we showed how the Ramsey allocation departs from the first-best in the long-run. We now analyze the cyclical properties of the model when the economy is hit by random shocks. We first present theoretical results which allow us to discuss the properties of the optimal labor tax schedule. In particular, we study the link between fluctuations in consumption/hours heterogeneity between households, market (in)completeness, and the volatility of labor taxes in the Ramsey allocation. Then, we rely on numerical simulations to study the response of key variables to the stochastic shocks present in the model.[18]

2.5.1 Heterogeneity, market (in)completeness, and optimal labor taxes

We first analyze the properties of the optimal labor tax schedule.[19] We start with the following proposition, which characterizes the optimal labor tax rate in the two-agent model presented above:

Proposition 2: When the planner has access to distortionary taxes and transfers, and financial markets are incomplete, the optimal labor tax rate can be expressed as:

$$\tau_t = 1 - \frac{(H_t^c)^{-1}\left[1 + (\Psi_t^{IM} - \Psi_{t-1}^{IM})\frac{b_{t-1}}{c_t^s}\right]}{H_t + (1+\phi)\left[1 - \frac{\lambda}{\omega}\theta_t^h \frac{h_t^n}{H_t^n}\right]\Psi_t^{IM}} \tag{2.23}$$

[18] Throughout the paper, we solve the model using linear perturbation techniques around the initial steady-state.

[19] We focus on taxes and not transfers, as it allows us to directly compare our results to the ones obtained in representative agent models that are common in the Ramsey literature, and in which the planner is usually not allowed to choose transfers optimally.

where $h_t^c \equiv \frac{c_t^h}{c_t^s}$; $h_t^n \equiv \frac{n_t^h}{n_t^s}$; $H_t^c \equiv \lambda h_t^c + 1 - \lambda$; $H_t^n \equiv \lambda \theta_t^h h_t^n + 1 - \lambda$; $H_t \equiv (\lambda \theta_t^h \frac{h_t^n}{h_t^c} + 1 - \lambda)/H_t^n$.

When markets are complete, the multiplier associated with the budget constraint is constant ($\Psi_t^{IM} = \Psi \ \forall \ t$), and the expression becomes:

$$\tau_t = 1 - \frac{(H_t^c)^{-1}}{H_t + (1+\phi)\Big[1 - \frac{\lambda}{\omega}\theta_t^h \frac{h_t^n}{H_t^n}\Big]\Psi}$$

Proof: We can rewrite the first order conditions given in section 2.3.1 (rearranging and using $\frac{v_{n,t}^s}{u_{c,t}^s} = \frac{v_{n,t}^h}{\theta_h u_{c,t}^h} = a_t(1\breve{}\tau))$ as follows:

$$\begin{aligned}
\Psi_t^2 a_t(1\breve{}\tau_t)\frac{u_{cc,t}^h}{u_{c,t}^h} &= -\lambda u_{c,t}^h + \lambda \Psi_t^1 + \Psi_t^3 u_{c,t}^s \\
\Psi_t^2 a_t(1\breve{}\tau_t)\frac{u_{cc,t}^s}{u_{c,t}^s} &= (1-\lambda)u_{c,t}^s - \Psi_t^1(1-\lambda) - u_{cc,t}^s \Psi_t^3 (c_t^h - \frac{\omega}{\lambda}T_t) \\
&\quad + \Psi_t^{IM} u_{cc,t}^s (a_t n_t - g_t - T_t) - u_{cc,t}^s b_{t-1}\Big[\Psi_t^{IM} - \Psi_{t-1}^{IM}\Big] \\
\Psi_t^2 a_t(1\breve{}\tau_t)\frac{v_{nn,t}^h}{v_{n,t}^h} &= -\lambda v_{n,t}^h + \Psi_t^{IM}(u_{c,t}^s a_t - v_{n,t}^s)\lambda\theta_t^h + \Psi_t^1 a_t \lambda \theta_t^h + \Psi_t^3 v_{n,t}^s \theta_t^h \quad (2.24) \\
\Psi_t^2 a_t(1\breve{}\tau_t)\frac{v_{nn,t}^s}{v_{n,t}^s} &= (1-\lambda)v_{n,t}^s - \Psi_t^{IM}\Big[(1-\lambda)(u_{c,t}^s a_t - v_{n,t}^s) + v_{nn,t}^s n_t\Big] \\
&\quad - \Psi_t^1(1-\lambda)a_t - \Psi_t^3 v_{nn,t}^s \theta_t^h n_t^h
\end{aligned}$$

Merging the expressions of the system (2.24), and using functional forms given in (2.17), we get:

$$\begin{aligned}
&\Big[\frac{c_t^h}{c_t^s}\lambda + 1 - \lambda\Big]\Big[\frac{n_t^h}{n_t^s}\Big(\lambda\frac{v_{n,t}^h}{u_{c,t}^h}u_{c,t}^h - \Psi_t^{IM}\lambda\theta_t^h a_t \tau_t u_{c,t}^s - u_{c,t}^s \Psi_t^3 \frac{v_{n,t}^s}{u_{c,t}^s}\theta_t^h a_t\Big) + (1-\lambda)\frac{v_{n,t}^s}{u_{c,t}^s}u_{c,t}^s \\
&\qquad - \Psi_t^{IM}\Big((1-\lambda)a_t\tau_t u_{c,t}^s - \frac{v_{n,t}^s v_{nn,t}^s}{n_t^s v_{n,t}^s} n_t n_t^s\Big) - \Psi_t^3 n_t^s \frac{v_{nn,t}^s}{v_{n,t}^s}\theta_t^h a_t \frac{n_t^h}{n_t^s} v_{n,t}^s\Big] \quad = \\
&\Big[\frac{n_t^h}{n_t^s}a_t\theta_t^h\lambda + (1-\lambda)a_t\Big]\Big[\frac{c_t^h}{c_t^s}(\lambda u_{c,t}^h - \Psi^3 u_{c,t}^s) + (1-\lambda)u_{c,t}^s + \Psi_t^{IM}u_{cc,t}^s(c_t - T_t) \\
&\qquad - \Big(\Psi_t^{IM} - \Psi_{t-1}^{IM}\Big)u_{cc,t}^s b_{t-1} - \Psi_t^3 u_{cc,t}^s (1-\tau_t) a_t \theta_t^h n_t^h\Big]
\end{aligned}$$

Dividing both sides by $u^s_{c,t}$ and a_t, we get:

$$\begin{aligned}
&\Big[\frac{c^h_t}{c^s_t}\lambda + 1 - \lambda\Big]\Big[\frac{n^h_t}{n^s_t}\Big(\lambda\theta^h_t(1\breve{}\,\tau)\frac{u^h_{c,t}}{u^s_{c,t}} - \Psi^{IM}_t\lambda\theta^h_t\tau_t - \Psi^3_t\theta^h_t(1-\tau_t)\Big) + (1-\lambda)(1\breve{}\,\tau_t) \\
&\qquad -\Psi^{IM}_t\Big((1-\lambda)\tau_t - \phi(1-\tau_t)(\lambda\frac{n^h_t}{n^s_t}\theta^h_t + 1 - \lambda)\Big) - \Psi^3_t\phi\theta^h_t\frac{n^h_t}{n^s_t}(1-\tau_t)\Big] \quad = \\
&\quad \Big[\frac{n^h_t}{n^s_t}\theta^h_t\lambda + 1 - \lambda\Big]\Big[\frac{c^h_t}{c^s_t}(\lambda\frac{u^h_{c,t}}{u^s_{c,t}} - \Psi^3_t) + (1-\lambda) - \Psi^{IM}_t(\lambda\frac{c^h_t}{c^s_t} + 1 - \lambda - \frac{T_t}{c^s_t}) \\
&\qquad\qquad + \Big(\Psi^{IM}_t - \Psi^{IM}_{t-1}\Big)\frac{b_{t-1}}{c^s_t} + \Psi^3_t\Big(\frac{c^h_t}{c^s_t} - \frac{\omega}{\lambda}\frac{T_t}{c^s_t}\Big)\Big]
\end{aligned} \tag{2.25}$$

Rearranging (2.25) and using $\Psi^3_t = \frac{\lambda}{\omega}\Psi^{IM}_t$, we get the tax expression given in (2.23). ■

As we can see from equation (2.23), when markets are incomplete changes in the optimal tax rate reflect changes in two types of variables: Ψ^{IM}_t, which denotes the multiplier on the government budget constraint (2.1) in the Lagrangian associated with the Ramsey problem, and variables related to household heterogeneity $(h^c_t, h^n_t, H^c_t, H^n_t, H_t)$. Notice that in the absence of heterogeneity these elements are all equal to one. In contrast, under complete markets, Ψ is constant and only $h^c_t, h^n_t, H^c_t, H^n_t, H_t$ affect the optimal allocation.[20]

In order to study the effects of heterogeneity on the optimal tax schedule, we provide the counterpart of equation (2.23) in the representative agent case, where only distortionary taxes are available to the planner.[21] In the Appendix, we show that in this case the labor tax expression is:

$$\tau^R_t = 1 - \frac{1 + (\Psi^R_t - \Psi^R_{t-1})\frac{b_{t-1}}{c_t}}{1 + (1+\phi)\Psi^R_t} \tag{2.26}$$

under incomplete markets, and $\tau^R = 1 - \frac{1}{1+(1+\phi)\Psi^R}$ when markets are complete.[22] Therefore, it turns out that the tax rate in the representative agent model is a special case of our two-agent economy, in which the calibration is such that all agents are Ricardian and there is no more heterogeneity in the model.[23]

[20]Note that, as mentioned above, the steady-state allocation is the same under complete and incomplete markets. Under incomplete markets, the multiplier Ψ is time-varying. When markets are complete, this multiplier is constantly equal to its steady-state value (see Aiyagari et al., 2002).

[21]Introducing lump-sum transfers/taxes in such a framework would imply a trivial response of fiscal variables: labor taxes would be constant at zero, and lump-sum taxes would finance the inter-temporal budget of the government, thereby allowing the government to complete the market.

[22]The constant optimal tax rate on labor is a well-known property of this class of model when the elasticity of labor supply is constant, as is the case under the functional form for individual preferences stated in (2.17).

[23]The tax rate in the representative agent model can be obtained from equation (2.23) by setting $\lambda = 0$, $h^c_t = h^n_t = H^c_t = H^n_t = H_t = 1$, and $c^s_t = c_t$.

The above property is useful to decompose the effect of heterogeneity and incomplete markets on the optimal labor tax. To simplify the exposition, we log-linearize expressions (2.23) and (2.26), to obtain:

$$\hat{\tau}_t^R = \tau \hat{\Psi}_t^R - \hat{\psi}_t^R \tag{2.27}$$

for the representative agent case, and

$$\hat{\tau}_t = \underbrace{\tau \hat{\Psi}_t - \hat{\psi}_t}_{\text{(a) IM}} + \underbrace{(1-\tau)(1-H^c H)\hat{\Psi}_t}_{\text{(b) IM + Heterogeneity}} + \underbrace{\hat{H}_t^c + (1-\tau)H^c H \hat{H}_t + \Big[1 - H^c\Big(1-\tau\Big)\Big(H + (1+\phi)\Psi\Big)\Big]\Big(\hat{\theta}_t^h + \hat{h}_t^n - \hat{H}_t^n\Big)}_{\text{(c) Heterogeneity}} \tag{2.28}$$

for the two-agent version. Variables with hats denote log-deviations from steady state ($\hat{x}_t = \log(\frac{x_t}{x})$)[24], and variables without time subscripts denote steady-state values. We also define $\psi_t \equiv 1 + (\Psi_t - \Psi_{t-1})\frac{b_{t-1}}{c_t^s}$, and similarly for ψ_t^R in the representative agent case.

We can distinguish two main components in the characterization of labor tax fluctuations given by equation (2.28).[25] The first component (elements a and b) is related to market incompleteness, and summarizes the co-movement between the labor tax and the multiplier on the government budget constraint Ψ_t. Note that, when comparing the equation with its representative agent version in (2.27), we can see that the introduction of heterogeneity implies an additional term related to market incompleteness (the component b). It implies that heterogeneity can amplify/dampen the effects of market incompleteness on tax volatility, depending on the sign of $1 - H^c H$. However, for plausible parameter values, this effect is likely to be small.

The second component, summarized in the term c, shows that the tax rate in the Ramsey allocation is also affected by variables related to household heterogeneity. It means that consumption and hours dispersion between households introduces an additional source of tax volatility in the two-agent model, compared to the representative agent counterpart.

Together, these two forces (market incompleteness and heterogeneity) imply that, for labor taxes to have low volatility, two conditions must be met. First, it is required that shocks affecting government deficits can be financed in a non-distorting way, i.e. through changes in lump-sum taxes, or through the accumulation of state-contingent claims when markets are complete. These two policies allow the government to finance its deficit while having little impact on households' welfare. Therefore, the volatility of the multiplier on the government budget constraint (Ψ_t) is low and little variations in labor taxes arise

[24]With the exception of $\hat{\tau}_t \equiv -\log\left(\frac{1-\tau_t}{1-\tau}\right)$. This way, $\hat{\tau}_t$ represents the approximated change in the tax rate in percentage points, rather the percentage change from its steady-state value. This makes results easier to interpret and facilitates the comparison between the representative agent and two-agent versions of the model, because these models do not feature the same steady-state tax rate.

[25]We should stress here that expressions (2.23) and (2.28) do not constitute optimal tax reaction functions for the fiscal authority. Instead, they describe relationship between taxes and other model variables which are satisfied along the Ramsey equilibrium path, and in this sense only they help to shed light of the mechanisms at play in the model.

from elements (a) and (b) of expression (2.28).

Second, it must be that shocks can be financed through a policy that limits variations in consumption and hours dispersion between households. In our model, the impact of fiscal policy on heterogeneity is influenced by the design of the sharing rule for transfers (the value of the parameter ω): if $\omega \approx \lambda$, variations in transfers imply little redistribution between households. In this case, variations in transfers are an efficient way to finance deficits in response to shocks that have little impact on heterogeneity, as they do not distort labor supply and do not exacerbate fluctuations in heterogeneity terms. For this reason, variations in transfers provide a good hedge against the aggregate shocks present in the model (government spending and TFP shocks). For values of ω away from λ, transfers imply redistribution and can therefore not be used in such a way. However, in this case they are efficient in bringing down fluctuations in heterogeneity arising from shocks that affect households unequally, such as the shocks to the productivity of hand-to-mouth households (θ^h). To shed light on these properties, the next section studies the response of taxes to shocks under different model specifications.

2.5.2 Impulse responses

Aggregate shocks Figure 2.7 displays the response of taxes to the two aggregate shocks present in the model (government spending, and TFP shocks), for the parametrization in which $\omega = \lambda$, i.e. where transfers are evenly distributed between the two households. We also provide in this figure the decomposition of the tax rate following equation (2.28). The main insight emerging from the figure is the following. Comparing the representative agent model (dashed black line) to the two-agent model (solid blue line), we observe that the tax response to any aggregate shock is stronger in the model featuring a representative agent. As explained above, fluctuations in the tax rate can be decomposed in two parts: a component related to market incompleteness, and a component related to heterogeneity. For each shock, we display the contributions of these two elements in the middle and bottom panels of Figure 2.7. As can be seen from the figure, the main difference between the representative agent model and the two-agent model lies in the weaker variation in the component related to market incompleteness. This result comes from the fact that, in the two-agent setting, the planner can use variations in transfers to finance her inter-temporal budget, and thus does not have to rely entirely on labor tax fluctuations. Because the use of transfers does not generate distortions, the shocks have a smaller effect on the Ψ multiplier, and we observe in the middle panels of the figure that the tax component linked to market incompleteness responds less strongly to shocks in the two-agent framework. This implies that the planner is able to rely on fluctuations in transfers to bring the economy closer to the complete market allocation.

Indeed, the solid blue lines (depicting the response of variables in the incomplete markets version of the two-agent framework), are close to the dotted cyan lines, which provide the response of variables in the complete markets version. Moreover, the planner can implement this transfer policy at low costs because, as can be seen from the bottom panels of the figure, the policy response is also associated with weak responses in the component associated with heterogeneity. Therefore, the policy implemented by the planner allows her to contain fluctuations in heterogeneity, while financing the deficit in a non-distorting way.

[Figure 2.7 approximately here]

The case of government spending shocks is a good illustration of this result. Following such a shock, the response of labor taxes is mute when $\omega = \lambda$, because financing the impact of the shock with transfers only allows the planner to spread the implied fiscal costs equally between households, in such a way that the dispersion in consumption and hours worked between them is left unaffected. Financing government spending shocks with transfers (or, in this case, lump-sum taxes) therefore implies that there is no variation in the multiplier Ψ (because transfers are non-distortive), and no variations in the variables summarizing heterogeneity. Both of these forces translate into no change in the tax rate following the shock.

To shed light on the mechanisms outlined in the previous paragraphs, we also provide in Figure 2.7 the tax response for a version of the two-agent model where only distortionary taxes are available.[26] The results are depicted in the dotted red lines. We can see that, in this setup, the tax response to shocks is similar to the one obtained in the representative agent model. This shows that, when transfers are not available and the labor tax is the main instrument allowing the government to finance the government budget, the resulting tax volatility implied by market incompleteness is stronger.

[Figure 2.8 approximately here]

In Figure 2.8 we consider the case where transfers are fully targeted towards hand-to-mouth households ($\omega = 1$). We can see from the figure that in this model version, the tax response is stronger than in the case where $\omega = \lambda$, which was displayed in the previous figure. This result arises from the fact that, under the present calibration, fluctuations in transfers cannot be used to finance the government budget without impacting heterogeneity between households. As a result, the planner is not able to use transfers to bring

[26] In this case, the tax expression, as stated in Proposition 2 and equation (2.28) is slightly altered. We provide the tax expression in this model version in Appendix 2.7.2.

the economy closer to the complete markets allocation, while keeping consumption and hours dispersion close to constant. This can be seen from comparing the solid blue lines with the dotted cyan lines which depict the response under complete markets: contrarily to the case where transfers are evenly distributed across households, when $\omega = 1$ the response of taxes under incomplete markets is much stronger than in the complete markets counterpart. Unevenly targeted transfers makes the policy of absorbing the shocks solely through variations in transfers sub-optimal. As a result, a bigger share of the intertemporal budget is financed through changes in the labor tax rate, which becomes more volatile. We can see from Figure 2.8 that, following government spending and technology shocks, the policy response implies little changes in variables describing heterogeneity, and makes fluctuations in the second component of taxes (bottom panels) very low. Indeed, most of the tax volatility implied by the shocks comes from the impact of market incompleteness (middle panels).

Shocks to hand-to-mouth productivity In Figure 2.9 we study the response of key variables to a positive shock to the individual productivity of hand-to-mouth households (θ^h). The top panels display the case where $\omega = \lambda$, while the bottom panels are obtained setting $\omega = 1$. The key takeaway from this figure is the following. We observe that in the two cases, and both under complete and incomplete financial markets, transfers drop following the shock. This can be explained by two factors: first, as hand-to-mouth agents become more productive, they rely less on transfers to finance their consumption, which leaves some room for the planner to reduce them; second, reducing transfers allows the planner to generate a negative wealth effect for these agents, which incentives them to increase their labor supply precisely at the time when their productivity is above average, thus generating efficiency gains. We observe that the fall in transfers is higher when $\omega = \lambda$: because in this case transfers are evenly distributed between agents, the planner needs to engineer a bigger reduction to produce the desired effect on hand-to-mouth agents' budget constraint.

[Figure 2.9 approximately here]

Turning to the response of the labor tax rate, we can see from the figure that, as was the case for aggregate shocks, the component related to market incompleteness is still the one explaining most of the tax change. Indeed, under complete markets (dotted cyan lines), the response of the labor tax is much weaker than under incomplete markets (solid blue lines), and the response of the incomplete markets component of the tax rate is much stronger than the one summarizing the effect of heterogeneity. Therefore, even though the shock considered here affects heterogeneity in a non-negligible way (even after accounting for the optimal transfer response), this effect does not have much influence on the volatility of labor taxes. This brings us to the conclusion that, in the model presented here, market incompleteness is the main component influencing the volatility of labor taxes, no matter which shock is considered.

Comparing the case where $\omega = \lambda$ to the one where $\omega = 1$, it can be observed that, under incomplete markets, the sign of the labor tax response is opposite: it increases in the first case, and decreases in the second. This is the case because, when variations in transfers affects the two households equally ($\omega = \lambda$), the total drop in transfers for HTM agents is weaker. This implies a lower negative wealth effect on these households, which then decide to increase their labor supply by a lesser amount. All in all, the aggregate labor supply drops, and so do labor tax revenues. This has the effect of tightening the government budget constraint (the multiplier Ψ increases), and calls for an increase in the labor tax rate. When transfers are fully targeted towards HTM households ($\omega = 1$), the drop in transfers is big enough to induce a labor supply response which has the effect of increasing total labor effort, and tax revenues. It loosens the government budget constraint, and implies a fall in the labor tax.

Let us also stress that, in the two cases considered in Figure 2.9, when markets are incomplete, the initial tax response when the shock occurs overshoots its response in the subsequent periods. This property is known in the optimal fiscal policy literature under the name of *interest rate twisting* (see e.g. Faraglia et al., 2019a): the Ramsey planner uses fluctuations in the tax rate to influence households' consumption path and therefore manipulate bond prices to influence borrowing conditions and use government debt as an active fiscal policy tool. In our model, because the bond is priced by Ricardian households, the planner seeks to influence this price through changing the consumption path of these agents only. Following a shock, she therefore uses the proper combination of labor taxes and transfers to induce a kink in the consumption process of Ricardian agents, while keeping the consumption path of hand-to-mouth households as smooth as possible.

Discussion To end this section, we want to stress the importance of the transfer sharing rule, summarized by the parameter ω, in shaping the optimal behavior of the labor tax rate. This parameter specifies the share of total transfers going towards hand-to-mouth households. As discussed above, the impact of ω on optimal policy depends on the type of shocks hitting the economy. When shocks affect individual households in a similar way (which is the case for government spending and productivity shocks in our model), the labor tax rate remains constant only when ω is close to λ. In this case, the planner uses transfers to finance its deficit, and such a policy does not impact consumption and hours heterogeneity. In contrast, when ω gets away from λ, as in our example assuming $\omega = 1$, transfers are redistributive and financing deficits using them can be welfare detrimental, as such a policy exacerbates consumption and hours heterogeneity. In this case the optimal policy features more tax volatility.

When the economy is hit by household-specific shocks (such as a shock to θ^h in our model), labor taxes are also less volatile when $\omega \approx \lambda$. However, in this case consumption heterogeneity between household is greater, as fiscal instruments do not allow the planner to reduce household heterogeneity.

2.5.3 Matching the empirical properties of transfers

In this section we assess whether our optimal fiscal policy model is able to match some of the key moments that summarize the business cyclical properties of macroeconomic and fiscal variables in the US. In particular, we study the co-movement between transfers, deficits, output and the market value of debt.

We are particularly interested in the ability of our model to match the following data properties. First, the negative correlation between transfers and GDP, and the positive correlation between transfers and the primary deficit, which have been described in Section 2.2. Being able to generate counter-cyclical transfers with our model will be possible (i) if there is less need for redistribution in expansions, meaning that transfers can be decreased without negatively impacting households' welfare; and (ii) if the government is willing to consolidate its budget in an expansion, and thereby decides to decrease transfers to generate fiscal surpluses. To generate the positive correlation between transfers and deficits, it is key that variations in transfers are not used to compensate for rising deficits implied, for instance, by a rise in government spending. Transfers should, on the contrary, be one of the main drivers behind changes in primary deficits.

Second, we are interested in the co-movement between deficits and the market value of debt (positive in the US data). Marcet and Scott (2009, MS) analyze the ability of simple optimal fiscal policy models to reproduce key facts related to the behaviour of government debt. They show that, in representative agent models, when markets are complete, the market value of debt falls in response to shocks that force the deficit to increase. Moreover, the optimal portfolio pays out more than the actual income shock to compensate for higher future deficits when shocks are persistent. This is what MS call the "over-insurance" effect. Hence, the co-movement between deficits and debt is negative when the government can issue state-contingent securities, which is at odds with what is observed empirically. In the incomplete markets case, the market value of debt cannot decrease when deficits rise, and the government issues more debt to absorb the shock. Therefore, there is a positive correlation between deficits and debt in the representative agent, incomplete markets benchmark.

Third, we are interested in the persistence of the market value of government debt. Throughout this section, we follow MS and use the *k-variance* to measure the persistence of government debt. The k-variance of a random variable x at horizon k is defined as:

$$Var_k(x) = \frac{Var(x_t - x_{t-k})}{kVar(x_t - x_{t-1})} \quad \text{for} \quad k = 1, 2, ...$$

This object is a measure of the persistence of a random variable: when a process reverts to its mean, its k-variance converges to zero; otherwise it takes a higher value. MS show that Ramsey models with complete markets imply no persistence of debt (the k-variance quickly converges to zero, which is at odds with the actual behaviour of US government debt), while incomplete markets models imply very persistent levels of debt, such that the k-variances obtained from these models usually overshoot their data counterpart.

In the previous sections, we have already seen that, in our two-agent, incomplete markets setting, the use of transfers to finance the government budget allows the planner to reduce tax distortions and brings the optimal allocation close to its complete markets counterpart. Such a property is both a threat and an opportunity for the ability of our model to reproduce the moments of interest. On the one hand, it might prevent the model to generate the desired co-movements between deficits, output and the market value of debt that are usually generated in a representative agent framework with incomplete markets. However, on the other hand, it might help the incomplete markets model to generate a lower persistence of debt, thereby overcoming the overshooting feature outlined by MS. The object of this section is therefore to study whether our two-agent model can help bringing the persistence of debt closer to its empirical counterpart while also doing a good job matching the other properties mentioned above.

The next subsection makes use of the impulse response functions of key fiscal variables to each of the shocks present in the model, under the calibration described in Table 2.2, to shed light on the ability of the model to match the moments described above. We then make use of the simulated method of moments to choose the model parameters in order to match a broad vector of moments computed from US data.

Figure 2.10 depicts the impulse response of the main fiscal variables, consumption and output for each of the three shocks present in the model, in the incomplete markets version of the framework with $\omega = \lambda$ (top panels), and $\omega = 1$ (bottom panels). This allows us to analyze the impact of shocks on the co-movement between variables, as described above.

Government spending shocks: The solid blue lines in Figure 2.10 depict the responses to a positive government spending shock. As described in the previous section, when the shock hits, households work more and the planner reduces transfers to finance the shock. When transfers are evenly distributed between households ($\omega = \lambda$), the combined effect of increased hours and decreased transfers enables the deficit to remain constant. Hence, while the co-movement between transfers and output is negative following a g shock, the co-movement between deficit and output is almost null.

When transfers are directed towards hand-to-mouth households only ($\omega = 1$), the deficit cannot be insulated from the shock. Indeed because of their effect on heterogeneity, the planner is not willing to use transfers to absorb the shock entirely. Therefore, it is sub-optimal to use variations in transfers to bring the economy close to the complete markets allocation. As such, government spending shocks imply positive co-movements between deficits and output and between deficits and the market value of government debt, the former being at odds with what we see in the data.

[Figure 2.10 approximately here]

TFP shocks: Dashed red lines of Figure 2.10 describe the response of variables to a positive TFP shock. When $\omega = \lambda$, transfers, deficits and output increase on impact. Then, as the shock fades away, transfers and deficits continue to increase, while output decreases to return to its steady-state value. Hence, the co-movement between these variables is positive.

When $\omega = 1$, transfers remain approximately constant. Then, there is no co-movement between transfers and output, and between transfers and deficits. Note that, in this case, the market value of debt and deficits have a positive co-movement, which is not the case when $\omega = \lambda$.

Shocks to the productivity of HTM agents: Dotted black lines of Figure 2.10 depict the response of variables to an increase in the productivity of hand-to-mouth households. Following the shock, we observe a big drop in transfers and an important increase in output. This is true for the two values of ω that are considered. Overall, output and transfers co-move negatively, and output and deficits too. We can also see from the graph that the response of deficits and the market value of debt is negative, implying a positive co-movement between the two variables, as is the case in the US data.

From this analysis of the impulse responses of fiscal variables to various shocks, we can draw two important conclusions. First, when transfers are unevenly targeted towards hand-to-mouth households ($\omega > \lambda$), our model produces moments that are getting closer to their empirical counterpart. Second, it appears that the only shock that enables the model to jointly match the empirical moments discussed in the preceding sections is the shock to the productivity of hand-to-mouth households θ^h. Hence, the values of ω and the properties of the θ^h shock are likely to be key to allow the model to match the behaviour of US data moments, a task that we undertake in the following section.

Choosing model parameters to target empirical moments To investigate the ability of our simple model to produce moments which are close to their empirical counterpart, we estimate some of the parameters using the simulated method of moments (SMM). In particular, we choose the share of transfers going to HTM households (ω), the inverse Frisch elasticity (ϕ), the steady-state productivity of hand-to-mouth agents (θ^h), as well as the AR coefficients and standard deviations of our three shock processes, to minimize the quadratic distance between the moments generated by the model and those observed in US data. We make use of quarterly US data for the period 1960Q1-2017Q3 to compute our moments.[27] We target 15 moments related to the cyclical properties of fiscal variables and heterogeneity between households (see Table 2.3 for the full list of moments). Our estimation methodology is explained in more detail in Appendix 2.7.2.

As the data moments are computed at a quarterly frequency, we assume a model period of one quarter. We set the discount factor to $\beta = 0.995$ (this implies a 2% steady-state annual real interest rate) and $\lambda = 0.3$, as was the case before. We set the steady-state levels of government spending-to-gdp and the market value of annual government debt-to-GDP to their empirical average of respectively 7.36% and 35.08%.

[Table 2.3 approximately here]

Our results are presented in Table 2.3. The upper part of the table displays the estimated values of model parameters. One result is worth mentioning here. The value we obtain for the parameter targeting the share of transfers going to HTM households is $\omega = 0.82$. This is higher than λ, which was calibrated to 0.3, which implies that most of the transfers are targeted towards hand-to-mouth households. We also observe that in steady-state transfers are positive (transfers-to-gdp are equal to 15.6%), which means that under the obtained parametrization, the planner chooses to redistribute resources towards HTM agents, which helps reducing consumption inequality. The value of ω is not far from its data counterpart (the average value of transfers targeted to households at the bottom 30% of the income distribution) which, as we mentioned in Section 2.2, is equal to 0.67. No data on the cross-sectional distribution of transfers were used to compute the targeted

[27] The sample period is reduced to 1984Q1-2017Q3 for the cross-sectional data, as the dataset we use only covers this timespan. The datasets we use are the ones described in Section 2.2: we make use of NIPA tables to compute aggregate statistics, and the CEX to get cross-sectional moments.

moments, so we see this as a way to stress the ability of our simple model to match key empirical facts.

In the lower part of Table 2.3, we compare the moments obtained from our model to the ones observed in the US data. We observe that our simple model does surprisingly well in matching the targeted moments. Indeed, each of the simulated correlation has the appropriate sign, with the exception of the correlation between the market value of debt and deficits, which is almost zero in the model. The absolute deviations of most of the model moments to their data counterpart are very small. Of particular interest are the aggregate fiscal moments described in Section 2.2: the correlation between transfers, deficits and GDP, which are displayed in the first three lines of the table. We see that, for these moments, the fit is particularly good. To understand this result, we provide in Figure 2.11 the variance decomposition of some of the key variables when the parameters are set to their estimated values. We can see from the figure that more than 90% of the variation in the labor tax rate, transfers, and consumption heterogeneity, is explained by the shocks to hand-to-mouth agents' productivity (θ_t^h). These shocks account for about 50% of variations in output, while TFP shocks account for about 46% of the fluctuations in this variable. The importance of shocks to the productivity of hand-to-mouth households in the estimated model explain why the model does so well in generating a co-movement of fiscal variables which is very close to the one observed in the US. Indeed, we have seen in the previous section that such a shock generates the desired co-movement between deficits, transfers, and GDP; then, given the prevalence of fiscal variables in the moments that we target, it is with little surprise that the estimated process for this shock is such that it plays an important role.

[Figures 2.11 and 2.12 approximately here]

In Figure 2.12 we display the k-variance of the market value of government debt in our model and in the data, up to an horizon of 40 quarters. The solid blue line depicts the k-variance for the baseline 2-agent, incomplete markets model, using our estimated parameters; the dotted black line provides the empirical counterpart, computed using US data.[28] We observe from the figure that the model generates a k-variance which implies too little persistence in the debt process compared to its empirical counterpart. This is in contrast with representative agents models, which typically overshoot this statistics, as reported in Marcet and Scott (2009). The k-variance obtained in the representative

[28]To compute the k-variance of US government debt, we use data for the period 1960Q1-2007Q4. This way, our sample period resembles the one used by Marcet and Scott (2009), which helps us comparing our results to the ones described in their paper.

agent version of the model is depicted by the dashed-dotted green lines in the figure.[29] These results confirm the fact that in our 2-agent framework, the use of transfers allow the planner to bring the economy closer to the complete markets allocation, thereby implying a lower persistence of government debt. Note that there is still a stark contrast between the baseline results, and the ones obtained assuming complete financial markets, which are displayed in the dashed red line of Figure2.12, and in which we observed that the k-variance of debt is below one at all horizons, and quickly reaches zero.

2.6 Conclusions

In this paper we extend some results of the optimal fiscal policy literature by considering a model which introduces a small degree of heterogeneity between households, and an additional instrument to the Ramsey planner: (un)targeted transfers. We make the following contributions.

First, we provide the condition under which the optimal labor tax is strictly positive in the steady-state allocation. We show that, when heterogeneity in consumption and hours between households is sufficiently high, and/or when transfers are targeted towards a given type of agent, it might be optimal for the planner to set positive labor taxes in order to free up resources and increase the value of transfers. Such a policy partially removes inequality between households, which, as we show, can be optimal even though it comes at the cost of reducing aggregate consumption and output.

Second, we derive analytical expressions for the optimal dynamic response of labor taxes under complete and incomplete financial markets. We find that the main driver of labor tax fluctuations comes from the incomplete markets assumption rather than household heterogeneity. When markets are incomplete, fluctuations in transfers are used by the planner to bring the economy closer to the complete markets allocation, featuring a very low tax volatility. The allocation gets closer to complete markets when the transfer rule is not targeted towards an agent type, and shocks are aggregate. When transfers are directed towards a given type of households, the planner faces a trade-off between minimizing labor tax distortions and reducing consumption and hours heterogeneity. Hence, it becomes less optimal to keep the labor tax close to constant.

Finally, in order to investigate the ability of our simple model to reproduce empirical moments, we study the joint behaviour of key macro and fiscal variables and estimate some of the model parameters using the simulated method of moments (SMM). We show

[29] We use the same model parameters as in our two-agent model to compute the statistics in the representative agent framework. Note that there are only two shocks (government spending and TFP) in this model version. We simply ignore the θ^h shock, and do not reestimate the variance of the shock processes to obtain our results.

that heterogeneity, and more specifically shocks to the productivity of hand-to-mouth agents, is key for the model to be able to reproduce the cyclical behaviour of transfers, deficits, and output.

2.7 Appendix

2.7.1 Tables and figures

TABLE 2.1: **Corr. matrix of fiscal variables in the US**

	GDP	**Deficit**	**Transfers**
GDP	1	-	-
Deficit	-0.7961	1	-
Transfers	-0.4462	0.6841	1

Notes: Data are from the NIPA database and cover the period 1984Q1-2013Q1. The frequency is quarterly. All variables are expressed in per-capita terms and are de-trended using the HP-Filter (Smoothing parameter: 1,600). Transfers correspond to our restricted definition, which covers unemployment benefits and other income assistance programs (see Section 2.2). Taking a broader definition of transfers - that is, government social benefits to persons - leads to even more counter-cyclical transfers. In this case, the correlation between GDP and transfers amounts to -0.7592.

TABLE 2.2: **Calibrated parameter values**

Parameter	**Description**	**Value**
β	Discount factor	0.95
λ	Share of hand-to-mouth households	0.30
ϕ	Inverse Frisch elasticity	1
g/y	Steady-state government spending to gdp	0.15
b/y	Steady-state government debt to gdp	0.6
θ^h	Steady-state productivity of HTM agents	1
ω	Share of transfers targeted to HTM agents	$\{\lambda; 1\}$

Notes: The table provides the assumed parameter values in the baseline specification of the model presented in Section 2.3.

Table 2.3: **Estimated parameters and implied moments**

Estimated parameters:

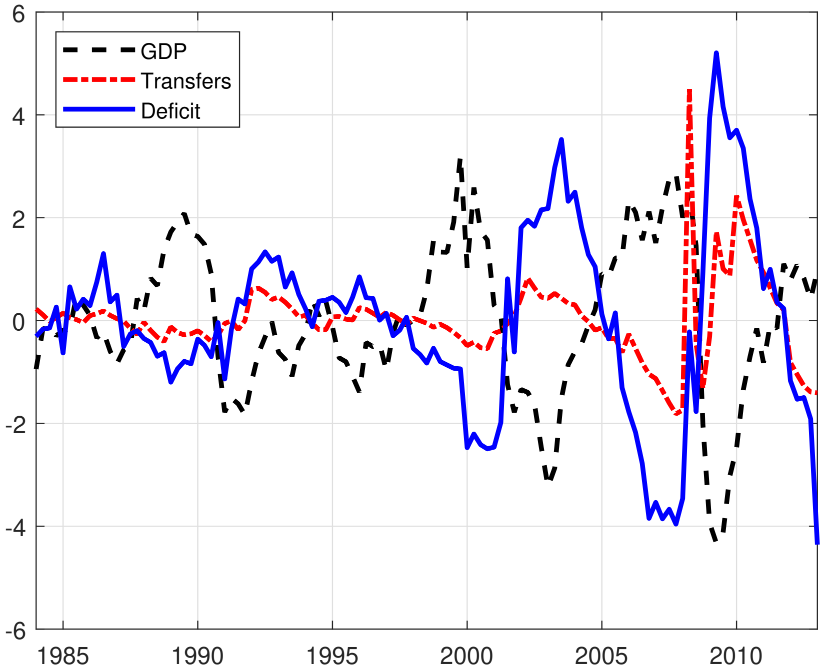
Parameter	Description	Value
ω	Share of transfers targeted to HTM agents	0.82043
ϕ	Inverse Frisch elasticity	1.9004
θ^h	Steady-state productivity of HTM agents	0.57591
Shocks		
ρ_g	AR coeff. gov. spending	0.7139
ρ_a	AR coeff. total factor productivity	0.93072
ρ_θ	AR coeff. relative productivity of HTM agents	0.60146
σ_g	Std, gov. spending	0.074173
σ_a	Std, total factor productivity	0.0052298
σ_θ	Std, relative productivity of HTM agents	0.052671

Implied moments:

	Data	Model
$corr(T, y)$	-0.6323	-0.59263
$corr(def, y)$	-0.631	-0.60137
$corr(T, def)$	0.7329	0.82561
$std(y)$	0.0146	0.016852
$std(g/y)$	0.0379	0.10109
$corr(mv/y, def)$	0.5065	-0.08311
$corr(g_t, g_{t-1})$	0.6836	0.68526
$corr(y_t, y_{t-1})$	0.8623	0.72385
$cor(mv_t, mv_{t-1})$	0.8752	0.98709
$mean(\theta^h n^h/n^s)$	0.2089	0.46055
$std(\theta^h n^h/n^s)$	0.0724	0.10122
$corr(\theta^h_t n^h_t/n^s_t, \theta^h_{t-1} n^h_{t-1}/n^s_{t-1})$	0.5085	0.57832
$mean(h^c)$	0.5582	0.88074
$std(h^c)$	0.0331	0.0045294
$corr(h^c_t, h^c_{t-1})$	0.5044	0.61323

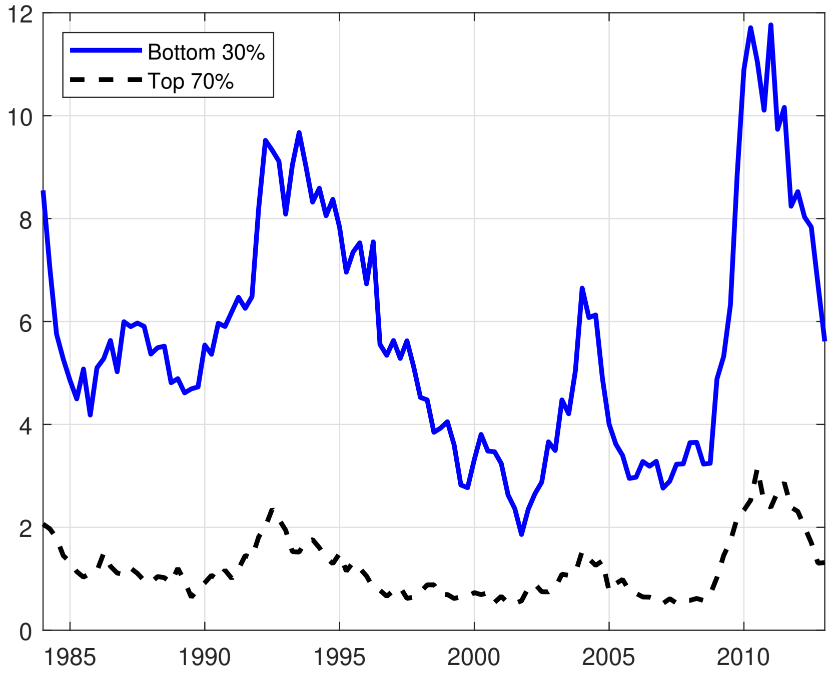
Notes: The table presents the results obtained in our Simulated Method of Moments (SMM) exercise. We choose the model parameters to match the set of moments presented in the bottom part of the table (the left column displays the values obtained from the US data for the period 1960Q1-20017Q3), while the right columns presents the values obtained from the model). The top part of the displays the values of estimated parameters. The remaining parameters are set to the value displayed in Table 2.2.

FIGURE 2.1: **Cyclical behaviour of fiscal variables in the US**



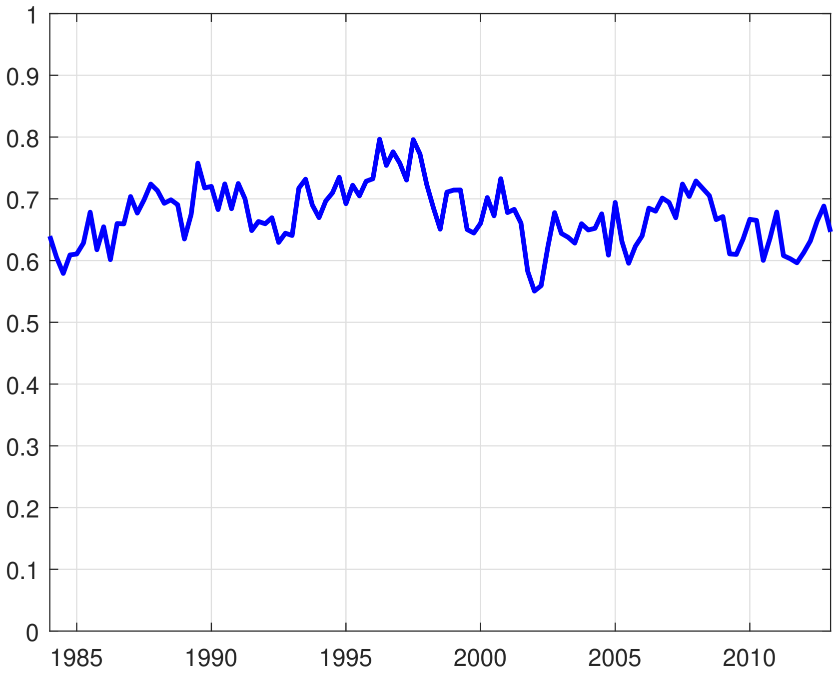
Notes: The figure plots the cyclical behaviour of real GDP (dashed black line), real deficit (solid blue line), and real transfers (dashed red line), at the quarterly frequency. Data are from the NIPA database. Variables are in per-capita terms and de-trended with HP-Filter.

