
Scanning gate imaging and tuning of quantum electronic transport in graphene

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Abstract

In graphene, charge carriers behave as massless (Dirac) fermions, yielding unprecedented electrical properties. In this thesis, the electronic transport of Dirac fermions is studied in graphene quantum point contacts thanks to low-temperature scanning gate microscopy (SGM), a technique consisting in recording the sample resistance while changing locally the charge carrier density with a biased metallic tip. The focus has been put on backscattering mechanisms and three main directions are explored. First, real space signatures of Klein tunneling, preventing the backscattering of Dirac fermions, are investigated by creating a movable pn junction with the SGM movable top gate. The experimental results are discussed in the light of simulations, revealing that the SGM conductance maps yield an image of the current density around and through the lens, with a direct evidence of Klein tunneling. Second, SGM characterizations are reported under a large magnetic field, in the quantum Hall regime where charge carriers flow in topologically protected quantum Hall edge channels (QHECs). The recorded SGM signatures indicate that counter-propagating QHECs, separated by a few hundreds of nanometers, exist along the same edges in graphene and that charge carriers backscattering is achieved by coupling these QHECs through the localized states of antidots located between them. Third, these antidots are shown to act as nano-sized Fabry-Perot interferometers. A simple model is used to reproduce the experimental results, showing that the signatures associated to two distinct regimes, namely the Aharonov-Bohm and Coulomb dominated regimes, can be explained in a single framework ignoring the Coulomb interactions.

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Introduction

Constraints are usually associated to a negative meaning, being a synonym of limitation and restriction. Though, the interest of games and music precisely originates from the established rules constraining the player, for instance. The emergence of fascinating properties out of constrain is also true for charge carriers in solid state physics, when they are confined to two, one or even zero dimensions (2D, 1D or 0D). Indeed, it favors regimes dominated by quantum mechanics and correlation, studied by the field of mesoscopic physics in systems between the macroscopic and the atomic scales.

From a two-dimensional electronic system (2DES), charge carriers can be further constrained to one dimension. A first method consists in defining a constriction geometry in the studied device, resulting in the formation of a 1D channel called quantum point contact (QPC). It is the archetypal mesoscopic device and its conductance has been found quantized due to the wave nature of charge carriers that are ideally transmitted through the QPC [1]. Another method consists in applying a strong perpendicular magnetic field to the 2DES. It confines charge carriers along the edges in 1D channels theoretically protected against backscattering. This is the quantum Hall effect (QHE).

However, the ideal properties exposed above are often impaired by the presence of impurities, causing charge carriers backscattering. Pinpointing these events in real space and studying the mechanisms at their origin has been proved possible thanks to scanning gate microscopy (SGM) [2, 3]. This technique consists in changing locally the electronic density of a 2DES with a movable biased top gate, while recording the device conductance. The ability to move the local gate is the key to access spatial information. For example, SGM has been used to highlight preferential electronic paths originating from a disorder potential [2] and the backscattering of electrons between counter-propagating quantum Hall edge channels (QHECs) when brought in close proximity of a QPC [4].

These last results were obtained in 2DESs formed at the interface between two semiconductors. Another way to confine charge carriers in a plane is reached with an intrinsic 2D material like graphene, a semimetal composed of carbon atoms in a periodic honeycomb lattice [5, 6]. This material offers

remarkable properties due to the massless nature of its charge carriers (electrons when n -doped and holes when p -doped with electrostatic gates). One of the implications of this characteristic is the absence of potential-induced backscattering, allowing charge carriers to be perfectly transmitted through a step-like pn junction, a phenomenon known as Klein tunneling [7, 8].

Nevertheless, these mechanisms are only true in the bulk of a graphene device as edges are known to strongly impair electrical transport. For instance, no conductance quantization is observed in etched-defined QPCs [9] and, in the QHE, backscattering has been highlighted along the device borders even when they are far apart from each other, which is prevented by a large insulating bulk in semiconductor-based 2DESs [10, 11]. The latter finding is problematic for some applications, like graphene quantum Hall interferometers (QHIs), the equivalence of optical interferometers where the role of monochromatic light beams is played by the QHECs [12, 13].

In this thesis, the transport of Dirac fermions is studied in graphene QPCs thanks to the SGM technique, with a focus on backscattering mechanisms. Three main axes are explored. At zero magnetic field, real space signatures of Klein tunneling, preventing backscattering, are investigated by creating a movable pn junction with the SGM top gate. In the QHE, SGM is used to pinpoint and characterize the backscattering causing the loss of QHECs protection. Finally, the modulation of backscattering and transmission through nano-size QHIs is characterized with SGM. The results of these experimental characterizations and their interpretation, preceded by a more extensive state-of-the-art, are presented in this manuscript according to the following structure:

Chapter 1 exposes a brief overview of the field of mesoscopic physics at zero magnetic field. First, it introduces key concepts as coherent electrical transport in the Landauer-Büttiker formalism, conductance quantification in QPCs or Coulomb blockade in quantum dots. Then, some remarkable transport properties of massless Dirac fermions in graphene are discussed. It includes the mechanisms preventing charge carriers backscattering, yielding Klein tunneling and Veselago lensing through pn junction, and the mechanisms impairing electrical transport like edge impurities.

Chapter 2 reviews the electrical transport in 2DESs under a large perpendicular magnetic field. First, it quickly describes the theoretical toolbox of QHE, exploited all along this thesis. Then, the concept of topological breakdown is introduced in the textbook framework of QHE and the microscopic mechanisms at its origin are illustrated with local probe measurements from the literature. Next, the state of knowledge about quantum Hall topological breakdown in the particular case of graphene is clearly defined. Finally, this chapter is closed with an introduction to Fabry-Pérot (FP) QHIs.

Chapter 3 covers the principal steps punctuating a typical experimental process. First, it presents the SGM setup used in this thesis and reviews the

main achievements obtained with this technique since the late 90s. Second, it portrays the two graphene QPC devices studied in this work, with a detour via the fabrication process of such samples. Third, it describes the tight-binding simulation formalism used to analyze the experimental data, with a particular focus on the KWANT package.

Chapter 4 is devoted to the electrical transport of Dirac fermions through a movable pn junction-based circular Veselago lens created in graphene thanks to the biased SGM tip. First, the experimental data, i.e., the device conductance as a function of the tip position, are exposed. They are then discussed under the light of simulations and reveal that the SGM conductance maps give an image of the current density around and through the lens, with a direct evidence of Klein tunneling. Finally, the major role of the QPC geometry is highlighted.

Chapter 5 discusses SGM data obtained in the quantum Hall regime, on the two graphene QPCs. It begins with the observation of peculiar SGM contrast, associated to backscattering, along the device edges, which is surprising in regards to the topological protection of QHECs. These signatures are explained by the presence of counterpropagating QHECs along the same graphene edges, coupled through antidots. This hypothesis is supported with simulations highlighting the mechanisms at the origin of the inter-QHECs backscattering. A second part is devoted to an observed asymmetry between electrons and holes in electrical transport and SGM measurements. It is explained by a different spatial configuration of QHECs in the devices for each type of charge carriers, originating from the effect of the metallic contacts.

Chapter 6 completes this thesis by showing that antidots, located between the counterpropagating QHECs in graphene, are FP QHIs free from Coulomb interactions, despite their unprecedented small sizes predicted to favor this regime. This is demonstrated in different experimental conditions and the influence of the reflection probabilities at the QHI beam splitters is analyzed with a simple analytical model that fits accurately the experimental results. A similar analysis is also presented for a QHI found in a semiconductor-based 2DES, where it originates from a dot coupling QHECs flowing along the opposite edges of a QPC.

chapter 1

Mesoscopic transport at zero magnetic field

In conventional electronics, the main challenge consists in reducing the transistors size while avoiding effects linked to the wave nature of charge carriers at the nanometer scale, such as tunneling effect causing current leakage in the digital *off* state. Mesoscopic physics, on the contrary, explores charge transport taking advantage of quantum mechanics in devices whose sizes range between the macroscopic and the atomic scales. This chapter gives a brief overview of this vast domain by discussing first the charge transport in semiconductor-based two dimensional systems and then in graphene, the material of interest in this thesis.

1.1 Transport in two dimensional electron systems

Confining charge carriers (electrons or holes) in a plane of the same typical thickness as their wavelength λ forms what is called a two-dimensional electron system (2DES). Different techniques exist to create such a system. For example, semiconductor-based 2DESs originate from a bending of the conduction (valence) band below (above) the chemical potential at the interface between two semiconductor materials (GaAs and AlGaAs for example) grown by molecular beam epitaxy [14].

In the 2DES, electrons motion is influenced by the periodic atomic potential of the crystal, resulting in a non-trivial dispersion relation of their energy E as a function of their momentum $\mathbf{p} = \hbar\mathbf{k}$, where $|\mathbf{k}| = 2\pi/\lambda$ is the wavevector and

$\hbar = h/2\pi$ with h the Plank constant. In semiconductor-based 2DESs, this relation can be approximated by a parabola $E = \hbar^2 \mathbf{k}^2 / 2m^*$ at low charge carriers density. Interestingly, it corresponds to the energy of a free particle of effective mass m^* , meaning that charge carriers are described by the Schrödinger equation

$$\frac{\hat{\mathbf{p}}^2}{2m^*} \psi = E\psi, \quad (1.1)$$

where $\hat{\mathbf{p}} = -i\hbar \nabla_{\mathbf{r}}$ is the momentum operator and $\psi = a \exp(i\mathbf{k} \cdot \mathbf{r})$ is the wavefunction with $\mathbf{r}$ the position in the conduction plane and a the amplitude. In a conductor, only charge carriers at the chemical potential (called the Fermi energy E_F at zero temperature) contribute to the electrical transport, their Fermi wavevector is given by

$$k_F = \sqrt{2m^* E_F}. \quad (1.2)$$

The corresponding wavelength is $\lambda_F = 2\pi/k_F$.

In this framework, coherent transport in 2DESs will be introduced as it conditions the apparition of quantum-dominated electrical transport in mesoscopic devices. Then, the Landauer-Büttiker formalism, useful to describe this regime, will be discussed and illustrated with the experimental measurement of conductance quantization in a quantum point contact. Finally, electrical transport dominated by electron-electron interactions will be discussed in the case of a quantum dot.

1.1.1 Coherent and ballistic transport

In real devices, impurities close to the 2DES induce a disorder potential $U(\mathbf{r})$, whose variation occurs on scales much larger than the inter-atomic distance, and the Schrödinger equation (1.1) becomes

$$\left(\frac{\hat{\mathbf{p}}^2}{2m^*} + U(\mathbf{r}) \right) \psi = E\psi. \quad (1.3)$$

1.1.1.1 Ballistic regime

The disorder potential introduced in Eq. (1.3), alongside other effects such as electron-phonon coupling, strongly impairs the transport of charge carriers compared to the ideal case described by Eq. (1.1). Indeed, these mechanisms are at the origin of "collision" events leading to charge carriers backscattering in the 2DES. One can then define the mean free path l_e as the average distance traveled by a charge carrier between two collisions. A device of size L is said to be in the ballistic regime when $l_e > L$. Otherwise, it is in the diffusive regime. Finding a device in one regime or the other depends on the density of impurities and on the temperature T that increases the number of electron-phonon collisions.

1.1.1.2 Coherent regime

When charge carriers move in a 2DES, their associated wavefunction accumulates a phase $\phi(\mathbf{r}, t)$ evolving in space and time. The conservation of this phase leads to interference phenomena that can dominate the electrical transport properties. However, coupling between the wavefunction and a fluctuating environment (electromagnetic noise for example) leads to phase decoherence, i.e., random phase change. One can then define the phase coherence length l_ϕ as the averaged distance on which a particle preserves its phase information. A device is said to be in the coherent regime when $l_\phi > L$ and it implies that transport is dominated by the wave nature of charge carriers.

1.1.2 Landauer-Büttiker formalism

The Landauer-Büttiker formalism offers a practical framework to deal with mesoscopic systems in the coherent regime introduced in previous section [15–18]. As illustrated in Fig. 1.1, such systems are modeled by a scattering region, corresponding to the studied device, connected by semi-infinite leads to inject and collect charge carriers, a role played by the electrical contacts in real devices.

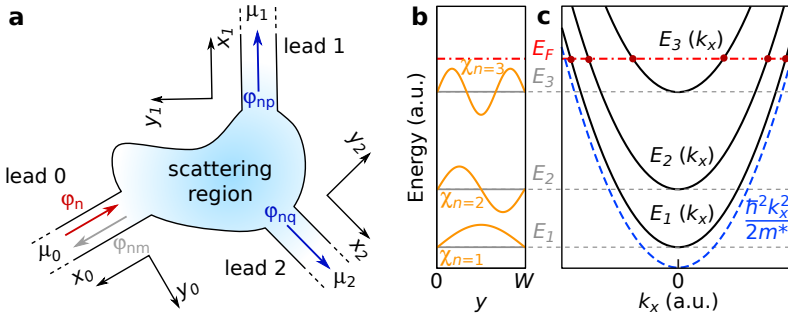


Figure 1.1: Illustration of the Landauer-Büttiker formalism. **a**, Schematic of a mesoscopic device composed of a scattering region connected to semi-infinite leads. Axes are associated to each lead with x being the charge transport direction. A forward mode ϕ_n in lead 0 is sent towards the scattering region and is reflected to the backward mode m in the same lead (ϕ_{nm}), with the reflection amplitude r_{nm} , and transmitted to mode p in lead 1 (ϕ_{np}) and to mode q in lead 2 (ϕ_{nq}) with the transmission amplitudes t_{np} and t_{nq} . **b**, Discrete energy levels E_n due to the confinement in the y direction in the leads. The associated wavefunction χ_n are shown in orange. **c**, Energy bands $E_n(k_x) = E_n + \hbar^2 k_x^2 / 2m^*$, where the second term is the parabolic energy dispersion of a free particle of effective mass m^* shown with a blue dashed line. A mode is created for each band crossing the Fermi energy E_F (red dot) for $k_x > 0$.

1.1.2.1 The leads

The leads are semi-infinite one-dimensional ribbons carrying current without backscattering (in a purely ballistic regime) towards and from the scattering region. Therefore, charge carriers behave as free particles along the transport

axis x_l in the l th lead ($l = 0, 1$ or 2 in Fig. 1.1a). In the y_l direction, the lateral confinement yields discrete energy levels E_n associated to the wavefunctions $\chi_n(y_l)$ of wavelength λ_n (Fig. 1.1b). Hence, the total energy forms parabolic bands expressed by $E_n(k_x) = E_n + \hbar^2 k_x^2 / 2m^*$ (Fig. 1.1c). A mode is created when the n th band crosses E_F . For a forward mode, i.e., towards the scattering region, the wavefunction is

$$\varphi_n(x_l, y_l) = \chi_n(y_l) \frac{1}{\sqrt{L}} \exp(-ik_x x_l), \quad (1.4)$$

where $k_{x,n}^2 = k_F^2 - (2\pi/\lambda_n)^2$, with k_F defined in Eq. (1.2), and L a normalization length.

1.1.2.2 The scattering region

When the n th mode in Eq. (1.4) enters the scattering region, it can either be backscattered towards the same lead or transmitted towards another lead. In the first case, the backward wavefunction writes

$$\varphi_{nm}(x_l, y_l) = r_{nm} \chi_m(y_l) \frac{1}{\sqrt{L}} \exp(ik_{x,m} x_l), \quad (1.5)$$

where r_{nm} is the reflection amplitude from mode n to mode m and necessarily obeys to $|r_{nm}| \leq 1$. In the second case, the transmitted wavefunction in lead $k \neq l$ is expressed as

$$\varphi_{np}(x_k, y_k) = t_{np} \chi_p(y_k) \frac{1}{\sqrt{L}} \exp(ik_{x,p} x_k), \quad (1.6)$$

where $t_{np} \leq 1$ is the transmission amplitude from mode n in lead l to mode p in lead k .

1.1.2.3 Conductance in a mesoscopic system

Now, we will see how transport properties can be derived from the wavefunctions in the leads, defined in the previous section. The starting point is the expression of the current density associated to the wavefunction φ_{np}^* in Eq. (1.6), given by

$$\mathbf{j}_{np} = g_s \frac{ie\hbar}{2m^*} \left(\varphi_{np}^*(x_k, y_k) \nabla \varphi_{np}(x_k, y_k) - \varphi_{np}(x_k, y_k) \nabla \varphi_{np}^*(x_k, y_k) \right), \quad (1.7)$$

where $-e$ is the electron charge and g_s is the spin degeneracy of the wavefunction φ_{np}^* (most of the time, $g_s = 2$). By integrating $\mathbf{j}_{np}$ along y_k and by considering that $\chi_p(y_k)$ is normalized in this direction, the current contribution of an energy interval dE along the transport axis is [14]

$$dI_{np} = g_s \frac{e}{\hbar} |t_{np}|^2 dE. \quad (1.8)$$

This expression does not depend on the energy because the decrease of group velocity with E compensates the increase of the density of states. One can further define $\mathcal{T}_{np} = |t_{np}|^2$ as the transmission probability since it is the ratio between the probability densities $|\varphi_{np}(x_k, y_k)|^2$ and $|\varphi_n(x_l, y_l)|^2$.

In order to have a non-zero current in the leads, an imbalance between the chemical potentials μ_l in lead l and μ_k in lead k should be imposed. It corresponds to the application of a voltage bias $V_{lk} = (\mu_l - \mu_k)/e$ between the leads (here, lead l is called the source and lead k the drain so that V_{lk} is the source-drain voltage denoted V_{SD} in this thesis). At finite temperature T , electrons in lead l are distributed in energy around the chemical potential according to the Fermi-Dirac statistics $f_l(E) = 1/(\exp((E - \mu_l)/k_B T) + 1)$, where k_B is the Boltzmann constant. From Eq. (1.8), the current is

$$I_{np} = g_s \frac{e}{h} \int_{-\infty}^{\infty} \mathcal{T}_{np} (f_l(E) - f_k(E)) dE. \quad (1.9)$$

When the source-drain voltage is small ($eV_{lk} \ll k_B T$) in Eq. (1.9), it enables the approximation [16]

$$f_l(E) - f_k(E) \approx \frac{\partial f_l(E)}{\partial E} (\mu_l - \mu_k) = \frac{\partial f_l(E)}{\partial E} eV_{lk}. \quad (1.10)$$

If the temperature is now lowered to zero ($T \rightarrow 0$), the source-drain voltage should be infinitesimal ($V_{lk} \rightarrow \delta V_{lk}$) to respect the approximation in Eq. 1.10. The system is then said to be at the equilibrium as the chemical potential in both leads are equal to the Fermi energy ($\mu_l \approx \mu_k = E_F$). In this framework, the conductance between leads l and k is given by $\sum_{n \in l} \sum_{p \in k} \delta I_{np} / \delta V_{lk}$, which yields [15–18]

$$G = g_s \frac{e^2}{h} \sum_{n \in l} \sum_{p \in k} \mathcal{T}_{np}, \quad (1.11)$$

provided that the discrete energies E_n and E_p (Fig. 1.1b) of the considered modes are below E_F . In this expression, e^2/h is the quantum of conductance.

1.1.3 The quantum point contact

A quantum point contact (QPC) in the coherent regime is the archetypal mesoscopic system as its electrical transport properties clearly highlight the wave nature of charge carriers. In semiconductor-based 2DEs, it is obtained with split gates located above the conducting plane, biased with a negative (positive) voltage to deplete the electron (hole) gas beneath them (Fig. 1.2a) [1, 19, 20]. It results in the formation of a one-dimensional channel caused by the confinement potential of the gates (black line in Fig. 1.2b). This potential can be approximated by a parabola at low charge carriers density. Different transverse modes appear in this channel at equidistant energies E_n (gray lines in Fig. 1.2b) and a wavefunctions χ_n along the y axis (Eq. (1.4)) is associated to each mode (Fig. 1.2b).

In 1988, van Wees *et al.* discovered that the conductance through a QPC formed with split gates (separated by 250 nm as shown in Fig. 1.2a) is quantized with plateaus at integer values of the quantum of conductance e^2/h (light blue curve in Fig. 1.2c) [1]. They have also showed that these plateaus are smoothed

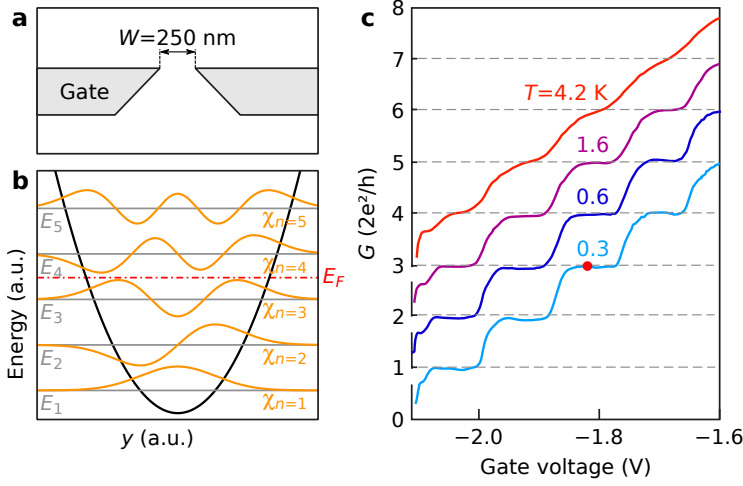


Figure 1.2: Quantized conductance in a quantum point contact (QPC). Adapted from [1]. **a**, Schematic of the QPC formed with split gates depleting the underneath 2DES when biased at a negative voltage. **b**, Parabolic confinement potential (black line) and the corresponding energy levels E_n (gray lines). The orange curves show the transverse wavefunctions χ_n . **c**, Conductance curves at different temperature (shifted by $2e^2/h$ for clarity). When the temperature is decreased, G exhibits clear steps at integer values of $2e^2/h$. Note that a small kink appears around $0.7 \times 2e^2/h$ and is called the 0.7 anomaly [21].

out when the temperature is increased (Fig. 1.2c) [19]. These observations are in full agreement with the Landauer-Büttiker formalism presented in the previous section. The QPC is a one-dimensional channel comparable to a lead and varying the split gates voltage towards less negative values adds a mode in the constriction each time one of the energy levels E_n is shifted below the Fermi energy E_F . The modes are spin-degenerated ($g_s = 2$) and contribute for $2e^2/h$ to the total conductance in Eq. (1.11), meaning they are perfectly transmitted through the QPC ($T_{np} = 1$). With three modes below E_F , the conductance is therefore $G = 6e^2/h$ (red dot in Fig. 1.2c). Lastly, when the temperature is raised, the Fermi-Dirac distribution widens in Eq. (1.9) and the steps are smoothed out.

1.1.4 Coulomb blockade

Transport through a one-dimensional channel has been discussed in the previous section in the absence of electron-electron interaction. When the dimensions are further reduced to form a zero-dimensional quantum dot (QD) tunnel-coupled to the source and the drain, these interactions can no longer be ignored [22]. Indeed, adding an electron in the dot has an energetic cost due to the Coulomb repulsion with the charges already confined in the QD. Hence, adding and removing electrons is achieved by changing the voltage V_{PG} of a plunger gate thanks to the capacitive coupling α_{PG} between the gate and the

dot (insets in Fig. 1.3a).

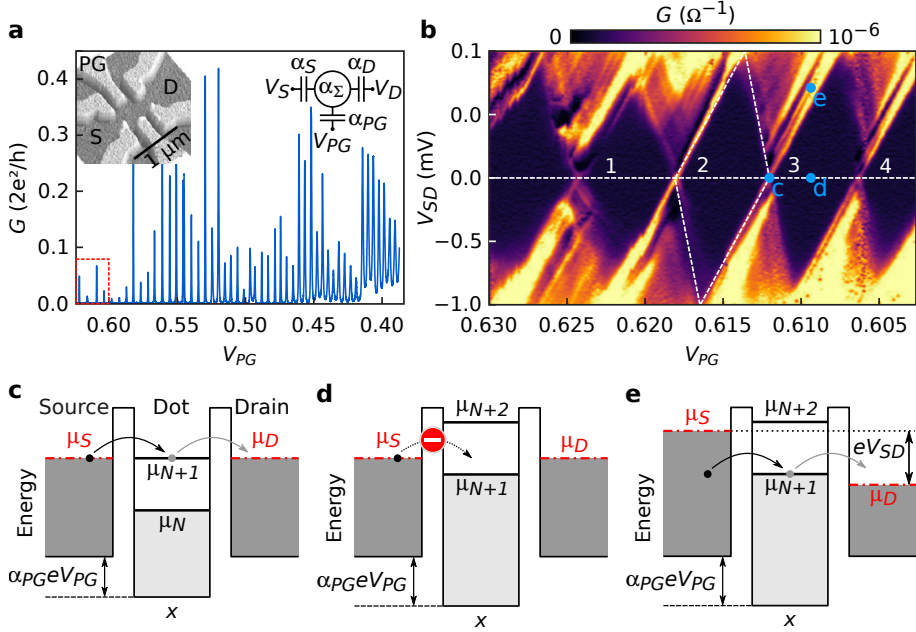


Figure 1.3: Transport through a quantum dot (QD). Adapted from [23]. **a**, Conductance through a QD as a function of the plunger gate voltage V_{PG} . The left inset shows an image of the sample with biased top gates depleting the underneath 2DES to create the QD. The right inset shows a capacitive model of the QD. **b**, Conductance map as a function of V_{PG} and the source-drain bias voltage V_{SD} (in the range indicated with a red dotted line in **a**). It features Coulomb diamonds (zero conductance regions in black). Each diamond is labeled with the number of electrons in the QD. **c-e**, Schematic of the energy profile along the transport axis x in the dot. The source and drain reservoirs at potential μ_S and μ_D are separated from the QD by tunnel junctions. The QD hosts discrete energy levels μ_N . Each situation illustrates a particular point of the map in **b**.

