



Mode sensing with the phase camera for gravitational-wave interferometers

Doctoral dissertation presented by

Ricardo de Abreu Silvério Cabrita

in fulfillment of the requirements for the degree of Doctor in Sciences.

Thesis Jury

Dr. Andrea Giammanco	President	Université Catholique de Louvain
Prof. Clément Lauzin	Supervisor	Université Catholique de Louvain
Prof. Giacomo Bruno	Supervisor	Université Catholique de Louvain
Prof. Joris van Heijningen	Supervisor	Universiteit Vrije Amsterdam
Prof. Matthieu Génévriez	Secretary	Université Catholique de Louvain
Dr. Matteo Tacca	Member	Nat. Inst. voor Subatomaire Fysica
Dr. Jérôme Degallaix	Member	Inst. de Phys. des 2 Infinis de Lyon

Private defense: 4th December, 2025

Public defense: 12th January, 2026

To my father.

Abstract

Gravitational-wave (GW) interferometers have detected over two hundred compact binary coalescences to date. This was achieved with long-baseline Michelson interferometers that employ hundreds of kW of optical power circulating in km-long optical cavities. In addition, squeezed light is injected into the interferometer in order to beat the standard quantum noise limit.

Optical absorption in the optics leads to thermal effects that shift the eigenmodes of the optical cavities, causing control issues. This shift also leads to mode mismatch which results in optical losses. In addition, high-power effects such as angular and parametric instabilities make the operation of optical cavities with high-circulating power notoriously difficult.

In turn, quantum noise reduction techniques are sensitive to optical losses and mode degradation. The combination of high-power with squeezed light makes mode sensing and control a crucial aspect of operating a gravitational-wave interferometer. This is particularly the case for next-generation detectors, for which some of the major planned upgrades include increasing circulating powers to the MW level and quantum noise reduction up to 10dB.

In this work, we use a spatio-temporal heterodyne wavefront sensor to tackle these challenges. This sensor is called the phase camera and is currently installed at the Virgo GW detector. It combines a heterodyne readout with beam scanning to achieve high-resolution amplitude and phase images of the laser beam wavefront.

This work is divided in two parts. One concerns the phase cameras of Virgo, their use for optical characterization as well as optical simulations of the Virgo detector. The second consists of two-table top experiments with a custom phase camera. In these, two novel mode sensing schemes are demonstrated that should help achieve the high-power goals of future GW detectors.

Associated Publications:

- [1] Ricardo Cabrita, Aaron Goodwin-Jones, Joris van Heijningen, Pavel Demin, Martin van Beuzekom, Matteo Tacca, Giacomo Bruno and Clément Lauzin, "Resonant enhanced detection of the higher-order modes of a locked cavity", Opt. Express 33, 30209-30220 (2025)
- [2] Aaron Goodwin-Jones, Ricardo Cabrita, Mikhail Korobko, Martin Van Beuzekom, Daniel D. Brown, Viviana Fafone, Joris Van Heijningen, Alessio Rocchi, Mitchell G. Schirowski, and Matteo Tacca, "Transverse mode control in quantum enhanced interferometers: a review and recommendations for a new generation", Optica 11, 273-290 (2024)

Conference Proceedings:

- [3] Ricardo Cabrita, Clément Lauzin, Giacomo Bruno and Joris van Heijningen, "Mode matching with a phase camera for gravitational-wave detectors", Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment, 1068, p.169806 (2024)
- [4] Ricardo Cabrita for the Virgo collaboration, "Sensing mode mismatch with the phase cameras at Advanced Virgo", GRAvitational-wave Science&technology Symposium 2022 (GRASS2022), Padova (Italy)

Contents

Introduction	1
I Gravitational-wave interferometry and mode sensing	7
1 Gravitational-wave interferometers	9
1.1 Gravitational waves: from concept to first detection	9
1.2 A new window into the cosmos	13
1.2.1 Gravitational-wave sources and science cases	18
1.3 How to build a gravitational-wave detector	22
1.3.1 Laser beams, higher-order modes and resonators	24
1.3.2 Dual-recycled Michelson interferometers	29
1.3.3 Limiting noises	34
1.4 Quantum noise reduction	42
1.4.1 Generating squeezed states	44
1.4.2 Frequency-dependent squeezing	46
1.5 High-power operation	48
1.5.1 Parametric instabilities	49
1.5.2 The megawatt goal of next-generation detectors	51
2 Mode sensing and control in gravitational-wave detectors	53
2.1 Heterodyne and homodyne detection	53
2.1.1 Detecting the GW signal	57
2.2 Longitudinal control and frequency stabilization	58
2.2.1 Beyond Pound-Drever-Hall	63
2.3 Angular control	66
2.4 Mode matching	69
2.4.1 Mode mismatch and quantum noise reduction	70
2.5 Thermal effects	72
2.6 Wavefront sensors for mode sensing	76
2.6.1 Hartman sensors	76

2.6.2	Resonant wavefront sensing	77
2.6.3	Mode weighting	79
2.6.4	Phase cameras	80
2.6.5	Radio-frequency modulation sensing schemes	86
2.7	Adaptive optics in GW detectors	87
2.7.1	Ring heaters	88
2.7.2	Central heating radius of curvature correction	89
2.7.3	CO ₂ lasers and compensation plates	89

II Virgo in O4 and the phase camera 91

3 Optical characterization and the Virgo phase cameras 93

3.1	The Virgo detector and O4 commissioning	94
3.1.1	Marginally stable recycling cavities	95
3.2	Phase camera commissioning	97
3.2.1	The single bounce configuration	99
3.2.2	Troubleshooting the beam shape	102
3.3	Phase camera observables and sideband monitoring	105
3.3.1	Power recycling gains	106
3.3.2	Dark port transfer function	107
3.3.3	Contrast defect	108
3.4	Thermal tuning of the power and signal recycling mirrors	109
3.4.1	CITF scan simulations	111
3.4.2	OptChar measurements	113
3.5	Mode mismatch in the arm cavities with the phase camera	115
3.5.1	Simulating the measurements	116
3.5.2	Measurements with the Virgo phase camera	119
3.6	Summary and overview	123

4 Simulating the O4b detector 125

4.1	The O4b detector in numbers	127
4.2	Optical simulations with OSCAR	131
4.3	The nominal detector	134
4.3.1	Impact of astigmatism	135
4.3.2	Common and differential mismatches	137
4.4	The hot O4b detector	142
4.4.1	Checking for consistency with the cold detector	146
4.5	Overview and outlook	147

III	Table-top experiments with the phase camera	149
5	A custom phase camera for table-top experiments	151
5.1	Developing a scanning phase camera	151
5.1.1	Hardware choices	152
5.1.2	Data acquisition and signal demodulation	154
5.1.3	Scanner set-up and control	157
5.1.4	Optical set-up and design constraints	159
5.2	Phase camera characterization	161
5.2.1	Amplitude information	162
5.2.2	Phase information	166
6	Mode-matching with a phase camera in reflection of a cavity	173
6.1	Context	173
6.2	Error signals with a single phase camera	174
6.2.1	Error signals proportional to ϵ_R and ϵ_w	175
6.3	Optical design and set-up of the table top experiment	178
6.3.1	Mode (mis)matching telescope	182
6.4	First measurements and perspectives	185
6.5	Summary and outlook	188
7	Resonant detection of the higher-order modes of a locked cavity	191
7.1	Context	191
7.2	Experimental set-up	193
7.2.1	Optical injection	195
7.2.2	Heterodyne detection with a phase camera	196
7.3	Results	197
7.3.1	Heterodyne scan and mode imaging	197
7.3.2	Sensitivity, noise and detection limits	202
7.3.3	Phase sensitive measurements	204
7.4	Summary and Outlook	205
IV	Conclusion	209
	Conclusion and Perspectives	211

V	Appendices	215
A	Phase camera beating: carrier and higher-order-mode sideband	217
A.0.1	Demodulation and phase	219
B	Simulating the etalon effect of the Virgo arm cavities	221
B.0.1	Summary and outlook	224
	Bibliography	227
	Acknowledgements	251

Acronyms

ADC Analog-to-Digital Converter

AOM Acousto-Optic Modulator

AR Anti-Reflective

BPD Bulls-eye Photodiode

CARM Common ARM Length

CD Contrast Defect

CH Central Heating

CHRoCC Central Heating Radius of Curvature

CITF Central Interferometer

CP Compensation Plate

DARM Differential ARM length

DAS Double Axicon System

DDS Direct Digital Synthesizer

DP Dark Port

DRMI Dual-Recycled Michelson Interferometer

EM End Mirror

EOM Electro-Optic Modulator

FSR Free Spectral Range

FWHM Full-Width Half Maximum

HOM Higher-Order Mode

HR Highly-Reflective

IM Input Mirror

IMC Input Mode Cleaner

MM Mode-Matching

MMi Mode Mismatch

OMC Output Mode Cleaner

OPL Optical Path Length

OPO Optical Parametric Oscillator

PD Photodiode

PDH Pound-Drever-Hall

PhC Phase Camera

PI Parametric Instability

POP Pick-Off Plate

PRC Power Recycling Cavity

PRCL Power Recycling cavity Length

PRM Power Recycling Mirror

PSD Power Spectral Density

QNR Quantum Noise Reduction

QPD Quadrant Photodiode

RF Radio Frequency

RMS Root Mean Square

RoC Radius of Curvature

RTL Round-Trip Losses

SB Side Band

SLM Spatial Light Modulator

SQL Standard Quantum Limit

SRC Signal Recycling Cavity

SRCL Signal Recycling cavity Length

SRM Signal Recycling Mirror

TCS Thermal Compensation System

ToF Time-of-Flight

Introduction

'In my entire scientific life, extending over forty-five years, the most shattering experience has been the realization that an exact solution of Einstein's equations of general relativity, discovered by the New Zealand mathematician Roy Kerr, provides the absolute exact representation of untold numbers of massive black holes that populate the universe. This "shuddering before the beautiful," this incredible fact that a discovery motivated by a search after the beautiful in mathematics should find its exact replica in Nature, persuades me to say that beauty is that to which the human mind responds at its deepest and most profound level.'

Subrahmanyan Chandrasekhar in *Truth and Beauty: Aesthetics and Motivations in Science* (1987)

Subrahmanyan Chandrasekhar is best known for the discovery of the upper limit to the mass of white dwarf stars, for which he received the Nobel prize in physics. In his book titled "Truth and Beauty: Aesthetics and Motivations in Science", Chandrasekhar "shuddered before the beautiful" of Roy Kerr and Albert Einstein's mathematical predictions regarding the nature of space-time. Almost 40 years after Chandrasekhar published these words, we now have the tools to glimpse into these "untold numbers" of black holes that populate our universe.

When two black holes rotate around each other, eventually colliding and merging, small ripples in the fabric of space-time are emitted. These small ripples, that propagate at the speed of light, are called gravitational-waves. Until the summer of 2025, the LIGO-Virgo-Kagra collaboration has detected gravitational-waves from the coalescence of 208 systems of binary black holes [5]. These detections have confirmed the predictions of Kerr regarding rotating black holes [6] as well as allowing for precision tests of Einstein's theory of General Relativity [7].

In fact, this work is concerned not with the black holes themselves, but with the machines that have made these detections possible. These machines employ laser light and km-scale Michelson interferometers that are sensitive enough to measure the ripples in space-time caused by these cataclysmic coalescences. In order to measure them it is necessary to measure changes in length in the order of 10^{-19} m, more than a thousand times smaller than the radius of a proton [8,9].

Gravitational-wave interferometers employ km-long Fabry-Perot cavities with kilowatts of circulating power. In addition, quantum-correlated light, called squeezed-light, is injected into the interferometer in order to beat the standard quantum noise limit [10].

Spherical Fabry-Perot resonators sustain transverse electro-magnetic eigenmode solutions which can be exactly represented by an orthonormal basis of polynomials [11]. The zeroth and first-order modes are used to control the longitudinal and angular degrees of freedom of these cavities, respectively. This knowledge comes from pioneers from the fields of atomic, molecular and optical physics (AMO) as well as GW interferometry, such as John Hall and Ronald Drever [12], Dana Z. Anderson [13] and Gary C. Bjorklund [14]. These techniques are already mature and enjoy high levels of automation [15].

Sensing and controlling the higher-order modes of optical cavities is however, not as well developed. This is especially relevant in the presence of high optical powers, which make cavity control notoriously difficult [16], suffering from different issues such as angular and parametric instabilities [17]. High circulating powers also lead to increased losses from scattering and also due to thermal transients from thermo-refractive effects in the cavity mirrors that shift the cavities eigenmodes. In turn, squeezed light manipulation is incredibly sensitive to optical losses and mode degradation [18]. The combination of high-power with squeezed light makes (higher-order) mode sensing and control a crucial aspect of operating a gravitational-wave interferometer. This is particularly the case for next-generation detectors, for which some of the major planned upgrades include increasing circulating powers to the MW level and squeezed light injection providing up to 10dB of quantum noise reduction [19]. Thermal compensation systems already exist, but in order to reach the goals of next-generation detectors novel sensing and control schemes are required.

The eigenmode basis of an optical cavity is characterized by the waist size and position of the circulating beam sustained by the cavity. When the waist size and position of an input beam is different from the one of the cavity, we talk of mode mismatch. This mode mismatch is a source of optical

loss and it is characterized by two quadratures, the waist size mismatch and the waist position mismatch. As the mirror of a cavity heats up due to high-circulating powers, the mirrors are deformed and the cavity eigenmode shifts changing the mode matching condition. This presents another challenge in current gravitational-wave detectors.

In this work, we combine the existing knowledge of cavity and laser control with wavefront sensing and adaptive optics in order to tackle these current technical challenges. In particular, this work focuses on using a phase camera as a wavefront sensor. The phase camera combines an heterodyne readout with beam scanning to achieve high-resolution amplitude and phase images of the laser beam wavefront at multiple heterodyne frequencies of interest [20]. The goal of this work is to contribute to the advancement of mode sensing and control in gravitational-wave detectors, and in particular answer the following questions:

- *By making use of the phase information, can a single phase camera be used to completely characterize the two mode mismatch quadratures.*
- *Can a phase camera be used to deliver in situ characterization of full eigenmodes of a cavity.*

The thesis starts with a description of the field of gravitational-wave interferometry in Chapter 1. After a brief historical overview of the development of gravitational-wave detectors, gravitational-wave sources and science cases are discussed. This is followed by an introduction on how gravitational-wave interferometers work and their limiting noises. The chapter ends with a focus on the motivations for this work, by introducing quantum noise reduction with squeezed light and the issues around high-power operation.

In Chapter 2, I introduce the concepts of heterodyne and homodyne readout and I review the state of the art on longitudinal and angular control of optical cavities. This is followed by a more detailed development of the issues surrounding higher-order mismatches and thermal effects due to high-power operation. The chapter ends with a review of the state-of-the art on wavefront sensors and adaptive optics for mode sensing and control in gravitational-wave detectors. This was the subject of a review article published in *Optica*, of which I was a co-author [2]. The overlapping Sections in Chapter 2, Sections 2.6 and 2.7 were adapted from the text of the published article.

The remainder of the thesis is concerned with work developed during the course of my PhD. This work is divided in two parts. The first concerns the

use of the phase camera at Virgo for optical characterization and optical simulations. The second concerns the development of two table-top experiments using the phase camera to show case two mode sensing schemes, one in reflection from a cavity and another in transmission.

In Chapter 3, I introduce the commissioning of the Virgo phase cameras ahead of the current observation run, 04 as well as two contributions to the optical characterization of the detector. One was the thermal tuning of the central interferometer and the other is an independent measurement of the mismatch in the arm cavities of Virgo using the phase camera. The later was the subject of a proceedings publication for the GRAvitational-waves Science&technology Symposium (GRASS 2022) [4].

In Chapter 4, I describe optical simulations of the Virgo detector as it were during the observation run 04. These simulations are concerned mostly with the mode matching and thermal working point of the detector and they show what we can do using the currently existing diagnostic channels. At the end of the chapter the importance of knowing the mode content in the detector in order to accurately simulate and model the detector is shown.

For the table-top experiments performed in the course of this work, it was required to develop a phase camera in-house similar to the ones installed at Virgo. The development and characterization of a custom phase camera for this purpose is described in Chapter 5.

In chapter 6, I detail a measurement scheme to characterize the mismatch in Fabry-Perot cavity by using a phase camera in reflection. This work builds directly upon the measurements done at Virgo with the phase camera. Optical simulations show the error signals that can be extracted from a single phase camera. These results were the subject of a proceedings publication for the 16th Pisa Meeting on Advanced Detectors (PMAD 2024) in a peer-reviewed journal [3]. The design of a table-top experiment to reproduce these error signals in the lab is described. Experimental preliminary results are also shown at the end of the Chapter.

The second table-top experiment is described in Chapter 7. This experiment consists of an optical injection scheme using an electro-optic modulator and a phase camera in transmission from the cavity. In this way it is possible to image the amplitude and phase of any higher-order mode in the cavity. These results were the subject of a peer-reviewed article [1].

The characterization of the higher-order modes of a cavity is relevant to predict issues with parametric instabilities as mentioned above. In addition this technique opens up a variety of applications concerning cavity mode sens-

ing, such as the on-line measurement of thermal transients in the presence of high circulating power and tracking the phase of higher-order modes. This technique can also find applications beyond the field of gravitational-wave detectors where high circulating power cavities are also crucial. Some of these are phase-electron microscopy [21], neutral beam injection systems for fusion reactors [22] and others [23].

The thesis ends with a reflection on the results of this work and their impact for current and future gravitational-wave detectors. As long as the reach of gravitational-wave interferometers keeps improving, so does our ability to peer into more and more of these "untold numbers" of black holes that populate our Universe.

Part I

Gravitational-wave interferometry and mode sensing

Gravitational-wave interferometers

1.1 Gravitational waves: from concept to first detection

The first mention of gravitational waves predates Einstein's theory of General Relativity (GR) [24] by 10 years. In 1905, Poincaré published in the *Comptes Rendus of the French Academy of Sciences* an article titled "*Sur la dynamique de l'électron*" [25]. In this work he summarized his findings on Special Relativity, theorizing that gravity was transmitted through waves, which Poincaré called gravitational waves (*onde gravifique*).

Einstein himself was skeptical at first about the existence of gravitational waves. In the case of electromagnetic waves, they arise most commonly from an electric dipole due to variations in the separation between the positive and negative charge. In GR, there is no such analogy as there is no negative mass. Nonetheless, and after significant back and forth [26], Einstein published the article "On gravitational waves", where the case for cylindrical gravitational waves was put forward [27].

In order to prove categorically the existence of gravitational waves it was necessary however to detect their effects. Another landmark on the way to the detection of gravitational waves, was the publication of "On the physical significance of the Riemann Tensor" by Felix A. E. Pirani in 1956 [28]. Typically, when working with GR, the coordinate system is chosen in order to simplify calculations. When considering real observations, however, the problem becomes how to choose an appropriate coordinate system where an observer may calculate an experimentally measurable quantity. Pirani created the mathematical formalism to do such thing, showing that a passing gravitational wave would cause particles to move back and forth.

Pirani's seminal work went unnoticed at first, and physicists were still discussing whether gravitational waves carried energy or not. In 1957, the Chapel Hill meeting focusing on gravity was held. Many renowned physicists attended this meeting, such as John Wheeler, Richard Feynman and Hugh Everett.

During the meeting, Feynman put forward a thought experiment that convinced most of the audience that gravitational waves carried energy and could indeed be detected. His argument is now known as the "sticky bead argument". Imagine two rings of beads surrounding a bar. The rings can slide freely on the bar, which is placed transversely to a passing gravitational wave. While the bar feels some longitudinal compressive stress due to the tidal forces from the gravitational wave, the bead rings slide back and forth. The friction between the rings and the bar would cause the bar to heat up, thus showing that gravitational waves carry energy and could, in principle, be detected.

Among the audience of the Chapel Hill conference was Joseph Weber, an engineer at the University of Maryland, who would, inspired by Feynman's argument, go on to build the first gravitational-wave (GW) detector.

In 1960, Weber published his idea for a GW detector [29]. The idea consisted of a resonant-mass detector, a cylindrical bar which would detect gravitational waves traveling. Weber finished building the first detector in 1966: a 3 ton, one-and-a-half-meter long cylindrical bar with a diameter of 66 cm [30]. The bar was isolated from vibrations via a wire suspension system and held inside a vacuum chamber. A belt of piezo electric transducers was used to detect changes in the length of the bar which would stretch and squeeze if a gravitational-wave near the mechanical mode of the bar would pass by.

In 1968, using one resonant-bar detector at the Argonne National Laboratory in Illinois and five resonant-bars at the University of Maryland, Weber claimed a coincident signal between them, announcing the detection of gravitational waves [31]. Weber's claims, however, implied that mass at the center of the Milky Way was being converted into gravitational waves in quantities five times larger than permitted by theoretical limits and astronomical observations [32]. The upper limit was derived from stellar dynamic considerations, as mass loss from gravitational radiation is expected to cause the galaxy to radially expand.

At this time, various other groups were operating their own detectors from various places such as the Bell-Labs in Rochester-Holmdel, University of Glasgow in Scotland, at IBM in Yorktown Heights, at the Observatoire de Paris in Meudon France and the Max Planck Institute in Munich. In these, several improvements were made such as cryogenically cooling the bars to reduce

thermal noise and improvements to the suspensions and vibration isolation. Despite these improvements, nothing around 0.7 kHz up to a little over 1 kHz¹ was ever detected and the technique was mostly abandoned.

Despite these false starts and null detections, in 1974 a discovery re-surged the interest in GW detection. Joseph Hooten Taylor and Alan Russel Hulse found a binary pulsar that strongly indicated that the accelerated masses that comprised the binary system were emitting GW radiation. From 1975 to 1979 changes to the Periastron (the point of closest proximity between the two stars) were measured. The observed advance of the Periastron was consistent with the GR predictions describing the loss of energy of the system through the emission of gravitational waves. The article was published in 1979 [33].

Despite this groundbreaking observation, a direct measurement of gravitational waves was still lacking. In the end, this would be achieved through the use of laser interferometry.

One of the first formal mentions of interferometry for GW detection was in the former USSR by Gertsenshtein and Pustovoid in 1962 [34]. Some years before, when juggling ideas to build the first GW detection, Weber, sketched, among other concepts, the idea of using laser interferometry, however, he ended up building resonant-bar detectors instead.

Finally, in 1978 the first GW interferometer was built by an old student of Weber, Robert L. Forward following the encouragement of Rainer Weiss [35]. The detector had 8.5 m arms and operated in the 1 to 20 kHz bandwidth range.

At the same time, Weiss started to build his own interferometer at MIT with military funding. The construction of the 1.5 m arm interferometer was upended shortly after due to a new law which prohibited military funding for projects of no military utility. This stopped the project and forced Weiss to look for other funding sources, writing a proposal to the NSF agency in particular. This proposal, which was approved in 1975, would prove to be the catalyst for the construction of the GW network currently in place. This chain reaction involving many different actors and false starts is detailed in Ref. [26].

One of the detectors that would spring from this proposal, is GEO600. The experimental group working on bar detectors at the Max Planck Institute in Garching got hold of Weiss's NSF proposal and, inspired by it, decided to build their own prototype. They contacted Weiss asking for advice and offering a job to one of his students. Many years later and after two prototypes (a 3 m one in 1975 and a 30 m one in 1981), GEO600 was born. After the rejection of a 3 km

¹These type of detectors operated only with a small bandwidth around the bar resonance, which would be around 1 kHz in most cases.

interferometer proposal, and in collaboration with a Glasgow group (initially led by R. Drever), a proposal for a 600 m interferometer in Germany was finally accepted in 1994.

Besides having taken part in the first GW search science run, GEO600 would become a crucial test bed for laser interferometry, having pioneered the use of squeezed states of light to reduce quantum noise [36]. Incidentally, it was also where many of the first adaptive optics used in GW detectors were first tested [2]. Much more recently, a proposal has been put forward to transform GEO600 into an interferometric dark matter observatory [37].

In 1985 during the Marcel Grossmann Meeting in Rome, the same meeting where the GEO proposal for a 3 km interferometer was first put forward, A. Brillet and A. Giazotto met. Brillet, from Orsay UPMC, Paris, had recently visited MIT where he met Weiss and became a supporter of interferometry for GW detection. Giazotto, from the University of Pisa, Italy, was working on suspension systems - super-attenuators from which interferometric mirrors could be suspended to isolate them from seismic perturbations. The two formed a partnership, initially having tried to join the Garching group in their 3 km proposal. Having been denied, they decided to start their own project. The name Virgo was chosen, after the galaxy cluster 18 Mpc from Earth, to highlight the goal of measuring extra-galactic gravitational waves. The project would only be funded in 1993 and 1994, by the French CNRS and the Italian INFN respectively.

The construction of Virgo, in Cascina, near Pisa, would only finish in 2003 [38]. Construction was planned to start in 1996 but it was delayed due to several complications, one of which was the acquisition of the land from around 50 land owners. Previous to that, in 2000 the CNRS and INFN assembled the European Gravitational Observatory (EGO) to oversee the construction of the site, assembly, and upgrades of Virgo and its operation.

On the other side of the Atlantic and in 1975, during a NASA conference, there was an important meeting between Weiss and K. Thorne. In this meeting, Weiss convinced Thorne of the potential of interferometry for GW detection. The latter decided to build a GW detector at Caltech, enlisting the help of the Glasgow group, headed by R. Drever. Disagreements between competing technologies for laser interferometry led to the appearance of two separate projects, one from MIT and another from Caltech, where Drever had moved to from Glasgow. The NSF however, with the help of Thorne, forced a collaboration between Drever and Weiss. The joint project submitted to the NSF was titled Laser Interferometer Gravitational-wave Observatory (LIGO).

The LIGO project was officially funded in 1988, with the goal of building two GW detectors. Due to frictions between Weiss and Drever and changes in management (Drever would leave the project in 1992), construction only started in 1993 and 1994 for the Hanford and Livingston sites, respectively.

After the construction, in 1997, B. C. Barish suggested the creation of two separate organizations. One was the LIGO laboratory, responsible for detector operation and laboratory administration, and the other was the LIGO scientific collaboration (LSC), responsible for the scientific and technological research and output. In 2007, a memorandum of agreement between the LIGO and Virgo collaborations was signed, which ensured joint analysis, data sharing and co-authorship of relevant publications.

Between 2002 and 2010, the initial LIGO detectors were in operation yet no GW was detected. From 2010 to 2015 the LIGO detectors underwent upgrades and the Advanced LIGO (aLIGO) was commissioned. Before starting observation, aLIGO underwent what is called an engineering run, where final tests are made while the detectors are already on-line.

On the 14th of September of 2015 a coincident observation of GW-like signal was made in both LIGO detectors (see Fig. 1.1). One month later, after enough statistics on the noise of the detectors was gathered it was possible to ensure the signal was real: the possibility of false detection was 1 in 203 000 years. Two black holes, with masses around 36 and 29 solar masses, had merged around 1.3 billion years before, forming a final black hole with a mass around 62 solar masses. The remaining three solar masses had been radiated as GWs. The first ever detected GW, labelled GW150914, was announced to the world in February 2016 and a new era of astronomy was born [39]. B. C. Barish, K. Thorne and R. Weiss would go on to receive the 2017 Nobel prize in physics for this discovery.

1.2 A new window into the cosmos

The Einstein field equation or tensor field equation of GR, relates the curvature of space-time to the distribution of matter, and it is typically written as [24]

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}, \quad (1.1)$$

where $R_{\mu\nu}$ is the Ricci tensor, R the Ricci curvature scalar, $T_{\mu\nu}$ the matter stress-energy tensor and G the gravitational constant. The left side of Eq. 1.1

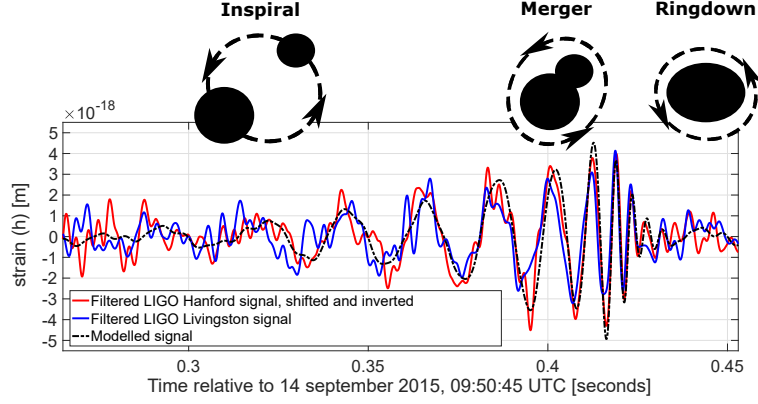


Figure 1.1: Strain amplitude in meters of the first GW ever detected, GW150914, measured by the LIGO Hanford and Livingston detectors. Comparison with the modelled signal is shown. The three main components of the event, inspiral, merger and ringdown are also shown schematically.

describes the structure of space-time through the metric tensor $g_{\mu\nu}$ and its first and second derivatives. The right side describes the stress and energy of the matter distribution. Eq. 1.1 tells us that the structure of space-time dictates the dynamics of mass distributions and vice-versa.

We can arrive at GWs from the Einstein field equation within the framework of linearised theory of GR. This is done by expanding the metric with a small perturbation, $h_{\mu\nu}$, around the flat Minkowskian metric tensor $\eta_{\mu\nu}$, such that

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1. \quad (1.2)$$

In this way, the Einstein field equation for a weak gravitational field can be written as [40, 41]

$$\square \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}, \quad (1.3)$$

where $\square$ is the D'Alembertian operator [40] and $\bar{h}_{\mu\nu}$ is the so called trace-reverse metric perturbation defined as

$$h = \eta_{\mu\nu} h^{\mu\nu} \quad \bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h. \quad (1.4)$$

Eq. 1.3 now resembles a standard wave equation. Note that in order to arrive at this equation, the follow gauge degree of freedom was fixed

$$\partial^\nu \bar{h}_{\mu\nu} = 0, \quad (1.5)$$

which is called the Lorentz gauge.

The Lorentz gauge does not fix the gauge of the framework entirely, however, and still allows for transformations of the type $x^\nu \rightarrow x^\nu + \varepsilon^\nu$. The so-called transverse-traceless (TT) gauge fixes the value of ε^ν such that the trace of h is 0 and consequently $\bar{h}_{\mu\nu} = h_{\mu\nu}$ and $h^0{}_i = 0$. In this way, h becomes

$$h_{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & h_{11} & h_{12} & 0 \\ 0 & h_{12} & -h_{11} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \quad (1.6)$$

Similarly to what happens with Maxwell's equations in electromagnetism, Eq. 1.3 can have solutions that take the form of waves that travel at the speed of light, c . This is easily seen far away from the source where the energy-stress tensor goes to 0, such that

$$\square h_\mu^\nu = \left(-\frac{1}{c^2} \frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x_i \partial x^i} \right) h_\mu^\nu = 0. \quad (1.7)$$

From inspection of Eqs. 1.6 and 1.7, it can be seen that the tidal deformations caused by the GW are transverse (in x and y) and the wave is time-dependent and propagates in the z direction. Additionally, $h_{\mu\nu}$ can be separated in two independent components, one dependent on h_{11} and another on h_{12} , which are in fact the two polarizations of GWs. Thus, it is common to write h as the sum of two functions as

$$h_{\mu\nu} = h_+(t - z/c) + h_\times(t - z/c), \quad (1.8)$$

where the $+$ and $\times$ are used to indicate the shape of each polarization. The TT gauge has an associated TT reference frame. In the TT frame, coordinates are invariant to the passage of a GW. Thus, in order to appreciate the effect of a GW on a ring of test masses, we need to use what is called the proper detector frame.

A treatment of the proper detector frame can be found in Ref. [42], but the crucial point is that the effect of a GW on the test masses can be well described by means of a Newtonian force. To simplify things, we can assume the ring of test masses is on the (x,y) plane, with the GW propagating in the z direction. In this case, the change to the x coordinates (δx) and to y (δy) as a function of

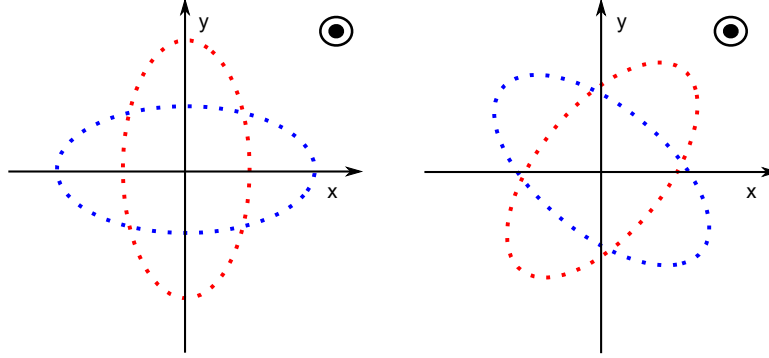


Figure 1.2: Effect of a passing GW on a ring of test masses, shown on the left for the plus polarization (h_+) and on the right for the cross polarization ($h_\times$). In blue the effect is shown for $\sin(\omega t) = -1$ and in red for $\sin(\omega t) = 1$ (refer to Eq. 1.9).

h_+ and $h_\times$ respectively, can be written as

$$\begin{cases} \delta x = -\frac{h_+}{2}x_0 \sin(\omega t) \\ \delta y = \frac{h_+}{2}y_0 \sin(\omega t) \end{cases} \quad \begin{cases} \delta x = \frac{h_\times}{2}x_0 \sin(\omega t) \\ \delta y = \frac{h_\times}{2}y_0 \sin(\omega t) \end{cases}, \quad (1.9)$$

where x_0 and y_0 are the test masses position at rest and ω is the angular frequency of the passing GW. The deformation on a ring of test masses is depicted in Fig. 1.2 for each cross and plus polarizations and for $\sin(\omega t) = -1$ (in blue) and for $\sin(\omega t) = 1$ (in red).

Fig. 1.2 makes evident the naming plus and cross of each polarization. Additionally, it appears both plus and cross deformations are the same modulo a 45° rotation and that both deformations are invariant under a 180° rotation.

The great difficulty in observing GWs arises due to the very small value of the gravitational constant and due to Einstein's weak equivalence principle, which states that inertial and gravitational mass are the same for all bodies.

Like in the case of electromagnetism, GW radiation arises from spatial changes to the gravitational field, or mass distribution. Due to the existence of positive and negative charges, dipole radiation is the typical example of electromagnetic radiation. However, there are no negative masses and the equivalence principle ensures that in the case of GWs there is only quadrupole radiation or higher. Since total mass is conserved there is no monopole radia-

tion, and since all bodies fall the same way in a gravitational field, the center of mass moves inertially and there is no dipole radiation².

In the TT gauge, the strain perturbation from passing GWs is given by

$$h_{\mu\nu} = \frac{2}{r} \frac{G}{c^4} (\ddot{Q}_{\mu\nu}), \quad (1.10)$$

where $\ddot{Q}_{\mu\nu}$ is the second time derivative of the mass distribution quadrupole tensor.

A quadrupole moment can be generated by two compact objects revolving around each other. Assume a fixed circular Keplerian orbit, where the orbital angular frequency is given by

$$\Omega = \pi f_{GW} = \sqrt{\frac{Gm}{R^3}}, \quad (1.11)$$

where R is the distance between the two objects and m is the sum of their masses m_1 and m_2 . We treat the two compact objects as point masses in the center-of-mass frame, with reduced mass

$$\mu = \frac{m_1 m_2}{m_1 + m_2}. \quad (1.12)$$

We also introduce the chirp mass³

$$M_c = \mu^{3/5} m^{2/5} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}. \quad (1.13)$$

With these, we can write Eq. 1.10 for a compact binary system as [43]

$$h = \frac{4}{rc^4} (GM_c)^{5/3} (\pi f_{GW})^{2/3}. \quad (1.14)$$

Plugging in the values in Eq. 1.14 for GW150914, for $f_{gw} = 100$ Hz and $r = 410$ Mpc away, a strain amplitude in the order of 10^{-21} is obtained.

²Savvy readers might point out that these conservation laws do not hold in the full theory of GR, where it is the stress-energy tensor which is conserved. However, it can still be shown that monopole and dipole GW radiation cannot exist through the use of quantum field theory. A theorem in Ref. [42] states that for a massless particle of mass S , in a multipolar expansion the lowest allowed multipole is $l=S$. So, for a graviton of spin 2, the monopole ($l=0$) and the dipole ($l=1$) vanish. Nonetheless, these assumptions hold in the linearized theory of GR and provide good physical intuition for the generation of GWs.

³The chirp mass M_c relates, to first order, the change in orbital evolution of a compact binary system as energy is lost to GW emission. It is so called because it also determines the frequency evolution of GW emission, which is similar to a chirp as it nears the merger.

1.2.1 Gravitational-wave sources and science cases

The LIGO-Virgo white paper on GW data analysis divides the sources of gravitational waves in four categories [44]: compact binary coalescence, burst, continuous waves and stochastic background.

Compact Binary Coalescence

There is a very large amount of binary systems in the Universe. However, only a fraction of these can be considered compact objects. In order to turn into a compact object, stars need to be massive enough during their main sequence period. The first binary system with a pulsar was in fact found by Joseph H. Taylor and Alan R. Hulse, who made the first indirect observation of GWs as mentioned in Section 1.1. Typically, the considered binary systems are made up of two black holes (BBH), two neutron stars (BNS) or one neutrons star and one black hole (NSBH).

The coalescence of compact objects can be understood in three stages (see also Fig. 1.1):

- **inspiral** - as the two compact objects slowly orbit around each other, GWs are slowly emitted. As the objects lose energy through GW emission, the orbit decays accordingly, the gravitational pull increases and the orbital frequency increases, thus increasing the frequency of the emitted GWs. Eventually, the orbital frequency becomes significantly larger than the objects rotational frequency.
- **merger** - as the orbital radius keeps reducing, eventually the last stable circular orbit is reached, and the objects acquire an inward radial motion until finally merging. This is the moment where the "loudest" GWs are emitted.
- **ringdown** - The remnants of the collision form a rotating Kerr black hole, during which GW emission is damped until stopping altogether.

With the end of the first part of the observation run 4 (O4a), a total of 218 detections were published in the Gravitational-Wave Transient Catalog (GWTC-4.0). This catalog is made of a collection of papers describing the methods for identifying and characterizing gravitational-wave transients, the detections themselves since the beginning of O1 until O4a, and the science made with these detection such as population studies and precision tests of GR [7]. Details on the catalog can be found in the introductory paper, which

references all the companion papers for more details on detections and science cases [5]. Table 1.1, reproduced from [45], shows all the BBH, NSBH and BNS detections per observation run and published catalogue.

Table 1.1: Transient GW detections, reproduced from [45].

Observing run	Catalogue	BBH	NSBH	BNS
O1	GWTC-1.0	3	0	0
O2	GWTC-1.0	7	0	1
O3a	GWTC-2.1	42	2	1
O3b	GWTC-3.0	31	4	0
O4a	GWTC-4.0	126	2	0

Historically, it was thought neutron stars would make the bulk of ground-based GW detection (see for example Ref. [41]). This is because at the time there were still many questions regarding black holes, and BNS were well known to exist. For this reason, the sensitivity of GW detectors is typically benchmarked in terms of BNS sensitivity.

BNS are particularly interesting events as they might provide insights into an otherwise inaccessible state of matter. Neutron stars are the densest observable objects in the universe, believed to consist of a superfluid and superconducting core. For this reason, the equation of state of neutron stars is an active area of research. Numerical results indicate that the stars composition should leave an imprint on the GW signal from their coalescence, particularly in the last stages of the inspiral and post merger remnants between 0.5 and 4 kHz [46].

Burst

Burst events are gravitational-wave transients, which also include the above mentioned compact binary coalescences, but are not restricted to them. Burst searches assume minimal information on the source and preform model independent searches that, for example, do not require a known waveform as is the case for BBH. Thus, burst events include core-collapse supernovae, long gamma-ray bursts and any other unknown or unpredicted sources of GW transients.

Continuous Waves

Very long-lasting and approximately monochromatic gravitational wave emission can be considered continuous. The most significant candidate for this kind of continuous wave signal are rapidly rotating neutron stars. Recall that GWs are produced by quadrupolar variations in the mass distribution, which would not be generated by a perfectly symmetric rotating star. However, non-axisymmetric deformation of the spinning neutron star should result in GW emission.

Continuous waves may provide limits on strain amplitudes from known pulsars such as Vela and Crab whose positions, rotation-frequency and spin-down are already known electro-magnetic measurements. Results from O3 constraint the equatorial ellipticity ϵ of millisecond pulsars to lower than 10^{-8} [47]. Taking into account the approximation $\epsilon \approx \Delta R/R$ [48], this means asymmetric "hills" in neutron stars should be below 0.1 mm. This level of ellipticity should lead to GW strain amplitudes around 10^{-25} .

These signals can also be used to provide information on known but uncharacterized astrophysical sources like galactic supernovae remnants or for broad-sky searches to look for undiscovered neutron stars [44]. Additionally, theorized axion clouds around black holes may also produce continuous waves [49].

Stochastic gravitational-wave background

A stochastic GW background can be formed from the superposition of all the uncorrelated events (transient or continuous) which are too weak to be resolved by the detectors individually. This is what is called an astrophysical background.

Alternatively, and perhaps more interestingly, a stochastic GW background might also be generated from cosmological sources or phenomena in the very early universe. These could be remnants from the Big Bang, inflationary or from primordial black holes or cosmic strings, which are hypothesized topological defects related to phase transitions in the early Universe. The energy scales potentially probed by measuring such early events are otherwise and currently inaccessible by any other means or experiments [50].

The nature of a potential stochastic GW background of cosmological origin will depend on the kind of sources it arises from. This could be an irreducible background from different inflation models, short duration phenomena from phase transitions in the first few instants of the universe or a persistent back-

ground from primordial sources still in existence today such as cosmic strings or primordial black hole binaries [51].

Multimessenger astronomy

While GWs signify a new channel to probe the universe, its combination with electromagnetic counter-parts offers a promising track for the future of astronomy and astrophysics.

The BNS event GW170817 hailed the beginning of what is called multimessenger astronomy. The BNS coalescence was followed by a short gamma-ray burst (GRB170817A), detected by the Fermi-GBM. The localization information provided by the LIGO-Virgo detection and from the Fermi-GBM triggered both ground and space-based telescopes to search for optical counterparts. About 10 hours in, the One-Meter, Two-Hemisphere team, in Chile, was the first one to discover the optical transient (SSS17a/AT 2017gfo) originating from the NGC4993 galaxy.

Combining the electromagnetic detection, which identified the host galaxy, with the GW detection, it is possible to use the GW170817 as a standard siren, analogous to an astronomical standard candle⁴ to give a completely independent measure of the Hubble-Lemaître constant [52]. Under the assumption of GR, the luminosity distance to the merger can be computed, and together with the recession velocity from the optical counter part, a Hubble constant value of $H_0 = 68_{-8}^{+12} \text{ kms}^{-1} \text{ Mpc}^{-1}$ was estimated. The considerable error bars stem from the low statistics of a single measurement. As more multi-messenger detections of BNS start to become available, the constraints on H_0 will start to improve.

The current state-of-the-art measurements of the Hubble constant come from Cosmic Microwave Background (CMB) measurements and direct measurements using Cepheid and Type 1A supernovae. The CMB measurements result in $H_0 = 67.4 \pm 0.5 \text{ kms}^{-1} \text{ Mpc}^{-1}$ [53] and $H_0 = 68.63 \pm 0.8 \text{ kms}^{-1} \text{ Mpc}^{-1}$ [54]. The direct measurements using the Hubble and James Webb telescope, result in $H_0 = 72.05 \pm 1.86(\text{stat}) \pm 3.10(\text{syst.}) \text{ kms}^{-1} \text{ Mpc}^{-1}$ [55]. This difference between CMB and direct measurements is known as the Hubble tension. An independent and direct measurement like the one possible with GWs could help solve the Hubble tension in the future.

⁴In astronomy, a standard candle is a source that has a known luminosity, such as Cepheid variables or Type 1A supernovae.

In addition to GW170817 and GRB170817A, a search for high-energy neutrinos was also performed by the ANTARES, Pierre Auger and ICECUBE observatories. No directionally coincident neutrinos were detected around a ± 500 ms window around the merger time neither within the first 14 days after the merger [56]. Future upgrades of ICECUBE and the addition of the KM3NET detectors will increase the likelihood of a future joint multimessenger detection including neutrinos [57].

Burst searches can also gain a lot from multimessenger searches due to the nature of the sources which usually come accompanied by electromagnetic counter-parts as is the case of supernovae.

1.3 How to build a gravitational-wave detector

As mentioned above, GW emission is quadrupolar in nature, and a detector for GW radiation needs to be sensitive to such deformations as shown in Fig. 1.2. In other words, at least two test masses are required in order to measure relative stretching and squeezing motion. In his 1965 article, Braginsky develops this rationale behind a GW detector [41]. He goes on to point the difficulty in measuring such small distances, even suggesting interferometry as a possible way of doing so. Regardless, the remainder of the article focuses on the physics of using extended bodies as detectors, such as Weber's resonant bars. However, as we saw in Section 1.1, interferometry would be the technology to finally detect GWs.

GW interferometers use Michelson interferometers for detection. Fig. 1.3 shows that the geometry of the Michelson interferometer is prime for GW detection. If the interferometer is aligned with the transverse profile of a GW, then one arm will compress and the other will extend as the wave passes. The GW strain amplitude h will impact the interferometer as

$$h(t) = \frac{\Delta L(t)}{L}, \quad (1.15)$$

where ΔL is the overall differential length between the two Michelson arms caused by the passing of a GW and L is the nominal length of the Michelson arms.

Despite its simplicity, from Eq. 1.15 we can already gauge that having larger arms will increase detector sensitivity, by increasing the experienced ΔL for the same strain amplitude h . On the other hand, the maximum signal

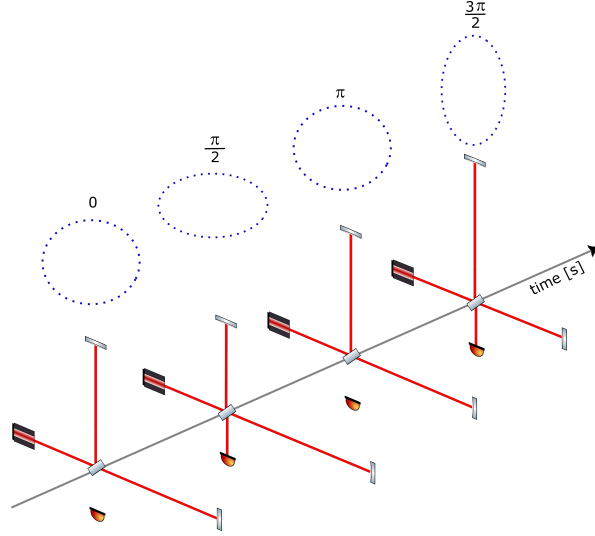


Figure 1.3: Schematic of the impact of a passing GW (plus polarization of Eq. 1.9) on a Michelson interferometer.

is reached when photons stay in the interferometer arms for half a period of the GW. This is because the maximum ΔL is reached at half the GW period as shown in Fig. 1.3, and if the photons stay less or longer in the interferometer arms, the signals is integrated away. This means that, optimally, the optical path length $2L$ should be half of the GW wavelength. For $f_{GW} = 100$ Hz, then the optimal arm length L should be

$$L = \frac{1}{4} \frac{c}{f_{GW}} = \frac{\lambda_{GW}}{4} \approx 750 \text{ km}, \quad (1.16)$$

where we recall that GWs travel at the speed of light and $\lambda_{GW} f_{GW} = c$.

Naturally, a Michelson interferometer of these dimensions is logistically unfeasible. The footprint of current ground-based GW interferometers is of 3 km in the case of Virgo, and 4 km in the case of the LIGO detectors. The effective optical length is increased by the use of optical resonators, or cavities, in the arms of the interferometer. These resonators simply consist of two partially reflective mirrors facing each other and separated by a distance L .

In this section, I will detail what goes into building a realistic GW detector, with a special focus on the optical and interferometric components. But first, I will introduce some key concepts related to laser beams and resonators.

1.3.1 Laser beams, higher-order modes and resonators

The same optical resonators as employed in current GW detectors were invented in 1899 and are most well known as Fabry-Perot (FP) interferometers [58]. They were initially conceived as spectroscopy devices but since 1960, when combined with a gain medium, also became the basis of laser devices.

If $\Phi(\nu)$ represents the single pass phase shift between the two mirrors for incident light (collimated) of frequency ν , we can write that [59]

$$2\Phi(\nu) = 2\pi\nu t_{RT} = 2\pi\nu \frac{2nL}{c}, \quad (1.17)$$

where t_{RT} is the cavity round-trip time, n the index of refraction of the cavity medium and L the cavity length.

Resonances inside the cavity occur at frequencies at which the forward and backward propagating beams exhibit constructive interference. This is the case when the round-trip phase shift $2\Phi(\nu)$ is equal to modulus 2π . Taking into account Eq. 1.17, this means that the resonance condition is at frequencies such that

$$\nu = q \frac{c}{2nL}, \quad (1.18)$$

where q is an integer number. As consequence, this also means that adjacent resonances are separated by 2π in phase. The corresponding spacing between adjacent resonances in frequency or wavelength, is called the Free Spectral Range (FSR) and can be derived, assuming $n = 1$ henceforth, as

$$FSR_\nu = \nu_{q+1} - \nu_q = \frac{c}{2nL}. \quad (1.19)$$

Thus, we can talk about the longitudinal modes of a FP resonator with mode index q , frequency ν_q and wavenumber k_q as

$$\nu_q = qFSR_\nu \Rightarrow k_q = \frac{2\pi qFSR_\nu}{c}. \quad (1.20)$$

These modes are shown in Fig. 1.4. The extent to which light builds up inside the resonator as a function of frequency, or phase, is given by the Airy distribution, which relates the circulating intensity to the input intensity as a function of the accumulated phase [59]

$$A_{circ} = \frac{I_{circ}}{I_{in}} = \frac{|E_{circ}|^2}{|E_{inp}|^2} = \frac{1}{|1 - r_1 r_2 e^{-i2\Phi}|^2}, \quad (1.21)$$

where r_1 and r_2 are the amplitude reflection coefficients of the cavity mirrors.

An important quantity of a resonator is the so called finesse $\mathcal{F}$, which relates the full-width half maximum (FWHM) of the cavity modes with respect to their frequency separation, such that $\mathcal{F} = FSR\nu/FWHM$. It can be seen that the finesse of a cavity as a function of mirror reflectivity can be written as

$$\mathcal{F} = \frac{\pi}{2} \left[\arcsin \left(\frac{1 - \sqrt{R_1 R_2}}{2 \sqrt[4]{R_1 R_2}} \right) \right]^{-1}, \quad (1.22)$$

where $R_i = r_i^2$ are the intensity reflectivities of the mirrors. This notion of finesse was derived initially from the Rayleigh criterion of spectral resolution, from using the FP resonator for spectroscopy. Despite the ubiquity of Eq. 1.22, it is important to note it breaks down at mirror reflectivities of 17.2% (assuming equal mirror reflectivities) [59]. Nonetheless, all of the cavities considered in this work are of considerable higher reflectivities. In the limit of high-reflectivity, using the small angle approximation we arrive at the usual simplified equation for high-finesse with the form

$$\mathcal{F} = \pi \frac{\sqrt[4]{R_1 R_2}}{1 - \sqrt{R_1 R_2}}. \quad (1.23)$$

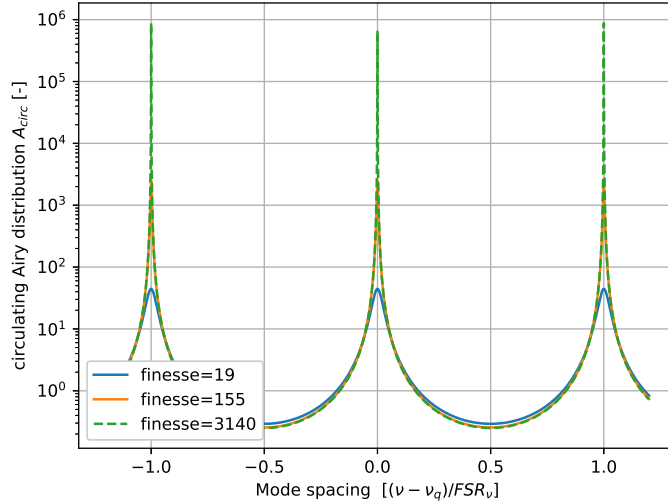


Figure 1.4: Plot of the cavity circulating Airy distribution as a function of the cavity round-trip phase (Eq. 1.21). The x-axis is normalized with respect to the FSR_ν .

Depending on the reflectivity of the mirrors and round-trip losses, cavities can be categorized as overcoupled, undercoupled or critically coupled. In critically coupled cavities, transmission of the mirrors are equal to the round-trip losses. For a cavity with no losses, this is achieved with equal reflectivity mirrors. In this case, light is injected into the cavity with maximum efficiency and all the resonant light leaks in transmission. In overcoupled cavities, the transmission of the mirrors is larger than the round-trip losses. In this case a larger fraction of the resonant light leaks from the side with the less reflective mirror. As we will see in Section 1.3.2, this is the case for the Virgo arm cavities which have an almost perfectly reflective end mirror. In turn, the input mirrors of the Virgo arm cavities have a transmission of about 1.4% and so, almost all of the light is reflected back to the beam splitter. In the undercoupled case, round-trip losses are larger than the mirror transmissions. Thus, there is no significant light build-up, and hence no significant resonant light reflection or transmission.

If at least one of the cavity mirrors is spherical, the resonator sustains different spatial or transverse modes. These modes, called the resonator eigenmodes, can be described in a polynomial orthonormal basis. The most typically used basis uses cartesian coordinates and is called the Hermite-Gaussian (HG) basis [11, 60], with $k = 2\pi/\lambda$ it is written as

$$HG_{mn}(x, y, z, q) = \sqrt{\frac{2}{\pi}} \sqrt{\frac{1}{2^{n+m} n! m!}} \frac{1}{w(z)} H_n \left(\frac{\sqrt{2}x}{w(z)} \right) H_m \left(\frac{\sqrt{2}y}{w(z)} \right) \exp \left[-i \frac{k(x^2 + y^2)}{2q(z)} + (1 + m + n)\varphi(z) \right], \quad (1.24)$$

where m, n are the mode indices, H_i the i th order Hermite polynomial and with the so called q beam parameter

$$q = z + i \frac{\pi w_0^2}{\lambda}, \quad \text{and} \quad \frac{1}{q} = \frac{1}{R(z)} - i \frac{\lambda}{\pi w(z)}, \quad (1.25)$$

where λ is the wavelength of light, w_0 is the waist of the beam and the z coordinate is defined such that it is 0 at the waist position. The phase $\varphi(z)$ is the so-called Gouy phase which, for the fundamental or zero-order mode, changes according to $\varphi(z) = \arctan(\text{Re}(q)/\text{Im}(q))$. Figure 1.5 (a) shows how the zeroth order and the first higher-order modes (HOMs) look like in this basis.

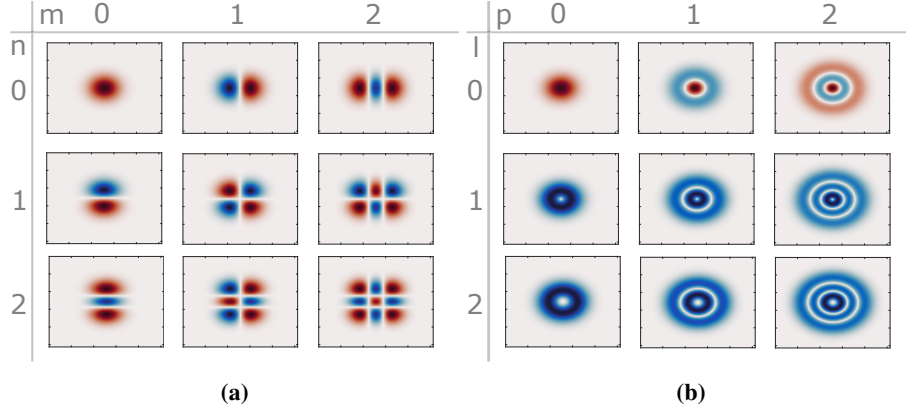


Figure 1.5: (a) First higher-order modes in Hermite-Gaussian basis. (b) First higher-order modes in Laguerre-Gauss basis. The images are shown with a signed amplitude in order to encode the phase information. The positive amplitudes are displayed in warm tones and represent the parts of the beam in phase with a zeroth order mode reference, and the cold tones represent the parts of the beam out of phase.

Another basis commonly used to describe the eigenmodes of a cavity is the Laguerre-Gauss (LG) basis, which uses cylindrical coordinates instead and reads as

$$LG_{pl}(r, \theta, z) = \sqrt{\frac{2}{\pi}} \sqrt{\left(\frac{p!}{(|l|+p)!}\right)} \frac{1}{w(z)} \exp \left[i(1+2p+|l|)\varphi(z) \right] \\ L_p^{|l|} \left(\frac{\sqrt{2}r}{w^2(z)} \right)^{|l|} \exp \left[-i \frac{kr^2}{2q(z)} + |l|\theta \right], \quad (1.26)$$

where $L_p^{|l|}$ is the Laguerre-Gauss polynomial with radial index p , which sets the number of radial fringes, and azimuthal index l that determines the width of the dark spot at the middle (see Fig. 1.5) as well as the spiral phase related to the propagation of a helical wavefront. The mode order is for LG modes is equal to $2p + |l|$.

Conveniently, lasers employ cavities and hence, the laser beam is well described by the same polynomial basis. Moreover, since the HG basis is a complete orthonormal basis, any arbitrary optical beam (under the paraxial assumption), $U(x, y, z)$ can be decomposed in a superposition of HG modes, for a given q , as

$$U(x, y, z) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} k_{mn} HG_{mn}(x, y, z, q), \quad (1.27)$$

where k_{mn} are the complex mode coefficients.

Typically, lasers emit light mostly in the 0th order or fundamental mode (which is the same for both HG and LG basis as seen in Fig. 1.5). Given that the spatial amplitude or intensity profile follows that of a Gaussian distribution, this fundamental mode is called the Gaussian mode. In this context, the waist w_0 in Eq. 1.25 refers to the radius of the beam at its focal point and $w(z)$ and $R(z)$ refer to the radius of the beam and wavefront radius of curvature at a certain point in the propagation axis z .

We say that a certain q beam parameter, or in other words a certain w_0 value fixes the basis of the beam or cavity. In fact, the w_0 of a cavity eigenmode is fixed by the cavity geometry, i.e. by the cavity length and radius of curvature (RoC) of the mirrors as

$$w_0 = \sqrt{\frac{\lambda L}{\pi}} \sqrt{\frac{g_1 g_2 (1 - g)}{(g_1 + g_2 - 2g)^2}}, \quad (1.28)$$

where the cavity g-factor is $g = g_1 g_2$, with

$$g_i = 1 - \frac{L}{R_i}, \quad (1.29)$$

for mirrors 1 and 2 respectively with R_1 and R_2 values of RoC, respectively.

For HOMs, the Gouy phase increases with mode order as seen in Eqs. 1.24 and 1.26. Thus, a general resonance condition valid for any HOM in the HG basis can be written as

$$\nu_q^{m+n} = q \frac{c}{2L} \frac{m+n+1}{\pi} \arccos \left[\sqrt{(1 - L/RoC_1)(1 - L/RoC_2)} \right]. \quad (1.30)$$

The resonance condition now depends not only on the length of the cavity, but also the mirrors RoC. The Gouy phase adds an extra π phase shift to the propagating beam as it goes through its focal point.⁵ The position and size of the focal point, or waist, inside the cavity depends on the cavity length and RoC of the mirror, and due to the Gouy phase, so does the resonance condition of the HOMs. Same order mode resonances with q and $q+1$ are nonetheless separated by one FSR ($c/2L$). We will see in later Sections and

⁵The Gouy phase was first observed by Louis Gouy in 1890 [61], its origin has intrigued physicists since. In recent developments, the Gouy phase has come to be understood as an example of the more general Berry phase, or geometrical phase, which is a characteristic of systems with a cyclic adiabatic evolution in their parameter space [62]. Other authors have shown that the transverse spatial confinement and the uncertainty principle are the physical origin for the Gouy phase shift [63].

Chapters that the resonance conditions for HOMs has a dramatic impact on GW interferometers.

When coupling a laser beam into a spherical optical resonator, the result of the wave interference can be understood as the incident wave being expanded into a series of eigenmodes of the resonator. Thus, for an input beam defined by mode $HG_{m'n'}$, the expansion is written as [11]

$$U(x, y, z) = HG_{m'n'}(x, y, z, q') = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} k_{m'n'mn} HG_{mn}(x, y, z, q), \quad (1.31)$$

where the ' superscript indicates the basis of the input beam and $k_{m'n'mn}$ the complex coupling coefficients between the $m'n'$ mode of the input beam to the mn mode of the cavity. For simplicity, here we assumed the same cartesian coordinates (x, y, z) for the input beam and cavity.

We can say that Eq. 1.31 describes the spatial basis change at the interface of the optical cavity. In general, if the input beam is described by multiple HOMs and a maximum number of HOMs N is fixed, then the basis change will be described by an $N \times N$ matrix. We will also see in Sections 2.2 and 2.4 that the low-order $k_{m'n'mn}$ coefficients can be related to transverse, angular and spatial mismatches of the input beam with respect to the cavity. We will see that these couplings can be exploited to actively control these degrees of freedom.

1.3.2 Dual-recycled Michelson interferometers

In this Section I address the general aspects of GW interferometers. However, when specificity should be required, I will fall back on the example of the Virgo detector, to which this work is related.

In a Michelson interferometer (refer back to Fig.1.3) light entering the two arms excites a stationary mode in the two arms. After the light recombines back at the beam splitter, the power at the output port is given by

$$P_{DP} = \frac{1}{4} P_i (R_1 + R_2) (1 + C \cos(\Delta\varphi)), \quad (1.32)$$

where P_i is the input power, R_1 and R_2 are the intensity reflectivities associated with mirrors 1 and 2 and $\Delta\varphi$ is the phase difference between the two waves arriving from each arm. The contrast C is given by

$$C = \frac{2\sqrt{R_1 R_2}}{R_1 + R_2}. \quad (1.33)$$

From Eq. 1.33 we can see the contrast is 1 if R_1 and R_2 are exactly equal. In this case, there is a certain value of $\Delta\varphi$ which will cause, through destructive interference, P_{DP} to be 0⁶. For this reason, this port, where detection occurs, is typically called the dark port (DP). In this case, by conservation of energy, all the recombined light exits back towards the input laser, hence why this is called the bright port.

The phase difference between the two waves at the beam splitter is

$$\Delta\varphi = 2k\Delta L, \quad (1.34)$$

where ΔL is the differential path length between the two arms and $k = 2\pi/\lambda$ the wave vector. Typically, the wavelength of light in GW detectors is 1064 nm. The differential path length ΔL could be induced, for example, by a passing GW according to Eq. 1.15. For this reason, the dark port is also called asymmetric port, since it responds to differences in phase. This is true not only for differences in phase due to the optical path length, but also differences in phase due to different spatial modes in each arm. In turn, the bright port is also called the symmetric port.

Consider that ΔL can be decomposed into a static path length between the two arms, i.e. the interferometer working point, and a smaller differential length from the GW strain

$$\Delta\varphi = \varphi_0 + \varphi_{GW}, \quad (1.35)$$

where $\varphi_0 = 2k\Delta L_0$ and $\varphi_{GW} = 2k\Delta L_{GW}$.

In this case, and assuming R_1 and R_2 are exactly 1, the power at the dark port becomes

$$P = \frac{1}{2}P_i(1 + \cos(\varphi_0) - \sin(\varphi_0)\varphi_{GW}), \quad (1.36)$$

where the trigonometric identity $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$ was used as well as the small angle approximation for φ_{GW} and the fact that $\varphi_0 \gg \varphi_{GW}$. So, with $\Delta L_{GW} = hL$, the power change from a passing GW

⁶Note that Eq. 1.32 implicitly assumes that the beam splitter splits the incoming light exactly in half and is lossless, a requirement for achieving a completely dark fringe. Often a compensator, made of the same material as the substrate of the beam splitter is also used, to compensate for the difference in path length between the two Michelson arms. This is not the case in GW detectors, as the interferometer arm length can be adjusted precisely to achieve the required path length. In fact, for control reasons as discussed in the Section 2.2, a macroscopic difference between the two Michelson arms called the Schnupp asymmetry is used.

can be written as

$$\Delta P_{\text{DP}} = \frac{1}{2} P_i \sin \varphi_0 (2khL). \quad (1.37)$$

This means that the most sensitive condition of the interferometer is at the mid-fringe, when $\varphi_0 = \pi/2$. However, the existence of noise and laser power fluctuations makes this not enough to allow the measurement of GWs. A description of how GWs are actually detected is given in Section 2.1.1.

We already saw before from Eq. 1.15 that a larger L leads to a larger interferometric signal. I also mentioned that this optical path length can be significantly increased by introducing optical resonators in the arms of the interferometer, as shown in Fig. 1.6. But the arm length only impacts the amount of $\Delta\varphi$ for a given strain and other parameters are crucial in order to build sensitive GW interferometers, such as having equal reflection from both arm cavities (or equal finesse) as well as injected power.

