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Modelling the Term Structure of Interest Rates in a Multicountry Setting

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To my family and friends

Abstract

Globalization has strengthened the linkage across economies and financial markets around the world. Against this background, this thesis builds alternative modelling frameworks to study movements in the term structure of interest rates in a context of global interdependencies. All frameworks are based on the arbitrage-free affine term structure model with unspanned risk (Joslin et al., 2014) and include macro-financial variables among the risk factors. Using data from several emerging economies, our models are also used to investigate the determinants of bond risk premia in recent decades, to appraise the importance of China's business cycle on Latin American yield curve dynamics, as well as to assess the impact of the global spread of the coronavirus on the term structure dynamics during the COVID-19 pandemic. We also guide the use of the functionalities of the *R* package *MultiATSM*, which we developed during the preparation of this thesis. Overall, our empirical results point to beyond-the-border economic forces as the primary source of domestic term structure fluctuations.

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Chapter 1

Introduction

The term structure of interest rates (or yield curve) relates bond yields to different investment maturities. The class of affine term structure models (ATSMs) has become a central tool in yield curve modelling since Duffie and Kan, 1996 published their seminal contribution. Supported by the principle of the absence of riskless arbitrage opportunities, this class of ATSMs produces closed-form solutions to zero-coupon bond yields of any maturity. Additionally, they successfully capture all the essential term structure developments in a setup that includes a few unobservable (latent) factors. Despite these appealing features, these early models are silent about the nature of the economic forces that cause interest rate fluctuations. Aiming at filling this gap, starting with Ang and Piazzesi, 2003, extensive literature has been devoted to understanding the dynamic interaction between the yield curve and the macroeconomy. In this strand of the literature, a recent work of reference is Joslin et al., 2014. Compared to previous macrofinance ATSMs such as the one of Ang and Piazzesi, 2003, the setup of Joslin et al., 2014 provides a more efficient model estimation process, preserves a good fit of the yield curve cross-section, and enables a straightforward economic interpretation of the risk factors of the model. Despite being handy for modelling single-country specifications, the framework of Joslin et al., 2014 becomes intractable for multicountry settings as the model dimension largely increases. Accordingly, this setup is inappropriate to account for increasing countries' interdependence caused by globalization. This thesis builds on Joslin et al., 2014 using different datasets from emerging markets to provide several multicountry extensions to the existing ATSMs. After a general introduction, Chapter 3 uses ATSMs to examine the drivers of bond risk premia in four emerging countries. Chapter 4 develops a more flexible ATSM to account for global interdependencies. Chapter 5 employs the methodology of Chapter 4 to analyse the determinants of bond yield movements in the context of the COVID–19 pandemic. Chapter 6 describes the functionalities of the *R* package *MultiATSM*, which was used to generate most of the model outputs of the previous chapters.

Chapter 2 presents the main assumptions required to construct an ATSM. In particular, we introduce the critical ingredients of ATSMs: the short-term interest rate, the dynamics of the risk factors under both the risk-neutral and the physical measures, the stochastic discount factor, and the market price of risk. Based on these elements, we show that we can derive the no-arbitrage condition of ATSMs in which the bond yields of any maturity can be expressed as an affine function of a set of risk factors. This mathematical result is used in the remaining chapters of this thesis.

Chapter 3 evaluates the global and domestic drivers of bond risk premia in Brazil, China, Mexico, and Russia. To this end, we estimate a standard ATSMs with unspanned risk developed by Joslin et al., 2014 for each country. We include the country-specific inflation rate, economic growth, and the nominal exchange rate against the U.S. dollar in the set of domestic risk factors, and we select a proxy measure for the worldwide economic activity, the Market Volatility Index (VIX), and a commodity price index in the group of global factors. We show that this modelling framework closely matches movements of realized risk premia and highlights that the global macro-financial factors are the main determinants of bond risk premia dynamics in these four economies.

In the model setup of Chapter 3, we allow global economic shocks to impact the term structure dynamics of each of the four countries, but we deter country-specific shocks to spread across borders. Despite being capable of reproducing appealing empirical features present in the data, the framework from Joslin et al., 2014 faces critical limitations when this setting is extended to accommodate multiple countries at once. The key reason is that the dynamics of the risk factors are assumed to evolve according to an unrestricted VAR model. Accordingly, as the dimensionality of the system grows exponentially with respect to the number of factors, a multicountry version of Joslin et al., 2014 fails to provide reliable statistical inference to test economic relationships in ATSMs. Therefore, studying domestic yield curves developments in a multicountry setting based on ATSMs calls for an alternative modelling specification.

This methodology is developed in Chapter 4. In particular, we extend Joslin et al., 2014 to efficiently account for global interdependencies. To this end, we propose a no-arbitrage ATSM with unspanned risk in which the joint risk factor dynamics follow a global vector-autoregressive (GVAR). We show that the GVAR-ATSM provides a parsimonious yield curve parametrization, which enables a fast estimation process, allows for meaningful statistical inference of economic relationships, and also generates accurate out-of-sample forecasting of bond yields. This chapter provides an empirical illustration of this setup on an economic cohort formed by China and three Latin American economies (Brazil, Mexico and Uruguay). We find evidence that financial investors account for the economic growth in China as an essential source of risk in Latin American bond markets.

In view of the promising results attained in Chapter 4, in Chapter 5 we apply our novel GVAR-ATSM to assess the impact of the worldwide coronavirus dissemination on the sovereign yield curves of emerging markets during the COVID–19 pandemic. We form an economic system that is composed of Brazil, India, Indonesia, and Russia and use data covering the period that roughly extends from the pandemic outbreak to an advanced stage of the rollout of the vaccination campaigns. In the group of domestic risk factors, we include proxy measures to capture the evolution of the coronavirus spread and the risk of sovereign default, in addition to some metrics of inflation and output gap. As global factors, our model features the output gaps of China and the U.S. economies, and the return on the S&P–500. Our results indicate that the (domestic and foreign) coronavirus-related factors accounted for a non-trivial share of the term structure fluctuations in these four countries during the period of analysis. Furthermore, we show that the cost of sovereign borrowing tends to increase when the rates of coronavirus transmission accelerate.

In Chapter 6, we present a detailed description of the use of the *R* package *MultiATSM* which is available from the Comprehensive *R* Archive Network (CRAN). This package provides several estimation routines for the different classes of ATSMs. The *MultiATSM* package was developed during the preparation of this thesis and was used to produce most of the model outputs reported throughout the thesis chapters.

Chapter 2

ATSMs: General Setup

The goal of this chapter is to present a general characterization of single-country ATSMs along the lines of Duffie and Kan, 1996. To avoid notational complexity, we omit country-specific markers to index the variables, as they all refer to the same country. We first present the main components of ATSMs, and, subsequently, we derive the no-arbitrage restrictions from this set of elements.

2.1 Building blocks

In general, ATSMs are formed from three key equations. The first is the short-term interest rate, r_t . By assumption, r_t is affine in N unobserved (or latent) factors, X_t :

$$r_t = \delta_0 + \delta_1' X_t. \quad (2.1)$$

The second element is the model state dynamics under the risk-neutral measure (the Q -measure). In this setting, the dynamics of X_t follows a VAR(1) model:

$$X_t = \mu_X^Q + \Phi_X^Q X_{t-1} + \Sigma_X \varepsilon_t^Q, \quad \varepsilon_t^Q \sim \mathcal{N}(0, I_N). \quad (2.2)$$

Finally, the state vector under the physical measure (the P -measure) also follows a VAR(1) process of the form

$$X_t = \mu + \Phi X_{t-1} + \Sigma_X \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, I_N). \quad (2.3)$$

At this point, it is worth highlighting the difference between the frameworks of Duffie and Kan, 1996 and Joslin et al., 2014. In Duffie and Kan, 1996 the state vectors under both the P and the Q -measures are N -dimensional and identical. In contrast, in Joslin et al., 2014 the state vector under the physical probability contains additional variables, usually consisting of financial and economic information, that are commonly referred

to as unspanned factors. Accordingly, the dimension of the state vector under the P -measure is $K = M + N$, where M is the dimension of the unspanned risk factors, whereas the vector size under the Q -measure is simply N .

To complete the setup, we need to ensure that the model parameters can be translated from the P to the Q -measures. To this end, we need to specify the functional forms of both the market price of risk and the Radon-Nikodym derivative. In the setup of Duffie and Kan, 1996, the former is modelled as a N -dimensional time-varying affine process of X_t :

$$\lambda_t = \lambda_0 + \lambda_1 X_t, \quad (2.4)$$

whereas the latter takes the form of

$$\Gamma_{t+1} = \exp(-1/2\lambda_t' \lambda_t - \lambda_t' \epsilon_{t+1}^Q). \quad (2.5)$$

2.2 No-arbitrage restrictions

Pricing bonds under the risk-neutral measure enables us to express the stochastic discount factor (SDF) as a function of the risk-free interest rate. Another convenient aspect of the Q -measure framework is that it can ensure the absence of riskless arbitrage opportunities, which, in turn, guarantees the existence of a unique SDF (Harrison and Kreps, 1979). Traditionally, the SDF is defined as

$$m_{t+1} = \exp(-r_t) \Gamma_{t+1}. \quad (2.6)$$

Substituting equations (2.1) and (2.5) into (2.6) yields

$$m_{t+1} = \exp(-1/2\lambda_t' \lambda_t - \lambda_t' \epsilon_{t+1}^Q - \delta_0 - \delta_1 X_t). \quad (2.7)$$

Moreover, in a market where the lack of arbitrage opportunities are ensured, the price at time- t of a zero-coupon bond must be equal to the expected price of this same bond one period ahead discounted by the no-arbitrage based SDF. Formally, we can write

$$P_t^{n+1} = \mathbb{E}_t(m_{t+1} P_{t+1}^n), \quad (2.8)$$

where n is the bond's time to maturity.

Next, we assume that bond prices are an exponentially affine process in the latent factors as follows:

$$P_t^n = \exp(\bar{A}_n + \bar{B}_n X_t). \quad (2.9)$$

If we plug equations (2.7) and (2.8) into equation (2.9), we can re-write the coefficients $\bar{A}_n$ and $\bar{B}_n$ in a recursive form:^{1 2}

$$\begin{cases} \bar{A}_{n+1} = \bar{A}_n + \bar{B}_n(\mu - \Sigma_X \lambda_0) + \frac{1}{2} \bar{B}_n \Sigma_X \Sigma_X' \bar{B}_n' - \delta_0, \\ \bar{B}_{n+1} = \bar{B}_n(\Phi - \Sigma_X \lambda_1) - \delta_1. \end{cases}$$

The model parameters from the P and the Q state dynamics are related via the market price of risk as follows: $\mu^Q = \mu - \Sigma_X \lambda_0$ and $\Phi^Q = \Phi - \Sigma_X \lambda_1$. Accordingly, the recursion formula of coefficients $\bar{A}_n$ and $\bar{B}_n$ can be re-expressed in terms of the Q -dynamics parameters as:

$$\begin{cases} \bar{A}_{n+1} = \bar{A}_n + \bar{B}_n \mu^Q + 1/2 \bar{B}_n \Sigma_X \Sigma_X' \bar{B}_n' - \delta_0, \\ \bar{B}_{n+1} = \bar{B}_n \Phi^Q - \delta_1. \end{cases} \quad (2.10)$$

To complete the derivation of no-arbitrage restrictions, we have to write down the formula of the price of a zero-coupon bond at time- t that pays a cash flow of 1 monetary unit in n periods in the future, namely:

$$P_t^n = \frac{1}{(1 + Y_t)^n}.$$

Applying the logarithmic transformation to both sides of the equation and rearranging terms leads to

$$y_t^n = -\frac{\ln(P_t^n)}{n},$$

where we use y_t^n to denote $y_t^n = \ln(Y_t)$.

Finally, we end up with the bond yields of any maturity as a function of the state variables as

$$y_t^n = A_n(\Theta_n) + B_n(\Theta_n)X_t, \quad (2.11)$$

where $A_n(\Theta_n) = -\frac{\bar{A}_n(\Theta_n)}{n}$ and $B_n(\Theta_n) = -\frac{\bar{B}_n(\Theta_n)}{n}$. Parameters $\bar{A}_n(\Theta_n)$ and $\bar{B}_n(\Theta_n)$ are given by (2.10).

¹See Ang and Piazzesi, 2003 for a detailed derivation.

²The boundary conditions of this recursive process are traditionally assumed to be $\bar{A}_1 = -\delta_0$ and $\bar{B}_1 = -\delta_1$.

Chapter 3

Bond Risk Premia in Emerging Markets: Evidence from Brazil, China, Mexico, and Russia ¹

Abstract

We employ an affine term structure model with no-arbitrage restrictions and unspanned risk factors to analyse the global and domestic determinants of bond risk premia in four major emerging markets (Brazil, China, Mexico, and Russia). Among the risk factors, we select national inflation and economic growth, and the country-specific nominal exchange rate against the US dollar as the variables related to the domestic economy. We include a measure of worldwide economic activity, the Market Volatility Index (VIX), and an aggregate price index of commodities in the group of global factors. Our model captures (long-term) movements of realized risk premia and indicates that global economic and financial factors play a relevant role in explaining country-specific bond risk premia dynamics. In contrast, domestic variables carry little explanatory power to rationalize risk premia developments in these four economies. We also provide evidence of heterogeneous responses of country-specific risk premia to global and domestic shocks.

Keywords: *Risk Premia, Term Structure of Interest Rates, International Finance, Emerging Markets.*

JEL Classification: *E43, E44, G15*

¹Joint work with Leonardo Iania and Marco Lyrio. This work is published as Iania et al., [2021](#).

3.1 Introduction

Emerging Markets (EM) economies have become a pivotal part of the world economy over the last decades. According to the 2020 International Monetary Fund (IMF) world's economic outlook, South Africa, Indonesia, Brazil, Russia, India, China, Korea, Turkey, and Mexico (the largest EM economies) have grown, on average, by 4.6% a year over the last two decades, a growth rate that is 2.6 percentage points higher than advanced economies.² This exceptional economic growth makes the largest EM economies account for around one-third of the world's Gross Domestic Product (GDP) in 2017, according to the Organization for Economic Cooperation and Development (OECD).

The prodigious economic growth of EM economies has been mirrored by a relatively modest economic performance and falling interest rates in industrialized economies. Those developments have led international investors to shift their attention to high-yield instruments denominated in EM currencies, such as government bonds (IMF, 2019). This trend, coupled with the increasing usage by EM countries of domestic government bonds to finance their activities, has amplified the size and importance of EM local currency debt.³ Against this background, understanding the drivers of government bond risk compensation (risk premium) in EMs is of primary importance for investors and policy-makers. Investors need to have a clear view of the risk factors affecting their portfolio returns, while policy-makers in EM governments are interested, for example, in assessing how national or international shocks affect their financing capacity.

Such concerns raise a number of questions. Are government bond risk premia in EM economies driven mainly by local or global factors? And do risk compensation in different EM economies react in a similar way to global shocks? We tackle these questions for four of the largest EM economies: Brazil, China, Mexico, and Russia, which together make 22% of world's GDP in 2017, expressed in PPP. The selected countries exhibit significant differences in terms of size (measured by GDP), degree of economic openness and dependence on commodity exports.⁴

²This growth rate is the simple average of the yearly growth rates of South Africa, Indonesia, Brazil, Russia, India, China, Korea, Turkey, and Mexico from 2000 to 2019. If we weight the yearly growth of those countries by their relative size, in terms of GDP, the average growth rate is around 6%. Source: 2020 IMF world's economic outlook and Organization for Economic Cooperation and Development.

³In February 2020, S&P Global estimates that the top 20 EMs are expected to issue a combined \$1.62 trillion of government debt in 2020, up 4% from 2019 and a historical high; see <https://www.spglobal.com/en/>.

⁴According to the World Bank, in 2018, the trade-to-GDP ratios of Brazil, China, Mexico, and Russia were 29%, 38%, 80%, and 51%, respectively. Furthermore, Brazil and Russia are considered as export-commodity-dependent countries while China and Mexico are not. A country is considered to be export-commodity-dependent when more than 60 per cent of its total merchandise exports are composed of commodities (UNCTAD, 2019). Although there are interest rate databases for other emerging economies, their coverage in terms of time series length and maturity spectrum is rather limited. For this reason, we do not include India and possibly other EM countries in our sample.

The quest for understanding the drivers of international bond risk premia dates back to Ilmanen, 1995, who finds that risk premia for long-maturity government bonds of six developed countries can be predicted by a small set of global instruments. In subsequent studies, scholars have explored the link between developed countries bond risk premia and different types of factors: (i) macroeconomic factors, in particular inflation uncertainty (see, for instance, Wright, 2011); (ii) factors based on a combination of forward rates, such as the Cochrane and Piazzesi, 2005 factor⁵ (see Sekkel, 2011 and Kessler and Scherer, 2009), or a world interest rate level factor as in Driessen et al., 2003; and (iii) global and local factors, where the global factor is linked to international business cycles, as in Dahlquist and Hasseltoft, 2013.

Relative to developed economies, the literature on EM government bond risk premia is scant. Some scholars have focused their attention on the local and international drivers of EM government spreads. For example, Ciarlone et al., 2009 find that, in the pre-Lehman's period, large part of the co-variation in EM economies spreads is due to a single factor proxying for financial market volatility.

Recently, Cepni et al., 2019 and Cepni and Güney, 2019 make several contributions adopting a panel setting. Cepni et al., 2019 highlight the combined importance of global and local factors in explaining bond risk premia, while Cepni and Güney, 2019 find that the influence of local factors on bond risk premia can vary, among other aspects, with the level of risk aversion. Finally, the importance of global factors as drivers of EM bond risk premia is further stressed by the out-of-sample exercise proposed by Cepni et al., 2020.

In order to study the determinants of bond risk premia, we rely on affine term structure models, whereby bond yields are expressed as a linear function of pricing factors. These models specify the stochastic discount factor (pricing kernel) as function of the risk free rate and a set of shocks for which investors ask time-varying risk compensation. The latter feature is crucial for obtaining the time-varying risk premia that are analysed throughout the article. We follow Joslin et al., 2011 and Joslin et al., 2014 and propose a no-arbitrage term structure model composed of two types of factors: (i) *spanned* factors (or yield portfolios). These variables are formed as a linear combination of yields and are responsible for modelling the cross-sectional features of the yield curve; and (ii) *unspanned* factors. These factors contain macroeconomic and financial information that contribute to explain the dynamics of bond excess returns over and above the information contained in bond yields.

Our set of unspanned factors include a total of six local and global economic and financial factors: (i) proxies for local economic activity and inflation, which are closely monitored

⁵The Cochrane and Piazzesi, 2005 factor is obtained by regressing 1-year excess returns across maturities at each time t on five forward rates. They show that a tent-shaped linear combination of forward rates predicts annual excess returns on one- to five-year maturity bonds with R^2 up to 0.44.

by central banks; (ii) the exchange rate, which is an important driver of local currency sovereign bond yields and risk premia in EM economies (Gadanecz et al., 2018); (iii) a proxy for global economic activity, which according to Dahlquist and Hasseltoft, 2013 is closely linked to bond risk premia; (iv) changes in global commodity prices, which might increase the risk of higher inflation in emerging countries (Mallick and Sousa, 2013); and (v) the VIX index, which is an important driver of market prices of risk in EM government bonds (Borri and Verdelhan, 2011). We investigate the relationship between each of these factors and the dynamics of bond risk premia through implied impulse response functions (IRFs), variance decompositions and historical decompositions.

We contribute to this emerging literature in three ways. First, we show that arbitrage-free dynamic term structure models with local and global factors generate risk premia (model-implied expected excess returns) that replicate the realized excess returns with a correlation between the two series ranging from +0.83 (Brazil) to +0.65 (China). Second, through the lenses of our model, we assess the importance of local and global factors on risk premia dynamics. The results highlight the importance of global economic activity for Brazil and Russia, and of global commodity prices for China. The VIX index is also relevant in the case of Brazil and China. None of the global or domestic factors seem important in the determination of Mexico's bond risk premia. Finally, we provide evidence of heterogeneous responses of country-specific risk premia to global shocks. For example, only in the case of China positive global demand and commodity prices shocks lead to statistically significant increases in bond risk premia.

The remainder of the article is organized as follows. Section 3.2 gives a brief overview of the evolution of affine term structure models and presents the framework adopted in this article. We describe the yield curve data and each of the local and global unspanned factors in Section 3.3. We present the empirical results in Section 3.4. First, we assess the fit of the model for the yield curve and bond risk premia. We then discuss the IRFs, variance and historical decompositions of bond risk premia for each of the four EM countries analysed. Section 3.5 presents our conclusions.

3.2 Econometrics framework

We first describe the main characteristics of the affine class of term structure models applied to default-free zero-coupon bonds and summarize the main advantages of the approach proposed by Joslin et al., 2011 and Joslin et al., 2014. We then present the technical structure of the model showing the dependence of bond risk premia on both spanned and unspanned factors. We also discuss briefly the estimation procedure of the model.

3.2.1 Affine yield curve models

The affine class of term structure models was introduced by Duffie and Kan, 1996. Such models rule out pure arbitrage opportunities and are both econometrically and numerically tractable. In this class of models, yields at all maturities are affine functions of the factors. This is due to the functional form assumed for the pricing kernel and the short-term interest rate.⁶

The factors in such models are, however, latent (unobservable) and do not have a direct economic meaning. A more satisfactory model should, therefore, also allow the identification of the economic forces behind movements in the yield curve and, as a consequence, in bond risk premia. Ang and Piazzesi, 2003 pioneered such approach with the inclusion of macroeconomic variables as factors in the standard affine term structure model.⁷

Despite the widespread use of this approach, it presents one important drawback. In such models, there is an implicit assumption that the macroeconomic factors can be expressed as a linear combination of bond yields, i.e. are spanned by bond yields. In other words, conditioning on the current yield curve, macroeconomic factors do not add information about their own future values and expected excess returns (bond risk premia). This assumption imposes counterfactual restrictions on the joint dynamics of bond yields and the macroeconomic factors and, moreover, is consistently rejected by standard regression analysis (Joslin et al., 2014).

To tackle this issue, Joslin et al., 2011 propose a normalization of affine models in which the pricing factors are observable portfolios of yields, i.e. linear combinations of yields. These are the *spanned* factors of the yield curve model. Joslin et al., 2014 complement this methodology and propose the introduction of macroeconomic variables as *unspanned* factors in otherwise standard affine models.⁸ Although the latter factors do not affect directly the shape of the yield curve, they may carry relevant information over and above the information contained in bond yields to forecast developments in the spanned factors and hence in excess bond returns. This methodology is suitable to assess the economic forces behind movements in bond risk premia and is, therefore, adopted in this article to study the risk premium dynamics of EM government bonds.

⁶Dai and Singleton, 2000 provide a complete characterization of the affine class of term structure models, specifying the conditions for admissibility and identification of such models.

⁷Other studies in this line of research include Bekaert et al., 2010, Dewachter and Lyrio, 2006 and Rudebusch and Wu, 2008, among others. In these papers, the factors either have a clear macroeconomic interpretation or are structural in nature. Gürkaynak and Wright, 2012 and Piazzesi, 2010 provide a detailed survey of the literature.

⁸This methodology has been used in different contexts by Bauer and Rudebusch, 2020 and Dewachter et al., 2015, among others.

3.2.2 The model

Following Joslin et al., 2014, we assume the existence of K unobserved factors, X_t , which under the risk-neutral probability measure evolve according to a VAR(1) model:

$$X_t = \mu_X^Q + \Phi_X^Q X_{t-1} + \Sigma_X \varepsilon_t^Q, \quad \varepsilon_t^Q \sim N(0, I_K), \quad (3.1)$$

where Σ_X is lower-triangular and Φ_X^Q is diagonal with distinct entries. Assuming the short-term rate r_t is affine on X_t ,

$$r_t = \delta_0 + \delta_1 X_t, \quad (3.2)$$

Dai and Singleton, 2000 show that the price of a zero coupon bond is given by:

$$P_t^{(n)} = E_t^Q[e^{\sum_{j=0}^{n-1} (-r_{t+j+1})}] = E_t^Q[e^{a^{(n)} + b^{(n)} X_t}], \quad (3.3)$$

where $a^{(n)}$ and $b^{(n)}$ solve the well-known Riccati difference equations:⁹

$$a^{(n)} = a^{(n-1)} + b^{(n-1)} \Phi_X^Q + \frac{1}{2} b^{(n-1)} \Sigma_X \Sigma_X' b^{(n-1)'} - \delta_0 \quad (3.4)$$

$$b^{(n)} = b^{(n-1)} \Phi_X^Q - \delta_1, \quad (3.5)$$

with initial conditions: $a^{(0)} = 0$ and $b^{(0)} = \mathbf{0}$. To obtain the loadings for the corresponding yield, we observe that the log of the bond price is $p_t^{(n)} = -n y_t^{(n)}$, hence:

$$y_t^{(n)} = -\frac{1}{n} a^{(n)} - \frac{1}{n} b^{(n)} X_t = a^{(n)} + b^{(n)} X_t, \quad (3.6)$$

Assuming a total of N maturities, we can express Eq. (3.6) as

$$Y_t = A + B X_t, \quad (3.7)$$

with $Y_t = [y_t^{(1)}, \dots, y_t^{(N)}]'$. Joslin et al., 2011 show that X_t can be rotated and linked to a set of spanned yield curve factors, $P_t = W Y_t$, with W being a full-rank matrix, leading to the following representation of the yield curve:

$$Y_t = A_P + B_P P_t, \quad (3.8)$$

with $A_P = I - B(WB)^{-1}WA$ and $B_P = B(WB)^{-1}$. Under the historical probability measure, spanned factors are related to U unspanned factors, M_t , via a VAR(1) model. Stacking P_t and M_t in a vector $Z_t = [M_t; P_t]$, we have that

$$Z_t = C^{\mathbb{P}} + \Phi^{\mathbb{P}} Z_{t-1} + \Sigma \varepsilon_{Z,t}^{\mathbb{P}}, \quad (3.9)$$

⁹See Ang and Piazzesi, 2003 for a detailed derivation.

where $\varepsilon_{Z,t}^{\mathbb{P}} = (\varepsilon_{M,t}^{\mathbb{P}}, \varepsilon_{P,t}^{\mathbb{P}})' \sim N(0, I)$ and Σ is a lower-triangular matrix. Equation (3.9) implies that the h -period expected value of Z_t (and hence of P_t and Y_t), $\mathbb{E}_t Z_{t+h}$, is a function of both P_t and M_t .

The h -period risk premium of a zero-coupon bond with maturity n , $rx_t^{(n,h)}$, is

$$rx_t^{(n,h)} = (\mathbb{E}_t[p_{t+h}^{(n-h)}] - p_t^{(n)}) - y_t^{(h)}, \quad (3.10)$$

where $p_t^{(n)} = -ny_t^{(n)}$ is the log of the bond price. Iterating forward Eq. (3.9) and given Eq. (3.7), we can express Eq. (3.10) as:

$$rx_t^{(n,h)} = A_{rx}^{(n)} + B_{rx}^{(n)} Z_t, \quad (3.11)$$

with

$$A_{rx}^{(n)} = nA_Z^{(n)} - A_Z^{(h)} - (n-h) \sum_{j=0}^{h-1} (\Phi^{\mathbb{P}})^j C^{\mathbb{P}}, \quad (3.12)$$

$$B_{rx}^{(n,h)} = nB_Z^{(n)} - (n-h)B_Z^{(n-h)}(\Phi^{\mathbb{P}})^h - B_Z^{(h)}, \quad (3.13)$$

where $A_Z^{(n)} = A_P^{(n)}$, and $B_Z^{(n)} = [B_P^{(n)} \mathbf{0}]$, with the zero vector is of dimension U . Given that $\Phi^{\mathbb{P}}$ is a full matrix, Eqs. (3.12) and (3.13) imply that bond risk premia are a function of both spanned and unspanned factors.

We estimate the model in two steps. First, we obtain estimates of $C^{\mathbb{P}}$ and $\Phi^{\mathbb{P}}$ via OLS (Eq. (3.9)). Subsequently, the remaining parameters, Φ_X^Q , δ_0 , and Σ , are estimated using maximum likelihood. We then obtain the confidence bands on (a function of) parameters of interest by wild bootstrapping the residuals of the model. For econometric identification, we assume that $\mu_X^Q = 0$, allowing us to interpret δ_0 as the long-run mean of the short rate under the $\mathbb{Q}$ -measure. We also choose δ_1 to be a K -dimensional row vector, all elements of which are one. Finally, we assume that under the risk-neutral probability measure the yield's portfolios follow a stationary VAR. This specification is necessary to avoid that yields diverge to minus infinity with maturity.¹⁰

3.3 Data

Our analysis covers the period from September 2005 to September 2017 and is composed of three sets of series: (i) the countries' term structures of interest rates; (ii) local macroeconomic factors; and (iii) global economic and financial factors. For each country's term structure, we use end-of-the-month zero-coupon yields with maturities of 1, 6, 12, 36, 60, and 120 months. For China, Mexico, and Russia, the data are from

¹⁰See Bauer and Rudebusch, 2020.

Bloomberg.¹¹ For Brazil, due to longer data availability, we select the yields on swap fixed-DI contracts¹² retrieved from B3, the Brazilian Stock Exchange and Over-the-Counter Market. All the yields refer to local currency-denominated contracts.

As local macroeconomic factors, we include proxies for inflation and local economic activity, which are closely monitored by central banks, and the nominal exchange rate, which is an important driver of local currency sovereign bond yields and risk premia in EM economies (Gadanecz et al., 2018). Specifically, (i) a proxy for inflation (π_t^j) is computed as the yearly growth rate of the Consumer Price Index (CPI) of country j and obtained from the IMF, (ii) a proxy for domestic economic activity (g_t^j) is obtained as the normalized country-specific leading indicator for the Gross Domestic Product (GDP), which is developed by the OECD and retrieved from the FRED, the Federal Reserve Bank of St. Louis, and (iii) the nominal exchange rate (s_t^j) is expressed as the price of one US dollar in terms of the local currency, also from the IMF.

We also include three global factors: (i) a proxy for real global economic activity (g_t^W), which accounts for the relationship between risk premia and global economic activity, as suggested by Dahlquist and Hasseltoft, 2013. This factor is captured by Lutz Kilian's measure (Kilian, 2009), available at his webpage;¹³ (ii) a proxy for changes in global commodity prices (π_t^W), which might increase the risk of higher inflation in emerging countries (Mallick and Sousa, 2013). This measure is given by the yearly growth of the global commodity price index; and (iii) a proxy for global financial market tensions (VIX_t), which is an important driver of market prices of risk in EM government bonds; see Borri and Verdelhan, 2011 and Zinna, 2013. This proxy is represented by the Chicago Board Options Exchange (CBOE) Market Volatility (VIX) Index. We obtain the last two factors from the FRED website. The choice of π_t^W and VIX_t as global factors is also motivated by Ciarlone et al., 2009. They extract a common factor capturing the variability of emerging market sovereign spreads which is not directly linked to the specific macroeconomic characteristics of any given country. This factor correlates strongly with the VIX index and the growth of commodities prices.

For each country j , our model includes two spanned, P_t^j , and six unspanned factors, M_t^j .¹⁴ The spanned factors are the first two principal components (PCs) of the country's bond yields. The unspanned factors are the country-specific and global factors listed

¹¹Since the Bloomberg data are for par yields, we bootstrap the original data to obtain zero-coupon yields. The 1-month yield for Russia, Mexico and China are obtained by estimating and fitting a Nelson-Siegel model.

¹²These are derivative instruments with the underlying values linked to Brazil's interbank lending rate. For a detailed explanation regarding the use of such instruments, see Vicente and Tabak, 2008.

¹³The Killian index is based on variations in global shipping freight rates and is tailored to capture fluctuations in the demand for industrial commodities in the global economy.

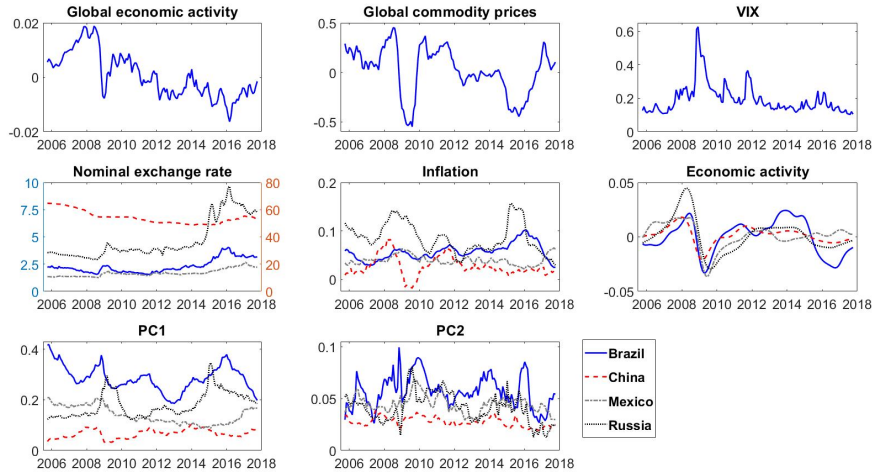
¹⁴Traditionally, term structure works resort to three principal components to account for roughly all the cross-sectional variation in bond yields, see e.g. Litterman and Scheinkman, 1991. As the inclusion of six unspanned factors renders the model dimension to be fairly large, we limit our framework to

above. We use a Cholesky identification scheme, whereby variables are ordered from the most exogenous (international factors, i.e. g^W , π^W , and VIX) to the least exogenous (national factors, i.e. s , π , and g):

$$M_t^j = [g_t^W, \pi_t^W, VIX_t, s_t^j, \pi_t^j, g_t^j]'; \quad P_t^j = [PC1_t^j, PC2_t^j]'. \quad (3.14)$$

Figure 3.1 shows the time series of all unspanned and spanned factors. The first row of subplots shows the unspanned international factors. We observe clearly a downward trend in global economic activity since its peak in May 2008 during the global financial crisis. We also see that the VIX index reached its peak in November 2008 and global commodity prices hit its lowest level in July 2009. The second and third rows of subplots depict the unspanned national factors and the spanned factors for each country, respectively.

Figure 3.1: Unspanned and spanned factors



Note: The figure depicts the time series of all eight factors used in the model: (i) Three global unspanned factors. Global economic activity is the Kilian’s measure (Kilian, 2009), the global commodity prices series is the yearly growth of the global commodity price index, and VIX is the CBOE Market Volatility Index; (ii) Three country-specific unspanned factors. The nominal exchange rate is expressed as the price of one US dollar in terms of the local currency (left-hand scale: Brazil and China; right-hand scale: Mexico and Russia). Inflation is the yearly growth rate of each country’s CPI, and domestic economic activity is the normalized country-specific leading indicator for the GDP developed by the OECD; and (iii) The country-specific spanned factors ($PC1$ and $PC2$), which are the first two principal components of the country’s bond yields. All series span from September 2005 to September 2017.

contain only two spanned factors. Yet, we show that the first two PCs explain at least 95% of yield variations in all countries (see Table 3.3). Previous multi-country term structure models, as the one of Jotikasthira et al., 2015, also make use of two principal components to fit bond yield cross-sections.

Tables 3.1 and 3.2 report a summary statistics of our set of local (Table 3.1) and global (Table 3.2) factors together with the 1-year realized excess returns for the 3- and 10-year bonds for each country. For each variable, we present the first four moments and the autocorrelations at 1 and 12 lags. We also present the correlation matrices for the factors and realized excess returns. For the correlation matrices, the numbers in bold are statistically significant at a 95% confidence level.

Focusing on the correlation matrices for each country, two general patterns emerge. First, while within country realized excess returns are highly correlated (the lowest correlation is +0.71 for the Mexican bonds), Table 3.2 indicates that the correlations among the realized excess returns across countries are characterized by a high degree of heterogeneity. Correlations range from -0.43 between the 10-year Russian bond ($er_R^{10,1}$) and the 3-year Chinese bond ($er_C^{3,1}$) to $+0.42$ between the 10-year Brazilian ($er_B^{10,1}$) and Mexican ($er_M^{10,1}$) bonds. Furthermore, the Brazilian realized excess returns seem to be disconnected from the Chinese and Russian counterparts (correlations are not statistically different from zero).

The second pattern is in line with the previous point. The correlations between local/global factors and realized excess returns are heterogeneous across countries. For example, we notice that realized excess returns on Chinese and Mexican (3-year maturity) bonds are positively correlated with global economic activity, while this correlation is negative for Russian bonds and insignificant for Brazilian ones. A similar heterogeneous pattern emerges if we consider local economic activity. The correlation between local economic activity and realized excess return is negative for all countries except China. Both patterns are in line with the extant evidence on realized excess return dynamics, i.e. realized excess return among maturities are highly correlated and tend to be counter-cyclical, see, e.g. Cochrane and Piazzesi, 2005 and Ludvigson and Ng, 2009), among others.

3.4 Empirical Results

In Section 3.4.1, we assess the fit of model-implied bond yields and risk premia. In the subsequent sections, we examine how shocks to each of the unspanned factors influence our estimates of bond risk premia. To this end, we analyse IRFs (Section 3.4.2), variance decompositions (Section 3.4.3), and historical decompositions (Section 3.4.4).

3.4.1 Fit of bond yields and risk premia

Our model delivers a good fit of the yield curve in each of the four countries. The coefficients of determination (R^2) are in all cases above 0.95, as reported in Table 3.3. To further illustrate this feature, we show in Fig. 3.2 the model fit for the 3- and 10-year

Table 3.1: Local factor statistics and realized excess returns

Brazil						China					
	s	π	g	$er_B^{3,1}$	$er_B^{10,1}$		s	π	g	$er_C^{3,1}$	$er_C^{10,1}$
μ	2.21	0.06	0.00	0.02	0.04	μ	0.07	0.03	0.00	0.01	0.01
σ	0.60	0.02	0.02	0.05	0.23	σ	0.01	0.02	0.01	0.02	0.07
$\tilde{\mu}^3$	1.56	0.73	-0.55	-0.14	-0.17	$\tilde{\mu}^3$	0.90	0.53	-0.72	0.64	0.03
κ	4.76	3.42	2.48	2.34	2.87	κ	2.56	3.32	4.21	3.18	2.50
ρ^1	0.97	0.97	0.97	0.92	0.89	ρ^1	0.98	0.95	0.98	0.94	0.93
ρ^{12}	0.42	0.33	-0.02	-0.41	-0.46	ρ^{12}	0.69	-0.21	-0.08	-0.31	-0.41
Correlation matrix						Correlation matrix					
s	1.00					s	1.00				
π	0.76	1.00				π	0.04	1.00			
g	-0.45	-0.22	1.00			g	0.13	0.72	1.00		
$er_B^{3,1}$	0.16	0.31	-0.37	1.00		$er_C^{3,1}$	-0.18	0.49	0.28	1.00	
$er_B^{10,1}$	0.26	0.35	-0.34	0.96	1.00	$er_C^{10,1}$	-0.21	0.37	0.31	0.87	1.00
Mexico						Russia					
	s	π	g	$er_M^{3,1}$	$er_M^{10,1}$		s	π	g	$er_R^{3,1}$	$er_R^{10,1}$
μ	0.13	0.04	0.00	0.01	0.04	μ	0.35	0.09	0.00	0.01	0.01
σ	0.02	0.01	0.01	0.02	0.08	σ	0.13	0.03	0.02	0.06	0.21
$\tilde{\mu}^3$	1.09	0.70	-1.10	-0.34	0.34	$\tilde{\mu}^3$	1.77	0.51	0.56	-0.21	-0.51
κ	3.94	3.42	4.55	3.07	2.88	κ	4.83	2.34	3.51	3.81	3.57
ρ^1	0.95	0.93	0.99	0.90	0.82	ρ^1	0.97	0.97	0.99	0.94	0.93
ρ^{12}	0.40	0.16	0.17	0.06	-0.12	ρ^{12}	0.49	0.01	0.14	-0.30	-0.33
Correlation matrix						Correlation matrix					
s	1.00					s	1.00				
π	-0.37	1.00				π	0.22	1.00			
g	-0.33	-0.37	1.00			g	-0.34	0.06	1.00		
$er_M^{3,1}$	-0.42	0.58	-0.41	1.00		$er_R^{3,1}$	0.29	0.17	-0.75	1.00	
$er_M^{10,1}$	-0.18	0.10	-0.37	0.71	1.00	$er_R^{10,1}$	0.36	0.15	-0.71	0.98	1.00

Note: The table presents four panels of statistics, one for each country. Each panel presents statistics for the local (domestic) unspanned factors (s , nominal exchange rate; π , inflation; and g , domestic economic activity), and for the 1-year realized excess returns of 3- and 10-year bonds of Brazil ($er_B^{3,1}$, $er_B^{10,1}$), China ($er_C^{3,1}$, $er_C^{10,1}$), Mexico ($er_M^{3,1}$, $er_M^{10,1}$), and Russia ($er_R^{3,1}$, $er_R^{10,1}$). The top panel presents the following statistics: (i) μ , average (level per annum); (ii) σ , standard deviation (level per annum); (iii) $\tilde{\mu}^3$, skewness; and (iv) κ , kurtosis; and (v) ρ^1 and ρ^{12} , autocorrelation coefficients of order 1 and 12 (monthly). The bottom panel presents the correlation matrix for the mentioned factors and realized excess returns. The numbers in bold are statistically significant at a 95% confidence level. The sample period is from September 2005 to September 2017.