FIGURE 2.2: **Transfers towards household income groups**



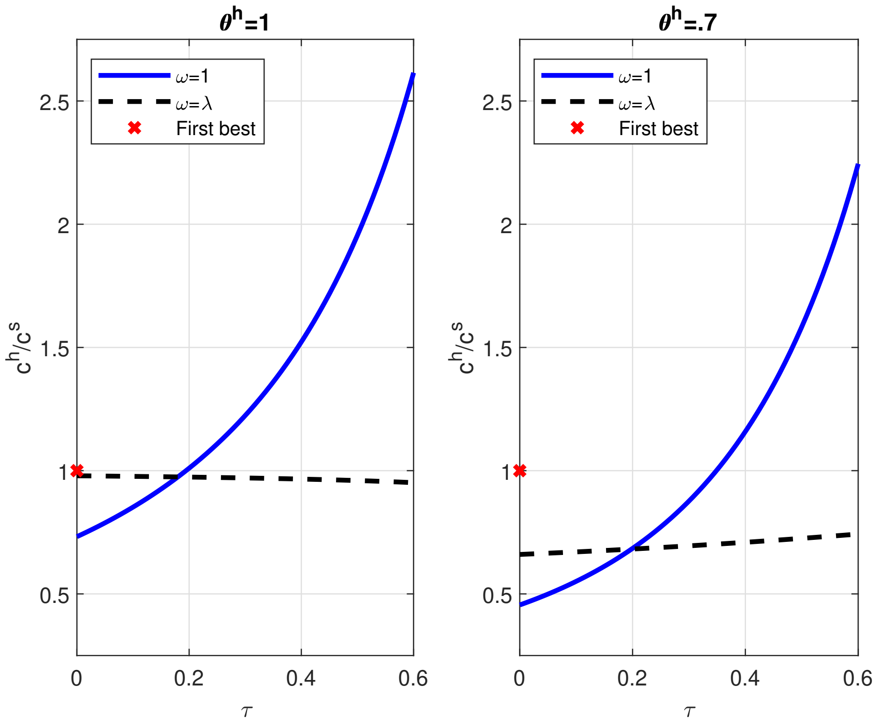
Notes: The figure plots the behaviour of transfers per capita for households at the bottom 30% of the income distribution (solid blue line), and the top 70% (dashed black line), at the quarterly frequency. Household-level data is aggregated from the CEX database.

FIGURE 2.3: **Share of transfers targeted towards low-income households**



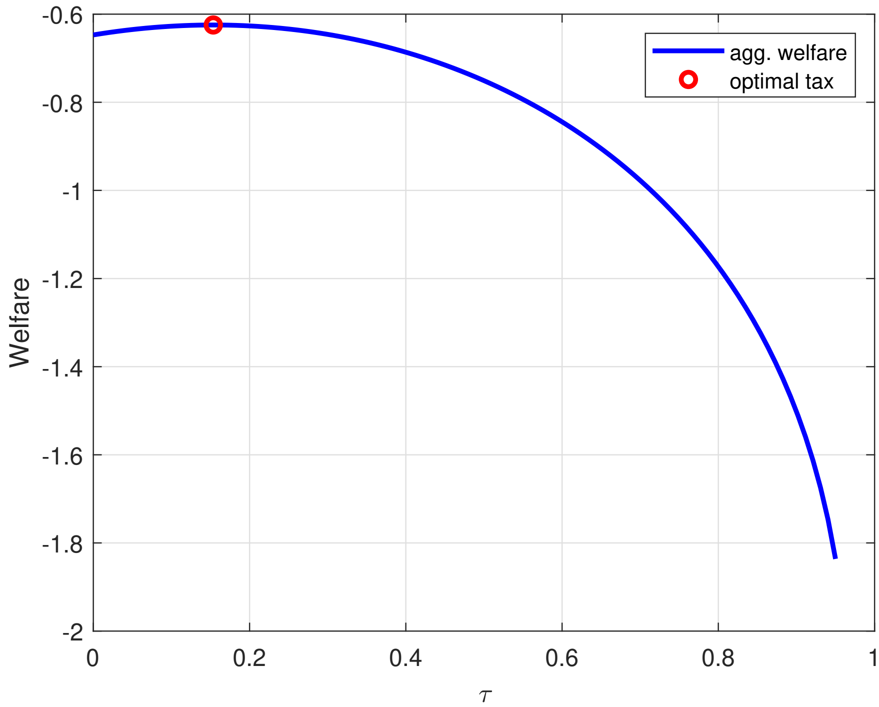
Notes: The figure plots the evolution of the share of total transfers targeted towards households at the bottom 30% of the income distribution (solid blue line), at the quarterly frequency. The series was constructed using household-level data from the CEX.

FIGURE 2.4: **Steady-state consumption inequality**



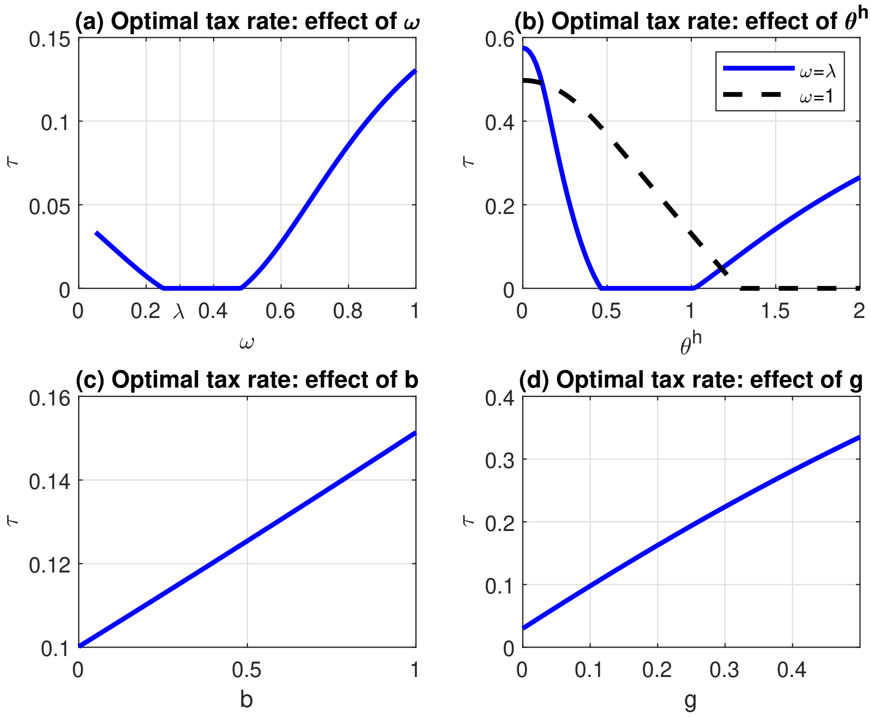
Notes: The figure depicts the steady-state consumption inequality, measured as the ratio of hand-to-mouth over Ricardian households' per capita consumption levels (c^h/c^s), as a function of the labor tax rate. The red cross depicts the first-best allocation. The dashed black line plots consumption inequality for the case of evenly targeted transfers ($\omega = \lambda$). The solid blue line presents consumption inequality for the case of transfers fully targeted towards hand-to-mouth households ($\omega = 1$).

FIGURE 2.5: **Steady-state welfare**



Notes: The solid blue line plots the value of aggregate welfare U in the steady-state of the model presented in Section 2.3, as a function of the labor tax rate. The red circle depicts the optimal the tax rate and associated welfare level. To produce this figure, we set $\theta^h = 1$, $\omega = 1$, and $b_{-1} = 0$. The remaining parameters are set to the baseline values provided in Table 2.2.

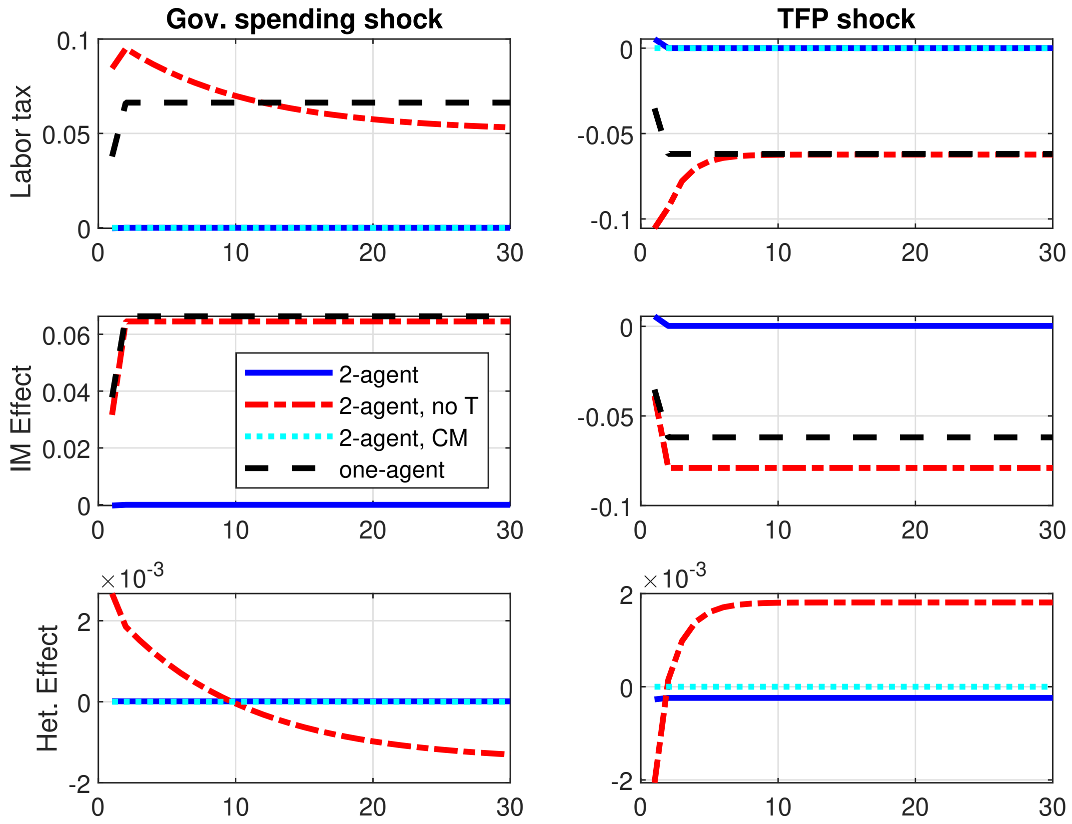
FIGURE 2.6: **Optimal long-run taxes: effect of key parameters**



Notes: The figure analyses the effect of key model parameters on the optimal labor tax rate in the steady-state of the baseline model. Panel (a) displays the effect of ω, the share of transfers targeted towards hand-to-mouth agents. Panel (b) depicts the effect of θ^h, the long-run value of the relative productivity of hand-to-mouth agents. Pnael (c) and (d) present, respectively, the effect of the long-run levels of the debt-to-gdp and government spending-to-gdp ratios. The remaining parameters are set to their baseline values, as presented in Table 2.2.

FIGURE 2.7: **Labor tax response under incomplete markets: one vs. two-agent models (1)**

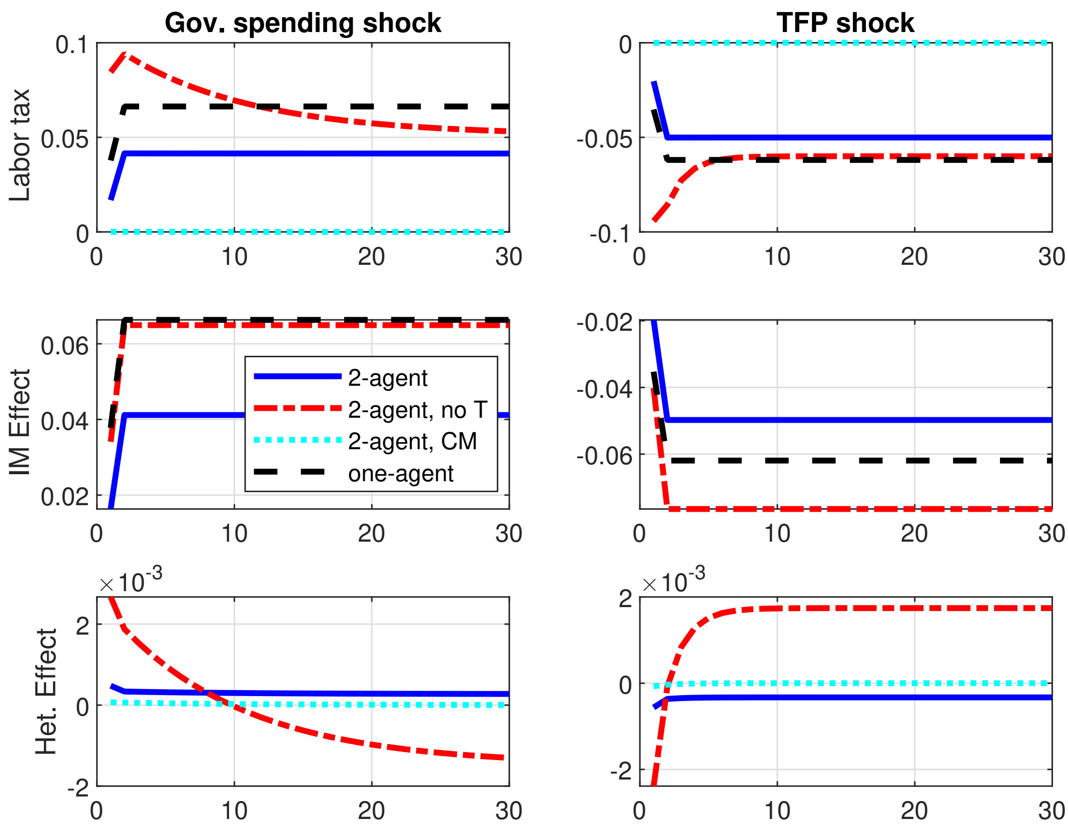
Case 1: $\omega = \lambda$



Notes: Response of the labor income tax to government spending (left panels) and TFP shocks (right panels), for the case of evenly targeted transfers ($\omega = \lambda$). The value of each shock is normalized to one. The top panels present the response of the optimal labor tax (in deviation from steady-state). The middle and bottom panels depict, respectively, the component of taxes related to market incompleteness and heterogeneity, following the decomposition presented in equation (2.28). The solid blue lines are for the baseline two-agent model under incomplete markets, and the dotted cyan lines for the complete markets counterpart. The dashed red lines display the responses for a 2-agent model version where only labor taxes are available to the planner. Finally, the dashed black lines present the results for a standard representative agent model without transfers.

FIGURE 2.8: **Labor tax response under incomplete markets: one vs. two-agent models (2)**

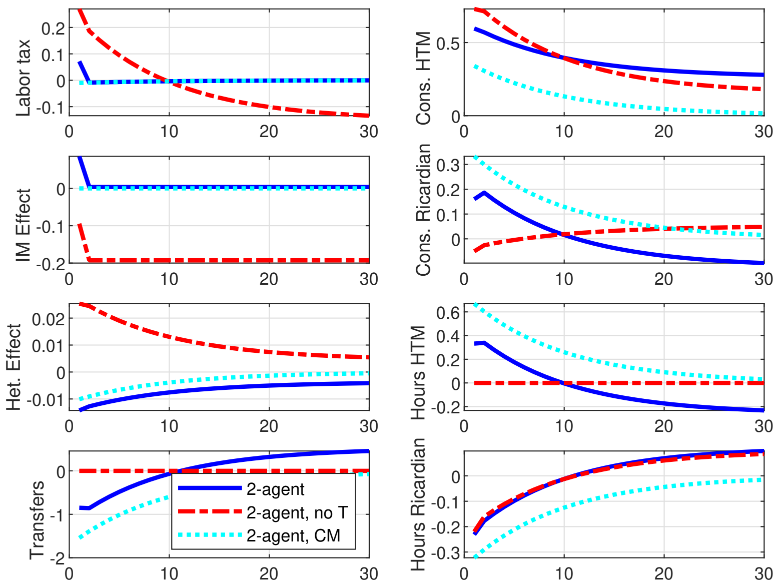
Case 2: $\omega = 1$



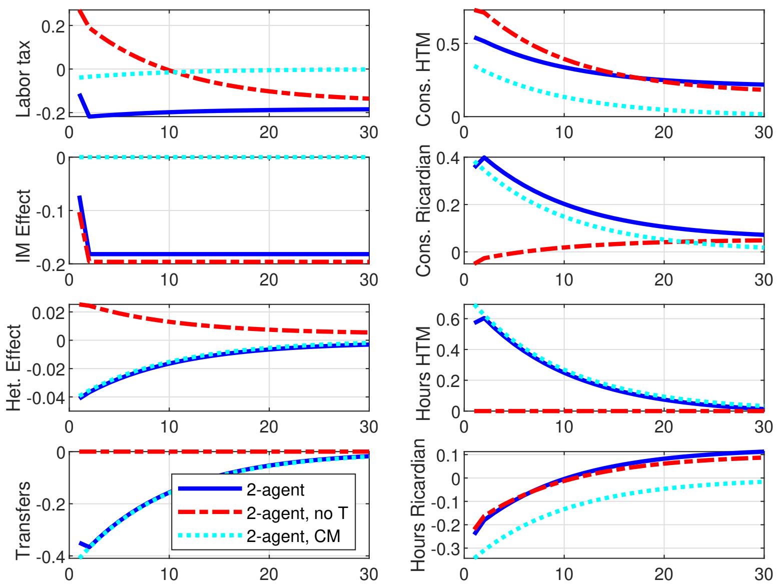
Notes: Response of the labor income tax to government spending (left panels) and TFP shocks (right panels), for the case where transfers are targeted towards hand-to-mouth agents ($\omega = 1$). The value of each shock is normalized to one. The top panels present the response of the optimal labor tax (in deviation from steady-state). The middle and bottom panels depict, respectively, the component of taxes related to market incompleteness and heterogeneity, following the decomposition presented in equation (2.28). The solid blue lines are for the baseline two-agent model under incomplete markets, and the dotted cyan lines for the complete markets counterpart. The dashed red lines display the responses for a 2-agent model version where only labor taxes are available to the planner. Finally, the dashed black lines present the results for a standard representative agent model without transfers.

FIGURE 2.9: **IRFs under incomplete markets: Shock to HTM productivity**

Case 1: $\omega = \lambda$



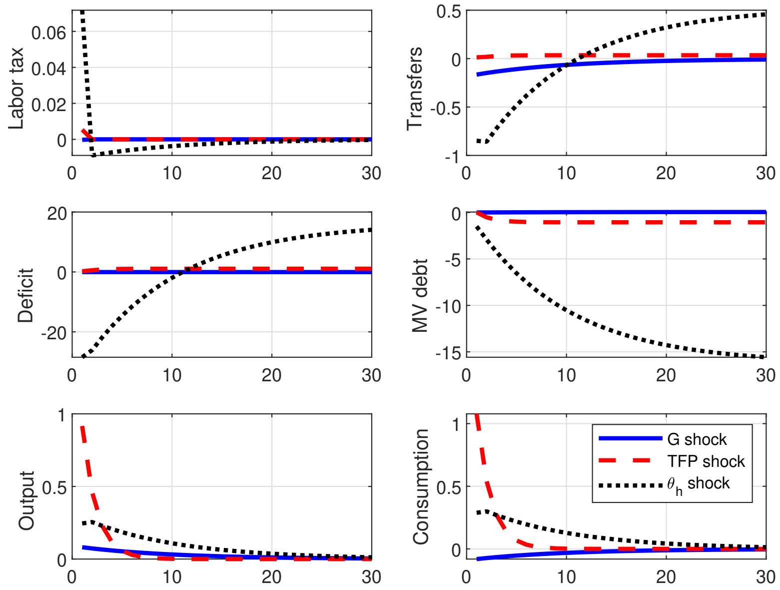
Case 2: $\omega = 1$



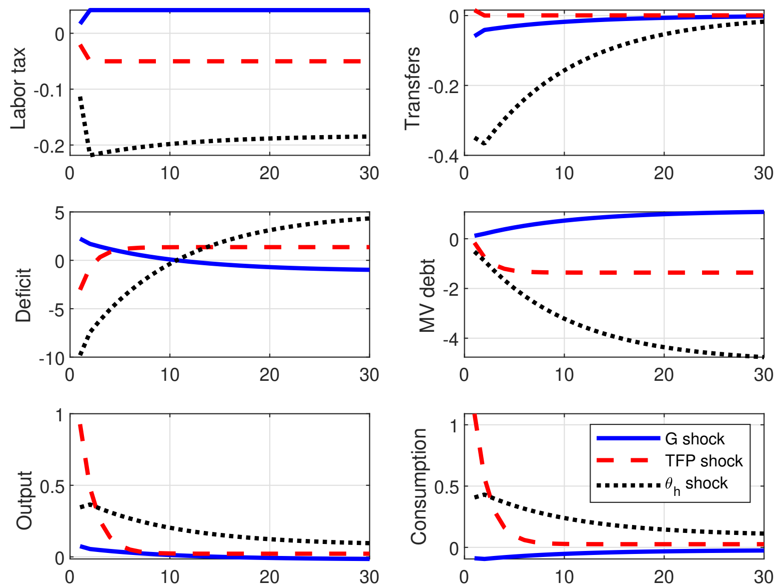
Notes: Response of key model variables to a θ^h shock, for the case of evenly targeted transfers ($\omega = \lambda$, top graphs) and transfers towards hand-to-mouth agents only ($\omega = 1$, bottom graphs). The value of the shock is normalized to one. The solid blue lines are for the baseline two-agent model under incomplete markets, and the dotted cyan lines for the complete markets counterpart. The dashed red lines display the responses for a 2-agent model version where only labor taxes are available to the planner.

FIGURE 2.10: **IRFs under incomplete markets: fiscal variables**

(a) **Case 1:** $\omega = \lambda$

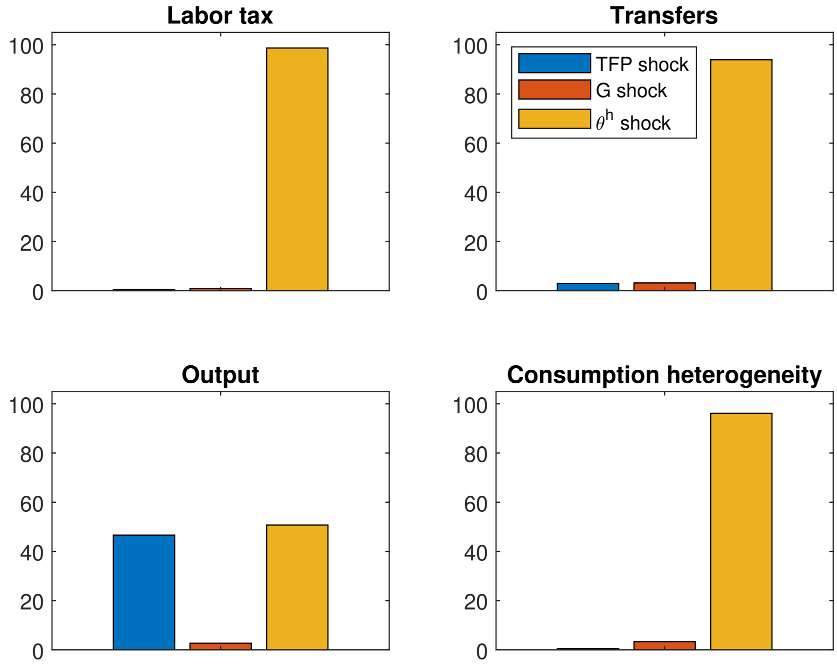


(b) **Case 2:** $\omega = 1$



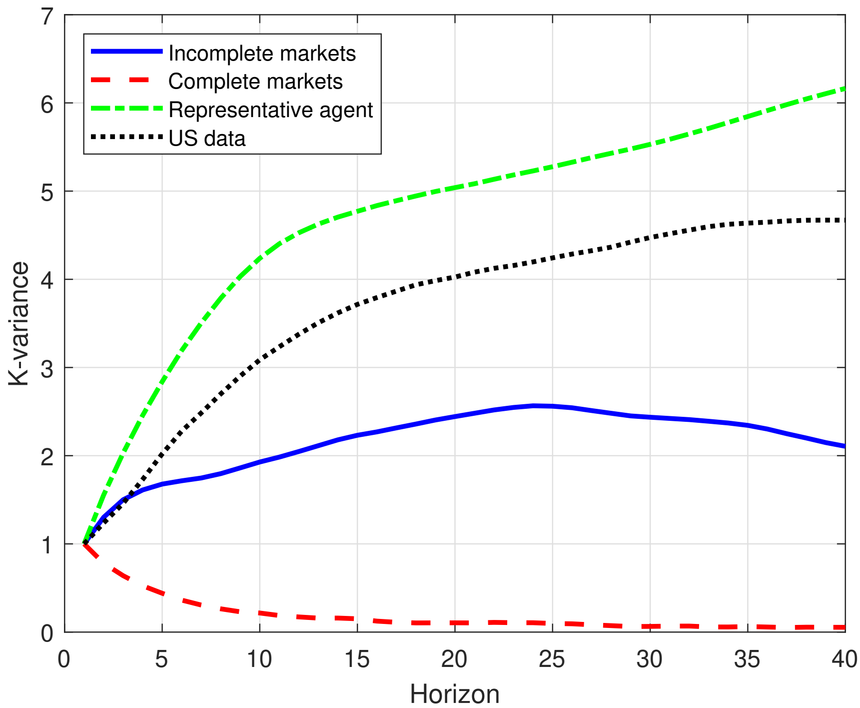
Notes: The figure presents the impulse response functions of key variables to the three shocks present in the model. The top panels display results for the case where $\omega = \lambda$, while the bottom panels set $\omega = 1$. The value of the remaining model parameters are displayed in Table 2.2. The solid blue lines display the responses to a government spending shock, the dashed red lines to the TFP shock, and the dotted black lines present results to a θ^h shock. The values taken by the shock is normalized to one in the three cases.

FIGURE 2.11: **SMM results: Variance decomposition**



Notes: The figure displays the variance decomposition of labor taxes, transfers, output and consumption heterogeneity, for the model estimated with the SMM (the parameter values are displayed in Table 2.3).

FIGURE 2.12: **SMM results: k-Variance of debt**



Notes: The figure displays the behaviour of the k-variance for the market value of debt in our estimated model. The solid blue line depicts the obtained values for our baseline 2-agent model with incomplete markets. The dashed red line provides the analogous result when we assume complete markets, and the dashed-dotted green line displays the representative agent counterpart, where only distortionary taxes are available. The dotted black line shows the series obtained from the US data in the period 1960Q1-2007Q4.

2.7.2 Model appendix

Complete markets In this section we provide additional details on the complete markets model studied in the main text and presented in Section 2.3.3. In this model version the Ricardian households' budget constraint can be written as:

$$c_t^s + \int_{s^{t+1}} q^{CM}(s^{t+1}) b^{s,CM}(s^{t+1})\, ds^{t+1} = (1-\tau_t) a_t n_t^s + b^{s,CM}(s^t) + T_t^s \tag{2.29}$$

The first order conditions give:

$$\frac{v_{n,t}^s}{u_{c,t}^s} = 1 - \tau_t \tag{2.30}$$

$$u_c^s(s^t) q^{CM}(s^{t+1}) = \beta f(s^{t+1}|s_t) u_c^s(s^{t+1}) \tag{2.31}$$

The second equation gives the pricing condition for each state contingent security $b^{CM}(s^{t+1})$. This equation can be aggregated over states s^{t+1} to obtain the usual Euler equation:

$$q_t^{CM} = \beta \int_{s^{t+1}} \frac{u_{c,t+1}^s}{u_{c,t}^s} f(s^{t+1}|s_t) ds^{t+1}$$

The Lagrangian associated to the Ramsey program (the complete markets equivalent of equation (2.10)) can be written as:

$$\begin{aligned}
\mathcal{L}^{CM} \quad = \quad & E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ \lambda \Big[u(c_t^h) - v(n_t^h) \Big] + (1-\lambda) \Big[u(c_t^s) - v(n_t^s) \Big] \\
& + \Psi^{CM} \Big[- u_{c,0}^s b_{-1} + (u_{c,t}^s a_t - v_{n,t}^s)(\lambda \theta_t^h n_t^h + (1-\lambda) n_t^s) - u_{c,t}^s (g_t + T_t) \Big] \\
& + \Psi_t^1 \Big[a_t \lambda \theta_t^h n_t^h + (1-\lambda) n_t^s a_t - \lambda c_t^h - (1-\lambda) c_t^s - g_t \Big] + \Psi_t^2 \Big[\frac{v_{n,t}^s}{u_{c,t}^s} - \frac{1}{\theta^h} \frac{v_{n,t}^h}{u_{c,t}^h} \Big] \\
& + \Psi_t^3 \Big[- u_{c,t}^s c_t^h + v_{n,t}^s \theta_t^h n_t^h + \frac{\omega}{\lambda} u_{c,t}^s T_t \Big]
\end{aligned}$$

The associated first order conditions are:

$$\begin{aligned}
c_t^h \quad &: \quad \lambda u_{c,t}^h - \lambda \Psi_t^1 + \frac{\Psi_t^2}{\theta_t^h} \frac{v_{n,t}^h}{(u_{c,t}^h)^2} u_{cc,t}^h - \Psi_t^3 u_{c,t}^s = 0 \\
c_t^s \quad &: \quad (1-\lambda) u_{c,t}^s + \Psi^{CM} \Big[u_{cc,t}^s a_t n_t - u_{cc,t}^s (g_t + T_t) \Big] \\
& \qquad - \Psi_t^1 (1-\lambda) - \Psi_t^2 \frac{v_{n,t}^s}{(u_{c,t}^s)^2} u_{cc,t}^s + \Psi_t^3 (-u_{cc,t}^s c_t^h + \frac{\omega}{\lambda} T_t u_{cc,t}^s) = 0 \\
n_t^h \quad &: \quad -\lambda v_{n,t}^h + \Psi^{CM} (u_{c,t}^s a_t - v_{n,t}^s) \lambda \theta_t^h + \Psi_t^1 a_t \lambda \theta_t^h - \frac{\Psi_t^2}{\theta_t^h} \frac{v_{nn,t}^h}{u_{c,t}^h} + \Psi_t^3 v_{n,t}^s \theta_t^h = 0 \\
n_t^s \quad &: \quad -(1-\lambda) v_{n,t}^s + \Psi^{CM} \Big[(1-\lambda)(u_{c,t}^s a_t - v_{n,t}^s) - v_{nn,t}^s n_t \Big] + \Psi_t^1 (1-\lambda) a_t + \Psi_t^2 \frac{v_{nn,t}^s}{u_{c,t}^s} + \Psi_t^3 v_{nn,t}^s \theta_t^h n_t^h = 0 \\
T_t \quad &: \quad -\Psi^{CM} u_{c,t}^s + \Psi_t^3 \frac{\omega}{\lambda} u_{c,t}^s = 0
\end{aligned}$$

Representative agent The representative agent model is a specific case of our two agent model where $\lambda = 0$. As such, the Lagrangian associated to the planner's optimization program in the incomplete markets setting is:

$$\begin{aligned}\mathcal{L} &= E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ u(c_t) - v(n_t) + \Psi_t^{IM} \Big[-u_{c,t} b_{t-1} + (u_{c,t} a_t - v_{n,t}) n_t) - u_{c,t} g_t + \beta u_{c,t+1} b_t \Big] \\ &\quad + \Psi_t^1 \Big[a_t n_t - c_t - g_t \Big] \Big\}\end{aligned}$$

where we make use of the representative agents' Euler equation $q_t = \beta \frac{E_t u_{c,t+1}}{u_{c,t}}$ and labor supply condition $(1 - \tau_t) a_t = \frac{v_{n,t}}{u_{c,t}}$ to substitute away the bond price q_t and the labor tax rate τ_t.

The optimality conditions associated to the planner's problem are the following:

$$\begin{aligned} c_t &: \; u_{c,t} + \Psi_t^{IM} u_{cc,t} \Big[-b_{t-1} + a_t n_t - g_t \Big] + \Psi_{t-1}^{IM} u_{cc,t} b_{t-1} - \Psi_t^1 = 0 \\ n_t &: \; -v_{n,t} + \Psi_t^{IM} \Big[(u_{c,t} a_t - v_{n,t}) - v_{nn,t} n_t \Big] + \Psi_t^1 a_t = 0 \\ b_t &: \; -E_t \Psi_{t+1}^{IM} u_{c,t+1} + \Psi_t^{IM} E_t u_{c,t+1} = 0 \end{aligned}$$

Eq. (2.26) is derived as follows. First, divide the above two first FOC by $u_{c,t}$ and make use of the labor supply condition to get:

$$\begin{aligned} 1 - \Psi_t^{IM} - \big(\Psi_t^{IM} - \Psi_{t-1}^{IM}\big) \frac{u_{cc,t}}{u_{c,t}} b_{t-1} - \frac{\Psi_t^1}{u_{c,t}} &= 0 \\ \tau_t - 1 + \Psi_t^{IM} \Big[\tau_t - \frac{v_{nn,t}}{v_{n,t}} n_t (1 - \tau_t) \Big] + \frac{\Psi_t^1}{u_{c,t}} &= 0 \end{aligned}$$

Then, using the functional form described in Eq. (2.17) and substituting away Ψ_t^1 using the above equations leads to Eq. (2.26).

Two-agent framework: first-best allocation The first best allocation in a two-agent framework is defined as the result of the maximization process of aggregate welfare subject to the feasibility constraint of the economy, i.e. the resource constraint. As such, the Lagrangian associated to the planner's optimization program is:

$$\begin{aligned}\mathcal{L} &= E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ \lambda \Big(u(c_t^s) - v(n_t^s) \Big) + (1 - \lambda) \Big((u(c_t^h) - v(n_t^h) \Big) \\ &\quad - \Psi_t^{RC} \Big[\lambda c_t^h + (1 - \lambda) c_t^s + g_t - \lambda a_t \theta_t^h n_t^h - (1 - \lambda) a_t n_t^s \Big] \Big\}\end{aligned}$$

The first order conditions associated to the planner's problem are the following:

$$c_t^s \ : \ u_{c,t}^s = \Psi_t^{RC} \qquad\qquad n_t \ : \ v_{n,t}^s = \Psi_t^{RC} a_t$$
$$c_t^h \ : \ u_{c,t}^h = \Psi_t^{RC} \qquad\qquad n_t \ : \ v_{n,t}^h = \Psi_t^{RC} \theta_t^h a_t$$

They imply that:

$$u_{c,t}^s = u_{c,t}^h \tag{2.32}$$
$$\frac{v_{n,t}^s}{u_{c,t}^s} = a_t \tag{2.33}$$
$$\frac{v_{n,t}^h}{u_{c,t}^h} = \theta_t^h a_t \tag{2.34}$$

In a decentralized economy, Eq (2.32) implies that hand-to-mouth households' consumption is equal to Ricardian households' consumption in the case of separable utility, as is assumed throughout the paper. Eq (2.33) and (2.34) indicate that the planner prefers to set labor taxes to zero. It also states that $\theta_t^h v_{n,t}^s = v_{n,t}^h$. Therefore, when $\theta_t^h < 1$, hand-to-mouth households work less than Ricardian households.

Additional results - Proposition 2B When the planner has access to distortionary taxes only and markets are incomplete, the optimal labor tax rate satisfies:

$$\tau_t = 1 - \frac{(H_t^c)^{-1}\Big[1 + (\Psi_t^{IM} - \Psi_{t-1}^{IM})\frac{b_{t-1}}{c_t^s}\Big](1-\lambda)}{\Big[H_t H_t^n + (1-\lambda)(1+\phi)\Psi_t^{IM}\Big] - (H_t^n - (1-\lambda))\Big[(1+\phi)(h_t^c)^{-1} - \frac{\phi}{H_t^c}\Big(1 + (\Psi_t^{IM} - \Psi_{t-1}^{IM})\frac{b_{t-1}}{c_t^s}\Big)\Big]} \tag{2.35}$$

where $h_t^c \equiv \frac{c_t^h}{c_t^s}$; $h_t^n \equiv \frac{n_t^h}{n_t^s}$; $H_t^c \equiv \lambda h_t^c + 1 - \lambda$; $H_t^n \equiv \lambda \theta_t^h h_t^n + 1 - \lambda$; $H_t \equiv (\lambda \frac{h_t^n}{h_t^c} + 1 - \lambda)/H_t^n$.

When markets are complete, the multiplier Ψ_t^{IM} is constant ($\Psi_t^{IM} = \Psi \ \forall \ t$), and the expression becomes:

$$\tau_t = 1 - \frac{(H_t^c)^{-1}(1-\lambda)}{\Big[H_t H_t^n + (1-\lambda)(1+\phi)\Psi\Big] - (H_t^n - (1-\lambda))\Big[(1+\phi)(h_t^c)^{-1} + \frac{\phi}{H_t^c}\Big]}$$

Proof: Using the equations of the system (2.24) and the functional forms given in (2.17), while setting $T_t = 0 \ \forall t$, we can define Ψ^3 as follows:

$$\Psi^3 = \frac{1}{(1+\phi)\theta^h h_t^n}\Big[H_t^n\Big(H_t - \frac{1}{H^c(1-\tau_t)}\Big) + \Psi_t^{IM}\Big(\frac{H_t^n \tau_t}{1-\tau_t} + \psi\frac{n}{n^s}\Big) - \frac{b_{t-1}}{c_t^s}(\Psi_t^{IM} - \Psi_{t-1}^{IM})\frac{1}{H_t^c(1-\tau_t)}\Big]$$

Replacing Ψ^3 by this expression in equation (2.25) leads to the tax expression given in equation (2.35).

Log-linearizing the equation for the incomplete market case leads to:

$$\begin{aligned}\hat{\tau}_t \ &= \ \Big[\frac{(1-\tau)}{(1-\lambda)}\Big(H^n - (1-\lambda)\Big)\phi - 1\Big]\hat{\psi}_t + (1-\tau)(1+\phi)H^c\Psi\hat{\Psi}_t \\ &+\hat{H}_t^c + \frac{(1-\tau)}{(1-\lambda)}H^n H^c H(\hat{H}_t + \hat{H}_t^n) - \frac{(1-\tau)}{(1-\lambda)}H^n\Big((1+\phi)\frac{H^c}{h^c} - \phi\Big)\hat{H}_t^n + \frac{(1-\tau)}{(1-\lambda)}\Big(H^n - (1-\lambda)\Big)\Big((1+\phi)\frac{H^c}{h^c}\hat{h}_t^c - \phi\hat{H}_t^c\Big)\end{aligned}$$

.

Simulated method of moments In Section 2.5.3 of the paper, we choose the model parameters to target empirical moments. In particular, we make use of the simulated method of moments (SMM), in which the structural model parameters are chosen to minimize the distance between the moments computed from numerical simulations of the model, and those observed in the data. More formally, we choose the vector of parameters which minimizes a quadratic loss function, thereby obtaining the estimator $\hat{\theta}$ that satisfies:

$$\hat{\theta} = \underset{\theta \in \Theta}{\text{argmin}} \Big(m(\theta) - m_{\text{us}}\Big)' \mathbf{W} \Big(m(\theta) - m_{\text{us}}\Big)$$

where $\mathbf{W}$ is a weighting matrix, which we set to the identity matrix in our estimation routine. m_{us} is the vector of moments computed from the US data, and $m(\theta)$ is a function mapping model parameters θ to a vector of simulated moments. We make use of a non-linear optimization routine to solve for the above problem.

2.7.3 Data Appendix

In this section we provide details on the construction of the data series used in Section 2.2, when we provide stylized facts for the US economy, and in Section 2.4 and 2.5 when we target empirical moments with our theoretical model.

Macro data

All aggregate data, i.e. government spending, transfers, deficits and the market value of government debt, are for the U.S and are observed at a quarterly frequency. They are computed based on National Income and Products Account (NIPA) collected by the Bureau of Economic Analysis. Real values are obtained using the GDP deflator.

All variables are in per capita terms. Each variable is divided by an index of the US population, constructed from the 'Pop Civilian noninstitutional population aged 16 years and over' series from the US Bureau of Labor Statistics.

Output is measured as real output per capita. We deflate nominal GDP (Table 1.1.5, Line 1) with the GDP deflator, and divide it with the population index.

Government spending is defined as the sum of consumption expenditure (Table 3.2 Line 25), gross government investment (Table 3.2 Line 45), net purchases of non-produced assets (Table 3.2 Line 47), minus consumption of fixed capital (Table 3.2 Line 48).

Tranfers: In this paper, we adopt a narrow definition of transfers. We are mainly interested in the components of transfers targeted towards households, and reflecting social insurance. We therefore define transfers as a subset of government social benefits to persons[30] (provided in section 2 of the NIPA): we compute them as the sum of unemployment insurance (Table 2.1 Line 21) and other benefits [31] (Table 2.1 Line 23).

Deficits are defined as government expenditures (Table 3.2 Line 42) minus government receipts (Table 3.2 Line 39) and interest payments (Table 3.2 Line 32).

The market-value of debt-to-GDP is taken from the Dallas Fed. We use series on the market value of marketable treasury debt. To construct quarterly series we use the stock of debt in the first month of each quarter.

Cross-sectional data

In order to construct our series on consumption, income and transfer receipts across household types, we make use of the Consumer Expenditure Survey (CEX).

The CEX consists of two separate surveys collected for the Bureau of Labor Statistics by the Census Bureau that provide detailed information about household consumption expenditures. It consists of a rotating panel of households that are selected to be representative of the US population every quarter. Each household is interviewed for a maximum of four consecutive quarters. However, we treat each wave as cross sectional.

Income: Each household reports information on income, hours worked and taxes paid over the twelve-month period preceding the interview. We compute the **earnings** of each household as the sum of wages and salaries plus two thirds of business and farm income earned by that household. **Income before taxes** (money income) includes the sum of wages, salaries, business and farm income earned by each member plus household financial income (including interest, dividends and rents) plus private transfers (including private pensions, alimony and child support) plus public transfers (including social security, unemployment compensation, welfare and food stamps). **Income after taxes** (disposable income) is computed as money income minus personal taxes (including federal, states and local income taxes), property taxes and other taxes such as vehicle personal property taxes.

[30]Government social benefits to persons are composed of: social security benefits (Table 2.1 Line 18) Medicare and Medicaid benefits (Table 2.1 Line 19 and 20, respectively), unemployment insurance (Table 2.1 Line 21), veterans' benefits (Table 2.1 Line 22) and other benefits (Table 2.1 Line 23).

[31]Other benefits include the main income assistance programs such as Supplemental Nutrition Assistance Program, Black lung benefits, Supplemental security income, and Direct relief. They also include housing subsidies and some education and childcare assistance programs.

Transfers: The CEX provides information about the following categories of private and public transfers received by the households:

1. **SSI**: Supplemental Security Income.
2. **WLF**: Amount received from public assistance or welfare including money received from job training grants such as Job Corps.
3. **UNEMP**: Amount received from unemployment compensation.
4. **FDSTMP**: Annual value of food stamps.
5. **OTHR**: Amount of income received from any other source such as Veteran's Administration (VA) payments, unemployment compensation, child support, or alimony.

These categories cover the amounts perceived in the past 12 months.

To comply with the narrow definition of transfers adopted in this paper, we define the individual transfer series as the sum of UNEMP and FDSTMP.

Real values: As is the case for aggregate variables, real values are obtained using the GDP deflator.

Transfers along income quintiles To complement the results displayed in Figure 2.3 (plotting the share of transfers towards housheolds at the bottom 30% and top 70% of the income distribution), we show in Figure 2.13 the same figure along income quintiles. We can see from the figure that the per capita amount of transfers received is decreasing along income quintile, thereby confirming the results displayed in the main text.

Indeed, most of the transfers are directed towards low income households. In particular, hand-to-mouth households receive on average more than 90% of Food Stamps benefits and around 40% of unemployment benefits [32].

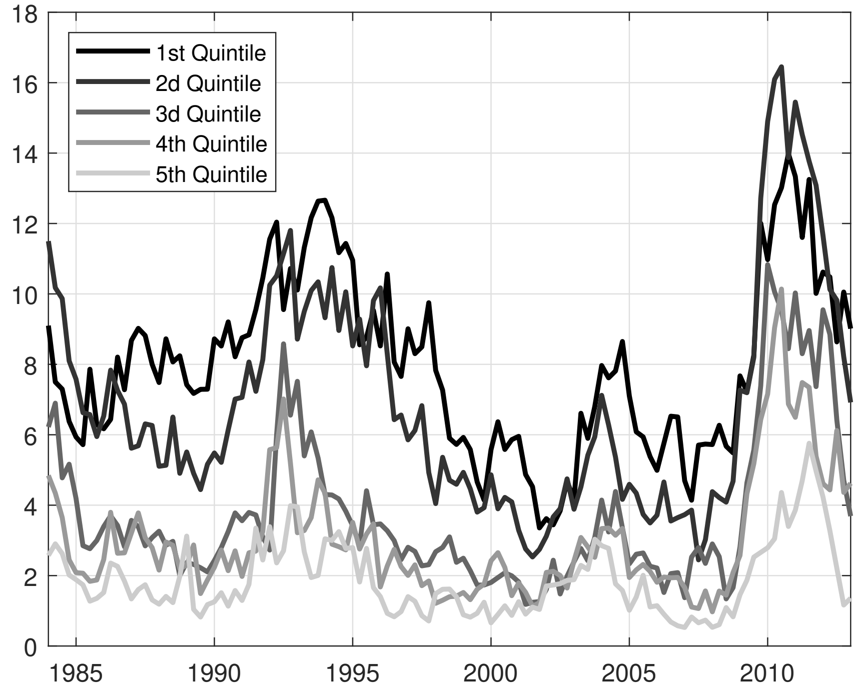
Inequality and the business cycle:

We provide here more details on how inequality evolves over the business cycle according to CEX data.