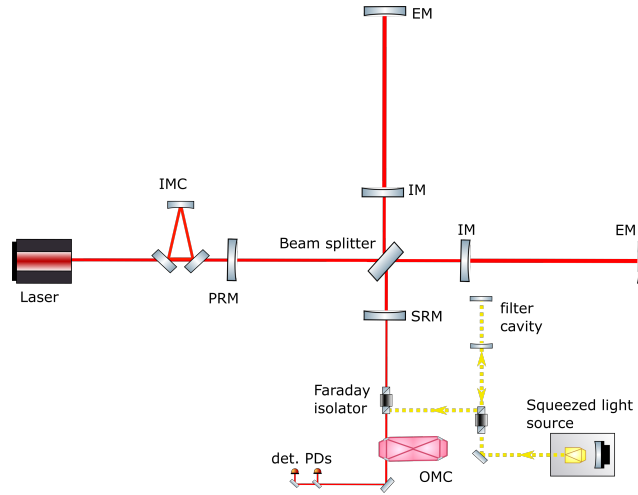


Figure 1.6: Optical schematic of a dual-recycled Michelson interferometer. The interferometer is enhanced by Fabry-Perot cavities composed of input (IM) and end mirrors (EM). A power recycling mirror (PRM) is placed on the bright port and a signal recycling mirror (SRM) is placed on the dark port. Input (IMC) and output mode cleaners (OMC) clean the mode of the input and output beams. Detection photodiodes (PDs) are placed after the OMC. The squeezed light source and filter cavity are used to surpass the standard quantum limit as discussed in Section 1.4.

The amount of injected power is limited by current laser technology. However, as mentioned above, most of the power in the interferometer (the common

laser mode) is reflected back to the laser. This power, can be re-used, or recycled, by introducing a partially reflective mirror between the beam splitter and the laser (see Fig. 1.6). This mirror is called the power recycling mirror (PRM). Between the PRM and the input mirrors of the Fabry-Perot cavities in each arm, a new cavity is formed called the power recycling cavity (PRC). The PRC has an optical gain of

$$G = \left(\frac{T_{\text{PR}}}{1 - R_{\text{PR}}R_{\text{FP}}(\nu)} \right), \quad (1.38)$$

where T_{PR} and R_{PR} are the intensity transmission and reflectivity of the PRM respectively. The reflectivity of the Fabry-Perot cavity, $R_{\text{FP}}(\nu)$ is shown to depend on the frequency of the light, ν (as seen in the previous Section, Eq. 1.21) to stress that the optical gain in the PRC depends on the resonance condition of the Fabry-Perot cavity. Naturally, the PRC has its own resonance condition, which is coupled to that of the arm cavity, as they form a coupled cavity.

Until now, the discussion in this section assumed implicitly the GW as a step function with a constant h . In practice we know that, as discussed in Section 1.2.1, GWs evolve in frequency spanning from a few Hz up to a few kHz. By introducing resonators in the interferometer arms we need to consider the frequency response of these resonators. The key figure of merit for this is the cavity pole frequency

$$f_{\text{pole}} = \frac{1}{4\pi\tau_s}, \quad (1.39)$$

where τ_s is the storage time of the cavity. It is commonly known as the ring-down time, as it is the exponential decay constant when input power is turned off. It can be written as [64]

$$\tau_s = \frac{L}{c} \frac{\sqrt{R_1 R_2}}{1 - R_1 R_2}. \quad (1.40)$$

The cavity pole frequency is the frequency at which the cavity response to length variations falls to -3 dB. Higher finesse cavities will have larger storage times and hence smaller pole frequencies. This connects to the intuition we built in previous Sections when noticing that larger interferometer arms were needed to approximate the GW half-period. In the other extreme, if the arms become too long, or the cavity finesse too high, the photons stay in the arms much longer than an half-period of the GW, and the signal is integrated away.

For example in the case of Virgo, with a length of 3 km the cavity pole is around 55 Hz.

This cavity pole affects the sensitivity of the detector for GWs above 55 Hz. To address this, the last round of upgrades to current GW detectors, introduced a partially reflective mirror between the beam splitter and the detection photodiodes (Fig. 1.6). This mirror is called the signal recycling mirror (SRM) as it increases the time some photons spend inside the interferometer and can be used to adjust the bandwidth response of the interferometer. The SRM and the input mirrors of the arm cavities make another cavity, the signal recycling cavity (SRC). The SRC is also coupled to the PRC and arm cavities, further complicating the interferometer configuration. With sufficient circulating power, the radiation-pressure force between the SRM and the Fabry-Perot arm cavities cause the optical fields to behave as a massive spring. This causes the small arm mirror motion to behave as free test-mass connected to the spring of the optical fields. This effect is called the optical spring and has very interesting impact on the quantum noise limit of the detector, but it can also cause trouble for control systems (see for example Chapter 3 of Ref. [65]).

Thus, GW interferometers are dual-recycled (i.e. power and signal recycled), Fabry-Perot enhanced, Michelson interferometers, often called DRMI for short. In addition, a few other cavities are shown in Fig. 1.6, namely the input mode cleaner (IMC) and the output mode cleaner (OMC). These cavities make use of the properties we discussed in Section 1.3.1 to filter the HOM content from the transmitted beam. The IMC is there to ensure a pure Gaussian beam is injected into the detector, and the OMC is there to filter the asymmetric HOMs from the signal, and leave only the asymmetric GW mode which matches the mode of the arm cavities.

The discussion on previous sections regarding the impact of a passing GW on a ring of test masses or on an interferometer, relies on the assumption of free-falling test masses. The main optics of the GW interferometer as shown in Fig. 1.6 are at the bottom of complex suspensions systems in order to approximate a free-falling test mass in the degree of freedom of interest. Additionally, current detectors can measure strain amplitudes in the order of $10^{-22} \text{ } 1/\sqrt{\text{Hz}}$, which taking into account the Fabry-Perot enhanced arm cavities, results in displacements in the order of $10^{-19} \text{ m } / \sqrt{\text{Hz}}$. In order to make such sensitive measurements, decoupling from the Earth's motion is also required. As discussed in the next section, these suspension systems are crucial to provide such decoupling.

1.3.3 Limiting noises

In order to be able to detect such small displacements, all possible sources of noise need to be well understood and mitigated, when possible. Figure 1.7 shows a projected noise budget for the first phase of Advanced Virgo+, the iteration of the Virgo detector that ran the fourth observation run. The figure shows the fundamental noises which intrinsically limit the sensitivity of the detector. In addition, we can also talk about technical noises which arise from the technology used, such as electronic noises, and from the infrastructure itself. Scattered light due to surface imperfections of the optics also enters in this category. I will not discuss technical noises here but a detailed overview can be found in Chapter 2.4.3 of Ref. [66] Below, I provide a brief summary of the fundamental noise sources that impact the detector sensitivity.

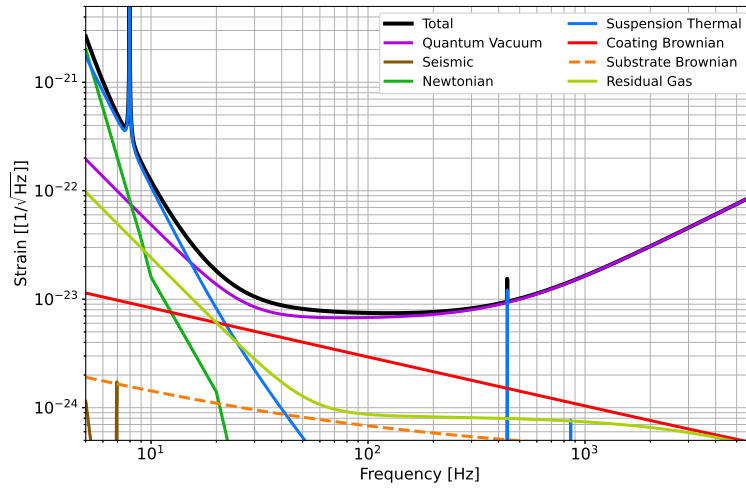


Figure 1.7: Advanced Virgo+ (Phase I) projected noise budget for 25W input and with no quantum noise reduction, produced with the GWINC [67] software in Python.

Seismic noise

Seismic noise refers to the direct coupling of seismic motion to the main optics of the interferometer. This kind of noise can impact the sensitivity of GW detectors in the low frequency end of the bandwidth. It originates from seismic and oceanic activity, wind and day-to-day human activity [68].

As mentioned in the previous section, in order to reach $10^{-23} \text{ 1}/\sqrt{\text{Hz}}$ strain sensitivity, mirror motion should be less than 10^{-19} m . In comparison the seismic ground motion at 10 Hz is in the order of 10^{-10} m [69]. This means that the suspended mirrors need to be decoupled from ground motion by at least 9 orders of magnitude. In Fig. 1.7 the seismic noise that is shown at the bottom left of the plot is already filtered by the suspension systems in place at Virgo.

As mentioned at the end of the previous section, this attenuation is reached by a combination of several pendula. Consider first a simple pendulum with length l and a mass, m , attached to it. The equation of motion of the attached mass is governed by

$$m\ddot{x} = -\frac{mg}{l}(x - x_{\text{sus}}), \quad (1.41)$$

where x is the motion of the attached mass, x_{sus} is the motion of the suspension point (which for an inverted pendulum represents ground motion) and g is Earth's gravity constant. We can write the transfer function between the motion of the mass and the suspension point as

$$\frac{x}{x_{\text{sus}}} = \frac{f_0^2}{f_0^2 - f^2}, \quad (1.42)$$

where f_0 is the resonance frequency of the pendulum

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{g}{l}}. \quad (1.43)$$

In Virgo a combination of an inverted pendulum with a chain of multi-stage pendulums is used. The suspension device, schematized in Fig. 1.8 is called the superattenuator. The inverted pendulum is used to pre-filter the horizontal motion at low frequencies, provide inertial damping and to position the full suspension system. The chain of pendulums is made up of a series of mechanical filters with blade springs in order to reduce the coupling of vertical ground motion into horizontal mirror motion, as this is the degree of freedom relevant for GW detection. The mirror is finally suspended at the last stage, from the so-called marionette which helps steer the mirror. More details on the superattenuator and the mechanical filters used can be found, for example, in Chapter 1.3 and 2.3 of Ref. [70].

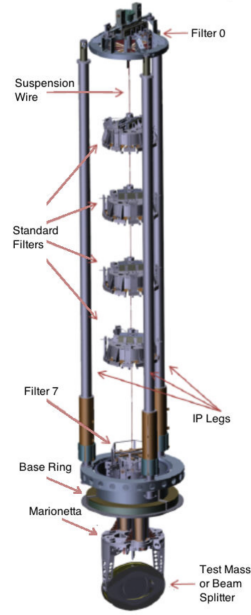


Figure 1.8: Schematic of the Virgo superatenuator from Ref. [70]. The inverted pendulum (IP) made up of three legs attached to a first passive filter. The standard filters are made of the blade springs described in the text. The test mass is suspended from the marionetta which steers and controls the mass.

Thermal noise

Two of the noise curves shown in Fig. 1.7, the suspension thermal noise and the Coating Brownian noise, share the same origin. These are instances of thermal noise which arise from energy exchanges between the thermal bath and the atoms that compose both the suspension wires and the high-reflectivity coatings of the optics. The fluctuation-dissipation theorem [71] relates the energy dissipation to spontaneous fluctuations of a given variable of the system in equilibrium, such as the position of the test masses in a GW detector.

For a given coordinate $x(\omega)$ where $\omega = 2\pi f$ is the angular frequency, the fluctuation-dissipation theorem states that the fluctuation of x is given by

$$x_{th}^2(\omega) = \frac{4k_B T}{\omega^2} \text{Re}\{Z^{-1}(\omega)\}, \quad (1.44)$$

where k_B is Boltzman's constant, T is the temperature and Z^{-1} is the system's mechanical admittance, which relates the speed acquired by the system in response to an applied force.

The wires that suspend the optics can be modeled as a structurally damped pendula with mass m and a composite elastic constant composed of the gravity restorative force k_g which is lossless and therefore real, and the complex elasticity constant k_{el} of the wires as

$$k = k_g + k_{el}(1 + i\varphi_\omega), \quad (1.45)$$

where φ_ω is the loss angle of the suspension wire which is related to the delay of the response of the system to an applied force. The gravity elastic constant is

$$k_g = \frac{Mg}{L}, \quad (1.46)$$

where M is the mirror mass and L the pendulum length. In turn, the wire elasticity constant is proportional to the Young's modulus of the wire material, number of wires and the moment of inertia of the wire cross-section. Assuming that k_g is much larger than k_{el} , Eq. 1.45 can be re-written as

$$k \approx k_g(1 + i\frac{k_{el}}{k_g}\varphi_\omega), \quad (1.47)$$

where the coefficient of the imaginary part of k can now be called the pendulum loss angle $\varphi_{pel} = k_{el}/k_g\varphi_\omega$. From this we can see the pendulum loss angle can be reduced by increasing the mirror mass.

From the typical motion of a structurally damped oscillator (with structural loss k of Eq. 1.46), the mechanical admittance can be computed arriving at a thermal suspension noise power spectral distribution (PSD) such as [72]

$$x_{th}^2(\omega) = \frac{4k_B T k_g \varphi_p}{\omega [(k_g - M\omega^2)^2 + k_g^2 \varphi_p^2]}. \quad (1.48)$$

Besides reducing the temperature, fiber thermal noise can be reduced by increasing mirror mass and the loss angle of fibers. Besides having mirrors weighting around 40 kg, current detectors employ ultra-low loss fused silica fibers ($\varphi_p < 10^{-9}$) to support the mirrors [73].

In the case of the Brownian coating noise, the random vibrations in the atoms of the mirror coatings translate to movements in the mirror surface, thus changing the length of the interferometer arms. There is also Brownian noise associated with the mirror substrate, but the Brownian coating noise is much more significant so I will not discuss the substrate thermal noise here.

Highly reflective coatings as those used in GW detectors are achieved with alternating layers of two materials with different refractive indices (one higher, another lower). Typically, the low refractive index material used is silica, SiO_2 , and the high refractive one is doped tantala, Ta_2O_5 [74]. While the loss angle of SiO_2 is in the order 5×10^{-9} , the loss angle of Ta_2O_5 is significantly higher, around 3×10^{-4} . For this reason, the losses of coatings are typically in the order of 10^{-4} [75].

The displacement power spectral distribution for the coating thermal noise can be written as [74]

$$x_{th}^2(\omega) = \frac{4k_B T}{\pi \omega E_{sub}} \frac{d}{w^2} \left(\frac{E_{sub}}{E_{coat}} \varphi_{\perp} + \frac{E_{coat}}{E_{sub}} \varphi_{\parallel} \right), \quad (1.49)$$

where w is the laser beam radius on the mirror, d is the coating thickness, E_{sub} is the substrate Young's modulus, E_{coat} the coating Young modulus and $\varphi_{\perp}$ and $\varphi_{\parallel}$ the coating loss angles perpendicular and parallel to the coating layers.

The coating thermal noise goes down with $1/w$, because the thermal noise is reduced when averaged over a larger surface area. This is why in GW detectors larger beam areas at the test masses are favored.

Newtonian noise

Changes to the local mass density distribution imply local changes to the gravitational field. The gravitational attraction between the test masses and said mass density distribution results in a kind of noise called Newtonian or Terrestrial gravity noise. These fluctuations of the local gravitational field can be introduced by subsurface seismic wave propagation resulting in ground motion as well as atmosphere fluctuations, temperature and air flux. The suspension systems mentioned above cannot suppress or filter this kind of noise.

According to Fig. 1.7, at low frequencies, Newtonian noise will soon become the limiting noise as thermal noise begins to improve with heavier test masses. For the planned future GW detectors, Newtonian noise will become a more significant problem given the goals of reaching significant strain amplitude down to the Hz level. Given the ground-based propagation of the seismic and infra-sound waves that cause this noise, one way to mitigate it is to go underground and build large enough tunnels so as to have the test masses away from mass distributions.

Currently, Newtonian noise is being monitored at the Virgo site using a collection of seismometers and microphones. The gathered data is being used to design Newtonian noise cancellation schemes [76]. Additionally, cancella-

tion schemes that make use of future detector topologies have also been proposed [77].

Quantum noise

The quantum nature of light, and in particular the uncertainty principle are at the origin of two noise sources, shot noise and radiation pressure noise. Below, I follow a treatment of quantum noise similar to the one in Chapter 1.4 of Ref. [78].

Shot noise arises from the quantum-mechanical fluctuations in the number and arrival time of photons at the output of the interferometer. A coherent state can be written in a Fock state basis. The inner product of such coherent state with a Fock state, gives the photon number probability distribution

$$P(n) = |\langle n|\alpha \rangle|^2 = \frac{\langle n \rangle^n e^{-\langle n \rangle}}{\sqrt{n!}}, \quad (1.50)$$

where $\langle n \rangle = \langle a^\dagger a \rangle = |\alpha|^2$ is the average photon number of the coherent state $|\alpha \rangle$ and $a^\dagger$ and a as the creation and annihilation operators respectively [79].

The probability distribution of Eq. 1.50 is a Poisson distribution. For $n \gg 1$, $P(n)$ can be approximated by a Gaussian distribution with standard deviation $\sqrt{\langle n \rangle}$. The average photon number at the output of the Michelson interferometer is

$$\left\langle \frac{dn}{dt} \right\rangle = \frac{\lambda}{2\pi\hbar c} P_{\text{DP}}, \quad (1.51)$$

where $\hbar$ is the reduced Planck's constant. Over a time interval τ , the standard deviation of the average photon number is

$$\sqrt{\langle n_\tau \rangle} = \sqrt{\left\langle \frac{dn}{dt} \right\rangle \tau} = \sqrt{\frac{\lambda P_{\text{DP}}}{2\pi\hbar c} \tau} = \sqrt{\frac{P_{\text{DP}}}{\omega\hbar}} \tau. \quad (1.52)$$

We can thus write the power fluctuation from shot noise for a time interval τ as

$$\Delta P_{\text{shot}} = \frac{\hbar\omega}{\tau} \sqrt{\langle n_\tau \rangle} = \sqrt{\frac{P_{\text{DP}}\hbar\omega}{\tau}}, \quad (1.53)$$

which, using Eq. 1.36 (minus the GW part), turns to

$$\Delta P_{\text{shot}} = \sqrt{\frac{P_{\text{in}}}{2\tau} \hbar\omega (1 + \cos(\varphi_0))}. \quad (1.54)$$

Using Eq. 1.37 for the power change at the detection port for a passing GW, we can write the SNR as

$$SNR_{\text{shot}} = \frac{\Delta P_{GW}}{\Delta P_{\text{shot}}} = \frac{1}{2} \sqrt{\frac{P_{in}}{\hbar\omega}} 2\tau \frac{\sin(\varphi_0)\varphi_{GW}}{\sqrt{1 + \cos(\varphi_0)}}. \quad (1.55)$$

Considering the mid-fringe condition for $\varphi_0 = \pi/2$, and with $\varphi_{GW} = (2\pi/\lambda)Lh$, the minimum detectable strain amplitude at SNR=1 is

$$h = \frac{\lambda}{2\pi L} \sqrt{\frac{2\hbar\omega}{P_{in}\tau}}. \quad (1.56)$$

The amplitude spectral density (ASD) can be obtained from the Fourier transform of the above equation, which upon further algebraic simplification yields

$$\tilde{h}_{\text{shot}}(f) = \frac{1}{L} \sqrt{\frac{2\hbar c\lambda}{\pi P_{in}}}, \quad (1.57)$$

where I added a $\sqrt{2}$ factor due to considering only the single sided spectra.

Note that shot noise has a flat frequency response and it scales down with increased input power. However, the ΔP_{GW} is not necessarily flat. The Eq. 1.37 I used for ΔP_{GW} does not take into account the frequency response of the Fabry-Perot arm cavities. If we take this into account, then the shot noise spectral density starts increasing at higher-frequencies as the cavity storage time integrates out some of the GW signal, decreasing ΔP_{GW} . This is why the right side slope of the quantum noise curve in Fig. 1.7 goes up with frequency.

Radiation pressure noise results from the back action of photons on the test masses. The quantum fluctuations of the photon arrival times at the mirrors cause fluctuations of the radiation pressure, leading to undesired displacement of the mirrors. The force exerted by the photons upon reflection on the interferometer arms, assuming optically lossless mirrors, is given by

$$F_{\text{rad}} = \frac{P_{in}}{c}, \quad (1.58)$$

where each arm mirror experiences $P_{in}/2$ due to the presence of the beam splitter. As we saw in regards to shot noise, the variation in power due to quantum fluctuations is

$$\Delta P_{in} = \frac{\hbar\omega}{\tau} \sqrt{\langle n_\tau \rangle} = \frac{\hbar\omega}{\tau} \sqrt{\frac{P_{in}\tau}{\omega\hbar}} = \sqrt{\frac{\hbar\omega P_{in}}{\tau}}. \quad (1.59)$$

Thus, the corresponding fluctuations in radiation pressure force are

$$\Delta F_{rad} = \frac{\Delta P_{in}}{c} = \sqrt{\frac{\hbar P_{in} 2\pi}{c\lambda}}. \quad (1.60)$$

Consider that the mirror behaves as an harmonic oscillator far from its resonance as it is suspended from the single pendulum in the last suspension stage. Then, the ASD of its displacement will be

$$\Delta L_{rad}(f) = \frac{\Delta F_{rad}(f)}{M(2\pi f)^2} = \frac{1}{Mf^2} \sqrt{\frac{\hbar P_{in}}{4\pi^2 c\lambda}}, \quad (1.61)$$

where M is the mirror mass. Finally, we can write the ASD of the strain amplitude

$$\tilde{h}_{rad}(f) = 2 \frac{\Delta L_{rad}(f)}{L} = \frac{1}{MLf^2} \sqrt{\frac{\hbar P_{in}}{\pi^2 c\lambda}}, \quad (1.62)$$

where the factor 2 was introduced to take into account that the power fluctuations in the two arms are anti-correlated, so one extra photon in one arm means one less photon in the other arm, as long as we ignore optical absorption and other sources of optical loss.

Note that in contrast to shot noise, radiation pressure noise scales up with increasing P_{in} . Additionally, even though the quantum noise is frequency-independent, the mirror response is not, thus the radiation pressure noise scales with f^{-2} , so it becomes more significant at lower frequencies, in opposition to shot noise. Finally, radiation pressure noise also scales down with mirror mass, so it is another reason, in addition to the suspension thermal noise, to favor heavy test masses in GW detectors.

The total quantum noise in the interferometer is the quadrature sum of both radiation pressure and shot noise

$$\tilde{h}_{qn}(f) = \sqrt{\tilde{h}_{rad}^2(f) + \tilde{h}_{shot}^2(f)}. \quad (1.63)$$

As mentioned above, shot noise decreases with increasing P_{in} while radiation pressure noise increases. If one minimizes the total quantum noise over the full detector bandwidth, the minimum noise reached, the so called standard quantum limit (SQL) is

$$\tilde{h}_{SQL}(f) = \sqrt{\frac{4\hbar}{Mf^2L^2}}. \quad (1.64)$$

Since this limit does not depend on any tunable parameter of the detector such as wavelength or optical power, it is a hard limit for the attainable sensitivity once the detector topology and test masses are fixed. It is, however, possible to go below this limit by employing squeezed states of light. In the next section I will briefly introduce squeezed states of light and summarize the key quantum noise reduction technique used in current detectors.

1.4 Quantum noise reduction

We have seen that both radiation pressure and shot noise arise from the uncertainty principle. In fact, the two types of quantum noise can be understood as two non-commuting, or non-correlated quadratures. This means that the SQL can be surpassed by introducing correlations between the radiation pressure and shot noise. This can be done by introducing squeezed light into the dark or antisymmetric port of the interferometer [10]. The optical spring effect of the SRM can also be exploited to achieve narrowband surpassing of the SQL, but this will not be discussed here. For more details see Ref. [80] and Chapter 2 of Ref. [81].

Here, I will focus on the production of frequency-dependent squeezed light. A brief but good introduction to the theory of squeezed states of light can be found, for example, in Chapter 5 (Complement 5A) of Ref. [79].

Coherent states have a minimum uncertainty which arises from Heisenberg's uncertainty principle. The annihilation operator a and the creation operator $a^\dagger$ of a coherent state satisfy the following commutation relation

$$[a, a^\dagger] = 1 \quad (1.65)$$

In phasor space, the uncertainties of the amplitude ($E_p = a + a^\dagger$) and phase ($E_Q = -i(a - a^\dagger)$) quadratures of a coherent state are thus related by the following uncertainty relation

$$\Delta E_Q \Delta E_P \geq 1. \quad (1.66)$$

The vacuum state itself has the same uncertainty relation in phasor space as shown in Fig. 1.9. This is to be expected as the coherent state is obtained by applying the displacement operator on a vacuum state. These vacuum fluctuations are behind spontaneous emission and allow explaining, for example, Lamb shifts in atoms [79].

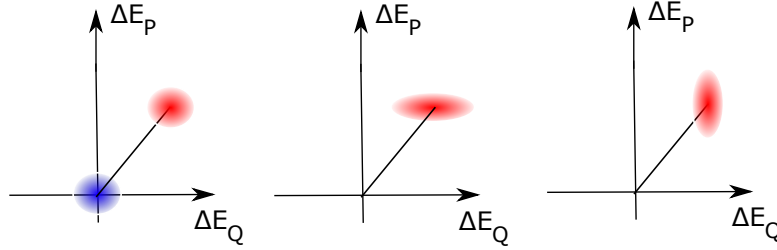


Figure 1.9: Phasor space schematic. In the left panel, a vacuum state (blue) and a coherent state (red) are shown. In the middle panel an amplitude squeezed state is shown and in the right panel a phase squeezed state.

A unitary squeezing operator can be obtained such that

$$S(R) = \exp \left[\frac{R(a^2 - a^{\dagger 2})}{2} \right], \quad (1.67)$$

where R is the squeezing factor. The squeezing operator transforms a coherent state into a squeezed state $|\alpha, R\rangle = S(R)|\alpha\rangle$. The variance of each quadrature of the squeezed state can be written as

$$(\Delta E_Q)^2 = e^{2R} \quad (\Delta E_P)^2 = e^{-2R}. \quad (1.68)$$

The opposing sign of the exponent shows that as one quadrature is squeezed and the noise is decreased below that of a coherent state, the other quadrature is anti-squeezed and the noise increases (see Fig. 1.9). Thus, the uncertainty relation of Eq. 1.66 remains unchanged for the squeezed states.

In Ref. [82] it is shown that the minimum discernible phase difference between the two arms of a Michelson interferometer, with an SNR of 1, is given by

$$\Delta\varphi_{min} = \frac{1}{\langle n \rangle} \Delta E_Q^{DP}, \quad (1.69)$$

where ΔE_P^{DP} is the standard deviation of the component entering the dark port of the interferometer, according to Eq. 1.68. Typically, no field is injected through the dark port, and as we saw, the vacuum state fluctuations are the same as that of the coherent state, in which case the SQL is retrieved with $\Delta\varphi_{min} = 1/\langle n \rangle$.

However, if a squeezed state is injected into the interferometer DP, a sensitivity improvement in the order of $e^{|R|}$ can be attained. This is the main idea

of quantum-noise reduction in GW detectors. Typically squeezing is characterized in dB units as $10\log_{10}(e^{|R|})$.

Note that a passing GW modulates the Michelson arm length with a certain frequency, thus modulating the phase quadrature. For this reason, the shot noise described in the previous section is affected by noise in the phase quadrature. Fluctuations of the amplitude quadrature directly impact radiation pressure noise. For this reason, reducing or increasing the fluctuations in each quadrature will impact both shot and radiation pressure noise accordingly. This motivates the need for frequency-dependent squeezing as discussed in Section 1.4.2.

1.4.1 Generating squeezed states

A clarification on nomenclature is due regarding the terms squeezed light and squeezed vacuum. These terms are often used interchangeably because they are essentially identical. Note that the vacuum fluctuations of a coherent state are inherited from the vacuum state itself. In the same manner, when a squeezed state is generated, it is not light itself that is squeezed, but the vacuum.

Squeezed states are typically generated using an optical parametric oscillator (OPO), which generate pairs of correlated down-converted sidebands. An OPO comprises a pumped non-linear crystal. The crystal is fed a pump signal at frequency ω_p as shown in Fig. 1.10. Through parametric down-conversion, two correlated photons are generated with frequencies ω_s and ω_i for the signal and idler photons respectively. Their frequency is related to the pump frequency by energy conservation such that $\omega_p = \omega_s + \omega_i$. In the degenerate case, which is the case in GW detectors, their frequency is

$$\omega_p = \omega_s = \omega_i/2. \quad (1.70)$$

The level of squeezing, R , can be seen to depend on the pump power P_p as

$$R(P_p) = \left(\frac{1 \pm \sqrt{P_p/P_t}}{1 \mp \sqrt{P_p/P_t}} \right)^2, \quad (1.71)$$

where P_t is a parameter representing properties of the OPO which is typically within a cavity, such as the crystal non-linear gain and the cavity losses.

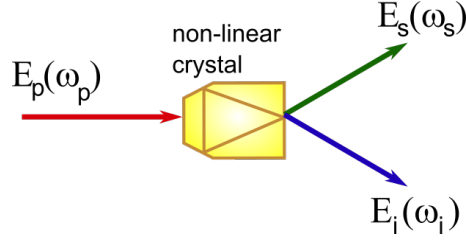


Figure 1.10: Schematic of the parametric down conversion process in an optical parametric oscillator. The input field is called the pump field with angular frequency ω_p and the two output fields are the signal and idler sidebands with angular frequency ω_s and ω_i respectively.

Here, it is useful to introduce the sideband model. A coherent state can be described in frequency space as a carrier and a collection of un-correlated quantum sidebands which are a pair of creation and annihilation operators in the ground state of a simple harmonic oscillator. All quantum sidebands have random amplitude and phase and so result in broadband white noise in both quadratures or in other words, shot noise. For a squeezed state, the quantum upper and lower sidebands will be correlated in either phase or amplitude across the frequencies over which squeezing is achieved. Currently squeezing sidebands spanning a few kHz, covering the bandwidth of GW detectors, have been demonstrated in the lab with squeezing gains as much as 10dB [83].

Given that squeezed states are typically generated with two pairs of sidebands, it is common to use the so-called two-photon formalism when describing the impact of squeezing on GW detectors. Horace P. Yuen was the first person to introduce the concept of two-photon states in quantum-optics when proposing squeezed states of light [84]. These concepts were later built into the two-photon formalism by Carlton Caves to study squeezing in the context of GW detectors [85].

The coherent states for signal α_s and idler α_i sidebands are defined as the eigenstates of the signal and idler annihilation operators as

$$a_{s/i} |\alpha_{s/i}\rangle = \alpha_{s/i} |\alpha_{s/i}\rangle, \quad (1.72)$$

where $\alpha_{s/i}$ is the corresponding eigenvalue for signal and idler sidebands. The key aspect of the two-photon formalism is the definition of new quadrature

operators in terms of the signal and idler photons as

$$a_1 = \frac{a_s + a_i^\dagger}{\sqrt{2}} \quad (1.73)$$

$$a_2 = \frac{a_s - a_i^\dagger}{i\sqrt{2}}, \quad (1.74)$$

where a_s and $a_s^\dagger$ are the annihilation and creation operators for signal photons and a_i and $a_i^\dagger$ the respective operators for the idler photons.

In turn, the squeezing operator of Eq. 1.67 is written in the two-photon formalism as

$$S_{si}(R) = \exp \left(R^* a_s a_i - R a_s^\dagger a_i^\dagger \right), \quad (1.75)$$

where now $R = r e^{2i\varphi}$, where r is the squeezing factor and φ the squeezing angle. Previously in Eq. 1.68 a squeezing angle of 0 and $\pi/2$ was implicitly assumed for the amplitude and phase quadratures, respectively.

In addition to the squeezing operator, we have two displacement operators $D_s(\alpha_s)$ and $D_i(\alpha_i)$ for the signal and idler sidebands. The squeezed state is obtained by applying the displacement operators on the two-photon vacuum state followed by the two-photon squeeze operator

$$|\alpha_s, \alpha_i, R\rangle = S_{si}(R) D_s(\alpha_s) D_i(\alpha_i) |0, 0\rangle. \quad (1.76)$$

The two-photon quadrature operators a_1 and a_2 can be written in matrix form as

$$a = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}. \quad (1.77)$$

An optical component acting on the squeezed field can be defined as a transfer function T in matrix form, and the new quadrature operators after the optical component, b , can be retrieved with $b = Ta$. This matrix notation will be used in the following section to describe the effect of a filter cavity on the squeezing quadratures.

1.4.2 Frequency-dependent squeezing

An OPO can generate squeezed light with a fixed squeezing angle, which can be adjusted to match the amplitude or phase quadrature of the interferometer signal. In O3, squeezed light was injected with a squeezed phase quadrature

achieving a quantum shot noise reduction of $3.2 \pm 0.1\text{dB}$ at high frequencies [86]. Due to the increase in the fluctuations of the amplitude quadrature, a slight increase of radiation pressure noise at low frequencies was observed [87]. For O4, one of the detector upgrades involved implementing frequency-dependent squeezing, which basically introduces a frequency-dependent rotation of the squeezing angle in order to achieve amplitude squeezing at low frequencies and phase squeezing at high frequencies [88].

The idea was first introduced in Ref. [10] by H. Jeffrey Kimble, from Caltech, following a request from K. Thorne. It is achieved by employing a detuned Fabry-Perot cavity, called filter cavity. Consider again the signal and idler sidebands, now with $\omega_s = \omega_0 + \Omega$ and $\omega_i = \omega_0 - \Omega$, with ω_0 the carrier frequency.

We can write the reflection from the filter cavity as a function of Ω . After some approximations valid at high finesse near resonance conditions, this can be written as [18]

$$r_{\text{FC}}(\Omega) = \frac{e^{-i\Phi(\Omega)r_{\text{in}}}}{r_{\text{in}}e^{-i\Phi(\Omega)} - 1}, \quad (1.78)$$

where r_{in} is the amplitude reflectivity of the input mirror and the round-trip phase $\Phi(\Omega)$ is

$$\Phi(\Omega) = (\Omega - \Delta\omega_{\text{FC}}) \frac{2L_{\text{FC}}}{c}, \quad (1.79)$$

where L_{FC} is the filter cavity length and $\Delta\omega_{\text{FC}} = \omega_{\text{FC}} - \omega_0$ is the cavity detuning away from the carrier frequency ω_0 .

A transfer function in terms of the sidebands can be transformed to that of the a_1 and a_2 quadratures by employing the following conversion matrix [10, 18, 89]

$$\mathcal{A}_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -i & i \end{bmatrix}. \quad (1.80)$$

Thus, the transfer function of the reflection of filter cavity in the two-photon picture will be given by

$$T_{\text{FC}} = \mathcal{A}_2 \begin{bmatrix} r_{\text{FC}}(\Omega) & 0 \\ 0 & r_{\text{FC}}^*(-\Omega) \end{bmatrix} \mathcal{A}_2^{-1}. \quad (1.81)$$

In order to have a more instructive version of Eq. 1.81, the cavity reflection r_{FC} is written in terms of sums and differences of the reflected sideband

amplitudes and phases [10, 89]. First the reflected amplitude ρ_{FC} and reflected phase α_{FC} are defined as

$$\rho_{\text{FC}}(\pm\Omega) = |r_{\text{FC}}(\pm\Omega)|, \quad (1.82)$$

$$\alpha_{\text{FC}}(\pm\Omega) = \arg[r_{\text{FC}}(\pm\Omega)]. \quad (1.83)$$

Then, the sum and difference amplitudes and phases can be defined as

$$\rho_{\text{p/m}} = \frac{\rho_{\text{FC}}(\Omega) \pm \rho_{\text{FC}}(-\Omega)}{2}, \quad (1.84)$$

$$\alpha_{\text{p/m}} = \frac{\alpha_{\text{FC}}(\Omega) \pm \alpha_{\text{FC}}(-\Omega)}{2}, \quad (1.85)$$

where the subscripts p and m are used to denote the sums and differences, respectively. Using these quantities, the transfer function of the filter cavity reflection can finally be re-written as [89]

$$T_{\text{FC}} = \rho_{\text{p}} e^{i\alpha_{\text{m}}} \begin{bmatrix} \frac{e^{i\alpha_{\text{p}}} + e^{-i\alpha_{\text{p}}}}{2} & \frac{e^{i\alpha_{\text{p}}} - e^{-i\alpha_{\text{p}}}}{2i} \\ \frac{e^{i\alpha_{\text{p}}} - e^{-i\alpha_{\text{p}}}}{2i} & \frac{e^{i\alpha_{\text{p}}} + e^{-i\alpha_{\text{p}}}}{2} \end{bmatrix} = \rho_{\text{p}} e^{i\alpha_{\text{m}}} R_{\alpha_{\text{p}}}. \quad (1.86)$$

Note that the matrix $R_{\alpha_{\text{p}}}$ is a rotation matrix with rotation phase α_{p} equal to

$$\theta = \frac{\alpha_{\text{FC}}(+\Omega) + \alpha_{\text{FC}}(-\Omega)}{2} = \arctan\left(\frac{2\gamma_{\text{FC}}\Delta\omega_{\text{FC}}}{\gamma_{\text{FC}}^2 - \Delta\omega_{\text{FC}}^2 \Omega^2}\right), \quad (1.87)$$

where γ is the cavity half-width half-maximum. When on resonance, $R_{\alpha_{\text{p}}}$ is equal to the identity matrix and there is no rotation. The rotation depends on the cavity linewidth detuning. In LIGO, filter cavities with 300 m of length and a Finesse of 6700 are used to induce a rotation of the squeezing ellipse around 30 Hz [88]. In Virgo, a filter cavity 285 m long and nearly 1000 finesse is used, resulting in a squeezing angle rotation at 25 Hz. In this way, squeezed sidebands below 30 Hz (LIGO) and 25 Hz (Virgo) are squeezed in amplitude to reduce radiation pressure noise and sidebands above are squeezed in phase to reduce shot noise.

1.5 High-power operation

We already discussed that increased power leads to a GW signal with increased SNR. Notwithstanding the technological limitations of current laser technologies, increasing input power is a straightforward way of increasing

detector sensitivity. However, high circulating optical power in the interferometer comes with its own set of complications.

Optical absorption of the coatings and substrates of the main optics of the interferometer are kept to extremely low levels. In the case of the substrates it can be as low as 0.1 ppm/cm and in the case of the coatings 0.3 ppm. However, with increased circulating power the absorbed power starts being significant and lead to what are called thermal effects. Absorption can occur at the level of the substrate and of the coating and lead to optical aberrations and thermal lensing effects.

These thermal effects change the cavities geometry, changing the g-factor and the HOM frequency spacing accordingly (Eqs. 1.29 and 1.30). We will see in Section 2.5 how thermal absorption impacts GW interferometers.

High-power operation of optical cavities can also lead to various types of instabilities mediated by the radiation pressure force impressed on the cavity mirrors by the kilowatts of stored photons. Angular instabilities [90] have been observed at the Gingin interferometer in Western Australia [91]. In this kind of instability, when the cavity is on resonance the two mirrors become coupled through radiation pressure force modifying the dynamics of the cavity angular control feedback system. In the next section, I will detail another kind instability, called parametric instability (PI) which can cause severe problems for the operation of high-power FP cavities.

1.5.1 Parametric instabilities

Parametric instabilities were first proposed by Braginsky et al. who theorized they could set an upper limit on the amount of circulating power inside a FP cavity [41]. This presents an issue for the future of GW detectors, but also for other areas of research where high-power FP cavities are crucial such as steady-state microbunching for the production of EUV radiation [23], in nuclear fusion with proposals for novel neutral beam heating systems [22], phase-contrast electron-microscopy [21] and dark matter detection [92].

PIs originate from the interaction between three modes, the optical fundamental mode, an optical HOM and the mechanical mode of the cavity mirror.

When the cavity is locked on the fundamental mode with frequency ω_{00} , any excited optical HOM will beat with the fundamental mode with frequency

$$\omega_{\text{beat}} = \omega_{00} - \omega_{mn} = \frac{c}{2L} \frac{m+n}{\pi} \arccos[\sqrt{g_1 g_2}]. \quad (1.88)$$

Typically in, GW detectors, ω_{beat} is in the acoustic range, where the mechanical modes of the test masses reside. If the frequency of a mechanical mode coincides with that of the optical beating then the optical HOM can be resonantly enhanced in the cavity. Through Brillouin scattering [93], energy is imparted from the optical beating into the mirror acoustic mode. If left unchecked, this can create a feedback loop pumping energy into the optical HOM and in turn feeding the mirror acoustic mode until the cavity is kicked out of resonance.

For the mechanical mode to become unstable, its dissipation rate needs to be smaller than the pumping rate of the radiation pressure force from the optical beating on the mirror. This can be quantified by the parametric gain $\mathcal{R}_0$. When $\mathcal{R}_0 > 1$ the rate of the radiation pressure force becomes higher than the dissipation rate of the mechanical mode and the mechanical mode amplitude grows. The parametric gain depends on the cavity configuration, but for a FP cavity it is

$$\mathcal{R}_0 = \frac{4P_{\text{circ}}\Lambda Q_{\text{mech}}Q_{\text{mn}}}{McL\omega_{\text{mech}}^2} \left(\frac{1}{1 + (\Delta\omega/\delta)^2} \right), \quad (1.89)$$

where P_{circ} is the circulating power in the cavity, Λ is the spatial overlap between the optical HOM beating and the mechanical mode of the mirror, Q_{mech} and Q_{mn} are the quality factors of the mechanical and optical modes respectively, M is the mirror mass and L the cavity length. The frequency difference between the mechanical mode and optical beating is given by $\Delta\omega/$ and δ is the the cavity half FWHM defined as

$$\delta = \frac{\omega}{2Q} = \frac{FSR}{2\mathcal{F}}. \quad (1.90)$$

Typically, in a GW detector the cavity length, mirror mass and acoustic modes of the mirrors are fixed. Ideally, we would also like to maximize circulating power. Thus, in order to minimize PIs, we would focus on reducing the quality factors of the optical and mechanical modes or increasing the frequency difference $\Delta\omega$. The latter can be achieved by thermal tuning of the RoC of the cavity mirrors to shift the HOM frequency away from the frequency of the mechanical mode [94]. In order to reduce the quality factor of acoustic modes, resonant [95] and electrostatic [96] damping actuators have been developed and implemented in current GW detectors. The optical HOM can also be directly targeted using optical feedback, where the same HOM is injected into the cavity with an opposing phase [97]. For future detectors, a recent approach

is to design FP cavities which are free of PIs provided the g-factors stay within a certain value [98].

The first experimental observation of a PI was made at Gingin in Western Australia [99]. Shortly after, LIGO Livingston also observed a PI on-site [100]. Ref. [101] details the first real-time imaging of a PI, caused by the fundamental and a first-order mode.

1.5.2 The megawatt goal of next-generation detectors

Current GW detectors continue to improve their sensitivity by increasing circulating power, increasing the size and mass of test masses and trying to improve coating thermal noise with new coatings. However, the current observatories are ultimately limited by their infrastructure, in particular arm length and seismic noise. For this reason, third-generation detectors are currently in the planning and design phase to increase BNS sensitivity by 1 order of magnitude.

Two flagship projects have been proposed, the Einstein Telescope (ET) in Europe, and the Cosmic Explorer (CE) in the United States. Both projects have very different approaches. CE aims to increase the scale of the current LIGO detectors, by employing 40 km long arm cavities and ramping up circulating power to 1.5 MW. [102].

In the case of ET, the current baseline design went with a very different approach. In order to increase sensitivity at low frequencies seismic and Newtonian noise needs to be minimised. To this end, the plan is to build the detector between 200 and 300 m underground [19]. The interferometer topology is also new, and would consist of a triangular arrangement with 60° angle instead of the 90° angle of the typical Michelson interferometer and 10 km arms. Three nested detectors in this configuration are planned, each detector composed of two triangular interferometers, one tuned for the low-frequency band of GW detection and the other for the high-frequency band.

The low-frequency detector plans to use cryogenics technology to get the temperature down to 10-20 K as well as 200 kg test masses to reduce thermal coating noise. In turn, the high-frequency detector aims to reach up to 3 MW of circulating power to increase sensitivity at high frequencies. In both cases, 10 dB of squeezing are required to achieve the sensitivity goals.

Besides ET and CE, other intermediate or prototype detectors are planned that will also make use of very high circulating powers. One such detector is the Neutron Star Extreme Matter Observatory (NEMO), which aims to use 4.5 MW of circulating power and 74.1 kg mirrors at around 130 K [103]. Another

one is the LIGO Voyager detector, proposing 3 MW of optical power with 200 kg mirrors at 123 K [104].

The work developed during my thesis is mostly concerned with addressing the issues related to high-power operation, namely the cavity eigenmode changes during thermal transients and PIs. In fact, with current thermal compensation techniques and control, LIGO has capped the amount of circulating power at around ≈ 300 kW, while the design goal was 750 kW [9]. Table 1.2 shows the design values for circulating power of current⁷ and future detectors, as well as the actual circulating power of operating detectors.

Table 1.2: Comparison of design circulating power for LIGO, Virgo and next-generation ET and CE detectors with the actually achieved power in current detectors.

Power [MW]	Detector			
	LIGO	Virgo	ET	CE
Design	0.75	0.15	3	1.5
Reached	0.35	0.096	NA	NA

On one hand, the amount of optical power reached is limited by the dynamic range or actuation power of the thermal compensation adaptive optics as we will discuss in Section 2.7. On the other, there is also a need for direct sensing of the cavity eigenmodes of the interferometer. This point is discussed in the review of Ref. [2] and will also become apparent in the discussion in Chapter 4. A part of this work involved two table-top experiments to address this, prototyping wavefront sensing schemes to characterize the cavity modes in reflection and transmission of a FP cavity. These experiments are addressed in Chapters 6 and 7.

⁷The design value for Virgo was computed taking into account an input power of 25 W and carrier gain of 42 set in the 2012 Advanced Virgo Technical Design report [105]. Previous to this document, the design input power was 125 W and as we will discuss in Chapter 3, the RoC of the power recycling mirror was manufactured with that in mind. Regardless, since this plan was replaced by the 25 W input since 2012, I considered this one as the design baseline for the Advanced Virgo circulating power.

Chapter 2

Mode sensing and control in gravitational-wave detectors

In this chapter, I review the state-of-the-art in regards to the sensing and control of cavity modes. When matching a cavity with an input beam, we can define 5 degrees of freedom - the longitudinal (Eq. 1.18), transverse offset, angular tilt and waist size and position mismatch.

I start by introducing the concepts of heterodyning and homodyning which are fundamental for most of the measuring schemes described later. Then, I detail longitudinal and angular control of cavities. These are well developed, mature techniques with high levels of automation. This knowledge was built by a combination of pioneers from the fields of atomic, molecular and optical physics (AMO) as well as GW interferometry, such as John Hall and Ronald Drever [12], Dana Z. Anderson [13] and Gary C. Bjorklund [14].

Sensing and controlling the waist size and position of beams and resonators, is however, not as well developed. I introduce the topic of mode mismatch and its importance for GW detectors as well as thermal defects in resonators as it pertains to its impact on the eigenmodes of a resonator in the presence of high optical powers.

Afterwards, I review the state-of-the-art of the wavefront sensors for measuring mode mismatch and finally I review the adaptive optics currently in use in GW detectors. For these two sections I rely heavily on the text of the review of Ref. [2] of which I am one of the co-authors. My contributions were especially significant in the sections detailed here, namely the review of wavefront sensors and thermal actuators in GW detectors.

2.1 Heterodyne and homodyne detection

Heterodyne and homodyne detection is a detection method through which a small-signal is combined with a strong local-oscillator signal before detection.

This combination with a stronger field allows for reaching the quantum shot noise limit even for low power signals. This is very useful for example for the detection of squeezed-states [106].

Optically, the signal and local-oscillator beams can be combined through interference at a beam splitter as shown in Fig. 2.1. Mathematically, consider the signal electrical field at the beam splitter with frequency ω

$$E_s = U_s(x, y)e^{i(\omega t)}, \quad (2.1)$$

where U_s is the (complex) transverse amplitude of the signal field, which could be described in terms of the HG polynomials introduced in Section 1.3.1. The electrical field for the local oscillator E_{LO} will have a similar form but with frequency ω' . The combined beam after the beam splitter will be

$$E_{BS} = t_{BS}E_s + ir_{BS}E_{LO} = t_{BS}U_s(x, y)e^{i(\omega t)} + ir_{BS}U_{LO}(x, y)e^{i(\omega' t)}, \quad (2.2)$$

where t_{BS} and r_{BS} are the amplitude transmission and reflection coefficients of the beam splitter respectively. The photodiode at the output of the beam splitter measures the intensity of the combined beam, which will be

$$I_{PD} = E_{BS}E_{BS}^* = t_{BS}^2|U_s|^2 + r_{BS}^2|U_{LO}|^2 + t_{BS}r_{BS}|U_s||U_{LO}|\cos(\Omega t), \quad (2.3)$$

where $\Omega = \omega - \omega'$ is the frequency difference between signal and local oscillator, the so-called heterodyne frequency. This frequency should be smaller than the photodiode detector bandwidth so that it is not integrated out as it happens with the optical frequencies. The component at Ω is called the heterodyne beat. This name is a reference to acoustics, where heterodyne frequencies are called beats, which are created by mixing two waves of close but different frequencies.

Typically, Ω is in the radio-frequency (RF) range. The optical beat at Ω , has the combined coherence properties of the signal and reference beams. In this way, heterodyne measurements can be used for example to characterize the linewidth, or monochromaticity, of lasers [107]. Additionally, if the frequency of the local oscillator signal is the same as that of the signal beam, then $\Omega = 0$ and we call it homodyne detection instead of heterodyne.

In order to be detected, the AC component of the photodiode signal, the Ω term in Eq. 2.3, can be fed to an electronic mixer as shown in the left panel of Fig. 2.1. Mathematically, the mixer action is equivalent to a multiplication

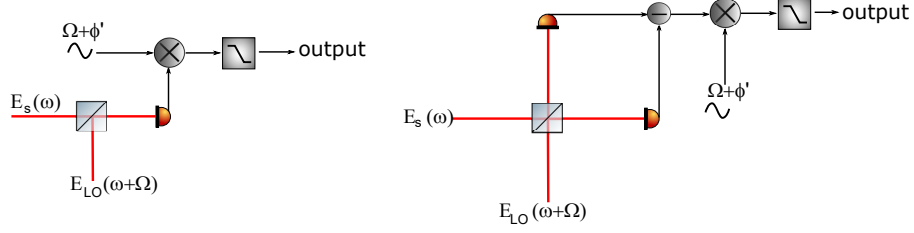


Figure 2.1: On the left, the schematic representation of heterodyne detection. A signal field E_s is interfered with a local oscillator field E_{LO} at a beamsplitter, resulting in the combined field of Eq. 2.2. The photodiode signal is mixed with a electrical reference and the output is low-pass filtered resulting in the detected signal of Eq. 2.5. On the right panel balanced detection is shown, where a second photodiode is added to the unused port and it is the difference signal that is detected. If the local oscillator frequency is the same as the signal, i.e. $\Omega = 0$, it is called homodyne detection and the mixing step is skipped as the signal of interest is already in the DC range.

between photodetector signal and an electrical local oscillator at Ω , yielding

$$t_{BS} r_{BS} |U_s| |U_{LO}| \cos(\Omega t) \sin(\Omega' t + \varphi') = \frac{t_{BS} r_{BS} |U_s| |U_{LO}|}{2} \{ \sin[(\Omega - \Omega')t - \varphi'] - \sin[(\Omega + \Omega')t + \varphi'] \}, \quad (2.4)$$

where φ' is the phase of the local oscillator with respect to the signal, which is called the demodulation phase. In order to extract the signal component at the frequency of interest, the electrical local oscillator is driven with the same frequency as the one of the optical heterodyne such that $\Omega' = \Omega$. The output of the mixer will then be

$$t_{BS} r_{BS} |U_s| |U_{LO}| \cos(\Omega t) \sin(\Omega t + \varphi') = \frac{t_{BS} r_{BS} |U_s| |U_{LO}|}{2} [\sin(\varphi') - \sin(2\Omega t + \varphi')]. \quad (2.5)$$

The 2Ω signal is removed by a low-pass filter as shown in the left panel of Fig. 2.1 and the signal of interest will be in the DC component of Eq. 2.5. The demodulation phase φ' needs to be tuned, in this case to $\pi/2$ to extract the maximum signal amplitude. Generally, for a random demodulation phase we can define the in-phase component I of the signal and the quadrature compo-

nent Q as

$$I = \frac{A_s A_{LO}}{2} \cos(\Omega t) \cos(\Omega t + \varphi') = \frac{A_s A_{LO}}{2} \cos(\varphi'), \quad (2.6)$$

$$Q = \frac{A_s A_{LO}}{2} \cos(\Omega t) \sin(\Omega t + \varphi') = \frac{A_s A_{LO}}{2} \sin(\varphi'), \quad (2.7)$$

where A_s and A_{LO} are the amplitudes of the optical signal beam and local oscillator respectively and where we have assumed that the $2\Omega t$ is removed by a low-pass filter.

It is shown in Ref. [108] that apart from the quantum fluctuations discussed in Section 1.4, excess phase and amplitude noise from the local oscillator affects both heterodyne and homodyne measurements. As shown on the right panel of Fig. 2.1, a second photodiode can be added to the unused beam splitter port. By subtracting the signal from both photodiodes this excess noise can be subtracted and the measurement can become shot noise limited [108]. This is called balanced detection, to highlight that noise maximum suppression is achieved when the split ratio of the beam splitter is exactly 50% and the quantum efficiency of both detectors is the same. Detection of squeezed states is done using balanced homodyne detection [106].

Another way to generate optical heterodyne beats is to use phase modulation. Typically this is achieved with an electro-optic modulator (EOM). An EOM consists of a crystal which exhibits a linear birefringence in response to an applied voltage. Typically, a sinusoidal wave of (radio-)frequency Ω is applied along the principal axis of the crystal. This causes the refractive index of the crystal to change following the applied signal, in this way modulating the phase of the beam crossing the crystal. The electrical field of the thus phase modulated beam can be written as

$$E_{\text{mod}} = E_0 e^{i(\omega t + \beta \sin(\Omega t))}, \quad (2.8)$$

where β is the so-called modulation depth, the amplitude of the applied modulation wave to the EOM. Eq. 2.8 can be evaluated following an expansion in terms of Bessel functions J_i of order i , such that

$$E_{\text{mod}} = E_0 \left[J_0(\beta) e^{i\omega t} + J_1(\beta) e^{i((\omega+\Omega)t)} - J_1(\beta) e^{i((\omega-\Omega)t)} \right] + \mathcal{O}(J_2(\beta)). \quad (2.9)$$

Disregarding the terms of Bessel functions of order 2 and above (which is valid for small values of β), the modulated beam can be understood as a carrier

at a frequency ω and two sidebands at $\omega \pm \Omega$. We will see in Section 2.2 these sidebands are crucial for the control of optical cavities in GW detectors.

Most of the wavefront sensing schemes introduced in the rest of this chapter use in some way or another some kind of heterodyning. Homodyning also plays an important role in GW detectors, below I describe how it is used for the readout of the strain amplitude signal.

2.1.1 Detecting the GW signal

As we saw in Chapter 1, a passing GW causes a stretching and squeezing of the interferometer arms, leading to a modulation of the amplitude of the signal at the dark port. Similar to phase modulation, amplitude modulation at a certain frequency can be understood as a pair of sidebands around the carrier. Thus, we can write the GW signal as a pair of sidebands at $\omega \pm \omega_{\text{GW}}$, where ω is the carrier frequency and ω_{GW} the GW frequency, typically in the audio range.

Initially, GW detectors operated at the dark fringe and employed a heterodyne read-out using the modulation and demodulation techniques described above in order to measure the GW signal. However, this readout scheme suffers from phase noise from the RF modulation. More fundamentally, it has been shown that heterodyne schemes suffer from twice the quantum noise of homodyne schemes [108, 109]. To avoid this, current GW detectors use a kind of homodyne detection scheme called DC-readout [110].

In the heterodyne readout, RF sidebands were used to generate an heterodyne beat with the GW sidebands, and in this way the very small GW sidebands were detected. In the DC-readout detuning, the carrier itself is used as the local oscillator. In order to do this, the difference in length of the north and west arm cavities is tuned in order to have a very small amount of the carrier fundamental mode leak into the dark port.

In addition to the advantages of using a homodyne readout as outlined above, in the DC-readout scheme, the phase noise of the local oscillator (carrier) is highly suppressed by the interferometer itself. This is due to the cavity pole of the combined power and arm cavities. If the arm cavities are approximately the same, the PRC and arm cavities can be approximated by a single, three mirror coupled-cavity, whose cavity pole is approximately

$$f_p = \frac{1}{2\pi} FSR_\nu \log \left(\frac{r_3 - r_1}{1 - r_1 r_3} r_2 \right), \quad (2.10)$$

where r_1 and r_2 are the reflectivities of the arm cavity input and end mirrors respectively, r_3 the reflectivity of the power recycling mirror and FSR_ν the FSR of the arm cavities.

For current GW detectors, this coupled cavity pole is around 1 Hz. For frequencies above that, phase noise is suppressed by the inverse of the carrier frequency, which is around 280 THz. Initial GW detectors employed the RF heterodyne readout because the frequency and phase noise of state-of-the-art lasers at the time was insufficient for DC readout. This highlights the importance of clean laser sources for GW detection.

2.2 Longitudinal control and frequency stabilization

We saw in Section 1.3.1 that optical cavities are resonant for specific frequencies (Eq. 1.18). One can make a laser beam resonate in a cavity by tuning its frequency, or alternatively, by finely tuning the length of the cavity. The first is to lock a laser to a cavity, the latter is to lock a cavity to a laser.

The most widely used technique to lock a laser to a cavity, or vice-versa, is the Pound-Drever-Hall (PDH) technique [12]. The genesis of this technique can be traced back to the fields of AMO (John Hall), GW interferometry (Ronald Drever) and microwaves (Robert Pound). In order to understand it, it is useful to start with the (amplitude) reflection coefficient of a FP cavity [111]

$$F_{\text{ref}}(\omega) = \frac{E_{\text{ref}}}{E_{\text{inp}}} = \frac{r(e^{i\omega/FSR_\nu} - 1)}{1 - r^2 e^{i\omega/FSR_\nu}}, \quad (2.11)$$

where, for simplicity, a lossless cavity was assumed and with equal reflectivities for input and end mirrors. With respect to the Airy Eq. 1.21, the round-trip phase was written in terms of the frequency ω and the cavity FSR_ν and in terms of the field amplitudes instead of field intensities. The intensity of the reflected beam is given by $|F(\omega)|^2$ and it follows the Airy function of Fig. 1.4. The input and cavity reflected beams are shown in the optical schematic of Fig. 2.2.

In the case of critical coupling, which for a lossless cavity is achieved with equal mirror reflectivity, no light is reflected on resonance. Now, consider a phase modulated beam such that sidebands are introduced at $\omega \pm \Omega$ around the carrier frequency ω . Here, we will consider an Ω large enough such that the

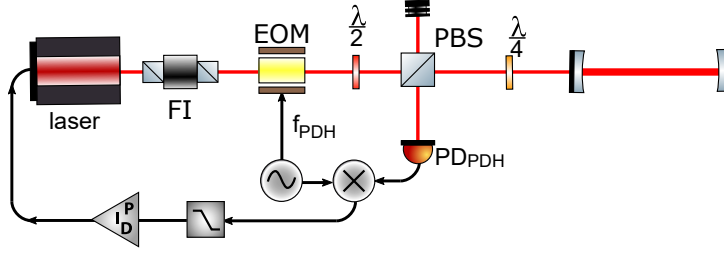


Figure 2.2: Optical scheme for PDH locking. An EOM introduces sidebands at $\omega \pm 2\pi f_{\text{PDH}}$. A polarizing beam splitter (PBS) ensures only light polarized in one direction is transmitted. Light reflected from the cavity, both carrier and sidebands (see text for more details) pass twice through the quarter-wave plate and undergo a 90° rotation of the polarization axis, thus being reflected at the PBS and onto the PDH photodiode. The resulting signal after the mixer (Eq. 2.13) is low-pass filtered and feedback to the laser current through a PID controller [112].

sidebands are entirely reflected from the cavity while the carrier is resonating in it.

The cavity reflected signal can be detected by a photodetector as shown in Fig. 2.2. The reflected power from the cavity will be

$$\begin{aligned} P_{\text{ref}} = & P_c |F(\omega)|^2 + P_s (|F(\omega + \Omega)|^2 + |F(\omega - \Omega)|^2) \\ & + 2\sqrt{P_c P_s} \left[\text{Re}\{F(\omega)F^*(\omega + \Omega) - F^*(\omega)F(\omega - \Omega)\} \cos(\Omega t) \right. \\ & \left. + \text{Im}\{F(\omega)F^*(\omega + \Omega) - F^*(\omega)F(\omega - \Omega)\} \sin(\Omega t) \right] + \mathcal{O}(2\Omega) \end{aligned} \quad (2.12)$$

Near resonance, the sidebands are completely reflected from the cavity, and $F(\omega \pm \Omega) \approx -1$. In this way, the $F(\omega)F^*(\omega + \Omega) - F^*(\omega)F(\omega - \Omega)$ is approximately only imaginary, and the cosine term of Eq. 2.12 vanishes. Thus, near resonance, the only remaining term at Ω is

$$P_{\text{ref}}^\Omega = 2\sqrt{P_c P_s} \text{Im}\{F(\omega)F^*(\omega + \Omega) - F^*(\omega)F(\omega - \Omega)\} \sin(\Omega t). \quad (2.13)$$

Demodulating the signal as described in the previous section (recall Eq. 2.5), we obtain the following error signal

$$\varepsilon = 2\sqrt{P_c P_s} \text{Im}\{F(\omega)F^*(\omega + \Omega) - F^*(\omega)F(\omega - \Omega)\}, \quad (2.14)$$

which is plotted in Fig. 2.3. In this way, through the interference of the carrier and sideband fields, we can retrieve from their heterodyne beating a zero crossing error signal around the cavity resonance.

We can see in the top plot of Fig. 2.3 that within the cavity mode, the error signal is linear (this is also shown mathematically in Ref. [111]), with a slope proportional to the inverse of the cavity linewidth $\text{FSR}_\nu/\mathcal{F}$. Within this locking range, the standard tools of control theory [112], such as PID control, can be used to act on the laser frequency or cavity length to keep one locked to the other. Outside of the cavity mode, the carrier is entirely reflected and there is no error signal.

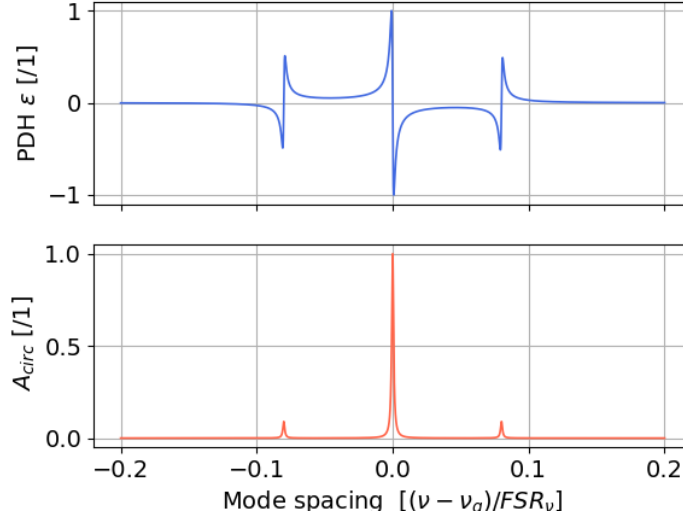


Figure 2.3: Top panel shows the PDH Error signal computed with Eq. 2.14 (normalized) for a sweep of the laser frequency around the cavity resonance. Bottom panel shows the (normalized) Airy distribution of the circulating field (Eq. 1.21), for the same frequency sweep.

Note that even though we used the example of a critically coupled cavity, PDH locking is valid for any kind of cavity, and the error signal of Fig. 2.3 is attainable as long the modulation frequency of the sidebands is sufficiently larger than the cavity linewidth. For lower sideband frequencies, a locking signal is also attainable but with a smaller slope as shown in Ref. [111]. As the sidebands get closer to the carrier, the storage time of the cavity needs to be taken into account in the error signal response.

Regarding the demodulation, the electronic local oscillator will have a certain phase with respect to the heterodyne beat signal. This phase needs to be adjusted so that they are both in phase and that Eq. 2.14 is retrieved. In fact,

while for high Ω the error signal is in the I component, for slow modulation, the signal is in the Q component (derivation in Ref. [111]).

Predating the original PDH article, Bjorklund showed the signals one obtains for both I and Q components after demodulation in the context of frequency modulation spectroscopy [14]. In this work, the signals are not obtained from a cavity, but from dispersion and absorption spectral features of atoms and molecules. In fact the same kind of heterodyne scheme of PDH is at the basis of frequency modulation spectroscopy. In this work, Bjorklund goes on to show how both I and Q signals look like for varying values of Ω with respect to the spectral feature linewidth. These results are valid irrespective of the nature of the spectral feature, either an absorption line or cavity resonance.

The PDH technique is ubiquitous in GW detectors in order to lock the multiple cavities to the ultra-stable laser employed. Multiple pairs of sidebands are used to extract the necessary number of error signals. In a typical GW interferometer, there are 5 longitudinal degrees of freedom. Refer to Fig. 2.4 for the distance definitions following that of Ref. [113]. Below I list each degree of freedom (DoF), with details for the Virgo case on how each error signal is obtained.

