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Aliasing in near-field array ambiguity functions: a spatial frequency-domain framework

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Next-generation communication and localization systems increasingly rely on extremely large-scale arrays (XL-arrays), which promise unprecedented spatial resolution and new functionalities. These gains arise from their inherent operation in the near-field (NF) regime, where the spherical nature of the wavefront can no longer be ignored. Consequently, characterizing the ambiguity function–which amounts to the matched beampattern– is considerably more challenging. Implementing extremely large-scale arrays with half-wavelength element spacing is costly and complex, and removing antennas introduces intricate aliasing structures, i.e., grating lobes. Whereas prior work has addressed the challenging modeling of grating lobes using approximations tailored to specific array geometries, this paper develops a general framework that reveals their fundamental origins and geometric behavior in NF ambiguity functions. Using a local spatial-frequency analysis of steering signals, we derive a systematic methodology to model NF grating lobes as aliasing artifacts, quantify their structure on the ambiguity function, and provide guidelines to design XL-arrays operating within aliasing-safe regions. We further connect the presented framework to established far-field principles. Finally, we demonstrate the practical value of the approach by deriving closed-form expressions for aliasing-free regions in canonical uniform linear arrays and uniform circular arrays.

With the recent advances towards 6G telecommunication networks and next-generation localization systems, the importance of XL-arrays has risen due to the increased spatial resolution and spectral efficiency they are anticipated to provide[1,2]. This evolution is characterized by dimensions that fundamentally modify the operating regime of arrays. Consequently, the spherical nature of the wavefronts can no longer be ignored. This invalidates the conventional far-field (FF) condition—defined using the Fraunhofer distance—as it relies on a planar wavefront approximation. The NF regime in which XL-arrays operate enables a single array to transmit or receive steering signals that encode both range and angular information. This feature supports advanced functionalities such as high-resolution positioning or beam pointing[3,4]. Nonetheless, the NF regime also gives rise to beampatterns with increased structural complexity, which complicates the analytical characterization of system performance[5–7].

XL-arrays can be categorized based on their structural configurations, ranging from large co-located arrays (composed of numerous densely spaced antennas) to fully distributed XL-arrays that spread their constituent antennas over wide areas[8]. The latter is typically encountered in cell-free multiple-input multiple-output (MIMO) systems. On the one hand, co-located XL-arrays commonly aim to meet the seminal half-wavelength antenna spacing criterion in order to avoid grating lobes. In the FF regime, this phenomenon is well understood as the undesired illumination of multiple angles with similar power levels[9]. On the other hand, extending array dimensions while maintaining half-wavelength spacing introduces significant challenges in terms of computational complexity and physical constraints. These limitations have motivated recent studies to investigate alternative structures (including sparse, modular, and distributed arrays) that relax this spacing requirement while maintaining a large array aperture[10–12]. Such designs require dealing with effects analogous to the grating lobes, but exhibiting significantly more intricate structure than their FF counterparts. The analysis of these effects constitutes the scope of this paper.

In practice, this paper studies these effects as they appear in the *ambiguity function (AF)*. The AF is a fundamental tool used to evaluate important indicators of localization performance, such as the resolution and the ambiguities[7,13]. It is equivalent to the matched beampattern obtained with the maximum ratio transmission (MRT) beamformer[14]. The AF evaluates the correlations between all possible pairs of signals associated with locations observed by the array. In theory, the perfect AF would reduce to a Dirac delta function, being non-zero only when matching twice the

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same location. In practice, however, finite-sized arrays introduce non-orthogonality between distinct steering signals, typically resulting in AFs with oscillatory behaviors. The width of the main lobe characterizes the intrinsic resolution of the system. It indicates the minimum separation required between two sources to produce weakly correlated signals. The secondary lobes generate estimation ambiguities. It corresponds to a risk of confusion between distinct source locations, which, although far from one another, exhibit correlated signals.

Severe additional estimation ambiguities arise from the presence of grating lobes. Mitigating their effect is essential to reach high performance in both single- and multi-source contexts. In the FF, it is often possible to obtain closed-form expressions of the AF with an angle-distance representation of the locations. As a result, the FF grating lobes are well-understood: antenna spacing exceeding half the wavelength introduces angular repetitions. Thanks to this understanding, their presence can be mitigated by restricting the source locations to an angular sector as seen by the array. Alternatively, several other techniques have been suggested in the literature to address grating lobe mitigation in the FF. In ref. 15, a sparse MIMO system composed of several linear subarrays transmitting at distinct frequencies is considered. The authors in ref. 16 reduce the grating lobes by leveraging variations in subarray spacing, orientation, and density. In ref. 17, the authors resolve angular repetitions using a compressed sensing approach, constrained to an angular sector.

To mitigate the effect of grating lobes in the NF regime, it is essential to mathematically characterize their geometric structure in the AF. This task is particularly challenging because the NF regime introduces nonlinear spatial phase variations in steering signals, which generally preclude the derivation of exact closed-form expressions of the AF. Recent approaches have largely relied on angle-distance domain analysis, leveraging *simplifying assumptions* to obtain approximate closed-form expressions of the AF for specific array topologies. For example, the authors in ref. 1 exploit the unique regular structure of modular XL-arrays and define a sub-array-based uniform spherical wave model that assumes an inner FF regime within each module. In ref. 12, the achievable beamwidth that maintains a low grating lobe level is investigated in a similar modular XL-array context. Additionally, Zhou et al.[11] exploit Fresnel approximations to evaluate the beampatterns for linear sparse arrays and extended coprime arrays.

While these approaches provide valuable insights, they rely on context-specific simplifications that limit their generality. Moreover, they exploit definitions of NF grating lobes that heuristically extend those used in the FF regime, thereby inherently lacking a formal theoretical foundation in the NF regime. In the FF regime, grating lobes admits three *equivalent* definitions:

1. Secondary lobes with the same amplitude as the main lobe.
2. The effect of pairs of distinct locations (or angles) that produce identical steering signals up to a constant phase shift.
3. The effect of locations yielding discrete steering signals affected by spatial aliasing.

In the NF, the first two definitions rarely (and presumably never) occur because the non-linear phase structure prevents exact repetitions of steering signals. This behavior is observed in refs. 1,8, 18 and in the AFs numerically computed in our paper. By contrast, defining the grating lobes as spatial *aliasing artifacts* remains formally valid in the NF regime. Building upon this definition, we develop a framework that is both theoretically grounded in the structural origins of grating lobes and directly consistent with established FF conventions.

Our novel framework provides the key advantage of determining grating lobe properties without requiring the challenging derivation of closed-form expressions for the AF. Exploiting a *spatial frequency-domain* representation of signals, we can identify the set of locations that generates the spectral folding associated with visible grating lobes in the AF. Furthermore, the abstract geometric formulation of our methodology enables its applicability to a wide range of array topologies.

As a result, our framework supports the systematic design of NF XL-array systems that operate robustly with respect to grating lobes. Indeed, once the structure of aliasing effects is understood, antenna positions and spacings can be related to regions of aliasing-free operations. Building on this principle, the present paper departs from prior studies and proposes a theoretical construction of aliasing-safe operational areas for XL-arrays.

To establish this new framework, we compressively capture the spatial spectral content of steering signals by exploiting the concept of *local* spatial frequencies. This concept has already been formalized in multiple studies conducted in the context of channel degrees of freedom[12,19,20]. Similarly, NF steering signals can be interpreted as *spatial chirp*. This concept, notably reported in refs. 21–23, aligns with the standard instantaneous frequency representation of chirp signals in the time domain.

This paper therefore extends our preliminary findings[24] and proposes a systematic methodology to characterize the aliasing structure that arises in the AF of XL-arrays operating in the NF regime. It also derives practical design guidelines for XL-arrays. Using a local spatial-frequency (or "chirp-based") representation of steering signals, the proposed framework is connected to established results in the FF regime.

Consequently, the contributions of this paper are:

- A mathematical formalism to define the NF grating lobes as the effect of "spatial aliasing artifacts"
- A novel framework enabling the systematic mathematical description of the geometric structures induced by these artifacts on the AF;
- Practical guidelines for designing arrays that operate within aliasing-safe regions;
- A compressive modeling of the steering signal spectrum that connects the NF analysis with FF results;
- Interpretable closed-form expressions describing the complex grating lobe structures for canonical uniform linear arrays (ULAs) and uniform circular arrays (UCAs).

The current paper adopts the following mathematical notations. The imaginary unit is $j = \sqrt{-1}$ and c is the speed of light. The operator $\otimes$ denotes the convolution product. The symbol; separating inputs in functions, as $f(x; y)$, indicates its functional dependence on the variable x and parameterization by y. Bold lowercase letters denote vectors, and caligraphic-style uppercase letters, like $\mathcal{P}$, are sets. The notations and $\|\boldsymbol{a}\|$ and $\angle\boldsymbol{a}$ respectively are the ℓ_2-norm and the angle of the vector $\boldsymbol{a}$. Given the vector $\boldsymbol{a}$ and the scalar factor b, the set $b \cdot \mathcal{P} + \boldsymbol{a}$ is identical to $\mathcal{P}$ but rescaled by b and translated by $\boldsymbol{a}$.

Results

The main contribution of this paper is the development of the proposed framework. Accordingly, the "Results" section is organized as follows.

The *System Model* section introduces the theoretical model considered in this paper. The *Aliasing Artifacts in the Ambiguity Function* section formalizes the concepts of the AF and aliasing artifacts, and presents the proposed framework to understand their structure. The *Local Spatial-Frequency Modeling* section introduces a spectral modeling of the steering signals that connects the proposed framework to established FF principles. This section also details the practical guidelines that originate from our framework. The final two sections apply the proposed theoretical framework to specific ULA and UCA topologies, providing closed-form expressions that explain the structure of the resulting grating lobes.

System model

The content of this paper is formulated for a canonical uplink scenario in which a static user equipment (UE), located in $\boldsymbol{x}$, transmits a unit-energy baseband signal $s(t)$ modulated onto a carrier with frequency f_c. As illustrated in Fig. 1, the transmitted signal is received by an XL-array. The antennas are located on the grid $\mathcal{Z}_G$ that discretizes the continuous-space domain $\mathcal{Z}$. Note that our system model allows $\mathcal{Z}_G$ to represent *any* discrete subset of $\mathcal{Z}$, and is therefore not restricted to uniform planar grids. The receiver performs network-based positioning, assuming that the source location $\boldsymbol{x}$ belongs to an *operating domain* $\mathcal{X} \subseteq \mathbb{R}^d$.

Following physical-optics principles, the antenna array element (AAE) located in $\boldsymbol{z} \in \mathcal{Z}$ measures, up to an additive white Gaussian noise

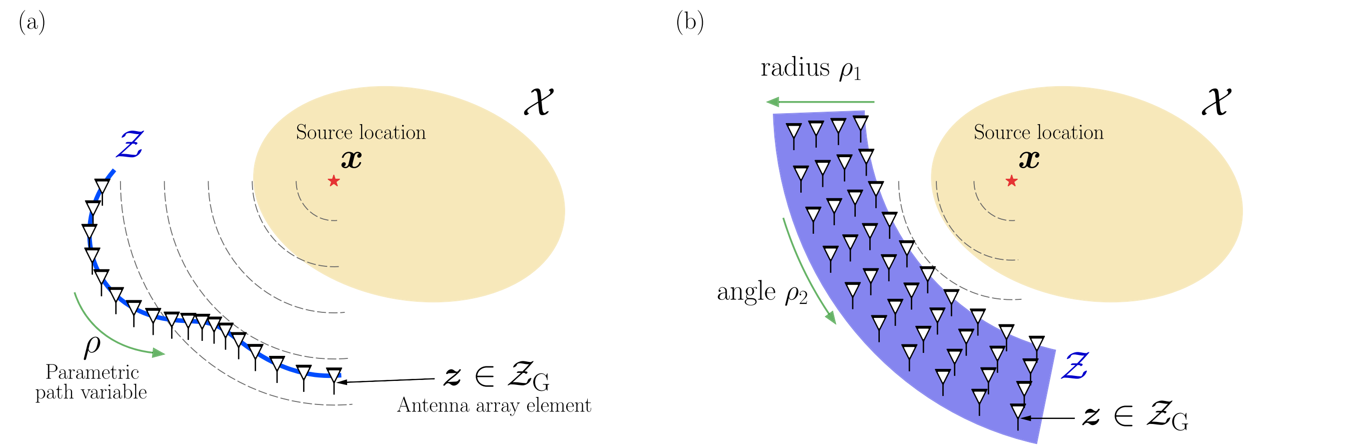


Fig. 1 | Schematic representation of the uplink scenario studied in this paper. a A case where the antenna array is defined over a curve $\boldsymbol{z}$ parameterized by a scalar path variable ρ. **b** A case where the antenna array is parameterized with two path variables. Representation of the simple case of the polar description.