The electrical transport through a QD is illustrated with the conductance curve in Fig. 1.3a, obtained by varying V_{PG} [23]. It displays a set of peaks separated by zero conductance regions. Figure 1.3b shows the evolution of these peaks when the source-drain bias voltage V_{SD} is raised. It features adjacent diamond-shaped regions of zero-conductance (in black in Fig. 1.3b) separated by regions of non-zero conductance (light shades in Fig. 1.3b) in the conductance map. These signatures can be understood with the schematics in Figs. 1.3c-e [14]:

- At zero V_{SD} , the QD is conducting when hosting N or $N + 1$ electrons is energetically equivalent. The $N + 1$ th electron can therefore hop in and out the QD and contributes to electrical transport. It is represented by an energy level μ_{N+1} aligned with the source and drain potential μ_S and μ_D in Fig. 1.3c. The spacing between two levels can be expressed by

$\mu_{N+1} - \mu_N = \Delta E + e^2/\alpha_\Sigma$, where ΔE is the quantum contribution coming from the size confinement and e^2/α_Σ is the Coulomb contribution with α_Σ the QD self-capacitance. Excepted in extremely small QDs, the quantum contribution is much smaller than the electrostatic term.

- When a change of voltage ΔV_{PG} shifts the levels by an energy $\alpha_{PG}e\Delta V_{PG}$, so that none of them is aligned with μ_S and μ_D , transport through the QD is forbidden (Fig. 1.3d). It is the Coulomb blockade regime explaining the regions of zero conductance in Figs. 1.3a,b.
- At finite V_{SD} , transport through the dot is observed when a level stands in the energy window eV_{SD} (Fig. 1.3e).

When looking at the schematics in Figs. 1.3c-e, one would expect the diamonds to be perfectly symmetric in V_{PG} and V_{SD} . Nevertheless, this is not the case in Fig. 1.3b where they are tilted. It is due to the capacitive coupling between the QD and the source and the drain (α_S and α_D in the right inset of Fig. 1.3a) that modifies the charging energy in the QD when V_{SD} is applied.

1.2 Dirac fermions in graphene

Graphene, produced and studied for the first time in 2004 [24], is a two-dimensional semi-metal composed of carbon atoms organized in a honeycomb lattice (Fig. 1.4a) [5]. Contrary to semiconductor-based 2DESs (section 1.1), the graphene band structure can be approximated at low charge carriers density by a conic energy dispersion (Figs. 1.4b,c) expressed as

$$E = \xi \hbar v_F |\mathbf{k}|, \quad (1.12)$$

where v_F is the group velocity and $\xi = \pm 1$ depending if E_F is in the conduction band ($\xi = +1$ when $E_F > 0$) or in the valence band ($\xi = -1$ when $E_F < 0$). This expression is identical to the energy of free massless fermions, with a velocity v_F instead of the speed of light. The Schrödinger equation (1.1) is therefore not valid in the case of graphene and the Dirac equation should be used instead. It writes

$$v_F \boldsymbol{\sigma} \cdot \hat{\mathbf{p}} \psi_K = E \psi_K, \quad (1.13)$$

where $\boldsymbol{\sigma} = (\sigma_x, \alpha_K \sigma_y)$ is the vector of the Pauli matrices σ_x and σ_y and K is the valley label with $\alpha_K = 1$ and $\alpha_{K'} = -1$ (Fig. 1.4b). The eigenstate of Eq. (1.13) in the reciprocal space is [5]

$$\psi_K(\mathbf{k}) = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\alpha_K \theta_{\mathbf{k}}/2} \\ \xi e^{i\alpha_K \theta_{\mathbf{k}}/2} \end{pmatrix}, \quad (1.14)$$

where $\mathbf{k} = (k_x, k_y)$ is the wavevector whose origin is one of the K points in Fig. 1.4b and $\theta_{\mathbf{k}} = \arctan(k_y/k_x)$ is the angle of the wavevector in the reciprocal space. The form of this wavefunction provides exceptional electrical properties to graphene, as will be detailed in the next section.

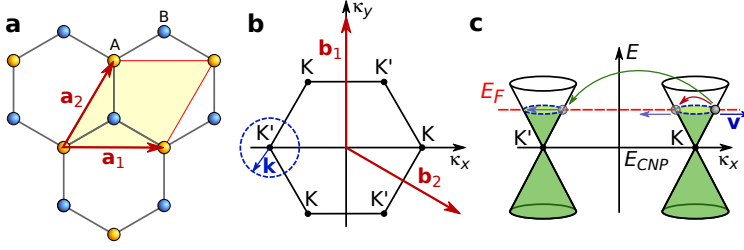


Figure 1.4: Graphene electrical properties in a nutshell. **a**, Schematic of the graphene lattice. The unit cell is shown with the yellow area and is defined by the two unit vectors $\mathbf{a}_1$ and $\mathbf{a}_2$. It contains two carbon atoms forming the sub-lattices A and B. **b**, Brillouin zone in the reciprocal space. **c**, The energy dispersion of graphene exhibits a conical evolution with the wavevector $\mathbf{k}$ centered at the K and K' points in **b**. It forms two distinct valleys. Intravalley scattering is illustrated with a red arrow and intervalley scattering is illustrated with a green arrow.

Before moving forward, it must be pointed out that the two dimensional nature of graphene allows to shift easily the band structure, compared to E_F , with electrostatic gates. When E_F stands at the intersection between the valence and the conduction bands, the charge carriers density is zero. This point is therefore called the charge neutrality point (CNP). Otherwise, when $E_F > E_{CNP}$, graphene is doped with electrons (green color code in this thesis) and when $E_F < E_{CNP}$, graphene is doped with holes (purple color code in this thesis). The charge carriers density is expressed from E_F as [5, 6]

$$n_c = \frac{k_F^2}{\pi} = \frac{4\pi E_F^2}{h^2 v_F^2} \quad (1.15)$$

1.2.1 Backscattering in graphene

This section discusses the mechanisms leading to the backscattering of Dirac fermions in graphene.

1.2.1.1 Effect of the disorder potential

In section 1.1.1, we have seen that the presence of a disorder potential $U(\mathbf{r})$ induces backscattering for charge carriers obeying to the Schrödinger equation. In the present section, the effect of such a potential is discussed in the case of graphene, with the Dirac equation (1.13) becoming

$$v_F \boldsymbol{\sigma} \cdot \hat{\mathbf{p}} \psi_K + U(\mathbf{r}) \psi_K = E \psi_K. \quad (1.16)$$

The probability to scatter a Dirac fermion from a state $\psi_K(\mathbf{k})$ to $\psi_K(\mathbf{k}')$ (Eq. (1.14)) is derived from the Fourier transform of the potential term $\tilde{U}(\mathbf{q})$ and depends on the angle θ between the wavevectors $\mathbf{k}$ and $\mathbf{k}'$ as [6, 25]

$$P(\theta) = |\tilde{U}(\mathbf{q})|^2 \frac{1 + \cos(\theta)}{2}. \quad (1.17)$$

This expression shows that intra-valley backscattering (red arrow in Fig. 1.4c) is forbidden in graphene as the probability to flip the wavevector from $\mathbf{k}$ to $\mathbf{k}' = -\mathbf{k}$ results in $P(\theta = \pi) = 0$. This remarkable property will be further discussed in the next section under the light of pseudospin conservation.

1.2.1.2 Pseudospin and chirality

In particle physics, the Pauli matrices operator σ in the Dirac equation is at the origin of the spin degeneracy. In Eq. (1.13), the presence of the same term yields what is called the pseudospin in graphene, linked to the two sublattices (Fig. 1.4). Similarly to particle physics, the chirality can be calculated by projecting the pseudospin on the normalized wavevector as

$$C = \frac{\sigma \cdot \mathbf{k}}{k} = \alpha_K \xi = \pm 1. \quad (1.18)$$

The chirality remains unchanged in the same band and the same valley. Furthermore, the charge carriers velocity can be expressed from the chirality as [25]

$$\mathbf{v} = v_F C \frac{\mathbf{k}}{k}. \quad (1.19)$$

In the K valley, $\mathbf{v}$ is therefore aligned with the momentum $\hbar\mathbf{k}$ in the conduction band while they are inverted in the valence band (and the opposite for the K' valley). Furthermore, $\mathbf{v}$ does not depend on the norm of $\mathbf{k}$.

The reason for the forbidden backscattering in Eq. (1.17) can now be interpreted in term of pseudospin as this quantity should be conserved [5, 6, 25, 26]. This requirement would be violated by changing $\mathbf{k}$ for $-\mathbf{k}$ in Eq. (1.18) as σ would also have to change to $-\sigma$ [25]. Despite the forbidden intra-valley backscattering, inter-valley backscattering is however made possible as it keeps σ unchanged. Nevertheless, it requires a long-range scattering event in the reciprocal space (Fig. 1.4b), only possible by the presence of a punctual impurity, as an atomic defect, whose size is around $1/|\mathbf{K} - \mathbf{K}'| \sim a$, where a is the lattice constant (Fig. 1.4a).

1.2.2 Effect of the strain fluctuations

While backscattering caused by a disorder potential is theoretically forbidden, it has been shown to originate from strain fluctuations [27]. Indeed, strain induces a variation of the interatomic distance in the plane, which is different than adding a potential term in the Hamiltonian of Eq. (1.16). In real devices, corrugation due to the substrate roughness is one of the principal sources of strain fluctuations. Hence, placing graphene on a Si/SiO₂ substrate strongly impairs the electrical properties compared to an hexagonal boron-nitride (hBN) substrate [28]. Indeed, hBN is an electrically insulating van der Waals material whose layers have the same lattice as graphene, and almost the same inter-atomic distance, with boron atoms on sublattice A and nitrogen atoms on sublattice B in Fig. 1.4a. The two-dimensional nature of this substrate makes it

extremely flat compared to Si/SiO₂ and therefore improves the conductivity of the graphene layer placed on top.

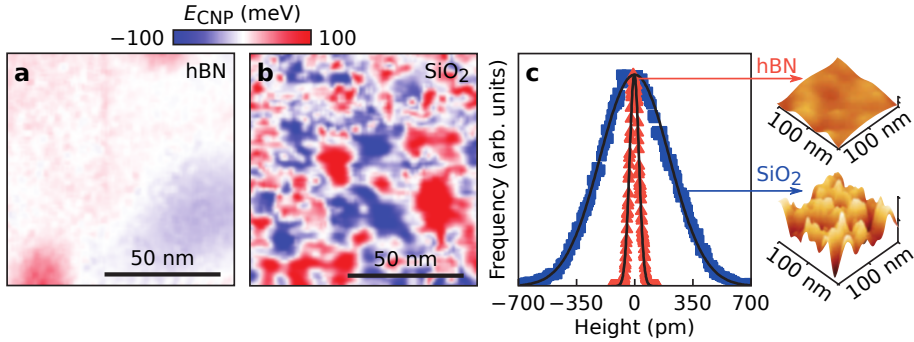


Figure 1.5: Charge puddles induced in graphene by the substrate corrugation. Adapted from [29]. **a**, Scanning tunneling microscopy map of the CNP energy in graphene on a hexagonal boron-nitride (hBN) substrate. **b**, Same as **a** for graphene on a Si/SiO₂ substrate. **c**, Distribution of the fluctuations height linked to the corrugation of the substrate (hBN in red and Si/SiO₂ in blue). The Schematics on the right show topography maps of graphene on hBN (top) and on Si/SiO₂ (bottom).

A comparison between the two substrates is shown in Figs. 1.5a,b by imaging the CNP energy E_{CNP} with a scanning tunneling microscope [29]. Graphene on hBN leads to small variations of E_{CNP} on distances around 100 nm (Fig. 1.5a) compared to the large variations of E_{CNP} on distances around 10 nm for graphene on Si/SiO₂ (Fig. 1.5b). By performing topographies on the same areas, the charge puddles have been associated to the substrate corrugation, much flatter with hBN than with Si/SiO₂ (Fig. 1.5c).

1.2.3 Veselago lensing

As mentioned at the beginning of section 1.2, gates can be used to dope graphene either with electrons (*n*-doping) or with holes (*p*-doping). By using two gates of opposite polarity, it is therefore possible to create a *pn* junction. One can then wonder what is the fate of Dirac fermions impinging perpendicularly such a junction, given that backscattering is prohibited due to pseudospin conservation (section 1.2.1.2).

1.2.3.1 Klein tunneling

Computing the transmission probability in this scenario tells us that charge carriers are perfectly transmitted through the *pn* junction. This phenomenon is known as Klein tunneling and has been predicted back in 1929 [30], with particles and antiparticles instead of electrons and holes that are their counterpart in graphene.

Figure 1.6a illustrates Klein tunneling for an abrupt *pn* junction created by the potential $U(x)$ shown with the gray line. The velocity $\mathbf{v}$ of an electron in

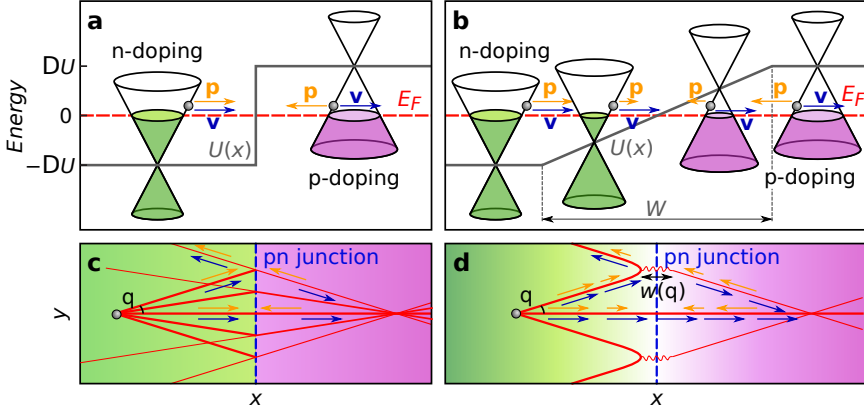


Figure 1.6: Veselago lensing through a pn junction. **a**, Abrupt potential step $U(x)$ along the transport axis with an electron (n) doping on the left and a hole (p) doping on the right. The momentum $\mathbf{p}$ is in the same direction as the velocity $\mathbf{v}$ for electrons and is in the opposite direction for holes. **b**, Same as **a** for a smooth pn junction. The momentum $\mathbf{p} = \hbar\mathbf{k}$ decreases continuously with $E_F - U(x)$ and is inverted on the other side of the junction but $\mathbf{v}$ remains the same. **c**, Charge carriers impinging the pn junction at finite angle θ are reflected and transmitted with a negative refraction index. **d**, For a smooth pn junction, charge carriers trajectories at angle θ are bent and a forbidden region of width $\omega(\theta)$ is formed. Transmission through this region is achieved by conventional tunneling.

the n region is given by Eq. (1.19) and is aligned with the momentum $\mathbf{p} = \hbar\mathbf{k}$ in the K valley. In the hole side, the velocity remains the same (such as the pseudospin) but the momentum is inverted to $\mathbf{p} = -\hbar\mathbf{k}$. Note that the electron does not become a hole in the p region [25], as often stated in the literature. Indeed, it would violate the charge conservation. When the electron goes through the junction, it fills one of the unoccupied states of an otherwise filled valence band, commonly called a hole state. It can therefore be seen as an *anti-hole* obeying to Eq. (1.19) in the valence band.

Based on the same reasoning, a smooth pn junction (Fig. 1.6b) also allows Klein tunneling. In this configuration, the momentum $\mathbf{p}$ is reduced as the CNP is shifted towards E_F (indeed, $E_F - E_{CNP} = \hbar v_F k_F$) but the velocity is kept constant (Fig. 1.6b). At the position of the pn junction, $\mathbf{p}$ vanishes and changes sign in the p region. This continuous change of $\mathbf{p}$ alongside the constant $\mathbf{v}$ explains the absence of backscattering. Klein tunneling has been highlighted in a real device with interference signatures in a npn junction [8].

1.2.3.2 Transmission at finite angles

The transmission of charge carriers through the pn junction as a function of their trajectory angle will now be discussed. In the case of an abrupt junction (Fig. 1.6c), trajectories impinging it at finite angle are partly reflected and partly transmitted. Interestingly, the transmission is comparable to that in a negative

refraction index material (whose properties have enabled the creation of an invisibility cloak [31]) and charge carriers are refocused in the other side of the junction. This phenomenon is known as Veselago lensing [32].

For smooth pn junctions, the potential gradient induces a progressive bending of charge carriers trajectories when their incident angle θ is increased (Fig. 1.6d). In this situation, a forbidden region appears and its width ω increases with θ [33]. The transmission through this region is achieved via conventional tunneling and is determined from the junction width W (Fig. 1.6b) by [33]

$$T(\theta) = \exp\left(-\pi k_F W \sin(\theta)^2\right). \quad (1.20)$$

Veselago lensing with smooth junctions has been evidenced experimentally in graphene with fixed gates in refs. [34, 35] and will be discussed in the case of a movable gate in chapter 4. The filtering induced by the junction smoothness has also been exploited to create a highly directional electron beam [36]. These lensing effects let envision a plethora of devices designed to manipulate Dirac fermions similarly to photons in an optical systems, a field known as graphene electron optics. Several achievements have already been demonstrated in this field, including the equivalent of an optic fiber [37] or the creation of optics-based transistors [38, 39]. The equivalent of a scanning electron microscope has also been proposed in this framework [40].

1.2.4 Influence of edges on transport

In section 1.2.1, we have seen that backscattering caused by potential fluctuations is prohibited in graphene. Importantly, this conclusion has been drawn with wavefunctions in a fully periodical lattice and is therefore only valid in the bulk of a device. Indeed, graphene edges are known to strongly affect the transport of charge carriers as illustrated with two experimental examples in this section.

1.2.4.1 Transport through a graphene QPC

Figure 1.7a shows the conductance G measured through a graphene QPCs (an optical image is shown in the inset of Figure 1.7a), by varying the bulk density n_c with a back gate [9]. Since the change of density modifies the wavevector k_F according to Eq. (1.15), the number of modes in the constriction should change as well. A steps-like evolution of G is therefore expected, similarly to the semiconductor-based QPC in Fig. 1.2c. However, no quantization is visible in Fig. 1.7a, except perhaps by the appearance of some kinks separated by $2e^2/h$ at large density (black arrows). Simulations, performed by introducing edge roughness (Figs. 1.7b,c) reproduce the experimental results as shown with the blue curve of G as a function of k_F in Fig. 1.7d. The chaotic fluctuations of this curve highlight that edge roughness causes a backscattering of the modes through the QPC that varies with the wavevector, with $\mathcal{T}_{np}(k_F) < 1$ in Eq. (1.11).

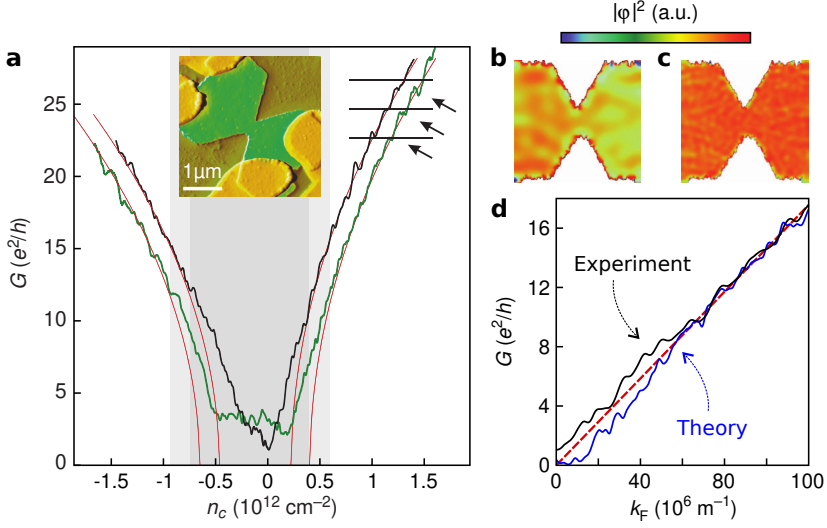


Figure 1.7: Absence of quantization in a graphene QPC. Adapted from [9]. **a**, Evolution of the conductance G as a function of the density n_c for two cooldowns of the same graphene QPC whose constriction is 230 nm-wide. Graphene is encapsulated between two hBN layers. The inset shows an optical photography of the sample (contacts in yellow and hBN/graphene/hBN stack in green). **b,c**, Simulated local density of state (LDOS) of two QPCs at different values of E_F . The LDOS exhibits larger fluctuations along the edges due to roughness. **d**, Comparison between the experimental (black) and the simulated (blue) conductance. Both exhibit chaotic fluctuations due to edge roughness.

1.2.4.2 Thermal dissipation along graphene edges

Figure 1.8 presents the results of another remarkable study highlighting the detrimental role of edges in graphene [41]. It has been performed with the experimental setup represented in Figs. 1.8a,b. A superconducting quantum interference device (SQUID) placed on a tip is scanned above a graphene sample (in black) encapsulated between hBN (in blue). This probe allows to measure the thermal dissipation with a high precision while the tip can be biased to induce a local gating beneath it [42]. Figure 1.8c displays the derivative of the thermal dissipation map when a DC current I_{dc} is sent through the device (Fig. 1.8b). The dissipation is highlighted by a large contrast in Fig. 1.8c and occurs essentially along the edges, except for three spots in the bulk labeled A, B and C. The observed dissipation has been assigned to the presence of atomic defects (Fig. 1.8d) for which the resonant interaction with charge carriers is tuned by the tip-induced potential [41]. It results in circular features centered on the probed defects (Fig. 1.8e), corresponding to the tip positions resulting in the same induced potential.

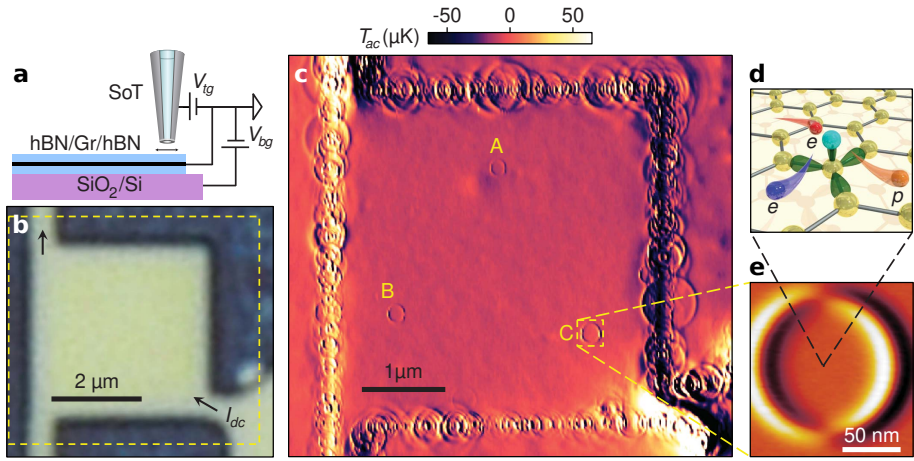


Figure 1.8: Thermal dissipation along graphene edges. Adapted from [41]. **a**, Side view of the experimental setup. A SQUID on tip (SoT) is scanned above a hBN/-grapene/hBN stack. **b**, Optical image of the sample. **c**, Derivative of the thermal dissipation map. It features contrast essentially located along the edges. **d**, Artist's view of the collision between an electron and an atomic defect, causing thermal dissipation. **e**, Zoom on one of the dissipation signature in the bulk.

chapter 2

Mesoscopic transport at high magnetic field

Modern physics tells us that electric and magnetic fields are two faces of the same coin: the electromagnetic force. The previous section has been focused on the transport of charge carriers in two-dimensional systems under the effect of an electric field. In this section, we will enlarge the scope by also considering the influence of magnetic field. First, the quantum Hall effect (QHE) will be briefly introduced, both for semiconductor-based heterostructures and for graphene. Then the case of graphene, with its particularities, will be further detailed, as it sustains the major part of this thesis. Finally, one of the most promising applications of the QHE, namely quantum Hall interferometers, will be introduced.

However, before getting into the substance of *quantum* Hall effect, it may be convenient for some readers to have a quick reminder of its classical counterpart, the Hall effect. The story goes back to 1879 when Edwin Hall discovered that a magnetic field B applied perpendicularly to the plane of an electrical conductor results in a voltage difference transverse to the electrical current direction [43]. This effect arises due to the Lorentz force yielding a charge accumulation along the device edges perpendicular to the transport axis. When divided by the current, the transverse voltage gives the Hall resistance denoted R_{xy} (see Fig. 2.1a) that is proportional to B . Next, we will see what is meant by the *quantum* version of this effect.

2.1 The quantum Hall effect

In this section, I review the key features of Quantum Hall effect that are useful for this thesis, without entering into the details of the mathematical developments. A more in-depth description of this regime can be found in refs [26] and [44].

One century after the discovery of the Hall effect, Klaus von Klitzing observed a peculiar behavior of R_{xy} in a metal-oxide-semiconductor field effect transistor, measured at low temperature ($T = 1.5$ K) and high perpendicular magnetic field ($B = 15$ T) [45]. In such a device, applying a sufficiently large voltage on the metallic gate leads to the formation of a thin electron gas in the semiconductor, at the interface with the oxide. By varying this voltage, R_{xy} exhibited some plateaus, coinciding with a vanishing longitudinal resistance R_{xx} (Fig. 2.1b). Interestingly, these plateaus were found at $R_{xy} = h/(2ne^2)$ where h is the Plank constant, e the charge of the electron and n is an integer. In other words, they only depend on fundamental constants. Therefore, this new phenomenon was directly acknowledged for its application in metrology and its discoverer was awarded the Nobel prize in 1985.