- **DARM** = $\frac{L_N - L_W}{2}$, differential arm length of the two FP arm cavities. The error signal comes from the DC-readout scheme discussed in Section 2.1.1¹. The DoF is locked by actuating differentially on the North and West end cavity mirrors.
- **CARM** = $\frac{L_N + L_W}{2}$, common arm length of the two FP arm cavities. The error signal comes from the I component of the heterodyne from 6 MHz sidebands. The DoF is locked by actuating commonly on the north and west end cavity mirrors.
- **MICH** = $l_N - l_W$, difference in the length of the small Michelson arms from the beam splitter to the input mirrors of the arm cavities. The error signal comes from the I component of the heterodyne from 56 MHz sidebands. The DoF is locked by acting jointly on the positioning of the signal and power recycling mirrors and on the beam splitter.

¹Note that the GW signal is encoded in the DARM degree of freedom. This means that for low frequencies where the DARM control loop takes effect, the GW signal is compensated by the feedback loop and needs to be recovered in post-processing.

- **PRCL** = $l_{PR} + \frac{l_N + l_W}{2}$, cavity length of the power recycling cavity. The error signal comes from the I component of the heterodyne from 8 MHz sidebands. The DoF is locked by acting on the positioning of the PRM.
- **SRCL** = $l_{SR} + \frac{l_N + l_W}{2}$, cavity length of the signal recycling cavity. The error signal comes from the Q component of the heterodyne from 56 MHz sidebands. The DoF is locked by acting on the positioning of the SRM.

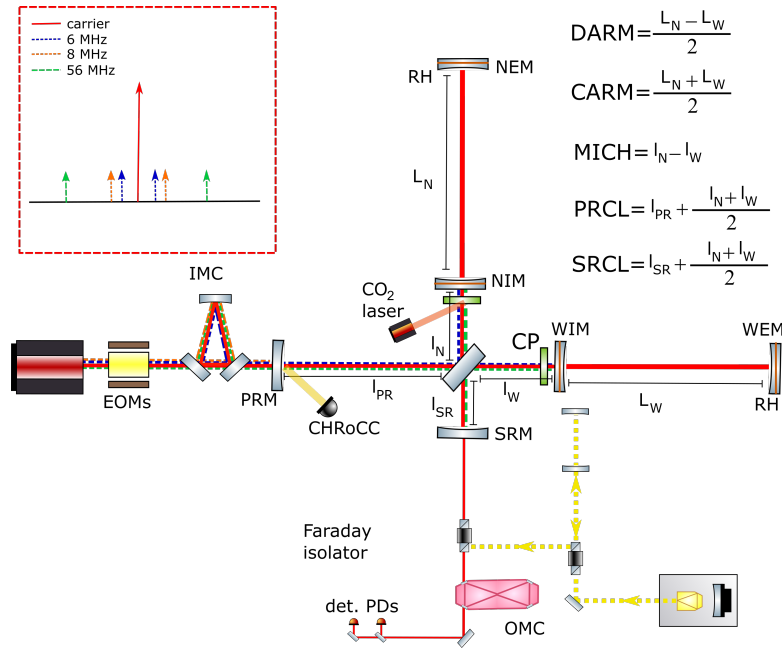


Figure 2.4: Simplified AdV schematic including the distances definitions for the various detector DoFs described in the text. Also showing the thermal actuators described in Section 2.7 - ring heaters (RH), compensation plates (CP) and CO₂ laser (only 1 is shown but there is another for the west CP) and the central heating radius of curvature correction (CHRoCC) to tune the power and signal recycling mirrors (only 1 shown).

The multiple degrees of freedom are locked iteratively through the so-called locking procedure. For example the central part of the interferometer, the power and signal recycling cavities, are locked before the FP arm cavities. In order to transmit part of the sidebands into the DP for control reasons, a macroscopic asymmetry in the lengths of the small Michelson (l_N and l_W) is introduced. This difference in length, called the Schnupp asymmetry is

22.9 ± 0.4 cm in Virgo [113]. More details on how a full GW interferometer is controlled can be found, for example, in Chapter 3.2 of Ref. [66].

2.2.1 Beyond Pound-Drever-Hall

Besides its widespread use in GW detectors, PDH locking is also crucial for many fields in optics such as cavity enhanced spectroscopy [114], optical clocks [115, 116], ultracold chemistry [117] and others such as quantum computers [118]. Due to its ubiquity in so many fields, several efforts have been made towards improving its bandwidth and noise performance. In this section, I briefly review the advances made in this field including both techniques with and without modulation in order to achieve the best phase locks.

Improving the lock bandwidth

Typically, one of the bottlenecks in the bandwidth of laser locking with PDH is in the actuator bandwidth. When acting on the length of a cavity, piezo transducers are normally used which have actuation bandwidths of a couple hundred kHz at most. When acting on the laser itself, for example on its current, a few MHz of bandwidth is easily available.

The issue of actuation bandwidth can be solved by using a different actuator. It has become common in some applications to act on the EOM itself, which typically has a bandwidth of tens of MHz or even GHz in some cases. This is what is done in Ref [119]. However, the bandwidth is ultimately limited by the time delay of the feedback loop itself. Recent work, has addressed this by using a feedforward technique [120]. In this work, the authors use the residual PDH error signal when the laser is locked to a cavity with the usual PDH lock, and use it for phase correction on a second EOM in a feedforward way. A delay fiber is used on the path to the feedforward EOM to account for the feedback loop delay.

Mitigating residual amplitude modulation

One issue with heterodyne schemes that depend on the use of sidebands is that of residual amplitude modulation (RAM). When using an EOM for sideband generation, RAM may appear as an undesired modulation at the same frequency as that of the generated sidebands. This can introduce spurious noise and offsets in the heterodyne signal used to lock a cavity. In Ref. [121] the

authors investigate the impact of RAM in the longitudinal control of GW detectors. In Ref. [115] RAM also appears as a non-negligible noise in an ultra-stable cavity for optical clocks.

When the polarization of the incident beam is misaligned with respect to the EOM crystal orientation the polarization axis of the transmitted beam is rotated. In turn, when interacting with polarizing optics as the beam propagates, amplitude modulation is generated. Small etalons (FP cavities) in the crystal due to non-zero reflection at the crystal faces will also lead to RAM, as the multiply reflected beam experiences a frequency dependent phase shift. Finally, temperature fluctuations that drive thermal expansion of the crystal and of the mounting opto-mechanics also lead to RAM by modulating the crystal birefringence.

Careful aligning of the input beam and EOM, precise temperature control and the use of anti-reflective (AR) coatings at the crystal surfaces can mitigate these issues. It has been shown that for some EOMs, a negative bias Voltage in addition to the modulation signal can significantly reduce the observed RAM.

Additionally, etalon effects on the optical path of the beam after the EOM, will also amplify the RAM. In Chapter 2.3 of Ref. [122] a careful and detailed account of how to mitigate these etalon effects in the optical path is given.

Another way to mitigate RAM is through active cancellation. Recent work has achieved 10^{-7} RAM cancellation with a photodiode monitoring the input beam and a PID controller with 500 kHz bandwidth to control the bias voltage of the EOM [123]. The active RAM cancellation reduces the locked laser fractional frequency noise by at least 2 orders of magnitude.

In Ref. [124], instead of using an EOM, the sideband modulated beam is generated by feeding a frequency-modulated triplet to the modulation input of an acoustic optic modulator (AOM). An AOM uses the acoustic-optic effect to diffract and shift the frequency (by an amount in the RF range) of the input beam. The frequency shift is the same as the one driving the crystal and so, by driving the crystal with a triplet, it is possible to create a frequency shifted beam with a carrier and two sidebands. The AOM does not have the same issues with RAM as an EOM and so much smaller values of RAM can easily be achieved. Additionally, RAM can be actively corrected directly at the triplet generation. Another interesting advantage of this technique, is that high sidebands and a low power carrier can be generated, without inducing higher-order sidebands. A lower power carrier can be interesting when a cavity is being used as a stable length reference and thermal absorption might cause problems. Thermal deformations are discussed in Section 2.5.

Modulation-free techniques

One way to avoid RAM entirely and other noises related to the laser modulation, is using modulation free techniques. For spherical resonators, this can be done using spatial mode interference, where HOMs play the role of the sidebands in the PDH. In Refs. [125, 126], the authors use the spatial interference of non-resonant first-order modes originating from small misalignments with the resonant carrier, to generate a PDH-like error signal. Detection is done using a quadrant photodiode (QPD) as shown in Fig 2.5.

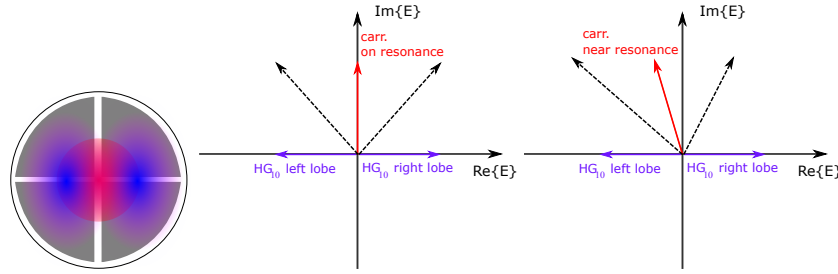


Figure 2.5: Left panel illustrates the measurement of resonant HG_{00} (red) with non-resonant HG_{10} (blue) at a QPD used for tilt-locking. Middle and right panel show the complex field amplitudes on and near resonance respectively. The dashed lines show the resultant field in each segmented half of the QPD. Subtracting one segmented half from the other, results in 0 on resonance, and non-zero near resonance, with opposite sign depending on which side of resonance the carrier is on.

The first-order mode, HG_{10} originates from misalignment (see the next Section 2.3). The complex amplitudes of each HG_{10} lobe and the resonant HG_{00} are shown in the middle and right panels of Fig. 2.5. The error signal can be extracted from the difference between the left and right segments of the QPD. On resonance, the resonant carrier adds up equally with each lobe and so the difference between the two segments is 0. Near resonance, the phase of the near-resonant carrier will be such that it adds up to one of the lobes depending on which side of resonance it is (note that the lobes are in anti-phase with each other). This gives origin to an error signal very similar to the one of PDH without the need for any kind of modulation. In Ref. [126], the authors use for the tilt locking error signal the double-pass beam from a triangular cavity. The cavity filtering allows to remove noise contributions from beam jitter and unwanted HOM contributions.

In Ref. [127], the authors extend this concept but use the beam ellipticity instead of the first-order modes from misalignment. In the presence of some

small astigmatism with an arbitrary angle, some power will scatter to second-order modes with an x shape. Detection can be made with a QPD the same as above. Now, however, the error signal is not dependent on misalignment drifts or pointing precision, as it depends only on the spatial profile of the input beam.

The method with beam ellipticity is still sensitive to beam shape fluctuations, but the authors address this by using the so-called polarization pre- and post-selection. Since the cavity is only resonant for one polarization, the authors perform a differential measurement between the two incident polarizations. The promptly reflected polarization serves as a reference to remove contributions from beam shape noise. In this way the authors claim a cavity locking stability around 1 order of magnitude below the RAM noise limit of Ref. [128].

Another way to lock a laser to a cavity is optical feedback locking, also known as self-locking or injection locking [129, 130]. This method is in a class of its own in that it is modulation free and is not limited by the dynamics of electronic feedback loops. For this reason, it is outside the scope of this review so I will only briefly describe the working principle.

It is a well known fact that retro-reflected light back into the laser has a de-stabilizing impact on laser emission spectrum. For this reason, most lasers come with installed optical isolators, and often for laser locking applications, additional isolation in the form of Faraday isolators is required. However, it can be shown that if the returning light is the resonant intracavity buildup from a cavity, it can lead to linewidth reduction of the emitted laser spectrum, and frequency locking to the cavity resonance. This effect, with some similarities to the principles behind external cavity diode lasers ([131]), is dependent on the amount of feedback, and on the phase of the returning light path [132].

With this method, very efficient linewidth reduction can be achieved with fast dynamics. Typically, it is combined with a slow feedback loop to control the return light optical path length for long term stability [133]. For now, this technique has mostly seen application in the field of spectroscopy.

2.3 Angular control

While the resonance condition fixes the longitudinal degree of freedom of a cavity, the input beam should also be aligned with the optical axis of the cavity. We can define two degrees of freedom related to this alignment, a transverse

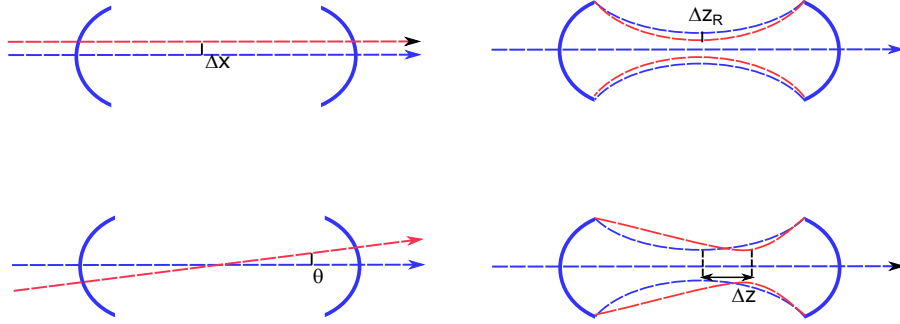


Figure 2.6: Mismatch degrees of freedom between an input beam and a resonator. The two panels on the left refer to transverse offset (top) and angular misalignments (bottom) discussed in Section 2.3. The two panels on the right refer to waist size (top) and position (bottom) mismatch discussed in Section 2.4.

displacement Δx of the input beam and an angular tilt θ of the input beam direction with respect to the cavity axis. Both of these are shown in Fig. 2.6.

D. Z. Anderson, showed that a Gaussian input beam, laterally offset at the waist in the x direction by Δx , may be described as (taking into account only 1 dimension and using the Hermite polynomials) [13]

$$\begin{aligned} U(x + \Delta x, y) &= A_x H_0(x + \Delta x)^2 + A_y H_0(y) \\ &= A_x \left(\frac{2}{\pi \omega_0^2} \right)^{\frac{1}{4}} e^{-\frac{(x+\Delta x)^2}{\omega_0^2}} + A_y \left(\frac{2}{\pi \omega_0^2} \right)^{\frac{1}{4}} e^{-\frac{y^2}{\omega_0^2}}, \quad (2.15) \end{aligned}$$

where H_0 is the zeroth order Hermite-Gauss polynomial introduced in Section 1.3.1, A_x and A_y are the field amplitudes in the x and y directions and ω_0 is the waist radius of the cavity (Eq. 1.28), because we are working in the cavity basis. Expanding the exponent and dropping the factors of second order and higher results in

$$\begin{aligned} U(x + \Delta x, y) &\approx A_x \left(\frac{2}{\pi \omega_0^2} \right)^{\frac{1}{4}} \left[1 + \frac{2x\Delta x}{\omega_0^2} \right] e^{-\frac{x^2}{\omega_0^2}} + A_y H_0(y) \\ &\approx A_x \left[H_0(x) + \frac{\Delta x}{\omega_0} H_1(x) \right] + A_y H_0(y), \quad (2.16) \end{aligned}$$

where H_1 is the first order Hermite-Gauss polynomial. This mean that a laterally offset input Gaussian beam is equivalent to the zeroth order of the cavity basis plus some first-order mode in proportion to the amount of offset. Similarly, an input Gaussian beam tilted by an angle θ , will be equivalent to, in the

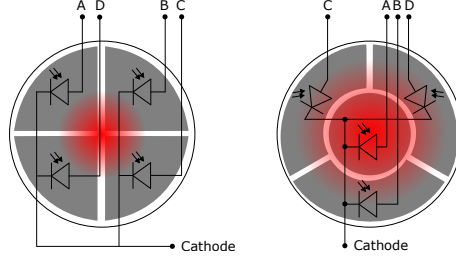


Figure 2.7: Frequently used segmented photodiodes, adapted from Fig. 2. of Ref. [2]. The left panel depicts a quadrant photodiode, and the right depicts a bulls-eye photodiode.

basis of the cavity [13],

$$U(x, y) = U(x', y) \cos(\theta)^{-1} \approx A_x \left[H_0(x) + i \frac{\theta \pi \omega_0}{\lambda} H_1(x) \right] + A_y H_0(y), \quad (2.17)$$

where the expression is valid for $(\theta \pi \omega_0)/\lambda \ll 1$. Noting that $\lambda/(\pi \omega_0)$ is the far-field beam divergence half-angle, it means the approximation is valid when the input beam tilt is much smaller than this angle. Note additionally that the coupling to H_1 scales in proportion to the far-field divergence angle. Both transverse offset and tilt can happen in either axis, and the above equations are changed simply by changing x with y .

Initially, D. Z. Anderson proposed measuring such mismatches in transmission of the cavity by modulating the input beam at the mode separation frequency, thus injecting the first-order modes of the sideband in the locked cavity [13]. Nowadays, both misalignments and transverse offsets are detected by combining the heterodyne techniques of PDH with a QPD (see left panel of Fig. 2.7), instead of a regular one [134]. This is typically called the Ward technique.

Instead of measuring the beat between the resonant carrier and sidebands, the Ward technique measures the beat between the sidebands and the promptly reflected first order modes. In order to do this, the QPD is used due to the shape of the H_{G10} and H_{G01} modes (refer to Fig. 1.5). For offsets and tilts in the x direction, the signals come from the sections $(A+D)-(B+C)$ and in the y direction from the sections $(A+B)-(D+C)$ (Fig. 2.7). Note the i factor in Eq. 2.17 and its absence in Eq. 2.16. This means the offset and misalignment signals are off by 90° in phase, and normally two QPDs are used, separated by 90° in Gouy phase, to measure separately each degree of freedom.

2.4 Mode matching

We saw in Section 1.3.1 that the cavity mode basis is defined by its geometry through the cavity g-factor and it is characterized by a certain beam waist ω_0 and waist position, z . Together, they make the cavity q beam parameter (Eq. 1.25). An input beam to be injected into the cavity can be characterized by its own q parameter. We will see that in order to maximize the coupling of input beam into the fundamental mode of the cavity, the input beam q needs to be matched to that of the cavity. Thus, we can write the mode matching condition as [135]

$$\mathcal{M} = i \left(\frac{q_2 - q_1}{q_2 - q_1^*} \right), \quad (2.18)$$

where q_2 and q_1 may be the parameters of any beams or cavities being matched. For the sake of the discussion here let's say q_1 is the parameter of the cavity and q_2 of the input beam. To first order, $\mathcal{M}$ can be written as

$$\mathcal{M} = i \frac{\epsilon_q}{2} + \mathcal{O}(2), \quad (2.19)$$

where we have defined the mismatch parameters ϵ_{z_R} and ϵ_z [11, 135]

$$\epsilon_q = \frac{q_2 - q_1}{z_{R,1}} = \frac{\Delta z}{z_{R,1}} + i \frac{\Delta z_R}{z_{R,1}} = \epsilon_z + i \epsilon_{z_R}, \quad (2.20)$$

where z_R refers to the Rayleigh range $\pi\omega_0^2/\lambda$ and Δz_R and Δz are the differences in Rayleigh range and waist position between beam parameters 1 and 2. Note that this approximation is valid only in the limit of small mismatch where $z_{R,1} \approx z_{R,2}$. Note as well that waist size and position mismatch are off by 90° , that is why they are called the two mismatch quadratures (shown schematically in the two right panels of Fig. 2.6).

The amount of mode mismatch can be characterized by the real parameter

$$M = |\mathcal{M}|^2, \quad (2.21)$$

which is equivalent to the fraction amount of the beam characterized by q_1 which is coupled to HOMs in the presence of mode mismatch.

As we discussed in Section 1.3.1, the input beam is decomposed in the cavity eigenmodes (Eq. 1.31). In the case of mode mismatch, Bayer-Helms showed that, in cartesian coordinates, all the coupling coefficients $k_{mnm\bar{n}}$ vanish except for the ones with even $m+n$ order (case 4 of Section III c) in Ref. [11]). In one dimension, using our notation, in the case of mode mismatch the

non-zero coefficients are given by [2, 135]

$$k_{n,\bar{n}+2} = \frac{\sqrt{(n+1)(n+2)}}{4} \left(\frac{i\Delta z - \Delta z_R}{\bar{z}_{R,1}} \right). \quad (2.22)$$

This means that for an input zeroth mode, power is scattered to second-order modes according to

$$k_{0,2} = \frac{1}{2\sqrt{2}} \left(\frac{i\Delta z - \Delta z_R}{\bar{z}_R} \right). \quad (2.23)$$

Under the assumption of no astigmatism (waist size and position are the same in both x and y axis), mode mismatch can be written in terms of cylindrical modes [13]. In this case, and according to Eq. 2.23, power scatters equally to the HG_{20} and HG_{02} modes, and the overall mode shape is that of the LG_{01} (see Fig. 1.5). Mathematically, for the waist size mismatch we have

$$U(r) \approx A [\text{LG}_{00}(r) + \varepsilon_{\omega_0} \text{LG}_{10}(r)], \quad (2.24)$$

where ε_{ω_0} represents the small fraction by which the input beam waist is mismatched $\omega_{0,2} = \omega_{0,1}(1 + \varepsilon_{\omega_0})$. And for the waist position mismatch

$$U(r) = A \left[\text{LG}_{00}(r) + i \frac{\Delta z}{2\bar{z}_{R,1}} \text{LG}_{10}(r) \right], \quad (2.25)$$

where the input beam waist is offset by Δz with respect to the cavity waist position (right bottom panel in Fig. 2.6).

Typically in GW detectors, optical cavities are locked on the laser fundamental mode. Any second-order modes from mode-mismatch are promptly reflected while the cavity is locked due to the different resonance condition of HOMs (recall Eq. 1.30). This means that in practice, mode mismatch is a source of optical loss when cavities are used.

2.4.1 Mode mismatch and quantum noise reduction

Mode mismatch in GW detectors can arise due to changes in the RoC of the various optics due to manufacturing tolerances, or due to thermal effects as we will see in the next Section. These can cause trouble during commissioning of the detector and adaptive optics techniques or beam telescope adjustments become necessary (see for the case of Virgo Sections 3.4 and 4.1).

Additionally, mode mismatch has a significantly detrimental impact on light squeezing. It impacts squeezed light injection in two ways, as a direct source of optical loss and through a squeezing dephasing mechanism.

Optical losses in general degrade the amount of squeezing achieved in a GW detector. Current detectors have achieved 6 dB of injected squeezing (suppression of quantum noise below the SQL) [36]. In turn, the record amount of squeezing is, up to now, 15 dB with a table-top set-up [136]. A source of optical loss can be conceptualized as a beam splitter, which reduces the field amplitude but also mixes in unsqueezed vacuum, decreasing the squeezing level. Thus, the amount of losses in the system, $\mathcal{L}$, dictate the amount of squeezing S achieved according to [137]

$$S = \beta^{-1}(1 - \mathcal{L}) + \mathcal{L}, \quad (2.26)$$

where $\beta = e^{2R}$ is the squeezing factor (recall Section 1.4). Fig. 2.8 shows the achievable squeezing as a function of the injected squeezing for various amounts of optical loss in the system.



Figure 2.8: Achievable squeezing for a given squeezing injection and loss. Reproduced from Ref. [2].

Optical loss can be due to absorption in the optics, beam clipping, contaminants in mirror surfaces and such. As mentioned above, mode mismatch is also a source of optical loss. However, besides contributing to squeezing degradation as a source of direct loss, mode mismatch is also a source of decoherence or phase mixing. This happens because as power is scattered to second-order modes (or any HOM) at a given mismatched interface, power can scatter back from the HOM to the fundamental mode at another mismatched interface. Due

to different Gouy phase accumulation, the HOM now scattered back to the fundamental mode will have accrued a different phase and its squeezing angle can be rotated with respect to the one of the fundamental mode. This introduces a frequency-dependent degradation of squeezing that is difficult to model as it depends on the optical path, on the HOM and on the different mismatched interfaces. A significant effort has been made towards modeling these effects in the context of the LIGO detectors [138].

In Ref. [18] the authors simulated the impact of mode-mismatch at different interfaces in a GW interferometer. It was observed that mode-mismatch between the squeezer and the filter cavity had the most detrimental impact (Fig. 3 of Ref. [18]). Around the resonance frequencies of the spatial modes from mode mismatch, an increased degradation of squeezing was observed due to the decoherence effect mentioned above. Losses in the filter cavity itself are non-trivial, as besides the impact of direct losses, there is another decoherence effect due to changes in the squeezing angle rotation induced by the presence of losses (see the discussion in Ref. [137] and the mathematical treatment in Ref. [89]).

Research, in simulation [18] and in the lab [139], has looked at squeezing the HOMs that couple into the detector to increase the robustness of squeezing to the presence of mode mismatch. In this work, however, I focus on sensing mode mismatch so that it can be mitigated by the use of adaptive optics or telescope adjustment.

2.5 Thermal effects

At hundreds of circulating kW of optical power, even with very low absorption materials and very low-loss coatings (a few tens of ppm in the case of current detectors), some significant power dissipation can be expected at the test masses of a GW interferometer. This dissipated power leads to temperature gradients in the coating and bulk of the optics which in turn lead to index gradients and geometric deformation of the mirrors.

The temperature profiles for bulk substrate and mirror coatings due to power absorption were derived analytically by Hello and Vinet for the static case [140] and for the thermal transient case [141]. The optical path length (OPL) due to the temperature T in the mirror can be written as (see Chapter 3.5 of Ref. [142])

$$\text{OPL} = n(T, L)d, \quad (2.27)$$

where d is the path length over which the refractive index, n , can be taken to be constant. The change in OPL, due to δT temperature changes around an average temperature T_0 will be

$$\begin{aligned}\delta OPL &= \frac{dOPL}{dt}(T_0)\delta T \\ &= d \frac{\partial n}{\partial x^\nu} \frac{dx^\nu}{dT} \Big|_{T_0} \delta T + d \frac{\partial n}{\partial T} \Big|_{T_0} \delta T + n(T_0) \frac{\partial d}{\partial T} \Big|_{T_0} \delta T. \quad (2.28)\end{aligned}$$

The first term is the elasto-optic effect which describes the stress on the refractive index due to the thermal expansion of the material. The second term is the thermo-optic effect which is the change in refractive index with temperature and the third term is the thermoelastic deformation which is due to the thermal expansion of the mirror.

Consider the mirror and the scheme of Fig. 2.9a. The reference planes p_1 and p_2 are there to facilitate treatment of the anti-reflective (AR) and highly-reflective (HR) sides (I have followed the treatment of Chapter 3.5.2 of Ref. [142]). The δOPL for the transmitted light and for the light reflected from the HR side will be

$$\delta OPL_t = \frac{\partial d_{p_1}}{\partial T} \Big|_{T_0} \delta T + n(T_0) \frac{\partial h}{\partial T} \Big|_{T_0} \delta T + h \frac{\partial n}{\partial T} \Big|_{T_0} \delta T + \frac{\partial d_{p_2}}{\partial T} \Big|_{T_0} \delta T \quad (2.29)$$

$$\delta OPL_{HR} = 2 \frac{\partial d_{p_2}}{\partial T} \Big|_{T_0} \delta T, \quad (2.30)$$

where h is the thickness of the mirror and d_{p_1} the distance from left reference plane to the AR surface and d_{p_2} the distance from the right reference plane to the HR surface. Note that I dropped the elasto-optic effect as it is negligible for fused silica and also the thermo-optic deformation in the case of the coating surfaces. If we assume the thermal expansion is axially symmetric, then $\partial d_{p_1}/\partial T \approx \partial d_{p_2}/\partial T$. Note also that propagation in air is opposite of propagation in the mirror, such that $\partial d_{p_1}/\partial T = \partial d_{p_2}/\partial T = -(\partial h/\partial T)/2$ (since the origin is conveniently placed at the middle of the mirror, recall Fig. 2.9a).

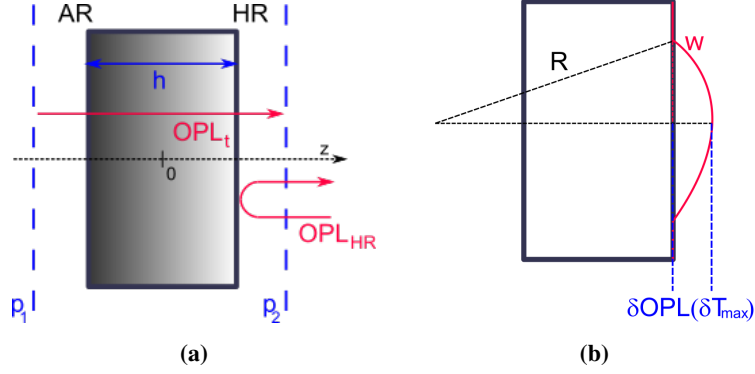


Figure 2.9: In (a) a schematic of the optical path lengths defined in Eq. 2.30 is shown. The z axis in the direction of propagation is shown, with the origin at the center of the mirror. In (b) a schematic of a thermal lens caused by the heat profile (computed in [140]) of a beam with radius w . Used to derive Eq. 2.33.

In this way, we can write

$$\delta OPL_t = (n(T_0) - 1) \frac{\partial h}{\partial T} \big|_{T_0} \delta T + h \frac{\partial n}{\partial T} \big|_{T_0} \delta T \quad (2.31)$$

$$\delta OPL_{HR} = - \frac{\partial n}{\partial T} \big|_{T_0} \delta T. \quad (2.32)$$

Eq. 2.31 represents the thermal deformation in a transmissive optical element such as the compensation plates introduced in Section 2.7. Eq. 2.32 represents the thermal deformation in reflection from a mirror.

In order to actually derive the final shape of differential OPL, δT needs to be replaced by the temperature profiles derived by Hello and Vinet [140]. If the mirror and coating surfaces are heated by a Gaussian beam, the maximum δT occurs at the center of the mirror and it goes down with decreasing power of the beam. In this way, the expected deformation from thermal absorption is that of a thermal lens (see schematic of Fig. 2.9b). Note the opposing sign for the differential OPL experienced by the transmitted and by the reflected beams. This means that transmitted and reflected beams experience a thermal lens with opposing sign.

We can estimate a change in the RoC of the optic due to the thermal lensing. This is easier to do assuming a mirror that is initially flat at rest temperature T_0 , and that the curvature is only significant up to beam radius w (see Fig.

2.9b), in that case, we can estimate the thermally induced change in RoC R as

$$w^2 + [R - \delta OPL_{HR}(\delta T_{max})]^2 = R^2$$

$$\Rightarrow R \approx \frac{w^2}{2\delta OPL_{HR}(\delta T_{max})}, \quad (2.33)$$

where $\delta OPL(\delta T_{max})$ is the change in OPL at the center of the beam and the approximation $\delta OPL(\delta T_{max})^2 \approx 0$ was used. Typically, the change in RoC expected in GW detectors is in the order of a few tens of meters. Bear in mind that the RoC of the test masses is in the order of 1.5 km.

The time-scale of these thermal effects can be estimated assuming the whole mirror of mass M heats up homogeneously up to the average temperature $\tilde{T}$ of a given temperature profile. In this way, the time needed for this change in temperature is given by

$$\Delta t = \frac{C_p M \tilde{T}}{P}, \quad (2.34)$$

where C_p is the specific heat of the bulk material and P the laser power. Given that the test masses in GW detectors weight 40 kg and are made of fused silica, the time transients are very slow. With a specific heat of $C = 745 \text{ J kg}^{-1} \text{ K}^{-1}$ for fused silica and for $\tilde{T} = 0.05 \text{ K}$ taken from Ref. [142], a time transient of 4 hours is obtained. This is similar to what is currently observed at Virgo.

In Ref. [21], in a near-concentric cavity with $\approx 130 \text{ kW}$ of circulating power, thermal expansion lasting a few hours was observed. In Ref. [23], a bow-tie cavity with 700 kW of circulating power, within a few seconds of locking the fundamental mode is converted into degenerated cavity modes due to the change in the cavity eigenmode. In both cases, a change in the cavity waist size is observed due to the change of the mirrors RoC (recall Eq. 1.28).

In O3, it was observed at Virgo that the cavity round-trip losses (RTL) increased with increasing input power. It was found that this was not from the change in mode matching between input beam but due to the presence of point absorbers [143]. Point absorbers are microscopic defects, typically present in the mirror coatings, with a significantly higher amount of absorption compared to the rest of the coating material. Due to their small size they can scatter power into HOMs whose resonance is close to that of the fundamental mode. These HOMs, typically of very high-order cover a much larger area and are subject to significant clipping losses, thus driving the observed increased RTL.

In preparation for O4, increased RTL were also observed with increasing input power [113]. Mitigation of point absorbers is an active area of research

[144, 145], and even though it is not the subject of this work, the detection of higher-order aberrations as discussed below and in Chapter 7 is relevant to this topic as well.

2.6 Wavefront sensors for mode sensing

In this section I will briefly review the state-of-the-art of wavefront sensors for mode sensing and in particular for mode mismatch. The text closely follows the review article of Ref. [2] of which I am one of the co-authors.

2.6.1 Hartman sensors

Hartmann Wavefront Sensor (HWS) are used by both LIGO and Virgo as described in [146]. These devices differ from the classical Shack-Hartmann sensors [147], common in other adaptive optics applications, as the micro-lens array is replaced with a plate of uniformly spaced pinholes, though the operating principle is the same.

A reference wavefront through the pinhole plate generates a given pattern on a CCD. When the wavefront changes, this change $\Delta\varphi$ can be inferred from the Δx change in the position of the bright spots according to (Fig. 2.10)

$$\frac{\partial \Delta\varphi}{\partial x} = \frac{\Delta x}{L}. \quad (2.35)$$

Forgoing the lens array allows for higher sensitivity as the effect of aberrations from imperfections in the lenses is removed, but also results in greatly reduced light collecting efficiency [146, 147].

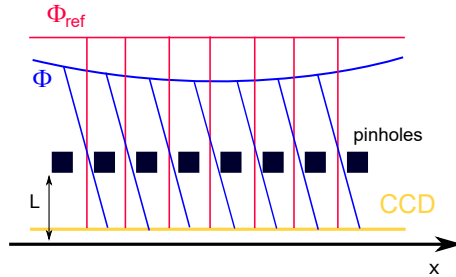


Figure 2.10: Schematic of a Hartman wavefront sensor (HWS). A plate of pinholes allow for comparing a wavefront with a reference according to how the bright pattern changes on a CCD screen.

In the GW detector implementation, probe beams of incoherent light are retro-reflected from the main optics and onto the HWS. The measured distortions of these probe beams are, hence, related to distortions in the actual test masses themselves.

Even though only changes to the wavefront can be measured, the HWS can be calibrated to the cold mirror surface maps, which are measured independently by a different method. As the mirror thermally deforms, the HWS can then provide information on the current absolute mirror surface to a high precision.

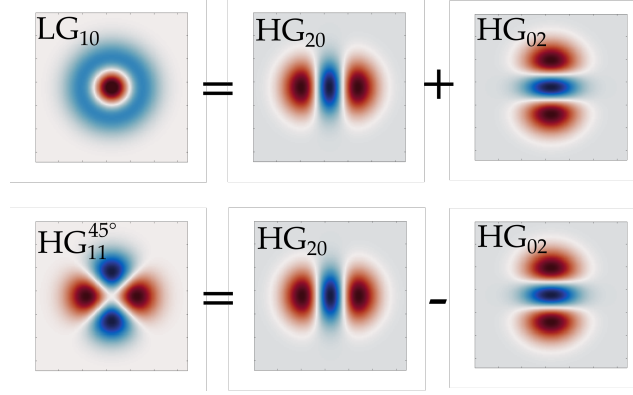
The Hartmann sensor is an indirect method of sensing mismatch. It can measure wavefront deformations, but cannot measure directly the input beam and cavity eigenmodes. It can however, track the changes in RoC of the optics, and is used to guide the thermal compensation system discussed in Section 2.7.

2.6.2 Resonant wavefront sensing

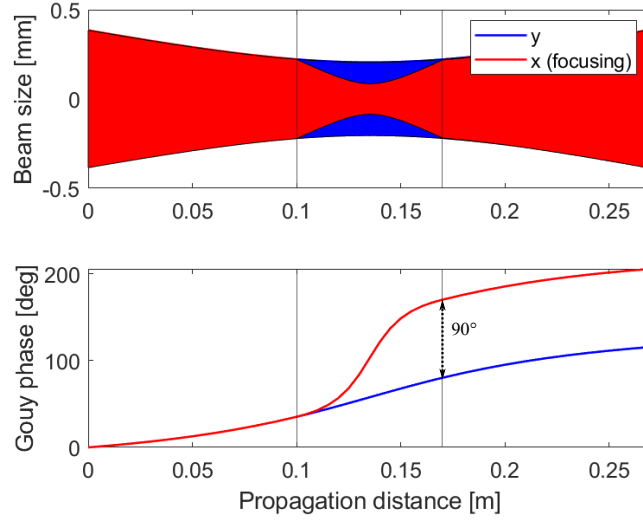
In the year 2000, Mueller and others proposed extending the alignment sensing and control schemes of Section 2.3, to mode matching control [148]. To do this, a bulls-eye photo-detector (BPD), shown schematically in Figure 2.7 has the appropriate segmented geometry to break the orthogonality between the LG_{10} mode and the reflected HG_{00} cavity locking sidebands. By using two BPDs, separated by 45° Gouy phase, it is possible to simultaneously gather information on ε_z & ε_{z_R} . One of the issues with this technique is that BPDs are difficult to source, furthermore, overlapping requirements on the accumulated Gouy phase & beam radius at the BPD mean that, likely, a custom BPD is required.

Mode converters

One solution to the above limitations is to sidestep the need for a BPD entirely by converting the LG_{10} mode associated with mode mismatch into a HG_{11} *pringle* mode which is more convenient for detection with a common place QPD. Because both HG and LG modes form a complete basis to describe laser beam modes, one can decompose LG_{10} into a combination of two HG modes: HG_{20} and HG_{02} [149]. Additionally, a 45° rotated HG_{11} mode can also be decomposed in the same combination of HG modes, minus a sign flip, as shown in Fig. 2.11a. So, by adding a $\pi/2$ phase shift along one axis, it is possible to convert LG_{10} into a 45° rotated HG_{11} .



(a) Decomposition of the LG_{10} and 45° rotated HG_{11} in the HG basis.



(b) Plot of the beam size and Gouy phase accumulation on the x and y axis as the beam travels through a mode converter.

Figure 2.11: Mode conversion can be used to make the LG_{10} mode detectable on a QPD. Image reproduced from [2]

Generally, this is done using two cylindrical lenses to create a confined region where the beam is astigmatic. Provided the lenses are positioned so that the beam is no longer astigmatic outside of the converter, the additional Gouy phase shift along the astigmatic axis remains constant after the converter and subsequent detection is possible on a QPD. Consider a beam (along the non-focusing axis) with Rayleigh range, z_R and beam-waist position, z_0 . The

desired Gouy phase shift will be achieved with cylindrical focal length [149],

$$f = \frac{z_R}{1 + 1/\sqrt{2}}. \quad (2.36)$$

The lenses must be placed at $z^\pm = z_0 \pm f\sqrt{2}$ to mode match the output beam while achieving the Gouy phase shift. Fig. 2.11b shows the beam size in the astigmatic and non-astigmatic axis, and the evolution of the Gouy phase shift for a mode converter using cylindrical lenses. Mode converters with the same principle have also been realised using spherical mirrors at a 45 ° angle [150].

Typically, mode converters are used in conjunction with heterodyne techniques to generate an error signal for mode mismatch. In [151], frequency-shifted sidebands in the HG₀₀ mode beat against the converted HG₁₁ on a QPD to generate an error signal. Again, by placing a beam splitter and two sets of QPDs separated by 45 degrees in Gouy phase, it is possible to get separate error signals for waist size and waist position mismatch respectively. Such a mode converter set-up is currently being used to mode match the filter cavity for frequency-dependent squeezing at Virgo [152]

One limitation of the mode converter is that it assumes an LG₁₀ mode and is thus incompatible with astigmatism. The frequency dependent source uses curved mirrors in off-axis telescopes in order to reduce optical losses [153], causing some small astigmatism by design. This astigmatism limits how much mode matching can be achieved with the current mode converter [152]. This issue, however, is also addressed in Ref. [152] where an updated scheme capable of detecting astigmatism is developed.

2.6.3 Mode weighting

We saw in Section 1.3.1 that any beam can be decomposed in a sum of either HG or LG modes (Eq. 1.27). The process of determining the parameter $|k_{nm}|$ for a given electric field is called modal weighting. In practice, it is the mode power $|k_{nm}|^2$ that one is able to measure. Measuring or estimating the complex mode weight k_{nm} is called modal decomposition, which I will refer in the context of phase cameras in the next section.

Diagnostic board

One of the ways to determine the modal weights is to perform a mode scan. To perform the mode scan, a special optical resonator is assembled which should

be stable, with clear separation between different modes. Usually, a cavity like the Virgo/LIGO output and input mode cleaners is used.

Kwee et al. developed a diagnostic breadboard based on the LIGO pre-mode cleaner, [154] that was able to perform the mode weighting along with measurement of the beam pointing and relative intensity noise [155]. This method was successfully used in 2015 to characterize thermal lensing in various materials [156] and has inspired follow-on work in the field of free space optical communications [157].

Spatial filters

An alternative approach is to use a spatial filter to compute the fraction of light in a particular mode. This is achieved by passing the beam through a basic optical convolution processor, consisting of a phaseplate, lens, aperture and photodiode. The on-axis intensity in the far-field is proportional to the overlap between the transmission function of the phase plate and the laser beam [158]

$$I(0, 0, z_{PD}) \propto k_{m,n}^2 / f^2 \lambda^2, \quad (2.37)$$

where f is the focal length of the lens used and λ the wavelength of the laser used. By using different phase plates or using spatial phase modulators [159], different $n+m$ modes can be weighted. There has also been work to develop phase transmission elements using computer generated holograms to [160].

Within the gravitational-wave community, mode weightings at the part-per-million level are required. A thorough tolerance analysis of spatial filters found that the mode precision is limited by: a) the relative positioning accuracy and b) finite aperture effects. These cause cross-coupling of unwanted modes [158]. Overcoming these limitations resulted in a mode (power) weighting precision of 4000 ppm. Further investigation used a meta-material spatial filter to achieve a mode (power) weighting precision of 0.6 ppm/ $\sqrt{\text{Hz}}$ [161]. The optical apparatus was fully integrated into the LIGO Control and Data System. Further work is still required before full integration in GW detectors is possible.

2.6.4 Phase cameras

The first phase camera was developed to image the differential wavefront of the GWD sidebands compared to the carrier [162]. In the first demonstration [163], a frequency offset reference beam was combined with the test beam

on a beamsplitter. The field was then reflected from a pair of galvanometer-driven steering mirrors onto a pinhole photodiode. By demodulating the photodiode signal at the difference between the reference field and the test fields, phase and amplitude maps could be obtained of the carrier and sidebands independently.

Since then, the initial design has seen significant development. Other designs have also been developed, such as the optical lock-in and time-of-flight phase cameras which I describe below. Before, I start by detailing scanning pinhole phase cameras, which follow the original design and are currently installed at the Virgo detector [142].

Scanning pinhole phase camera

This type of phase camera consists of a pinhole and a fast photodiode (so that the heterodyne beatings of interest can be detected) combined with a 2-axis piezo scanner (see left panel of Fig. 2.12). The piezo scanner moves the beam over the (much smaller) photodiode, and data is acquired for each pixel synchronously.

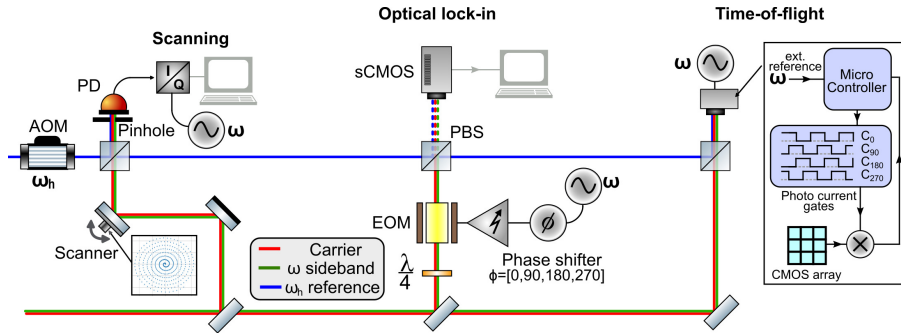


Figure 2.12: Adapted from Fig. 5 of Ref. [2]. From left to right, scanning pinhole phase camera, optical lock-in camera and time-of-flight camera.

Data acquisition is made with high-dynamic range ADCs connected to a FPGA which digitally demodulates the signal at each beat frequency of interest. Demodulation is done for both I and Q quadratures, and for each pixel and beat frequency, the amplitude can be reconstructed according to

$$A = \sqrt{I^2 + Q^2}, \quad (2.38)$$

and the phase according to

$$\Phi = \arctan\left(\frac{Q}{I}\right). \quad (2.39)$$

In this way, and with knowledge of the scanner position for each data point, amplitude and phase images can be reconstructed for all demodulated frequencies. A frequency-shifted reference beam can be used to generate extra heterodyne frequencies for the carrier and each sideband (see Fig. 2.13). This allows imaging separately the carrier and each sideband.

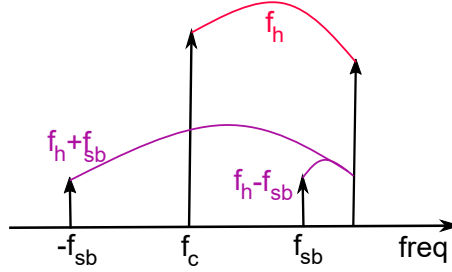


Figure 2.13: Heterodyne beats with the addition of a frequency shifted (by f_h Hz) beam. Independent heterodynes with each upper and lower sideband can be retrieved.

The phase camera can operate in a 2-beam scanning configuration, where both signal and reference beams are scanned, or in a 1-beam scanning configuration, where only the test beam is scanned and the reference beam beats with the test beam at a beam splitter placed after the scanner. The 2-beam scanning configuration is shown in Fig. 2.14b.

The 1-beam scanning configuration is shown in Fig. 2.14a. Because the signal beam wavefront is tilted with respect to the reference beam, there is a fringe interference effect which introduces some visibility loss. The effect is smaller for smaller scanning angles, so 1-beam scanning typically uses larger distances between scanner and photodiode. The fringing also introduces a spatial offset (because of the varying scanning angles during acquisition) to the phase signal. Qualitative details are given in Section 5.1.4 because 1-beam scanning was the configuration that was implemented in the lab.

In 2-beam scanning, larger scanning angles can be used since the reference and signal beams are scanned together. However, SNR is decreased away from the center of the beam at the Gaussian skirt. So, if the signal beam has

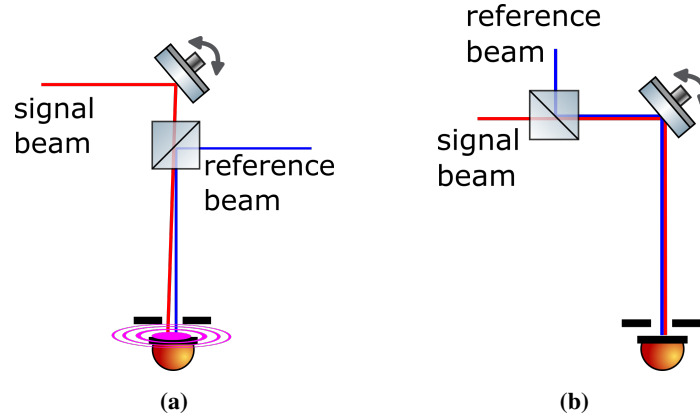


Figure 2.14: Simplified schematic of the 1-beam scanning phase camera configuration on panel (a). The tilting of one wavefront with respect to another results in fringes shown in purple. Simplified schematic of the 2-beam scanning phase camera configuration on panel (b).

significant HOM content, the intensity needs to be calibrated to take this into account.

The phase cameras currently installed at Virgo use the 1-beam scanning configuration because there is enough space to keep the angles small and high SNR is important. Current technology allows for simultaneously imaging 11 sidebands (upper and lower sideband at 5 demodulation frequencies plus the carrier). Each image has 2^{14} pixels, with maximum phase resolution $\sim \lambda/1600$ (at the centre of the beam) acquired in under a second. The beam is scanned following an Archimedean spiral at a constant velocity. This makes for a smooth actuation of the scanner while avoiding changes in scanning frequency [20]. The spiral scanning pattern is shown in the left panel of Fig. 2.12.

Due to the slow scanning speed when compared to environmental phase fluctuations, this resolution is only achievable for differential phase images between carrier and sidebands. In the case of Virgo, these fluctuations come from the propagation of the reference beam in the optical fiber and also the residual motion of the benches themselves. The subtraction of common phase noise also removes the spatial phase offset due to 1-beam scanning.

Optical lock-in phase camera

These phase cameras use amplitude modulation to optically demodulate the RF beat down to ~ 100 Hz where it can be measured with a standard CCD camera.

The amplitude modulation is achieved with a Pockels cell & waveplates, which first modulate the polarisation, and a Polarising Beam Splitter (PBS) which converts this into amplitude modulation (see the middle part of Fig. 2.12).

The Pockels cell is driven at the desired sideband frequency to modulate the polarization of the input beam, with large voltages ~ 1 kV being required to achieve adequate modulation depths. Amplitude & phase maps are acquired by subsequently stepping the phase of the amplitude modulation through $\varphi = [0, \pi/2, \pi, 3\pi/2]$ and recording images. The four images taken are then digitally processed into amplitude & phase maps.

The technique was proposed in [164], where a 2 Mpx camera is used to produce images with sensitivity up to -62 dBc after 2 s of averaging. Subsequently, the camera has been trialled at LIGO [165] and used to image the HOM causing a parametric instability [101]. However, the camera can only image one sideband at a time.

Time-of-flight phase camera

A phase camera can also be realized using time-of-flight (ToF) cameras and their in-pixel demodulation capabilities. The operating principle of ToF cameras is that a set of infrared emitters are amplitude modulated with a RF. The back-reflected light is collected by a pixel array, of which each pixel has typically four collection nodes. Each node is triggered to collect charges at 4 different phases, synchronized with the RF modulation signal applied to the emitters. The voltage difference between the different nodes is a function of the phase of the captured light. By taking the ratio of the charges collected in each node, the distance can be estimated.

The idea to use the ToF camera as a phase camera, proposed by the Advanced Virgo team (section 7.7.1.3 in Ref. [105]), is to replace the RF modulated source with the beam coming from the interferometer, as shown in right panel of Fig. 2.12. The nodes of each pixel are clocked at the sideband RF (or at the heterodyne shifted frequency if a reference beam is used), similar to the optical demodulation approach discussed in Section 2.1. This means that only one spectral component can be interrogated at a time. Meanwhile, up to 100 MHz demodulation with -62 dBc sensitivity has been demonstrated [166].

Spatial light modulator phase camera

Spatial light modulator phase cameras allow for scanless imaging of heterodyne frequencies. The signal beam is reflected from a SLM, and it can be

recombined or not with a reference beam. The wavefront of the signal beam is modulated by the SLM, encoding the spatial information on the phase of the beam [167]. The beam is focused on a single photodiode, and by demodulating the signal the image can be reconstructed with knowledge of the SLM encoding. Since the image is obtained simultaneously and without moving parts, sensitivity to environmental noise is much reduced compared with the scanning phase camera. However, the issue is reduced spatial resolution and the need for concurrent demodulation for each pixel. For an SLM with 9 pixels, 9 concurrent demodulations must take place. As this technique is less developed than the others, it was not included in Fig. 2.12 or Table 2.1.

Summary

Table 2.1 compares the performance of scanning, optical lock-in and ToF phase cameras. The minimum beam-size change detectable depends mostly on the number of pixels. The spatial mode weighting precision was computed taking into account the smallest beam size change and Eq. 2.23.

Table 2.1: Phase camera parameter comparison reproduced from [2]. The values presented were extracted or derived from [20, 164, 166, 168] respectively. The detectable beam diameter change assumes the beam diameter on the camera is 1/3 of the aperture, the detectable change is $3/(\text{Num. Pixels})$. Spatial (mode weight) precision is computed from the spot size change using Eq. 2.23. The sensitivity for the scanning phase camera was computed taking into account the optimal power in the reference beam [168].

	scanning	optical lock-in	time-of-flight
Pixels (px)	128x128	2048x2048	320x240
Frame rate (fps)	1 (max. 10)	10 (max. 100)	max. 60
Sensitivity (dBc/px)	-61 (1 fps)	-62 (0.5 fps) -72 (120x128 px, 1 fps)	-62 (1 fps) -50 (7 fps)
beam $\Delta w/w$ (%)	2.3	0.15	1.1
Mode weight (ppm)	16500	1100	7800
Phase RMSE (nm)	0.7	Not available	0.1
Bandwidth	250 MHz	100 MHz	100 MHz
Num. demodulations	11	1	1

In Ref. [169], a scanning pinhole phase camera, similar to the ones installed at Virgo, was combined with a polarizing beam splitter and two fast photodiodes to characterize the birefringence of a sample.

Because phase cameras can measure amplitude and phase of the entire beam, they can actually be used to decompose the beam in terms of the complex coefficients k_{mn} . Some initial tests for modal decomposition with the Virgo phase cameras are described in chapter 6.14 of Ref. [142]. More recently, I also evaluated the mode of the Virgo arm cavities using modal decomposition of the phase camera images as discussed in Section 3.5. Another work has explored using neural networks to determine the mode decomposition of the beam [170]. This last work was done in the context of the optical lock-in camera. The same phase camera has also been used to make the first image of PI-causing HOM [101].

2.6.5 Radio-frequency modulation sensing schemes

The first RF Beam modulation proposal paper is detailed in Ref. [135]. The premise is to generate a frequency offset beam-shape modulation using an AOM and a phase-plate to generate second-order modes ². In this way, the carrier is at $HG_{0,0}^{\omega_0}$ and the modulation is at $HG_{2,0}^{\omega_0+\Omega} + HG_{0,2}^{\omega_0+\Omega}$.

The error signal can be measured in reflection of the cavity by a photodiode as is the case for PDH locking. On a photodiode where the beam radius is much smaller than the active area, only modes of the same order will produce beat notes due to the orthogonality of the HG modes. Therefore, in the presence of a mismatch, there will be a beat with frequency Ω between the second-order mode of the carrier and the second-order single sideband. The two mode-matching quadratures z and z_R can be read out with appropriate choices of demodulation phase as mentioned above for the other sensing schemes.

In contrast to generating the modulation with an AOM and phaseplate, several authors have begun exploring the possibility of using an electro-optic lens. Initial results were shown by two independent groups in 2020 [171, 172]. Proof-of-principal work was presented in 2023 [173].

Another approach is to forego the phaseplate and inject only the frequency offset beam. Thus relying on the mode mismatch to excite the higher-order

²In Ref. [135] in the table-top demonstration of this scheme a Mach-Zender interferometer and mismatched cavity locked on the transverse second-order modes is used to generate the necessary modes. However, the proposed scheme for GW detectors uses a phase-plate for the same goal.

mode. If the modulation frequency is equal to twice the mode separation, $\Omega = 2\nu_m$, then the sideband second-order modes from the cavity mismatch will co-resonate with the carrier in the locked cavity. In general, these do not produce a beat as they are different modes, however on a finite area photodiode or on a BPD a beat note will be produced. The scheme is an extension of the 1984 Anderson alignment sensing proposal [13] and a proposal for using it in linear and coupled cavities was recently published [174] (see Fig. 2.15).

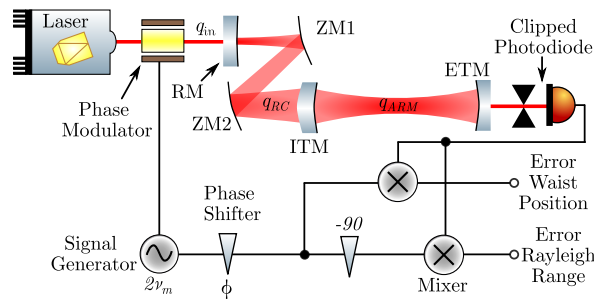


Figure 2.15: Reproduced from [174], used with permission.

In Ref. [174], the authors showed that, provided that the FWHM of the recycling cavity is larger than $2\nu_m$, then the beat in transmission of the coupled cavity will be proportional to the mismatch between the first and second cavity.

The table-top experiment described in Chapter 7 combines the same frequency modulation principles with a scanning-pinhole phase camera in transmission of a cavity in order to measure modes of arbitrary order in a locked cavity.

2.7 Adaptive optics in GW detectors

Adaptive optics, the process by which optical aberrations are corrected by active elements, finds use in various fields such as astronomy, microscopy and vision-science (such as retina imaging) [2]. From developments in these fields, many adaptive optics technologies already exist, but are unsuitable for use in a GW due to requirements on scattered light. For example, SLM are excluded from the main beam path due to large angle scattering. Also, the achievable polishing of ultra-thin mirrors makes many classes of deformable mirrors unsuitable. An exception for this, is the development of deformable mirrors with compression bias to use in the squeezing path of the interferometer [175].

Below, I will focus the discussion on the thermal actuators used for the core interferometer (shown in Fig. 2.4), as these are the more well developed in the field of GWs and the most relevant for this work in particular. These actuators are, for the most part, used to compensate radially symmetrical deformations (thermal lensing) in recycling and arm cavities of the interferometer. In addition to these, there is also work being done at Virgo to address point absorbers (discussed in Section 2.5) using matrix heaters [176,177]. The idea is to use the matrix heater to suppress the resonances of HOMs excited by point absorbers.

2.7.1 Ring heaters

Ring heaters can be used to compensate for thermal lensing and also to correct for small radii of curvature errors. While thermal lensing mainly occurs at the centre of a mirror, because that is where the laser beam deposits its energy, the ring heater heats the outside barrel of the mirror.

Fused silica has both a positive refractive index change with increasing temperature, $dn/dT > 0$ and a positive coefficient of thermal expansion, $dL/dT > 0$. The recycling cavity eigenmodes are affected by both dn/dT and dL/dT of the arm cavity input mirror (as the sidebands go through the IM substrate and come back out). The arm cavity eigenmode is only affected by dL/dT on both test masses (recall Eq. 2.30). Therefore the heating at the edge creates a more convex mirror, offsetting the central heating caused by absorption of the Gaussian laser beam.

The heaters are made of two pyrex rings surrounded by a polished copper shield, which reflects the radiation onto the test mass. The pyrex ring is heated from a conductive wire wrapped around [178, 179]. Because the ring heater acts by reducing the thermal gradient, it can only induce a reduction of the radius of curvature of a mirror. For a typical fused silica mirror with a nominal radius of curvature around 1.5 km, the ring heater at maximum power is able to reduce the radius of curvature by 100 m [178, 179].

In Virgo, ring heaters are used to tune the radii of the input and end mirrors of the arm cavities (shown in Fig. 2.4). Ring heaters are also used to tune the recycling mirrors (both PR and SR) and the optics of the filter cavity for the injection of the squeezing [179].

Recently, Richardson et al. have investigated extending the use of the ring heater concept to the front surface of the test mass. Referred to as a FROnt Surface Type Irradiator (FROSTI), it would allow a non-spherical radius of curvature to be created by thermally tuning a spherical optic [180]. This non-

spherical tuning of the arm cavity mirrors could be used to change the resonance condition of very high order optical modes away from the fundamental mode.

2.7.2 Central heating radius of curvature correction

Ring heaters can only reduce the radius of curvature. While this allows to compensate for thermal effects (thermo-optic and thermo-elastic), there may be situations where it is necessary to increase the radius of curvature of an optic. For example, to compensate for deviations from nominal specifications. The central heating radius of curvature correction (CHRoCC) projects a heat pattern onto the center of an optic, increasing the radius of curvature. The device is a black body emitter inside a heat shield, together with an ellipsoidal reflector used to project the heat into the target optic [181].

Currently, two CHRoCC devices are installed at Virgo (see Fig. 2.4) and being used to increase the tuning possibilities of the power and signal recycling mirrors in Advanced Virgo. In Section 3.4 I detail my role in the tuning of the recycling mirrors using the CHRoCC during the commissioning for O4.

2.7.3 CO₂ lasers and compensation plates

Compensation plates (CP) are dedicated fused silica transmission optics which are used in combination with CO₂ lasers to add thermal correction for the thermal lensing in the arm cavities (see Fig. 2.4). The interferometer is much less sensitive to noise on the CP than noise on the test mass, and so it is preferred to use a dedicated optic for this than to shine the CO₂ laser directly on the test masses [182].

CO₂ lasers emits at a wavelength of $10.6\mu\text{m}$, which is strongly absorbed by fused silica. The absorbed light changes the optical path length through the material and tuning the intensity of the laser adjusts the change to optical path length [178, 183].

The CO₂ laser and CP can be used as either a positive or negative lens [178, 183]. When shining a Gaussian beam at the center of the compensation plate, the temperature rises in the middle and the radius of curvature of the optic is increased. This mode of operation is called central heating.

In Virgo, the aberrations due to the average coating absorption, which are mostly radially symmetric, are corrected for by shining two annuli-shaped ring patterns on the compensation plates. The optimal compensation heating pattern was estimated using a FEA simulation [105]. These two ring-shaped in-

tensity patterns are obtained with two axicon lenses. A combination of waveplates and lenses is used to control the intensity and thickness of each ring respectively. This set-up is called the Double Axicon System (DAS) [183].

Finally the CO_2 laser can also be used in combination with a scanning system [184, 185], to compensate for non-axi-symmetric aberrations. The beam is moved with a constant speed over the compensation plate and the intensity is adjusted for each location according to a correction map. Additionally, work is ongoing at Virgo [186] to use the modified Gerchberg-Saxton algorithm [187] and a commercially available deformable mirror to shape the CO_2 laser beam and develop novel heating pattern beams.

Part II

Virgo in O4 and the phase camera

Chapter 3

Optical characterization and the Virgo phase cameras

In Section 2.6.4 we reviewed the multiple types of phase cameras, a summary comparison is presented in Table 2.1. In this chapter I will discuss the Virgo phase cameras, which are of the scanning pinhole type, how they were used in preparation for O4 and some of my contributions for the optical characterization of the detector.

In Section 3.1 I dive into the specifics of the Virgo detector. Even though the working principle is the same as other ground-based detectors, as discussed in Section 1.3.2, we will see the current configuration of Virgo suffers from marginally stable recycling cavities which increases the requirements on minimizing the mode-mismatch of the sidebands used to control the interferometer.

In Section 3.2 I detail the tests carried out during the commissioning of the phase cameras ahead of O4. A brief overview of some of the issues encountered during this time is included. Typically, these issues are scattered over various reports and elogs, so it is useful to collect them here for future record. Next, in Section 3.3 we review how the phase camera was used to monitor the sidebands in the recycling cavities during the commissioning period and the useful detector observables that can be extracted from the phase camera signals. Most of these observables are crucial for the characterization of the Virgo detector, and were published in the Collaboration optical characterization paper of Ref. [113].

The remainder of the chapter is dedicated to the characterization of mode matching in the Virgo detector. In Section 3.4 I detail my contribution to the thermal tuning of the Signal and Power recycling cavities. Here, the phase camera was used to calibrate the measurements done with the arm cavity scans, in order to have consistent scans with each other. It serves as a good example of how the thermal tuning can be conducted in the absence of on-line error signals and the complexity that ensues.

Mode mismatch of the arm cavities is addressed in 3.5 where I show a proof-of-concept measurement with the phase camera. This is the first application of the phase camera at Virgo for mode mismatch characterization and it was the subject of a proceedings publication for the 2022 GRAvitational - wave Science&technology Symposium (GRASS). Even though some open questions remain, we will see the potential of direct measurements of the cavity and beam eigenmodes with respect to more indirect methods as the one used in Section 3.4. A more developed version of this technique and its test in a table-top experiment is given in Chapter 6.

I conclude the Chapter with a small summary and outlook where we review the advantages of a wavefront sensor such as the phase camera for GW detectors, but also its limitations. The discussion will set the context and motivation for the remaining Chapters of this thesis.

3.1 The Virgo detector and O4 commissioning

In August 2017 the Advanced Virgo detector joined the GW detector network for O2. With each observation run, the detector is upgraded with the aim of increasing the sensitivity. Since the end of O3, Virgo is receiving a two-phase upgrade to the so-called Advanced Virgo plus (AdV+). Here I discuss the phase 1 of this upgrade, henceforth called AdV for simplicity, or alternatively, the Virgo detector.

The AdV+ Phase I upgrade was aimed at reducing the quantum noise in the detector. The main changes with this upgrade was the addition of a signal recycling mirror and the installation of frequency-dependent squeezing, as described in Section 1.4. With the addition of the SRM, the AdV+ detector becomes a dual-recycled Michelson interferometer (DRMI as detailed in Section 1.3.2), similar to the LIGO detectors [9]. However, one thing that distinguishes the Virgo detector is that the power and signal recycling cavities are marginally stable. The consequences of this and a more detailed explanation of marginally stable cavities is given in the next Section.

Historically, this configuration is the result of infrastructure and budget limitations. As the detector has undergone upgrades, the issue of marginally stable cavities has become more significant. With each iteration of the detector, the goal is to increase input power. In turn, this leads to increased constraints on the thermal compensation system to keep the recycling cavities stable and the HOMs to a minimum. With the addition of the SRM and the increased input power, the issue became worse during the O4 commissioning.

For these reasons, the TCS had a central role in the commissioning of AdV in preparation for O4. Additionally, the phase camera was especially relevant as a diagnostic tool because of its ability to directly image the mode basis of the carrier and sidebands in the recycling cavities.