(AWGN), the baseband equivalent signal

$$s_{\mathrm{R}}(t,\boldsymbol{z};\boldsymbol{x}) = \frac{e^{-jk_c\|\boldsymbol{z}-\boldsymbol{x}\|}}{\|\boldsymbol{z}-\boldsymbol{x}\|} s\left(t - \frac{\|\boldsymbol{z}-\boldsymbol{x}\|}{c}\right), \tag{1}$$

where $k_c = 2\pi f_c/c = 2\pi/\lambda_c$.

To simplify mathematical developments, we use a normalized formulation with respect to the total received energy

$$\mathcal{E}(\boldsymbol{x}) = \int_{\mathcal{Z}} \|\boldsymbol{z}-\boldsymbol{x}\|^{-2}\, d\boldsymbol{z}. \tag{2}$$

Using the above, we can now write

$$s_{\mathrm{R}}(t,\boldsymbol{z};\boldsymbol{x}) = \mathcal{E}^{\frac{1}{2}}(\boldsymbol{x})\, h(\boldsymbol{z};\boldsymbol{x})\, s\left(t - \frac{\|\boldsymbol{z}-\boldsymbol{x}\|}{c}\right), \tag{3}$$

where the wave propagation coefficients, or the *steering signal*,

$$h(\boldsymbol{z};\boldsymbol{x}) = a(\boldsymbol{z};\boldsymbol{x}) \exp(-j\phi(\boldsymbol{z};\boldsymbol{x})), \tag{4}$$

represents the energy-normalized attenuation and phase received at the antenna located in position $\boldsymbol{z}$, parameterized by the given source location $\boldsymbol{x}$, with

$$\phi(\boldsymbol{z};\boldsymbol{x}) = k_c \|\boldsymbol{z}-\boldsymbol{x}\|, \tag{5}$$

$$a(\boldsymbol{z};\boldsymbol{x}) = \left(\mathcal{E}^{\frac{1}{2}}(\boldsymbol{x})\,\|\boldsymbol{z}-\boldsymbol{x}\|\right)^{-1}. \tag{6}$$

By definition, the steering signal $h(\boldsymbol{z};\boldsymbol{x})$ has unit energy regardless of the value of $\boldsymbol{x}$.

The current paper assumes a narrowband signal $s(t)$, therefore focusing exclusively on the wave propagation coefficients described by $h(\boldsymbol{z};\boldsymbol{x})$. Consideration of wideband aspects and extensions to moving UEs is left for future work.

In the NF regime, the phase $\phi(\boldsymbol{z};\boldsymbol{x})$ varies *non-linearly* across $\boldsymbol{z}$, enabling beam pointing at a specific location. While the *continuous-space* steering signal $h(\boldsymbol{z};\boldsymbol{x})$ achieves this focusing, its *discrete-space* counterpart (obtained from the finite set of antennas in $\mathcal{Z}_G$) may introduce grating lobes, resulting in localization ambiguities. As conventional FF interpretations of the grating lobes ignore the non-linear phase structure, they do not apply in the NF. This is a gap our framework addresses in the following sections.

This paper adopts a generalized parametric model based on a *bijective function* v mapping a compact parametric domain $\mathcal{P} \in \mathbb{R}^q$ onto the antenna domain $\mathcal{Z}$, such that

$$\mathcal{Z} = \{v(\rho) : \rho \in \mathcal{P}\}. \tag{7}$$

For example, Fig. 1a shows a 1D curved domain $\mathcal{Z}$ parameterized by the path variable ρ. Figure 1b shows a case where ρ is a polar description of $\boldsymbol{z}$. This parameterization enables us to

(i) represent a wide range of array topologies within a single framework, including canonical ULAs, UCAs, and paving the way for more complex structures,
(ii) assume only uniform Cartesian sampling of the parametric domain $\mathcal{P}$, denoted by $\mathcal{P}_G$, without loss of generality in the definition of $\mathcal{Z}_G$.

Any grid $\mathcal{Z}_G$ can be described by a uniform Cartesian grid $\mathcal{P}_G$ via a bijection v, preserving the generality of our approach. This mapping allows grating lobes originating from any set of locations $\mathcal{Z}_G$ to be analyzed using standard Fourier theory on $\mathcal{P}_G$.

For clarity, this paper expresses its results only for $q = 1$, as represented in Fig. 1a, with Δ denoting the sampling step generating $\mathcal{P}_G$ from $\mathcal{P}$. The extension to $q > 1$ can be obtained by following the steps of this paper with multi-dimensional Fourier transforms, in which the spectral folding must be considered separately in each dimension.

By substituting $\boldsymbol{z} = v(\rho)$ in the steering signal model, the analysis focuses on the signal $h(v(\rho);\boldsymbol{x})$ as a function of ρ.

Aliasing artifacts in the ambiguity function

In this section, we define the AF for the NF sensing scenario introduced in the *System Model* section. We establish a formalism to characterize the NF grating lobes as the result of an aliasing process. To illustrate the methodology, we present a toy example consisting of a horizontal ULA of length $1000\lambda_c$ centered at the origin. Defining the trivial transformation $v_{\mathrm{toy}}(\rho) := [\rho, 0]^{\top}$, we build the grid $\mathcal{Z}_G$ by taking values of ρ in the uniform grid $\mathcal{P}_G$ with a spacing $\Delta = 10\lambda_c$. For example, with a high carrier frequency f_c = 60GHz, this corresponds to a co-located 5m-long linear array with a 5cm antenna spacing. Alternatively, at a lower carrier frequency f_c = 200MHz, it can also correspond to a *distributed* array composed of 100 access points spaced 15m apart. We fix the user location in $\boldsymbol{x} = [0, 600\lambda_c]$, centered in front of the array, in its NF operational region.

Near-field ambiguity function. The AF is an important tool for evaluating the structural properties of a sensing system. Motivated by a maximum likelihood estimation approach in the presence of AWGN, it is defined as the noise-free matched-filter response of the sensing system for a *candidate* position $\tilde{\boldsymbol{x}}$, while the *true* source's position is $\boldsymbol{x}$. For a

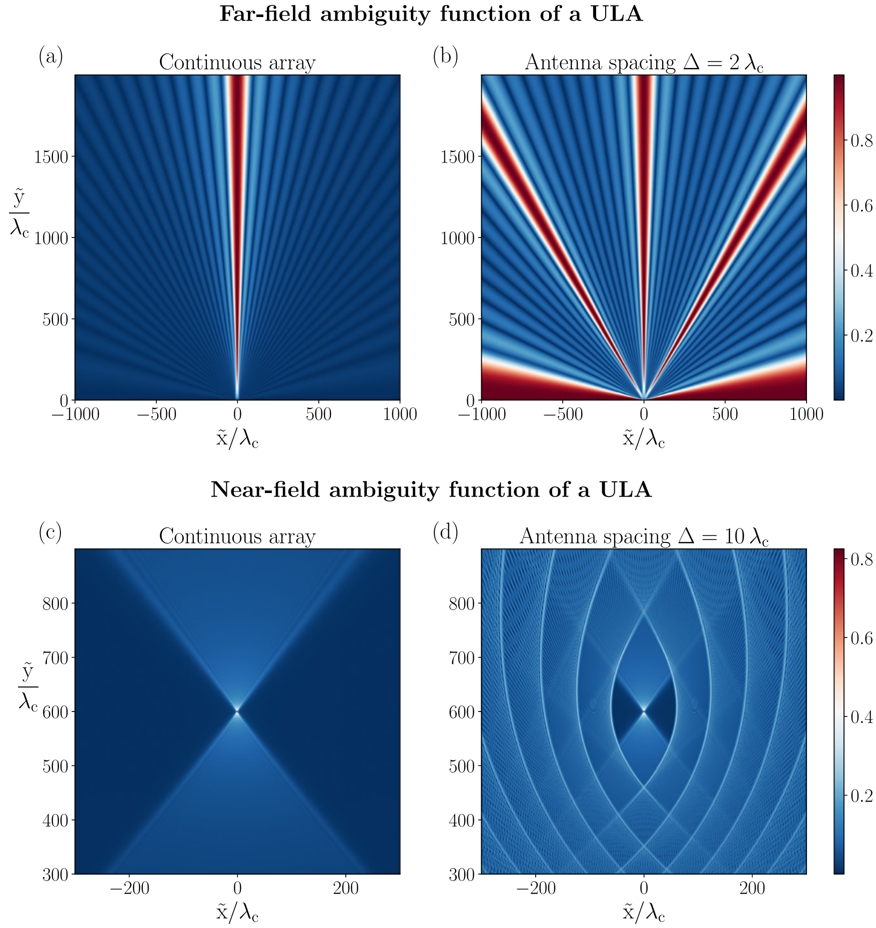


Fig. 2 | Ambiguity functions for (top) a horizontal ULA with length $20\lambda_c$ operating in the FF, and (bottom) the ULA from our toy example in the NF, as a function of $\widetilde{\boldsymbol{x}} = [\widetilde{\mathrm{x}}, \widetilde{\mathrm{y}}]^\top$ given the fixed location $\boldsymbol{x} = [\mathrm{x}, \mathrm{y}] = [0, 600]\lambda_c$. It shows in (**a–c**) the continuous-space $|A(\widetilde{\boldsymbol{x}}, \boldsymbol{x})|$ and in (**b–d**) the discrete-space $|A_S(\widetilde{\boldsymbol{x}}, \boldsymbol{x})|$, with Δ chosen for improved visibility of the grating lobes.

wideband localization system, its standard expression is

$$\mathcal{A}(\widetilde{\boldsymbol{x}}, \boldsymbol{x}) := \sum_{\boldsymbol{z} \in \mathcal{Z}_G} \int_{-\infty}^{\infty} \overline{s_R}^*(t, \boldsymbol{z}; \widetilde{\boldsymbol{x}})\, \overline{s_R}(t, \boldsymbol{z}; \boldsymbol{x})\, dt, \tag{8}$$

where $\overline{s_R}(t, \boldsymbol{z}; \boldsymbol{x}) = \mathcal{E}^{-\frac{1}{2}}(\boldsymbol{x}) s_R(t, \boldsymbol{z}; \boldsymbol{x})$.

In practice, the AF is inherently tied to the probability of erroneously detecting $\widetilde{\boldsymbol{x}}$ instead of the true location $\boldsymbol{x}$. When multiple sources must be located, it also quantifies the mutual interference between signals received from the different UEs. Higher values of $\mathcal{A}(\widetilde{\boldsymbol{x}}, \boldsymbol{x})$ indicate a higher probability of failing to resolve the two locations. This situation typically arises, by definition, when $\widetilde{\boldsymbol{x}}$ and $\boldsymbol{x}$ are separated by less than the array's intrinsic resolution. However, the grating lobes reveal pairs of *further apart* locations that can also be unresolvable. This emphasizes the need to study their properties.

In the narrowband settings of this paper, the AF reduces to the correlation function between possible pairs of steering signals, providing the theoretical signature of the antenna array. For the continuous steering signal, it is expressed as

$$A(\widetilde{\boldsymbol{x}}, \boldsymbol{x}) := \int_{\mathcal{P}} g(\rho; \widetilde{\boldsymbol{x}}, \boldsymbol{x})\, d\rho, \tag{9}$$

where we define the *matched signal* as

$$g(\rho; \widetilde{\boldsymbol{x}}, \boldsymbol{x}) := h^*(\nu(\rho); \widetilde{\boldsymbol{x}})\, h(\nu(\rho); \boldsymbol{x}), \tag{10}$$

for the simplicity of the notations throughout this section. Considering the discrete-space steering signal observed by the antennas in $\mathcal{Z}_G$, the discrete-space AF is expressed as

$$A_S(\widetilde{\boldsymbol{x}}, \boldsymbol{x}) := \Delta \sum_{\rho \in \mathcal{P}_G} g(\rho; \widetilde{\boldsymbol{x}}, \boldsymbol{x}). \tag{11}$$

The AF behaves differently in the FF and NF regimes. Figure 2 illustrates this contrast by showing continuous- and discrete-space AFs for ULAs operating in both contexts. Compared to its FF counterpart, the NF AF achieves a high resolution in both angle and range.