Table 3.2: International factor statistics and realized excess returns

	g^W	π^W	VIX	$er_B^{3,1}$	$er_B^{10,1}$	$er_C^{3,1}$	$er_C^{10,1}$	$er_M^{3,1}$	$er_M^{10,1}$	$er_R^{3,1}$	$er_R^{10,1}$
μ	0.00	0.01	0.20	0.02	0.04	0.01	0.01	0.01	0.04	0.01	0.01
σ	0.01	0.25	0.09	0.05	0.23	0.02	0.07	0.02	0.08	0.06	0.21
$\tilde{\mu}^3$	0.33	-0.49	2.30	-0.14	-0.17	0.64	0.03	-0.34	0.34	-0.21	-0.51
κ	2.24	2.41	9.41	2.34	2.87	3.18	2.50	3.07	2.88	3.81	3.57
ρ^1	0.95	0.96	0.88	0.92	0.89	0.94	0.93	0.90	0.82	0.94	0.93
ρ^{12}	0.52	-0.13	0.23	-0.41	-0.46	-0.31	-0.41	0.06	-0.12	-0.30	-0.33
Correlation matrix											
g^W	1.00										
π^W	0.58	1.00									
VIX	0.07	-0.19	1.00								
$er_B^{3,1}$	-0.03	0.19	0.24	1.00							
$er_B^{10,1}$	-0.07	0.14	0.12	0.96	1.00						
$er_C^{3,1}$	0.25	0.17	0.14	0.10	0.10	1.00					
$er_C^{10,1}$	0.23	0.10	-0.00	-0.07	-0.01	0.87	1.00				
$er_M^{3,1}$	0.23	0.16	0.40	0.20	0.14	0.26	0.15	1.00			
$er_M^{10,1}$	0.12	0.12	0.11	0.40	0.42	0.13	0.16	0.71	1.00		
$er_R^{3,1}$	-0.29	-0.58	0.27	0.02	0.00	-0.41	-0.31	0.05	0.24	1.00	
$er_R^{10,1}$	-0.35	-0.61	0.20	0.03	0.02	-0.43	-0.31	-0.06	0.20	0.98	1.00

Note: The table presents statistics for the international (global) unspanned factors (g^W , global economic activity; π^W , global commodity price indicator; and VIX , Market Volatility Index), and for the 1-year realized excess returns of 3- and 10-year bonds of Brazil ($er_B^{3,1}$, $er_B^{10,1}$), China ($er_C^{3,1}$, $er_C^{10,1}$), Mexico ($er_M^{3,1}$, $er_M^{10,1}$), and Russia ($er_R^{3,1}$, $er_R^{10,1}$). The top panel presents the following statistics: (i) μ , average (level per annum); (ii) σ , standard deviation (level per annum); (iii) $\tilde{\mu}^3$, skewness; (iv) κ , kurtosis; and (v) ρ^1 and ρ^{12} , autocorrelation coefficients of order 1 and 12 (monthly). The bottom panel presents the correlation matrix for the mentioned factors and realized excess returns. The numbers in bold are statistically significant at a 95% confidence level. The sample period is from September 2005 to September 2017.

Table 3.3: Fit of the yield curves

Country	Bond maturity (month)					
	1	6	12	36	60	120
Brazil	0.98	0.99	0.96	0.99	0.99	0.95
China	0.99	1.00	0.99	0.98	0.98	0.97
Mexico	0.99	1.00	0.99	0.98	0.99	0.98
Russia	1.00	1.00	0.99	0.99	1.00	0.98

Note: The table reports the coefficient of determination (R^2) of model-implied bond yields in fitting realized bond yields for each country separately. The sample period is from September 2005 to September 2017.

bond yields. We observe that the error terms are close to zero throughout the sample period. As such results are standard in the literature, we focus our attention on the fit of bond risk premia.

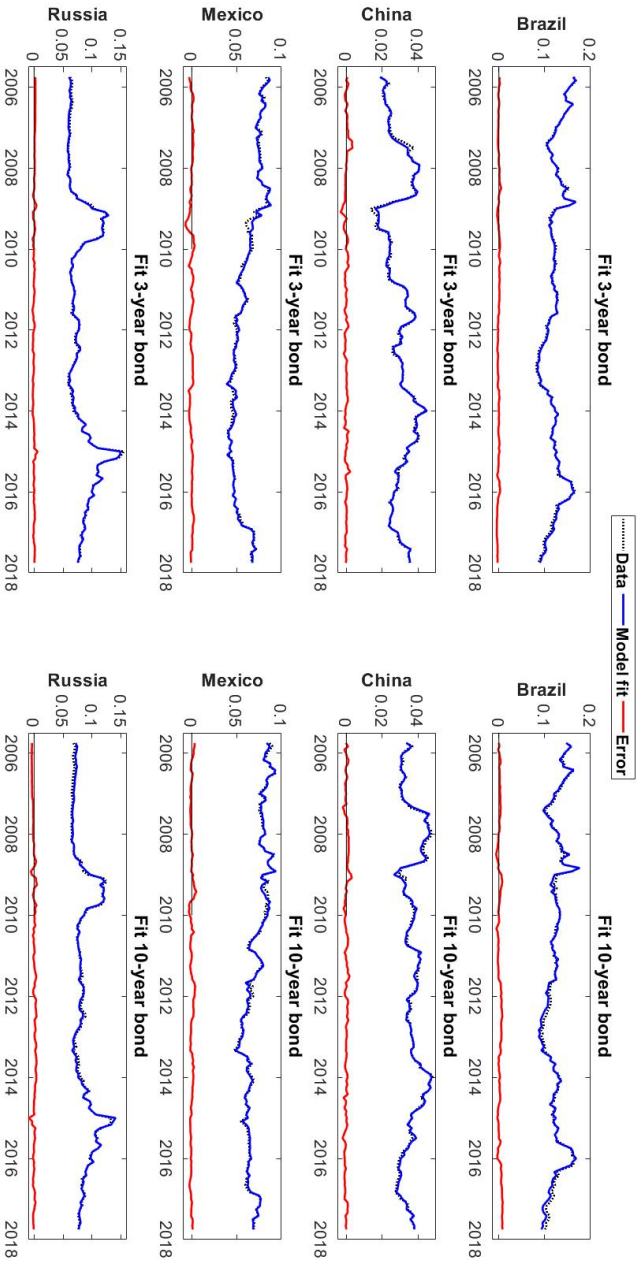
In Table 3.4, we summarize the statistics for the model-implied 1-year bond risk premia, i.e. risk premia are computed for a holding period of 1 year. Figure 3.3 plots the risk premia on a 3- and 10-year bond against the realized excess returns. Note that the realized excess return (black dotted line) ends 12 months before the end of our sample period as the realized excess return is computed based on data at $t + 12$ months. In contrast, the risk premium (blue solid line) are computed until the end of the sample period.

From Table 3.4, we observe that for all countries both the mean (μ) and standard deviation (σ) of the risk premia increase with maturity. The largest risk premia are, on average, from Brazil and Mexico and the most volatile from Brazil and Russia. This same Table reports the ratio between the standard deviation of bond risk premia and the standard deviation of realized excess returns (σ/σ^{er}). For all countries, this ratio is below 1 and decreases with maturity. Across countries, this ratio ranges from 0.87 (Mexico, 36-month bond) to 0.52 (China, 120-month bond).

Figure 3.3 shows that bond risk premia are in line with realized excess returns, despite the higher volatility of the latter. Although realized excess returns are more volatile than bond risk premia, our model captures the low-frequency movements of such returns. The correlation between these two series for all countries and bond maturities ranges from +0.65 to +0.83 (see Table 3.4). We believe this is a striking result. As mentioned before, realized excess returns are obtained using information at time $t + 12$ months, while risk premia are model-implied estimates based on information available at time t only.

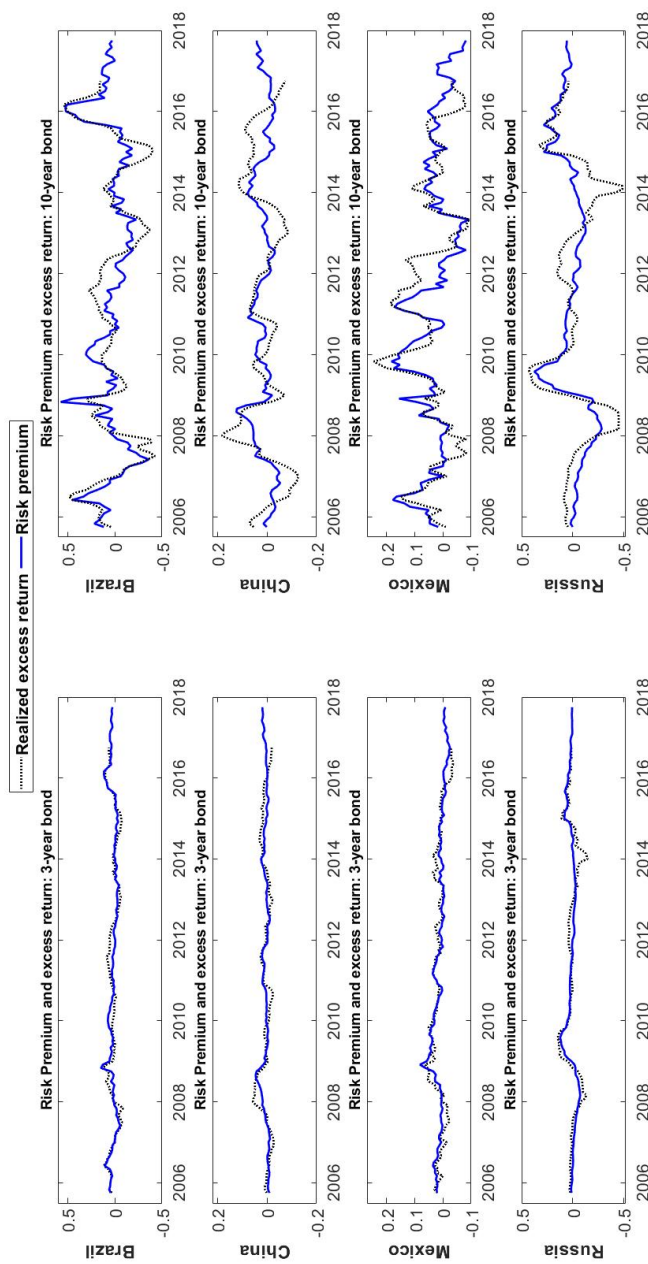
Overall, the correlation between risk premia and realized excess returns decreases with maturity. This is an expected result due to the pattern exhibited by the ratio σ/σ^{er} described above. In other words, the higher the volatility of realized excess returns

Figure 3.2: Fit of the 3- and 10-year bond yield



Note: The figure compares the model-implied yield (blue solid line) on a 3- and 10-year bond with the respective realized bond yield (black dotted line) for each country. The figure also shows the fitting errors (red solid line) around the zero line. The yields are given in level per annum. All series span from September 2005 to September 2017.

Figure 3.3: Fit of the 1-year bond risk premium on a 3- and 10-year maturity bond



Note: The figure compares the model-implied risk premium (blue solid line) on a 3- and 10-year bond with the realized excess return (black dotted line) for each country. The holding period is 1 year. The risk premia and realized excess returns are given in level per annum. All series span from September 2005 to September 2017.

Table 3.4: Bond risk premia statistics

Maturity	Brazil			China			Mexico			Russia		
	36	60	120	36	60	120	36	60	120	36	60	120
μ	1.91	2.73	3.86	0.60	1.01	1.50	1.18	2.09	3.30	0.95	1.17	1.19
σ	3.71	7.38	16.75	1.12	1.95	3.56	1.81	3.10	6.04	3.85	7.19	13.46
σ/σ^{er}	0.78	0.75	0.72	0.60	0.57	0.52	0.87	0.83	0.77	0.67	0.66	0.65
Corr.	0.78	0.81	0.83	0.71	0.69	0.65	0.77	0.76	0.74	0.82	0.79	0.77
R^2	0.61	0.66	0.68	0.49	0.46	0.41	0.59	0.58	0.55	0.65	0.61	0.58

Note: The table reports summary statistics for the 1-year risk premium of bonds with maturities of 36, 60, and 120 months: (i) μ , average bond risk premia (in annual percentage terms); (ii) σ , standard deviation of bond risk premia (in annual percentage terms); (iii) σ/σ^{er} , standard deviation of bond risk premia over the standard deviation of realized excess returns; (iv) Corr., correlation between risk premia and realized excess returns; and (v) R^2 , coefficient of determination of bond risk premia in fitting realized excess returns. The sample period is from September 2005 to September 2017.

relative to risk premia, the lower the fitting performance of the model. This pattern is not observed in the case of Brazil: despite a higher volatility of realized excess returns for long-term bonds the performance of the model improves with maturity.

3.4.2 Impulse response functions

Figure 3.4 plots the median response of the 1-year risk premium on a 10-year maturity bond to a one standard deviation-positive shock in each of the factors. The confidence intervals are obtained from a bootstrap procedure for a level of significance equal to 66%. The horizontal axes express the time horizon (up to 24 months in the future) and the vertical axes (risk premia responses) are given in level per annum.

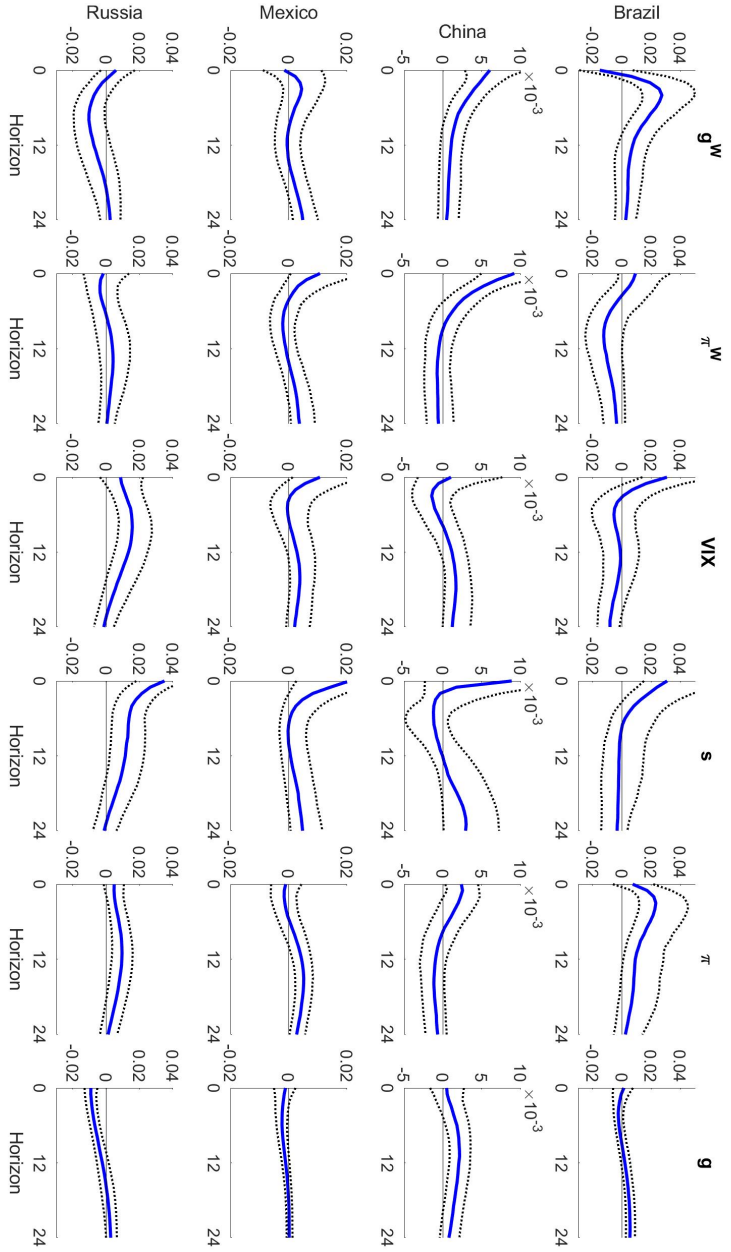
The IRFs are computed based on a Cholesky decomposition with the ordering of the variables defined in Eq. 3.14. The same ordering (from left to right) is used in each row of subplots for each country.

In line with intuition, positive shocks to global economic activity (g^W) induce an instantaneous reduction in the bond risk premium in Brazil and Mexico (marginally). These responses are, however, not statistically significant. In the case of Russia, this response is negative and statistically significant between 6 and 9 months. On the contrary, for Brazil, this response becomes positive and statistically significant between 2 and 10 months. And, for Mexico, the same happens after about 21 months. For China, the instantaneous effect is positive being statistically significant up to about 9 months. A lack of a clear counter-cyclical pattern of EM bond risk premia may reflect the fact that the sovereign bonds of these economies are not perceived as riskless investments during periods of global recessions.

Positive global commodity price (π^W) shocks trigger a statistically significant increase in the Chinese bond risk premium of around 90 basis points. The response of China to shocks on π^W (and g^W) might be linked to possible fears of internal inflationary pressure following those global shocks. Furthermore, unlike other EM economies, China is a large commodity importer and not exporter (UNCTAD, 2019). Hence, an increase in risk-premium is expected. Brazil and Mexico also show a positive response but not statistically significant. The opposite effect is observed for Russia, where positive shocks to π^W decrease the required compensation for risk. Although this response is not statistically significant, it is probably due to the country's heavy reliance on commodity exports. As a matter of fact, a large body of literature documents that the international prices of commodities have nontrivial macroeconomic implications for developing economies (Fernández et al., 2018).

A positive shock to the VIX index causes an increase in the bond risk premium in all countries. This response is statistically significant in the cases of Brazil and Russia. For Brazil, the risk premium increases by around 3%, while for Russia it peaks after about 6

Figure 3.4: Impulse response functions: response of the 1-year risk premium on a 10-year maturity bond to each unspanned factor



Note: The figure shows the median responses (solid lines) of the 1-year risk premium on a 10-year bond for each country to a one standard deviation shock to each of the unspanned factors. Unspanned factors: g^W , global economic activity; π^W , global commodity price indicator; VIX , Market Volatility Index; s , nominal exchange rate (domestic currency per US dollar); and g , domestic economic activity. We make use of a Cholesky decomposition where the ordering of the variables is the one just listed and also presented in each row of subplots from left to right. The dotted lines show the confidence intervals obtained from a bootstrap procedure and constructed for a level of significance equal to 66%. The horizontal axis is given in months and the risk premium responses (vertical axis) are given in level per annum. All series span from September 2005 to September 2017.

months. These results point to a higher sensitivity of those two bond markets to global financial market turbulence.

An exchange rate (s) devaluation shock drives risk premium up in all countries. The initial increase is statistically significant for Brazil and Russia, where it increases by 3% and almost 4%, respectively. Intuitively, as part of a country's liabilities is denominated in foreign currency, a weakening of the domestic currency increases the total debt burden. This positive increment in the debt service might lead investors to require an extra compensation for holding domestic assets.

In line with the findings of Wright, 2011, inflation (π) shocks increase bond risk premium for all countries in a statistically significant way. The highest impact is observed for Brazil where it reaches a maximum of 200 basis points after about 3 months. The reaction of China's risk premium is statistically significant on impact but with the lowest value among the four countries, about 25 basis points. Mexico and Russia exhibit a maximum of 50 and 90 basis points after about 15 and 11 months, respectively. This indicates that following inflationary pressures investors require a positive risk premium to hold nominal bonds of long maturities.

Finally, as expected, a positive shock to the local economic activity factor (g) triggers, in general, a negative response of the risk premium on impact in all countries. Russia is the most prominent case with a decrease in the risk premium of about 30 basis points being statistically significant until almost 1 year. For Brazil, the negative effect is significant around 4 months but positive and significant after 1 year. Although the Chinese risk premium decreases on impact, it only becomes significant after 6 months when is then positive. Finally, the effect on Mexico's risk premium is not significant.

The responses of the risk premium on bonds of other maturities are qualitatively very similar. To illustrate this, we present in the Appendix (Fig. 3.A.1) the response of the 1-year risk premium on a 3-year maturity bond.

3.4.3 Variance decompositions

We now perform a variance decomposition exercise in order to identify the main drivers behind movements in bond risk premia. This technique allows us to estimate the proportion of risk premium movements attributable to each factor (unspanned and spanned). Table 3.5 shows the variance decomposition of the 1-year risk premium on a 10-year bond. We list the contribution of each factor to the forecast variance for a horizon of 1, 12, 60, and 120 months ahead. Each row in the table adds up to 100%. The ordering of the variables is the same as the one used for the IRFs.

Table 3.5 shows that, in all cases except Russia, the proportion of forecast variance accounted for by the six unspanned factors increases with the forecasting horizon. Global

Table 3.5: Variance decomposition of the 1-year risk premium on a 10-year bond

Brazil								
Horizon (mth)	g^W	π^W	VIX	s	π	g	$PC1$	$PC2$
1	0.20	0.02	0.20	0.17	0.00	0.00	0.21	0.20
12	0.29	0.06	0.09	0.13	0.14	0.00	0.15	0.14
60	0.28	0.11	0.13	0.10	0.16	0.03	0.09	0.10
120	0.29	0.10	0.15	0.09	0.18	0.04	0.07	0.08

China								
Horizon (mth)	g^W	π^W	VIX	s	π	g	$PC1$	$PC2$
1	0.05	0.23	0.05	0.01	0.06	0.00	0.16	0.44
12	0.12	0.19	0.07	0.02	0.05	0.02	0.07	0.46
60	0.10	0.20	0.08	0.02	0.07	0.06	0.06	0.41
120	0.10	0.20	0.08	0.02	0.07	0.06	0.06	0.41

Mexico								
Horizon (mth)	g^W	π^W	VIX	s	π	g	$PC1$	$PC2$
1	0.08	0.01	0.01	0.01	0.00	0.00	0.43	0.46
12	0.04	0.02	0.01	0.01	0.01	0.02	0.45	0.44
60	0.16	0.06	0.04	0.07	0.13	0.02	0.28	0.24
120	0.16	0.06	0.05	0.07	0.13	0.02	0.27	0.24

Russia								
Horizon (mth)	g^W	π^W	VIX	s	π	g	$PC1$	$PC2$
1	0.04	0.19	0.05	0.32	0.00	0.06	0.05	0.29
12	0.19	0.11	0.10	0.12	0.04	0.08	0.28	0.08
60	0.21	0.06	0.11	0.11	0.06	0.05	0.33	0.07
120	0.21	0.06	0.11	0.11	0.06	0.05	0.33	0.07

Note: The table reports the contribution of each factor to the forecast variance of the 1-year risk premium on a 10-year bond for each country. We use a Cholesky decomposition with the variables ordered from the most exogenous (international factors) to the least exogenous (national factors), as follows. Unspanned factors: g^W , global economic activity; π^W , global commodity price indicator; VIX , Market Volatility Index; s , nominal exchange rate (domestic currency per US dollar); π , inflation; and g , domestic economic activity. Spanned factors: $PC1$ and $PC2$, first and second principal components of the country's bond yields, respectively. The forecasting horizons considered are 1, 12, 60, and 120 months. Each row in the table, therefore, adds up to 100%.

economic activity is the most important unspanned factor driving movements in bond risk premia for all countries except China. This factor is especially important for Brazil, having a consistent impact across all horizons. In the case of China, global commodity prices have the largest effect on risk premia. This factor is also relevant for Russia. The VIX index, the exchange rate, and inflation also have a significant influence in the forecast variance of risk premia for Brazil and Russia. Interestingly, domestic economic activity seems to have very little impact on bond risk premia for all countries.

For Mexico, the spanned factors account for most of the forecast variance for all forecasting horizons (89% for one-month ahead to 51% for a 10-year horizon). In the case of China, latent factors dominate the forecast variance for short horizons (1 and 12 months) but macro and financial factors slightly dominate medium to long horizons. Among those, global commodity prices stand out for having an important contribution (around 20%) at all horizons.

For Brazil and Russia, the unspanned factors are responsible for most of the forecast variance at all forecasting horizons, reaching 85% in the case of Brazil for a 10-year horizon. With respect to Brazil, global economic activity is overall the most important factor driving movements in bond risk premia. The VIX index and the exchange rate also play a significant role in the short horizon, while domestic inflation appears relevant for longer horizons. For Russia, the exchange rate and global commodity price shocks dominate at a short horizon, while global economic activity is predominant at long horizons.

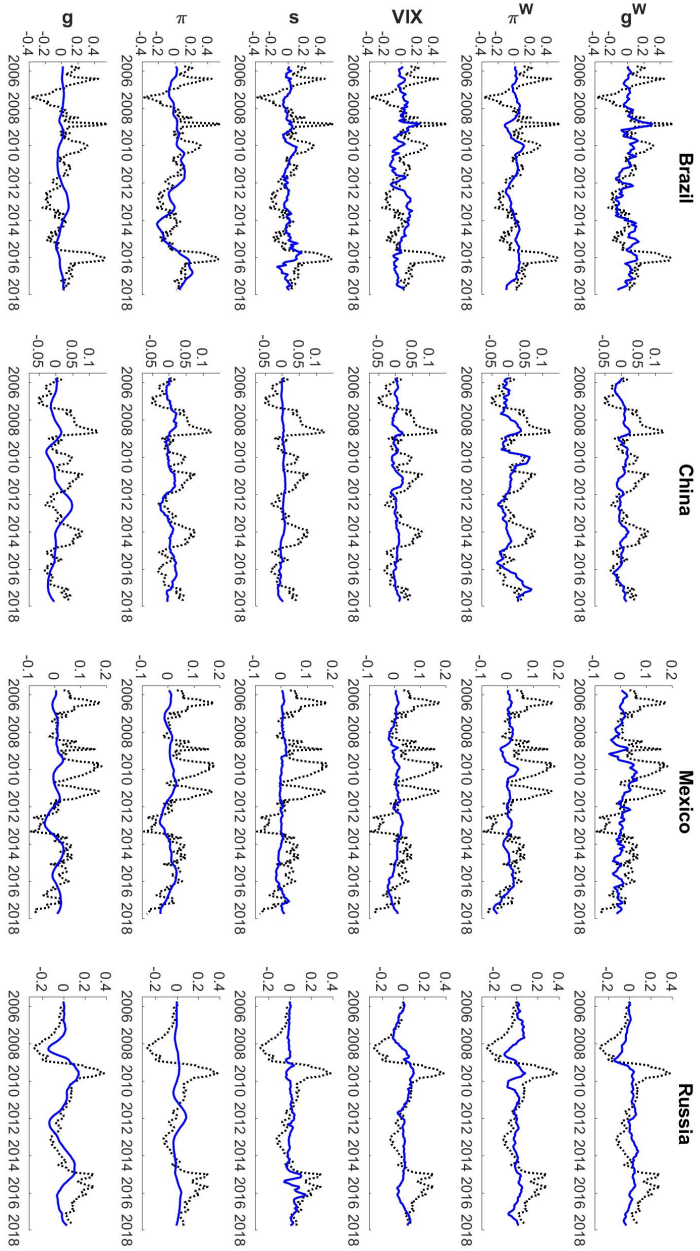
In the Appendix 3.A (Table 3.A1), we present the variance decomposition of the 1-year risk premium on a 3-year bond. The qualitative results are mostly the same. One main difference is the presence of inflation instead of global commodity prices as the most important unspanned factor in the case of China.

3.4.4 Historical decompositions

The variance decomposition discussed above highlights the importance of specific factors for each country. In order to visualize the contribution over time of each of those factors, we perform a historical decomposition of the 1-year risk premium on a 10-year bond for each country over our sample period. Each column in Fig. 3.5 shows the contribution of each unspanned factor for a particular country. The dotted line in each subplot shows the total risk premium over time and the solid line the contribution of the factor under consideration. We focus our discussion on the main spikes of the risk premium in each country.

For Brazil, we analyse two spikes. The first one occurred in October 2008, at the end of the global financial crisis. Not surprisingly, the sharp increase in the risk premium is mainly driven by variables external to the country. Global economic activity and the

Figure 3.5: Historical decomposition of the 1-year risk premium on a 10-year maturity bond



Note: The figure shows the historical contribution of each unspanned factor to the 10-year bond risk premium for each country. Unspanned factors: g^W , global economic activity; π^W , global commodity price indicator; VIX , Market Volatility Index; s , nominal exchange rate (domestic currency per US dollar); π , inflation; and g , domestic economic activity. The risk premium (black dotted line) and its components (red solid line) are given in level per annum. All series span from September 2005 to September 2017.

VIX index represent 37% and 27% of the absolute contribution to the total bond risk premium in that period. The second spike occurred at the beginning of 2016 (January). During this period, Brazil experienced a deep economic recession combined with high inflation and a depreciated domestic currency (see also Fig. 3.1). The exchange rate reached its weakest level against the dollar during that period. Global commodity prices were also still recovering from its bottom value in August 2015. As a consequence, we find that domestic inflation, global commodity prices, and the exchange rate are responsible for a substantial part of the bond risk premium (above 50% in absolute terms).

In the case of China, we focus on the main spike in the 10-year risk premium occurred in July 2008, also during the global financial crisis. Global commodity prices were almost at its peak, reached in June 2008 (see Fig. 3.1). The increase in the risk premium during this period, therefore, can be largely attributed (30%) to the high level of commodity prices in the world market. Global economic activity is also responsible for a substantial part (around 20%) of the risk premium.

With respect to Mexico, our historical decomposition shows the predominant contribution of latent factors. This can also be seen in the three spikes observed in the risk premium during our sample period. In the first spike (May 2006), we observe some contribution of global commodity prices and the VIX index (less than 10% each in absolute terms). Around the second spike (Oct 2009), at the end of the global financial crisis, global economic activity and global commodity prices play significant roles. In the third spike (Feb 2011), we observe a significant contribution (20% in absolute terms) from the global economic activity.

Finally, for Russia, we focus on the spike of July 2009. Local economic activity is responsible for a significant part of the 10-year bond risk premium (close to 30% in absolute terms) during this period. The VIX index and global commodity prices also have a significant impact on bond risk premium (almost 40% jointly in absolute terms).

The historical decomposition of the risk premium for bonds with other maturities provide qualitatively similar results. As an illustration, Fig. 3.A.2 in the Appendix 3.A shows the 1-year risk premium on a 3-year bond for the countries in our sample.

3.5 Concluding Remarks

The increasing importance of EM economies as a source of global economic growth calls for an understanding of the dynamics of local government debt market. This market is a major source of financing for local governments and offers to international investors financial instruments with relatively high returns. Understanding the dynamics of these markets is hence pivotal.

We examine the importance of global and local factors in the determination of bond risk premia in major EM economies, i.e. Brazil, China, Mexico, and Russia. Global economic and financial factors are key drivers of movements in risk premia. Global economic activity, financial tension and the exchange rate play a significant role for Brazil and Russia. Global commodity prices influence Russian bond risk premia and are a leading factor for Chinese ones. The Mexican bond market seems more resilient to global and financial shocks.

Policy-makers and investors might benefit from our results. For investors, the diverse reaction of local risk premia to global shocks might create potential diversification opportunities. For policy-makers, our results show that, depending on the shock, some large countries might require closer monitoring than others.

Our results open up new avenues of research. Two are the following: First, in this article, we model each bond market separately. It would be interesting to develop a model that jointly prices government bonds of different emerging countries by means of global and local factors. The advantage of such a model would be that one can potentially analyse spillover effects among countries. Second, in this article we look at one aspect of government bond markets, i.e. risk premia. However, it would be interesting to look at more policy-related measures such as the expected evolution of short-term rates and term premia. These lines of research will be the topic of future work.

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Disclosure statement

No potential conflict of interest was reported by the author(s).

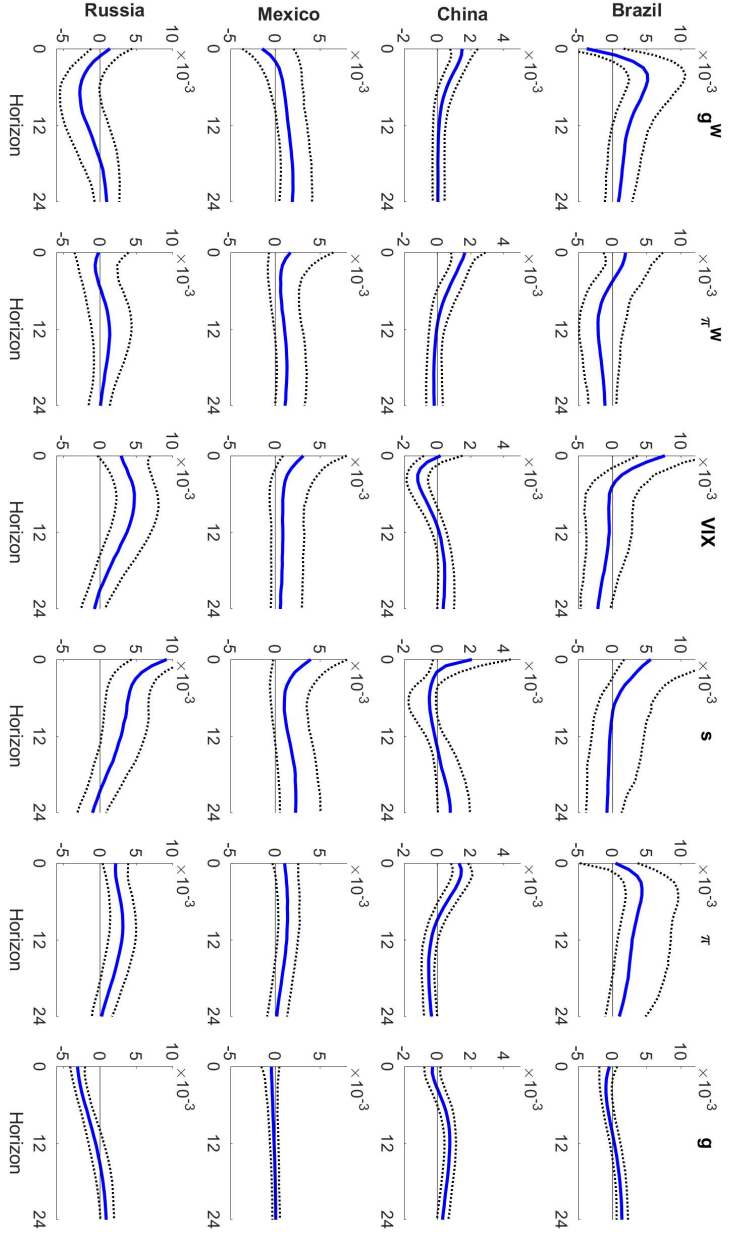
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Appendices

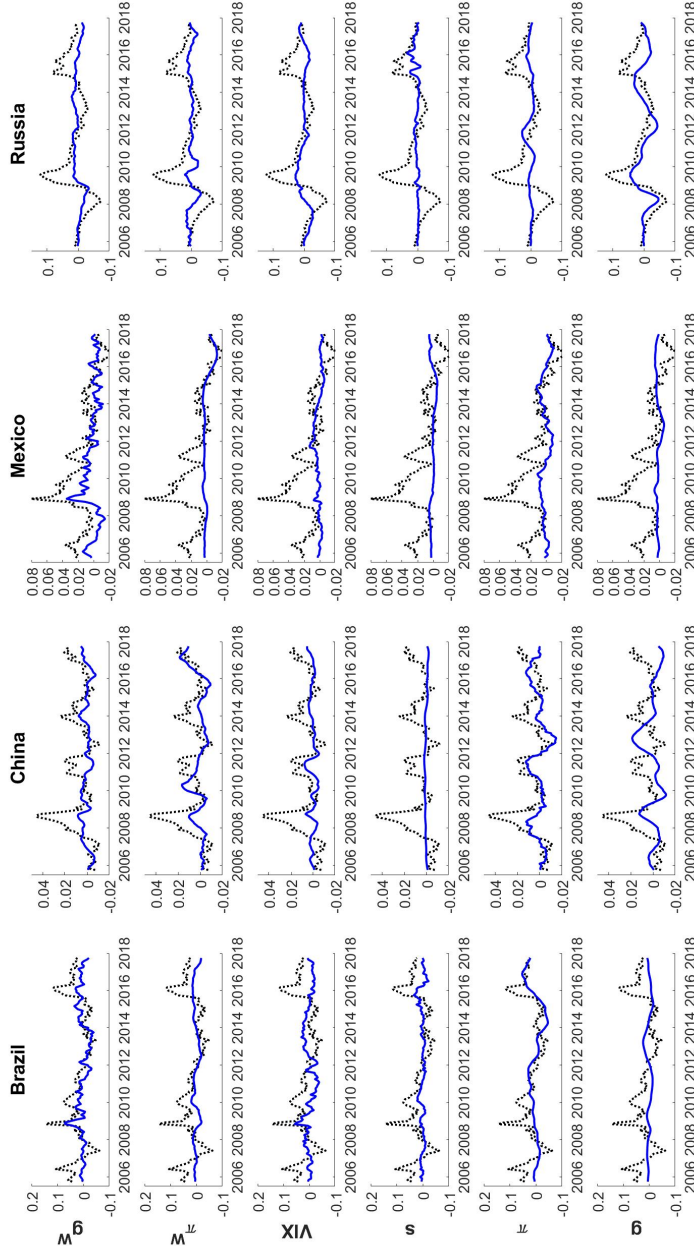
3.A Figures and Tables

Figure 3.A.1: Impulse response functions: response of the 1-year risk premium on a 3-year maturity bond to each unspanned factor



Note: The figure shows the median responses (solid lines) of the 1-year risk premium on a 3-year bond for each country to a one standard deviation shock to each of the unspanned factors. Unspanned factors: g^W , global economic activity; π^W , global commodity price indicator; VIX , Market Volatility Index; s , nominal exchange rate (domestic currency per US dollar); π , inflation; and g , domestic economic activity. We make use of a Cholesky decomposition where the ordering of the variables is the one just listed and also presented in each row of subplots from left to right. The dotted lines show the confidence intervals obtained from a bootstrap procedure and constructed for a level of significance equal to 66%. The horizontal axis is given in months and the risk premium responses (vertical axis) are given in level per annum. All series span from September 2005 to September 2017.

Figure 3.A.2: Historical decomposition of the 1-year risk premium on a 3-year maturity bond



Note: The figure shows the historical contribution of each unspanned factor to the 3-year bond risk premium for each country. Unspanned factors: g^W , global economic activity; π^W , global commodity price indicator; VIX , Market Volatility Index; s , nominal exchange rate; π , inflation; and g , domestic economic activity. The risk premium (black dotted line) and its components (red solid line) are given in level per annum. All series span from September 2005 to September 2017.