3.1.1 Marginally stable recycling cavities

It can be seen using the ABCD matrix formalism that a cavity is geometrically stable for [188] $0 \leq g \leq 1$ for $g = g_1 g_2$ as defined in Eq. 1.29. In stable resonators whose g-factor verifies this condition, the resonator transverse modes are well defined and can be described using the polinomyal basis introduced in Section 1.3.1. Outside of this condition, the resonator is unstable and there are no defined cavity eigenmodes. This leads to unbound divergent beams in the cavity and loss of light. When $g = 0$ or $g = 1$, we say the cavity is marginally stable, and as we will see, the resonance of HOMs as defined in Eq. 1.30 is degenerate with the fundamental mode resonance.

The round-trip Gouy phase, for a Fabry-Perot cavity, can be written in terms of the cavity g-factor as [189]

$$\Phi_{\text{RT}} = 2 \arccos(\pm 1 \times \sqrt{g}), \quad (3.1)$$

where the + or - sign is given by the sign of g_1 , which should have the same sign as g_2 for stable cavities. This yields a cavity round-trip Gouy phase shift between 0 and 180° for $g_1 > 0$, and between 180 and 360° for $g_1 < 0$. For HOMs with m and n transverse mode indices, the round-trip Gouy phase is simply

$$\Phi_{mn} = (m + n + 1)\Phi_{\text{RT}}. \quad (3.2)$$

Following Eq. 3.1, we can see that at the edges of stability, when marginally stable, the round-trip Gouy phase is equal to 0 for $g = 1$, and equal to 180° for $g = 0$. Recalling Eq. 1.30, for round-trip phase smaller than 180° , the mode separation of the cavity is

$$\Delta f_{mn} = \frac{(m + n + 1)\Phi_{\text{RT}}}{2\pi} FSR_\nu. \quad (3.3)$$

This means that for a marginally stable cavity with $g = 1$, the mode separation is 0 and all the HOMs resonate alongside the fundamental mode. In the case of the Virgo recycling cavities, the round-trip Gouy phase is 0.39° taking into

account the nominal mirror RoC values shown in Table 3.1. In frequency, this is equal to a mode separation of 13.57 kHz.

Table 3.1: Nominal parameters of the power and signal recycling cavities regarding their (marginal) stability.

	IM RoC [m]	EM RoC [m]	length [m]	$\mathcal{F}$	linewidth [kHz]
PRC	1430	-1420	11.97	96	130.39
SRC	1430	-1420	11.97	12	1050

How much an HOM will resonate in the recycling cavities does not depend only on round-trip Gouy phase, but also on the cavity linewidth. For the PRC, with a linewidth of 130.39 kHz and with a mode separation 13.57 kHz, we can consider that modes order 10 and above will have negligible gain in the cavity. For the SRC, however, with a linewidth of 1.05 MHz, we can consider that all HOMs will be resonantly enhanced by the cavity.

As an example, take the 56 MHz sideband, which resonates in both the power and signal recycling cavities. We can consider that the first and second-order modes of the 56 MHz sideband are promptly reflected from the arm cavities given that the FSR of the arm cavities is ~ 50 kHz, almost 4 times the mode separation of the recycling cavities. In this way we can use Eq. 1.21 which describes the Airy enhancement factor of a FP cavity to estimate the gain of the sideband HOMs for the power and signal recycling cavities. Doing this, we see that in the PRC, the gain of the sideband first-order and second-order modes is $\sim 85\%$ and $\sim 58\%$ of the gain of the fundamental mode, respectively. For the SRC these values become $\sim 99.7\%$ and $\sim 98.88\%$ for the first and second-order modes.

This enhancement of HOMs has a variety of negative impacts in the interferometer. First, as discussed in Section 2.4.1, circulating HOMs degrade the achievable level of squeezing. While Ref. [18] evaluates the impact of mode-mismatch in a LIGO-like interferometer, the same has not been done in the context of a marginally stable DRMI as is the case of Virgo. Secondly, the presence of HOMs in the sidebands, which are used to PDH lock the multiple cavities, has been shown to introduce offsets in the control signals, degrading the working point of the interferometer [65]. As discussed in Ref. [190], sideband HOMs are also thought to be the culprit behind a bi-stability of the working point of the recycling cavities.

For this reason, it becomes important to minimise HOMs in the central interferometer (CITF), composed of the PRM, the arm cavity input mirrors and the SRM. In Section 3.4 I describe my contribution to the thermal tuning of the PRM and SRM in order to improve the sideband mode matching and decrease the amount of sideband HOMs in the recycling cavities.

Another consequence of marginal stability is that small deviations in the radius of curvature of the cavity mirrors can cause the cavity to become unstable. Hence, tuning and controlling the thermal working point of the interferometer is crucial for the operation of AdV. In this regard, I show in Section 3.5 how the phase camera might be useful to characterise the mode mismatch of the arm cavities.

3.2 Phase camera commissioning

Two phase cameras, as detailed in Ref. [20] and Section 2.6.4, were installed and commissioned at the Virgo detector ahead of O3 [142]. During O4, the phase cameras remained in the same locations as in O3 which can be seen in Fig. 3.1. One monitors the B4 beam in the power recycling cavity and another monitors the B1p beam, just before the output mode cleaner (OMC).

A pick-off plate (POP) samples about ≈ 300 ppm of the PRC circulating beam [142]. This is the B4 beam which goes into the suspended power recycling bench (SPRB), which is also under vacuum. In turn, SPRB is connected to the external power recycling bench (EPRB), this one in air, where the B4 PhC is installed. Both benches are shown in Fig. 3.1.

After the SRM the beam is split into two. One part goes directly into the suspended detection bench 2 (SDB2), which is under vacuum. This is the B1p beam, which contains the full mode content of the dark port. It propagates from SDB2 to the external detection bench (EDB), now in air, where the B1p PhC is installed. The other part goes through the OMC and is the beam that is used for detection of the GW signal, following the DC readout method introduced in Section 2.1.1. The beam paths and benches are shown in Fig. 3.1.

The OMC filters out all the HOMs present in the dark port. Only the fundamental mode from the arm cavities is transmitted and goes into the detection photodiodes. The fundamental mode contains the GW strain signal. After the OMC, the beam also goes into SDB2, where the readout optics are installed (shown in Fig. 3.1).

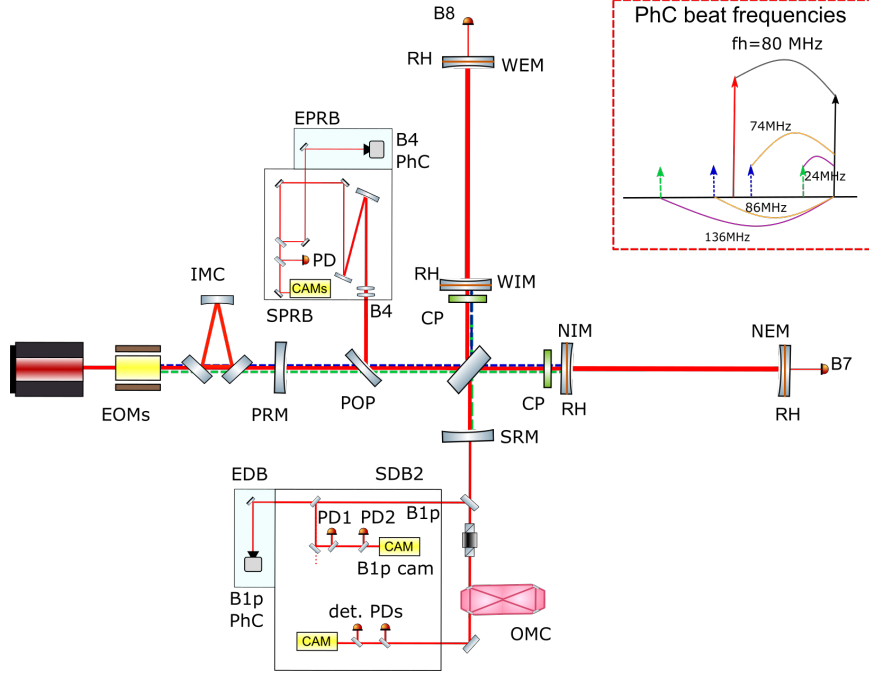


Figure 3.1: Simplified optical schematic of AdV. A pick-off plate (POP) samples a fraction of the power recycling beam, called B4 beam. The B4 beam goes into the, under vacuum, suspended power recycling bench (SPRB). A fraction of this beam goes into the external power recycling bench (EPRB) where the B4 PhC is installed. The beam after the SRM is sampled into the, under vacuum, suspended detection bench 2 (SDB2). This is the B1p beam, a fraction of which goes into the external detection bench (EDB) where the B1p phase camera is installed. The transmission of the north arm cavity with north input mirror (NIM) and north end mirror (NEM) is sampled by photodiode B7. Similarly to the west arm cavity, there is the B8 photodiode.

Recall from Section 2.2 that three sidebands are used to control the main interferometer. These are the 6, 8 and 56 MHz sidebands (also shown in Fig. 3.1). As we saw, the 8 MHz sideband is used to lock the PRC and thus does not resonate in it. For this reason, this sideband is not measurable by the B4 and B1p phase cameras. Both the 6 MHz and 56 MHz sidebands resonate in the PRC but not in the arm cavities, while the 56 MHz also resonates in the SRC. Both of these sidebands are measured by both phase cameras.

As detailed in section 2.6.4, the phase camera is able to measure and image carrier and upper and lower sidebands separately. In this case, a total of 5 beat frequencies are measured between the carrier and the lower and upper components of the 6 and 56 MHz sidebands. The different beatings with the

80 MHz frequency-shifted reference beam are shown in the inset on the top right corner in Fig. 3.1.

The phase cameras are installed in a 1-beam scanning configuration, as discussed in Section 2.6.4. The images are acquired following an Archimedean spiral as shown in Fig. 2.12 and reconstructed into a 128x128 pixel grid. The images are afterwards compressed to a 100x100 grid to conform to the requirement of the Virgo data acquisition system. Examples of such images, for amplitude and phase, are shown in the inset of Fig. 3.2. The phase images shown are differential, between carrier and sidebands, in order to remove common phase noise due to the reference beam propagation and the residual motion of the benches where the phase cameras are installed, as discussed in Section 2.6.4 and Refs. [20, 142].

More details about the installation of the two phase cameras ahead of O3 and their commissioning at the time can be found in Ref. [142]. Concerning the phase cameras, the main change from O3 to O4 was the addition of the SRM before the B1p PhC. In any case, with the break between runs, the optics had to be re-aligned and their proper functioning validated. In the remainder of this section, I detail the main checks done using the single bounce configuration as well as the main issues encountered during this time.

3.2.1 The single bounce configuration

The main checks of the phase camera were done with the interferometer in the so-called single bounce configuration, which allows for the input beam to reach the phase cameras.

When the interferometer is in the single bounce configuration (Fig. 3.2), only the input mirror of one of the cavities is aligned and both end mirrors are misaligned. Additionally, both power and signal recycling mirrors are misaligned such that no reflection from either reaches the phase cameras. In this way, both phase cameras see the input beam that bounces off the input mirror that is aligned (either the WIM as shown in the image or the NIM).

Since the optical path for carrier and sidebands is the same in this configuration, the sidebands should be perfectly superposed to the carrier. Additionally, the relative phase difference between carrier and sidebands should simply be a flat offset due to the difference in wavelength between each of the sidebands and the carrier. The inset of Fig. 3.2 shows such an example, displaying both amplitude and phase images of the B1p beam. We can validate how well the

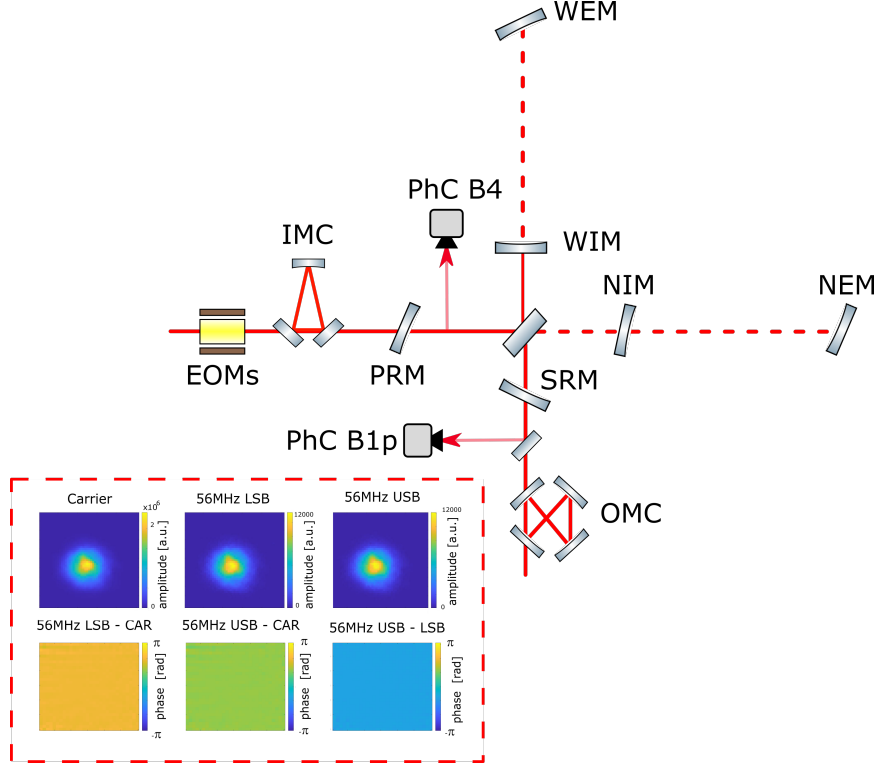


Figure 3.2: Simplified single bounce configuration from the west input mirror. Optics are intentionally misaligned as part of the configuration, see main text for more details. One PhC looks at the beam picked-off after the PRM, called B4, while the other looks at the beam picked-off after the signal SRM, called B1p. Both see the input beam in this configuration. Dashed lines indicate light that doesn't reach the phase cameras. Both suspended and external benches are not depicted in this figure.

carrier mode overlaps with the mode of the sidebands through the complex overlap integral. The integral is defined for two beams $E_1(\vec{x})$ and $E_2(\vec{x})$, and it is given by

$$\gamma(E_1, E_2) = \frac{|\int U_1 U_2^* dx dy|}{\sqrt{\int U_1 dx dy \int U_2 dx dy}}, \quad (3.4)$$

where $U_i(x, y, z)$ is the complex field amplitude as defined in Eq. 1.27, for a beam that propagates in the z direction. Note that the overlap is computed for the transverse plane and does not depend on z . If the mode content of the two beams is the same, then $\gamma = 1$. In the single bounce configuration as shown in the inset of Fig. 3.2, the overlap between carrier and sidebands is typically close to 0.99, computed using both the amplitude and phase information.

In this configuration, the phase camera can be used to measure the modulation depth of the sidebands. This is one way to validate that the power reconstruction of the phase camera is accurate. Additionally, it also serves as an independent measurement of the modulation depth, which is otherwise made by a scan of the reference cavity or of the arm cavities. The modulation depth, β , is a function of the power ratio between carrier and sideband:

$$\beta = 2\sqrt{\frac{P_{carr}}{P_{sb}}}. \quad (3.5)$$

The power in carrier and sidebands is taken by summing over the respective amplitude images and squaring this value. Table 3.2 shows a comparison of the modulation depth values obtained with the phase cameras in single bounce and with a scan of the arm cavities. The table reproduces the same values as published in [113], to which I contributed by computing the modulation depth values from the phase camera data.

Table 3.2: Modulation depth measurements, values reproduced from [113].

sideband	FSR scan	B1p PhC	B4 PhC
56 MHz USB	0.189	0.193	0.197
56 MHz LSB	0.183	0.193	0.197
6 MHz USB	0.201	0.206	0.210
6 MHz LSB	0.202	0.206	0.201

The disagreement between the two methods is at most 6.3 %, while the disagreement between the two phase cameras is 2 %. The difference between the two phase cameras can be due to small changes in the frequency dependence of the amplifying electronics chain of each phase camera, since upper and lower sideband are measured at different beating frequencies (eg. 24 MHz and 136 MHz for the 56 MHz upper and lower sidebands respectively). The frequency dependence of each phase camera was previously calibrated in order to have upper and lower sidebands balanced, but there could be differences at the % level between the two phase cameras. Some RAM as discussed in Section 2.2 could also affect these measurements.

Apart from the phase camera checks, the single bounce configuration is also as a reference for several other measurements. This is the case of the sideband PRC gains and dark port transfer functions as detailed in section 3.3, and also of the contrast defect as detailed in Ref. [113].

3.2.2 Troubleshooting the beam shape

One of the great advantages of the phase camera is the ability to image independently the beam profile of carrier and all upper and lower sidebands. Even though this ability presents great opportunities for monitoring and mitigating higher-order aberrations, it becomes crucial to validate the shapes observed by the phase camera.

In this section I present the efforts during commissioning to understand and characterize the beam shapes observed in the B4 and B1p beams and the challenges encountered.

B4 beam

Fig. 3.3 shows the carrier beam amplitude image measured by the phase camera in B4 in different interferometer configurations. The shape of the images suggest that there is some astigmatism as well as some other sort of aberration. Moreover, it is interesting to see how the beam shape changes in different detector configurations. When the CITF is locked, the expectation is that the shape of the carrier remains similar to the shape in the single bounce configuration since at this stage the carrier does not resonate in the PRC. This is because at this stage the arms are unlocked, and the carrier is set to only resonate in the PRC when it has the 2π round-trip phase shift from resonating in the arm cavities. It can be seen in Fig. 3.3 that indeed the CITF locked beam looks qualitatively similar, albeit slightly more aberrated, to the beam in single bounce.

When the carrier resonates in the arm cavities, the phase shift experienced by the resonant carrier makes it so that it resonates as well in the PRC. With arms locked, we can see the beam shape changed considerably in the third panel of Fig. 3.3. The arm cavities are over coupled, so most of the light reflected by the arm cavity is the resonant reflection, carrying the mode of the cavity. In other words, the mode of the PRC becomes dominated by the arm cavity mode, this is why the beam becomes more Gaussian with the arms locked. If the mode matching was perfect, then no change in the beam shape would be observed.

Already during the O3 commissioning, a problem of astigmatism in the B4 beam was identified [142]. The problem remained in the commissioning for O4 and was thoroughly investigated by the detector team. The results of this investigation can be found in Ref. [191]. Below, I give a summary of

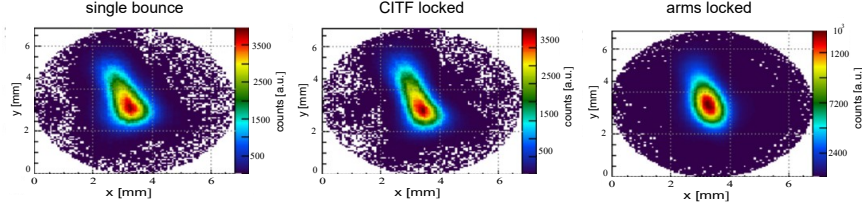


Figure 3.3: Carrier amplitude image measured by the B4 phase camera in 3 different detector configurations: single bounce, only the CITF is locked, full interferometer lock with the arm cavities locked on resonance.

conclusions arrived at, as it is relevant for understanding the morphology of the B4 PhC images.

It was observed that a regular camera monitoring the B4 beam sees a similar shape as that seen by the B4 PhC. Moreover, it was seen that the shape of the B4 beam changes whether the SPRB is under vacuum or not. The tests reported in Ref. [191] go on to show that the beam is indeed astigmatic. Additionally the presence of a second, lower power spurious beam, was also confirmed. This ghost beam could account, at least partly, for the departure from Gaussian shape.

Finally, it was noted that the observed astigmatism at the cameras depends on the beam parameter of B4. This explains the different shapes under vacuum and in air, as the pumping process and a thermal lens in the view port are changing the focusing of the B4 beam.

In the rightmost panel of Fig. 3.3 we can see the beam appears less aberrated, given the observation just described, this could be due to two reasons. One is that if the beam is not perfectly mode matched, the beam parameter of the carrier is different once the arm cavities are locked. This change in beam parameter, should change the astigmatism observed in the B4 cameras. Additionally, if part of the aberrations in the left and middle panels of Fig. 3.3 are due to a spurious beam, this would become negligible once the arm cavities are locked and the power of the carrier is increased by about 40 times inside the PRC.

Even though the source of astigmatism was not found, the position of one of the lenses on SPRB was adjusted in order to minimize the astigmatism seen on the cameras. More importantly, a separate analysis found that the B4 beam aberration has a negligible impact on the longitudinal controls of the interferometer [192]. The aberrations do, however, make it more difficult to interpret the phase camera images in the context of studying the PRC or arm cavity

eigenmodes. For this reason, the analysis of Section 3.5 was done using the B1p PhC.

B1p beam

Initially, the single bounce beam, as seen by the phase camera on B1p, looked as that seen on the inset in Fig. 3.2. The image started appearing degraded in May 2022 after some alignment work in the EDB. In August, the parking position of the PRM was changed because it was impacting the longitudinal lock signals. After this, the B1p image became dominated by vertical fringes as seen on the top panel of Fig. 3.4a.

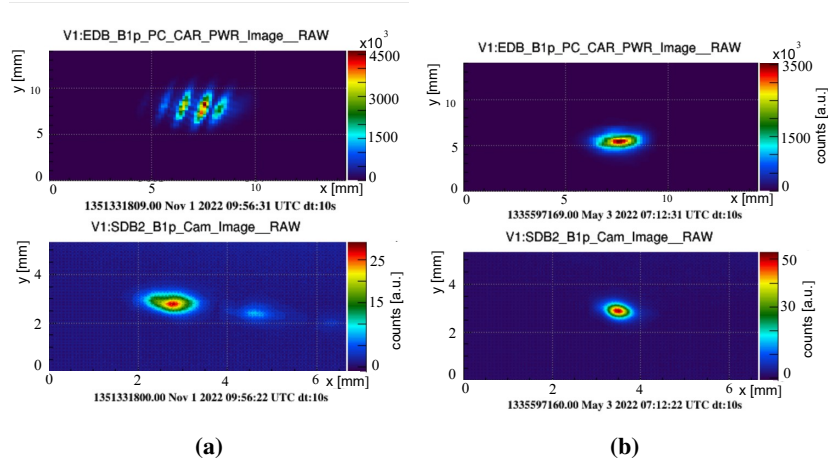


Figure 3.4: Example of the fringes seen at the B1p PhC (top panel) and comparison with a regular camera overlooking the same port (bottom panel), where a spurious beam can be seen (a). Example of the image at the B1p PhC before the fringe pattern problem started, the spurious beam is not observable in the regular camera (b).

Even after re-aligning the phase camera, the observed fringe pattern persisted. One hypothesis was that a spurious beam was interfering with the B1p beam creating the interference observed. This seems to be the case according to Fig. 3.4a. We can see on the bottom a small beam on the regular B1p camera. It could be that the spurious beam does not interfere on the path to the regular camera, but ends up interfering with the main beam on the way to the B1p PhC.

In Fig. 3.4b, corresponding to data from the beginning of May 2022 before the issues started, there are no fringes in the B1p PhC and neither a spurious

beam in the B1p camera. I posted these observations in a Virgo logbook post in November 2022 [193].

Despite the changes in the B1p beam shape, no significant impact on the carrier and sidebands power or modulation depth was observed as reported in [194]. Analysis of the contrast of the fringes suggests the culprit spurious beam should have about 75% of power of the main beam. The spurious beam observed in Fig. 3.4a, appears much dimmer than that, however. A possible explanation for this discrepancy could be that the spurious beam gets clipped on the path to the regular B1p camera.

At the end of November 2022, the PRM parking position was changed by about 40 μ rad in x and y. As shown in Fig. 3.5, the fringe pattern disappeared from the B1p PhC and the spurious beam disappeared from the regular camera. This indicates that the spurious beam and fringe pattern are related to the parking position of the PRM. Since the vacuum could not be broken at this time, no further investigations were possible. The PRM had to be returned to its fringe inducing position for the remainder of the commissioning and the following observation run, as this was the best for the stable operation of the interferometer.

Note that the analysis of Section 3.5 was done using B1p Phc images taken before the PRM parking position was changed and the fringe issues started.

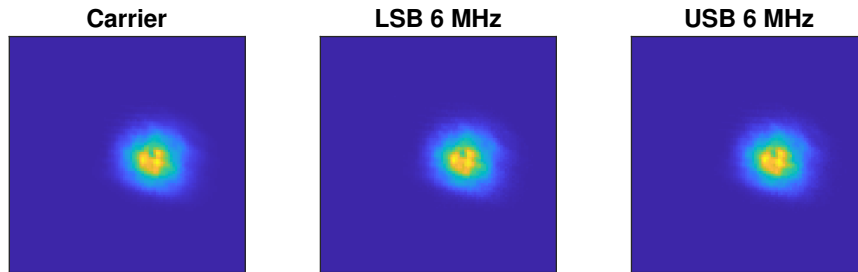


Figure 3.5: Example of the B1p phase camera amplitude images with a different PRM parking position (November 2022).

3.3 Phase camera observables and sideband monitoring

The phase camera allows us to monitor how the upper and lower sideband powers are varying during the interferometer locking procedure. It also allows

for comparing different interferometer locks, as different parameters are tuned. Additionally, the amplitude images can also be inspected to make sure there are no problematic HOMs and to ensure the overlap between the sideband and carrier modes.

In addition to the amplitude information, the phase camera can also measure and image the phase of each beat component. Due to phase noise in the fiber that propagates the reference beam (as detailed in Section 2.6.4) the absolute phase images are not available. Instead, and in order to remove the common phase noise, the phase cameras generate differential phase images between the carrier and sidebands.

From the power in the upper and lower sidebands, a new observable, called the sideband imbalance, can be defined as

$$I_{\text{SB}} = \frac{P_{\text{USB}} - P_{\text{LSB}}}{P_{\text{USB}} + P_{\text{LSB}}}. \quad (3.6)$$

Below, I review the other observables that we can retrieve from the phase camera which are crucial for the optical characterization of the detector.

3.3.1 Power recycling gains

When the CITF is locked, the sidebands resonate in the PRC and we can talk about the sideband recycling gain. In this case, the circulating power for a sideband will be given by:

$$P_{\text{circ}}^{\text{SB}} = G_{\text{rec}}^{\text{SB}} P_{\text{in}}^{\text{SB}}, \quad (3.7)$$

where $G_{\text{rec}}^{\text{SB}}$ is the recycling gain of a given sideband and $P_{\text{in}}^{\text{SB}}$ its respective input power. The phase camera intensity signals are in units of counts (cts), associated to the square of the sum of the pixels of the amplitude images. These could be converted into power with an unknown conversion coefficient $\alpha(f)$, which depends on the frequency of the optical beating in question.

For the input power, the single bounce can be used as a reference. The power measured by the phase camera in single bounce will be given by:

$$P_{\text{ref}}^{\text{SB}} = P_{\text{in}}^{\text{SB}} T_{\text{PRM}} T_{\text{BS}} R_{\text{IM}} T_{\text{BS}}, \quad (3.8)$$

where T_{PRM} is the transmission of the PRM, T_{BS} the transmission of the beamsplitter and R_{IM} the reflectance of the arm cavity input mirror which is being used for the single bounce configuration. All of these values can be found in the AdV optical characterization article [113].

Combining the two above equations yields:

$$G_{\text{rec}} = \frac{P_{\text{circ}}^{\text{SB}}}{P_{\text{ref}}^{\text{SB}}} T_{\text{PRM}} R_{\text{IM}} T_{\text{BS}}^2. \quad (3.9)$$

Since the $\alpha(f)$ coefficient is the same irrespective of the detector configuration, it drops out of Eq. 3.9 and the phase camera signals in counts can be used directly to calculate the sideband recycling gains.

While the carrier recycling gain can be computed with the transmission signals of the arm cavities, the phase camera is currently the only way to measure the recycling gain of the sidebands.

3.3.2 Dark port transfer function

Similarly to what can be done with the B4 PhC to compute the power recycling gains, the same can be done for the DP.

For the DP it makes more sense to speak of transfer function instead of gain, since the power there is a fraction of the input power. This transfer function can be written as:

$$Tf_{\text{DP}} = \frac{P_{\text{DP}}}{P_{\text{ref}}} T_{\text{PRM}} R_{\text{IM}} T_{\text{BS}}^2 T_{\text{SRM}}, \quad (3.10)$$

where T_{SRM} is the transmission of the SRM. This transfer function is very similar to Eq. 3.9. The differences are that the B1p PhC is used for the reference signals in single bounce and for the DP signal while the interferometer is locked as well as the addition of the T_{SRM} factor. Similarly to the power recycling gains, the phase camera values in counts can be used directly.

It is interesting to calibrate these values in terms of actual power in the DP because there are independently calibrated photodiodes monitoring the same beam. Assuming the power in the sidebands is negligible, this can be done for the carrier as

$$P_{\text{DP}}^{\text{carr}} = P_{\text{in}} T_{\text{DP}}^{\text{carr}}. \quad (3.11)$$

The same can be done for the sidebands, taking into account their modulation depth as reported in Table 3.2:

$$P_{\text{DP}}^{\text{SB}} = P_{\text{in}} J_1(\beta) T_{\text{DP}}^{\text{SB}}, \quad (3.12)$$

where J_1 is a Bessel function of order 1 and β the modulation depth of the corresponding SB. The top panel of Fig. 3.6 shows the carrier and sideband

power calibrated in this way as measured by the B1p PhC taking into account an input power of 17 W.

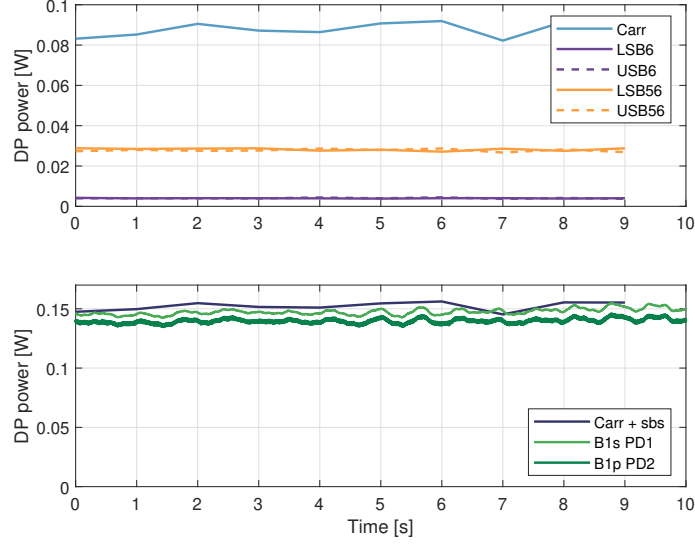


Figure 3.6: DP carrier and sideband powers using the B1p PhC and the single bounce configuration (top panel). Comparison between phase camera calibration and a similar calibration for the B1s PD1 and B1p PD2 photodiodes (bottom panel).

This calibration can be verified by comparing with the power calibrated photodiodes PD1 of the B1s beam and PD2 of the B1p beam. This comparison is shown in the bottom panel of Fig. 3.6. Note that the two photodiodes measure the total power (carrier + sidebands) at the DP in the carrier and sidebands. Averaged over 10 seconds, the thus calibrated B1p PhC measures 152 mW while the B1p photodiode measures 140 mW and the B1s 148 mW. The disagreement between the measurements is at most 3%, which is reasonable considering the optical losses in the different optical paths to each photodiode were not taken into account.

3.3.3 Contrast defect

The contrast defect is a measure of how much excess power is reaching the dark port. It can be written as the ratio of the power at the dark port and the circulating power in the interferometer,

$$CD = 2 \frac{P_{DP}}{P_{circ}}, \quad (3.13)$$

where the factor 2 is to take into account the beamsplitter and the fact that the circulating power is normally measured in the B4 beam (see Fig. 3.1).

The CD can be measured with the phase camera, once again using the B1p and B4 PhC references in single bounce as well as the B1p and B4 phase camera powers when the interferometer is locked. Thus, with the phase camera measured powers it is written as

$$CD = \frac{\frac{P_{B1p}^{DRMI}}{P_{B1p}^{ref}}}{\frac{P_{B4}^{DRMI}}{P_4^{ref}}}, \quad (3.14)$$

where the convention of Ref. [113] was followed, with the superscript DRMI indicating that the interferometer is locked, and the ref superscript indicating the reference power measured in single bounce configuration.

3.4 Thermal tuning of the power and signal recycling mirrors

During commissioning, it was necessary to tune the radius of curvature of both the power and signal recycling mirrors. The PRM RoC was designed to be 1477 m, longer than the nominal value, to account for the thermal deformation of the POP induced by the main beam at the initial goal power of 125 W. However, this goal input power was never achieved, which led the PRM to be mismatched with respect to the arm cavities. The SRM RoC at 1444 m was also shorter than it should be in order to match with the arm cavities.

As we saw in Section 3.1.1, it is crucial for the control of the interferometer to minimise the presence of HOMs in the sidebands as much as possible. For this reason, having a good mode matching of the sidebands in both power and signal recycling cavities is very important. The tuning of the RoC of the recycling mirrors was done using the CHRoCC actuators described in Section 2.7.

In the case of the PRM, the CHRoCC was installed so as to induce a focusing lens in the POP. This is equivalent to reducing the RoC of the PRM. For the SRM, the CHRoCC shines directly on the SRM, increasing its RoC to equalize it with the RoC of PRM. This adjustment of the RoC of both recycling mirrors implies a later adjustment of the central heating on the compensation plates to adjust the mode matching of the recycling cavities with the arm cavities. However, here I only report on the CHRoCC tuning.

In order to measure the mode matching of the sidebands in the recycling cavities, we can use the second-order mode of the sidebands as a figure of merit. We can measure the sideband second-order modes with a scan of the arm cavity length while the CITF is kept locked, as proposed in Ref. [195]. For this, the arm cavities can only be scanned around the second-order mode of the sidebands. Otherwise, if the scan goes through the fundamental mode of the sidebands, it will spoil the PDH error signal and unlock the CITF.

In order to do this scan, the auxiliary laser system (ALS) was used [196]. The ALS uses two green beams injected from each arm cavity end mirror to facilitate the cavity lock acquisition, since the finesse is lower at the ALS wavelength of 532 nm (half of the main laser wavelength at 1064 nm).

First the arm cavities are locked using the green beam with a certain CARM offset so that they are away from resonance. Afterwards the CITF is locked using the signals from the 8 and 56 MHz sidebands as discussed in Section 2.2. Once the CITF is locked, both arm cavities are scanned simultaneously using the CARM signal of the green beam and changing the CARM offset slowly. More details regarding the arm scans can be found in the optical characterization paper [113]. An example scan is shown in Fig. 3.7. The ramp signal is proportional to the CARM offset value.

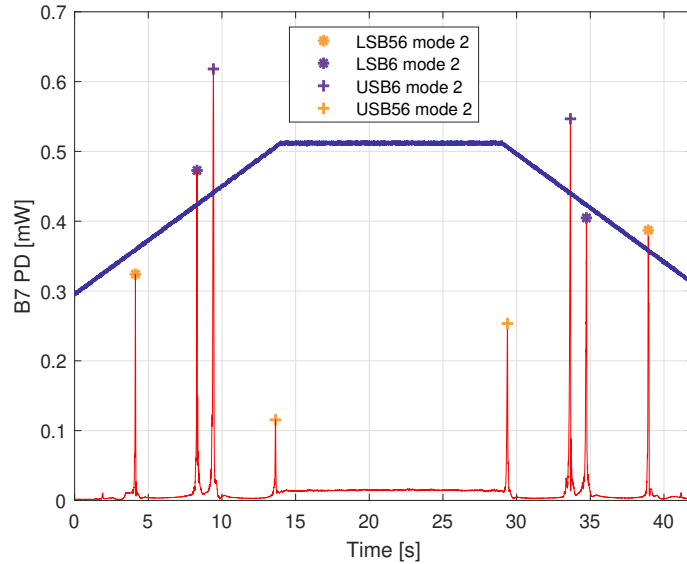


Figure 3.7: Example of the (north) arm cavity scan around the second-order peaks of the 6 and 56 MHz sidebands. Signal is measured at the B7 photodiode.

3.4.1 CITF scan simulations

Before the measurements described above were done, it was necessary to make some studies with optical simulations. The goal of this simulation study was to verify how the sideband mismatch changes with the PRM and SRM RoC detuning and to check if scanning the OMC could also give similar information. These simulations were done in the software OSCAR [197].

While the real measurements involve scanning the cavities, in OSCAR cavity scans take very long to compute. This is the case because for every detuning of the cavity length the circulating fields need to be computed.

As discussed in section 1.3.1, when an electric field is incident on an optical resonator, light scatters from the field into the resonant modes of the cavity. Recalling Eq. 1.31, the amount of scattering is given by the inner product of the cavity eigenmode and the incident field [11]. Experimentally, the cavity needs to be scanned in order to make each mode resonate in the cavity and to measure their power. In simulation, however, we can compute the inner product of each eigenmode with the incident field directly.

To compute the mode mismatch in OSCAR we can simply compute the inner product of the incident field with the modes of order 2 in the cavity basis. The amount of mode mismatch is computed as the ratio between the amount of second-order modes vs. the amount of the 00 mode:

$$\text{MMi} = \frac{\text{HG}_2}{\text{HG}_{00}} * 100 \quad [\%], \quad (3.15)$$

where the notation for Hermite-Gaussian (HG) modes introduced in section 1.3.1 was used and HG_2 corresponds to the sum of all HG_{mn} modes with $m+n=2$ in the basis of the cavity.

In the simulation a CARM offset is applied to the arm cavities to bring them out of resonance and the resonant fields in the CITF are computed. Next, the RoC of the PRM is scanned, while keeping the SRM mode matched. The amount of mode mismatch evaluated in the PRC and at the dark port after the SRM is computed for each scanned point. The resulting mode mismatches for the 6 and 56 MHz sidebands are shown in Fig. 3.8.

We can see from the top panel of Fig. 3.8 that the sidebands in the PRC are very sensitive to changes in the RoC of the PRM. In the middle panel, the mode mismatch for the 56 MHz sideband as measured in the dark port is shown. This was not computed for the 6 MHz sideband as it does not resonate in the SRC.

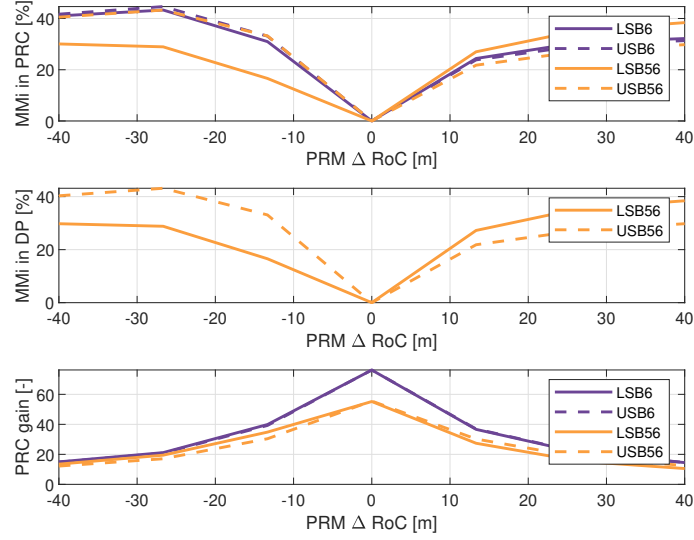


Figure 3.8: Sideband mode mismatch in the PRC (top panel), sideband mode mismatch in the SRC (middle panel) and sideband PRC gain (bottom panel) with respect to the tuning of the PRM RoC.

The middle panel of Fig. 3.8 shows us that we can also use a scan of the OMC to measure the mode mismatch in the sidebands as it follows the same trend as seen in the PRC (in the panel above).

Finally, the bottom panel shows the decrease in the PRC gain of the sidebands with respect to the change in the RoC of the PRM. This was done to provide an expectation for the drop in sideband gain in the PRC ahead of the measurements.

In addition to scanning the RoC of the PRM, the same simulation was done for the RoC of the SRM. The results are shown in figure 3.9.

From the figure, we see that the 6 MHz sideband is virtually unaffected by detuning of the SRC RoC. This is to be expected since the 6 MHz sideband does not resonate in the SRC. For the 56 MHz sideband the amount of mode mismatch induced is significantly smaller than that from PRC detuning. This can be attributed to the difference in finesse between the power and signal recycling cavities, as shown in Table 3.1.

Despite the small impact of the SRM mismatch, a good matching of the SRC with respect to the arm cavities is still important because the GW signal is carried by the mode of the arm cavities. However, this was not the topic of these simulations.

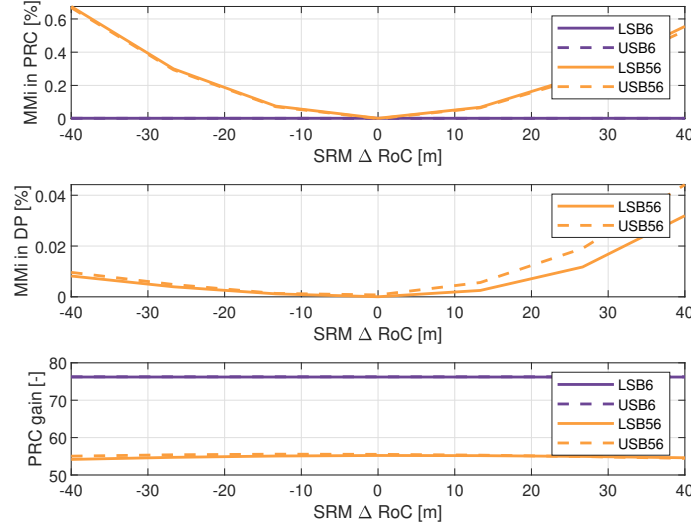


Figure 3.9: Sideband mode mismatch in the PRC (top panel), sideband mode mismatch in the SRC (middle panel) and sideband PRC gain (bottom panel) with respect to the tuning of the SRM RoC.

3.4.2 OptChar measurements

The data to characterize the CHRoCC tuning of both PRM and SRM was acquired over the course of four OptChar shifts. A report on the subsequent analysis of the data can be found in Ref. [198].

For the analysis, only data from the north arm cavity (B7 PD) was used. For reasons unclear, the power in the west arm B8 PD was very low. Simulations in OSCAR done by J. Degallaix [199], show that the second-order peaks are highly suppressed when only one arm is scanned at a time. It is not clear why a similar effect was observed in practice, since both arms were being scanned simultaneously. It could be possible that a specific DARM offset makes it so that the modes resonate mostly in one of the arm cavities during the scan, but this was not further explored.

The RoC of the PRM was tuned by changing the voltage applied on the CHRoCC, as shown in the x axis of Fig. 3.10. Three sets of measurements were taken for three different working points of the RoC of the SRM: 1431, 1436 and 1426 m. The change of the SRM RoC in response to the SRM CHRoCC voltage was calibrated previously by the TCS team.

Fig. 3.10 shows the measured second-order modes for each sideband. Note that the sideband circulating power in the PRC varies from lock to lock and this has an impact on the amount of power measured in the cavity scans. For

this reason, the B4 PhC sideband signals were used to normalize the values of the scans, such that they are comparable to each other in between scans.

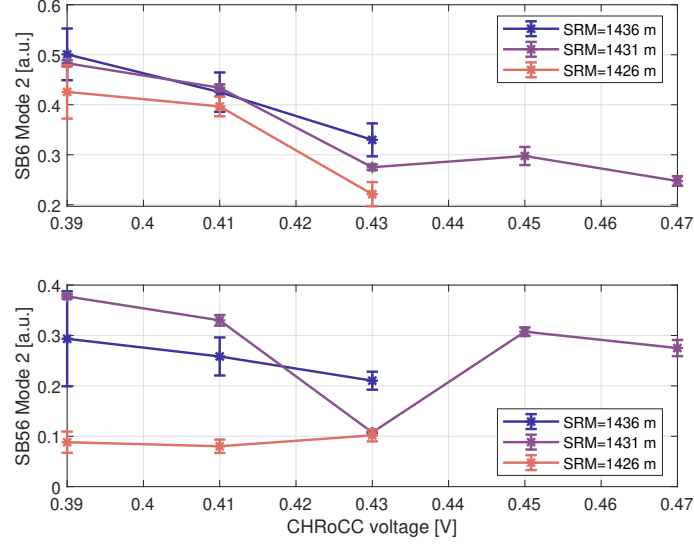


Figure 3.10: Second-order mode peaks of the 6 and 56 MHz sidebands measured with the north arm cavity scans, for different values of the PRM CHRoCC voltage and different RoC of the SRM. The peak heights were normalized taking into account the B4 phase camera signals, so that the signals were comparable from lock to lock.

The best overall working point appears to be at 0.43 V CHRoCC voltage and SRM RoC=1426 m. Analysis of scans of the OMC done in parallel [200], point to the same optimal working point. Something to note is that the power in the recycling cavity was not maximized for this working point. However, the recycling gain also depends on the quality of the interferometer lock itself, which is sensitive to technical noise, other than mode mismatch.

From the simulation results presented in Fig. 3.8, we expect the sideband imbalance (Eq. 3.6) for the 6 MHz sideband to be negligible. However, experimentally, a significant imbalance of the 6 MHz sideband was measured, about 0.16 for the optimal point. Considering the mode mismatch is not zero even in the best working point, an hypothesis is that the imbalance is caused by asymmetric lensing between the two arm cavities, which was not included in the simulation above. Asymmetric mismatch in the arm cavities is discussed in Section 4.3.2.

The CHRoCC tuning values for both the PR and SR mirrors were set by the analysis just presented. Any asymmetric defects remaining were compensated

with the use of the CO₂ lasers, compensation plates and ring heaters. These were optimized during the course of the commissioning period and even well into the observation run by the TCS.

3.5 Mode mismatch in the arm cavities with the phase camera

We saw in Section 3.1.1 the importance of minimising HOMs in the presence of marginally stable cavities. While in the previous Section we discussed the mode matching of the recycling cavities, this Section concerns the mode matching of the arm cavities. In addition and contrary to the previous Section, the work presented here is a direct measurement of the mode mismatch of the arm cavities.

Such a direct measurement is made possible by the use of the phase camera and is the first of its kind at Virgo. Some of these results were published in the proceedings of the GRAVitational-waves Science & technology Symposium (GRASS) 2022, with the title "Sensing mode mismatch with the phase cameras at Advanced Virgo" [4]. Additional results were presented posteriorly in [201] and in [202] and are also presented here.

The measurements are taken in two steps. The first measurement corresponds to the input beam measured in single bounce configuration by the phase camera. In the second measurement, the end mirror of the cavity is aligned and the cavity is locked to the laser and the reflected beam from the locked cavity is measured by the phase camera.

Because the cavity is overcoupled, the reflected carrier beam is dominated by the resonant carrier mode of the locked cavity. The idea is to compare the amplitude and phase of the input beam with the mode of the cavity. If the input beam is matched to the cavity, the size of the beam and wavefront radius of curvature should be the same. Additionally, any second-order mode due to mode mismatch will also be promptly reflected from the cavity input mirror and be present in the reflected beam measured by the phase camera. This is useful because the phase of these second-order modes carries important information for the sensing of mode mismatch, as will become clear in the next section.

Note that since the sidebands do not resonate in the cavity, this could be done by comparing the sideband to the carrier mode, without the need for the single bounce measurement. However, the single bounce measurement is

done to check that everything is working properly, and that prior to the cavity alignment and lock, that the carrier has the same mode of the sidebands.

3.5.1 Simulating the measurements

In order to understand what phase camera observables can be used, I simulated the Virgo arm cavity in OSCAR. Since in the simulation the input beam can be compared directly to the resonantly reflected mode of the cavity, the single bounce configuration was not part of the simulations.

The mode mismatch was introduced in the simulation by detuning independently the input beam radius ($w(z)$) and wavefront curvature ($R(z)$), which, as we saw in Section 1.3.1, are related to the complex parameter q of the input beam by:

$$\frac{1}{q} = \frac{1}{R(z)} - i \frac{\lambda}{\pi w(z)^2}, \quad (3.16)$$

where λ is the laser wavelength and the beam propagates in the z direction.

The introduced mismatch was quantified by taking the ratio between the second-order and fundamental modes content in the reflected beam from the cavity. The amount of mode mismatch was computed as in the simulations of Section 3.4, by computing the inner product between the reflected beam from the cavity and the second-order modes in the cavity basis.

For phase camera observables, we inspect the radius of the reflected carrier beam with respect to the radius of the input beam. The relative beam radius as

$$\Delta r = \frac{r_{\text{ref}} - r_{\text{inp}}}{r_{\text{inp}}} \cdot 100 [\%]. \quad (3.17)$$

Another phase camera observable is the differential phase cross section between carrier and sidebands. Since the sidebands are not resonating in the cavity, this is equivalent to taking the differential phase between the input beam and the cavity mode beam.

No additional misalignments or defects were introduced in the simulation, so as to be sure to isolate the impact of mode mismatch on the phase camera signals. Figure 3.11 shows how these observables change with individual detuning of the input beam parameters.

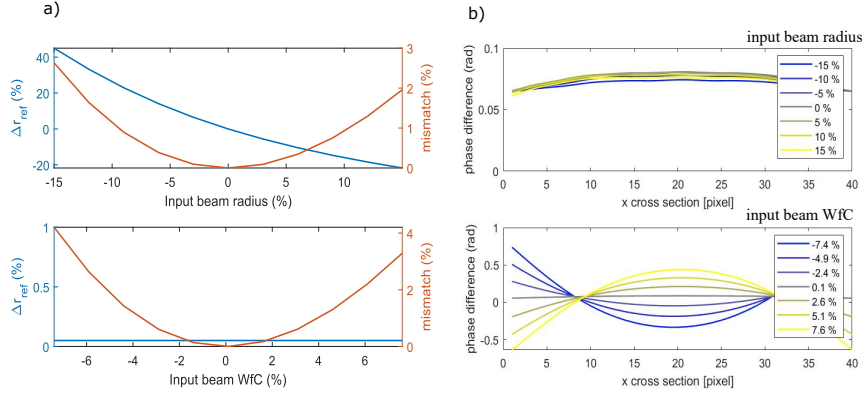


Figure 3.11: AdV arm cavity was simulated and input beam radius and wavefront curvature (WfC) were detuned independently. Panel a) shows the relative reflected carrier radius as defined in equation 3.17, panel b) shows differential phase cross section between the 56 MHz upper sideband and carrier.

It can be seen that, for small values of mismatch, the radius of curvature of the reflected carrier beam is sensitive to the detuning of the input beam radius only, and the differential phase cross section is sensitive to the detuning of the input beam wavefront curvature, with negligible coupling between the two. It was seen that as larger detunings are applied to either input beam parameter, both radius and phase cross section of the reflected beam are affected more significantly. Larger mismatches were not plotted, however, since typical measured mismatches of the arm cavities are of the order of a couple percent [113].

Additionally, it can be seen that for both cases of changes to either the input beam radius or wavefront curvature it is possible to infer what is the sign of the detuning applied. In the case of detuning the input beam radius, the sign of the signal is given by whether the radius of reflected beam is larger or smaller than the input beam. In the case of detuning the input beam wavefront curvature, the sign of the signal is given by whether the curvature of the differential phase cross is positive or negative curvature.

Looking at the mode mismatch curve in the left plot of Fig. 3.11 we can see that the amount of mode mismatch is not sensitive to the direction of the detuning. In fact, for approximately the same amount of mismatch, it is possible to have beams with very different radius. It was checked that in either case, the mode content of the beam is dominated by the fundamental and second-order modes. The different size in beams with the same mode content in terms of power is due to the phase of the second-order mode with respect to the phase

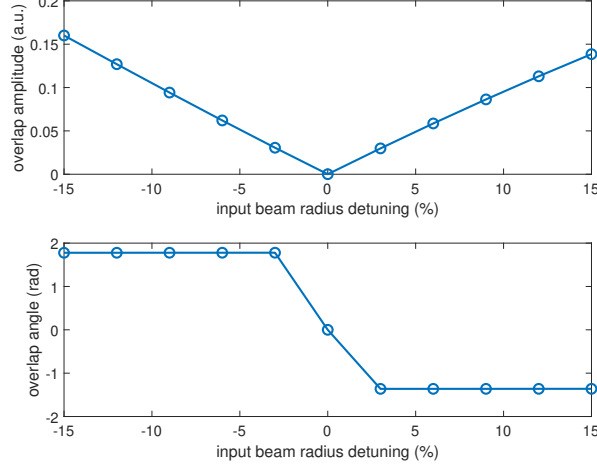


Figure 3.12: Amplitude (top) and angle (bottom) of the complex field overlap between the reflected carrier beam from the cavity and the second order mode in the cavity eigenmode basis.

of the fundamental one. This can be clearly seen by computing the complex overlap of LG_{01} with the reflected carrier beam:

$$\gamma(E_{\text{ref}}, LG_{10}) = \frac{\int E_{\text{ref}} \cdot LG_{10}^* dx dy}{\sqrt{(\int |E_{\text{ref}}|^2 dx dy \cdot \int |LG_{10}|^2 dx dy)}}, \quad (3.18)$$

where E_{ref} is the complex field amplitude of the carrier beam reflected from the cavity, and LG_{10} is defined in the basis of the cavity. Normally, the norm of the overlap is computed, giving a value between 0 and 1 which corresponds to how much the two fields have the same power mode content. It is a known result that the mode mismatch coupling to the fundamental mode has a complex amplitude (see for example [13]). With this in mind, one can also look at the angle of the complex overlap, which will give us a measure of the overall phase difference between the two field distributions. Doing this in OSCAR as the input beam radius is detuned, resulted in Fig. 3.12.

It can be seen in Fig. 3.12, that while the amplitude of the overlap is proportional to the mismatch, the angle is constant for detuning except for a π phase shift at 0 beam radius detuning. This phase shift corresponds to a sign change, which explains why the reflected beam is larger for a smaller input beam and smaller for a larger input beam.

3.5.2 Measurements with the Virgo phase camera

The measurements presented here were taken on April 1st 2022 [201]. First, two 5-minute measurements were taken in the single bounce configuration, one bouncing from the north input mirror, and the other from the west input mirror. Then, another 5 minutes of data were taken first with the north arm cavity aligned and locked followed by the west arm cavity aligned and locked.

Only the B1p PhC data was used, because of the issues with the B4 beam shape reported in Section 3.2.2. Note that these measurements occurred before the fringe issues with the B1p beam shape started, as discussed in Section 3.2.2.

Amplitude information

A bidimensional astigmatic gaussian fit was applied to each sideband and carrier image from which the radius in the horizontal (r_x) and vertical (r_y) direction were extracted. First for the single bounce measurement, and then for the measurement with a single cavity locked. The change in r_x and r_y when the cavity is locked with respect to the input beam measured in single bounce was computed according to equation 3.17 for each x and y direction, where r_{ref} is the radius measured when the cavity is locked and r_{inp} is the radius measured in single bounce. This was done for each sideband and carrier.

As expected, the radius of the sidebands does not change appreciably if the arm cavity is locked, while the reflected carrier beam was seen to have increased. For the north arm, r_x increased by 1.6% and r_y by 13%, while for the west arm, r_x increased by 1.1% and r_y by 10.1%. There is a clear astigmatism in either case when the cavity is locked. Additional simulation efforts presented in [201] show that the observed astigmatism when the cavity is locked could be due to a misalignment of the end mirror of the cavity. Simulations on the same measurement scheme made posteriorly and published in [3], show that in the presence of astigmatic mismatch the reflected carrier beam will have different radius along the x and y axis values corresponding to the amount of mismatch in each axis. However, measurements from the optical characterization paper show an astigmatism in the cavity in the order of 2 m at most over a RoC of ~ 1.5 km. This small amount of astigmatism is unlikely the cause of the significant difference between r_x and r_y observed here.

Phase information

For the input beam measured in single bounce, the differential phase between carrier and sidebands is flat. This can be seen in the inset of Fig. 3.2.

However, when the arm cavities are locked, some wavefront curvature can be seen as shown in the bottom panel of Fig. 3.13. Although the differential phase cross section appears to have positive curvature, the phase suffers from some aberration other than the curvature, which is more clear in the WA.

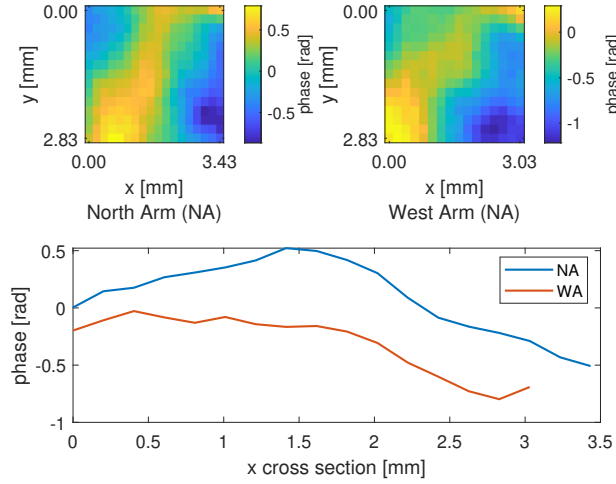


Figure 3.13: Differential phase images and cross sections between the 56 MHz upper sideband and carrier for both locked north arm and west arm cavities.

Besides being a possible explanation for the beam astigmatism, I showed in [201] that a tilt of the cavity end mirror could also reproduce, at least in part, the observed shape in the phase images. Fig. 3.14a and 3.14b show the simulated differential phase in the presence of mode mismatch of the input beam RoC and a tilt on the EM and the measured phase image. This result was obtained with a $1.4 \mu\text{rad}$ tilt on the cavity EM, pointing again to the presence of some misalignment in the cavity.

These results can be used to match the simulation and find the input beam mode mismatch in terms of radius and wavefront RoC. It was assumed that the change in r_y was only due to mode mismatch and the supposed misalignment of the cavity EM was disregarded. First, the input beam radius was changed in order to obtain a similar radius increase as the one measured. Secondly,

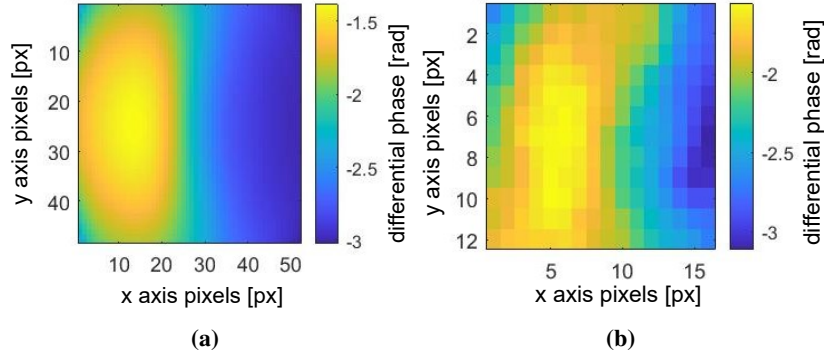


Figure 3.14: Comparison of simulated differential phase map between carrier and sideband (a) and the corresponding measured differential phase map (b). In the simulation, besides some mode mismatch, a $1.4 \mu\text{rad}$ tilt of the EM was added to induce the observed shape.

the input beam wavefront was changed in order to obtain the same amount of mode mismatch as that measured by the FSR scan method. The simulation was done including the cold mirror maps for each north and west IM.

For the north arm, this resulted in a reflected carrier beam 12.98 % larger than the input beam and 2.02 % mismatch, when injecting an input beam with radius 6.45 % smaller and wavefront curvature 5.5 % larger than the mode-matched case. For the west arm, this resulted in a reflected carrier beam 10.8 % larger and 1.22 % mismatch, when injecting an input beam with radius 5.7 % smaller and wavefront curvature 4% larger than the modematched case.

From this we can infer that the input beam is smaller than it should be in order to be mode matched. An independent analysis of the input beam using a different method and optical simulations reached the same conclusion [203].

Modal decomposition

In order to validate if there is some misalignemnt in the locked cavity we can perform modal decomposition of the phase camera images and look for the presence of first-order modes.

Here, the modal decomposition was done using only the amplitude images. The parameters to be optimized by the modal decomposition were the beam rotation, and the coefficients of the HG modes up to the order included in the fit. The basis for the modal fit was the size of the HG_{00} mode fitted from the single bounce images.

The modal decomposition is an optimization problem where the cost function is the residual between the generated image in terms of the HG coefficients of Eq. 1.31 and the real image. The minimisation of the cost function was done with the Constrained Optimization By Linear Approximation (COBYLA) algorithm [204]. Fig. 3.15 shows an example of the measured amplitude image with the fitted contours overlaid on it.

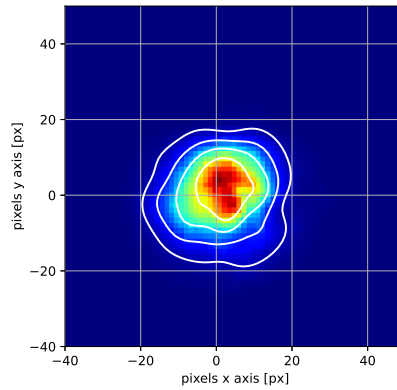


Figure 3.15: The carrier amplitude image measured by the phase camera in reflection of the north arm cavity is shown in color. The white contour lines shows the result of the fitted image obtained with modal decomposition.

Table 3.3 shows the obtained results of the modal decomposition. The COBYLA algorithm allows for including constraints in the fit. According to the FSR scans of the arm cavities shown in [113], we know that the cavity mode is dominated by the fundamental mode. For this reason, I ran two fits with 90% and 95% lower bound limits on the amount of fundamental mode. Additionally, it is necessary to choose the maximum order of the HOMs included in the fit. In order to do this, I ran multiple fits with increasing maximum mode order to see how the fit residuals evolved. Doing this showed that including HOMs above order 5 starts degrading the quality of the fit, so the maximum mode order was set to 5.

Unfortunately, both fits give different solutions with respect to the amount of first-order modes, but with comparable residuals. One solution to address this ambiguity could be to use the phase information in the fit as well. However, the absolute phase images are not available.

Table 3.3: Higher-order mode results of the modal decomposition of the north and west arm carrier beams respectively. Fit A was done with a 95% lower bound on the amount of fundamental mode, and Fit B with a lower bound of 90%.

	NA		WA	
	Fit A	Fit B	Fit A	Fit B
Residual [%]	0.43	0.35	0.39	0.48
HG ₂ [%]	2.6	2.8	1.8	1.4
HG ₀₁ [%]	0.1	2.4	0.0	0.7
HG ₁₀ [%]	0.0	0.0	1.7	0.7
HG _{M>2} [%]	0.4	0.6	0.5	0.7

In any case, both the modal decomposition and the Δr analysis show that the north arm cavity has a larger mode mismatch than the west arm. This result is consistent with 2% mismatch in the north arm and 1.2% mismatch in the west arm measured at the time with FSR scans.

3.6 Summary and overview

In this Chapter we saw the importance of TCS and wavefront sensing in Virgo in particular due to the marginal stability of the recycling cavities. In that regard, I presented the results of thermal tuning of the power and signal recycling mirrors with view of improving the mode matching of the sidebands.

We also reviewed the multiple uses of the phase cameras at Virgo and saw the first direct measurement of mode mismatch in the arm cavities using the phase camera. Some questions remain regarding the mode content of the reflected beam from the cavity, which were not entirely put to rest even with the use of modal decomposition. For this, the use of the phase information would be crucial in order to obtain a unique modal decomposition fit.

Because of the issues with the B4 beam shape and the B1p beam fringes discussed in Section 3.2.2, it was not possible to take these kinds of measurements and do this kind of analysis more often. In order to further explore this technique as a way to characterize mode mismatch, we developed a table-top experiment with a phase camera monitoring the reflected beam from a cavity. This work is developed and reported in Chapter 6.

Chapter 4

Simulating the O4b detector

Simulation studies are used to inform the optical design of the interferometer as well as to support the design of the sensing and control schemes. Additionally, simulations help during the commissioning process to deal with deviations from the design parameters due to manufacturing tolerances and unforeseen complications. We saw in the previous chapter, in Section 3.4, how useful simulations can be, for example, to prepare the calibration shifts for the thermal tuning of the power and signal recycling mirrors.

In Chapter 3 of Ref. [65] a simulation regarding the impact of optical HOMs in PDH locking of linear and coupled cavities is presented. In the context of Virgo, the work shows that the error signals for marginally stable cavities are more sensitive to the presence of HOMs. In Ref. [205], simulations were used to study the impact of angular control noise in the Virgo detector, including the coupling of the angular and longitudinal degrees of freedom. These two works were simulated using the Finesse software [206].

Due to several issues related to the addition of the signal recycling cavity and in particular the marginal stability of the recycling cavities as discussed in Section 3.1.1, the Virgo detector only joined the second part of the fourth observation run (O4) in May 2023, the so-called O4b run. Initially, the upgrade plans for the next scientific run O5, targeted the mitigation of thermal noise. The plan was to install larger mirrors with improved mirror coatings with even lower losses [105]. Given the issues with the marginally stable cavities during the O4 commissioning, the priorities for O5 have since then shifted and the focus is now on implementing stable recycling cavities by introducing focusing elements. In this context, simulations have been done in OSCAR to evaluate the impact of astigmatism in the losses of the planned stable recycling cavities [207, 208].

Even though the plan is to shift towards stable recycling cavities, it is very important to verify how well we understand the current working point of the Virgo detector. As discussed in Ref. [66], accurate modeling of the detector

is crucial to build an accurate noise budget of the detector and to design the sensing and control schemes.