With the large antenna spacing used in Fig. 2b, d, the discrete-space AFs display grating lobes. Their well-understood angular structure in the FF regime appears in Fig. 2b, whereas Fig. 2d shows the more complex geometry observed in the NF regime. By developing on the *aliasing* nature of the NF grating lobes, this paper mathematically characterizes their origin and their intricate geometrical structure. Although the resolutions of the main lobe and the grating lobes are additional important factors affecting both localization and communication performance, extending our framework to include this aspect is left for future studies.

We finally highlight the mathematical connection between the narrowband AF and the concept of beampattern, which is commonly used to study the performance of communication systems. The beampattern is classically defined as the set of correlations between each steering vector and a *generic* beamformer, i.e., one not restricted to the set of steering vectors. As discussed in ref. 14, the AF in (11) corresponds precisely to the beampattern obtained with beamforming weights tailored for the specific MRT (or maximum ratio combining (MRC)) strategy.

Aliasing artifacts. We can now develop the spectral folding process that generates "aliasing artifacts" in $A_S(\widetilde{\boldsymbol{x}}, \boldsymbol{x})$, thereby defining the NF grating

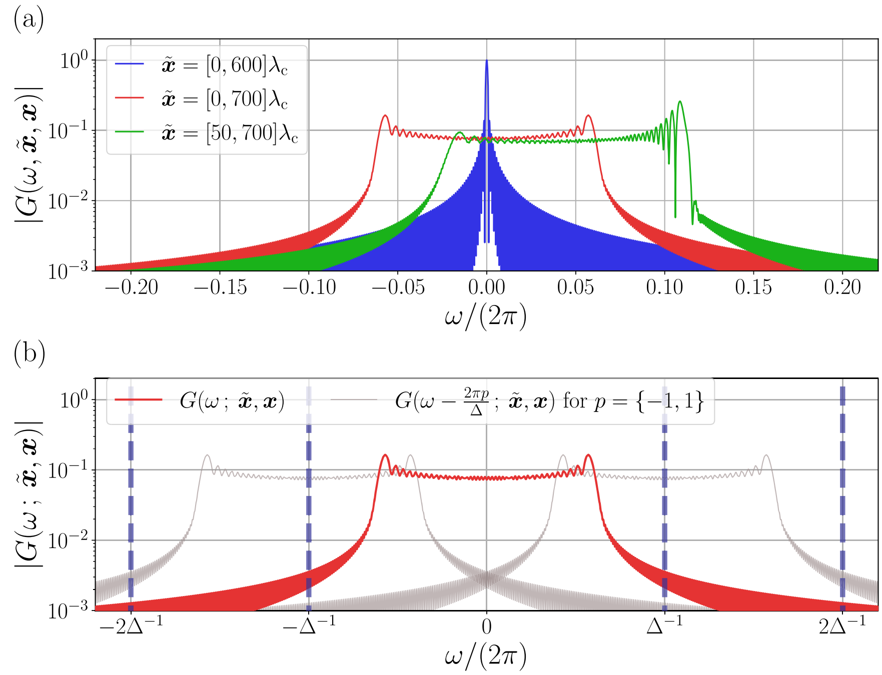


Fig. 3 | Matched spectrum's amplitude $|G(\omega; \tilde{x}, x)|$ corresponding to the toy example. a Comparison of the spectra for multiple tested locations $\tilde{x}$, given $x = [0, 600]\lambda_c$. **b** Repetition of the matched spectrum, due to sampling the matched signal with spacing Δ.

lobes. To this end, we exploit the mathematical connection between (9) and (11). The continuous-space AF in (9) can be interpreted as evaluating the Fourier transform (FT), $G(\omega; \tilde{x}, x)$, of the matched signal $g(\rho; \tilde{x}, x)$ for $\omega = 0$. Indeed, given that

$$G(\omega; \tilde{x}, x) = \int_{\mathcal{P}} g(\rho; \tilde{x}, x) e^{-j\omega\rho}\, d\rho, \tag{12}$$

we have

$$A(\tilde{x}, x) = G(0; \tilde{x}, x). \tag{13}$$

Figure 3a illustrates the spectra $G(\omega; \tilde{x}, x)$ for several tested locations $\tilde{x}$. The reference spectrum obtained when $\tilde{x} = x$ exhibits a unit-height peak at $\omega = 0$, yielding $A(x, x) = 1$. As $\tilde{x}$ becomes further away from x, the spectral energy in $G(\omega; \tilde{x}, x)$ is both frequency-spread and shifted, lowering its value in $\omega = 0$ and thus decreasing $A(\tilde{x}, x)$.

The discrete-space AF, given in (11), analogously amounts to the FT of the discretized matched signal, in $\omega = 0$. Uniformly sampling g causes repetitions in its spectrum, which directly lead to

$$A_S(\tilde{x}, x) = \sum_{p \in \mathbb{Z}} G\left(\frac{2\pi}{\Delta} p; \tilde{x}, x\right) \tag{14}$$

$$= A(\tilde{x}, x) + \sum_{p \in \mathbb{Z}\setminus\{0\}} G\left(\frac{2\pi}{\Delta} p; \tilde{x}, x\right). \tag{15}$$

The right-side summation of (15) isolates the aliasing artifacts affecting $A_S(\tilde{x}, x)$ relatively to the continuous-space reference $A(\tilde{x}, x)$. These aliasing terms are strictly zero when the spectrum $G(\omega; \tilde{x}, x)$ spans frequencies that are lower, in absolute value, than the sampling frequency $2\pi/\Delta$. This observation is similar to the Shannon-Nyquist sampling theorem, but is relaxed by a factor of 2. This factor stems from the specificity that aliasing artifacts appear in $A_S(\tilde{x}, x)$ only when spectral folding affects G in $\omega = 0$. Folding at other frequencies has no consequence on the discrete-space AF.

Formally, defining the maximum frequency content, coined the "strict band limit", of the matched signal $g(\rho; \tilde{x}, x)$, as

$$\overline{K_0}(\tilde{x}, x) = \max\{|\omega| : |G(\omega; \tilde{x}, x)| > 0\}, \tag{16}$$

this *sufficient* condition is given in Lemma 1.

Lemma 1. Given a pair of locations $\tilde{x}$ and x, if the band limit satisfies

$$\overline{K_0}(\tilde{x}, x) \le 2\pi\Delta^{-1}, \tag{17}$$

then the equality $A_S(\tilde{x}, x) = A(\tilde{x}, x)$ holds.

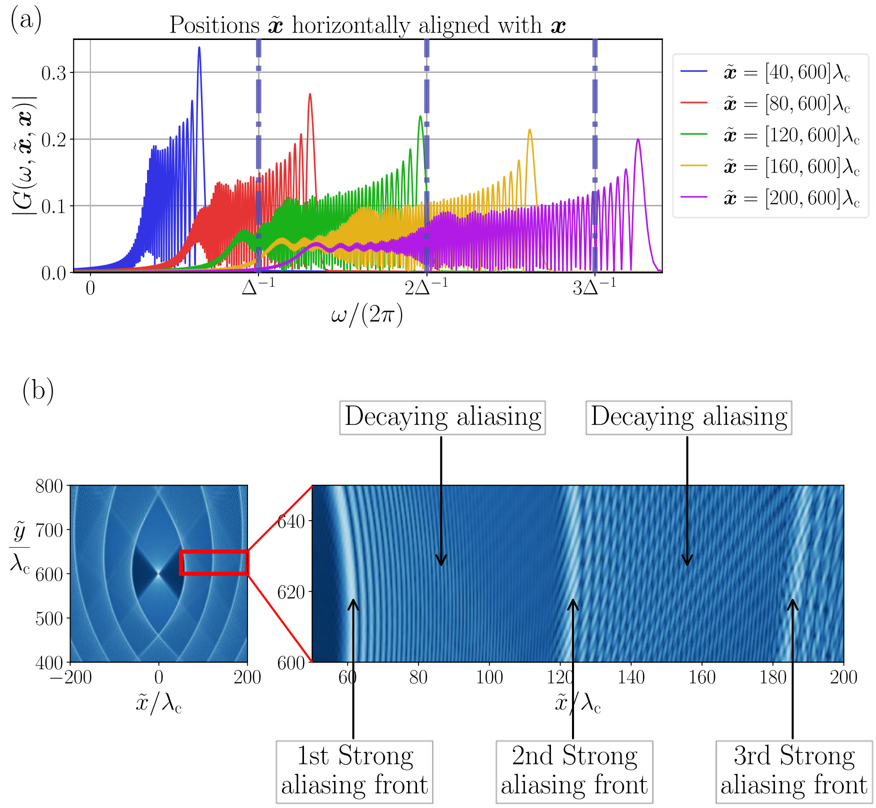
Proof. By construction of (16), the condition (17) directly nullifies all the terms in the right-side summation of (15).

Reciprocally, for a given location x, aliasing artifacts can appear in the discrete-space AF for the tested locations $\tilde{x}$ that do *not* satisfy (17). As a result, Lemma 1 is the key to determining the set of $\tilde{x}, x$ that are unaffected by grating lobes in $A_S(\tilde{x}, x)$.

In practice, however, the strict band limit used in Lemma 1 is typically infinite, as illustrated in Fig. 3. Instead, a common practice is to define the band limit in a *soft* manner, using a small tolerance threshold ϵ rather than a strict zero in (16). Yet, this intuitive approach lacks consistency with established FF conventions, and is sensitive to the choice of ϵ. As detailed in the *Local Spatial-Frequency Modeling* section, we address this matter by introducing a chirp-based compressive representation of the spectral content of $G(\omega; \tilde{x}, x)$. This representation bridges our NF framework with the classical FF interpretation of grating lobes, and supports design guidelines for XL-arrays.

Figure 3b illustrates the spectral repetitions for $\tilde{x} = [0, 700]\lambda_c$, at every multiple of $\frac{2\pi p}{\Delta}$. Although significant folding occurs in some part of the spectrum, the zero-frequency is barely affected, resulting in no visible artifacts in the discrete-space AF in $\tilde{x} = [0, 700]\lambda_c$ (see Fig. 2b). Intuitively, Lemma 1 states that if the spectrum's "significant" support remains entirely within the $\frac{2\pi}{\Delta}$ limits, then $A_S(\tilde{x}, x)$ remains aliasing-free. Importantly, outside the specific FF case, the AF itself is *not* directly repeated by the sampling

Fig. 4 | Representation of the spreading of the spectrum $G(\omega;\widetilde{\boldsymbol{x}},\boldsymbol{x})$ and its effect on the discrete-space AF displayed in Fig. 2. **a** Spectrum for multiple locations $\widetilde{\boldsymbol{x}}$ given $\boldsymbol{x}=[0,600]\lambda_c$. **b** Zoom in Fig. 2b showing the distinct aliasing regions in the resulting discrete-space AF.



of g. Rather, the geometry of the artifacts altering $A_S(\widetilde{\boldsymbol{x}},\boldsymbol{x})$ is dictated, in each direction around $\boldsymbol{x}$, by the spread and shift behaviors of G toward the $\frac{2\pi}{\Delta}$ boundaries as $\widetilde{\boldsymbol{x}}$ moves away from $\boldsymbol{x}$.

By observing the spectra in Fig. 3, we note that the band edges exhibit sharp peaks that are a few dB higher than the bands' centers. When $\widetilde{\boldsymbol{x}}$ reaches a value such that one peak crosses the critical frequency $\frac{2\pi}{\Delta}$, a strong aliasing front appears in $A_S(\widetilde{\boldsymbol{x}},\boldsymbol{x})$. As $\widetilde{\boldsymbol{x}}$ continues to move away from $\boldsymbol{x}$, the spectrum keeps spreading its energy, thereby lowering its amplitude at $\frac{2\pi}{\Delta}$. Figure 4a illustrates this progressive spectral spreading. This results in a decaying aliasing amplitude as we enter deeper into the aliased region of the AF. When the spectrum spreads sufficiently to cross $\frac{4\pi}{\Delta}$, a second strong aliasing front appears and then decays again. This process repeats for all $p\geq 1$ as illustrated in Fig. 4b, which zooms on the aliasing patterns we observed in Fig. 2b.