Table 3.A1: Variance decomposition of the 1-year risk premium on a 3-year bond

Brazil								
Horizon (mth)	g^W	π^W	VIX	s	π	g	$PC1$	$PC2$
1	0.26	0.01	0.30	0.13	0.00	0.00	0.16	0.13
12	0.30	0.04	0.16	0.12	0.14	0.01	0.13	0.11
60	0.31	0.09	0.17	0.09	0.18	0.03	0.06	0.06
120	0.30	0.09	0.19	0.08	0.19	0.04	0.05	0.05

China								
Horizon (mth)	g^W	π^W	VIX	s	π	g	$PC1$	$PC2$
1	0.08	0.08	0.02	0.00	0.23	0.04	0.54	0.02
12	0.14	0.16	0.14	0.01	0.22	0.03	0.18	0.12
60	0.11	0.17	0.13	0.01	0.22	0.08	0.15	0.14
120	0.11	0.17	0.13	0.01	0.22	0.08	0.15	0.14

Mexico								
Horizon (mth)	g^W	π^W	VIX	s	π	g	$PC1$	$PC2$
1	0.34	0.00	0.05	0.00	0.05	0.00	0.35	0.21
12	0.23	0.01	0.06	0.01	0.21	0.01	0.31	0.16
60	0.32	0.06	0.05	0.11	0.18	0.01	0.20	0.08
120	0.31	0.07	0.06	0.11	0.17	0.01	0.19	0.07

Russia								
Horizon (mth)	g^W	π^W	VIX	s	π	g	$PC1$	$PC2$
1	0.03	0.19	0.03	0.24	0.02	0.13	0.13	0.24
12	0.18	0.08	0.12	0.06	0.07	0.10	0.36	0.04
60	0.20	0.05	0.11	0.10	0.08	0.06	0.36	0.05
120	0.20	0.05	0.10	0.11	0.08	0.06	0.36	0.05

Note: The table reports the contribution of each factor to the forecast variance of the 1-year risk premium on a 3-year bond for each country. We use a Cholesky decomposition with the variables ordered from the most exogenous (international factors) to the least exogenous (national factors), as follows. Unspanned factors: g^W , global economic activity; π^W , global commodity price indicator; VIX , Market Volatility Index; s , nominal exchange rate (domestic currency per US dollar); π , inflation; and g , domestic economic activity. Spanned factors: $PC1$ and $PC2$, first and second principal components of the country's bond yields, respectively. The forecasting horizons considered are 1, 12, 60, and 120 months. Each row in the table, therefore, adds up to 100%.

Chapter 4

A Multicountry Model of the Term Structures of Interest Rates with a GVAR¹

Abstract

Extant multicountry affine term structure models (ATSMs) are able to handle global interdependence at the unfeasible cost of increasing model dimensionality. To close this gap, this paper proposes a no-arbitrage *ATSM* in which the risk factor dynamics follow a global vector-autoregressive (*GVAR*). *GVAR – ATSM* renders a parsimonious yield curve parametrization, which allows for a fast estimation process, enables meaningful statistical inference of economic relationships, and produces accurate bond yields out-of-sample forecasting. To empirically illustrate our novel *ATSM*, we build a markedly integrated economic system composed of three Latin American economies and China. We find that, consequent to its prominent role in the worldwide economy, China's economic stances have nonnegligible impacts on Latin American yield curve dynamics.

Keywords: *Term Structure of Interest Rates, Global Financial Market, GVAR*

JEL Classification: : C58, E44, G15.

¹Joint work with Bertrand Candelon. The working paper version of this chapter is available as Candelon and Moura, [2021](#).

4.1 Introduction

The interplay between the term structure of interest rates and economic unfolding is of key relevance to economic players. For central banks, the yield curve is an important tool of monetary policy transmission (Evans and Marshall, 2007), whereas for financial investors, the term structure reflects the expectations and risk-return assessments of the future macroeconomic outlook (Ang et al., 2006). The linkage between macroeconomic fundamentals and the yield curve is at the core of affine term structure models (henceforth *ATSMs*).² Recently, *ATSMs* have been developed to accommodate growing countries' interdependence induced by globalization. However, these models hardly succeed to properly account for cross-border shocks due to their high complexity and the large numbers of parameters to be estimated. This paper sheds new light on this field by means of a novel parsimonious *ATSM* representation that accurately captures the effects of shocks for globalized economies. This new framework makes important progress toward model tractability and therefore increasing the speed of the estimation process of large-scale *ATSMs* while improving statistical inference and the forecasting abilities of *ATSMs*.

The backbone of our term structure framework is the influential work of Joslin et al., 2014 (henceforth *JPS – ATSM*). In essence, *JPS – ATSM* assumes the absence of arbitrage opportunities and considers linear state space representations of the yield curve dynamics. In particular, this model representation simultaneously combines the traditional yield curve factors (e.g., level, slope, and curvature)³ alongside economic and financial variables. The former, labeled spanned factors, grant a good fit for the cross-section of bond yields. The latter, termed unspanned factors, shape the time evolution of the term structure and contribute to explaining the linkage between bond yields and the real economy. With respect to early referenced macrofinance *ATSMs*, *JPS – ATSM* enables a more tractable and accurate estimation of the model parameters while preserving good explanatory power of the yield cross-section and providing a clear economic interpretation of their risk factors.

Over the last two decades, many papers have employed *ATSMs* to evaluate the role played by domestic macroeconomic fundamentals (usually inflation and real economic growth) in the evolution of the shape of the U.S. term structure.⁴ Recently, *ATSMs* assimilated the effects of a globalized economy. To this end, aside from domestic underlying factors, foreign aspects are considered to steer term structure developments. Based

²This class of models includes, for instance, the works of Ang and Piazzesi, 2003, Bikbov and Chernov, 2010, and Wright, 2011, Bauer et al., 2014, among many others.

³See, for example, Hordahl et al., 2006, Rudebusch and Wu, 2008, Bekaert et al., 2010, and Huse, 2011 for alternative structural interpretations for these factors.

⁴See, for instance, Ang and Piazzesi, 2003, Rudebusch and Wu, 2008, and Joslin et al., 2014, among others.

on the *ATSM* developed by Joslin et al., 2014, Jotikasthira et al., 2015 (henceforth *JLL – ATSM*) extended the unspanned macroeconomic risk setting to accommodate multiple countries. As such, the *JLL – ATSM* framework enables us to determine the relative contributions of domestic idiosyncratic and foreign shocks to explaining the yield curve dynamics.

As in the traditional *ATSM* literature, *JLL – ATSM* assumes that the joint dynamics of the risk factors evolve according to a first-order *VAR*. This feature of the model raises several important limitations. First, the inclusion of a few supplementary countries in the economic system could quickly render the model unreasonably large. This issue results, thus, in tractability issues leading to timely (when feasible) estimation. Second, the identification scheme of structural shocks requires a large set of zero restrictions on both the feedback and the variance-covariance matrices. As a consequence, the assessment of the relationship between any two variables of the system becomes undermined. Third, as is standard for reduced form *VARs*, the nature of the linkages among the units of an economic system are unspecified. Accordingly, these models are silent on the sources of transmission of cross-border shocks.

This paper takes a step toward closing these gaps by proposing an alternative model for multicountry no-arbitrage *ATSMs*. In particular, we assume that the dynamics of the risk factors follow a global vector-autoregressive (henceforth, *GVAR*). The *GVAR* approach is a tool for modelling interactions in multidimensional economic systems. Along the lines of Chudik and Pesaran, 2016, our *GVAR* can be split into two components. The first, commonly referred to as the marginal model, is a standard *VAR* that describes the dynamics of global economic factors. The second, denoted by *VARX** models, relates to small-scale country-specific *VARs* augmented by weighted cross-section averages of foreign variables. These weights capture the degree of interdependence across countries.

The transition matrix is a central pillar of *GVAR* specifications. It is precisely this peculiarity of the model that enables us to address the key shortcomings of the *JLL – ATSM*. First, the inclusion of an explicit transmission structure allows for transparent control of the mechanism of propagation of shocks, hence providing a normative result. Second, imposing a structure of interdependence among countries greatly reduces the overall number of model parameters and thus boosts the accuracy of the estimates. In addition to promoting econometric tractability, our *GVAR – ATSM* does not require economic-based controversial restrictions and provides more robust economic interpretations. We also show that, due to this parsimonious feature of our proposed model, the required estimation time is approximately 3 to 5 times shorter than analog multicountry versions of the *JPS – ATSM* and the *JLL – ATSM*.⁵

⁵Arguably, the alternative *ATSM* from Adrian et al., 2013 can yield almost instantaneous model estimates. However, this framework is subject to criticism for overlooking the internal consistency condition of the pricing factors (see Joslin et al., 2011).

We illustrate our new *GVAR – ATSM* on data of a group of four emerging economies: China, Brazil, Mexico, and Uruguay. The reason for forming this specific economic system is twofold. First, as bond yields in emerging economies are unbind to the nominal zero lower bound, our *GVAR – ATSM* enables us to address current major economic prospects.⁶ In particular, we intend to assess the extent to which the recent emergence of China as a global economic player has impacted the bond market dynamics of Latin American markets.⁷ Second, the choice of a group of Latin American countries guarantees the existence of substantial interconnectedness in the system. Indeed, as we proxy our measure of interconnectedness by trade flow weights⁸ and geographical proximity consistently strengthens trading linkages (Disdier and Head, 2008), a system including three regionally concentrated countries (Brazil, Mexico, and Uruguay), and the world's leading exporter (China)⁹ establishes a notable degree of economic integration.¹⁰ Previous studies have investigated the determinant of sovereign bond yields and risk premia in the context of emerging markets.¹¹ However, to the best of our knowledge, this work is the first on emerging economies based on a flexible framework that enables the assessment of cross-country interactions.

We appraise our empirical results in three dimensions. First, we compare the out-of-sample forecasting performance of bond yields for both *GVAR – ATSM* and *JLL – ATSM*. We show that *GVAR – ATSM* produces more accurate forecast. For instance, for the case of Mexico, the superior forecasting performance of the *GVAR – ATSM* is observed in almost 90% of the analyzed scenarios. The strength of these results is

⁶Besides Jotikasthira et al., 2015, other macrofinance multicountry *ATSMs* based on advanced economies are Kaminska et al., 2013 Rios, 2017, Abbritti et al., 2018, and Meldrum et al., 2018. All of these works limit the period of analysis prior to 2008-2009 in order to avoid model estimation complications that turn up from the proximity of interest rates to the zero lower bound. In fact, it is widely documented that the performance of standard *ATSMs* in terms of model fit and forecasting is undermined when nominal short-term rates are constrained to the zero level (see, e.g., Christensen and Rudebusch, 2016).

⁷In a related work, Cesa-Bianchi et al., 2012 study the effects of the recent prominent insertion of China in the world trade on the transmission of international business cycle to Latin America.

⁸This choice is consistent with referenced GVAR works, see, for instance, the seminal work of Pesaran et al., 2004. A more formal statistical approach to test for the validity of transition matrices in *GVAR* setups is described in Candelon, Luisi, et al., 2020.

⁹See the Direction of Trade Statistics (DTS) database released from the International Monetary Fund (IMF).

¹⁰We do not include the U.S. economy in this economic system, since the sample period under investigation coincides, to a large extent, with the years of zero short-term rates. Instead, the model includes global economic factors which highly correlate with the stance of the U.S. economy. Furthermore, it is important to emphasize that the degree of interconnection between China and the Latin American economies has markedly strengthened in recent years. In terms of trade, China was the largest partner of Brazil and Uruguay and the second largest partner of Mexico in 2019 (DTS-IMF database).

¹¹For instance, Chernov et al., 2020 studied five Asian-Pacific economies using a large-scale panel *VAR* that, nevertheless, requires a fair number of identification restrictions. Iania et al., 2021 employed a *JPS – ATSM* to assess the role of global economic shocks on the risk premia dynamics of four major emerging markets from different regions of the world.

corroborated by the fixed-smoothing asymptotics tests proposed in Coroneo and Iacone, 2020. Second, we use generalized impulse response functions (henceforth *GIRFs*) to investigate the yield curve responses of Latin American economies to an economic growth shock from China. We observe that this same shock induces a flattening in the yield curves of Brazil and Mexico and a steepening in the case of Uruguay. We claim that, for the first two countries, these results reflect the responses of the domestic monetary authorities to inflationary pressures, whereas for the latter, they relate to liquidity reasons. We also show that our model consistently generates narrower confidence intervals and is statistically indistinguishable from those of the *JLL – ATSM*. Third, based on generalized forecast error variance decompositions (henceforth *GFEVDs*), we compute the shares of domestic and foreign shocks that explain the volatility of the yield curve evolutions. Overall, we observe that both *GVAR – ATSM* and *JLL – ATSM* point to foreign factors as the main source of bond yield fluctuations, while domestic developments are less important. Our work therefore corroborates the findings of a fast growing literature highlighting strong international co-variations in risky asset prices around the globe, a phenomenon referred to as the Global Financial Cycle (Miranda-Agrippino and Rey, 2020).¹²

This paper proceeds as follows. Section 4.2 presents the basic methodological features of the single and multicountry *ATSMs*. The *GVAR* extension is presented in Section 4.3. The empirical results are presented in Section 4.4, and Section 4.5 concludes.

4.2 Early *ATSMs* with unspanned economic risks

In this section, we outline the frameworks of *JPS – ATSM* and *JLL – ATSM*. One convenient aspect of these setups is that they enable a clear split of the yield curves into cross-sectional and time series dimensions. In light of this characteristic of the models, we present the one economy *JPS – ATSM* in Section 4.2.1. Next, we expose the specific features of the risk factor dynamics from the multicountry *JLL – ATSM* in Section 4.2.2. We defer to Section 4.3 the presentation *JLL – ATSM*’s cross-section representation due to its straight overlap with our *GVAR – ATSM*. In Section 4.2.3, we point out some limitations of these two setups.

¹²In particular, Miranda-Agrippino and Rey, 2020 show that a single global factor related to the global risk appetite can explain roughly 25% in asset price dynamics around the world.

4.2.1 The seminal ATSM with unspanned macroeconomic risk (*JPS – ATSM*)

Cross-sectional dimension of the term structure

The cross-sectional setting of the *JPS – ATSMs* is based on two central equations. The first assumes that the country i short-term interest rate, $r_{i,t}$, is an affine function of N unobserved (latent) country-specific factors, $X_{i,t}$:

$$r_{i,t} = \delta_{i,0} + \delta'_{i,1} X_{i,t}, \quad (4.1)$$

where $\delta'_{i,1}$ is a N -dimensional vector with all entries equal to one.

The second equation refers to the state dynamics under the risk-neutral measure (the Q -measure). By assumption, $X_{i,t}$ evolves according to a zero-mean maximally flexible affine $VAR(1)$

$$X_{i,t} = \Phi_{i,X}^Q X_{i,t-1} + \Sigma_{i,X} \varepsilon_{i,t}^Q, \quad \varepsilon_{i,t}^Q \sim \mathcal{N}(0, I_N), \quad (4.2)$$

where $\Phi_{i,X}^Q$ is a diagonal matrix, the elements of which are real and distinct eigenvalues, λ_i^Q .

Based on equations (4.1) and (4.2), Dai and Singleton, 2000 showed that the country-specific yield of a zero-coupon bond with maturity of n periods, $y_{i,t}^{(n)}$, is affine on $X_{i,t}$. For notational simplicity, we collect J yields into the vector $Y_{i,t} = [y_{i,t}^{(1)}, \dots, y_{i,t}^{(J)}]'$ such that

$$Y_{i,t} = A_X(\delta_{i,0}, \lambda_i^Q, \Sigma_{i,X}) + B'_X(\lambda_i^Q) X_{i,t}, \quad (4.3)$$

where the forms of $A_X(\delta_{i,0}, \lambda_i^Q, \Sigma_{i,X})$ and $B'_X(\lambda_i^Q)$ are restricted to preclude arbitrage opportunities in the bond market of country i .

Joslin et al., 2011 showed that a rotation from latent factors $X_{i,t}$ to portfolios of yields, the spanned factors $P_{i,t}$, leads to an observationally equivalent term structure representation. This invariant transformation assumes that N portfolios of yields are perfectly priced and observed without errors, while the remaining $J - N$ portfolios are priced and observed imperfectly. Specifically, the spanned factors are computed as $P_{i,t} = V_i Y_{i,t}$, where V_i is a full-rank matrix. Accordingly, $Y_{i,t}$ is an affine function of $P_{i,t}$ as follows:

$$Y_{i,t} = A_P(\delta_{i,0}, \lambda_i^Q, \Sigma_{i,X}, V_i) + B'_P(\lambda_i^Q, V_i) P_{i,t}. \quad (4.4)$$

State dynamics of the term structure

The model state dynamics incorporate N spanned factors and M financial-economic variables, the unspanned factors $M_{i,t}$. Formally, the model state vector, $Z_{i,t}$, is formed

as $Z_{i,t} = [M'_{i,t}, P'_{i,t}]'$, and it follows a standard unrestricted Gaussian $VAR(1)$ under the physical measure (the P -measure)

$$Z_{i,t} = C_i^{\mathbb{P}} + \Phi_i^{\mathbb{P}} Z_{i,t-1} + \Sigma_i \varepsilon_{Z,t}^{\mathbb{P}}, \quad \varepsilon_{Z,t}^{\mathbb{P}} \sim \mathcal{N}(0, I_K), \quad (4.5)$$

where $K = M + N$ is simply the dimension of $Z_{i,t}$.

It is worth specifying the role played by unspanned factors in term structure developments. Although unspanned factors do not directly appear as an element of the pricing setting, they impinge on the dynamics of the spanned factors and ultimately affect bond yields through equation (4.4).

4.2.2 *JLL – ATSM* state dynamics

In *JLL – ATSM*, the economic system comprises the global economy, one worldwide large (dominant) economy, and another set of smaller economies. We denote by C the number of countries in this system, and we index the dominant country by D .

Risk factor construction

JLL – ATSM resorts to a series of projections to build the risk factors present in the state dynamics. This approach aims at forming domestic variables, the dynamics of which are trimmed from the influence of either the other countries, the global economy, or both.

The domestic spanned factors are built in two steps. First, for all countries of the economic system, $P_{i,t}$ is projected on $M_{i,t}$ in this same country. Second, for the nondominant economies, the residuals of the regressions obtained in the first stage are further projected onto the orthogonalized spanned factors of the dominant country. Stated simply, we have

$$P_{i,t} = b_i M_{i,t} + P_{i,t}^e, \quad (4.6)$$

$$P_{i,t}^e = c_i^D P_{D,t}^e + P_{i,t}^{e*}, \quad (4.7)$$

where $P_{i,t}^e$ ($P_{i,t}^{e*}$) corresponds to the (non-)dominant country i time series residuals.

The composition of the unspanned factors pursues a logic similar to that of the spanned factors. For the dominant economy, $M_{D,t}$ is projected onto the G global economic factors, M_t^W , and for the other economies, the residuals of the first regression are used in the orthogonalization process of the nondominant economies

$$M_{D,t} = a_D^W M_t^W + M_{D,t}^e, \quad (4.8)$$

$$M_{i,t} = a_i^W M_t^W + a_i^D M_{D,t}^e + M_{i,t}^{e*}. \quad (4.9)$$

Risk factor dynamics and model restrictions

The risk factors of the economic system comprise G global economic factors and K country-specific (including both spanned and unspanned) factors for each of the C countries. Accordingly, the total number of variables of this cohort total to $F = KC + G$.

The risk factors of interest include orthogonalized spanned and unspanned factors. More specifically, the state vector is formed by $Z_t^e = (M_t^{W'}, M_{D,t}^{e'}, P_{D,t}^{e'}, M_{2,t}^{e*'}, P_{2,t}^{e*'} \dots M_{C,t}^{e*'}, P_{C,t}^{e*'})'$, and its dynamics under the P -measure evolve as a $VAR(1)$

$$Z_t^e = C_Y^e + \Phi_Y^e Z_{t-1}^e + \Sigma_Y^e \varepsilon_{Z,t}^e, \quad \varepsilon_{Z,t}^e \sim \mathcal{N}(0, I_F). \quad (4.10)$$

The feedback matrix, Φ_Y^e , and the variance-covariance matrix, $\Sigma_Y^e \Sigma_Y^{e'}$, are formed by $(KC+G)^2$ entries each. It is therefore clear that the inclusion of supplementary countries and/or domestic factors can dramatically increase the number of model parameters. To mitigate the overparametrization issue, Jotikasthira et al., 2015 imposed a set of zero-restrictions on the feedback matrix, Φ_Y^e , and on the Cholesky factor of the variance-covariance, Σ_Y^e . We briefly comment below on the features of these two matrices.¹³

The structure of Φ_Y^e allows the global and dominant economies to directly feedback to each other, and both also directly impact the dynamics of the other economies. In contrast, shocks from smaller economies are assumed not to spread across borders by any means. Moreover, within each country, spanned and unspanned factors can freely respond to each other.

For the matrix Σ_Y^e , no cross-country shock correlations are possible, but the correlations are possible from the dominant country to the smaller economies and from the world to the rest of the system. Furthermore, the shocks across groups of spanned and unspanned factors of the same economy are bound to be uncorrelated.

4.2.3 Limitations of *JPS – ATSM* and *JLL – ATSM*

A well-known shortcoming of *VAR* models is that the dimensionality of the system can largely increase with the inclusion of a few additional factors. The relevance of this drawback is amplified in the context of *ATSMs* estimated along the lines of Joslin et al., 2014. The reason is that the estimation process of the variance-covariance matrix requires the use of numerical optimization techniques (see Section 4.3.3).

To illustrate the required estimation computational burden of several *ATSMs*, we present in Table 4.1 the number of parameters in the variance-covariance matrix that require estimation for specifications containing different numbers of countries. The entries of this same table are computed for models featuring two global unspanned factors

¹³See Appendix 4.A for a more detailed design of Φ_Y^e and Σ_Y^e .

($G = 2$), two domestic unspanned factors ($M = 2$), and three domestic spanned factors ($N = 3$). The label *JPS – ATSM* refers to a model in which the risk factor dynamics are completely unrestricted, whereas *JLL – ATSM* contemplates a restricted *VAR* along the lines previously described. The *GVAR – ATSM* is our setup detailed in Section 4.3.

Table 4.1: Number of estimated parameters in the variance-covariance matrix

Model label	Number of countries			
	3	4	5	10
JPS-ATSM	153	253	378	1378
JLL-ATSM	81	133	198	835
GVAR-ATSM	48	63	78	153

Note: The entries in the table are computed for an economic system containing two global unspanned factors ($G = 2$), two domestic unspanned factors ($M = 2$), and three domestic spanned factors ($N = 3$).

It is possible to verify that *JPS – ATSM* and *JLL – ATSM* quickly become intractable as the number of countries of the economic system increases.¹⁴ The construction of the variance-covariance of a five-country model requires the (numerical) estimation of 378 parameters in the *JPS – ATSM* and almost 200 in the *JLL – ATSM*, more than double of the *GVAR – ATSM*. It is also worth noting that a ten-country *GVAR – ATSM* depends upon the estimation of as many parameters as a *JPS – ATSM* formed by three economies.

In addition to building on a heavily parameterized *VAR* system, we consider that *JLL – ATSM*'s set of restrictions imposed in this same *VAR* are open to criticism in two further dimensions. First, the authors resort to Cholesky decomposition as the identification strategy of the structural shocks of the model. This approach has been widely criticized because of the dependence of the results on the ordering of the countries in the *VAR* (see, e.g., Bikbov and Chernov, 2010). Moreover, this choice implies the unrealistic assumption that all of the variables of the dominant economy are more exogenous than any other factor from the other economies in the system. Second, the structure of matrices Φ_Y^e and Σ_Y^e fully disrupts the transmission channel of cross-border shocks among nondominant economies – an undesirable feature of models designed to study multicountry specifications.

¹⁴We further illustrate the degree of complexity of these models by presenting the required estimation time for a four-country system in Section 4.4.2.

4.3 Multicountry *ATSM* with unspanned economic risk and a *GVAR* (*GVAR* – *ATSM*)

We divide the exposition of our multicountry *GVAR* – *ATSM* into four sections. In Section 4.3.1, we introduce the joint yield curve cross-sections, whereas in Section 4.3.2, we display the *GVAR* risk factor dynamics. In Section 4.3.3, we describe the model estimation procedure of our entire *GVAR* – *ATSM*. Finally, in Section 4.3.4, we highlight the *GVAR* – *ATSM* sources of computational efficiency gain compared to the other *ATSM*s.

4.3.1 The cross-section of the multicountry yield curves

The cross-sectional dimension of our multicountry *ATSM* follows the same structure present in *JLL* – *ATSM*. Specifically, we encompass the sets of country-specific yields, spanned factors, and intercepts from equation (4.4) into, respectively, $Y_t = [Y'_{1,t}, Y'_{2,t}, \dots, Y'_{C,t}]'$, $P_t = [P'_{1,t}, P'_{2,t}, \dots, P'_{C,t}]'$, and $A_P(\Theta) = [A'_P(\Theta_1), A'_P(\Theta_2), \dots, A'_P(\Theta_C)]'$, where $\Theta_i = \{\delta_{i,0}, \lambda_i^Q, \Sigma_{i,X}, V_i\}$ for each country i . Furthermore, we assume that the spanned factors of one country have no contemporaneous effect on the term structure of any other country. In this sense, $B_P(\Theta)$ is block diagonal such that $B'_P(\Theta) = \text{diag}(B'_P(\Theta_1), B'_P(\Theta_2), \dots, B'_P(\Theta_C))$. Therefore, we represent the multicountry term structure for all C countries as

$$Y_t = A_P(\Theta) + B'_P(\Theta)P_t. \quad (4.11)$$

4.3.2 The *GVAR* model

The *VARX** and the marginal model

The *VARX** builds on standard small-scale local *VAR* models augmented by foreign-specific variables, the *star* variables. These variables are a weakly exogenous weighted average of foreign variables. Formally, the star variables from country i are formed into $Z_{i,t}^{*'} = \sum_{j=1}^C w_{i,j} Z'_{j,t}$, where the weights $w_{i,j}$ s measure the degree of interdependence of country i with country j .¹⁵ In this setting, the choice of $w_{i,j}$ is crucial since it determines the transmission channels among countries.

In addition to $Z_{i,t}^*$, the *VARX** model for country i includes the K domestic risk factors and the G unspanned global factors. We assume that $Z_{i,t}$ follows a *VARX**(1, 1, 1) under the P -measure of the form

$$Z_{i,t} = C_i^X + \Phi_i^X Z_{i,t-1} + \Phi_i^{X*} Z_{i,t-1}^* + \Phi_i^{XW} M_{t-1}^W + \Sigma_i^X \varepsilon_{i,t}^X, \quad \varepsilon_{i,t}^X \sim \mathcal{N}(0, I_K). \quad (4.12)$$

¹⁵The $w_{i,j}$ s are normalized by $\sum_{j=1}^C w_{i,j} = 1$ with $w_{ii}=0$ for all i . A possible extension is to allow for time-dependent weights to account for time-varying interdependence (see Chudik and Pesaran, 2016).

Without loss of generality, we do not include contemporaneous terms for ease of exposition. The marginal model describes the joint dynamics of the global factors. We consider that M_t^W evolves according to a standard $VAR(1)$

$$M_t^W = C^W + \Phi^W M_{t-1}^W + \Sigma^W \varepsilon_t^W, \quad \varepsilon_t^W \sim \mathcal{N}(0, I_G). \quad (4.13)$$

In this arrangement, the dynamics of the global variables are limited to their own developments. To more closely match *JLL-ATSM*, this current setting can be extended to accommodate the influence of a dominant country in the world economy. One possible way to pursue this implementation is to incorporate the risk factors from the dominant country into the system (4.13).¹⁶

The *GVAR* representation

GVAR models are formed from stacking estimated *VARX**s by means of the country-specific link matrices, W_i s. Specifically, to construct W_i , we collect country i 's domestic and foreign variables in a $2K$ -dimension vector $z_{i,t}$, $z_{i,t} = [Z'_{i,t}, Z'^*_{i,t}]'$ and all endogenous country-specific variables of the system from the *VARX**'s in a KC -dimension vector z_t , $z_t = [Z'_{1,t}, Z'_{2,t}, \dots, Z'_{C,t}]'$. As such, W_i is a $2K \times KC$ dimensional matrix that satisfies the identity $z_{i,t} \equiv W_i z_t$.¹⁷ Based on these inputs, a generic first-order *GVAR* representation (excluding the dynamics of the global risk factors) under the physical measure, has the following structure:

$$z_t = C^X + G_1 z_{t-1} + \Sigma^X \varepsilon_t^X, \quad \varepsilon_t^X \sim \mathcal{N}(0, I_{KC}) \quad (4.14)$$

where $C^X = [C_1^{X'}, C_2^{X'}, \dots, C_C^{X'}]'$, $\Sigma^X = \text{diag}(\Sigma_1^X, \Sigma_2^X \dots \Sigma_C^X)$, and $\varepsilon_t^X = [\varepsilon_{1,t}^{X'}, \varepsilon_{2,t}^{X'} \dots \varepsilon_{C,t}^{X'}]'$. G_1 is a KC -dimensional squared matrix and is formed as

$$G_1 = \begin{bmatrix} \Phi_1 W_1 \\ \Phi_2 W_2 \\ \vdots \\ \Phi_C W_C \end{bmatrix}$$

for $\Phi_i = [\Phi_i^X, \Phi_i^{X*}]$ and $i = 1, 2, \dots, C$.

To complete our setup, we add the global economic variables to the state vector as follows $Z_t = [M_t^{W'}, z_t]'$. As a result, we assume that Z_t follows a *GVAR*(1) as:

$$Z_t = C_y + \Phi_y Z_{t-1} + \Sigma_y \varepsilon_{y,t}, \quad \varepsilon_{y,t} \sim \mathcal{N}(0, I_F), \quad (4.15)$$

¹⁶In the marginal model, any number of dominant countries can be included simultaneously. Clearly, the appropriate set of relevant dominant economies depends on the economic context under investigation.

¹⁷The link matrices have standard forms to *GVAR* models. See, for instance, Chudik and Pesaran, 2016 for the detailed structure of these matrices.

where $C_y = [C^{W'}, C^{X'}]'$, $\varepsilon_{y,t} = [\varepsilon_t^{W'}, \varepsilon_t^{X'}]'$ and $\Sigma_y = \text{diag}(\Sigma^W, \Sigma^X)$. The matrix Φ_y is a composite of the feedback matrices from both the marginal and the $VARX^*$'s models of all countries. More specifically,

$$\Phi_y = \begin{bmatrix} \Phi^W & 0_{G \times KC} \\ \Phi^{X^W} & G_1 \end{bmatrix},$$

$$\text{where } \Phi^{X^W} = \begin{bmatrix} \Phi_1^{X^W} \\ \Phi_2^{X^W} \\ \vdots \\ \Phi_C^{X^W} \end{bmatrix} \text{ has dimensions } KC \times G.$$

4.3.3 Estimation procedure

Our estimation strategy builds on the single-country methodology of *JPS – ATSM*. This approach offers some important advantages compared to earlier reference macrofinance term structure models, such as that of Ang and Piazzesi, 2003. In particular, the framework of Joslin et al., 2014 allows for a simplified estimation procedure that, nevertheless, ensures efficient parameter estimates. The reason is that the *JPS – ATSM* setting enables a convenient split of the likelihood function between the parameters governing the P - and Q -measures. As a result, the estimation of the physical and risk-neutral parameters can be performed rather independently, largely reducing the required computational burden during the estimation process. Our estimation approach, hence, follows a two-step procedure.

In the first step, parameters of the *GVAR* model are obtained following the Chudik and Pesaran, 2016 approach. For this purpose, we use ordinary least squares to separately estimate each country's $VARX^*(1, 1, 1)$ and the marginal model. Subsequently, we use the estimates of C_i^X , Φ_i^X , $\Phi_i^{X^*}$, Φ_i^W , C^W , Φ^W , and Σ^W to build the *GVAR* parameters C_y and Φ_y presented in equation (4.15).

In the second step, we use maximum likelihood to convene the country-specific risk-neutral parameters $\Phi_{i,X}^Q$, $\delta_{i,0}$, and $\Sigma_{i,X}$, present in $A_P(\Theta)$ and $B_P(\Theta)$ of equation (4.11).¹⁸ Joslin et al., 2014 showed that the estimation of equation (4.5) by least squares generates variance-covariance estimates close to the global optimum. As such, on the basis of the estimates from the first estimation step, we employ the $N \times N$ matrices at the bottom right of Σ_i^X as the starting values of $\Sigma_{i,X}$ in the second stage.¹⁹ Due to the absence of closed-form expressions, $\Phi_{i,X}^Q$ and $\Sigma_{i,X}$ must be evaluated numerically.

¹⁸The *JPS – ATSM* setting also requires the estimation of the variance of the errors from the country-specific $J - N$ portfolio of yields observed with errors. Like Jotikasthira et al., 2015, we assume that these parameters are equal at the country level, but they can differ across countries.

¹⁹Note that, by definition, the bottom right $N \times N$ part of Σ_i^X is equivalent to $\Sigma_{i,X}$.

4.3.4 *GVAR*: source of computational advantages

The source of computational advantage from the *GVAR* – *ATSM* results from the dimension of a *GVAR* being invariant to the number of countries present in the cohort. More specifically, *GVAR* is built from small-scale setups (the individual countries *VARX**s and the marginal models); hence, its dimension depends exclusively on the quantity of global and each country’s risk factors, i.e., $K + G$. In contrast, in *JLL* – *ATSM*, the full dimension of the system is the length state vector, $F = CK + G$. In this sense, it is clear that the *GVAR* – *ATSM* bears substantial estimation efficiency gains over the *JLL* – *ATSM* once the number of countries of the system becomes large, as we illustrated in Table 4.1.

4.4 *GVAR* – *ATSM*: an empirical illustration for Latin American countries

We study an economic system consisting of China and three Latin American economies: Brazil, Mexico, and Uruguay. For the estimation of *JLL* – *ATSM*, we choose China as the dominant economy due to its recent prominent economic role in this region.^{20 21} For the sake of generality, we opt to assign no dominant country for the *GVAR* – *ATSM*. We choose the country-specific state vector ($Z_{i,t}$) to contain three spanned ($N = 3$) and two unspanned domestic factors ($M = 2$). The spanned factors can thus be interpreted as level ($L_{i,t}$), slope ($S_{i,t}$), and curvature ($C_{i,t}$) (see, for instance, Litterman and Scheinkman, 1991). As unspanned factors, in accordance with the standard choice in the macrofinance term structure literature, we include the domestic rates of inflation ($\pi_{i,t}$) and of economic activity growth ($g_{i,t}$), in addition to their global counterparts, π_t^W and g_t^W . Therefore, the complete state vector Z_t is formed by $Z_t = [M_t^{W'}, M_{1,t}', P_{1,t}', M_{2,t}', P_{2,t}' \dots M_{C,t}', P_{C,t}']'$ such that $M_t^W = [g_t^{W'}, \pi_t^{W'}]'$, $M_{i,t} = [g_{i,t}', \pi_{i,t}']'$ and $P_{i,t} = [L_{i,t}', S_{i,t}', C_{i,t}']'$. For *JLL* – *ATSM*, the variables with the index $i = 1$ represent those of the dominant economy.

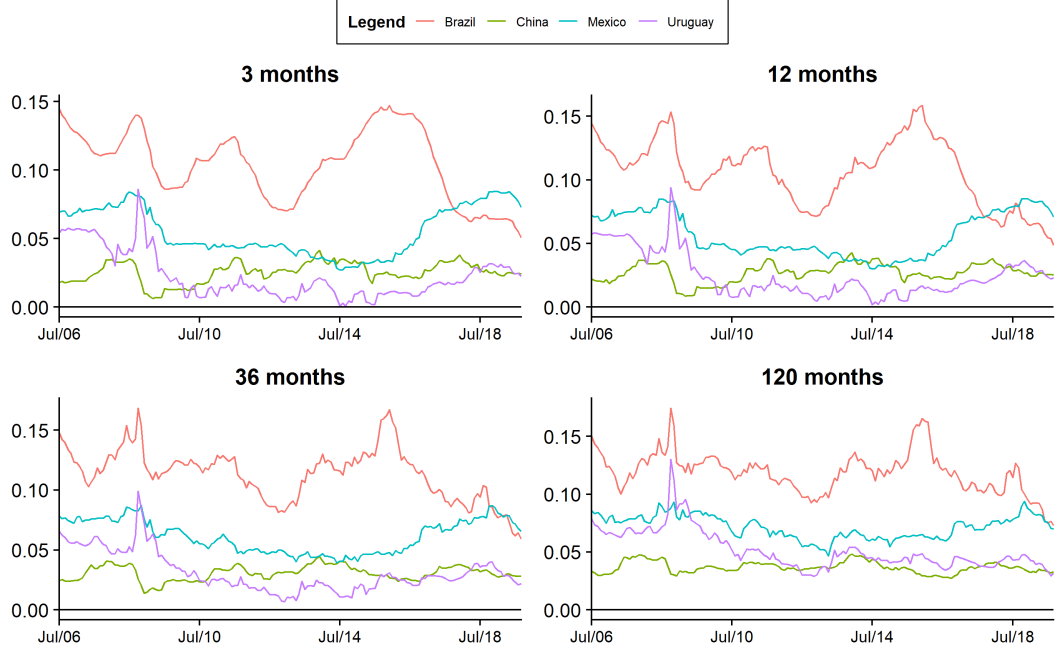
In Figure 4.1, we show the time series of bond yields for the four countries of our sample. We observe that, contrary to advanced economies in the recent years, each emergent country’s short-term interest rates were far from zero throughout the period

²⁰In comparison to the U.S. economy, China currently enjoys more preponderant bilateral trade relations with two of the three Latin American economies in our sample. By way of illustration, according to the DTS-IMF database, in 2019, China’s (U.S. ’s) exports and imports accounted for approximately 10% (62%), 25% (15%), and 22% (8%) of the total trade of Mexico, Brazil, and Uruguay, respectively.

²¹We have also estimated an alternative version of the model in which the π_t^W and g_t^W are the domestic rates of inflation and economic growth of the U.S. economy, respectively. Our major empirical results remain unchanged, indicating a high degree of correlation between the global factor and the U.S. economy. For sake of space, we report only the results from *GIRFs* in Appendix 4.A, see Figure 4.A.1.

under study. It is therefore obvious that the implementation of a term structure model based on the framework of Joslin et al., 2014 becomes appropriate.

Figure 4.1: Time series of yields



Note: The graphs show the monthly historical evolution of domestic yields corresponding to maturities of 3, 12, 36 and 120 months. Rates are annualized. The sample period is from July 2006 to September 2019.

4.4.1 Data

We use monthly data ranging from July 2006 to September 2019. Our dataset contains four categories of series: *(i)* global economic indicators; *(ii)* domestic economic indices; *(iii)* country-specific term structure of interest rates; and *(iv)* bilateral trade flows.

Our measure of global inflation is the year-over-year variation in the Organisation for Economic Cooperation and Development (OECD) aggregated Consumer Price Index (*CPI*) containing all of its member countries. As a proxy variable of worldwide economic growth, we use Lutz Kilian's index of global real economic activity, available at the author's webpage.²²

At the country level, we retrieve the country-specific CPIs from the IMF to build the domestic inflation factors. For the composition of the economic growth variables, we collect the monthly GDP leading indicator compiled by the OECD and available at the

²²See Kilian, 2009 and Kilian and Zhou, 2018 for a detailed methodological explanation about the compilation of this index.

FRED.²³ To construct all domestic unspanned factors, $\pi_{i,t}$ and $g_{i,t}$, we calculate the year-on-year changes of the respective indicator.

The term structure of each country comprises end-of-month zero-coupon yields from maturities of 3, 6, 12, 36, 60 and 120 months. To broaden the sample span, we adopt different sorts of datasets. For China, Mexico, and Uruguay, we use government bond yields. For the two former countries, data are denominated in domestic currency and are collected from Bloomberg, whereas for the latter, the yields are U.S. dollars denominated and are retrieved from the Uruguayan stock exchange (BEVSA). For Brazil, we collect swap fixed-DI contracts,²⁴ quoted in domestic currency from Brazil's Stock Exchange and Over-the-Counter Market (B3).

We build the transition matrix, W , from the trade flows between each pairwise combination of countries in the sample. Specifically, we define the strength of dependence of country i with respect to country j , w_{ij} , as $w_{ij} = \frac{I_{ij}+E_{ij}}{\sum_{j=1}^C I_{ij}+E_{ij}}$, where I_{ij} (E_{ij}) corresponds to the value of goods imports (exports) from economy i to j . Further, to compute I_{ij} and E_{ij} , we total over the entire sample period the yearly series of bilateral trade flows released by the IMF.²⁵ Table 4.2 reports the complete transition matrix.²⁶

Table 4.2: Transition matrix (W)

	China	Brazil	Mexico	Uruguay
China	-	0.655	0.316	0.030
Brazil	0.827	-	0.123	0.050
Mexico	0.860	0.133	-	0.008
Uruguay	0.381	0.550	0.069	-

Note: Each entry corresponds to the share of the bilateral trade flows between any two countries over the total amount of one country's trade flows with the other economies in the system. Trade flows are computed as the yearly sum of exports and imports from the period spanning from 2006 to 2019. Trade flow values are free on board and quoted in U.S. dollars. All rows total one.

Figure 4.2 represents the time series of all of the built-up risk factors used in our model. This group of variables includes the global economic factors and country-specific spanned (level, slope, and curvature) and unspanned (inflation and economic growth) factors.