Earnings of hand-to-mouth households appear to be slightly more pro-cyclical than the earnings of Riccardian households. Indeed, as shown in Table 2.4 the correlation between *hand to mouth* households' earned income and GDP amounts to 0.34, while it amounts to 0.13 for Ricardian households. Moreover, the volatility of the (log-)earnings of hand-to-mouth households appears to be four times higher than the one associated with the earnings of Riccardian households.

[32] Moreover, hand-to-mouth households received in average more than 90% of the total amount of the

FIGURE 2.13: **Average transfers per household along the income distribution**



Notes: The figure plots the behaviour of transfers per capita for households belonging to each quintile of the income distribution at the quarterly frequency. Transfers to households of the first quintile of the income distribution (that is the bottom 20%) are drawn with the darker black line, while transfers to households of the fifth quintile of the income distribution (that is the top 20%) are drawn in the lightest gray line. Data is from the CEX database.

public assistance and welfare programs benefits, more than 80% of the total amount of SSI benefits and around 60% of other transfers

Table 2.4: **Correlation between Earnings and GDP**

x	**Corr(x,GDP)**	**Std(x)**
Earned Income, bottom 30%	0.34	0.05
Earned Income, top 70%	0.13	0.01

Notes: The table displays the correlation between the cyclical components of households log-earnings and GDP for low and high income households. The table also displays the standard deviation of households earnings for both groups. Data on GDP are from the NIPA database. Data on Earned Income come from the CEX. Since income data are accounted as the amount perceived over the twelve-month period preceding the interview in the CEX, GDP is computed as a moving average over 4 consecutive quarters. All variables are logged, and de-trended with the HP-Filter.

Chapter 3

Optimal Monetary Policy with and without Debt

Co-authored with Rigas Oikonomou (UCLouvain), Romanos Priftis (Bank of Canada) and Lukas Vogel (European Commission).

Abstract
We propose a novel framework of optimal monetary policy in the case where debt sustainability may be a constraint for the central bank. We assume that a monetary authority solves a Ramsey problem under commitment, subject to the competitive equilibrium constraints, and including the 'consolidated budget constraint'. Depending on the specification of fiscal policy, the consolidated budget maybe (ir)relevant to the planner's program: If fiscal policy cannot generate sufficient surpluses to finance debt then optimal monetary policy must ensure debt sustainability and takes into account the constraint. In contrast, if fiscal policy can finance debt through taxes, then the constraint is slack. In the case where optimal policy exhibits 'debt concerns' monetary policy becomes subservient to fiscal policy, and needs to partially give up on the objective of stabilizing inflation and the output gap to satisfy the consolidated budget. We show analytically that this regime is isomorphic to a "passive monetary /active fiscal policy" regime (e.g.Leeper, 1991). Instead, under 'no debt concerns', monetary policy is "active" and fiscal policy 'passive'. During liquidity trap (LT) episodes optimal passive monetary policy with debt is ineffective in stabilizing inflation: Committing to keep interest rates low for a long period leads to deflation rather than to an increase inflation rates. Conversely, under no debt concerns, monetary policy is very effective in stabilizing the macroeconomy.

3.1 Introduction

Since the 2008-9 recession governments in developed economies have accumulated large stocks of debt. High debt levels require bold fiscal adjustments to ensure solvency of government budgets. However, in many countries it is questionable whether fiscal authorities will be able to generate the large required surpluses to finance debt. This problem is likely to be exacerbated in the coming years; large (current and projected) deficits aimed at mitigating the effects of COVID 19 as well as anemic economic growth will likely worsen the fiscal situation in advanced economies.

At high debt levels, when fiscal authorities have little margin to adjust taxes sufficiently, debt becomes an important constraint for monetary policy. How should policy be designed under such circumstances? A sizable literature adopting the Ramsey approach to optimal policy has relied on *nonlinear models* to draw conclusions about the processes of inflation and interest rates when the 'planner' takes into account debt sustainability. Schmitt-Grohé and Uribe (2004), Faraglia et al. (2013), Lustig, Sleet, and Yeltekin (2008), Eggertsson and Woodford (2006), Jung, Teranishi, and Watanabe (2005), Bhattarai, Eggertsson, and Gafarov (2015) and Adam and Billi (2006, 2008) are well known examples of this work. Because these models are non-linear and characterize optimal policies through first order conditions they are not easy to use in order to draw practical conclusions about the conduct of policy. It is, for instance, not straightforward to use the optimality conditions from these models to arrive to explicit interest rate rules, that could guide policy responses to macroeconomic conditions, or that can be made comparable to actual policies identified in (estimated) DSGE models.

On the DSGE literature side, a widely used framework suitable to analyze the interactions between monetary and fiscal policies is that of Leeper (1991) (see also Bianchi and Ilut (2017), Bianchi and Melosi (2017, 2019) for recent extensions to medium scale DSGE models). In this framework monetary policy is summarized in simple transparent interest rate rules. However, the policy rules employed are ad hoc and so monetary policy is not designed to be optimal.

In this paper we propose a tractable framework of Ramsey optimal monetary policy in the case where debt sustainability may be a constraint for the central bank. The model gives rise to interest rates rules in closed form, which enables us to develop analytical insights and draw sharp conclusions about the conduct of policy. Moreover, our theory can be easily embedded in a medium scale DSGE model and estimated with data.

Our framework is developed in Section 2 and assumes that a Ramsey planner (the Fed) under commitment sets allocations to minimize the deviations of inflation, output and interest rates from their respective target levels, subject to the standard set of dynamic equations which define the competitive equilibrium. In the first version of the Ramsey policy equilibrium, in which we assume that the Fed's policies are also driven by *debt sustainability concerns*, we assume that this set also includes the *consolidated budget constraint* which determines the value of net debt in the hands of the private sector. Taxes are distortionary and, moreover, tax policy is assumed to be exogenous to the planner's problem; it follows a simple rule which determines the tax rate as a function of lagged debt, a standard assumption in the DSGE literature (e.g. Leeper, 1991).

Our first analytical result derives the interest rate rule that emerges from optimal policy in this model. We show that interest rates follow a standard Taylor rule (respond to inflation, output growth etc) but also the central bank engages in 'forward guidance'. In our model forward guidance is expressed as the sum of two components: The first component represents commitment to keep interest rates low at the exit from a LT episode (e.g. Eggertsson et al. (2003); Eggertsson and Woodford (2006)) and is captured by the lags of the Lagrange multiplier attached to the occasionally binding zero lower bound (ZLB) constraint. The second component measures the impact of past shocks to the consolidated budget constraint, capturing the planner's commitment to "twist interest rates" in order to satisfy the intertemporal budget (e.g. Lustig et al. (2008), Faraglia et al. (2019b), henceforth FMOS). Since debt is distortionary, ours is a model of optimal policy under incomplete markets (see e.g. Aiyagari et al. (2002, FMOS) and the Lagrange multiplier on the consolidated budget constraint is a state variable. Interest rate twisting is captured by the lags of this multiplier.

The second version of Ramsey policy we consider is one where the consolidated budget does not enter into the constraint set. In this case, forward guidance emerges only from the impact of the occasionally-binding ZLB and the model does not feature any interest rate twisting effects from shocks to the intertemporal budget. By switching on and off the consolidated budget from the Ramsey program in this way, we are able to contrast the properties of equilibria where monetary policy is concerned with debt sustainability with equilibria where monetary policy has none of such concerns.

In section 3 we study the properties of these two versions of our model in the neighborhood of the steady state, that is assuming that the zero lower bound is non-binding. We first focus on the monetary/fiscal interactions to evaluate what types of fiscal policies justify taking into account the consolidated budget in optimization and what types do not. We establish a link between our model and Leeper's (1991) famous analysis on 'passive /active' policies. We firstly show analytically that in the "debt concerns" model the rational expectations equilibrium is (locally) unique if taxes do not respond strongly to the deviations of government debt from its steady state level and debt becomes an explosive process. In contrast, in the case of "no debt concerns", when the consolidated budget does not enter in optimization, taxes need to strongly adjust to the debt level and debt becomes a mean reverting process. In the widely used terminology of Leeper (1991), determinacy under debt concerns requires an 'active' fiscal policy whereas under no debt concerns fiscal policy is 'passive'.

We then turn to characterize monetary policy into active/passive in each of the versions of our optimal policy program. Leeper's classification hinges on the response of interest rates to inflation and this is not so straightforward to map into our model especially in the presence of interest rate twists. However, using a mixture of analytical results and simulations, we show that the policy rule under debt concerns is equivalent to a standard 'passive money' rule, exhibiting anemic responses of interest rates to inflation. The no debt concerns policy is equivalent to an active monetary policy. This finding, which shows that Leeper's analysis can be made consistent with an optimal Ramsey policy framework, is to our knowledge new to the literature.

More importantly, this finding enables us to easily apprehend our optimal policy model's properties. A large body of work has been devoted to solving and estimating 'passive money' models and a standard feature of these models is that they magnify the impact of shocks on interest rates output and inflation; the macroeconomic volatility predicted by these models tends to be higher.[1] This is precisely what happens under debt concerns; the monetary authority (partially) gives up on the goal of stabilizing interest rates, output and inflation, to satisfy the intertemporal budget; this leads to higher volatility. In contrast, volatility is much lower under no debt concerns, where monetary policy focuses on stabilizing macroeconomic variables.

In Section 4 of our paper we turn to the properties of optimal policies in the liquidity trap. Our key finding is that, under debt concerns, monetary policy faces a worse tradeoff and is generally ineffective in stabilizing output and inflation. Moreover, forward guidance, promising to keep interest rates low for a long period, does not lead to an increase inflation and may even lead to a sharp deflation. In contrast, under no debt concerns monetary policy is effective, and promising lower future rates increases inflation at the onset of a liquidity trap.

What explains the ineffectiveness of forward guidance under debt concerns? It is now well known, that under passive monetary policy an exogenous drop in interest rates can (eventually) make inflation turn negative; this is what Sims (2011) and Cochrane (2018) refer to as the 'stepping on a rake'. Our debt concerns model has this property, and as we show, promising to lower the interest rate in the future, ultimately leads to persistently negative inflation.

In other words, the standard argument in favor of forward guidance being useful to engineer inflation in the future, which then translates through the Phillips curve into

[1]See e.g. Bianchi and Melosi (2017) and Bianchi and Ilut (2017).

higher inflation at the onset of the LT fails in the debt concerns model. Nevertheless, the planner may still find optimal to promise interest rate cuts, since when these are filtered through the consolidated budget they may increase inflation today and the cost of reversing this increase in the future through deflation. If the goal of the monetary authority is to avoid a very sharp price drop at the onset of the liquidity trap, then forward guidance can be useful. Even so, liquidity traps under debt concerns are much more deflationary than under no debt concerns, suggesting that the overall planner's tradeoff worsens.

Section 5 presents several extensions of our framework. Firstly, we embed our optimal policy framework into a medium-scale DSGE model. We do so for robustness, to show that our results carry through in a model which provides a more accurate image of the US economy. Our quantitative model extends the baseline with preferences exhibiting habit formation, shocks to TFP, markup shocks, government transfers, realistic fiscal rules etc. Our key results continue to hold: Debt concerns magnifies macroeconomic volatility and when the economy experiences a large negative demand shock that drives interest rates to the ZLB the model predicts a worse tradeoff than under no debt concerns. Secondly, we explore the robustness of our findings towards introducing lump sum taxation and varying the maturity of debt and also compare our results to the benchmark Ramsey equilibrium which assumes coordinated monetary and fiscal policies. More substantively, we show that our framework can be extended to allow for regime fluctuations, as recent literature has assumed (e.g. Bianchi and Ilut (2017) and Bianchi and Melosi (2017)). The appendix extends further this analysis. A final Section concludes the paper.

3.2 Theoretical Framework

We begin with a simple setup of the Ramsey policy equilibrium which allows us to illuminate key forces driving interest rates in our model. We consider two alternative specifications of the baseline model. In the first version of Ramsey policy we let the planner choose interest rates, inflation and output, subject to the consolidated budget constraint (of the Fed and the government). We refer to this model as the 'debt concerns' (DC) model. The aim is to capture a scenario under which the planner (Fed) takes into account the sustainability of debt. We show analytically that in this case the optimal interest rate rule has three components: First, a standard Taylor rule component, which links interest rates to realized inflation, output growth and lagged values of interest rates. Second, a component which represents commitment to keep interest rates low at the exit from a LT episode (e.g. Eggertsson et al. (2003); Eggertsson (2006)). Third, a component which measures the impact of past promises made by the planner to alter interest rates in the face of shocks to the consolidated intertemporal budget constraint. The third component is an "interest twisting effect" which emerges because debt is distortionary and long-term as in FMOS.

In the second setup of policy, in which the planner has 'no debt concerns' (NDC) she does not account for the consolidated budget constraint in optimization. The optimal policy rule in this case is identical to debt concerns, in terms of the first two components, but features no interest rate twisting impacts.

Throughout this section we focus on monetary policy, assuming that fiscal policy is a simple tax rule of the form:

$$\hat{\tau}_t = \rho_\tau \hat{\tau}_{t-1} + (1 - \rho_\tau)\phi^R_{\tau,b}\hat{b}_{t-1,\delta} + \epsilon_{\tau,t} \tag{3.1}$$

where $\hat{\tau}_t$ denotes the tax rate and $\hat{b}_{t-1,\delta}$ is debt (both in deviation from steady state values). $\epsilon_{\tau,t}$ is a shock to the tax rate. (3.1) is a standard rule linking taxes to lagged taxes and debt (e.g. leeper1991equilibria). Coefficient $\phi^R_{\tau,b}$ measures the feedback effect of debt issued in $t-1$ on the tax rate in t. Index $R \in \{DC, NDC\}$ is used to denote that taxes will follow a different process under DC and NDC.

In Section 3 where we will examine the interactions between monetary and fiscal policies, we will discuss what types of fiscal policies justify taking into account the consolidated budget in optimization and what types do not. In particular we will illustrate that when fiscal policy is 'active' (e.g. Leeper, 1991) and taxes do not strongly adjust to satisfy intertemporal solvency of debt, then monetary policy needs to account for the constraint. Otherwise if taxes adjust to debt, then monetary policy should optimize as in the no debt concerns model. As we will explain any other combination of policies, will lead to indeterminacy of the equilibrium.

3.2.1 The baseline model

The building blocks of our model are a standard NK Phillips curve, an IS (Euler) equation which prices a short nominal bond, and the consolidated budget constraint which determines the value of (net) debt held by the private sector. The model is a simplified version of the New-Keynesian (NK) model that we will later employ in estimation and which we formally describe in Section 5 and in the appendix. For brevity, we summarize here the competitive equilibrium in the linearized version of the model. For detailed derivations we refer the reader to the appendix.

Let $\hat{x}$ denote the log deviation of variable x from its steady state value, $\overline{x}$. The competitive equilibrium equations are the following:

$$\hat{\pi}_t = \kappa_1 \hat{Y}_t + \kappa_2 \hat{\tau}_t - \kappa_3 \hat{G}_t + \beta E_t \hat{\pi}_{t+1}, \tag{3.2}$$

where $\kappa_1 \equiv -\frac{(1+\eta)\overline{Y}}{\theta}(\gamma_h + \sigma\frac{\overline{Y}}{\overline{C}}) > 0$, $\kappa_2 \equiv -\frac{(1+\eta)\overline{Y}}{\theta}\frac{\overline{\tau}}{(1-\overline{\tau})} > 0$, $\kappa_3 \equiv -\frac{(1+\eta)}{\theta\overline{Y}}\sigma\frac{\overline{G}}{\overline{C}} > 0$,

$$\hat{i}_t = E_t\left(\hat{\pi}_{t+1} - \hat{\xi}_{t+1} + \hat{\xi}_t - \sigma\left[\frac{\overline{Y}}{\overline{C}}(\hat{Y}_t - \hat{Y}_{t+1}) - \frac{\overline{G}}{\overline{C}}(\hat{G}_t - \hat{G}_{t+1})\right]\right) \tag{3.3}$$

$$\hat{i}_t \geq -\frac{1}{\beta} + 1 \equiv -i^* \tag{3.4}$$

$$\begin{aligned}
&\frac{\beta\overline{b}_\delta}{1-\beta\delta}\hat{b}_{t,\delta} + \overline{b}_\delta\sum_{j=1}^{\infty}\beta^j\delta^{j-1}\left[E_t\left(-\sigma(\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j} - \frac{\overline{G}}{\overline{C}}\hat{G}_{t+j}) - \sum_{l=1}^{j}\hat{\pi}_{t+l} + \hat{\xi}_{t+j}\right)\right] \\
&+\frac{\overline{\tau}(1+\eta)\overline{Y}}{\eta}\left((\gamma_h+1)\hat{Y}_t + \hat{\xi}_t + \frac{\hat{\tau}_t}{1-\overline{\tau}}\right) - \overline{G}\left(\hat{G}_t - \sigma\frac{\overline{Y}}{\overline{C}}\hat{Y}_t + \sigma\frac{\overline{G}}{\overline{C}}\hat{G}_t + \hat{\xi}_t\right) + \overline{b}_\delta\sigma\left(\frac{\overline{Y}}{\overline{C}}\hat{Y}_t - \frac{\overline{G}}{\overline{C}}\hat{G}_t\right) - \overline{b}_\delta\hat{\xi}_t \\
&= \frac{\overline{b}_\delta}{1-\beta\delta}(\hat{b}_{t-1,\delta} - \hat{\pi}_t) + \delta\overline{b}_\delta\sum_{j=1}^{\infty}\beta^j\delta^{j-1}E_t\left(-\sigma(\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j} - \frac{\overline{G}}{\overline{C}}\hat{G}_{t+j}) - \sum_{l=1}^{j}\hat{\pi}_{t+l} + \hat{\xi}_{t+j}\right)
\end{aligned} \tag{3.5}$$

(3.2) is the Phillips curve at the heart of our model. $\hat{Y}_t$ is the output gap and $\hat{\tau}$ denotes a distortionary tax levied on the labor income of households. $\hat{G}_t$ denotes government spending in t. Parameters $\eta < 0$ and $\theta > 0$ govern the elasticity of substitution across differentiated products and the degree of price stickiness respectively.[2]

(3.3) is the log-linear Euler equation which prices a short term nominal asset. $\hat{\xi}$ is a standard preference shock which affects the relative valuation of current vs. future utility by the household. A drop in $\hat{\xi}$ makes the household relatively patient, willing to substitute current for future consumption. (3.4) is the ZLB constraint on the short-term nominal interest rate.

(3.74) is the consolidated budget constraint. We assume that debt is issued in a perpetuity bond with decaying coupons where δ denotes the decay factor. Short debt is in zero net supply. Finally, parameter σ denotes the inverse of the intertemporal elasticity of substitution and γ_h is the inverse of the Frisch elasticity of labor supply. Taxes are set according to rule (3.1).

[2]We assume price adjustment costs as in Rotemberg (1982). θ governs the magnitude of these costs. When θ equals zero, prices are fully flexible.

3.2.2 Ramsey Policy under Debt Concerns

The Ramsey policy chooses inflation, output and interest rate sequences to maximize the following objective

$$-\frac{1}{2}\sum_{t=0}^{\infty}\beta^t E_0\Big\{\hat{\pi}_t^2+\lambda_Y\hat{Y}_t^2+\lambda_i\hat{i}_t^2\Big\} \tag{3.6}$$

subject to equations (3.2) to (3.74). λ_Y and λ_i govern the relative weights attached to output gap and interest rate stabilization by the planner.

We further assume that tax policies cannot be controlled by the planner. This essentially means that rule (3.1) is taken as given and therefore $\hat{\tau}_t$ is not a choice variable.[3] As it is standard we solve for optimal policies with a Lagrangian. Letting $\psi_{\pi,t}$ be the multiplier attached to the Phillips curve constraint, $\psi_{i,t}$, $\psi_{ZLB,t}$ and $\psi_{gov,t}$ the analogous multipliers attached to the Euler equation, the ZLB constraint and the consolidated budget respectively, the Lagrangian can be written as:

$$\begin{aligned}
&E_0\sum_{t=0}^{\infty}\beta^t\Bigg[-\frac{1}{2}\Big\{\hat{\pi}_t^2+\lambda_Y\hat{Y}_t^2+\lambda_i\hat{i}_t^2\Big\}\\
&+\psi_{gov,t}\Bigg(\frac{\beta\bar{b}_\delta}{1-\beta\delta}\hat{b}_{t,\delta}+\beta\bar{b}_\delta\sum_{j=1}^{\infty}\beta^{j-1}\delta^{j-1}\Big[E_t\Big(-\sigma(\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j}-\frac{\overline{G}}{\overline{C}}\hat{G}_{t+j})-\sum_{l=1}^{j}\hat{\pi}_{t+l}+\hat{\xi}_{t+j}\Big)\Big]+\overline{S}\hat{S}_t-\frac{\bar{b}_\delta}{1-\beta\delta}\hat{b}_{t-1,\delta}\\
&-\bar{b}_\delta\sum_{j=0}^{\infty}\beta^j\delta^j E_t\Big[-\sigma\Big(\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j}-\frac{\overline{G}}{\overline{C}}\hat{G}_{t+j}\Big)-\sum_{l=0}^{j}\hat{\pi}_{t+l}+\hat{\xi}_{t+j}\Big]\Bigg)+\psi_{\pi,t}\Big(\hat{\pi}_t-\kappa_1\hat{Y}_t-\kappa_2\hat{\tau}_t+\kappa_3\hat{G}_t-\beta E_t\hat{\pi}_{t+1}\Big)\\
&+\psi_{i,t}\Bigg(\hat{i}_t-E_t\Big(\hat{\pi}_{t+1}-\hat{\xi}_{t+1}+\hat{\xi}_t-\sigma\Big[\frac{\overline{Y}}{\overline{C}}(\hat{Y}_t-\hat{Y}_{t+1})-\frac{\overline{G}}{\overline{C}}(\hat{G}_t-\hat{G}_{t+1})\Big]\Big)\Bigg)+\psi_{i,t}\Big(\hat{i}_t+i^*\Big)\Bigg]
\end{aligned} \tag{3.7}$$

where $\overline{S}\hat{S}_t \equiv \Bigg[-\overline{G}\Big(\hat{G}_t(1+\sigma\frac{\overline{G}}{\overline{C}})-\sigma\frac{\overline{Y}}{\overline{C}}\hat{Y}_t+\hat{\xi}_t\Big)+\frac{\overline{\tau}\overline{Y}(1+\eta)}{\eta}\Big((1+\gamma_h)\hat{Y}_t+\frac{\hat{\tau}_t}{1-\overline{\tau}}+\hat{\xi}_t\Big)\Bigg]$ is the surplus of the government multiplied by marginal utility.

The first order conditions for the optimum are given by:

$$-\hat{\pi}_t+\Delta\psi_{\pi,t}-\frac{\psi_{i,t-1}}{\beta}+\frac{\bar{b}_\delta}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l}=0 \tag{3.8}$$

$$-\lambda_Y\hat{Y}_t-\psi_{\pi,t}\kappa_1+\sigma\frac{\overline{Y}}{\overline{C}}(\psi_{i,t}-\frac{\psi_{i,t-1}}{\beta})+\sigma\frac{\overline{Y}}{\overline{C}}\bar{b}_\delta\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l}+\omega_Y\psi_{gov,t}=0 \tag{3.9}$$

$$-\lambda_i\hat{i}_t+\psi_{i,t}+\psi_{ZLB,t}=0 \tag{3.10}$$

$$\frac{\beta\bar{b}_\delta}{1-\beta\delta}\Big(\psi_{gov,t}-E_t\psi_{gov,t+1}\Big)=0 \tag{3.11}$$

$$\psi_{ZLB,t}\geq 0 \qquad \text{and} \qquad (\hat{i}_t+i^*)\psi_{ZLB,t}=0 \tag{3.12}$$

where $\omega_Y \equiv \overline{G}\sigma\frac{\overline{Y}}{\overline{C}}+\frac{\overline{\tau}(1+\eta)}{\eta}\overline{Y}(1+\gamma_h)$. (3.8) is the FONC with respect to $\hat{\pi}_t$; (3.9), (3.10), (3.11) are first order conditions with respect to $\hat{Y}_t$, $\hat{i}_t$ and $\hat{b}_{t,\delta}$ respectively. (3.12) is the complementary slackness condition for the ZLB constraint.

[3] We will also not allow the planner to influence $\hat{\tau}$ through the choice of debt. In section (3.3.2) we will discuss how letting the planner influence taxes through debt affects our results.

Optimal interest rate policy Combining the first order conditions (3.8), (3.9) and (3.10) and following the argument of Giannoni and Woodford (2003) we can derive analytically the optimal interest rate rule from the Ramsey program. The following proposition summarizes the result:

Proposition 1. *Assume that the central bank has 'debt concerns'. The optimal interest rate rule is:*

$$\hat{i}_t = \max\{\mathcal{T}_t + \mathcal{D}_t + \mathcal{Z}_t, -i^*\} \tag{3.13}$$

$$\mathcal{T}_t \equiv \phi_\pi \hat{\pi}_t + \phi_Y \Delta \hat{Y}_t + \phi_i \hat{i}_{t-1} + \frac{1}{\beta}\Delta \hat{i}_{t-1}$$

with $\phi_\pi = \frac{\kappa_1 \overline{C}}{\lambda_i \sigma \overline{\overline{Y}}}$, $\phi_i = (1 + \frac{\kappa_1}{\beta\sigma}\frac{\overline{C}}{\overline{\overline{Y}}})$ and $\phi_Y = \frac{\lambda_Y \overline{C}}{\sigma \lambda_i \overline{\overline{Y}}}$,

$$\mathcal{D}_t = -\frac{\overline{C}}{\overline{\overline{Y}}}\frac{\kappa_1}{\lambda_i \sigma}\frac{\overline{b}_\delta}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l \Delta\psi_{gov,t-l} - \frac{\overline{b}_\delta}{\lambda_i}\sum_{l=0}^{\infty}\delta^l\Big(\Delta\psi_{gov,t-l} - \Delta\psi_{gov,t-l-1}\Big) - \frac{\overline{C}\omega_Y}{\overline{\overline{Y}}\sigma\lambda_i}\Delta\psi_{gov,t}$$

and

$$\mathcal{Z}_t = -\frac{1}{\lambda_i}(1 + \frac{1}{\beta} + \frac{\kappa_1}{\sigma\beta}\frac{\overline{C}}{\overline{\overline{Y}}})\psi_{ZLB,t-1} + \frac{1}{\beta\lambda_i}\psi_{ZLB,t-2}.$$

Proof See appendix.

As equation (3.13) shows there are three distinct components in the interest rate rule. The first, $\mathcal{T}_t$, is a standard Taylor rule component which links interest rates to inflation, output growth and lagged values of the interest rate. The impact of these variables on the interest rate policy rule depends on the weights λ_i, λ_Y which capture the output and interest rate stabilization objectives of the planner. It also depends on the structural parameters σ, κ_1 and β.

The second component, $\mathcal{Z}_t$, is a pull factor which relates interest rates to the "bindness" of the ZLB constraint. If for example $\hat{i}_{t-1} = -i^*$ then (from complementary slackness) we have $\psi_{ZLB,t-1} > 0$. In this case $\mathcal{Z}_t$ becomes negative and so the planner keeps the interest rate in t lower than the value implied by the Taylor rule component (and partially reverses the effect in $t+1$). The impact of this channel on the level of interest rates depends on λ_i, with higher values of λ_i leading to a lower impact, since the planner's objective to minimize the deviation of the interest rate from its target becomes stronger.

Finally, the term $\mathcal{D}_t$ measures the impact of changes in the value of the multiplier $\psi_{gov,t}$

on the optimal interest rate path. To analyze this term, use equation (3.11). We have:

$$\psi_{gov,t+1} = \psi_{gov,t} + \epsilon_{t+1,G} \tag{3.14}$$

In other words, the multiplier $\psi_{gov,t}$ is a random walk, and $\epsilon_{t+1,G}$ denotes a mean zero, i.i.d shock to the value of the multiplier. We can now write:

$$\mathcal{D}_t = -\frac{\overline{C}}{\overline{Y}}\frac{\kappa_1}{\lambda_i\sigma}\frac{\bar{b}_\delta}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l\epsilon_{t-l,G} - \frac{\bar{b}_\delta}{\lambda_i}\sum_{l=0}^{\infty}\delta^l\left(\epsilon_{t-l,G} - \epsilon_{t-l-1,G}\right) - \frac{\overline{C}\omega_Y}{\overline{Y}\sigma\lambda_i}\epsilon_{t,G} \tag{3.15}$$

which relates $\mathcal{D}_t$ to current and past shocks to the value of the multiplier.

What do these shocks capture? Notice that since (real) debt can either be financed through distortionary taxes or through distortionary inflation, ours is a standard model of optimal policy under incomplete markets (e.g. Aiyagari et al. (2002), Schmitt-Grohé and Uribe (2004), Lustig et al. (2008), Faraglia et al. (2013), FMOS among others). As is well known, in these models shocks to the economy translate to changes in the excess burden of distortions, the multiplier ψ_{gov} that measures the magnitude of these distortions behaves like a random walk, since the planner wants to spread evenly the costs across periods. In the presence of long term debt ($\delta > 0$) the sequence of shocks $\{\epsilon_{t-l,G}\}_{l=0}^{\infty}$ influences interest rates because all the lags of these variables enter into the state vector, as the FONC reveal.

To clarify further the role played by the sequence $\{\epsilon_{t-l,G}\}_{l=0}^{\infty}$ we iterate forward on constraint (3.74) to get:

$$E_t\sum_{j=0}^{\infty}\beta^j\overline{S}\hat{S}_{t+j} = \frac{\bar{b}_\delta}{1-\beta\delta}\hat{b}_{t-1,\delta} + \bar{b}_\delta\sum_{j=0}^{\infty}\beta^j\delta^jE_t\left[-\sigma\left(\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j} - \frac{\overline{G}}{\overline{C}}\hat{G}_{t+j}\right) - \sum_{l=0}^{j}\hat{\pi}_{t+l} + \hat{\xi}_{t+j}\right] \tag{3.16}$$

(3.16) is the *intertemporal consolidated budget constraint* linking the present discounted value of the fiscal surplus to the real value of debt outstanding in t. Notice also that (3.16) is equivalent to (3.74) in terms of the Ramsey policy.[4] Consider the impact of a shock which lowers the LHS of (3.16) relative to the RHS. This may, for example, occur following a shock which lowers taxes. In response to this shock the constraint tightens, the value of the multiplier ψ_{gov} increases. Therefore, $\epsilon_{t,G} > 0$. To satisfy the constraint, the monetary authority needs to engineer a drop in the real payout of debt (the RHS of (3.16)) either through increasing future inflation and /or increasing future output when $\sigma > 0$. Note that under commitment it is feasible to make such promises about the future course of economic variables. The terms that enter in $\mathcal{D}_t$ in equation (3.15) are essentially

[4] See for example Aiyagari et al. (2002).

the promises made by the planner to manipulate inflation and output, and hence also manipulate the interest rate, in response to shocks to the consolidated budget which have occurred in the past. Following FMOS, we label this impact an 'interest rate twisting' effect of optimal policy.

3.2.3 Assuming No Debt Concerns

We now consider an alternative setup in which the planner maximizes (3.6) subject to (3.2), (3.3) and (3.4) and leaves the consolidated budget (3.74) outside the optimal policy program. (3.74) continues to hold, and as we shall later show, it will be satisfied in equilibrium given optimal policies and the tax rule (3.1) (more on this below). In this version of the model the multiplier $\psi_{gov,t}$ is obviously dropped from the list of model variables. The Lagrangian for this program is otherwise similar to (3.7) and for the sake of brevity we omit it.

Letting superscript NDC denote the equilibrium under 'no debt concerns', the first order conditions[5] become:

$$-\hat{\pi}_t + \Delta\psi_{\pi,t}^{NDC} - \frac{\psi_{i,t-1}^{NDC}}{\beta} = 0$$

$$-\lambda_Y \hat{Y}_t - \psi_{\pi,t}^{NDC}\kappa_1 + \sigma\frac{\overline{Y}}{\overline{C}}(\psi_{i,t}^{NDC} - \frac{\psi_{i,t-1}^{NDC}}{\beta}) = 0$$

$$-\lambda_i \hat{i}_t + \psi_{i,t}^{NDC} + \psi_{ZLB,t}^{NDC} = 0$$

The above equations give rise to the following interest rate rule:

Proposition 2. *Assume that the planner does not account for the consolidated budget in optimization. The optimal interest rate rule is given by*

$$\hat{i}_t = \max\{\mathcal{T}_t^{NDC} + \mathcal{Z}_t^{NDC}, -i^*\} \tag{3.17}$$

$$\mathcal{T}_t^{NDC} \equiv \phi_\pi \hat{\pi}_t + \phi_Y \Delta\hat{Y}_t + \phi_i \hat{i}_{t-1} + \frac{1}{\beta}\Delta\hat{i}_{t-1}$$

and

$$\mathcal{Z}_t^{NDC} = -\frac{1}{\lambda_i}(1 + \frac{1}{\beta} + \frac{\kappa_1\overline{C}}{\sigma\beta\overline{Y}})\psi_{ZLB,t-1}^{NDC} + \frac{1}{\beta\lambda_i}\psi_{ZLB,t-2}^{NDC}.$$

Proof. See appendix.

[5] See appendix for a more detailed description of the planner's program.

Several comments are in order. First, notice that the expressions for components $\mathcal{T}_t^{NDC}$ and $\mathcal{Z}_t^{NDC}$ in (3.17) are essentially the same as the expressions for $\mathcal{T}_t$ and $\mathcal{Z}_t$ in the optimal policy (3.13) of the DC model. The difference between the two policies is that (3.17) omits the term $\mathcal{D}_t$, which appears in the debt concerns version of optimal policy. This may seem to imply that the two policies are similar, at least in the case where the shocks to the consolidated budget are not large, so that $\mathcal{D}_t$ is close to zero. In Section 3 we will show that this is not the case. Even in the absence of large shocks the monetary policy under DC has considerably different properties and effects than the policy under NDC.

Second, as we have seen, both $\mathcal{Z}_t$ ($\mathcal{Z}_t^{NDC}$) and $\mathcal{D}_t$ are functions of lagged state variables (ψ_{ZLB} and ψ_{gov}) and these lagged variables summarize the effects of shocks that have occurred in the past. Since their values have been revealed in past periods, and apply to policy in t, we can interpret them as forward guidance in optimal monetary policy.

A recent literature using DSGE models has utilized monetary policy rules with forward guidance, similar to the rules in Propositions 1 and 2. In Laséen and Svensson (2009), Campbell and Justiniano (2012), Campbell, Fisher, Justiniano, and Melosi (2017), Del Negro and Sims (2015), De Graeve, Ilbas, and Wouters (2014) and others, interest rate policy is modelled in the following form:

$$\hat{i}_t^{DSGE} = \max\{\mathcal{T}_t^{DSGE} + \sum_{l=0}^{M} \epsilon_{t-l}^{l,DSGE}, -\hat{i}^*\} \tag{3.18}$$

where $M > 0$. $\mathcal{T}_t^{DSGE}$ is a Taylor rule, a function of inflation, output growth and lags of the interest rate. ϵ_t^0 is a standard monetary policy shock and $\left(\epsilon_{t-1}^1, ..., \epsilon_{t-M}^M\right)$ are 'forward guidance shocks'. The policy rules in Propositions 1 and 2 are indeed similar to equation (3.18); the difference, however, is that the shocks $\left(\epsilon_{t-1}^1, ..., \epsilon_{t-M}^M\right)$ are exogenous, whereas the state variables which enter in $\mathcal{Z}_t$ ($\mathcal{Z}_t^{NDC}$) and $\mathcal{D}_t$ are endogenous functions of fundamental shocks to preferences, taxes, spending etc. [6]

[6] As discussed previously, several papers have studied optimal monetary policies using frameworks broadly similar to ours. See for example Eggertsson et al. (2003); Eggertsson and Woodford (2006), Lustig et al. (2008) and Faraglia et al. (2013) among others. These papers also give rise to endogenous FG. For example in Faraglia et al. (2013) and Lustig et al. (2008) the presence of long term debt implies that an element such as $\mathcal{D}_t$ is part of optimal policy. Moreover, optimal policy in models with occasionally binding zero lower bound constraints should include a term of the form $\mathcal{Z}_t$.

However, the focus of these papers is mainly theoretical and, building on nonlinear models, they do not derive simple interest rate rules comparable to the recent DSGE literature, as we do here. Moreover, since many of these models bring together monetary and fiscal policies under one authority, 'forward guidance' is not only promises made to manipulate future inflation and interest rates, but also promises to manipulate taxes. This makes difficult to evaluate which of the recommendations of these models are relevant for monetary policy and which not.

3.2.4 Discussion

Thus far we derived optimal interest rate rules in the two environments: Under debt concerns the planner takes into account the consolidated budget constraint, whereas under no debt concerns she does not have to satisfy the constraint. We now provide some more context behind our modelling assumptions.

First, the reader may be asking herself if in the NDC case the optimal program is not complete, if the full structure of the macroeconomy is not taken into account. As was discussed previously, coefficient $\phi^R_{\tau,b}$ in the fiscal policy rule will be different in the two scenarios; we will later show (Section 3) that DC corresponds to the case where fiscal policy does not generate sufficient surpluses to finance debt, and therefore monetary policy has to satisfy the consolidated budget constraint and NDC to the case where fiscal surpluses are sufficient to satisfy the intertemporal budget. Thus, from the point of view of the planner, under NDC, the consolidated budget constraint is already satisfied by the fiscal authority. It then holds that $\psi_{gov} = 0$.

Next, one may think that if a central bank is concerned with stabilizing debt, then the real value of debt should appear in the objective of the planner rather than as part of the constraint set. We claim that these alternatives yield similar policy outcomes: If the central bank's loss function invoked a penalty in the case where the RHS of (3.74) is not equal to the LHS (the real value of debt is not compensated by the surpluses) then monetary policy will adjust inflation and output to satisfy intertemporal budget just like in our DC program. The reason that we do not pursue such modelling here, is that adding the intertemporal budget as a constraint seems to us much more credible setup of policy. Cochrane (2001) solves a similar problem assuming that a Ramsey planner that minimizes the volatility of inflation has to satisfy the constraint. Thus our assumptions are in line with previous work in the literature. Moreover, we will later show that, in the case where fiscal policy does not generate surpluses to finance debt and when the planner fails to take into account the intertemporal budget, the rational expectations equilibrium is not unique and therefore solutions that lead to explosive paths of inflation or deflation cannot be ruled out. Therefore, a planner that is concerned about stabilizing inflation and output, should also be concerned about satisfying the constraint.

The reader may also be asking whether objective (3.6) is meant to represent a second order approximation to the household's utility. Though our analysis below will extend to various calibrations of parameters λ_Y, λ_i (including $\lambda_i = 0$ which would result from a second order approximation in a cashless economy like ours) we do not consider that the monetary authority's objective has to be identical to the objective of the household. If it were, then minimizing the volatility of distortionary taxes would also be part of the welfare criterion and our policy problem would look like a joint optimal monetary and fiscal policy problem (as for example in Schmitt-Grohé and Uribe (2004) and Siu (2004) and numerous others). We thus assume that the monetary authority has its own objective, which focuses on stabilizing inflation, output and the interest rate, and this loss function will not, in general, coincide with the loss function of the household. [7]

3.3 Monetary and Fiscal Policy Interactions

In this section we study the properties of the models in the neighborhood of the steady state, when the ZLB does not bind. We first ask, what types of fiscal rules are compatible with an equilibrium where the central bank has debt concerns and what types of rules are compatible with no debt concerns. We establish a link between the DC/NDC models and Leeper's (1991) seminal analysis on monetary/fiscal policy interactions. Our key result is that in the case of debt concerns, monetary policy becomes subservient to fiscal policy, and behaves similarly to the passive monetary policy model defined in Leeper's work. Fiscal policy is 'active' and does not respond to government debt. The opposite holds under no debt concerns.

We then study the effects of disturbances on macroeconomic variables. The main finding of this analysis is that under debt concerns, macroeconomic volatility increases, inflation, output and interest rates are more exposed to both demand and supply disturbances. This property is a standard feature of 'passive money models'. In our optimal policy framework under debt concerns, it arises because the planner needs to partially give up on the goal of stabilizing inflation, output and interest rates to satisfy the consolidated budget constraint.

3.3.1 Determinacy Under Optimal Policies

Fiscal Policy Consider the case where the ZLB constraint is non-binding. Since ours is a linear model approximated around a non-stochastic steady state with a positive nominal interest rate, equilibria with a non-binding ZLB constraint can occur if shocks are not too big to drive the economy far away from steady state. We have: $\hat{i}_t = \mathcal{T}_t + \mathcal{D}_t$ and $\hat{i}_t^{NDC} = \mathcal{T}_t^{NDC}$. To derive analytical results we first consider a simplistic setup letting $\lambda_Y = \sigma = \delta = \rho_\tau = \overline{G} = 0$. We further assume that tax shocks are the only source of uncertainty in the economy.[8] Under these assumptions and using equations (3.9) and

[7] Analogously, implicit in rule (3.1) is the assumption that the fiscal authority aims to smooth taxes. The objective of the fiscal authority is however, not explicitly modelled here.

[8] Our findings in this section do not hinge on the nature of shocks that hit the economy. We assume only tax shocks and set $\overline{G} = 0$ to simplify the algebra; otherwise we could assume more than one disturbance in the model and all results described below would carry through.

(3.10) to substitute out $\psi_{\pi,t}$ and $\psi_{i,t}$ in the debt concerns model we get:

$$-\hat{\pi}_t - \frac{\lambda_i \hat{i}_{t-1}}{\beta} + \bar{b}_\delta \widetilde{\eta} \Delta\psi_{gov,t} = 0 \tag{3.19}$$

where $\widetilde{\eta} = \left(1 + \frac{(1-\beta)(1+\gamma_h)}{\kappa_1}\right)$.[9] $\psi_{gov,t}$ is again a martingale and so $E_t\Delta\psi_{gov,t+1} = 0$. From the Euler equation we have $\hat{i}_t = E_t\hat{\pi}_{t+1}$.

Using the Phillips curve to substitute $\hat{Y}_t = \frac{\hat{\pi}_t - \beta E_t\hat{\pi}_{t+1} - \kappa_2\hat{\tau}_t}{\kappa_1}$ the consolidated budget constraint can be written as follows:

$$\beta\bar{b}_\delta\hat{b}_{t,\delta} - \beta\bar{b}_\delta\widetilde{\eta}E_t\hat{\pi}_{t+1} + (1-\beta)\bar{b}_\delta\left(\frac{1}{1-\overline{\tau}} - \frac{\kappa_2}{\kappa_1}(1+\gamma_h)\right)\hat{\tau}_t = \bar{b}_\delta\hat{b}_{t-1,\delta} - \bar{b}_\delta\widetilde{\eta}\hat{\pi}_t \tag{3.20}$$

Finally, using $\frac{\kappa_2}{\kappa_1} = \frac{\overline{\tau}}{(1-\overline{\tau})\gamma_h}$ and the tax rule (3.1) we have:

$$\hat{b}_{t,\delta} - \widetilde{\eta}E_t\hat{\pi}_{t+1} = \underbrace{\frac{1}{\beta}\left[1 - \frac{(1-\beta)\phi^{DC}_{\tau,b}}{(1-\overline{\tau})\gamma_h}\Big(\gamma_h - \overline{\tau}(1+\gamma_h)\Big)\right]}_{\widetilde{\epsilon}_{DC}}\hat{b}_{t-1,\delta} - \frac{\widetilde{\eta}}{\beta}\hat{\pi}_t - \underbrace{\frac{1}{\beta}\frac{(1-\beta)}{(1-\overline{\tau})\gamma_h}\Big(\gamma_h - \overline{\tau}(1+\gamma_h)\Big)}_{A}\epsilon_{\tau,t} \tag{3.21}$$

Coefficient $\widetilde{\epsilon}_{DC}$ is a key object in determining the dynamics of government debt. In the case where $\widetilde{\epsilon}_{DC} > 1$ government debt becomes an explosive process, whereas if $\widetilde{\epsilon}_{DC} < 1$, debt is a stationary process. Notice that the magnitude of $\widetilde{\epsilon}_{DC}$ hinges on the size of the feedback effect of lagged debt on taxes, $\phi^{DC}_{\tau,b}$, and also on the steady state level of taxes and the Frisch elasticity $\frac{1}{\gamma_h}$, since these quantities influence the responsiveness of aggregate hours and tax revenues to shocks when taxes are distortionary.