We note that the width of these aliasing fronts reflects the width of the prominent peak in the spectra shown in Fig. 2a. Analyzing these widths, however, lies beyond the scope of the current study, which is focused on determining the boundaries of the aliasing-free regions.

As discussed earlier, the spectra observed in these figures illustrate the necessity to define a soft band limit that gives a practical meaning to Lemma 1. Since $|G(\omega;\widetilde{\boldsymbol{x}},\boldsymbol{x})|$ never durably reaches zero, we highlight that strictly speaking, non-zero aliasing artifacts arise for any sampling step, even finer than $\frac{\lambda_c}{2}$. This has already been stated, for example, in ref. 25. This principle also holds in the FF regime, where a sufficiently small sampling step that avoids grating lobes still deteriorates the theoretical cardinal sine-shaped AF into a Dirichlet kernel-shaped one (A *Dirichlet kernel* denotes the $\frac{\sin(Nx)}{\sin(x)}$ function.).

Consequently, the next section presents our approach to define a soft band limit that enables deriving closed-form expressions of the aliasing structure in the NF regime and remains consistent with the FF conventions.

Local spatial-frequency modeling

To deal with the theoretical impossibility of satisfying (17), we aim to replace its right term by a softer alternative. Instead of an intuitive ϵ-tolerant band, we leverage the theoretical findings reported in ref. 26, and follow the preliminary results elaborated in ref. 24. More precisely, we exploit a chirp-based representation of the matched function g. This approach:

1. provides a compressive spectral representation of $G(\omega;\widetilde{\boldsymbol{x}},\boldsymbol{x})$;
2. enables *closed-form expressions* of the aliasing artifact fronts in the discrete-space AF;
3. reduces to known results in the FF regime, hence providing a generalized framework.

Chirp-based analysis of aliasing artifacts. Let us define notations to separate the matched signal by its phase and amplitude as

$$g(\rho;\widetilde{\boldsymbol{x}},\boldsymbol{x})=\alpha(\rho)\exp(-j\xi(\rho)), \tag{18}$$

where we set the amplitude and phase functions

$$\alpha(\rho):=a(\nu(\rho);\widetilde{\boldsymbol{x}})\,a(\nu(\rho);\boldsymbol{x}), \tag{19}$$

$$\xi(\rho):=k_c(\|\,\nu(\rho)-\boldsymbol{x}\,\|-\|\,\nu(\rho)-\widetilde{\boldsymbol{x}}\,\|). \tag{20}$$

In the above, the dependencies on $\widetilde{\boldsymbol{x}}$ and $\boldsymbol{x}$ were omitted for readability. We interpret the non-linear phase content $e^{-j\xi(\rho)}$ as a spatial "chirp", by reference to time chirp signals that are commonly used, for example, in radar systems. Formally, a *spatial* (resp. *time*) chirp is defined by a varying *local wave number* (resp. *instantaneous frequency*). As notably seen in refs. 19, 20, this local wave number is defined by the derivative $\dot{\xi}(\rho)$ of $\xi(\rho)$. Figure 5a illustrates the chirp nature of the matched function, characterized by a slowly varying amplitude and a rapidly varying non-linear phase.

Assuming a high-frequency source signal (i.e., a large k_c), the phase factor $e^{-j\xi(\rho)}$ oscillates much faster than $\alpha(\rho)$. Consequently, $\alpha(\rho)$ has a negligible convolutive effect on the support of the matched spectrum, compared to the pure phase contribution. More precisely, given $G_\xi(\omega)$ and

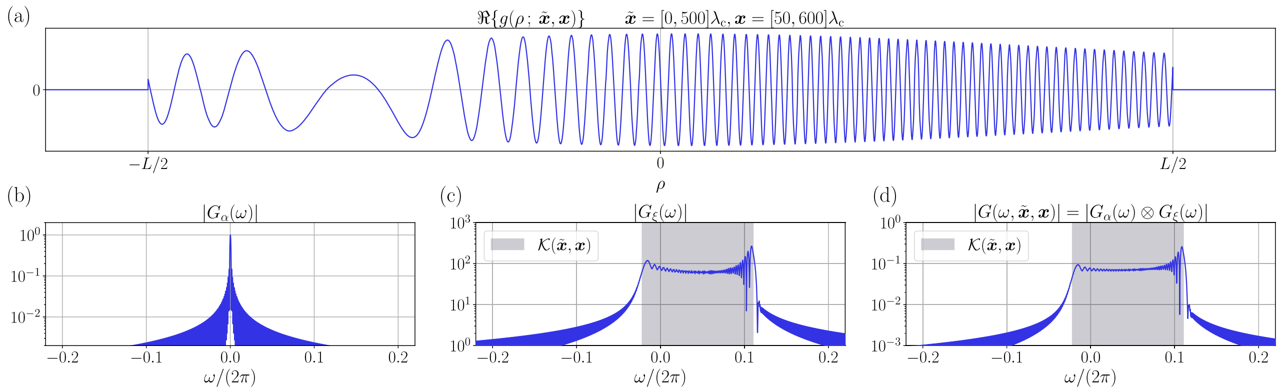


Fig. 5 | Representation of the chirp-structured matched signal $g(\rho;\widetilde{x},x)$ for $x=[0,600]\lambda_c$ and $\widetilde{x}=[50,700]\lambda_c$. **a** Real part of $g(\rho;\widetilde{x},x)$. **d** Corresponding spectrum, decomposed by (**b**) the amplitude's spectrum and **c** the phase factor's spectrum.

$G_\alpha(\omega)$ denoting the FTs of, respectively, $e^{-j\xi(\rho)}$ and $\alpha(\rho)$, we have

$$G(\omega;\widetilde{x},x)=\frac{1}{2\pi}G_\xi(\omega)\otimes G_\alpha(\omega). \quad (21)$$

The convolution in (21) implies that the band limit of $G(\omega;\widetilde{x},x)$ equals that of $G_\xi(\omega)$ plus the bandwidth of $G_\alpha(\omega)$. As illustrated in Fig. 5, the typically slow variations of $\alpha(\rho)$ (compared to λ_c) yields a narrowband spectrum $G_\alpha(\omega)$ whose bandwidth can be neglected. We thus reduce our study to the content of $G_\xi(\omega)$ as a good indication of $G(\omega;\widetilde{x},x)$.

As supported by the theoretical findings detailed in ref. 26, the set of local spatial frequencies observed by the array, defined as

$$\mathcal{K}(\widetilde{x},x):=\{\dot{\xi}(\rho):\rho\in\mathcal{P}\}, \quad (22)$$

constitutes a good indication of the "most active" spatial frequency (wave number) content in the chirp's spectrum. This observation has also been reported through the concept of "local spatial bandwidth"[19]. Capitalizing on this approach, Definition 1 provides a practical expression of the spatial band limit, which matches its significant spectral content in practical cases. Said differently, the set of local frequencies $\mathcal{K}(\widetilde{x},x)$ yields a compressive spectral modeling of the matched signal.

Definition 1. (Soft Band Limit). The soft band limit associated with the locations $\widetilde{x}$ and x is the maximal *local* frequency of the matched signal $g(\rho;\widetilde{x},x)$. It is thereby defined as

$$K(\widetilde{x},x):=\max_{\rho\in\mathcal{P}}|\dot{\xi}(\rho)|. \quad (23)$$

Definition 1 can now replace the impractical strict band limit $\overline{K_0}(\widetilde{x},x)$ in Lemma 1. This enables us to define a region in which $A_S(\widetilde{x},x)$ is free of significant aliasing.

Definition 2. (Aliasing-free region). The "aliasing-free region (AFR)" associated with the source location x is the set of all the *tested* locations $\widetilde{x}$ that yield no aliasing when evaluating $A_S(\widetilde{x},x)$. Mathematically, it is expressed as

$$\mathcal{S}(x):=\{\widetilde{x}:K(\widetilde{x},x)\le 2\pi\Delta^{-1}\}. \quad (24)$$

The aliasing front, for instance observed in Fig. 2b, is given by the boundary of $\mathcal{S}(x)$, denoted by $\partial\mathcal{S}(x)$. This boundary is the contour line of the function $K(\widetilde{x},x)$ at the level $2\pi\Delta^{-1}$.

Importantly, aliasing-free region (AFR) is a *location-dependent* concept as each source location x leads to a possibly different AFR. By contrast, Definition 3 proposes a global concept that only depends on the structure of the array. Still, both concepts depend on the sampling step Δ, as seen in (24).

Definition 3. (Aliasing-Safe operating Domain). The operating domain $\mathcal{X}$ is an "aliasing-safe operating domain (ASOD)" if all the pairs $\widetilde{x},x\in\mathcal{X}$ yield no aliasing in $A_S(\widetilde{x},x)$. Mathematically, this is achieved if and only if, for all $x\in\mathcal{X}$,

$$\mathcal{X}\subseteq\mathcal{S}(x). \quad (25)$$

We note that (25) is equivalent to stating that all possible pairs of locations $\widetilde{x},x\in\mathcal{X}$ are such that $\widetilde{x}\in\mathcal{S}(x)$. Under this condition, no grating lobe appears in the AF restricted to $\mathcal{X}$. Also, ASODs are typically not unique for a given spacing Δ.

These definitions lead to design guidelines to avoid estimation ambiguities over an operating domain $\mathcal{X}$.

1. Given an array geometry $\mathcal{Z}$, obtain the soft spatial band limit $K(\widetilde{x},x)$.
2. Derive the AFR $\mathcal{S}(x)$ as a function of both x and the spacing Δ.
3. Using Definition 3, derive the value of Δ required to ensure that the domain $\mathcal{X}$ is an ASOD, i.e., meets (25).

Whereas steps 1) and 2) are analytical, step 3) likely requires the development of dedicated numerical methods, extending beyond the theoretical scope of this paper. The remainder of this paper, therefore, concentrates on the analysis of AFRs.

Properties of aliasing-free regions. We begin by extending a classical FF result to the NF regime. Theorem 1 exploits modeling (23) to state that no aliasing can arise in the discrete-space AF as soon as the half-wavelength spacing between antennas is maintained. In the FF regime, which permits no range resolution from the steering signal, this principle reduces to its standard angular interpretation, guaranteeing no angular grating lobes with a half-wavelength antenna spacing.

Theorem 1. Given $z_i:=v(i\Delta)$ denoting the i-th antenna's locations from the grid $\mathcal{Z}_G$, if

$$\|z_{i+1}-z_i\|\le\frac{\lambda_c}{2} \quad (26)$$

is met for all $i\in\{1,\ldots,N\text{-}1\}$, then the AFR is $\mathbb{R}^d$ for all x. As a result, any $\mathcal{X}\subseteq\mathbb{R}^d$ is an ASOD.

Proof. See the Supplementary Information file.

We emphasize the significance of this result, as Theorem 1 extends beyond the conventional understanding of the FF rules. It demonstrates that for *any* set of antenna locations $\mathcal{Z}_G$, if at least one path connecting all these locations can be drawn such that the consecutive spacings are smaller than half the wavelength, then no aliasing occurs, neither in the NF nor the FF regimes.

As previously discussed, this theorem does not imply that the continuous and discrete AFs, $A(\tilde{\boldsymbol{x}}, \boldsymbol{x})$ and $A_S(\tilde{\boldsymbol{x}}, \boldsymbol{x})$, can be identical. Instead, it supports that using $K(\tilde{\boldsymbol{x}}, \boldsymbol{x})$ to define the soft band limit yields a NF modeling of aliasing artifacts that matches the conventional FF modeling of the grating lobes.

Besides its theoretical reach, the sufficient condition of Theorem 1 can be impractical to effectively design XL-arrays. By extending the array dimensions while increasing carrier frequencies, engineers face physical constraints that prevent meeting a half-wavelength spacing in some applications[8–10]. This further motivates the role of our framework, focused on deriving aliasing-safe operating domains, given $\Delta > \lambda_c/2$.

While finding practical expressions for the AFRs is sometimes challenging, we can bound it by another one that is easier to compute. This principle, which we exploit in the next sections, is formalized by Properties 1 and 2.