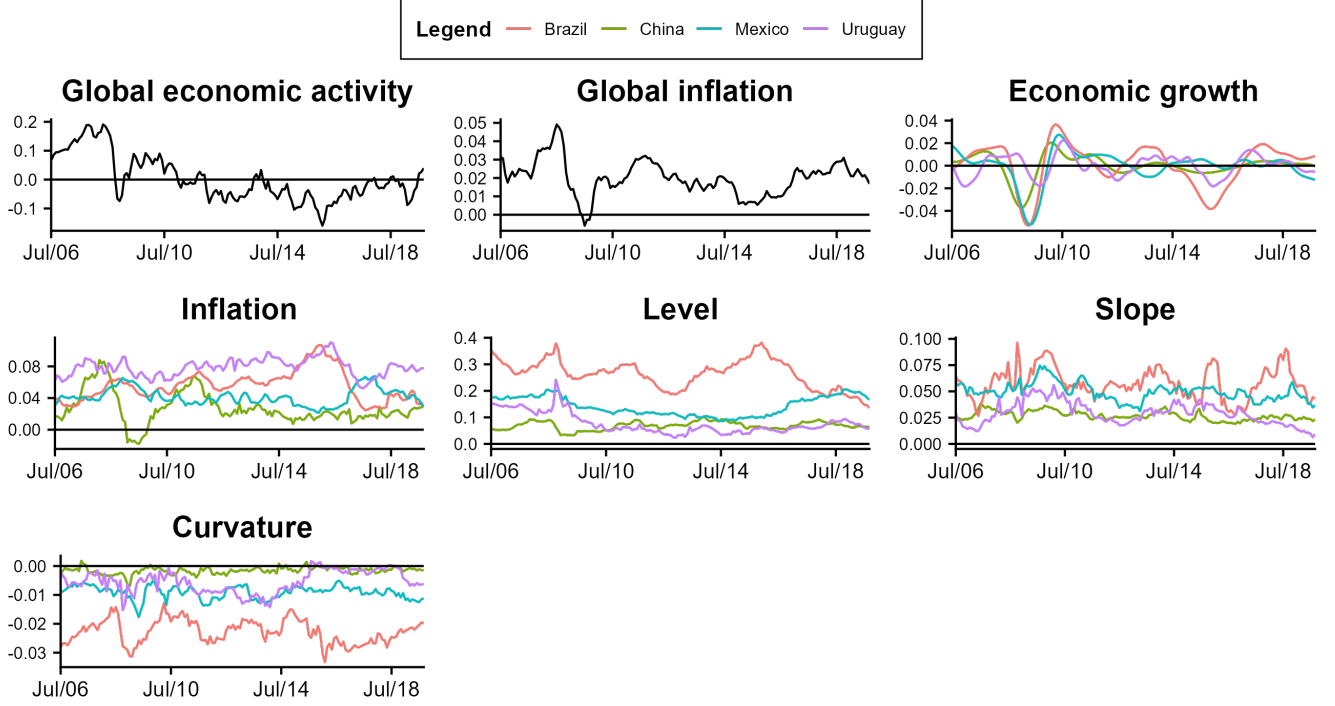
²³The latest index is not available for Uruguay. In such case, we construct a monthly time series from the cubic interpolation of quarterly GDP data, to which we apply the Hodrick-Prescott filter to remove the GDP trend (see Hodrick and Prescott, 1997).

²⁴Swap fixed-DI contracts are derivative securities indexed to the overnight rate of Brazil's interbank loan market.

²⁵The values of goods, imports and exports are free on board in U.S. dollars.

²⁶Note that $W = [w'_1, w'_2, \dots, w'_C]'$, where $w_i = [w_{i,1}, w_{i,2}, \dots, w_{i,C}]$ for $i = 1, 2, \dots, C$.

Figure 4.2: Time series of risk factors



Note: The graphs show the historical evolution of the spanned and unspanned factors. Global economic activity is the index compiled by Kilian, 2009. Global inflation is the year-over-year variation of the *OECD*'s aggregated CPI containing all of its member countries. Domestic inflation and economic growth are, respectively, the year-over-year variation of each country's CPI from the IMF and the GDP leading indicator from the *OECD*. For Uruguay, economic growth is the yearly growth rate of a monthly index built using cubic interpolation of the quarterly GDP and the Hodrick-Prescott filter. Level, slope, and curvature are the first three principal components of each country's yield set. The sample period is from July 2006 to September 2019.

4.4.2 Empirical results

Estimation time

In the preliminary investigation presented in Table 4.1, we showed that *GVAR-ATSM* requires the estimation of fewer model parameters than the other frameworks. Accordingly, it is reasonable to expect that the estimation time of our framework is shortened. To illustrate this fact, in Table 4.3, we present a comparison of the necessary estimation burden for several *ATSMs*. In Appendix 4.5, we summarize some general features of each of the models in Table 4.A.2.

In Table 4.3, all of the listed setups incorporate the complete set of risk factors, $F = 22$ (five domestic factors for each of the four economies and the two global ones). Columns λ^Q and Σ present the overall number of eigenvalues and the distinct entries of the variance-covariance matrix estimated under each representation, respectively. Each

model is estimated 5 times using a standard computer.²⁷ The estimation time intervals correspond to the minimum and maximum numbers of minutes necessary to estimate each of these models.

In this same table, we construct two versions of the *JPS – ATSM*. In the variant labeled *JPS – ATSM*, the parameters governing the *Q*-dynamics are obtained for each of the four countries separately. In the other version, designated *JPS – ATSM adapted*, we accommodate the estimates of these same parameters for all of the economies jointly so that we can measure the time savings in the estimation.

In the original *JLL – ATSM* form, the elements of the Σ matrix are exclusively computed alongside the other parameters that are part of the *P*-dynamics. This procedure greatly reduces the computational estimation time since Σ does not undergo the numerical optimization process from the risk-neutral dynamics part of the likelihood function.²⁸ As such, for the purpose of rendering the computational time comparable across the other classes of models, we build an alternative version of the *JLL – ATSM* in which the maximization of the variance-covariance matrix is part of the two estimation steps described in Section 4.3.3. We assign the label *JLL – ATSM adapted* to this variant of the model,²⁹ whereas the model identified simply by *JLL – ATSM* follows the same estimation procedure as in the original paper.

This experiment shows a significant gain in terms of time of the *GVAR – ATSM* compared to the selected alternative specifications. Interestingly, the estimation time is shortened by approximately a factor of 3 to 5 times relative to the adapted multicountry *JPS – ATSM* and *JLL – ATSM* counterparts. Notably, the time-saving benefit is further amplified once statistical inference is performed through simulation methods which use multiple model estimations, such as bootstrapping or Monte Carlo. Furthermore, it is worth emphasizing that the necessary estimation times for both *GVAR – ATSM* and *JLL – ATSM* are reasonably similar, although the latter model incorporates a substantially greater number of estimated parameters.

4.4.3 Out-of-sample forecasting performance

The *GVAR – ATSM* provides an almost perfect in-sample fit to the cross-section of bond yields.³⁰ In fact, it is well-documented that the first three principal components of

²⁷The processor used for estimation is an Intel(R) Core(TM) i5 – 8365U CPU cadenced at 1.6 GHz.

²⁸Jotikasthira et al., 2015 argued that, although this procedure is not fully efficient, it has little impact on the authors' main empirical findings.

²⁹In the general *JLL – ATSM*, the estimation of Σ under the physical dynamics also requires the use of numerical optimization techniques (see Appendix 4.B). For this estimation step, we interrupt the optimization of Σ once the absolute tolerance on the function value can no longer be improved by 0.01.

³⁰To save space, these results are not reported but are available from the authors upon request.

Table 4.3: Time required for the model estimation

Model label	F	λ^Q	Σ	$\lambda^Q + \Sigma$	Required estimation time (interval, in minutes)
JPS-ATSM	22	3	253	256	14.5 - 15
JPS-ATSM adapted	22	12	253	265	10.5 - 12
GVAR-ATSM	22	12	63	75	4 - 4.5
JLL-ATSM	22	12	133	145	3.8 - 4.2
JLL-ATSM adapted	22	12	133	145	20 - 22

Note: F represents the dimension (overall sum of domestic and foreign factors) of the system. λ^Q corresponds to the number of eigenvalues governing the Q -dynamics. For the *JPS-ATSM*, λ^Q is the number of country-specific eigenvalues, whereas for the other models λ^Q is the overall sum of eigenvalues from all countries in the economic system. Σ denotes the required number of parameter estimates for the variance-covariance matrix. The model label *JLL-ATSM adapted* refers to a model in which Σ from *JLL-ATSM* is estimated in the two steps described in Section 4.3.3. All models are estimated for four countries (China, Brazil, Mexico, and Uruguay), two global unspanned factors (economic growth, and inflation), two domestic unspanned factors (economic growth, and inflation), and three domestic spanned factors (level, slope, and curvature). The sample covers the period from July 2006 to September 2019. Each model is estimated 5 times. The bounds of the required estimation time intervals correspond to the minimum and maximum numbers of minutes necessary to estimate each one of the models. A personal computer with a processor Intel(R) Core(TM) i5 - 8365U CPU cadenced at 1.6 GHz is used for the estimations.

domestic interest rates can nearly span the whole term structure. As this is a standard result in the literature, we now investigate the out-of-sample forecasting performance of the *GVAR-ATSM* relative to *JLL-ATSM*. Arguably, no-arbitrage restrictions in *ATSMs* do not contribute to improving bond yield forecasts with regard to standard factor models (Duffee, 2011). In this sense, we do not claim that the *GVAR-ATSM* forecasts outperform any other term structure model. Rather, the goal of this section is to assess the extent to which our less parameterized framework can render more accurate predictions than competing multicountry *ATSMs*. Throughout this section, we use the root-mean-square error (*RMSE*) as our general metrics of forecasting performance.

To generate the out-of-sample forecast, we proceed as follows. First, we estimate each of the *GVAR-ATSM* and *JLL-ATSM* conditional on a restricted information set that spans the period from July-2006 to October-2013. Next, for each country, we produce the bond yield forecasts for the forecast horizons of 1, 3, 6, and 12 periods ahead. Finally, we compare these predictions with the actual observed values to obtain the forecast errors. We repeat this procedure iteratively, including the observation of the next month.³¹ In Table 4.4, we report the model *RMSEs* for the bond yield maturities

³¹To ensure that the global number of forecast errors is identical across all forecast horizons, the last array of yield forecasts is generated for the information set containing data until September-2018.

3, 12, 36, and 120 months of each economy. The values of the table are expressed in percentage points.

Table 4.4: Out-of-sample root mean square errors (RMSE)

	China				Brazil				Mexico				Uruguay			
	3 M	12 M	60 M	120 M	3 M	12 M	60 M	120 M	3 M	12 M	60 M	120 M	3 M	12 M	60 M	120 M
$h = 1$																
GVAR-ATSM	0.273	0.253	0.159	0.15	0.225	0.463	0.635	0.685	0.245	0.263	0.339	0.311	0.342	0.381	0.311	0.365
JLL-ATSM	0.262	0.248	0.151	0.179	0.505	0.653	0.809	0.866	0.324	0.313	0.382	0.371	0.35	0.343	0.296	0.387
$h = 3$																
GVAR-ATSM	0.401	0.383	0.231	0.215	0.418	0.739	0.952	1.019	0.375	0.392	0.515	0.499	0.552	0.597	0.482	0.553
JLL-ATSM	0.4	0.391	0.238	0.284	1.03	1.138	1.289	1.342	0.588	0.555	0.663	0.677	0.488	0.486	0.442	0.605
$h = 6$																
GVAR-ATSM	0.471	0.46	0.279	0.274	0.63	1.014	1.232	1.294	0.501	0.521	0.65	0.628	0.718	0.765	0.616	0.668
JLL-ATSM	0.475	0.476	0.307	0.373	1.587	1.623	1.688	1.714	0.895	0.843	0.945	0.96	0.623	0.645	0.597	0.779
$h = 12$																
GVAR-ATSM	0.541	0.527	0.324	0.328	0.882	1.338	1.527	1.564	0.617	0.637	0.761	0.724	0.846	0.893	0.73	0.747
JLL-ATSM	0.537	0.541	0.373	0.449	2.182	2.162	2.108	2.086	1.226	1.148	1.233	1.253	0.777	0.827	0.786	0.962

Note: The table presents the out-of-sample root mean square errors ($RMSEs$) for the country-specific bond yields with maturities 3, 12, 36, and 120 months (M). The forecast horizons (h) are 1, 3, 6, and 12 months. The information set of the first (last) forecasts contains data from July/2006 to October/2013 (September/2018). The values of the $RMSE$ are expressed in percentage points.

An overall analysis of Table 4.4 highlights three common features about the forecasts properties of $GVAR - ATSM$ and $JLL - ATSM$. First, both models capture more accurately the future bond yield fluctuations in China and least precisely in Brazil. Second, except in the case of China, the two $ATSMs$ render more precise forecasts for the short-end than for the long-end of the yield curves. Third, unsurprisingly, the forecast precision of both frameworks worsens with the forecast horizons. More specifically, for the one-month ahead predictions, the $RMSEs$ vary between 0.15% and 0.87%, whereas, for the twelve-periods forecast, the prediction errors exceed 2% in the worst cases.

From Table 4.4, it is also noticeable that $GVAR - ATSM$ consistently generates smaller forecast errors than the $JLL - ATSM$. To formally verify whether there is evidence of different predictive accuracy between these two competing frameworks, we make use of the fixed- b and fixed- m statistical tests developed by Coroneo and Iacone, 2020. In essence, these tests consider fixed-smoothing asymptotics for the Diebold-Mariano test (Diebold and Mariano, 1995) to correct for potential large size distortions in small samples.³²

³²The fixed- b and fixed- m tests are based on different asymptotic procedures for the limit distribution of the loss differential between two alternative forecast models. Specifically, the fixed- b relies on the assumption that the bandwidth-to-sample size ratio is constant to build the weighted autocovariances estimate of the long-run variance. The fixed- m estimates the long-run variance using a weighted periodogram estimate with a constant truncation parameter m that is independent of the sample size.

In Table 4.5, we report the outcomes for the fixed- b and fixed- m tests. In this table, the tests are performed on the same set of maturities, forecast horizons, and evaluation period as those in Table 4.4. The losses associated with the forecast error are assumed to be quadratic, and the loss differential is computed by subtracting the (quadratic) forecast errors of the $JLL - ATSM$ from the ones of the $GVAR - ATSM$. As such, negative (positive) values of the test statistics indicate evidence in favor of the superior accuracy performance of the $GVAR - ATSM$ ($JLL - ATSM$). The null hypotheses of the tests are that the predictive ability of the two methods are equal. The entries marked with one (two) asterisk(s) indicate two-sided significance at 10% (5%) significance level.

Table 4.5: Comparative predictive accuracy test (full sample)

	China				Brazil				Mexico				Uruguay			
	3 M	12 M	60 M	120 M	3 M	12 M	60 M	120 M	3 M	12 M	60 M	120 M	3 M	12 M	60 M	120 M
$h = 1$																
Fixed-b	0.994	0.433	0.782	-1.54	-4.714**	-4.11**	-2.442**	-2.846**	-4.196**	-3.125**	-1.562	-1.686	-0.309	0.967	0.46	-0.511
Fixed-m	1.098	0.448	0.646	-1.3	-5.601**	-4.881**	-2.284*	-2.51**	-4.561**	-4.064**	-1.453	-1.564	-0.3	0.815	0.454	-0.501
$h = 3$																
Fixed-b	0.042	-0.322	-0.326	-1.633	-3.66**	-3.073**	-1.622	-1.957*	-4.126**	-3.629**	-2.349**	-2.568**	1.233	1.74	0.648	-0.751
Fixed-m	0.039	-0.296	-0.259	-1.355	-3.413**	-2.777**	-1.538	-1.793	-4.209**	-3.871**	-2.157*	-2.434*	1.015	1.414	0.589	-0.708
$h = 6$																
Fixed-b	-0.107	-0.369	-0.756	-1.576	-2.88**	-2.312*	-1.175	-1.31	-3.911**	-3.583**	-2.72**	-3.059**	1.278	1.239	0.203	-0.961
Fixed-m	-0.095	-0.329	-0.594	-1.312	-2.596**	-2.116*	-1.147	-1.269	-3.9**	-3.625**	-2.543**	-2.9**	1.044	1.008	0.17	-0.84
$h = 12$																
Fixed-b	0.06	-0.262	-0.895	-1.524	-2.45**	-1.959*	-0.987	-1.021	-3.751**	-3.554**	-3.271**	-3.75**	0.572	0.434	-0.423	-1.156
Fixed-m	0.052	-0.221	-0.709	-1.287	-2.211*	-1.834	-0.98	-1.019	-3.633**	-3.46**	-3.08**	-3.492**	0.459	0.348	-0.343	-0.984

Note: Fixed- m and fixed- b test statistics of comparative predictive accuracy from Coroneo and Iacone, 2020. The forecast error losses are quadratic. Positive entries indicate higher average loss for the forecast produced by the $GVAR - ATSM$. The null hypotheses are that both models have equal predictive accuracy. Entries with one (two) asterisk(s) show two-sided significance at 10% (5%) significance level. The evaluation period of the tests is from October-2013 to September-2018.

Table 4.5 shows that the comparative performance of the $GVAR - ATSM$ is best for the entire segments of the term structure in Mexico and for the short-end in Brazil. In most of such cases, the evidence of superior forecast accuracy from the $GVAR - ATSM$ is supported at 5% significance level. In contrast, $JLL - ATSM$ tends to produce smaller forecast errors for the 3-months maturity bond in China and for the bonds with maturity up to 36 months in Uruguay, but these results are not statistically significant even at 10%.³³

It is worth stressing that among all 128 cases reported in Table 4.5, the fixed- m and fixed- b tests indicate that $GVAR - ATSM$ produces more accurate forecasts in approximately 37%, whereas the predictions from the $JLL - ATSM$ are superior in no case. Furthermore, when we include the remaining observed yields of our sample (i.e.

³³As a robustness check, we restrict the evaluation period to the first half of the original forecasts period (from October-2013 to March-2016). The main results remain similar, as presented in Table 4.A.3 in Appendix 4.A.

the bonds with 6 and 36-month maturities) into the analysis, the percentage of cases in favor of the $GVAR - ATSM$ increase to 38.5%, while the proportion of favorable cases to the $JLL - ATSM$ remain unchanged (not reported).

GIRFs and GFEDVs analyses

We qualitatively assess our $GVAR - ATSM$ based on *GIRFs* and *GFEDVs* along the lines of Pesaran and Shin, 1998.³⁴ These approaches do not require the identification of shocks according to some canonical procedure. Rather, they account for correlated shocks using the historically observed distribution of the errors. As such, in contrast to the Cholesky identification restrictions employed in $JLL - ATSM$, *GIRFs* and *GFEDVs* from Pesaran and Shin, 1998 provide empirical results that are invariable to the ordering of the variables. This feature of *GIRFs* and *GFEDVs* is imperative in a fairly large system like the one under study, in which the risk factors can be ordered in more than 10^{21} alternative forms.

The current setting enables the appraisal of 22 risk factor shocks. Accordingly, we are able to address a rich set of factor responses to myriad domestic and foreign shocks. To save space, we focus on the *GIRFs* of the term structures of Brazil, Mexico, and Uruguay to an economic growth shock in China.³⁵ We display these results in Figure 4.3.

From Figure 4.3, we observe that a shock to Chinese output growth has similar impacts on the bond yields of Brazil and Mexico. Specifically, we observe that all yields shift upwards and that short maturities are more responsive than long maturities, implying a flattening of the yield curves of these countries. This effect could result from active monetary policy responses to inflationary pressures arising from a positive foreign demand shock.

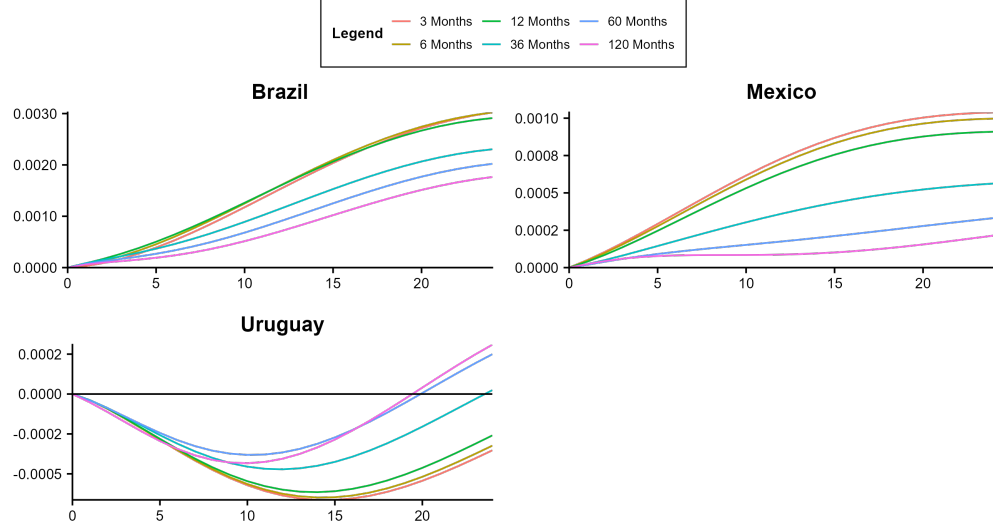
The impact of the Chinese economic growth shock on the Uruguayan term structure has a distinct pattern: the long end of the yield curve drops less than the short end, resulting in a steeper yield curve. Since Uruguay yields data are denominated in U.S. dollars, it is less likely that they reflect the reaction of the monetary authority to demand shocks. Rather, U.S. dollar inflows might push bond yields downward through the risk compensation channel in two dimensions. First, since Uruguay is a small economy, modest amounts of capital inflows might substantially improve the capability of the government to commit to its debt repayment, leading to a reduction in default risk

³⁴We present in Appendix 4.C the analytical derivations of the *GIRFs* and *GFEDVs*.

³⁵In Appendix 4.A, we further report the cross-border yield curve responses to the economic growth shocks of the other countries. All of the remaining outcomes of the other shocks are available from the authors upon request.

(Reinhart et al., 2003). Second, because Uruguay's bond market is fairly illiquid,³⁶ financial investors might require a smaller liquidity premium to hold Uruguayan bonds when the country experiences inflows of U.S. dollars.

Figure 4.3: Generalized impulse responses from the *GVAR – ATSM* – Shock to economic growth in China



Note: The x -axes are expressed in months and the bond yields responses (y -axes) are given in level per annum. The size of the innovation is equal to one standard deviation.

In Figure 4.4, we qualitatively compare the *GIRFs* from the *GVAR – ATSM* and the *JLL – ATSM* at different ranges of the maturity spectrum. We use a residual-based bootstrap procedure to estimate the confidence bounds at a level of significance of 95%.³⁷ For the *JLL – ATSM*, the *GIRFs* are computed for the shock on the orthogonalized Chinese economic growth.

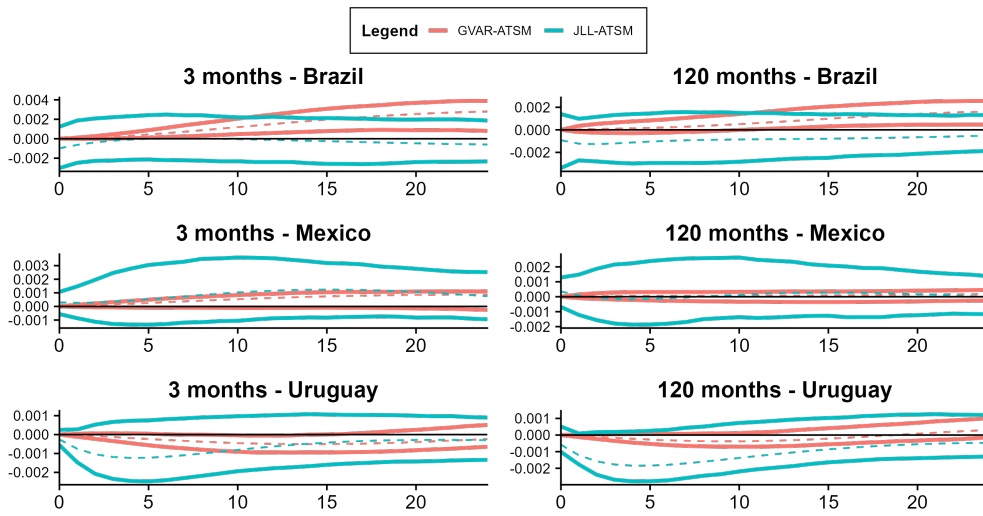
Three points deserve attention. First, for all cases, the *GIRFs* obtained with the *GVAR – ATSM* are statistically indistinguishable from those of the *JLL – ATSM* at this level of significance. Second, we show that the confidence intervals of *GVAR – ATSM* are substantially narrower than those obtained from *JLL – ATSM*. A similar pattern emerges from the *GIRFs* of the three other domestic economic shocks reported in Appendix 4.A. These results corroborate that our more parsimonious representation produces more accurate model outputs, while *JLL – ATSM*'s bulky model parametrization leads to vague statistical inferences about variable responses to shocks in general.

³⁶For the sake of illustration, in 2013, Uruguay was ranked 98th out of 183 countries on the IMF's Financial Market Depth index, which measures the development level of domestic financial markets in terms of size and liquidity (see Sviryzdenka, 2016).

³⁷We detail our bootstrap methodology in Appendix 4.D.

Third, $GVAR - ATSM$ indicates that the Chinese output growth shock is, at times, statistically significant for Brazil and Mexico. This finding constitutes an interesting result since it sheds new light on the relevant international channels of shock transmission to these countries. While previous studies have documented U.S. economy shocks as an important driver of economic fluctuation in Latin America,³⁸ less attention has been paid to investigating spillover effects from China (a remarkable exception is the work from Cesa-Bianchi et al., 2012). In this sense, it is somewhat surprising to verify that, despite the recent strengthening of the trade relations among these countries, China's economic growth might already have nontrivial effects on the asset price dynamics of Latin American economies.

Figure 4.4: Generalized impulse responses of the $GVAR - ATSM$ and the $JLL - ATSM$ – Shock to economic growth in China



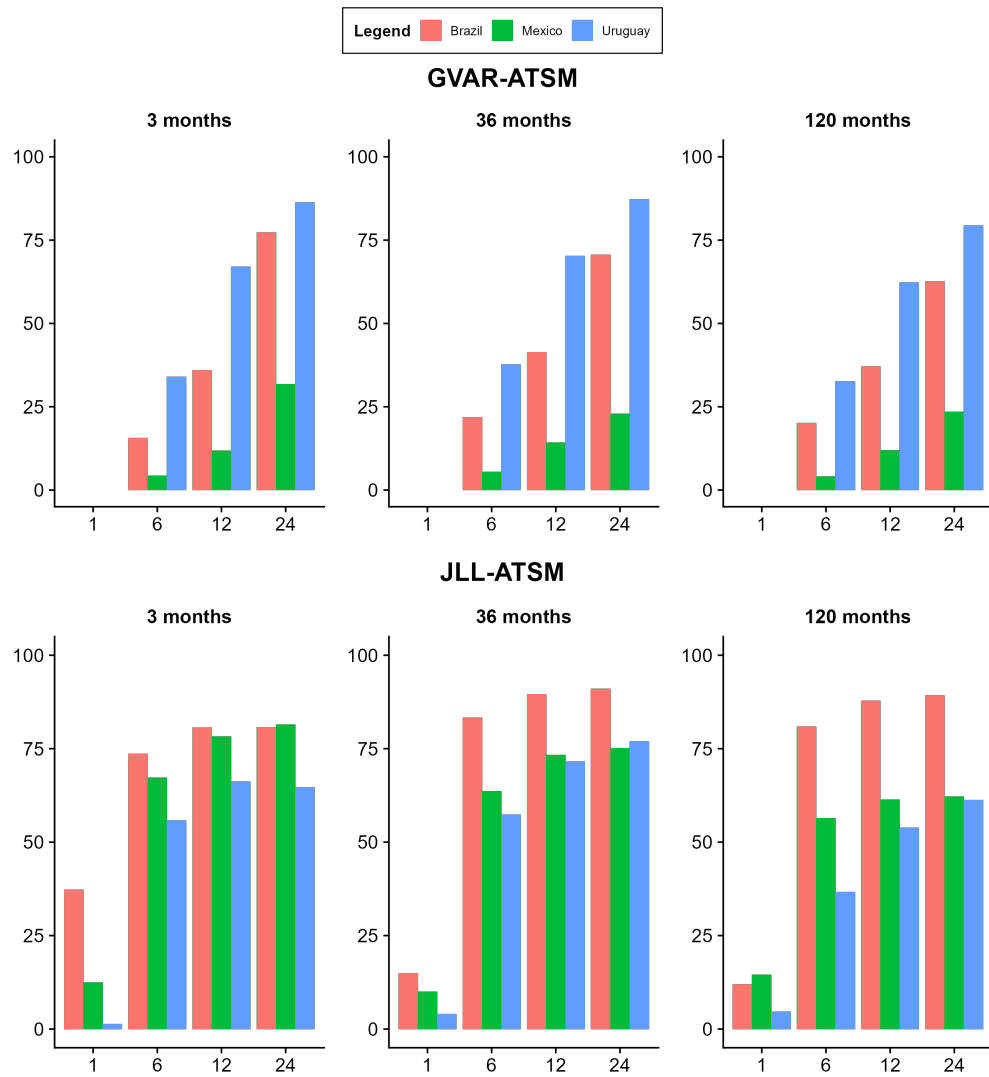
Note: The dashed lines correspond to the point estimates based on the real data. The continuous lines are the 95% confidence bounds computed by bootstrapping with 1000 draws. For $JLL - ATSM$, the yield responses are computed for the orthogonalized version of the Chinese economic growth shock. The x -axes are expressed in months and the bond yield responses (y -axes) are given in level per annum. The size of the innovation is equal to one standard deviation.

In Figure 4.5, we report the $GFEVDs$. In particular, we compute the shares of contributions of domestic and foreign shocks that explain the volatility of bond yield shocks for different segments of the maturity spectrum. Two aspects are worth emphasizing. First, overall, our empirical results point to foreign shocks as the main drivers of bond yield volatility for the three Latin American economies in the $JLL - ATSM$ and for Brazil and Uruguay (for long forecast horizons) in the $GVAR - ATSM$. This feature of the models emphasizes that cross-border developments play a key role in explaining the dynamics of domestic term structures. Second, for both models and all of the countries,

³⁸See, for instance, Canova, 2005 and Mackowiak, 2007 for referenced works.

the importance of foreign shocks increases with the forecast horizon. Among all cases, the largest shares of foreign shocks are captured by both models after 24 months for Brazil and Uruguay. In such cases, the shares of foreign shocks reach values greater than 80%.

Figure 4.5: Generalized forecast error variance decomposition of yields – Share of foreign shocks



Note: The bars represent the sum of the proportion of the cross-border shocks (global and foreign economies' spanned and unspanned factors) for each economy. The x -axes show the number of forecast periods ahead in months. The y -axes are expressed in percentages. For $JLL-ATSM$, the outputs are computed based on the orthogonal version of the risk factors.

4.5 Conclusions

This paper builds a multicountry *ATSM* in which the dynamics of the risk factors evolve according to a *GVAR* model. Compared to extant multicountry frameworks, *GVAR* – *ATSM* better captures the interdependence among countries within economic systems since it requires the specification of a cross-border transmission channel. Our setting also offers a more parsimonious term structure model representation, which grants a faster estimation process while providing more economically meaningful empirical results and better predictive power.

Our *GVAR* – *ATSM* is estimated on a system composed of four emerging markets (Brazil, China, Mexico, and Uruguay) for which the trading linkages are relevant. Based on the analysis of impulse responses and variance decompositions, we show that the Chinese output growth shock, alongside other cross-border developments, has an important influence on the yield curve dynamics of these three Latin American countries. This result underscores the important dependence of Latin America on worldwide economic events. Furthermore, our findings support that China has recently assumed a major role in the emerging markets of Latin America. While Canova, 2005, among other earlier studies, pointed to the U.S. economy as an important source of shock transmission to Latin America, we present evidence that financial investors seem to account for economic developments in China as a relevant factor of risk. We also show that *GVAR* – *ATSM* outperforms the referenced multicountry *JLL* – *ATSM* in terms of out-of-sample forecasting performance of bond yields.

The *GVAR* – *ATSM* paves the way to a broad range of empirical questions related to the interactions among term structures and the macroeconomy around the globe. An interesting extension of our setting is to integrate economies whose interest rates are constrained by an effective lower bound. Such a framework certainly raises major estimation challenges and thus is left for future research.

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Disclosure statement

No potential conflict of interest was reported by the author(s).

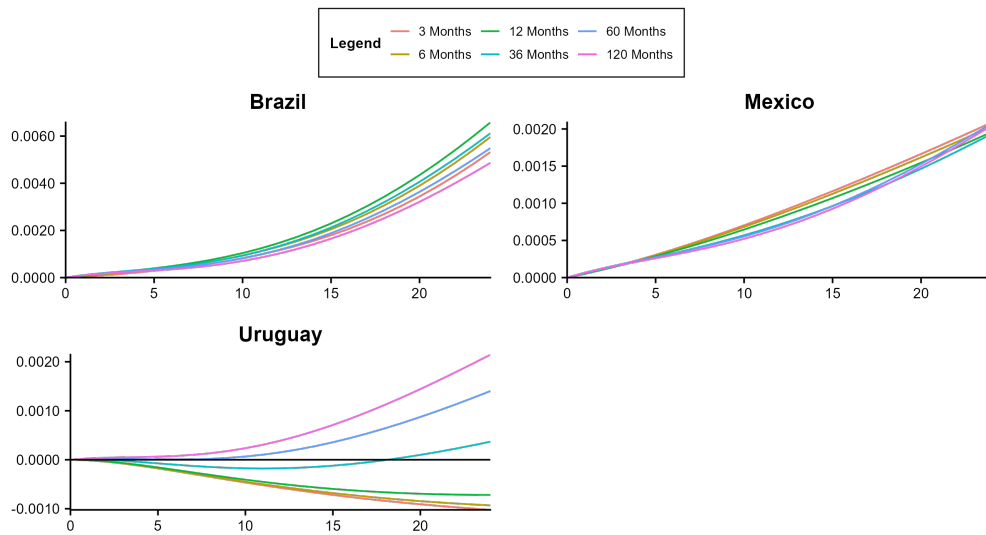
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Appendices

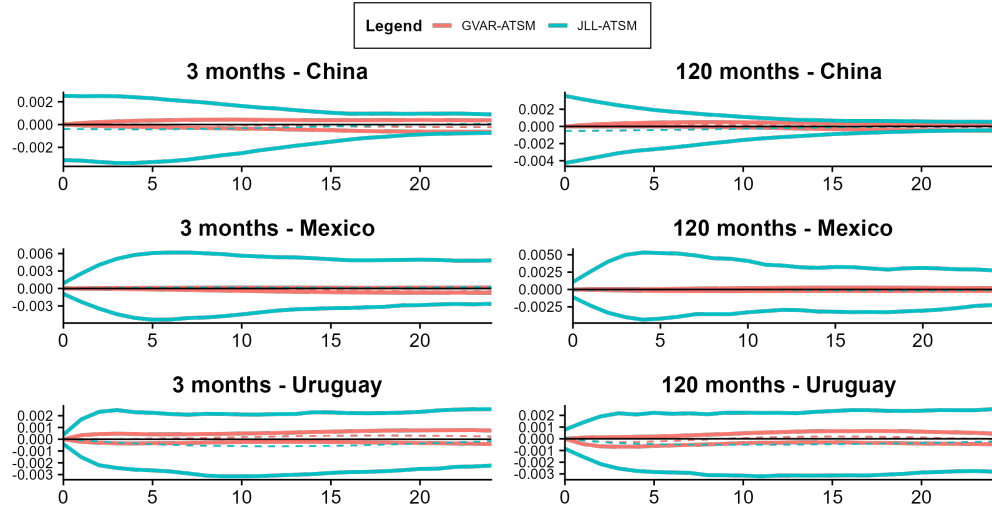
4.A Figures and Tables

Figure 4.A.1: Generalized impulse responses from the *GVAR – ATSM* – Shock to economic growth in China (model with U.S. variables)



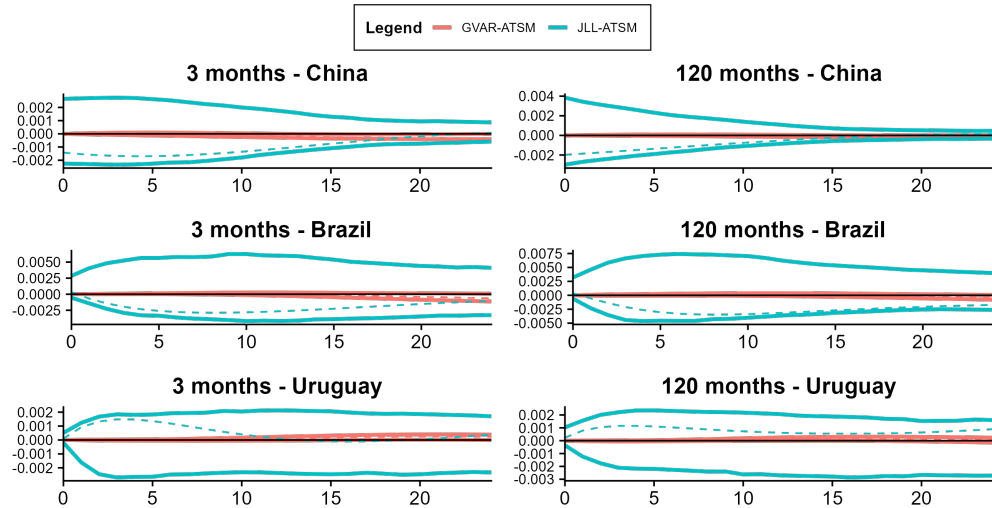
Note: Model in which the π_t^W and g_t^W are given by the year-over-year variations of the CPI and the OECD's GDP leading indicator in the U.S. economy, respectively. The x -axes are expressed in months and the bond yields responses (y -axes) are given in level per annum. The size of the innovation is equal to one standard deviation.

Figure 4.A.2: Generalized impulse responses of the *GVAR – ATSM* and the *JLL – ATSM* – Shock to economic growth in Brazil



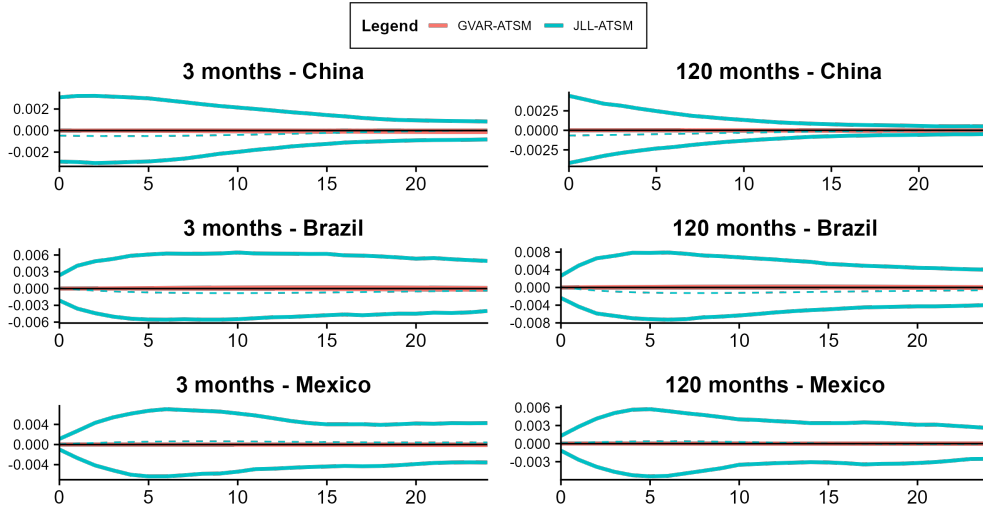
Note: The dashed lines correspond to the point estimates based on the real data. The continuous lines are the 95% confidence bounds computed by bootstrapping with 1000 draws. For *JLL – ATSM*, the yield responses are computed for the orthogonalized version of the Brazilian economic growth shock. The x -axes are expressed in months and the bond yield responses (y -axes) are given in level per annum. The size of the innovation is equal to one standard deviation.