The work I will present in this Chapter was done in the context of simulating the thermal working point of the interferometer and to see how well it can reproduce the power recycling gains of carrier and sidebands and the CD measured on-site. The recycling gains are a figure of merit for the quality of the interferometer lock and the CD is directly related to the sensitivity of the detector to GWs. The lowest the CD is, as defined in Section 3.3, the more sensitive is the detector to the GW-induced sidebands we wish to detect.

In Ref. [209] simulations were done in OSCAR in order to reproduce the observed CD and to test the use of an aperture to minimise HOMs in the SRC. In the end, instead of using an aperture, the signal recycling mirror was slightly misaligned in order to reduce the gain of HOMs and thus reducing the CD. In the simulations I will present, the impact of tilting the signal recycling mirror on the CD is evaluated as well as the impact of the thermal working point of the detector and the arm cavity astigmatism.

As discussed in Section 2.5 and in Chapter 3, the circulating power in the arm cavities changes the cavity eigenmodes as the optics are distorted as they heat up. While the role of the TCS is to actuate and compensate for this change in the thermal working point, we saw in Section 3.4 that it is not trivial to characterize exactly the eigenmodes of the interferometer. Here, the thermal working point was simulated in OSCAR using the RoC of the various optics and hot and cold cavity g-factors published in the Optical Characterization paper of the Virgo collaboration [113]. The simulation results, in particular the carrier and sideband recycling gains and the CD were compared with the detector values measured on-site as published also in Ref. [113].

In section 4.1 I detail the optical parameters of the Advanced Virgo detector according to Ref. [113] that were used in the simulation. In the next section I expand on the topic of optical simulations and in particular on the OSCAR simulation code.

I start presenting the simulation results starting from the nominal detector in Section 4.3, where the impact of astigmatism and common and differential mode mismatches are evaluated. In Section 4.4 I present and discuss the results of simulating the hot detector.

I conclude the Chapter with a summary of the conclusions and a discussion on how well we can describe the thermal working point of the detector with the information we have. In addition, improving our knowledge of the impact of

thermal effects in the detector can also help achieving higher circulating power in future upgrades.

4.1 The O4b detector in numbers

In preparation for O4, the marginally stable cavities caused many issues for the commissioning of the detector. In particular, on-site studies show that imperfect mode matching of the recycling cavities and circulating higher-order modes caused a bi-stability in the locking of the recycling cavities [190]. This causing unpredictable jumps of the working point of the interferometer. Additionally, the issues with the recycling cavities limited significantly the achievable input power. In the end, the detector was operated at **17 W** input power.

The sensitivity of the Advanced Virgo detector during O4b was limited to about **55 Mpc** for binary neutron star (BNS) mergers¹. This value is significantly lower than the initial target sensitivity between 88 and 115 Mpc. Fig. 4.1 shows an example sensitivity curve during the O4b science run.

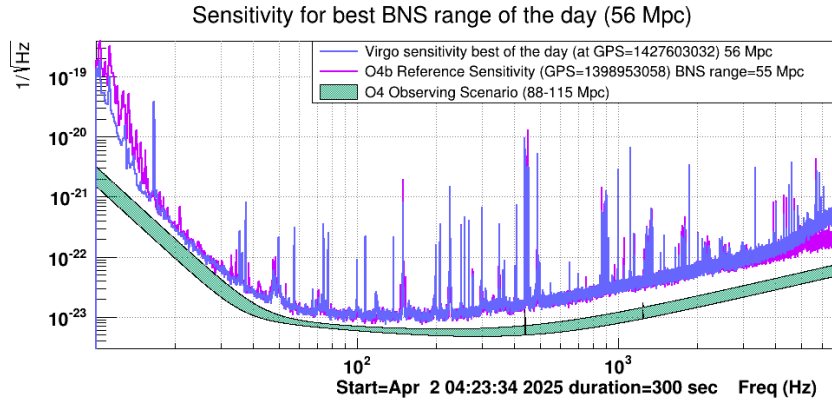


Figure 4.1: Example sensitivity for binary neutron star mergers of AdV during the O4b run, taken from the Virgo Interferometer Monitoring webpage.

The O4 sensitivity was limited not only by the issues mentioned above, but also by an unknown technical noise affecting what is called the bucket of the sensitivity curve, around 100 Hz. This so called mystery noise was fitted to have a frequency dependence of $1/f^{2/3}$. A detailed study of possible sources of this noise and their mitigation, can be found in Ref. [210].

¹For this calculation two neutron stars of 1.4 solar masses are considered. The GW is assumed to be optimally aligned with the interferometer.

Despite the lack of success in finding the source of this mystery noise, it was observed that misaligning the SRM mitigates its impact. The SRM misalignment reduces the amount of resonant sideband extraction, and by virtue of the marginally stable cavities, also the amount of circulating HOMs on the antisymmetric side (dark port) of the interferometer. This has a positive impact on the detector sensitivity. For this reason the interferometer was operated with a misaligned SRM during the entirety of the O4b run. The impact of SRM misalignment is discussed further in section 4.4.

Because of the marginal stability of the recycling cavities, or in other words the high g -factor of these cavities (Section 3.1.1), it is very important to know with precision the RoC of all the main optics. Table 4.1 lists a summary of the measured characteristics of the main optics of the detector as well as their intended design RoC values. The optics were characterized at Laboratoire des Matériaux Avancés (LMA) in Lyon, the same laboratory responsible for making and applying the reflective and anti-reflective coatings to the main optics [211]. Fig. 4.2 shows the parameters of the optics superposed on a simplified optical schematic of AdV, as well as the remaining optical parameters of the detector discussed in the rest of this section.

Table 4.1: Comparison of measured RoC and design RoC of the main interferometer optics. Measured RoC and transmission taken from Ref. [113].

Mirror	Design	Measured in [113]	
	RoC [m]	RoC [m]	Transm.
PRM	1430	1477	4.835 [%]
SRM	1430	1443	39.6 [%]
North IM	1420	1425	1.377 [%]
West IM	1420	1425	1.375 [%]
North EM	1683	1695	4.4 [ppm]
West EM	1683	1696	4.3 [ppm]

As discussed in Chapter 3, the power recycling gain of the sidebands is an important figure of merit of the interferometer, and an indicator of how well the input beam, power and arm cavities are mode matched. For this reason, the recycling gains change during the locking procedure and as the interferometer heats up. An example of the dynamics of the recycling gains as a function of the power in the arm cavities during the locking procedure is shown in Fig.

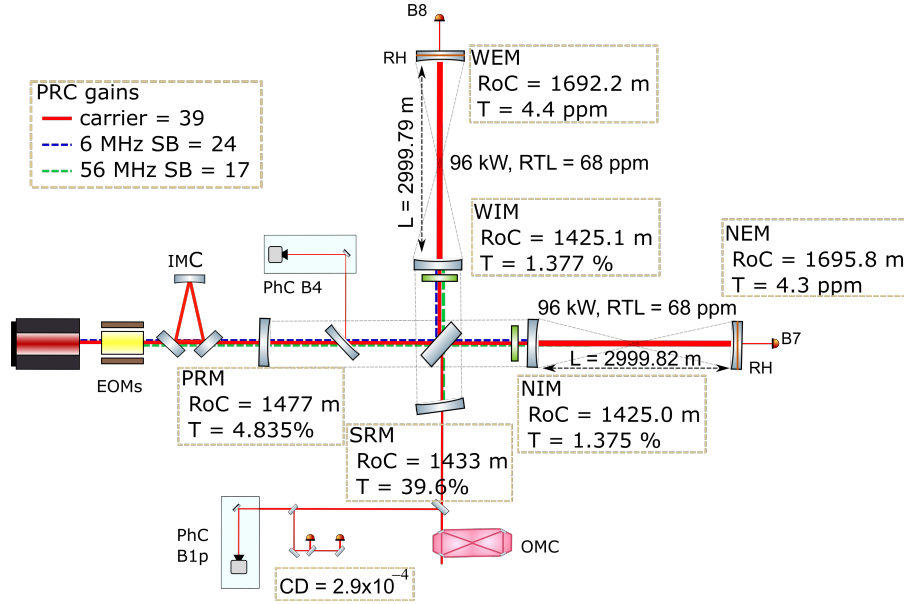


Figure 4.2: Simplified optical schematic of AdV with the respective optical parameters taken from [113] superposed on top. The SRM is misaligned as discussed in the text (exaggerated in the image).

4.3. We define the cold gains when only the CITF is locked (or just after locking the arm cavities), and the hot gains when the arm cavities have heated up and the science mode, or observation starts. In between, the power builds up in the arm cavities as they are slowly brought to resonance in a process that is called CARM reduction. At this time a small drift of the gains can already be observed. We will try to reproduce the drop in sideband gains in the simulations of Section 4.4.

Table 4.2 compares the design carrier and sideband recycling gains with the measured gains for the cold and hot interferometer as published in Ref. [113]. These values were measured with the phase camera following the description in Section 3.3.1.

Table 4.2: Design, cold and hot recycling gains reproduced from Ref. [113].

	G_{carr}	$G_{6\text{MHz}}$	$G_{56\text{MHz}}$
Design	42	72	53
Cold state	42	48	26
Hot state	39	24	17

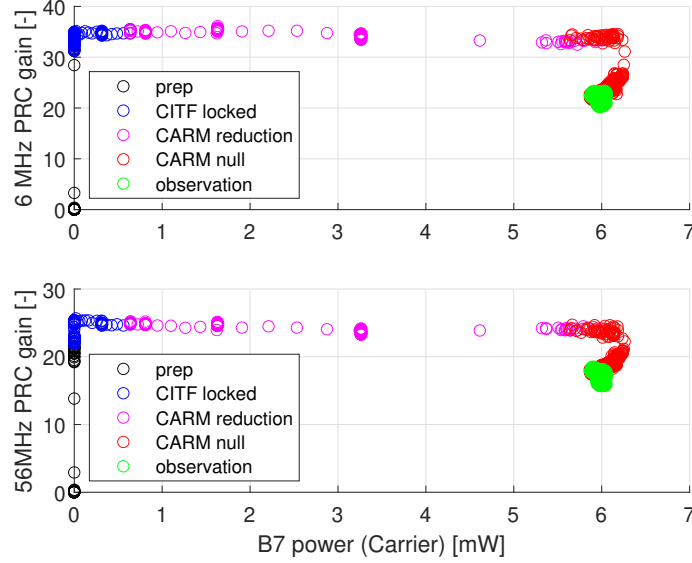


Figure 4.3: Sideband recycling gains vs. power in the arm cavities. Data points are colored according to the corresponding step in the locking procedure. The points in blue, when only the CITF is locked, are considered the cold gains. The points in green, after the thermal transient and when the detector is in observation mode, are considered the hot gains.

The values in Table 4.2 show only a fixed snapshot of the recycling gains. In practice, these values vary depending on the quality and conditions of the interferometer lock. I used the B4 phase camera signals and a single bounce reference to compute the hot and cold gains from November 2024 to February 2025. In this way, for the 6 MHz sideband I measured a cold gain of 35.6 ± 3.4 and a hot gain of 21.9 ± 1.3 . For the 56 MHz, I obtained a cold gain of 23.3 ± 1.9 and a hot gain of 16.4 ± 0.8 . While the 56 MHz gain is within $\sim 10\%$ of that of Table 4.2, the 6 MHz gain is as much as $\sim 25\%$ away, in particular for the cold state, from the values of Ref. [113]. The error bar, or variability of the 6 MHz gain, is also higher from lock to lock. This might indicate that its value is not only a function of the thermal working point of the interferometer, which should remain more or less the same during this period, but probably also of the quality of the lock and control signals.

From the carrier power recycling gain, the circulating power in the arm cavities can be computed according to the following equation [113]

$$P_{\text{circ}} = \frac{1}{2} P_{\text{in}} G_{\text{PR}} G_{\text{FP}}, \quad (4.1)$$

where G_{PR} and G_{FP} are the gains of the PRC and the arm cavity respectively. In this way, at 17 W input power, the arm cavity circulating power is $96 \pm 1 \text{ kW}$.

Using the FSR scans of the arm cavities it is also possible to measure the FSR, or the length of each cavity. This was done in Ref. [113] by comparing the distance (normalized by the FSR) from the 56 MHz sideband to the carrier TEM00. Using the exact frequency value of the 56 MHz sideband, the lengths of $2999.8203 \pm 0.0004 \text{ m}$ and $2998.7884 \pm 0.0005 \text{ m}$ were retrieved for the north and west arm respectively.

The arm cavity round-trip loss (RTL) is another important optical figure of the GW detector. Losses in the interferometer have a direct impact in the recycling gains and can degrade the control signals used to keep the interferometer locked. As discussed in Section 2.4.1, optical losses in the interferometer also reduce the amount of injected squeezed light. Finally, characterizing the cavity RTL confirms the enduring quality of the mirror test masses. These losses can be due to absorption in the substrates and coatings of the optics, beam clipping and scattering from mirror defects. In Ref. [113] an average value of 68 ppm RTL was measured for each west and north arm cavities for 17 W of input power.

Finally, the contrast defect, or CD, was measured using the phase camera as described in Section 3.3 and a value of 2.9×10^{-4} was obtained, with the SRM misaligned. Misaligning the SRM diminishes the contrast defect. Before misaligning the SRM, the contrast defect is about twice as much. A lower contrast defect results in better detector sensitivity.

4.2 Optical simulations with OSCAR

The fields circulating in a complex interferometer with multiple optical cavities such as AdV cannot be computed analytically. Already for computing the fields inside a lossy finite-aperture optical resonator numerical methods are needed to solve the resulting integral equations [188]. The problem becomes even more complicated when considering realistic non-perfect mirrors and thermal aberrations. For this reason, optical simulations are used to study the interferometer response.

Two main classes of optical simulation codes are used in the GW community: modal methods and Fast Fourier Transform (FFT) methods. Modal methods expand the electric field in a Hermite-Gaussian or Laguerre-Gaussian mode basis (see section 1.3.1). This means that the proper mode basis needs

to be selected as well as the maximum number of higher-order modes to be included so as to describe the wavefront aberrations with sufficient precision and detail. Computation time scales with the number of higher-order modes. Thus, simulation of highly aberrated beams can take long, while quasi-ideal interferometers are very quick to simulate (which is very useful in the early stages of design). Currently, the most common modal code in use is Finesse [206]. Its ability to compute the frequency response of the interferometer makes it especially suited to design and validate the control schemes for the various degrees of freedom of the interferometer.

In FFT codes an electric field is decomposed in a series of plane waves via the Fourier transform. The plane waves are propagated across a medium and the resulting electric field is recovered with the inverse Fourier transform.

A commonly used FFT code in the Virgo collaboration is the Matlab code OSCAR, short for Optical Simulation Containing Ansys results. The name already gives an indication of one of the advantages of FFT codes. Ansys is a Finite-Element Methods (FEM) simulation package, used in the GW community to simulate the effect of thermal deformation in the mirrors. In OSCAR, any wavefront distortion is characterized by an induced optical path length difference and so, it is able to include any arbitrary mirror profile, including thermal effects as well as micro-surface roughness imperfections without added computational time or convergence issues. This makes OSCAR ideally suited to compute the impact of thermal effects in stationary solutions of Fabry-Perot cavities or complex interferometers such as those used in GW detection. Fig. 4.4 shows such a mirror map with the surface imperfections measured at LMA, the so-called cold maps.

In OSCAR, The electric field is discretized in a 2D $N \times N$ grid of a given size. Then, the beam can be propagated in 3 steps. First, the beam is decomposed into a sum of plane waves through the FFT. Under paraxial approximation, propagation of a plane wave over a distance d corresponds simply to a phase shift. Thus, the fast-Fourier transformed electric field is multiplied by a phase shift corresponding to the propagated distance by a 2D point by point multiplication. Lastly, the propagated beam is recomputed back from the plane waves with the inverse FFT.

Any wavefront distortion is simply a 2D phase shift map, corresponding to some equivalent optical path length applied to the electric field again with a 2D point by point multiplication. This is the case for any lens, mirror (spherical or not) or any other aberrated surface.

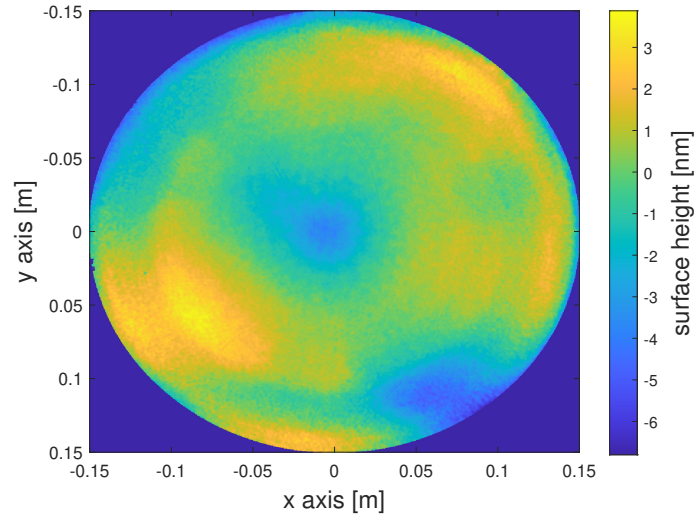


Figure 4.4: Mirror surface map of the west input arm, measured at LMA. These are often called cold maps because they are imperfections of the mirror itself in the absence of thermally induced effects.

To compute the steady-state circulating fields in an optical cavity, OSCAR first computes the necessary phase shift for the beam to resonate in the cavity. Afterwards, an iterative method is used to compute the circulating fields inside the cavity, propagating back and forth the circulating field until its value converges within a set error margin. Higher-finesse cavities need a higher number of iterations to compute the circulating fields.

Simulating the dual recycled Michelson interferometer is more complicated than a single Fabry-Perot cavity. First, the resonance condition for the power recycling cavity is found. This is done by propagating the beam through the PRM and beamsplitter into the north and west arm cavities. The resonant fields for each arm are computed as described above, and the small Michelson distance between the beamsplitter and the arms is microscopically adjusted to have both reflected fields from each arm add up coherently. Lastly, the distance between the PRM and beamsplitter is microscopically adjusted to ensure that the coherent field from the Michelson arms adds up coherently with the input beam into the PRC.

The resonant condition of the signal recycling field is found by applying a large dark fringe offset to the Michelson, in order to have the fundamental mode of the arms leak into the dark port. Then, the length of the SRC is detuned microscopically to find the highest circulating power.

Finally, in a similar iterative process as described above, all the circulating fields in the detector are computed, starting first with the reflected, transmitted and circulating beams of the arm cavities, and then for the power and signal recycling cavities in sequence.

Irrespective of any mode mismatch of the input beam and different optics, the inclusion of thermal effects and mirror surface defect maps (such as the one shown in Fig. 4.4) the process to compute the steady state beams in the detector remains the same.

In idealized conditions, the discretization of the field makes the FFT code more computationally intensive than the modal methods, which are analytic in nature. However, the FFT code becomes more advantageous computationally when aberrated beams need to be considered, as the modal method needs to include more and more HOMs into the computation. Thus, the FFT code is especially suited to compute the wavefront of the carrier and sideband beams in the steady state of the interferometer at any desired point. The simulated wavefront can be compared with the the phase camera images in the Virgo interferometer. As I am interested in simulating a realistically (and significantly) aberrated interferometer, all of the results in this chapter were obtained using OSCAR.

4.3 The nominal detector

The starting point of simulating the Advanced Virgo detector is the nominal detector, which is an idealized version of the interferometer. In this configuration we expect to obtain the design carrier and sideband power recycling gains, shown in the top of Table 4.2. The design carrier gain assumes a minimum but non-zero amount of round-trip losses. In OSCAR, I introduce losses in the north and west end mirrors until a carrier PRC gain of 42 is obtained. The round-trip losses corresponding to the design carrier gain are 55 ppm per arm cavity.

The sidebands recycling gain is set by design. In OSCAR, the exact sideband frequencies of the 6 and 56 MHz sidebands are adjusted in order to have their PRC gain match the design one.

In the nominal configuration, the RoC and transmission of all the main optics are the same as shown in Table 4.1 and the input beam is perfectly mode matched to the interferometer. No mirror maps are used, and in this way, the contrast defect obtained is $4 \cdot 10^{-7}$ with the SRM aligned. This value is about

700 times smaller than the contrast defect measured in the real detector with the SRM misaligned.

The nominal detector configuration is useful to test how particular defects impact the detector in isolation, before moving on to simulate a more realistic detector in the following sections.

4.3.1 Impact of astigmatism

The FSR scans of the Virgo arm cavities in Ref. [113] were seen to have a double resonance for the first and second-order modes which can be attributed to astigmatism of the cavity. The cavity g-factor was fitted for each of these resonances and, for each arm, the astigmatism was estimated from the difference between the two obtained g-factors. Fig. 4.5a shows a schematic of the two cavity eigenmodes and g-factors due to astigmatism. Note that from the cavity FSR scans, the g-factor $g = g_1 g_2$ is fitted, but no information is retrieved on each g_1 and g_2 independently. This means that the measured astigmatism is over an equivalent cavity mirror RoC. Note as well that no information on the orientation of the astigmatism can be retrieved in this way, as we cannot know from the FSR scan which second-order peak corresponds to the major axis and which to the minor axis.

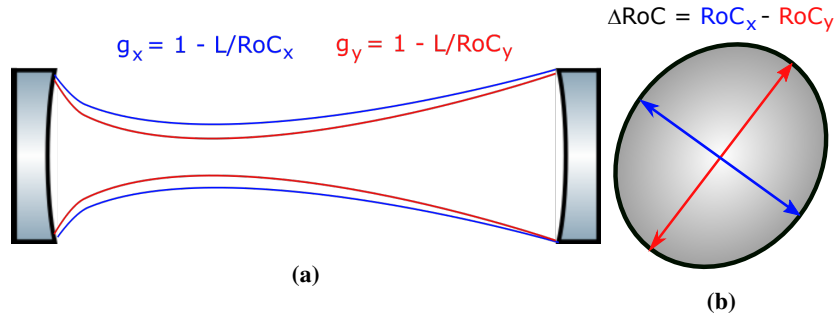


Figure 4.5: (a) Schematic of the two cavity eigenmodes due to astigmatism. In blue the beam profile across the x axis is shown and in red the profile across the y axis. (b) Astigmatism definition as the difference between the RoC over the x and y axis. Notice that the orientation of the astigmatism ellipse is unknown from the FSR scans as described in the text.

Thus, in Ref. [113] an astigmatism of $\Delta RoC = 1.8$ m was measured for the north arm and $\Delta RoC = 2.4$ m for the west arm. This astigmatism is defined as the difference in RoC between the major and minor axis as shown

in Fig. 4.5b. Despite these measurements, during the characterization of the optics at LMA, no significant astigmatism was measured. However, the FSR scan measurements are indubitable in this regard. The measured astigmatism could be explained by gravity-induced stress from the mirror suspension and also from small asymmetry in the ring heaters around the mirrors or simply small manufacturing errors that evaded measurement at LMA.

Measurements done with the ring heaters on and off reported in Ref. [212], show that the ring heaters could be responsible for about 0.5 m astigmatism. Note that while ring heaters are installed on both the input and end mirrors of each cavity, only the end mirror ring heaters are turned on.

In order to understand the impact of astigmatism in the detector, I first add a 2 m difference in the RoC in one of the axes of the north input mirror. In this way, the CD goes from $4 \cdot 10^{-7}$ to about $1 \cdot 10^{-4}$, with the SRM aligned. The impact on the CD is the same whether the astigmatism is in the x or y direction. The only difference is that for astigmatism in the x axis, the dark port beam becomes dominated by the HG_{20} mode and for astigmatism in the y direction by the HG_{02} mode.

A 2 m astigmatism over a RoC of 1420 m, corresponds to about 0.15% astigmatism. Because it is so small, no impact is observed in the PRC itself. The dark port, however, is sensitive to all the differential defects and higher-order modes in the interferometer.

Adding the same amount of astigmatism, 2 m, in the north end mirror (and none in the input mirror) results in a CD of $2 \cdot 10^{-4}$. In terms of mode content at the dark port, there is a 1% increase in the amount of HG_{02} .

It is not clear why there is a slightly larger CD and amount of HG_{02} for the astigmatic end mirror instead of astigmatic input mirror. In terms of the cavity eigenmode, the astigmatism added to the input mirror moves the waist (in the astigmatic axis) away from input mirror, while the astigmatism added to the end mirror moves the waist towards the input mirror. This seems to result in a slightly different mode coupling. Comparing the HG_{02} mode in the two cases, there is not an exact overlap between them.

Added astigmatism in the west arm has a similar effect as that in the north arm. However, the same astigmatism, in the same axis, on both arms cancel each other out, and the mode content at the dark port remains unchanged. This means that the orientation of the astigmatism in each arm cavity is important when simulating its impact on the detector, since it will have a bigger or smaller impact depending on the relative orientation. However, with the FSR scan method mentioned above it is not possible to have any information on the

orientation of the astigmatism. For this reason, I tested in the simulation the best and worst case scenarios to see their impact on the CD.

Besides not measuring the astigmatism orientation, the FSR scans also cannot distinguish between astigmatism in the input and end mirror. However, the ring heater test mentioned above points to about 0.5 m astigmatism induced from the ring heater, so I have included this astigmatism in the simulation as well. Table 4.3 shows the results of different astigmatism orientation on the CD, with and without the tilt of the SRM.

Table 4.3: OSCAR Simulation results for the CD with and without an SRM tilt for different astigmatism configurations. The orientation of the astigmatism, either x or y, is shown in parenthesis. The worse and best case scenarios were tested.

CD		astigmatism [m]			
SRM align	SRM tilted	NIM	NEM	WIM	WEM
$1.2 \cdot 10^{-5}$	$1.1 \cdot 10^{-6}$	1.8 (x)	0	2.4 (x)	0
$2.5 \cdot 10^{-4}$	$4.5 \cdot 10^{-5}$	1.8 (y)	0	2.4 (x)	0
$2.3 \cdot 10^{-5}$	$1.8 \cdot 10^{-6}$	0	1.8 (x)	0	2.4 (x)
$4.7 \cdot 10^{-4}$	$8.7 \cdot 10^{-5}$	0	1.8 (y)	0	2.4 (x)
$1.2 \cdot 10^{-5}$	$1.1 \cdot 10^{-6}$	1.3 (x)	0.5 (x)	1.9 (x)	0.5 (x)
$2.9 \cdot 10^{-4}$	$5.5 \cdot 10^{-5}$	1.3 (y)	0.5 (y)	1.9 (x)	0.5 (x)

The results shown in Table 4.3 show that the astigmatism cannot be ignored when simulating the real detector, due to its impact on the CD. Given the real value of the CD of $2.9 \cdot 10^{-4}$ when the SRM is tilted, we can see that in the worst case the measured astigmatism can contribute up to one third of the observed CD.

4.3.2 Common and differential mismatches

Typically, we can distinguish between common and differential mode mismatch in the coupled cavity formed by the PRC and the arm cavities. The common mode mismatch affects both arms of the Michelson equally. This can be, for example, a change in the RoC of the PRM away from the design value, or a similar lens effect in both north and west arm compensation plates. The differential mismatch can be due to a different change in the RoC of the north and west input mirrors, or a different lens effect in each north and west arm compensation plates.

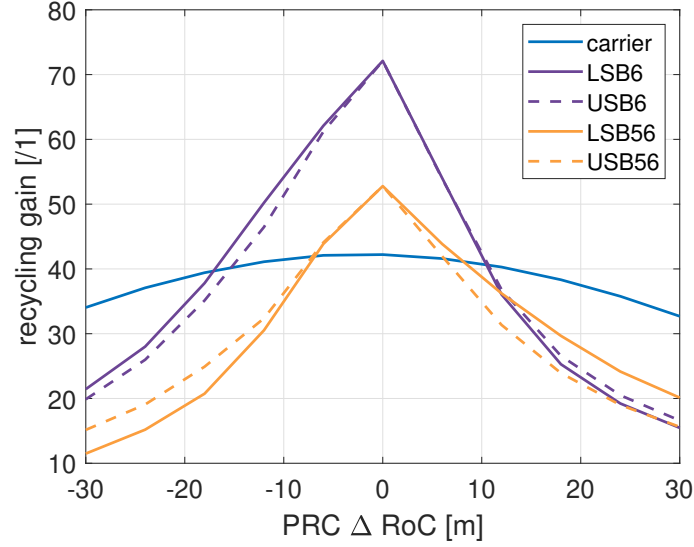


Figure 4.6: Change in the (power) recycling gain of sidebands and carrier as the RoC of the PRM is detuned. Simulated in OSCAR.

Typically, both common and differential mismatches will impact the recycling gains, but the differential mismatches will also increase the power at the dark port, hence worsening the CD.

First, we start with the common mismatch. Fig. 4.6 shows the impact of a non-ideal RoC of the PRM on the power recycling gains. This figure is similar to the last panel of Fig. 3.8, but here I have also included the carrier recycling gain. The most important point here is that the carrier is much less affected by mismatch of the PRM because it is mostly dominated by the mode of the arm cavities. The sidebands, however, as we had already seen in Section. 3.4, are more sensitive to mismatch in the recycling cavity.

The north arm compensation plate has been measured to have a significant lens, a so called cold lens. We use the word cold to indicate that the lens is there irrespective of the circulating power in the interferometer, as opposed to thermal lenses arising from optical absorption (see Section 2.7). While the transmission of the compensation plate suggests a 70 km lens, a study made on-site at the detector measured a cold lens closer to 30 km [213].

Thus, it makes sense to investigate the impact of asymmetric lensing in the detector. This was done in the simulation by introducing a lens in each arm at a time. In place of the compensation plate, the lens is introduced in the mirror

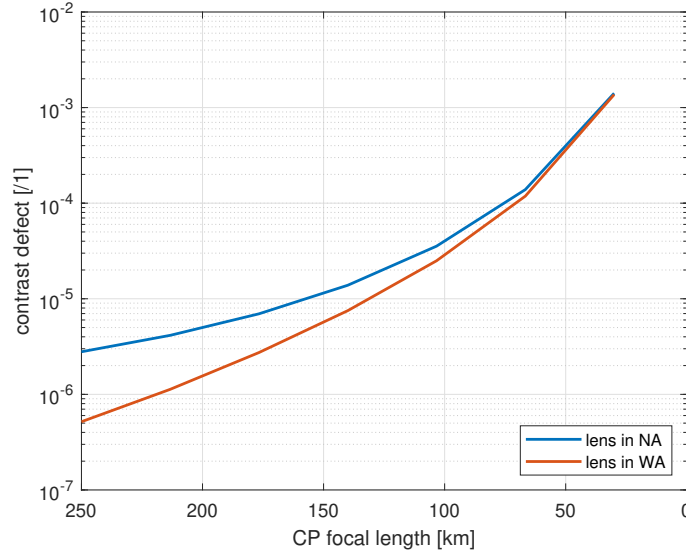


Figure 4.7: CD values for a cold lens in the compensation plate of either the north or west arms of decreasing focal length.

map of the anti-reflective side of the respective input mirror, but the results can be interpreted in light of a cold lens in the compensation plate.

A systematic scan of the focal length of the thermal lens from 30 km up to 250 km was done for each arm at a time. Fig. 4.7 shows the CD for the two positions of the thermal lens, with the SRM aligned. The impact on the CD starts being quite significant for focal lengths smaller than 50 km.

In addition, Fig. 4.8 shows the PRC gains for the 6 and 56 MHz gain for the thermal lens in each arm. Already with a 250 km lens, the sidebands gains are negatively impacted. Note that the cold lens of the north arm compensation plate should be compensated by the DAS of the TCS (see Section 2.7 for more details on the DAS). However, given its significant impact on the CD and PRC gains, even a residual asymmetric lens might cause trouble to the detector.

Since both common and asymmetric effects impact the PRC gains, this is not a good figure of merit to characterize the asymmetric lensing, much less to determine the position of the lens and where the correction needs to be applied. However, this could potentially be done using the sideband images. Figure 4.9 shows the B4 and B1p images for the 6 MHz sideband for a thermal lens of 70 km in the north (upper panels) and west (bottom panels) arms respectively. From these images, we can see the B1p images are sensitive to where the ther-

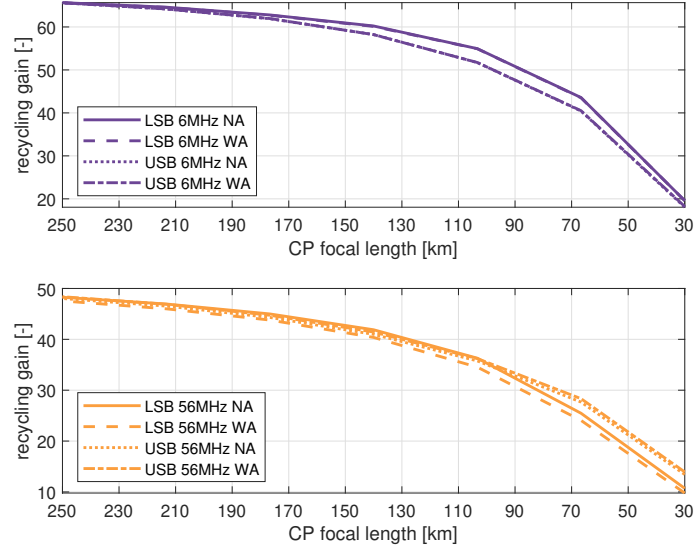


Figure 4.8: Sideband (power) recycling gains for a cold lens in the compensation plate of either the north or west arms of decreasing focal length.

mal lens is positioned. This is to be expected as the dark port, or asymmetric port is by design sensitive to differences between the two arms.

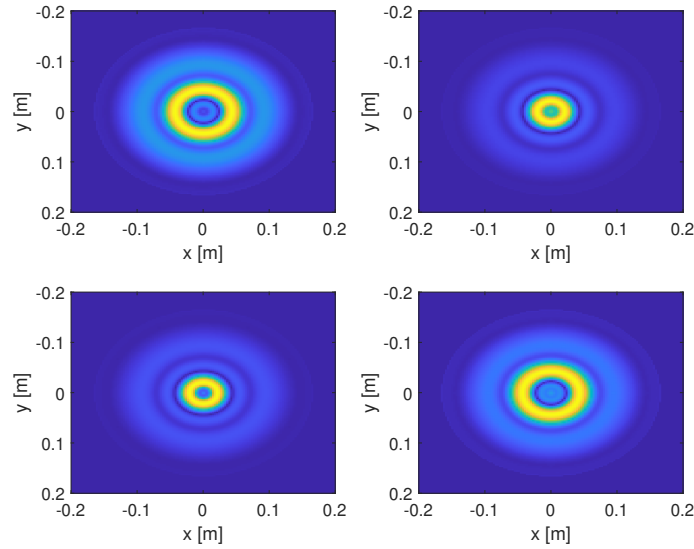


Figure 4.9: Upper and lower 6 MHz sidebands at the dark port (B1p beam) for a 70 km lens in the north arm in the upper panels and a 70 km lens in the west arm for the bottom panels. Left side panels show the 6 MHz lower sideband in both cases, and right side panels show the upper sideband.

Figure 4.10 shows the same images but for the 56 MHz sideband. In this case, even though upper and lower sideband have different modes due to the asymmetric mismatch, one cannot really tell them apart as the modes are not consistent for the two cases. This is the case because the 56 MHz sideband resonates in the SRC, coupling further the symmetric and asymmetric portions of the beam.

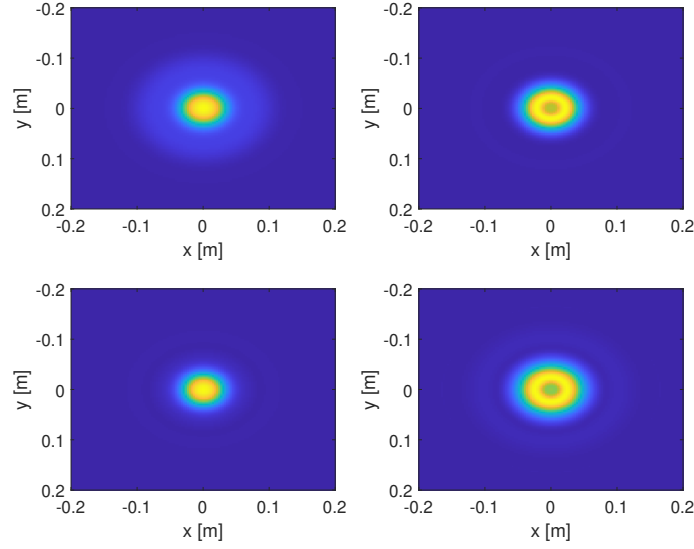


Figure 4.10: Upper and lower 56 MHz sidebands at the dark port (B1p beam) for a 70 km lens in the north arm in the upper panels and a 70 km lens in the west arm for the bottom panels. Left side panels show the 56 MHz lower sideband in both cases, and right side panels show the upper sideband.

In summary, the power recycling gains, of the sidebands in particular, are a good figure of merit to optimize the common mode matching in the PRC. The sideband images at the dark port can be used to determine the position of the cold lens. This is especially the case for the 6 MHz sideband images. Finally, the asymmetric mismatches have a significant impact on the CD while the common mismatches do not. Even a residual asymmetric lens that is not entirely corrected by the TCS can cause trouble and should be included in the simulation of the real detector. Note that here we do not discuss mismatch of the SRC since, as shown in Fig. 3.8, this has negligible impact on the sidebands and CD.

4.4 The hot O4b detector

We are now ready to simulate the full realistic interferometer that was in operation during the O4b run. The first step is introducing in the OSCAR code the RoC of all the main optics. Even though Table 4.1 has the RoC values as measured at LMA, these are not necessarily the ones we should use in the simulation. When the interferometer is fully locked, the high circulating power in the arm cavities (96 kW) heats up the test masses. The increased absorption at the test masses causes their RoC to change and thus the cavity g-factor changes accordingly.

In Ref. [113], the change in the g-factor of the arm cavities was measured when the detector goes from cold to hot. In order to do this, the interferometer is locked for at least one hour, so that the cavities heat up and the thermal transient is over. Afterwards, the interferometer is purposefully unlocked and FSR scans of the arm cavities are started right away.

Even though it is not possible to measure the thermal transient of the arm cavities while they are locked, with this method it is possible to monitor the cavities as the mirrors cool down. From this measurement, a decrease of 1.5 m per mirror was estimated for both the north and west arm cavities when hot.

Additionally, the g-factor of the cavities was measured with the end mirror ring heater on and off to measure its effect on the end mirror RoC. This measurement estimated a -24.6 ± 0.9 m change for the NEM due to the ring heater and -27.3 ± 0.6 m for the WEM.

Due to the astigmatism in the arm cavities two (cold) g-factors were measured in Ref. [113], one for each x and y axis. Using these cold g-factors as well as the changes induced by the ring heater, I recovered the cold RoC of each cavity mirror for each astigmatic orientation, arriving at slightly different values than those measured at LMA which are shown in Table 4.1. Since the measurements at LMA are made using a reference surface [211], it is possible that there could be a sub-percent systematic error in the measurement. Additionally, the mirrors are suspended in a different way with respect to how they were at LMA, which could also justify a difference of a few meters.

To stay consistent with the study of Ref. [212], I used the astigmatism values on the last row on Table 4.3 for the simulation: +1.3 m and +0.5 m along the y axis for the NIM and NEM and +1.9 m and +0.5 m along the x axis for the WIM and WEM. Given that we cannot know the orientation of the astigmatism, I fixed it arbitrarily but with opposite directions between the north and west arm cavities, to have the worst-case scenario (as discussed in Section

4.3.1). In this way, the cold g-factors for the north arm cavity for each x and y axis are written as

$$g_x^{\text{NA}} = \left(1 - \frac{L_{\text{NA}}}{1425 - 5.24}\right) \left(1 - \frac{L_{\text{NA}}}{1695 - 5.24 - 24.7}\right) = 0.8920 \quad (4.2)$$

$$g_y^{\text{NA}} = \left(1 - \frac{L_{\text{NA}}}{1425 - 6.18}\right) \left(1 - \frac{L_{\text{NA}}}{1695 - 6.18 - 24.7}\right) = 0.8943, \quad (4.3)$$

where $L_{\text{NA}} = 2999.8203$ m is the length of the north arm cavity. Similarly, the cold g-factors of the west arm are written as

$$g_x^{\text{WA}} = \left(1 - \frac{L_{\text{WA}}}{1425 - 5.1}\right) \left(1 - \frac{L_{\text{WA}}}{1696 - 5.6 - 27.3}\right) = 0.8949 \quad (4.4)$$

$$g_y^{\text{WA}} = \left(1 - \frac{L_{\text{WA}}}{1425 - 4.35}\right) \left(1 - \frac{L_{\text{WA}}}{1696 - 4.35 - 27.3}\right) = 0.8919, \quad (4.5)$$

where $L_{\text{WA}} = 2999.7884$ m is the length of the west arm cavity.

Ref. [113] indicates -1.5 m RoC change per mirror when the arm cavities heat up. However, as also mentioned in the article, this does not take into account the different size of the beams at the input and end mirrors. Taking this into account, the thermal effect is about 40% smaller at the EM. Thus, scaling accordingly the RoC difference with respect to the experienced thermal effect, the input mirror should experience -1.735 m in RoC, and the end mirror -1.235 m. Finally, the hot g-factors can be written as

$$\begin{aligned} g_x^{\text{NAhot}} &= \left(1 - \frac{L_{\text{NA}}}{1425 - 5.24 - 1.765}\right) \left(1 - \frac{L_{\text{NA}}}{1695 - 5.24 - 24.7 - 1.765}\right) \\ &= 0.8956 \end{aligned} \quad (4.6)$$

$$\begin{aligned} g_y^{\text{NAhot}} &= \left(1 - \frac{L_{\text{NA}}}{1425 - 6.18 - 1.235}\right) \left(1 - \frac{L_{\text{NA}}}{1695 - 6.18 - 24.7 - 1.5}\right) \\ &= 0.8979, \end{aligned} \quad (4.7)$$

and

$$g_x^{\text{WAhot}} = \left(1 - \frac{L_{\text{WA}}}{1425 - 5.6 - 1.765}\right) \left(1 - \frac{L_{\text{WA}}}{1696 - 5.1 - 27.3 - 1.765}\right) = 0.8985 \quad (4.8)$$

$$g_y^{\text{WAhot}} = \left(1 - \frac{L_{\text{WA}}}{1425 - 4.35 - 1.235}\right) \left(1 - \frac{L_{\text{WA}}}{1696 - 4.35 - 27.3 - 1.235}\right) = 0.8955. \quad (4.9)$$

The denominators of the hot g-factors of Eqs. 4.6, 4.7, 4.8 and 4.9 were the values used in the simulation for the input and end mirrors of the north and west arms respectively.

For the power and signal recycling mirrors the RoC of Table 4.1 were used as starting points. In addition, the SRM is tilted by 2 μrad in the y direction, to mimic what is done in the real detector, as discussed above.

The observables we want to match with the simulation are the CD and the power recycling gains of carrier and sidebands. Having fixed the mirrors RoC as defined above, I tuned the following parameters of the simulation in order to match the simulation with the real detector:

- PRM RoC - As discussed in Section 3.4, the RoC of the PRM was decreased using the CHRoCC to get it closer to the design value of 1430 m. However, we do not know exactly what is the RoC achieved, and so this was left as a parameter of the simulation in order to introduce common mode mismatch. As discussed in Section 3.4, this detuning of the PRM also takes into account any possible mismatch of the input beam parameter, as both have a similar effect in the interferometer.
- Mirror cold map scaling - in addition to the astigmatism which worsens the contrast defect, I had to further aberrate the detector to reach the CD levels measured on-site. This was done by increasing the amplitude the cold maps in the arm cavities by re-scaling them. This has the effect of changing the mode of the north and west arm cavities asymmetrically, thus increasing the contrast defect.
- Arm cavity round-trip loss - the RTL was tuned, by adding losses to the cavities end mirrors by convention, in order to adjust the carrier gain to the value measured on-site.

At this point, I assumed the asymmetric lensing of the cold plates was entirely compensated by the DAS of the TCS (recall the DAS and compensation plates introduced in Section 2.7).

In this way the simulation was matched to the real detector with a PRM RoC of 1446.5 m (1430+16.5) and a total of 65 ppm RTL in each arm cavity, versus the 68 ppm estimated by the OptChar group in Ref. [113]. The RTL value in the simulation includes 29 ppm introduced in each cavity end mirror plus an estimated 72 ppm losses from the cold maps in the two cavities. The cold maps of the west arm mirrors were scaled to roughly twice of the original value, in order to achieve the CD value of $2.9 \cdot 10^{-4}$. This scaling was already included in the computation of the RTL. Table 4.4 shows a comparison of the simulated and real detector.

Table 4.4: Comparison of the simulated detector in OSCAR with the recycling gains and CD of the real detector as published in Ref. [113].

	G_{carr}	$G_{6\text{MHz}}$	$G_{56\text{MHz}}$	CD	CD SRM tilted
Sim.	39	24	18.6	$1.3 \cdot 10^{-3}$	$2.9 \cdot 10^{-4}$
Meas.	39	24.0 ± 0.7	17.0 ± 0.9	$0.6 \cdot 10^{-3}$	$2.9 \cdot 10^{-4}$

While the matching between the simulation and the real detector is close, it seems we cannot reproduce the gap in recycling gain between the 6 and 56 MHz sideband compared to what is measured on-site. The tuning of the PRM RoC was done to match the 6 MHz sideband gain and the 56 MHz gain is still $\sim 9\%$ higher than measured on-site. Introducing asymmetric lensing in one of the arms did not improve this.

Something else of note is the drop in CD when tilting the SRM. The tuning of the CD was done with the SRM tilted, so comparing the CD without the tilt works as a consistency check with the real detector. In the simulation, the CD drops by more than 4 times, while in the real detector we observe a drop of about 2 times. If we assume the actual tilt of the SRM is slightly smaller, we can achieve in the simulation the same CD after tilting the SRM by $1.2 \mu\text{rad}$ without a scaling of the cold maps. In this case, the CD only drops by 3 times with the SRM tilt, which is closer to what is measured on-site.

In order to better study the impact of tilting the SRM on the CD, I did the modal decomposition of the beam after the SRM with and without tilt up to mode order 20. The power per mode with the SRM aligned and misaligned is shown in Fig. 4.11.

In the figure we can see that when the SRM is tilted, the power of the even modes is decreased very significantly, but the power of the odd modes increases. This is because while the mirror misalignment decreases the enhancement factor of the SRC, it also excites odd modes due to its misalignment. This different impact on different modes indicates that in order to accurately simulate the impact of the SRM tilt accurate knowledge of the mode content at the dark port is necessary.

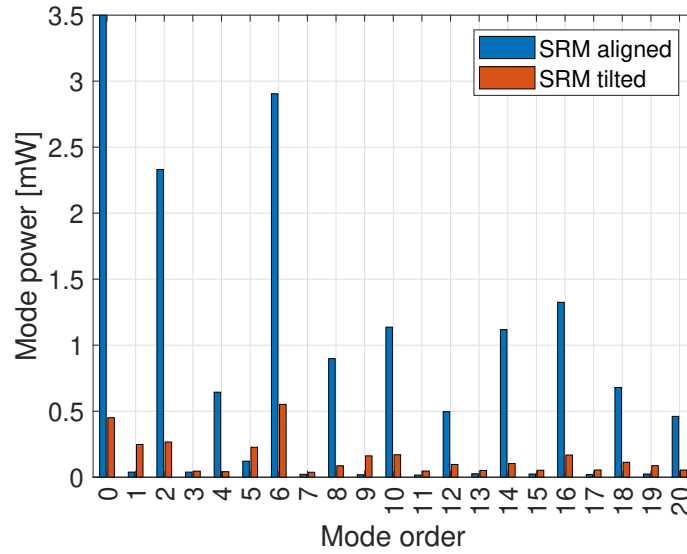


Figure 4.11: Power at the dark port per modes of same order, up to order 20. Values are shown with the SRM aligned and misaligned, for the case of 17 W input power. Simulation done in OSCAR with the hot detector parameters described in the text.

4.4.1 Checking for consistency with the cold detector

Despite the good matching between simulation and the real detector shown in Table 4.4 the thermal working point of the CITF was adjusted arbitrarily to match the recycling gains by tuning the PRM. Different working points, including for example cold lens in the compensation plates, or a different distribution of the astigmatism and different PRM RoC could achieve the same recycling gains.

Another way to check the validity of the hot working point is to check the cold recycling gains with the same configuration. In terms of the thermal working point of the CITF, the only difference should be the -1.5 m RoC in

the arm cavities. We assume there is no significant thermal distortion in the PRC as the circulating power there should be lower than 1 kW, as compared to the 96 kW in the arm cavities. With this assumption, I removed the -1.5 m contribution from the cavity mirrors, and added an offset to the arm cavities to bring them out of resonance. Everything else remains the same. Recomputing the recycling gains for this cold configuration, we obtain 29.1 and 23.6 for the 6 and 56 MHz recycling gains respectively. Table 4.5 shows a summary of the hot and cold interferometer comparison.

Table 4.5: Comparison between the cold and hot recycling gains in the OSCAR simulation, Ref. [113] and the values measured with phase camera during O4b.

	6 MHz SB		56 MHz SB	
	cold	hot	cold	hot
Simulation	27.5	24	22.5	18.6
measurement [113]	48	24.0 ± 0.7	26	17.0 ± 0.9
PhC meas.	35.6 ± 3.4	21.9 ± 1.3	23.3 ± 1.9	16.4 ± 0.8

While the 56 MHz gain is close to that measured in the real detector, the 6 MHz gain is 43% smaller than the value of Ref. [113] and 23% smaller than the values measured with the PhC here. It seems that we cannot reproduce the drop in the 6 MHz gain with just the g-factor change of the arm cavities.

4.5 Overview and outlook

This simulation work shows that optical parameters measured by the Virgo OptChar group and published in Ref. [113] can, with just a couple of free parameters, describe much of the detector behavior with what regards the sidebands recycling gains and the CD, which directly impacts detector sensitivity. This means that mode matching and the thermal working point have a large impact on the performance of the Virgo detector. Even so, mode mismatch alone is not enough to describe the gain of the 56 MHz sideband, which is off with respect to the measurement of Ref. [113] by 8.5%. The simulations of the Appendix B show that a mistuned etalon of the input mirror substrates could account for this difference.

Even though there is a good match between simulation and the real detector in terms of recycling gains in the hot state, the drop in the 6 MHz sideband after the thermal transient that is observed on-site is difficult to account for just

in terms of the cavity matching. Future work should investigate the magnitude and impact of a possible thermal lens in the beam splitter. Additionally, there could be other dynamics of the various control loops that keep the interferometer locked that impact especially the 6 MHz sideband and are not taken into account in the OSCAR simulation. This would explain also the larger variability of the 6 MHz sideband gain observed on-site.

Additionally, we have seen that just from these parameters alone we cannot uniquely determine the thermal working point of the detector. This warrants the need for a direct measurement of the cavity eigenmodes as, for example, is discussed in the review article of Ref. [2].

Additionally, we have seen that the change in CD when tilting the SRM is not exactly reproduced. In fact, it is shown in Fig. 4.11 that the impact of tilting the SRM on the amount of power at the dark port depends on the mode content. Hence, fully matching the simulation to the real detector requires knowledge of the mode content at the dark port. Here, the phase camera can be crucial since it can be used to perform modal decomposition of the carrier beam.

In the next part of this thesis I present two table-top experiments with the aim of using the phase camera to address some of these issues. There, the phase camera is used to measure mode mismatch (Chapter 6) and the higher-order mode content (Chapter 7) of optical cavities. These experiments are the first step in showing how the phase camera can be used in GW detectors to fully characterize the eigenmodes of the interferometer.

Part III

Table-top experiments with the phase camera

Chapter 5

A custom phase camera for table-top experiments

In this chapter, I detail the development of a custom phase camera, similar to the ones used at Virgo as described in section 2.6.4 and in References [20, 142]. The developed phase camera was used in two distinct table-top experiments. One using the phase camera in reflection from a cavity to measure mode mismatch is detailed in Chapter 6. The other, detailed in Chapter 7, uses the phase camera in transmission of a cavity to detect the higher-order modes of the cavity while it is locked.

In Section 5.1, I start by giving an overview of the hardware choices and practical implementation of all aspects of the developed phase camera. Next, in Section 5.2, I present in detail the characterization of the phase camera with respect to the amplitude and phase information, respectively. It is crucial to validate that the phase camera is measuring both amplitude and phase correctly.

5.1 Developing a scanning phase camera

The development of our custom phase camera happened in collaboration with the phase camera group at Nikhef. Following the COVID-19 crisis, due to electronic shortages, Nikhef was unable to provide the original ADC board and acquisition electronics used in the Virgo phase camera. Thus, the greatest motivation to opt for developing a custom phase camera was the ability to choose readily available commercial options for all the components. Additionally, building the knowledge in-house has its own advantages in terms of adaptability, customization and control.

5.1.1 Hardware choices

As discussed in section 2.6.4, the main components of a phase camera are a fast photodiode, a piezo scanner to scan the laser beam over the photodiode, an ADC and a field programmable gate array (FPGA) for data acquisition and control. If a reference beam is used, an AOM is also necessary. Table 5.1 lists all of the main pieces used in our custom phase camera. Below, I detail each piece of equipment and the reasoning for its choice.

Table 5.1: Main phase camera components.

	model	company
photodiode + amplifier	N/A	custom Nikhef
piezo scanner	S-335.2SHM1	PI
scanner driver	E-727.3SDAP	PI
FPGA + ARM CPU	ALPHA250-4	Koheron
AOM	MT40-IIR80-Fio-SM	AA Optoelectronic

Fast photodiode

Nikhef provided a custom fast InGaAs photodiode and amplifier electronics. It is slightly different from the Virgo photodiode since it has a diameter of 70 instead of 55 μrad . A larger area means a larger capacitance and, hence a slower response time and slower bandwidth. The transimpedance amplifier, model HMC799LP3E from Hittite Microwave Corporation, now part of Analog Devices, has a 700 MHz bandwidth. The overall photodiode, following experimental tests, was deemed to have a bandwidth larger than 250 MHz.

Piezo scanner and driver

The original phase camera used the tip/tilt platform from Physik Instrumente (PI) model S-334. This model has since been replaced by the S-335, so we acquired from the same company the tip/tilt platform model S-335.2SHM1. The scanner already came with a half-inch silver mirror installed. Overall, it has the same characteristics as the previous model, with 35 mrad range and similar bandwidth but improved resolution. It has 1 μrad closed-loop resolution against the 5 μrad resolution of the old model. The maximum operation frequency recommended by PI is one third of the mechanical resonance of the

scanner plus mirror system. With a half-inch mirror, this resonance frequency should be, according to the manual, around $1.60 \pm 0.32\text{kHz}$. Therefore, the maximum open-loop control frequency should be just over 500 Hz.

The scanner of the original Virgo phase camera is driven by an analog driver from PI, due to bandwidth constraints. However, after exchanging notes about our desired application, we were advised by the technical sales team at PI to get the E-727.3SDAP digital driver. It does not impact negatively the actuation bandwidth and it has the necessary amplification, strain gauge readout, closed and open-loop operation in a single compact piece of equipment. Another advantage of this driver, as discussed in section 5.1.3, is the ability to control the scanner via Serial Peripheral Interface (SPI).

FPGA and CPU

The phase camera requires digital demodulation of the photodiode signal in the two quadratures. This is normally implemented on an FPGA. Additionally, the FPGA must connect to an analog-to-digital converter (ADC), and finally the demodulated data needs to be accessed by some sort of computer. The ALPHA250-4 board, from Koheron, provides all of the above in a compact design. It is based around the Zynq 7020 System on a Chip (SoC) from AMD providing a mixed processor + FPGA architecture. This means it has the functionalities of an FPGA, but it can also run an operative system, such as Linux, on the ARM processor. We acquired the option with $4 \times \text{ADC}$ front ends, each with 250 Msps sample rate and 14 bit resolution. The noise level on the ADC front end is below 20 nV above 300kHz. Another advantage of the ALPHA250 board, is that it is commercially available and requires no custom parts, so it is more easily accessible and ideal for prototyping.

AOM

One disadvantage of the ALPHA250 board with respect to the original Virgo design, is the loss in ADC bandwidth. The Virgo design uses the ISLA214P50 ADC board from Intersil, which has 500 Msps against the 250 Msps of the ALPHA250 board. In order to make up for the loss in bandwidth, we use an AOM (MT40-IIR80-Fio-SM from AA Optoelectronic) for the reference beam which provides a 40 MHz shift instead of the 80 MHz of the Virgo phase camera. A larger frequency shift may be useful to help separate close sidebands, but this is not really an issue for the table-top experiments developed in this work. In this way we lower the ADC requirements for data acquisition.

In the next section, I describe with more detail the data acquisition and demodulation with the ALPHA250 board.

5.1.2 Data acquisition and signal demodulation

The programming of the ALPHA250 board was done by Dr. Demin, who works as a research software engineer at CP3. I worked with him to define the phase camera requirements in terms of the necessary signal demodulation as well as the scanner control. The programming of the ALPHA250 board is a mix of hardware programming, as usual in FPGAs, with software programming of the ARM processor.

The FPGA was programmed in Vivado, where the signal acquisition and demodulation architecture, shown in Fig. 5.1, was implemented. The PhC photodiode signal is acquired by the ADC0 on the ALPHA250 frontend and is mixed with 7 local oscillators for demodulation in both I and Q quadratures for each of the 7 frequencies. All local oscillators are implemented with Direct Digital Synthesis (DDS) using a combination of a look-up table with 2048 quarter-wave period samples of the cosine waveform and Taylor series correction in order to obtain 24 bit sine and cosine wavefunction at the required frequency. The demodulation frequencies are software defined, but a typical demodulation scheme is shown in Fig. 5.2.

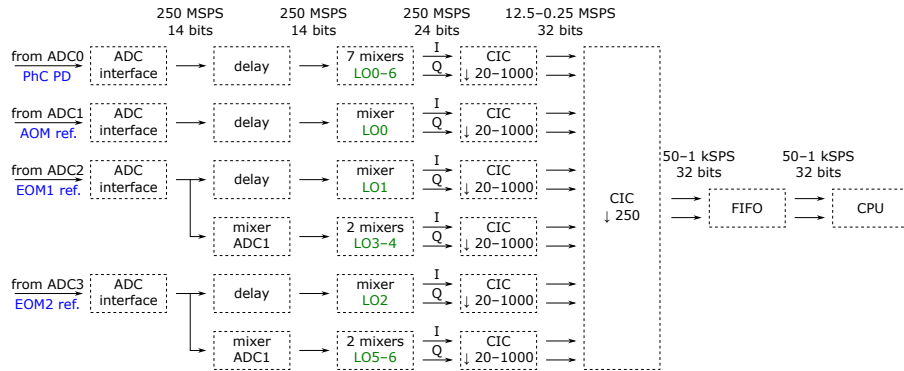


Figure 5.1: FPGA block diagram showing the acquisition and demodulation pipeline from the ADC frontend to the CPU. Seven local oscillators LO0-LO6 are used for demodulation as shown in Fig. 5.2. Low pass filtering is achieved with two sets of cascaded integrator-comb (CIC) filters to achieve the required level of decimation. More details are given in the text.

The remaining ADC inputs are used for the AOM and EOM electrical references respectively. These signals are mixed with the same local oscillators in order to obtain correction signals to offset a time dependence of the demodulated phase signal as discussed in References [20, 142] and in Section 5.2.2.

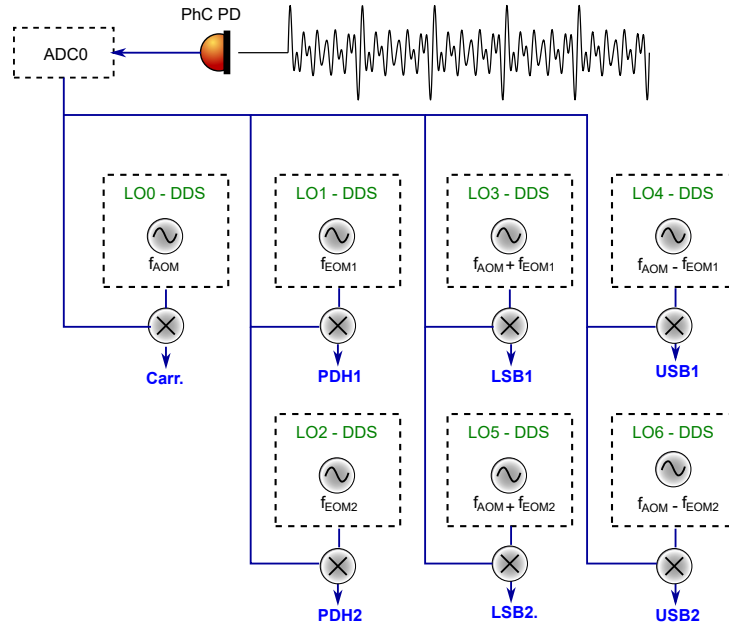


Figure 5.2: Block diagram depicting the demodulation of the photodiode signal with the 7 local oscillators. The demodulation frequencies are software defined, but a typical example is shown in order to demodulate the optical beatings of the carrier and each sideband with the reference beam, as well as the sideband only signals (sometimes referred in the text as the PDH signal).

Finite input response (FIR) filters are used after demodulation to filter out any unwanted signal contamination. In this case, the FIR filters are implemented in two stages of cascaded integrator-comb (CIC) filters. Two stages are necessary to reach the required level of decimation for the low-pass filtering.

A server, written in C, runs on the ARM processor and is responsible for communicating with an outside process through sockets and for interfacing with the scanner driver as well. The server accepts the following commands from the outside:

- set scanning rate - rate at which the position of the scanner is updated.

- set sampling rate - rate at which the demodulated data is read from the FPGA registers.
- set up to 7 demodulation frequencies - Even though the frequencies are set by the client, the typical values in terms of the AOM and EOM frequencies are shown in Fig. 5.2.
- clear coordinate buffer - where the scanner positions to be scanned over are stored.
- add coordinates to the buffer.
- set position - command the scanner to go to a single position in x and y.
- run scan - goes through a full scan, corresponding to the coordinates loaded in the buffer, at the defined scanning rate. Additionally, it saves the demodulated data, at the defined sampling rate, and in the end of the scan, sends it back to the client.

It is clear from the above list of commands that the operation of the phase camera is highly modular. This means that from the client side it is possible to configure the desired demodulation frequencies, to load any scanning pattern needed, or even control the scanner in a step-by-step manner. This modularity is very useful for prototyping. It will be clear from Chapters 6 and 7 how the phase camera can be used in very different ways without any changes to the underlying programming of the ALPHA250 board.

In turn, I developed the client side of the phase camera in Python. A class `ScannerConn` wraps the basic commands the server side expects. This class has the main following commands/functions:

- connect and disconnect from the ALPHA250 board.
- configure - a single command that configures the scan and sample rate, and also sets 4 demodulation frequencies taking into account the AOM and EOM frequencies: f_{AOM} for the carrier frequency, f_{EOM} for the PDH beat, $f_{\text{AOM}} - f_{\text{EOM}}$ for the USB and $f_{\text{AOM}} + f_{\text{EOM}}$ for the LSB. Three more demodulation frequencies could be used if another pair of sidebands is used. These frequencies are received by the server running on the ALPHA250 board, and generated by the DDS of each local oscillator as described above.
- save coordinates - save any scanning pattern to the ALPHA250 board.

- scan - tell the board to start scanning and wait to receive the demodulated data.
- read data - helper function to take data from buffer.
- move to given coordinate.
- go home - command for the scanner to go to the centered position, at 17.5 mrad in both x and y.

The demodulated data is sent to the client as a complex number, $a + ib$, for each demodulation frequency, as shown in Fig. 5.2, plus the corresponding correction signals demodulated from the electrical references as shown in Fig. 5.1. The amplitude is simply the module of this complex number, $\sqrt{a^2 + b^2}$, and the phase is its angle, $\arctan(a/b)$.

5.1.3 Scanner set-up and control

Scanner control through the FPGA is done via SPI. This is very convenient as there is no need to go through any digital-to-analog converters (DAC) which have their limitations in terms of bandwidth and bit resolution. In this way, we are only limited by the bandwidth and resolution of the scanner itself. The readout of the gauge sensors is also done through SPI but it has not been implemented yet on the FPGA. For now, the sensors can be monitored via a PC connected by a USB cable to E727 driver and using the PI control software.

Both the previous scanner model, used in the Virgo phase camera, and the more recent model used in our custom version, come equipped with gauge sensors. These sensors can be used to read the position of the scanner or for closed-loop operation, where a PID feedback loop calibrated by the manufacturer ensures better repeatability, at the cost of a small loss in angular resolution. Thus, both models allow for open and closed-loop operation of the piezos.

For the Virgo phase camera it was opted to use the closed-loop operation to guarantee long-term stability. The old scanner model used in the Virgo phase camera has 5 μrad resolution in closed loop, while the new model we are using has a 1 μrad closed-loop resolution against 0.1 μrad in open-loop. Despite the loss of resolution, we opt for closed-loop operation as well, for similar reasons.

Another reason to do so is due to the gauge sensor readout not being implemented yet. In closed-loop we guarantee the correct coordinates are reached.

Otherwise, in open-loop, the gauge sensors would have to be used in order to ensure the image is correctly reconstructed every time.

Scanning angles and reference frames

In the discussion that follows, I have distinguished between mechanical and optical angles. Mechanical angle refers to the actuation angle applied to the scanner, while the optical angle refers to the deflection angle experienced by the incident beam on the mirror. Because of Snell's law of specular reflection, the optical angle is twice the mechanical angle. This is shown in Fig. 5.3a for the yaw angle, where the reflection angle experienced by the incident beam is twice the angle between the home position normal vector of the mirror, and the normal of the mirror after rotation, $\theta_{Yopt} = 2\theta_Y$.