To identify which values of $\widetilde{\epsilon}_{DC}$ give rise to a unique rational expectations equilibrium use the Euler equation together with (3.19) to write

$$\frac{\lambda_i\hat{i}_t}{\beta} = -E_t\hat{\pi}_{t+1} + \bar{b}_\delta\widetilde{\eta}E_t\Delta\psi_{gov,t+1} = -\hat{i}_t, \tag{3.22}$$

where the last equality makes use of the martingale property of $\psi_{gov,t+1}$. From (3.22) we get $\hat{i}_t = 0$ when $\lambda_i \geq 0$. Therefore, we have $\hat{\pi}_t = \bar{b}_\delta\widetilde{\eta}\Delta\psi_{gov,t}$ and with this we can write (3.21) as

$$\hat{b}_{t,\delta} = \widetilde{\epsilon}_{DC}\hat{b}_{t-1,\delta} - \frac{\bar{b}_\delta\widetilde{\eta}^2}{\beta}\Delta\psi_{gov,t} - A\epsilon_{\tau,t} \tag{3.23}$$

Equation (3.23) together with the martingale property, $E_t\Delta\psi_{gov,t+1} = 0$, form the system of equations that needs to be resolved to find the values of $\hat{b}_{t,\delta}$ and $\Delta\psi_{gov,t}$.

[9] Under $\sigma = \overline{G} = 0$ we have $\omega_Y = \frac{\overline{\tau}(1+\eta)}{\eta}\overline{Y}(1+\gamma_h)$. From the budget constraint we get $\frac{\overline{\tau}(1+\eta)}{\eta}\overline{Y} = (1-\beta)\bar{b}_\delta$.

It is now easy to show that the solution to this system is unique only when $\widetilde{\epsilon}_{DC} > 1$. To see this, assume that $\widetilde{\epsilon}_{DC}$ is less than one and solve equation (3.23) backwards to obtain:

$$\hat{b}_{t,\delta} = -\sum_{j=0}^{\infty} \widetilde{\epsilon}_{DC}^{j} \left(\frac{\overline{b}_{\delta}\widetilde{\eta}^{2}}{\beta} \Delta\psi_{gov,t-j} + A\epsilon_{\tau,t-j} \right) \tag{3.24}$$

which determines the debt level at t as function of the lagged shocks $\Delta\psi_{gov,t-j}$ and $\epsilon_{\tau,t-j}$. Notice that the value of $\Delta\psi_{gov,t-j}$ is not pinned down in this model; the martingale property $E_{t-j-1}\Delta\psi_{gov,t-j} = 0$ does not determine a unique value for this object.

In the unique rational expectations equilibrium, $\widetilde{\epsilon}_{DC} > 1$, and (3.23) is solved forward to give

$$\hat{b}_{t-1,\delta} = E_t \sum_{j=0}^{\infty} \frac{1}{\widetilde{\epsilon}_{DC}^{j+1}} \left(\frac{\overline{b}_{\delta}\widetilde{\eta}^{2}}{\beta} \Delta\psi_{gov,t+j} + A\epsilon_{\tau,t+j} \right) = \frac{1}{\widetilde{\epsilon}_{DC}} \left(\frac{\overline{b}_{\delta}\widetilde{\eta}^{2}}{\beta} \Delta\psi_{gov,t} + A\epsilon_{\tau,t} \right) \tag{3.25}$$

From (3.25) the equilibrium satisfies $\Delta\psi_{gov,t} = -\frac{\beta}{\overline{b}_{\delta}\widetilde{\eta}^{2}} A\epsilon_{\tau,t}$ and therefore $\hat{b}_{t,\delta} = 0$ for all t[10]

Now consider the case of no debt concerns. We have $\psi_{gov,t} = 0$ for all t and the consolidated budget constraint can be written as:

$$\hat{b}_{t,\delta}^{NDC} = \widetilde{\epsilon}_{NDC} \hat{b}_{t-1,\delta}^{NDC} - A\epsilon_{\tau,t} \tag{3.26}$$

(where $\widetilde{\epsilon}_{NDC}$ is essentially given by the same expression as $\widetilde{\epsilon}_{DC}$, but $\phi_{\tau,b}^{DC}$ is replaced with $\phi_{\tau,b}^{NDC}$). A unique equilibrium can be found when $\widetilde{\epsilon}_{NDC} < 1$ so that (3.26) is solved backwards. We summarize the above findings in the following proposition:

Proposition 3. *Assume that parameters λ_Y, $\overline{G}$, δ and σ equal zero. Define*

$$\widetilde{\epsilon}_R \equiv \frac{1}{\beta}\left[1 - \frac{(1-\beta)\phi_{\tau,b}^{R}}{(1-\overline{\tau})\gamma_h}\left(\gamma_h - \overline{\tau}(1+\gamma_h)\right)\right]$$

for $R \in \{DC, NDC\}$. Assume that the planner takes into account the consolidated budget constraint. Determinacy of the equilibrium requires $\widetilde{\epsilon}_{DC} > 1$. In contrast in the no debt concerns case determinacy requires $\widetilde{\epsilon}_{NDC} < 1$.

The above condition extends the analysis of Leeper (1991) to the case of distortionary taxes and in a model where monetary policy is optimal. When $\widetilde{\epsilon}_R < 1$ (i.e. in the no debt concerns model) fiscal policy is sufficiently responsive to debt levels, and so taxes adjust

[10]In other words, debt does not explode if it equals zero for all t.

Notice that if this fails we have: $\Delta\psi_{gov,t} = \frac{\beta}{\overline{b}_{\delta}\widetilde{\eta}^{2}}\left(\widetilde{\epsilon}_{DC}\hat{b}_{t-1,\delta} - A\epsilon_{\tau,t}\right)$ and therefore, $E_{t-1}\Delta\psi_{gov,t} = \frac{\beta}{\overline{b}_{\delta}\widetilde{\eta}^{2}}\widetilde{\epsilon}DC\hat{b}_{t-1,\delta} \neq 0$. Thus $\hat{b}_{t,\delta} = 0$ is the only path consistent with the random walk.

to guarantee the sustainability of debt. In the sense of Leeper (1991) fiscal policy is "passive". In contrast, in the model with debt concerns, fiscal policy needs to be "active" for the rational expectations equilibrium to be unique or, to put it differently, taxes should not respond aggressively to deviations of debt from its steady state value.

The above findings, that determinacy requires an explosive debt process in the debt concerns model, and a mean reverting process under no debt concerns, do not hinge on the assumptions made in Proposition 3. They hold more generally, for positive values of λ_Y, σ, δ, and $\overline{G}$, and, therefore, also hold when monetary policy follows the rules derived in Propositions 1 and 2.

Monetary Policy: an example affirming Leeper. What do these optimal interest rate rules tell us about whether monetary policy is active/passive? Leeper's classification hinges on the response of interest rates on inflation. When interest rates respond strongly to inflation monetary policy is active and conversely, when they do not respond strongly, it is passive. Admittedly, Leeper's analysis is not easy to map into our optimal policy framework: Even if we can show that $\mathcal{T}^{NDC}$ defines an active monetary policy, it is not obvious how we could then show that $\hat{i}_t = \mathcal{T} + \mathcal{D}$ defines a passive policy, especially when $\mathcal{T}^{NDC} = \mathcal{T}$, as was previously illustrated. If anything, since $\mathcal{D}$ is a moving average of mean zero innovations to the Lagrange multiplier it would seem that interest rates in the debt concerns model will on average be equal to $\mathcal{T}$ and therefore also equal to $\mathcal{T}^{NDC}$.

This is however not the case. The innovations in $\mathcal{D}$ are not orthogonal to inflation; they are functions of the same fundamental shocks (in preferences, spending, taxes etc) which drive inflation and so they are correlated with inflation (and with the remaining variables in $\mathcal{T}$). In principle we can express $\mathcal{D}$ as a function of inflation, lagged values of interest rates etc. We can thus map the optimal policy into Leeper's analysis.

In the next subsection we will use the numerical solution of the model to approximate $\mathcal{D}$ as a function of the variables in $\mathcal{T}$. To derive an analytical solution we assume $\lambda_Y = \sigma = 0$ as in the previous paragraph, however now let $\delta \geq 0$ and $\lambda_i = 0$. Note that under these assumptions we cannot derive an optimal policy rule directly from the first order conditions; however, we can find a Taylor rule that implements the allocation under optimal policy.

Assume that the policy rule is of the form:

$$\hat{i}_t = \widetilde{\phi}_\pi \hat{\pi}_t \tag{3.27}$$

Consider first the no debt concerns case; we can show that in the presence of only tax shocks in this model, we will have $\hat{i}_t = \hat{\pi}_t = 0$.[11] Combining (3.27) with the Euler equation

[11] See below in subsection 3.3.3 where we look at the effect of tax shocks.

$\hat{i}_t = E_t\hat{\pi}_{t+1}$ we get the following difference equation in inflation:

$$\hat{\pi}_t\widetilde{\phi}_\pi = E_t\hat{\pi}_{t+1}$$

Standard results yield that the equilibrium is unique if and only if $\widetilde{\phi}_\pi > 1$.

Now consider the case of debt concerns. The equilibrium under optimal policy satisfies:

$$\hat{\pi}_t = \frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t} + \frac{\bar{b}_\delta}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l} \qquad \hat{i}_t = \frac{\bar{b}_\delta\delta}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l} \tag{3.28}$$

Assuming that parameter values are such that $\frac{\omega_Y}{\kappa_1} \approx 0$ and considering a rule of the form (3.27) that can implement (3.28) we have:

$$\widetilde{\phi}_\pi\frac{\bar{b}_\delta}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l} = \delta\frac{\bar{b}_\delta}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l} \tag{3.29}$$

hence $\widetilde{\phi}_\pi = \delta < 1$.

This simple example shows that whereas the no debt concerns model requires a coefficient $\widetilde{\phi}_\pi$ which exceeds unity (the standard condition for 'active' monetary policy), the debt concerns model features 'passive' monetary policy under an interest rate rule of the form (3.27).

We summarize the above in the following proposition:

Proposition 4. *Assume $\lambda_Y = \sigma = \lambda_i = 0$. The optimal policy is a rule of the form (3.27). In the case of no debt concerns $\widetilde{\phi}_\pi > 1$ and monetary policy is 'active'. Under debt concerns, $\widetilde{\phi}_\pi = \delta < 1$, and monetary policy is 'passive'.*

Monetary Policy: Numerical Examples For more plausible calibrations of the model the above properties can be demonstrated using numerical simulations. In Table 3.1 we show the coefficients we obtain from Taylor rule approximations of optimal policy in the debt concerns model under various model specifications.[12] More precisely, we approximate the optimal policies using rules of the form:

$$\hat{i}_t = \widetilde{\phi}_\pi\hat{\pi}_t + \widetilde{\phi}_Y\hat{Y}_t + \widetilde{\phi}_i\hat{i}_{t-1} + \widetilde{\phi}_{\Delta i}\Delta\hat{i}_{t-1} \tag{3.30}$$

which do not include object $\mathcal{D}_t$.[13] The coefficients $\widetilde{\phi}$ are such that the model with rule (3.30) produces impulse responses to shocks close to the analogous objects in the optimal

[12] The calibration of model parameters $\eta, \theta, ...$ follows Table 3.2 (see subsection 3.2). However, we consider here a wider range of values for $\sigma, \lambda_i, \lambda_Y$ than what is reported in this table. The values we consider are reported in text.

[13] Equivalently we approximate $\mathcal{D}_t$ as a function of the RHS variables in (3.30).

policy model.[14]

In the top panel of Table 3.1 we assume $\lambda_Y = 0$ and thus set $\widetilde{\phi}_Y = 0$ to isolate our focus on coefficient $\widetilde{\phi}_\pi$. Each of the columns of the table corresponds to a different calibration of parameters λ_i and σ. Consider the first of these columns which sets $\lambda_i = 0.5$ and $\sigma = 1$. For each parameter ($\widetilde{\phi}_\pi$, $\widetilde{\phi}_i$ etc) we report two sets of numbers: The top numbers correspond to the estimates of (3.30). The bottom numbers (reported in parenthesis) correspond to the coefficients in $\mathcal{T}$ derived in Proposition 1. Notice that the estimates of (3.30) yield a much weaker response of interest rates to inflation ($\widetilde{\phi}_\pi = 0.104$ vs. 0.537 in the analytic solution) and also the coefficients on $\hat{i}_{t-1}$ and $\Delta\hat{i}_{t-1}$ are much lower than their analytic solution counterparts. (3.30) is basically a 'passive money' rule.

The same pattern emerges in the remaining model specifications. For each of the calibrations considered in Columns 2 to 4, the estimates of $\widetilde{\phi}_\pi$, $\widetilde{\phi}_i$ and $\widetilde{\phi}_{\Delta i}$ are low. The bottom panel of the table considers the case $\lambda_Y > 0$ and thus also reports the estimates of coefficient $\widetilde{\phi}_Y$. The response to inflation in (3.30) continues to be weaker than in the exact solution of the model. The estimated output coefficient $\widetilde{\phi}_Y$ is also weaker.

The above patterns hold for many alternative calibrations of the model which for brevity we leave outside the table.

[Table 3.1 About Here]

3.3.2 Discussion: How could Optimal Policy be Indeterminate?

Determinacy in our model requires the combination of DC with active fiscal policy or NDC with passive policy. Policy configurations such as DC /passive or NDC/active, induce indeterminacy of the equilibrium and we cannot rule out paths featuring large inflation rates or deflation, stemming from shifts in expectations.

How could optimal policy be indeterminate? Shouldn't solving the DC program be sufficient, in the sense that when fiscal policy is active we endogenously reach the outcome $\psi_{gov,t} = 0$, for all t? The crucial assumption we made here is that policies are uncoordinated and when it solves the DC program, monetary policy does not take into account the tax rule (it treats taxes as exogenous). In contrast, as we will show now, if the monetary

[14] Computing coefficients through OLS regressions (that is fitting (3.30) to simulated data from the debt concerns model) does not (generally) provide a good fit. It may also give us coefficients which are not consistent with a unique equilibrium. This pattern is consistent with the findings of Cochrane (2011) and Schmitt-Grohé and Uribe (2004).

Obviously, the impulse responses of the model under (3.30) do not perfectly match the responses under optimal policy. In the case where $\delta > 0$ we need several lags of interest rates (and also lags of inflation and output) to produce a perfect fit. We minimize the distance between the two models. We obtain a perfect fit only when we set $\delta = 0$.

authority did take into account the tax schedule, solving the DC program would indeed be sufficient.

Though in our model fiscal policy is not optimal, let us pretend that (3.1) derives from optimization of some objective (presumably one that prioritizes tax smoothing). The monetary/fiscal interactions we described in the previous paragraph can thus be seen as the outcome of an uncoordinated Nash game, the two equilibria correspond to the configurations active/passive and passive/active, as in Leeper (1991).

Now consider the outcome of a different game. The monetary authority acts as Stackelberg leader, taking into account the response of fiscal policy in optimization. Suppose that this response is still described by (3.1). To derive the optimality conditions of the monetary policy problem we now need to simply replace taxes in the DC program with (3.1). The only FONC that changes is (3.11) which now becomes:

$$\frac{\beta\bar{b}_\delta}{1-\beta\delta}\Big(\psi_{gov,t}-E_t\psi_{gov,t+1}\Big)-\beta\phi_{\tau,b}^{DC}(1-\rho_\tau)E_t\Big(\sum_{j=0}^{\infty}(\rho_\tau\beta)^j(\kappa_2\psi_{\pi,t+1+j}-\frac{\overline{\tau}\overline{Y}(1+\eta)}{\eta(1-\overline{\tau})}\psi_{gov,t+1+j})\Big)=0 \tag{3.31}$$

The dynamics of $\psi_{gov,t}$ in (3.65) are obviously different than in (3.11). Crucially, (3.65) no longer defines a random walk. To see how this matters let us go back to the example of subsection 3.3.1. Under the assumptions we made there we can write (3.65) as:

$$\beta\bar{b}_\delta\Big(\psi_{gov,t}-E_t\psi_{gov,t+1}\Big)-\beta\bar{b}_\delta\phi_{\tau,b}^{DC}(1-\beta)\Big(\frac{\overline{\tau}}{\gamma_h(1-\overline{\tau})}-1\Big)E_t\psi_{gov,t+1}=0$$

(see appendix) giving rise to the following dynamics of the multiplier:

$$\psi_{gov,t}=\Big[1+\phi_{\tau,b}^{DC}(1-\beta)\Big(\frac{\overline{\tau}}{\gamma_h(1-\overline{\tau})}-1\Big)\Big]E_t\psi_{gov,t+1}=0 \tag{3.32}$$

Notice that $\left(\frac{\overline{\tau}}{\gamma_h(1-\overline{\tau})}-1\right)<0$ otherwise the economy is on the wrong side of the Laffer curve. Therefore, for all $\phi_{\tau,b}^{DC}>0$, (3.32) has a unique non-explosive solution $\psi_{gov,t}=0$. The case $\phi_{\tau,b}^{DC}=0$ brings us back to the random walk we had previously.

We just showed that there is a way to define the Ramsey program so that optimal monetary policy is never indeterminate. This requires that the monetary authority internalizes the tax schedule. Then the passive monetary/active fiscal configuration of policy obtains in the case when taxes do not respond at all to debt. (In)determinacy of the equilibrium is no longer an issue.

Since our goal is to build a framework for optimal monetary policy which can be easily mapped into the DSGE literature on monetary/fiscal interactions (i.e. Leeper's framework), we will continue with the assumption of uncoordinated policies. We leave to future work to study the case where the monetary authority can act as leader and internalize the tax rule.

3.3.3 The effects of shocks

We have seen that in the debt concerns model fiscal policy is 'active' and debt becomes a non-stationary process. Monetary policy is subservient to fiscal policy and it can be represented (in reduced form) with a Taylor rule which has the standard features of passive money models. The opposite holds under no debt concerns: the policy mix becomes active monetary/passive fiscal.

These findings are crucial to understand the working of the model and the effects of economic shocks which we now study. A standard prediction of passive money models is that they magnify macroeconomic volatility. Shocks that hit the economy lead to larger fluctuations in inflation and output (see for example Bianchi and Ilut, 2017). In our optimal policy model this will also be the case. When optimization is subject to the consolidated budget the planner needs to partially give up on the goal of stabilizing inflation, output and interest rates to satisfy the constraint.

To show this clearly, we first derive the impulse responses analytically, in a simplified setup assuming, as previously, $\lambda_Y = \lambda_i = \sigma = 0$ but now let $\overline{G}, \rho_\tau > 0$. We consider shocks at date 0 which change the values of parameters $\{G_0, \xi_0, \tau_0\}$ assuming that after $t = 0$ there are no further shocks to the economy. Under these assumptions we derive in the appendix the optimal paths of inflation and interest rates and the path of the multiplier $\psi_{gov,t}$.

The appendix arrives at the following expressions for $\hat{\pi}_t$ and $\hat{i}_t$ under DC:

$$\hat{\pi}_t = \frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,0}\mathcal{I}_{t=0} + \frac{\overline{b}_\delta \delta^t}{1-\beta\delta}\Delta\psi_{gov,0} \tag{3.33}$$

$$\hat{i}_t = -\rho_\xi^t(\rho_\xi - 1)\hat{\xi}_0 + \frac{\overline{b}_\delta \delta^{t+1}}{1-\beta\delta}\Delta\psi_{gov,0} \tag{3.34}$$

for $t = 0, 1, 2...$ and where $\mathcal{I}_{t=0}$ takes the value 1 at $t = 0$ and 0 otherwise. Moreover, $\Delta\psi_{gov,0}$ is a linear function of $\{\hat{G}_0, \hat{\xi}_0, \hat{\tau}_0\}$. We have:

$$\upsilon_{gov}\Delta\psi_{gov,0} = \upsilon_G\hat{G}_0 + \upsilon_\tau\hat{\tau}_0 + \upsilon_\xi\hat{\xi}_0 \tag{3.35}$$

where $\upsilon_{gov}, \upsilon_G, \upsilon_\xi > 0$ and $\upsilon_\tau < 0$ and therefore partial derivatives satisfy $\frac{\partial\Delta\psi_{gov,0}}{\partial\hat{G}_0} > 0$, $\frac{\partial\Delta\psi_{gov,0}}{\partial\hat{\tau}_0} < 0$ and $\frac{\partial\Delta\psi_{gov,0}}{\partial\hat{\xi}_0} > 0$.

We also show in the appendix that in the no debt concerns model $\hat{i}_t = -\rho_\xi^t(\rho_\xi - 1)\hat{\xi}_0$ and $\hat{\pi}_t = 0$ for all t.

Preference Shocks We consider first the impact of a preference shock which lowers the value $\hat{\xi}_0$. The previous derivations show that in the no debt concerns equilibrium this shock has no effect on inflation. This result is standard: Since the planner does not want to smooth the nominal interest rate, i.e. $\lambda_i = 0$, $\hat{i}_t$ drops one for one with the real rate thus fully stabilizing inflation.

Now consider the case of the 'debt concerns' model. Since $\frac{\partial \Delta\psi_{gov,0}}{\partial \hat{\xi}_0} > 0$, a drop in $\hat{\xi}_0$ lowers $\Delta\psi_{gov,0}$. From (3.33) and (3.34) we see that inflation turns negative after the shock and the nominal interest rate drops below the real rate, $-\rho_\xi^t(\rho_\xi - 1)\hat{\xi}_0$.

What is going on? Notice that a preference shock has two impacts on the consolidated budget: First, it increases real bond prices and hence increases the real payout of government debt (the RHS of equation (3.16)); second, it increases the present value of surpluses (LHS of (3.16)) which finance the debt. Since the planner has to satisfy the intertemporal constraint she will either use inflation or use deflation to adjust the real payout of debt so that the constraint is satisfied with equality. If the first effect is stronger, and debt increases more than the surpluses, then inflation is optimal. In contrast, if the second effect dominates, then it is optimal to make inflation negative. In our analytic solution the second effect is stronger for all values $0 \leq \delta < 1$ and so inflation becomes negative in response to the preference shock.[15]

Figure 3.1 shows the above responses over 40 periods.[16] The left column of the figure shows the case of the preference shock; the solid line represents the case of the debt concerns model and the dashed line shows no debt concerns. The top two panels show the responses of inflation and interest rates, the bottom two panels show output and the market value of debt.

[Figure 3.1 About Here]

[15]When $\delta = 1$, long bonds are consols, and preference shocks have zero impact on the consolidated budget. To see this, simply use (3.16) assuming that in equilibrium under the preference shock $\hat{Y}_t = \hat{b}_{t,\delta} = \hat{\pi}_t = \hat{G}_t = \hat{\tau}_t = 0$ for all t. Then (3.16) becomes:

$$E_t \sum_{j=0}^{\infty} \beta^j \left(\frac{\overline{\tau}\overline{Y}(1+\eta)}{\eta} - \overline{G} \right) \hat{\xi}_{t+j} = \overline{b}_\delta \sum_{j=0}^{\infty} \beta^j \delta^j E_t \hat{\xi}_{t+j}$$

Since $\frac{\overline{\tau}\overline{Y}(1+\eta)}{\eta} - \overline{G} = (1-\beta)\overline{b}_\delta$ the above equation is satisfied when $\delta = 1$.

This result, that preference shocks are neutral when long bonds are consols, is consistent recent papers on optimal government debt management (e.g. Debortoli, Nunes, and Yared (2017) and Bhandari, Evans, Golosov, and Sargent (2017a)). See Section 5 where we briefly discuss optimal portfolios in our setup.

[16]The values we assign to the model's parameters are presented in Table 3.2. The top panel of the table reports values for parameters which are common across the two versions of the model, the bottom panel reports the value that we assign to $\phi_{\tau,b}$ in each version separately. In the debt concerns model we set $\phi_{\tau,b} = 0.0$ and in the case where the Fed has no debt concerns we set $\phi_{\tau,b} = 0.07$. $\phi_{\tau,b} = 0.07$ is chosen so that debt has a near unit root, consistent with the empirical evidence on US government debt presented in Marcet and Scott (2009). The cutoff for determinacy is 0.065.

Equilibrium output under debt concerns is given by:

$$\hat{Y}_t = \frac{1}{\kappa_1}\left(\frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,0}\mathcal{I}_{t=0} + \bar{b}_\delta \delta^t \Delta\psi_{gov,0} - \kappa_2 \rho_\tau^t \hat{\tau}_0\right)$$

(see appendix). Output drops in response to the preference shock because $\Delta\psi_{gov,0}$ turns negative. Under no debt concerns, output remains roughly constant after the shock.[17]

Fiscal Shocks The middle and right columns of Figure 3.1 study the responses to a positive spending shock and a negative tax shock respectively. In both cases the consolidated budget constraint tightens because the present value of the government's surplus falls and so $\Delta\psi_{gov,0} > 0$. From (3.33) and (3.34), interest rates rise in response to the shocks in the debt concerns model and inflation rises above its steady state value. Positive inflation ensures the solvency of the consolidated budget. In contrast, under no debt concerns inflation does not respond to these shocks because debt rises and taxes eventually adjust to make the surplus positive.

[Table 3.2 About Here]

Responses under $\lambda_i, \lambda_Y, \sigma > 0$ In Figure 3.2 we assume $\lambda_i = \lambda_Y = 0.5$ and $\sigma = 1$. Now inflation and output change in response to the shocks in the no debt concerns model: Because the planner wants to smooth interest rates, following a preference shock, $\hat{i}_t$ does not drop one for one with the real rate to fully stabilize inflation; moreover, since the planner also wants to smooth output, she optimally adjusts inflation in order to partially offset fiscal shocks.

Contrasting the responses of the two models, it is clear that the qualitative patterns outlined previously remain. In the debt concerns model inflation becomes negative in response to a preference shock, and positive in response to spending and tax shocks. The (numerical) partial derivatives $\frac{\partial \Delta\psi_{gov,0}}{\partial \hat{G}_0}, \frac{\partial \Delta\psi_{gov,0}}{\partial \hat{\tau}_0}, \frac{\partial \Delta\psi_{gov,0}}{\partial \hat{\xi}_0}$ have the same sign as before. This again implies that interest rates drop more in the debt concerns model following a negative shock to preferences and they rise (relative to the no debt concerns model) in response to shocks to the fiscal variables.

[17] In the NDC scenario output changes are driven by taxes. From the Phillips curve we get:

$$\hat{Y}_t^{NDC} = -\frac{\kappa_2}{\kappa_1}\hat{\tau}_t^{NDC} \tag{3.36}$$

where $\kappa_1 \equiv -\frac{(1+\eta)\overline{Y}}{\theta}\gamma_h$, $\kappa_2 \equiv -\frac{(1+\eta)\overline{Y}}{\theta}\frac{\overline{\tau}}{(1-\overline{\tau})}$.

Under no debt concerns a negative shock in preferences makes taxes drop. Even though the market value to GDP initially increases (due to the increase in bond prices) fewer bonds need to be issued and so $\hat{b}_{t,\delta}$ is lower along the optimal path.

In the debt concerns case, taxes do not adjust to debt, since we have assumed $\phi_{\tau,b} = 0$. The real market value of debt increases due to the negative inflation.

[Figure 3.2 About Here]

The responses shown in Figures 3.1 and 3.2 reveal that shocks have a larger impact on macroeconomic variables under debt concerns than in the case of no debt concerns. As discussed previously, higher volatility is a standard prediction of models where monetary policy is passive. In our optimal policy model higher volatility derives from the fact that the planner needs to partially give up on the objective to stabilize macroeconomic variables in order to satisfy the consolidated budget.

Varying the value of $\phi^{R}_{\tau,b}$ Are the responses of optimal policy to shocks sensitive towards assuming different values of $\phi^{DC}_{\tau,b}$? What happens if we increase $\phi^{DC}_{\tau,b}$ from the value of zero which we assumed in the previous experiments, to a value that is closer to the determinacy cutoff? In the appendix we report results for a wider range of values and show that the optimal responses shown in Figures 3.1 and 3.2 do not change. In this way we illustrate that the model defines two distinct regions where the equilibrium is unique, and optimal policy behavior changes drastically only when we switch from one region to the other, not when we vary the fiscal policy parameter within a region.

3.4 Optimal Policies at the ZLB

The previous section investigated the properties of optimal policy in the case where interest rates are not constrained by the ZLB. We now turn to the dynamics of the economy when interest rates hit the ZLB. As it is common in the literature, we consider a shock which lowers the value of $\hat{\xi}_0$ sufficiently so that the ZLB binds. Standard results imply that this shock puts downward pressure on prices and output, leading to deflation and to a recession during the LT. Monetary policy can then commit to keep interest rates low for a long period, and stabilize the macroeconomy (e.g. Eggertsson et al., 2003). We show in this section that in the case of debt concerns, the above well known result does not apply; forward guidance-committing to keep interest rates low- does not stabilize inflation during the LT and may even lead to deflation. We explain that this model property is due to what Sims (2011) and Cochrane (2018) label the 'Stepping on a rake': in an equilibrium with passive monetary policy, lowering the nominal interest rate leads (eventually) to a drop in the price level.

In order to demonstrate transparently the effects of FG, we first use a model where FG is exogenous, as in the recent DSGE papers cited previously. This allows us to demonstrate how FG shocks impact inflation and output under passive money and active money rules, keeping the timing and duration of the shocks constant across the two regimes. We show that in the case of passive monetary policy, not only can inflation turn negative in response to a FG shock, but also FG is much less powerful than when policy is active.

Under optimal policy the timing and the duration of FG are endogenous and generally they are different between the debt concerns and no debt concerns models. In the analytical example we construct, FG can last for much longer in the case of debt concerns and the duration of FG hinges on the magnitude of the effect of the preference shock to the intertemporal budget. We show that in spite of keeping interest rates longer at zero, when monetary policy becomes subservient to fiscal policy, FG can lead to negative inflation.

3.4.1 FG under passive and active monetary policies

Consider an economy where the Phillips curve, the Euler equation and the budget constraint are given by (3.2), (3.3) and (3.74) respectively and fiscal policy follows (3.1). Also assume no preference shocks, for the moment. Monetary policy follows an interest rate rule of the form

$$\hat{i}_t = \widetilde{\phi}_\pi \hat{\pi}_t + \mathcal{Z}_t \tag{3.37}$$

where $\mathcal{Z}_t$ is a standard (exogenous) FG shock, as in the DGSE literature discussed previously, which is revealed to private agents in period 0. We study the impact of a shock in period $\bar{t}$, $\mathcal{Z}_{\bar{t}} < 0$, on equilibrium inflation, output and interest rates in the case where monetary policy is active ($\widetilde{\phi}_\pi > 1$) and in the case where it is passive ($\widetilde{\phi}_\pi < 1$).

'Stepping on a Rake' : an analytic example Consider first the case where $\widetilde{\phi}_\pi > 1$ and assume further that $\sigma = 0$. Combining (3.37) with the Euler equation and iterating forward on the resulting difference equation gives:

$$\hat{\pi}_t = -\sum_{j\geq 0} (\frac{1}{\widetilde{\phi}_\pi})^{j+1} \mathcal{Z}_{t+j}. \tag{3.38}$$

According to (3.38) a shock $\mathcal{Z}_{\bar{t}} < 0$ lowering the nominal interest rate, increases the value of inflation in periods $0, 1, ..., \bar{t}$. (The effect is zero after period $\bar{t}$.)

Consider now the case of passive policy, assuming wlog that $\widetilde{\phi}_\pi = \delta < 1$. Equilibrium inflation satisfies:

$$\hat{\pi}_t = \sum_{j\geq 0}^{t-1} \delta^j \mathcal{Z}_{t-j-1} + \delta^t \hat{\pi}_0. \tag{3.39}$$

which implies that a shock $\mathcal{Z}_{\bar{t}} < 0$ will lower the value of $\hat{\pi}_{\bar{t}-1}, \hat{\pi}_{\bar{t}}, \hat{\pi}_{\bar{t}+1}, ...$ Under passive policy, therefore, the FG shock seems to have the opposite impact: promising to lower the interest rate reduces inflation, starting one period before the shock arrives.

What is going on? Though (3.38) fully characterizes the path of inflation in response to an anticipated FG shock in $\bar{t}$ (and this conforms with the common intuition that lower rates produce higher inflation), under passive policy (3.39) does not fully reveal how inflation will behave, because we do not have the initial condition $\hat{\pi}_0$. To fully characterize inflation we need to make use of the intertemporal consolidated budget constraint at $t = 0$.[18] Under the assumptions made in this section we can show that government surpluses are constant in the absence of shocks. The date zero constraint is:

$$-\frac{\bar{b}_\delta}{1-\beta\delta}\sum_{t\geq 0}(\beta\delta)^t\hat{\pi}_t \approx 0 \tag{3.40}$$

which basically says that the drop in inflation in $\bar{t}-1$ and onwards needs to be compensated by a rise in inflation between periods 0 and $\bar{t}-2$. When the monetary authority announces $\mathcal{Z}_{\bar{t}} < 0$, $\hat{\pi}_0$ increases to satisfy (3.40) and since $\hat{i}_0 = \delta\hat{\pi}_0$, the nominal interest rate will also increase. This will lead to a further increase in prices in period 1 and positive interest rates subsequently, until inflation switches sign in $\bar{t}-1$.

This pattern is essentially the stepping on a rake of Sims (2011) and Cochrane (2018). When monetary policy is passive, and debt is long term, a drop in the interest rate leads to a rise in inflation first (the conventional effect), only to reverse the sign of inflation after a few periods.

Effects of FG under $\sigma > 0$ The top graphs of panels a) and b) Figure 3.3 demonstrate the responses of inflation and interest rates in the case where $\sigma = 1$. The solid lines correspond to the case where $\bar{t} = 4$. The dashed lines set $\bar{t} = 8$. The left plots show the behavior of interest rates and the right plots trace the responses of inflation.

[Figure 3.3 About Here]

Notice that just like in our analytical example with $\sigma = 0$, in both models and under both timings of the shock, inflation rises on impact when the shock is announced and remains above steady state value until the shock occurs. However, after period $\bar{t}$ inflation turns negative in the passive money model, whereas under active money, it returns to steady state.

[18]Since we assume perfect foresight the date 0 constraint is sufficient. The intertemporal constraints at $t \geq 1$ will be satisfied since $\hat{b}_{t\geq 0}$ can be chosen as a residual (see FMOS).

We also see from the figure that interest rates rise between the announcement of the shock and date $\bar{t}$. Clearly, monetary policy attempts to ward off the inflationary effects of the shock. Under passive policy, the rise in interest rates is somewhat weaker and eventually, when inflation turns negative, interest rates do so as well.

We now ask: What would be the impact of the shock, if the planner were to eliminate the rise in interest rates between 0 and $\bar{t}-1$? In the bottom graphs of panels a) and b) of Figure 3.3 the planner announces $\hat{i}_0, \hat{i}_1, .., \hat{i}_{\bar{t}-1} = 0$ along with $\mathcal{Z}_{\bar{t}} < 0$. Notice that now inflation rises considerably under active policy (bottom right of panel a)). Moreover, the initial rise in inflation is larger, the higher is $\bar{t}$. This is the so called 'FG puzzle' (see for example Del Negro et al. (2012) and McKay, Nakamura, and Steinsson (2016)).

In contrast, under passive policy the rise in inflation is moderate and when $\bar{t} = 8$ inflation rises (slightly) less initially than when $\bar{t} = 4$. Moreover, since inflation becomes negative after few periods the cumulative response of the price level to FG is weak. Overall, FG exerts a much weaker effect in the passive money model.[19]

To reiterate, we showed that FG in a passive money model has moderate effects. First, because of the stepping on a rake, inflation at some point turns negative following a promise to lower interest rates. Second, because the power of FG is limited, a longer horizon $\bar{t}$ does not imply a larger initial impact on inflation. These results we drawn from a model where monetary policy is not optimal, which enabled us to vary exogenously the path of interest rates. We next study optimal policy when interest rates respond endogenously to a shock to preferences which drives the economy to a LT.

3.4.2 Optimal FG policies

An analytical example Assume that the preference shock occurs in period 0, lowering the value of $\hat{\xi}_0$. The shock is i.i.d. and moreover, for simplicity, assume that after period 0 there are no further shocks to the economy. We set $\hat{\xi}_0 < -\hat{i}^*$. The shock is large enough for the ZLB to bind.

Consider first the case where $\sigma = \lambda_Y = \lambda_i = 0$. Under these assumptions a rule of the form (3.37) implements optimal policy (now the sequence $\mathcal{Z}$ is, however, endogenous).

[19] A recent literature has explored alternative mechanisms to reduce the power of FG in New Keynesian models (see for example McKay et al (2015) and references therein). Our result that the impact of FG under passive money is limited should be of interest.

The appendix shows that the optimal paths of inflation and interest rates are given by:

$$\hat{\pi}_t = \begin{cases} \frac{\bar{b}_\delta}{1-\beta\delta}\Delta\psi_{gov,0} & t=0 \\ -\hat{i}^* - \hat{\xi}_0 & t=1 \\ \max\{-\hat{i}^*, \frac{\bar{b}_\delta \delta^t}{1-\beta\delta}\Delta\psi_{gov,0}\} & t \geq 2 \end{cases} \qquad \hat{i}_t = \begin{cases} -\hat{i}^* & t=0 \\ \max\{-\hat{i}^*, \frac{\bar{b}_\delta \delta^{t+1}}{1-\beta\delta}\Delta\psi_{gov,0}\} & t \geq 1 \end{cases} \tag{3.41}$$

in the case of the debt concerns model and

$$\hat{\pi}_t^{NDC} = \begin{cases} 0 & t = 0 \; and \; t \geq 2 \\ -\hat{i}^* - \hat{\xi}_0 & t=1 \end{cases} \qquad \hat{i}_t^{NDC} = \begin{cases} -\hat{i}^* & t=0 \\ 0 & t \geq 1 \end{cases} \tag{3.42}$$

in the no debt concerns model. Notice that according to (3.41) and (3.42) the two models predict positive inflation $t = 1$, (both equal to $-\hat{i}^* - \hat{\xi}_0$, trivially otherwise the ZLB would be violated) and different inflation levels at $t = 0$ and $t \geq 2$. In the case of no debt concerns, inflation returns to its steady state value from period 2 onwards. Under debt concerns inflation and interest rates continue being different from zero, the sign of the responses hinges on the sign of the term $\Delta\psi_{gov,0}$ which measures the impact of the LT shock on the consolidated budget. In the case where $\Delta\psi_{gov,0} < 0$, following a negative shock to preferences, the inflation rate becomes negative. In contrast, if $\Delta\psi_{gov,0} > 0$, inflation turns positive after the shock.

What does FG do in the optimal policy model? Using (3.42) and the rule (3.37) we can show that the optimal path for $\mathcal{Z}^{NDC}$ is:

$$\mathcal{Z}_1^{NDC} = \widetilde{\phi}_\pi(\hat{i}^* + \hat{\xi}_0) < 0 \quad \text{and} \quad \mathcal{Z}_t^{NDC} = 0, \quad t \neq 1.$$

In other words, FG keeps the interest rate low in period 1, which enables inflation to rise in that period. This is the standard result of Eggertsson et al. (2003).

Now consider the case of DC. We can show that:

$$\mathcal{Z}_t = \begin{cases} \max\{-\hat{i}^*, \frac{\bar{b}_\delta \delta^{t+1}}{1-\beta\delta}\Delta\psi_{gov,0}\} + \delta(\hat{\xi}_0 + \hat{i}^*) < 0 & t = 1 \\ \max\{-\hat{i}^*, \frac{\bar{b}_\delta \delta^{t+1}}{1-\beta\delta}\Delta\psi_{gov,0}\} + \delta\hat{i}^* < 0 & t > 1, i_{t-1} = -\hat{i}^* \\ 0 & \text{othws} \end{cases} \tag{3.43}$$

Let us consider $\Delta\psi_{gov,0} < 0$ as in the previous section. For the sake of the exposition let us also (momentarily) assume $\frac{\bar{b}_\delta \delta}{1-\beta\delta}\Delta\psi_{gov,0} < -\hat{i}^* < \frac{\bar{b}_\delta \delta^2}{1-\beta\delta}\Delta\psi_{gov,0}$. Under this condition the shock is large enough to make the ZLB bind in period 0, but from period 1 onwards the interest rate is given by $\frac{\bar{b}_\delta \delta^{t+1}}{1-\beta\delta}\Delta\psi_{gov,0}$.

From (3.43) it is clear that $\mathcal{Z}_1$ is negative. Moreover, it holds that $\mathcal{Z}_{\geq 2} = 0$. Therefore, FG lowers the interest rate in period 1 only. We can show that $\hat{\pi}_t$ is given by:

$$\hat{\pi}_t = \delta^{t-1}\underbrace{\left(-\hat{i}^* - \hat{\xi}_0\right)}_{\hat{\pi}_1} + \delta^{t-2}\mathcal{Z}_1, \qquad t = 2, 3, , ...$$

Since $\mathcal{Z}_1 < 0$ this equation suggests that the impact of FG is to lower inflation after $t = 1$. In this example FG under debt concerns has exactly the opposite effect than under NDC: It reduces inflation.

From (3.41) and (3.43) it is also easy to apprehend how optimal policy behaves when a large shock hits, so that $\Delta\psi_{gov,0}$ becomes sufficiently negative to make the ZLB bind for several periods. It is evident that the longer is the commitment to keep interest rates at ZLB, the more persistent is deflation during the LT. In other words, in response to large shocks the planner optimally announces a path that keeps interest rates at the ZLB over a longer horizon, however, this extends negative inflation over to more periods.

Why are these responses optimal then? It may thus seem counterintuitive why the monetary authority would consider engaging in FG in the first place, if all that it accomplishes is to extend deflation. To understand the tradeoff facing the planner, it is purposeful to take a step back and consider the optimal choice of the duration of FG in this model. Suppose that the policy is to hold interest rates at the ZLB until period $\bar{t} - 1 > 1$. After $\bar{t}$ the planner sets $\hat{i}_t = \delta\hat{\pi}_t$ which is the optimal policy when the ZLB will not further bind. It is simple to show that inflation will be equal to $-i^*$ between periods 2 and $\bar{t}$, then it will be $\hat{\pi}_t = -i^*\delta^{t-\bar{t}}$, in $t = \bar{t}+1, \bar{t}+2, ...$

What then remains is to specify inflation in period 0, when the shock hits. In the appendix we show that

$$\hat{\pi}_0 = \frac{1}{\tilde{\bar{\lambda}}}\left[\left(\bar{b}_\delta - \overline{S}\right)\hat{\xi}_0 + \frac{\beta\delta\bar{b}_\delta}{1-\beta\delta}\left(i^* + \hat{\xi}_0\right) + i^*\frac{(\beta\delta)^2\bar{b}_\delta}{(1-\beta\delta)^2}\left(1 - (\beta\delta)^{\bar{t}-1}\right) + i^*\frac{(\beta\delta)^{\bar{t}+1}\bar{b}_\delta\delta}{(1-\beta\delta)(1-\beta\delta^2)}\right] \quad (3.44)$$

where $\widetilde{\lambda}_1 \equiv \frac{\overline{\tau}(1+\eta)}{\eta}\overline{Y}\frac{(1+\gamma_h)}{\kappa_1} + \frac{\overline{S}}{1-\beta}$. According to (3.69) $\hat{\pi}_0$ increases in the duration $\bar{t}$.

To understand how the planner will set $\bar{t}$ we once again need to consider the effect of the preference shock on the intertemporal budget constraint. If indeed the present value of surpluses rises relative to the market value of debt (as we assumed in the previous paragraph with $\Delta\psi_{gov,0} < 0$), then inflation must turn negative to satisfy intertemporal budget balance. Since debt is long term, this can be through having negative inflation after period 2, as well as through having deflation in period 0. There is thus a tradeoff: future deflation will reduce the drop in the price level in period 0. (3.69) essentially tells us what $\hat{\pi}_0$ has to be so that given the duration $\bar{t}$ and the implied path of prices, the intertemporal constraint is satisfied. Then from equation (3.41) it is obvious that the planner will find optimal to set $\hat{\pi}_0 < 0$.

To reiterate, FG mitigates deflation at the onset of the LT, through producing persistent deflation afterwards. Overall, the tradeoff facing the planner (relative to NDC) worsens; having to satisfy the intertemporal budget in the DC regime leads to a larger cumulative drop in the price level.

3.4.3 Numerical Examples

The example of the previous subsections is of course a very particular case. Assuming $\sigma = 0$ means that output growth exerts no influence on the real rate. Inflation in $t = 1$ has to equal $-(\hat{i}^* + \hat{\xi}_0)$ under both debt concerns and no debt concerns, otherwise the ZLB is violated. In the case where $\sigma > 0$ we can have deflation at the onset of the LT: since output drops, positive expected output growth will increase the real rate, 'allowing' inflation to turn negative without violating the ZLB. It is thus important to contrast the properties of the two models under more plausible calibrations, when $\sigma, \lambda_Y, \lambda_i > 0$, to see whether the findings of the previous paragraph generalize.

This is done in Figure 3.4 which shows the responses of inflation, output, interest rates and debt under the two versions of the model assuming the parameter values of Table 2. The solid (blue) line shows responses in the debt concerns model. The dashed line shows responses under no debt concerns.

Qualitatively, the pattern of adjustment of inflation and interest rates resembles the analytical paths derived previously. (LTs last longer now in both models because we assume a persistent shock). Under no debt concerns inflation turns positive during the period when the interest rate is at the ZLB and gradually goes back to zero after the economy escapes from the LT, as interest rates also gradually return to their steady state value.[20] When the shock hits, aggregate output drops, and the planner promises to gradually increase output and ultimately engineer a boom as the economy escapes from

[20] The fact that the adjustment of inflation is gradual is due to the persistent shock and also due to the assumption $\sigma > 0, \lambda_Y > 0$.