Figure 4.A.3: Generalized impulse responses of the *GVAR – ATSM* and the *JLL – ATSM* – Shock to economic growth in Mexico



Note: The dashed lines correspond to the point estimates based on the real data. The continuous lines are the 95% confidence bounds computed by bootstrapping with 1000 draws. For *JLL – ATSM*, the yield responses are computed for the orthogonalized version of the Mexican economic growth shock. The x -axes are expressed in months and the bond yield responses (y -axes) are given in level per annum. The size of the innovation is equal to one standard deviation.

Figure 4.A.4: Generalized impulse responses of the $GVAR - ATSM$ and the $JLL - ATSM$ – Shock to economic growth in Uruguay



Note: The dashed lines correspond to the point estimates based on the real data. The continuous lines are the 95% confidence bounds computed by bootstrapping with 1000 draws. For $JLL - ATSM$, the yield responses are computed for the orthogonalized version of the Uruguayan economic growth shock. The x -axes are expressed in months and the bond yield responses (y -axes) are given in level per annum. The size of the innovation is equal to one standard deviation.

Table 4.A.1: Parameters structure under the P-dynamics (JLL-ATSM)

	Φ_Y^e							Σ_Y^e						
	M^W	M_D^e	P_D^e	M_i^{e*}	P_i^{e*}	M_j^{e*}	P_j^{e*}	M^W	M_D^e	P_D^e	M_i^{e*}	P_i^{e*}	M_j^{e*}	P_j^{e*}
M^W	X	X	X					X						
M_D^e	X	X	X					X	X					
P_D^e	X	X	X							X				
M_i^{e*}	X	X	X	X	X			X	X		X			
P_i^{e*}	X	X	X	X	X					X		X		
M_j^{e*}	X	X	X			X	X	X	X				X	
P_j^{e*}	X	X	X			X	X			X				X

Note: Relationships among the risk factors from the $JLL - ATSM$ as of the feedback matrix (Φ_Y^e) and the Cholesky factorization term (Σ_Y^e). The variable labels M and P correspond to the unspanned and spanned factors, respectively. The superscripts e and e^* indicate orthogonalized variables according to the procedure described in the Section 4.2.2. The table shows the responses from the factors listed in the rows to shocks from the factors placed in the columns. The entries marked with X indicate the absence of restrictions, whereas empty entries denote zero restrictions. Factors indexed by W and D correspond to the global economy and the dominant country, respectively. The remaining variables indexed by i and j represent any two smaller economies.

Table 4.A.2: Summary of model estimation features

	<i>P</i> -dynamics				<i>Q</i> -dynamics		Σ		Dom. Eco.	
	Individual		Joint		Individual	Joint	<i>P</i>	<i>P</i> and <i>Q</i>		
	UR	R	UR	R <i>JLL</i> <i>GVAR</i>						
JPS-ATSM	x				x			x		
JPS-ATSM adapted			x			x		x		
JLL-ATSM				x		x	x		x	
JLL-ATSM adapted				x		x		x	x	
GVAR-ATSM					x	x		x		

Note: The table reports features of the several *ATSMs* with regard to: (i) the estimation of the *P*-dynamics parameters (*P*-dynamics columns); (ii) the estimation of the *Q*-dynamics parameters (*Q*-dynamics columns); (iii) the estimation of the variance-covariance matrix (Σ columns); and (iv) the presence of a dominant economy (*Dom. Eco.* column). Under the *P*- and *Q*-dynamics, the *VAR*(1) part of each *ATSM* can be estimated either on a country-by-country basis (*Individual* columns) or jointly for all countries in the economic system (*Joint* columns). Furthermore, under the *P*-measure, the dynamics of the risk factors can be unrestricted (*UR* columns) or restricted (*R* columns). For the restricted cases, the constraints are those present in the *JLL – ATSM* and in the *GVAR – ATSM*. The estimation of the Σ matrix can be performed exclusively along with the other parameters of the *P*-dynamics (*P* column) or jointly under both *P*- and *Q*-parameters (*P and Q* column). The entries marked with *X* indicate that the mentioned characteristic is present, whereas empty entries denote the absence of this same feature.

Table 4.A.3: Comparative predictive accuracy test (sub-sample)

	China				Brazil				Mexico				Uruguay			
	3 M	12 M	60 M	120 M	3 M	12 M	60 M	120 M	3 M	12 M	60 M	120 M	3 M	12 M	60 M	120 M
<i>h</i> = 1																
Fixed-b	1.471	0.741	2.135*	-1.973	-3.486**	-2.566**	-1.873	-2.038*	-2.699**	-2.213*	-0.459	-0.417	-0.961	-0.144	1.506	0.45
Fixed-m	1.371	0.731	1.877	-1.693	-2.964**	-2.136*	-1.822	-2.14*	-2.6**	-2.095*	-0.439	-0.391	-0.932	-0.133	1.329	0.395
<i>h</i> = 3																
Fixed-b	0.808	0.25	0.417	-1.837	-2.76**	-2.141*	-1.282	-1.397	-3.127**	-3.314**	-1.563	-1.468	-0.075	0.477	0.378	-0.243
Fixed-m	0.758	0.235	0.339	-1.56	-2.407*	-1.751	-1.17	-1.329	-2.738**	-3.049**	-1.448	-1.365	-0.069	0.43	0.333	-0.217
<i>h</i> = 6																
Fixed-b	0.626	0.223	-0.19	-1.631	-2.202*	-1.829	-1.112	-1.133	-3.161**	-3.268**	-2.287*	-2.398*	-0.415	-0.476	-0.458	-0.869
Fixed-m	0.594	0.212	-0.163	-1.379	-1.936	-1.509	-0.975	-1.007	-2.613**	-2.741**	-1.977*	-2.089*	-0.38	-0.436	-0.406	-0.794
<i>h</i> = 12																
Fixed-b	0.829	0.432	-0.317	-1.441	-1.91	-1.671	-1.068	-1.056	-3.18**	-3.234**	-2.942**	-3.112**	-0.912	-1.151	-1.256	-1.351
Fixed-m	0.792	0.417	-0.271	-1.206	-1.687	-1.409	-0.935	-0.929	-2.624**	-2.678**	-2.445*	-2.616**	-0.81	-1.032	-1.119	-1.242

Note: Fixed-*m* and fixed-*b* test statistics of comparative predictive accuracy from Coroneo and Iacone, 2020. The forecast error losses are quadratic. Positive entries indicate higher average loss for the forecast produced by the *GVAR – ATSM*. The null hypotheses are that both models have equal predictive accuracy. Entries with one (two) asterisk(s) show two-sided significance at 10% (5%) significance level. The evaluation period of the tests is from October-2013 to March-2016.

4.B P -dynamics model estimation for the $JLL - ATSM$

In $JLL - ATSM$, the risk factor dynamics under the P -measure follow a standard structural VAR with restrictions on both the feedback and the variance-covariance matrices. As such, the estimation is performed in two steps, preserving the constraints detailed in Table 4.A.1.

In the first step, we use restricted least squares to obtain C_Y^e and Φ_Y^e . Conditional on these estimates, the second step involves the maximum-likelihood estimation of Σ_{Y^e} assuming that v_t^e follows a Gaussian distribution of the form $\mathcal{N} \sim (0, \Sigma_{Y^e} \Sigma_{Y^e}')^e$. At this stage, due to the absence of a closed-form solution for $\Sigma_{Y^e} \Sigma_{Y^e}'^e$, the estimation requires the use of numerical optimization methods.

4.C $GIRFs$ and $GFEVDs$

To construct the $GIRFs$ and $GFEVDs$, we recast the $JLL - ATSM$ state-space representation in two stages. First, we re-express equation (4.11) as an affine function of the set of non-orthogonal risk factors

$$Y_t = A_Z(\Theta) + B_Z'(\Theta)Z_t,$$

where $A_Z(\Theta) = A_P(\Theta)$ and $B_Z'(\Theta)$ have nonzero entries for the coefficients of the country-specific yields associated with their respective spanned factors and have zero entries elsewhere. Second, we rewrite the state dynamics of equation (4.10) as:

$$Z_t^e = C_Y^e + \Phi_Y^e Z_{t-1}^e + v_t^e.$$

where $v_t^e = \Sigma_Y^e \varepsilon_{Z,t}^e$ and $\varepsilon_{Z,t}^e \sim \mathcal{N}(0, I_F)$. Denoting the variance-covariance matrix of this VAR process by Σ^e , it follows that $\Sigma^e = \Sigma_Y^e \Sigma_Y^{e'}$ and that $v_t^e \sim N(0, \Sigma^e)$. To complete the setup, the Π matrix links Z_t to Z_t^e from $Z_t = \Pi Z_t^e$, where Π contains the loadings of the orthogonalization procedure of Z_t (equations (4.6) through (4.9)).

To compute the $GIRFs$ and $GFEVDs$ of the country yields, we extend the results of Pesaran and Shin, 1998 for a $VAR(1)$ to the state-space form of $JLL - ATSM$. For the $GIRFs$, the h -period in the future responses of all factors to a one-standard deviation shock to the risk factor j is stored in the vector:

$$\psi_j^{Z,e}(h) = \frac{(\Phi_Y^e)^h \Sigma^e e_j}{\sigma_j},$$

where e_j is a $F \times 1$ vector containing unity on the j^{th} entry and zeros elsewhere, and σ_j is the standard deviation of the j^{th} variable. Using $Z_t = \Pi Z_t^e$, it is straightforward to see

that the responses of the yields to a one-standard deviation shock to the orthogonalized risk factor j are

$$\psi_j^{Y,e}(h) = B_Z(\Theta)\Pi\psi_j^{Z,e}(h).$$

For the *GFEVDs*, within the *VAR*(1) setting, the contribution of the shock in the j^{th} variable to explain the h -step ahead forecast error variance of the j^{th} variable is

$$\theta_{ij}^{Z,e}(h) = \sigma_j^{-2} \sum_{l=0}^h \frac{(e_i'(\Phi_Y^e)^l \Sigma^e e_j)^2}{e_i'(\Phi_Y^e)^l \Sigma^e (\Phi_Y^e)^{l'} e_i}.$$

Similarly, defining $\Psi_l = B_Z(\Theta)\Pi(\Phi_Y^e)^l$, we use

$$\theta_{kj}^{Y,e}(h) = \sigma_j^{-2} \sum_{l=0}^h \frac{(e_k' \Psi_l \Sigma^e e_j)^2}{e_k' \Psi_l \Sigma^e \Psi_l' e_k}$$

to determine the contribution of the shock in the j^{th} variable to the variance of the h -step ahead forecast error of the k^{th} yield. To render the results comparable across countries and different forecast horizons, we normalize $\sum_{k=1}^{CJ} \theta_{kj}^{Y,e}(h) = 1$ for every h and k .

4.D Bootstrap procedure

To construct the confidence intervals of the *GIRFs* for both the *GVAR – ATSM* and *JLL – ATSM*, we build on the residual bootstrap procedure proposed by Cesa-Bianchi, 2015 for *VAR* models. Our approach features five main steps described as follows. First, holding fixed the *P*-dynamics real-data estimates for the intercepts, K_0 , and for the feedback matrix, K_1 , we draw and resample a set of shocks, ε_t^{**} , from their historical distribution to compute some artificial time series of the risk factors, Z_t^{**} :

$$Z_t^{**} = K_0 + K_1 Z_{t-1}^{**} + \varepsilon_t^{**}$$

where Z_t^{**} contains all applicable unspanned (global, domestic, and/or foreign) and spanned factors $Z_t^{**} = [M_t^{**'}, P_t^{**'}]'$. The parameters from the *P*-dynamics are then estimated on Z_t^{**} . We discard the draws that lead to nonstationary systems or otherwise proceed to the next step. We set $Z_1^{**} = Z_1$ to initialize the autoregressive process.

Second, holding fixed the real-data estimates of the intercepts, A_i , and the slope coefficients, B_i , from the yield-affine equation, we build the country-specific artificial time series of bond yields, $Y_{i,t}^{**}$, from its country-specific artificial time series of spanned factors, $P_{i,t}^{**}$, and some resampled shock, $u_{i,t}^{**}$:

$$Y_{i,t}^{**} = A_i + B_i P_{i,t}^{**} + u_{i,t}^{**}$$

where A_i and B_i are derived from no-arbitrage conditions.

Third, we compute the country-specific weight matrix V_i^{**} compatible with the generated artificial series as $P_{i,t}^{**} = V_i^{**}Y_{i,t}^{**}$. Fourth, on the basis of Z_t^{**} , Y_t^{**} and every country's V_i^{**} , we re-estimate the entire model to obtain the new set of estimates for K_0^{**} , K_1^{**} , A_i^{**} , and B_i^{**} and the remaining parameters of a traditional *ATSM*. Finally, conditional on this new parameter set, we produce point estimate *GIRFs*. We repeat this overall procedure for 1,000 draws, from which the 95 % confidence intervals are extracted.

Chapter 5

The COVID-19 Pandemic and the Term Structures of Interest Rates: the Case of Emerging Markets¹

Abstract

In view of the growing importance of global financial economic interdependence, this paper investigates the impact of the global spread of the *SARS – CoV – 2* viruses on the sovereign yield curve of four major emerging economies (Brazil, India, Indonesia, and Russia). Based on the framework of Candelon and Moura, 2021, we make use of arbitrage-free affine term structure models in which the joint dynamics of the risk factors follow a global vector autoregressive process. In terms of the domestic risk factors, our model features the credit default swap and the detrended measures of the COVID–19 reproduction rate, inflation rate, and economic growth. We also include the output gaps of the Chinese and U.S. economies and the return on the S&P–500 as global risk factors. We find that domestic and foreign coronavirus-related factors explain around 7% to 16% of yield curve movements in these four countries during the period March/20 - September/21. Additionally, we show that an increase in the global rate of coronavirus transmission induces a rise in the cost of sovereign borrowing. Finally, we provide mixed evidence about the responses of the term structure slopes to coronavirus-related shocks.

Keywords: *Term Structure of Interest Rates, COVID-19 Pandemic, Global Financial Market, Economic Connectedness*

JEL Classification: E43, E44, G01, G13, G15.

¹Joint work with Bertrand Candelon.

5.1 Introduction

The outbreak of the COVID–19 pandemic profoundly impacted the functioning of economies around the globe. To curb the propagation of the coronavirus, public authorities implemented stringent measures, such as lockdowns, causing a sudden and disruptive effect on firms’ production abilities and labour mobility. To avoid further deepening of the economic crisis, governments incurred large fiscal deficits to support populations and provide a stimulus to foster economic recovery. Along the same lines, central banks adopted the most accommodating monetary policies since the Great Financial Crisis. Against this gloomy economic backdrop, we explore how the global dissemination of the COVID–19 and its consequent economic unfolding have impacted the shape of the sovereign term structure of the interest rate in major emerging markets.

The economic literature points to several possible (theoretical) channels through which the coronavirus pandemic impinges on the yield curve dynamics of emerging markets. The first one is associated with the fiscal policy stance. As pointed out by Arellano et al., 2020 and Augustin et al., 2021, sharp increases in the level of public indebtedness may easily trigger solvency concerns in fiscally constrained governments such as the cases of emerging markets in the context of the pandemic. The second channel relates to monetary policy reasons. Specifically, long-lasting low levels of policy rates may lead financial investors to revise long-term inflation expectations upwards and, hence, demand a higher premium to invest in nominal bonds with long maturities. Third, Zaremba et al., 2022 document a potential liquidity channel, where, in periods of high economic uncertainty, investors tend to rebalance their fixed income portfolios towards shorter-term and more liquid securities, implying an increase in the yield curve’s slope. The fourth channel is linked to *flight-to-safety*. As also argued by Zaremba et al., 2022, the outbreak of the COVID–19 crisis could have increased investors’ demand for safe assets, such as government bonds, relative to risky assets. As a result, this process could be responsible for the flattening of the term structure of several countries. Fifth, Zaremba et al., 2022 emphasize that behavioral-related factors such as the *ostrich effect*, the *information overload* effect, and bad experiences could also drive the flattening of the curves.

To further investigate the relevance of these relationships, we employ arbitrage-free affine term structure model (*ATSM*) along the lines of Candelon and Moura, 2021. This modelling framework extends the single-country *ATSM* with unspanned risk from Joslin et al., 2014 to a multicountry setting that accurately accounts for cross-country interdependences. Specifically, the setup from Candelon and Moura, 2021 sets the joint dynamics of the risk factors to follow a global vector autoregressive (*GVAR*) model.² The *GVAR*-based *ATSM* (henceforth, *GVAR – ATSM*) has a more tractable and

²See Chudik and Pesaran, 2016 for a comprehensive description of *GVAR* setups.

parsimonious model parametrization than previous multicountry *ATSMs* such as the one developed by Jotikasthira et al., 2015. As a result, the *GVAR* – *ATSM* framework enables more economically meaningful statistical inferences. Our methodological approach differs from Candelon and Moura, 2021 in one aspect: due to the lack of a sound economically based justification on how macroeconomic aggregates impact the dissemination of COVID–19, we restrict the dynamics of the coronavirus-related risk factors to be exclusively influenced by its own domestic and foreign developments.

As in Joslin et al., 2014, our set of risk factors is formed from two groups: the spanned and the unspanned factors. The former variables are constituted as portfolios of bond yields and are used for modelling the cross-sectional dimension of the domestic term structures. The latter relate to macroeconomic and financial variables that impact the dynamics of the yield curves. As unspanned factors, our model features a total of four local and three global economic and financial variables. For the country-specific risk factors, in line with traditional macrofinance *ATSMs*, we include measures of domestic economic activity and inflation, the variables closely overseen by central banks. Furthermore, we add proxy variables based on the credit default swap (CDS) and the COVID–19 reproduction rate to describe, respectively, the market participant perception of sovereign default risk and the expected future evolution of the coronavirus spread in each country. Regarding global unspanned factors, our model contains measures of the output gaps in the U.S. and China, the world’s two largest economies. The importance of global economic activity in explaining bond yield dynamics was highlighted by previous studies such as Dahlquist and Hasseltoft, 2013 and Abbritti et al., 2018. Finally, we include the return on the Standard and Poor’s 500 stock market index, which significantly correlates with sovereign bond markets across the world (Cappiello et al., 2006).

We estimate our model on data from three out of the four BRIC countries (Brazil, India, and Russia) and Indonesia.^{3 4} In addition to accounting for roughly 8% of the world’s GDP in 2020 (source: The World Bank) and enjoying prominent economic growth in recent decades, these emerging economies have been fairly exposed to international trade and capital markets.⁵ In view of this substantial dependence on global economic de-

³China’s COVID–19-related data have been widely criticized for lacking trustworthiness. To illustrate this, as of late March/22, the number of coronavirus confirmed cases (deaths) in China was approximately 100 (3.21) per million people, which represents a 610 (242)-fold smaller rate than the cumulative sum of the whole world (source: *Our World in Data* database from Ritchie et al., 2020). Therefore, we exclude China from the analysis. Instead, we include Indonesia as an additional representative of a large Asian economy.

⁴While there are term structure databases for other emerging markets, their coverage in terms of the range of maturities is fairly restricted for the period under study. Therefore, we limit our sample to these four economies.

⁵In terms of the degree of economic openness of these countries, we emphasize that, in 2020, the trade-to-GDP ratios of Brazil, India, Indonesia, and Russia were 32%, 38%, 33%, and 46%, respectively

velopments, we could expect financial market participants to severely penalize financial asset prices in these four economies.

To evaluate the outcomes of our model, we make use of generalized impulse response functions (*GIRFs*) and generalized forecast error variance decompositions (*GFEVDs*). Anticipating our results, we highlight three main empirical features. First, we find that during the period under analysis, domestic and foreign coronavirus-related factors rationalize 7% to 16% of yield curve movements in the four emerging markets. Second, we find that the worsening of the international COVID–19 pandemic landscape triggers an increase in the costs of sovereign borrowing. This result could reflect the concern of financial market participants about the sustainability of public debt in these countries. Third, we provide heterogeneous evidence of changes in the slope of the country-specific yield curves as a result of coronavirus shocks. Specifically, our results indicate a flattening of the curves in Indonesia and India and a steeper slope in the Brazilian and Russian term structures.

This paper relates to a recent, but already extensive, literature that assesses the impact of the COVID–19 pandemic on financial market developments. While previous works have broadly focused on different asset classes, less attention has been devoted to the study of bond markets.⁶ Two noteworthy exceptions are Esteves and Sussman, 2020 and Zaremba et al., 2022. The former finds that country-specific COVID–19 death rates explain the changes in the 10-year bond spreads between the U.S. and 52 other countries, whereas the latter shows that the dissemination of coronavirus cases leads to a widening of the domestic term spreads (the difference between long and short maturity bonds) in a sample covering 30 economies, both advanced and emerging. Our work differs from these other studies in that we place emphasis on the joint dynamics of short-, medium-, and long-maturity bond yields across economies, and we exclusively focus on large emerging economies. Furthermore, our period of analysis spans from late March/20 to late September/21. As such, in contrast to previous studies that focus on the time around the pandemic outbreak, our sample enables us to assess the impact of coronavirus-related shocks on the yield curves over a longer timeframe that encompasses different stages of the pandemic.

This article is sketched as follows. Section 5.2 details our *GVAR* – *ATSM* framework as well as its estimation process. Section 5.3 presents the dataset used in the estimation of the model. The empirical results are reported and discussed in Section 5.4, and Section 5.5 concludes.

(source: The World Bank).

⁶The papers of Albuquerque et al., 2020, Alfaro et al., 2020 and Mazur et al., 2021 are some examples of studies on equity markets, whereas Iyke, 2020 and Fasanya et al., 2021 focus on foreign exchange, and Bakas and Triantafyllou, 2020 and Corbet et al., 2020 focus on commodities markets.

5.2 The model

In this section, we present our empirical approach that closely follows the *GVAR – ATSM* framework developed in Candelon and Moura, 2021. To this end, we divide the exposition as follows. In Sections 5.2.1 and 5.2.2, we present the cross-sectional and the time series dimensions of our setup, respectively. Finally, in Section 5.2.3, we describe the model estimation procedure of our entire *GVAR – ATSM*.

5.2.1 Multicountry term structure cross-section

Along the lines of Joslin et al., 2011, we build the yield curve cross-section of country i from two standard equations. First, we assume that the country i short-term interest rate, $r_{i,t}$, is affine in N country-specific latent factors, $X_{i,t}$. Second, we consider that $X_{i,t}$ evolves as a zero-mean maximally flexible Gaussian $VAR(1)$ under the risk-neutral (the Q -measure). Specifically,

$$r_{i,t} = \delta_{i,0} + \iota X_{i,t}, \quad (5.1)$$

$$X_{i,t} = \Phi_{i,X}^Q X_{i,t-1} + \Sigma_{i,X} \varepsilon_{i,t}^Q, \quad \varepsilon_{i,t}^Q \sim \mathcal{N}(0, I_N), \quad (5.2)$$

where ι is a 1-vector of size N , $\Phi_{i,X}^Q$ is a diagonal with real and distinct eigenvalues elements, λ_i^Q , and $\Sigma_{i,X}$ is a lower-triangular matrix. From these two equations, Duffie and Kan, 1996 demonstrate that the zero-coupon bond yield with maturity n of country i , $y_{i,t}^{(n)}$, is affine in $X_{i,t}$:

$$y_{i,t}^{(n)} = A_{n,X}(\Psi_i) + B'_{n,X}(\Psi_i) X_{i,t}, \quad (5.3)$$

where $A_{n,X}(\Psi_i)$ and $B_{n,X}(\Psi_i)$ follow from the standard solution to the Riccati difference equations and $\Psi_i = (\delta_{i,0}, \lambda_i^Q, \Sigma_{i,X})$.

To complete the cross-section characterization of the yield curve of a specific country, as in Joslin et al., 2011, we rotate $X_{i,t}$ to the first N principal components of yields, the spanned factors $P_{i,t}$. This approach is an observationally equivalent no-arbitrage representation, which allows a simplified model estimation process. Formally, we define $P_{i,t} = V_i Y_{i,t}$, where $Y_{i,t}$ is a vector comprising the J observed yields from country i , $Y_{i,t} = [y_{i,t}^{(1)}, \dots, y_{i,t}^{(J)}]'$, and V_i is a full-rank matrix of weights. As such, we can re-express the country's i time- t term structure as an affine function of the observable spanned factors:

$$Y_{i,t} = A_P(\Theta_i) + B'_P(\Theta_i) P_{i,t}, \quad (5.4)$$

where $\Theta_i = (\Psi_i, V_i)$.

We set the multicountry cross-sectional yield curve representation for the C economies present in the model as follows. First, we stack the country-specific bond yields, spanned factors, and intercepts from equation (5.4) into $Y_t = [Y'_{1,t}, Y'_{2,t}, \dots, Y'_{C,t}]'$, $P_t =$

$[P'_{1,t}, P'_{2,t}, \dots, P'_{C,t}]'$, and $A_P(\Theta) = [A'_P(\Theta_1), A'_P(\Theta_2), \dots, A'_P(\Theta_C)]'$, respectively. Second, we impose a block-diagonal structure on the slope coefficients such that $B'_P(\Theta) = \text{diag}(B'_P(\Theta_1), B'_P(\Theta_2), \dots, B'_P(\Theta_C))$. As a result, domestic spanned factors exert no contemporaneous effect on foreign term structures. Accordingly, we can write

$$Y_t = A_P(\Theta) + B'_P(\Theta)P_t. \quad (5.5)$$

5.2.2 Multicountry term structure time series: the GVAR

Under the historical probability measure (the P -measure), the N spanned factors relate to M domestic ($M_{i,t}$) and G global ($M_{i,t}^W$) unspanned factors by means of a $GVAR(1)$ model. We divide the exposition of the $GVAR$ framework into two parts. First, we present the two building blocks of this setup: the $VARX^*$ and the marginal models. The former aims to capture the joint dynamics of the domestic (spanned and unspanned) factors, whereas the latter casts global economic developments. Second, we jointly lay out these two models into the single $GVAR$ representation.

$VARX^*$ is based on standard small-scale local VAR models extended by the *star* and global variables. The star variables are weakly exogenous weighted averages of foreign factors formed as $Z_{i,t}^{*'} = \sum_{j=1}^C w_{i,j} Z'_{j,t}$, where $Z_{j,t}$ is a vector of domestic factors $Z_{j,t} = [M'_{j,t}, P'_{j,t}]'$ with dimension $K = M + N$, and $w_{i,j}$ is a metric of connectedness of country i with country j . By assumption, $w_{i,j}$'s are normalized by $\sum_{j=1}^C w_{i,j} = 1$ with $w_{ii}=0$ for all i . In our setup, we assume that $Z_{i,t}$ follows a $VARX^*(1, 1, 1)$ of the form

$$Z_{i,t} = C_i^X + \Phi_i^X Z_{i,t-1} + \Phi_i^{X*} Z_{i,t-1}^{*'} + \Phi_i^{XW} M_{t-1}^W + \Sigma_i^X \varepsilon_{i,t}^X, \quad \varepsilon_{i,t}^X \sim \mathcal{N}(0, I_K), \quad (5.6)$$

while the marginal model is a standard $VAR(1)$ including solely the global unspanned factors:

$$M_t^W = C^W + \Phi^W M_{t-1}^W + \Sigma^W \varepsilon_t^W, \quad \varepsilon_t^W \sim \mathcal{N}(0, I_G). \quad (5.7)$$

$GVAR$ models are set from country-specific link matrices, W_i s, that convene the individual $VARX^*$'s. Formally, W_i follows from the identity $z_{i,t} \equiv W_i z_t$, where $z_{i,t}$ is a $2K$ -dimension vector that gathers country i 's domestic and star variables, $z_{i,t} = [Z'_{i,t}, Z_{i,t}^{*'}]'$, and z_t collects every $Z_{i,t}$ of the system into $z_t = [Z'_{1,t}, Z'_{2,t}, \dots, Z'_{C,t}]'$.⁷

We form the model F -dimensional state vector by stacking the G global economic variables on top of the CK country-specific risk factors, as $Z_t = [M_t^{W'}, Z'_{1,t}, Z'_{2,t}, \dots, Z'_{C,t}]'$.

⁷Presumptively, the link matrices have dimension $2K \times CK$. See Chudik and Pesaran, 2016 for a precise description of the structure of these matrices.

Accordingly, we specify the dynamics of the risk factors to evolve as a $GVAR(1)$ of the following form:

$$Z_t = C_y + \Phi_y Z_{t-1} + \Sigma_y \varepsilon_{y,t}, \quad \varepsilon_{y,t} \sim \mathcal{N}(0, I_F), \quad (5.8)$$

where $C_y = [C^{W'}, C_1^{X'}, C_2^{X'} \dots C_C^{X'}]'$, $\varepsilon_{y,t} = [\varepsilon_t^{W'}, \varepsilon_{1,t}^{X'}, \varepsilon_{2,t}^{X'} \dots \varepsilon_{C,t}^{X'}]'$ and $\Sigma_y = \text{diag}(\Sigma^W, \Sigma_1^X, \Sigma_2^X \dots \Sigma_C^X)$. The matrix Φ_y is a function of the link matrices and the parameters from the marginal and $VARX^*$'s models. More specifically, $\Phi_y = \begin{bmatrix} \Phi^W & 0_{G \times CK} \\ \Phi^{X^W} & G_1 \end{bmatrix}_{F \times F}$,

$$\text{where } \Phi^{X^W} = \begin{bmatrix} \Phi_1^{X^W} \\ \Phi_2^{X^W} \\ \vdots \\ \Phi_C^{X^W} \end{bmatrix}_{CK \times G} \quad \text{and } G_1 = \begin{bmatrix} \Phi_1 W_1 \\ \Phi_2 W_2 \\ \vdots \\ \Phi_C W_C \end{bmatrix}_{CK \times CK}, \quad \text{for } \Phi_i = [\Phi_i^X, \Phi_i^{X^*}] \text{ and } i = 1, 2, \dots, C.$$

Model restrictions

In line with the economic literature, we allow the COVID–19 risk factor to freely impact the other domestic factors through the $VARX^*$ models presented in Equation 5.6. In fact, previous works documented, for instance, that the stringent lockdown measures at the onset of the pandemic had forceful harmful effects on major macroeconomic aggregates such as inflation and economic activity (see, e.g., Jaravel and O'Connell, 2020, Caselli et al., 2020).

In contrast, there is less clear evidence about the direction of the effect and the channels through which economic conditions affect the evolution of sanitary developments. Accordingly, to avoid controversial economic outcomes, we opt to impose a set of zero restrictions to curb the effect of macroeconomic aggregates on the evolution of the country-specific COVID–19 variable. The first array of restrictions is included in the feedback matrices of every country $VARX^*$ model. In particular, we allow the dynamics of domestic COVID–19 to be exclusively impacted by its own past developments and from the lagged values of the foreign COVID–19. Furthermore, in terms of the variance-covariance matrix, we bind the shocks of the COVID–19 risk factors to be uncorrelated with the other variables of the model. In appendix 5.A, we present the exact forms of matrices Φ_i^X , $\Phi_i^{X^*}$, $\Phi_i^{X^W}$, and Σ_i^X .

5.2.3 Estimation procedure

Our estimation approach is based on Joslin et al., 2014 for a one-country framework. Due to the particular features of this setting, the authors show that the model parameters governing the physical and risk-neutral measures can be estimated nearly separately

while ensuring an efficient and swift estimation process. As such, as in Joslin et al., 2014, our estimation procedure features two steps.⁸

In the first step, we follow the Chudik and Pesaran, 2016 methodology to compute the P -dynamics parameters from the $GVAR$ model. Specifically, we estimate each country's $VARX^*$ and the marginal models by restricted least squares, and we use these estimates to form the $GVAR$ of equation (5.8). In the second step, we use maximum likelihood to obtain the set of domestic risk-neutral parameters present in equation (5.5).⁹

5.3 Data

Our sample includes four emerging countries (Brazil, India, Indonesia, and Russia). The frequency of our database is weekly. As a general procedure, we work with end-of-week (Sunday) data. For financial series released exclusively during business days, we collect the data of the corresponding Friday, or, contingently, Thursday when the former information is unavailable. Due to the lack of availability of coronavirus-related data in early dates, our sample starts on March 22, 2020, and ends on September 26, 2021. This time frame thus covers the very early stages of the global COVID–19 pandemic, when the worldwide spread of the coronavirus became severe, to a period in which the progress in the roll-out of vaccination against the COVID–19 rendered a more certain future economic outlook. We describe below in more detail the dataset used in the estimation of our $GVAR - ATSM$.¹⁰

For each economy, we obtain from Bloomberg zero-coupon government bond yields with maturities of 12, 24, 36, 60, 72, and 120 months. We estimate a Nelson-Siegel model (Nelson and Siegel, 1987) to gap-fill the occasional missing values in the time series. In Figure 5.1, we report the complete set of time series of yields of our sample. To form the domestic spanned factors, we compute the first two principal components of the set of yields of each country.¹¹ Accordingly, the spanned factors are interpreted as the

⁸Candelon and Moura, 2021 show that this model representation leads to a substantially faster estimation process than traditional $ATSM$ multicountry frameworks.

⁹This model setup also depends upon the $(J - N)$ variance terms from the country-specific portfolio of yields observed with errors. We set these parameters to be equal for a particular country but allow them to be different across economies.

¹⁰As an illustration, the onset of our sample closely matches the famous speech given by Tedros Adhanom, Director-General of the World Health Organization, on March 11, 2020, in which he declared that the world was facing the "first pandemic caused by a coronavirus", whereas the sample end date coincides with the week when at least 30 % of the population in each of the countries under examination had been partially vaccinated (see the *Our World in Data* database from Ritchie et al., 2020).

¹¹In contrast to traditional $ATSM$ s that use three principal components to fit the cross-section of bond yields, we limit our framework to two country-specific spanned factors, since our sample time series length is rather short relative to the model dimension. In fact, the option for two principal components is in line with previous multicountry term structure works, such as Jotikasthira et al., 2015 and Iania et al., 2021.

standard level ($L_{i,t}$) and slope ($S_{i,t}$) yield curve factors.¹²

The model includes two traditional country-specific unspanned factors: the local rate of inflation ($\pi_{i,t}$) and the growth rate of the output gap ($g_{i,t}$). Since these two variables are closely overseen by central banks, they are commonly noted in the literature as the most relevant macroeconomic forces that steer interest rate fluctuations (see, e.g., Ang and Piazzesi, 2003, Joslin et al., 2014). Our model also contains the CDS ($CDS_{i,t}$) as a proxy for sovereign default risk in these four countries. Furthermore, we add a metric to capture the state of the COVID-19 epidemiological situation in each country ($COVID_{i,t}$). This factor enables us to assess the extent to which COVID-19-related shocks impinge on the dynamics of the country term structures, the primary goal of our study. In terms of the type of variable, we consider the reproduction number (the so-called R rate) by Arroyo-Marioli et al., 2021 to be a more adequate measure than the number of new COVID-19 cases. The reason is twofold. First, the direct new case data are more prone to measurement error, as they are to a great extent influenced by changes in the quantity of testing undertaken during the period under study (see, e.g., Brodeur et al., 2021).¹³ Second, unlike the number of new COVID-19 cases, the R rate is a more suitable candidate of a forward-looking variable. As such, it embeds the expectations of financial investors about the future dissemination of the coronavirus. Another possibility would be to use the number of new deaths. However, this measure is also likely to be largely influenced by the testing as the number of deaths depends on the number of diagnosed cases.¹⁴ Therefore, among the three options available, the R -rate is our preferred proxy measure for keeping track of the evolution of the pandemic.¹⁵

To construct $\pi_{i,t}$, we first collect the monthly consumer price index (CPI) from the Organisation for Economic Co-operation and Development (*OECD*) for all countries. Next, we obtain daily interpolated observations via spline interpolation. From the resulting daily time series, we compute the year-over-year variation, extract the last observation of the week and use the HP filter (Hodrick and Prescott, 1997) to remove the inflation trend. To compute $g_{i,t}$, we employ the HP filter with the weekly tracker of

¹²See the seminal work of Litterman and Scheinkman, 1991.

¹³In fact, one advantage of the R rate from Arroyo-Marioli et al., 2021 is that the method is accurate even when the number of infected people is poorly measured. Specifically, this methodology computes the country-specific R rate as $R_t = 1 + \frac{gr(I_t)}{\gamma}$ where $gr(I_t)$ and γ are, respectively, the growth rate of infected individuals between two consecutive periods and the transition rate from infected to recovered people. As such, as changes in the testing capacity are likely negligible between two consecutive weeks, we expect $gr(I_t)$ and, hence, R_t to be fairly invariant to the number of testing over time.

¹⁴To add to this challenge, identifying the causes of death can be particularly difficult when the patient suffers from several comorbidity factors.

¹⁵As a robustness check, we try estimating our model with $COVID_{i,t}$ as the detrended number of deaths per million of inhabitants, and our main empirical results remain consistent. These results are available upon request from the authors.

the GDP yearly growth rate compiled by the *OECD*. Our $CDS_{i,t}$ is simply the 5-year country-specific CDS from Bloomberg. Finally, to form our $COVID_{i,t}$ variables, we apply the HP filter to the R rate from the *Our World in Data* database (Ritchie et al., 2020). The use of the HP filter enables us to remove short-term fluctuations in the data and, therefore, work with a steadier measure of the COVID–19 trend evolution in each country.

Our model features two types of global unspanned factors: (i) weekly measures of the output gap from the U.S. (g_t^{US}) and China (g_t^{CH}), the two world largest economies. The former is the *OECD* weekly tracker and is computed as its domestic counterparts. This same indicator is unavailable for China. In this case, we collect the monthly composite leading indicator from the *OECD* and extract the end-of-week year-over-year variation of the daily spline-interpolated series. (ii) the year-over-year variation from the Standard and Poor’s 500 stock market index ($S\&P - 500$) retrieved from the FRED (SP_t). $S\&P - 500$ is a forward-looking measure that captures the expected future financial performance of the largest companies listed on stock exchanges in the U.S. economy. In fact, Cappiello et al., 2006 find evidence of a significant degree of correlations between the volatility of the returns in the U.S. stock exchanges and several sovereign bond markets around the globe during periods of financial turmoil.

Therefore, our model state vector (Z_t) contains two spanned factors ($N = 2$), in addition to four domestic ($M = 4$) and three global ($G = 3$) unspanned factors. Formally, the state vector is formed by $Z_t = [M_t^{W'}, M'_{1,t}, P'_{1,t}, M'_{2,t}, P'_{2,t} \dots M'_{C,t}, P'_{C,t}]'$, where $M_t^{W'} = [g_t^{US}, g_t^{CH}, SP_t]$, $M'_{i,t} = [g_{i,t}, \pi_{i,t}, CDS_{i,t}, COVID_{i,t}]$, and $P'_{i,t} = [L_{i,t}, S_{i,t}]$. The complete time series of the built-up risk factors are reported in Figure 5.2.

Finally, in line with the standard choice in the *GVAR* literature, we use trade flows as a proxy measure of interconnections between countries.¹⁶ In particular, we quantify the strength of connectedness of country i with country j (w_{ij}) as the share of trade between these two economies over the total amount of trade that country i carries with all the countries of the economic system. Formally, w_{ij} is computed as $w_{ij} = \frac{I_{ij} + E_{ij}}{\sum_{j=1}^C I_{ij} + E_{ij}}$, where I_{ij} and E_{ij} are the values in U.S. dollars of imported and exported goods between economies i and j over the years 2015 and 2020, namely, $I_{ij} = \sum_{t=2015}^{2020} I_{ij,t}$ and $E_{ij} = \sum_{t=2015}^{2020} E_{ij,t}$. The sum of trade flows over recent years enables us to smooth the disruptive effect in world trade caused by the pandemic outbreak in 2020 and therefore obtain a contemporary average measure of economic interconnectedness. All trade flow data are collected from the yearly International Monetary Fund (IMF) database. We summarize the connectedness measures in Table 5.1.