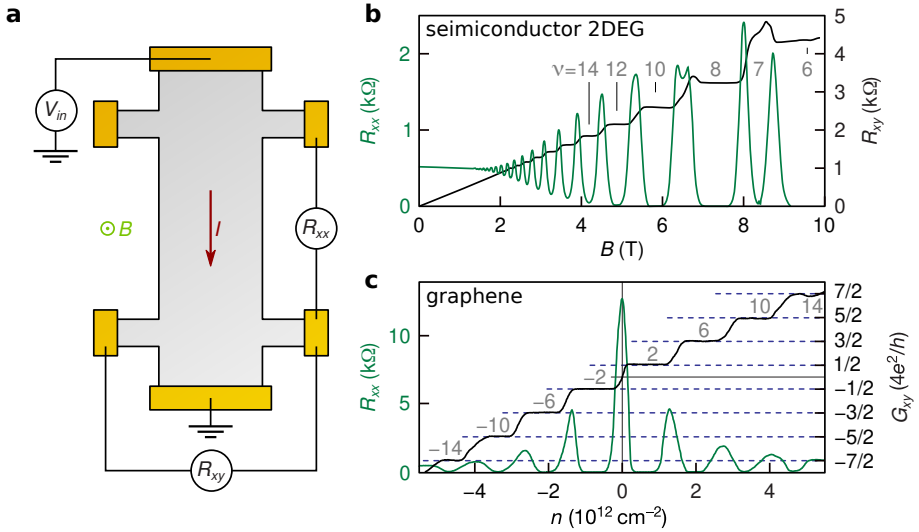


Figure 2.1: Observation of the quantum Hall effect (QHE). **a**, Schematic of a Hall bar, the archetypal device geometry to measure (quantum) Hall effect. It allows to measure both the longitudinal resistance R_{xx} and the transverse (Hall) resistance R_{xy} under a perpendicular magnetic field B . **b**, R_{xx} (green) and R_{xy} (black) evolution with B in a semiconductor-based 2DES holding in a InGaAs/InAlAs heterostructure. The filling factors are indicated in gray. Adapted from [4]. **c**, R_{xx} (green) and the Hall conductance G_{xy} (black) evolution with the charge carriers density n in graphene. $G_{xy} = R_{xy}^{-1}$ when $R_{xx} = 0$. The filling factors are indicated in gray. Adapted from [46].

Similarly to Klaus von Klitzing's experiment, varying the magnetic field at fixed charge carrier density also leads to a quantization of R_{xy} . This is illustrated in Fig. 2.1b where the longitudinal (black) and Hall (green) resistances are recorded in a InGaAs/InAlAs heterostructure at 100 mK. QHE has also been studied in graphene and presents some particularities due to the relativistic nature of the charge carriers in this material [46]. The conductance plateaus are indeed found at half integer values $G_{xy} = 4(n + 1/2)e^2/h$ when the carriers density is varied (Fig. 2.1c). The next sections aim to present the current state of knowledge explaining the QHE both in semiconductor-based 2DESs and in graphene.

2.1.1 Landau levels

2.1.1.1 In semiconductor-based 2DESs

Understanding the evolution of R_{xx} and R_{xy} in Fig. 2.1b requires a quantum formalism involving the magnetic field. For a semiconductor-based 2DES, this is achieved by performing the Peierls substitution in the Hamiltonian of Eq. (1.3) [47]. In that framework, the momentum operator is modified as $\hat{\mathbf{p}} \rightarrow \hat{\mathbf{p}} + e\mathbf{A}$ where $\mathbf{A}$ is the vector potential related to the magnetic field by $\mathbf{B} = \nabla \times \mathbf{A}$. The Schrödinger equation then writes

$$\left(\frac{(\hat{\mathbf{p}} + e\mathbf{A})^2}{2m^*} + U(\mathbf{r}) \right) \psi = E_n \psi. \quad (2.1)$$

By neglecting the potential term $U(\mathbf{r})$ in Eq. (2.1), that will be discussed in the next section, the eigenvalues of the Hamiltonian are quantized as [48]

$$E_n = \frac{e\hbar B}{m^*} \left(n + \frac{1}{2} \right), \quad (2.2)$$

where $n \geq 0$ is an integer. These energies are called the *Landau Levels* (LLs) and evolve linearly with B (Fig. 2.2a).

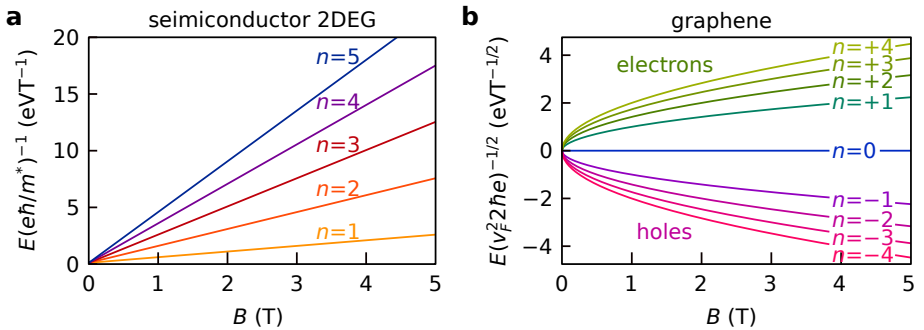


Figure 2.2: Evolution of the Landau levels (LLs) with the magnetic field. **a**, In a semiconductor-based 2DES, the LLs evolve linearly with B according to Eq. (2.2). **b**, In graphene, the LLs follow a $\sqrt{B}$ evolution according to Eq. (2.3).

2.1.1.2 In graphene

In graphene, the development leading to Eq. (2.2) does not hold since charge carriers obey to the Dirac equation. Nevertheless, a similar reasoning can be applied with the Dirac Hamiltonian in Eq. (1.13) and yields energy levels expressed as [26]

$$E_n = \pm v_F \sqrt{2e\hbar} \sqrt{nB} \quad (2.3)$$

where the $+$ sign stands for electrons, the $-$ sign for holes and $n \geq 0$. Interestingly, the $n = 0$ LL is located at the CNP and is filled both with electrons and holes. Moreover, contrary to Eq. (2.2), the LLs do not evolve linearly with B (Fig. 2.2b).

2.1.1.3 Zeeman splitting

So far, the LLs described in Eqs. (2.2) and (2.3) are degenerated in spin. At large magnetic field, however, each LL can be split due to Zeeman effect [49] and these expressions become [14]

$$E_n^\pm = E_n \pm \frac{1}{2} g^* \mu_B B, \quad (2.4)$$

where E_n is the n th LL given in Eq. (2.2) or (2.3), g^* is the Landé g -factor depending on the considered material and $\mu_B = e\hbar/2m^*$ is the Bohr magneton.

2.1.2 Quantum Hall edge channels

This section aims to make the link between the LLs illustrated in Fig. 2.2 and the presence of R_{xy} plateaus coinciding with a vanishing R_{xx} in the transport measurement shown in Figs. 2.1b,c.

2.1.2.1 A naive picture

As a starting point, we will look at the LLs evolution at the device edges. To do so, let us come back to Eq. (2.1) and, in particular, to the potential term $U(\mathbf{r})$ that has been neglected to obtain the LLs energies in Eq. (2.2). Two cases can be distinguished when looking at the variation of this term on a length scale of the order of the magnetic length, defined as

$$l_B = \sqrt{\frac{\hbar}{eB}}. \quad (2.5)$$

1. When $U(\mathbf{r})$ varies slowly over a distance l_B ($U(x + l_B) - U(x) \ll \hbar\omega_C$), it can be approximated as a constant term in Eq. (2.1) and the energy levels are given by

$$E_n(\mathbf{r}) = E_n + U(\mathbf{r}). \quad (2.6)$$

It means that the LLs of Eq. (2.2) follow the potential landscape. If we consider a real device (Fig. 2.3a made from a semiconductor-based 2DES for example), a smooth confinement potential at the edges (black

dashed line in Fig. 2.3b) will cause the LLs to bend and to cross the Fermi energy (Fig. 2.3c). This leads to the formation of one-dimensional and unidirectional quantum Hall edge channels (QHECs) depicted along one edge in Fig. 2.3d and in the whole sample in Fig. 2.3a.

2. When $U(\mathbf{r})$ varies quickly over a distance l_B ($U(x + l_B) - U(x) \gtrsim \hbar\omega_C$), it can no longer be approximated as a constant term in Eq. (2.1). This is the case of graphene, for example, where the confinement potential raises abruptly at the physical edges. In that situation, QHECs also appear along the device borders. The simulated wavefunctions corresponding to each of these QHECs are shown in Fig. 2.4. As in Fig. 2.3, the number of QHECs is given by the number of LLs below the Fermi energy. In graphene, however, along with the spin degeneracy, each LL is also valley-degenerated (K and K' in Fig. 1.4) and therefore yields two different wavefunctions. For $n = 0$, one of the valleys contributes for holes and the other for electrons. Importantly, the width of the QHECs wavefunctions is on the order of the magnetic length defined in Eq. (2.5).

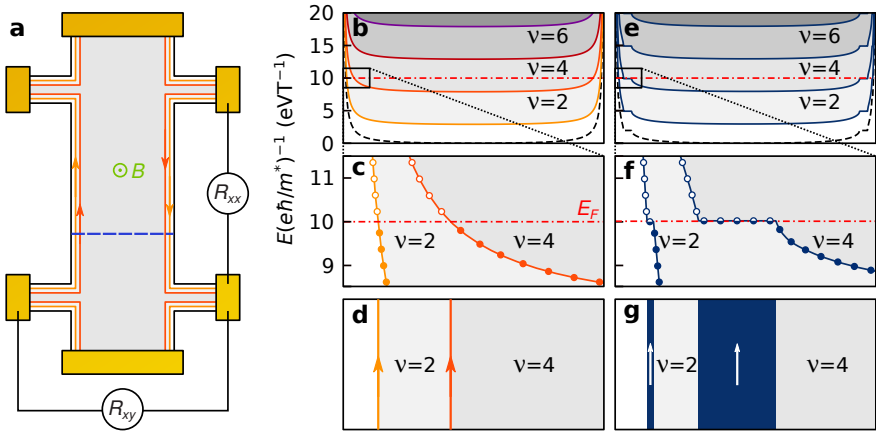


Figure 2.3: Formation of quantum Hall edge channels (QHECs). **a**, Schematic of a Hall bar under a perpendicular magnetic field B . QHECs run clockwise along the device borders. **b**, Evolution of the Landau levels (LLs) along the blue dashed line in **a**. The LLs follow the confinement potential landscape (black dashed line) that rises at both edges. **c**, Zoom on **b** where the LLs cross the Fermi energy E_F . The colored circles represent the filled states whereas the white circles represent the empty states. **d**, Schematic of the one dimensional QHECs appearing where the LLs cross E_F . **e-g**, Same as **b-d** by considering electrostatics. With a self-consistent treatment, the confinement potential is found to form plateaus where the LLs cross E_F . It results in compressible strips where the states are partially filled (represented with half-filled circles in **f**).

2.1.2.2 Compressible and incompressible strips

Figures 2.3a-d illustrate the formation of one-dimensional QHECs in the quantum Hall (QH) regime. Despite its benefits in term of visualization, this picture

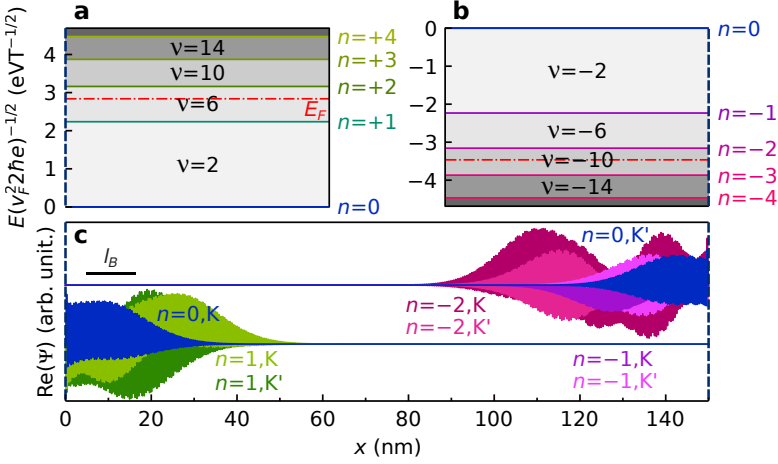


Figure 2.4: Quantum Hall edge channels (QHECs) in graphene. **a,b**, Landau levels (LLs) in a 150 nm-wide graphene nanoribbon for electrons (**a**) and holes (**b**). **c**, Simulated QHECs wavefunctions associated to the each LL below the Fermi energy E_F . Each LL is valley-degenerated (K and K'), resulting in two QHECs per LL, excepted for the $n = 0$ LL where the valley K contributes for electrons and the valley K' contributes for holes. The width of the wavefunctions are in the order of the magnetic length l_B . The results have been obtained with the Kwant package (section 3.3.2)

is too naive as it completely ignores the effective width of the QHECs in a real system for two reasons. The first one has already been discussed in Fig. 2.4 and concerns the wavefunctions associated to the QHECs, whose width is $\sim l_B$. The second reason is electrostatics. In 1992, Chklovskii *et al.* have shown with a self-consistent electrostatic treatment that charges in the 2DES rearrange around the locations where LLs cross the Fermi energy to form finite-width conducting regions, the *compressible strips*, separated by insulating regions, the *incompressible strips* (Figs. 2.3e-g) [50, 51]. Experimentally, compressible strips have been imaged with different scanning probe techniques in a semiconductor-based 2DES [52–54] and in graphene [55].

2.1.2.3 Edge channels and quantized conductance

The formation of QHECs is the key to understand the transport measurement presented in Figs. 2.1b,c. Indeed, each QHEC contributes for $g_s g_v e^2/h$ to G_{xy} , where e^2/h is the quantum of conductance introduced in section 1.1.2.3, $g_s = 2$ the spin degeneracy (that can be lifted by Zeeman splitting, as discussed in section 2.1.1.3) and g_v the valley degeneracy ($g_v = 1$ in a semiconductor-based 2DES and $g_v = 2$ in graphene). It explains the evolution of G_{xy} (and R_{xy}) in plateaus. This quantization has profound implications that will be further detailed in section 2.1.3.

Furthermore, the one-dimensional nature of the QHECs, combined with the insulating bulk (see Fig. 2.3a), implies that the backscattering of charge carriers

flowing in the QHECs is prohibited. It explains why a zero R_{xx} coincides with the G_{xy} plateaus in Figs. 2.1b,c. It remains however to describe the mechanism causing the backscattering at the transition between two plateaus. It is the subject of section 2.2 for the general considerations about this mechanism and section 2.3 for the particular case of graphene.

2.1.3 Filling factors



In this section, I have chosen to introduce the *filling factor* by departing from the framework of topological insulators. Indeed, QH effect is the first studied manifestation of this new state of matter. However, I will stick to a short qualitative picture of this vast domain because it is both beyond the scope of this thesis and of my theoretical skills.

2.1.3.1 The Chern number in the TKNN formalism

In the previous section, we have seen that each non-degenerated QHEC contributes for e^2/h to the total Hall conductance, no matter the confinement potential shape along the edges. This property, simple at first sight, hides fundamental considerations shared by a new class of materials: *the topological insulators*. The research on these materials takes its roots in 1982 when Thouless, Kohmoto, Nightingale and den Nijs (TKNN) studied theoretically the Hall conductance of a crystal where the magnetic length l_B is similar to the lattice parameter a (that determines the scale on which the atomic potential $U(\mathbf{r})$ varies) [56]. As discussed in section 2.1.2.1, it makes the computation of the LLs energy non-trivial and it was already known since 1976 that it leads to a fractal evolution of the LLs with B (each LL being split in sub-energy bands), called the Hofstadter's butterfly [57]. The contribution of TKNN was to show that G_{xy} exhibits plateaus at Ce^2/h when the Fermi energy stands in one of the energy spectrum sub-gaps [56]. In that expression, C is an integer called the TKNN invariant. Since then, their formalism has been generalized for any band structure and C takes the name of Chern number [58]. This appellation makes reference to the field of topology, in mathematics, from which the calculation of this invariant originates.

2.1.3.2 The topological insulators

Next, an intuitive picture will be used to illustrate the topological character of the Chern number. First, let us consider the group of conventional insulators (CIs), characterized by the presence of an energy gap between the valence and the conduction bands. They form a topological class in the sense that any CI band structure can be obtained by deforming continuously the band structure of another CI, without closing the gap. CIs are also topologically equivalent to the vacuum. On the contrary, the band structure of a topological insulator (TI) can not be deformed towards a CI band structure without closing the gap. It implies that states will cross the Fermi energy at the interface between a TI and

a CI (or vacuum). The Chern number measures the number of states crossing the Fermi energy [58].

Mathematically, the band structure of a CI is equivalent to a sphere while that of a TI is equivalent to a g -holes torus, where g is equivalent to the Chern number. Indeed, the sphere can not be deformed continuously to the torus.

2.1.3.3 Integer filling factors

The above discussion reminds the outcome of section 2.1.2, where the QHECs were shown to appear along the borders of the sample, due to the crossing between the LLs and the Fermi energy. Indeed, a 2DES in the QH regime is a topological insulator where each LL (labeled n) has a Chern number $C_n = 1$. For historical reasons, the total Chern number is called the filling factor and is defined as

$$\nu = \sum_{n=0}^N C_n = N, \quad (2.7)$$

where N is the number of undegenerated LLs below the Fermi energy. This filling factor labels each insulating region in Figs. 2.3 and 2.4. In a semiconductor-based 2DES, they take the values $\nu = 2N$, because each LL is spin-degenerated while in graphene, $\nu = 4(N + 1/2)$ because each LL is spin- and valley-degenerated and the $n = 0$ LL is split among holes (negative N) and electrons (positive N). Following the TKNN formalism, the value of the conductance (resistance) plateaus in Figs. 2.1b,c can be understood in term of filling factor with $G_{xy} = \nu e^2/h$.

2.1.3.4 Continuous filling factors

In the previous section, the filling factor has been defined from the Chern number and is therefore an integer. However, things would have been too simple if only one definition existed. Indeed, it is common practice to use a continuous filling factor in the literature. In this definition, the filling factor evolves with the magnetic field and its value is given by the position of the Fermi energy E_F compared to the LLs. When E_F is located perfectly in between two LLs, the filling factor takes the integer value given by Eq. (2.7). In the case of semiconductor-based 2DESs, it implies that $E_n = E_F$ in Eq. (2.2) for $n = (\nu - 1)/2$ with $\nu = 2N$ (spin degeneracy). Therefore, the continuous filling factor is given by

$$\nu = \frac{2m^* E_F}{e\hbar B}. \quad (2.8)$$

For graphene, the continuous filling factor is obtained from Eq. (2.3) for $n = \nu/4$ with $\nu = 4(N + 1/2)$ and writes

$$\nu = \pm \frac{2}{e\hbar B} \left(\frac{E_F}{v_F} \right)^2. \quad (2.9)$$

These definitions are particularly useful to substitute B or E_F for the filling factor when plotting transport measurements. By doing so, the different QH

states are identified more easily and measurement obtained by varying E_F at different B can be directly compared on the same scale.

To close this section, a more intuitive picture of the filling factor will be drawn from Eqs. (2.8) and (2.9) by considering the relation between E_F and the charge carrier density $n_c = N_c / A$, where N_c is the number of charge carriers in the device area A . This relation is $n_c = m^* E_F / \pi \hbar^2$ for semiconductor-based 2DESs [14] and $n_c = E_F^2 / \pi \hbar^2 v_F^2$ for graphene [59]. By defining the number of magnetic flux quanta ϕ_0 as $N_\phi = BA / \phi_0$, Eqs. (2.8) and (2.9) become simply

$$\nu = \frac{N_c}{N_\phi}. \quad (2.10)$$

When the filling factor is an integer, it therefore gives the number of charge carriers per magnetic flux quantum.

2.1.4 Beyond quantum Hall effect

In this section, I briefly present the QH effect's *family* with some examples found in two-dimensional materials (graphene-based or others). It is not of central interest in this thesis but the experimental characterization exposed in chapter 5 could also reveal microscopic breakdown mechanisms in these systems as they have a lot of similarities with the integer QH effect. Furthermore, there will be further mentions of the fractional QH effect later in the manuscript.

2.1.4.1 The fractional quantum Hall effect

In 1982, Tsui *et al.* discovered a new QH plateau, coinciding with a vanishing R_{xx} , at a filling factor $\nu = 1/3$ in an ultra-clean GaAs-AlGaAs-based 2DES [60]. This state originates from the opening of a gap at $1/3$ filling of the lowest LL (spin polarized due to Zeeman splitting - section 2.1.1.3). One year later, Laughlin proposed a first model to explain this puzzling observation by taking into account the electron-electron Coulomb interactions [61]. It led him to a ground state given by the wavefunction

$$\psi_{\nu=1/(2s+1)} = \prod_{j < k} (z_j - z_k)^{2s+1} \exp \left(-\frac{1}{4} \sum_l |z_l|^2 \right), \quad (2.11)$$

where $\nu = 1/(2s+1)$ is the filling factor with s an integer (here $s = 1$) and $z_j = x_j - iy_j$ is the coordinate of the j -th electron in the complex notation.

Interestingly, the probability density function $|\psi_{\nu=1/(2s+1)}|^2$ vanishes for $z_j = z_k$, meaning that two electrons can not occupy the same position. Hence, the wavefunction in Eq. (2.11) minimizes the Coulomb energy. Furthermore, when the j -th electron performs a loop around the k -th electron, it acquires a phase $2\pi m$ similar to the Aharonov-Bohm phase of an electron encircling m

magnetic flux quanta [14]. From this observation, electrons are described as *attached* to $2s$ (here 2) magnetic flux quanta to form new ground state excitations called *composite fermions* [44, 62, 63], described by Eq. (2.11) and holding a fractional charge $1/(2s + 1)$ [64–67]. In this framework, each composite fermion experiences an effective magnetic field

$$B_{\text{eff}} = B - B_{\nu=1/2s}, \quad (2.12)$$

where $B_{\nu=1/2s}$ is the magnetic field at $\nu = 1/2s$. Indeed, from Eq. (2.10), this filling factor corresponds to a ratio of one electron for two magnetic flux quanta, i.e., the ingredients of a composite fermion. The $\nu = 1/3$ state in a magnetic field B can therefore be described as an effective $\nu_{\text{eff}} = 1$ state in a magnetic field B_{eff} where charge carriers are composite fermions instead of electrons or holes. Note that for $B = B_{\nu=1/2}$, $B_{\text{eff}} = 0$ and the state $\nu = 1/2$, at the very middle of the lowest LL, is a metallic state.

The advantage of the composite fermions picture is its ability to explain the zoo of fractional filling factors that has been discovered since 1982 [68] and that can be generalized under the relation

$$\nu = \frac{\nu_{\text{eff}}}{2s\nu_{\text{eff}} + 1}, \quad (2.13)$$

where ν_{eff} is the effective filling factor in the field B_{eff} for a composite fermion holding $2s$ magnetic flux quanta. However, the development leading to Eq. (2.13) is only valid in the lowest LL. In the third LL ($n = 2$), fractional states with an even denominator, namely $\nu = 5/2$ and $7/2$, were found. As will be discussed in section 2.4.4, these states could offer groundbreaking properties [69].

The series of fractions given by Eq. (2.13) is illustrated in Fig. 2.5c with a longitudinal conductance G_{xx} vs ν obtained in a graphene Corbino. With this geometry, the device is free from edges. Since there is no QHEC, G_{xx} directly reflects the conductivity in the bulk. The large number of the well-defined fractional QH states obtained with Corbino geometries [70, 71], has never been reached with the usual Hall bar geometry in graphene. It highlights the detrimental role played by edges in this material.

2.1.4.2 The quantum spin Hall effect

In 2005, Kane and Mele studied a toy model of graphene where they introduced a spin-orbit coupling [72]. It leads to the opening of a gap at the K and K' points. However, the two gaps are not topologically equivalent and the obtained band structure can not be deformed continuously to the topologically trivial gaped band structure obtained with a potential imbalance between the two sublattices. As described in section 2.1.3.2, it means that conducting states should appear along the edges. In particular, Kane and Mele showed that these states are two counterpropagating spin-polarized QHECs leading to a Hall conductance $G_{xy} = 2e^2/h$. This new phase of matter has therefore been named *quantum spin Hall effect*.

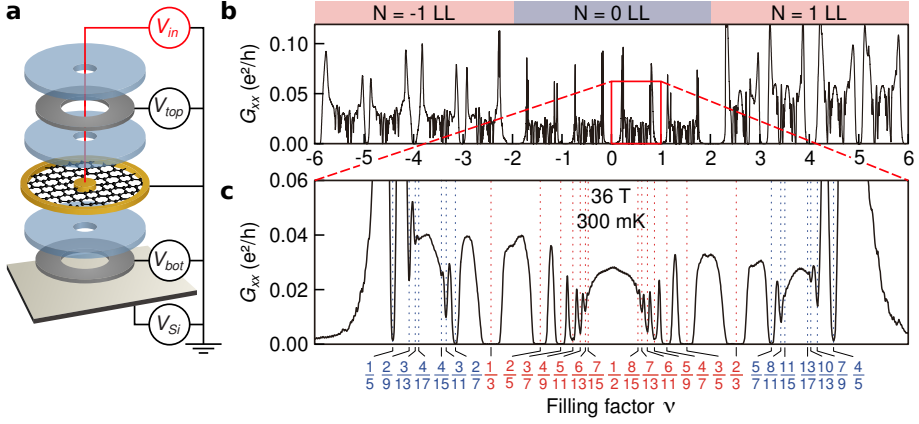


Figure 2.5: Transport through a Corbino in the quantum Hall regime. Adapted from [71]. **a**, Schematic of the Corbino device consisting in a graphene disc, contacted at its center and its perimeter with metallic contacts (in gold), encapsulated between hBN layers (in blue). Graphene charge carriers density is controlled with graphite bottom and top gates biased at V_{bot} and V_{top} respectively. **b**, Longitudinal conductance G_{xx} measured between the inner and outer contacts of graphene at $B = 36$ T and $T = 300$ mK. **c**, Zoom on **b**. The dips in G_{xx} correspond to the fractional states concomitant with an insulating bulk.

The quantum spin Hall effect (QSHE) has never been observed in graphene alone because its spin-orbit coupling is negligible but signatures of QSHE have been found in graphene on WS_2 thanks to proximity effects [73, 74]. It has also been observed in 2017 in another 2D material: the monolayer WTe_2 [75, 76]. Notably, the effect survives up to $T = 100$ K.