In the analytical example of the previous subsection the planner lowered interest rates only in $t = 1$, in the no debt concerns model, and this led to a rise in inflation in that period. This path was optimal, as opposed to extending lower rates beyond period 1, because inflation beyond period 1 is not useful to satisfy the ZLB. The optimal path implies a sharp rise in $\hat{Y}_1$ and then $\hat{Y}_{t\geq 2} = 0$.

When the planner wants to smooth output she commits to keep interest rates at the ZLB for longer, generating high inflation today through expectations of high inflation tomorrow. This mitigates the output rise in period 1.

the LT. These are standard properties of the New Keynesian model's response to the LT shock (see Eggertsson et al., 2003).

[Figure 3.4 About Here]

In the case of debt concerns, we see that following the preference shock in period 0, inflation drops sharply. It turns positive for few periods, but subsequently becomes negative again. As the bottom left panel shows, interest rates remain longer at the ZLB. This leads to a drop in the inflation rate below its steady state value, rather than a rise, consistent with our results in the previous subsection.

3.4.4 Discussion

The key result we obtain in this section is that when monetary policy is subservient to fiscal policy, the tradeoff facing the planner in a LT worsens. We now place this finding within the literature on optimal policy in a LT.

Several papers (see e.g. Eggertsson (2006), Bhattarai et al. (2015), Burgert and Schmidt (2014) amongst numerous others) have considered models of jointly optimal monetary and fiscal policies in a LT assuming that the planner cannot commit to allocations. The findings of these papers suggest that in response to a LT shock, the planner will find optimal to reduce taxes (increase spending in the case where it is not exogenous to the program) and increase government debt. Since debt is a state variable in these models and higher debt levels lead to higher inflation, this accomplishes to increase inflation at the exit from the LT episode. Through the Phillips curve higher future inflation translates into higher inflation in the LT. Thus in these models when monetary policy takes into account the consolidated budget constraint, the planner's tradeoff improves.

There are two main differences here. First, we assume that taxes (and spending) are exogenous to the planner's program and second we assume commitment. Had we assumed that the planner can control the tax rate, the optimal policy response would be to lower taxes temporarily, and this would tighten the intertemporal budget. [21] Equation (3.41) then says that instead of deflation a LT could lead to inflation, when $\Delta\psi_{gov,0} > 0$.[22] In our model where fiscal variables are exogenous this could only happen by chance, if a negative tax shock (equivalently a positive spending shock) hits the economy in period 0.

[21] Note that this would then be similar to the response of optimal policy in Eggertsson and Woodford (2006) who consider jointly optimal policies under commitment.

[22] Note that in the case of the jointly optimal monetary/fiscal program the equations we derived in this section remain. See Section 6 where we describe this model.

The absence of commitment would also reverse our conclusions. With commitment, the level of debt is not a state variable influencing inflation and thus a higher debt level in the LT will not result to a rise in future inflation.[23] In the NDC case, assuming no commitment, worsens the outcome of policy considerably. This has been well documented in the literature.

3.5 Robustness and Extensions

In this section we explore the robustness of our findings towards varying several model features. First, we show that our findings generalize to a medium scale DSGE model. Second, we consider the case where taxes are lump sum and separately the case of jointly optimal monetary and fiscal policies assuming the planner can set the tax rate along with inflation, output and the interest rate. Third, we investigate how varying the maturity of debt affects our findings. Lastly, we briefly explore an extension of our model which allows for regime fluctuations.

3.5.1 A Medium Scale DSGE Model

We fit an augmented version of the theoretical framework developed in the previous sections, to US data. We augment the previous setup with habit formation, a trend in TFP, and shocks to markups, TFP and to the consolidated budget. Moreover, we enrich the fiscal block following recent DSGE modelling (e.g.Bianchi and Ilut, 2017); we assume that besides spending, governments have to finance transfers to the private sector which we model as a stochastic process with two components (a trend and a cyclical component). Also, we augment the tax rule so that taxes now adjust, to other macroeconomic variables, besides debt, in order to capture the endogenous response of taxes to the cycle.

Demonstrating robustness of our findings towards introducing these additional model features is important. The previous sections highlighted that in the debt concerns model the response of the intertemporal constraint of the government to shocks is key to understand the behavior of monetary policy. Yet, our model thus far has relied on simple fiscal rules for tractability. Here we use the data to identify more realistic fiscal rules and study optimal policy in an model which provides a more accurate image of the US economy.

We will provide a brief overview of the model here and of the output that we get from estimation. We will then use the output to repeat the experiments of the previous sec-

[23] The fact that debt is not a state variable here shouldn't be surprising. In a one off shock case only the debt inherited in period 0, $\hat{b}_{\delta,-1}$, matters. The sequence $\hat{b}_{\delta,t\geq 0}$ can be recovered given optimal commitment policies to satisfy the intertemporal consolidated budget constraints.

tions studying the responses of policy to shocks. Further details on the output and our modelling assumptions are contained in the appendix.

The model Household preferences are of the following form:

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left(\log(C_t - \Omega C^a_{t-1}) - \chi \frac{h_t^{1+\gamma_h}}{1+\gamma_h} \right)$$

where C_t denote the consumption of the household, ΩC^a_{t-1} is an external habit stock, where $0 < \Omega < 1$ and C^a_{t-1} denotes the average level of consumption in $t-1$. The household derives disutility from exerting labor effort h_t. Parameters χ and γ_h govern the household's preferences over leisure. As previously, ξ_t is a preference shifter which impacts the relative discounting of current and future utility flows.

The household maximizes utility subject to the flow budget constraint:

$$P_tC_t + P_{t,L}B_{t,\delta} + P_{t,S}B_{t,S} = (1-\tau_t)W_th_t + P_tTr_t + B_{t-1,S} + (1+\delta P_{t,L})B_{t,\delta} + P_tDiv_t$$

$B_{t,\delta}$ is a long government bond. $P_{t,L}$ is the price of the asset. $B_{t,S}$ denotes the quantity of short term (one-period) debt which is in zero net supply. W_t denotes the nominal wage and P_t is the price level. Div_t is real dividends paid by monopolistically competitive firms and Tr_t denotes lump-sum transfers given to the household by the fiscal authority. The household maximizes utility subject to the flow budget constraint. For brevity the first order conditions are stated in the appendix.

We assume that output is produced by a continuum of monopolistically competitive firms which operate technologies with labor as the sole input. Aggregate output is produced by a representative, perfectly competitive, final-good producer that aggregates the intermediate products of firms according to $Y_t = \left(\int_0^1 Y_t(j)^{\frac{1+\eta_t}{\eta_t}} dj \right)^{\frac{\eta_t}{1+\eta_t}}$. η_t is a (time varying) parameter that governs the elasticity of substitution. The production function of the generic good firm j is $Y_t(j) = A_t h_t(j)^{1-\alpha}$; A_t denotes the level of TFP in the economy.

Firms face price adjustment costs as in Rotemberg (1982). The cost function of firm j is: $AC_t(j) = \frac{\theta}{2}(\frac{P_t(j)}{P_{t-1}(j)} - \pi)^2 Y_t$. $\theta \geq 0$ again governs the degree of price stickiness. π is the steady state level of gross inflation. In the appendix we show that our model gives rise to the following (non-linear) New-Keynesian Phillips Curve:

$$\theta(\pi_t - \overline{\pi})\pi_t = (1+\eta_t)(1 - \frac{MC_t}{P_t}) + \beta\theta E_t \frac{C_t - \Omega C^a_{t-1}}{C_{t+1} - \Omega C^a_t} \frac{Y_{t+1}}{Y_t} (\pi_{t+1} - \overline{\pi})\pi_{t+1}.$$

MC_t denotes marginal costs of production.

The log growth rate of TFP evolves according to the following stochastic process

$$\ln\left(\frac{A_t}{A_{t-1}}\right) \equiv a_t = (1-\rho_a)\gamma + \rho_a a_{t-1} + \epsilon_{a,t}$$

Parameter γ denotes the steady-state growth rate of the economy.

The government levies distortionary taxes and issues debt to finance spending G_t and transfers T_t. The flow budget constraint of the government is:

$$P_{t,L}B_{t,\delta} = (1+\delta P_{t,L})B_{t-1,\delta} + P_t(G_t + Tr_t) - \tau_t W_t h_t + \Lambda_t$$

Notice that we augment the flow budget with an exogenous shock variable Λ_t capturing fiscal variables that we have left outside the model.

Moreover, we assume that taxes follow an exogenous rule relating the current tax rate to the lagged value of debt. In log linear form this rule is:

$$\hat{\tau}_t = \rho_\tau \hat{\tau}_{t-1} + (1-\rho_\tau)\left[\phi_{\tau,b}\hat{b}_{\delta,t-1} + \phi_{\tau,y}(\hat{Y}_t - \hat{Y}_t^n) + \phi_{\tau,g}(g^{-1}\hat{g}_t + \hat{tr}_t^*)\right] + \epsilon_{\tau,t} \tag{3.45}$$

Notice that we now allow aggregate output (in deviation from its natural level $\hat{Y}_t^n$) and the levels of spending and transfers to impact directly the tax schedule in period t. Variable $\hat{tr}_t^*$ represents the long-term component of transfers, and as in Bianchi and Ilut (2017) evolves exogenously as an AR(1) process:

$$\hat{tr}_t^* = \rho_{tr^*}\hat{tr}_{t-1}^* + \epsilon_{tr^*,t} \tag{3.46}$$

The difference $\hat{tr}_t - \hat{tr}_t^*$ evolves according to:

$$\hat{tr}_t - \hat{tr}_t^* = \rho_{tr}(\hat{tr}_{t-1} - \hat{tr}_{t-1}^*) + (1-\rho_{tr})\phi_{tr,y}(\hat{Y}_t - \hat{Y}_t^n) + \epsilon_{tr,t} \tag{3.47}$$

We further assume that $\epsilon_{tr,t}$ and $\epsilon_{tr^*,t}$ are i.i.d.

Finally, the remaining exogenous variables are assumed to follow:

$$\hat{x}_t = \rho_x \hat{x}_{t-1} + \epsilon_{x,t}$$

for $\hat{x} \in \{\hat{G}, \hat{a}, \hat{\Lambda}, \hat{\xi}, \hat{\eta}\}$ and where $\epsilon_{x,t}$ is an i.i.d shock to variable x.

Estimation In estimating our model we maintain the assumption that monetary policy is optimal; this allows us to recover directly from the data parameters λ_i, λ_Y along with other model parameters. Relative to the model of sections 2 to 4 we now specify a slightly different objective assuming that the planner seeks to maximize

$$-\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left[\hat{\pi}_t^2+\lambda_Y\left(\hat{Y}_t-\hat{Y}_t^n\right)^2+\lambda_i\left(\hat{i}_t-\hat{i}_{t-1}\right)^2\right] \tag{3.48}$$

We made this choice since (3.48) has been used in previous papers estimating DSGE models with optimal monetary policy.[24]

We fit the optimal policy model to US observations using data from the period 1980Q1-2008Q4.[25] The macroeconomic series that we employ in estimation are: output, inflation, the federal funds rate, tax revenues, total government expenditures, government spending, and the market value of government debt. The details on these variables, together with the measurement equations we employ to link the series to model variables, are spelled out in the appendix.

The task of estimating the model is complicated because we have two model versions we can choose from, DC and NDC. To choose which version we want to estimate we have to decide whether assuming no debt concerns describes more accurately US monetary policy since the 1980s when our sample begins, or if over this period, monetary policy was subservient to fiscal policy as in the DC model. Our choice is guided by the findings reported in Bianchi and Ilut (2017) who find that from the 1980s and until 2008 monetary policy was active and fiscal policy passive. Thus, we estimate our structural optimal policy framework assuming no debt concerns.

In the appendix we report our choices over the prior distributions (which are relatively loose and in line with previous papers in the literature) as well as the values of parameters that we fix in estimation. We also show the output that we get, showing that our estimates of the posterior distribution are in line with the relevant literature.[26]

The effects of shocks Figures 3.5 and 3.6 show the responses of inflation, interest rates, output and debt-to-GDP to each of the shock processes in the model. Figure 3.5 considers the case of a preference shock (left panel), a spending shock (middle left panel), a tax shock (middle right panel), and a markup shock (right panel). Figure 3.6 shows

[24]See for example Debortoli and Lakdawala, 2016). Our results below do not hinge crucially on this choice (we get very similar findings when we assume a loss function as in (3.6)).

[25]We stop before the first quarter of 2009 because as it is well known, the short term interest rate in that quarter was at the ZLB. Accounting for this in estimation implies that standard estimation techniques (e.g. Smets and Wouters (2007) do not apply.

[26]For example we find a Phillips curve that is relatively flat (the mean estimate of κ is 0.017) and the distribution of the habit parameter Ω is centered around roughly 0.5. Moreover, the estimated response of taxes to the lagged value of debt is low ($\phi_{\tau,b}=0.068$ at the mean). These values are very close to the analogous objects reported in Bianchi and Ilut (2017). Our estimates estimates of the coefficients λ_i and λ_Y are also in range of recent estimates of optimal Ramsey models with US data (e.g. DDebortoli and Lakdawala, 2016).

the responses to a shock to the government budget, a shock to TFP, and shocks to the business cycle and long-term components of transfers (respectively from left to right).[27]

[Figures 3.5 and 3.6 About Here]

Under no debt concerns, inflation is basically unaffected by shocks to preferences, spending, taxes, transfers, and shocks to the budget. Shocks to TFP exert a small influence on inflation, but mainly markup shocks are the key driving force behind inflation variability. This model property can be explained as follows: First, preference and spending shocks exert only a minor influence because these shocks are persistent and also because monetary policy is optimal. Due to high persistence these shocks do not provoke large movements to the real rate. Given welfare losses derive from the volatility of interest rate growth in (3.48), the planner can adjust permanently the nominal interest rate by a few basis points in response to the shocks, without impinging substantial welfare losses. Tax shocks on the other hand, affect the inflation output tradeoff through their influence on the Phillips curve, but our estimates of the model suggest that this effect is not large. This also applies to shocks to the consolidated budget which can lead to changes in taxes. Given these observations, the property that markup shocks are the key driving force behind inflation dynamics in the optimal policy model is not surprising: Monetary policy is very effective in stabilizing inflation against other types of shocks.[28]

In contrast, as can be seen from Figures 3.5 and 3.6, under debt concerns inflation responds strongly to all types of shocks, including shocks in fiscal variables and also the effects of markup shocks and TFP shocks are now larger. Now shocks exert an influence on the consolidated budget and the planner will use inflation (or deflation) to adjust real debt accordingly. Following a negative TFP shock tax revenues will drop (since real wages decrease) and this will tighten the intertemporal constraint. Analogously, a markup shock, increases profits (not taxed) and lowers real wages, thus reducing overall government revenues. This induces the planner to engineer a larger rise in inflation to ensure debt sustainability.

Consistent with our previous findings, debt concerns, introduces a new channel (the consolidated budget) via which shocks can affect macroeconomic variables and it magnifies macroeconomic volatility.

We now turn to study optimal policies in response to a shock that drives the economy in a LT. Rather than simply asking the model to draw a large enough preference shock

[27]Notice that the impact effects of each of the shocks on the model variables are now measured in percentage points. Therefore, 1 is a 1 percent increase of a variable relative to the balanced growth path, 0.1 is a 0.1 percent increase, etc.

[28]This finding is also not out of line with the DSGE literature. For example, Fratto and Uhlig (2020), Hall (2011), Michaillat and Saez (2014) all find that markup shocks drive inflation in the US.

from the posterior distribution, we ask the model to recover the shocks which, in the first quarter of 2009, brought the US economy to the LT. Thus we first use the data to recover smoothed shocks so that our (NDC) model can fit the macroeconomic time series in 2009Q1, and then analyze the effect of the preference shock separately in the DC and NDC models. [29]

Figure 3.7 plots the paths of inflation (top left), interest rate (top right), debt and output growth (bottom left and right respectively). Since the responses are generated using the preference shock we recovered from the data, the horizonal axis is dated. The solid lines show again the case of debt concerns, the dashed lines show the no debt concerns model. For comparison we also show in the dotted (black) lines, the (filtered) data observations.

[Figure 3.7 About Here]

Consider first the responses in the no debt concerns model. Notice that the model generates a positive inflation rate in response to the preference shock; the inflation rate drops to around 1 per cent in the beginning of 2009, and very fast reaches the steady state level (around 2 percent). Thus inflation is stable in the LT. Second, the model predicts that interest rates remain at the ZLB for several quarters after 2009. Third, as can be seen from the middle right panel, the model predicts that output growth recovers rapidly in 2009 and subsequently is stabilized close to the steady state rate of TFP growth.

In contrast, the DC model predicts that inflation turns negative in 2009 and continues being negative until the final quarter of 2010. Interest rates remain at the ZLB until the end of 2013 and finally, output growth is more negative at the onset of the LT episode than in the NDC model.

These predictions are clearly in line with our previous findings, that when monetary policy is subservient to fiscal policy the planner's tradeoff in a LT worsens. Debt concerns leads to a sharp drop in the price level, and as we saw, output growth is less stable than in the case of no debt concerns. Moreover, the duration of the LT is longer. We conclude that the theoretical results in Section 4, hold also in the larger scale model we employ in this section.

3.5.2 Lump Sum Taxes

Our analysis throughout Sections 2 to 5 assumes that the government levies distortionary taxes on labour income. This assumption is realistic and also standard in models of

[29] For brevity we describe in detail in the appendix the procedure that we follow to recover the shocks. Basically we use a standard Kalman filter augmented to account for piecewise linear solutions, allowing us to deal with an occasionally binding ZLB constraint.

optimal policy under incomplete markets. In numerous DGSE models, however, taxes are lump sum. We show that our findings are robust towards this assumption.

Note that assuming lump sum taxes essentially amounts to setting $\kappa_2 = 0$ in equation (3.2). Moreover, the surplus becomes $\overline{S}\hat{S}_t = \left[(\overline{\tau} - \overline{G})\left(\hat{\xi}_t - \sigma\frac{\overline{Y}}{\overline{C}}\hat{Y}_t + \hat{\xi}_t\right) + \overline{\tau}\hat{\tau}_t - \overline{G}\hat{G}_t\right]$. Notice that these changes have basically no impact on the monetary policies derived in Propositions 1 and 2. Thus, the categorization of monetary policy into passive/active, is the same when we introduce lump sum taxation.

In contrast, the conditions under which fiscal policy is active/passive do change. For instance when we use the assumptions of subsection 3.3.1 and assume lump sum taxation we get:

Proposition 5. Determinacy under lump sum taxes *Assume that taxes are lump sum and follow a rule of the form (3.1). Also assume that parameters λ_Y, $\overline{G}$, δ σ and ρ_τ have values as in subsection 3.3.1 (Proposition 3). The equilibrium under DC (NDC) is uniquely determined if*

$$\frac{1}{\beta}\left[1-(1-\beta)\phi^R_{\tau,b}\right] > 1 \ (< 1)$$

Proof: See appendix.

Clearly the regions in which fiscal policy is active/passive defined in Proposition 5 differ from the analogous objects in Proposition 3. However, the principle that fiscal policy is active when taxes strongly respond to debt, and passive otherwise continues to hold.

Given this finding, it is possible to rederive all results of Section 3 with the assumption that taxes are lump sum. Rather than showing additional derivations in the appendix we resort to simulations to study the responses to shocks and compare them with the case where taxes are distortionary. We find that the responses are very similar in both the debt concerns and no debt concerns models. We thus conclude that our findings continue to hold when we introduce lump sum taxation in our framework.

3.5.3 Jointly optimal policies

We now extend our analysis to consider the case where the planner can set taxes along with inflation, interest rates and output. As discussed previously, many papers have studied jointly optimal monetary and fiscal policies, using models which are broadly similar to ours but mainly resorting to global solution methods to approximate equilibria numerically. We investigate whether in our linear model, an optimal allocation with coordinated policies is close to the debt concerns equilibrium or to the no debt concerns outcome.

To allow taxes to be set optimally we abandon fiscal rule (3.1). We further assume that distortionary taxation enters into the loss function (3.6) so that the planner seeks to minimize tax volatility. We assign a weight $\lambda_\tau \geq 0$ to this objective. The optimal policy is otherwise similar to the debt concerns model; the planner optimizes subject to constraints (3.2) to (3.74). It is easy to show that the first order condition with respect to $\hat{\tau}_t$ gives

$$-\lambda_\tau \hat{\tau}_t + \psi_{gov,t} \frac{(1+\eta)\overline{\tau}\overline{Y}}{\eta(1-\overline{\tau})} - \psi_{\pi,t}\kappa_2 = 0$$

The remaining FONC (for $\hat{\pi}_t, \hat{Y}_t, ...$) for this program are unchanged relative to Section 2.

Consider assumptions $\sigma = \lambda_Y = 0$. We can show (see appendix) that optimal taxes are given by

$$\hat{\tau}_t = \psi_{gov,t} \frac{1}{\lambda_\tau} \frac{(1+\eta)\overline{\tau}\overline{Y}}{\eta(1-\overline{\tau})} [1 - \frac{\overline{\tau}\gamma_h}{1+\gamma_h}] \tag{3.49}$$

Moreover, since $[1 - \frac{\overline{\tau}\gamma_h}{1+\gamma_h}]$ exceeds 0, from (3.49) taxes adjust upwards whenever the consolidated budget tightens and vice versa. According to (3.11) taxes follow a random walk.

Is fiscal/monetary policy active/passive? Optimal monetary policy follows a rule of the form:

$$\hat{i}_t = (1+\delta)\hat{\pi}_t - \frac{\overline{b}_\delta}{1-\beta\delta} \sum_{l=0}^{\infty} \delta^l \Delta\psi_{gov,t-l} \tag{3.50}$$

The impact of the last (interest rate twisting) term on the RHS is key. When this term is zero then monetary policy is active, otherwise it is passive. Therefore, ultimately, the behavior of policy depends on whether shocks to the consolidated budget exert a significant influence, so that the multiplier $\psi_{gov,t}$ displays volatility. In the case where $\lambda_\tau = 0$ taxes will adjust to fully absorb shocks to the consolidated budget and we can show that $\psi_{gov,t} = 0$ for all t. In this case fiscal policy is passive and monetary is active. In contrast, when λ_τ approaches infinity, taxes are constant and equal to the steady state value. Under this scenario, fiscal policy is active and monetary policy is passive.

Intermediate cases cannot be characterized analytically. It is evident that the model has both forces, passive and active policies. Notice that this is consistent with previous findings in the literature (e.g. Leeper and Leith (2016), Schmitt-Grohé and Uribe (2004)).

To find out which of the two regimes is closer to equilibrium policies in the coordinated policies outcome we need to resort to numerical simulations. In the appendix, we consider alternative values of λ_τ and study impulse responses to shocks. We find that even moderate values for the weight λ_τ brings us close to the debt concerns outcome.

Lastly, consider the case where taxes are lump sum. Since in this case taxes are essentially a slack variable in the government budget constraint we have $\psi_{gov,t} = 0$ at the optimum. The optimal mix of monetary/fiscal policies is thus 'active/passive'.

3.5.4 The role debt maturity

We provided several analytical expressions in previous sections, showing how monetary and fiscal policy rules vary with parameter δ. The maturity of debt is an important variable here since long term bonds allow the planner to spread inflation over more periods. Hence, the policy responses to shocks depend on whether the government issues short debt ($\delta = 0$) or long term bonds $\delta > 0$. This property is well known (see for example Lustig et al., 2008) and for brevity we leave it to the appendix to show impulse responses of key model variables under alternative calibrations of δ. It is however, important to highlight here that none of our previous results, regarding the characterization of monetary and fiscal policies hinges on the exact value of δ. Thus, broadly speaking, our analysis is robust towards changing the value of this parameter.

The assumption we made in this paper, that governments issue debt in the form of a single bond that pays decaying coupons is standard in DSGE models. A recent literature, however, studying optimal debt management in macroeconomic models (see for example Angeletos (2002), Buera et al. (2004), Faraglia et al. (2019a), Debortoli et al. (2017), Bhandari et al. (2017a) among others) allows governments to issue multiple bonds simultaneously and investigates how the maturity structure of debt can be targeted to make debt sustainable. If we included in the model an optimizing debt management authority, as recent papers in debt management do, would our conclusions be affected?

In subsection 3.2.1 (footnote 15) we gave an example where issuing a flat maturity structure ($\delta = 1$) completely eliminates the impact of preference shocks on the consolidated budget. We had $\psi_{gov} = 0$ and therefore the debt concerns outcome became identical to the no debt concerns outcome. This provided an example where debt management overrides fiscal/monetary policies and the distinction between active and passive policies becomes less meaningful. Analogously, the work of Angeletos (2002) and Buera et al. (2004) showed that governments that want to hedge against spending shocks, can issue long term debt and accumulate assets in short maturity. Presumably, if an optimizing debt management authority followed this strategy in our model, the impact of spending shocks to the consolidated budget would also be zero.

In both these cases debt management can 'complete the market', so that intertemporal consolidated budgets become irrelevant to the planner's program. Recent papers in the debt management literature have argued however, that such an outcome is practically unattainable, since governments face frictions when issuing debt. Lustig et al. (2008) find it is implausible to assume that governments invest in private assets, Faraglia et al. (2019a) argue that transaction costs associated with repurchasing long bonds limit the scope of hedging against fiscal shocks and finally, Bhandari et al. (2017a) argue that debt management has too few instruments to deal with a large numbers of shocks. In all of these papers markets are 'incomplete' (as in our model) and thus intertemporal budget constraints matter in equilibrium. Under incomplete markets, we have $\psi_{gov} \neq 0$ and the results of this paper continue to hold.

Clearly, the interplay between debt management, fiscal and monetary policies is interesting and worthwhile exploring in future research.

3.5.5 Switching Across Regimes

In this final section of the paper we discuss an extension of our framework which allows for regime fluctuations. A recent stream of papers (e.g. Davig and Leeper (2007), Bianchi and Ilut (2017), Bianchi and Melosi (2017, 2019)) considers models where the policy mix can oscillate between 'active/passive' and 'passive/active', randomly through time. To complete our analysis, we discuss here how to model shifts across the DC and NDC equilibria when monetary policy is optimal. Since a full theory of recurrent shifts is beyond the scope of this paper, we assume that they are one off events. Thus we will consider the case where starting from the DC model, there is a sudden and unexpected shift to the NDC equilibrium and vice versa. This is sufficient to demonstrate that our framework can encompass regime fluctuations, but also it allows us to show transparently key aspects of the transition between the two equilibria. In the appendix we provide a more complete treatment, allowing for recurrent shifts across DC and NDC and derive optimal interest rate rules.

As we have seen, the optimal rules in Propositions 1 and 2, differ when $\mathcal{D}_t \neq 0$. When the fiscal authority does not guarantee solvency of the intertemporal budget, the multiplier ψ_{gov} differs from zero and conversely, when $\phi^R_{\tau,b}$ is sufficiently high, entailing a strong response of taxes to debt, then $\psi_{gov} = 0$, in the NDC regime. Thus, switching from the DC to the NDC equilibrium, requires changing the value of $\phi^R_{\tau,b}$, and setting $\psi_{gov} = 0$ when the switch occurs.

An important aspect of this transition (from DC to NDC) has to do with the promises $\mathcal{D}$ that the planner made in past periods. These interest rate twists are time inconsistent, in the sense that if the planner is allowed to reoptimize when the switch occurs, she will set $\mathcal{D} = 0$.

Formally, suppose that in period $\bar{t}$ the feedback tax rule coefficient changes from $\phi_{\tau,b}^{DC}$ to $\phi_{\tau,b}^{NDC}$. Since now fiscal policy generates sufficient surpluses to finance debt, the consolidated budget constraint is, from the point of view of the planner, slack. The continuation Ramsey program solves:

$$\max -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t E_{\bar{t}}\Big\{\hat{\pi}_{t+\bar{t}}^2 + \lambda_Y \hat{Y}_{t+\bar{t}}^2 + \lambda_i \hat{i}_{t+\bar{t}}^2\Big\} \tag{3.51}$$

subject to the Phillips curve and the Euler equation (assuming we are away from the ZLB). It can be easily showed that the solution to this program is a policy rule of the form $\hat{i}_{t+\bar{t}} = \mathcal{T}_{t+\bar{t}}^{NDC}$ for $t = 0, 1, 2....$ The planner sets $\mathcal{D}_{\bar{t}} = 0$ when reoptimization occurs. An alternative setup in which the planner 'remembers' promises made in the past and in which the solution to the continuation problem in $\bar{t}$ does not set $\mathcal{D}_{\bar{t}} = 0$ is the following: The planner maximizes

$$-\frac{1}{2}\sum_{t=0}^{\infty}\beta^t E_{\bar{t}}\Big\{\hat{\pi}_{t+\bar{t}}^2 + \lambda_Y \hat{Y}_{t+\bar{t}}^2 + \lambda_i \hat{i}_{t+\bar{t}}^2\Big\} + \underbrace{\Big(-\psi_{gov,\bar{t}-1} + \sum_{l=1}^{\infty}\delta^l \Delta\psi_{gov,\bar{t}-l}\Big)\sum_{t=0}^{\infty}(\delta^t\beta^t)E_{\bar{t}}\Big(\frac{\bar{b}_\delta}{1-\beta\delta}\hat{\pi}_{t+\bar{t}} + \sigma\frac{\overline{Y}}{\overline{C}}\bar{b}_\delta \hat{Y}_{\bar{t}+t}\Big)}_{P}$$

subject to (3.2) and (3.3). Adding the second order term P in the continuation program forces the planner to keep with past promises.[30] It can be easily checked that the FONC with respect to $\hat{Y}_{\bar{t}+t}$ and $\hat{\pi}_{\bar{t}+t}$ are essentially equations (3.8) and (3.9) when $\psi_{gov,t+\bar{t}} = 0, t > 0$. The solution to the above program is a policy rule of the form:

$$\begin{aligned}\hat{i}_{t+\bar{t}} = \mathcal{T}_{t+\bar{t}} - \frac{\overline{C}}{\overline{Y}}\frac{\kappa_1}{\lambda_i\sigma}\frac{\bar{b}_\delta}{1-\beta\delta}\Big(-\delta^t\psi_{gov,\bar{t}-1} + \sum_{l=1}^{\infty}\delta^{l+t}\Delta\psi_{gov,\bar{t}-l}\Big)\\ -\frac{\bar{b}_\delta}{\lambda_i}\Big(-\delta^t\psi_{gov,\bar{t}-1} + \sum_{l=1}^{\infty}\delta^{l+t}\Big(\Delta\psi_{gov,\bar{t}-l} - \Delta\psi_{gov,\bar{t}-l-1}\Big)\Big) + \frac{\overline{C}\omega_Y}{\overline{Y}\sigma\lambda_i}\psi_{gov,\bar{t}-1}\mathcal{I}_{t=0}\end{aligned} \tag{3.52}$$

which is basically the interest rate rule in Proposition 1, with multipliers $\psi_{gov,\bar{t}} = \psi_{gov,\bar{t}+1} = ...$, set to zero. Since the economy was in the DC regime before $\bar{t}$, the multipliers $\psi_{gov,\bar{t}-l}, l > 0$ will generally be different from zero.

Optimal policy under rule (3.52) and optimal policy when the planner 'forgets' past promises, will in general have different effects on the macroeconomy. First, obviously, because interest rate commitments made between period 0 and $\bar{t}-1$ will have a bearing on the path of interest rates from date $\bar{t}$ onwards and thus impact macroeconomic variables. Second, because of these extra terms that capture previous commitments, monetary

[30] See e.g. FMOS. Alternatively, we could assume optimal policy from a 'timeless perspective' to force the planner to keep with past promises.

policy may ‘partially’ be in the passive regime even after date $\bar{t}$, for a while, until the influence of these terms fades out.

To evaluate this we turn to a numerical example. We consider an ‘optimal disinflation scenario’: At date 0 the planner inherits a high debt level, and for the first $\bar{t}-1$ periods the economy is in the DC regime. Fiscal policy is active, unable to generate sufficient surpluses to ensure debt sustainability and inflation rises to make debt sustainable. In period $\bar{t}$, unexpectedly, fiscal policy becomes passive, and monetary policy switches to NDC. We consider the impact of the switch in policy on macroeconomic variables, in the case where the planner remembers past commitments and in the case where she reneges on them.

An Optimal Disinflation Consider for simplicity the calibration $\sigma = \lambda_Y = \lambda_i = 0$. Under these assumptions, rule (3.52) becomes

$$\hat{i}_{t+\bar{t}} = (1+\delta)\hat{\pi}_{t+\bar{t}} - \frac{\bar{b}_\delta}{1-\beta\delta}\left(-\delta^t\psi_{gov,\bar{t}-1} + \sum_{l=1}^{\infty}\delta^{t+l}\Delta\psi_{gov,\bar{t}-l}\right) \tag{3.53}$$

In the case where monetary policy abandons past promises at $\bar{t}$, the interest rate rule becomes $\hat{i}_{t+\bar{t}} = (1+\delta)\hat{\pi}_{t+\bar{t}}$. Between period 0 and $\bar{t}-1$, optimal policy is of the form $\hat{i}_t = \delta\hat{\pi}_t$.

The top panels of Figure 3.8 plot the responses of inflation, output interest rates and debt when we assume that initially debt is 30% above steady state and set $\bar{t} = 10$. Between periods 1 and 9 (in the DC regime) inflation turns positive; this reduces the real payout of debt and makes the intertemporal budget solvent. Nominal interest rates rise, since inflation and interest rates are positively correlated in the DC model. The dashed red line shows the case where the planner remembers past commitments in the NDC regime and the solid (blue) line the case where she sets $\mathcal{D}_{\bar{t}} = 0$. To make the effects transparent, we show the behavior of macroeconomic variables over 20 model periods.

[Figure 3.8 About Here]

Notice first that the two scenarios give the same outcome between 0 and $\bar{t}-1$. This is so because the switch in period $\bar{t}$ is fully unanticipated. After period $\bar{t}$, the two models behave quite differently, with the solid blue graphs suggesting that debt, inflation, output and interest rates are all stabilized when the planner reneges on past commitments. In contrast, the dashed red lines show that inflation turns negative, the real value of debt grows and gradually reverts back to the pre switch level and, finally, interest rates drop below their steady state value.

To understand these results, notice that when fiscal policy becomes active, the government's future surpluses increase. Standard arguments imply that then, inflation can turn negative in order to stabilize the intertemporal budget (so that debt does not continue decreasing rapidly). In the case where the policy rule is of the form $\hat{i}_{t+\bar{t}} = (1+\delta)\hat{\pi}_{t+\bar{t}}$, the planner would lower interest rates aggressively in response to deflation. This accomplishes to stabilize prices. Debt continues to be higher than its steady state level, since taxes remain low, when $\rho_\tau > 0$ as is assumed in the Figure.

In contrast in the case monetary policy is of the form (3.53) it resembles a passive money policy. The term $-\frac{\bar{b}_\delta}{1-\beta\delta}\left(-\delta^t\psi_{gov,\bar{t}-1} + \sum_{l=1}^{\infty}\delta^{t+l}\Delta\psi_{gov,\bar{t}-l}\right)$ exceeds 0 and this makes interest rates less responsive to deflation. In equilibrium, prices drop and interest rates turn negative.

The bottom panels of Figure 3.8 perform the above experiment under the calibration $\sigma = \lambda_Y = 1$ and $\lambda_i = 0.5$. The results are essentially the same. The above results suggest that in the case where regime switching is not accompanied by re-optimization, an optimal policy which aims at stabilizing inflation has the opposite impact and instead destabilizes inflation.

Note that these findings are partly driven by the assumption that regime changes are unanticipated one off events: If regime changes were recurrent events, then setting lagged multipliers to zero when there is a switch towards NDC, would introduce an element of lack of commitment that would possibly make policy less effective in stabilizing debt during a DC regime, since the private sector would expect interest rate commitments not to be kept. On the other hand, keeping with past commitments leads to deflation in the NDC regime. This will also affect the planner's ability to reduce debt. The interplay between these forces is left to explore in future work.

In the appendix we extend further this analysis. We firstly derive optimal interest rate rules in the case of recurrent Markov switches across regimes. We then show that how in this model, in which the history of regimes needs to remembered by the planner, the state vector can be condensed so that standard solution techniques are utilized.

3.6 Conclusion

A large literature on optimal monetary and fiscal policies has relied on non-linear models to characterize optimal policies. These models cannot be mapped to DSGE models on monetary/fiscal interactions. We offer a novel framework that brings together these strands of literature and analyze *optimal monetary policy with and without debt.*

In our framework a Ramsey planner (the central bank) sets allocations under commitment to minimize the deviations of inflation, output and interest rates from their respective target levels. When monetary policy exhibits 'debt concerns' then optimization is subject to the consolidated budget constraint; otherwise it is not.

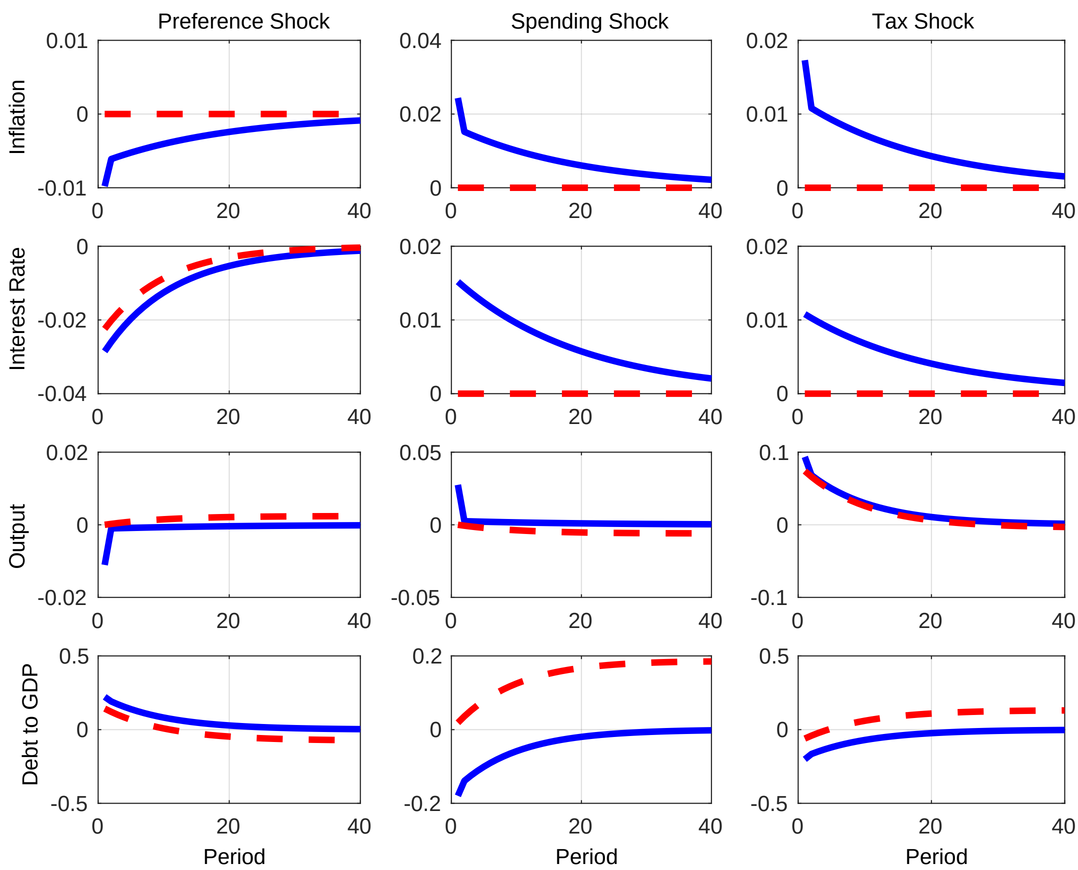
Our model is tractable and admits an analytical solution for interest rate rules. In the case where debt is taken into account in optimization, monetary policy becomes subservient to fiscal policy and resembles a "passive money" policy. Fiscal policy is "active" and taxes do not finance debt. In contrast, under 'no debt concerns', monetary policy is "active" and fiscal policy is 'passive' and ensures debt sustainability.

We analyse the effectiveness of monetary policy in stabilizing the macroeconomy under each of the two setups considered. We find that under debt concerns, policy is much less effective in dealing with shocks and macroeconomic volatility increases. During liquidity trap episodes, the tradeoff facing the planner worsens. These findings continue to hold in a medium scale version of our model that we estimate with US data.

3.7 Appendix

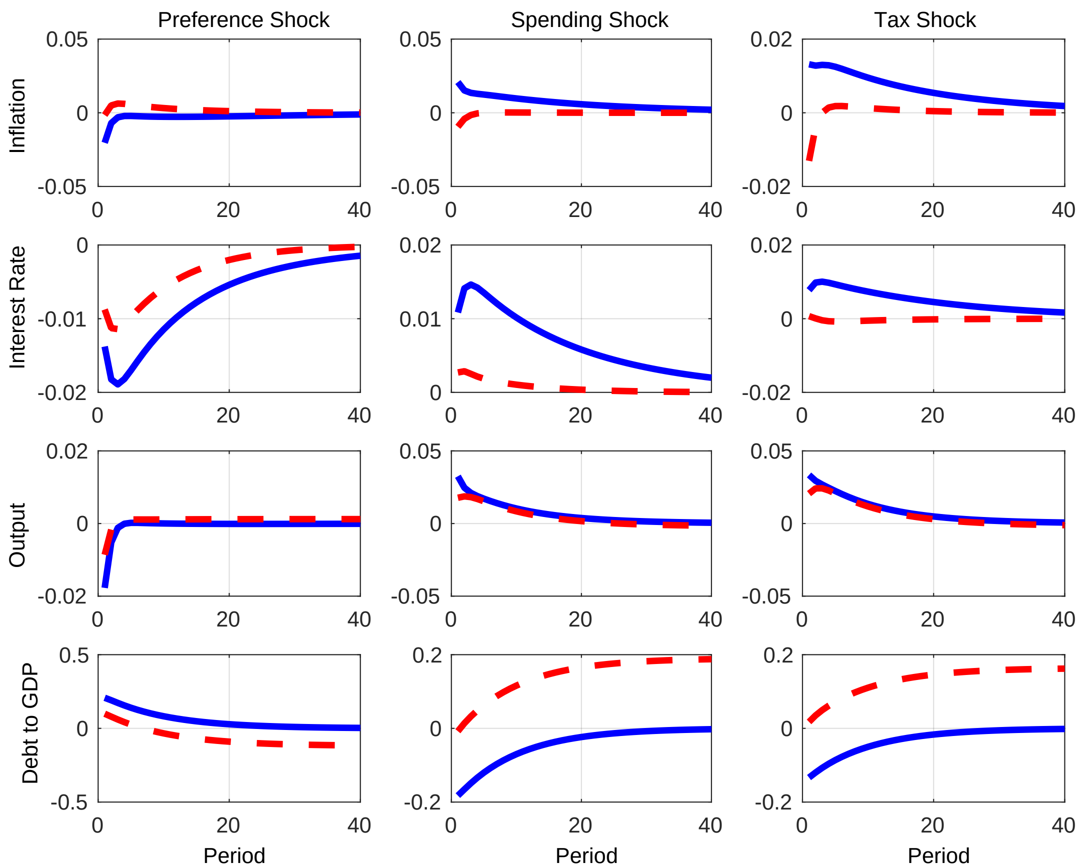
3.7.1 Tables and Figures

FIGURE 3.1: **Optimal Policies away from the ZLB: $\lambda_Y = \lambda_i = 0$.**



Notes: The figure plots the response of model variables to a shock in preferences (left panels), spending (middle panels) and taxes (right panels). The solid (blue) line represents the debt concerns model and the dashed (red) line the no debt concerns model. We set $\lambda_Y = \lambda_i = 0$ in the planner's objective. See Section 3.3.3 for details.