When programming the scanner, the client inputs the scanning pattern or scanning commands in terms of the scanning pitch and yaw angles. The relation between the scanning yaw angle, θ_Y , and the covered distance at the plane of the photodiode is given by (see Fig. 5.3a)

$$d_x = L_{sp}\theta_{Yopt} = L_{sp}2\theta_Y, \quad (5.1)$$

where L_{sp} is the distance from the scanner mirror to the photodiode. Note that the small angle approximation was used, since the mirror rotation range is limited to 35 mrad. In the case of pitch, there is a projection effect [20] due to the non-zero incident angle which reduces the vertical angle by $\cos(\theta_{in})$

$$dy = L_{sp}2\theta_P \cos(\theta_{in}), \quad (5.2)$$

where in this case, the incident angle is always kept to $\approx 45^\circ$ and the correction is $\approx 1/\sqrt{2}$.

The scanner, however, is not actuated in pitch and yaw. Instead, it uses actuation angles θ_1 and θ_2 which are rotated by 45° with respect to the pitch and yaw as depicted in Fig. 5.3b. In this way, the pitch and yaw angles of the scanner can be written as

$$\theta_P = \frac{1}{\sqrt{2}}(\theta_1 - \theta_2) \quad \theta_Y = \frac{1}{\sqrt{2}}(\theta_1 + \theta_2). \quad (5.3)$$

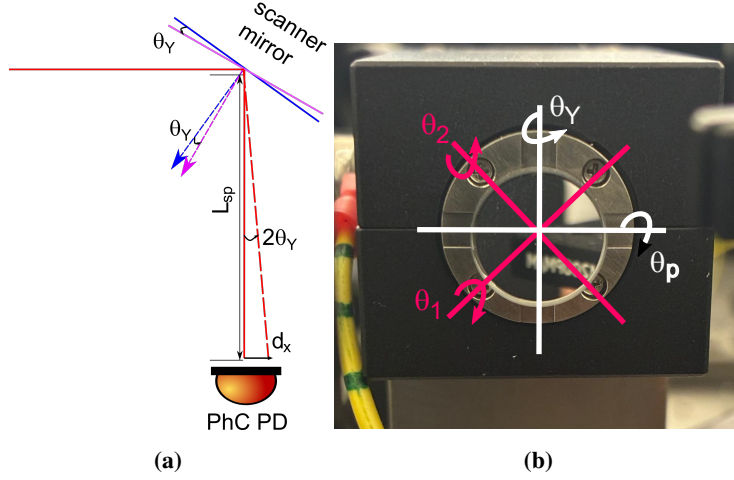


Figure 5.3: On panel (a), the scanning angles (distinguishing the mechanical scanning angle from the optical angle) are shown, including the distances dx and L_{sp} of Eq. 5.1. The normal to the mirror is represented by a dashed vector. On panel (b) the actuation angles of the scanner are shown versus the pitch and yaw angles. Their relation is given by Eqs. 5.4 and 5.5.

Using Eqs. 5.3 and subtracting and summing the expressions for θ_P and θ_Y , we can write the actuation angles in terms of the pitch and yaw angles

$$\theta_1 = \frac{\sqrt{2}}{2} (\theta_P - \theta_Y) \quad \theta_2 = \frac{\sqrt{2}}{2} (\theta_P + \theta_Y). \quad (5.4)$$

Additionally, a 17.5 mrad offset is added since the 0 position of the mirror is at this value plus the pitch projection correction with a value of $1/\sqrt{2}$ as mentioned above. A phase camera user provides the scanning pattern or scanner commands in terms of the θ_P and θ_Y and these are converted into θ_1 and θ_2 actuation angles in the ScanConn Python class according to

$$\theta_1 = \theta_P - \frac{\sqrt{2}}{2} \theta_Y + 17.5 \quad \theta_2 = \theta_P + \frac{\sqrt{2}}{2} \theta_Y + 17.5 \quad [\text{mrad}]. \quad (5.5)$$

5.1.4 Optical set-up and design constraints

In this section I describe the optical set-up of the phase camera and in particular the design constraints that motivate the choices for the optical set-up.

As discussed in section 2.6.4, a scanning phase camera can be set-up in the one-beam or two-beam scanning configuration [20]. In the one-beam scanning the scanner is placed before the beam-splitter and only the test-beam is scanned (Fig. 5.4), whilst in two-beam scanning the scanner is after the beam-splitter

and both test and reference beams are scanned together. The advantages and disadvantages of each configuration were already discussed in section 2.6.4, but in summary, the one-beam scanning offers better SNR in a reduced space, but suffers from fringes and a spatial phase offset due to the tilted wavefront of the test beam with respect to the reference beam. The two-beam scanning configuration has no offset on the phase, but has diminished visibility at the Gaussian skirt and needs intensity calibration.

The installed Virgo phase cameras are set-up in a one-beam scanning configuration due to the limitations in available space and the sensitivity requirements. For this reason, we implemented the one-beam scanning configuration for our table-top experiments since we would like for the techniques developed in the lab to be directly applicable to the Virgo phase cameras.

In one beam scanning, the test beam is angled with respect to the reference beam as it is scanned and the tilting of the wavefront causes spatial fringes. We will see this sets a constraint on the optical set-up. The fringe gap can be written as a function of the angle θ between the two interfering wavefronts of wavelength λ as

$$\Lambda = \frac{\lambda}{2 \sin(\theta)}. \quad (5.6)$$

We should make sure that the fringe gap is larger than the photodiode diameter d_{PD} . Or, as done in Ref. [20], the requirement can be set for twice the photodiode diameter $\Lambda > 2d_{PD}$. We can write this condition in terms of θ , in the small angle approximation as

$$\theta < \frac{\lambda}{4d_{PD}}. \quad (5.7)$$

For $d_{PD} = 70\mu\text{rad}$, this condition results in a maximum angle of 5.5 mrad. Eq. 5.7 can be rephrased in terms of the distance between scanner and photodiode using Eq. 5.1, resulting in a lower limit for the distance between the scanner and phase camera photodiode as

$$L_{sp} > \frac{d_m d_{PD}}{\lambda}, \quad (5.8)$$

where d_m is the image diameter corresponding to the maximum θ_Y angle.

There is also an upper limit on the distance between scanner and photodiode if we set a constraint for the minimum distance between adjacent pixels. Similarly to reference [20], setting the uncertainty between adjacent points to

10% of d_{PD} , the maximum allowed distance becomes

$$L_{sp} = \frac{d_m}{2 \cdot 10 \cdot N \cdot \Delta\theta_s}, \quad (5.9)$$

where N is the number of pixels across the x or y axis, and $\Delta\theta_s$ is the scanner resolution, which remember, is $1 \mu\text{rad}$ in closed-loop.

For an image diameter of 4 mm, this results in a maximum distance between scanner and photodiode of 2 m. From the fringe visibility requirement we arrive at 18.06 cm lower limit on the distance. Thus we can write

$$0.18 \text{ m} < L_{sp} < 2.00 \text{ m}. \quad (5.10)$$

5.2 Phase camera characterization

Figure 5.4 shows the test optical set-up to characterize the phase camera in the one-beam scanning configuration.

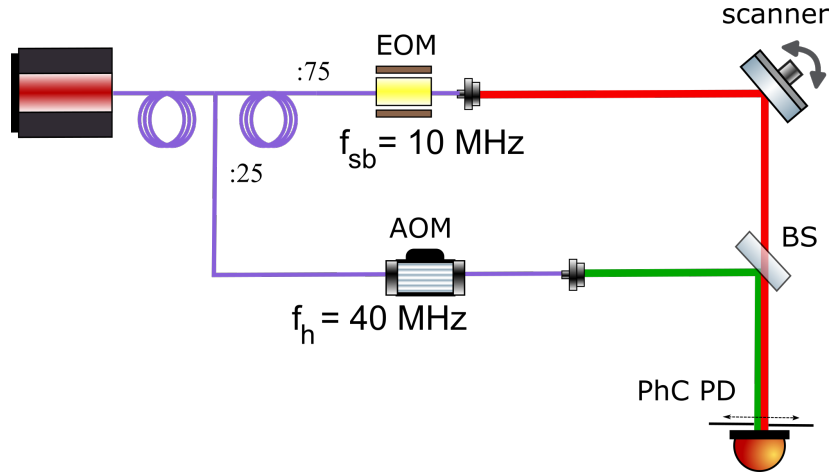


Figure 5.4: Simplified optical schematic of the set-up to characterize the phase camera. The laser beam goes through a 25:75 fiber splitter. The 75% light goes into an EOM (MPX-LN-0.1 from Exail) which is modulated at 10 MHz to introduce sidebands in the test beam. The beam goes on to open air and is steered by the PI scanner, through a beam splitter and into the PhC photodiode. The 25% light goes into a fibered AOM (MT40-IIR80 from AA Optoelectronic), modulated at 40 MHz in order to frequency shift the beam by the same amount. The reference beam goes into open air via the F280APC-1550 collimator from Thorlabs and is reflected by the beam splitter, where it interferes with the test beam.

The nominal distance between the scanner and the phase camera photodiode was set to 25 cm. A maximum scan angle of 4 mrad was used in either direction, resulting in a scanning diameter of 4 mm at most. In order to validate the proper working of the scanner a regular camera was installed in place of the phase camera photodiode. The camera used was Beamage 4M-IR from Te Lintelo Systems, which uses a phosphorous coating to detect light in the short-wave infrared range (1550 nm).

The Beamage camera can track the centroid position of the beam. This was used to validate the scanning angles and distances. Multiple yaw and pitch values from the Python client were sent. By measuring how much the beam moved on the camera, and by using equations 5.1 and 5.2 I verified the correct actuation angles were sent to the scanner by checking that the right L_{sp} value is retrieved. This is also a precise way of estimating L_{sp} , which was measured to be 25.190 ± 0.003 cm. This error bar takes into account a small discrepancy between the measurements with θ_Y and θ_P . This discrepancy induces a fictitious ellipticity of 98%, corresponding to 2% astigmatism in favor of a larger y axis. This is most likely due to the $\cos \theta_{in}$ factor in Eq. 5.2, which assumes the incident angle is exactly 45° . Taking the discrepancy into account, an actual incident angle of $43.74 \pm 0.54^\circ$ is estimated.

If the phase camera is to be used to measure astigmatism, understanding these effects are essential. Otherwise, relative measurements can be used as was done in Chapter 6.

5.2.1 Amplitude information

It might seem odd to refer to the phase camera generated images as "amplitude images", since photodiodes measure intensity. However, it can be shown that, in the one-beam configuration, and provided the reference beam is large enough, this is indeed the case.

For a test beam with only a carrier and a spatially overlapped reference beam, the intensity at the PhC PD, can be written as

$$I_{PD}(x, y, t) = E_c(x, y, t)E_{ref}(x, y, t)^* + E_c(x, y, t)^*E_{ref}(x, y, t) + |E_c(x, y, t)|^2 + |E_{ref}(x, y, t)|^2, \quad (5.11)$$

where E_c and E_{ref} refer to the time-dependent transverse profile at the x, y plane of the PD of the carrier and reference beam electric fields respectively. In

the development that follows the DC terms are dropped since we are interested in the optical beating between test and reference beam only.

The reference beam with a diameter of ≈ 2.9 mm is much larger than the photodiode, which has a diameter of $70 \mu\text{m}$. Since it is not scanned, we can approximate the E_{ref} portion that falls on the photodiode by a plane wave. Taking the x, y origin to be at the PD coordinates, we can write

$$E_{\text{ref}}(0, 0, t) = A_{\text{ref}} \exp(i2\pi f_{\text{ref}}t + i\varphi_{\text{ref}}), \quad (5.12)$$

where A_{ref} is the reference beam electric field amplitude, f_{ref} its frequency and φ_{ref} its phase.

The test beam in turn, is scanned over the photodiode, and so, E_c will follow the usual Gaussian beam distribution

$$E_c(x, y, t) = A_c \exp\left(\frac{-r^2}{\omega^2}\right) \exp(i2\pi f_c t + i\varphi_c), \quad (5.13)$$

where the same definitions were used as for the reference beam. Plugging this in Eq. 5.11, without the DC terms, results in

$$I_{\text{beat}} = 2A_c A_{\text{ref}} \exp\left(\frac{-r^2}{\omega^2}\right) \cos(-2\pi f_h t + (\varphi_c - \varphi_{\text{ref}})), \quad (5.14)$$

where $f_h = f_{\text{ref}} - f_c$ is the heterodyne frequency of the AOM, ω the beam radius assuming a perfectly stigmatic beam and $r^2 = x^2 + y^2$.

The spatial profile of Eq. 5.14 goes with $\exp(-r^2/\omega^2)$, same as the Gaussian amplitude of Eq. 5.13, even though it is an intensity in units of watts. The typical intensity for a Gaussian beam scales with $\exp(-2r^2/\omega^2)$. It is a known result that the FWHM of the Gaussian intensity profile is related to the beam radius by

$$\omega = \frac{\text{FWHM}}{-2 \ln(0.5)} = 0.8493218 \cdot \text{FWHM}. \quad (5.15)$$

A similar relation can be derived for a Gaussian amplitude profile such as that of Eq. 5.14. Ignoring the constant and time-dependent terms, we set $r = \text{FWHM}$ and $I_{\text{carr}} = 0.5$ obtaining

$$0.5 = \exp\left(\frac{-\text{FWHM}^2}{\omega^2}\right), \quad (5.16)$$

solving for ω results in

$$\omega = \frac{\text{FWHM}}{-4 \ln(0.5)} = 0.6006 \cdot \text{FWHM}. \quad (5.17)$$

By comparing the phase camera generated images with the regular camera, we can both validate the image reconstruction and that indeed the phase camera measures the amplitude profile. Image 5.5a shows an example of a measured image for the carrier, demodulated at $f_h = 40$ MHz. On top of the image the white contours of a 2D Gaussian fit are shown. The image is generated by scanning the beam cross section over the x axis for multiple y steps such that a 100x100 pixel grid is obtained. This is different from the smooth Archimedean spiral used in the Virgo phase camera [20], but the cross sectional scanning facilitates phase unwrapping as discussed in the next section.

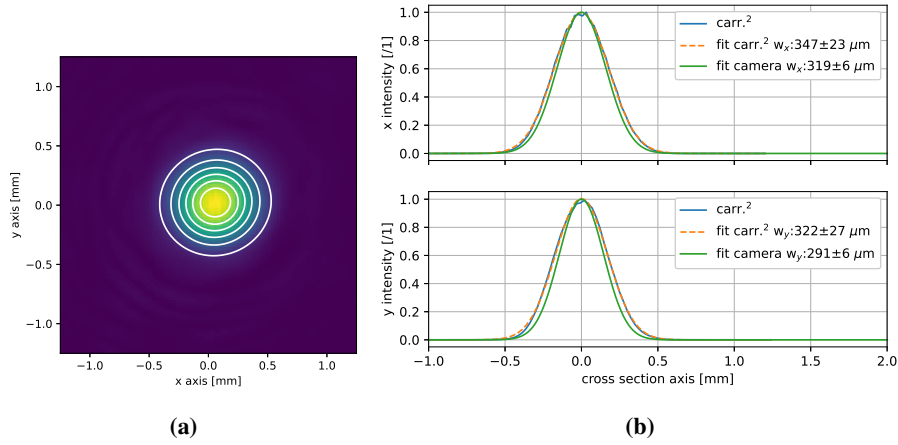


Figure 5.5: On panel (a) the carrier (demodulated at 40 MHz) amplitude signal, reconstructed on a 100x100 grid is shown. The white contours refer to a 2D Gaussian fit performed on the image. Panel (b) shows a comparison of the cross section of the images acquired with the PhC and with a regular camera. Two different FWHMs are obtained because the regular camera measures an intensity profile while the PhC measures an amplitude profile. More details are given in the text.

In figure 5.5b the x and y cross sections of both the phase camera image and the test beam measured on the Beamage camera are shown. The amplitude cross sections from the phase camera were squared to more easily compare visually with the intensity profile of the Beamage camera. Note that for the Beamage measurement the reference beam is covered so that only the test beam is measured.

The beam radius can be retrieved with Eq. 5.15 from the Beamage FWHM, and with Eq. 5.17 from the FWHM from the PhC cross section. In order to compare the beam radius measured with the two methods, the Beamage camera image was convoluted with a square window of $70\text{ }\mu\text{m}$ to take into account the size of the phase camera photodiode. The impact of considering the size of the photodiode was seen to be within 1%, but it was included in the comparison nonetheless. Doing this, the phase camera measured $\omega_x = 347 \pm 23\text{ }\mu\text{m}$ and $\omega_y = 322 \pm 4\text{ }\mu\text{m}$ while the regular camera measured $\omega_x = 319 \pm 6\text{ }\mu\text{m}$ and $\omega_y = 291 \pm 6\text{ }\mu\text{m}$. This results in a disagreement between the two of 8% and 10% for the x and y axis respectively.

The phase camera values already include the incident angle correction for the y axis mentioned in the previous section. In this way, the phase camera measures an ellipticity of 92.9% and the regular camera one of 91.2%, resulting in a disagreement between the two of 1.9%. The difference in the ellipticity error and absolute radius measurement points to some unknown systematic errors in the calibration, which does not affect the morphology of the image up to 2%.

In the analysis done with the phase camera in Chapters 6 and 7, either relative changes in radius are measured or the morphology of the beam is used to validate the mode basis. Thus, the absolute value of the radius is not so relevant, but more so that the image is reconstructed without introducing extra features. With the above analysis, we have validated the correct image reconstruction and that indeed the phase camera measures an amplitude profile. Fig. 5.6 shows such examples of amplitude images for carrier, upper and lower sidebands.

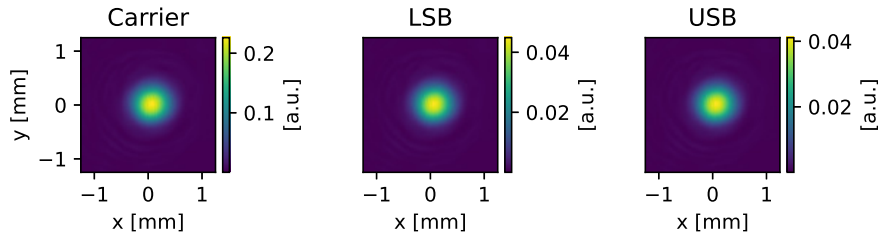


Figure 5.6: PhC amplitude images for carrier (demodulated at 40 MHz) and the 10 MHz upper and lower sidebands (demodulated at 30 and 50 MHz respectively).

5.2.2 Phase information

According to the literature [20, 142] there are three aspects that have an impact on the measured phase and that need to be understood and compensated or corrected for. The first is a time-dependent change of the measured phase, which is due to differences between the local oscillator used for demodulation and the signal used for the AOM or EOM. The second is phase-noise acquired by the reference beam as it propagates in the fiber. This is currently an issue at Virgo, as discussed in section 2.6.4. The third is only relevant for the one-beam scanning configuration, and is related to the tilting of the wavefront, which adds an additional phase offset as the beam is scanned over the photodiode.

Phase time dependence

In order to verify the first aspect and to check that we can correct for it, we made phase measurements without a reference beam and for a single pixel, hence, without scanning. Without a reference beam, only the beating between sidebands and carrier can be measured. As discussed in section 2.2, ideally the upper and lower sidebands cancel each other out due to opposite phase and nothing should be measured. However, in practice, small RAM and sideband imbalances cause this beating signal to be non-zero.

I took several measurements for different sideband modulation frequencies. One single pixel was acquired over 16 ms and demodulated at the respective sideband frequency. The results of these measurements are shown in Fig. 5.7.

For a given pixel with coordinates $\tilde{x}_p$, we can write the time-varying phase after demodulation as:

$$\Phi(\tilde{x}_p, t) \propto \alpha 2\pi f_{SB} t, \quad (5.18)$$

where, for simplicity and since only one pixel is being recorded, only the time-dependent phase is shown, not the spatial component which will depend on the transverse profile of the beam. Fitting the slopes of the curves for different sideband frequencies shown in Fig. 5.7, a single α value of $1.25 \cdot 10^{-7}$ was computed. This value can be attributed to the clock drift of the ALPHA250 board, and it can be corrected by taking the AOM and EOM electrical reference signals. These signals are demodulated for the carrier (at f_{AOM} MHz) and PDH signal correction (at f_{EOM} MHz), and also mixed and demodulated at $f_{AOM} + f_{EOM}$ and $f_{AOM} - f_{EOM}$ MHz so that phase references at all beat frequencies of interest are recorded. These phase references are then subtracted

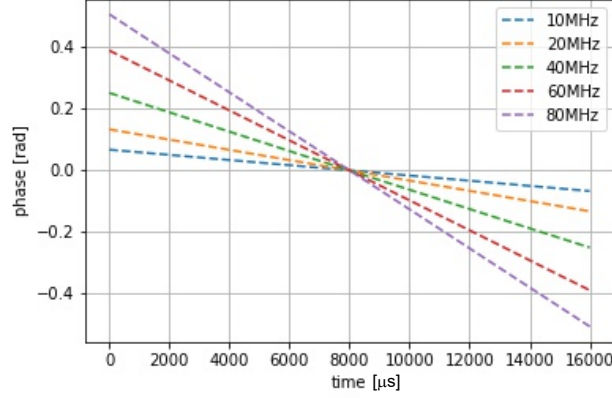


Figure 5.7: Drift over time of the demodulated phase for the single pixel beating between carrier and sidebands for increasing modulation (and demodulation) frequency. Using the slopes of each curve, the curve of Eq. 5.18 was fitted in order to obtain $\alpha = 1.25 \cdot 10^{-7}$, which corresponds to the fractional frequency ($\Delta\nu/\nu$) clock drift of the ALPHA-250 board.

from the equivalent demodulated signals from the photodiode (Fig. 5.2) to remove the time-dependence of Eq. 5.18.

Phase noise for a single pixel

With the time-dependent correction in place, we can now evaluate the phase noise of the phase camera for a single pixel. As mentioned before, and discussed in Section 2.6.4, the reference beam acquires phase noise as it propagates in the fiber. This is due to thermal, seismic and acoustic noise coupling into the fiber. Additionally, there is also noise from the mechanical vibrations of the optics, for which the test beam is also sensitive to.

Fig. 5.8 shows the phase noise of the carrier and lower sideband for a single pixel, acquired with a test beam in the configuration of Fig. 5.4, over 400 ms. Other than the seismic noise mitigation of the optical table, no other isolation, thermal or otherwise, was implemented.

Despite the large phase drifts, it can be seen that both carrier and sideband phases follow the same trend. This means the phase noise is common and can be mostly removed by taking the difference of the two phase signals. The differential phase signal, also shown in Fig. 5.8 is thus much less noisy. For multiple measurements, a differential phase noise between carrier and side-

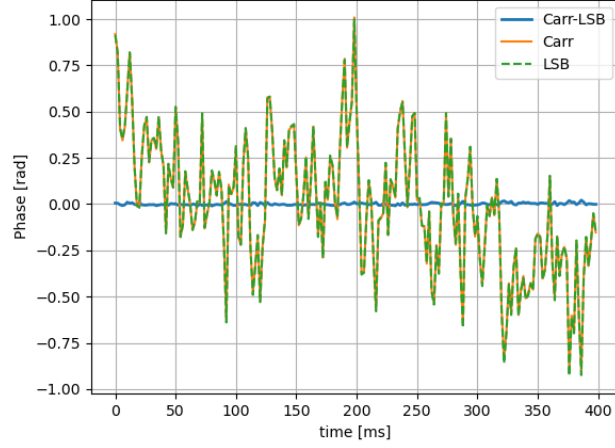


Figure 5.8: Demodulated phase for the beatings between carrier and sidebands with the reference beam. Data was acquired without scanning, for a single pixel. The Carr.-LSB curve shows the phase difference between the acquired signals, resulting in the subtraction of all the common phase noise.

band between 6 and 9 mrad was observed. It is comparable to the best sensitivity reported in [20] of 4 mrad, which was obtained in a two-beam scanning configuration for the differential phase at the center of the beam.

In order to understand the phase noise limits in the absence of a reference beam, the PDH beating could potentially be used. However, since the beating strength is very low and dominated by RAM, this is not ideal. To test the phase noise of a single beam with one carrier and a single sideband, the setup shown in Fig. 5.9 was implemented. Here, an AOM is used instead of an EOM for the sideband generation. The idea came from Ref. [124], where the authors input a triplet to an AOM to generate the PDH sidebands. Here, we modulate the AOM at 40 and 30 MHz. The frequency shifted thus has a carrier and a single sideband beating at 10 MHz. Since the sideband frequency is 10 MHz, a third signal with the same frequency is generated to serve as the electrical reference to remove the phase time dependence as discussed above. A programmable DDS from Analog Devices, AD9959 evaluation board, was used to generate the modulation sine waves.

Figure 5.10 shows the phase noise for the AOM generated 10 MHz beating, acquired for a single pixel and for increasing acquisition times. For 10 ms acquisition it is just below 0.2 mrad, and at most it is below 1 mrad.

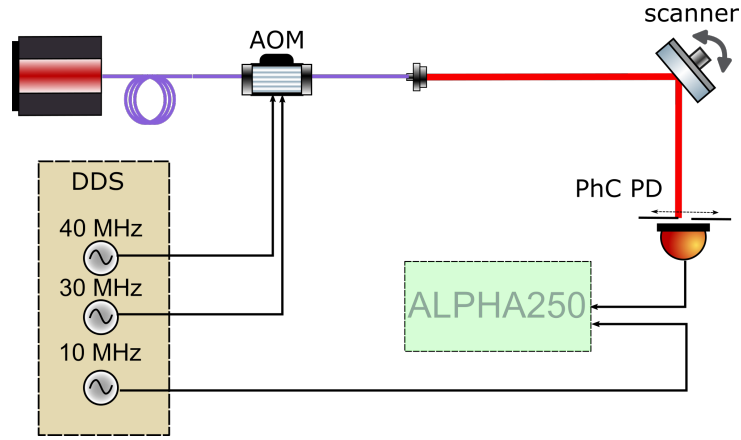


Figure 5.9: Optical schematic of test set-up to generate a single sideband. A doublet modulation is applied to an AOM, the frequency shifted beam has a carrier and a 10 MHz sideband. A 10 MHz electrical reference is also generated to remove the phase time dependence as discussed above.

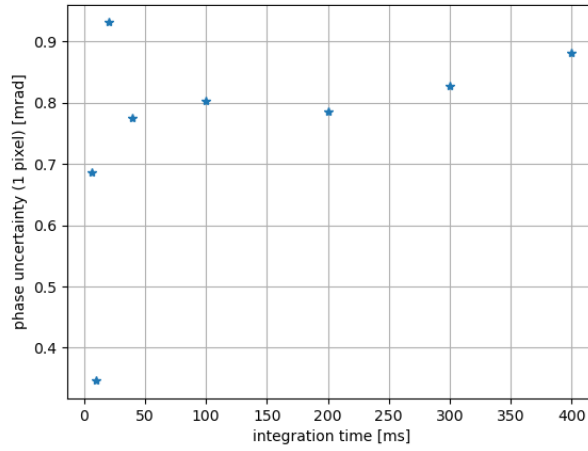


Figure 5.10: Acquired phase noise for a single pixel for increasing integration time. Measurement obtained with the set-up of Fig. 5.9 for a single sideband beating.

Also with AOM set-up of Fig. 5.9, another test was done to validate that the phase camera is indeed retrieving the optical phase between the two optical frequencies. The phase of the 30 MHz modulation applied to the AOM was changed, from 0 to 2π radians. A 10 MHz signal was also generated in the DDS to serve as the electrical reference for the time-dependent correction.

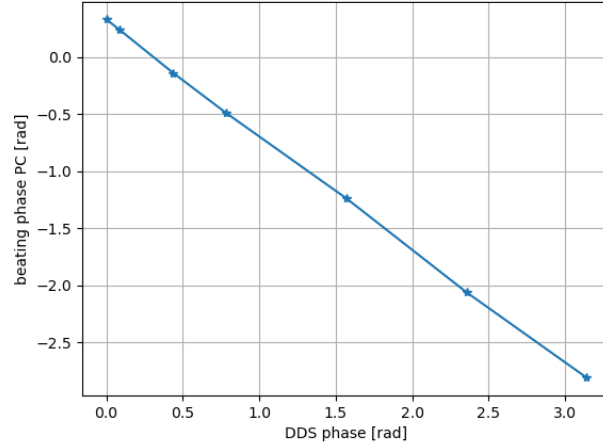


Figure 5.11: With the set-up of Fig. 5.9 we obtain the demodulated phase at the PhC as the sideband optical phase is changed by changing the modulation phase on the DDS.

The result of the measurement is shown in Fig. 5.11. The applied phase is imprinted on the frequency-shifted beam, changing the phase of the 10 MHz beating accordingly and this phase is measured correctly by the phase camera.

Differential phase images and phase unwrapping

After considering the phase acquisition for a single pixel we can move on to the phase images. As mentioned above, in one-beam scanning a spatial offset is added to the phase information because the test beam wavefront is tilted with respect to the reference beam as it is scanned over the photodiode.

Additionally, as we have just seen, the phase noise from the reference beam is also quite significant. For both of these reasons, and similar to what is done at Virgo, we only work with differential phase images. Such images are shown in Fig. 5.12. One image is generated for the carrier-LSB $\Delta\varphi$, another for the carrier-USB $\Delta\varphi$ and another for the $\Delta\varphi$ between the two upper and lower sidebands.

A mask was applied to the phase images, so as to not display the phase outside of the beam. The mask was applied for pixels with a power at least 20 times smaller than the maximum of the beam. This is done because for

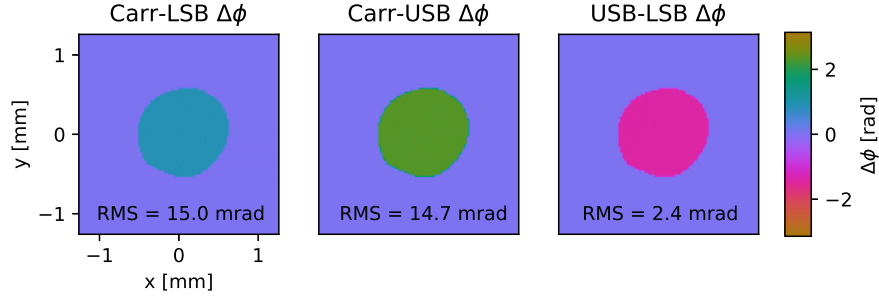


Figure 5.12: Differential phase images between carrier and LSB, carrier and USB and between USB and LSB. The time dependence correction is included before subtracting the phase images, and the phase unwrapping is done at the end following the Itoh algorithm [214], as described in the text.

pixels close to the noise floor, the noise starts contributing significantly to the measured phase, which becomes meaningless.

The measured phase is computed as the $\arctan(I/Q)$ and so it goes from $-\pi$ to π . Thus, the phase needs to be unwrapped in order to form a continuous phase map without 2π phase jumps. Before applying the corrections mentioned above, the signals are unwrapped, but they need to be unwrapped again after the differential phase is generated. Phase unwrapping in 2D is not trivial, and it can be especially complicated in the presence of noise and/or undersampling [215].

We have characterized the phase noise per pixel to be below 9 mrad, so this should not prove a problem to the unwrapping. We should also not be in a regime of undersampling, meaning from one pixel to the next there should be no large phase jumps. This is not a problem for an approximately Gaussian beam as is the case here. Note, however, it could potentially be an issue for very high-order modes if a small number of pixels is being used.

A simple 2D phase unwrapping algorithm is the Itoh algorithm [214]. It consists simply of sequentially unwrapping first the rows and then the columns or vice-versa. In this case, because the image is acquired as a collection of row cross-sections, the unwrapping is applied in the same sense: first to the rows and afterwards to the columns. In this way, images as those shown in Fig. 5.12 can be obtained.

Since carrier and sidebands are well spatially overlapped and carry the same mode, the only difference in the phase is due to the small difference in the optical path due to slightly different k vectors. This results in the flat phase offsets shown in Fig. 5.12. As the modes of the carrier and sidebands change,

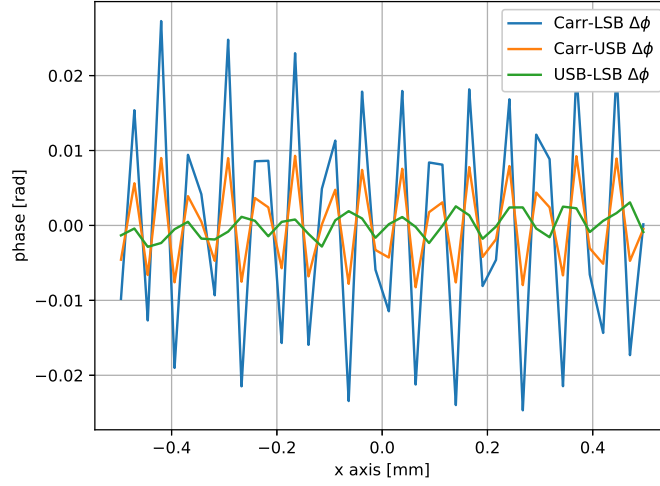


Figure 5.13: Zoom in on the $\Delta\phi$ images by plotting the X cross section of the beam area. The y axis was normalized and offset to 0 to facilitate comparison between the three signals.

due to the presence of a cavity for example, features start to appear in the phase images as shown in Chapters 6 and 7.

The RMS phase noise over the phase image between carrier and sidebands is up to 15 mrad. Fig. 5.13 zooms in on the cross section to allow for comparing visually the phase noise of each image. The y axis was normalized and offset to 0 to facilitate comparison between the three differential phases.

The measured RMS noise is higher than the one pixel noise measured (between 6 and 9 mrad). The increase in noise could be attributed to scanning procedure and its pointing precision. Note, however, the reduced phase noise of the differential phase image between the two sidebands. At 2.4 mrad, it is significantly lower than the carrier-sideband $\Delta\phi$. It is not clear if this is due to excessive noise of the carrier or some common phase noise in the sidebands originating, for example, from RAM.

Chapter 6

Mode-matching with a phase camera in reflection of a cavity

In Section 3.5 I described a first attempt at using the phase camera in reflection from the Virgo arm cavities to characterize the mode mismatch. In this Chapter, this technique is further developed through optical simulations and brought into the lab in a table-top experiment.

I start by introducing the broader context for this table-top experiment in Section 6.1. Then, In Section 6.2, I present the optical simulations made in OSCAR, which go a step further than what was detailed in Section 3.5 by showing the phase camera error signals in terms of waist size and position mismatch (for the theory background see Section 2.4). The work presented in this Section was the subject of a proceedings peer-reviewed publication of the 16th Pisa Meeting on Advanced Detectors [3], and I have reproduced part of the text and figures of the proceedings in this Section.

Next, in Section 6.3, I detail the optical design of the table-top experiment and in particular the design of a mode matching telescope to probe the two mismatch quadratures. In Section 6.4 I present preliminary results with the table-top experiment and I end the chapter with a summary and a discussion of the next-steps regarding this project.

6.1 Context

Recall the error signal for mode-mismatch that we defined in Section 2.4

$$\epsilon_q = \frac{\Delta z_{1,2}}{z_{R,1}} + i \frac{\Delta z_{R1,2}}{z_{R,1}} = \epsilon_z + i\epsilon_{z_R}, \quad (6.1)$$

where $\Delta z_{1,2}$ and $\Delta z_{R1,2}$ are the difference in the waist position and Rayleigh

range between the beam parameters q_1 and q_2 . We call ϵ_z the waist position error signal and ϵ_{zR} the waist size error signal.

In Section 2.6 we reviewed the wavefront sensors relevant for sensing mode mismatch. As discussed there, current sensing schemes require two sensors to measure each waist position and waist size quadrature independently.

Most of the mode mismatch sensing schemes utilize RF wavefront sensing to extract the phase between the fundamental and the LG_{01} mode. Because the LG_{01} has cylindrical symmetry, many of these schemes are insensitive to astigmatism.

In Ref. [216], the authors use an optical lock-in camera as described in Section 2.6 to measure mode-mismatch. The authors use digital processing of the images to use the phase camera as an high-resolution photodiode, using digital masking to extract signals sensitive to mode mismatch and to astigmatism as well. However, in order to measure the two quadratures at the same time, two lock-in amplifiers are required.

Takeno et al. developed a variation of the diagnostic breadboard approach discussed in Section 2.6.4, where the phase and amplitude of deviations from a perfect Gaussian mode could be imaged. These deviations can be measured with a CCD camera in reflection from the locked cavity. A Mach-Zender interferometer is also used, using the cavity transmitted signal as the reference in order to distinguish opposite phase signs of the wavefront distortion [217].

The measurement scheme developed in this chapter has some similarities to the above two references. It employs a phase camera instead of a CCD in reflection from a cavity and it also relies on Mach-Zender interferometry through the inclusion of a reference beam. With respect to Ref. [217], the frequency-shifted reference beam and fast photodiode allows to detect separately the upper and lower sidebands and the carrier as detailed in Section 2.6.4.

Our proof-of-concept scheme requires only a single phase camera to measure both mismatch quadratures simultaneously, as opposed to the scheme of Ref. [216]. Additionally, it is sensitive to astigmatism by design. The scheme is appropriate in the limit of small mode mismatch, where the cavity reflected beam can still be approximated by a Gaussian beam as it depends on a direct comparison of the radius and RoC of the carrier and sidebands.

6.2 Error signals with a single phase camera

The phase camera images the amplitude and phase of the beam in the x,y plane. From the amplitude we have access to the beam radius, w , and from the phase

to the RoC, R . These quantities are related to q , via [60]

$$\frac{1}{q} = \frac{1}{R} - i \frac{\lambda}{\pi w^2}. \quad (6.2)$$

Instead of Eq. 2.18, we can define the mode mismatch in the q reciprocal space, where optimal mode matching is achieved when $1/q_2 - 1/q_1 = 0$. Thus, we can define a value ϵ'_q , that equals 0 when q_1 and q_2 are perfectly mode matched to one another, as

$$\epsilon'_q = -\frac{\Delta R}{R_2 R_1} + i \frac{\lambda}{\pi} \frac{\Delta w^2}{w_2^2 w_1^2} = -\epsilon_R + i \epsilon_w, \quad (6.3)$$

with $\Delta R = R_2 - R_1$ and $\Delta w^2 = w_2^2 - w_1^2$. In Fig. 6.1 we compare ϵ_R and ϵ_w with ϵ_Z and ϵ_{z_R} . As expected, both ϵ_R and ϵ_w couple to both mismatch quadratures. However, ϵ_R couples more strongly to waist size mismatch while ϵ_w does so to waist position mismatch.

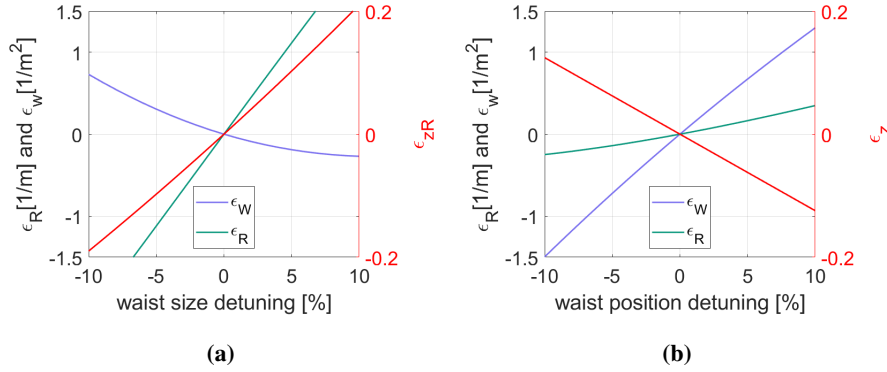


Figure 6.1: Computation of ϵ_R and ϵ_w from eq. 6.3 by detuning a) waist size and b) waist position relative to $q_1 = 0.0426 + i0.0343$.

6.2.1 Error signals proportional to ϵ_R and ϵ_w

Optical simulations of the set-up presented in Fig. 6.2 were done in OSCAR. Simulations start with the beam after the fiber collimator shown in the figure, where the same q parameter used in Fig. 6.1 is used for the input beam. The cavity is composed of two fused silica plano-concave mirrors with 0.1 m radius of curvature and a separation of 7.5 cm in air, resulting in an FSR of 1.999 GHz. The cavity is overcoupled with a finesse of ~ 800 . These param-

eters are similar to the ones of the table-top experiment described in the next Section. The reflected beam from the cavity is monitored just before the input mirror. The beam is propagated in the z axis, and the transverse amplitude and phase images are defined in the transverse x, y plane. Note that in Fig.6.5 the impact of the placement of the phase camera is evaluated.

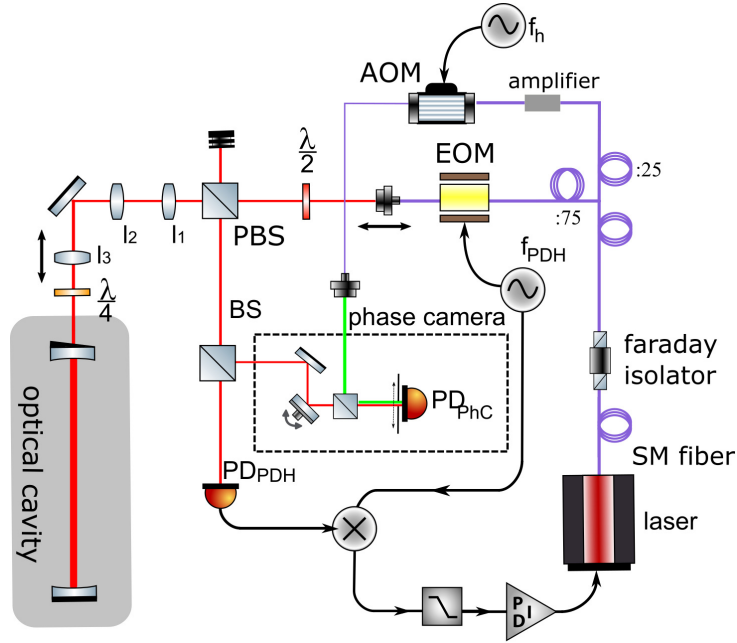


Figure 6.2: Optical schematic of the table-top experiment to validate the simulated error signals derived in Section 6.2. A phase camera is placed in reflection of the cavity to monitor mode mismatch of the input beam with respect to the cavity. A fiber splitter distributes 75% of the power into the test beam going into the cavity and 25% into the phase camera reference beam. The three lens telescope (composed of l_1 , l_2 and l_3) is described in Section 6.3.1, the double arrows show the positions of the translation stage to induce mode mismatch. The laser is locked to the cavity via PDH lock.

While the carrier is kept resonant with the cavity, the sidebands are away from resonance and experience a perfectly reflective mirror. In this way, the sidebands are promptly reflected and carry the same spatial mode as that of the input beam. Part of the reflected carrier field shares the mode of the cavity from the resonant leakage field. The rest of the carrier is from prompt reflection, mainly the second-order mode from mode mismatch. We validate through simulations that if other HOMs are negligible and the total reflected field is

approximately Gaussian, the error signals can be retrieved by comparing the carrier with the sidebands.

The error signals derived from the simulation are not exactly ϵ_w and ϵ_R , but proportional to them, as they only contain the numerators of Eq. 6.3. These are the error signals one would expect to obtain in a table-top experiment, from a differential measurement between carrier and sidebands as described below.

The error signal proportional to ϵ_R , err_R , is extracted by subtracting the phase image of the carrier from one of the sidebands. In the presence of mode mismatch, the cross section of the differential phase is curved, this is shown in Fig. 6.3. When perfectly mode matched, the RoC of the carrier and sideband wavefronts is the same. In this case, the phase difference between carrier and sidebands is flat and no curvature is observed, just an offset from the different optical path length. The difference in the RoC of the carrier and sideband wavefronts ΔR could be fitted from this cross section, but alternatively we can simply take the signed amplitude of the differential phase cross section.

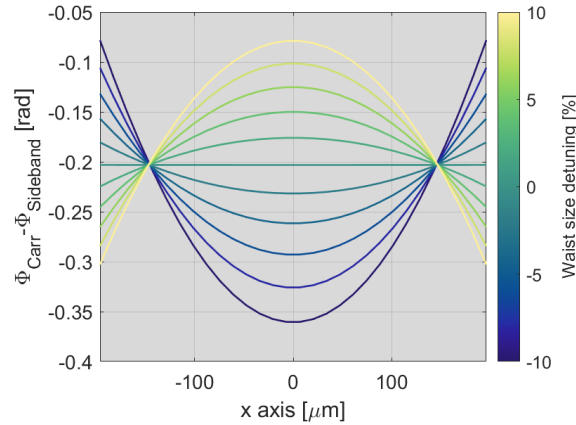


Figure 6.3: Cross section along the x axis of the differential phase between carrier and (upper) sideband for different values of waist size detuning. Simulated in OSCAR.

In fig 6.4a we plot the resulting err_R while detuning the waist size of the input beam in the simulation. We also check the cross coupling of err_R with respect to the other quadrature. The presence of position mismatch causes a small offset in the zero-crossing of the error signal.

The error signal proportional to ϵ_w , err_w , is obtained by subtracting the fitted radius of the carrier from the fitted radius of one of the sidebands. In Fig. 6.4b we plot the resulting err_w while detuning the waist position of the input

beam in the simulation. We also check the cross coupling of err_W with respect to the other quadrature. As expected from panel (b) of Fig. 6.1, the offsets are larger in this case.

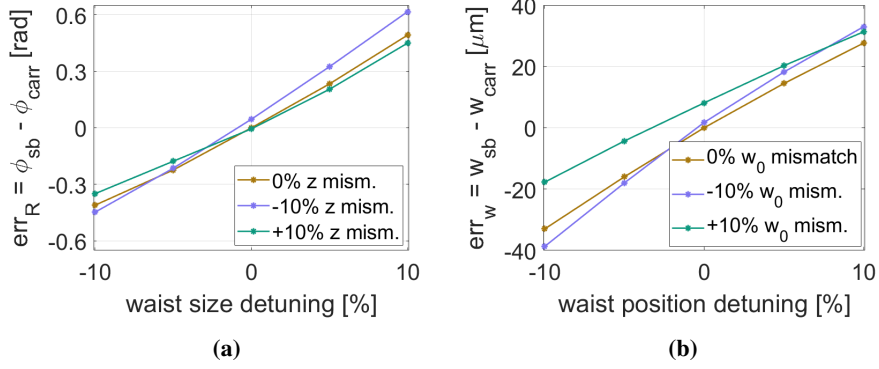


Figure 6.4: Simulated phase camera error signals (a) err_R vs. waist size, (b) err_W vs. waist position, and the impact of coupling to the opposing quadrature.

The offsets are well within the considered actuation range. Thus, the signals can be used in two feedback loops, one for each mismatch quadrature, converging to zero mode mismatch. Both error signals can be extracted separately for the x and y axis, making this scheme also sensitive to astigmatism.

Finally, we have investigated how the error signals depend on the placement of the phase camera. Fig. 6.5 compares err_W when the phase camera is in the near and far field. We can see the phase camera placement impacts the slope of the error signal with respect to the amount of mismatch, so this needs to be taken into account when designing the measurement scheme with the phase camera.

6.3 Optical design and set-up of the table top experiment

In order to validate the results of the previous section, I designed and set-up the table-top experiment following the scheme of Fig. 6.2. The phase camera used is described in Chapter 5 and it was used in a 1-beam scanning configuration, as in Virgo, and with a reference beam. In our case, the reference beam is shifted by 40 MHz by a fibered AOM. A fiber splitter distributes 75% of the

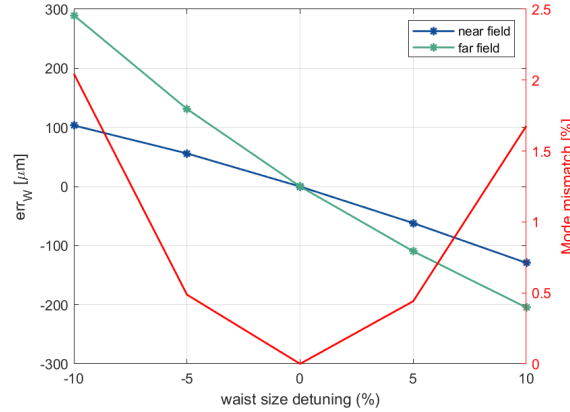


Figure 6.5: Simulated error signal based on the radius difference between carrier and sidebands evaluated in the near and far field (after a lens) for the same waist size detuning.

power to the test beam that is going into the cavity and 25% into the reference beam. A fiber amplifier on the way to the AOM allows to increase the power in the reference beam and improve the phase camera signals as discussed in Ref. [142].

The laser used was a DFB laser at 1550 nm, EP1550-0-DFB-B01-FM from Eblana, with an instantaneous linewidth as low as 100 kHz. The cavity was adapted from the design detailed in Chapter 5.9 of Ref. [218]. The cavity geometry is the same as the one simulated in the previous section. It consists of two half-inch mirrors pressed against a 7.5 cm long ultra-low expansion glass (ULE) spacer (see Fig. 6.6) with two mechanical mounts. For this set-up, the cavity mount was adapted for horizontal placement in the optical table (previously it was vertically placed inside a vacuum chamber). We adapted the center piece so that the cavity fits on a 2-inch Thorlabs KS2 kinematic mirror mount. This mount allows for more freedom of alignment of the cavity with respect to the optical axis, in addition to the input beam steering mirrors. With respect to how it was used previously, the finesse of the cavity was also increased significantly.

Both cavity mirrors have a 0.1 m RoC and the cavity g-factor is 0.0625. The input and end mirrors have a reflectivity of 99.35% and 99.90% respectively, resulting in an overcoupled cavity with 830 finesse. Thus, in the absence of additional round-trip-losses, the nominal cavity linewidth (FWHM) is 2.4 MHz. The cavity mode separation frequency is about 353 MHz, just short of 18% of the cavity FSR. Fig. 6.7 shows a typical cavity scan measured with an oscillo-

scope in the presence of a small amount of mode mismatch and misalignment (recall Section 2.3). The peaks in the measured cavity scan were fitted with the *lmfit* [219] library to obtain the mode positions in units of $1/\text{FSR}$. These mode positions were used to fit the cavity length and mirrors RoC using Eq. 1.30. In this way, a length of 7.65 cm was found to the ULE spacer and a RoC of 0.0999 was found for the mirrors RoC (which is within the 0.5% tolerance from the manufacturer). The fitted mode positions are shown in the plot. We can see the matching is not perfect, this could be due to nonlinearities in the frequency tuning of the laser which was not calibrated.

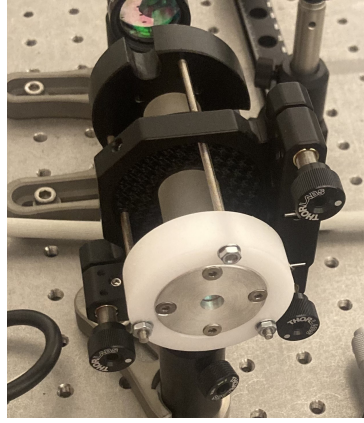


Figure 6.6: Picture of the ULE cavity used in the table-top experiment. The cavity is mounted on a kinematic mirror mount (KS2 from Thorlabs). The mirrors are pressed and kept parallel to each other by the mechanical mounts and the spacer screws.

The photodiode used for PDH locking was PD100-SWIR1 from Qubig, with 100 MHz bandwidth, 1.1 A/W sensitivity and 11 kV/A of transimpedance gain for the AC output (which was the one used for locking). The PDH sidebands were generated at 30 MHz with a fibered EOM MPX-LN0.1 from Exail. The feedback loop was closed with the Fast Servo Controller from Moglabs. The fast PID loop, with up to 35 MHz bandwidth, was used. The locking bandwidth was limited to 2 MHz, however, through the bandwidth of the modulation input of the laser driver used, QubeDL from ppQ Sense.

The experimentally measured cavity mode in transmission of the cavity and the PDH error signal in reflection are shown in Fig. 6.8. The signals are obtained by scanning slowly the current of the laser, thus scanning its frequency

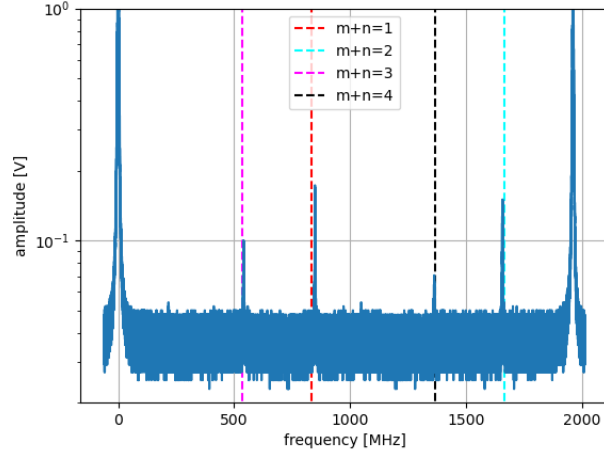


Figure 6.7: ULE Cavity mode scan as the laser frequency is tuned, measured in transmission of the cavity. The mode positions and cavity g-factor were fitted as described in the text.

while measuring the signals. An offset was added to the error signal, as well as a rescaling, for visualization purposes.

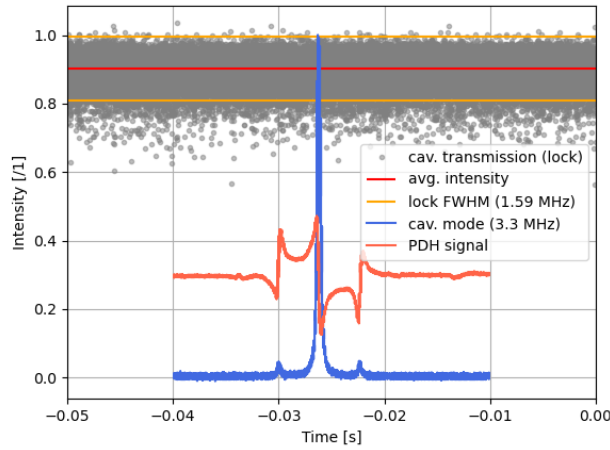


Figure 6.8: Cavity fundamental mode (measured in transmission) along with 30 MHz sidebands and corresponding PDH error signal. Also shown is the cavity intensity in transmission while the laser is locked.

The carrier mode was fitted with a Voigt profile using `lmfit` [219], and using the sideband frequency, 30 MHz, a cavity linewidth of 3.3 MHz was estimated. This results in a measured finesse of 606, instead of the 830 nominal value. The excess linewidth could be due to additional round-trip losses in the cavity and some small non-linearity in the frequency scanning of the laser.

Regarding the PDH lock, the transmitted intensity when the cavity is locked is also shown in the Fig. 6.8. Using the transmitted intensity and with knowledge of the cavity mode, we can compute the locking range in Hz. To do this, I computed the standard deviation of the intensity, σ , and from it the FWHM as $2\sqrt{2\ln 2}\sigma$. By seeing where the average intensity minus the FWHM of the lock crosses (bottom orange line in Fig. 6.8) the cavity mode in blue, we can convert the variation in intensity to a variation in frequency. In this way I estimated a lock FWHM of 1.59 MHz.

One would expect a tighter lock given that the instantaneous linewidth of the laser is between 400 and 100 kHz and the feedback bandwidth is 2 MHz. However, it is possible that high-frequency electronic noise is coupling to the DFB and that a higher bandwidth would be required to address it, as done in Ref. [119] for example. Otherwise, it could also be a limitation of the PID transfer function. The transfer function was optimized by tuning the proportional gain and integrator and differentiator corner frequencies while monitoring the spectrum of the transmitted intensity after acquiring the lock. It could also be that a higher gain would be required at lower frequencies for a tighter lock, this could be achieved with double or triple integrators. Often, two separate controllers for lower and higher frequencies are used, acting on two different actuators (laser temperature and current for example). Either of these were not possible in our configuration and the issue was not explored further.

6.3.1 Mode (mis)matching telescope

A mode matching telescope consists of a collection of lenses or other focusing elements (such as curved mirrors) in order to match the waist size and position of the input beam to the waist size and position of the cavity. As shown in Fig. 6.2, after the EOM, the fibered beam is coupled to open air through a fiber collimator (CFC11A-C from Thorlabs). In order to measure the input beam q parameter, I measured the beam radius with a CCD camera (Beamage 4M-IR, same as used in Section 5.2) at multiple distances from the collimator output. In order to retrieve q parameter of the input beam, the data points were fitted

according to [60]

$$w^2(z) = \omega_0^2 \left[1 + \left(\frac{\lambda z}{\pi \omega_0^2} \right)^2 \right], \quad (6.4)$$

where z was defined relative to waist position, such that it is negative before, 0 at the waist, and positive after it. For each datapoint, the average of the x and y radii were considered. From the fit we can estimate the waist has a radius of 0.2327 mm and is 32.3 cm away from the collimator. The resulting fit is shown in Fig. 6.9. In turn, the cavity waist is located at the middle of the cavity with a size of 0.1545 mm following from Eq. 1.28. In order to be matched to the cavity eigenmode, the input beam (just before the cavity input mirror) should have a waist size of 0.1299 mm and distance to the waist of 4.26 cm.

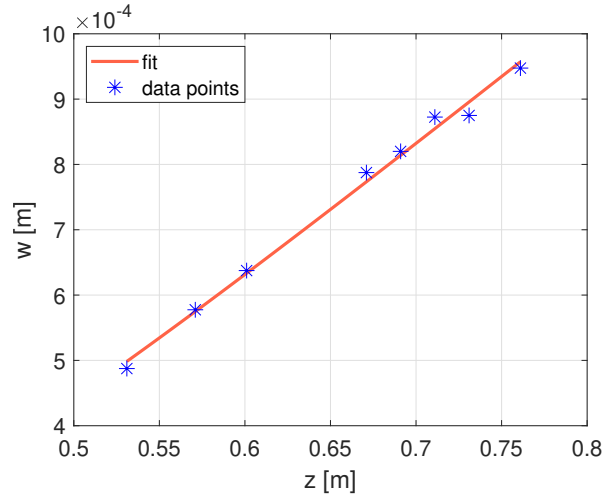


Figure 6.9: Radius of the input beam measured at different locations after the fiber collimator output in the far field. Fit (with Eq. 6.4) shown in red.

Our requirement for the telescope is not, however, simply to mode match precisely the input beam to the cavity eigenmode, but also that it should be possible to tune the waist size and position in a controlled way. In particular, we would like the two quadratures to be decoupled from each other as much as possible when acting on the positions of certain elements of the telescope. In order to do this, I designed a three-lens telescope, similar to what is done in Ref. [220] but adapted to this specific case.

The first two lenses of the telescope, l_1 and l_2 are used to form a nearly afocal system. The third lens (with focal length f_3) will act on the nearly

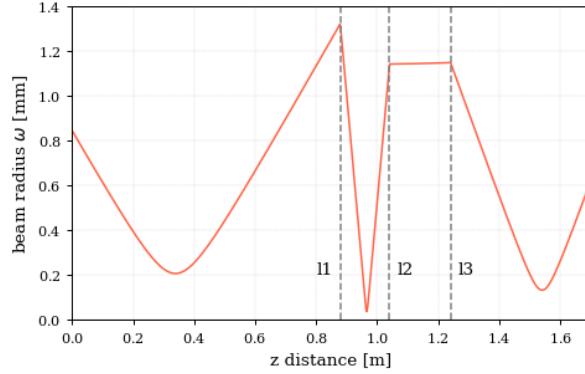


Figure 6.10: Figure produced using the ABCD matrix formalism, using Finesse2 (pykat) [206]. For this figure, the distance z starts at the fiber collimator output with $z=0$.

collimated beam, $q_3 = z_3 + z_{R,3}$, such that

$$z_3 = f_3. \quad (6.5)$$

This means that changing the position of l_3 will change the waist position by the same amount with very little impact on waist size as the beam is nearly collimated. Thus, we have a way of acting only on the waist position quadrature.

The collimated beam is achieved by using two lenses with the same focal length. If the back focal point of l_2 is placed coincident with the waist position of q_1 (the beam after l_1), then the output beam, q_2 , will be afocal. The waist size at the end of the telescope will be mostly a function of the waist size between l_1 and l_2 .

The actual positions of each lens and their focal lengths were optimized using a script, which matches the output beam of the telescope with the mode-matched beam to the cavity. The beam after the telescope is obtained with the ABCD matrix formalism [60]. The resulting lens positions and the propagated beam size through the telescope, simulated in Finesse 2 (pykat), is shown in Fig. 6.10. The focal lengths used were 0.075 m for l_1 and l_2 and 0.300 m for l_3 .

If l_1 is placed in the far-field of the input beam ($q_0 = z_0 + z_{R,0}$) and in the case where $z_0 \gg z_{R,0}$ and $z_0 \gg f_1$, then the waist size is determined by the position of l_1 , with no impact on the waist position [220]. In our case, however, these approximations are not entirely valid. Even though there is freedom to adjust l_1 , we are limited by the divergence of the input beam, which limits

the relation between z_0 and $z_{R,0}$. For the configuration shown in Fig. 6.10, we have $z_0 = 0.879 \text{ m} \gg z_{R,0} = 0.0846 \text{ m}$. In this way, even though we can tune the waist size by moving l_1 , there will be some coupling to the waist position quadrature. Figure 6.11 shows the waist size and position tuning when moving the positions of l_1 and l_3 .

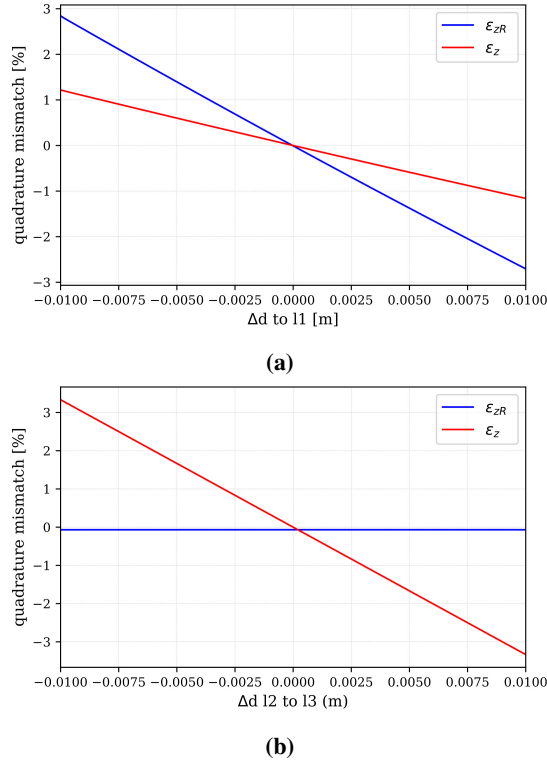


Figure 6.11: Induced waist size (ε_{zR}) and position (ε_z) from tuning the position of l_1 and l_3 . Simulated in Finesse2 (pykat).

Note that in order to change the distance to l_1 without changing the distance between l_1 and l_2 , it is the input beam collimator that needs to be moved. For this reason, the injection collimator was mounted on a translation stage.

6.4 First measurements and perspectives

The goal of this experiment is to reproduce the error signals predicted in Fig. 6.4 in the lab. In order to do this, it is necessary to estimate the waist size and position mismatch independently of the phase camera signals. The most

straightforward way of doing this is calibrating the mode matching telescope by measuring the beam parameter after the telescope for each tuning of the translation stages of the collimator and l_3 .

I tried doing so using the 4M-IR camera, as done for the input beam shown in Fig. 6.9. However, I obtained uncertainties in the waist position at the cm level. This is a limitation of the achievable precision with the 4M-IR camera, mainly because the lack of dynamic range which requires changing the exposure time as the beam size changes during data acquisition. This change of exposure time impacts the estimated beam size, thus impacting the beam fit. A better precision should be achievable using scanning slit or pinhole beam profiler. Additionally, the mode mismatch quadratures could be independently measured with another wavefront sensor, such as a mode-converter, in order to benchmark the measurements with the phase camera.

Regardless, preliminary results with the phase camera already show that we can correlate an improvement in the waist position as the carrier size approximates the size of the sidebands, i.e. as err_W approximates 0. This is shown in Fig. 6.12 for three positions of the translation stage of l_3 , the difference in radius between the carrier and sidebands is recorded. The mode mismatch was fitted from FSR scans similar to the one of Fig. 6.7, taken for each position of the translation stage.

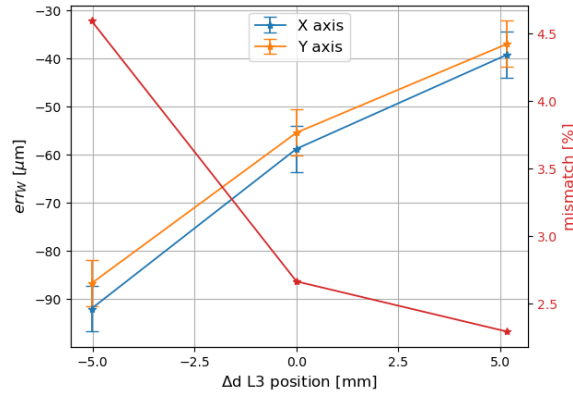


Figure 6.12: Difference between carrier and sideband radius (err_W) for 3 different positions of the translation stage of l_3 , obtained with the set-up of Fig. 6.2. For each datapoint, the corresponding mode mismatch, taken from the ratio $\text{HG}_{mn}/\text{HG}_{00}$ with $m + n = 2$ from the FSR scans, is also shown.