2.1.4.3 The quantum anomalous Hall effect

The quantum anomalous Hall effect is another example of QH effect emerging without external magnetic field, a possibility already envisioned by Haldane in 1988 [77]. This effect requires a material which is both a ferromagnet and a topological insulator. Such conditions have been found together in 2013 in topological thin films [78] and in 2019 in a magic angle twisted bilayer graphene aligned with a hBN lattice [79, 80]. In this last system, both the spin-orbit couplings of graphene and hBN are negligible, suggesting that the observed ferromagnetism is purely orbital.

2.2 Topological breakdown

The previous section has remained focused on the topological state characterized by a quantized Hall conductance ($G_{xy} = \nu e^2/h$) concurrent with vanishing longitudinal resistance ($R_{xx} = 0$). However, the deviation of G_{xy} and R_{xx} from these values, referred to as topological breakdown in this thesis, has been left aside so far. In the QH regime, the transition between two successive plateaus

in Figs. 2.1b,c is the textbook case of topological breakdown and will be discussed in this section in the case of semiconductor-based 2DESs under the light of three mechanisms: the percolation through the bulk, the coupling of counterpropagating QHECs through antidots and the QHECs equilibration with the contacts.

The breakdown of a QH state sometimes refers to the backscattering of charge carriers through the bulk enabled by a strong source-drain DC voltage ($|V_{sd}| \gg 0$), i.e., exceeding the energy difference between two LLs. This definition is not used in this thesis as all the measurements are performed at or close to the equilibrium ($V_{sd} \sim 0$).

2.2.1 Percolation through the bulk

2.2.1.1 Topological breakdown and conducting bulk

The topological breakdown of a QH state, characterized by the deviation of R_{xx} from zero, implies the formation of a conducting path between the counterpropagating QHECs flowing along the opposite edges of a sample (left and right sides in Fig. 2.1). Hence, the backscattering of charge carriers towards the injection contact is made possible. In other words, topological breakdown originates from the loss of a perfectly insulating bulk.

The link between the QHECs topological protection and the conducting state of the bulk is established in Fig. 2.6 by comparing microwave impedance microscopy (MIM) maps (Figs. 2.6b-d) obtained on a semiconductor-based 2DES island (inset of Fig. 2.6a) with reference R_{xx} and G_{xy} curves measured in a Hall bar made from the same 2DES (Fig. 2.6a) [81]. In this technique, the imaginary component of the signal reflected at the tip (MIM-Im) increases when the scanned area becomes more conductive. At the transition between two Hall plateaus, the MIM-Im map exhibits a large signal (light blue shades) in the whole bulk which indicates that it is entirely conducting. On the contrary, a well defined QH state ($R_{xx} = 0$) is accompanied with an insulating bulk and conducting edges (Fig. 2.6c). Finally, Fig. 2.6d shows the map obtained at the onset of the plateau and the conducting region along the edges is strongly enlarged. These results are consistent with the measurements obtained from other scanning probe techniques [53, 54].

The results of Fig. 2.6 can be understood in the scope of compressible and incompressible states introduced in section 2.1.2.2. The observed conducting bulk would simply arise from the alignment between one of the LLs and the Fermi energy, causing the formation of a compressible state in the whole sample. However, with such a picture, it remains to explain the width of the R_{xx} peaks in Figs. 2.1b,c and 2.6a. Furthermore, microwave microscopy can not resolve details below ~ 100 nm. A more convincing picture of the topological breakdown through the bulk has been provided by other scanning

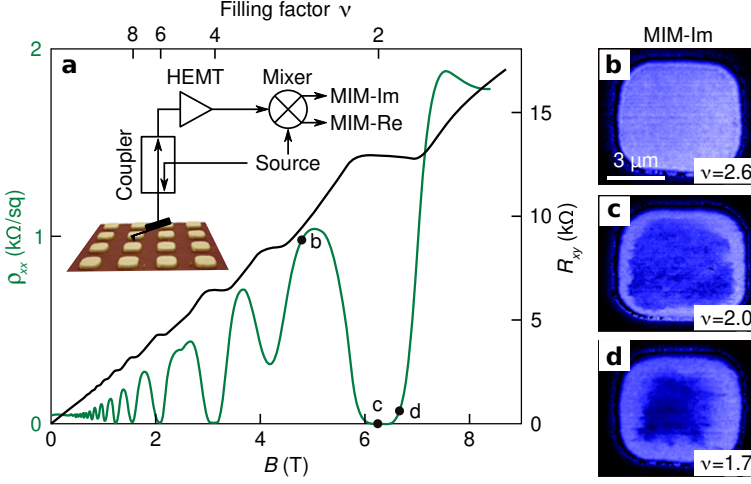


Figure 2.6: Imaging the edge and bulk conductivity. Adapted from [81]. **a**, Resistivity ρ_{xx} (green curve) and Hall Resistance R_{xy} (black curve) as a function of B measured in a Hall bar built from a GaAs/AlGaAs 2DES at $T = 2.3$ K. The inset shows the schematic of a microwave microscope measuring an array of isolated islands (light brown) carved out in the same GaAs/AlGaAs sample. The reflected signal coming from the tip is demodulated in an imaginary component (MIM-Im) and a real component (MIM-Re). **b-d**, MIM-Im maps obtained by scanning the tip above one island for the magnetic fields indicated in **a**. A large MIM-Im signal (light blue) indicates a large conductivity. The observation of a conductive bulk (**b**) occurs when ρ_{xx} is large in **a** while conducting edges with an insulating bulk (**c**) are observed when $\rho_{xx} \sim 0$ in **a**. At the onset of the R_{xy} plateau in **a**, the conducting region along the edges enlarges (**d**).

probe measurements performed in the QH regime, offering a better spatial resolution as discussed in next section.

2.2.1.2 Imaging percolation through quantum Hall liquids

Figures 2.7a-g display a set of high resolution maps obtained by scanning tunneling spectroscopy performed on a 2DES in the QH regime [82]. The measured signal (transconductance dI/dV between the sample and the tip) probes the local density of states (LDOS) in the bulk. When varying the sample bias voltage, the patterns of available states (large LDOS) evolve cyclically between a few isolated islands (Figs. 2.7a,b,f,g) and a dense network (Figs. 2.7c-e), with recursive features observed after one cycle (white arrows in Figs. 2.7a,b,f,g) [82, 83]. Obviously, the bulk is not evenly conducting or insulating and the inhomogeneous LDOS in Figs. 2.7a-g can be understood when considering the effect of disorder, illustrated in Fig. 2.8.

In particular, Fig. 2.8a presents a map of a simulated inhomogeneous potential, associated with disorder, with edges on both sides of the considered device region (blue dashed lines). This disorder induces a broadening of the LLs (Fig. 2.8b) [14, 48]. Indeed, assuming that the LLs follow the potential landscape

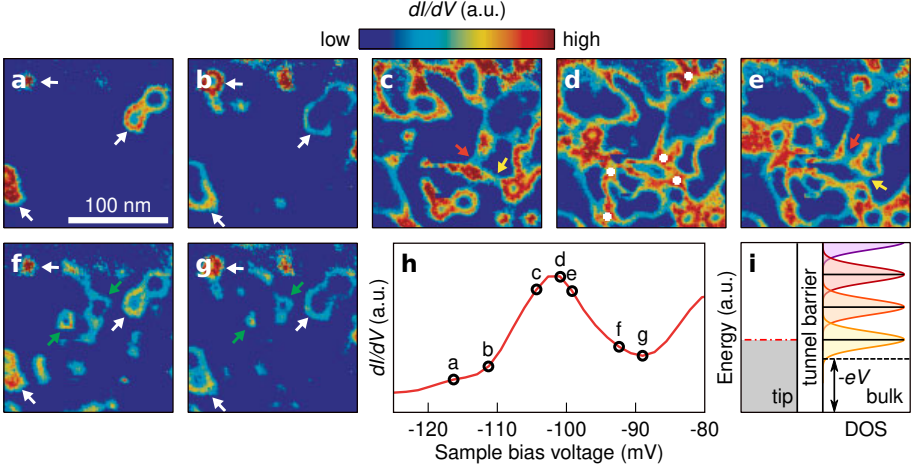


Figure 2.7: Scanning tunneling spectroscopy of a compressible bulk. Adapted from [82]. **a-g**, dI/dV maps obtained by scanning the tip above a 2DES in the QH regime for different sample bias voltages V (shown in **h**) at $B = 6$ T. I is the tunneling current between the tip and the sample. The dI/dV signal is proportional to the LDOS. The white arrows indicate recursive patterns in the LDOS when varying V , associated to the same potential inhomogeneities. The yellow and red arrows indicate saddle points where tunneling of charge carriers is made possible. **h**, Spatially averaged dI/dV signal with the sample bias voltage V . The large signal indicates that the tip potential is aligned with a LL (here the $n = 0$ LL at $V \sim -100$ mV). **i**, Schematic of the tunnel barrier between the tip and the sample. The tip potential is indicated with a red dashed line on the left and the LLs DOS is shown on the right.

(Figs. 2.8c-f), they fluctuate in an energy window given by the disorder amplitude. It results in different QHECs configurations, given by the position of the LLs according to the Fermi energy E_F . First, when E_F is in the energy gap between two LLs (Figs. 2.8c,e), QHECs appear along the edges (the *extended states*), as in Fig. 2.3, but also form isolated islands (the *localized states*) in the bulk (Figs. 2.8g,i and j,l in the compressible picture). It explains the experimental maps in Figs. 2.7a,b,f,g. Second, when E_F is aligned with one of the LLs (Fig. 2.8d), it results in a QHECs network connecting the extended states along the opposite edges (Figs. 2.8h,k in the compressible picture), similarly to the experimental maps in Figs. 2.7c-e. The peak in the averaged dI/dV curve in Fig. 2.7h therefore corresponds to an alignment between the tip potential and the lowest LL (Fig. 2.8i), and the width of the peak is linked to the disorder amplitude (width of the LLs in Fig. 2.8i).

In the picture exposed above, the network in Fig. 2.8k, often referred to as *QH liquid*, forms conduction channels between the counterpropagating extended states at the opposite device edges. Scanning charge accumulation [83, 84] and scanning single-electron transistor [85] techniques have shown that the transport of charge carriers in the QH liquid occurs via percolation through localized states interconnected via tunneling paths. These paths are associated

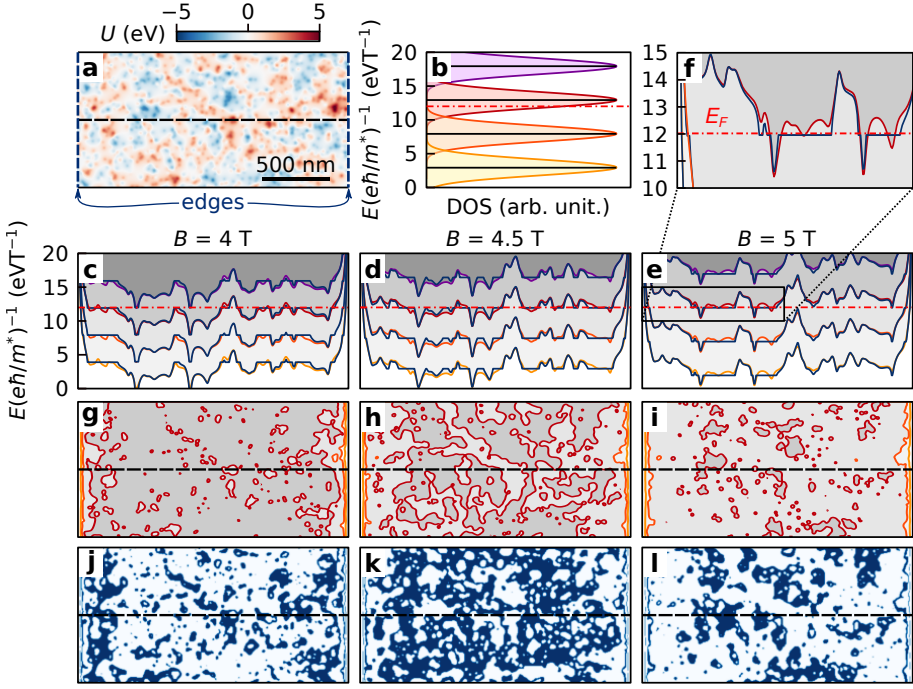


Figure 2.8: Percolation through the bulk. **a**, Simulated disordered potential landscape U obtained by randomly placing Lorentzian functions with amplitudes of ± 1.5 eV and radii of 16 nm. The edges of the sample are shown with blue dashed lines. **b**, Broadened LLs (with colors corresponding to Fig. 2.2a) due to the disordered potential. **c-e**, Evolution of the LLs with (in dark blue) and without (colors of **b**) the effect of electrostatics, along the black dashed line in **a** for $B = 4$ T (**c**), 4.5 T (**d**) and 5 T (**e**). The confinement potential of Fig. 2.3b has been added on top of the potential U . **f**, zoom on **e** that highlights the flattened LLs at the crossing with the Fermi energy E_F due to electrostatics. **g-i**, QHECs appearing where the LLs cross E_F (see the corresponding panels above). The colors correspond to the filling factor indicated in Fig. 2.2. **j-l**, Same as **g-i** when electrostatics is taken into account. The compressible strips are shown in dark blue and the incompressible strips in white.

with saddle points in the inhomogeneous potential landscape associated to disorder (yellow and red arrows in Figs. 2.7c-e).

2.2.1.3 Back to the transport measurements

The topological breakdown caused by percolation of charge carriers through a conducting bulk allows to understand the transition between two plateaus in the transport curves of Fig. 2.1, ignored in the discussion of section 2.1.2.3. The R_{xx} peaks are the logical consequence of the percolation that enables the backscattering and their width is related to the disorder amplitude in the sample.

2.2.2 Coupling through antidots

In the previous section, we have seen that the topological breakdown of a QH state occurs by the percolation of charge carriers through a network of localized states in the bulk. In the following, we will focus on the case where a single localized state couples two QHECs brought in close proximity.

2.2.2.1 Artificial antidot

Since the early 90's, several experiments have studied the coupling between counterpropagating QHECs through a localized state pinned at the location of a patterned antidot [86–90] (Fig. 2.9a). An antidot is associated to a QHEC loop in which charge carriers are confined, formed as one or more LLs cross the Fermi energy due to a bump in the potential landscape (inset of Fig. 2.9b). This configuration has been achieved artificially in a GaAs/AlGaAs heterostructure with negatively-biased gates (in blue in Fig. 2.9a) depleting the underneath high-mobility 2DES. Hence, the QHECs are tunnel-coupled to the central antidot (red dashed lines in Fig. 2.9a). The resulting R_{xx} curve is shown in Fig. 2.9b. It exhibits two interesting features, detailed hereafter.

For a magnetic field $B < 0.95$ T, the magnetoresistance forms a plateau at $R_{xx} = 0.05h/e^2$. This observation is explained by the perfect backscattering of one of the QHECs towards the opposite edge of the device on both sides of the constriction. It leads to the formation of a region of filling factor ν_C in the vicinity of the constriction and two regions of filling factor ν_B in the bulk (Fig. 2.9a). In this configuration, the magnetoresistance is given by [91, 92]

$$R_{xx} = \frac{h}{e^2} \left(\frac{1}{\nu_C} - \frac{1}{\nu_B} \right). \quad (2.14)$$

In Fig. 2.9b, the $0.05h/e^2$ plateau corresponds to $\nu_B = 5$ and $\nu_C = 4$.

For $B > 0.95$ T, R_{xx} exhibits sharp oscillations, whose amplitude increases with B . As will be further discussed in section 2.4, constructive interferences in the antidot occur each times an integer number of quantum of magnetic fluxes $\phi_0 = h/e$ are enclosed in the antidot. Therefore, the oscillation period should be $\Delta B \sim \phi_0/A$, where A is the antidot area considered constant. However, this relation misses two considerations:

- The area A also increases with B (as the LL in the inset of Fig. 2.9b is shifted upwards in energy). Hence, the tunnel paths between the antidot and the QHECs in the constriction are strengthened, leading to an increase of the oscillations amplitude in Fig. 2.9b.
- The measured oscillations period also depends on the number of QHECs transmitted through the constriction and turns out to be $\Delta B \sim \phi_0/\nu_C A$ [90].

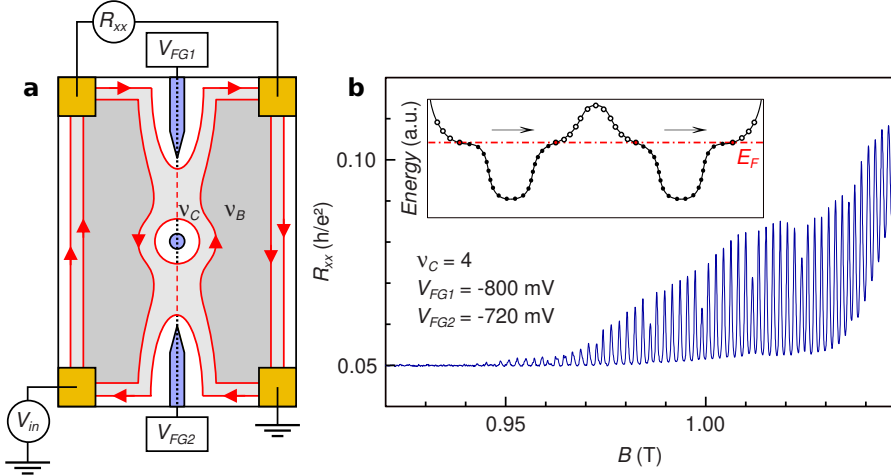


Figure 2.9: Transport through an artificial antidot. Adapted from [90]. **a**, Schematic of the device created in a GaAs-GaAlAs heterostructure. Top gates (in blue) deplete the buried 2DES and lead to the formation of a constriction with, at the middle, an antidot in the QH regime. The QHEC in the antidot is tunnel-coupled (red dashed lines) to the QHECs in the constriction. Backscattered QHECs on both side of the constriction separate the device in a bulk region (ν_B) and a constriction region (ν_C). **b**, R_{xx} curve as a function of B . Inset: schematic of the upper LL along the black dotted line in **a**. The filled states are represented with black circles and the empty states with white circles. QHECs appear where the LL crosses the Fermi energy E_F (red dashed line).

To explain the last observation, one must consider the electron-electron interactions. They imply that an energy cost should be paid to add N charge carriers in the localized state associated to the antidot. In the QH regime, this cost is given by the charging energy [93–96]

$$U(N) = \frac{1}{2C_\Sigma} \left(qN - q\nu_c \frac{\delta\phi}{\phi_0} + C_g V_g \right)^2, \quad (2.15)$$

where C_Σ is the antidot self-capacitance, q the electron ($-e$) or hole ($+e$) charge, $\delta\phi$ the flux variation in the antidot and $C_g V_g$ the charge imbalance induced by the gate voltage V_g coupled to the antidot by the capacitance C_g . When the system is at equilibrium, charge-discharge events can occur when the antidot charging energy is equal for N and $N + 1$ charge carriers. In other word, the antidot is conducting when $U(N) = U(N + 1)$. When this condition is not met, the antidot experiences Coulomb blockade (section 1.1.4), preventing any current to go through it.

The resonance condition $U(N) = U(N + 1)$ has been studied in detail thanks to scanning tunneling spectroscopy (STS) performed on antidots in graphene lying on top of hBN [97, 98]. Antidots have been created by injecting charges in the underneath hBN layer to locally induce a hill or a dip of potential in graphene (Figs. 2.10a-c). In the QH regime, it results in compressible

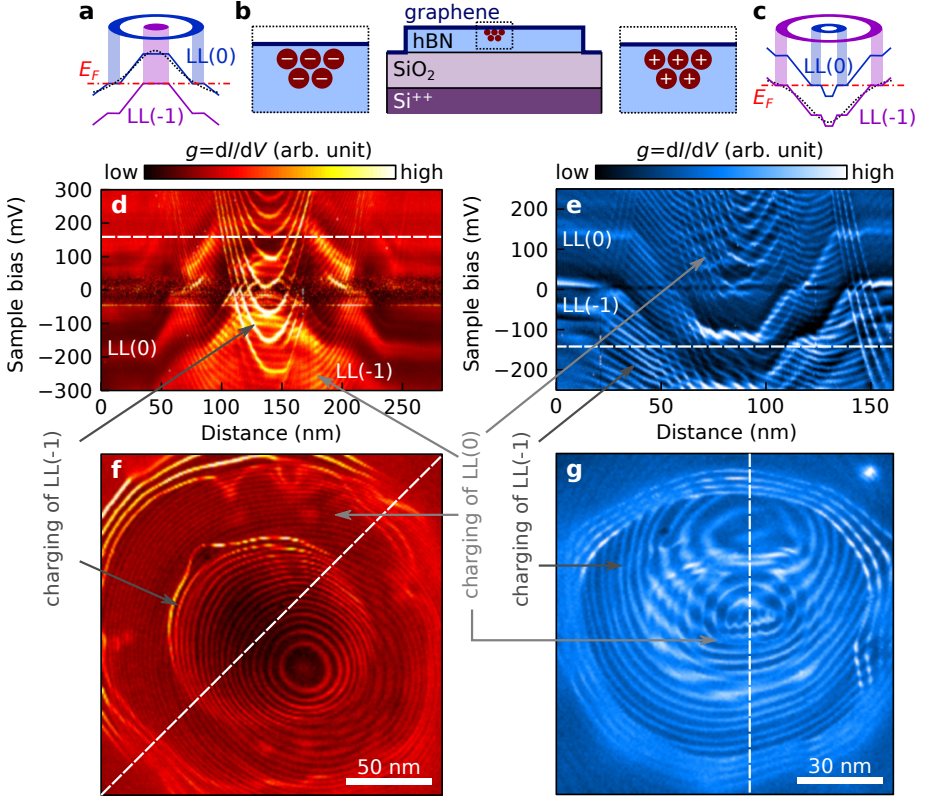


Figure 2.10: Scanning tunneling spectroscopy of an antidot. Adapted from [97]. The data have been obtained with $B = 8$ T at $T = 4.3$ K. **a**, Schematic of the potential (black dotted line), the $n = -1$ LL (in purple) and $n = 0$ LL (in blue) due to the presence of negative charges. It results in compressible rings (shown on top) where the LLs cross the Fermi energy E_F . **b**, Schematic of the sample where charges have been introduced in the hBN located underneath the graphene layer. **c**, Same as **a** for positive charges. **d,e**, Scanning tunneling spectroscopies (tunneling conductance $g = dI/dV$ between the tip and the sample as a function of the tip position and the sample bias) for negative (**d**) and positive (**e**) charges in hBN. The tip is scanned along the white dashed lines in **f,g**. **f,g**, Scanning tunneling microscopy maps obtained by measuring g above the antidot.

rings where the LLs cross the Fermi energy (Figs. 2.10a,c). Spectroscopies have been obtained by measuring the tunneling conductance $g = dI/dV$ between the STS tip and the sample, as a function of the tip position and the sample bias (Figs. 2.10d-e). The obtained maps feature two remarkable signatures. First, narrow peaks in g draw two sets of parabolic lines in the map, with different periodicities along the sample bias-axis. It corresponds to the resonance conditions reached when a charge is added in the $n = -1$ or $n = 0$ LL. These resonances form concentric circles in the $x - y$ maps in Figs. 2.10f-g. Second, the background presents broad peaks in g , evolving independently from the resonances. They correspond to the LLs evolution along the antidot potential and feature plateaus associated to the compressible strips [98].

2.2.2.2 Natural antidot

Antidots, coupling counterpropagating QHECs brought in close proximity by a constriction, have also been shown to appear naturally in devices, due to fluctuations in the potential landscape [4, 99, 100]. Scanning gate microscopy (SGM), presented in details in section 3.1, has been used to explain oscillations in the R_{xx} curve measured in an InGaAs/InAlAs quantum ring (Fig. 2.11a). The resulting SGM maps exhibit concentric features in the vicinity of one of the constrictions (Figs. 2.11b-d). They are the signatures of a localized state (here associated to an antidot) pinpointed by the center of the rings. The evolution of those signatures with temperature (Fig. 2.11e) indicate that at least part of the antidots operate in the CB regime. Note that the presence of different temperature dependence (Fig. 2.11e) indicates that other mechanisms may be at play, beyond CB (further discussed in chapter 6).

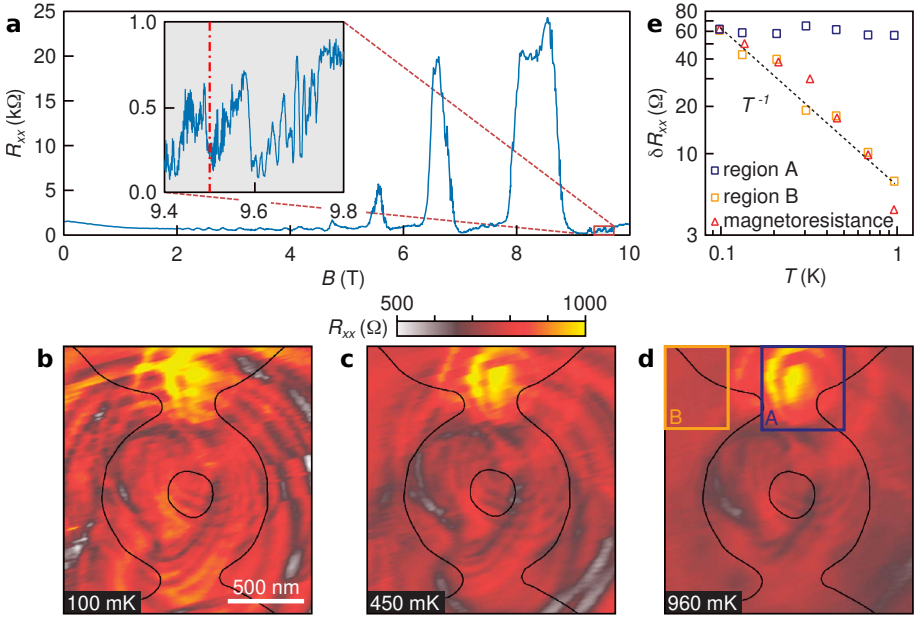


Figure 2.11: Scanning gate microscopy to pinpoint natural antidots. Adapted from [4]. **a**, R_{xx} curve as a function of B in a nGaAs/InAlAs quantum ring. Inset: zoom on the curve that exhibits high-frequency oscillations in B . **b-d**, SGM map of the quantum ring at different temperatures T . The large R_{xx} concentric rings (region A in **d**) persist when T is raised. **e**, T -evolution of the R_{xx} variations (δR_{xx}) in the zones A and B in **d**.