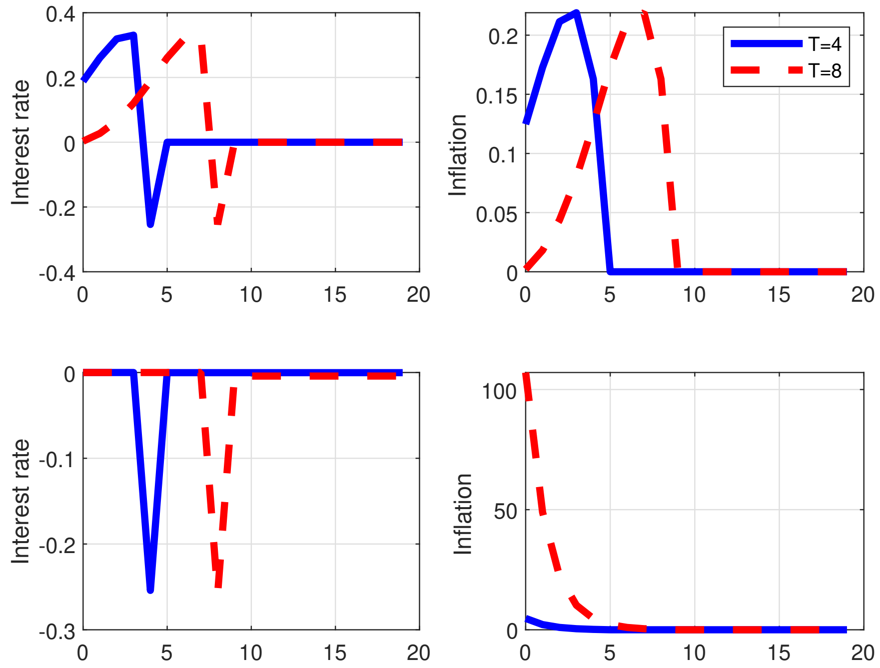
FIGURE 3.2: **Optimal Policies away from the ZLB: $\lambda_Y = \lambda_i = 0.5$, $\sigma = 1$.**



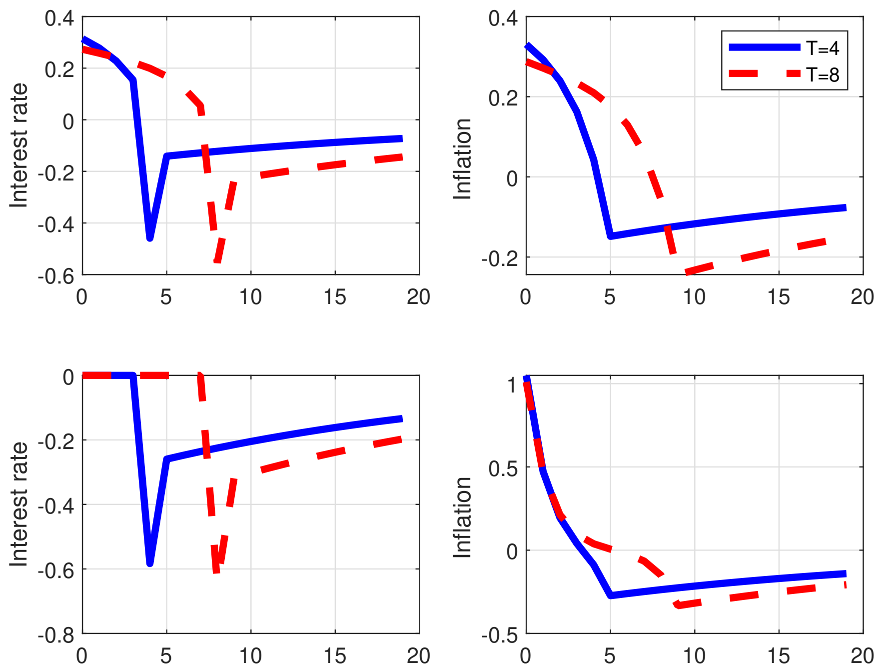
Notes: The figure plots the response of model variables to a shock in preferences (left panel), spending (middle panels) and taxes (right panels). The solid (blue) line represents the debt concerns model and the dashed (red) line the no debt concerns model. We set $\lambda_Y = \lambda_i = 0.5$ in the planner's objective. See Section 3.3.3 for details.

FIGURE 3.3: **Forward Guidance in the model with Taylor rules**

a) Active Money

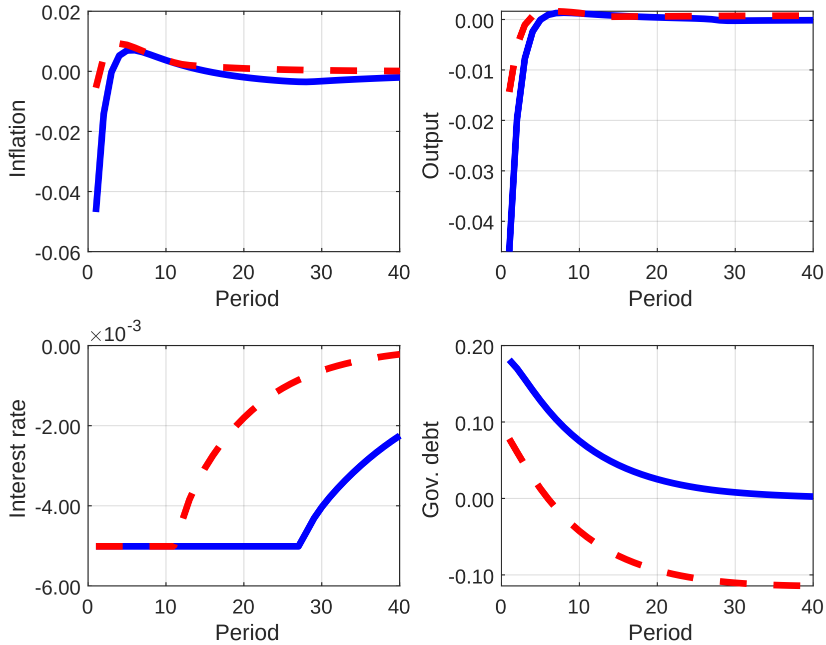


b) Passive Money



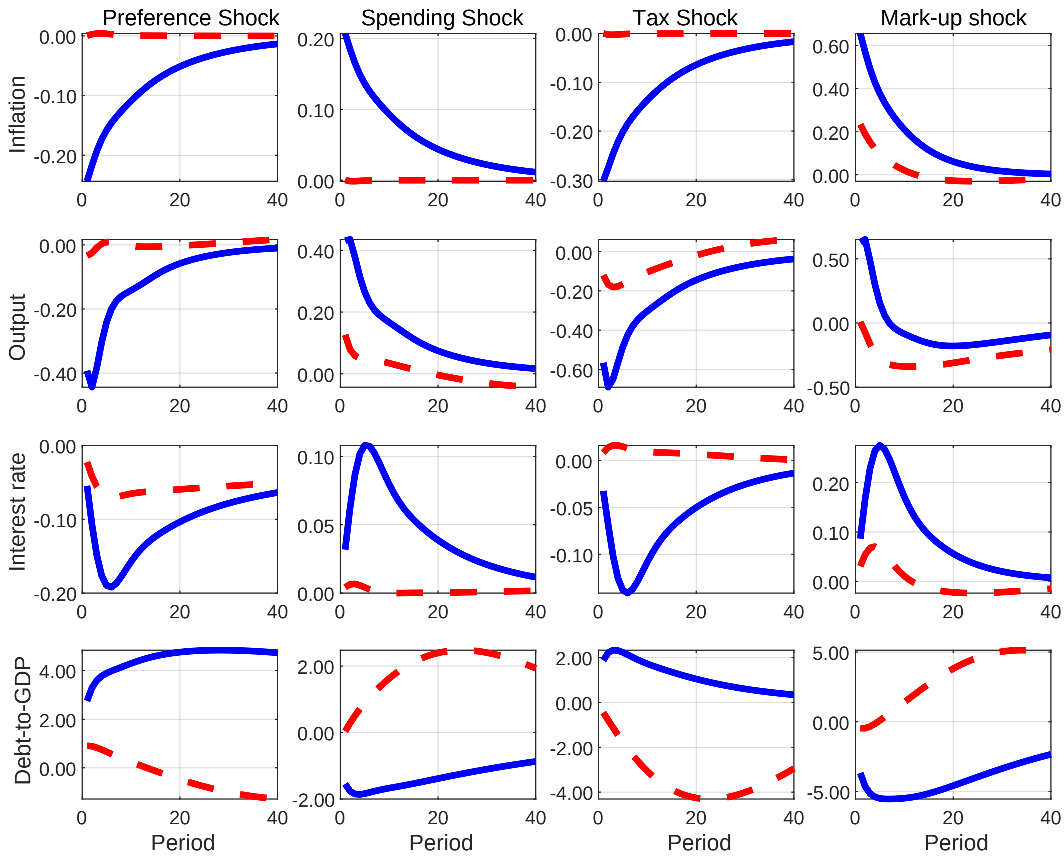
Notes: The figure plots the response of model variables to a FG shock. We assume that monetary policy follows rule (3.37) and $\sigma = 1$. The solid lines assume that the planner commits to lower interest rates in period $\bar{t} = 4$. The dashed lines set $\bar{t} = 8$.
In the bottom graphs of panels a) and b) of the Figure the planner announces that interest rates will be kept at zero between periods 0 and $\bar{t} - 1$ and also announces lower rats in period $\bar{t}$.

FIGURE 3.4: **Optimal Policies at the ZLB.**



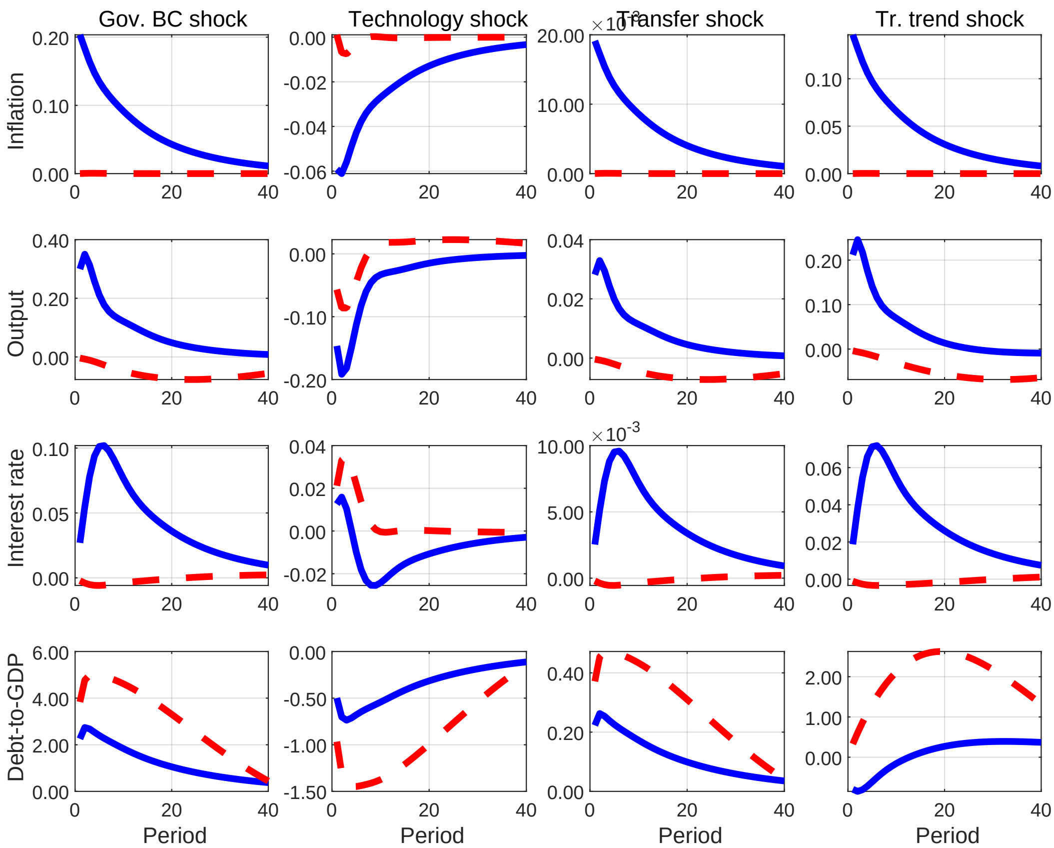
Notes: The figure plots the response of model variables to a preference shock which drives the economy to the LT. The solid (blue) line represents the debt concerns model and the dashed (red) line the no debt concerns model. We set $\lambda_Y = \lambda_i = 0.5$ in the planner's objective.

FIGURE 3.5: **Responses to Shocks: Quantitative Model**



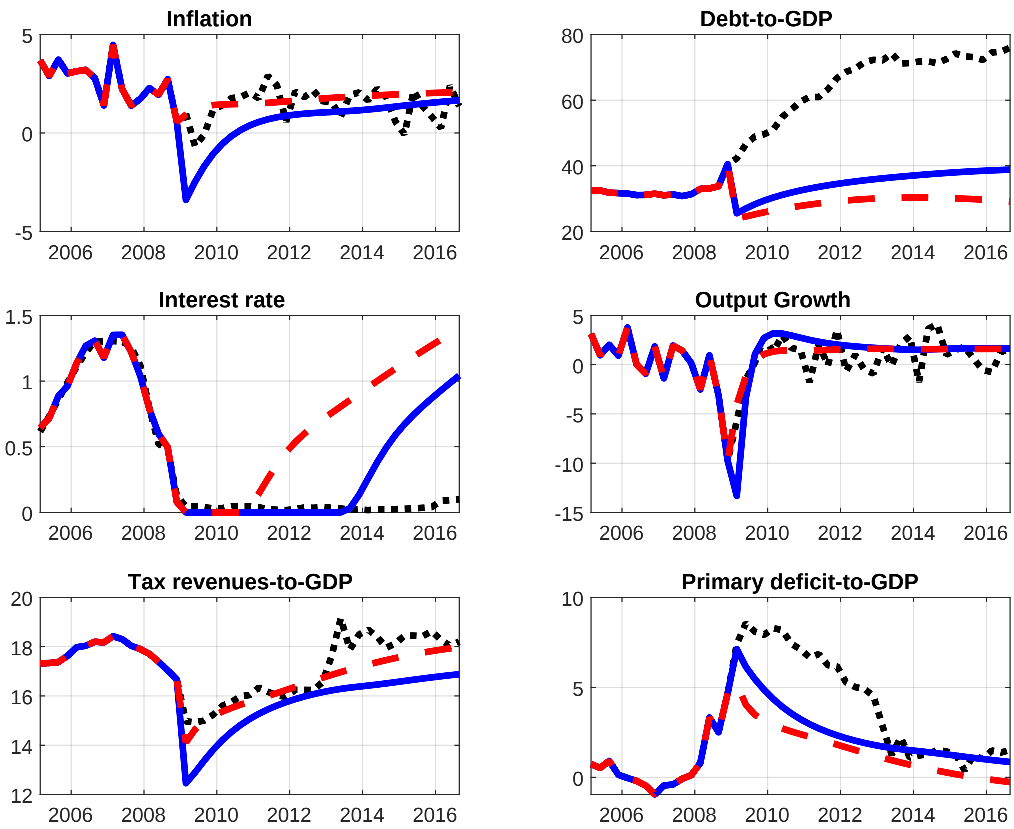
Notes: The figure plots the responses of model variables to economic shocks in the quantitative model of Section 5. To construct the impulse response functions we apply the estimates of the model reported in that section. The size of each shock is 1 standard deviation from its posterior distribution estimate. The left panels show the responses to a preference shock, the middle left panels the responses to a spending shock, the middle right panels the responses to a tax shock, and the right panels the responses to a markup shock. The solid (blue) line represents the debt concerns model and the dashed (red) line the no debt concerns model.

FIGURE 3.6: **Responses to Shocks: Quantitative Model**



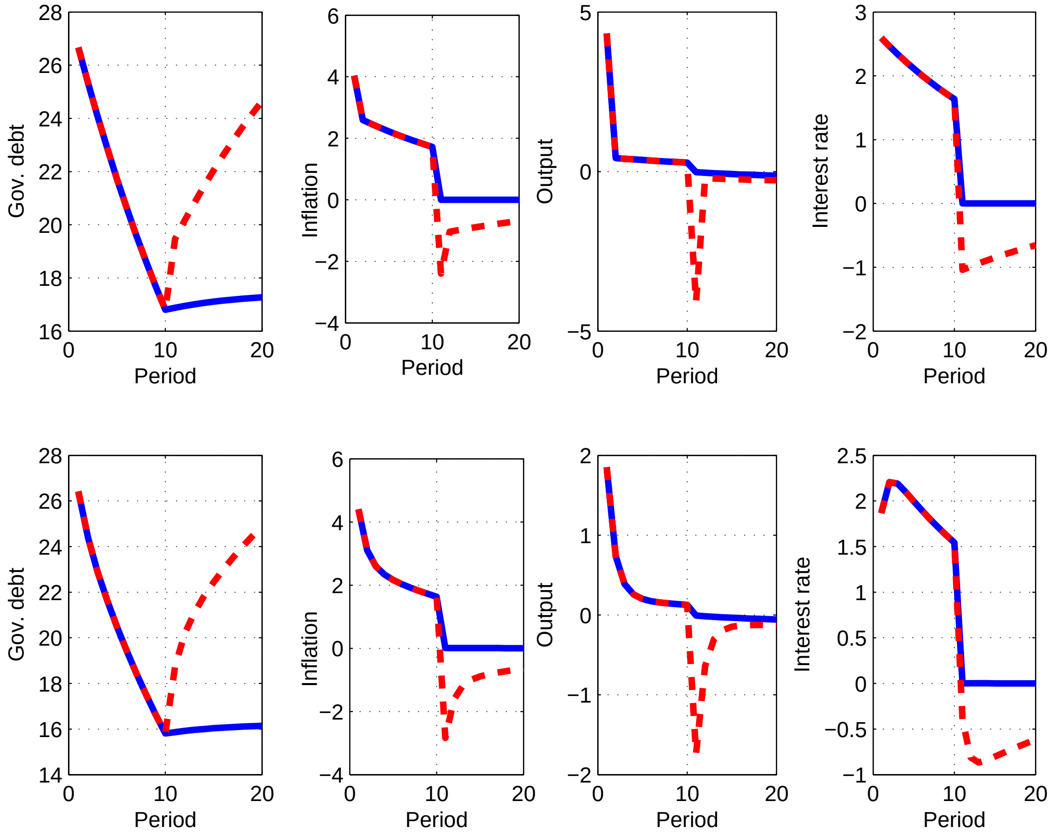
Notes: The figure plots the responses of model variables to economic shocks in the quantitative model of Section 5. To responses are constructed in the same fashion as those shown in Figure 3.5. The left panels show the responses to a shock to the government budget constraint, the middle left panels the responses to a TFP shock, the middle right panels the responses to a transfer shock, and the right panels the responses to the long-run components of transfers. The solid (blue) line represents the debt concerns model and the dashed (red) line the no debt concerns model.

FIGURE 3.7: **Optimal Policy in response to a LT shock.**



Notes: The figure plots forecasts of inflation (top left), market value of debt-to-GDP (top right), interest rates (middle left), output growth (middle right), tax revenues-to-GDP (bottom left) and the primary deficit-to-GDP ratio (bottom right) in the models, in the counter-factual case where all fiscal variables (government debt, government spending, taxes and transfers) are set to their steady-state value at the end of period 2009Q1. The solid (blue) line represents the debt concerns model. The dashed (red) line is the no debt concerns model. The data is the same as in Figure 3.7.

FIGURE 3.8: **Switching from DC to NDC**



Notes: The Figure plots the behavior of macroeconomic variables, when there is an unanticipated change from DC to NDC. The top panels simulate the economy when $\sigma = \lambda_Y = 0$. The bottom panels assume $\sigma = \lambda_Y = 1$. The solid lines represent the case where the planner reneges on promises $\mathcal{D}$ when the shift to the NDC regime occurs. The dashed lines assume the planner 'remembers' $\mathcal{D}$ after the switch.

TABLE 3.1: **Model Implied Taylor Rules**

Parameter	**Coefficients of Taylor Rule**			
	$\sigma = 1$		$\sigma = 2$	
	$\lambda_i = 0.5$	$\lambda_i = 1$	$\lambda_i = 0.5$	$\lambda_i = 1$
$\widetilde{\phi}_\pi$	0.104	0.070	0.080	0.055
	(0.537)	(0.268)	(0.268)	(0.134)
$\widetilde{\phi}_i$	0.870	0.905	0.897	0.923
	(1.270)	(1.270)	(1.135)	(1.135)
$\widetilde{\phi}_{\Delta i}$	0.509	0.528	0.585	0.599
	(1.005)	(1.005)	(1.005)	(1.005)
$\widetilde{\phi}_\pi$	0.271	0.209	-0.039	-0.024
	(0.537)	(0.268)	(0.268)	(0.134)
$\widetilde{\phi}_Y$	0.052	0.003	0.2305	0.156
	(0.800)	(0.800)	(0.200)	(0.200)
$\widetilde{\phi}_i$	0.629	0.750	0.705	0.765
	(1.270)	(1.270)	(1.135)	(1.135)
$\widetilde{\phi}_{\Delta i}$	0.0904	0.252	-0.272	-0.232
	(1.005)	(1.005)	(1.005)	(1.005)

Notes: The table reports model implied Taylor rules $\hat{i}_t = \widetilde{\phi}_\pi \hat{\pi}_t + \widetilde{\phi}_Y \Delta \hat{Y}_t + \widetilde{\phi}_i \Delta \hat{i}_{t-1} + \widetilde{\phi}_{\Delta i} \Delta \hat{i}_{t-1}$ under debt concerns. (See Section 3.3). The top panel assumes $\lambda_Y = 0$ the bottom panel sets $\lambda_Y = 0.5$. Each of the columns reports the values of parameters $(\widetilde{\phi}_\pi, \widetilde{\phi}_Y, \widetilde{\phi}_i, \widetilde{\phi}_{\Delta i})$ under alternative calibrations of λ_i and σ. The numbers in parentheses report the values we obtain from the analytical solution of the model (object $\mathcal{T}$).

Table 3.2: **Calibration**

Parameters Common Across Models		
Parameter	Value	Label
β	0.995	Discount factor
λ_Y	$\{0, 0.5\}$	Loss function - weight on output
λ_i	$\{0, 0.5\}$	Loss function - weight on interest rate
θ	17.5	Price Stickiness
η	-6.88	Elasticity of Demand
σ	$\{0, 1\}$	Inverse of IES
γ_h	1	Inverse of Frisch Elasticity
ρ_τ	0.9	Persistence of Taxes
ρ_ξ	0.9	Persistence of ξ
ρ_G	0.9	Persistence of Spending
$\bar{b}_\delta$	0.1321	Debt Level
$\overline{\tau}$	0.2545	Tax Rate
$\overline{Y}$	1	Output
$\overline{G}$	0.1	Spending

Parameters Not Common Across Models		
Parameter	Value	Label
$\phi_{\tau,b}$	0.00 0.07	Tax rule coefficients

Notes: The table reports the values of model parameters assumed in the numerical experiments in Section 3. β notes the discount factor chosen to target a steady state real interest rate of 2 percent. λ_Y and λ_i are the weights on output and interest rates in the objective of the planner. Parameter η is calibrated to target markups of 17 percent in steady state. θ is calibrated as in Schmitt-Grohé and Uribe (2004). Finally, the steady state level of debt is assumed equal to 60 percent of GDP (at annual horizon), and the level of public spending is 10 percent of aggregate output which is normalized to unity in steady state.
The bottom panel of the Table reports the value of the coefficient $\phi_{\tau,b}$ in the tax policy rule (3.1). As discussed in text we set $\phi_{\tau,b} = 0.07$ in the no debt concerns model to have a determinate equilibrium. In the debt concerns case we set $\phi_{\tau,b} = 0.00$ to find a unique equilibrium. See text for further details.

3.7.2 The small-scale New-Keynesian model: Further Derivations and a Markov Switching model

This section contains the proofs of Propositions 1, 2 and 5 and additional derivations in the small-scale New-Keynesian model of Sections 2-4 of the paper. It also extends the analysis of Section 6.4 (Markov switching model).

Proofs of Propositions and Further Derivations

Proof of Proposition 1.

From the first order conditions of the planner's program (equations (8) to (12) in text) we have:

$$\kappa_1 \Delta\psi_{\pi,t} = -\lambda_Y \Delta\hat{Y}_t + \sigma\frac{\overline{Y}}{\overline{\overline{C}}}\left(\lambda_i \Delta\hat{i}_t + \Delta\psi_{ZLB,t} - \frac{\lambda_i \Delta\hat{i}_{t-1} + \Delta\psi_{ZLB,t-1}}{\beta}\right)$$
$$+\sigma\frac{\overline{Y}}{\overline{\overline{C}}}\bar{b}_\delta \sum_{l=0}^{\infty} \delta^l(\Delta\psi_{gov,t-l} - \Delta\psi_{gov,t-l-1}) + \omega_Y \Delta\psi_{gov,t}$$

and so equation (8) in text becomes

$$-\hat{\pi}_t + \frac{1}{\kappa_1}\left[-\lambda_Y \Delta\hat{Y}_t + \sigma\frac{\overline{Y}}{\overline{\overline{C}}}\left(\lambda_i \Delta\hat{i}_t + \Delta\psi_{ZLB,t} - \frac{\lambda_i \Delta\hat{i}_{t-1} + \Delta\psi_{ZLB,t-1}}{\beta}\right)\right.$$
$$\left.+\sigma\frac{\overline{Y}}{\overline{\overline{C}}}\bar{b}_\delta \sum_{l=0}^{\infty} \delta^l(\Delta\psi_{gov,t-l} - \Delta\psi_{gov,t-l-1}) + \omega_Y \Delta\psi_{gov,t}\right] - \frac{\lambda_i \hat{i}_{t-1} + \psi_{ZLB,t-1}}{\beta} + \frac{\bar{b}_\delta}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l \Delta\psi_{gov,t-l} = 0 \tag{3.54}$$

Rearranging (3.54) we get:

$$\frac{\sigma\lambda_i}{\kappa_1}\frac{\overline{Y}}{\overline{\overline{C}}}\hat{i}_t = \hat{\pi}_t + \frac{\lambda_Y \Delta\hat{Y}_t}{\kappa_1} + \left(\frac{\sigma\lambda_i}{\kappa_1}\frac{\overline{Y}}{\overline{\overline{C}}} + \frac{\lambda_i}{\beta}\right)\hat{i}_{t-1} + \frac{\sigma\lambda_i \frac{\overline{Y}}{\overline{\overline{C}}}\Delta\hat{i}_{t-1}}{\kappa_1\beta} - \frac{\overline{Y}}{\overline{\overline{C}}}\frac{\sigma}{\kappa_1}\psi_{ZLB,t} + \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{\overline{C}}}\left(1 + \frac{1}{\beta} + \frac{\overline{C}}{\overline{\overline{Y}}}\frac{\kappa_1}{\beta\sigma}\right)\psi_{ZLB,t-1}$$
$$-\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{\overline{C}}}\bar{b}_\delta \sum_{l=0}^{\infty}\delta^l(\Delta\psi_{gov,t-l} - \Delta\psi_{gov,t-l-1}) + \frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t} + \frac{\bar{b}_\delta}{(1-\beta\delta)}\sum_{l=0}^{\infty}\delta^l \Delta\psi_{gov,t-l} \tag{3.55}$$

We have two cases. 1. $\psi_{ZLB,t} > 0$ so that the ZLB binds. In this case it follows trivially that $\hat{i}_t = -i^*$. 2. $\psi_{ZLB,t} = 0$. In this case rearranging (3.55) we can easily get (13) in Proposition 1.■

Proof of Proposition 2.

The proof is trivial: Following the steps in the proof of Proposition 1 and setting $\psi_{gov,t} = 0$ for all t we can arrive to the optimal policy in the no debt concerns model. ■

Proof of Proposition 5.

In the case of lump sum taxes we can write the budget constraint as:

$$\beta \bar{b}_\delta \hat{b}_{t,\delta} - \beta \bar{b}_\delta E_t \hat{\pi}_{t+1} + \bar{b}_\delta (1-\beta)\hat{\tau}_t = \bar{b}_\delta \hat{b}_{t-1,\delta} - \bar{b}_\delta \hat{\pi}_t \tag{3.56}$$

Using the tax rule, letting $\rho_\tau = 0$ and rearranging we have:

$$\hat{b}_{t,\delta} - E_t \hat{\pi}_{t+1} + = \frac{[1-(1-\beta)\phi^R \tau, b]}{\beta} \hat{b}_{t-1,\delta} - \frac{1}{\beta}\hat{\pi}_t - \frac{(1-\beta)}{\beta}\epsilon_{\tau,t} \tag{3.57}$$

Following the argument of subsection 3.1.1 we have $\hat{\pi}_t = \bar{b}_\delta \Delta\psi_{gov,t}$. With the martingale property we can write the budget constraint as:

$$\hat{b}_{t,\delta} + = \frac{[1-(1-\beta)\phi^R \tau, b]}{\beta} \hat{b}_{t-1,\delta} - \frac{\bar{b}_\delta \Delta\psi_{gov,t}}{\beta} - \frac{(1-\beta)}{\beta}\epsilon_{\tau,t} \tag{3.58}$$

Clearly, $\frac{[1-(1-\beta)\phi^R \tau, b]}{\beta} > 1$ in the DC model so that the equilibrium is unique. Under NDC $\psi_{gov,t} = 0$ and $\frac{[1-(1-\beta)\phi^R \tau, b]}{\beta} < 1$ is required for a unique equilibrium. ■

Analytical expressions for impulse responses shown in Section 3.

We derive analytically coefficients υ_ξ, υ_τ and υ_G in Section 3- 'The effects of shocks'. These coefficients govern the impulse responses to shocks in preferences, taxes and spending in the debt concerns model.

We assume $\lambda_Y = \lambda_i = \sigma = 0$ and focusing on the case where the ZLB does not bind, the first order conditions become:

$$-\hat{\pi}_t + \Delta\psi_{\pi,t} + \frac{\bar{b}_\delta}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l \Delta\psi_{gov,t-l} = 0$$
$$-\psi_{\pi,t}\kappa_1 + \omega_Y \psi_{gov,t} = 0$$

Rearranging we get

$$\hat{\pi}_t = \frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t} + \frac{\bar{b}_\delta}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l \Delta\psi_{gov,t-l} \tag{3.59}$$

the optimal inflation path in this model.

Under perfect foresight (from period 0 onwards) we can drop conditional expectations and the multiplier ψ_{gov} satisfies $\psi_{gov,t} = \psi_{gov,t+1}$. Therefore, $\Delta\psi_{gov,t} = 0$ for $t \geq 1$ and $\Delta\psi_{gov,0} \neq 0$ since a shock hits in period 0. Therefore, (3.59) becomes:

$$\hat{\pi}_t = \frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,0}\mathcal{I}_{t=0} + \frac{\overline{b}_\delta}{1-\beta\delta}\delta^t\Delta\psi_{gov,0} \tag{3.60}$$

as is claimed in text. Moreover, from the Euler equation we have

$$\hat{i}_t = \hat{\pi}_{t+1} - \hat{\xi}_{t+1} + \hat{\xi}_t = \frac{\overline{b}_\delta}{1-\beta\delta}\delta^{t+1}\Delta\psi_{gov,0} - \rho_\xi^t(\rho_\xi - 1)\hat{\xi}_0$$

From the Phillips curve we obtain the following expression for $\hat{Y}_t$:

$$\hat{Y}_t = \frac{1}{\kappa_1}\left[\hat{\pi}_t - \beta\hat{\pi}_{t+1} - \kappa_2\hat{\tau}_t + \kappa_3\hat{G}_t\right] \tag{3.61}$$

which, after noting that taxes do not respond to lagged debt in this version of the model and $\kappa_3 = 0$, becomes:

$$\hat{Y}_t = \frac{1}{\kappa_1}\left[\frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,0}\mathcal{I}_{t=0} + \overline{b}_\delta\delta^t\Delta\psi_{gov,0} - \kappa_2\rho_\tau^t\hat{\tau}_0.\right] \tag{3.62}$$

Now let us turn to the intertemporal budget constraint to find an analytical expression for $\Delta\psi_{gov,0}$ as a function of the shocks. Notice that since there is perfect certainty after period 0, the date 0 intertemporal constraint (equation (18) in text) is sufficient for the equilibrium (see FMOS). Given the parameterization of the model and assuming for simplicity $\hat{b}_{-1,\delta} = 0$ we have

$$\sum_{t=0}^{\infty}\beta^t\left[-\overline{G}\left(\hat{G}_t + \hat{\xi}_t\right) + \frac{\overline{\tau}\overline{Y}(1+\eta)}{\eta}\left((1+\gamma_h)\hat{Y}_t + \frac{\hat{\tau}_t}{1-\overline{\tau}} + \hat{\xi}_t\right)\right] = \overline{b}_\delta\sum_{t=0}^{\infty}\beta^t\delta^t E_t\left[\hat{\xi}_t - \sum_{l=0}^{t}\hat{\pi}_l\right] \tag{3.63}$$

Substituting out the expressions for output and inflation, the LHS of the above equation becomes:

$$-\hat{G}_0\overline{G}\sum_{t=0}^{\infty}\beta^t\rho_G^t + \hat{\tau}_0\frac{\overline{\tau}\overline{Y}(1+\eta)}{\eta}\left(\frac{1}{1-\overline{\tau}} - (1+\gamma_h)\frac{\kappa_2}{\kappa_1}\right)\sum_{t=0}^{\infty}\beta^t\rho_\tau^t$$
$$+\frac{\overline{\tau}\overline{Y}(1+\eta)}{\eta}\frac{1}{\kappa_1}\sum_{t=0}^{\infty}\beta^t\left[\frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,0}\mathcal{I}_{t=0} + \overline{b}_\delta\delta^t\Delta\psi_{gov,0}\right] + \left(-\overline{G} + \frac{\overline{\tau}\overline{Y}(1+\eta)}{\eta}\right)\sum_{t=0}^{\infty}\beta^t\rho_\xi^t\hat{\xi}_0 =$$
$$\overline{b}_\delta\sum_{t=0}^{\infty}\beta^t\delta^t\rho_\xi^t\hat{\xi}_0 - \overline{b}_\delta\sum_{t=0}^{\infty}\beta^t\delta^t\left(\frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,0}\mathcal{I}_{t=0} + \sum_{l=0}^{t}\frac{\overline{b}_\delta}{1-\beta\delta}\delta^l\Delta\psi_{gov,0}\right)$$

Rearranging all $\Delta\psi_{gov,0}$ terms to the LHS of the above equation we get:

$$\underbrace{\left[\frac{\overline{\tau}\overline{Y}(1+\eta)}{\eta}\frac{1}{\kappa_1}\left(\frac{\omega_Y}{\kappa_1}\sum_{t=0}^{\infty}\beta^t+\overline{b}_\delta\delta^t\right)+\overline{b}_\delta\sum_{t=0}^{\infty}\beta^t\delta^t\left(\frac{\omega_Y}{\kappa_1}+\sum_{l=0}^{t}\frac{\overline{b}_\delta}{1-\beta\delta}\delta^l\right)\right]}_{\upsilon_{gov}}\Delta\psi_{gov,0}$$

on the LHS where $\upsilon_{gov}>0$. On the RHS we have:

$$\underbrace{\frac{\overline{G}}{1-\beta\rho_G}}_{\upsilon_G}\hat{G}_0\underbrace{-\frac{1}{1-\beta\rho_\tau}\frac{\overline{\tau}\overline{Y}(1+\eta)}{\eta}\left(\frac{1}{1-\overline{\tau}}-(1+\gamma_h)\frac{\kappa_2}{\kappa_1}\right)}_{\upsilon_\tau}\hat{\tau}_0\underbrace{+\overline{b}_\delta\frac{\beta(1-\rho_\xi)(1-\delta)}{(1-\beta\rho_\xi)(1-\beta\delta)(1-\beta\rho_\xi\delta)}}_{\upsilon_\xi}\hat{\xi}_0$$

The coefficient υ_ξ is positive and becomes zero when $\delta=1$. Coefficient υ_G is positive.

Further Derivations of the analytical model in Section 3.

We now derive the expressions shown in section 3.2, in the case where the planner internalizes the tax schedule. From the FONC of the planner we have

$$\frac{\beta\overline{b}_\delta}{1-\beta\delta}\left(\psi_{gov,t}-E_t\psi_{gov,t+1}\right)-\beta\phi^{DC}_{\tau,b}(1-\rho_\tau)E_t\left(\sum_{j=0}^{\infty}(\rho_\tau\beta)^j(\kappa_2\psi_{\pi,t+1+j}-\frac{\overline{\tau}\overline{Y}(1+\eta)}{\eta(1-\overline{\tau})}\psi_{gov,t+1+j})\right)=0 \tag{3.64}$$

Assume that $\sigma=\lambda_Y=\delta=\rho_\tau=\lambda_i=0$. From the planner's FONC we get $\psi_{\pi,t}=\frac{\omega_Y}{\kappa_1}\psi_{gov,t}$. With this we can write:

$$\beta\overline{b}_\delta\left(\psi_{gov,t}-E_t\psi_{gov,t+1}\right)-\beta\phi^{DC}_{\tau,b}E_t\left(\kappa_2\frac{\omega_Y}{\kappa_1}-\frac{\overline{\tau}\overline{Y}(1+\eta)}{\eta(1-\overline{\tau})}\right)\psi_{gov,t+1}=0 \tag{3.65}$$

Since $\kappa_2\frac{\omega_Y}{\kappa_1}=-\frac{(1+\eta)}{\theta(1-\overline{\tau}}\overline{Y}\overline{\tau}\frac{\overline{\tau}\frac{(1+\eta)}{\eta}\overline{Y}(1+\gamma_h)}{-\frac{(1+\eta)}{\theta}\overline{Y}\gamma_h}=\frac{(1+\eta)}{\eta}\overline{Y}\frac{\overline{\tau}^2}{1-\overline{\tau}}\frac{(1+\gamma_h)}{\gamma_h}$ what is inside the parenthesis in the second term in (3.65) can be written as:

$$\left(\kappa_2\frac{\omega_Y}{\kappa_1}-\frac{\overline{\tau}\overline{Y}(1+\eta)}{\eta(1-\overline{\tau})}\right)=\frac{(1+\eta)}{\eta}\overline{Y}\overline{\tau}\left(\frac{\overline{\tau}}{1-\overline{\tau}}\frac{(1+\gamma_h)}{\gamma_h}-\frac{1}{1-\overline{\tau}}\right) \tag{3.66}$$

Noting that when $\overline{G}=0$, it holds that $\frac{(1+\eta)}{\eta}\overline{Y}\overline{\tau}=\overline{S}=(1-\beta)\overline{b}_\delta$ and also that $\left(\frac{\overline{\tau}}{1-\overline{\tau}}\frac{(1+\gamma_h)}{\gamma_h}-\frac{1}{1-\overline{\tau}}\right)=\left(\frac{\overline{\tau}}{\gamma_h(1-\overline{\tau})}-1\right)$ we obtain the result derived in text.

Derivations of the analytical model in Section 4.

We now derive the expressions contained in Section 4 (Optimal policies in the liquidity trap).

Under the assumptions made in text about parameters $\sigma, \lambda_Y, \lambda_i$ and the timing of the shock to preferences, it straightforward to show that the FONC in the DC model become:

$$-\hat{\pi}_t + \frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,0}\mathcal{I}_{t=0} + \frac{\overline{b}_\delta}{1-\beta\delta}\delta^t\Delta\psi_{gov,0} - \frac{\lambda_i\hat{i}_{t-1}}{\beta} + \frac{\psi_{ZLB,t-1}}{\beta} = 0 \tag{3.67}$$

and

$$-\hat{\pi}_t^{NDC} - \frac{\lambda_i\hat{i}_{t-1}^{NDC}}{\beta} + \frac{\psi_{ZLB,t-1}^{NDC}}{\beta} = 0 \tag{3.68}$$

in the no debt concerns model.

Focus first on the NDC equilibrium. According to (3.68) $\hat{\pi}_0^{NDC} = 0$ clearly. Moreover, since the shock is large we have: $\hat{i}_0^{NDC} = \hat{\pi}_1^{NDC} - \hat{\xi}_0 = -\hat{i}^*$. This gives the value of $\hat{\pi}_1^{NDC}$. From $t = 1$ onwards, lets guess (and then verify) that $\hat{i}_t^{NDC} = \hat{\pi}_{t+1}^{NDC} > -\hat{i}^*$. From (3.68) we have $-\hat{\pi}_{t+1}^{NDC} - \frac{\lambda_i\hat{\pi}_{t+1}^{NDC}}{\beta} = 0$ and so inflation equals zero from period 2 onwards. Clearly this gives $\hat{i}_t^{NDC} = 0$ for $t \geq 0$ consistent with the guess.

Now consider the debt concerns model. From (3.67) we have: $\hat{\pi}_0 = \frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,0}\mathcal{I}_{t=0} + \frac{\overline{b}_\delta}{1-\beta\delta}\Delta\psi_{gov,0}$. Moreover, from the binding ZLB $\hat{i}_0 = -\hat{i}^*$, we have $\hat{\pi}_1 = -\hat{i}^* - \hat{\xi}_0$. For $t > 1$ it must be $\hat{\pi}_t = -\hat{i}^*$ if $\hat{i}_{t-1} = -\hat{i}^*$, however,

$$-\hat{\pi}_t + \frac{\overline{b}_\delta}{1-\beta\delta}\delta^t\Delta\psi_{gov,0} - \frac{\lambda_i\hat{\pi}_t}{\beta} = 0$$

if $\hat{i}_{t-1} > -\hat{i}^*$. Setting $\lambda_i = 0$ gives equation (39) in text. Given $\hat{i}_t = \hat{\pi}_{t+1}$ it is simple to obtain the sequence of interest rates. Finally, from $\mathcal{Z}_t = \hat{i}_t - \widetilde{\phi}_\pi\hat{\pi}_t$ when $\hat{i}_{t-1} = -\hat{i}^*$ it is simple to derive the path of $\mathcal{Z}_t$ shown in (41).

We also showed in Section 4 that given the duration of FG $\overline{t}$ the optimal inflation in period 0 is:

$$\hat{\pi}_0 = \frac{1}{\tilde{\lambda}}\left[\left(\overline{b}_\delta - \overline{S}\right)\hat{\xi}_0 + \frac{\beta\delta\overline{b}_\delta}{1-\beta\delta}\left(i^* + \hat{\xi}_0\right) + i^*\frac{(\beta\delta)^2\overline{b}_\delta}{(1-\beta\delta)^2}\left(1-(\beta\delta)^{\overline{t}-1}\right) + i^*\frac{(\beta\delta)^{\overline{t}+1}\overline{b}_\delta\delta}{(1-\beta\delta)(1-\beta\delta^2)}\right] \tag{3.69}$$

To derive this expression we first trace the path of inflation in periods 1,2,... We have:

$$\hat{\pi}_1 = -i^* - \hat{\xi}_0$$
$$\hat{\pi}_t = -i^*, \quad t = 2,3,...,\overline{t}$$
$$\hat{\pi}_t = -i^*\delta^{t-\overline{t}}, \quad t = \overline{t},\overline{t}+1,...$$

The intertemporal constraint of the government in period 0 can then be written as:

$$\sum_{t\geq 0}\beta^t\left(\overline{R}(1+\gamma_h)\hat{Y}_t+\overline{S}\hat{\xi}_t\right)=-\bar{b}_\delta\sum_{t\geq 0}(\beta\delta)^t\sum_{l=0}^{t}\hat{\pi}_t+\bar{b}_\delta\hat{\xi}_0$$

where $\overline{R}$ is the steady state revenue of the government. Using the Phillips curve the first term on the LHS can be written as: $\sum_{t\geq 0}\beta^t\overline{R}(1+\gamma_h)Y_t=\hat{\pi}_0\overline{R}\frac{(1+\gamma_h)}{\kappa_1}$. The second term is simply $\overline{S}\hat{\xi}_0$.

The first term on the RHS can be written as:

$$-\bar{b}_\delta\sum_{t\geq 0}(\beta\delta)^t\sum_{l=0}^{t}\hat{\pi}_t=-\frac{\bar{b}_\delta}{1-\beta\delta}\hat{\pi}_0-\frac{\bar{b}_\delta\beta\delta}{1-\beta\delta}\hat{\pi}_1-\bar{b}_\delta\sum_{t=2}^{\infty}(\beta\delta)^t\sum_{l=2}^{\min\{t,\bar{t}\}}\hat{\pi}_t-\bar{b}_\delta\sum_{t\geq\bar{t}+1}(\beta\delta)^t\sum_{l=\bar{t}+1}^{t}\hat{\pi}_t$$

and making use of the path of inflation we can write this as:

$$-\frac{\bar{b}_\delta}{1-\beta\delta}\hat{\pi}_0+\frac{\bar{b}_\delta\beta\delta}{1-\beta\delta}(i^*+\hat{\xi}_0)+i^*\frac{(\beta\delta)^2\bar{b}_\delta}{(1-\beta\delta)^2}\left(1-(\beta\delta)^{\bar{t}-1}\right)+\underbrace{i^*\bar{b}_\delta\sum_{t\geq\bar{t}+1}(\beta\delta)^t\sum_{l=\bar{t}+1}^{t}\delta^{l-\bar{t}}}_{=i^*\frac{(\beta\delta)^{\bar{t}+1}\bar{b}_\delta\delta}{(1-\beta\delta)(1-\beta\delta^2)}}$$

Rearranging these expressions we get the result.

The small-scale New-Keynesian model

We now derive the Lagrangian functions used in Section 2 of the paper. We also (for completeness) present the nonlinear equations for the simplistic NK model used in Section 2. As stated in text this model is a special case of the quantitative model of Section 5.