Property 1. (Inclusion principle I). Two arrays resulting from the sampling of $\mathcal{P}$ with respective sampling steps $\Delta_1 \leq \Delta_2$ yield respective AFRs, $\mathcal{S}_1(\boldsymbol{x})$ and $\mathcal{S}_2(\boldsymbol{x})$ that satisfy

$$\mathcal{S}_2(\boldsymbol{x}) \subseteq \mathcal{S}_1(\boldsymbol{x}). \tag{27}$$

for all $\boldsymbol{x} \in \mathbb{R}^d$.

Proof. Since $\Delta_1 \leq \Delta_2$, the condition in (24) is softer when using the sampling step Δ_1 rather than Δ_2. Thus any $\tilde{\boldsymbol{x}} \in \mathcal{S}_2(\boldsymbol{x})$ also meets $\tilde{\boldsymbol{x}} \in \mathcal{S}_1(\boldsymbol{x})$.

Property 2. (Inclusion principle II). Given two arrays resulting from the sampling of, respectively, $\mathcal{P}_1$ and $\mathcal{P}_2$, with the same sampling step Δ, the inclusion

$$\mathcal{P}_1 \subseteq \mathcal{P}_2, \tag{28}$$

implies that their respective AFRs, $\mathcal{S}_1(\boldsymbol{x})$ and $\mathcal{S}_2(\boldsymbol{x})$ satisfy

$$\mathcal{S}_2(\boldsymbol{x}) \subseteq \mathcal{S}_1(\boldsymbol{x}), \tag{29}$$

for all $\boldsymbol{x} \in \mathbb{R}^d$.

Proof. Given (28), $K_2(\tilde{\boldsymbol{x}}, \boldsymbol{x})$ is the solution of a *relaxed* version of the maximization problem (23) that yields $K_1(\tilde{\boldsymbol{x}}, \boldsymbol{x})$. Therefore, for all $\tilde{\boldsymbol{x}}, \boldsymbol{x}$, we have $K_2(\tilde{\boldsymbol{x}}, \boldsymbol{x}) \geq K_1(\tilde{\boldsymbol{x}}, \boldsymbol{x})$. Consequently

$$K_2(\tilde{\boldsymbol{x}}, \boldsymbol{x}) \leq 2\pi\Delta^{-1} \Rightarrow K_1(\tilde{\boldsymbol{x}}, \boldsymbol{x}) \leq 2\pi\Delta^{-1}. \tag{30}$$

This directly proves (29) by definition of the AFR.

While Property 1 states that densifying the array with extra antennas improves (widens) the AFRs, Property 2 implies that extending the size of the array deteriorates (narrows) the AFRs. Conversely, we deduce that truncating an antenna array can only *widen* its AFRs. This possibly counterintuitive conclusion stems from the fact that aliasing artifacts alter the AF as soon as *one* local spatial frequency is greater than $2\pi\Delta^{-1}$. Extending the array provides more opportunities for this condition to be met in at least one location in the array. This contrasts with the natural intuition from the FF regime, where the matched signal maintains a constant local spatial frequency along the array. In that case, extending the array has no consequence on the aliasing, but solely improves the system's resolution. Similarly, in the NF regime, narrower arrays are not universally preferable since large arrays also offer enhanced spatial resolution for beam pointing. A fundamental trade-off therefore follows between the resolution and the ambiguities caused by aliasing artifacts. Additionally, while wider arrays exhibit aliasing artifacts closer to $\boldsymbol{x}$ in the AF, their amplitude, which remains to be evaluated, might be lower. This motivates future studies addressing aliasing not only by presence, but also by amplitude.

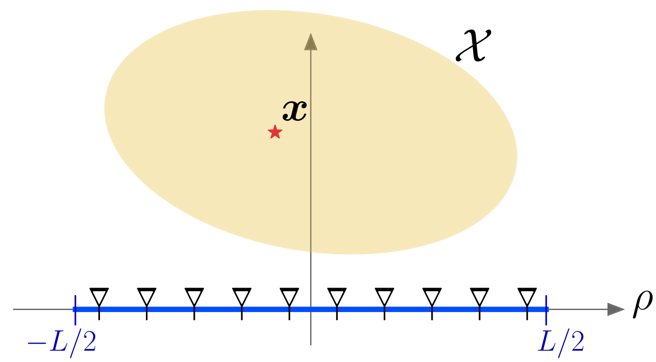


Fig. 6 | Schematic representation of a ULA operating in the NF regime.

Finally, while the above properties affect AFRs, Property 3 extends their conclusions to ASODs.

Property 3. (Inclusion principle III). When two arrays are providing AFRs such that

$$\mathcal{S}_2(\boldsymbol{x}) \subseteq \mathcal{S}_1(\boldsymbol{x}) \tag{31}$$

holds for all $\boldsymbol{x} \in \mathbb{R}^d$, any operating domain $\mathcal{X}$ that is an ASOD for the array "2" is also an ASOD for the array "1".

Proof. Combining the definition (25) with (31), we obtain that if $\mathcal{X}$ is an ASOD for the array "2", then for all $\boldsymbol{x} \in \mathcal{X}$, we have $\mathcal{X} \subseteq \mathcal{S}_2(\boldsymbol{x}) \subseteq \mathcal{S}_1(\boldsymbol{x})$.

The next two sections present applications of the different steps of our methodology to two canonical array topologies, i.e., the ULA and the UCA.

Uniform linear array the NF regime

Using the theoretical framework presented in the previous sections, we further develop the specific case of a ULA. Without loss of generality, we consider the zero-centered and horizontal ULA, similar to the toy example we previously used. Its geometry, shown in Fig. 6, is trivially described with $\nu(\rho) = [\rho, 0]^\top$ for $\rho \in \mathcal{P}_L := [-L/2, L/2]$, where L denotes its length.

This section derives the expression of the soft band limit, which we denote here by $K_L(\tilde{\boldsymbol{x}}, \boldsymbol{x})$. It also derives the structure of the corresponding AFR, denoted by $\mathcal{S}_L(\boldsymbol{x})$. Whereas the preliminary work in ref. 24 proposed an approximation of $K_L(\tilde{\boldsymbol{x}}, \boldsymbol{x})$, whose validity was limited by the scope of the Fresnel approximation, the current paper derives its exact expression.

Property 2 motivates the study of the aliasing structure of the *infinite-length* ULA as a *worst-case reference*. Indeed, this property guarantees

$$\mathcal{S}_\infty(\boldsymbol{x}) \subseteq \mathcal{S}_L(\boldsymbol{x}), \tag{32}$$

for all finite length L. As a result, $\mathcal{S}_\infty(\boldsymbol{x})$ provides a pessimistic representation of the AFR. Moreover, since $\mathcal{P}_\infty = \mathbb{R}$, the resulting unconstrained computation (23) of the soft band limit is simplified.

For these reasons, we begin this section by applying our methodology to the infinite ULA. We also investigate under which conditions the inclusion (32) is tight with equality. Those are the cases where finite-size ULAs behave as infinite in terms of grating lobes. Next, we provide the general expression of $K_L(\tilde{\boldsymbol{x}}, \boldsymbol{x})$, for $L < \infty$, and study the geometry of the resulting AFR.

Infinite-length ULA. Let us now derive the closed-form expression of the infinite-length ULA's band limit, $K_\infty(\tilde{\boldsymbol{x}}, \boldsymbol{x})$ for all 2D locations,

$$\boldsymbol{x} = [\mathrm{x}, \mathrm{y}]^\top \text{ and } \tilde{\boldsymbol{x}} = [\tilde{\mathrm{x}}, \tilde{\mathrm{y}}]^\top, \tag{33}$$

in the upper half-plane ($\mathrm{y} > 0$ and $\tilde{\mathrm{y}} > 0$). By symmetry, our conclusions directly extend to the other half-plane.

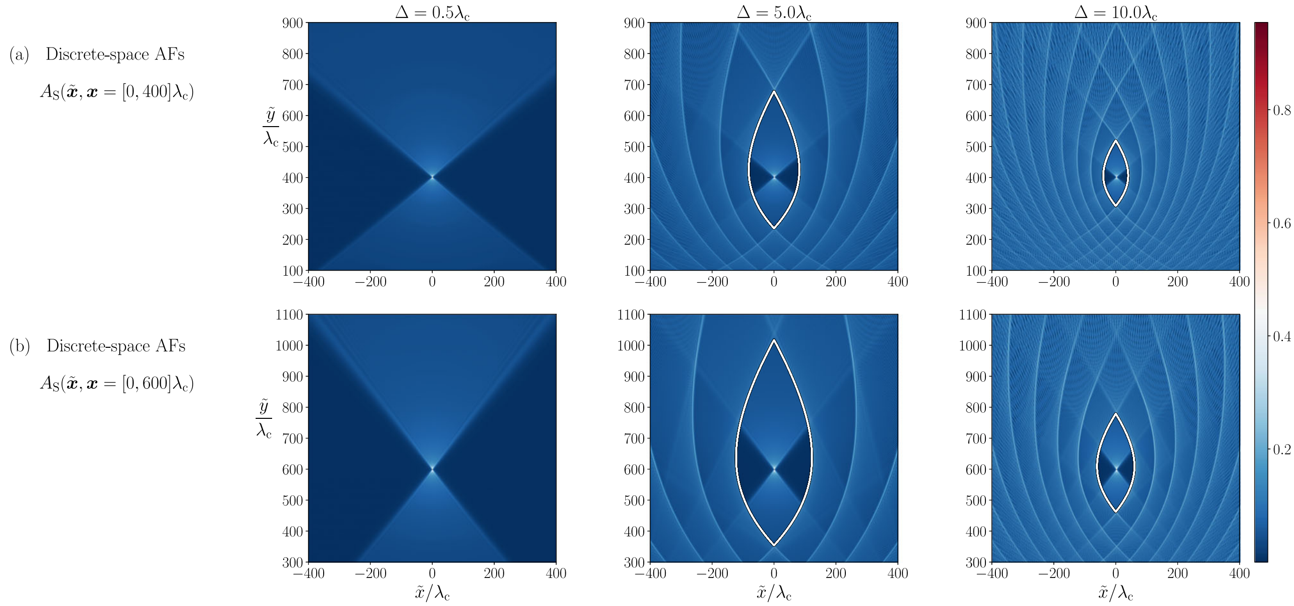


Fig. 7 | Discrete-space AFs obtained for a horizontal ULA of length $L = 1000\lambda_c$. a With $x = [0, 400]\lambda_c$ and **b** with $x = [0, 600]\lambda_c$. The white eye depicts the contour line where $K_\infty(\tilde{x}, x) = 2\pi\Delta^{-1}$, i.e., $\partial\mathcal{S}_\infty(x)$, increasing in size as the distance y increases and shrinking as Δ increases.

This band limit's expression is formalized in Theorem 2. Interestingly, it is entirely driven by two scalar variables

$$u := \left(\frac{\tilde{\mathrm{y}}}{\mathrm{y}}\right)^{\frac{2}{3}} \text{ and } v := \frac{\tilde{\mathrm{x}} - \mathrm{x}}{\mathrm{y}}, \tag{34}$$

reducing it from four degrees of freedom to two. To simplify notations, we also set $w := \frac{v}{u^2 - 1}$.

Theorem 2. (Band limit of an infinite-length ULA). The infinite-length ULA yields a soft band-limit (Definition 1) that is described by the closed-form expression

$$K_\infty(\tilde{x}, x) = k_c \frac{|(u-1)\beta + v|}{\sqrt{u(\beta^2 + 1)}}, \tag{35}$$

where

$$\beta := \mathrm{sign}(w) u \sqrt{(u+1)^{-1} + w^2} - w. \tag{36}$$

Proof. See the Supplementary Information file.

Inspecting the expression (35) for specific locations $\tilde{x}$ that are aligned with the source location x yields simplifications formulated in Corollaries 1 and 2. These particular cases are similar to the spatial bandwidth expressions obtained in ref. 19.

Corollary 1. When $\tilde{x}$ and x are horizontally aligned, such that $\mathrm{y} = \tilde{\mathrm{y}}$, the band limit expression (35) is reduced to

$$K_\infty(\tilde{x}, x) = k_c \frac{|v|}{\sqrt{v^2/4 + 1}}. \tag{37}$$

Proof. See the Supplementary Information file.