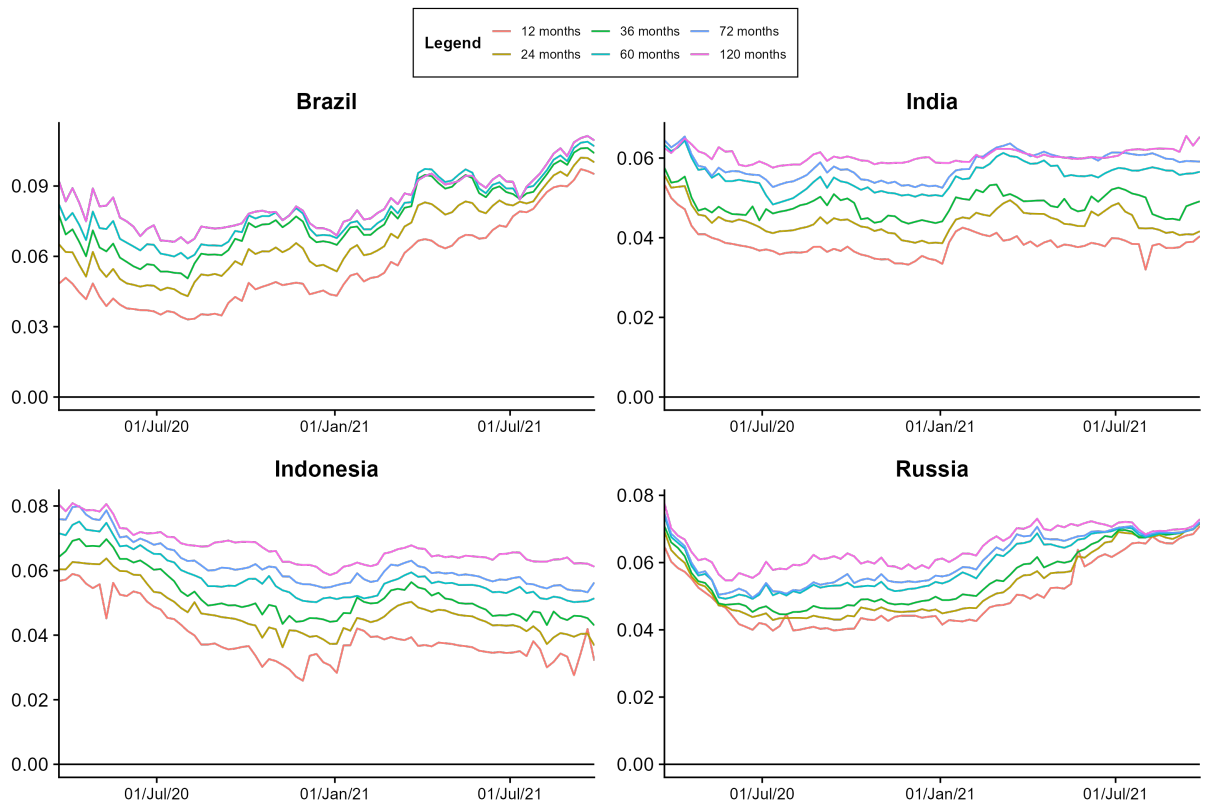
¹⁶See, e.g., the works of Pesaran et al., 2004, Chudik and Fratzscher, 2011, and Eickmeier and Ng, 2015, among others.

Table 5.1: Connectedness matrix

	Brazil	India	Indonesia	Russia
Brazil	-	0.461	0.215	0.325
India	0.215	-	0.543	0.242
Indonesia	0.147	0.749	-	0.104
Russia	0.283	0.564	0.153	-

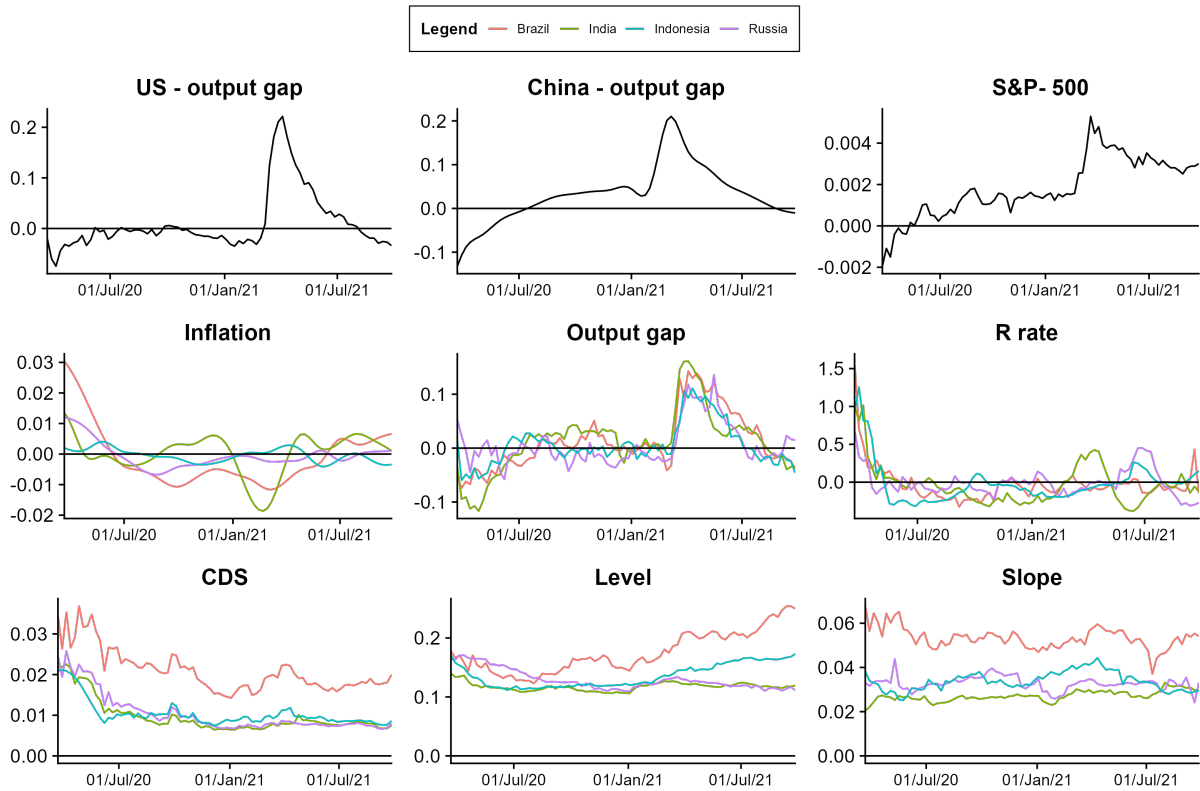
Note: Each cell shows the share of the bilateral trade flows across countries. Trade flows are defined as the yearly sum of exports and imports in U.S. dollars from the period covering the years 2015 to 2020. All rows total one.

Figure 5.1: Time series of bond yields



Note: The graphs show the weekly time series of domestic bond yields. Rates are per annum. The sample period is from March 22, 2020, to September 26, 2021.

Figure 5.2: Time series of risk factors



Note: The graphs show the weekly time series of the spanned and unspanned factors. For the output gap, the set of domestic series and the set for the U.S. economy are the detrend versions of the weekly tracker of the GDP yearly growth rate from the *OECD*. For China, weekly series of output gaps are the yearly growth rate from the detrended version of the spline-interpolated monthly composite leading indicator from the *OECD*. *S&P* – 500 is the year-over-year variation of the Standard and Poor’s 500 stock market index. Inflation series are the yearly growth rates of the domestic *CPIs*. *R* rates are the domestic detrended versions of the reproduction number. CDS is the credit default swap scaled down by a factor of 100. Level and slope are the first two principal components of each country’s bond yield set. All rates in the vertical axes (y-axes) are expressed in levels per year. The sample period is from March 22, 2020, to September 26, 2021.

In Table 5.2, we present the correlation across the term structures and the *R* rate. In terms of the correlation structure across the yield curves, the top of Table 5.2.A confirms the feature that bond yields of different maturities tend to move in close lockstep over time at the country level. We can also observe that the correlation is commonly positive and strong across countries. Regarding the interrelationship among the country-specific *R* rates, Table 5.2.B indicates that, unsurprisingly, the diffusion of the coronavirus across economies is fairly synchronized, even in the cases of countries that are largely geographically distant. For the *R* rate and the bond yield developments, the bottom of Table 5.2.A shows that these variables correlate positively and in a nontrivial manner. Such empirical features of the data suggest that the worsening of the global epidemiolog-

ical situation is associated with periods of interest rate hikes. In the following sections, we delve deeper into the nature of this structural relationship.

Table 5.2: Correlations across term structures and detrended R rates

<i>Panel A: Bond yields and the detrended R rates</i>								
	Brazil		India		Indonesia		Russia	
	12 M	120 M	12 M	120 M	12 M	120 M	12 M	120 M
Bond yields								
Brazil - 12 M	1.000							
Brazil - 120 M	0.928	1.000						
India - 12 M	0.061	0.227	1.000					
India - 120 M	0.570	0.661	0.488	1.000				
Indonesia - 12 M	-0.408	-0.217	0.638	0.169	1.000			
Indonesia - 120 M	-0.452	-0.209	0.545	0.164	0.853	1.000		
Russia - 12 M	0.869	0.878	0.335	0.680	-0.073	-0.082	1.000	
Russia - 120 M	0.793	0.800	0.416	0.546	-0.197	-0.188	0.801	1.000
R rate								
Brazil	0.047	0.250	0.720	0.360	0.522	0.475	0.360	0.305
India	-0.006	0.194	0.689	0.397	0.598	0.596	0.216	0.309
Indonesia	-0.043	-0.031	0.433	-0.036	0.186	0.211	0.222	0.309
Russia	0.206	0.341	0.676	0.451	0.311	0.416	0.519	0.486
<i>Panel B: R rates</i>								
	Brazil		India		Indonesia		Russia	
Brazil	1.000							
India	0.648		1.000					
Indonesia	0.440		0.209		1.000			
Russia	0.676		0.528		0.530		1.000	

Note: The table reports the correlation structure (i) among country-specific bond yields, (ii) among the domestic detrended R rates, and (iii) between bond yields and the detrended R rate. The maturities of the country-specific bond yields are 12 and 120 months (M).

5.4 Empirical results

We divide our empirical analysis into two dimensions. First, in Sections 5.4.1, we use *GIRFs* to assess the impact of a COVID–19 related-shock on the term structure dynamics of the various emerging economies. Next, in Sections 5.4.2, we employ *GFEVDs* to investigate the importance of a coronavirus shock in explaining the variations of the yield curves. We use the *R* package *MultiATSM* (Moura, 2022b) to generate all the empirical results presented in the paper.¹⁷

Our *GIRFs* and *GFEVDs* are computed according to the procedure of Pesaran and Shin, 1998. It is worth stressing that the use of these generalized versions of impulse responses and forecast error decomposition is particularly convenient in a rather large model such as ours. The key reason is that, contrary to the standard Cholesky-based impulse responses and the forecast error decompositions, our *GIRFs* and *GFEDVs* produce results that are robust to the ordering of the risk factors.¹⁸

5.4.1 *GIRFs*

In Figures 5.4 - 5.7, we plot the point estimates of the yield curves' responses to the one-standard-deviation shocks on the country-specific *R* rates.¹⁹ Each figure represents the *GIRFs* of a specific shock to each one of the four economies of our sample. All shocks are positive and, therefore, reflect an expected deterioration in the dissemination of the coronavirus. To ease the interpretation of the results, we express the value of the term structure responses in percentage points rather than in levels.

The analysis of the *GIRFs* reveals several interesting patterns. First, overall, we observe that the term structures tend to respond positively to both domestic and foreign coronavirus-related shocks. This effect implies that the sovereign borrowing cost amplifies as the worsening of the pandemic unfolds. This result could reflect the market perception of an increased sovereign default risk, as documented by previous works (see, e.g., Arellano et al., 2020, Hevia and Neumeyer, 2020, Augustin et al., 2021, and Cevik and Öztürkkal, 2021). To shed light on this conjecture, in Figure 5.3, we employ our model-based *GFEVDs* to assess the shares of the contributions of the 5-year CDS in the forecast error variances of the 12, 60, and 120-month bond yields in each country. This same figure shows that the sovereign default risk proxied by the domestic CDS may account for up to 15% in both Brazil and Indonesia, and around 10% in Russia of

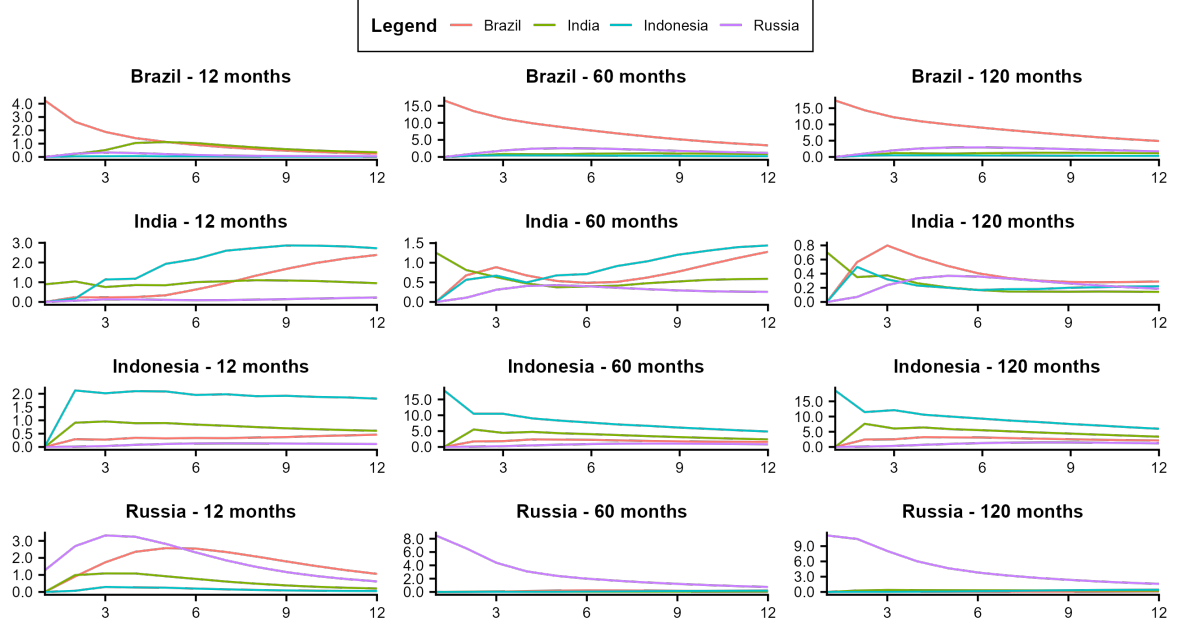
¹⁷See also Moura, 2022a.

¹⁸In fact, the *GIRFs* and *GFEDVs* as in Pesaran and Shin, 1998 do not involve the identification of structural shocks as in traditional methods. Instead, these procedures are constructed from the historical distribution of the observed errors and, hence, allow the shocks to be correlated.

¹⁹The model outcomes for the remaining shocks can be obtained using the code script available in the *MultiATSM* package.

bonds yield fluctuations for the longest maturities. In contrast, in India, the long-term sustainability of public debt does not seem to raise major concerns among financial market participants.

Figure 5.3: Generalized forecast error variance decomposition – 5-year CDS shares



Note: The x -axes are expressed in weeks, and the y -axes are given in percentage points.

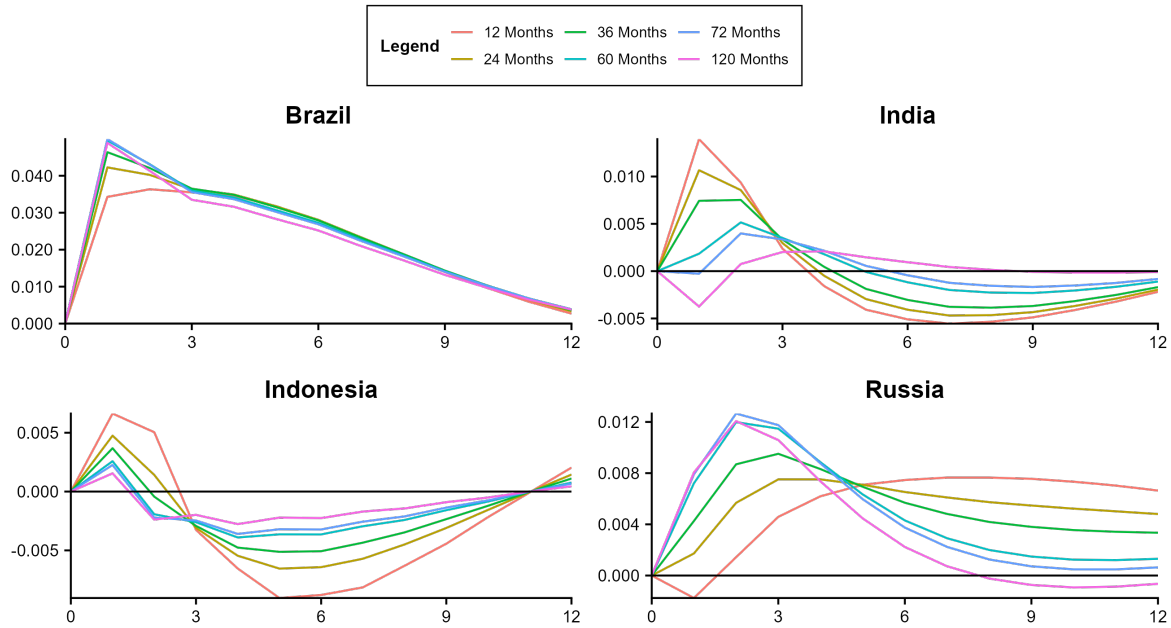
Second, in line with our intuition, bond yields are at least as responsive to domestic coronavirus shocks as their foreign counterparts. The sole exception is the case of Indonesia, where the R rate shock originating from India triggers stronger reactions than the domestic shock. The response of Indonesian yields might be attached to a high degree of dependence on India, as reported in Table 5.1.

Third, relative to the size of the bond yield responses, we observe that, in general, the short part of each yield curve is roughly similar to the longer maturity part (10-year maturity). This empirical feature suggests that financial investors likely expect the economic consequences of the COVID–19 crisis to have significant long-lasting effects, instead of being concentrated in the short-run.

Fourth, our $GVAR - ATSM$ provides mixed evidence concerning the changes in the slopes of the term structures induced by the several R rate shocks. In particular, we note that the yield curve(s) (i) flatten in India (except for its own R rate shock) and Indonesia, (ii) slopes up in Brazil (except for the Indonesian R rate shock) and Russia. This variability of effects across countries is in line with the economic narratives described in Zaremba et al., 2022. Specifically, on the one hand, Zaremba et al., 2022 list three

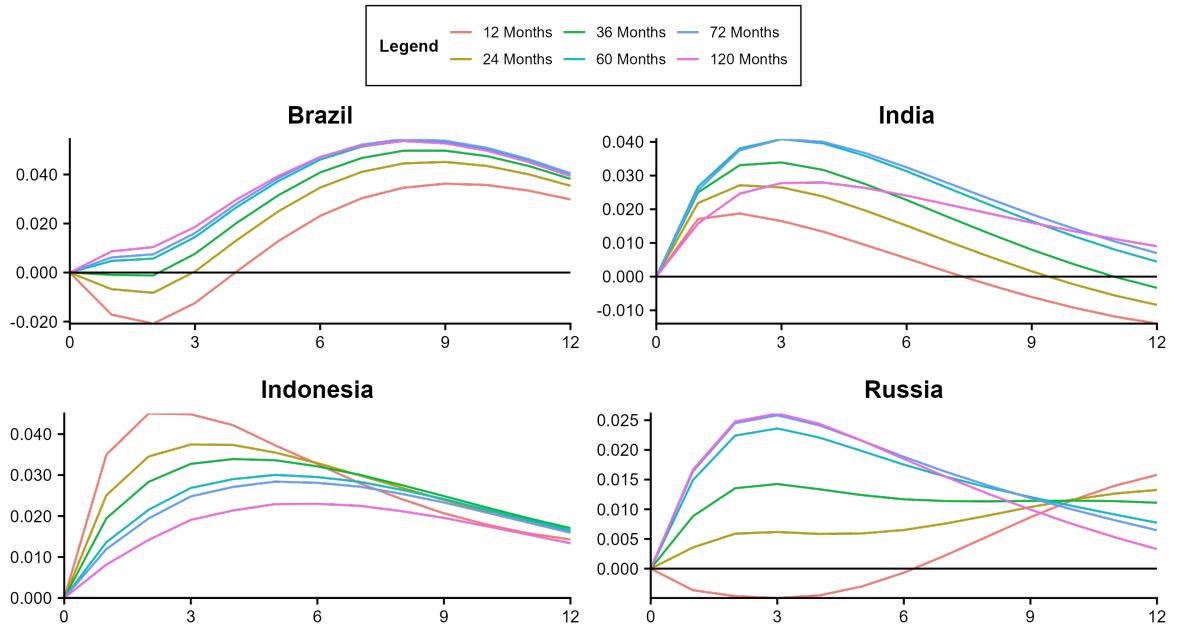
primary arguments to justify the potential widening of the spreads between short and long maturity yields. The first reason stems from the fact that market participants would require higher compensation for bearing larger default risk over long time horizons. The second explanation relates to a portfolio rebalancing strategy. In particular, in uncertain times of economic crisis, bondholders are keen to tilt their portfolio exposures towards more liquid assets such as short-term bonds (see also Goyenko et al., 2011).²⁰ Third, coronavirus-related shocks could increase the term structure slopes due to inflation risk premia. In fact, central banks' acute monetary easing at the onset of the pandemic could produce strong future inflationary pressures, leading investors to require higher returns to hold nominal long-maturity bonds (see also Wright, 2011). On the other hand, Zaremba et al., 2022 point to the so-called *flight-to-safety* effect as the main reason to explain the flattening of the term structures in emerging markets. The main argument is as follows. During periods of financial unrest, market participants perceive government bonds as the only safe asset. As such, financial investors' demand for bonds of all maturities increases, implying a flattening of the yield curves.

Figure 5.4: Generalized impulse responses – Shock to the R rate in Brazil

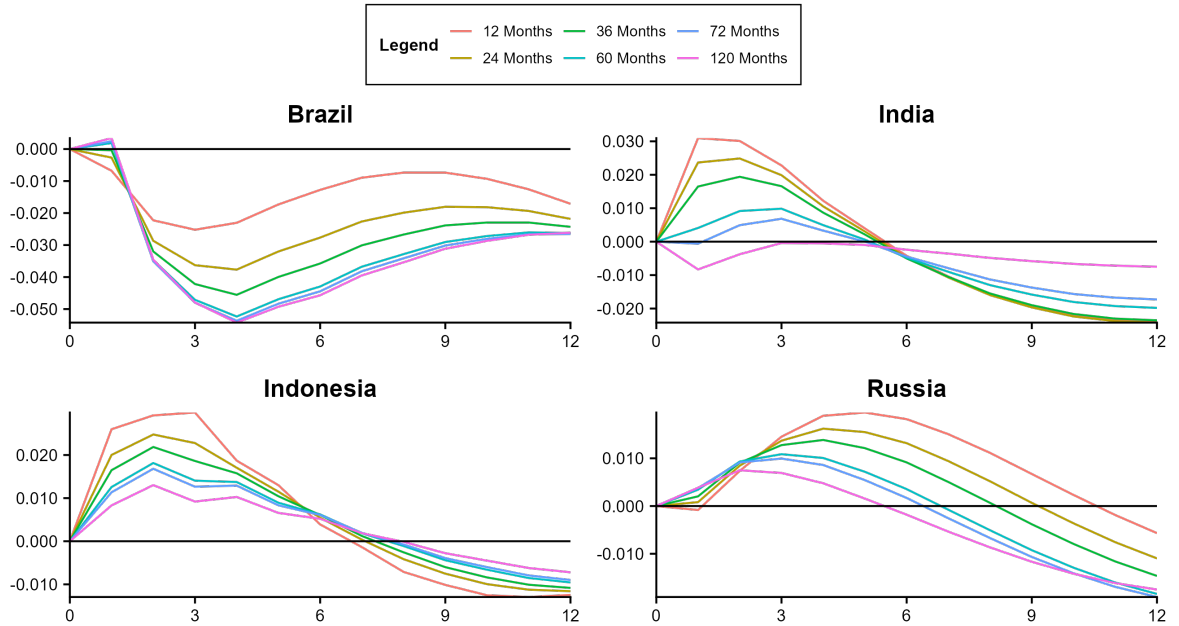


Note: The x -axes are expressed in weeks, and the bond yield responses (y -axes) are given in percentage points per annum. The size of the innovation is equal to one standard deviation.

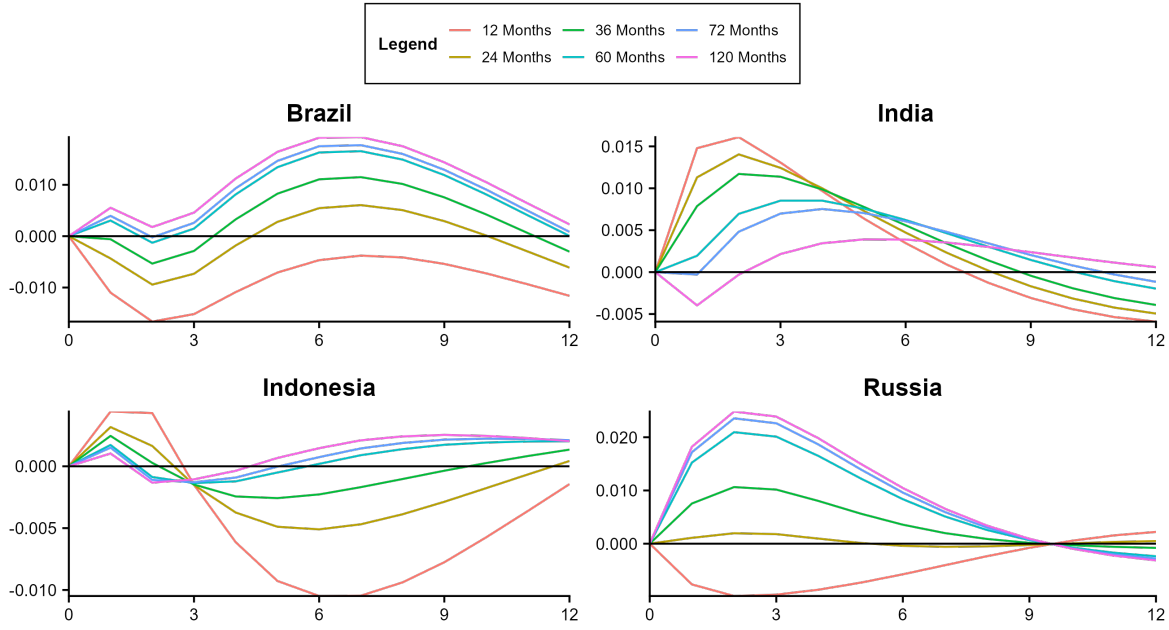
²⁰As a matter of fact, Gubareva, 2021 finds evidence that several months after the pandemic outbreak, bond market liquidity in 30 emerging markets had not fully recovered to the prior levels of March/20.

Figure 5.5: Generalized impulse responses – Shock to the R rate in India

Note: The x -axes are expressed in weeks, and the bond yield responses (y -axes) are given in percentage points per annum. The size of the innovation is equal to one standard deviation.

Figure 5.6: Generalized impulse responses – Shock to the R rate in Indonesia

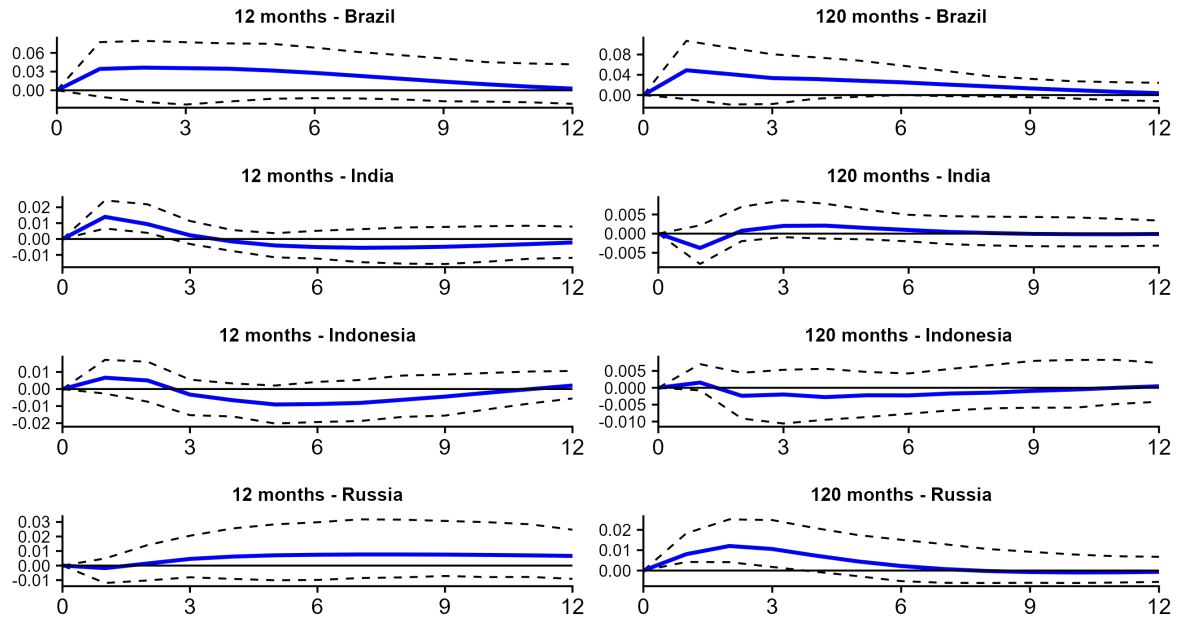
Note: The x -axes are expressed in weeks, and the bond yield responses (y -axes) are given in percentage points per annum. The size of the innovation is equal to one standard deviation.

Figure 5.7: Generalized impulse responses – Shock to the R rate in Russia

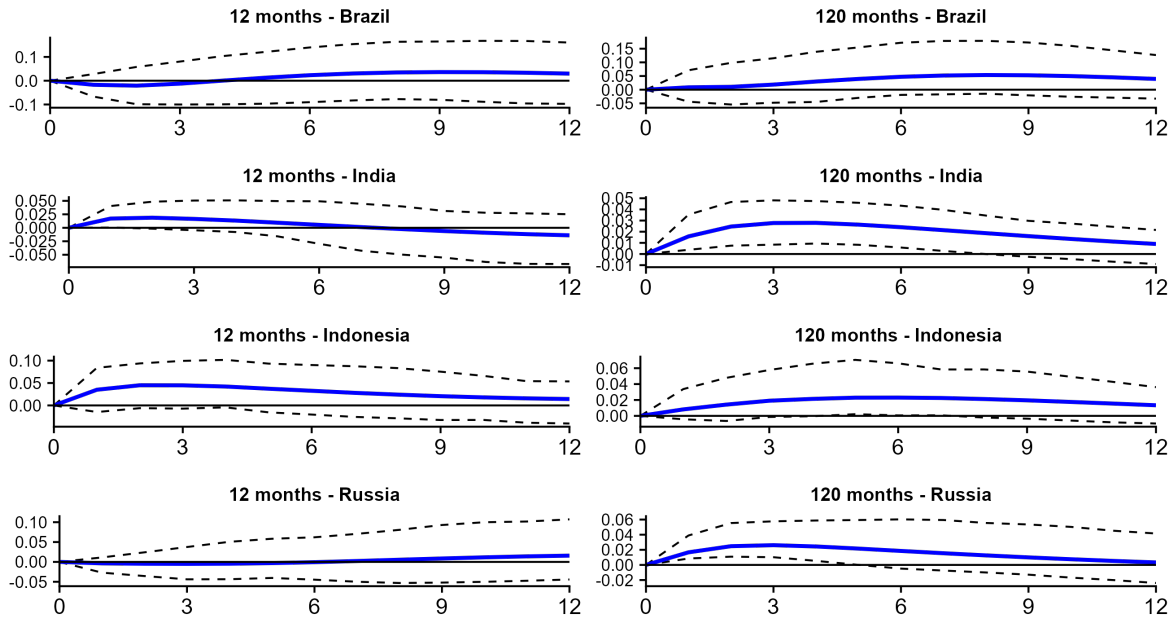
Note: The x -axes are expressed in weeks, and the bond yield responses (y -axes) are given in percentage points per annum. The size of the innovation is equal to one standard deviation.

To assess the statistical significance of our previous results, in Figures 5.8 - 5.11, we add to the point estimate *GIRFs* the 95% confidence intervals to the same set of R shocks. To save space, we only report the results for the 12- and 120-month maturity bonds for each country. The confidence intervals are obtained using a standard residual bootstrap method as detailed in Candelon and Moura, 2021.

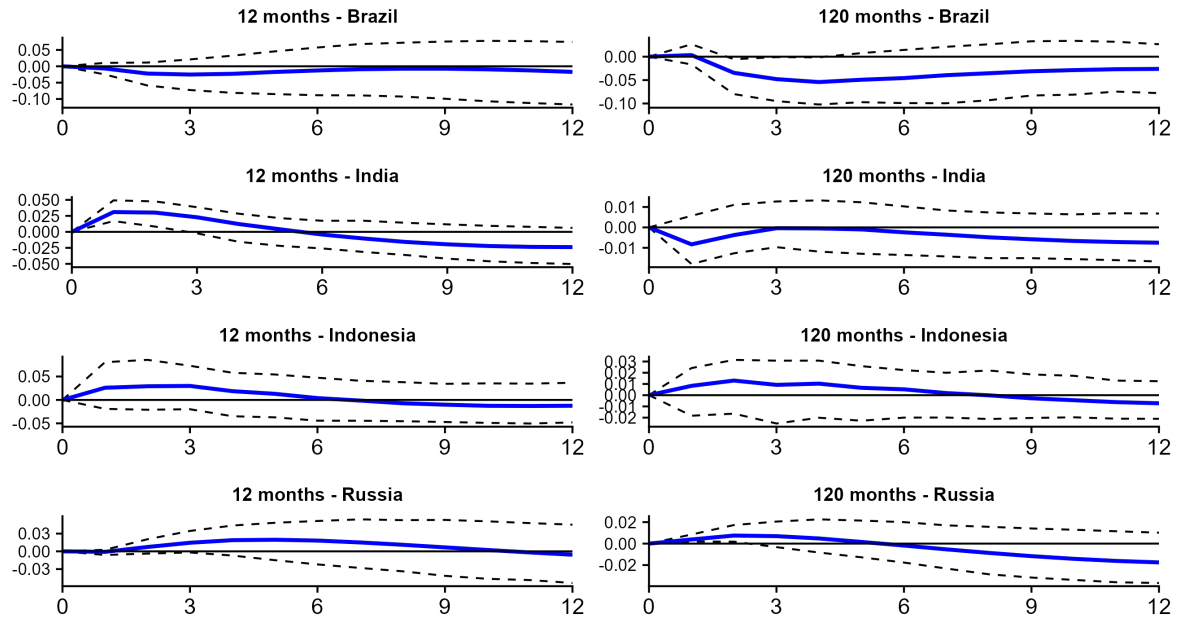
Overall, we can observe that, for all statistically significant cases, the selected bond yields respond positively to an expected deterioration of the all-around epidemiological situation. Moreover, our model shows that the effect of an R rate shock may last up to 8 weeks (response of the 120-month maturity bond from India to its own shock) and reach an order of magnitude of up to 0.15% (response of the 120-month maturity bond from Brazil to an Indian shock). Furthermore, it is interesting to verify that a shock originating in each country is relevant for at least one other economy. In particular, such foreign transmission effects are statistically significant for the cases in which the shocks stream from (i) Brazil and Indonesia to India and Russia, (ii) India to Russia, and (iii) Russia to India. Allegedly, the large geographical distances among countries in some cases could suggest a weak degree of interdependence among them and, hence, an independent evolution of their financial markets. Nonetheless, in the context of a pandemic driven by a fast-spreading virus such as the coronavirus, it is reasonable to expect that macro-financial indicators must respond critically to the degree of dissemination of the virus.

Figure 5.8: Generalized impulse responses – Shock to the R rate in Brazil

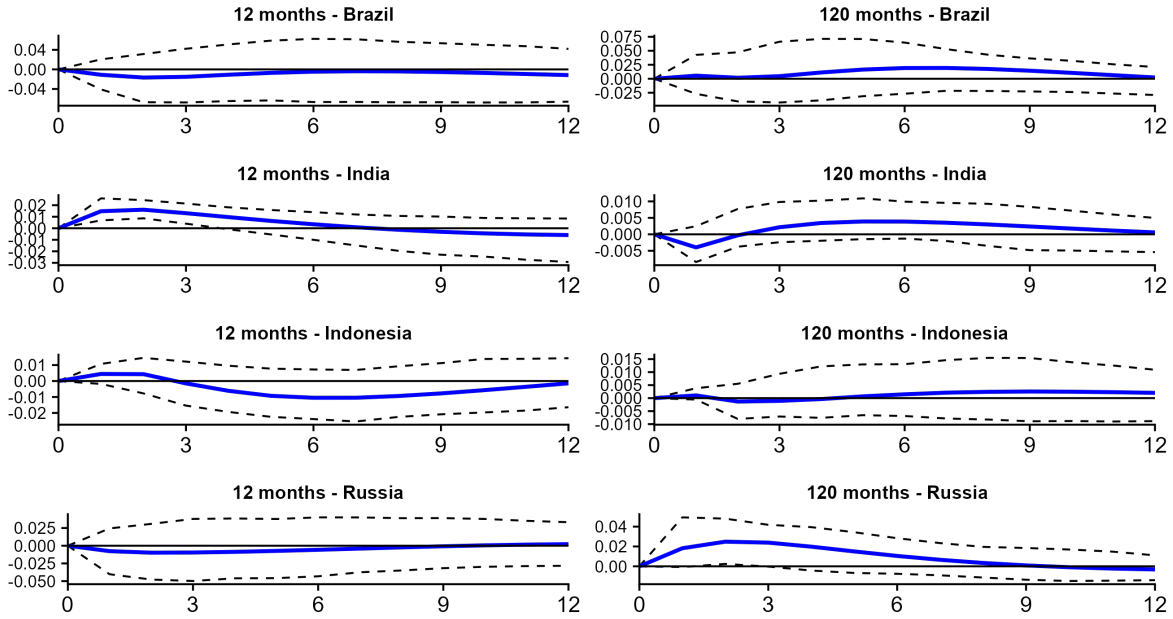
Note: The x -axes are expressed in weeks, and the bond yield responses (y -axes) are given in percentage points per annum. Blue continuous lines are the point estimates, and dashed black lines correspond to the bands of the 95% confidence interval. Confidence bands were generated using a residual bootstrap procedure with 100 replications. The size of the innovation is equal to one standard deviation.

Figure 5.9: Generalized impulse responses – Shock to the R rate in India

Note: The x -axes are expressed in weeks, and the bond yield responses (y -axes) are given in percentage points per annum. Blue continuous lines are the point estimates, and dashed black lines correspond to the bands of the 95% confidence interval. Confidence bands were generated using a residual bootstrap procedure with 100 replications. The size of the innovation is equal to one standard deviation.

Figure 5.10: Generalized impulse responses – Shock to the R rate in Indonesia

Note: The x -axes are expressed in weeks, and the bond yield responses (y -axes) are given in percentage points per annum. Blue continuous lines are the point estimates, and dashed black lines correspond to the bands of the 95% confidence interval. Confidence bands were generated using a residual bootstrap procedure with 100 replications. The size of the innovation is equal to one standard deviation.

Figure 5.11: Generalized impulse responses – Shock to the R rate in Russia

Note: The x -axes are expressed in weeks, and the bond yield responses (y -axes) are given in percentage points per annum. Blue continuous lines are the point estimates, and dashed black lines correspond to the bands of the 95% confidence interval. Confidence bands were generated using a residual bootstrap procedure with 100 replications. The size of the innovation is equal to one standard deviation.

