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Arithmetic and the standard electroweak theory

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Abstract

We propose the relations, $1/e - e = 3$ and $\tan 2\theta_W = 3/2$, where e is the positron charge and θ_W is the weak angle. Present experimental data support these relations to a very high accuracy. We suggest that some duality relates the weak isospin and hypercharge gauge groups of the standard electroweak theory.

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In the $SU(2)\otimes U(1)$ electroweak theory [1], two parameters are necessary to determine the gauge sector. Those parameters are associated to the coupling strengths of the weak isospin and hypercharge gauge groups, and are not related among themselves in the electroweak theory. In the following, we choose them to be α [2] and $\cos\theta_W$:

$$\alpha \equiv \frac{e^2}{4\pi} = [137.03599944(57)]^{-1} \tag{1}$$

where e is the positron charge, and

$$\cos\theta_W \equiv \frac{m_W}{m_Z} = 0.8820 \pm 0.0009, \tag{2}$$

with [3]

$$\begin{aligned} m_Z &= 91.1867 \pm 0.0020 \text{ GeV} \\ m_W &= 80.43 \pm 0.08 \text{ GeV}. \end{aligned}$$

In the following we consider simple relations satisfied by the positron charge e and the weak angle θ_W, which reproduce the experimental values of these quantities to a high accuracy.

First let us consider the equation

$$y - \frac{1}{y} = 3. \tag{3}$$

The two solutions of Eq.(3) are

$$y_\pm = \frac{3 \pm \sqrt{13}}{2} = \begin{cases} 3.302775638 \\ -0.302775638, \end{cases}$$

with $y_+ = -(y_-)^{-1}$, as it is immediately seen by inspection of Eq.(3). We note that y_+ is a continuous fraction given by

$$y_+ = 3 + \frac{1}{3 + \frac{1}{3+\frac{1}{3+\frac{1}{3+\cdots}}}}.$$

Let us define a positron charge $\bar{e}$ and a weak angle $\bar{\theta}_W$ which we *assume* to satisfy the following relation

$$\frac{1 - \tan\bar{\theta}_W}{1 + \tan\bar{\theta}_W} = \bar{e} = -y_- = (y_+)^{-1} = 0.302775638\ . \tag{4}$$

$\bar{e}$ can to be compared with

$$e = (4\pi\alpha)^{1/2} = 0.302822121 \tag{5}$$

as obtained from Eq.(1). We obtain

$$\frac{e}{\bar{e}} = 1.00015 \ ,$$

i.e., $\bar{e}$ reproduces to an impressive accuracy the value of the positron charge.

From Eq.(4), we can evaluate the weak angle $\bar{\theta}_W$,

$$\tan\bar{\theta}_W = \frac{1-\bar{e}}{1+\bar{e}} = 0.535184,$$

or equivalently:

$$\sin\bar{\theta}_W = \frac{1-\bar{e}}{\sqrt{2(1+\bar{e}^2)}} = 0.471858 \ , \tag{6}$$

$$\cos\bar{\theta}_W = \frac{1+\bar{e}}{\sqrt{2(1+\bar{e}^2)}} = 0.881674 \ . \tag{7}$$

Again, the impressive agreement between Eqs. (7) and (2) provides support for the assumption contained in Eq. (4).

By combining Eqs. (3) and (4), we can get very simple relations to determine the absolute values of $\bar{e}$ and $\bar{\theta}_W$, namely

$$\frac{1}{\bar{e}} - \bar{e} = 3 \tag{8}$$

and

$$\tan 2\bar{\theta}_W = \frac{3}{2} \ . \tag{9}$$

These two equations summarize our main results.

A relation between the positron and the down quark electric charges can also be obtained by introducing $\bar{e}_d = -\bar{e}/3$ into Eq. (8). We obtain:

$$\frac{1}{\bar{e}_d} = 1 - \left(\frac{1}{\bar{e}}\right)^2 .$$

The quark fractional charge is given in term of the square of electron charge.

It is worthwhile to consider now the running coupling $\alpha(q^2)$ defined as [4]

$$\alpha(q^2) = \frac{\alpha}{1 + \frac{\Sigma_\gamma(q^2)}{q^2} - \Sigma'_\gamma(0)} \ .$$

The expressions of the photon self-energy $\Sigma_\gamma(q^2)$, at the one-loop level, can be found in appendix A of Ref. [4]. At the electron mass scale we obtain:

$$\begin{aligned} \alpha^{-1}(m_e^2) &= \alpha^{-1} + \frac{17}{9\pi} - \frac{1}{\sqrt{3}} \\ &= 137.0599011 \ . \end{aligned} \tag{10}$$

Then

$$e(m_e^2) \equiv \left(4\pi\alpha(m_e^2)\right)^{1/2} = 0.302795715, \tag{11}$$

can be compared with $\bar{e}$ given in Eq.(4):

$$\frac{e(m_e^2)}{\bar{e}} = 1.000066.$$

The agreement between $e(m_e^2)$ and $\bar{e}$ is even more impressive than between e and $\bar{e}$. Indeed, $\bar{e}$ coincides with the running positron charge at the scale $\sqrt{q^2} \approx 1.27 m_e$.

Defining

$$\bar{\alpha} \equiv \frac{\bar{e}^2}{4\pi} = [137.0780788]^{-1} \tag{12}$$

with $\bar{e}$ given in Eq.(4), we find that

$$\alpha = \bar{\alpha}\left(1 + \frac{16\bar{\alpha}}{121\pi}\right) = (137.0360012)^{-1} \ , \tag{13}$$

provides a relation between α and $\bar{\alpha}$ to an excellent accuracy.

To conclude, let us comment that Eq.(3) belongs to the class of equations

$$y - \frac{1}{y} = n \qquad (n \text{ is a real number}). \tag{14}$$

When $n = 1$, $y_+ = \frac{1+\sqrt{5}}{2}$ is the famous golden number [5]. For any n, the two solutions of Eq.(14) exhibit the property $y_+ y_- = -1$ (and, on the other hand, $y_+ + y_- = n$), with

$$y_\pm = \frac{n \pm \sqrt{n^2+4}}{2}.$$

Notice that y_+ can be represented by a simple continuous fraction

$$y_+ = n + \cfrac{1}{n + \frac{1}{n+\frac{1}{n+\cdots}}} \ .$$

Because Eq.(14) implies

$$y_\pm^2 = ny_\pm + 1 \ ,$$

then

$$y_\pm^{m+2} = ny_\pm^{m+1} + y_\pm^m. \tag{15}$$

From Eq.(15), we get by recurrence, the Fibonacci series [5], when $n = 1$ and m is an integer. With $n = 3$, we get $\bar{e}^{m+2} + 3\bar{e}^{m+1} = \bar{e}^m$, relating the different terms in the series expansions of Q.E.D. Note that the dual relation $y_+y_- = -1$ reminds the relation between the magnetic charge g of the Dirac monopole [6] and the electric charge e : $ge = \frac{\text{integer}}{2}$, if the integer is equal to –2. The dual property of the solutions to Eq. (3) and Eq. (4) together with the excellent agreement between Eqs. (8)–(9) and experimental data suggest that some duality may be hidden relating the weak isospin and hypercharge gauge groups of the standard electroweak theory.

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