Corollary 2. When $\tilde{x}$ and x are vertically aligned, such that $\mathrm{x} = \tilde{\mathrm{x}}$, the band limit expression (35) is reduced to

$$K_\infty(\tilde{x}, x) = k_c \frac{|u-1|}{\sqrt{1 + u + u^2}}. \tag{38}$$

Proof. See the Supplementary Information file.

Using Theorem 2, we obtain the AFR of the infinite-length ULA,

$$\mathcal{S}_\infty(x) = \{\tilde{x} : K_\infty(\tilde{x}, x) \leq 2\pi\Delta^{-1}\}. \tag{39}$$

To illustrate this region, Fig. 7 presents numerically computed AFs for long ULAs with $L = 1000\lambda_c$, considering multiple antenna spacings Δ and two distinct source locations x. Although the simulated arrays have a finite length, the results in Fig. 7 are generated in a regime that exactly yields $\mathcal{S}_L(x) = \mathcal{S}_\infty(x)$, as further explained in subsequent discussions on finite ULAs. We can therefore assume that these long ULAs behave as effectively infinite with respect to the grating lobes.

In that figure, the white eye-shaped contours depict the boundaries $\partial\mathcal{S}_\infty(x)$, computed using the expression (35) in (39). Their apparent correspondence with the actual aliasing front observed in the AFs illustrates that our methodology, based on *local* spatial frequency content of the matched signal, correctly captures the aliasing structure appearing in such AFs.

We now elaborate on the properties of $\mathcal{S}_\infty(x)$. First, we note that $K_\infty(\tilde{x}, x)$ only depends on u and v. Their definition in (34) translates an x-shift and y-scaling invariance. This effect is detailed in Property 4.

Property 4. (Invariance). Given a spacing Δ, the AFR around the true location x is identical to the "ULA-eye", defined as

$$\mathcal{U} := \mathcal{S}_\infty([0, 1]^\top), \tag{40}$$

up to a translation—centering it in x— and a rescaling by a factor y. That is,

$$\mathcal{S}_\infty(x) = \mathrm{y} \cdot \mathcal{U} + \begin{bmatrix} \mathrm{x} \\ 0 \end{bmatrix}. \tag{41}$$

Proof. See the Supplementary Information file.

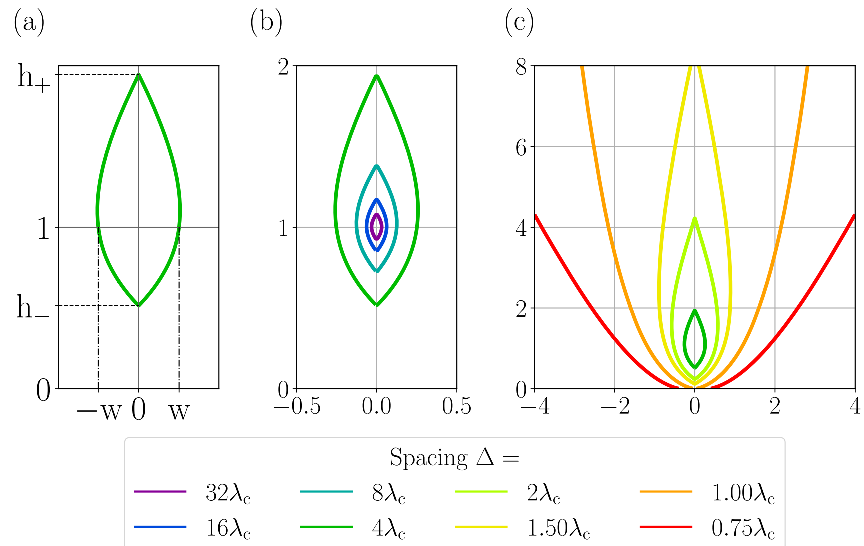


Fig. 8 | Geometry of the ULA-eyes. **a** representation of its width w, upper and lower aperture h_+, h_-. **b**, **c** Examples of ULA-eyes for multiple antenna spacing Δ.

Property 4 is an important result enabling us to restrict, without loss of generality, the study of $\mathcal{S}_\infty(\boldsymbol{x})$ to the one of the "ULA eye" $\mathcal{U}$, whose sole remaining dependency is with the antenna spacing Δ. Subsequently, we describe the aperture of the eye as a function of Δ in Properties 5 and 6.

Property 5. (Symmetry). If $\widetilde{\boldsymbol{x}} \in \mathcal{U}$, then its x-symmetric location $[-\widetilde{\mathrm{x}}, \widetilde{\mathrm{y}}]^\top \in \mathcal{U}$.

Proof. See the Supplementary Information file.

Property 6. (Eye's aperture). The vertical and horizontal aperture of the ULA eye, characterized by w, h_+, and h_- as depicted in Fig. 8a, are given by

$$w = \frac{2}{\sqrt{4\delta^2 - 1}} \tag{42}$$

$$h_+ = \begin{cases} \left[\frac{2\delta^2+1+\sqrt{12\delta^2-3}}{2(\delta^2-1)}\right]^{\frac{3}{2}} & \text{if } \delta > 1 \\ \infty & \text{otherwise} \end{cases}, \tag{43}$$

$$h_- = \begin{cases} \left[\frac{2\delta^2+1-\sqrt{12\delta^2-3}}{2(\delta^2-1)}\right]^{\frac{3}{2}} & \text{if } \delta > 1 \\ 0 & \text{otherwise} \end{cases}, \tag{44}$$

where

$$\delta = \frac{\Delta}{\lambda_c}. \tag{45}$$

Proof. See the Supplementary Information file.

Figure 8b, c shows the eyes for multiple spacings Δ. In accordance with Property 1, increasing this spacing shrinks the eye. Smaller eyes exhibit near vertical-symmetry, while larger ones elongate upward, until $\lambda_c/2 < \Delta \le \lambda_c$. In that case, expressions (43) and (44) saturate, causing the eye to degenerate. The degenerate eyes (Fig. 8c) never close vertically and are squeezed on the bottom. When $\Delta \le \lambda_c/2$, the eye no longer has boundaries because, as stated in Theorem 1, we have $\mathcal{U} = \mathbb{R}^2$.

Finite-length ULA. When the length L is finite, the expressions for the soft band limit and the resulting AFR become more sophisticated. Theorem 3 provides the composite expression of $K_L(\widetilde{\boldsymbol{x}}, \boldsymbol{x})$, which exhibits three regimes that we explain in this section.

Theorem 3. (Band limit of an finite-length ULA). The ULA of length L yields a soft band-limit (Definition 1) that is described by the closed-form expression

$$K_L(\widetilde{\boldsymbol{x}}, \boldsymbol{x}) = \begin{cases} K_\infty(\widetilde{\boldsymbol{x}}, \boldsymbol{x}) & \text{if } |\mathrm{y}\beta + \mathrm{x}| \le \frac{L}{2} \\ \max\{B_1(\widetilde{\boldsymbol{x}}, \boldsymbol{x}), K^s(\widetilde{\boldsymbol{x}}, \boldsymbol{x})\} & \text{if } |\mathrm{y}\beta + \mathrm{x}| > \frac{L}{2} \\ & \text{and } |\mathrm{y}\beta^s + \mathrm{x}| \le \frac{L}{2} \\ \max\{B_1(\widetilde{\boldsymbol{x}}, \boldsymbol{x}), B_{-1}(\widetilde{\boldsymbol{x}}, \boldsymbol{x})\} & \text{otherwise} \end{cases}, \tag{46}$$

where, for $q \in \{-1, 1\}$,

$$B_q(\widetilde{\boldsymbol{x}}, \boldsymbol{x}) = k_c |\cos(\theta_q(\widetilde{\boldsymbol{x}})) - \cos(\theta_q(\boldsymbol{x}))|, \tag{47}$$

$$K^s(\widetilde{\boldsymbol{x}}, \boldsymbol{x}) = k_c \frac{|(u-1)\beta^s + v|}{\sqrt{u((\beta^s)^2 + 1)}}, \tag{48}$$

and where

$$\theta_q(\boldsymbol{x}) := \angle(\boldsymbol{x} - [q \operatorname{sign}(w)\frac{L}{2}, 0]^\top), \tag{49}$$

$$\beta^s := -\operatorname{sign}(w) u \sqrt{(u+1)^{-1} + w^2} - w. \tag{50}$$

Proof. See the Supplementary Information file.

The first regime of (46) highlights the equivalence $K_L(\widetilde{\boldsymbol{x}}, \boldsymbol{x}) = K_\infty(\widetilde{\boldsymbol{x}}, \boldsymbol{x})$ for all pairs $\widetilde{\boldsymbol{x}}, \boldsymbol{x}$ such that

$$|\mathrm{y}\beta + \mathrm{x}| \le \frac{\mathrm{L}}{2}. \tag{51}$$

This can be explained since by construction of Theorem 2's proof, the variable β is linked to the value of ρ that sees the highest local frequency,

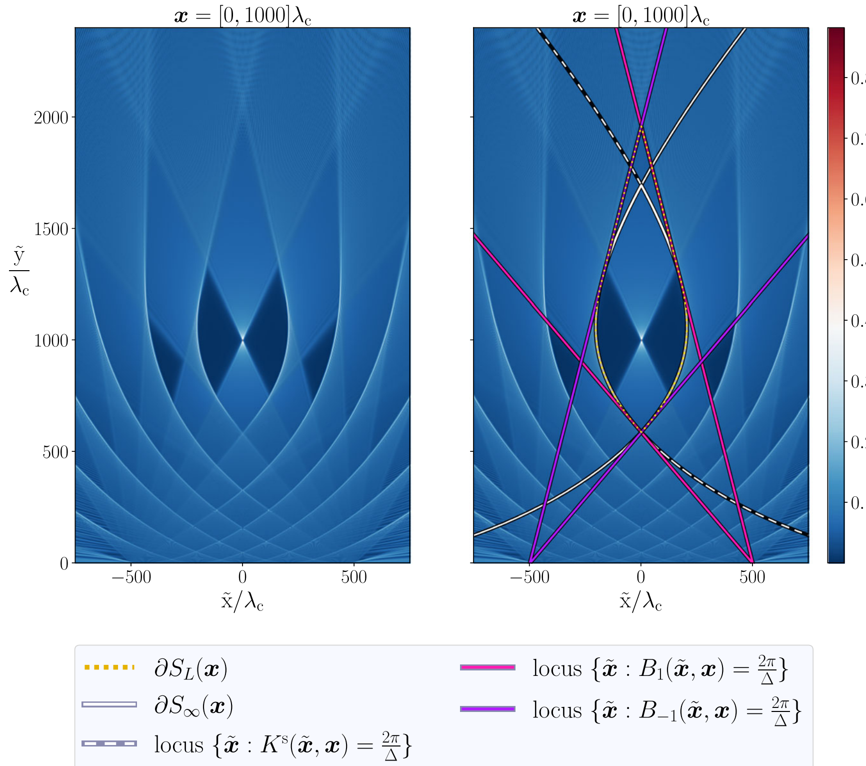


Fig. 9 | Ambiguity function $A_S(\tilde{\boldsymbol{x}}, \boldsymbol{x} = [0, 1000]\lambda_c)$ for a ULA with length $L = 1000\lambda_c$ and antenna spacing $\Delta = 5\lambda_c$; (left) without annotations and (right) with annotations corresponding to the three regimes of the band limit.

through

$$\mathrm{y}\beta + \mathrm{x} = \arg\max_{\rho\in\mathbb{Z}} |\mathring{\xi}(\rho)|. \tag{52}$$

Therefore, under (51), the constraint $\rho \in [-L/2, L/2]$ does not exclude the global maximizer of the unconstrained maximization that underpins $K_\infty(\tilde{\boldsymbol{x}}, \boldsymbol{x})$. Corollary 3 directly follows.

Corollary 3. Consider an infinite-length ULA and its truncation of length L. For a given location $\boldsymbol{x}$, if for all $\tilde{\boldsymbol{x}} \in \mathcal{S}_\infty(\boldsymbol{x})$ the condition (51) holds, then

$$\mathcal{S}_L(\boldsymbol{x}) = \mathcal{S}_\infty(\boldsymbol{x}). \tag{53}$$

Proof. This is a direct consequence of Theorem 3.