2.2.3 Equilibration with the contacts

In theoretical works studying the transport of charge carriers in the QH regime, the contacts are usually modeled as the smooth extension of the 2DES (see Fig. 2.12a). In practice, however, obtaining well-defined G_{xy} plateaus with a vanishing R_{xx} requires a proper equilibration between the QHECs and the

metallic contacts, which is far from trivial. Indeed, the interface between the 2DES and the contacts yields a complex evolution of the potential between the two different materials [101–103]. At $B = 0$, a proper interface design leads to ohmic contacts but in the QH regime, the apparition of QHECs complexifies the problem and two cases can be distinguished. In the first (Fig. 2.12b), a compressible strip close to the contact favors the transfer of charge carriers from and to the contact whereas in the second (Fig. 2.12c), the presence of an incompressible strip between the contact and the outer compressible strip constitutes a tunneling barrier, detrimental to the transfer of charge carriers. The latter configuration has two consequences:

1. Dissipation occurs between the contacts and the QHECs, as imaged in a beautiful experiment using the fountain effect of superfluid helium to pinpoint heat sources in a 2DES [104]. Importantly, increasing B can lead to an enlargement of the incompressible strip and a total decoupling between the QHECs and the contacts.
2. The presence of the incompressible strip can prevent the equilibration of the chemical potential between the contacts and the QHECs. Hence, the measured R_{xx} and G_{xy} deviate from the value that would be measured in case of proper equilibration of the QHECs with the contacts.

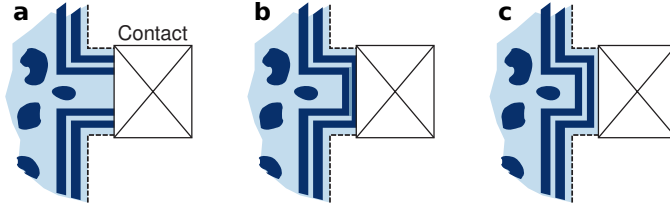


Figure 2.12: Schematics of the QHECs in the vicinity of a metallic contact. Adapted from [103]. **a**, Ideal picture where the compressible strips (dark blue) enter the contact. **b**, Due to a shifted potential along the contact, the compressible strips do not enter the contact. In this case, the compressible strip is close to the contact. **c**, Same as **b** but with the presence of an incompressible strip (light blue) between the contact and the outer compressible strip that impair the equilibration.

2.3 Upstream modes in graphene

Up to now, the topological breakdown of QH states has been discussed for semiconductor-based 2DESs only (excepted in Fig. 2.10). The mechanisms described in this framework constitute the textbook picture in the field of quantum Hall effect. In this section, we will see that the topological breakdown in graphene does not necessarily obey to the picture elaborated previously. Interestingly, the difference does not originate from fundamental properties (discussed in section 2.1.1) but from the device architecture inherent to graphene.

2.3.1 Fringing fields

In the widespread device architecture (see section 3.2), graphene is deposited on a highly doped silicon (Si) substrate covered by a thin (generally 300 nm) silicon oxide (SiO_2) insulating layer (Fig. 2.13a). Hence, the Si can be used as a back gate, when biased at a voltage V_{bg} , to tune the charge carrier density in graphene. Such a configuration leads to a strong charge accumulation along the graphene edges (CA@GE) (black curve in Fig. 2.13b), as shown by Silvestrov *et al.* [105] by approximating the Si back-gate as infinite. Their conclusion relies on an analytical model taking the screening of charge carriers into account with the "image charges" technique (Fig. 2.13a). Importantly, they pointed out that this increase of charge carriers density at the edges should lead to the formation of both downstream QHECs (the same as in section 2.1.2) and upstream QHECs flowing in the opposite direction (red in Fig. 2.13b). Indeed, some LLs (gray dashed line in Fig. 2.13b) cross twice the Fermi energy by following the potential. The model also captures the formation of compressible and incompressible strips (see section 2.1.2.2).

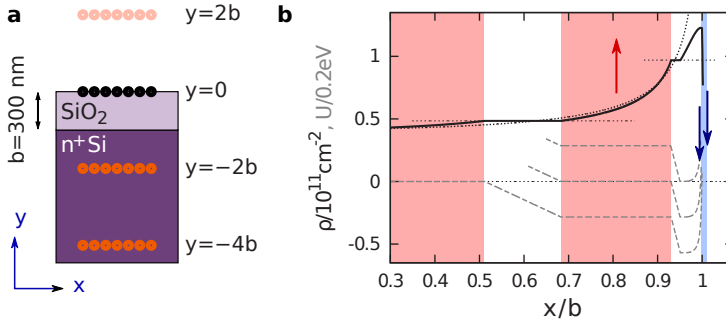


Figure 2.13: Increase of the charge carriers density at the graphene edges. Adapted from [105]. **a**, Lateral view of a graphene device. The graphene plane is represented with black dots, the insulating SiO_2 back gate is shown in light purple and the conducting Si back gate is shown in dark purple. The image charges introduced to take the screening into account are shown with red dots. **b**, Density (plain (dotted) black curve with (without) B) and LLs (gray dashed curves) as a function of the distance x (with the edge at $x = 1$). Downstream (upstream) modes are shown with light red (blue) compressible strips.

A more intuitive way to visualize the origin of the CA@GE is proposed in Fig. 2.14a. The electric field lines between the biased back gate and the graphene plane have been computed with a finite element software (COMSOL) [106]. It highlights the presence of fringing fields originating from the back gate regions remote from the graphene plane and more dense at the graphene edges (orange lines in Fig. 2.14a). The resulting charge carrier density profile is shown for two different values of V_{bg} in Fig. 2.14b. As for the analytical model described previously, they feature an increase of density along the edges. These curves will be further discussed in the next section.

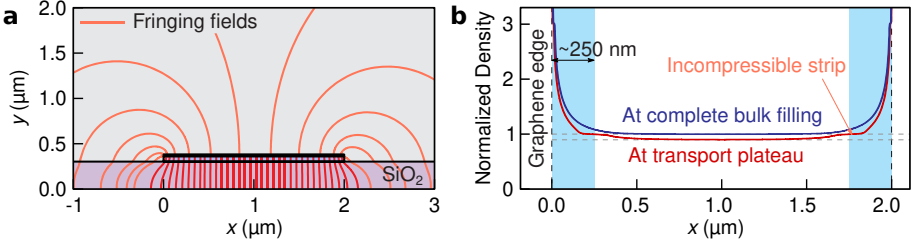


Figure 2.14: Numerical simulation of fringing fields. Adapted from [106]. **a**, Electrical field lines computed with a recursive finite elements method. The fringing fields are illustrated with light red lines. **b**, Normalized charge carriers density profiles in the case of an insulating bulk (blue curve) and a conducting bulk (red curve). The region hosting QHECs is depicted in light blue and is ~ 250 nm wide. When the bulk is conducting, it is separated from the QHECs by an incompressible strip.

2.3.2 Topological protection despite a conducting bulk

In section 2.2.1.1, the simultaneity of the QH topological breakdown with the presence of a conducting bulk has been highlighted with MIM measurements on a semiconductor-based 2DES. The same experiment has been performed in

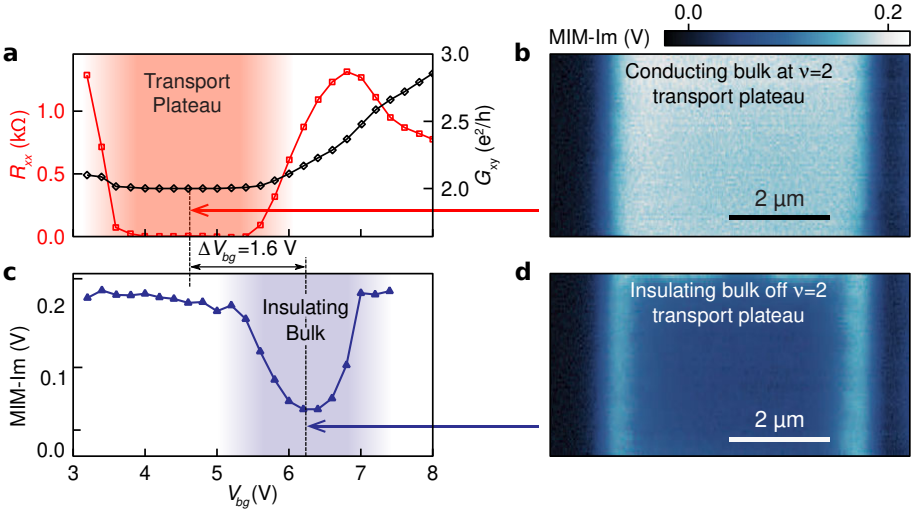


Figure 2.15: Microwave microscopy in graphene. Adapted from [106]. **a**, R_{xx} (red curve) and G_{xy} (black curve) vs V_{bg} measured at $T = 4.7$ K and $B = 9$ T in a graphene sample encapsulated between hBN layers (section 3.2.1.1). The G_{xy} plateau, corresponding to the QHECs topological protection, is highlighted in red. **b**, Map of the MIM-Im signal measured in the transport plateau in **a**. The large signal in the bulk indicates it is conducting. **c**, Averaged MIM-Im signal in the bulk. The dip in the curve indicate that the bulk is insulating. **d**, Map of the MIM-Im signal measured when the bulk is insulating (off the transport plateau). The large signal along the edges indicate the presence of QHECs.

graphene by Cui *et al.* and their results, published in 2016, are not consistent with the textbook picture observed in semiconductor-based 2DES. Figure 2.15 shows a comparison between the transport measurement (R_{xx} and G_{xy} curves) and the MIM signal as a function of V_{bg} . Interestingly, it is found that the bulk can be conducting at the G_{xy} plateau (Fig. 2.15b) while it can be insulating outside the plateau (Fig. 2.15d). It results in a shift ΔV_{bg} between (i) the plateau in the transport measurement (Fig. 2.15a) and (ii) the small averaged MIM signal in the bulk, indicating it is insulating (Fig. 2.15c).

To explain these puzzling results, they authors invoke the presence of fringing fields and the associated CA@GE. From numerical simulations, they observe that the bulk, while conducting, can be decoupled from the region hosting the QHECs by a large incompressible strip (red curve in Fig. 2.14). Therefore, the QHECs can be topologically protected despite the presence of the conducting bulk. On the contrary, when the bulk is insulating, topological breakdown can occur due to the presence of both downstream and upstream modes in the QHECs region (see Fig. 2.13). It will be the subject of the next section.

The presence of upstream QHECs as the cause for the topological breakdown is actually discussed in the supplemental materials of ref. [106]. In the main paper, the authors present another hypothesis, based on a model proposed in ref. [103] where current flows in the incompressible strips. To be honest, I am not convinced at all by this theory for epistemic reasons: first, in solid state physics, the movement of charge carriers is conditioned to the presence of compressible states. I don't see why it would be different in the QH regime. Second, there is, to my knowledge, no strong experimental evidence to reject the QHECs picture in favor of a more *exotic* theory.

Nevertheless, the interpretation becomes more complex when looking at Fig. 2.16 (data from the same paper [106]), that displays the evolution of the shift ΔV_{bg} (Fig. 2.15) with V_{bg} for two different samples. Indeed, it exhibits a clear asymmetry between electrons (positive V_{bg}) and holes (negative V_{bg}). For each type of charge carrier, ΔV_{bg} evolves linearly with V_{bg} , which is expected since the charge density along the edges should be proportional to the density in the bulk. However, the slope is different between the electron and the hole side. In the first sample (Fig. 2.16a), ΔV_{bg} remains almost constant for holes while the slope is $\eta_e = 9.7\%$ for electrons. In the second sample (Fig. 2.16b), the behavior is inverted with a larger slope for holes ($\eta_h = 9.1\%$) than for electrons ($\eta_e = 4.0\%$). To explain this asymmetry, the authors assume that unipolar states, associated to impurities located near the edges, hold some of the accumulated charges and decrease the accumulation profile. The nature of these impurities strongly depends on the fabrication techniques (section 3.2) used to build the sample.

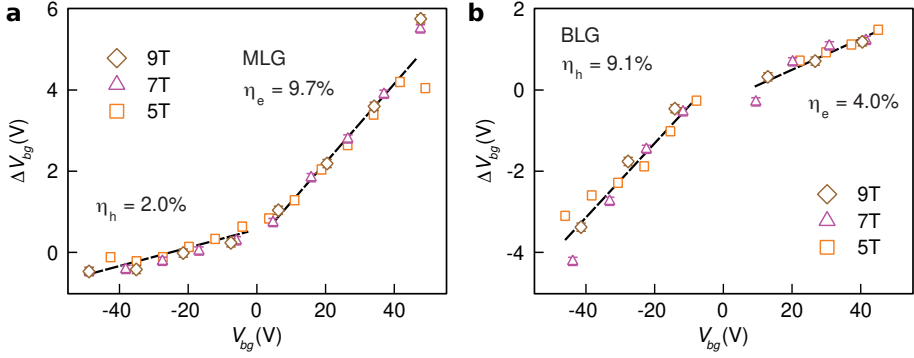


Figure 2.16: Electron-hole asymmetry. Adapted from [106]. **a**, ΔV_{bg} vs V_{bg} for the first sample (monolayer graphene). **b**, same as **a** for the second sample (bilayer graphene).

2.3.3 Dissipation along the edges

The presence of upstream QHECs as the origin of topological breakdown in graphene has been highlighted in 2019 by Marguerite *et al.* from SoT microscopy (section 1.2.4.2) combined with SGM (section 3.1.2) [10]. By scanning the biased tip above the sample, they were able to tune locally the thermal dissipation (light shades in Fig. 2.17a) and the backscattering (larger R_{xx} in Fig. 2.17b) to pinpoint where both phenomena occur in the device.

Surprisingly, the SGM map in Fig. 2.17b indicates that backscattering around integer filling factor occurs all along the edges, at a distance of ~ 200 nm from them, without any contrast in the bulk. The SoT microscopy map in Fig. 2.17a shows thermal dissipation for the same tip positions but also along the very edges of the device, where no SGM contrast is visible in Fig. 2.17b. To explain these data, the authors invoke the presence of counterpropagating QHECs along the same graphene edges, as discussed in section 2.3.1. In this model, charge carriers backscattering occurs via elastic tunneling ($\dot{W}$) from the down- to the upstream QHECs (Figs. 2.17c,d) and is tuned when the biased tip approaches the QHECs. On the contrary, thermal dissipation occurs via inelastic scattering ($\dot{Q}$) due to the presence of impurities, leading to the emission of phonons (Figs. 2.17c,d). This mechanism occurs both in down- and upstream QHECs.

The results in Fig. 2.17 are obtained for holes and are consistent with a CA@GE. However, measurements in the electron side indicate that holes are accumulated along the edges, and not electrons. The authors therefore suggest that the increase of density along the edges originates from negatively charged impurities [10, 11]. Nevertheless, all their data were obtained at low B (and therefore low density in the bulk), in a not well defined QH regime (R_{xx} does not fall to zero). In the next section, the possible origin of CA@GE is discussed based on other studies.

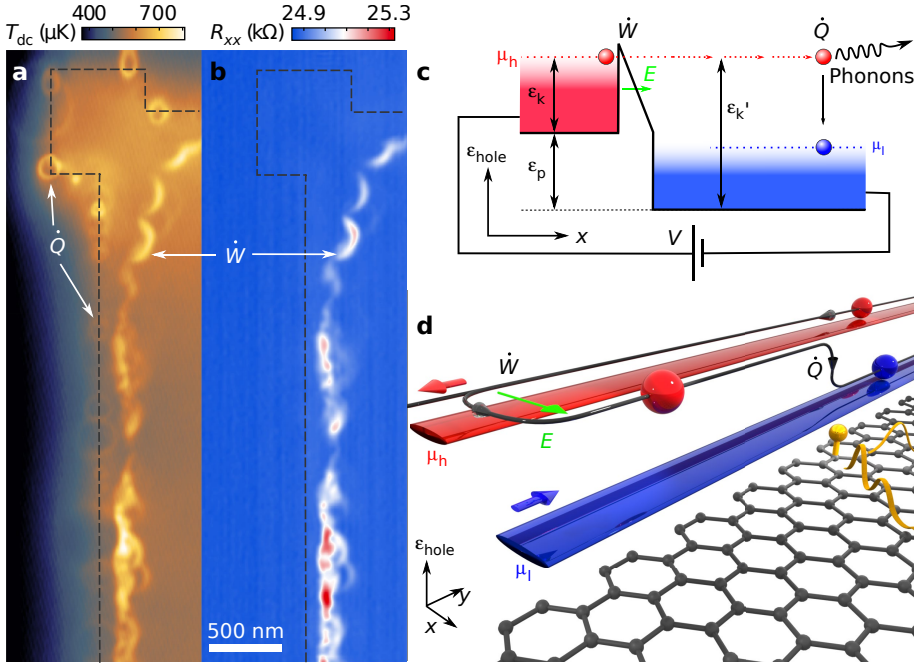


Figure 2.17: Imaging backscattering and thermal dissipation in graphene QH regime. Adapted from [10]. **a**, SoT microscopy map of the thermal dissipation obtained for $B = 1$ T, $\nu = -1.9$, $T = 4.2$ K and $V_{tip} = 1.6$ V. Graphene edges are shown with black dashed lines. **b**, SGM map of R_{xx} in the same conditions. The tip-induced backscattering is indicated with larger R_{xx} (red shades). **c** Schematic of the tunnel barrier between the downstream (red) and upstream (blue) QHEC. $\dot{W}$ corresponds to the elastic tunneling and $\dot{Q}$ to the inelastic scattering. **d**, Artist's view of the down- (red) and upstream (blue) QHECs at the edge of the graphene device. The inelastic scattering is caused by impurities (in yellow).

2.3.4 Debate around the origin of upstream modes

In the two previous sections, we have looked into experimental data demonstrating that topological breakdown in graphene is decorrelated from the state of the bulk and originates from the coupling of counterpropagating QHECs along the same device edges. The presence of upstream QHECs comes from the formation of a CA@GE. While theoretical calculations (section 2.3.1) assign this CA@GE to fringing fields, refs. [10, 11, 106] evoke the presence of charge impurities at the edges to explain an electron-hole asymmetry in their data. This section aims to review and discuss the influence of fringing fields and impurities in the formation of a CA@GE.

First, SGM data obtained by Chae *et al.* at $B = 0$ T in graphene on SiO₂ exhibit an increase of conductivity along the edges, proportional to the absolute value of V_{bg} and therefore vanishing at the charge neutrality point [107]. This observation suggests a CA@GE caused by fringing fields. On the contrary, Dou

et al. performed a similar experiment on a graphene-hBN moirée lattice and reported the presence of conducting channels at the charge neutrality point consistent with a hole doping along the edges [108]. In that case, the CA@GE should originate from the presence of impurities.

Second, Vera-Marun *et al.* estimated the CA@GE by measuring the capacitance of a graphene (and bilayer graphene) device as a function of the perpendicular magnetic field. [109]. By increasing B , the charge carriers are confined along the edges in a region of width $\sim l_B$ (Eq. (2.5)) which exhibits a different capacitance than in the bulk. Their data are consistent with simulations based exclusively on electrostatics, with a $\sim 100\%$ increase of the measured capacitance between zero and large B . Barraud *et al.* conducted a similar study but found an increase of only 20% when increasing B [110]. They concluded that the charge density profile is not given solely by electrostatics. Finally, always in a similar experiment conducted more recently, Caridad *et al.* observed that devices with large edge disorder exhibit a constant charge homogeneous density in the whole sample (no CA@GE then) while low edge disorder leads to a CA@GE originating from fringing fields [111, 112].

From these different studies, it appears that the exact evolution of the potential landscape leading to the CA@GE remains largely unknown. It indeed depends on the type and the amount of edge disorder introduced during the fabrication process. A large level of impurities can indeed suppress the effect of fringing field while, in some cases, introduce a constant doping along the edges. Further studies are therefore needed to characterize accurately the influence of edge disorder on the CA@GE.

2.4 The quantum Hall interferometers

In two-dimensional electronic systems, quantum Hall interferometers (QHIs) are the counterpart of optical interferometers, with the coherent QHECs playing the role of monochromatic light beams [113]. In this framework, Fabry-Perot (FP) [114] and Mach-Zehnder [115] QHIs have been successfully implemented. In graphene, the extra valley degeneracy and the possibility to mix electron and hole-filled QHECs with pn junction opens the path to unprecedented realizations [116–122]. This section will mainly focus on FP QHIs.

2.4.1 Fabry-Perot quantum Hall interferometers

As in optics, FP QHIs are obtained from two beam-splitters (semi-reflective interfaces) put in series as shown for a graphene QHI in Fig. 2.18a. These beam-splitters take the form of quantum point contacts (QPCs), created thanks to biased split-gates, that bring the interfering QHECs in close proximity. The split-gates voltage allows to tune the reflection amplitude r_i and the transmission amplitude t_i of charge carriers in each QHEC at the i -th QPC (Fig. 2.18b). In the first QPC, the wavefunction ψ_{QHI} at the output is obtained from the input

wavefunction ψ_{in} as

$$\psi_{\text{QHI}} = t_1 \psi_{\text{in}}. \quad (2.16)$$

In the second one, the output wavefunction ψ_{out} is obtained from the same reasoning, with the transmission amplitude t_2 .

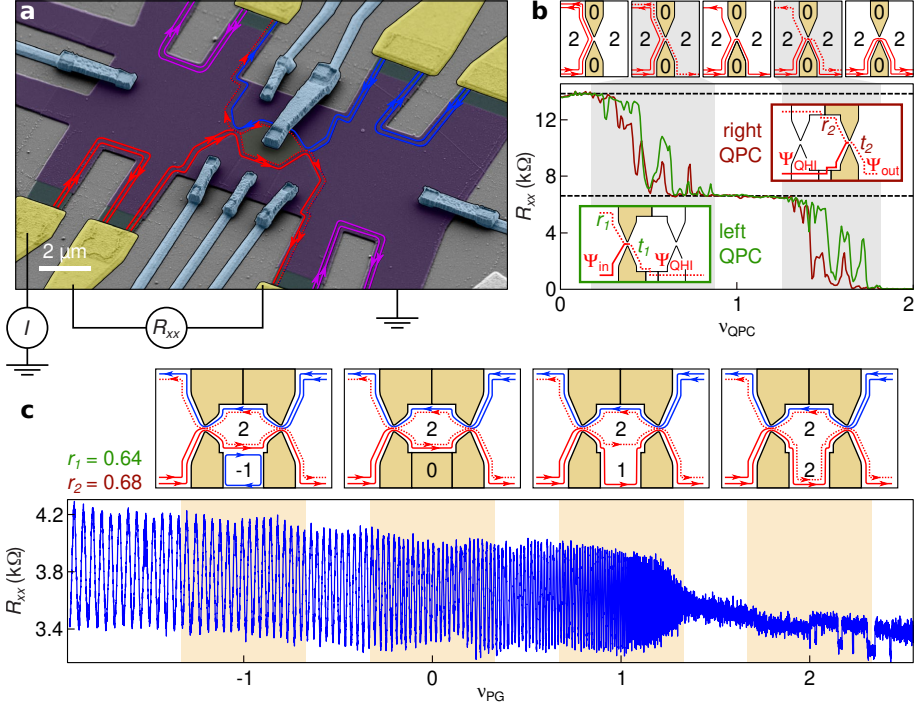


Figure 2.18: Fabry-Perot QHI in graphene. Adapted from [13]. The measurements are obtained at $T = 60$ mK with $B = 8$ T. **a**, Colored scanning electron microscopy image of the QHI. The device is created from a graphite-hBN-graphene-hBN-graphite stack. The top graphite layer (in dark purple) is patterned in several gates biased by the mean of metallic contacts (light blue). The line contacts used for the transport measurements in the graphene layer are shown in yellow. The QHECs are illustrated with red, blue and light purple arrows. **b**, Evolution of R_{xx} as the left (in green) or the right (in Bordeaux) split gate voltage is changed while keeping the other at zero. The voltage is converted in the filling factor ν_{QPC} at the middle of the QPC. The integer part of ν_{QPC} indicates the number of QHECs perfectly transmitted through the QPC. A partially transmitted QHEC at the i -th QPC is characterized by the reflection amplitudes r_i and transmission amplitude t_i . **c**, Evolution of R_{xx} by varying the plunger gate voltage, converted in the filling factor ν_{PG} of the region below the gate. A schematic corresponding to each integer value of ν_{PG} is shown above the curve. R_{xx} exhibits oscillations that vanishes when the QHECs flow along the edges of the sample ($\nu_{\text{PG}} > 1$).

We can now consider the combined effect of both QPCs on transport in a coherent FP QHI. To do so, Eq. (2.16) should be transformed to include the

wavefunction carried by the QHECs reflected at the second QPC. It yields

$$\psi_{\text{QHI}} = t_1 \psi_{\text{in}} + r_2 r_1 \exp(i2\varphi_{\text{QHI}}) \psi_{\text{QHI}}, \quad (2.17)$$

where φ_{QHI} is the phase accumulated along one arm of the QHI (to ease the development, the length of both arms is considered equal). Following the same logic, the wavefunction of the right QHEC is given by

$$\psi_{\text{out}} = t_2 \exp(i\varphi_{\text{QHI}}) \psi_{\text{QHI}}. \quad (2.18)$$

From Eqs. (2.17) and (2.18), the transmission probability through the QHI is defined as $T = |\psi_{\text{out}}|^2 / |\psi_{\text{in}}|^2$. To develop this expression, one must define the reflection probabilities at each beam splitter as $R_i = |r_i|^2 = 1 - |t_i|^2$. The transmission probability then writes [12]

$$T(\varphi_{\text{QHI}}) = \frac{(1 - R_1)(1 - R_2)}{1 + R_1 R_2 - 2\sqrt{R_1 R_2} \cos(2\varphi_{\text{QHI}})}. \quad (2.19)$$

The transmission through a QHI is therefore modulated by the phase $2\varphi_{\text{QHI}}$ accumulated by charge carriers along one complete turn of the QHI. In the next section, the value of this phase will be exposed in the scope of the Aharonov-Bohm (AB) effect. I will be followed by a discussion on the charging mechanisms, linked to Coulomb effect, and their influence on the transport signatures.