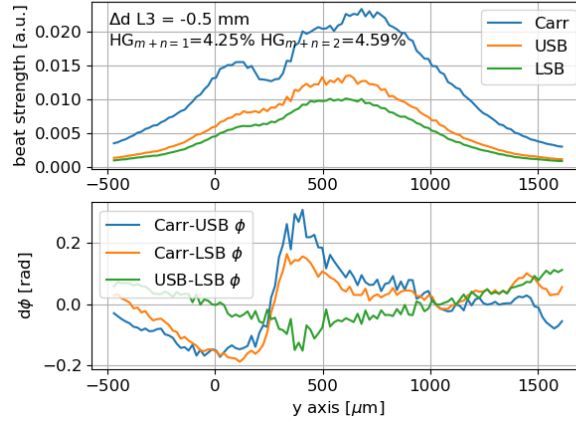
We can see in Fig. 6.12 that as err_W approximates 0, so does the mode mismatch. As the slope of the mode mismatch changes from position 0 to 5 mm, so does the slope of the error signal. We never reach, however, the 0 crossing of the error signal. It seems that a larger translation would be required for that. However, in Fig. 6.4 we saw that the waist size mismatch can add an offset to the 0 crossing of the err_W signal. We can investigate the presence of waist size mismatch by looking at the phase images (i.e. at err_R).

Fig. 6.13 shows the y cross sections taken with the phase camera for two of the data points of Fig. 6.12. On the bottom panel, we have the cross sections for the smallest amount of mode mismatch. Although the phase is not flat, it is difficult to derive an error signal as we would expect something curved (recall Fig. 6.3). Although an analysis of phase noise was made in Section 5.2.2, this did not include the presence of an optical cavity. For example, what is the impact of small phase drifts of the PDH lock during image acquisition. Another issue might be the presence of small amounts of other HOMs other than LG_{01} due to the mismatch of the modes.

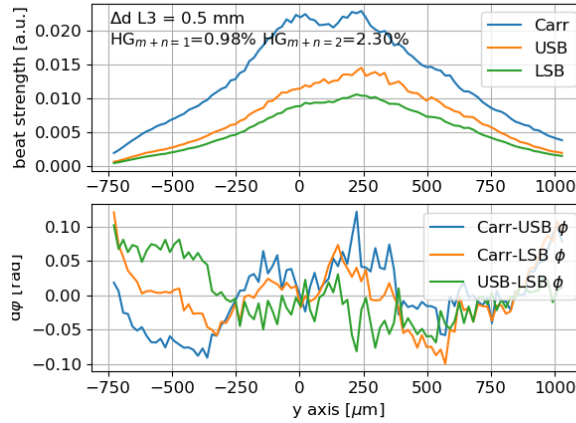
In the top panel of Fig. 6.3, we can see the differential phase in the presence of a significant amount of first-order modes in addition to the mode mismatch modes. There is a bump in the amplitude cross section, and a sign shift in the phase can be seen. This sign shift is not evident in the sidebands, which we do not expect to carry any HOM.

I investigated the impact of misalignments in the error signals in simulation. Fig. 6.14 shows the amplitude of the carrier and differential phase cross sections in the nominal case, with mode mismatch, tilt and a mix of both. Indeed it seems that the presence of HG_{10} (or HG_{01} depending on the direction of the tilt) leads to the appearance of this kind of features in both the amplitude and phase information.

While the impact of other HOMs in the error signals needs to be studied further, it could also potentially have its uses. This information could be used to generate error signals for misalignment as well, similar to what is done in Ref. [216] where control signals for both misalignment and mode mismatch were derived.



(a)



(b)

Figure 6.13: Amplitude and differential phase cross sections for carrier and sidebands. Panel (a) is for $\Delta d l_3 = -0.5 \text{ mm}$ and panel (b) for $\Delta d l_3 = 0.5 \text{ mm}$. Recorded with the phase camera as shown in Fig. 6.2.

6.5 Summary and outlook

In summary, we have seen through simulations how a single phase camera in reflection from a cavity can be used to generate error signals for both mode mismatch quadratures. For this, a reference beam is essential in order to disentangle the promptly reflected sidebands from the resonant carrier.

I have also shown the design and setup of a table-top experiment to validate the simulation results. I presented preliminary results which, albeit promising,

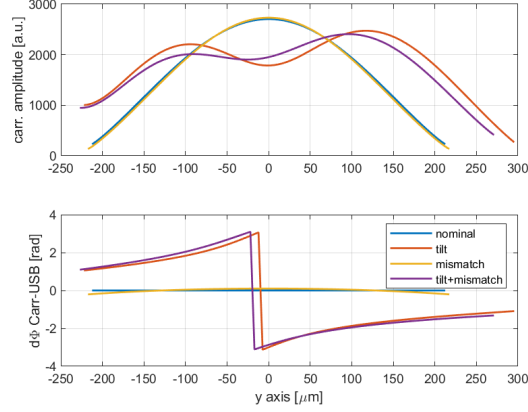


Figure 6.14: Simulated (in OSCAR) amplitude and phase cross sections in the presence of misalignment, mode mismatch and a mix of both.

raise new questions, especially regarding the impact of other HOMs in the carrier beam. While this may open other doors for wavefront sensing, it also presents itself as a challenge for the mode mismatch signals derived in this chapter. More work is also needed to calibrate the mode matching telescope or measure independently both mismatch quadratures with an alternative method for validation and benchmarking.

Resonant detection of the higher-order modes of a locked cavity

In this chapter I detail a table-top experiment that combines optical injection with an EOM and a phase camera in transmission of a cavity. By doing so, it is possible to image the HOMs of a cavity while it is locked on the fundamental mode of a laser. From scanning the EOM, the mode frequency positions for any mode of arbitrary order can be ubiquitously determined. This ability to determine the spatial and frequency properties of any HOM of a cavity while it is locked is extremely relevant to address the issues with high-power operation of GW detectors discussed in Section 1.5.

The results of this experiment were published in an Optics Express article titled "Resonant enhanced detection of the higher-order modes of a locked cavity" [1]. Below, I reproduce the text and figures of this publication with a few changes to stay consistent with the rest of the document. I conclude with a small summary and an outlook on the future perspectives for this work.

7.1 Context

As briefly mentioned at the end of Chapter 1, optical resonators with high circulating powers are crucial in a broad range of fields, such as phase-contrast electron-microscopy [21], dark matter detection [92], gravitational-wave (GW) detectors [8, 9], steady-state microbunching for the production of EUV radiation [23] and even in nuclear fusion with proposals for novel neutral beam heating systems [22, 221]. In GW detectors, increased optical powers lead to an increase in detection sensitivity, particularly at high detection frequencies [222]. For this reason, next-generation GW detectors plan to have circulating powers in the order of the MW in km-long Fabry-perot cavities [19].

Similar optical powers are required in order to perform efficient photoneutralization of deuterium for nuclear fusion [22]. In the case of phase contrast electron-microscopy, hundreds of GW/cm^2 are needed in order to have the laser beam act as a phase plate for the electron beam [21].

Operation of optical resonators with such high optical powers can suffer from issues with thermal absorption in the optics (Section 2.5) and power-induced instabilities (Section 1.5).

In this work, we are particularly interested in the use of high optical powers for gravitational-wave detection. Current detectors were designed to store up to 750 kW of intra-cavity power [9]. Operation at this level of circulating power puts strict constraints on the absorption in the optics of the resonator. Even so, absorption in the substrate and coating of the optics, especially at the center where the beam is incident (recall Section 2.5), leads to thermo-refractive effects and changes in the radius of curvature. These effects modify the g-factor of the cavity as it warms up. This thermally induced mismatch is currently a limitation in the achievable intracavity power in GW interferometers. Despite existing thermal actuation (Section 2.7), methods to sense and accurately control the mismatch remain elusive, with no mode control feedback loop in place [2].

High circulating powers in cavities can also lead to other complications, such as the PIs discussed in Section 1.5. Existing strategies to control and mitigate PIs include shifting the frequency of the optical HOM away from the frequency of the problematic acoustic mode by thermal tuning [94], electrostatic damping [96], or damping the quality factor of problematic acoustic modes [95]. Another technique involves the suppression of the optical HOM by using an electro-optic modulator (EOM) to inject the same HOM with opposing phase into the cavity [97]. This has been shown for a first-order optical mode. However, this technique is limited due to difficulties in measuring and tracking the complex PI amplitude, especially for high-order modes. In reference [101] the first real-time imaging of a PI, caused by the fundamental and a first-order mode, was made.

The idea of HOM injection goes back as far as 1984 in the proposal by D. Z. Anderson of measuring first-order modes in transmission of a cavity for alignment sensing [13]. As discussed in Section 2.6, the work of Ref. [174] develops in simulation a similar concept with second-order modes in order to measure mode mismatch in linear and coupled cavities. In reference [21], and the supplemental material of [223], an EOM was scanned around the mode separation frequency in order to derive a measurement of the waist size in a

near concentric cavity for phase contrast electron microscopy. Reference [224] uses an EOM in a similar way, scanning its modulation frequency to measure HOMs up to order 2 in a v-shaped cavity locked with optical feedback.

It is clear from the challenges of operating high circulating powers that on-line monitoring of the cavity eigenmodes is needed. Here, we combine the above mentioned injection scheme with a scanning pinhole phase camera (described in Chapter 5) to measure the amplitude and phase of the eigenmodes of a locked linear cavity. This is done by changing the modulation of an EOM to match the mode frequency of a given HOM. More details are given in section 7.2.1. The resulting heterodyne beating between the injected HOM and fundamental mode is then measured in transmission of the cavity using the phase camera. The advantage of using a phase camera is the ability to easily produce spatial maps of both the amplitude and phase of any cavity mode of arbitrary order.

We demonstrate the capability of this method to monitor the g-factor of a cavity while it is locked. We also analyze the relative power of the different cavity modes taking into account the overlap integral between the fundamental and the sideband higher-order mode to retrieve the calibrated mode powers.

Finally, we measure the optical phase between the carrier fundamental mode and the injected sideband HOM. The ability to measure the phase makes this technique suitable for use in alignment and mode matching control [3, 174, 216] and PI suppression via optical feedback [97].

7.2 Experimental set-up

The optical schematic of the experimental set-up is shown in Fig. 7.1. It is composed of an optical cavity and two heterodyne detection schemes, one in reflection of the cavity and another in transmission. The cavity is composed of an invar bar and two mirrors mounted on K05S2 polaris mounts, separated by ≈ 320 mm, resulting in a free spectral range (FSR) of ≈ 468.4 MHz. The two mirrors have a radius of curvature (RoC) of 500 mm with a nominal reflection of 99.15 ± 0.15 %, resulting in a nominal finesse $F \approx 368^{+79}_{-55}$ and a linewidth $\Delta\nu \approx 1.27^{+0.23}_{-0.22}$ MHz. The laser used is a butterfly packaged DFB laser from Eblana, EP1550-0-DFB-B01-FM, with a linewidth as low as 100 kHz. The input beam after the fiber collimator is the same as shown in Fig. 6.9.

The reflection detection scheme, shown in the orange colored area of Fig. 7.1, is used to frequency lock the fundamental mode of the laser to the cavity using the PDH locking scheme (recall Section 2.2). The error signal is ob-

tained with a first pair of radio-frequency (RF) sidebands at $f_{\text{PDH}} = 50$ MHz, generated by an EOM (EOM1 in Fig. 7.1). The carrier and PDH sideband beating is measured at a fast photodiode (150 MHz bandwidth). Throughout the text we call f_0 the carrier fundamental frequency which is locked to the cavity.

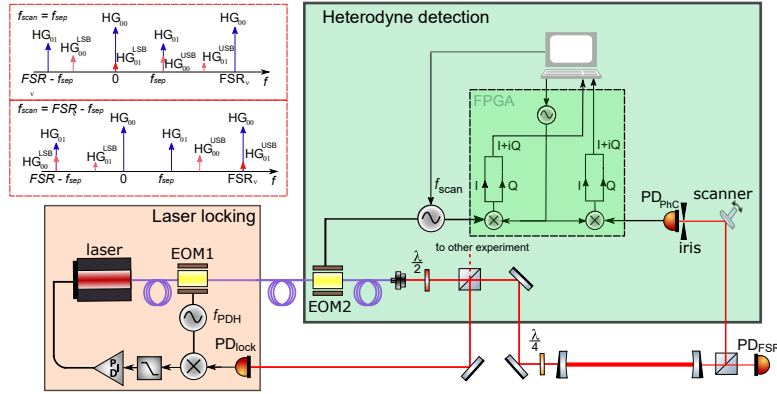


Figure 7.1: Optical schematic of the experiment. The orange colored area shows the optical and electronic components to PDH lock the laser to the cavity resonance. The green colored area shows the sideband HOM injection and heterodyne measurement with a phase camera in transmission of the cavity. More details given in sections 7.2.1 and 7.2.2. The top left inset shows a diagram of how the sideband HOMs (red arrows) are injected into the cavity, co-resonating with the carrier (blue arrows). More details given in section 7.2.1.

As discussed in Section 1.3.1, a spherical optical resonator decomposes the input beam into a series of its own eigenfunctions or mode basis according to complex coupling coefficients $k_{m'n',mn}$ (Eq. 1.31). Here, we assume the orthonormal HG basis, in order to allow for some small amount of astigmatism. The modes, in the basis of the cavity, are written here as HG_{mn} , where m and n are the mode numbers of the 1D Hermite-Gauss polynomial in the x and y transverse coordinates, respectively, and $\text{HG}_{m'n'}$ in the basis of the input beam. We assume the input beam is a Gaussian beam perfectly described in its own basis by the fundamental mode $\text{HG}_{0'0'}$. Hence, the transverse input beam decomposed in the basis of the cavity can be written as [11]:

$$U(x, y) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} k_{0'0',mn} \text{HG}_{mn}(x, y), \quad (7.1)$$

where the coupling coefficients are such that:

$$\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} |k_{0'0',mn}|^2 = 1. \quad (7.2)$$

For simplicity, in the remainder of the text, we omit the input beam indices when writing the coupling coefficients and write them as k_{mn} . In a high-power cavity such as is the case in a GW detector, HOMs can be excited from point absorbers in the mirror coatings and the non-ideal surface of the cavity mirrors. In this experiment, in order to excite the HOMs, the input beam is significantly mode-mismatched and slightly misaligned with respect to the cavity.

Finally, note that in linear or Fabry-Perot cavities and in the limit of very small or no astigmatism, HG modes of same $m + n = M$ order resonate together. Thus, when we can't distinguish between same order modes in the cavity we write them as HG_M with overall coupling coefficient k_M and refer to them as cavity modes.

7.2.1 Optical injection

A second pair of sidebands is generated via a second EOM (EOM2 in Fig. 7.1) driven by a waveform generator at frequency f_{scan} (Fig. 7.1). The transverse modes of the generated sidebands will resonate along with the carrier for certain key frequencies. The beating between the carrier HG_{00} and the co-resonating sideband transverse HOM can be measured in transmission of the cavity, with a heterodyne readout demodulated at f_{scan} . By scanning the EOM modulation frequency, a spectrum of the HOMs of the cavity can be retrieved from the lower sideband (LSB), and another copy of the HOM spectrum at $\text{FSR}_{\nu} - \text{HOM}_{\nu}$ can be retrieved from the upper sideband (USB). We exploit this in order to scan the full FSR of the cavity (≈ 468 MHz) despite the 250 MHz limit on the acquisition bandwidth, by having the LSB probe the first half of the FSR from 0 to 250 MHz, and the USB probe the second half from $\text{FSR}_{\nu} - 250$ MHz to FSR_{ν} .

As an example, the upper left inset in Fig. 7.1 illustrates what happens when the EOM is driven at the mode separation frequency of the cavity, $f_{\text{scan}} = f_{\text{sep}} = \text{FSR}_{\nu} / \pi \arccos(1 - L/RoC) \approx 179$ MHz. In this case, the lower sideband fundamental mode is at -179 MHz from the carrier fundamental mode HG_{00} , while the sideband first order mode is exactly at the carrier fundamental mode frequency, f_0 . Alternatively, when the EOM is driven at $f_{\text{scan}} = \text{FSR}_{\nu} - f_{\text{sep}}$, the upper sideband fundamental mode is ≈ 179 MHz away from

the next HG_{00} resonant frequency. In this case, the sideband first order mode frequency is exactly at $f_0 + \text{FSR}_\nu$ and thus co-resonating with the locked carrier. Note that the coupling of the sideband to a HG_{mn} mode is given by the cavity itself, as is the case for the carrier.

The impact of the extra sideband modulation scan on the PDH locking was tested and it was observed that it is largely unaffected except for frequencies exactly at $f_{\text{scan}} = n f_{\text{PDH}}$, with n a natural number. Note that for longer cavities with an FSR smaller than the RF range, as is the case in GW detectors, this is never an issue.

7.2.2 Heterodyne detection with a phase camera

In order to measure the beating between the carrier and the sideband transverse HOM it is necessary to break the orthogonality between the modes. For this purpose, reference [224] uses a masking blade in a translation stage, while in reference [97] a quadrant photodiode, which works better for first order modes, is used. In reference [174] the author simulates the use of a clipped photodiode for measuring the beating between the resonant carrier and second order modes.

In this work, we used our scanning pinhole phase camera described in Chapter 5. In this case a reference beam is not necessary as the beating to be measured is composed of a carrier and a single sideband. The device, composed mainly of a piezo scanning mirror and a fast pinhole photodiode, is shown on the right of Fig. 7.1 in the green area.

The pinhole photodiode clips the beam, breaking the orthogonality between the fundamental and the higher-order mode. The piezo scanner allows moving the beam over the photodiode, either to build an image of the beating or to select the pixel with the best signal-to-noise ratio (SNR).

The photodiode signal is mixed digitally with a local oscillator in the FPGA with a software defined frequency, which is changed as the EOM2 modulation frequency is swept. Digital demodulation allows for acquiring the I and Q quadratures of the optical beating. A computer receives the beating quadratures as a complex number $I + iQ$ and the amplitude and phase information are simply $\sqrt{I^2 + Q^2}$ and $\arctan(I/Q)$ respectively.

7.3 Results

After PDH-locking the laser frequency to the HG_{00} of the cavity, a DC voltage can be measured at PD_{FSR} , resulting from the resonant carrier HG_{00} transmitted through the cavity end mirror. Next, f_{scan} is swept and the AC signal of PD_{PhC} is digitally demodulated by the FPGA board. For each step of f_{scan} (from 5 to 249 MHz), the beam is scanned over the photodiode along the x and y cross section and the amplitude and phase are recorded. A full scan with 300 demodulation frequencies and 100 pixels across each x and y cross sections takes about 300 seconds. The amplitude of the measured beating is corrected using the calibrated frequency response of the ADC as well as the calibrated frequency response of the EOM (more details are given in the supplemental document of Ref. [1]).

7.3.1 Heterodyne scan and mode imaging

Fig. 7.2 compares the recorded heterodyne amplitude spectrum with the typical cavity FSR scan. The theoretical mode positions, $f_{m+n} = (m+n)f_{sep}$, are shown. For the upper sideband HOMs, the mode positions are given by $FSR_{\nu} - f_{m+n}$. For this reason, the resonant spectrum is shown in duplicate in the figure to facilitate comparison with the regular FSR scan in the middle panel of Fig. 7.2. On the left-hand side the frequencies probed by the lower-sideband (LSB) are highlighted, and on the right-hand side the frequencies probed by the upper-sideband (USB). The right side is mirrored for scanning frequencies $FSR_{\nu} - f_{scan}$ in order to match the FSR scan.

The PDH sidebands also get modulated by EOM2 and these sideband-of-sideband HOMs will, despite the very small amplitude, also be injected into the cavity. For completion, these HOMs are tagged with dashed grey lines in Fig. 7.2.

In addition to the scan, at each resonant peak mode frequency up to mode order 10, a 100x100 matrix of the beat amplitude is taken. Each image takes about 30 seconds to record. These mode images, shown in the bottom of Fig. 7.2, can be used to verify the mode order and basis. Even if not necessary for our application, an interesting perspective for this work is to combine the images provided by this technique with mode decomposition. This is largely unexplored but can be critical for the future of quantum-enhanced GW interferometers as discussed in reference [2].

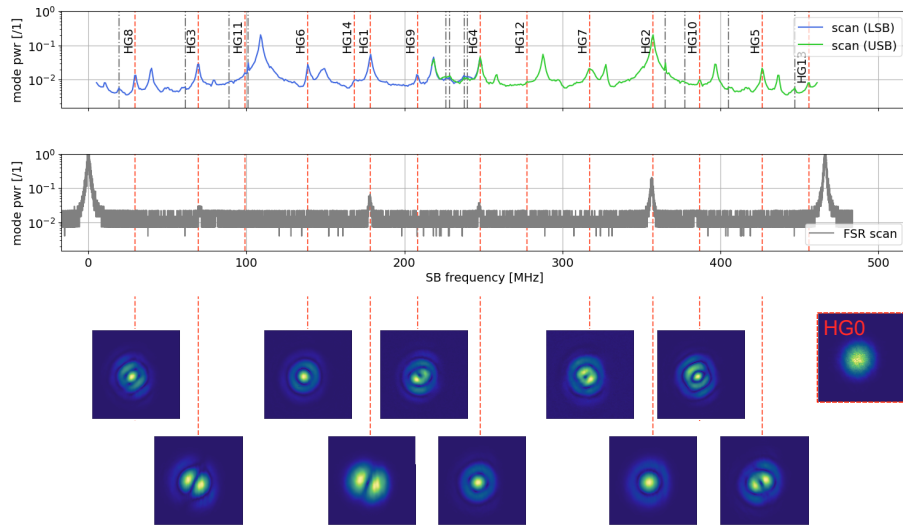


Figure 7.2: Comparison of the resonant heterodyne scan (top panel) with a regular FSR scan (middle panel). The red dashed lines show the injected sideband HOMs. The grey dashed lines tag HOMs from the sideband HOMs of the PDH sidebands. The frequency scale on the x-axis is directly obtained from the scanning of EOM2. The y-scale calibration is explained in section 7.3.1. Additionally, the amplitude images of all measured modes are shown in the bottom panel. The shape of the fundamental mode is also shown in the red-dashed inset, obtained with 500 kHz sidebands.

Extracting the cavity parameters

One of the key advantages of this heterodyne scan is that the frequency scale is obtained directly, unlike a cavity scan where an intermediate parameter, such as crystal temperature or piezo voltage is tuned.

The value of the FSR was found with a fitting procedure using the mode positions obtained with the heterodyne scan. The frequencies of modes 1 through 10 were fitted, using the following equation for modes probed by the LSB:

$$f_{\text{LSB}}(m+n) = \text{FSR}_\nu \frac{m+n}{\pi} \arccos(1 - L/\text{RoC}), \quad (7.3)$$

and, for the modes probed by the USB:

$$f_{\text{USB}}(m+n) = \text{FSR}_\nu - \text{FSR}_\nu \frac{m+n}{\pi} \arccos(1 - L/\text{RoC}), \quad (7.4)$$

where both L and RoC are the parameters to be fitted, and $\text{FSR}_\nu = c/2L$ with c the speed of light. Note that the mode positions depend only on the overall cavity g -factor and cannot unambiguously determine the RoC of input and end mirrors. Here we only consider the effective RoC of the cavity mirrors, $(R_{\text{IM}} + R_{\text{EM}})/2$. The mode positions were obtained from the measured spectrum by fitting the peaks with a Lorentzian using the Python library `lmfit` [219]. The stability of the mode positions depends on the stability of the invar bar cavity and changes in the refractive index of air due to pressure and temperature variations. In practice, the largest change in relative mode positions observed was on the order of 1 MHz over a couple of weeks.

A simultaneous fit of both LSB and USB modes was done, having the length and RoC as free parameters. While the RoC was allowed to change within twice the manufacturer tolerances, no bounds were set on the length parameter. The fitted RoC found was $0.501992 \text{ m} \pm 70 \text{ } \mu\text{m}$ and the fitted cavity length $0.321518 \text{ m} \pm 30 \text{ } \mu\text{m}$. The fitted FSR is then $466.213 \text{ MHz} \pm 43 \text{ kHz}$ and the cavity g -factor is ($g = 1 - L/\text{RoC}$) 0.35951 ± 0.00017 .

The x-axis of the regular FSR scan in Fig. 7.2 was converted to frequency by using the fitted FSR value mentioned above and the distance between consequent fundamental modes.

Table 7.1 shows the difference between the measured mode frequencies and the fitted mode frequencies, which were obtained with Eq. (7.3) and Eq. (7.4) using the fitted cavity length and RoC detailed above. The fitted full width at half-maximum (FWHM) of the cavity modes are also shown. The FWHM

values fitted for modes of order 7 and 10 were affected by the low SNR of the measured peaks.

Table 7.1: Fitted mode frequencies obtained with Eq. (7.3) and Eq. (7.4). The measured mode frequencies and FWHM were obtained from the heterodyne scan with a Lorentzian fit of each measured peak. The difference between measured and fitted mode frequencies is also shown.

Modes	Fitted [MHz]	Meas. [MHz]	FWHM [MHz]	RTL [%]
1 (LSB)	178.482	178.492 ± 0.015	1.75 ± 0.07	0.66
3 (LSB)	69.429	69.394 ± 0.014	1.83 ± 0.08	0.77
6 (LSB)	138.878	138.789 ± 0.021	1.57 ± 0.09	0.42
8 (LSB)	29.601	29.691 ± 0.020	1.68 ± 0.16	0.57
9 (LSB)	208.064	208.183 ± 0.042	1.74 ± 0.26	0.64
2 (USB)	109.053	109.098 ± 0.010	1.74 ± 0.04	0.65
4 (USB)	218.280	218.196 ± 0.010	1.76 ± 0.05	0.67
5 (USB)	39.698	39.703 ± 0.016	1.70 ± 0.08	0.60
7 (USB)	148.932	148.801 ± 0.107	4.9 ± 1.40	NA
10 (USB)	79.176	79.407 ± 0.159	3.4 ± 0.68	NA

Because of the different spatial distributions different modes can experience different cavity round-trip losses and thus, will experience a different cavity finesse or linewidth. An average finesse of $F = 271 \pm 11$ is obtained, which is lower than the lower bound on the nominal finesse of 313. The difference could be due to excess optical losses in the cavity due to dust and mirror surface roughness. For this reason, an estimate of the cavity round trip losses (RTL) is also shown in Table 7.1, assuming the mirror reflectance remains at the nominal value of 99.15%. An average RTL of $0.6 \pm 0.3\%$ was computed, where the uncertainty takes into account the manufacturer's uncertainty of 0.15% in the mirror reflectance. No RTL was computed for modes 7 and 10 because considering the large fitted FWHM of these modes the nominal reflection assumption does not hold.

Since different modes interrogate different parts of the mirror, they acquire the noise (e.g. [225]) and losses (e.g. [224]) associated with the different mirror areas. The different RTL in Table 7.1 could be useful for mirror characterization in terms of noise and loss mapping.

Mode power calibration and relative intensities

The y-axis in Fig. 7.2 is the ratio of the mode heights with respect to fundamental mode height. For the regular FSR scan this can be done directly. For the heterodyne scan, however, this is not the case.

First, note that the phase camera is not measuring directly the sideband HOM, but the optical beating between the carrier HG_0 and the sideband HOM HG_M . For a sideband modulated at $\Omega = 2\pi f_{scan}$, resulting in the injection of sideband HOMs of $m + n = M$ order, the intensity of the optical beating at the phase camera photodiode can be written as (its derivation can be found Appendix A):

$$I_{inj}^{\Omega}(x, y) = J_0(\beta)J_1(\beta)\text{Re}[k_0HG_0k_M^*HG_M^*] \cos(\Omega t + \Delta\varphi_{M0}) + J_0(\beta)J_1(\beta)\text{Im}[k_0HG_0k_M^*HG_M^*] \sin(\Omega t + \Delta\varphi_{M0}), \quad (7.5)$$

where $\Delta\varphi_{M0}$ is the phase between the sideband HOM and the carrier fundamental mode, J_0 and J_1 are bessel functions of the first kind of order 1 and 0, respectively, and β is the modulation depth applied to EOM2. After demodulation, the $(\sqrt{I^2 + Q^2})$ amplitude of the overlap integral is obtained. In order to extract the HOM from the overlap integral, we can divide it by the HG_0 . Thus, when performing the heterodyne scan, the obtained x and y cross sections at each frequency were divided by the x and y cross section of the HG_0 mode. The latter was obtained by modulating EOM2 at 500 kHz, which falls well within the cavity linewidth, thus injecting the HG_0 of the sideband into the cavity, which is then measured by the phase camera.

In order to validate this calibration and since the heterodyne scan cannot probe directly the fundamental mode, we compare the relative intensities of modes order 2 and 1 with their relative intensities from the FSR scan. For the regular FSR scan, the HG_2/HG_1 ratio is 4.15 ± 0.19 , while for the heterodyne scan the HG_2/HG_1 ratio is 3.98 ± 0.11 , where the latter is just within the error bar of the first.

Finally, taking into account the HG_2 and HG_1 FSR scan mode powers, we calibrate the heterodyne scan in terms of mode power as well.

While the heterodyne scan probes HOMs up to order 10, the FSR scan only probes HOMs up to order 4. Note that because of the relatively low finesse of the cavity, there is always some amount of sideband leaking through the cavity, and a direct comparison of the noise floors of the two scans is not straightfor-

ward. Additionally, the difference in the oscilloscope and the acquisition board ADCs (8 and 14 bit, respectively) makes it an unfair comparison.

In the next section, we provide a quantitative analysis of the noise floor of the heterodyne scan and its sensitivity limits.

7.3.2 Sensitivity, noise and detection limits

The amplitude of the injected sideband HOM of order $m+n=M$ will depend on the EOM modulation depth and the (complex) coupling from the HG_{00} to the HOMs in the cavity basis, k_{mn} . Assuming the EOM frequency matches the HOM resonant frequency, the amplitude of the sideband HOM in transmission of an (approximately) impedance matched cavity, can be approximated by (see the supplement document of Ref. [1]):

$$E_{SBtransm}^M \approx \frac{2T}{(2T^2 + \mathcal{L})} |k_M| J_1(\beta) \frac{E_0}{\sqrt{2}}, \quad (7.6)$$

where E_0 is the amplitude of the input beam to the EOM, T is the power transmission of both input and output mirrors and $\mathcal{L}$ the cavity round-trip losses.

We can express the power in the transmitted sideband HOM in decibels relative to the transmitted carrier, dBc:

$$10 \log_{10} \left(\frac{|k_M|^2}{|k_0|^2} \frac{P_{SB}}{P_{carr}} \right) \quad [\text{dBc}]. \quad (7.7)$$

From a regular FSR scan we can extract the coupling coefficient ratio for a given order M . Additionally, the sideband to carrier ratio is known from the EOM calibration for a given modulation depth in use. In this way, Eq. (7.7) can be used to calibrate the y scale of the noise measurements. This was done using the relation between modes of order 2 measured with the heterodyne scan and the HG_2/HG_0 measured with the FSR scan. The EOM was calibrated for a given modulation depth and over the range of modulated frequencies in order to retrieve the power ratio between sideband and carrier, which already takes into account power lost to higher-order sidebands. The details of this calibration are given in the supplemental document of Ref [1].

The top panel of Fig. 7.3 shows the noise levels calibrated in this way. The thickness of the lines is given by the uncertainty of the calibration, dominated by the uncertainty of the EOM calibration.

Two noise measurements were made. The first was taken by demodulating the photodiode signal at the various frequencies while scanning the resonant

carrier HG_{00} over the photodiode but without the HOM sideband. This measurement allows us to estimate the phase noise carried by the locked carrier, corresponding to the noise floor of the heterodyne scan. The second is the ADC noise, which was measured by demodulating the signal without a photodiode connected to the ADC board. These noise levels set the sensitivity limits that can be achieved.

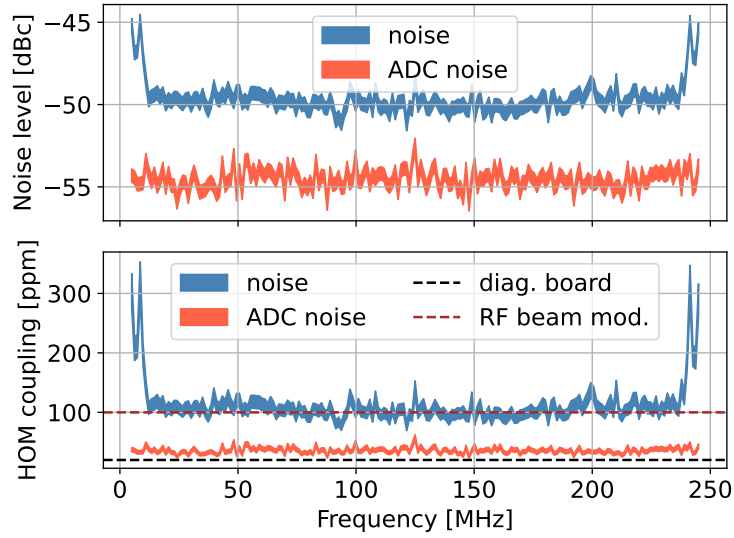


Figure 7.3: Noise levels of the resonant heterodyne scan in dBc (top panel) and also converted to HOM coupling, assuming 10% sidebands (bottom panel). For comparison, the two best sensitivities reported in [2] are shown as well: diagnostic board [155] and RF beam modulation [135].

The dBc units can be converted to mode coupling $|k_{mn}|$ for a given value of modulation depth. The bottom panel of Fig. 7.3 shows the mode coupling sensitivity limit assuming a measurement made with 10% sidebands. This is a comparable value to the best sensitivities reported in [2] (and discussed in Section 2.6). These are, for example, the 20 ppm (RMS) with a diagnostic board based [155] and 100 ppm (RMS) with radio-frequency beam modulation [135], both shown in Fig. 7.3 for comparison.

As the finesse of the cavity increases, the locked laser frequency noise should go down and the blue line in Fig. 7.3 should approach the ADC noise level. Note as well that the ADC hardware used here is different from the Virgo phase camera, for which a -61 dBc limit was reported [2, 20].

7.3.3 Phase sensitive measurements

In this section, we show both that the phase of the injected sideband HOM can be controlled and that the phase camera can determine the optical phase between the sideband HOM and the carrier HG_{00} .

The phase of the injected HOM can be changed by changing the modulation phase that is applied to the EOM. In this measurement, we sweep the phase of the modulating sine wave applied to EOM2. In turn, the phase of the optical beating between the carrier and the sideband HOM is measured by the phase camera, and given by the arctan (I/Q) as explained in section 7.2.2.

The results of these measurements are shown in Fig. 7.4, which were repeated for modes up to order 8 (except mode order 7 which had low SNR).

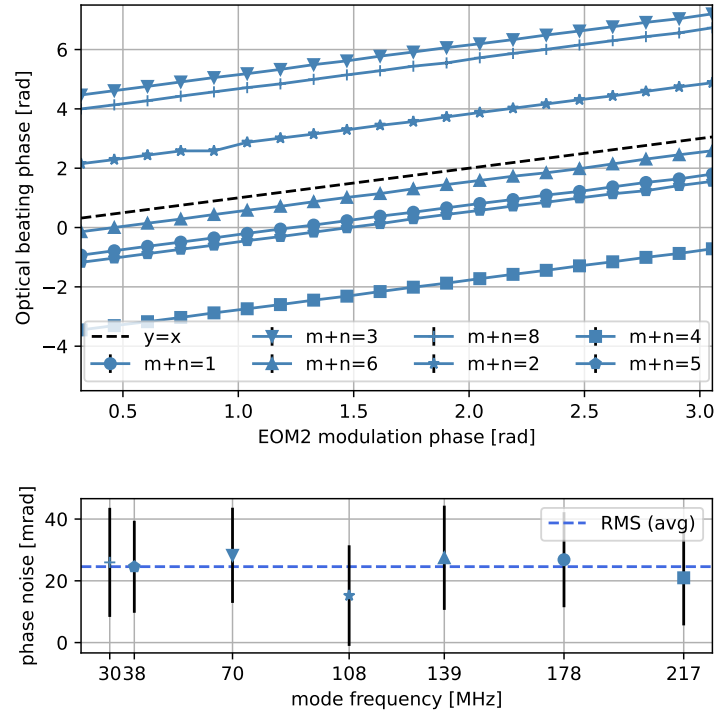


Figure 7.4: Measured phase of the optical beating between carrier and sideband HOM as function of the modulation phase of EOM2. The bottom panel shows the phase noise as a function of the mode frequency.

Each datapoint is the average of 100 ms of data and the noise of each datapoint is also stored. The average noise value in mrad for each mode is shown

in the bottom panel of Fig. 7.4, as well as the overall RMS average which has a value of ≈ 25 mrad.

The measured beating phase will depend on ΔR , the difference in RoC between the carrier and sideband HOM, the difference in accumulated Gouy phase ΔG , and the optical path difference ΔL , but also in the phase difference ΔK between the coupling coefficients k_0 and k_M and the optical phase difference imprinted on EOM2 $\Delta\varphi_{M0}$ (derivation can be found in Appendix A.0.1):

$$\Phi_{\text{beat}} = \Delta R - \Delta G + \Delta L + \Delta K + \Delta\varphi_{M0}. \quad (7.8)$$

Even though the carrier and sideband are spatially overlapped and same basis modes share the same RoC, in this case we can expect a small but non-zero ΔR . This is due to the fact that the fundamental and HOM probe different parts of the mirror with slightly different (non-ideal) RoCs. Additionally, due to the difference in frequency, the slight difference in wavevector will also lead to a small but non-zero ΔL .

As shown in Fig. 7.4, the measured optical beating phase follows the modulating phase applied to EOM2, as expected since we only change $\Delta\varphi_{M0}$. Note, however, the phase noise is about one order of magnitude higher than that reported in [20]. This could come from unwanted variations in the other phase affecting parameters. This is especially the case for ΔK . Small mechanical and thermal instabilities in the cavity or in the input beam pointing precision will couple into the alignment and mode mismatch and affect the phase difference between the HOM k couplings. This could also explain the spurious small dip in phase observed for the second order mode in Fig. 7.4.

In this way, we have shown the ability to measure the phase of a cavity circulating HOM as well as to control the phase of the injected sideband HOM, both of which are crucial for PI suppression via optical feedback.

7.4 Summary and Outlook

In this work we combined an optical sideband injection scheme with a phase camera heterodyne readout in order to measure and characterize the full eigenmode basis of a locked cavity. Monitoring the changes of the eigenmodes of a cavity as it heats up will be crucial for achieving the MW goal of intracavity power in future GW detectors. This is especially so if this measurement can be used to close the loop in the thermal actuators used in GW detectors.

We have shown that this set-up can measure HOM couplings $|k_{nm}|$ as low as 100 ppm. This limit is given by the phase/frequency noise of the locked laser and it should go down with both cavity linewidth and with the improvement of laser frequency noise. Thus, even lower sensitivity limits can be expected if applied in a GW detector.

Additionally, monitoring the linewidth of the injected modes can be used to retrieve an online map of the RTL as the cavity heats up. It has been observed that increased RTL can be related to mirror damage from high circulating powers [23]. Future work should further develop this technique in combination with the phase images of the HOMs (some examples are shown in Fig. 7.5) to map any degradation and thermal deformation in the cavity mirrors subject to high optical powers.

We have also successfully measured the optical beating phase between the injected sideband HOM and the resonating carrier for modes up to order 8. This shows that a phase camera in transmission of a cavity can be used to perform PI suppression for modes of arbitrary order. The phase sensitive measurements can also be used to automatically correct for misalignments and mode mismatch using the transmission signal.

Finally, future work should focus on assessing the viability of implementing this technique at current and future GW detectors. This would involve understanding how the technique would look like in the presence not only of the arm cavities, but also the power and signal recycling cavities (partly similar to what is done in Ref. [174] for the coupled cavity case).

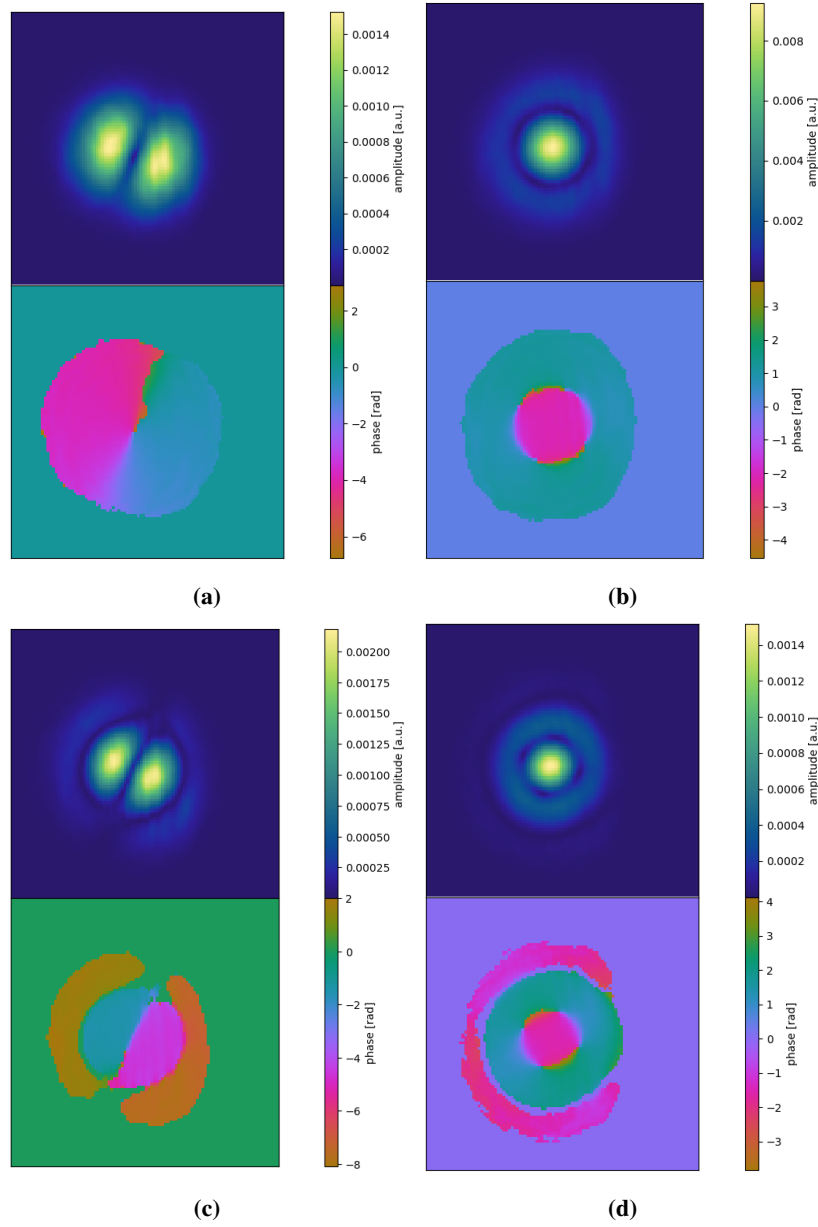


Figure 7.5: Amplitude and phase images for modes of order 1 (a), 2 (b), 3 (c) and 4 (c). Recorded as described in Section 7.3.1.

Part IV

Conclusion

Conclusion and Perspectives

Fabry-Perot cavities with km-long footprints and hundreds of kW of circulating optical powers are at the heart of GW interferometers. In addition, squeezed light injection allows for reaching sensitivities below the quantum noise limit. In this way, sensitivity to differential length changes on the order of 10^{-19} m are already within reach [8, 9]. This sensitivity has enabled detection of hundreds of compact binary coalescences up to now [5]. Many of these events come from as far as hundreds or even thousands of Mpc away, allowing us a glimpse of the Universe at unprecedented redshift.

Operation of optical cavities with high circulating powers is notoriously difficult, suffering from different issues such as angular and parametric instabilities [17], thermal transients due to thermo-refractive effects in the cavity mirrors that shift the cavities eigenmodes as well as increased losses and scattering. In turn, squeezed light manipulation is incredibly sensitive to optical losses and mode degradation [2, 18]. These issues are reviewed in Chapter 1 (Sections 1.5) and Chapter 2 (Sections 2.4.1 and 2.5).

The combination of these issues makes mode sensing and control a crucial aspect of operating a GW interferometer. This is particularly the case for next-generation detectors, for which some of the major planned upgrades include increasing circulating powers to the MW level and increased squeezing injection up to 10dB [19] (recall Section 1.5.2). This work aims to tackle these challenges by combining the existing knowledge of cavity and laser control with wavefront sensing. In particular, this work focuses on the use of the phase camera as a wavefront sensor to address these issues.

The phase camera combines a heterodyne readout with beam scanning to achieve high-resolution amplitude and phase images of the laser beam wavefront at multiple beat frequencies of interest. Two such phase cameras are already installed at Virgo, the European GW detector. During my PhD I had the chance to be part of the Virgo collaboration and contribute to the instrumentation effort through the optical characterization (OptChar) and optical simulation and design (OSD) working groups. This work is described in Chapters 3 and 4.

The optical characterization work of Chapter 3 allowed me to gain experience regarding the use of the phase camera for optical characterization and its importance for the GW detector commissioning and operation. The measurements with the phase camera taken during my stay at Virgo and detailed in Section 3.5 went on to inspire the work of Chapter 6 and were also the subject of a proceedings publication for the GRAvitational-waves Science&technology Symposium (GRASS 2022) [4].

Through the simulation work of Chapter 4 I was able to build my working knowledge of the interferometer and the result is a significant contribution to simulating the realistic detector. The difficulty in simulating the thermal working point of the detector brings to the forefront the need for direct measurement of the eigenmodes of the different cavities. This need motivates the work developed in the lab, at UCLouvain, and the two table-top experiments which were also the subject of this thesis.

In particular, each of these experiments answers each of the scientific questions we would like to address with this work:

- In Chapter 6 we see how a single phase camera can be used to determine completely the two quadratures of mode mismatch for a single cavity.
- In Chapter 7 a phase camera is used to characterize the eigenmodes of a cavity while it is locked.

For these experiments, we developed a custom phase camera, similar to the ones installed at Virgo. This is detailed in Chapter 5.

In Chapter 6, the custom phase camera is placed in reflection from a cavity in order to derive error signals for mode matching from the reflected beam from the cavity. As discussed in Section 2.4, mode mismatch can be understood in terms of two orthogonal quadratures, waist size and position mismatch. Most of the proposed sensing schemes need two sensors to measure both quadratures as discussed in Section 2.6. Our proposed scheme, however, allows for complete characterization of the mode mismatch with a single phase camera. The simulation work detailing the concept was the subject of a conference proceedings publication in a peer-reviewed journal for the 16th Pisa Meeting on Advanced Detectors (PMAD 2024) [3]. Experimental preliminary results are also shown in Section 6.4 to support the simulated predictions.

In Chapter 7, an optical injection scheme using an EOM and a phase camera in transmission are used to image the amplitude and phase of any HOM in a locked cavity. The nature of the injection scheme and the heterodyne readout

in transmission make it so that the mode frequency positions of all the cavity modes can be ubiquitously determined. The on-line monitoring of the mode frequencies allows for monitoring the cavity g-factor (cavity length and mirror RoC) while the cavity is locked. This means the cavity eigenmode can be monitored during a thermal transient. This opens the door for new ways to control the thermal actuators described in Section 2.7 and to address the current limitations in stored cavity power. Additionally, we also showed the ability to track the phase of any HOM with this sensing scheme. This can be crucial for PI mitigation using optical feedback as described in Ref. [97]. This work was the subject of a peer-reviewed publication in Optics Express [1]. This technique can also find applications beyond GW detection, but in other fields where high circulating power cavities are crucial. Examples of these are phase-electron microscopy [21], neutral beam injection systems for fusion reactors [22] and optical trapping of atoms [226].

Regarding the future perspectives for the work developed here, the results mentioned above set some of the ground for the future of mode sensing and control in GW detectors. In order for mode sensing and control to reach the maturity of current longitudinal and angular controls, multiple wavefront sensors capable of directly measuring the eigenmodes of the various cavities and circulating beams, should be deployed.

In particular, phase cameras could be installed both in reflection and in transmission of the cavities in a GW detector. This would allow combining the techniques developed in Chapter 6 and Chapter 7. Along with the ideas developed in Ref. [174], where simulations show that the mismatch in a coupled cavity can be measured in transmission, it should be possible to uniquely and independently determine the eigenmodes of the input beam and of the various cavities.

While this work focused on the use of the scanning pinhole phase camera, other sensors, especially the optical lock-in and ToF cameras could achieve similar results. Some of these can offer different advantages, especially in terms of spatial resolution and mode weighting precision (recall Table 2.1). However, the scanning phase camera remains with the significant advantage of being able to demodulate multiple beat signals at the same time.

Beyond the field of GW detectors, the results of Chapter 7 open new avenues of research. Besides its interest for high-power cavities, as already mentioned, the technique could also be interesting for the development of ultra-stable cavities in general. In practice, it opens the door for time- and spatially-resolved measurements of the cavity parameters (length and RoC).

The ability to measure different HOMs with different spatial distributions, should allow for on-line measuring of the coating thermal noise of the cavity mirrors (see Ref. [225]). By increasing the injected sideband frequency up to the cavity FSR, it would also be possible to measure directly length variations of the cavity. This is the principle behind Noise-immune cavity-enhanced optical heterodyne molecular spectroscopy (NICE-OHMS) [227]. Disentangling these various sources of noise in cavities while in operation, would have a significant impact on the development of ultra-stable cavities for optical clocks [115, 116]. While smaller cavities present a challenge due to their large FSRs, recent work has achieved a phasemeter with GHz bandwidth [228], showing this limitation should be possible to overcome.

Future GW detectors, such as the Einstein Telescope, hope to increase detector sensitivity by one order of magnitude. One of the planned upgrades of these detectors is to increase circulating powers up to 3 megawatt [19]. Current detectors have been unable, however, to increase the circulating power above a few hundreds of kilowatts. It is my hope that this work might contribute to bridging the gap between the current state of affairs in terms of high-power operation and the ambitious goals of future detectors. In particular, I hope the novel wavefront sensing schemes developed in this work will also inspire the design of novel control strategies based on the higher-order mode content of circulating beams. In this way, that this work might contribute to the advancement of mode sensing and control in GW interferometry and beyond.

Part V

Appendices

Appendix A

Phase camera beating: carrier and higher-order-mode sideband

In Ref. [20], the authors derive the expressions for the beating between carrier and reference beam as well as the phase signal after demodulation. Here, I follow a similar derivation but for the phase camera signal between the carrier and a HOM sideband as obtained from the measurements described in Chapter 7.

Recall Eq. 1.24, which describes transverse electrical field distribution of a beam in terms HG modes. Since all the results derived here are valid for any value of z , for simplicity we drop the z dependency and write only $HG_{mn}(x, y)$. If we consider the spatial coupling of the input beam electrical field to a cavity, we can write the input laser beam decomposed in the cavity basis as:

$$HG_{0'0'}(x, y) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} k_{0'0',mn} HG_{mn}(x, y). \quad (\text{A.1})$$

We assume that the input beam is in the fundamental mode of its own basis, which is here denoted by the prime superscript. The complex scattering coefficient from this fundamental mode to a higher-order mode (HOM) in the cavity basis is given by $k_{0'0',mn}$.

If we phase modulate the input beam, at certain key Ω frequencies, the sideband HOMs with a certain order $m+n$ will co-resonate in the cavity with the fundamental mode of the carrier and the beating. Hence, it is useful to write the phase modulated input beam that will be injected in the cavity. This includes the fundamental mode of the carrier with frequency ω , and for now, a

sideband HOM at $\omega + \Omega$ frequency, with $\Omega = 2\pi f_{\text{scan}}$:

$$E_{\text{inj}}(x, y, t) = k_{00}J_0(\beta)HG_{00}(x, y)e^{i(\omega t + \varphi_c)} + k_{mn}J_1(\beta)HG_{mn}(x, y)e^{i((\omega + \Omega)t + \varphi_{\text{sb}})}, \quad (\text{A.2})$$

where β is the modulation depth of the sideband, which depends on the voltage applied to the electro-optic modulator (EOM) and J_0 and J_1 are zero and first order Bessel functions, respectively. Note that for simplicity, the input beam indices were dropped.

Thus, the resonant or injected intensity will be given by:

$$I_{\text{inj}}(x, y) = E_{\text{inj}}E_{\text{inj}}^* = J_0(\beta)^2|k_{00}|^2|HG_{00}|^2 + J_1(\beta)^2|k_{mn}|^2|HG_{mn}|^2 + J_0(\beta)J_1(\beta)k_{00}HG_{00}k_{mn}^*HG_{mn}^*e^{-i(\Omega t + \varphi_{\text{sb}} - \varphi_c)} + J_0(\beta)J_1(\beta)k_{mn}HG_{mn}k_{00}^*HG_{00}^*e^{i(\Omega t + \varphi_{\text{sb}} - \varphi_c)}, \quad (\text{A.3})$$

where the $HG_{00} = HG_{00}(x, y)$ and $HG_{mn} = HG_{mn}(x, y)$ abbreviations were made for simplicity.

For the optical beating we are only concerned with the Ω terms, so we can write the beating intensity as:

$$I_{\text{inj}}^\Omega(x, y) = J_0(\beta)J_1(\beta)k_{00}HG_{00}k_{mn}^*HG_{mn}^*\cos(\Omega t + \Delta\varphi) + iJ_0(\beta)J_1(\beta)k_{00}HG_{00}k_{mn}^*HG_{mn}^*\sin(\Omega t + \Delta\varphi) + J_0(\beta)J_1(\beta)k_{mn}HG_{mn}k_{00}^*HG_{00}^*\cos(\Omega t + \Delta\varphi) - iJ_0(\beta)J_1(\beta)k_{mn}HG_{mn}k_{00}^*HG_{00}^*\sin(\Omega t + \Delta\varphi), \quad (\text{A.4})$$

where $\Delta\varphi = \varphi_{\text{sb}} - \varphi_c$. Factoring out the cosines and sines and doing some complex number algebra, we finally arrive at:

$$I_{\text{inj}}^\Omega(x, y) = J_0J_1\text{Re}[k_{00}HG_{00}k_{mn}^*HG_{mn}^*]\cos(\Omega t + \Delta\varphi) + J_0J_1\text{Im}[k_{00}HG_{00}k_{mn}^*HG_{mn}^*]\sin(\Omega t + \Delta\varphi). \quad (\text{A.5})$$

A point of subtlety here is that for linear or Fabry-Perot cavities and in the limit of very small or no astigmatism, modes of the same order $m+n$ are degenerate and resonate together. This will be the case as long as the difference in accumulated Gouy phase in the two axis is less than 2π times the cavity linewidth. For this reason, we can talk about cavity modes which will be the

sum of all the HG_{mn} with same $m+n$ order. We denote this cavity modes as HG_M , using a single index for the sum of the x and y orders and with coupling coefficient k_M . In this way, we can write:

$$I_{inj}^\Omega(x, y) = J_0(\beta)J_1(\beta)Re[k_0HG_0k_M^*HG_M^*] \cos(\Omega t + \Delta\varphi_{M0}) \\ + J_0(\beta)J_1(\beta)Im[k_0HG_0k_M^*HG_M^*] \sin(\Omega t + \Delta\varphi_{M0}). \quad (A.6)$$

Due to the orthogonality of the HG modes, the integrated intensity of equation A.6 over the entire beam is zero. However, the phase camera uses a pinhole photodiode and only measures a small portion of the beam at a time. If the intermodal phase does not vary significantly over the beam, then the integral is non-zero. In appendix B of reference [174], the authors workout the integral for the fundamental and second-order modes and plot how the aperture size impacts the measured signal.

A.0.1 Demodulation and phase

A piezo scanner allows for moving the beam over the pinhole photodiode while data is acquired. Each pixel is digitally demodulated with a sine and cosine at Ω frequency so as to obtain the two quadrature signals:

$$I = J_0(\beta)J_1(\beta)Re[k_0HG_0k_M^*HG_M^*] \cos(\Omega t + \Delta\varphi_{M0}) \cos(\Omega t) \quad (A.7)$$

$$Q = J_0(\beta)J_1(\beta)Im[k_0HG_0k_M^*HG_M^*] \sin(\Omega t + \Delta\varphi_{M0}) \sin(\Omega t) \quad (A.8)$$

Using the rules of cosine and sine multiplication, this results in:

$$I = \frac{J_0(\beta)J_1(\beta)}{2} Re[k_0HG_0k_M^*HG_M^*] \cos(\Delta\varphi_{M0}) + 2\Omega \text{ term} \quad (A.9)$$

$$Q = \frac{J_0(\beta)J_1(\beta)}{2} Im[k_0HG_0k_M^*HG_M^*] \sin(\Delta\varphi_{M0}) + 2\Omega \text{ term}. \quad (A.10)$$

The 2Ω terms are removed by a low pass filter and the data is retrieved from the FPGA as a complex number of the form $I + iQ$.

The amplitude signal is given by the modulus of this complex number $\sqrt{I^2 + Q^2}$, while the phase is the angle:

$$\Phi_{beat} = \arctan \left(\frac{Im[k_0HG_0k_M^*HG_M^*] \sin(\Delta\varphi_{M0})}{Re[k_0HG_0k_M^*HG_M^*] \cos(\Delta\varphi_{M0})} \right). \quad (A.11)$$

Similar to [20], from the HG modes we obtain a phase difference from different radius of curvature ΔR , Gouy phase ΔG and optical path length ΔL :

$$\Phi_{\text{beat}} = \Delta R - \Delta G + \Delta L + \Delta K + \Delta\varphi_{M0}, \quad (\text{A.12})$$

where ΔK depends on the phase between the coupling coefficients K_0 and k_M and $\Delta\varphi_{M0}$ is the original phase between the carrier and sideband introduced above. The latter can be changed by changing the phase of the modulating sinewave applied to the EOM.

In this case, since carrier and sideband share the same basis (are spatially overlapped), ΔR is equal to zero. Different HOMs of the same basis share the same radius of curvature, as implicit in equation 1.24. However, when coupled to a spherical cavity there will be a small non-zero ΔR because the fundamental and HOM probe different parts of the mirror with slightly different (non-ideal) RoC. Due to the difference in frequencies, the slight difference in wavevector will also lead to a small but non-zero ΔL .

Appendix B

Simulating the etalon effect of the Virgo arm cavities

The Advanced Virgo arm cavities have an enhancement factor of ~ 285 , which given the cavities nominal length of 3 km results in an effective path length on resonance of around 855 km. As discussed in Section 1.3.2, this very long effective baseline is one of the reasons behind the very high sensitivity of GW detectors.

However, it can be seen that the sensitivity of a GW interferometer relies on the two arm cavities having nearly the same finesse, or mirror reflectivity. Asymmetries in the finesse of the arm cavities can couple common noises into the readout signal, such as laser frequency or intensity noise, and spoil the sensitivity [229].

For this reason, and contrary to what is normally done in table-top set-ups, the input mirrors of the arm cavities have parallel faces and no wedges. The parallel faces in the input mirror allow for a very low finesse¹ etalon in the mirror substrate between the AR and HR sides of the mirrors. This is possible because the AR side reflectivity, although small (~ 60 ppm), is not zero [230]. The end mirrors of the arm cavities have a wedge and thus does not allow for a substrate etalon, fixing the reflectivity of the mirror to the manufacturing value. Recall that the reflectivities of the Virgo optics are shown in Table 4.1.

The etalon in the substrate is akin to a change in the reflectivity of the mirror. Thus, by adjusting the etalon effect in the input mirror of the arm cavities, one can adjust the overall finesse of the cavity. In Virgo, the input mirror etalon is adjusted through thermal expansion by adjusting the temperature of the mirror substrate [230].

¹Recall Section 1.3.1 where we saw that Eq. 1.22 for the Airy finesse breaks down for small mirror reflectivity. So technically, these are etalons with a very small amount of round-trips, but the interference effects of the substrate resonator can still have significant impact in the GW interferometer as we will see.

In this Appendix, I present optical simulations of the Virgo detector concerning the etalon effect of the arm cavities. The simulations were done in OSCAR, using the same nominal configuration as the one used in Section 4.3. The goal is to see how the input mirror etalon impacts the carrier and sideband gains as well as the power in the dark port. In order to do this, the substrate length of each north and west input mirrors was scanned. This resulted in a 2D grid for each combination of north and west input substrate length detuning. The simulations were performed assuming 60 ppm of reflectivity in the AR coating of each input mirror.

Fig. B.1 shows how the enhancement factor, or cavity gain, of the north and west arm cavities changes with the detuning. We can see from the figure that each arm finesse is only sensitive to the detuning of its input mirror substrate, despite being coupled through the PRC. The arm gain can be detuned by up to 2.8% with the substrate etalon, which is similar to what obtained in Ref. [230].

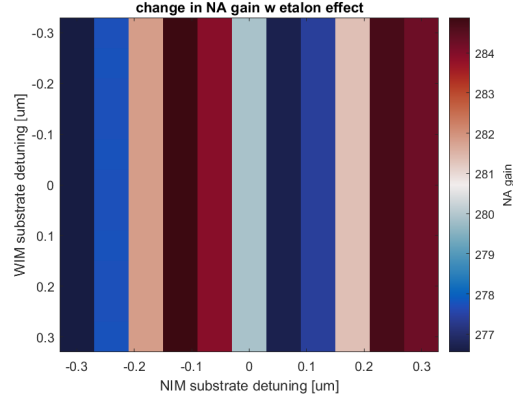
Fig. B.2 shows the change in the carrier PRC gain as the two substrate lengths are detuned. The impact in the PRC carrier gain is smaller than in the arm gains, a maximum change of 1.77% is achieved. More over, we can see in the figure that the lowest carrier PRC gain matches the detunings where both the west and north arm gains are at a maximum. In turn, the highest carrier PRC gain are achieved for detunings where both the west and north arm gains are at a minimum. This is to be expected if we look at the equation for the reflected power from an overcoupled cavity (Eq. 31 of Ref. [64])

$$P_{\text{ref}} = (1 - G_{\text{cav}} \times \mathcal{L}_{\text{RTL}})P_{\text{in}}, \quad (\text{B.1})$$

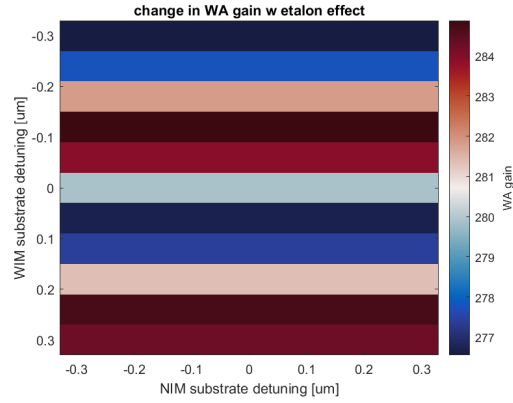
where P_{in} is the input power to the cavity, $\mathcal{L}_{\text{RTL}}$ the cavity round-trip losses and G_{cav} the cavity gain.

The result of Eq. B.1 is intuitive if we think of an overcoupled cavity. The arm cavity increase signifies an increase of mirror reflectivity, which in turn signifies a decrease in coupling losses. As coupling losses decrease and get closer to the round-trip or intrinsic losses, the amount of reflected light diminished as we approach the condition of critical coupling.

The change in the 6 MHz PRC gain with the input mirror etalon, is at most 0.4%. As it is so small, no figure is shown for this. The 56 MHz sideband PRC gain is, however, the most sensitive to the etalon detuning. As shown in Fig. B.3, the 56 MHz PRC gain can change as much as 12% with the detuning.



(a)



(b)

Figure B.1: (a) North arm cavity gain as the substrate length of the north and west input mirrors is detuned. (b) Same thing for the west arm cavity gain.

Fig. B.4, shows the change in the amount of carrier at the dark port. As the figure of merit the DP transfer function as defined in Section 3.3 is used. We can see from the figure that the amount of carrier at the dark port can increase as much as 4 times with the substrate etalon. We can expect a similar impact on the contrast defect.

In order to evaluate the impact of the etalon detuning on the noise in the detector, audio sidebands at 100 Hz were added to the input beam to the interferometer. These sidebands are within the typical frequency range of the GWs measured in a detector. By looking at the change in power of the audio sideband in the dark port, we can evaluate how the interferometer noise is affected.

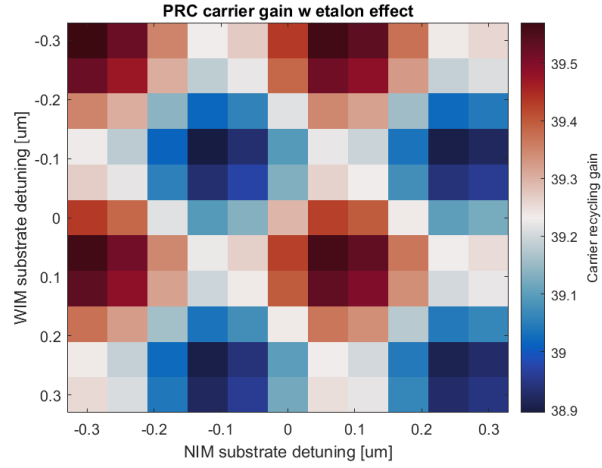


Figure B.2: Carrier gain in the PRC as the substrate length of the north and west input mirrors is detuned.