Despite the non-trivial expression of β (which depends on both $\boldsymbol{x}$ and $\tilde{\boldsymbol{x}}$), we can intuitively anticipate that (51) is typically satisfied when $\boldsymbol{x}$ and $\tilde{\boldsymbol{x}}$ are sufficiently close to each other and to the array's center, compared to the length L. Such close-by locations see the array as effectively infinite.

Conversely, when this condition is not met, the band limit expression is mainly driven by its third regime, $K_L(\tilde{\boldsymbol{x}}, \boldsymbol{x}) = \max\{B_1(\tilde{\boldsymbol{x}}, \boldsymbol{x}), B_{-1}(\tilde{\boldsymbol{x}}, \boldsymbol{x})\}$. The second regime describes a transition between the two others. In the third regime, the highest local frequency occurs at one of the array edges, revealing, in (47), the well-known difference of directional cosines with angles taken relative to the given edge.

Expression (46) can now be exploited to compute $\mathcal{S}_L(\boldsymbol{x})$. Unlike Fig. 7, which focused on the "near-infinite array" regime, Fig. 9 shows results with a further source location, causing $\mathcal{S}_L(\boldsymbol{x}) \neq \mathcal{S}_\infty(\boldsymbol{x})$. The bottom part of the boundary $\partial\mathcal{S}_L(\boldsymbol{x})$ matches $\partial\mathcal{S}_\infty(\boldsymbol{x})$ as it corresponds to locations $\tilde{\boldsymbol{x}}$ that meets condition (51). The top part, however, is attained in the third regime of the band limit, causing $\partial\mathcal{S}_L(\boldsymbol{x})$ to instead follow the directional cosine lines (in pink).

Figure 10 shows similarly annotated AFs for multiple source locations $\boldsymbol{x}$, illustrating how the corresponding AFRs vary in shape while consistently adhering to the lines marking the regimes we detailed in this section.

Connections with the far-field regime. To conclude our analysis of ULAs, we examine the behavior of the AFR expression as the scenario approaches the FF regime. Figure 11 presents results similar to those in Fig. 10, where the source location $\boldsymbol{x}$ is progressively moved further from a ULA of length $200\lambda_c$. The right-side subfigure illustrates the case where $\boldsymbol{x}$ is in the FF as it reaches the Fraunhofer distance. As the distances become much larger than the array length L, two effects emerge: (i) the "directional cosine" regime becomes the sole active regime in the boundary $\partial\mathcal{S}_L(\boldsymbol{x})$, and (ii) the cosines in (47) measured from both array edges converge. The combination of these two effects connects our model with the typical angular repetition of the FF regime.

Distributed uniform circular array

Circular arrays constitute a natural reference for theoretical studies on XL-arrays[14,27], due to their inherent isotropic geometry providing a 360-degree coverage. As such, this topology can be considered as the canonical example of the "perfectly" distributed XL-array. This simple geometry enables the derivation of closed-form expressions, exploiting the Fourier-Bessel series for its associated beampattern and ambiguity function, demonstrating a resolution with an order of magnitude comparable to the carrier wavelength[14,24]. While such results are associated with complete circles of continuous antenna elements, this section exploits our framework to understand the aliasing structure that appears in the discrete-space AF of UCAs.

In all generality, a UCA is described by an antenna domain $\mathcal{Z}$ that forms an arc on the zero-centered circle of radius R_{uca} surrounding the operating domain $\mathcal{X}$, as shown in Fig. 12. The parametric path variable ρ

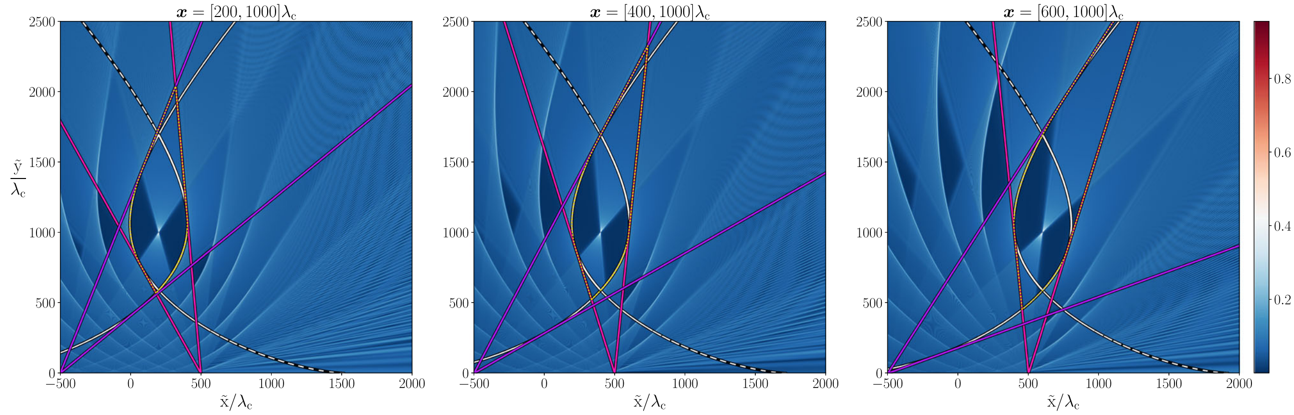


Fig. 10 | Ambiguity functions for a ULA with length $L = 1000\lambda_c$ and antenna spacing $\Delta = 5\lambda_c$ for multiple source locations x that yield distinct AFR shapes, combining the three regimes explained in Theorem 3. The legend is identical to Fig. 9.

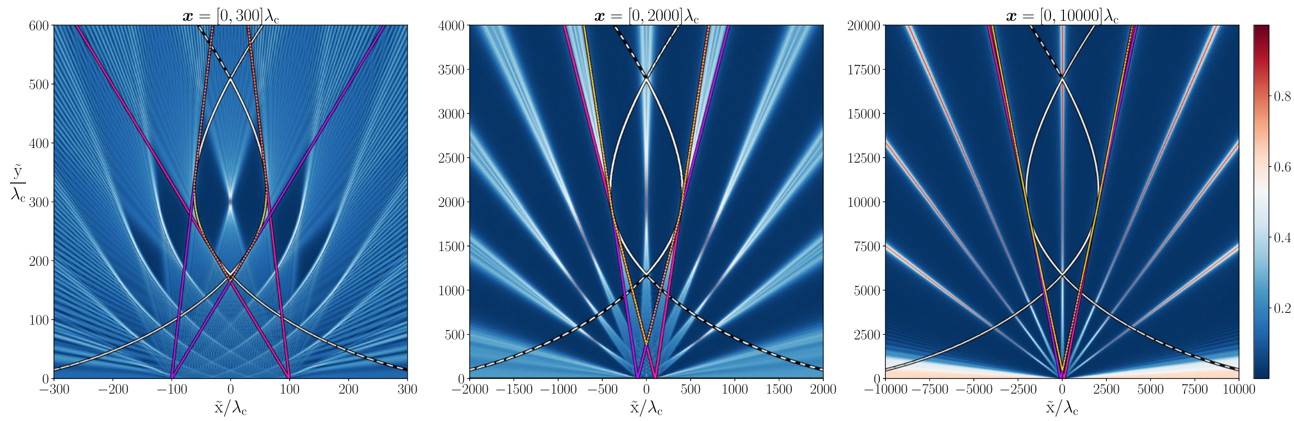


Fig. 11 | Ambiguity functions for a ULA with length $L = 200\lambda_c$ and antenna spacing $\Delta = 5\lambda_c$ for multiple source locations x that progressively tends towards the FF conditions. The location $\boldsymbol{x} = [0, 10000]\lambda_c$ is in the FF according to the Fraunhofer criterion. The legend is identical to Fig. 9.

denotes here an angle such that

$$\nu(\rho) = R_{\text{uca}} \begin{bmatrix} \cos(\rho) \\ \sin(\rho) \end{bmatrix}, \tag{54}$$

and we set the domain $\mathcal{P} = [-\psi, \psi]$ with $0 \leq \psi \leq \pi$ to enable us to consider either the full circle with $\psi = \pi$ or arcs with $\psi < \pi$. The antenna spacing Δ corresponds, in the context of this section, to an *angular* spacing, expressed in radians.

To develop further the expression of the phase (20), we restrict the focus of this section to an infinitely large array with $R_{\text{uca}} \to \infty$ surrounding the operating domain $\mathcal{X}$. Practically speaking, the resulting study provides good representations of the aliasing structure one observes for a non-infinite radius satisfying

$$R_{uca} \gg \max\{\| \boldsymbol{x} \| : \boldsymbol{x} \in \mathcal{X}\}. \tag{55}$$

Under the infinite-radius condition, the matched signal's amplitude, given in (19), is asymptotically constant, causing $G_\alpha(\omega)$ to be a Dirac delta. As a result, $G(\omega; \widetilde{\boldsymbol{x}}, \boldsymbol{x})$ and $G_\xi(\omega)$ are exactly equals, up to a constant coefficient.

The phase of this signal, given in (20), remains non-linear across the circle $\mathcal{Z}$, hence preserving the NF nature of this scenario. However, in the reciprocal point of view, each specific antenna's location $\boldsymbol{z} = \nu(\rho)$ is seen in the FF of the domain $\mathcal{X}$. Said differently, the domain $\mathcal{X}$ is infinitely small compared to the distance separating it from $\boldsymbol{z}$. Consequently, this condition validates the modeling in ref. 14, implying that for all $\boldsymbol{z} \in \mathcal{Z}$ and all $\widetilde{\boldsymbol{x}}, \boldsymbol{x} \in \mathcal{X}$, the matched signal phase reads

$$\xi(\rho) = k_c R \cos(\rho - \theta), \tag{56}$$

where we define

$$R := \| \boldsymbol{x} - \widetilde{\boldsymbol{x}} \| \text{ and } \theta := \angle(\boldsymbol{x} - \widetilde{\boldsymbol{x}}), \tag{57}$$

as visualized in Fig. 12. The resulting band limit is straightforwardly derived in Theorem 4.

Theorem 4. The infinite-radius UCA yields a soft band limit (Definition 1) that is described by the closed-form expression

$$K_{uca}(\widetilde{\boldsymbol{x}}, \boldsymbol{x}) = k_c R\, \Omega(\theta), \tag{58}$$

where $\Omega(\theta)$ is the *visual aperture* of the array at the angle θ. We define it as

$$\Omega(\theta) := \max_{\rho \in [-\psi, \psi]} |\sin(\rho - \theta)|. \tag{59}$$

Proof. Using (56), the local spatial frequency is trivially expressed as $\dot{\xi}(\rho) = -k_c R \sin(\rho - \theta)$. Developing the definition (23) with this expression directly provides (58).

Let us note that (59) possesses a closed-form expression that we provide here for the sake of clarity. If there is an integer n such that

$$|\theta + (2n+1)\frac{\pi}{2}| \in [-\psi, \psi], \tag{60}$$

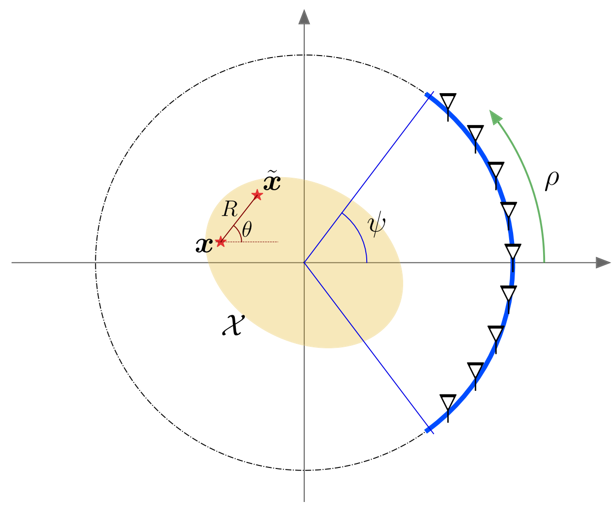


Fig. 12 | Schematic representation of a UCA.

then the visual aperture takes its maximum value $\Omega(\theta) = 1$. Otherwise, the constraints in (59) are active, implying that

$$\Omega(\theta) = \max\{|\sin(\psi+\theta), |\sin(\psi-\theta)|\}. \tag{61}$$

Figure 13 shows how expression (58) of the band limit captures the support of $G(\omega; \widetilde{\boldsymbol{x}}, \boldsymbol{x})$ given a UE located in $\boldsymbol{x} = 0$, and multiple tested locations $\widetilde{\boldsymbol{x}}$. The complete UCA with $\psi = \pi$ exhibits an angular invariance, implying only symmetric spectra, as seen in Fig. 13. This invariance no longer holds for incomplete circles. Therefore, it results in asymmetric spatial spectra (Fig. 13b), except for the particular case of horizontal positional difference $\widetilde{\boldsymbol{x}} - \boldsymbol{x}$ (Fig. 13c), assuming an array oriented as depicted in Fig. 12.