Households Households maximize

$$E_0\sum_{t=0}^{\infty}\beta^t\xi_t\left(\frac{C_t^{1-\sigma}}{1-\sigma}-\chi\frac{h_t^{1+\gamma_h}}{1+\gamma_h}\right)$$

subject to

$$P_tC_t+P_{t,s}B_{t,S}+P_{t,L}B_{t,\delta}\leqslant(1-\tau_t)W_th_t+P_tD_t+B_{t-1,S}+(1+\delta P_{t,L})B_{t-1,\delta}$$

where C_t denotes consumption and h_t denotes hours worked. D_t represents firms' profits redistributed to households, and P_t denotes the aggregate price level. $B_{t,\delta}$ is a long-term government bond, a perpetuity with coupon payments decaying at the rate $0\leqslant\delta<1$

and price $P_{t,L}$. $B_{t,s}$ denotes short-term bonds with price is $P_{t,S}$. We assume that short debt is in zero net supply. ξ_t is the preference shock.

The first order conditions of the household's problem are:

$$\begin{aligned}
P_{t,s}\xi_t C_t^{-\sigma} &= \beta E_t \xi_{t+1} \frac{C_{t+1}^{-\sigma}}{\pi_{t+1}} \\
P_{t,L}\xi_t C_t^{-\sigma} &= \beta E_t \xi_{t+1} \frac{C_{t+1}^{-\sigma}}{\pi_{t+1}}(1+\delta P_{t+1,L}) \\
h_t^{\gamma_h} C_t^{\sigma} &= (1-\tau_t)\frac{W_t}{P_t}
\end{aligned}$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$ is the gross inflation rate.

Firms The problem solved by firms is described in more detail in Appendix 3.7.3, which is dedicated to the estimated model. As discussed in text we assume that firms are monopolistically competitive and maximize profits subject to price adjustment costs as in Rotemberg (1982). The firms' technology is linear in labour and thus $Y_t(j) = h_t(j)$ where $j \in [0,1]$ denotes the generic firm. Optimal pricing gives rise to the following (non-linear) New-Keynesian Phillips curve:

$$\theta(\pi_t - \pi)\pi_t = 1 + \eta(1 - \frac{W_t}{P_t}) + \beta\theta E_t \frac{C_t^{\sigma}}{C_{t+1}^{\sigma}} \frac{Y_{t+1}}{Y_t}(\pi_{t+1} - \pi)\pi_{t+1} \tag{3.70}$$

η is a parameter which governments the elasticity of substitution across intermediate products. θ governs the price adjustment costs faced by firms.

Fiscal policy Government spending G_t evolves exogenously according to an AR(1) process in logs:

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \epsilon_{g,t}$$

where $\hat{g}_t \equiv \log G_t - \log \overline{G}$.

The (distortionary) labor tax is set by the fiscal authority according to the simple rule described in text (in log-linear form). The flow government budget constraint can be written as:

$$P_{t,L} b_{t,\delta} = (1+\delta P_{t,L})\frac{b_{t-1,\delta}}{\pi_t} + G_t - \tau_t h_t w_t \tag{3.71}$$

where $b_{t,\delta} \equiv \frac{B_{t,\delta}}{P_t}$ denotes real long-term government debt.

Log-linear model Making use of the labor supply condition $h_t^{\gamma_h} C_t^{\sigma} = (1-\tau_t)\frac{W_t}{P_t}$, as well as the resource constraint $h_t = Y_t = C_t + G_t$ to dispense with W_t, C_t and h_t, we get the following linear New Keynesian Phillips Curve:

$$\hat{\pi}_t = \kappa_1 \hat{Y}_t + \kappa_2 \hat{\tau}_t - \kappa_3 \hat{G}_t + \beta E_t \hat{\pi}_{t+1} \tag{3.72}$$

where κ_1, κ_2 and κ_3 are defined in text.

Defining $i_t \equiv -\log P_{t,S}$, log-linearizing the Euler equation for short bonds (and again making use of the resource constraint $Y_t = C_t + G_t$) we get:

$$\hat{i}_t = E_t\left(\hat{\pi}_{t+1} - \hat{\xi}_{t+1} + \hat{\xi}_t - \sigma\left[\frac{Y}{C}(\hat{Y}_t - \hat{Y}_{t+1}) - \frac{G}{C}(\hat{G}_t - \hat{G}_{t+1})\right]\right) \tag{3.73}$$

Finally, making use of the Euler equation for long bonds and iterating forward and making use of the resource constraint we get $P_{t,L} = \sum_{j\geq 1} E_t \beta^j \delta^{j-1} \frac{\xi_{t+j}}{\xi_t} \frac{(Y_{t+j} - G_{t+j)}^{-\sigma}}{(Y_t - G_{t)}^{-\sigma} \prod_{l=1}^{j} \pi_{t+l}}$. Using this to substitute out $P_{L,t}$ from the government budget constraint we obtain:

$$\sum_{j\geq 1} E_t \beta^j \delta^{j-1} \frac{\xi_{t+j}}{\xi_t} \frac{(Y_{t+j} - G_{t+j)}^{-\sigma}}{(Y_t - G_{t)}^{-\sigma} \prod_{l=1}^{j} \pi_{t+l}} b_{t,\delta} =$$
$$(1 + \delta \sum_{j\geq 1} E_t \beta^j \delta^{j-1} \frac{\xi_{t+j}}{\xi_t} \frac{(Y_{t+j} - G_{t+j)}^{-\sigma}}{(Y_t - G_{t)}^{-\sigma} \prod_{l=1}^{j} \pi_{t+l}}) \frac{b_{t-1,\delta}}{\pi_t} + G_t - \tau_t h_t w_t$$

Log-linearizing this equation we get:

$$\begin{aligned} &\frac{\beta \bar{b}_\delta}{1-\beta\delta} \hat{b}_{t,\delta} + \bar{b}_\delta \sum_{j=1}^{\infty} \beta^j \delta^{j-1} \left[E_t\left(-\sigma(\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j} - \frac{\overline{G}}{\overline{C}}\hat{G}_{t+j}) - \sum_{l=1}^{j} \hat{\pi}_{t+l} + \hat{\xi}_{t+j}\right)\right] \\ &+ \frac{\overline{\tau}(1+\eta)\overline{Y}}{\eta}\left((\gamma_h + 1)\hat{Y}_t + \hat{\xi}_t + \frac{\hat{\tau}_t}{1-\overline{\tau}}\right) - \overline{G}\left(\hat{G}_t - \sigma\frac{\overline{Y}}{\overline{C}}\hat{Y}_t + \sigma\frac{\overline{G}}{\overline{C}}\hat{G}_t + \hat{\xi}_t\right) + \bar{b}_\delta \sigma\left(\frac{\overline{Y}}{\overline{C}}\hat{Y}_t - \frac{\overline{G}}{\overline{C}}\hat{G}_t\right) - \bar{b}_\delta \hat{\xi}_t \\ &= \frac{\bar{b}_\delta}{1-\beta\delta}(\hat{b}_{t-1,\delta} - \hat{\pi}_t) + \delta \bar{b}_\delta \sum_{j=1}^{\infty} \beta^j \delta^{j-1} E_t\left(-\sigma(\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j} - \frac{\overline{G}}{\overline{C}}\hat{G}_{t+j}) - \sum_{l=1}^{j} \hat{\pi}_{t+l} + \hat{\xi}_{t+j}\right) \end{aligned} \tag{3.74}$$

Optimal monetary policy The central bank minimizes the loss function:

$$\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(\hat{\pi}_t^2 + \lambda_i \hat{i}_t^2 + \lambda_Y \hat{Y}_t^2\right)$$

subject to equations (3.72), (3.73) and (3.74), and the zero lower bound constraint in the case of **debt concerns**. In the case of **no debt concerns** we drop (3.74) from optimization.

The Lagrangian associated to the central bank's optimization problem under debt concerns is:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ -\frac{1}{2}\Big(\hat{\pi}_t^2 + \lambda_i \hat{i}_t^2 + \lambda_Y \hat{Y}_t^2\Big) + \psi_{\pi,t}\Big(\hat{\pi}_t - \kappa_1 \hat{Y}_t - \kappa_2 \hat{\tau}_t + \kappa_3 \hat{G}_t - \beta E_t \hat{\pi}_{t+1}\Big)$$
$$+\psi_{i,t}\Big(\hat{i}_t - \hat{\pi}_{t+1} + \hat{\xi}_{t+1} - \hat{\xi}_t + \sigma\Big[\frac{Y}{C}(\hat{Y}_t - \hat{Y}_{t+1}) - \frac{G}{C}(\hat{G}_t - \hat{G}_{t+1})\Big]\Big)$$
$$+\psi_{gov,t}\Bigg(\frac{\beta \bar{b}_\delta}{1-\beta\delta}\hat{b}_{t,\delta} + \bar{b}_\delta \sum_{j=1}^{\infty} \beta^j \delta^{j-1} \Bigg[E_t\Bigg(-\sigma(\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j} - \frac{\overline{G}}{\overline{C}}\hat{G}_{t+j}) - \sum_{l=1}^{j}\hat{\pi}_{t+l} + \hat{\xi}_{t+j}\Bigg)\Bigg]$$
$$+\frac{\overline{\tau}(1+\eta)\overline{Y}}{\eta}\Big((\gamma_h+1)\hat{Y}_t + \hat{\xi}_t + \frac{\hat{\tau}_t}{1-\overline{\tau}}\Big) - \overline{G}\Big(\hat{G}_t - \sigma\frac{\overline{Y}}{\overline{C}}\hat{Y}_t + \sigma\frac{\overline{G}}{\overline{C}}\hat{G}_t + \hat{\xi}_t\Big) + \bar{b}_\delta \sigma\Big(\frac{\overline{Y}}{\overline{C}}\hat{Y}_t - \frac{\overline{G}}{\overline{C}}\hat{G}_t\Big) - \bar{b}_\delta \hat{\xi}_t$$
$$-\frac{\bar{b}_\delta}{1-\beta\delta}(\hat{b}_{t-1,\delta} - \hat{\pi}_t) - \delta \bar{b}_\delta \sum_{j=1}^{\infty} \beta^j \delta^{j-1} E_t\Big(-\sigma(\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j} - \frac{\overline{G}}{\overline{C}}\hat{G}_{t+j}) - \sum_{l=1}^{j}\hat{\pi}_{t+l} + \hat{\xi}_{t+j}\Big)\Bigg) + \psi_{zlb,t}\Big(\hat{i}_t + i^*\Big)\Bigg\}$$

Taking first order conditions it is simple to arrive to the expressions provided in text. Under no debt concerns the Lagrangian is:

$$\mathcal{L}^{NDC} = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ -\frac{1}{2}\Big(\hat{\pi}_t^2 + \lambda_i \hat{i}_t^2 + \lambda_Y \hat{Y}_t^2\Big) + \psi_{\pi,t}^{NDC}\Big(\hat{\pi}_t - \kappa_1 \hat{Y}_t - \kappa_2 \hat{\tau}_t + \kappa_3 \hat{G}_t - \beta E_t \hat{\pi}_{t+1}\Big)$$
$$+\psi_{i,t}^{NDC}\Big(\hat{i}_t - \hat{\pi}_{t+1} + \hat{\xi}_{t+1} - \hat{\xi}_t + \sigma\Big[\frac{Y}{C}(\hat{Y}_t - \hat{Y}_{t+1}) - \frac{G}{C}(\hat{G}_t - \hat{G}_{t+1})\Big]\Big) + \psi_{zlb,t}^{NDC}\Big(\hat{i}_t + i^*\Big)\Bigg\}$$

Once again it is easy to verify that the FONC are the equations shown in text.

A Markov Switching Model

In Section 6.4 we considered a model with regime fluctuations. For simplicity, we derived interest rate rules assuming one-off shifts across regimes. We now derive interest rate rules under recurrent regime shifts.

As discussed in text, a complete theory of Markov transitions across regimes, is beyond the scope of this paper. Nevertheless, we claim that such modelling is essentially an extension of the above analysis. We provide a brief sketch here, showing what optimal interest rate rules would look like in a model which allows for recurrent regime fluctuations. We leave to future work the implementation of this extension.

Let us assume that the economy switches across regimes according to some transition matrix Π_{R_{t+1},R_t} the elements of which are constant through time. Consider first the case where the planner 'forgets' past commitments when the $R_t = NDC$ is realized. Denote by R^t the history of realized regimes and let $\underline{t} = \max\{0 \leq k \leq t : R_k = NDC\}$ given R^t (the last time process R was NDC given the history). The monetary policy rule can be written as:

$$\hat{i}_t = \begin{cases} \mathcal{T}_t & \text{if } R_t = NDC \\ \\ \mathcal{T}_t + \mathcal{D}_{\underline{t},t} & \text{if } R_t = DC \end{cases} \tag{3.75}$$

where $\mathcal{D}_{\underline{t},t} \equiv -\frac{\overline{C}}{\overline{Y}}\frac{\kappa_1}{\lambda_i\sigma}\frac{\bar{b}_\delta}{1-\beta\delta}\sum_{l=0}^{t-\underline{t}+1}\delta^l\Delta\psi_{gov,t-l} - \frac{\bar{b}_\delta}{\lambda_i}\sum_{l=0}^{t-\underline{t}+1}\delta^l\Big(\Delta\psi_{gov,t-l} - \Delta\psi_{gov,t-l-1}\Big) - \frac{\overline{C}\omega_Y}{\overline{Y}\sigma\lambda_i}\Delta\psi_{gov,t}$ and where $\psi_{gov,\underline{t}-2} = \psi_{gov,\underline{t}-1} = \psi_{gov,\underline{t}-2} = 0$.

According to (3.75) the multipliers ψ_{gov} influence optimal policy when the economy is in the DC regime, but cease to exert an influence when $R = NDC$. When the monetary policy switches to active, the planner sets all multipliers (including their lags) to zero.

Consider now the case where the planner remembers past promises. The optimal policy rule can be written as:

$$\hat{i}_t = \mathcal{T}_t - \frac{\overline{C}}{\overline{Y}}\frac{\kappa_1}{\lambda_i\sigma}\frac{\bar{b}_\delta}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l}(R_{t-l}) - \frac{\bar{b}_\delta}{\lambda_i}\sum_{l=0}^{\infty}\delta^l\Big(\Delta\psi_{gov,t-l}(R_{t-l}) - \Delta\psi_{gov,t-l-1}(R_{t-l-1})\Big) - \frac{\overline{C}\omega_Y}{\overline{Y}\sigma\lambda_i}\Delta\psi_{gov,t}(R_t) \tag{3.76}$$

where we assume that $\psi_{gov,t}(R_t) = 0$ if $R_t = NDC$.

A final remark closes this subsection. Note that the above policy rules together with equations (1) to (5) in text and the law of motion of the multiplier ψ_{gov} form a system of equations that can be resolved to characterize competitive equilibria in models with regime fluctuations. Given that the policy rules keep track of the history of regimes R it may not be evident that the models have a (first order) Markov structure. Close inspection of equation (3.76) reveals that we can write the term $\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l}$ recursively as:

$$\Psi_{gov,t} \equiv \sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l} = \Delta\psi_{gov,t} + \delta\Psi_{gov,t-1}$$

and thus term $\mathcal{D}_t$ in 3.76 becomes

$$\mathcal{D}_t = -\frac{\overline{C}}{\overline{Y}}\frac{\kappa_1}{\lambda_i\sigma}\frac{\bar{b}_\delta}{1-\beta\delta}\Psi_{gov,t} - \frac{\bar{b}_\delta}{\lambda_i}\Delta\Psi_{gov,t} - \frac{\overline{C}\omega_Y}{\overline{Y}\sigma\lambda_i}\Delta\psi_{gov,t}$$

It is thus sufficient to keep track of $\Psi_{gov,t-1}, \psi_{gov,t-1}$ in the state vector to solve this model. In the case of rule (3.75) one simply needs to set $\Psi_{gov,t-1} = 0$ whenever $R_t = NDC$.

Additional results and figures

Effect of $\phi_{\tau,b}$ We report here the impulse response functions of main model variables for a broader set of values for $\phi_{\tau,b}$, the parameter representing the strength of the fiscal surplus' response to rising level of debts. We display the results obtained for $\phi_{\tau,b} \in \{0, 0.03, 0.06\}$ in the debt concerns model. Figure 3.9 presents results under the calibration which sets $\lambda_Y = \lambda_i = \sigma = 0$, the counterpart of Figure 1 in the paper. Figure 3.10 displays the results obtained when we set $\lambda_Y = \lambda_i = 0.5$ and $\sigma = 1$, as in Figure 2 in the main text. In each figure, the solid blue lines display IRFs when $\phi_{\tau,b} = 0$, the dash-dotted black lines and dotted light blue lines set, respectively, $\phi_{\tau,b} = 0.03$ and $\phi_{\tau,b} = 0.06$. The dashed red lines provide the responses for the baseline 'no debt concerns' calibration.

Lump-sum taxes We now show that the properties of our model do not change when we assume lump sum taxes, as was claimed in text. Figure 3.11 presents the impulse responses of key variables to shocks. The dashed (red) lines show the case of lump-sum taxes whereas the solid (blue) lines correspond to distortionary labor taxes. The debt concerns model is in panel a and the no debt concerns model in panel b.

[Figure 3.11 approximately here]

As is evident from the figures the responses under lump sum taxes are similar to the responses with distortionary taxes. Thus assuming lump sum taxation does not affect our conclusions. We found that this also holds for the case where preference shocks are large enough to lead the economy to the LT. For brevity we left this case outside the figures.

Jointly optimal policies We now present the impulse responses to shocks in a model with jointly optimal policies assuming different values for parameter λ_τ (see text).

Figure 3.12 assumes $\sigma = 1$, while Figure 3.13 considers the case $\sigma = 0$ (see analytical results in text). In each case we display the impulse response of variables to a negative preference shock for $\lambda_\tau \in \{0, 1, 1.5\}$, and compare the results to the (no) debt concerns scenarios considered in the text.

[Figures 3.12 and 3.13 Here]

The role of debt maturity To study the role played by the maturity of government debt in the model, we show in Figure 3.14 the impulse responses assuming different values for the parameter δ. On top of the baseline specification where $\delta = 0.95$ (solid blue line), we present results for the two extreme cases $\delta = 0$ (dashed red lines), and $\delta = 1$ (dotted black lines).

[Figure 3.14 approximately here]

3.7.3 The estimated model

In Section 5 of the paper, we estimate a New-Keynesian DSGE model broadly similar to Bianchi and Melosi (2017) and Bianchi and Ilut (2017). We depart from these papers in assuming the presence of distortionary labor taxes, and also in solving the optimal policy problem instead of using standard Taylor rules for monetary policy. We provide a concise description of our model here.

Household preferences are of the following form:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left(\log(C_t - \Omega C_{t-1}^a) - \chi \frac{h_t^{1+\gamma_h}}{1+\gamma_h} \right)$$

where C_t is again consumption and C_t^a is an external habit stock ($0 < \Omega < 1$ determines the degree of habit formation) as discussed in text. Households maximize utility subject to

$$P_t C_t + P_{t,S} B_{t,S} + P_{t,L} B_{t,\delta} \leqslant (1-\tau_t) W_t h_t + P_t D_t + B_{t-1,S} + (1 + \delta P_{t,L}) B_{t-1,\delta}$$

The first order conditions give us Euler equations for short and long-term government bonds, and the labor supply condition:

$$\frac{1}{C_t - \Omega C_{t-1}^a} = \beta R_t E_t \frac{1}{\pi_{t+1}} \frac{1}{C_{t+1} - \Omega C_t^a} \tag{3.77}$$

$$\frac{1}{C_t - \Omega C_{t-1}^a} P_{L,t} = \beta E_t \frac{1 + \delta P_{L,t+1}}{\pi_{t+1}} \frac{1}{C_{t+1} - \Omega C_t^a} \tag{3.78}$$

$$\chi h_t^{\gamma_h} (C_t - \Omega C_{t-1}^a) = (1-\tau_t) \frac{W_t}{P_t} \tag{3.79}$$

Production takes place in monopolistically competitive firms which operate technologies with labour as the sole input. The final good is a CES aggregate of the intermediate goods $Y_t(j)$:

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{1+\eta_t}{\eta_t}} dj \right)^{\frac{\eta_t}{1+\eta_t}} \tag{3.80}$$

where η_t is a time-varying process governing the elasticity of substitution between differentiated goods. Firms set prices to maximize profits subject to the demand curve

$$Y_t(j) = \Big(\frac{P_t(j)}{P_t}\Big)^{\eta_t} Y_t \tag{3.81}$$

and given price adjustment costs. The dynamic profit maximization program is:

$$\begin{aligned}
\max_{P_t(j)} \quad & E_t \sum_{s=0}^{\infty} Q_{t,t+s}\Big(\frac{P_{t+s}(j)}{P_{t+s}} Y_{t+s}(j) - \frac{MC_{t+s}(j)}{P_{t+s}} Y_{t+s}(j) - AC_{t+s}(j)\Big) \\
\text{s.t.} \quad & Y_{t+s}(j) = \Big(\frac{P_{t+s}(j)}{P_{t+s}}\Big)^{\eta_t} Y_{t+s} && (3.82) \\
& AC_{t+s}(j) = \frac{\theta}{2}\Big(\frac{P_{t+s}(j)}{P_{t+s-1}(j)} - \overline{\pi}\Big)^2 Y_{t+s} && (3.83)
\end{aligned}$$

where $Q_{t,t+s} \equiv \beta^s E_t \frac{C_t - \Omega C^a_{t-1}}{C_{t+s} - \Omega C^a_{t+s-1}}$ is the discount factor of households and MC_{t+s} denotes marginal costs of production. $AC_{t+s}(j)$ in equation (3.83) is the quadratic price adjustment cost incurred by the firm.

Focusing on a symmetric equilibrium the first order condition from the firms's dynamic programs gives us the following economy wide non-linear Phillips Curve:

$$\theta(\pi_t - \overline{\pi})\pi_t = 1 + \eta_t(1 - \frac{MC_t}{P_t}) + \beta\theta E_t \frac{C_t - \Omega C^a_{t-1}}{C_{t+1} - \Omega C^a_t} \frac{Y_{t+1}}{Y_t}(\pi_{t+1} - \overline{\pi})\pi_{t+1} \tag{3.84}$$

The production of $Y_t(j)$ is determined according to

$$Y_t(j) = A_t h_t(j)^{1-\alpha} \tag{3.85}$$

where A_t is TFP, whose growth rate (in logs) follows an AR(1) process:

$$\ln\Big(\frac{A_t}{A_{t-1}}\Big) \equiv a_t = (1-\rho_a)\gamma + \rho_a a_{t-1} + \epsilon_{a,t}$$

Parameter γ denotes the steady-state growth rate if A_t.

Given the above, (real) marginal costs can be expressed as:

$$\frac{MC_t(j)}{P_t} = \frac{W_t}{P_t}\frac{1}{(1-\alpha)A_t h_t(j)^{-\alpha}}$$

Fiscal policy The government levies distortionary taxes and issues debt to finance spending, and transfers to the private sector. The flow budget constraint is:

$$P_{t,L}B_{t,\delta} = (1 + \delta P_{t,L})B_{t-1,\delta} + P_t(G_t + TR_t) - \tau_t W_t h_t + \Lambda_t \tag{3.86}$$

where G_t and TR_t denote respectively government purchases of goods and services and lump-sum transfers rebated to households. As discussed in text, Λ_t is an exogenous shock to the budget constraint that captures features of the government expenditures and policies which are not explicitly modelled.

We follow Bianchi and Ilut (2017) and express the government budget constraint as a fraction of output Y_t. Define $b_{\delta,t} \equiv \frac{P_{L,t}B_{\delta,t}}{P_t Y_t}$ as the real market value of debt-to-GDP ratio and $R_t^L \equiv \frac{1+\delta P_{L,t+1}}{P_{L,t}}$ is the holding period return on the long bond. Also let $g_t \equiv \frac{1}{1-\frac{G_t}{Y_t}}$ and $tr_t \equiv \frac{TR_t}{Y_t}$. Using $\frac{W_t h_t}{P_t Y_t} = \frac{MC_t}{P_t} \equiv mc_t$, we can write the (real) flow government budget constraint as:

$$b_{\delta,t} = \frac{R_{t-1}^L b_{\delta,t-1}}{\pi_t}\frac{Y_{t-1}}{Y_t} + 1 - g_t^{-1} + tr_t - \tau_t mc_t \tag{3.87}$$

Equilibrium In the symmetric equilibrium all firms set the same price and produce equal output, hence $Y_t(j) = Y_t$ and $h_t(j) = h_t$ $\forall j \in [0,1]$. We can therefore write the aggregate production function as

$$Y_t = A_t h_t^{1-\alpha} \tag{3.88}$$

and the (economy-wide) marginal costs of production are

$$\frac{MC_t}{P_t} = \frac{W_t}{P_t}\frac{1}{(1-\alpha)A_t h_t^{-\alpha}} \tag{3.89}$$

The resource constraint of the economy is

$$Y_t = C_t + G_t \tag{3.90}$$

and rescaling in terms of $g_t = \frac{1}{1-\frac{G_t}{Y_t}}$ we have

$$Y_t = C_t\, g_t \tag{3.91}$$

Detrending Since TFP grows over time we rescale model variables to make them stationary. We define $c_t \equiv \frac{C_t}{A_t}$ and $y_t \equiv \frac{Y_t}{A_t}$. The model equations can be written as follows:

$$\frac{1}{c_t - \Omega c^a_{t-1} e^{-a_t}} = \beta R_t E_t \frac{1}{\pi_{t+1}} \frac{e^{-a_{t+1}}}{c_{t+1} - \Omega c^a_t e^{-a_{t+1}}} \tag{3.92}$$

$$\frac{1}{c_t - \Omega c^a_{t-1} e^{-a_t}} = \beta E_t \frac{1 + \delta P_{L,t+1}}{\pi_{t+1}} \frac{e^{-a_{t+1}}}{c_{t+1} - \Omega c^a_t e^{-a_{t+1}}} \tag{3.93}$$

$$\theta(\pi_t - \pi)\pi_t = 1 + \eta_t(1 - mc_t) + \beta\theta E_t \frac{(c_t - \Omega c^a_{t-1} e^{-a_t})}{(c_{t+1} - \Omega c^a_t e^{-a_{t+1}})} \frac{y_{t+1}}{y_t} (\pi_{t+1} - \pi)\pi_{t+1} \tag{3.94}$$

$$mc_t = \frac{\chi y_t^{\frac{\gamma_h + \alpha}{1-\alpha}}}{1 - \tau_t} (c_t - \Omega c^a_{t-1} e^{-a_t}) \tag{3.95}$$

$$y_t = c_t g_t \tag{3.96}$$

$$b_{\delta,t} = \frac{R^L_{t-1} b_{\delta,t-1}}{\pi_t} \frac{y_{t-1}}{y_t} e^{-a_t} + 1 - g_t^{-1} + tr_t - \tau_t mc_t \tag{3.97}$$

$$R^L_t = \frac{1 + \delta P_{L,t+1}}{P_{L,t}} \tag{3.98}$$

where $R_t \equiv P^{-1}_{t,s}$. Notice that we made use of the production function (3.88) and the optimal labour supply condition (3.79) to dispense with W_t and h_t.

Equations (3.92) to (3.98) (together with the ZLB) provide a full description of the competitive equilibrium of the model economy. Dropping time subscripts we obtain steady state relationships. Define $\overline{x}$, the steady-state value of variable x_t. We can write:

$$\overline{R} = \frac{\overline{\pi} e^{\gamma}}{\beta} \qquad \overline{mc} = \frac{1 + \eta}{\eta}$$

$$\overline{P}_L = \frac{\beta e^{-\gamma}}{\overline{\pi} - \beta\delta e^{-\gamma}} \qquad \overline{c} = \frac{\overline{y}}{\overline{g}}$$

$$\overline{R}_L = \frac{1 + \delta\overline{P}_L}{\overline{P}_L}$$

The value of $\overline{Y}$ is normalized to 1. The steady-state values of inflation (π), the market value of government debt-to-GDP (b_δ), government spending-to-GDP (g) and taxes (τ) are all estimated from the data. Parameters χ and the value of tr are adjusted accordingly to satisfy the steady-state equations.

Log-linearized equations Define $\hat{x}_t \equiv \log x_t - \log \overline{x}$ as the log deviation of variable x from its steady-state value. We will apply a log linear transformation to all model variables except to the labor tax rate, transfers and shocks to the government budget constraint in which case we use a linear approximation (i.e. $\hat{x}_t = x_t - \overline{x}$, for $x = \tau, tr, \Lambda$).

Linearizing the system of equations (3.92)-(3.98) gives:

$$(1+\Omega e^{-\gamma})\hat{c}_t = E_t(\hat{c}_{t+1}+\hat{\xi}_{t+1}) - (\hat{i}_t - E_t\pi_{t+1}) + \Omega e^{-\gamma}(\hat{c}_{t-1} + E_t\hat{a}_{t+1} - \hat{a}_t) + (1-\Omega e^{-\gamma})(\hat{\xi}_t - E_t\hat{\xi}_{t+1}) \tag{3.99}$$

$$\hat{i}_t = E_t\hat{r}_{L,t+1} \tag{3.100}$$

$$\hat{\pi}_t = \kappa\hat{mc}_t + \beta E_t\hat{\pi}_{t+1} + \hat{\eta}_t \tag{3.101}$$

$$\hat{b}_{L,t} = \frac{\overline{b_L}}{\beta}(\hat{r^L}_t - \hat{\pi}_t - \hat{a}_t + \hat{y}_{t-1} - \hat{y}_t) + \frac{\hat{b}_{L,t-1}}{\beta} - (1-\nu)(\hat{\tau}_t + \tau\hat{mc}_t) + g^{-1}\hat{g}_t + \hat{tr}_t + \hat{\Lambda}_t \tag{3.102}$$

$$\hat{y}_t = \hat{c}_t + \hat{g}_t \tag{3.103}$$

$$\hat{mc}_t = \frac{1}{1-\Omega e^{-\gamma}}(\hat{c}_t - \Omega e^{-\gamma}(\hat{c}_{t-1} - \hat{a}_t)) + \frac{\hat{\tau}_t}{1-\tau} + \frac{\alpha+\phi}{1-\alpha}\hat{y}_t \tag{3.104}$$

$$\hat{r^L}_t = \delta R^{-1}\hat{p}_{L,t} - \hat{p}_{L,t-1} \tag{3.105}$$

Fiscal variables follow simple feedback rules. We adopt the following specifications which are very similar to Bianchi and Ilut (2017) and Bianchi and Melosi (2017):

$$\hat{\tau}_t = \rho_\tau\hat{\tau}_{t-1} + (1-\rho_\tau)\Big[\phi_{\tau,b}\hat{b}_{\delta,t-1} + \phi_{\tau,y}(\hat{y}_t - \hat{y}_t^n) + \phi_{\tau,g}(g^{-1}\hat{g}_t + \hat{tr}_t^*)\Big] + \epsilon_{\tau,t} \tag{3.106}$$

for the labour income tax rate, assuming that taxes have a first order autoregressive component and respond to the lagged value of debt and to the deviation of output from its natural level.

The variable $\hat{tr}_t^*$ is the long-term component in government transfers, and evolves exogenously according to:

$$\hat{tr}_t^* = \rho_{tr^*}\hat{tr}_{t-1}^* + \epsilon_{tr^*,t} \tag{3.107}$$

Moreover, the deviation of $\hat{tr}_t$ from its long-run trend $\hat{tr}_t^*$ follows

$$\hat{tr}_t - \hat{tr}_t^* = \rho_{tr}(\hat{tr}_{t-1} - \hat{tr}_{t-1}^*) + (1-\rho_{tr})\phi_{tr,y}(\hat{y}_t - \hat{y}_t^n) + \epsilon_{tr,t}. \tag{3.108}$$

Lastly government spending (as a fraction of GDP) follows a simple AR(1) process:

$$\hat{g}_t = \rho_g\hat{g}_{t-1} + \epsilon_{g,t} \tag{3.109}$$

The shocks $\epsilon_{\tau,t}$, $\epsilon_{tr^*,t}$, $\epsilon_{tr,t}$ and $\epsilon_{g,t}$ are all assumed to be i.i.d.

The natural level of output $\hat{y}_t^n$, is defined as the level of output that would prevail under flexible prices. It satisfies the following:

$$\Big(\frac{1}{1-\Omega e^{-\gamma}} + \frac{\alpha+\gamma_h}{1-\alpha}\Big)\hat{y}_t^n = \frac{1}{1-\Omega e^{-\gamma}}\hat{g}_t + \frac{\Omega e^{-\gamma}}{1-\Omega e^{-\gamma}}(\hat{y}_{t-1}^n - \hat{g}_{t-1} - \hat{a}_{t-1}) - \frac{1}{1-\tau}\hat{\tau}_t \tag{3.110}$$

The remaining shock processes (markups, preferences, technology growth, and shocks to the government budget) are all assumed to follow AR(1) processes:

$$\begin{aligned}\hat{\eta}_t &= \rho_\eta \hat{\eta}_{t-1} + \epsilon_{\eta,t} \\ \hat{\xi}_t &= \rho_\xi \hat{\xi}_{t-1} + \epsilon_{\xi,t} \\ \hat{a}_t &= \rho_a \hat{a}_{t-1} + \epsilon_{a,t} \\ \hat{\Lambda}_t &= \rho_\lambda \hat{\Lambda}_{t-1} + \epsilon_{\lambda,t}\end{aligned}$$

where $\epsilon_{\eta,t}$, $\epsilon_{\xi,t}$, $\epsilon_{a,t}$, $\epsilon_{\lambda,t}$ are i.i.d shocks.

Optimal monetary policy As discussed in text in the quantitative model we assume that the central bank minimizes the following loss function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \Big(\hat{\pi}_t^2 + \lambda_i (\hat{i}_t - \hat{i}_{t-1})^2 + \lambda_y (\hat{y}_t - \hat{y}_t^n)^2 \Big)$$

subject to equations (3.99) to (3.105), and taking as given the behaviour of fiscal variables and the other exogenous processes. We assign Lagrange multipliers $\Psi_{1,t}$ to equation (3.99), $\Psi_{2,t}$ to equation (3.100), and so on until $\Psi_{7,t}$ to equation (3.105). The multiplier on the ZLB constraint $\hat{i}_t \geqslant -\log(R)$ is denoted $\Psi_{zlb,t}$. The first order conditions of the central bank's optimization problem in the case of **debt concerns** can be written as:

$$\partial \hat{c}_t: \quad -(1+\Omega e^{-\gamma})\Psi_{1,t} + \frac{\Psi_{1,t-1}}{\beta} + \beta \Omega e^{-\gamma} E_t \Psi_{1,t+1} + \Psi_{5,t} + \frac{\Psi_{6,t}}{1-\Omega e^{-\gamma}} - \frac{\beta \Omega e^{-\gamma}}{1-\Omega e^{-\gamma}} E_t \Psi_{6,t+1} = 0 \tag{3.111}$$

$$\partial \hat{y}_t: \quad -\lambda_y (y_t - y_t^n) - \Psi_{5,t} + \frac{\alpha + \gamma_h}{1-\alpha} \Psi_{6,t} + \frac{b_\delta}{\beta} \Psi_{4,t} - b_\delta E_t \Psi_{4,t+1} = 0 \tag{3.112}$$

$$\partial \hat{i}_t: \quad -\lambda_i (i_t - i_{t-1}) + \beta \lambda_i (E_t i_{t+1} - i_t) - (1-\Omega e^{-\gamma})\Psi_{1,t} - \Psi_{2,t} + \Psi_{zlb,t} = 0 \tag{3.113}$$

$$\partial \hat{r}_{L,t}: \quad \beta^{-1} \Psi_{2,t-1} - \frac{b_\delta}{\beta} \Psi_{4,t} - \Psi_{7,t} = 0 \tag{3.114}$$

$$\partial \hat{\pi}_t: \quad -\pi_t + (1-\Omega e^{-\gamma})\beta^{-1} \Psi_{1,t-1} - \Psi_{3,t} + \Psi_{3,t-1} + \frac{b_\delta}{\beta} \Psi_{4,t} = 0 \tag{3.115}$$

$$\partial \hat{mc}_t: \quad \kappa \Psi_{3,t} - \Psi_{6,t} + (1-\nu)\tau \Psi_{4,t} = 0 \tag{3.116}$$

$$\partial \hat{p}_{L,t}: \quad \delta R^{-1} \Psi_{7,t} - \beta E_t \Psi_{7,t+1} = 0 \tag{3.117}$$

$$\partial \hat{b}_{\delta,t} \quad \Psi_{4,t} - E_t \Psi_{4,t+1} = 0 \tag{3.118}$$

Under **no debt concerns**, the government budget constraint (3.102) is removed from the constraint set and the planner does not optimize with respect to $\hat{b}_{\delta,t}$. This is equivalent to removing equation (3.118) from the system, and setting $\Psi_{4,t} = 0\ \forall t$.

Estimation

Data

The dataset we use to estimate our structural model contains series on GDP, inflation, interest rates, government spending, total government expenditures, tax revenues, and the market value of government debt. All the data are for the U.S and are observed at a quarterly frequency for the period 1980Q1-2008Q4. For the shock filtering exercise we extended the dataset to 2016Q4 to include the Great Recession period.

Unless stated otherwise, all data are taken from the National Income and Products Account (NIPA) collected by the Bureau of Economic Analysis. Real values are obtained using the GDP deflator. The construction of fiscal data series follows Leeper et al. (2010, 2015) and Bianchi and Ilut (2017).

Output (GDP_t)**:** Nominal GDP (Table 1.1.5, Line 1) is deflated using the GDP deflator, and divided by the Population Index (constructed using the Civilian Noninstitutional Population series (CNP16OV) from the Bureau of Labor Statistics). The data used for estimation is the quarterly growth rate of the resulting series.

Inflation ($INFL_t$)**:** The quarterly inflation rate is defined as the growth rate of the GDP deflator (Table 1.1.4 Line 1).

Nominal interest rate (FFR_t)**:** The quarterly nominal interest rate used for estimation is the Fed Funds rate, retrieved from the FRED database.

Government spending-to-GDP ratio (GOV_t)**:** Government spending is defined as the sum of consumption expenditure (Table 3.2 Line 25), gross government investment (Table 3.2 Line 45), net purchases of non-produced assets (Table 3.2 Line 47), minus consumption of fixed capital (Table 3.2 Line 48).

Total government expenditures-to-GDP (EXP_t)**:** Total government expenditures are taken as the sum of government spending (defined above) and transfers. Transfers are defined as the sum of current transfer payments (Table 3.2 Line 26), subsidies (Table 3.2 Line 36), capital transfer payments (Table 3.2 Line 46), minus current transfer receipts (Table 3.2 Line 19) and capital transfer receipts (Table 3.2 Line 42).

Tax revenues-to-GDP (TAX_t)**:** Tax revenues are defined as current receipts (Table 3.2 Line 42) minus current transfer receipts (Table 3.2 Line 19).

Market-value of debt-to-GDP ($MVDEBT_t$)**:** The data for the market value of U.S government debt is taken from the Dallas Fed. We use the series on the market value of marketable treasury debt. To construct quarterly series we use the stock of debt in the first month of each quarter.

Measurement equations The equations linking model variables to the data series used for estimation are:

$$\begin{aligned}
GDP_t &= 100\gamma + \hat{y}_t - \hat{y}_{t-1} + \hat{a}_t \\
INFL_t &= 100\log(\Pi) + \hat{\pi}_t \\
FFR_t &= 100\log(R) + \hat{i}_t + \epsilon_{m,t} \\
TAX_t &= (1-\nu)(100\tau + \hat{\tau}_t + \tau\hat{mc}_t) \\
MVDEBT_t &= 0.25(100b_\delta + \hat{b}_{\delta,t}) \\
GOV_t &= 100\log(g) + \hat{g}_t \\
EXP_t &= 100\Big[\frac{\beta-1}{\beta}b_\delta + (1-\nu)\tau\Big] + g^{-1}\hat{g}_t + \hat{tr}_t
\end{aligned}$$

where $\epsilon_{m,t}$ is an i.i.d measurement error shock to the observed interest rate series which we introduce in estimation following Debortoli and Lakwadala (2016).

Estimation output

As is typical we proceed with estimation by first selecting prior distributions for the parameters we wish to estimate and picking values for parameters that we want to fix in estimation. Table 3.3 summarizes the calibrated values of the parameters that we fix and the right side of Table 3.4 reports our choice of prior distributions for the parameters we estimate with Bayesian techniques. The priors are in line with previous papers in the literature (see e.g. Bianchi and Ilut (2017)) and are relatively loose.

[Tables 3.3 and 3.4 About Here]

We fix the values of the labor share, α, the elasticity of labor supply, $\frac{1}{\gamma_h}$, the demand elasticity parameter, η, the decay factor of the long term bond, δ. We assume $\alpha = 0.66$ and $\gamma_h = 1$. The decay factor δ is set to 0.95 which gives us an average maturity of 5 years, consistent with US data. The steady state value of η is such that markups are 15 percent. Finally, we normalize the steady-state value of output to unity.

The left side of Table 3.4 reports the posterior estimates of the model parameter distributions. According to the values reported in the table, the Phillips curve is relatively flat (the mean estimate of κ is 0.017) and the distribution of the habit parameter Ω is centered around roughly 0.5. Moreover, the estimated response of taxes to the lagged value of debt is low ($\phi_{\tau,b} = 0.068$ at the mean). These values are very close to the analogous objects reported in Bianchi and Ilut (2017).

Finally, notice that the estimates of the coefficients λ_i and λ_Y are in range of recent estimates of optimal Ramsey models with US data (see e.g. Debortoli and Lakdawala (2016) and references therein).

Figures 3.15 and 3.16 plot the prior and posterior distributions of estimated model parameters. The dotted red lines and solid blue lines represent respectively prior and posterior densities.

[Figures 3.15, 3.16 and 3.17 approximately here]

Figure 3.17 contains the analogous plot with prior and posterior densities for the standard deviations of exogenous shock processes.

[Figures 3.18 approximately here]

Figure 3.18 shows the smoothed shocks obtained from the Kalman filter for parameters at the means of the posterior distributions, for the estimation period 1980Q1-2008Q4.

3.7.4 Solution method

We describe here the numerical algorithms that we used to (i) construct the piecewise linear solution of the model which accounts for the occasionally binding zero lower bound constraint and (ii) get filtered shocks for the case where the ZLB binds in the data. Before doing this, we lay out in the next section a general formulation of the different (log-linear) versions of our model that will help to describe the methodology.

General model formulation Our debt concerns model can be written in state space form as

$$A^{DC} E_t X_{t+1} + B^{DC} X_t + C^{DC} X_{t-1} + D^{DC} \epsilon_t = 0 \tag{3.119}$$

in the case where the ZLB is not binding. The vector X_t contains all the endogenous variables of our model, and the vector ϵ_t all the stochastic disturbances. Matrices A, B, C, D (which depend on model parameters) describe the (log-linear) structural equations of the model. The analogous matrices when the ZLB is binding are written with tildes, and the model in this case is written as:

$$\widetilde{A}^{DC} E_t X_{t+1} + \widetilde{B}^{DC} X_t + \widetilde{C}^{DC} X_{t-1} + \widetilde{D}^{DC} \epsilon_t + \widetilde{E}^{DC} = 0 \tag{3.120}$$

The vector $\tilde{E}$ is added because we approximate the solution around the steady state of the baseline model, which features a positive nominal interest rate.

The analogous expressions in the model without debt concerns are:

$$A^{NDC} E_t X_{t+1} + B^{NDC} X_t + C^{NDC} X_{t-1} + D^{NDC} \epsilon_t = 0 \tag{3.121}$$

$$\widetilde{A}^{NDC} E_t X_{t+1} + \widetilde{B}^{NDC} X_t + \widetilde{C}^{NDC} X_{t-1} + \widetilde{D}^{NDC} \epsilon_t + \widetilde{E}^{NDC} = 0 \tag{3.122}$$

The piece-wise linear solution method described below will give us a solution to the system of equations of the form

$$X_t = \mathcal{E}_t X_{t-1} + \mathcal{F}_t \epsilon_t + \mathcal{G}_t \tag{3.123}$$

for time varying matrices $\mathcal{E}_t$, $\mathcal{F}_t$ $\mathcal{G}_t$. In what follows we describe how we get to (3.123).

Constructing the path of variables given shocks and initial conditions Our quantitative experiments presented in the paper recover shocks and initial conditions to match US data under the no debt concerns regime and subsequently simulate the behavior of the model in the case where NDC persists forever and in the case of a permanent switch towards debt concerns.[31] We describe here a general version of our algorithm, used to compute the reduced-form matrices in (3.123), and which is based on the Occbin algorithm of Guerrieri and Iacoviello (2015), however, extended to allow for unanticipated switches across regimes. To make the exposition more general we allow for switches to occur in every period, assuming however that these switches are unanticipated and expected to persist forever by private agents.

[31] In the latter case structural parameters of the model such as the parameters concerning the tax rule change.