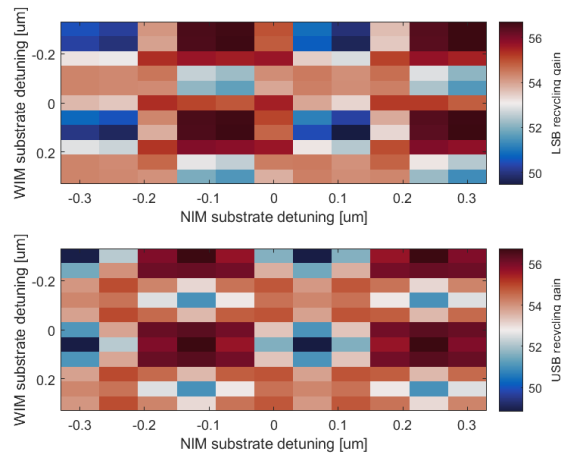


Figure B.3: PRC gain of the 56 MHz upper and lower sidebands as the substrate length of the north and west input mirrors is detuned.

This is shown in Fig. B.5 and we can see the amount of the 100 Hz sideband in the DP can change by up to 14 times.

B.0.1 Summary and outlook

We have seen how even with a very small AR reflectivity, the etalon in the input mirror of the Virgo arm cavities can impact the finesse of these cavities by up

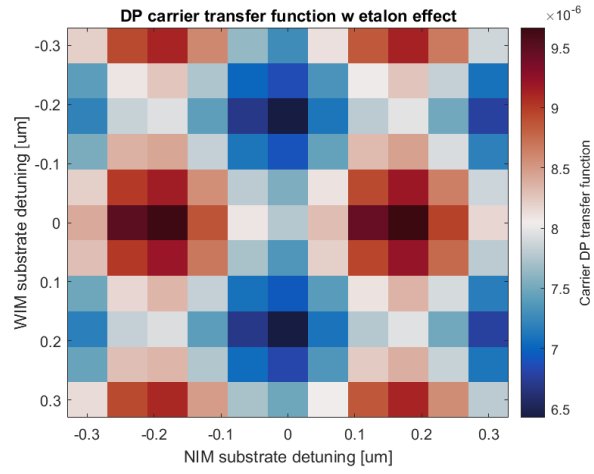


Figure B.4: Carrier dark port transfer function (Section 3.3) as the substrate length of the north and west input mirrors is detuned.

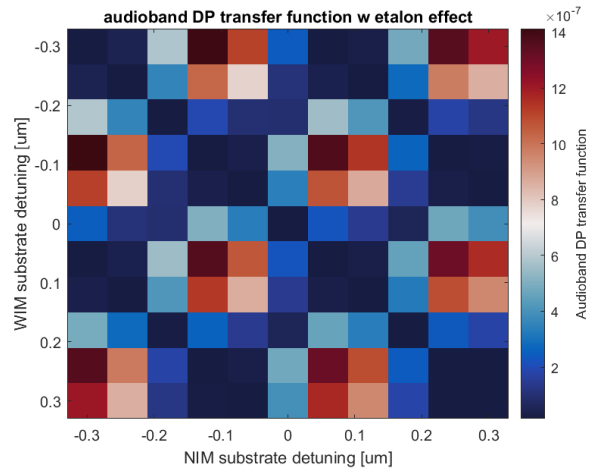


Figure B.5: Dark port transfer function of 100 Hz sideband as the substrate length of the north and west input mirrors is detuned.

to 2.8%. We have shown that the behavior of the carrier in the PRC follows the behavior of the overcoupled arm cavities. In turn, the 56 MHz PRC gain was seen to be impacted by as much as 12%. This could be enough to explain the difference between the 6 and 56 MHz gains obtained in the simulations of Section 4.4, which don't seem to be explained by mode mismatch alone.

Regarding the power at the dark port, the etalon effect has a non-negligible effect. Even though the north and west arm etalons are tuned to minimize noise at the dark port, it could be interesting to see how Figs. B.4 and B.5 change when realistic mirror maps are included in the simulation.

Bibliography

- [1] R. Cabrita, A. Goodwin-Jones, J. Van Heijningen, P. Demin, M. Van Beuzekom, M. Tacca, G. Bruno, and C. Lauzin, “Resonant enhanced detection of the higher-order modes of a locked cavity,” *Optics Express*, vol. 33, no. 14, pp. 30209–30220, 2025.
- [2] A. W. Goodwin-Jones, R. Cabrita, M. Korobko, M. V. Beuzekom, D. D. Brown, V. Fafone, J. V. Heijningen, A. Rocchi, M. G. Schiowski, and M. Tacca, “Transverse mode control in quantum enhanced interferometers: a review and recommendations for a new generation,” *Optica*, vol. 11, pp. 273–290, Feb 2024.
- [3] R. Cabrita, C. Lauzin, G. Bruno, and J. van Heijningen, “Mode matching with a phase camera for gravitational-wave detectors,” *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, vol. 1068, p. 169806, 2024. doi: <https://doi.org/10.1016/j.nima.2024.169806>.
- [4] R. Cabrita, “Sensing mode mismatch with the phase cameras at advanced virgo,” *Zenodo*, June 2022. doi: <https://doi.org/10.5281/zenodo.6758929>.
- [5] T. L. S. Collaboration, the Virgo Collaboration, the KAGRA Collaboration, A. G. Abac, I. Abouelfettouh, F. Acernese, and et al., “GWTC-4.0: An Introduction to Version 4.0 of the Gravitational-Wave Transient Catalog,” 2025.
- [6] R. P. Kerr, “Gravitational field of a spinning mass as an example of algebraically special metrics,” *Physical review letters*, vol. 11, no. 5, p. 237, 1963.
- [7] R. Abbott, H. Abe, F. Acernese, K. Ackley, N. Adhikari, R. X. Adhikari, V. K. Adkins, V. B. Adya, C. Affeldt, D. Agarwal, et al., “Tests

of general relativity with gwtc-3,” *Physical Review D*, vol. 112, no. 8, p. 084080, 2025.

- [8] F. Acernese *et al.*, “Advanced Virgo: a second-generation interferometric gravitational wave detector,” *Class. Quantum Grav.*, vol. 32, no. 2, p. 024001, 2015.
- [9] J. Aasi *et al.*, “Advanced LIGO,” *Class. Quant. Grav.*, vol. 32, p. 074001, 2015.
- [10] H. J. Kimble, Y. Levin, A. B. Matsko, K. S. Thorne, and S. P. Vyatchanin, “Conversion of conventional gravitational-wave interferometers into quantum nondemolition interferometers by modifying their input and/or output optics,” *Physical Review D*, vol. 65, no. 2, p. 022002, 2001.
- [11] F. Bayer-Helms, “Coupling coefficients of an incident wave and the modes of a spherical optical resonator in the case of mismatching and misalignment,” *Applied optics*, vol. 23, no. 9, pp. 1369–1380, 1984.
- [12] R. W. Drever, J. L. Hall, F. V. Kowalski, J. Hough, G. Ford, A. Munley, and H. Ward, “Laser phase and frequency stabilization using an optical resonator,” *Applied Physics B*, vol. 31, no. 2, pp. 97–105, 1983.
- [13] D. Z. Anderson, “Alignment of resonant optical cavities,” *Appl. Opt.*, vol. 23, pp. 2944–2949, Sep 1984.
- [14] G. C. Bjorklund, M. Levenson, W. Lenth, and C. Ortiz, “Frequency modulation (FM) spectroscopy: theory of lineshapes and signal-to-noise analysis,” *Applied Physics B*, vol. 32, no. 3, pp. 145–152, 1983.
- [15] G. D. Meadors, K. Kawabe, and K. Riles, “Increasing LIGO sensitivity by feedforward subtraction of auxiliary length control noise,” *Classical and Quantum Gravity*, vol. 31, no. 10, p. 105014, 2014.
- [16] A. Rocchi, E. Coccia, V. Fafone, V. Malvezzi, Y. Minenkov, and L. Sperandio, “Thermal effects and their compensation in Advanced Virgo,” in *Journal of Physics: Conference Series*, vol. 363, p. 012016, IOP Publishing, 2012.
- [17] V. Braginsky, S. Strigin, and S. P. Vyatchanin, “Parametric oscillatory instability in Fabry-Perot interferometer,” *Physics Letters A*, vol. 287, no. 5-6, pp. 331–338, 2001.

- [18] D. Töyrä, D. D. Brown, M. Davis, S. Song, A. Wormald, J. Harms, H. Miao, and A. Freise, “Multi-spatial-mode effects in squeezed-light-enhanced interferometric gravitational wave detectors,” *Phys. Rev. D*, vol. 96, p. 022006, Jul 2017.
- [19] H. Lück *et al.*, “ET design report update 2020,” 2020. Design Report Update 2020 for the Einstein Telescope.
- [20] K. Agatsuma, L. van der Schaaf, M. van Beuzekom, D. Rabeling, and J. van den Brand, “High-performance phase camera as a frequency selective laser wavefront sensor for gravitational wave detectors,” *Opt. Express*, vol. 27, pp. 18533–18548, Jun 2019.
- [21] C. Turnbaugh, J. J. Axelrod, S. L. Campbell, J. Y. Dioquino, P. N. Petrov, J. Remis, O. Schwartz, Z. Yu, Y. Cheng, R. M. Glaeser, *et al.*, “High-power near-concentric fabry–perot cavity for phase contrast electron microscopy,” *Review of scientific instruments*, vol. 92, no. 5, 2021.
- [22] D. Fiorucci and A. Fassina, “Overview of photo-neutralization techniques for negative ion-based neutral beam injectors in future fusion reactors,” *The European Physical Journal D*, vol. 76, no. 8, p. 141, 2022.
- [23] X. Liu, X.-Y. Lu, Q.-L. Tian, Z.-L. Pan, X.-J. Deng, L.-X. Yan, R.-K. Li, W.-H. Huang, C.-X. Tang, R. Chiche, *et al.*, “Prototype optical enhancement cavity for steady-state microbunching,” *Review of Scientific Instruments*, vol. 95, no. 10, 2024.
- [24] E. A., “Integration neherungsweise feldgleichungen der der gravitation,” *Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.)*, vol. 22, pp. 688–696, 1916.
- [25] P. H., “Sur la dynamique de l’électron,” in *Proc. acad. sci*, vol. 140, pp. 1504–1508, 1905.
- [26] J. L. Cervantes-Cota, S. Galindo-Uribarri, and G. F. Smoot, “A brief history of gravitational waves,” *Universe*, vol. 2, no. 3, p. 22, 2016.
- [27] A. Einstein and N. Rosen, “On gravitational waves,” *J. Frank. Inst.*, vol. 223, pp. 43–54, 1937.
- [28] F. A. Pirani, “On the physical significance of the Riemann tensor,” *Acta Physica Polonica*, vol. 15, pp. 389–405, 1956.

- [29] J. Weber, “Detection and generation of gravitational waves,” *Physical Review*, vol. 117, no. 1, p. 306, 1960.
- [30] J. Weber, “Observation of the thermal fluctuations of a gravitational-wave detector,” *Physical Review Letters*, vol. 17, no. 24, p. 1228, 1966.
- [31] J. Weber, “Evidence for discovery of gravitational radiation,” *Physical Review Letters*, vol. 22, no. 24, p. 1320, 1969.
- [32] D. Sciama, G. Field, and M. Rees, “Upper limit to radiation of mass energy derived from expansion of Galaxy,” *Physical Review Letters*, vol. 23, no. 26, p. 1514, 1969.
- [33] J. H. Taylor, L. A. Fowler, and P. M. McCulloch, “Measurements of general relativistic effects in the binary pulsar PSR1913+ 16,” *Nature*, vol. 277, no. 5696, pp. 437–440, 1979.
- [34] M. Gerstenshtein and V. Pustovoi, “On the detection of low frequency gravitational waves,” *Soviet Physics-JETP*, vol. 16, no. 2, pp. 433–435, 1963.
- [35] R. L. Forward, “Wideband laser-interferometer gravitational-radiation experiment,” *Physical Review D*, vol. 17, no. 2, p. 379, 1978.
- [36] J. Lough, E. Schreiber, F. Bergamin, H. Grote, M. Mehmet, H. Vahlbruch, C. Affeldt, M. Brinkmann, A. Bisht, V. Kringel, *et al.*, “First demonstration of 6 db quantum noise reduction in a kilometer scale gravitational wave observatory,” *Physical Review Letters*, vol. 126, no. 4, p. 041102, 2021.
- [37] J. Heinze, A. Gill, A. Dmitriev, J. Smetana, T. Yan, V. Boyer, D. Martynov, H. Grote, J. Lough, A. Ejlli, *et al.*, “DarkGEO: a large-scale laser-interferometric axion detector,” *New Journal of Physics*, vol. 26, no. 5, p. 055002, 2024.
- [38] T. Accadia, F. Acernese, M. Alshourbagy, P. Amico, F. Antonucci, S. Aoudia, N. Arnaud, C. Arnault, K. Arun, P. Astone, *et al.*, “Virgo: a laser interferometer to detect gravitational waves,” *Journal of instrumentation*, vol. 7, no. 03, p. P03012, 2012.
- [39] B. P. Abbott, R. Abbott, T. D. Abbott, M. R. Abernathy, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, *et al.*,

“Observation of gravitational waves from a binary black hole merger,” *Physical review letters*, vol. 116, no. 6, p. 061102, 2016.

- [40] L. D. Landau, E. Lifshitz, and C. Holbrow, “The classical theory of fields,” 1963.
- [41] V. B. Braginskii, “Gravitational radiation and the prospect of its experimental discovery,” *Uspekhi Fizicheskikh Nauk*, vol. 86, pp. 433–446, 1965.
- [42] K. S. Thorne, C. W. Misner, and J. A. Wheeler, *Gravitation*. Freeman San Francisco, 2000.
- [43] M. Maggiore, *Gravitational waves: Volume 1: Theory and experiments*, vol. 1. Oxford university press, 2008.
- [44] “The lsc-virgo white paper on gravitational wave data analysis and astrophysics (summer 2019 edition).”
- [45] D. Williams, “How many gravitational waves have we seen? (summer 2025 update).” <https://daniel-williams.co.uk/2025/08/26/how-many-part-2.html>. Accessed: 2025-09-01.
- [46] S. Bernuzzi, “Neutron star merger remnants,” *General Relativity and Gravitation*, vol. 52, p. 108, Nov. 2020.
- [47] R. Abbott, T. Abbott, S. Abraham, F. Acernese, K. Ackley, A. Adams, C. Adams, R. X. Adhikari, V. B. Adya, C. Affeldt, *et al.*, “Gravitational-wave constraints on the equatorial ellipticity of millisecond pulsars,” *The Astrophysical journal letters*, vol. 902, no. 1, p. L21, 2020.
- [48] G. Ushomirsky, C. Cutler, and L. Bildsten, “Deformations of accreting neutron star crusts and gravitational wave emission,” *Monthly Notices of the Royal Astronomical Society*, vol. 319, no. 3, pp. 902–932, 2000.
- [49] R. Abbott, H. Abe, F. Acernese, K. Ackley, N. Adhikari, R. Adhikari, V. Adkins, V. Adya, C. Affeldt, D. Agarwal, *et al.*, “All-sky search for gravitational wave emission from scalar boson clouds around spinning black holes in ligo o3 data,” *Physical Review D*, vol. 105, no. 10, p. 102001, 2022.

- [50] R. Caldwell, Y. Cui, H. Guo, V. Mandic, A. Mariotti, J. M. No, M. J. Ramsey-Musolf, M. Sakellariadou, K. Sinha, L. Wang, *et al.*, “Detection of early-universe gravitational-wave signatures and fundamental physics,” *General relativity and gravitation*, vol. 54, no. 12, p. 156, 2022.
- [51] C. Caprini and D. G. Figueroa, “Cosmological backgrounds of gravitational waves,” *Classical and Quantum Gravity*, vol. 35, no. 16, p. 163001, 2018.
- [52] B. P. Abbott, R. Abbott, T. D. Abbott, F. Acernese, and *et al.*, “A gravitational-wave standard siren measurement of the Hubble constant,” *Nature*, vol. 551, p. 85–88, Oct. 2017.
- [53] N. Aghanim, Y. Akrami, F. Arroja, M. Ashdown, J. Aumont, C. Baccigalupi, M. Ballardini, A. J. Banday, R. Barreiro, N. Bartolo, *et al.*, “Planck 2018 results-i. overview and the cosmological legacy of planck,” *Astronomy & Astrophysics*, vol. 641, p. A1, 2020.
- [54] A. Adame, J. Aguilar, S. Ahlen, S. Alam, D. Alexander, M. Alvarez, O. Alves, A. Anand, U. Andrade, E. Armengaud, *et al.*, “Desi 2024 v: Full-shape galaxy clustering from galaxies and quasars,” *Journal of Cosmology and Astroparticle Physics*, vol. 2025, no. 09, p. 008, 2025.
- [55] W. L. Freedman, B. F. Madore, T. J. Hoyt, I. S. Jang, A. J. Lee, and K. A. Owens, “Status report on the chicago-carnegie hubble program (cchp): Measurement of the hubble constant using the hubble and james webb space telescopes,” *The Astrophysical Journal*, vol. 985, no. 2, p. 203, 2025.
- [56] I. Antares, A. Pierre, C. Virgo, A. Albert, M. Andre, M. Anghinolfi, M. Ardid, J. Aubert, J. Aublin, T. Avgitas, *et al.*, “Search for high-energy neutrinos from binary neutron star merger GW170817 with ANTARES, IceCube, and the Pierre Auger Observatory,” *THE ASTROPHYSICAL JOURNAL*, vol. 850, no. L35, pp. 1–18, 2017.
- [57] F. S. Greus and A. L. Sánchez, “Multimessenger astronomy with neutrinos,” *Universe*, vol. 7, p. 397, Oct. 2021.
- [58] C. Fabry, “Theorie et applications d’une nouvelle methods de spectroscopie intereferentielle,” *Ann. Chim. Ser. 7*, vol. 16, pp. 115–144, 1899.

- [59] N. Ismail, C. C. Kores, D. Geskus, and M. Pollnau, “Fabry-Pérot resonator: spectral line shapes, generic and related Airy distributions, linewidths, finesses, and performance at low or frequency-dependent reflectivity,” *Optics express*, vol. 24, no. 15, pp. 16366–16389, 2016.
- [60] H. Kogelnik and T. Li, “Laser beams and resonators,” *Applied Optics*, vol. 5, pp. 1550–1567, Oct 1966.
- [61] L. G. Gouy, *Sur une propriété nouvelle des ondes lumineuses*. Gauthier-Villars, 1890.
- [62] M. V. Berry, “Quantal phase factors accompanying adiabatic changes,” *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences*, vol. 392, no. 1802, pp. 45–57, 1984.
- [63] S. Feng and H. G. Winful, “Physical origin of the Gouy phase shift,” *Optics letters*, vol. 26, no. 8, pp. 485–487, 2001.
- [64] J. Degallaix, “A tale of two cavities,” tech. rep., Universite Claude Bernard-Lyon I, Institut de Physique des 2 Infinis de Lyon . . . , 2021.
- [65] R. Maggiore, *Adventures in gravitational wave detector control: A round-trip journey*. PhD thesis, Vrije Universiteit Amsterdam, 2024.
- [66] E. Tapia San Martín *et al.*, *Capturing Gravitational Waves in a World of Noise*. PhD thesis, Vrije Universiteit Amsterdam, 2024.
- [67] J. Rollins and K. Kuns, “Gwinc python gravitational wave interferometer noise calculator.” <https://pypi.org/project/gwinc/>, 2024.
- [68] M. G. Beker, *Low-frequency sensitivity of next generation gravitational wave detectors*. PhD thesis, Vrije Universiteit Amsterdam, 2013.
- [69] B. Boom, *Acceleration sensing at the nano-g level: Development and characterisation of low-noise microseismometers for next generation gravitational wave detectors*. PhD thesis, Vrije Universiteit Amsterdam, 2020.
- [70] J. van Heijningen, *Turn up the bass!: Low-frequency performance improvement of seismic attenuation systems and vibration sensors for next generation gravitational wave detectors*. PhD thesis, Vrije Universiteit Amsterdam, 2018.

- [71] R. Kubo, “The fluctuation-dissipation theorem,” *Reports on progress in physics*, vol. 29, no. 1, p. 255, 1966.
- [72] P. R. Saulson, “Thermal noise in mechanical experiments,” *Physical Review D*, vol. 42, no. 8, p. 2437, 1990.
- [73] A. Heptonstall, M. Barton, A. Bell, A. Bohn, G. Cagnoli, A. Cumming, A. Grant, E. Gustafson, G. Hammond, J. Hough, *et al.*, “Enhanced characteristics of fused silica fibers using laser polishing,” *Classical and Quantum Gravity*, vol. 31, no. 10, p. 105006, 2014.
- [74] J. Steinlechner, “Development of mirror coatings for gravitational-wave detectors,” *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 376, no. 2120, p. 20170282, 2018.
- [75] G. M. Harry, A. M. Gretarsson, P. R. Saulson, S. E. Kittelberger, S. D. Penn, W. J. Startin, S. Rowan, M. M. Fejer, D. Crooks, G. Cagnoli, *et al.*, “Thermal noise in interferometric gravitational wave detectors due to dielectric optical coatings,” *Classical and Quantum Gravity*, vol. 19, no. 5, p. 897, 2002.
- [76] F. Badaracco, J. Harms, A. Bertolini, T. Bulik, I. Fiori, B. Idzkowski, A. Kutynia, K. Nikliborc, F. Paoletti, A. Paoli, *et al.*, “Machine learning for gravitational-wave detection: surrogate Wiener filtering for the prediction and optimized cancellation of Newtonian noise at Virgo,” *Classical and Quantum Gravity*, vol. 37, no. 19, p. 195016, 2020.
- [77] J. Harms and H. J. Paik, “Newtonian-noise cancellation in full-tensor gravitational-wave detectors,” *Physical Review D*, vol. 92, no. 2, p. 022001, 2015.
- [78] K. Goda, *Development of techniques for quantum-enhanced laser-interferometric gravitational-wave detectors*. PhD thesis, Massachusetts Institute of Technology, 2007.
- [79] G. Grynberg, A. Aspect, and C. Fabre, *Introduction to quantum optics: from the semi-classical approach to quantized light*. Cambridge university press, 2010.
- [80] A. Buonanno and Y. Chen, “Laser-interferometer gravitational-wave optical-spring detectors,” *Classical and Quantum Gravity*, vol. 19, no. 7, p. 1569, 2002.

- [81] H. Miao, *Exploring macroscopic quantum mechanics in optomechanical devices*. Springer Science & Business Media, 2012.
- [82] C. M. Caves, “Quantum-mechanical noise in an interferometer,” *Physical Review D*, vol. 23, no. 8, p. 1693, 1981.
- [83] J. Heinze, B. Willke, and H. Vahlbruch, “Observation of squeezed states of light in higher-order Hermite-Gaussian modes with a quantum noise reduction of up to 10 dB,” *Physical Review Letters*, vol. 128, no. 8, p. 083606, 2022.
- [84] H. P. Yuen, “Two-photon coherent states of the radiation field,” *Physical Review A*, vol. 13, no. 6, p. 2226, 1976.
- [85] C. M. Caves and B. L. Schumaker, “New formalism for two-photon quantum optics. i. quadrature phases and squeezed states,” *Physical Review A*, vol. 31, no. 5, p. 3068, 1985.
- [86] F. Acernese, M. Agathos, L. Aiello, A. Allocca, A. Amato, S. Ansoldi, S. Antier, M. Arène, N. Arnaud, S. Ascenzi, *et al.*, “Increasing the astrophysical reach of the advanced virgo detector via the application of squeezed vacuum states of light,” *Physical review letters*, vol. 123, no. 23, p. 231108, 2019.
- [87] F. Acernes, M. Agathos, L. Aiello, A. Ain, A. Allocca, A. Amato, S. Ansoldi, S. Antier, M. Arène, N. Arnaud, *et al.*, “Quantum back-action on Kg-scale mirrors: observation of radiation pressure noise in the advanced Virgo detector,” *Physical Review Letters*, vol. 125, no. 13, p. 131101, 2020.
- [88] D. Ganapathy, W. Jia, M. Nakano, V. Xu, N. Aritomi, T. Cullen, N. Kijbunchoo, S. Dwyer, A. Mullavey, L. McCuller, *et al.*, “Broadband quantum enhancement of the ligo detectors with frequency-dependent squeezing,” *Physical Review X*, vol. 13, no. 4, p. 041021, 2023.
- [89] P. Kwee, J. Miller, T. Isogai, L. Barsotti, and M. Evans, “Decoherence and degradation of squeezed states in quantum filter cavities,” *Physical Review D*, vol. 90, no. 6, p. 062006, 2014.
- [90] J. A. Sidles and D. Sigg, “Optical torques in suspended Fabry–Perot interferometers,” *Physics Letters A*, vol. 354, no. 3, pp. 167–172, 2006.

- [91] J. Liu, V. Bossilkov, C. Blair, C. Zhao, L. Ju, and D. Blair, “Angular instability in high optical power suspended cavities,” *Review of Scientific Instruments*, vol. 89, no. 12, 2018.
- [92] J. Heinze, A. Gill, A. Dmitriev, J. Smetana, T. Yan, V. Boyer, D. Martynov, and M. Evans, “First results of the laser-interferometric detector for axions (LIDA),” *Physical Review Letters*, vol. 132, no. 19, p. 191002, 2024.
- [93] Brillouin, L., “Diffusion de la lumière et des rayons X par un corps transparent homogène,” in *Annales de physique*, vol. 9, pp. 88–122, 1922.
- [94] J. Degallaix, C. Zhao, L. Ju, and D. Blair, “Thermal tuning of optical cavities for parametric instability control,” *Journal of the Optical Society of America B*, vol. 24, no. 6, pp. 1336–1343, 2007.
- [95] S. Gras, P. Fritschel, L. Barsotti, and M. Evans, “Resonant dampers for parametric instabilities in gravitational wave detectors,” *Physical Review D*, vol. 92, no. 8, p. 082001, 2015.
- [96] C. Blair, S. Gras, R. Abbott, S. Aston, J. Betzwieser, D. Blair, R. DeRosa, M. Evans, V. Frolov, P. Fritschel, *et al.*, “First demonstration of electrostatic damping of parametric instability at advanced ligo,” *Physical review letters*, vol. 118, no. 15, p. 151102, 2017.
- [97] V. Bossilkov, J. Liu, C. Blair, C. Zhao, and L. Ju, “Demonstration of the parametric instability suppression through optical feedback,” *Physical Review D*, vol. 109, no. 10, p. 102006, 2024.
- [98] J. Zhang, C. Zhao, L. Ju, and D. Blair, “Designing arm cavities free of parametric instability for gravitational wave detectors,” *Classical and Quantum Gravity*, vol. 37, no. 7, p. 075015, 2020.
- [99] C. Zhao, L. Ju, Q. Fang, C. Blair, J. Qin, D. Blair, J. Degallaix, and H. Yamamoto, “Parametric instability in long optical cavities and suppression by dynamic transverse mode frequency modulation,” *Physical Review D*, vol. 91, no. 9, p. 092001, 2015.
- [100] M. Evans, S. Gras, P. Fritschel, J. Miller, L. Barsotti, D. Martynov, A. Brooks, D. Coyne, R. Abbott, R. X. Adhikari, *et al.*, “Observation

of parametric instability in Advanced LIGO,” *Physical review letters*, vol. 114, no. 16, p. 161102, 2015.

- [101] M. Schiworski, V. Bossilkov, C. Blair, D. Brown, A. Jones, D. Ottaway, and C. Zhao, “Observing the optical modes of parametric instability,” *Optics Letters*, vol. 47, no. 7, pp. 1685–1688, 2022.
- [102] D. Reitze, R. X. Adhikari, S. Ballmer, B. Barish, L. Barsotti, G. Billingsley, D. A. Brown, Y. Chen, D. Coyne, R. Eisenstein, *et al.*, “Cosmic explorer: the US contribution to gravitational-wave astronomy beyond LIGO,” *arXiv preprint arXiv:1907.04833*, 2019.
- [103] K. Ackley, V. Adya, P. Agrawal, P. Altin, G. Ashton, M. Bailes, E. Baltinas, A. Barbuio, D. Beniwal, C. Blair, *et al.*, “Neutron Star Extreme Matter Observatory: A kilohertz-band gravitational-wave detector in the global network,” *Publications of the Astronomical Society of Australia*, vol. 37, p. e047, 2020.
- [104] R. X. Adhikari, K. Arai, A. Brooks, C. Wipf, O. Aguiar, P. Altin, B. Barr, L. Barsotti, R. Bassiri, A. Bell, *et al.*, “A cryogenic silicon interferometer for gravitational-wave detection,” *Classical and Quantum Gravity*, vol. 37, no. 16, p. 165003, 2020.
- [105] T. V. Collaboration, “Advanced Virgo Technical Design Report,” 2012. VIR–0128A–12 - internal Virgo document.
- [106] G. Breitenbach, S. Schiller, and J. Mlynek, “Measurement of the quantum states of squeezed light,” *Nature*, vol. 387, no. 6632, pp. 471–475, 1997.
- [107] J.-Q. Chen, C. Chen, J.-J. Sun, J.-W. Zhang, Z.-H. L., L. Qin, Y.-Q. Ning, and L.-J. Wang, “Linewidth measurement of a narrow-linewidth laser: principles, methods, and systems,” *Sensors*, vol. 24, no. 11, p. 3656, 2024.
- [108] H. P. Yuen and V. W. Chan, “Noise in homodyne and heterodyne detection,” *Optics letters*, vol. 8, no. 3, pp. 177–179, 1983.
- [109] S. Personick, “BSTJ brief: an image band interpretation of optical heterodyne noise,” *The Bell System Technical Journal*, vol. 50, no. 1, pp. 213–216, 1971.

- [110] S. Hild, H. Grote, J. Degallaix, S. Chelkowski, K. Danzmann, A. Freise, M. Hewitson, J. Hough, H. Lück, M. Prijatelj, *et al.*, “DC-readout of a signal-recycled gravitational wave detector,” *Classical and Quantum Gravity*, vol. 26, no. 5, p. 055012, 2009.
- [111] E. D. Black, “An introduction to Pound–Drever–Hall laser frequency stabilization,” *American journal of physics*, vol. 69, no. 1, pp. 79–87, 2001.
- [112] G. F. Franklin, J. D. Powell, A. Emami-Naeini, *et al.*, *Feedback control of dynamic systems*, vol. 10. Pearson Upper Saddle River, NJ, 2010.
- [113] the Virgo collaboration, “Optical characterization of the advanced virgo gravitational wave detector for the o4 observation run,” *Applied Optics*, vol. 64, no. 17, pp. 4710–4726, 2025.
- [114] D. Gatti, A. Gotti, R. and Gambetta, M. Belmonte, G. Galzerano, P. Laporta, and M. Marangoni, “Comb-locked lamb-dip spectrometer,” *Scientific reports*, vol. 6, no. 1, p. 27183, 2016.
- [115] Z. Wang, Z. Ma, W. Wei, J. Chang, J. Zhang, Q. Wu, W. Yuan, K. Deng, Z. Lu, and J. Zhang, “Noise characterization of an ultra-stable laser for optical clocks,” *Review of Scientific Instruments*, vol. 95, no. 5, 2024.
- [116] J. M. Robinson, E. Oelker, W. R. Milner, W. Zhang, T. Legero, D. G. Matei, F. Riehle, U. Sterr, and J. Ye, “Crystalline optical cavity at 4 K with thermal-noise-limited instability and ultralow drift,” *Optica*, vol. 6, no. 2, pp. 240–243, 2019.
- [117] B. Yan, S. A. Moses, B. Gadway, J. P. Covey, K. Hazzard, A. M. Rey, D. S. Jin, and J. Ye, “Observation of dipolar spin-exchange interactions with lattice-confined polar molecules,” *Nature*, vol. 501, no. 7468, pp. 521–525, 2013.
- [118] H. Levine, A. Keesling, A. Omran, H. Bernien, S. Schwartz, A. S. Zibrov, M. Endres, M. Greiner, V. Vuletić, and M. D. Lukin, “High-fidelity control and entanglement of Rydberg-atom qubits,” *Physical review letters*, vol. 121, no. 12, p. 123603, 2018.
- [119] A. Hallal, S. Bouhier, and F. Bondu, “Synthesis of a 30-Hz linewidth wave tunable over 500 GHz,” *IEEE Transactions on Microwave Theory and Techniques*, vol. 65, no. 4, pp. 1367–1371, 2016.

- [120] Y.-X. Chao, Z.-X. Hua, X.-H. Liang, Z.-P. Yue, L. You, and M. Khoon Tey, “Pound–Drever–Hall feedforward: laser phase noise suppression beyond feedback,” *Optica*, vol. 11, no. 7, pp. 945–950, 2024.
- [121] K. Kokeyama, K. Izumi, W. Z. Korth, N. Smith-Lefebvre, K. Arai, and R. X. Adhikari, “Residual amplitude modulation in interferometric gravitational wave detectors,” *Journal of the Optical Society of America A*, vol. 31, no. 1, pp. 81–88, 2013.
- [122] F. M. J. Cozijn, *High-Precision Rovibrational Saturation Spectroscopy of H₂, HD and HT*. PhD thesis, Vrije Universiteit Amsterdam, 2024.
- [123] A. L. Parke and M. Schioppo, “Three hundred microsecond optical cavity storage time and 10^{-7} active RAM cancellation for 10^{-19} laser frequency stabilization,” *Optics Letters*, vol. 50, no. 10, pp. 3405–3408, 2025.
- [124] D. Kedar, Z. Yao, I. Ryger, J. L. Hall, and J. Ye, “Synthetic FM triplet for AM-free precision laser stabilization and spectroscopy,” *Optica*, vol. 11, pp. 58–63, Jan 2024.
- [125] D. Shaddock, M. Gray, and D. McClelland, “Frequency locking a laser to an optical cavity by use of spatial mode interference,” *Optics letters*, vol. 24, no. 21, pp. 1499–1501, 1999.
- [126] N. Chabbra, A. R. Wade, E. Rose Rees, A. J. Sutton, A. Stochino, R. L. Ward, D. A. Shaddock, and K. McKenzie, “High stability laser locking to an optical cavity using tilt locking,” *Optics Letters*, vol. 46, no. 13, pp. 3199–3202, 2021.
- [127] F. Diorico, A. Zhutov, and O. Hosten, “Laser-cavity locking utilizing beam ellipticity: accessing the 10^{-7} instability scale relative to cavity linewidth,” *Optica*, vol. 11, no. 1, pp. 26–31, 2024.
- [128] D. Matei, T. Legero, C. Grebing, S. Häfner, C. Lisdat, R. Weyrich, W. Zhang, L. Sonderhouse, J. M. Robinson, F. Riehle, *et al.*, “A second generation of low thermal noise cryogenic silicon resonators,” in *Journal of Physics: Conference Series*, vol. 723, p. 012031, IOP Publishing, 2016.

- [129] J. Morville, S. Kassi, M. Chenevier, and D. Romanini, “Fast, low-noise, mode-by-mode, cavity-enhanced absorption spectroscopy by diode-laser self-locking,” *Applied Physics B*, vol. 80, no. 8, pp. 1027–1038, 2005.
- [130] K. M. Manfred, L. Ciaffoni, and G. A. D. Ritchie, “Optical-feedback cavity-enhanced absorption spectroscopy in a linear cavity: model and experiments,” *Applied Physics B*, vol. 120, no. 2, pp. 329–339, 2015.
- [131] S. C. Doret, “Simple, low-noise piezo driver with feed-forward for broad tuning of external cavity diode lasers,” *Review of Scientific Instruments*, vol. 89, no. 2, 2018.
- [132] P. Laurent, A. Clairon, and C. Breant, “Frequency noise analysis of optically self-locked diode lasers,” *IEEE Journal of Quantum Electronics*, vol. 25, no. 6, pp. 1131–1142, 1989.
- [133] P. Buch and P. Kohns, “Optically self-locked semiconductor laser with servo control for feedback phase and laser current,” *IEEE journal of quantum electronics*, vol. 27, no. 7, pp. 1863–1868, 1991.
- [134] E. Morrison, B. J. Meers, D. I. Robertson, and H. Ward, “Automatic alignment of optical interferometers,” *Applied Optics*, vol. 33, no. 22, pp. 5041–5049, 1994.
- [135] A. A. Ciobanu, D. D. Brown, P. J. Veitch, and D. J. Ottaway, “Mode matching error signals using radio-frequency beam shape modulation,” *Applied Optics*, vol. 59, no. 31, pp. 9884–9895, 2020.
- [136] H. Vahlbruch, M. Mehmet, K. Danzmann, and R. Schnabel, “Detection of 15 dB squeezed states of light and their application for the absolute calibration of photoelectric quantum efficiency,” *Physical review letters*, vol. 117, no. 11, p. 110801, 2016.
- [137] M. Korobko, “Quantum technologies for the einstein telescope,” *Galaxies*, vol. 13, no. 1, p. 11, 2025.
- [138] L. McCuller, S. Dwyer, A. C. Green, H. Yu, K. Kuns, L. Barsotti, C. Blair, D. Brown, A. Effler, M. Evans, *et al.*, “Ligo’s quantum response to squeezed states,” *Physical Review D*, vol. 104, no. 6, p. 062006, 2021.

- [139] S. Steinlechner, N.-O. Rohweder, M. Korobko, D. Töyrä, A. Freise, and R. Schnabel, “Mitigating mode-matching loss in nonclassical laser interferometry,” *Physical review letters*, vol. 121, no. 26, p. 263602, 2018.
- [140] P. Hello and J.-Y. Vinet, “Analytical models of thermal aberrations in massive mirrors heated by high power laser beams,” *Journal de Physique*, vol. 51, no. 12, pp. 1267–1282, 1990.
- [141] P. Hello and J.-Y. Vinet, “Analytical models of transient thermoelastic deformations of mirrors heated by high power cw laser beams,” *Journal de Physique*, vol. 51, no. 20, pp. 2243–2261, 1990.
- [142] L. van der Schaaf, *The phase cameras of Advanced Virgo*. PhD thesis, Vrije Universiteit Amsterdam, 2020.
- [143] A. Alloca, A. Chiummo, P. Ruggi, and H. Yamamoto, “Transient power drop in dark fringe lock acquisition during the commissioning before o3,” 2019. VIR-1047A-19 - internal Virgo document.
- [144] A. F. Brooks, G. Vajente, H. Yamamoto, R. Abbott, C. Adams, R. X. Adhikari, A. Ananyeva, S. Appert, K. Arai, J. S. Areeda, *et al.*, “Point absorbers in Advanced LIGO,” *Applied Optics*, vol. 60, no. 13, pp. 4047–4063, 2021.
- [145] S. R. Goode, M. Schiowski, D. Brown, E. Thrane, and P. D. Lasky, “You only thermoelastically deform once: point absorber detection in LIGO test masses with YOLO,” *Optics Express*, vol. 33, no. 8, pp. 17601–17616, 2025.
- [146] A. F. Brooks, T.-L. Kelly, P. J. Veitch, and J. Munch, “Ultra-sensitive wavefront measurement using a Hartmann sensor,” *Opt. Express*, vol. 15, pp. 10370–10375, Aug 2007.
- [147] A. Chernyshov, U. Sterr, F. Riehle, J. Helmcke, and J. Pfund, “Calibration of a Shack–Hartmann sensor for absolute measurements of wavefronts,” *Appl. Opt.*, vol. 44, pp. 6419–6425, Oct 2005.
- [148] Mueller, Q.-Z. Shu, R. Adhikari, D. B. Tanner, D. Reitze, D. Sigg, N. Mavalvala, and J. Camp, “Determination and optimization of mode matching into optical cavities by heterodyne detection,” *Opt. Lett.*, vol. 25, pp. 266–268, Feb 2000.

- [149] M. W. Beijersbergen, L. Allen, H. Van der Veen, and J. Woerdman, "Astigmatic laser mode converters and transfer of orbital angular momentum," *Optics Communications*, vol. 96, no. 1-3, pp. 123–132, 1993.
- [150] R. Uren, S. Beecher, C. R. Smith, and W. A. Clarkson, "Method for Generating High Purity Laguerre-Gaussian Vortex Modes," *IEEE JOURNAL OF QUANTUM ELECTRONICS*, vol. 55, July 2019.
- [151] F. Magana-Sandoval, T. V., D. Vander-Hyde, J. R. Sanders, and S. W. Ballmer, "Sensing optical cavity mismatch with a mode-converter and quadrant photodiode," *Physical Review D*, vol. 100, p. 102001, November 2019.
- [152] A. Grimaldi, "Mode matching sensing frequency dependent squeezing source for Advanced Virgo Plus," *PhD thesis, Universita di Trento*, 2022.
- [153] P. Hello and C. N. Man, "Design of a low-loss off-axis beam expander," *Applied Optics*, vol. 35, no. 15, pp. 2534–2536, 1996.
- [154] N. Uehara, E. K. Gustafson, M. M. Fejer, and R. L. Byer, "Modeling of efficient mode-matching and thermal-lensing effect on a laser-beam coupling into a mode-cleaner cavity," in *Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series* (U. O. Farrukh and S. Basu, eds.), vol. 2989 of *Presented at the Society of Photo-Optical Instrumentation Engineers (SPIE) Conference*, pp. 57–68, May 1997.
- [155] P. Kwee, F. Seifert, B. Willke, and K. Danzmann, "Laser beam quality and pointing measurement with an optical resonator," *Review of Scientific Instruments*, vol. 78, p. 073103, 2007.
- [156] C. Bogan, P. Kwee, S. Hild, S. H. Huttner, and B. Willke, "Novel technique for thermal lens measurement in commonly used optical components," *Opt. Express*, vol. 23, pp. 15380–15389, Jun 2015.
- [157] M. L. Mah and J. J. Talghader, "Decomposition of aberrated or turbulent wavefronts into a spatial mode spectrum using optical cavities," *Applied Optics*, 2019.
- [158] A. W. Jones, M. Wang, C. M. Mow-Lowry, and A. Freise, "High dynamic range spatial mode decomposition," *Optics Express*, vol. 28, pp. 10253–10269, Mar. 2020.

- [159] B. R. Boruah, “Dynamic manipulation of a laser beam using a liquid crystal spatial light modulator,” *American Journal of Physics*, vol. 77, pp. 331–336, 04 2009.
- [160] T. Kaiser, D. Flamm, S. Schröter, and M. Duparré, “Complete modal decomposition for optical fibers using CGH-based correlation filters,” *Optics Express*, vol. 17, pp. 9347–9356, May 2009.
- [161] A. W. Jones, M. Wang, X. Zhang, S. J. Cooper, S. Chen, C. M. Mow-Lowry, and A. Freise, “Metasurface-enhanced spatial mode decomposition,” *Phys. Rev. A*, vol. 105, p. 053523, May 2022.
- [162] J. Betzwieser, *Analysis of spatial mode sensitivity of a gravitational wave interferometer and a targeted search for gravitational radiation from the Crab pulsar*. PhD thesis, Massachusetts Institute of Technology, 2007.
- [163] K. Goda, D. Ottaway, B. Connelly, R. Adhikari, N. Mavalvala, and A. Gretarsson, “Frequency-resolving spatiotemporal wave-front sensor,” *Optics Letters*, vol. 29, pp. 1452–1454, 2004.
- [164] H. T. Cao, D. D. Brown, P. J. Veitch, and D. J. Ottaway, “Optical lock-in camera for gravitational wave detectors,” *Opt. Express*, vol. 28, pp. 14405–14413, May 2020.
- [165] D. D. Brown, H. T. Cao, P. V., and D. Ottaway, “Optical lock-in camera: LHO O3 results and imaging error signals,” Tech. Rep. LIGO-G2001604, LIGO Scientific Collaboration, Sept. 2020. Talk presented at LVK Conference Sep 2020.
- [166] E. Muñoz, V. Srivastava, S. Vidyant, and S. W. Ballmer, “High frame-rate phase camera for high-resolution wavefront sensing in gravitational-wave detectors,” *Phys. Rev. D*, vol. 104, p. 042002, Aug 2021.
- [167] D. T. Ralph, P. A. Altin, D. S. Rabeling, D. E. McClelland, and D. A. Shaddock, “Interferometric wavefront sensing with a single diode using spatial light modulation,” *Appl. Opt.*, vol. 56, pp. 2353–2358, Mar 2017.
- [168] L. van der Schaaf, K. Agatsuma, M. van Beuzekom, M. Gebyehu, and J. van den Brand, “Advanced virgo phase cameras,” *Journal of Physics: Conference Series*, vol. 718, no. 7, p. 072008, 2016.

- [169] S. Tanioka, T. Pearce, and K. Kokeyama, “Development of a polarimetry method toward in situ substrate birefringence characterization of ground-based gravitational wave detectors,” *Review of Scientific Instruments*, vol. 96, p. 074503, 07 2025.
- [170] M. G. Schiowski, D. D. Brown, and D. J. Ottaway, “Modal decomposition of complex optical fields using convolutional neural networks,” *J. Opt. Soc. Am. A*, vol. 38, pp. 1603–1611, Nov 2021.
- [171] M. Diaz-Ortiz and P. Fulda, “An electro-optic lens concept for mode mismatch sensing,” Conference Presentation G2001575, LIGO Scientific Collaboration, Sept. 2020. Presented at the LIGO-Virgo-KAGRA Biannual Conference.
- [172] M. Bazzan, M. Carlassara, G. Ciani, L. Conti, A. Grimaldi, M. Valentini, and J. Zendri, “Test of LG10 RF high order mode sensing technique for laser-cavity mode-matching,” Conference Presentation LIGO-G2001619, Virgo Scientific Collaboration, Sept. 2020. Presented at the LIGO-Virgo-KAGRA Biannual Conference.
- [173] G. Chiarini and G. Ciani, “Mode matching sensing through RF Higher Order Modulation method,” in *Gravitational-Wave Advanced Detector Workshop*, 2023. Available at https://agenda.infn.it/event/32907/contributions/200854/attachments/105798/148800/gwadv_poster.pdf.
- [174] A. W. Goodwin-Jones, H. Zhu, C. Blair, D. D. Brown, J. van Heijningen, L. Ju, and C. Zhao, “Single and coupled cavity mode sensing schemes using a diagnostic field,” *Optics Express*, vol. 31, no. 21, pp. 35068–35085, 2023.
- [175] H. T. Cao, S. W. S. Ng, M. Noh, A. Brooks, F. Matichard, and P. J. Veitch, “Enhancing the dynamic range of deformable mirrors with compression bias,” *Optics Express*, vol. 28, no. 26, pp. 38480–38490, 2020.
- [176] M. Cifaldi, E. Cesarini, V. Fafone, M. Lorenzini, D. Lumaca, Y. Minenkov, I. Nardecchia, A. Rocchi, C. Taranto, and T. Zelenova, “Mitigation of point absorbers in the advanced virgo plus core optics: the actuator design,” conference presentation, VIRGO Scientific Collaboration, June 2022. Publically available at <https://agenda.infn.it/>

event/30681/contributions/170401/attachments/
92004/125181/PosterGRASS_v3.pdf.

- [177] M. Cifaldi, E. Cesarini, V. Fafone, M. Lorenzini, D. Lumaca, Y. Minenkov, I. Nardecchia, A. Rocchi, and C. Taranto, “Mitigation of Point Absorbers in the Advanced Virgo plus core optics: the actuator design,” GWADW23 conference presentation, VIRGO Scientific Collaboration, May 2023. Publically available at [https://agenda.infn.it/event/32907/contributions/201031/attachments/105644/148535/GWADW2023_Cifaldi%20\(1\).pdf](https://agenda.infn.it/event/32907/contributions/201031/attachments/105644/148535/GWADW2023_Cifaldi%20(1).pdf).
- [178] A. F. Brooks, B. Abbott, M. A. Arain, G. Ciani, A. Cole, G. Grabeel, E. Gustafson, C. Guido, M. Heintze, A. Heptonstall, *et al.*, “Overview of Advanced LIGO adaptive optics,” *Applied Optics*, vol. 55, no. 29, pp. 8256–8265, 2016.
- [179] I. Nardecchia, Y. Minenkov, M. Lorenzini, L. Aiello, E. Cesarini, D. Lumaca, V. Malvezzi, F. Paoletti, A. Rocchi, and V. Fafone, “Optimized radius of curvature tuning for the virgo core optics,” *Classical and Quantum Gravity*, vol. 40, p. 055004, feb 2023.
- [180] H. T. Cao, R. Anderson, T. Rosauer, A. Wilkin, A. Brooks, and J. Richardson, “Development Status of HOM Ring Heater,” Conference Presentation LIGO-G2201439, LIGO Scientific Collaboration, Sept. 2022. Presented at the LIGO-Virgo-KAGRA Biannual Conference.
- [181] T. Accadia, F. Acernese, M. Agathos, A. Allocca, P. Astone, G. Ballardin, F. Barone, and *et al.*, “Central heating radius of curvature correction (CHRoCC) for use in large scale gravitational wave interferometers,” *Classical and Quantum Gravity*, vol. 30, February 2013.
- [182] J. Degallaix, B. Slagmolen, C. Zhao, L. Ju, and D. Blair, “Thermal lensing compensation principle for the ACIGA’s High Optical Power Test Facility Test 1,” *General Relativity and Gravitation*, vol. 37, no. 9, pp. 1581–1589, 2005.
- [183] L. Aiello, E. Cesarini, V. Fafone, M. Lorenzini, Y. Minenkov, I. Nardecchia, A. Rocchi, and V. Sequino, “Thermal compensation system in advanced and third generation gravitational wave interferometric detectors,” *Journal of Physics: Conference Series*, vol. 1226, p. 012019, may 2019.

- [184] R. Lawrence, M. Zucker, P. Fritschel, P. Marfuta, and D. Shoemaker, “Adaptive thermal compensation of test masses in advanced LIGO,” *Classical and Quantum Gravity*, vol. 19, pp. 1803–1812, Apr. 2002.
- [185] M. L., “Realization and characterization of a scanning system for optical asymmetric aberrations in Advanced Virgo,” *Master Thesis, University of Rome Tor Vergata*, 2013.
- [186] C. Taranto, E. Cesarini, M. Cifaldi, V. Fafone, M. Lorenzini, D. Lumaca, Y. Minenkov, I. Nardecchia, and A. Rocchi, “Mitigation of non-axisymmetric optical aberrations in Advanced Virgo plus,” Conference Presentation VIR-0373A-23, VIRGO Scientific Collaboration, May 2023. Presented at the GWADW23 Conference.
- [187] T. Zhao and Y. Chi, “Modified Gerchberg–Saxton (GS) algorithm and its application,” *Entropy*, vol. 22, no. 12, p. 1354, 2020.
- [188] K. H and L. T, “Laser Beams and Resonators,” *Applied Optics*, vol. 10, p. 004, 1966.
- [189] K. Arai, “On the accumulated round-trip Gouy phase shift for a general optical cavity,” 2013. LIGO-T1300189–v1, internal LIGO document.
- [190] P. Spinicelli and T. V. collaboration, “Lessons learned from Advanced Virgo+ ISC commissioning,” 2023. VIR-0402A-23 - internal Virgo document.
- [191] A. Daumas, “B4 beam aberration study,” 2023. Virgo internal document, VIR-0361A-23.
- [192] R. Flaminio, “Two problems: Contrast defect and b4 offset,” 2022. Virgo internal document, VIR-0972A-22.
- [193] R. Cabrita, “57646,” 2022. Virgo logbook entry: <https://logbook.virgo-gw.eu/virgo/?r=57646>.
- [194] R. Cabrita, J. Degallaix, and J. van Heijningen, “Single bounce studies: fringe pattern and modulation depth,” 2023. Virgo internal document, VIR-0158A-23.
- [195] E. Capocasa, R. Flaminio, C. de Rossi, M. Barsuglia, M. Mantovani, M. Boldrini, P. Spinicelli, and J. Degallaix, “VIR-0717A-22,” 2022. internal Virgo document.

- [196] K. Izumi, *Multi-Color Interferometry for Lock Acquisition of Laser Interferometric Gravitational-wave Detectors*. PhD thesis, PhD thesis, University of Tokyo, Tokyo, Japan, 2012.
- [197] J. Degallaix, “OSCAR: A MATLAB based package to simulate realistic optical cavities,” *SoftwareX*, vol. 12, p. 100587, 2020.
- [198] J. D. R. Cabrita, “VIR-0820A-22,” 2022. internal Virgo document.
- [199] J. Degallaix, “VIR-1166A-22,” 2022. Virgo internal document.
- [200] M. Was, “56766,” 2022. Virgo logbook entry: <https://logbook.virgo-gw.eu/virgo/?r=56766>.
- [201] R. Cabrita, J. Dégallaix, and J. van Heijningen, “Phase camera/oscar update: Phase images in the single arm lock,” 2022. Virgo internal document, VIR-1123B-22.
- [202] R. Cabrita, “Phase camera modal decomposition - arm cavity,” 2023. Virgo internal document, VIR-0543A-23.
- [203] V. Hui and E. Tournefier, “CP and input beam parameters from the analysis of MM measurements and tunings,” 2022. Virgo internal document, VIR-1168A-22.
- [204] M. Powell, “Direct search algorithms for optimization calculations,” *Acta numerica*, vol. 7, pp. 287–336, 1998.
- [205] R. Maggiore, A. Freise, J. W. Perry, C. M. Mow-Lowry, P. Ruggi, M. Mantovani, J. Casanueva Diaz, D. Brown, E. N. Tapia San Martín, M. Tacca, *et al.*, “Angular control noise in Advanced Virgo and implications for the Einstein Telescope,” *Physical Review D*, vol. 111, no. 10, p. 102003, 2025.
- [206] D. Brown, “Finesse3,” 2023. LIGO-Virgo-KAGRA Scientific Collaboration.
- [207] J. Degallaix, “Astigmatism cross-check in long stable recycling cavities,” 2023. VIR-0960A-23, internal Virgo document.
- [208] V. Hui, R. Bonnand, J. Degallaix, R. Flaminio, and E. Tournefier, “Simulation of PR stable cavity losses due to astigmatism with OSCAR,” 2023. VIR-1132A-23, internal Virgo document.

- [209] P. Coutin, V. Hui, J. Degallaix, and E. Tournefier, “Contrast defect and simulation of effect of SR aperture,” 2023. VIR-0892A-23, internal Virgo document.
- [210] M. Was, “Update on 1/f 2/3 mystery noise current understanding and next steps,” 2024. Virgo internal document, VIR-1000A-24.
- [211] J. Degallaix, C. Michel, B. Sassolas, A. Allocca, G. Cagnoli, L. Balzarini, V. Dolique, R. Flaminio, D. Forest, M. Granata, *et al.*, “Large and extremely low loss: the unique challenges of gravitational wave mirrors,” *Journal of the Optical Society of America A*, vol. 36, no. 11, pp. C85–C94, 2019.
- [212] C. Jacquet and A. Goodwin-Jones and J. Dégallaix, “OSD meeting - Astigmatism measurement and simulation,” 2025. Virgo internal document, VIR-0706A-25.
- [213] I. Nardecchia, “CP cold lens measurement and DAS power evaluation method,” 2022. Virgo internal document, VIR-0556A-22.
- [214] I. Kazuyoshi, “Analysis of the phase unwrapping algorithm,” *Appl. Opt.*, vol. 21, pp. 2470–2470, Jul 1982.
- [215] D. Ghiglia and M. Pritt, *Two-dimensional phase unwrapping: Theory, algorithms, and software*. Wiley-Interscience, 1998.
- [216] D. Brown, H. T. Cao, A. Ciobanu, P. Veitch, and D. Ottaway, “Differential wavefront sensing and control using radio-frequency optical demodulation,” *Optics Express*, vol. 29, no. 11, pp. 15995–16006, 2021.
- [217] K. Takeno, N. Ohmae, N. Mio, and T. Shirai, “Determination of wavefront aberrations using a Fabry P  rot cavity,” *Optics Communications*, vol. 284, no. 13, pp. 3197 – 3201, 2011.
- [218] R. Glorieux, *High-resolution spectroscopic investigation of the systematic increase of vibrational excitation in water-containing van der Waals complexes*. PhD thesis, UCL-Universit   Catholique de Louvain, 2023.
- [219] M. Newville, R. Otten, A. Nelson, T. Stensitzki, A. Ingargiola, D. Allan, A. Fox, F. Carter, and M. Rawlik, “LMFIT: Non-Linear Least-Squares Minimization and Curve-Fitting for Python,” July 2024.

- [220] M. Granata, *Optical development for second-and third-generation gravitational-wave detectors: stable recycling cavities for advanced virgo and higher-order Laguerre-Gauss modes*. PhD thesis, Université Paris-Diderot-Paris VII; Università degli Studi di Roma Tor Vergata, 2011.
- [221] D. Bresteau, C. Blondel, and C. Drag, “Saturation of the photoneutralization of a H^- beam in continuous operation,” *Review of Scientific Instruments*, vol. 88, p. 113103, 11 2017.
- [222] D. Martynov, H. Miao, H. Yang, F. H. Vivanco, E. Thrane, R. Smith, P. Lasky, W. E. East, R. X. Adhikari, A. Bauswein, *et al.*, “Exploring the sensitivity of gravitational wave detectors to neutron star physics,” *Physical Review D*, vol. 99, no. 10, p. 102004, 2019.
- [223] O. Schwartz, J. J. Axelrod, S. L. Campbell, C. Turnbaugh, R. M. Glaeser, and H. Müller, “Laser phase plate for transmission electron microscopy,” *Nature methods*, vol. 16, no. 10, pp. 1016–1020, 2019.
- [224] M. Beaumont, I. Ventrillard, and D. Romanini, “Optical cavity spectroscopy using heterodyne detection with optical feedback laser frequency locking,” *Appl. Opt.*, vol. 63, pp. 2227–2233, Mar 2024.
- [225] S. Gras and M. Evans, “Direct measurement of coating thermal noise in optical resonators,” *Phys. Rev. D*, vol. 98, p. 122001, Dec 2018.
- [226] A. Singh, L. Maisenbacher, Z. Lin, J. J. Axelrod, C. D. Panda, and H. Müller, “Dynamics of a buffer-gas-loaded, deep optical trap for molecules,” *Physical Review Research*, vol. 5, no. 3, p. 033008, 2023.
- [227] F. M. Cozijn, P. Dupré, E. J. Salumbides, K. Eikema, and W. Ubachs, “Sub-Doppler frequency metrology in HD for tests of fundamental physics,” *Physical review letters*, vol. 120, no. 15, p. 153002, 2018.
- [228] S. C. Subrahmanya, C. Darsow-Fromm, and O. Gerberding, “On the development of an RFSOC-based ultra-fast phasemeter with GHz bandwidth,” *IEEE Transactions on Instrumentation and Measurement*, 2024.
- [229] K. Somiya, Y. Chen, S. Kawamura, and N. Mio, “Frequency noise and intensity noise of next-generation gravitational-wave detectors with RF/DC readout schemes,” *Physical Review D—Particles, Fields, Gravitation, and Cosmology*, vol. 73, no. 12, p. 122005, 2006.

- [230] J. Brooks, M. Mantovani, A. Allocca, J. Casanueva Diaz, V. Dattilo, A. Masserot, and P. Ruggi, “Temperature control for an intra-mirror etalon in interferometric gravitational wave detector Fabry–Perot cavities,” *Galaxies*, vol. 8, no. 4, p. 80, 2020.

Acknowledgements

*‘Eles não sabem, nem sonham,
que o sonho comanda a vida.’*

António Gedeão (Rómulo de Carvalho) in Pedra Filosofal (1956)

As Rhoj Bashkar says in ‘A realist theory of Science’, the applied scientist must be adept at ‘thinking of different levels at once, recognizing clues, piecing together diverse bits of information and assessing the likely outcomes of various courses of action’. I daresay this is quite the case for an experimental physicist. For this reason, I must start by thanking my brother Gonçalo: for all that you taught me during my first job as an engineer. Even though I was already slowly realizing I wanted to go into physics, the time I spent working alongside you laid the foundations on which I still stand upon today.

Going chronologically, I want to thank the dark matter group at LIP Coimbra for taking me in and making me join the LUX-ZEPLIN (LZ) Dark Matter collaboration as I was finishing the Master in Physics in the University of Coimbra. I really enjoyed my time with you, and it reinforced my wish to pursue physics. In particular, I want to thank Prof. Isabel Lopes for all her support, which made a big difference, and also Vladimir Solovov and Cláudio Pascoal for their supervision and guidance, which helped me mature as a scientist and prepared me for the PhD. I must also thank Guilherme Pereira, my colleague in the LZ slow control group. I am also thankful to all the people and friends that I made during my time in the department of physics in Coimbra and in the LZ collaboration, it was indeed a great time.

Making my way into Belgium, I am immensely grateful to my PhD supervisors, Clément Lauzin, Joris van Heijningen and Giacomo Bruno for having believed in me to take on this PhD project. Clément, thank you for pushing me into the lab and really helping me become a better experimental scientist, for the attention to details and also for all the discussions and exchange of ideas. Joris, thank you for all the support during this PhD journey, for all that you taught me and for showing me Virgo, it was a crucial experience and a won-

derful time. Giacomo, thank you for all the support in particular in the last few years and also for your thought provoking questions and comments.

I would like to also extend my gratitude to my thesis jury committee. To Andrea Giamanco, for presiding the jury and all the help in these last few weeks. To Matthieu Génévriez, for the fresh look on the thesis, and all the nice discussions over the last few years. To Matteo Tacca, for your support during my time at Virgo, and for all the help surrounding the phase camera as well. To Jérôme Degallaix for sparking my curiosity in all things optics and for always having time to answer my questions.

I want to give my thanks to the original gravitational-wave instrumentation group at UCLouvain, Federica, Elvis, Wang Yi and Joris. I have nice memories from that first year. I also want to thank Morgane, who joined us later. Despite splitting your time with Liège, you've been a great colleague and friend. As for the latest additions to the group, thank you to Charlotte, Swapnil and Aaron. Thank you, Aaron, for including me in the review article that brought so many of us together, and you know how glad I am that you have joined to take the lead in the group, and bring so many new ideas into the mix. Thank you as well to Antoine, Federico and Andrew, you were there from the beginning, and all the other colleagues from the data-analysis end of the gravitational-wave group. Thank you to Justin, and Soumen and everyone for answering my (many) questions about data analysis!

Regarding my (in)formal education in optics, I want to thank Robin and Alexis. To Robin, thank you for teaching me all the tricks for cleaning and aligning optics and optical cavities. To Alexis, thank you for imparting all your knowledge in experimental work and always knowing where everything is in the lab. Thank you to Samir Kassi for all the help setting up our first Pound-Drever-Hall cavity lock scheme. Later on came Alexandr. Thank you for your absolute expertise in troubleshooting things and the bottomless well of practical knowledge. I also want to thank Joffrey for all the useful tips and advice. To Baptiste Fabre, thank you for sharing your knowledge on optics, both in and outside the classroom. I must also thank all the summer intern students that I worked with, Thomas, Romain, Jennifer, Brieuc, Malo, Yaru and Cheyanne.

During my PhD I was part of two research poles, CP3 and NAPS. I am thankful to all the friends I made in CP3, Karlijn, Paola, Tommasu, Ishan, Maxime, Matteo, Stavros, Leonardo, Jonathan, Francisco, Per, Chris, Clémentine and Simone, to name just a few! The CP3 Christmas dinners and outing days were always great fun. Thank you as well to Elvis and Ishan for giving me

a place to stay when I needed. Regarding the CP3 administration, I want to express my gratitude to Carinne Mertens, Stéphanie Landrain and more recently Agnessa Dresin, without which we would not be able to function.

I would also like to thank all the friends I made at NAPS. Besides Robin, Alexis, Alexandr and Joffrey that I have already mentioned thank you to Benjamin, Eduardo, Simon, Arthémise, Alisée, Miriam, Juan, Fernando, Brian and all the others. The summer barbecue and the IMCN concerts were always one of the highlights of the year. Thank you as well to Laurence Lenoir, for being always kind and helpful.

Regarding the development of the phase camera, I must thank once again Matteo Tacca and Martin van Beuzekom from Nikhef for all the support provided, and for letting me borrow a scanner to get started. On the side of CP3, my deepest thanks to Pavel Demin, for all the discussions and patience and your incredible expertise behind the phase camera FPGA. I must also thank Xavier Urban for the support when faced with long delays with the scanner delivery and also for your immense knowledge and know-how. My thanks to the electronics team as well, to Marc Daman for your incredible expertise and kindness, and to Guillaume, Arnaud, Alain, Raphael and Quentin. On the mechanics side, I must thank Thomas Corbo and Thomas Schiltz as well as Hervé Laurent and all the team at the workshop. Thank you Hervé for all the help and your patience with my low-level of French.

Between March and April 2022, I had the chance to stay at the European Gravitational Observatory to work in Virgo. I must thank all the people I met and the friends I made while there, in particular Enzo, Yuefan, Antonella, Mathieu, Alex, Mathyn and Julia. I also want to thank the Fonds National de la Recherche Scientifique (FNRS) for the financial help to make this stay possible. I am also grateful for all the conferences I was able to attend, and all the people I met in this way, to name just a few, Haidar, Francesco and Enrico.

Because not everything is about work, I want to thank all the friends I made while living in Belgium. In particular I want to thank Paola, Marco, Charlotte, Clara, Mara and Michael. You have made these last few years, and life in Brussels, truly enjoyable. In that sense, I also want to thank all of my friends back home. Part of the pleasure of having adventures away, is coming back home and having people to come back to.

To conclude, I want to thank all of my family, from the youngest to the oldest, for being home. In particular, I want to thank my parents, for your unwavering belief in me for as long as I can remember, and also for the many lessons that have only come in later years. I must thank Gonçalo and Sofia

for letting me use their house as working and writing studio when staying in Portugal in the last year. Last but not least, I want to thank Maria for her warm companionship during this Belgium adventure. I wouldn't choose anyone else to do it with, and it is mostly thanks to you that this entire journey even happened. Thank you for your patience, kindness and sense of humour. I will always cherish the memories of this time together.