2.4.2 The Aharonov-Bohm regime

The development exposed in this section is inspired by the supplementary note XIV of ref. [12] (a really well written paper by the way).

2.4.2.1 Accumulated phase in the QHI

The phase φ_{QHI} in Eq. (2.19) evolves due to two contributions:

1. The Aharonov-Bohm effect tells us that charge carriers flowing in a potential vector $\mathbf{A}$ accumulate a phase [123]

$$\varphi_{\text{AB}} = \frac{e}{\hbar} \oint_{\Gamma} \mathbf{A} \cdot d\mathbf{l} = 2\pi \frac{\Phi}{\phi_0}, \quad (2.20)$$

where $\Phi = BA$ is the magnetic flux through the area A enclosed in the path Γ . Here, A is the surface of the QHI and Γ is the path described by the QHECs.

2. A change in the charge carriers energy ϵ , compared to the Fermi energy, induces in turn a change of wavevector $\Delta k = \epsilon / \hbar v$, where v is the charge carriers velocity in the QHECs. The induced phase shift is found by integrating this value along the path Γ , which gives [124]

$$\varphi_{\epsilon} = \frac{2L\epsilon}{\hbar v}, \quad (2.21)$$

where L is the length of one arm of the QHI, and $2L$ is its perimeter.

From Eqs. (2.20) and (2.21), the phase accumulated along one of the QHI arm is

$$\varphi_{\text{QHI}}(\epsilon, \Phi) = \pi \frac{\Phi}{\phi_0} + \frac{L\epsilon}{\hbar v} + \varphi, \quad (2.22)$$

where φ is a constant phase. Combining Eqs. (2.19) and (2.22) allows to understand the evolution of R_{xx} in Fig. 2.18c. By varying the plunger-gate voltage, the area A of the QHI changes, inducing oscillations in the transmission $T(\Phi = BA, \epsilon = 0)$, which is inversely proportional to R_{xx} .

2.4.2.2 Conductance in an open QHI

The link between transport measurements and the transmission in the QHI will be analyzed for the particular case where the reflection probabilities are small $R_1, R_2 \ll 1$ in Eq. (2.18), corresponding to open QPCs. The following approximation can be made

$$T(\Phi, \epsilon) \approx 1 - R_1 - R_2 + 2\sqrt{R_1 R_2} \cos \left(2\pi \frac{\Phi}{\phi_0} + \frac{2L\epsilon}{\hbar v} + \varphi \right). \quad (2.23)$$

Next, the effect of a bias voltage applied between both sides of the QHI will be studied. To do so, the potential inside the QHI is taken as a reference and is equal to zero. The potential drop at each QPC is eV^+ on left and $-eV^-$ on the right. Here, only the case of an identical voltage drop at both QPCs will be considered, with $V^+ = V^- = V/2$. The current through the QHI then writes

$$I = \frac{e}{h} \int_{-eV/2}^{eV/2} T(\Phi, \epsilon) d\epsilon. \quad (2.24)$$

When including in this expression the oscillations contribution of Eq. (2.23) (without the term $1 - R_1 - R_2$ that is of low interest here), it leads to

$$I_{\text{osc}} = \frac{2\pi e v}{L} \sqrt{R_1 R_2} \left(\sin \left(2\pi \frac{\Phi}{\phi_0} + \frac{eLV}{\hbar v} + \varphi \right) - \sin \left(2\pi \frac{\Phi}{\phi_0} - \frac{eLV}{\hbar v} + \varphi \right) \right).$$

The differential conductance through the QHI is then given by a product of cosines

$$G_{\text{osc}} = \frac{dI_{\text{osc}}}{dV} = \frac{2e^2}{h} \sqrt{R_1 R_2} \cos \left(2\pi \frac{\Phi}{\phi_0} + \varphi \right) \cos \left(2\pi \frac{eLV}{\hbar v} \right). \quad (2.25)$$

2.4.2.3 Source-drain bias spectroscopies

An experimental illustration of the latter theoretical result, yielding a checkerboard pattern indicative of the oscillatory behavior of G as a function of Φ and V [12, 13, 99, 121, 125–128], is shown in Fig. 2.19a [127]. It presents the differential conductance as a function of the source-drain bias voltage V_{sd} and the magnetic field, in a QHI (blue QHECs in Fig. 2.19b) built by depleting a GaAs/AlGaAs 2DES with top gates (in black in Fig. 2.19b). The smoothly varying background of the conductance has been removed to only display

the oscillation contribution. Figure 2.19c shows a map obtained by fitting Eq. (2.25) on the data of Fig. 2.19a with the following parameters : $A = 18 \mu\text{m}^2$, $L = 8.2 \mu\text{m}$ and $v \sim 1.5 \cdot 10^5 \text{ m/s}$. Both the experimental and fitted maps exhibit a decrease of the visibility of the oscillations vs V_{sd} . The origin of the phase randomization mechanism leading to this behavior is debated in the literature and a consensual explanation is still missing [129–132]. In Fig. 2.19c, the visibility decay has been modeled by multiplying Eq. (2.25) by an exponential fitted to the experimental data of Fig. 2.19a.

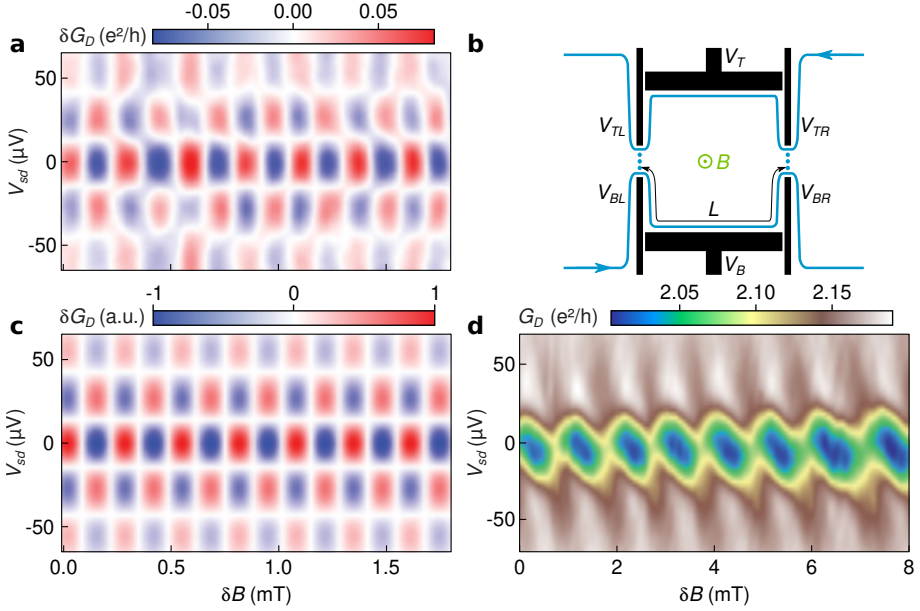


Figure 2.19: Source-drain spectroscopy of a FP QHI. Adapted from [127]. **a**, Oscillation contribution δG_D to the differential conductance as a function of V_{sd} and the variation of the magnetic field δB near $B = 0.47 \text{ T}$, obtained in a QHI of area $A = 18 \mu\text{m}^2$. **b**, Schematic of the QHI created by depleting the underneath GaAs/AlGaAs-based 2DES with top gates (in black). The QHECs are shown with blue arrows. The reflection at the QPCs is controlled with the voltages V_{TL} and V_{BL} at the left and V_{TR} and V_{BR} at the right. **c**, Simulation map obtained from **a**. **d**, Differential conductance G_D as a function of V_{sd} and δB in a QHI of area $A = 2 \mu\text{m}^2$ near $B = 3.77 \text{ T}$.

2.4.3 The Coulomb-dominated regime

In the previous section, the transport of charge carriers through a QHI has been studied in the scope of a pure AB regime. In the following, the influence of electron-electron interactions and the resulting Coulomb-dominated (CD) regime will be discussed, based on the theoretical framework of ref. [113]. On this basis, the condition for a QHI to be in the AB or CD regime will be outlined and experimental strategies to lower Coulomb interactions will be reviewed.

2.4.3.1 CD regime: a matter of capacitive effects

Coulomb charging effects in the QH regime has already been tackled in section 2.2.2, in the case of antidots. At this stage, reconsidering antidots is interesting because they turn out to be good building blocks for QHIs. Indeed, they are nothing more than a QHEC loop, where charge carriers can interfere, coupled to two extended QHECs via tunneling paths playing the role of QPCs. However, their transport properties are totally different from those of an AB QHI characterized by the sinusoidal oscillations described in Eq. (2.25). Due to Coulomb blockade, charge carriers can hop on and hop off an antidot only when its charging energy is identical for N and $N + 1$ charges (Eq. (2.15)). It results in narrow peaks in the conductance, when this condition is fulfilled, and a zero conductance otherwise.

It therefore remains to explain why some QHIs are found in a pure AB regime (Fig. 2.19a) while others, like antidots for example, are dominated by Coulomb interactions. The answer lies in the significance of capacitive effects (central in Eq. (2.15) while ignored in Eq. (2.25)), objectified by the parameter [113]

$$\xi = \frac{C_{eb}}{C_{eb} + C_b}, \quad (2.26)$$

where C_{eb} is the capacitive coupling between the QHECs and the localized states in the bulk, and C_b is the bulk capacitance. The QHI operates in the AB regime when $\xi \rightarrow 0$ while it is in the CD regime when $\xi \rightarrow 1$. According to ref. [113], this condition is independent from the regime of weak or strong reflection at the QPCs.

The fact that the strength of the reflection probabilities does not determinate whether the QHI operates in the AB or the CD regime has unsettled me for a while as I find the discussion in ref. [113] not really clear. Indeed, at zero magnetic field, the capacitive coupling between a localized state and the leads is key to observe or not Coulomb blockade [14]. I therefore thank professor Gwendal Fève for having given me a better insight into the condition to observe the CD regime. The key is indeed the Coulomb energy needed to add charges in the localized states encircled by the QHI and capacitively coupled to it, and not the energy needed to add charges in the QHI itself. From this perspective, the discussion of ref. [113] makes much more sense.

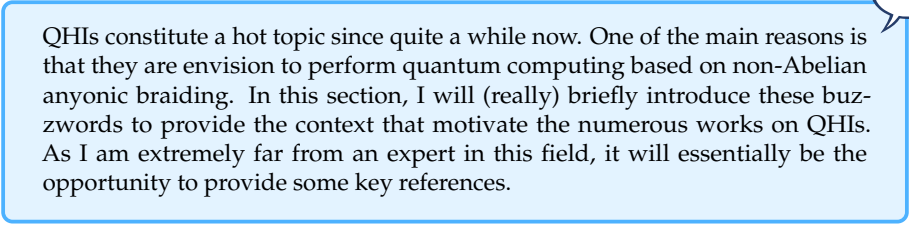
In Eq. (2.26), C_{eb} is proportional to the QHI perimeter ($C_{eb} \propto \sqrt{A}$) and C_b is proportional to the QHI area ($C_b \propto A$). Therefore, by decreasing the QHI size, the coefficient C_b becomes negligible compared to C_{eb} in Eq. (2.26) and the CD regime dominates. The determinant influence of the QHI size has been highlighted by several studies [96, 127, 133]. In particular, Fig. 2.19d, obtained with a similar device than in Fig. 2.19a but with an area $A = 2 \mu\text{m}^2$ instead of $17 \mu\text{m}^2$, displays Coulomb diamonds, the archetypal signatures of a CD

regime, instead of a checkerboard pattern [127]. This dichotomy will be further discussed in chapter 6.

2.4.3.2 Strategies to lower Coulomb interactions

We have just seen that decreasing the area of a QHI favors the CD regime. However, large QHIs suffer from a disadvantageous decoherence length on QHI perimeter ratio [134]. Recently, strategies have been elaborated to limit the effect of Coulomb charging in QHIs of small size. Electrically conductive planes have been added close to the semiconductor-based 2DES to screen the Coulomb effect of localized states in the bulk and lower their coupling with QHECs. With such architecture, the QHI minimal size at which the Coulomb dominated (CD) regime supplants AB effect has been substantially decreased [128] (typically down to a μm lateral size). In the same vein, graphene devices using graphite back-gates provide similar advantages [12, 13].

2.4.4 The Holy Grail: non-abelian anyons braiding



QHIs constitute a hot topic since quite a while now. One of the main reasons is that they are envisioned to perform quantum computing based on non-Abelian anyonic braiding. In this section, I will (really) briefly introduce these buzzwords to provide the context that motivates the numerous works on QHIs. As I am extremely far from an expert in this field, it will essentially be the opportunity to provide some key references.

In section 2.1.4.1, we have seen that electron-electron interactions form composite fermions at the origin of QH states at fractional filling factors. This fractional QHE at magnetic field B has been shown equivalent to an integer QHE with free particles possessing fractional charges in an effective magnetic field B_{eff} , where $B_{eff} = 0$ corresponds to an alignment between E_F and the considered LL (at $\nu = (2n - 1)/2$, with n the LL label) and, hence, to a metallic state. Surprisingly, however, QH states have been observed at the filling factors $\nu = 5/2$ and $\nu = 7/2$ [135, 136]. Different theories exist to explain the emergence of these states but the more widespread assumes that they are superconducting states caused by the formation of Cooper pairs with two $e/4$ -charged pseudoparticles (instead of one $e/2$ -charged particle as expected with half integer values of ν) [44, 136, 137]. It is called a Pfaffian state, predicted to be non-Abelian. It means that exchanging (or "braiding" [138, 139]) twice the positions of two particles (Fig. 2.20a) leads to a different state than the original one.

The latter property has been envisioned by Das Sarma *et al.* to perform quantum computing [140]. To do so, they propose an experiment based on a double FP QHI (Fig. 2.20b) that should both allow to infer the nature of the $\nu = 5/2$ state and, if it is indeed a Pfaffian state, should allow to flip a qubit with an unprecedented low error rate as it would be topologically protected against decoherence (section 1.1.1.2). The idea is to place the two $e/4$ -charged pseudoparticles of the Cooper pair in two different antidots (section 2.2.2)

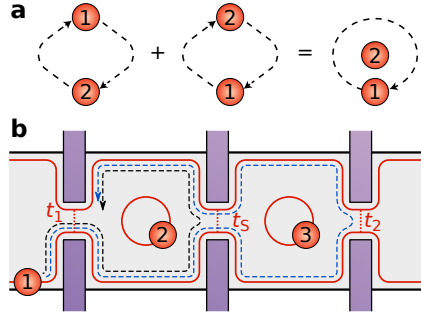


Figure 2.20: Particle braiding in a FP QHI. Adapted from [139]. **a**, In two dimensions, exchanging twice the positions of two particles is equivalent to encircle one of the particle with the other one. **b**, Double FP QHI proposed by Das Sarma *et al.* to perform qubits manipulation with non-Abelian anyons [140]. The black path corresponds to the double exchange illustrated in **a** and is used to flip the qubit state. The blue path allows to read the qubit state.

enclosed in both sides of the QHI (Fig. 2.20b). In this situation, the qubit can be in the state $|-\rangle$ or $|+\rangle$. Each state leads to a different measured conductance value $\sigma_{xx} \propto |t_1 \pm it_2|^2$ through the QHI, by making interfering $e/4$ -charged pseudoparticles with the blue path in Fig. 2.20b. The state of the qubit can then be flipped by braiding the pseudoparticles 1 and 2 with the black path in Fig. 2.20b.

chapter 3

Setting up an experiment

This chapter gives an overview of the main steps performed to produce the SGM outcomes exposed in this thesis. First, the experimental setup will be briefly presented (more details can be found in [141, 142]) and typical SGM results from the literature will be reviewed. Then, the construction of graphene samples will be quickly explained and the devices studied in this work will be portrayed into more details. Finally, the KWANT package, an experimenter-friendly tight-binding code used to analyze the SGM results, will be introduced.

3.1 Low temperature scanning gate microscope

SGM can be summarized as a technique intended to extract local information on the electrical transport in low-dimensional systems by the use of a movable electrically-biased gate. This definition will be clarified through the present section with a glance to our experimental setup and some examples taken from the literature.

3.1.1 The Fridge

The experimental setup of my research group is called "The Fridge". For the curious readers, I must precise that, despite its name and its geographical location, it has never been used to store beers (as far as I know at least). More seriously, a $^3\text{He}/^4\text{He}$ dilution circuit allows to cool the sample down to ~ 100 mK and a superconducting coil gives the possibility to apply a magnetic field up to 17 T. Combined with the SGM local gating, it offers a wonderful playground to explore a wide variety of phenomena in the field of mesoscopic physics.

3.1.1.1 The liquefier circuit

Figure 3.1 displays a schematic of the Fridge. The global system is illustrated on the left side of Fig. 3.1 with its main features listed hereafter by referencing to the corresponding numbers and letters under parenthesis in the figure. The core of the system is the inner vacuum space (IVS) (1) which is under high vacuum. The IVS is immersed into a liquid helium bath (2) at a temperature of about 4.2 K. The superconducting magnet (3) used to apply a magnetic field up to 17 T is also immersed in the bath. The helium vapors are collected and condensed in the liquefier (4). The gaseous helium flux is controlled thanks to the cryo-valve (5) and an inlet allows to insert gaseous helium into the system (6). The liquefaction is achieved by condensing helium on a cold plate (7), naively represented here. Liquid helium then flows through a tube (8) back in the bath. To limit heat flow towards the bath, it is contained in a tank (9) which is isolated from the outside with an ultrahigh vacuum spacing (10) and a liquid nitrogen bath (11) at a temperature of about 77 K. The IVS is further surrounded by an epoxy tube called "la chaussette" (12). Different plates (13) act as heat screens to limit heat flow by radiation and convection. The helium vapors produced in the chaussette are collected via (14) and are sent back to the main bath. The IVS is pumped via (15). The dilution circuit (right schematic in Fig. 3.1), used to lower the sample temperature far below liquid helium temperature, is located inside the IVS. Its working principle will be discussed in the next section.

3.1.1.2 The dilution circuit

Achieving a temperature below that of liquid helium is made possible by taking advantage of the ^3He dilution in ^4He , which is an endothermic process. The dilution circuit designed for this purpose is illustrated in the right schematic of Fig. 3.1. The first step consists in injecting ^3He (in red) (a) towards the IVS where it is liquefied in the condenser (b) thermally anchored to the 1K pot (c). This small bath is filled with ^4He (in blue) via a tube (d) immersed in the main bath (2) and the flow in the tube is controlled by a needle valve (e). The temperature of the 1K pot is lowered down to ~ 1.6 K by pumping on it (f) with the 65m^3 pump (g) (the collected helium is sent back to the liquefier (4)). After the condenser, the liquid ^3He goes through two ultra-thin impedances (h) used to regulate its flow through the dilution circuit. Then, it goes through

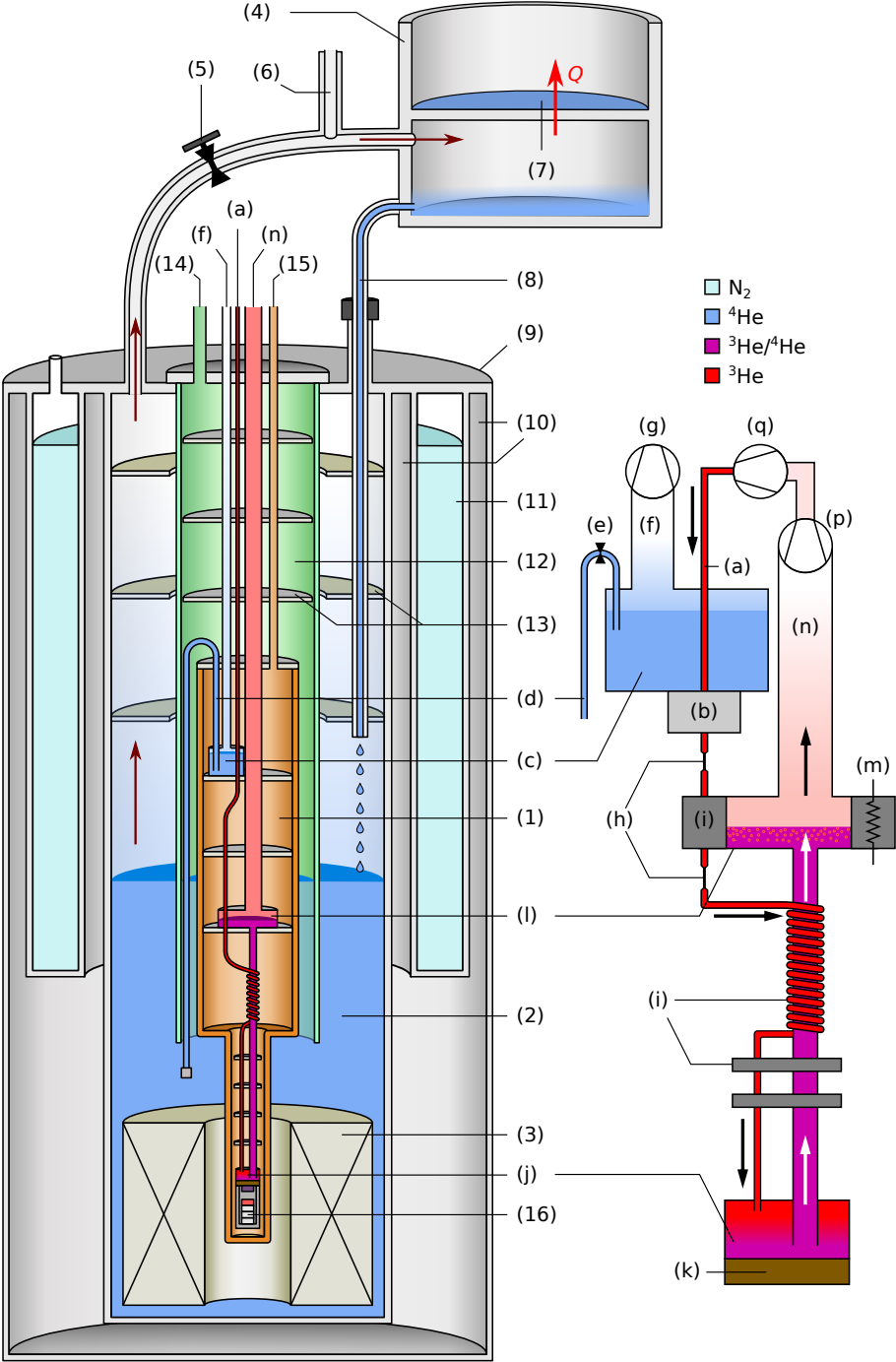


Figure 3.1: Schematic of the dilution refrigerator. (Description in section 3.1.1.)

different heat exchangers (i) where it is further cooled down before entering the mixing chamber (j). There, it is diluted in the ^3He – ^4He mixture (in purple), which is an endothermic process that lowers the temperature of the sample anchored to the chamber (k). ^3He then diffuses thanks to osmotic pressure towards the still (l) where its concentration is lower than in the mixing chamber because it is evaporated much more efficiently than ^4He due to a larger vapor pressure. The evaporation is eased thanks to a resistive heater (m). The ^3He vapors are pumped outside the system (n) by the ROOTS (secondary) pump (p) and the Rotary (primary) pump (q). It is then injected back in the loop (a).

3.1.1.3 Electrical measurements

The electrical measurements on the studied sample are performed using the lock-in technique. Most of the data exposed in this thesis were obtained with a MFLI lock-in from Zurich Instruments®. The amplitude of the input AC signal v_{ac} was chosen to be smaller than the electronic temperature (whose lower bound is $T_e \sim 100$ mK due to electrical noise) such that $ev_{ac} < k_B T_e$. When the temperature of the mixing chamber T_{MC} was above 100 mK, we considered that $T_e = T_{MC}$. Furthermore, DC voltages are applied on the gates with Stahl® precision DC sources. Finally, the temperature was measured in different locations of the Fridge by measuring the resistance of calibrated thermometers.

3.1.2 Local gating with an AFM tip

SGM relies on the use of a movable local gate that commonly takes the form of a sharp atomic force microscopy (AFM) tip. This section explains how the gating is realized in practice.

3.1.2.1 SGM mapping

Figure 3.2a illustrates the setup used to displace the tip in the experiment presented in the previous section. It consists in a stack of three piezo-motors and a scanner from Attocube®. The motors are used to displace the tip by discrete steps (typically a few micrometer by step) along either the x , y or z direction by applying voltage pulses on the piezos. The scanner allows to displace continuously the tip along the x , y and z axes by tuning the voltage applied on the piezos associated to each direction.

Note that a current leakage in the piezo moving the scanner along the x -axis limited the scanning range in the SGM characterizations performed on sample G1 (section 3.2.3) presented in chapters 5 and 6. Indeed, the stack is located at the bottom of the IVS, in front of the sample ((16) in Fig. 3.1) and the thermal dissipation caused by the leakage heated the mixing chamber, changing both the sample temperature and the mechanical properties of the system (including the tip-device distance).