Let X_0 and $\{\epsilon_t\}_{t=1}^T$ respectively denote the initial conditions and shocks that we feed to the model. $t=1$ is 2009Q1. T denotes the end of sample in 2016Q4. Define the variable $\mathcal{I}_t \in \{NDC, DC\}$ for $t=1,...,T$, which indicates the regime in period t. Our Occbin algorithm is the following:

Given initial conditions and shocks $\left\{X_0, \{\epsilon_t\}_{t=1}^T\right\}$, and given a sequence of regimes $\{\mathcal{I}_t\}_{t=1}^T$:

1. Set $t=1$

2. - If $\mathcal{I}_t = DC$, use the structural matrices described in equations (3.119) and (3.120). If $\mathcal{I}_t = NDC$, use the matrices described in (3.121) and (3.122).
   - Given current initial condition X_{t-1} and the vector of shocks ϵ_t, and assuming no further shocks after t:

     (a) Guess a sequence of ZLB regimes: $ZLB_t = \{zlb_t, zlb_{t+1|t}, ..., zlb_{t+L|t}\}$, where $zlb_{t+j|t}$ is an indicator function taking the value 1 if the ZLB is binding in $t+j$, conditional of no further shocks occurring between t and $t+j$. Also let T^* denote the last period the ZLB binds where $T^* \in \{t, t+1, ..., t+L\}$.

     (b) Construct matrices $\mathcal{E}_t$, $\mathcal{F}_t$ and $\mathcal{G}_t$ by backward induction starting from T^* and continuing until t. We have:

$$
\begin{aligned}
\mathcal{E}_{T^*|t} &= \overline{\mathcal{E}}^{\mathcal{I}_t} \\
\mathcal{G}_{T^*|t} &= 0 \\
\mathcal{E}_{t+j|t} &= \begin{cases} -(\tilde{A}^{\mathcal{I}_t}\mathcal{E}_{t+j+1|t} + \tilde{B}^{\mathcal{I}_t})^{-1}\tilde{C}^{\mathcal{I}_t} & \text{for } 0<j<T^*, \text{ if } zlb_{t+j|t}=1 \\ -(A^{\mathcal{I}_t}\mathcal{E}_{t+j+1|t} + B^{\mathcal{I}_t})^{-1}C^{\mathcal{I}_t} & \text{for } 0<j<T^*, \text{ if } zlb_{t+j|t}=0 \end{cases} \\
\mathcal{G}_{t+j|t} &= \begin{cases} -(\tilde{A}^{\mathcal{I}_t}\mathcal{E}_{t+j+1|t} + \tilde{B}^{\mathcal{I}_t})^{-1}(\tilde{A}^{\mathcal{I}_t}\mathcal{G}_{t+j+1|t} + \tilde{E}^{\mathcal{I}_t}) & \text{for } 0<j<T^*, \text{ if } zlb_{t+j|t}=1 \\ -(A^{\mathcal{I}_t}\mathcal{E}_{t+j+1|t} + B^{\mathcal{I}_t})^{-1}A^{\mathcal{I}_t}\mathcal{G}_{t+j+1|t} & \text{for } 0<j<T^*, \text{ if } zlb_{t+j|t}=0 \end{cases} \\
\mathcal{E}_t &= \begin{cases} -(\tilde{A}^{\mathcal{I}_t}\mathcal{E}_{t+1|j} + \tilde{B}^{\mathcal{I}_t})^{-1}\tilde{C} & \text{if } zlb_t=1 \\ -(A^{\mathcal{I}_t}\mathcal{E}_{t+1|j} + B^{\mathcal{I}_t})^{-1}C & \text{if } zlb_t=0 \end{cases} \\
\mathcal{F}_t &= \begin{cases} -(\tilde{A}^{\mathcal{I}_t}\mathcal{E}_{t+1|j} + \tilde{B}^{\mathcal{I}_t})^{-1}\tilde{D} & \text{if } zlb_t=1 \\ -(A^{\mathcal{I}_t}\mathcal{E}_{t+1|j} + B^{\mathcal{I}_t})^{-1}D & \text{if } zlb_t=0 \end{cases} \\
\mathcal{G}_t &= \begin{cases} -(\tilde{A}^{\mathcal{I}_t}\mathcal{E}_{t+1|j} + \tilde{B}^{\mathcal{I}_t})^{-1}(\tilde{A}^{\mathcal{I}_t}\mathcal{G}_{t+1|t} + \tilde{E}^{\mathcal{I}_t}) & \text{if } zlb_t=1 \\ -(A^{\mathcal{I}_t}\mathcal{E}_{t+1|j} + B^{\mathcal{I}_t})^{-1}A^{\mathcal{I}_t}\mathcal{G}_{t+1|t} & \text{if } zlb_t=0 \end{cases}
\end{aligned}
$$

     where $\overline{\mathcal{E}}^{\mathcal{I}_t}$ is the reduced form matrix associated with the linear solution of model $\mathcal{I}_t$ where the ZLB never binds, obtained with standard perturbation methods, and $\mathcal{E}_{t+j|t}, \mathcal{F}_{t+j|t}, \mathcal{G}_{t+j|t}$ denote reduced form matrices for period $t+j$, conditional on no further shocks between t and $t+j$.

(c) Simulate the model with the obtained matrices, and infer a new sequence of binding regimes ZLB_t^{new} by evaluating the ZLB constraint $\hat{i}_t \geqslant -i^*$.
- If $ZLB_t^{new} = ZLB_t$, CONTINUE.
- Otherwise, update $ZLB_t = ZLB_t^{new}$ and go back to step (b).

3. If $t < T$, set $t = t + 1$ and go back to 2. Otherwise, STOP.

A couple of remarks end this subsection. Note that the above algorithm is suitable to compute the impulse responses for a more general class of experiments than what we show in text. We could use the above algorithm to study cases where the regime fluctuates between DC and NDC. However, in the exercise we showed in text we assume that the regime is determined once and for all in in 2009Q1, when the US economy first hits the ZLB. We thus set $\mathcal{I}_{t\geq 1} = NDC$ when the no debt concerns regime persists after 2009 and $\mathcal{I}_{t\geq 1} = DC$ in the case of a permanent switch towards debt concerns in 2009. As discussed in text to show how the economy responds to the LT shock we isolate our focus on the preference shock that we identify in 2009Q1, setting all other shocks to zero. Moreover, we set $\epsilon_{t\geq 1} = 0$.

Using the above algorithm we performed a number of other (unreported) experiments, whereby we considered the effect of a LT shock after 2009Q1. Our model's predictions did not change.

For more details on the Occbin solution method, we refer the reader to Guerrieri and Iacoviello (2015).

Filtering exercise: getting historical shocks and initial conditions We now describe the algorithm used to obtain the smoothed shocks and variables in the quantitative exercise of the Great Recession period. Our algorithm is based on the method used inKollmann, Pataracchia, Raciborski, Ratto, Roeger, and Vogel (2016), and described in Ratto and Giovannini (2016), which is an extension of the standard filtering algorithms for DSGE models to allow for occasionally binding constraints. The method combines Occbin to get the piecewise linear solution of the model given shocks, and the diffuse Kalman filter to get shocks given a piecewise linear solution.

In this exercise we also consider the (unexpected) switches between no debt concerns and debt concerns regimes as being exogenously given. The algorithm goes as follows:

Given a sequence of regimes $\{\mathcal{I}_t\}_{t=1}^T$, where $\mathcal{I}_t \in \{NDC, DC\}$:

1. Provide an initial guess for the binding ZLB constraint:

$$\mathcal{ZLB} = \{ZLB_t\}_{t=1}^T$$

where $ZLB_t = \{zlb_t, zlb_{t+1|t}, ..., zlb_{t+J|t}\}$, and $zlb_{t+j|t} \in \{1, 0\}$ is (as before) an indicator function taking the value 1 if the zlb is expected to bind in $t+j$, conditional of no further shocks occurring between t and $t+j$.[32]

2. Construct reduced form matrices $\mathcal{E}_t$, $\mathcal{F}_t$ and $\mathcal{G}_t$ using the methodology described in the previous section, given the guess provided in 1.

3. Feed the piecewise linear solution to a Kalman filter to obtain initial conditions X_0 and smoothed shocks $\{\epsilon_t\}_{t=1}^T$.

4. Given $\left\{X_0, \{\epsilon_t\}_{t=1}^T\right\}$, get a sequence of ZLB regimes using the occbin algorithm described above. Store the results in

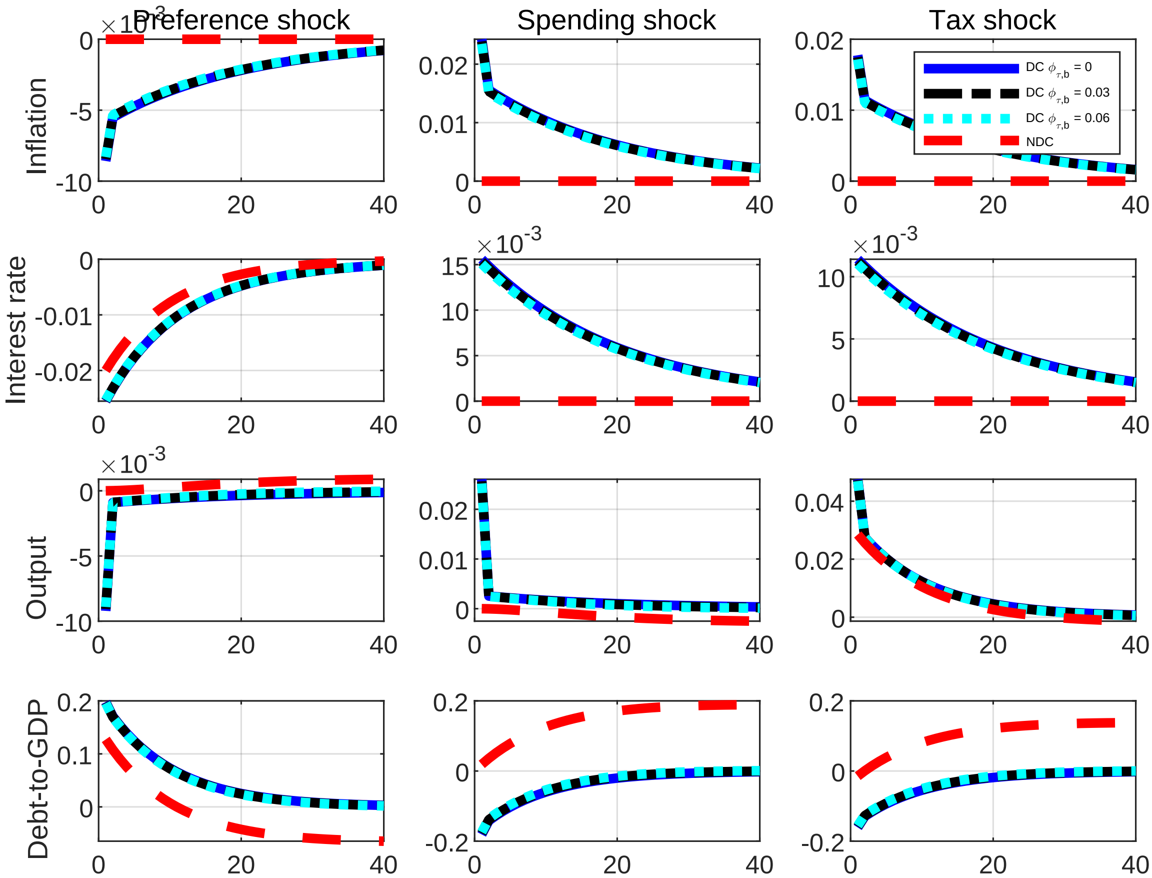
$$\mathcal{ZLB}^{new} = \{ZLB_t^{new}\}_{t=1}^T$$

where ZLB_t^{new} is defined analogously to ZLB_t in point 1.

5. Compare the sequence of regimes $\mathcal{ZLB}^{new}$ to $\mathcal{ZLB}$.
   - If they are identical, STOP.
   - If they differ, update the guess $\mathcal{ZLB} = \mathcal{ZLB}^{new}$ and go back to step 2.

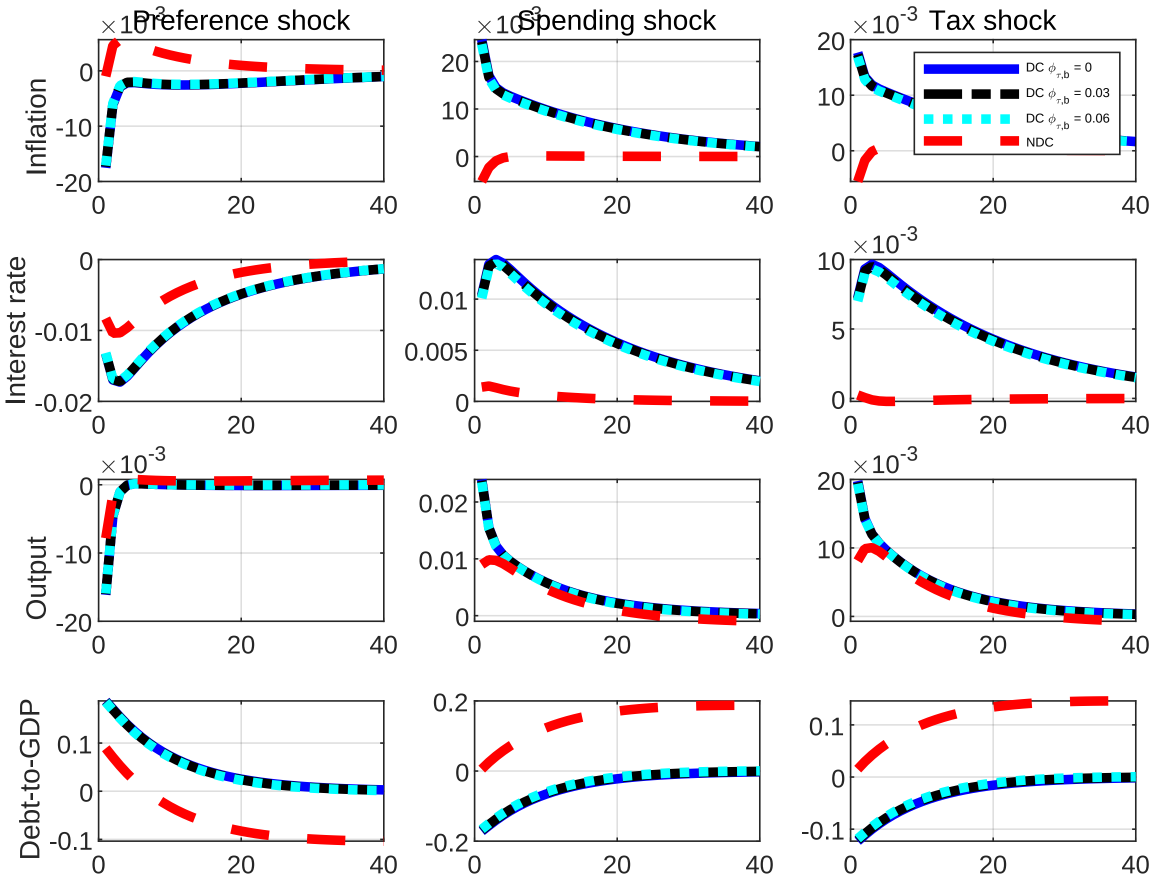
[32]Here, zlb_t is treated separately from $zlb_{t+j|t}$, for $j \geqslant 1$, in order to impose that, when the ZLB is observed to be binding in the data, it has to be that it is binding in the model. More formally, we set $zlb_t = 1(i_t^{US} \geqslant i^{ZLB})$, where i_t^{US} is the (annualized) US fed funds rate used in estimation, and we set i^{ZLB} to 0.05%. Note that this assumption differs from Kollmann et al. (2016) who let the algorithm choose whether the current (shadow) interest rate satisfies the constraint or not.

FIGURE 3.9: **Optimal Policies: Varying $\phi^{DC}_{\tau,b}$.**



Notes: The figure plots the response of model variables to a shock in preferences (left panels), spending (middle panels) and taxes (right panels) under various calibrations of $\phi^{DC}_{\tau,b}$. We set $\lambda_Y = \lambda_i = 0$ in the planner's objective.

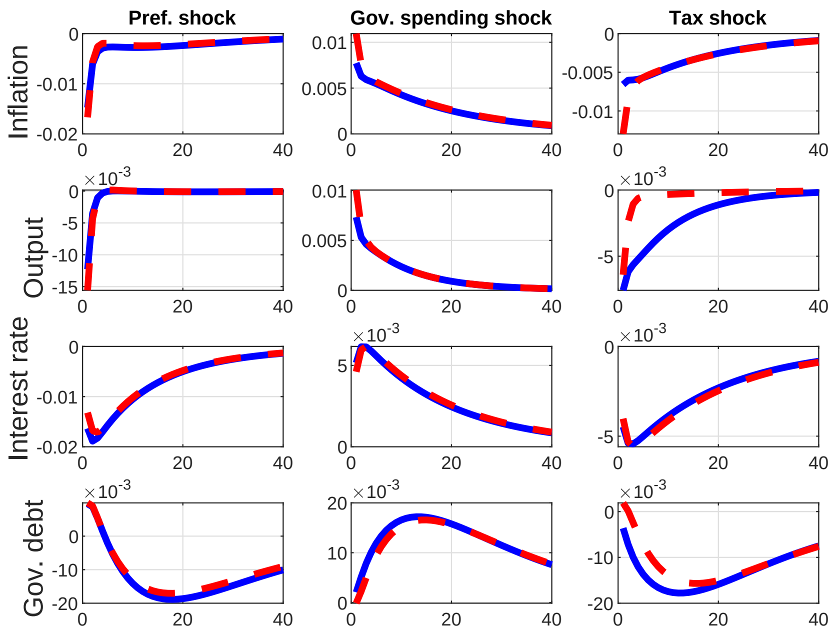
FIGURE 3.10: **Optimal Policies: : Varying $\phi^{DC}_{\tau,b}$ ($\lambda_Y = \lambda_i = 0.5$, $\sigma = 1$).**



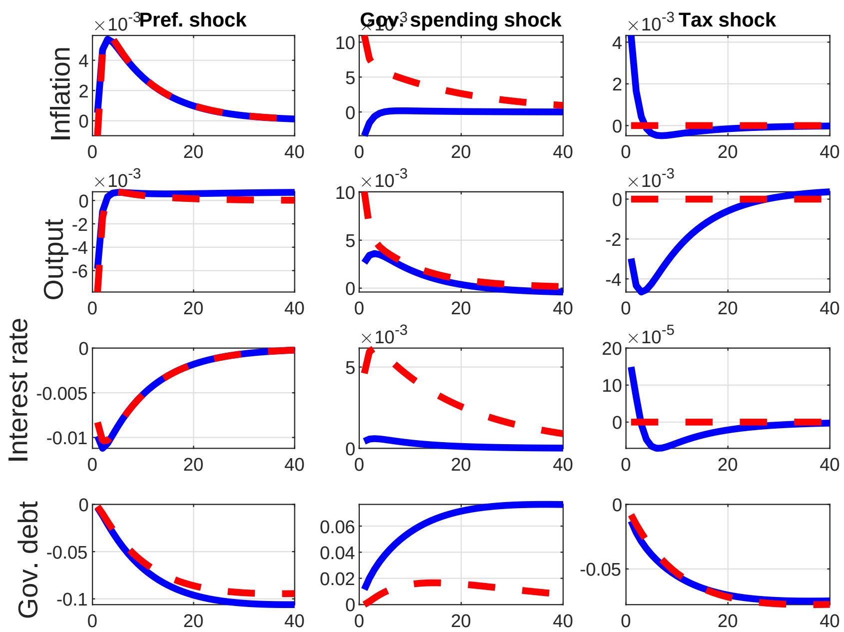
Notes: The figure plots the response of model variables to a shock in preferences (left panels), spending (middle panels) and taxes (right panels) under various calibrations of $\phi^{DC}_{\tau,b}$. We set $\lambda_Y = \lambda_i = 0.5$ in the planner's objective.

FIGURE 3.11: **Impulse responses: Lump-sum taxes**

a) Debt concerns

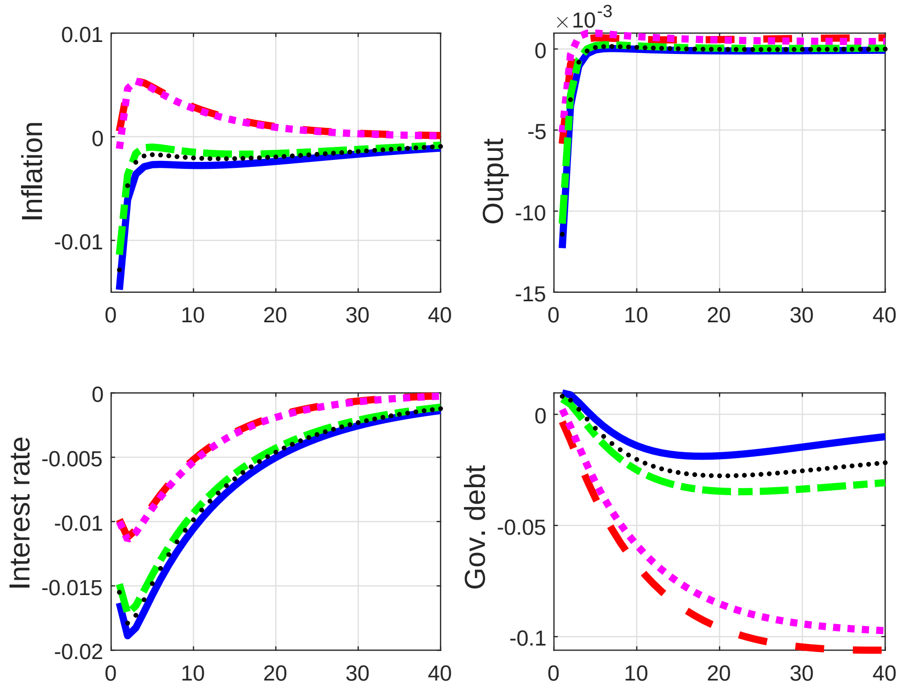


b) No debt concerns



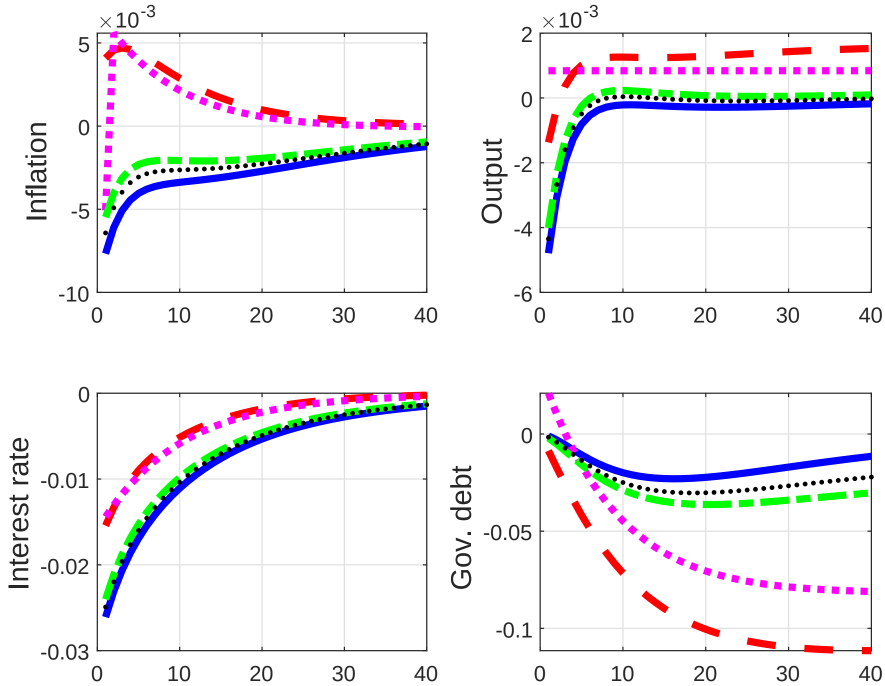
Notes: Solid blue lines: Distortionary taxes, Dashed red lines: Lump-sum taxes

FIGURE 3.12: **Impulse responses: Jointly optimal policies:** $\sigma = 1$



Notes: Solid blue line: DC, Dashed red line: NDC, Dotted pink line: Jointly optimal policies with $\lambda_\tau = 0$, Dash-dotted green line: Jointly optimal policies with $\lambda_\tau = 0.5$, Dotted black line: Jointly optimal policies with $\lambda_\tau = 1$.

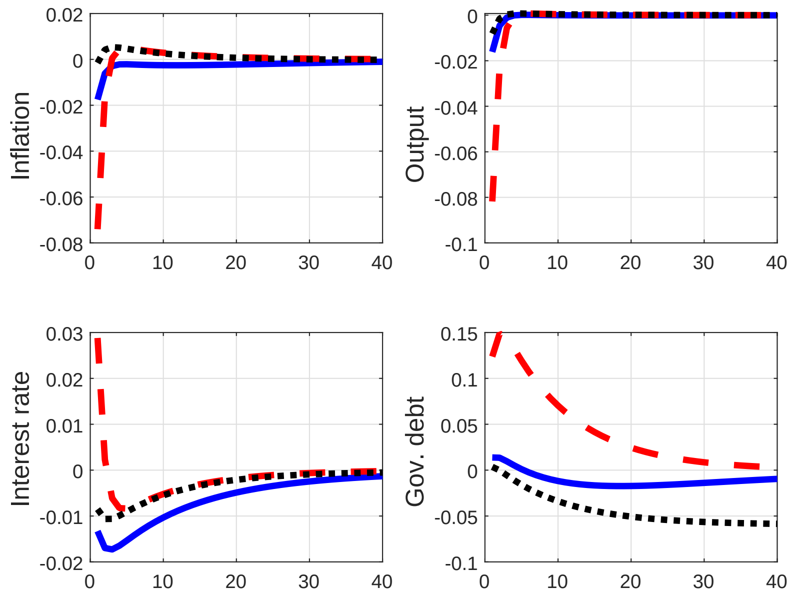
Figure 3.13: **Impulse responses: Jointly optimal policies** ($\sigma = 0$)



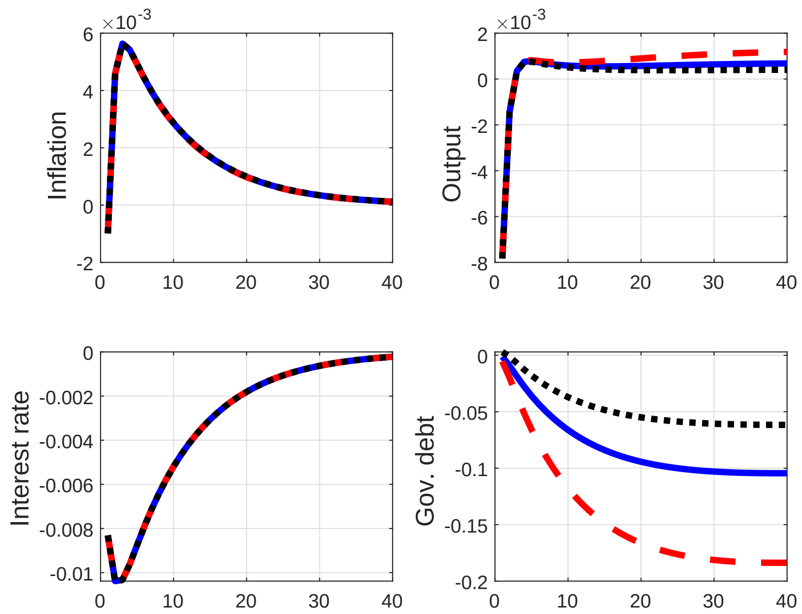
Notes: Solid blue line: DC, Dashed red line: NDC, Dotted pink line: Jointly optimal policies with $\lambda_\tau = 0$, Dash-dotted green line: Jointly optimal policies with $\lambda_\tau = 0.5$, Dotted black line: Jointly optimal policies with $\lambda_\tau = 1$.

FIGURE 3.14: **Impulse responses: Impact of debt maturity**

a) Debt concerns

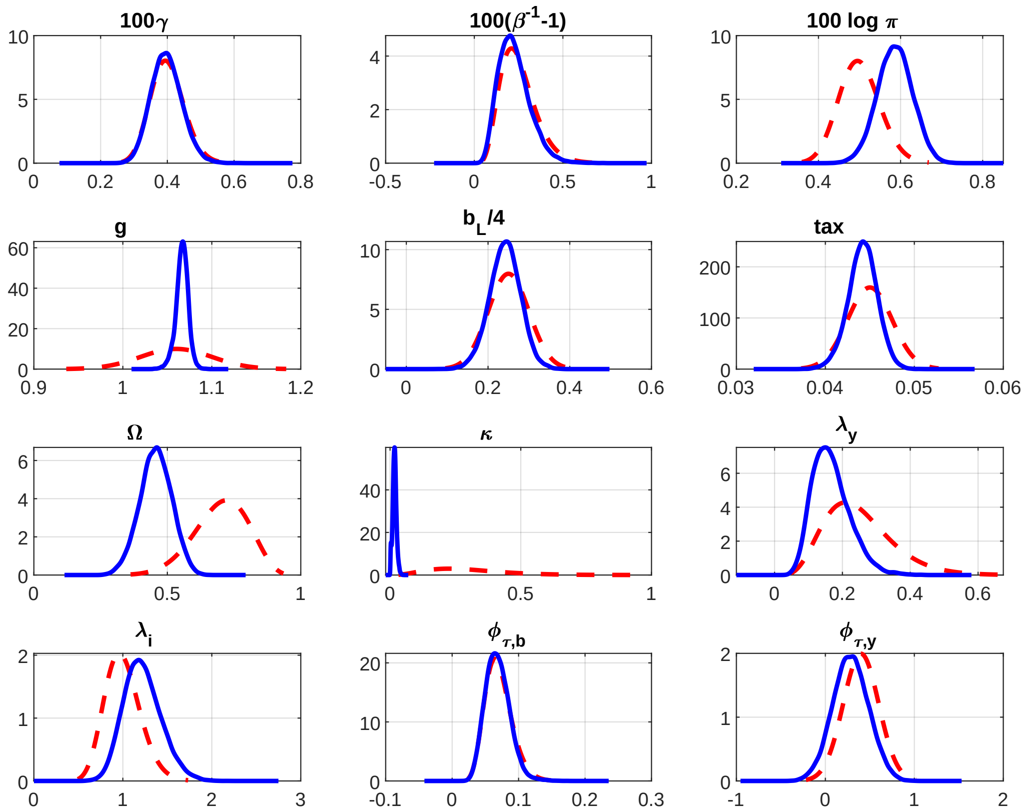


b) No debt concerns



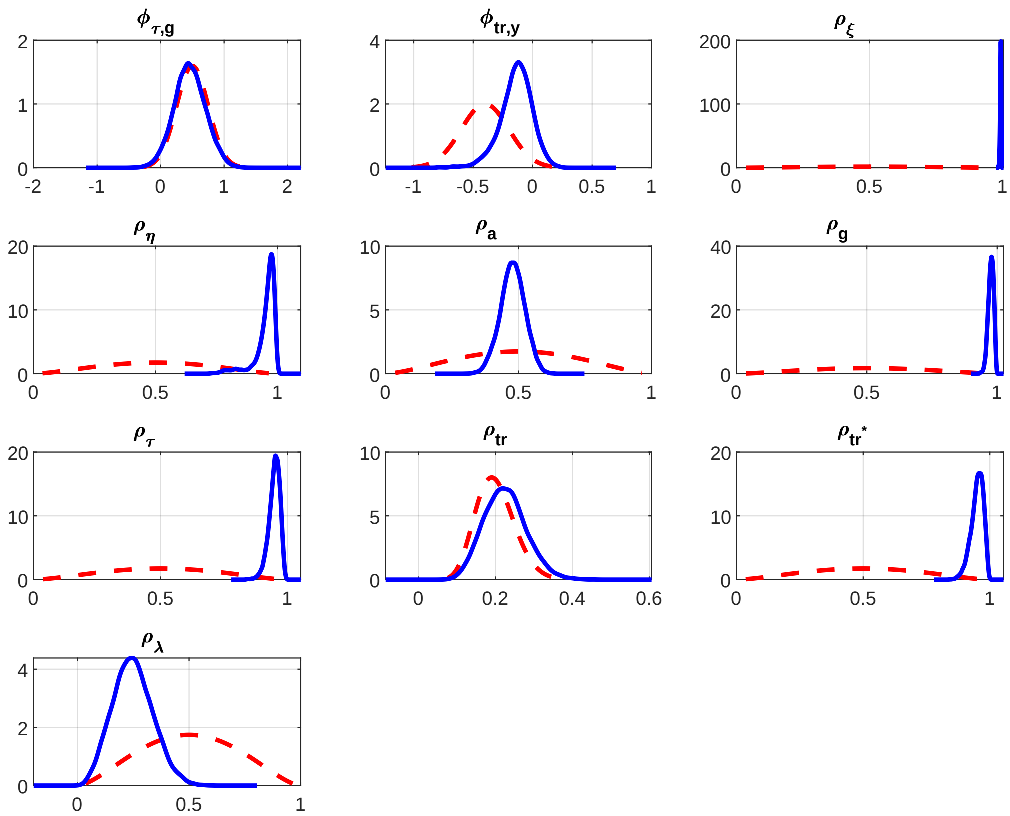
Notes: Solid blue lines: $\delta = 0.95$ (baseline), Dashed red lines: $\delta = 0$, Dotted black lines: $\delta = 1$.

Figure 3.15: **Posterior distributions – Parameters (1)**



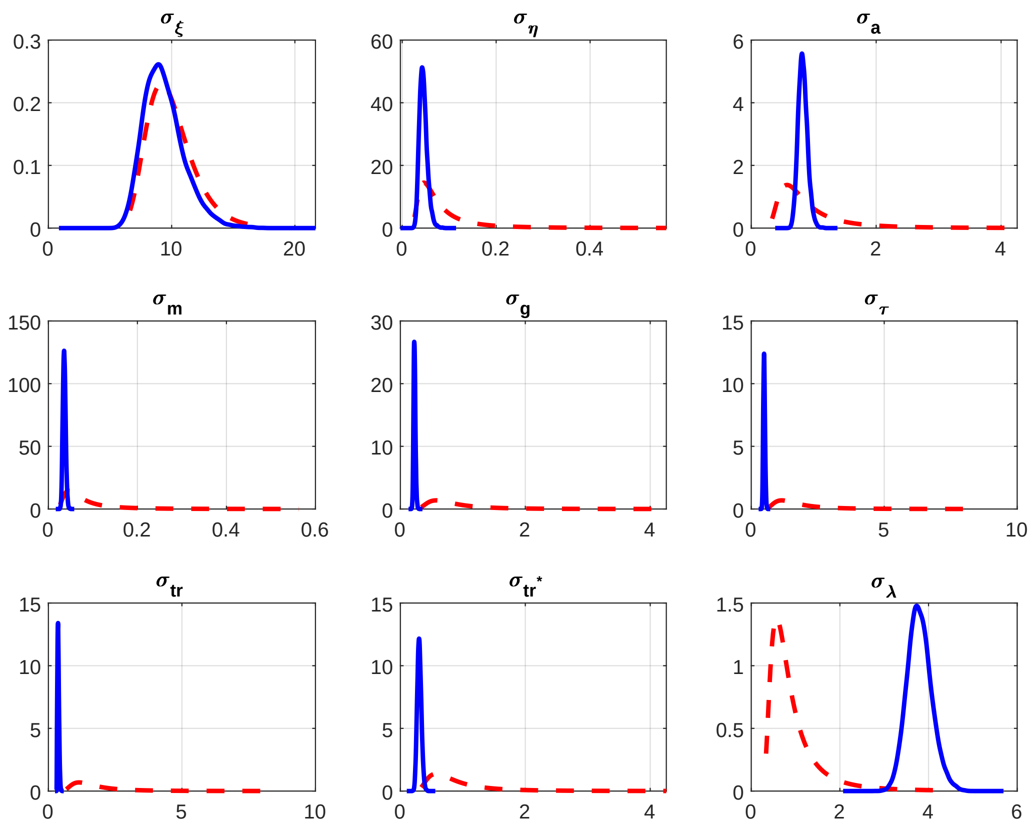
Notes: Solid blue lines represent posterior distributions of model parameters, obtained from the simulation of two chains of the Metropolis Hastings algorithm with 100,000 parameter draws. Dotted red lines give the prior distributions.

FIGURE 3.16: **Posterior distributions – Parameters (2)**



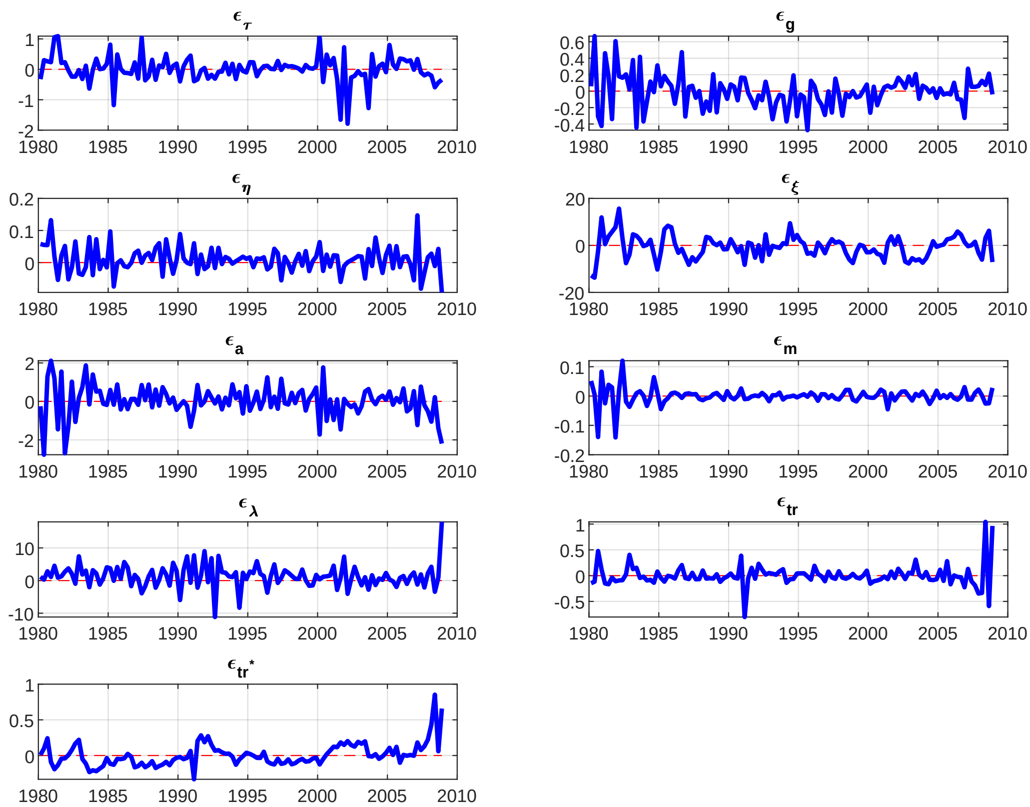
Notes: Solid blue lines show the posterior distributions of model parameters, obtained from the simulation of two chains of the Metropolis Hastings algorithm with 100,000 parameter draws. Dotted red lines show the prior distributions.

FIGURE 3.17: **Posterior distributions – Standard deviations of shocks**



Notes: Solid blue lines show the posterior distribution of the standard deviations of the i.i.d shock processes of the model, obtained from the simulation of two chains of the Metropolis Hastings algorithm with 100,000 parameter draws. Dotted red lines show the prior distributions.

FIGURE 3.18: **Smoothed shocks**



Notes: The figures provide the smoothed shock processes of the no debt concerns model with parameters at the mean of the posterior distribution, over the estimation sample 1980Q1-2008Q4. Initial conditions and shocks were recovered using the Kalman filter.

TABLE 3.3: **Calibrated parameters**

	Parameter	Value
y	steady state output (normalization)	1
$1-\alpha$	labor share	0.66
δ	decaying rate of coupon bonds	0.95
η	demand Elasticity	-7.66
γ_h	inverse of Frisch elasticity	1

Notes: The table reports model parameters whose values we fix in estimation. See text for details.

TABLE 3.4: **Estimated parameters**

	Parameter	Posterior		Prior		
		mean	90 % interval	distrib	par A	par B
Quarterly trends						
100γ	growth rate	0.397	[0.318 ; 0.469]	G	0.4	0.05
$100(\beta^{-1} - 1)$	discount rate	0.226	[0.089 ; 0.363]	G	0.25	0.1
Households and firms						
$100 \log \pi$	inflation	0.586	[0.517 ; 0.656]	G	0.5	0.05
g	g-to-GDP	1.066	[1.055 ; 1.077]	N	1.06	0.04
$b_L/4$	debt-to-GDP	0.241	[0.177 ; 0.301]	N	0.25	0.05
tax	taxes-to-GDP	0.044	[0.042 ; 0.047]	N	0.045	0.0025
Ω	habits	0.456	[0.359 ; 0.554]	B	0.7	0.1
κ	slope NKPC	0.017	[0.003 ; 0.028]	G	0.3	0.15
Central bank preferences						
λ_i	i.r smoothing	0.167	[0.08 ; 0.253]	G	0.25	0.1
λ_Y	y smoothing	1.218	[0.875 ; 1.557]	G	1	0.2
Fiscal rules						
$\phi_{\tau,b}$	τ response to b	0.068	[0.039 ; 0.097]	G	0.07	0.02
$\phi_{\tau,y}$	τ response to y	0.283	[-0.033 ; 0.622]	N	0.4	0.2
$\phi_{\tau,g}$	τ response to g	0.458	[0.037 ; 0.856]	N	0.5	0.25
$\phi_{tr,y}$	tr response to y	-0.141	[-0.356 ; 0.078]	N	-0.4	0.2
Shocks, persistence						
ρ_ξ	preference	0.993	[0.989 ; 0.996]	B	0.5	0.2
ρ_η	markup	0.95	[0.902 ; 0.996]	B	0.5	0.2
ρ_a	tfp	0.477	[0.4 ; 0.552]	B	0.5	0.2
ρ_g	gov. spending	0.976	[0.96 ; 0.993]	B	0.5	0.2
ρ_τ	tax rate	0.948	[0.914 ; 0.983]	B	0.5	0.2
ρ_{tr}	transfers	0.224	[0.133 ; 0.311]	B	0.2	0.05
ρ_{tr^*}	transfers trend	0.95	[0.912 ; 0.988]	B	0.5	0.2
ρ_λ	government b.c	0.247	[0.096 ; 0.389]	B	0.5	0.2
Shocks, standard deviations						
σ_τ	tax rate	9.365	[6.826 ; 11.963]	IG	10	2
σ_g	gov. spending	0.045	[0.031 ; 0.058]	IG	0.1	2
σ_η	markup	0.825	[0.703 ; 0.944]	IG	1	1
σ_ξ	preference	0.036	[0.03 ; 0.041]	IG	0.1	2
σ_a	tfp	0.226	[0.202 ; 0.251]	IG	1	1
σ_m	int. rate, m.e	0.484	[0.429 ; 0.535]	IG	2	2
σ_λ	government b.c	0.367	[0.318 ; 0.413]	IG	2	2
σ_{tr}	transfers	0.303	[0.248 ; 0.356]	IG	1	1
σ_{tr^*}	transfers trend	3.793	[3.328 ; 4.228]	IG	1	1

Notes: The table reports the prior and posterior distributions of the estimated parameters. The first column reports the mean of the posterior of each parameter, obtained from Monte-Carlo simulations of the posterior distribution using the MH algorithm. The second column reports the 90% HPD intervals obtained from the same draws. The third column indicates the assumed prior distribution (B: beta, G: gamma, IG: inverse gamma, N: normal). The fourth and fifth columns report the first and second moments of the priors.

Conclusions

This thesis contributes to the field of macroeconomics. It is composed of three chapters that investigate the role of household heterogeneity and the design of optimal policies in shaping macroeconomic outcomes.

Chapter 1 develops a model where households are heterogeneous to shed light on a new mechanism giving endogenously rise to pro-cyclical prices and purchases of used durable goods. Chapter 2 investigates the properties of optimal taxes and transfers in a framework where household heterogeneity is accounted for, and describes how the introduction of heterogeneity alters the prescriptions of an otherwise standard optimal fiscal policy framework. Chapter 3 provides a new optimal monetary policy framework in which the central bank may have concerns about the sustainability of government debt; the chapter bridges the gap between the optimal policy literature and the DSGE literature on monetary and fiscal policy interactions.

Put together, these three chapters contribute to enlarge the set of tools at our disposal for macroeconomic analysis, by investigating new properties of widely used models, and extending their scope. As is often the case with academic research, some of the results presented in the thesis call for further investigation. In this sense, it opens the route for future research prospects, to which I aim to contribute.

Léon Walras, the father of general equilibrium analysis, was reported to have said that "if one wants to harvest promptly, one should plant carrots and salads; if one has the ambition to plant oak trees, one must be wise enough to say: [posterity] will owe me this shade."[33] This is our collective responsibility as researchers to plant the seeds of a better world, as little as they can be. This is my ambition to help make these seeds grow into oaks, not salads.

[33] As reported in De Vroey (2016).

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