5.4.2 GFEVDs

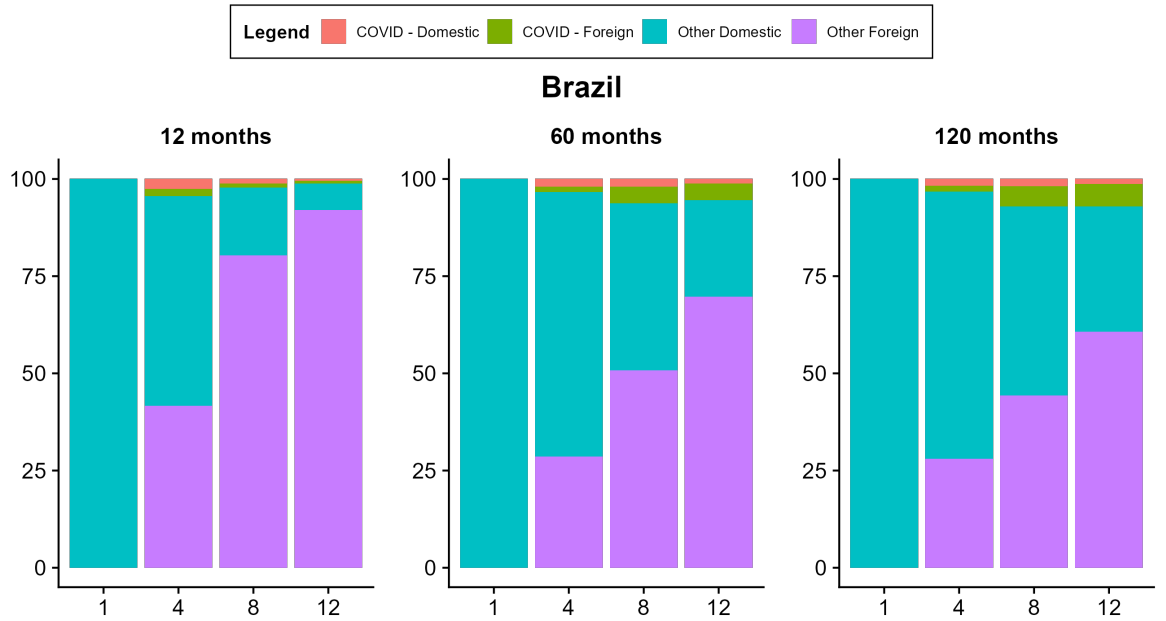
We report our *GFEVDs* in Figures 5.12- 5.15. For each country, we select bond yields from the short (12 months), medium (60 months), and long (120 months) segments of the maturity spectrum and collect the risk factors in four groups. These groups are labelled as follows: (i) *COVID - domestic*, which includes exclusively the R rate factor of the respective country; (ii) *COVID - Foreign*, featuring the R rates from all the other countries of the economic system; (iii) *Other Domestic*, formed by the domestic rates of output gap, detrended inflation and the CDS; and (iv) *Other Foreign*, comprising the global unspanned factors in addition to the CDS and the rates of output gap and detrended inflation of the remaining economies.

These same figures reveal that coronavirus-related shocks explain a nontrivial share of the yield curve variations during the period of analysis. In particular, we observe that, the total share of COVID–19 shocks (sum of domestic and foreign groups) may account for up to 7%, 16%, 13%, and 11% of the term structure variations in Brazil, India, Indonesia, and Russia, respectively. It is worth highlighting the heterogeneity of results across countries and maturities in terms of the *COVID* shares. While domestic *COVID* prevails for the shortest (medium and longest) maturity (maturities) in Brazil

(India), foreign *COVID* preponderate for the other cases. This result confirms the intuition that, in an economically integrated world facing a global pandemic, individual economies are reasonably subject to beyond-the-border sanitary shocks.

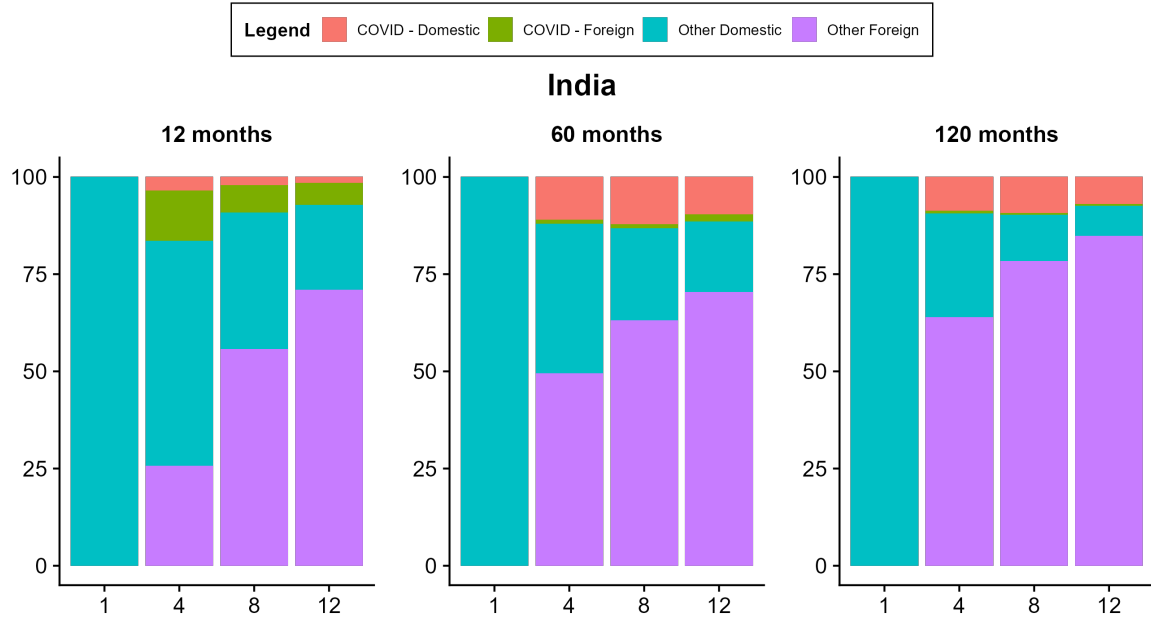
In terms of economic-related risk factors, we observe that the shares of the foreign factors increase with the time horizon and might explain at least 33% at the 12-weeks forecast horizon (Indonesia’s 12-month bond). In contrast, domestic economic developments seem to have limited power to explain the yield curve dynamics during the coronavirus pandemic. This relatively greater importance of external economic factors over the internal elements was also documented by recent term structure works such as Abbritti et al., 2018, Iania et al., 2021, and Candelon and Moura, 2021.

Figure 5.12: Generalized forecast error variance decomposition – Brazil



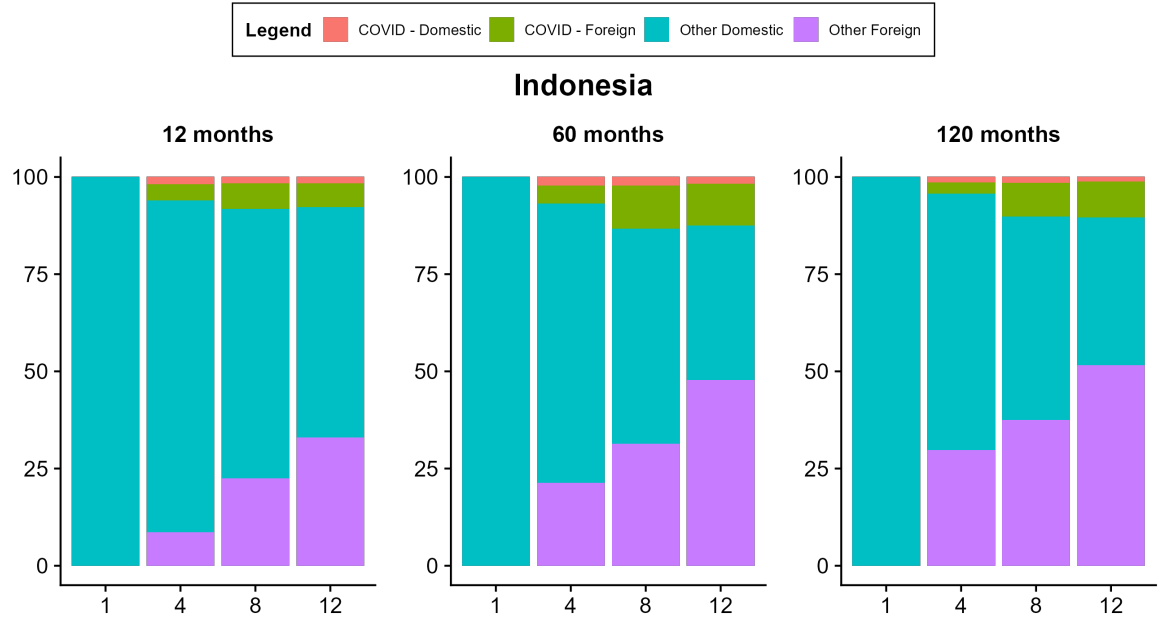
Note: The bars indicate the sum of the proportion explained by the shocks of the following four groups of risk factors: (i) *COVID - Domestic* includes exclusively the domestic *R* rate factor; (ii) *COVID - Foreign* features the *R* rates from all the other economies; (iii) *Other Domestic* is composed of the domestic rates of output gap and detrended inflation; and (iv) *Other Foreign* is composed of the global unspanned factors and the rates of output gap and detrended inflation of the remaining economies. The *x*-axes are expressed in weeks, and the *y*-axes are given in percentage points.

Figure 5.13: Generalized forecast error variance decomposition – India



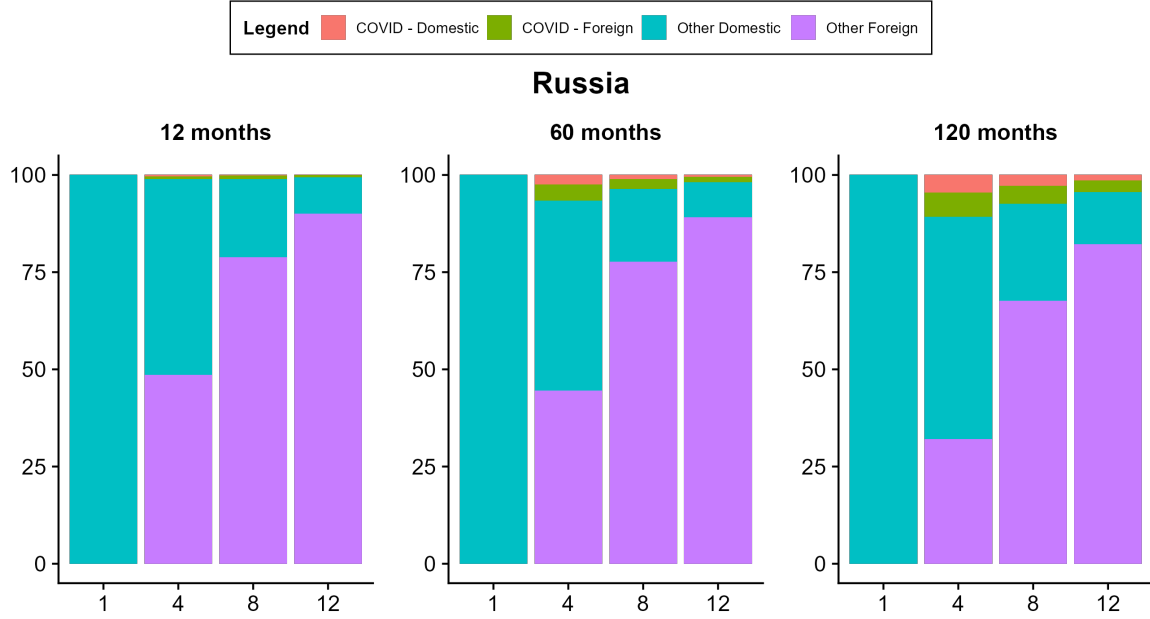
Note: The bars indicate the sum of the proportion explained by the shocks of the following four groups of risk factors: (i) *COVID - Domestic* includes exclusively the domestic R rate factor; (ii) *COVID - Foreign* features the R rates from all the other economies; (iii) *Other Domestic* is composed of the domestic rates of output gap and detrended inflation; and (iv) *Other Foreign* is composed of the global unspanned factors and the rates of output gap and detrended inflation of the remaining economies. The x -axes are expressed in weeks, and the y -axes are given in percentage points.

Figure 5.14: Generalized forecast error variance decomposition – Indonesia



Note: The bars indicate the sum of the proportion explained by the shocks of the following four groups of risk factors: (i) *COVID - Domestic* includes exclusively the domestic R rate factor; (ii) *COVID - Foreign* features the R rates from all the other economies; (iii) *Other Domestic* is composed of the domestic rates of output gap and detrended inflation; and (iv) *Other Foreign* is composed of the global unspanned factors and the rates of output gap and detrended inflation of the remaining economies. The x -axes are expressed in weeks, and the y -axes are given in percentage points.

Figure 5.15: Generalized forecast error variance decomposition – Russia



Note: The bars indicate the sum of the proportion explained by the shocks of the following four groups of risk factors: (i) *COVID - Domestic* includes exclusively the domestic R rate factor; (ii) *COVID - Foreign* features the R rates from all the other economies; (iii) *Other Domestic* is composed of the domestic rates of output gap and detrended inflation; and (iv) *Other Foreign* is composed of the global unspanned factors and the rates of output gap and detrended inflation of the remaining economies. The x -axes are expressed in weeks, and the y -axes are given in percentage points.

5.5 Concluding remarks

Financial markets around the globe reacted with panic to the inception of the coronavirus pandemic and the ensuing dismal worldwide economic outlook. In this context of financial distress, we assess the impact of coronavirus-related shocks on the term structure dynamics of four major emerging economies (Brazil, India, Indonesia, and Russia). To this end, we employ a no-arbitrage *ATSM* where the dynamics of the risk factors evolve as a *GVAR*. This flexible framework enables us to capture the interdependencies across economies and financial markets in this economic cohort. As such, we are able to appraise bond yield responses to a rich set of domestic and beyond-the-borders sanitary shocks.

Our key empirical findings are threefold. First, we show that during the year and a half that followed the pandemic outbreak, the domestic and foreign coronavirus dissemination rates might account for 7%-16% of interest rate fluctuations in these four countries. Our model also highlights that external financial and economic variables play a significant role, whereas domestic economic factors are less relevant. Second, we show that the

acceleration in the transmission rate of COVID–19 cases leads to an increase in the cost of sovereign borrowing in these four countries, partially reflecting markets’ perception of a heightened sovereign default risk. Finally, we provide mixed evidence about the effects of coronavirus-related shocks on the slope of the term structures. While in Indonesia and India the yield curves tend to flatten, in Brazil and Russia, the term structure becomes steeper. In the former cases, the reaction could be linked to investors’ demand for higher compensation in terms of default risk, inflation risk premia, and adjustment in investment strategies focused on short-term liquid assets. In the latter, the bond yield responses could be triggered by the predominance of the flight-to-safety effect.

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Appendices

5.A Model restrictions

To ease the exposition, we use $\mathcal{M}_{i,t}$ to denote the vector containing the domestic unspanned factors excluding the COVID-related risk factor measure ($COVID_{i,t}$). As such, we represent the domestic unspanned vector by $M'_{i,t} = [\mathcal{M}_{i,t}, COVID_{i,t}]$ and the vector including the complete set of domestic risk factors by $Z_{i,t} = [\mathcal{M}'_{i,t}, COVID'_{i,t}, P'_{i,t}]'$. Along the lines described in Section 5.2.2, we detail in Table 5.A.1 the array of restrictions imposed on the $VARX^*$ model of each country. The empty entries indicate the presence of zero restrictions.

Table 5.A.1: Structure of the matrices from the $VARX^*$ models

	Φ_i^X			$\Phi_i^{X^*}$			$\Phi_i^{X^W}$	Σ_i^X		
	$\mathcal{M}_{i,t-1}$	$COVID_{i,t-1}$	$P_{i,t-1}$	$\mathcal{M}_{i,t-1}^*$	$COVID_{i,t-1}^*$	$P_{i,t-1}^*$	$M_{i,t-1}^W$	$\mathcal{M}_{i,t}$	$COVID_{i,t}$	$P_{i,t}$
$\mathcal{M}_{i,t}$	x	x	x	x	x	x	x	x		
$COVID_{i,t}$		x			x				x	
$P_{i,t}$	x	x	x	x	x	x	x	x		x

Note: Relationships among the risk factors from the $VARX^*$ models described in Equation 5.6. The table shows the responses from the factors listed in the rows to shocks from the factors placed in the columns. The matrices Φ_i^X , $\Phi_i^{X^*}$, and $\Phi_i^{X^W}$ capture the responses of the domestic risk factors to the lagged values of, respectively, their own developments, the star variables and the global unspanned factors. Σ_i^X is the Cholesky factorization term. The cells including a X express a non-zero block of a matrix.

Chapter 6

*MultiATSM: An R Package for Arbitrage-free Multicountry Affine Term Structure of Interest Rates Models with Unspanned Macroeconomic Risk*¹

Abstract

The R package **MultiATSM** provides several estimation routines and additional outputs for eight classes of affine term structure of interest rates models (ATSMs). All the ATSMs from this package build on the single-country unspanned macroeconomic risk framework by Joslin et al., 2014. The **MultiATSM** package also features alternative multicountry extensions based on the settings of Jotikasthira et al., 2015, which imposes the existence of a dominant (global) economy, and Candelon and Moura, 2021, where the joint dynamics of the risk factors are captured by a GVAR setup. For each ATSM, the **MultiATSM** package produces a set of model outputs that include: (i) the graphical representations from the model fit, the orthogonalized and generalized versions of impulse response and forecast error variance decomposition from bond yields and risk factors; (ii) a number of bootstrap procedures for constructing confidence intervals, and (iii) out-of-sample forecasting of bond yields.

Keywords: *term structure of interest rates models, macrofinance, international finance, financial-economic connectedness, R, MultiATSM*

¹The working paper version of this chapter is available as Moura, 2022a.

6.1 Introduction

The term structure of interest rates (or yield curve) describes the relationship between bond yields and different investment maturities. The workhorse in yield curve modelling is referred to as affine term structure models (ATSMs). Based on the assumption of absence of arbitrage opportunities, ATSMs provide a simple framework to assess how markets price different sources of risk and generate predictions for the price of any bond.²

Early ATSMs have gained popularity in the academic literature due to their capability of capturing nearly all term structure movements.³ Despite producing accurate statistical descriptions of the yield curve, this class of models provides little insight into the underlying economic forces that steer interest rate fluctuations. In view of this limitation, a large body of research has emerged to study the interplay between the term structure and macroeconomic developments.⁴

The unspanned economic risk framework developed by Joslin et al., 2014 (henceforth JPS, 2014) is a recent and referenced macrofinance ATSM. In essence, this model assumes the absence of arbitrage opportunities and considers a linear state-space representation of the yield curve dynamics. With respect to previous macrofinance ATSMs, JPS (2014) offer a more tractable estimation approach that simultaneously combines the traditional yield curve factors (spanned factors) along with economic and financial variables (unspanned factors).

The work of Joslin et al., 2014 lays the foundations for the modelling frameworks from the R package **MultiATSM** (Moura, 2022b), which is available from the Comprehensive R Archive Network (CRAN) at <https://CRAN.R-project.org/package=MultiATSM>. Aside from the original single-country framework of JPS (2014), this same package also features the multicountry extensions of Jotikasthira et al., 2015 (henceforth JLL, 2015) and Candelon and Moura, 2021 (henceforth CM, 2021). In total, the package provides several functions to construct 8 types of ATSMs that include the original versions and variations of these three setups. In addition to providing complete estimation routines for these models, the **MultiATSM** package generates a myriad of other outputs. Specifically, in terms of graphical representations, one can obtain the model fit of the country term structures in addition to both the orthogonalized and generalized versions of the (i) impulse response functions of bond yields on shocks to the risk factors and (ii) forecast error variance decomposition of bond yields and risk factors. The confidence intervals of this set of outputs can be computed via residual-based, block, or wild bootstrap procedures. Furthermore, the package can also be used to produce out-of-sample forecasts

²See Piazzesi, 2010 and Gürkaynak and Wright, 2012 for a comprehensive summary of this literature.

³This line of research includes, for instance, Vasicek, 1977, Duffie and Kan, 1996, and Dai and Singleton, 2002.

⁴See, for instance, the seminal works of Ang and Piazzesi, 2003 and Rudebusch and Wu, 2008.

of bond yields. The goal of this paper is to provide detailed guidance on the use of the **MultiATSM** package.

There are a few notable packages for term structure modelling in R. **YieldCurve** (Guireri, 2015) and **fBonds** (Setz, 2017) provide a collection of functions to compute term structures based on the frameworks of Nelson and Siegel, 1987 and Svensson, 1994. These yield curve methods have gained the attention of financial practitioners because they offer a parsimonious model parametrization while granting a good fit of the term structure. However, these approaches are subject to criticism for allowing arbitrage opportunities, a feature that can be avoided with ATSMs. Moreover, the functionalities of **YieldCurve** and **fBonds** are limited to the estimation of the model parameters and the yield curve fit. As such, other popular model outputs, such as the ones available at **MultiATSM**, are not contemplated in these packages. Relatedly, the R package **EWS** (Hasse and Lajaunie, 2021) uses information from the short and the long-end of the term structure (yield spreads) to forecast future recessions. Yet, this package does not intend to model the whole maturity spectrum of term structures.

The remainder of the paper is as follows. Section 6.2 provides the theoretical foundations from the ATSMs of the **MultiATSM** package, and Section 6.3 provides a detailed description of the characteristics of each ATSM. In the subsequent sections, the focus is on the description of the various necessary pieces to implement ATSMs in practice. Specifically, Section 6.4 describes the dataset available in this package and the set of functions that are useful to retrieve data from Excel files. Section 6.5 exposes the necessary inputs that have to be specified by the user. In Section 6.6 the estimation procedure is detailed. Section 6.7 shows how to use the **MultiATSM** package to estimate ATSMs from scratch, as well as to closely reproduce some empirical features of academic papers. To ease the exposition, throughout the next sections, the database used by CM (2021) is employed to illustrate the use of the various functions available in this package.

6.2 ATSMs with unspanned economic risks: theoretical background

In this section, we outline the arbitrage-free ATSMs with unspanned macroeconomic risk frameworks. One convenient aspect of these setups is that they enable a clear split of the yield curves into cross-sectional (governed by the Q -dynamics model parameters) and time series (formed from the P -dynamics parameters) dimensions. In light of this characteristic of the models, we present the single and the multicountry Q -dynamics model dimensions in Section 6.2.1. Next, we expose the specific features of the risk factor dynamics under the P -measure of the various restricted and unrestricted VAR settings in Section 6.2.2.

6.2.1 Cross-sectional dimension of the term structure (Q -dynamics)

Single-country specifications (individual Q -dynamics models)

ATSM cross-sectional settings are based on two central equations. The first one assumes that the country i short-term interest rate, $r_{i,t}$, is an affine function of N unobserved (latent) country-specific factors, $X_{i,t}$:

$$r_{i,t} = \delta_{i,0} + \delta_{i,1}^\top X_{i,t}. \quad (6.1)$$

The second equation refers to the state dynamics. By assumption, for each country i , the dynamics of the unobserved factors follow a maximally flexible affine VAR(1) model under the Q -measure:

$$X_{i,t} = \mu_{i,X}^Q + \Phi_{i,X}^Q X_{i,t-1} + \Sigma_{i,X} \varepsilon_{i,t}^Q, \quad \varepsilon_{i,t}^Q \sim \mathcal{N}(0, I_N). \quad (6.2)$$

Supported by Equations (6.1) and (6.2), Dai and Singleton, 2000 show that the country-specific yield of a zero-coupon bond with maturity of n periods, $y_{i,t}^{(n)}$, is affine in $X_{i,t}$:

$$y_{i,t}^{(n)} = A_{i,n}(\Theta_n) + B_{i,n}(\Theta_n) X_{i,t}, \quad (6.3)$$

where $A_{i,n}(\Theta_n)$ and $B_{i,n}(\Theta_n)$ are restricted to preclude arbitrage opportunities in this bond market. For notational simplicity, we collect J yields into the vector $Y_{i,t} = [y_{i,t}^{(1)}, y_{i,t}^{(2)}, \dots, y_{i,t}^{(J)}]^\top$, the J intercepts into $A_X(\Theta_i) = [A_{i,1}(\Theta_1), A_{i,2}(\Theta_2), \dots, A_{i,J}(\Theta_J)]^\top$, and the N slope coefficients into a $J \times N$ matrix $B_X(\Theta_i) = [B_{i,1}^\top(\Theta_1), B_{i,2}^\top(\Theta_2), \dots, B_{i,N}^\top(\Theta_N)]^\top$. Accordingly, we outline the country's i yield curve cross-section dimension as:

$$Y_{i,t} = A_X(\Theta_i) + B_X(\Theta_i) X_{i,t}. \quad (6.4)$$

It follows from Equations (6.1) and (6.2) that the parameter set $\Theta_i = \{\mu_{i,X}^Q, \Phi_{i,X}^Q, \Sigma_{i,X}, \delta_{i,0}, \delta_{i,1}\}$ fully characterizes the cross-section of country's i term structure. Importantly, Dai and Singleton, 2000 show that this system is not identified without the imposition of restrictions.⁵ To circumvent this problem, JPS (2014) employ the three sets of (minimal) restrictions proposed by Joslin et al., 2011. Firstly, they impose the latent factors to be zero-mean processes, namely $\mu_{i,X}^Q = 0$. Secondly, they choose $\delta_{i,1}$ to be a N -dimensional vector with all entries equal to one. Lastly, $\Phi_{i,X}^Q$ is a diagonal matrix, the elements of which are the real and distinct eigenvalues, λ_i^Q , of the matrix of eigenvectors of $\Phi_{i,X}^Q$.

⁵The authors demonstrate that $X_{i,t}$ and any invertible affine transformation of $X_{i,t}$ have observationally equivalent representations. The model restrictions imposed by Joslin et al., 2011 ensure that no additional invariant rotation is possible.

Joslin et al., 2011 also demonstrate that a rotation from latent factors $X_{i,t}$ to portfolios of yields, the spanned factors $P_{i,t}$, leads to an observationally equivalent term structure representation. This invariant transformation assumes that N portfolios of yields are perfectly priced and observed without errors, while the remaining $J - N$ portfolios are priced and observed imperfectly. Specifically, the spanned factors are computed as $P_{i,t} = V_i Y_{i,t}$, for some full-rank matrix V_i . Supported by this definition, we can rewrite the vector of country-specific yields as an affine function of $P_{i,t}$ as follows:⁶

$$Y_{i,t} = A_P(\Theta_i) + B_P(\Theta_i)P_{i,t}. \quad (6.5)$$

The rotation from $X_{i,t}$ to $P_{i,t}$ is convenient for at least two reasons. First, in contrast to $X_{i,t}$, $P_{i,t}$ is directly observable, and its N time series elements can be interpreted as the traditional yield curve factors.⁷ Second, the model representation in terms of $P_{i,t}$ allows for an opportune decomposition of the likelihood function that makes the model estimation tractable and the interpretation of each parameter straightforward.

Multicountry specifications (joint Q -dynamics models)

To build the multicountry yield curve cross-section, we simply stack the sets of country yields, spanned factors, and intercepts from Equation (6.5) into, respectively, $Y_t = [Y_{1,t}^\top, Y_{2,t}^\top, \dots, Y_{C,t}^\top]^\top$, $P_t = [P_{1,t}^\top, P_{2,t}^\top, \dots, P_{C,t}^\top]^\top$, and $A_P(\Theta) = [A_P^\top(\Theta_1), A_P^\top(\Theta_2), \dots, A_P^\top(\Theta_C)]^\top$. Additionally, we set $B_P(\Theta)$ as block diagonal, $B_P(\Theta) = \text{diag}(B_P(\Theta_1), B_P(\Theta_2), \dots, B_P(\Theta_C))$. Accordingly, we can write

$$Y_t = A_P(\Theta) + B_P(\Theta)P_t. \quad (6.6)$$

6.2.2 Time series dimension of the term structure (P -dynamics)

In the modelling frameworks available at the **MultiATSM** package, the risk factor dynamics under the P -measure may include N domestic spanned variables ($P_{i,t}$), in addition to M domestic ($M_{i,t}$) and G global unspanned factors (M_t^W). The dynamics of these same risk factors may evolve as either an unrestricted or a restricted VAR models. While the former refer to JPS related settings, the latter feature the GVAR and JLL frameworks.

It is worth stressing the role of unspanned factors in the yield curve developments. Although unspanned factors are absent in the cross-section dimension of the models,

⁶The loadings of $Y_{i,t}$ are computed as $A_P(\Theta_i) = I - B_X(\Theta_i)(V_i B_X(\Theta_i))^{-1} V_i A_X(\Theta_i)$ and $B_P(\Theta_i) = B_X(\Theta_i)(V_i B_X(\Theta_i))^{-1}$.

⁷For instance, for $N = 3$ and V_i being the weight matrix that results from a principal component analysis, the portfolios of yields $P_{i,t}$ are commonly referred to as the level, slope, and curvature factors (see Section 6.6.1 for a more detailed discussion).

they influence the dynamics of the spanned factors which, in turn, affect directly bond yields.

JPS-based models

The country-specific state vector, $Z_{i,t}$, is formed from stacking the global and domestic (unspanned and spanned) risk factors: $Z_{i,t} = [M_t^{W^\top}, M_{i,t}^\top, P_{i,t}^\top]^\top$. In JPS-based setups, $Z_{i,t}$ follows a standard unrestricted Gaussian VAR(1):

$$Z_{i,t} = C_i^{\mathbb{P}} + \Phi_i^{\mathbb{P}} Z_{i,t-1} + \Sigma_i \varepsilon_{Z,t}^{\mathbb{P}}, \quad \varepsilon_{Z,t}^{\mathbb{P}} \sim \mathcal{N}(0, I_{G+M+N}). \quad (6.7)$$

JLL-based models

In JLL-based frameworks, the economic system comprises the global economy, one worldwide large (dominant) economy,⁸ and another set of smaller economies. We denote by C the number of countries in this economic system and by D the dominant economy.

In these models, the state vector is formed from a number of linear projections. This approach is intended to build purely country-specific risk factors, *i.e.*, risk factors that are free from the influence of the variables from other countries and/or from the global economy.

The construction of the domestic spanned factors follows a two-step procedure. First, for each economy i , $P_{i,t}$ is projected on $M_{i,t}$ of this same country

$$P_{i,t} = b_i M_{i,t} + P_{i,t}^e, \quad (6.8)$$

where the residuals $P_{i,t}^e$ are orthogonal to the economic fundamentals of the country i .

Second, for the non-dominant economies, $P_{i,t}^e$ is additionally projected on the orthogonalized spanned factors of the dominant country as follows:

$$P_{i,t}^e = c_i^D P_{D,t}^e + P_{i,t}^{e*}, \quad (6.9)$$

where $P_{i,t}^{e*}$ corresponds to the non-dominant country i set of residuals.

The design of the domestic unspanned factors also features two steps: for the dominant economy, $M_{D,t}$ is projected on the global economic factors

$$M_{D,t} = a_D^W M_t^W + M_{D,t}^e \quad (6.10)$$

and, for the other economies, the residuals of the previous regression are used to compute

$$M_{i,t} = a_i^W M_t^W + a_i^D M_{D,t}^e + M_{i,t}^{e*}. \quad (6.11)$$

⁸Noticeably, the model type *JLL NoDomUnit* is the only exception (see the detailed description of the different ATSMs in Section 6.3).

Accordingly, the state vector is formed by $Z_t^e = [M_t^{W^\top}, M_{D,t}^{e^\top}, P_{D,t}^{e^\top}, M_{2,t}^{e*^\top}, P_{2,t}^{e*^\top} \dots M_{C,t}^{e*^\top}, P_{C,t}^{e*^\top}]^\top$ and its dynamics evolve as a restricted VAR(1) of the following form:

$$Z_t^e = C_Y^e + \Phi_Y^e Z_{t-1}^e + \Sigma_Y^e \varepsilon_{Z,t}^e, \quad \varepsilon_{Z,t}^e \sim \mathcal{N}(0, I_F). \quad (6.12)$$

where $F = CK + G$ and the set of restrictions imposed in Φ^e and Σ_Y^e are detailed in Jotikasthira et al., 2015.

GVAR-based models

In the **MultiATSM** package, the GVAR setup is formed from two parts: the $VARX^*$ and the marginal models. The former captures the developments from the domestic (spanned and unspanned) factors, whereas the latter describes the joint dynamics of the global economy.⁹

$VARX^*$ models are small-scale local VAR augmented by the *star* and global factors. The star factors are a weakly exogenous weighted average of foreign variables constructed as follows:

$$Z_{i,t}^{*\top} = \sum_{j=1}^C w_{i,j} Z_{j,t}^\top, \quad \sum_{j=1}^C w_{i,j} = 1, \quad w_{ii} = 0 \quad \forall i \in \{1, 2..C\}, \quad (6.13)$$

where $Z_{j,t}$ is a vector of domestic factors $Z_{j,t} = [M_{j,t}^\top, P_{j,t}^\top]^\top$ with dimension $K = M + N$ and $w_{i,j}$ measures the degree of connectedness of country i with country j .

In terms of functional form, these models assume the structure $VARX^*(p, q, r)$ where p , q , and r are the number of lags from, respectively, the domestic, the star, and the global risk factors. In its current version, the **MultiATSM** package provides the estimates for the case $p = q = r = 1$. In such a case, the dynamics of $Z_{i,t}$ is described as a $VARX^*$ of the following form:

$$Z_{i,t} = C_i^X + \Phi_i^X Z_{i,t-1} + \Phi_i^{X*} Z_{i,t-1}^* + \Phi_i^{XW} M_{t-1}^W + \Sigma_i^X \varepsilon_{i,t}^X, \quad \varepsilon_{i,t}^X \sim \mathcal{N}(0, I_K). \quad (6.14)$$

The marginal model is a standard unrestricted VAR(1) featuring exclusively the global factors:

$$M_t^W = C^W + \Phi^W M_{t-1}^W + \Sigma^W \varepsilon_t^W, \quad \varepsilon_t^W \sim \mathcal{N}(0, I_G). \quad (6.15)$$

GVAR models require the specification of country-specific link matrices, W_i s, to unify the individual $VARX^*$ s. Formally, W_i s are computed as

$$\begin{bmatrix} Z_{i,t} \\ Z_{i,t}^* \end{bmatrix}_{2K \times 1} \equiv W_i \begin{bmatrix} Z_{1,t} \\ Z_{2,t} \\ \vdots \\ Z_{C,t} \end{bmatrix}_{CK \times 1} \quad (6.16)$$

⁹See the work of Chudik and Pesaran, 2016 for a thorough description of GVAR models.

Lastly, we compose the GVAR state vector by gathering the global economic variables and the country-specific risk factors, as $Z_t = [M_t^{W^\top}, Z_{1,t}^\top, Z_{2,t}^\top, \dots, Z_{C,t}^\top]^\top$. As such, we can form a first order GVAR process as

$$Z_t = C_y + \Phi_y Z_{t-1} + \Sigma_y \epsilon_{y,t}, \quad \epsilon_{y,t} \sim \mathcal{N}(0, I_F), \quad (6.17)$$

where $C_y = [C^{W^\top}, C_1^{X^\top}, C_2^{X^\top}, \dots, C_C^{X^\top}]^\top$, $\epsilon_{y,t} = [\epsilon_t^{W^\top}, \epsilon_{1,t}^{X^\top}, \epsilon_{2,t}^{X^\top}, \dots, \epsilon_{C,t}^{X^\top}]^\top$, $\Sigma_y = \text{diag}(\Sigma^W, \Sigma_1^X, \Sigma_2^X, \dots, \Sigma_C^X)$, and

$$\Phi_y = \begin{bmatrix} \Phi^W & 0_{G \times CK} \\ \Phi^{X^W} & G_1 \end{bmatrix}_{F \times F}, \quad (6.18)$$

where $\Phi^{X^W} = \begin{bmatrix} \Phi_1^{X^W} \\ \Phi_2^{X^W} \\ \vdots \\ \Phi_C^{X^W} \end{bmatrix}_{CK \times G}$ and $G_1 = \begin{bmatrix} \Phi_1 W_1 \\ \Phi_2 W_2 \\ \vdots \\ \Phi_C W_C \end{bmatrix}_{CK \times CK}$, for $\Phi_i = [\Phi_i^X, \Phi_i^{X^*}]$ and $i = 1, 2, \dots, C$.

6.3 The ATSMs available at the MultiATSM package

In its current version, the **MultiATSM** package can generate the outputs of 8 different classes of ATSMs. Overall, the ATSMs differ in terms of the form of the P and Q dynamics, the model estimation procedure, and the presence (or not) of a dominant economy. In Table 6.1, we summarize the general features of each model available in the package. We briefly comment below on the characteristics of the different frameworks.

As exposed in Section 6.2, under both the P and the Q -dynamics, the ATSMs can be cast on a country-by-country basis - as in JPS (2014) - or jointly for all the countries of the economic system - as in JLL(2015) and CM (2021). Further, under the P -measure, the risk factor dynamics follow a VAR(1) model of some sort (restricted or unrestricted). In terms of the model estimation process, most of the ATSMs follow the method described in Joslin et al., 2014. Specifically, the authors show that, due to the peculiar features of their specification, efficient estimation of the parameters governing the Q and the P -measures can be carried out rather independently. The only exception is the variance-covariance matrix term, Σ , which is a common element in both the P and the Q likelihood functions.

The ATSMs in which the estimation is performed individually for each country are labeled as *JPS*, *JPS jointP* and *GVAR sepQ*. In the *JPS* setup, the set of risk factors includes exclusively each country's domestic factors and the global unspanned factors,

Table 6.1: Summary of model estimation features

	<i>P</i> -dynamics				<i>Q</i> -dynamics		Σ		Dom. Eco.
	Individual		Joint		Individual	Joint	<i>P</i>	<i>P</i> and <i>Q</i>	
	UR	R	UR	R					
				<i>JLL</i>					

Unrestricted VAR									
<i>JPS</i>	x				x			x	
<i>JPS jointP</i>			x		x			x	
<i>VAR jointQ</i>			x			x		x	
Restricted VAR (GVAR)									
<i>GVAR sepQ</i>				x	x			x	
<i>GVAR jointQ</i>				x		x		x	
Restricted VAR (JLL)									
<i>JLL original</i>			x		x		x		x
<i>JLL NoDomUnit</i>			x		x		x		
<i>JLL jointSigma</i>			x		x			x	x

Note: Under the P -measure, the dynamics of the risk factors can be unrestricted (UR columns) or restricted (R columns). The set of restrictions present in the JLL-based and GVAR-based models are described in Jotikasthira et al., 2015 and Candelon and Moura, 2021, respectively. The estimation of the Σ matrix is done either exclusively with the other parameters of the P -dynamics (P column) or jointly under both P - and Q -parameters (P and Q column). The entries featuring X indicate that the referred characteristic is part of the model.

whereas *JPS jointP* and *GVAR sepQ* also incorporate domestic risk factors of the other countries of the economic system. Noticeably, the difference between *JPS jointP* and *GVAR sepQ* stems from the restrictions imposed under the P -dynamics.

In terms of the multicountry frameworks, some aspects are worth highlighting. As for the models based on the setup of JLL (2015), the version *JLL original* follows closely the seminal work of JLL (2015), *i.e.*, it is assumed an economic cohort containing a worldwide dominant economy and a set of smaller countries, in addition to the estimation of the Σ matrix be performed exclusively under the P -measure.¹⁰ The two other alternative versions assume the absence of a dominant country (*JLL NoDomUnit*) and the estimation of Σ under both the P and Q measures (*JLL jointSigma*), as in the standard JPS (2014) model. As for the remaining multicountry ATSMs, it is considered that the dynamics of the risk factors under the P -measure evolve according to a GVAR model. The version labeled *GVAR jointQ* is the one presented in CM (2021).

¹⁰Jotikasthira et al., 2015 claim that, although this method is not fully efficient, it makes a limited impact empirically.