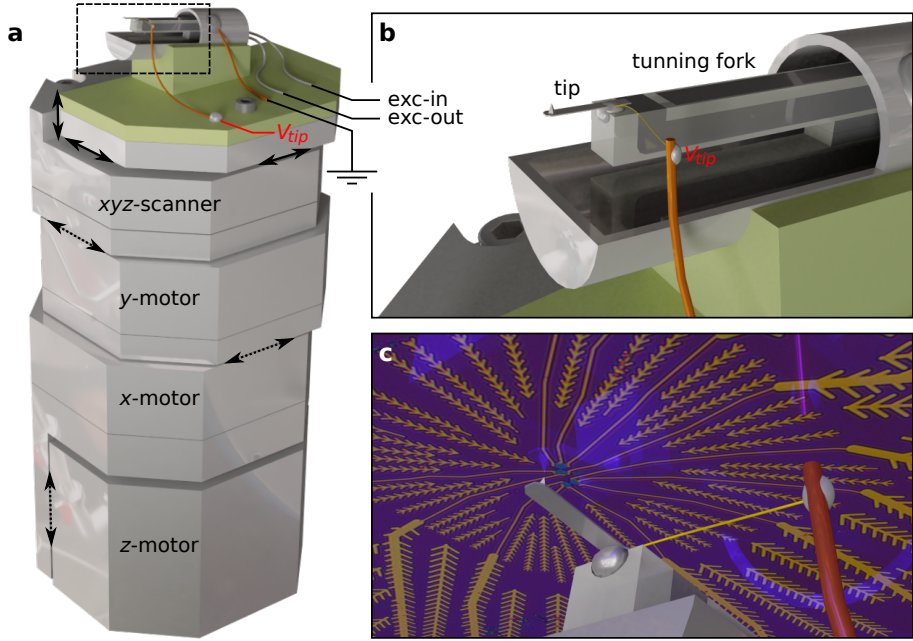


Figure 3.2: Schematic of the system used to move the SGM tip. **a**, A stack of three motors (x , y and z) and a xyz -scanner is used to move the tip in the space. **b**, The tip is glued on a tuning fork on which is applied an excitation $exc-in$ in **a**. A thin gold wire connected to the tip is used to bias it at V_{tip} . **c**, Schematic of the tip close to the device. The gold arrows are used to find the device by performing topography.

Figure 3.2b (zoom in Fig. 3.2a) illustrates how the tip is mechanically attached to the scanner. It is glued with silver epoxy to a watch tuning fork (whose casing has been opened), itself glued to an epoxy holder screwed to the scanner. The tuning fork is essential because it is used to measure the force experienced by the tip in the topography (AFM) mode [143]. To do so, an AC excitation ($exc-in$ in Fig. 3.2a) is sent to the tuning fork, at its resonance frequency, and the phase of the collected current ($exc-out$ in Fig. 3.2a) is monitored and kept constant by adjusting the frequency of $exc-in$. Indeed, the resonance frequency changes when a force is applied on the tip. Topography is then achieved by scanning the tip in contact with the device while changing its height (z -axis) to keep the measured frequency shift constant, at the value of a chosen set point (generally around 300 mHz).

In the SGM mode, the tip is moved away from the sample surface by a given distance (generally between 10 and 100 nm along the z direction) and the tip is biased with a voltage V_{tip} . To do so, a thin gold wire is glued on both sides to the tip and to a copper wire used as support (Figs. 3.2b,c). Figure 3.2c gives a realistic illustration of the tip close to a sample seen through an optical microscope. The gold arrows have been deposited on the surface to find the device in topography mode, i.e., recording the movement of the z -scanner

while scanning in x and y direction, and maintaining the tuning fork amplitude, phase and frequency shift signals constant using feedback loops. The mirroring purple surface represents a polished 300 nm-thick SiO_2 layer covering a silicon substrate.

3.1.2.2 The tip-induced potential

When biased in close proximity of a device, the AFM tip induces a local change of charge carriers density in the 2DES. By considering the screening of the tip potential by the 2DES, the effective tip-induced potential follows a Lorentzian function, as confirmed by several experimental studies [144, 145] and further discussed in next chapters.

3.1.3 Imaging electron flow and transport in localized states

This section explains how SGM allows to retrieve information on transport properties of a mesoscopic device at the local scale. The archetypal SGM measurement consists in recording the conductance (or resistance) through the device while scanning the biased tip above the sample surface. It results in conductance maps where the contrast is induced by a modification of the charge carriers flow in the sample when the tip position is changed.

Before illustrating the above discussion by two characteristic examples (including one presented in almost each thesis about SGM that I couldn't avoid due to the beauty of the data), I must insist that no current flows between the tip and the sample. Indeed, a lot of people make the confusion with scanning tunneling microscopy that is a quite different scanning probe technique.

3.1.3.1 Imaging coherent charge flow

Figure 3.3 is adopted from one of the founder paper on SGM, published in 2001 by Topinka *et al.* [2]. It has been obtained by scanning a negatively biased (depleting) tip above a semiconductor-based 2DES in the vicinity of a QPC defined with top split gates biased to deplete the electron gas beneath them. The data have been recorded at a temperature $T = 1.7$ K. Figures 3.3a,b show maps of conductance measured through the QPC for each tip position. Two main features can be highlighted in these maps. First, paths of reduced conductance (from light blue to red) depart from the QPC. It corresponds to the tip positions blocking the electron flow through the 2DES and backscattering them towards the QPC. It highlights the fact that the disorder potential constrains the electrical current through preferential paths. Second, oscillations perpendicular to the imaged electron flow are visible in the map (white arrows in Fig. 3.3a). They are the consequence of the wave nature of electrons that interfere either constructively or destructively when they are backscattered by the tip-induced potential towards the QPC (Fig. 3.3b). The distance between two oscillations is therefore linked to the Fermi wavelength (section 1.1).

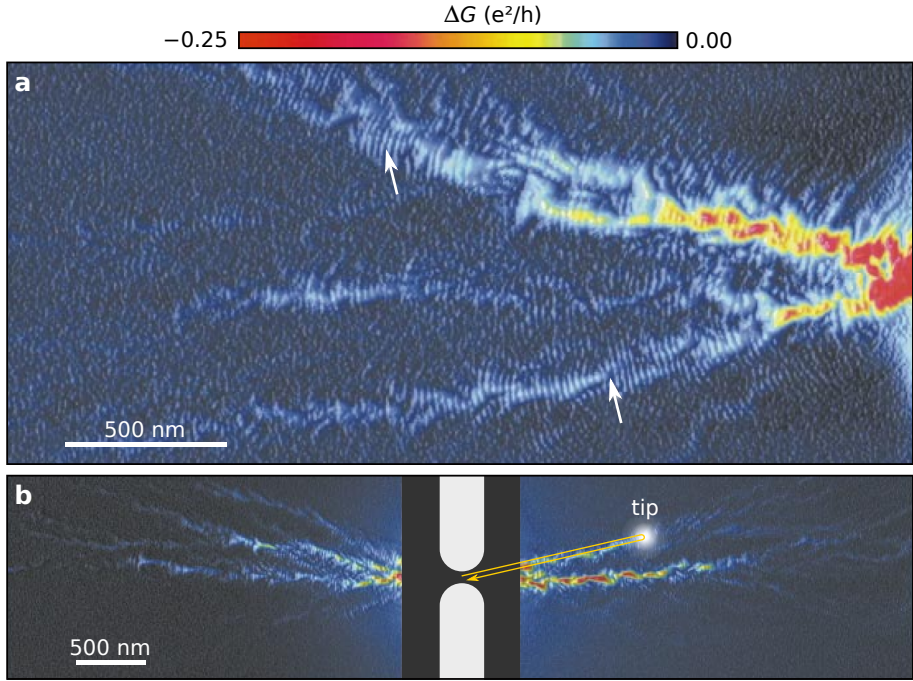


Figure 3.3: Imaging electron flow through a QPC. Adapted from [2]. **a**, Map of the conductance variation ΔG measured by scanning the tip above a 2DES buried in a GaAs/AlGaAs heterostructure. The map features reduced conductance paths departing from the constriction (located on the right). The white arrows indicate interference fringes linked to the wave nature of electrons. **b**, Same as **a** for another QPC. The position of the metallic split gates are shown in light gray. The interfering path between the tip-induced potential (in white) and the QPC, at the origin of the interference fringes in **a**, is illustrated in orange.

The electrical branched flow illustrated in Fig. 3.3 has been studied in more details in other experimental [146–152] and theoretical [153–155] studies and has been compared to the thermoelectric transport in a QPC [156]. Charge carriers trajectories experiencing magnetic focusing have also been imaged with SGM in semiconductor-based 2DESs [157] and in graphene [158]. Lastly, the technique has been used to characterize the ballistic injection of charge carriers in mesoscopic devices [159].

SGM also gives access to the LDOS associated to the different modes in a QPC [160] as well as the LDOS in semiconductor-based 2DESs [161, 162] and in graphene [163, 164] quantum devices, associated to resonant states (or quantum "scars"). At low magnetic field, oscillations due to Aharonov-Bohm effect have been imaged in real space in such quantum rings [165].

Finally, at large magnetic field, SGM has been used to characterize the coupling between QHECs (section 2.2.2) brought in close proximity in semiconductor-

based 2DESs [4, 99, 100, 166–171] and in graphene [10, 11, 122]. Figures 2.11 and 2.17 presents some key results of SGM applied to the QH regime.

A first conclusion can be drawn at this point: SGM requires to constrain the charge carriers flow through a region whose width is of the same length scale as the tip perturbation. Otherwise, no SGM contrast can be expected due to a large number of conduction channels circumventing the tip. It explain why a lot of remarkable SGM results have been obtained in the vicinity of a QPC or in quantum rings.

3.1.3.2 SGM on systems featuring electron-electron interactions

Other systems of interest where SGM provides precious information are quantum dots experiencing Coulomb blockade (section 1.1.4). Figure 3.4a shows an SGM map measured on a graphene quantum dot [172]. It features concentric rings of high conductance (light shades) separated by regions of low conductance (dark shades) when the tip is scanned in the vicinity of the dot. The origin of these rings is illustrated in Figs. 3.4b,c. Changing the distance $\mathbf{r}$ between the dot and the tip (biased at V_{tip}) modifies the capacitive coupling $\alpha_{tip}(\mathbf{r})$ between the two, and hence the induced potential $\alpha_{tip}eV_{tip}$ in the dot. A low conductance is found when no discrete energy state stands between the potential μ_S of the source and μ_D of the drain (Coulomb blockade in Fig. 3.4b). On the contrary, a high conductance is measured when a discrete energy levels allows the conduction from the source to the drain (Fig. 3.4c). This kind of signature has been highlighted in several studies [144, 173–179].

Based on the above discussion, the rings in Fig. 3.4a can be understood as the loci of tip positions yielding the same induced potential in the dot. Naively, we would expect that these rings are circular, which is however disproved in Fig. 3.4a. The reason is the presence of metallic side gates (in yellow) that induce a screening of the tip potential [172] and lead to different values of $\alpha_{tip}(\mathbf{r})$ despite identical dot-tip distances $|\mathbf{r}|$.

Electron-electron interactions dominating charge transport have been unraveled with SGM in other systems. Indeed, the contribution from Kondo effect has been revealed thanks to this technique in QPCs at low electron density, which explains the 0.7 and zero-bias anomalies observed in such a system at ultra-low temperature [21, 180].

3.1.3.3 A wide variety of SGM measurements

In the two previous section, we have seen two examples of SGM maps obtained by measuring the conductance while scanning the tip above the device. Nevertheless, SGM offers a plethora of characterization possibilities. The constant is the measurement of transport through the studied device (conductance, resistance or thermopower). Otherwise, it can be recorded by varying the tip position and/or the tip voltage in combination with a change of source-drain

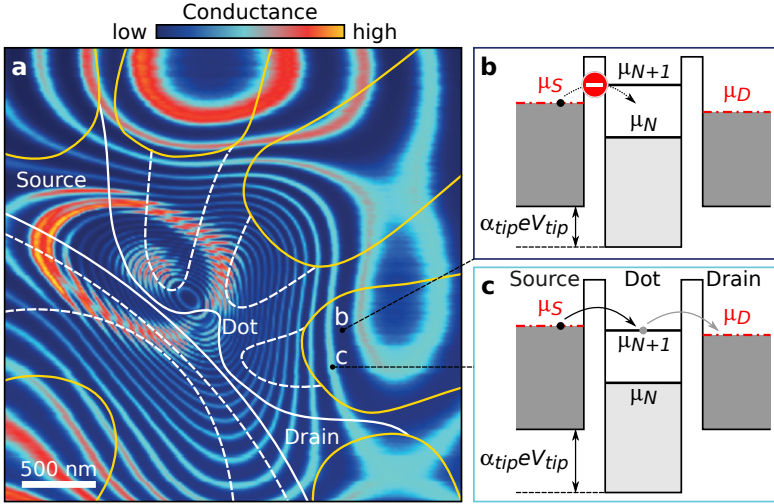


Figure 3.4: SGM on a graphene quantum dot. Adapted from [172]. **a**, SGM map of the conductance through a dot (whose edges are shown with plain white lines) with a finite source-drain bias voltage $V_{sd} = 0.5\text{mV}$. The graphene is doped with electrons and the $V_{tip} = -1\text{V}$. The device is surrounded by graphene side gates (white dashed lines) contacted with metallic pads (yellow lines). The SGM map features concentric rings alternating between low and high conductance and centered in the vicinity of the dot. **b**, Schematic representing Coulomb blockade and corresponding to the regions of low conductance in **a**. **c**, Schematic representing a conducting quantum dot corresponding to the regions of high conductance in **a**.

bias or voltage applied on a side or back gate. These different possibilities are exploited in the next chapters.

3.2 Device fabrication

I must confess that I am not a specialist of the process of device fabrication. By chance, I received some working graphene samples built by Sowmya Somanchi in Christoph Stampfer's group (RWTH) at the beginning of my thesis. I take the opportunity to thank them again for their precious work. In the present section, I will quickly present how such samples are built as it will be necessary for section 5.2. It is also the occasion to present the stacking station that I have developed with Sébastien Toussaint as we were quite proud of its performances. To keep on with acknowledgments, I thank him for the great moments spent when developing this station.

3.2.1 Encapsulating graphene between hBN

In 2010, Andrei Geim and Konstantin Novoselov won the Nobel prize for having produced and studied graphene for the first time in 2004 [24]. This

achievement has been made possible thanks to... Scotch[®] tape. They used it to exfoliate repeatedly graphite (being simply a stacking of graphene planes) until they found a single graphene layer on the Si/SiO₂ slice used as final substrate. Since 2004, the quality of graphene samples has been strongly improved by encapsulating it between hBN layers (section 1.2.2). This section shows how hBN-graphene-hBN stacks are built.

3.2.1.1 Realization of a hBN-graphene-hBN stack

Figure 3.5 illustrates the different steps needed to build a stack (as first developed in Cory Dean's group (Columbia University) in ref. [181]). First, exfoliated hBN and graphene flakes, deposited on Si/SiO₂ substrates, should be carefully selected. Then, a polypropylene carbonate (PPC) stamp is used to pick up the top hBN layer (Fig. 3.5a). This hBN flake is deposited on the graphene flake (Fig. 3.5b) and the substrate is slightly heated to soften the PPC and improve the contact between hBN and graphene. The PPC stamp is then lifted with the hBN/graphene stack attached to it (Fig. 3.5c). Finally, this stack is deposited on the bottom hBN layer (Fig. 3.5d) and the substrate is heated at high temperature to melt the PPC in contact with it and ensure that the three layers stays on the substrate when the rest of the stamp is removed (Fig. 3.5e). A last step consists in removing the PPC residues in a ultra-high vacuum oven.

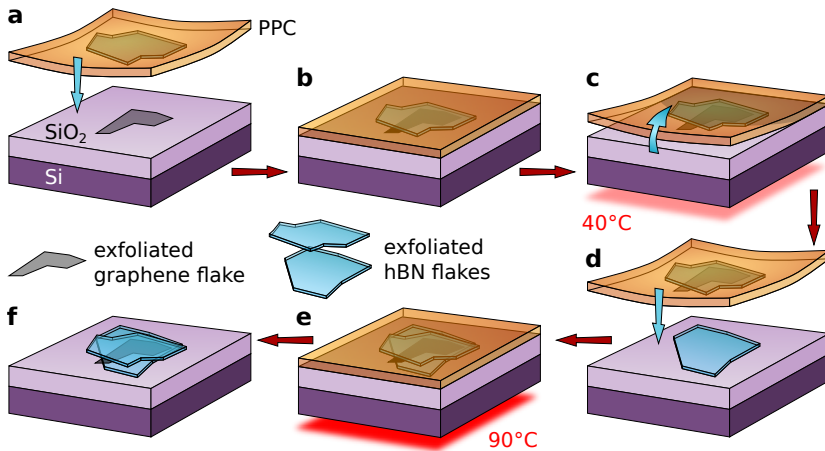


Figure 3.5: Schematic of the stacking process of an encapsulated graphene device. a-f, Different steps of the stacking process. The PPC is illustrated in orange, the Si/SiO₂ substrate in purple, the graphene flake in grey and the hBN flakes in blue.

3.2.1.2 A home-made transfer station

In practice, the different steps illustrated in Fig. 3.5 are realized with a stacking station as the one shown in Fig. 3.6. In this home-made setup, an aluminum plate (1) is put on top of a copper plate (2) and both are separated by a Peltier module (3). The sample (4) sticks to the aluminum plate by suction thanks to a

small hole connected to vacuum (5). The temperature of the aluminum plate, and therefore of the sample, is tuned by applying a voltage V on the Peltier module. Depending on the voltage polarity, the heat flow (Q) direction can be inverted between the aluminum and the copper plate. Hence, the copper plate should act as a thermal reservoir kept at constant temperature. This is ensured by its larger mass-related thermal inertia compared to that of the lighter aluminum plate and, more importantly, by a water circuit carved into the plate (6). Water at room temperature is injected (blue arrows) in the circuit via a silicon tube and collected in the same way at the output. This design allows to reach a temperature of 100°C and then to go down below 0°C in a few minutes only. The temperature of the sample is measured thanks to a platinum-based resistor inserted into the aluminum plate (7). It is taken as an input by a Omron[®] controller that switches on and off the Peltier voltage to follow the given temperature set-point.

The PPC stamp (8) used to transfer graphene and hBN flakes (Fig. 3.5) is fixed to a microscope glass slide (9). A Polydimethylsiloxane (PDMS) pad (10) is placed between the glass slide and the PPC to give it a domed shape. Hence, it touches the sample on a precise point, defined by adapting its xyz -position thanks to a micromanipulator (11). The contact area between PPC and the sample is varied by the z -position-dependent pressure and by the temperature that softens the polymer. Finally, the sample can be rotated thanks to a radial micromanipulator (12) and displaced in the xy plane thanks to linear micromanipulators (13).

3.2.2 Line contacts

We have seen in section 3.2.1.1 how to build a hBN-graphene-hBN stack. It remains to explain how the desired device can be carved out from this stack and how charge carriers can be injected and collected to perform electrical measurements.

The device geometry is defined with a lithographically patterned polymer mask deposited on top of the stack (Fig. 3.7a). After an etching process with fluorine-based gases [181, 182], only the material below the mask remains (Fig. 3.7b). This mask is then chemically removed to be replaced by a new one defining the shape of the metallic contacts. Since graphene is encapsulated, its interface with the deposited metal is one-dimensional (Fig. 3.7c) and is improved with beveled device borders (Figs. 3.7c-e), obtained by properly tuning the etching parameters in Fig. 3.7b [181]. Importantly, an adhesion layer should be first deposited (Chromium in Fig. 3.7e) before depositing the main metal used for the contacts (Palladium and gold in Fig. 3.7e), in order to lower the resistance of the interface with graphene.

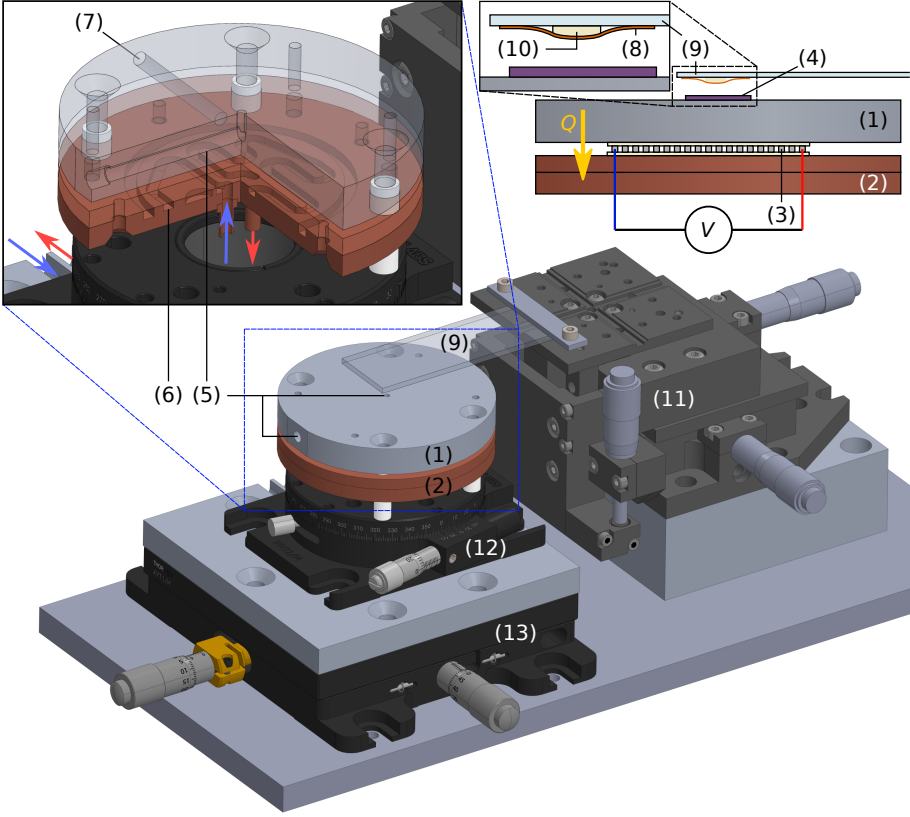


Figure 3.6: Scaled schematic of a home-made transfer station. The upper left scheme shows a zoom of the aluminum heater plate (1) and copper thermal reservoir (2), with a cross-section and transparency to visualize the vacuum feedthrough (5), the water circuit (6) and the thermometer access (7). The upper right scheme shows a side view of the plates separated by the Peltier module (3). The sample (4) sticks to the Aluminum plate by suction. The PPC stamp (8) is put in front of the sample by being attached to a glass slide (9). A PDMS pad (10) gives a domed shape to it. The lower scheme shows the whole setup. Micromanipulators (11), (12) and (13) allows to move and rotate the sample and the glass slide.

3.2.3 Graphene quantum point contacts

As mentioned at the beginning of section 3.2, we received samples from Christoph Stampfer's group (RWTH Aachen). These devices consist in encapsulated graphene QPCs similar to that of Fig. 1.7. Among the four received QPCs, two were unusable due to a back-gate leakage for the first and bad contacts for the second. The two others are named Sample G1 and G2 and are presented in this section.

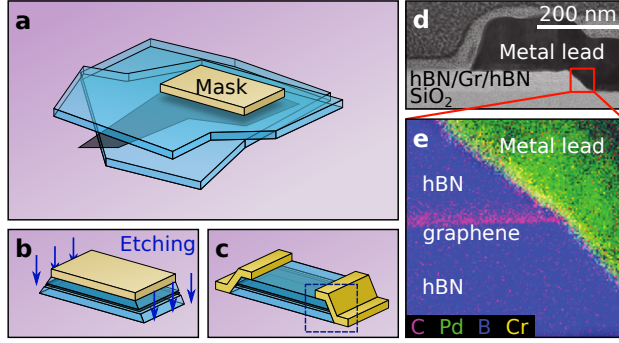


Figure 3.7: Line contacts in the graphene device. Adapted from [181]. **a**, A shaped polymer mask is deposited on the hBN-graphene-hBN stack. **b**, The regions not protected by the mask are etched away. **c**, Metallic contacts are deposited at the device edges. **d**, Scanning electron microscopy showing a side view of the stack. **e**, Imaging of the different species in the contact by electron energy loss spectroscopy. The metal lead is constituted by an adhesion layer of 1 nm of Chromium (yellow) covered by 15 nm of Palladium (green) and 60 nm of gold (black).

3.2.3.1 Presentation of the samples

Figure 3.8a displays a schematic of sample G1. It is built from a hBN-graphene-hBN stack patterned into a QPC geometry. The above hBN is ~ 20 nm-thick and the bottom hBN is ~ 30 nm-thick. Four line contacts allows to inject current through the sample while measuring the resulting voltage drop. It gives access to the longitudinal resistance R_{xx} (and hence the conductance $G = 1/R_{xx}$) by being free from the contact resistance. A back-gate is biased at V_{bg} to tune the global charge carriers density and an SGM tip is biased at V_{tip} to change locally the density (section 3.1.2). The AFM topography of the device performed in-situ is shown in Fig. 3.8b. The constriction width is ~ 250 nm and the whole device dimensions are $\sim 3 \times 3 \mu\text{m}^2$. Figure 3.8c shows an optical photography of sample G1.

Sample G1 went through 9 cooldowns. Chapter 4 presents results measured without magnetic field during the first cooldown whereas chapters 5 and 6 discuss results obtained in the QH regime during the 9th cooldown.