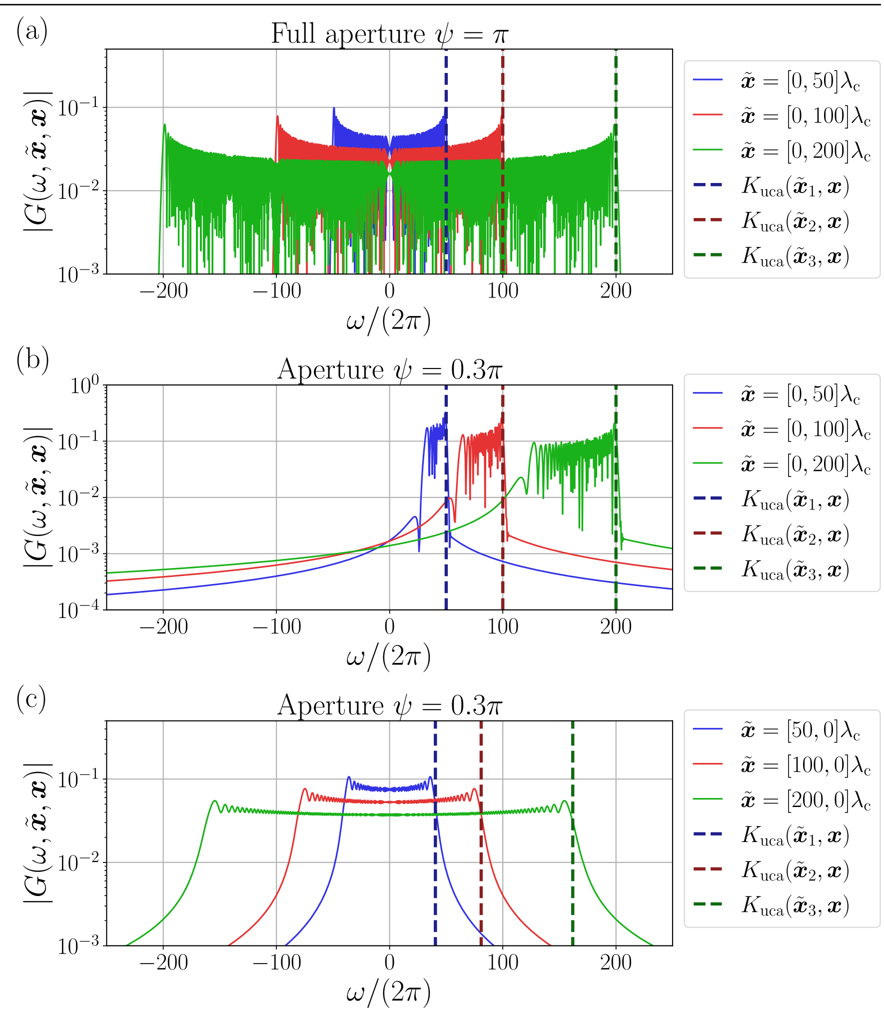
We can now study the resulting AFR of the UCA, given the angular spacing Δ. It follows the interpretable expression:

$$\mathcal{S}_{\mathrm{uca}}(\boldsymbol{x}) := \{\widetilde{\boldsymbol{x}} : K_{\mathrm{uca}}(\widetilde{\boldsymbol{x}}, \boldsymbol{x}) \leq 2\pi\Delta^{-1}\} \tag{62}$$

$$= \left\{(R, \theta) : R \leq \frac{\lambda_c}{\Delta\Omega(\theta)}\right\}. \tag{63}$$

Expression (63) informs us that the aliasing front appears around $\boldsymbol{x}$, in each direction θ, at a distance $\frac{\lambda_c}{\Delta\Omega(\theta)}$. Interestingly, as soon as the UCA draws a *half-circle*, i.e., when $\psi \geq \frac{\pi}{2}$, condition (60) is met, and hence the visual aperture provides its maximum value $\Omega(\theta) = 1$ for all θ. As a result, the aliasing front draws a perfect circle around $\boldsymbol{x}$.

Fig. 13 | Spectrum $G(\omega; \widetilde{\boldsymbol{x}}, \boldsymbol{x} = \boldsymbol{0})$ for a UCAs with $R_{\mathrm{uca}} = 10000\lambda_c$. a With $\psi = \pi$. **b** With $\psi = 0.3\pi$ and vertical position differences. **c** With $\psi = 0.3\pi$ and horizontal position differences.



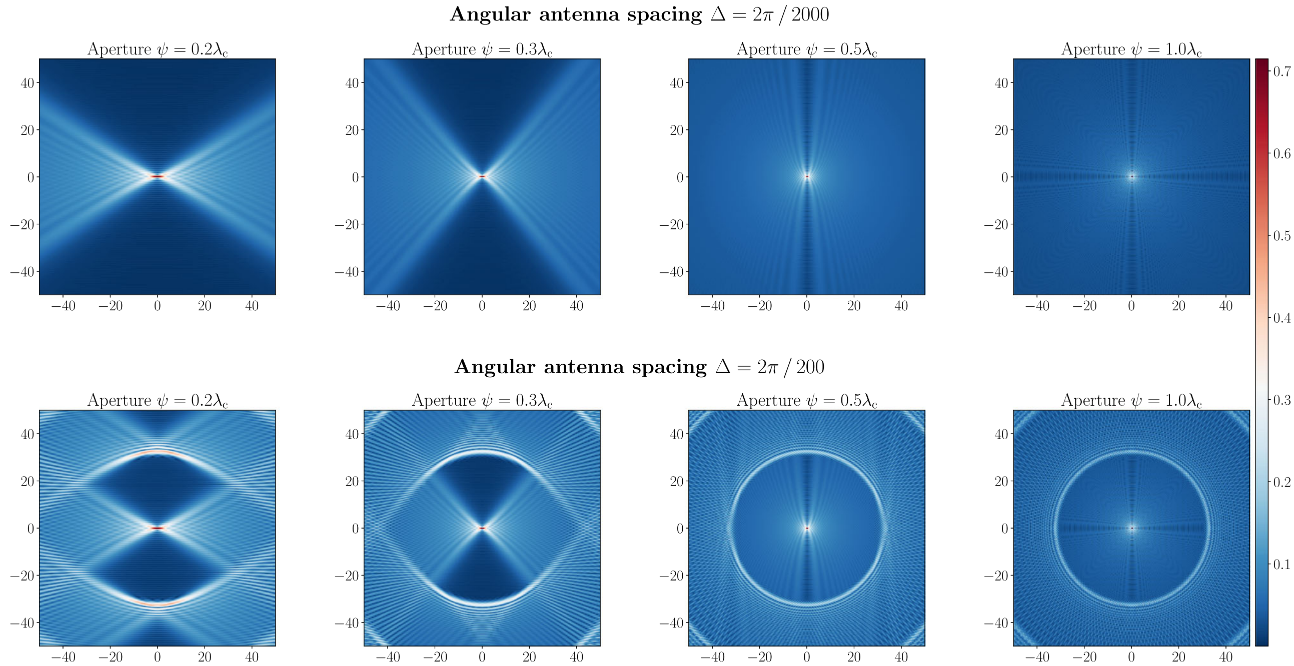


Fig. 14 | Discrete-space AFs, $|A_S(\tilde{\boldsymbol{x}}, \boldsymbol{x} = \boldsymbol{0})|$, for a UCA of radius $R_{\text{uca}} = 10,000\lambda_c$ and multiple apertures ψ. (Top) uniform angular spacing $\Delta = 2\pi/2000$. (Bottom) $\Delta = 2\pi/200$.

By contrast, when $\psi < \frac{\pi}{2}$, then $\Omega(\theta) = 1$ only for angles satisfying (60). These are angles such that the line passing through $\boldsymbol{x}$ that is normal to them crosses the array. In other angles, the visual aperture follows (61), resulting in a locally linear aliasing front in the AF.

These effects can be observed in Fig. 14, which shows discrete-space AFs computed for UCAs with multiple apertures λ_c and either $\Delta = (2\pi)/2000$ or $\Delta = (2\pi)/200$ as angular antenna spacing. The denser array acts as the aliasing-free reference. We indeed observe a circular aliasing front of radius $\frac{\lambda_c}{\Delta} = 31.83\lambda_c$ with alternating fronts and decaying zones repeating at every multiple of this radius, This pattern aligns with the analysis we presented in Fig. 4. When $\psi < \frac{\pi}{2}$, the circle stretches as the visual aperture increases. The portion of the front that fits the circle exactly corresponds to the angular aperture of the array (i.e., an arc of 2ψ radians). These two arcs—upper and lower—are oriented perpendicularly to the array. Notably, the resulting eye-shaped region always includes the full-aperture disc, in accordance with Property 2. That is, because any arc of aperture 2ψ inherently represents a truncation of the whole circle.

Discussion

This paper has introduced a novel theoretical framework to address the challenging characterization of NF grating lobes. In the proposed approach, we have formally defined these lobes as aliasing artifacts affecting AFs. Their structural origin has been further explained by leveraging a local spatial-frequency representation of the steering and matched signals. This enabled us to extend the conventional FF interpretation of grating lobes while consistently explaining their observed behavior in the NF regime.

Exploiting this understanding, this paper has provided a systematic methodology to derive *AFRs*. The boundaries of those regions analytically define the geometry of the NF grating lobes. Studying them is the key to designing ASODs. These domains guarantee no aliasing when restricting a given array to operate in this domain. Deriving methods to mathematically determine ASODs therefore constitutes a potential next step that directly follows the current work.

This paper illustrated its practical value in deriving closed-form expressions of AFRs for both ULAs and UCAs. In the context of ULAs, the present study has also detailed the connection between its NF findings and established FF knowledge. This result further illustrated how this work can potentially bridge this gap in a unified and coherent framework.

Importantly, the proposed framework is currently restricted to explaining the geometrical *presence* of aliasing effects–and hence grating lobes– in the AFs of NF arrays. As a key feature of the NF regime, these lobes are usually weaker in amplitude than the main lobe. This allows XL-arrays to theoretically function beyond the AFR. Expanding the present analysis to include aliasing amplitudes is therefore critical to fully assess the attainable performance of XL-arrays. Moreover, subsequent work extending our findings to include the resolution of both the main and grating lobes represents a key step toward a comprehensive characterization of the AF properties.

Future work directions also include exploiting the flexibility of our approach to determine the attainable AFRs for more sophisticated array topologies that meet the demands of practical applications. This is expected to be achievable because the current methodology exploits a parametric representation theoretically capable of describing any antenna configuration. However, applying this representation to highly complex topologies may require adapting the present framework to assess the practical significance of the grating lobes. This challenge emphasizes the importance of including the amplitude characterization of the aliasing artifacts in the analysis.

Finally, this work is expected to be adaptable to study the coupled space-time-frequency AFs arising when considering wideband systems and Doppler effects. This adaptability paves the way for additional extensions, defining a general methodology to study the NF grating lobes in wideband non-static scenarios.

Data availability

No datasets were generated or analyzed during the current study.

Code availability

All relevant code supporting the document is available here: https://forge.uclouvain.be/GillesMonnoyer/NPJWT-Aliasing-NF-AF. For any additional request, please refer to G.M., gilles.monnoyer@uclouvain.be.

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Author contributions

G.M. and L.V. came to the idea of investigating the impact of array discretization on the ambiguity function through the approach of spatial frequencies, and the formal link between AF and spectral content. G.M. introduced the AFR and ASOD notions. G.M. achieved the theoretical results for the ULA. L.V. derived the results for the UCA. All numerical results were generated by G.M., as well as the first version of the manuscript. J.L, L.D., and B.S. highly contributed to the ordering of ideas, the identification of the best ways to present them, and to the improvement of the text.

Competing interests

The authors declare no competing interests.

Additional information

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