6.4 Risk factors dataset

6.4.1 Package dataset

MultiATSM package contains the four datasets used in CM (2021). The first set of data comprises several time series of zero-coupon bonds yields from four emerging markets: China, Brazil, Mexico, and Uruguay. It is worth noting that this package requires *(i)* for estimation purposes, the maturities of the bond yields to be the same for all countries (although the function `DataForEstimation()` handles different maturities across countries as inputs, the outputs generated by this same function are common bond yields for all the economies); *(ii)* bond yields to be expressed in percentage terms (and not in basis points) per annum. **MultiATSM** package does not support a routine to bootstrap zero-coupon yields from coupon bonds. As such, this data manipulation procedure, if necessary, must be handled by the user herself.

```
R> data("CM_Yields")
```

The second group of data concerns the time series of the risk factors. Specifically, along the terminology defined in JPS (2014), this dataset contains *(i)* country-specific spanned factors and *(ii)* an array of country-specific and global unspanned factors (namely, some measure of economic growth and inflation). Similar to the case of the bond yields data, the measures of economic and financial variables must be constructed by the user.

```
R> data("CM_Factors")
```

The two last blocks of data are necessary for the estimation of the GVAR-based models. The trade flows database presents the sum of the value of all goods imports and exports between any two countries of the sample on a yearly basis since 1948. All the values are free on board and are expressed in U.S. dollars. This data are used to construct the transition matrix from the GVARs models.

```
R> data("CM_Trade")
```

The GVAR factors database casts country-specific lists of all the factors that are used in the estimation of each country's VARX. A specific function for computing the star risk factors is detailed in Section 6.6.2.

```
R> data("CM_Factors_GVAR")
```

6.4.2 Importing data from Excel files

This package also offers an automatized procedure to extract data from Excel files and, subsequently, prepare the risk factors database that are directly used in the estimation of the models. The use of the package functions requires that the databases *(i)* are

constructed in separate Excel files for the unspanned factors and the term structure data. Additionally, for the GVAR-based models, the measures of interdependence must be compiled in a supplementary file; (ii) contain, in each Excel file, one separate tab per country. Further, in the case of the unspanned factors' database, one must include a separate tab for the global variables; and (iii) have identical variable labels across all the tabs within each file. For illustration, see the Excel file available in the package. One example of a list of inputs to be provided is

```
R> Initial_Date <- "2006-09-01"
R> Final_Date <- "2019-01-01"
R> DataFrequency <- "Monthly"
R> GlobalVar <- c("GBC", "VIX")
R> DomVar <- c("Eco_Act", "Inflation", "Com_Prices", "Exc_Rates")
R> N <- 3
R> Economies <- c("China", "Mexico", "Uruguay", "Brazil", "Russia")
R> ModelType <- "JPS"
```

Based on these inputs, one can construct the variable *ZZfull* which contains the complete set of the model risk factors.

```
R> FactorLabels <- LabFac(N, DomVar, GlobalVar, Economies, ModelType)
R> ZZfull <- DataForEstimation(Initial_Date, Final_Date, Economies, N,
+                               FactorLabels, ModelType, DataFrequency)
```

6.5 Required user inputs

6.5.1 Basic inputs used in the model estimation

In order to estimate any ATSM, the user needs to specify several general model inputs, namely:

- Model type: a string vector containing the label of the model to be estimated (as per described in Table 6.1);
- Frequency of the data: a string vector that specifies the frequency of the time series data. The available options are: *Annually*, *Quarterly*, *Monthly*, *Weekly*, *Daily Business Days*, and *Daily All Days*;
- Economies: a string vector containing the names of the economies which are part of the economic system;
- Number of spanned factors (N): number of country-specific spanned factors. In this package, N is assumed to be equal across countries;

- Global variables: a string vector containing the labels of the global unspanned factors used in the model estimation;
- Domestic variables: a string vector containing the labels of the domestic unspanned factors used in the model estimation;
- Bond yield set of maturities: a numerical vector containing the maturities of the yields used in the model estimation. The units of this vector must be expressed in years. Alternatively, the user can rely on the `Maturities()` routine to extract the bond maturities from the time series of bond yields (see the documentation of the `Maturities()` function). In this case, the user must specify the unit in which the label of the bond yields are expressed, *e.g.*, if the label of the 10-years bond is selected as *Y120M* (*Y10y*), then the option *Month* (*Year*) must be selected;
- Factor labels: a string-list based which contains the labels of all the variables present in the model. This input is particularly useful for constructing the plots. The routine `LabFac()` provides the desired list of factor labels;
- Stationarity constraint under the Q -dynamics: a binary variable which takes value equal to 1 if the user wishes the largest eigenvalue under the Q -measure to be strictly smaller than 1. Otherwise, this same variable must be set to 0;
- Output label: a single element string-vector which contains the name that appears in the file name that stores the model outputs.

One possible example of the basic model inputs is

```
R> ModelType <- "JPS"
R> DataFrequency <- "Monthly"
R> Economies <- c("China", "Brazil", "Mexico", "Uruguay")
R> N <- 3
R> GlobalVar <- c("GBC", "CPI_OECD")
R> DomVar <- c("Eco_Act", "Inflation")
R> mat <- c(0.25, 0.5, 1, 3, 5, 10)
R> FactorLabels <- LabFac(N, DomVar, GlobalVar,
+                        Economies, ModelType)
R> StationarityUnderQ <- 0
R> OutputLabel <- "Model_demo"
```

6.5.2 GVARinputs and JLLinputs

Some additional inputs are required if the user intends to estimate the GVAR-ATSM or JLL-ATSM-related setups. For a convenient use, the extra inputs should be stored in a separate list for each model. The GVAR model list of inputs must contain:

- Economies: a string vector containing the names of the economies as described above;
- GVAR list of risk factors: a list of risk factors sorted by country in addition to the global variables. See `DatabasePrep()`;
- VARX type: a string vector containing the desired estimated form of the VARX*. Two possibilities are available. The *unconstrained* form estimates the model without any constraints, using standard OLS regressions for each model equation. The *constrained: Spanned Factors* form prevents foreign spanned factors to impact any domestic risk factor in the feedback matrix, whereas *constrained: '* followed by the name of the risk factor restricts this same factor to be influenced only by its own lagged values and the lagged values of its own star variables. In the last two cases, the VARX* is estimated by restricted least squares.

```
R> GVARinputs <- list()
R> GVARinputs$Economies <- Economies
R> GVARinputs$GVARFactors <- FactorsGVAR
R> GVARinputs$VARXtype <- "constrained: Inflation "
```

Furthermore, the GVAR estimation requires the specification of the necessary inputs to build the transition matrix (see Section 6.6.2 for more details). One example of this list of inputs is

```
R> t_First <- "2000"
R> t_Last <- "2015"
R> W_type <- "Sample Mean"
```

Concerning the JLL-ATSM frameworks, the list of inputs includes:

- JLL type: a string vector containing the label of the JLL model to be estimated (as described in Table 6.1);
- Economies: a string vector containing the names of the economies as described above;

- Dominant Country: a string vector containing the name of the economy which is assigned as the dominant country (applicable for the *JLL original* and *JLL jointSigma* models) or *None* (applicable for the *JLL NoDomUnit*).
- Wish the estimation of the sigmas matrices: this is a binary variable that assumes a value equal to 1 if the user wishes the estimation of all JLL sigma matrices (*i.e.*, variance-covariance and the Cholesky factorization matrices) and, 0 otherwise. As the sigma matrices must be evaluated numerically, the estimation process can take several minutes;
- Sigma of the non-orthogonalized variance-covariance matrix: to save time, the user may provide the variance-covariance matrix from the non-orthogonalized dynamics. Otherwise, this input should be set as *NULL*.

One possible composition for the *JLLinputs* is as follows:

```
R> JLLinputs <- list()
R> ModelType <- "JLL original"
R> JLLinputs$Economies <- Economies
R> JLLinputs$DomUnit <- "China"
R> JLLinputs$WishSigmas <- 1
R> JLLinputs$SigmaNonOrtho <- NULL
R> JLLinputs$JLLModelType <- ModelType
```

6.5.3 Additional inputs for numerical and graphical outputs

Once the model parameters from the ATSM have been estimated, the **MultiATSM** package enables the numerical and graphical compilation of the following additional outputs:

- model fit of the bond yields;
- orthogonalized impulse response functions (IRFs);
- orthogonalized forecast error variance decompositions (FEVDs);
- generalized impulse response functions (GIRFs);
- generalized forecast error variance decompositions (GFEVDs).

The methods used in the computation of the orthogonalized and generalized outputs are described in Pesaran and Shin, [1998](#).

Furthermore, for the IRFs, GIRFs, FEVDs, or GFEVDs, a horizon of analysis has to be specified, *e.g.*,

```
R> Horiz <- 100
```

For the graphical outputs, the user must indicate the desired types of graphs in a string-based vector. Available options are: *Fit*, *IRF*, *FEVD*, *GIRF*, and *GFEVD*. One example is

```
R> DesiredGraphs <- c("Fit", "GIRF", "GFEVD")
```

Moreover, the user must select the types of variables of interest (yields, risk factors, or both) and, for the JLL type of models, whether the orthogonalized version should be additionally included. These variables take value 1 if the graphs of interest are to be generated and 0, otherwise.

```
R> WishGraphRiskFac <- 0
R> WishGraphYields <- 1
R> WishOrthoJLLgraphs <- 0
```

The function `InputsForOutputs()` can provide some guidance for customizing the features of the wished outputs. Conditional to these settings, individual folders are created in the user's temporary directory to store the different types of the desired graphical outputs.

Bootstrap settings

If the user intends to generate confidence intervals through some bootstrap procedure, an additional list of inputs is required. Specifically:

- *methodBS*: the desired bootstrap procedure. Available options are (i) standard residual bootstrap (*bs*); (ii) wild bootstrap (*wild*) and block bootstrap (*block*). If the latest is selected, then the block length must be indicated;
- *ndraws*: the number of bootstrap draws;
- *pctg*: the confidence level expressed in percentage points.

```
R> Bootlist <- list()
R> Bootlist$methodBS <- "block"
R> Bootlist$BlockLength <- 4
R> Bootlist$ndraws <- 3
R> Bootlist$pctg <- 95
```

Out-of-sample forecast settings

To perform out-of-sample forecasts of bond yields, the following list-based features have to be detailed:

- *ForHoriz*: number of periods ahead that the forecasts are to be generated;
- *t0Sample*: time-dimension index of the first observations belonging to the information set;
- *t0Forecast*: time-dimension index of the last observation of the information set used to perform the first forecast set.

```
R> ForecastList <- list()
R> ForecastList$ForHoriz <- 12
R> ForecastList$t0Sample <- 1
R> ForecastList$t0Forecast <- 70
```

6.6 Model estimation

Having gathered bond yields and other economic time series data, the estimation of the ATSM of interest is carried out in three steps. Firstly, one needs to provide the number of country-specific spanned factors to be included in the global ATSM. Secondly, it is decided on the form of the risk factor dynamics under the P -measure. Finally, the user needs to select general features for the model optimization.

6.6.1 Spanned Factors

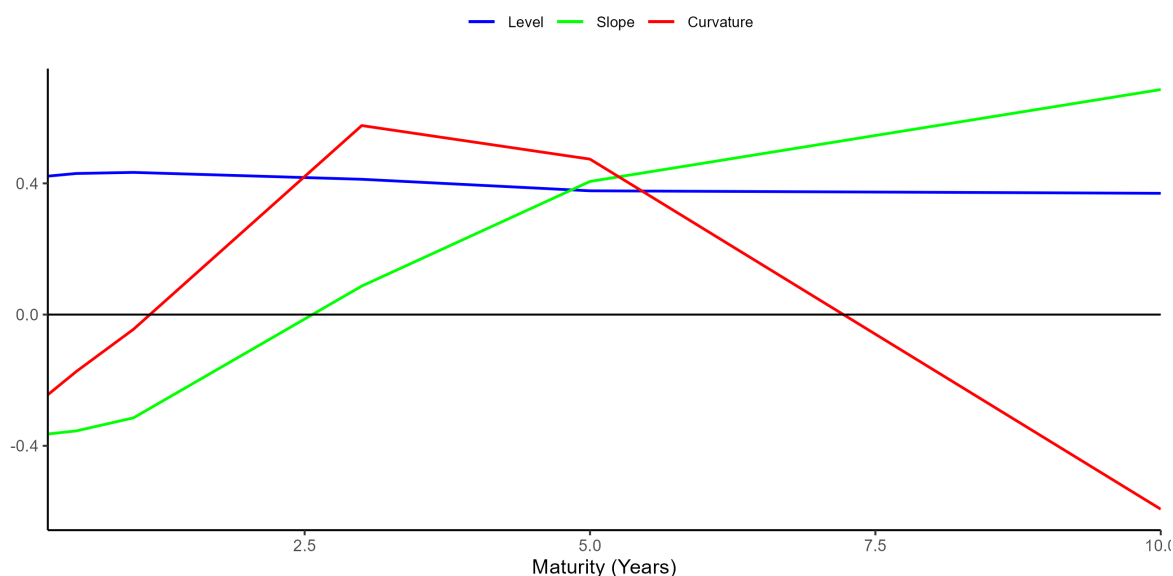
The spanned factors are yield-related variables that are used to fit the cross-section dimensions of the term structures. Typically, the spanned factors of country i , $P_{i,t}$, are computed as the N first principal components (PCs) of the set of observed bond yields. Formally, $P_{i,t}$ is constructed as $P_{i,t} = w_i Y_{i,t}$ where w_i is the PC weight matrix and $Y_{i,t}$ is a country-specific column-vector of yields with increasing maturities.

For $N = 3$, the spanned factors are traditionally interpreted as level, slope, and curvature. The nature of such interpretability follows from the features of the PC weight matrix as illustrated below:¹¹

```
R> w <- pca_weights_one_country(Yields, Economy = "Uruguay")
```

¹¹See the seminal work of Litterman and Scheinkman, 1991 for a supplementary discussion.

Figure 6.1: Yield loadings on the spanned factors



Note: The figure shows the traditional yield curve factors (level, slope, and curvature). These factors are computed as the three first principal components of the set of observed bond yields.

In matrix w , each row stores the weights used for constructing each spanned factor. The entries of the first row are linked to the composition of the level factor in that they load rather equally on all yields. Accordingly, high (low) values of the level factor indicate an overall high (low) value of yields across all maturities. In the second row, the weights monotonically increase with the maturities, and, therefore, they capture the degree of steepness (slope) of the term structure. High slope factor values imply a steep yield curve, whereas low ones entail flat (or, possibly, downward) curves. In the third row, the weights of the curvature factor are presented. The name of this factor follows from the fact that the weights have a more pronounced effect on the middle range maturities of the curve. These concepts are graphically illustrated in Figure 6.1.

To directly obtain the time series of the country-specific spanned factors, the user can simply use the `Spanned_Factors()` as follows:

```
R> data('CM_Yields')
R> Economies <- c("China", "Brazil", "Mexico", "Uruguay")
R> N <- 2
R> SpaFact <- Spanned_Factors(Yields, Economies, N)
```

6.6.2 The P -dynamics estimation

As presented in Table 6.1, the dynamics of the risk factors under the P -measure evolve according to a VAR(1) model, which may be fully unrestricted (as in the case of the JPS-related models) or somewhat restricted (as in the GVAR and JLL frameworks). Below, the usage of each of these model dimensions is illustrated.

VAR

Using the `VAR()` function of this package requires simply selecting the appropriate set of risk factors for the desired estimated model. For instance, for the models *JPS jointP* and *VAR jointQ*, the estimation of the P -dynamics parameters is obtained as

```
R> data("CM_Factors")
R> PdynPara <- VAR(RiskFactors, VARtype = "unconstrained")
```

whereas the estimation of a *JPS* model for China is

```
R> FactorsChina <- RiskFactors[1:7,]
R> PdynPara <- VAR(FactorsChina, VARtype = "unconstrained")
```

In both cases, the outputs generated are the vector of intercepts and both the feedback and variance-covariance matrices.

GVAR

As exposed in Section 6.2.2, the estimation of a GVAR model requires defining a measure of interdependence among the countries of the economic system. This information is reported in the transition matrix, the entries of which reflect the degree of interconnection of two entities of this same economic system. In this package, the illustration of the transition matrix is based on the average of the cross-border trade flow weights for the period spanning the years from 2006 to 2019. Note that each row sums up to 1.

```
R> data("CM_Trade")
R> t_First <- "2006"
R> t_Last <- "2019"
R> Economies <- c("China", "Brazil", "Mexico", "Uruguay")
R> type <- "Sample Mean"
R> W_gvar <- Transition_Matrix(t_First, t_Last, Economies,
+                             type, DataPath = NULL, TradeFlows)
R> round(W_gvar, digits= 4)
```

	China	Brazil	Mexico	Uruguay
China	0.0000	0.6549	0.3155	0.0296

Brazil	0.8269	0.0000	0.1234	0.0497
Mexico	0.8596	0.1326	0.0000	0.0078
Uruguay	0.3811	0.5498	0.0691	0.0000

Having defined the form of the transition matrix, one can complete the *GVARinputs* variable along the lines discussed in Section 6.5.2.

```
R> data("CM_Factors_GVAR")
R> GVARinputs <- list()
R> GVARinputs$Economies <- Economies
R> GVARinputs$GVARFactors <- FactorsGVAR
R> GVARinputs$VARXtype <- "unconstrained"
R> GVARinputs$Wgvar <- W_gvar
R> N <- 3
R> GVARpara <- GVAR(GVARinputs, N)
```

A separate routine is provided for computing the star variables used in the estimation of the VARX* models.

```
R> data("CM_Factors")
R> StaFac <- StarFactors(RiskFactors, Economies, W_gvar)
```

JLL

Calculating the *P*-dynamics parameters in the form proposed by JLL (2015) requires the following inputs: (i) the time series of the risk factors in non-orthogonalized form; (ii) the number of country-specific spanned factors, and (iii) the specification of the *JLLinputs* as presented in Section 6.5.2. See, for instance:

```
R> data("CM_Factors")
R> N <- 3
R> JLLinputs <- list()
R> ModelType <- "JLL original"
R> JLLinputs$Economies <- Economies
R> JLLinputs$DomUnit <- "China"
R> JLLinputs$WishSigmas <- 1
R> JLLinputs$SigmaNonOrtho <- NULL
R> JLLinputs$JLLModelType <- ModelType
R> JLLpara <- JLL(RiskFactors, N, JLLinputs)
```

6.6.3 ATSM estimation

The model estimation involves defining the model inputs from the log-likelihood function (LLK) and, subsequently, the parameters used in its optimization process. The structure proposed in this part of the code is in a great extent based on the term structure package by Le and Singleton, [2018](#).

The log-likelihood function

For the sake of simplicity, we illustrate the construction of an LLK based on a *VAR jointQ* model. For both the JLL and GVAR-based models, one must further specify the *JLLinputs* or *GVARinputs* along the lines described in Section [6.5.2](#). We divide the exposition into four parts:

1. Inputs to be specified by the user (see Section [6.5](#))

```
R> data("CM_Yields")
R> data("CM_Factors")
R> ModelType <- "VAR jointQ"
R> Yields <- Yields
R> ZZ <- RiskFactors
R> Economies <- c("China", "Brazil", "Mexico", "Uruguay")
R> mat <- Maturities(Yields, Economies, UnitYields = "Month")
R> DataFrequency <- "Monthly"
R> GlobalVar <- c("GBC", "CPI_OECD")
R> DomVar <- c("Eco_Act", "Inflation")
R> N <- 3
```

2. Generate the *Factor Labels* list

```
R> FactorLabels <- LabFac(N, DomVar, GlobalVar, Economies,
+                          ModelType)
```

3. Prepare the inputs for LLK

```
R> ATSMInputs <- InputsForMLEdensity(ModelType, Yields, ZZ,
+                                     FactorLabels, mat, Economies,
+                                     DataFrequency)
```

4. Set the objective function

```
R> f <- Functionf(ATSMInputs, Economies, mat, DataFrequency,
+                 FactorLabels, ModelType)
```

Optimization parameters

JPS (2014) requires the estimation of a set of parameters containing 6 elements, namely: the risk-neutral long-run mean of the short rate ($r0$), the risk-neutral feedback matrix ($K1XQ$), the variance-covariance matrix (SSZ) from the VAR processes, the standard deviation of the errors from the portfolios of yields observed with error (se), in addition to the intercept ($K0Z$) and the feedback ($K1Z$) matrices of the physical dynamics. Each of these parameters must be cast in an individual list that should contain the (i) the starting value of the parameter (if any); (ii) the variable label followed by a ':' and a type of constraint; (iii) a lower bound (if any), and (iv) an upper bound (if any). The variable labels of $r0$, se , $K0Z$ and $K1Z$ should be preceded by the symbol @ as a manner to account for the fact that the solutions of these parameters are known in closed-form. For the remaining ones ($K1XQ$ and SSZ), Le and Singleton, 2018 provide standardized routines (already incorporated in this package) to set good initial values so that the optimization process runs faster.

The type of constraint for the parameters $r0$, se , $K0Z$, and $K1Z$ is typically *bounded* and is used for bounded matrices. For $K1XQ$ and SSZ , the **MultiATSM** package currently provides the following options:

- $K1XQ$: *Jordan* (or *Jordan MultiCountry*) for a matrix of Jordan type for single (multi) country specifications. These labels can be extended by '*; stationary*', if one wishes to impose the largest eigenvalue of the risk-neutral feedback matrix to be strictly smaller than 1;
- SSZ : *psd* for a positive semi-definite matrix; *diag* for a diagonal matrix; *BlockDiag* for a block diagonal matrix (typical from the GVAR-based models) and *JLLstructure* for the models containing the restrictions along the lines of JLL (2015).

One example for the list of parameters specification is

```
R> K1XQinputs <- list(NULL, "K1XQ: Jordan", NULL, NULL)
R> SSZinputs <- list(NULL, "SSZ: psd", NULL, NULL)
R> r0inputs <- list(NULL, "@r0: bounded", NULL, NULL)
R> seinputs <- list(NULL, "@se: bounded", 1e-6, NULL)
R> K0Zinputs <- list(NULL, "@K0Z: bounded", NULL, NULL)
R> K1Zinputs <- list(NULL, "@K1Z: bounded", NULL, NULL)
```

To complete the optimization setting, the user needs to specify the level of convergence tolerance (usually, $1e - 4$ is a reasonable value) and the vector *OptRun*. The first element of this vector must be filled by the word *iter off*, if the user wishes to switch off the printouts of the numerical optimization routines, or simply by *iter* otherwise. The second element of the vector concerns the algorithm that is used in the optimization:

the option *fminunc only* only uses `fminunc()`, whereas *fminsearch only* only applies `fminsearch()`. If *OptRun* is formed exclusively by *iter off*, then both `fminunc()` and `fminsearch()` are employed in the optimization.

6.7 Example of full implementation of ATSMs

This section presents an example of how to use the **MultiATSM** package to fully implement ATSMs. We use the *JPS* framework to illustrate the example. The implementation steps are described below.

1. Defining user inputs along the lines presented in Section 6.5

a) Load database data

```
R> data ("CM_Factors")
R> data ("CM_Factors_GVAR")
R> data ("CM_Trade")
R> data ("CM_Yields")
```

b) Decide on general model inputs

```
R> ModelType <- "JPS"
R> StationarityUnderQ <- 0
R> Economies <- c("China", "Brazil", "Mexico", "Uruguay")
R> GlobalVar <- c("GBC", "CPI_OECD")
R> DomVar <- c("Eco_Act", "Inflation")
R> N <- 3
R> OutputLabel <- "Test"
R> DataFrequency <- "Monthly"
R> UnitMatYields <- "Month"
```

i. Decide on specific model inputs

A. GVAR-based models (transition matrix and VARX inputs)

```
R> if (ModelType == "GVAR sepQ" ||
+     ModelType == "GVAR jointQ"){
+   t_First <- "2006"
+   t_Last <- "2019"
+   W_type <- "Sample Mean"
+   VARXtype <- "unconstrained"
+ }
```

B. JLL-based models

```
R> if (ModelType == "JLL original" ||
+      ModelType == "JLL NoDomUnit" ||
+      ModelType == "JLL jointSigma"){
+      DominantCountry <- "China" }
```

c) Decide on the settings of the numerical outputs

```
R> Horiz <- 20
R> DesiredGraphs <- c("IRF", "FEVD")
R> WishGraphRiskFac <- 0
R> WishGraphYields <- 1
R> WishOrthoJLLgraphs <- 0
```

d) Bootstrap settings

```
R> WishBootstrap <- 1
R> Bootlist <- list()
R> Bootlist$methodBS <- "wild"
R> Bootlist$BlockLength <- 4
R> Bootlist$ndraws <- 30
R> Bootlist$pctg <- 95
```

e) Out-of-sample forecast features

```
R> WishForecast <- 1
R> ForecastList <- list()
R> ForecastList$ForHoriz <- 12
R> ForecastList$t0Sample <- 1
R> ForecastList$t0Forecast <- 145
```

2. Minor preliminary work. From this point of the code onward, no changes are needed from the user

```
R> C <- length(Economies)
R> FactorLabels <- LabFac(N, DomVar, GlobalVar, Economies,
+                          ModelType)
R> mat <- Maturities(Yields, Economies,
+                    UnitYields = UnitMatYields)
R> ZZ <- RiskFactors
```

a) Build the GVARinputs

```
R> if (ModelType == "GVAR sepQ"
+      || ModelType == "GVAR jointQ"){
```

```

+   GVARinputs <- list()
+   GVARinputs$Economies <- Economies
+   GVARinputs$GVARFactors <- FactorsGVAR
+   GVARinputs$VARXtype <- VARXtype
+   GVARinputs$Wgvar <- Transition_Matrix(t_First, t_Last,
+                                           Economies, W_type,
+                                           DataPath = NULL,
+                                           Data = TradeFlows)
+ } else { GVARinputs <- NULL }

```

b) Build the JLLinputs

```

R> if (ModelType == "JLL original"
+     || ModelType == "JLL NoDomUnit"
+     || ModelType == "JLL jointSigma"){
+   JLLinputs <- list()
+   JLLinputs$Economies <- Economies
+   JLLinputs$DomUnit <- DominantCountry
+   JLLinputs$WishSigmas <- 1
+   JLLinputs$SigmaNonOrtho <- NULL
+   JLLinputs$JLLModelType <- ModelType
+ } else {
+   JLLinputs <- NULL
+ }

```

3. Obtaining initial guesses for $K1XQ$ and SSZ , build the objective function and perform the model optimization according to the procedure described in Section 6.6

```

R> ModelParaList <- list()
R> for (i in 1:C){
+   if (( ModelType == "GVAR jointQ" || ModelType == "VAR jointQ"
+       || ModelType == "JLL original" || ModelType == "JLL NoDomUnit"
+       || ModelType == "JLL jointSigma" ) & i > 1 ){break}
+
+   ATSMInputs <- InputsForMLEdensity(ModelType, Yields, ZZ,
+                                       FactorLabels, mat,
+                                       Economies, DataFrequency,
+                                       JLLinputs, GVARinputs)
+
+   K1XQ <- ATSMInputs$K1XQ

```

[illegible]

```

+                               Economies , ModelType ,
+                               JLLinputs ,
+                               GVARinputs)$Summary}
+   }

```

4. Generate the numerical and graphical outputs

```

R> InputsForOutputs <- InputsForOutputs(ModelType, Horiz ,
+                                       DesiredGraphs ,
+                                       OutputLabel ,
+                                       StationarityUnderQ ,
+                                       WishGraphYields ,
+                                       WishGraphRiskFac ,
+                                       WishOrthoJLLgraphs ,
+                                       WishBootstrap , Bootlist ,
+                                       WishForecast ,
+                                       ForecastList)

```

a) Fit, IRFs, FEVDs, GIRFs and GFEVDs

```

R> NumericalOutputs <- NumOutputs(ModelType, ModelParaList ,
+                                 InputsForOutputs ,
+                                 FactorLabels , Economies)

```

b) Bootstrap

```

R> Bootstrap <- Bootstrap(ModelType, ModelParaList ,
+                          NumericalOutputs , mat, Economies ,
+                          InputsForOutputs , FactorLabels ,
+                          DataFrequency , varargin ,
+                          JLLinputs , GVARinputs)

```

c) Out-of-sample forecasting

```

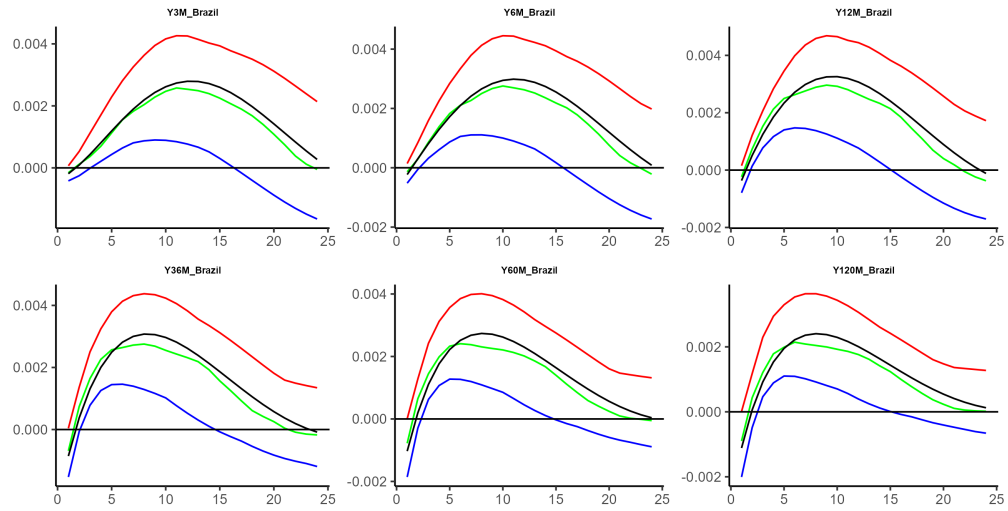
R> Forecasts <- ForecastYields(ModelType, ModelParaList ,
+                               InputsForOutputs , FactorLabels ,
+                               Economies , DataFrequency ,
+                               JLLinputs , GVARinputs)

```

Out of illustration, we present some of the model graphical outputs in Figures [6.2](#) and [6.3](#). In particular, the former shows the IRFs for Brazilian bonds to a global

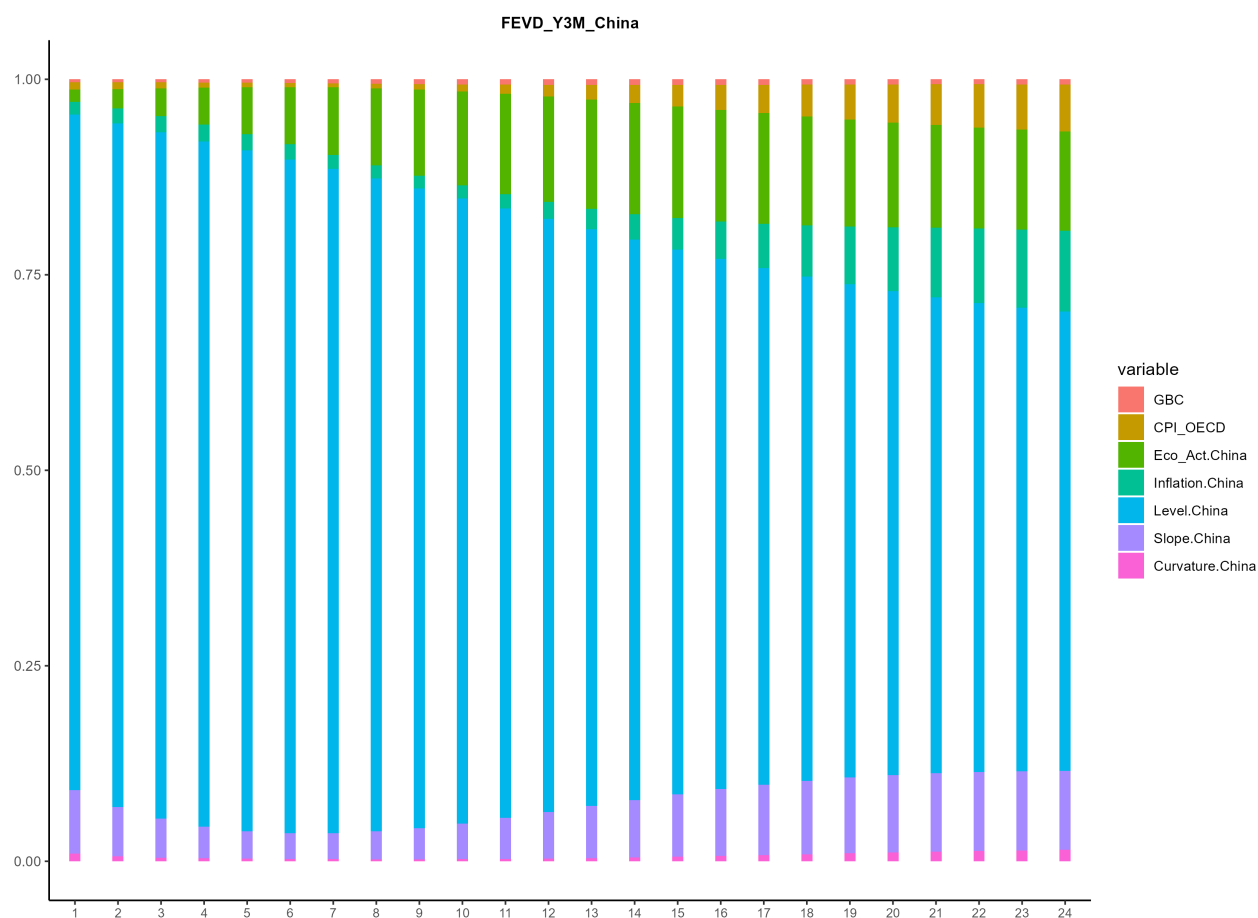
economic growth shock and the latter exhibits the FEVD of the 3-month bond yield in China.

Figure 6.2: IRFs from bond yields from Brazil - shock on global economic growth



Note: IRFs from the Brazilian bond yields to global economic growth shock. The maturities of bond yields are 3, 6, 12, 36, 60, and 120 months. The size of the shock is a one-standard-deviation. Red, green, and blue lines are, respectively the upper bound, median, and the lower bounds of the 95% confidence intervals. The black lines are the point estimates. The dataset is the same used in Candelon and Moura, [2021](#).

Figure 6.3: FEVD - 3-month maturity bond yield in China



Note: FEVD from the Chinese bond yield with a maturity of 3 months. The dataset is the same used in Candelon and Moura, [2021](#).

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Disclosure statement

No potential conflict of interest was reported by the author(s).

Appendices

6.A Paper replications

In this section, we use the **MultiATSM** package to replicate some of the empirical results presented in the original papers of JPS (2014) and CM (2021).

6.A.1 Joslin, Priebisch and Singleton (2014)

To the best of our knowledge, the original dataset used in Joslin et al., 2014 is not publicly available. Nevertheless, there exists an alternative source of data that closely matches JPS (2014) in terms of the construction method and the sample period. This database was constructed by Bauer and Rudebusch, 2017 (henceforth BR, 2017) and is available at Bauer's website.¹² In that paper, BR (2017) investigate whether macrofinance term structure models better suit the unspanned macro risk framework by JPS (2014) or the earlier traditional spanned settings such as the one of Ang and Piazzesi, 2003. Accordingly, BR (2017) intend to replicate some of the empirical results reported in JPS (2014). The R-code used by BR (2017) is also available on Bauer's webpage.

Supported by BR (2017)'s dataset, the code below uses the **MultiATSM** package to estimate the key model parameters from the ATSM of JPS (2014).

1. Load database data and set the model inputs

```
R> data("BR_jps_gro_R3")
R> ModelType <- "JPS"
R> StationarityUnderQ <- 0
R> Economies <- c("U.S.")
R> GlobalVar <- c()
R> DomVar <- c("GRO", "INF")
R> N <- 3
R> DataFrequency <- "Monthly"
R> UnitMatYields <- "Month"
R> mat <- c(0.25, 0.5, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10)
```

2. Minor preliminary work

```
R> C <- length(Economies)
R> M <- length(DomVar)
R> FactorLabels <- LabFac(N, DomVar, GlobalVar,
+                        Economies, ModelType)
```

¹²See <https://www.michaeldbauer.com/publication/spanning-puzzle/>.

The tables below compare the ATSM parameter estimates generated from BR (2017) and the **MultiATSM**. Overall, one can note that the differences in the estimates are economically modest.

Table 6.2: Q -dynamics parameters

	MultiATSM	BR (2017)
$r0$	0.00055	-0.00016
λ_1	0.99670	0.99682
λ_2	0.91488	0.95945
λ_3	0.91488	0.87174

Note: λ 's are the eigenvalues from the risk-neutral feedback matrix and $r0$ is the long-run mean of the short rate under the Q -measure.

Table 6.3: P -dynamics parameters

		$K0Z$		$K1Z$			
			PC1	PC2	PC3	GRO	INF
<i>BR (2017)</i>							
	PC1	0.07811	0.93691	-0.01307	-0.02181	0.10457	0.10033
	PC2	0.02100	0.00582	0.97814	0.17031	-0.16719	-0.04016
	PC3	0.10047	-0.01037	-0.00625	0.78346	-0.03987	0.04369
	GRO	0.06904	-0.00483	0.01801	-0.11117	0.88177	-0.00247
	INF	0.04996	0.00185	0.00642	-0.05920	0.02767	0.98593
<i>MultiATSM</i>							
	PC1	0.07811	0.93691	-0.01307	-0.02181	0.10457	0.10033
	PC2	0.02100	0.00582	0.97814	0.17031	-0.16719	-0.04016
	PC3	0.10047	-0.01037	-0.00625	0.78346	-0.03987	0.04369
	GRO	0.06904	-0.00483	0.01801	-0.11117	0.88177	-0.00247
	INF	0.04996	0.00185	0.00642	-0.05920	0.02767	0.98593

Note: $K0Z$ is the intercept and $K1Z$ is the feedback matrix from the P -dynamics.

Table 6.4: Portfolio of yields with errors

	MultiATSM	BR (2017)
se	0.0000546	0.0000550

Note: se is the standard deviation of the portfolio of yields observed with errors.

It is relevant to highlight that the code script above makes use of the principal component weights provided by BR (2017). Such a matrix is simply a scaled-up version of the one provided by the function `pca_weights_one_country()`. Accordingly, despite

the numerical differences on the weight matrices, both methods generate time series of spanned factors that are perfectly correlated. Another difference between the two approaches relates to the construction form of the LLK: while in the BR (2017) code the LLK is expressed in terms of a portfolio of yields, the **MultiATSM** package generates this same input directly as a function of observed yields (*i.e.*, both procedures lead to equivalent LLK up to the Jacobian term).

6.A.2 Candelon and Moura (2021)

We now provide the lines of code that replicate the empirical results presented in Candelon and Moura, 2021. Specifically, setting *ModelType* as *GVAR jointQ* generates the outputs of the model labeled as *GVAR-ATSM*, whereas *JLL original* builds the results for the *JLL-ATSM*. To save space, we report only the overall model features outlined in step 1 of Section 6.7. The remaining part of the code is identical to steps 2 through 4 described in that same section.

1. Load database data and the general and specific model inputs:

```
R> data("CM_Factors")
R> data('CM_Factors_GVAR')
R> data('CM_Trade')
R> data('CM_Yields')
R> ModelType <- "GVAR jointQ"
R> StationarityUnderQ <- 0
R> Economies <- c("China", "Brazil", "Mexico", "Uruguay")
R> GlobalVar <- c("GBC", "CPI_OECD")
R> DomVar <- c("Eco_Act", "Inflation")
R> N <- 3
R> OutputLabel <- "CM_2021"
R> DataFrequency <- "Monthly"
R> UnitMatYields <- "Month"
R> if (ModelType == 'GVAR jointQ'){
+   t_First <- "2006"
+   t_Last <- "2019"
+   W_type <- 'Sample Mean'
+   VARXtype <- "unconstrained"
+ }
R> if (ModelType == "JLL original"){DominantCountry <- "China"}
```

2. Defining the settings of the numerical outputs, bootstrap, and the out-of-sample forecast:

```
R> Horiz <- 25
R> DesiredGraphs <- c("Fit", "GIRF", "GFEVD")
R> WishGraphRiskFac <- 0
R> WishGraphYields <- 1
R> WishOrthoJLLgraphs <- 1
R> WishBootstrap <- 0
R> Bootlist <- list()
R> Bootlist$methodBS <- "bs"
R> Bootlist$BlockLength <- c()
R> Bootlist$ndraws <- 1000
R> Bootlist$pctg <- 95
R> WishForecast <- 0
R> ForecastList <- list()
R> ForecastList$ForHoriz <- 12
R> ForecastList$t0Sample <- 1
R> ForecastList$t0Forecast <- 90
```

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Modelling the Term Structure of Interest Rates in a Multicountry Setting

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Globalization has strengthened the linkage across economies and financial markets around the world. Against this background, this thesis builds alternative modelling frameworks to study movements in the term structure of interest rates in a context of global interdependencies. All frameworks are based on the arbitrage-free affine term structure model with unspanned risk (Joslin, Pribsch, and Singleton, 2014) and include macro-financial variables among the risk factors. Using data from several emerging economies, our models are also used to investigate the determinants of bond risk premia in recent decades, to appraise the importance of China's business cycle on Latin American yield curve dynamics, as well as to assess the impact of the global spread of the coronavirus on the term structure dynamics during the COVID-19 pandemic. We also guide the use of the functionalities of the *R* package *MultiATSM*, which we developed during the preparation of this thesis. Overall, our empirical results point to beyond-the-border economic forces as the primary source of domestic term structure fluctuations.

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