Figure 3.9a shows a schematic of sample G2, similar to the one of sample G1 in Fig. 3.8a. The thicknesses of the hBN layers are the same for both samples. The presence of six contacts allow to measure the Hall conductance G_{xy} . The optical photography of the device is shown in Fig. 3.9b. As for sample G1, the constriction is ~ 250 nm-wide but sample G2 is larger than sample G1 with dimensions of $\sim 6 \times 8 \mu\text{m}^2$. Finally, Fig. 3.9c displays an electrostatic force microscopy (EFM) map. This technique consists in mapping the force applied on the electrically-biased tip, through the frequency shift Δf (section 3.1.2), related to the electrostatic interactions with the substrate. When the tip lies above the grounded graphene or above the remote polarized back gate,

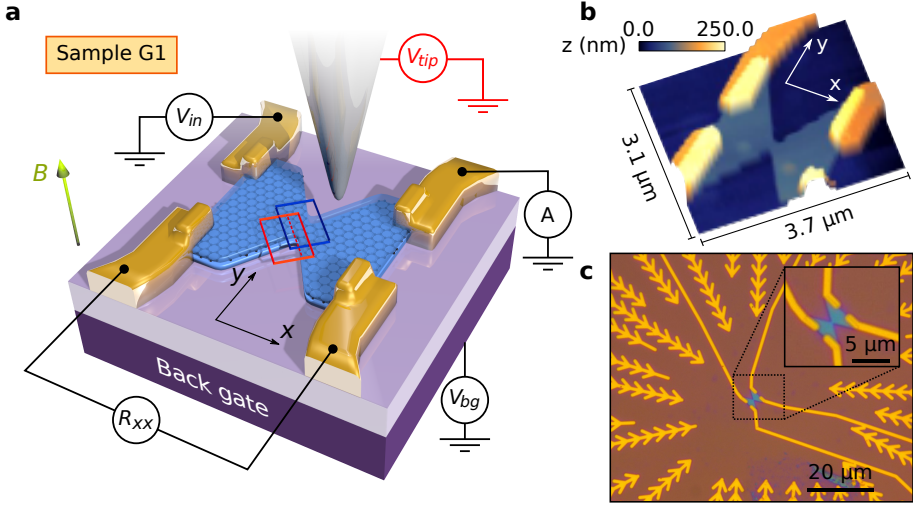


Figure 3.8: Sample G1. **a**, Artist's view of the experimental setup. Graphene is encapsulated between two hBN layer (in blue) and R_{xx} is measured via four line contacts (in gold). The device is deposited on a conducting $n^{++}\text{Si}$ substrate (dark purple) biased at V_{bg} with an insulating 300 nm-thick SiO_2 on top (light purple). The SGM tip is shown in silver and is biased at V_{tip} . A perpendicular magnetic field B can be applied (green arrow). **b**, AFM topography realized with the tip in **a**. **c**, Optical photography of the device (in blue). The contacts and the arrows used to find the device are in yellow.

both metallic plane located at different distances from the tip, the resulting electrostatic interaction is different and results in the contrast shown in Fig. 3.9c.

3.2.3.2 Landau fan diagram

As mentioned in previous section, the back-gate voltage V_{bg} in Figs. 3.8a and 3.9a tunes the global charge carriers density n_c in the sample. The proportional factor between the two values is the lever arm defined $\alpha = n_c/V_{bg}$. It has been extracted for both samples by recording the longitudinal resistance R_{xx} while varying V_{bg} (the Fermi energy E_F is shifted) and the magnetic field B (the energy gap between the Landau levels (LLs) depends on B as explained in section 2.1.1). In a textbook representation of quantum Hall effect, R_{xx} is maximal when a E_F is aligned with one of the LL (section 2.2.1.3). It results in stripes of local R_{xx} maxima in the $V_{bg} - B$ map located along the straight lines

$$B_n = \frac{\pi\hbar}{2en} \alpha (V_{bg} - V_{bg,0}), \quad (3.1)$$

where n are the LLs labels (negative for holes and positive for electrons) and $V_{bg,0}$ is the back gate voltage corresponding to charge neutrality (+1.2 V for sample G1 and +7.4 V for sample G2). By converting n to the integer effective filling factor $\nu_{eff} = 4(n + 1/2)$ (section 2.1.3.4), stripes of local R_{xx} minima (E_F

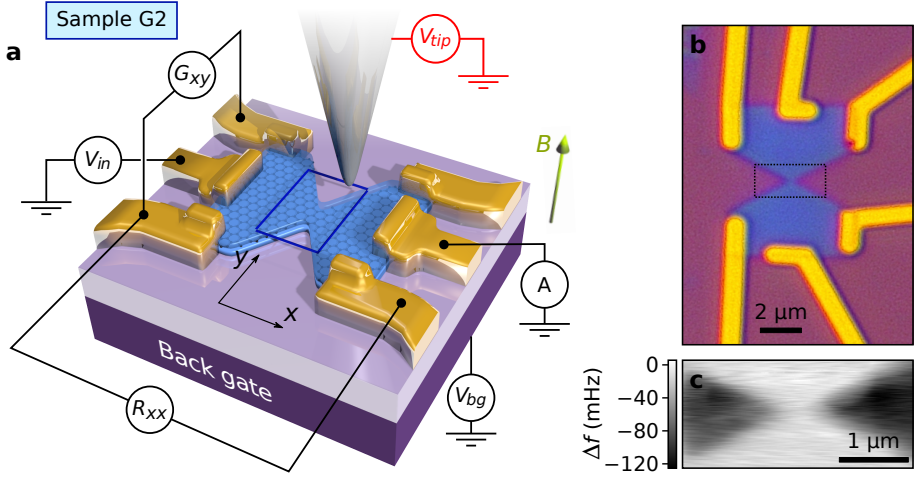


Figure 3.9: Sample G2. **a**, Artist's view of the experimental setup, similar to Fig. 3.8. The presence of six contacts also allows to measure the Hall conductance G_{xy} . **b**, Optical photograph of the device. **c**, Electrostatic force microscopy (frequency shift Δf as a function of the tip position) measured in the region corresponding to the dotted rectangle in **b**. The map has been obtained with $V_{tip} = 3$ V, $V_{bg} = -10$ V and $z_{tip} = 55$ nm.

stands between two LLs) therefore appear at

$$B_n = \frac{2\pi\hbar}{ev_{eff}}\alpha(V_{bg} - V_{bg,0}), \quad (3.2)$$

Fits to the R_{xx} minima stripes were performed for both samples as shown with blue dotted lines in Fig. 3.10 for sample G1 and with red dotted lines in Fig. 3.11 for sample G2. For both samples, it gives $\alpha \sim 9.4 \times 10^{14} \text{ m}^{-2}\text{V}^{-1}$.

Nevertheless, this method assumes that the charge carriers density is constant across the sample. Here, we compare the filling factor value obtained from the fits to the expected theoretical values determined from a simple capacitance model, i.e., assuming that the charge carriers density remains constant in the whole sample. This theoretical values is given by

$$\alpha_{\text{theo}}^{-1} = \left(\frac{\epsilon_{\text{SiO}_2}\epsilon_0}{z_{\text{SiO}_2}} \right)^{-1} + \left(\frac{\epsilon_{\text{hBN}}\epsilon_0}{z_{\text{hBN}}} \right)^{-1}, \quad (3.3)$$

where $\epsilon_{\text{SiO}_2} = 3.9$ and $\epsilon_{\text{hBN}} = 3$ are respectively the SiO_2 and the hBN relative permittivities, $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$ is the vacuum permittivity, $z_{\text{SiO}_2} = 300$ nm is the SiO_2 thickness and $z_{\text{hBN}} = 30$ nm is the hBN thickness. We then find

$$\alpha_{\text{theo}} = 6.5 \times 10^{14} \text{ m}^{-2}\text{V}^{-1}. \quad (3.4)$$

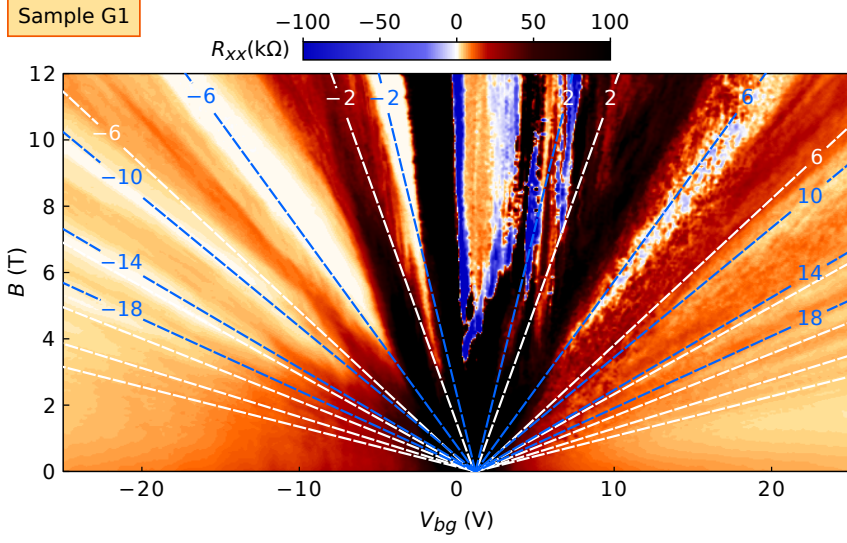


Figure 3.10: Fan diagrams for sample G1. R_{xx} map as a function of V_{bg} and B for sample G1. The blue dashed lines follow the couples (V_{bg}, B) for which, in a textbook representation of quantum Hall effect, the Fermi energy E_F is located perfectly between two LLs in the bulk (R_{xx} exhibits local minima along these lines). These minima coincide with the filling factors ν that label each line. The white dashed lines indicate the couples (V_{bg}, B) giving an alignment between the LLs and E_F calculated from a simple capacitive model.

It appears that the value of α obtained with a fit is larger than the theoretical one, resulting in steeper slopes in the Fan diagram (white lines in Fig. 3.10 and gray lines in Fig. 3.11). The reason for this shift is explained in section 5.1.4.2.

3.2.3.3 The electrons curse

In both samples, measurements at large magnetic field in the electron side were impossible. In sample G1, large R_{xx} fluctuations are visible for positive V_{bg} at $B > 4$ T in Fig. 3.10. It coincides with a drop of the collected current to zero between $\nu = 0$ and $\nu = +4$, leading to diverging R_{xx} , probably due to the loss of electrical contact between the metallic leads and the graphene plane. For $\nu > +4$, we recovered all the current we injected in the sample but the voltage V_{xx} exhibited large chaotic fluctuations that we associate to poor equilibration of the metallic contacts with the QHECs. Such an asymmetric and perturbed behavior has already been reported in [183] and is relatively common in the case of encapsulated graphene devices. An interpretation of these observations is given in section 5.2.4. In sample G2, a leak between the electron gas and the back gate for $V_{bg} > +11$ V prevented us to make measurements with electron-type charge carriers (The charge neutrality point in this sample being at +7.4 V).

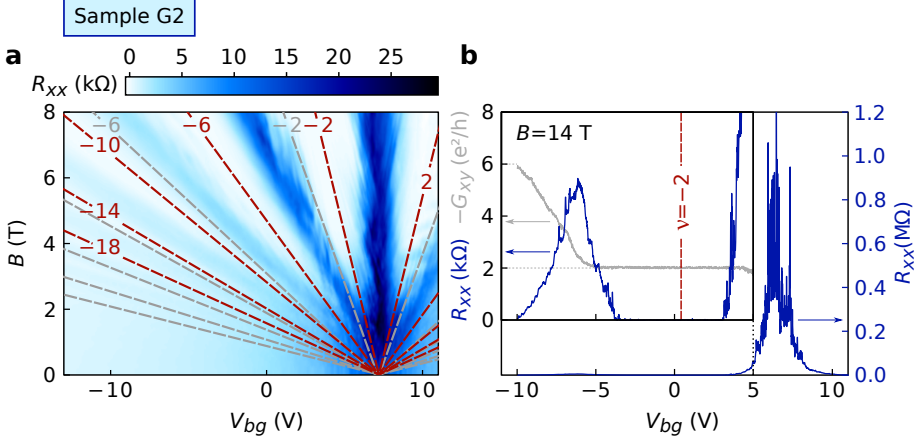


Figure 3.11: Fan diagrams for samples G2. **a**, Same as Fig. 3.10 with the same B and V_{bg} scales. The range in V_{bg} is limited by a back-gate leakage for $V_{bg} < -13$ V and $V_{bg} > 11$ V. The red lines are the integer filling factors fitted to the minima of R_{xx} and the gray lines are obtained with the theoretical capacitive model. **b**, R_{xx} (blue curve) vs V_{bg} at $B = 14$ T. Inset: zoom on the curve with $R_{xx} < 8$ kΩ (blue curve) and $-G_{xy}$ (grey curve) on the same V_{bg} scale. G_{xy} exhibits the expected plateau at $-2e^2/h$ in the $\nu = -2$ QH state. The lowest LL ($n = 0$) is much more insulating than the second LL ($n = -1$).

3.3 Results analysis with the Kwant package

Sections 3.1 and 3.2 give an overview of the main ingredients required for an SGM experiment. Nevertheless, the results obtained with this technique are usually far from trivial to interpret. Indeed, they require to make the link between a global data, the conductance for example, and the local effect of the tip perturbation on this data. Therefore, developing a theoretical model is most of the time mandatory to test the hypotheses elaborated to explain the data. In this thesis, the KWANT package [184], a tight-binding code designed to simulate charge transport in mesoscopic devices, has been used for this purpose and will be presented in the present section.

3.3.1 Tight-binding model for graphene

In next chapters, all the simulations are performed in graphene devices. The tight-binding model will therefore be introduced in the scope of this material.

3.3.1.1 The tight-binding Hamiltonian

As the tight-binding method is used to explore the electronic properties of materials in the quantum mechanics framework, one should first define the Hamiltonian of the studied system. By considering only the orbital combination of the closest neighboring atoms, it can be expressed in the second quantization

formalism as [59]

$$\hat{\mathcal{H}} = \sum_i U(\mathbf{r}_i) \hat{c}_i^\dagger \hat{c}_i - \sum_{\langle i,j \rangle} t \hat{c}_i^\dagger \hat{c}_j, \quad (3.5)$$

where $t = 2.7$ eV is the hopping parameter between neighboring atoms in graphene [5], $\langle i, j \rangle$ is the restriction over the nearest neighboring atoms indices, $\hat{c}_i^\dagger$ and $\hat{c}_i$ are respectively the creation and annihilation operators on site i and $U(\mathbf{r}_i)$ is the potential on site i . For an honeycomb lattice as graphene, the energy dispersion at low energies is linear as a function of the wavevector $\mathbf{k}$ (section 1.2) and the energy dispersion writes

$$E = \pm \frac{3at}{2} |\mathbf{k}|, \quad (3.6)$$

where $a = 1.42$ Å is the inter-atomic distance of graphene (Fig. 1.4a). In this expression, the ratio between E and $|\mathbf{k}|$ is a constant directly proportional to the Fermi velocity $v_F = 3at/2\hbar$. This approximation of the original band structure is however valid provided that the Fermi energy remains in the range $-t/2 \lesssim E_F \lesssim t/2$.

3.3.1.2 Charge carriers density

In a real graphene device, applying a back-gate voltage shifts the potential landscape $U(\mathbf{r})$ compared to the Fermi energy E_F . It results in a change of charge carriers density $n(\mathbf{r})$. From the same reasoning, the density in a simulated system is expressed from the Fermi energy and onsite potential as [59]

$$n(\mathbf{r}_i) = \frac{\text{sgn}(E_F - U(\mathbf{r}_i))}{\pi} \left(\frac{E_F - U(\mathbf{r}_i)}{\hbar v_F} \right)^2, \quad (3.7)$$

where $\text{sgn}(E_F - U(\mathbf{r}_i)) = 1$ for electrons ($U(\mathbf{r}) < E_F$) and $\text{sgn}(E_F - U(\mathbf{r}_i)) = -1$ for holes ($U(\mathbf{r}) > E_F$). With this relation, simulations can be properly compared with experimental results provided that V_{bg} has correctly been converted into the charge carriers density (section 3.2.3.2).

3.3.1.3 Scaling the system

When the tight-binding method is used to simulate a real device, it is most of the time impossible to have as much atoms as in the experiment. Thankfully, it is possible to apply a scaling on a to reduce the number of atoms in the simulation while keeping the density $n(\mathbf{r}_i)$ in Eq. (3.7) unchanged for the same Fermi energy E_F . To do so, v_F should remains constant. It implies to transform the hopping parameter as well and the scaled system is built with [59]

$$a_{\text{scaled}} = a \times s \quad \text{and} \quad t_{\text{scaled}} = t/s, \quad (3.8)$$

where s is the scaling factor. Importantly, this transformation does not affect the simulation output if E_F remains in the linear regime of the band structure. The condition introduced in section 3.3.1.1 then becomes $-t_{\text{scaled}}/2 \lesssim E_F \lesssim t_{\text{scaled}}/2$. Furthermore, at low E_F , one should insure that the Fermi wavelength $\lambda_F = 2\pi/k_F$ remains much larger than the inter-atomic distance a_{scaled} to avoid sampling issues.

3.3.1.4 Adding the magnetic field

A magnetic field B perpendicular to the graphene plane is introduced in the tight-binding Hamiltonian of Eq. (3.5) by modifying the hopping parameter t as

$$t \longrightarrow t \exp \left(-j \frac{e}{h} B (x_i - x_j) \frac{y_i + y_j}{2} \right), \quad (3.9)$$

where (x_i, y_i) are the Cartesian coordinates of site i at position $\mathbf{r}_i$. In graphene, the scaling transformations of Eq. (3.8) also keeps LL energies unchanged while varying s at constant magnetic field B . Indeed, these energies are given by $E_n = v_F \sqrt{2\hbar e} \sqrt{nB}$ (Eq. (2.3)) where v_F is kept constant.

3.3.2 Simulating a device with KWANT

The KWANT package is based on the Landauer-Büttiker formalism introduced in section 1.1.2.3. This section explains how a system is created, based on the example shown in Fig. 3.12, and what are the available outputs of the simulations.

3.3.2.1 Creation of the system

The first step to create a system is to define a unit cell and a lattice symmetry. Here, it is the one of graphene (Fig. 3.12). The scattering region is then created by adding the atoms in a defined region (the blue dashed line in Fig. 3.12). An onsite potential value is associated to each atom and the hopping parameters are defined between pairs of atoms (the closest neighbors in Fig. 3.12). Finally, semi-infinite leads are attached to the scattering region along the translation symmetries of the lattice (the red and pink atomic chains show the two first unit cells of lead 0 and 1 in Fig. 3.12). They are used to send and collect plane waves to and from the scattering region.

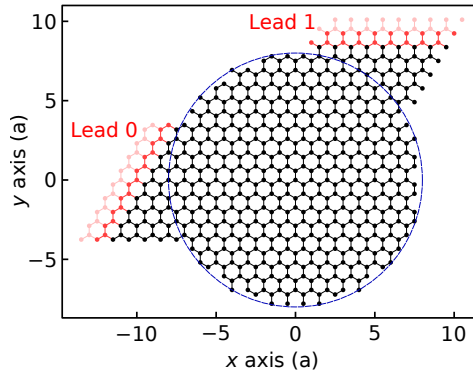


Figure 3.12: Schematic of a system built with the KWANT package. It is built with a lattice featuring a honeycomb structure as in graphene. A scattering region in a defined geometry (here a circle of radius $R = 8a$ shown with a blue dashed line) and two semi-infinite leads of width $8a$ following the crystal orientation.

Based on this system, KWANT builds an Hamiltonian matrix for the whole system that takes the form [184]

$$H = \begin{pmatrix} \ddots & \vdots & \vdots & \vdots \\ \cdots & H_L & V_{LL} & 0 \\ \cdots & V_{LL}^\dagger & H_L & V_{LS} \\ \cdots & 0 & V_{LS}^\dagger & H_S \end{pmatrix}. \quad (3.10)$$

In this expression, H_S is the Hamiltonian matrix of the scattering region, built with the onsite parameters U_i on the diagonal and the hopping terms t_{ij} off-diagonal (Eq. (3.5)). The terms H_L are the Hamiltonian of each unit cell in the leads (the chain of atoms repeated periodically in Fig. 3.12). Each of these terms consider all the leads. With two leads (Fig. 3.12), it writes

$$H_L = \begin{pmatrix} H_{L0} & 0 \\ 0 & H_{L1} \end{pmatrix},$$

where H_{L0} is the Hamiltonian for lead 0 and H_{L1} the one for lead 1. Finally, the term V_{LS} connects the scattering region to the leads and the terms V_{LL} connect the neighboring unit cells in the leads. Therefore, they are only composed of off-diagonal terms.

Based on the Hamiltonian of Eq. (3.5), The wavefunction of the system is defined as [184]

$$\psi = (\cdots, \psi^L(2), \psi^L(1), \psi^S), \quad (3.11)$$

where ψ^L are the wavefunctions in the leads unit cells and ψ^S that of the scattering region.

3.3.2.2 The output in KWANT

From Eqs. (3.5) and (3.11), the Schrödinger equation $H\psi = E\psi$ can be resolved. It gives access to:

- The wavefunction in the leads taking the form

$$\psi_n^L(i) = \phi_n^{in}(i) + \sum_m S_{mn} \phi_n^{out}(i) + \sum_p S_{pn} \phi_n^{ev}(i), \quad (3.12)$$

where n is the label of the n -th mode in the i -th unit cell of the lead, $\phi_n^{in}(i)$ is the incoming plane wave, defined by a current

$$\langle I \rangle = 2\text{Im} \left\{ \left\langle \phi_n^{in}(i) \right| V_L \left| \phi_n^{in}(i-1) \right\rangle \right\} = 1,$$

$\phi_n^{out}(i)$ is the outgoing plane wave, defined by a current $\langle I \rangle = -1$, S_{mn} is the scattering coefficient between mode m and n , and $\phi_n^{ev}(i)$ is an evanescent component of the wave defined by a current $\langle I \rangle = 0$.

- The wavefunction in the scattering region ϕ_n^S associated to a mode in an injection lead. It is matched to the plane wave in the leads.

In Kwant, S_{mn} in Eq. (3.12) and the scattering wavefunction ϕ_n^S are the available outputs. The scattering coefficient S_{mn} incorporates the definition of r_{nm} in Eq. (1.5) and t_{nm} in Eq. (1.6) and therefore gives access to the transmission $\mathcal{T}_{nm} = |S_{mn}|^2$ and to the conductance defined in Eq. (1.11) as

$$G = \frac{g_s e^2}{h} \sum_{n \in a, m \in b} |S_{mn}|^2$$

where g_s is the spin degeneracy and a and b are the labels of two leads. The scattering wavefunction, on its side, allows to compute the LDOS and the local current density.

chapter 4

Veselago lensing

In section 1.2.3, we have seen that charge carriers in graphene can flow through a pn junction by experiencing a refocusing called Veselago lensing. This chapter reports the characterization of a movable circular Veselago lens created with an SGM polarized tip in a graphene QPC (device G1 in section 3.2.3) [185].

4.1 Creation of a movable Veselago lens with SGM

The creation of the lens is achieved by doping the whole device with a back-gate voltage V_{bg} and by applying a sufficiently large tip voltage V_{tip} to invert the charge carriers polarity below the tip (Fig. 4.1a). In this configuration, a smooth circular pn junction (blue dashed line in Fig. 4.1a) is created at the transition between the electron and hole-doped regions. In the following, we show that mapping the variation of the device conductance G as a function of tip position yields images related to the charge flow through the lens. The reference curve of G without the influence of the tip is shown in Fig. 4.1b. Finally, the sample exhibits ballistic transport on a scale larger than the radius of the circular pn junction defined by the tip. This is corroborated by the observation of Fabry-Pérot interference in the npn and pnp configurations, highlighted in the same sample in ref. [186].

4.1.1 Tip potential characterization

The biased tip locally changes the carriers density n , in an isotropic way, leading to a Lorentzian evolution of the density modulation Δn , centered at the tip position. When a pn junction is induced, the Lorentzian profile sharpness has a

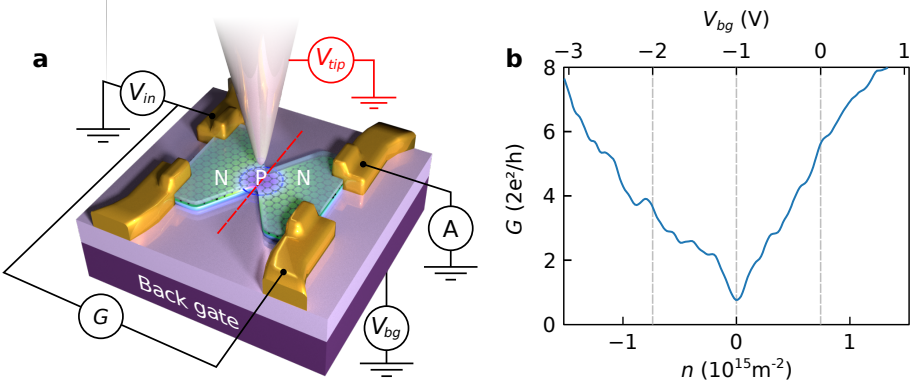


Figure 4.1: Creation of a Veselago lens in a graphene QPC. **a**, Artist's view of the studied sample. A movable Veselago lens (circular pn junction shown with a blue dotted circle) is formed by inverting locally the type of charge carriers with the tip (voltage V_{tip}), compared to the bulk polarized with the back-gate voltage V_{bg} . **b**, Longitudinal conductance G as a function of V_{bg} (converted into charge carriers density in the bottom axis). This curve is obtained without the influence of the tip.

crucial influence on Dirac fermion semiclassical trajectories in its vicinity (section 1.2.3). It is therefore important to extract this profile from our experimental data. For this, we scan the tip along a line perpendicular to the constriction transport axis (red dashed line in Figs. 4.1a and 4.3a), at a distance $z_{tip} = 90$ nm from the graphene plane, and record the device resistance $R_{xx} = 1/G$ as a function of V_{bg} (Fig. 4.2a). The locus of maxima (dark shades) in the resulting map draws a Lorentzian shape centered on the constriction. This locus corresponds to the value of V_{bg} required to locally reach the charge neutrality point (CNP) at the center of the constriction for each tip position y_{tip} . In other words, it gives the change of global density n required to compensate the tip-induced local change of density Δn in the constriction. Therefore, the Lorentzian shape in Fig. 4.2a directly maps the tip-induced potential.

From the above discussion, the modulation of tip density Δn follows a Lorentzian evolution given by

$$\Delta n(\mathbf{r}_{tip}) = \frac{\Delta n_{max}}{1 + \left(\frac{\mathbf{r}_{tip} - \mathbf{r}}{R_{tip}} \right)^2}, \quad (4.1)$$

where $\mathbf{r}_{tip}$ is the $x - y$ position of the tip (in the graphene plane), $\mathbf{r}$ is the position where the density modulation is evaluated (here the center of the constriction) and R_{tip} is the half-width at half maximum (HWHM). The value of Δn_{max} can be extracted from the amplitude ΔV_{bg}^{max} in Fig. 4.2a giving the difference between the maximal CNP shift in V_{bg} (at $y_{tip} = 0$) and the CNP value V_{CNP} without tip influence ($|y_{tip}| = \infty$). This is done by converting

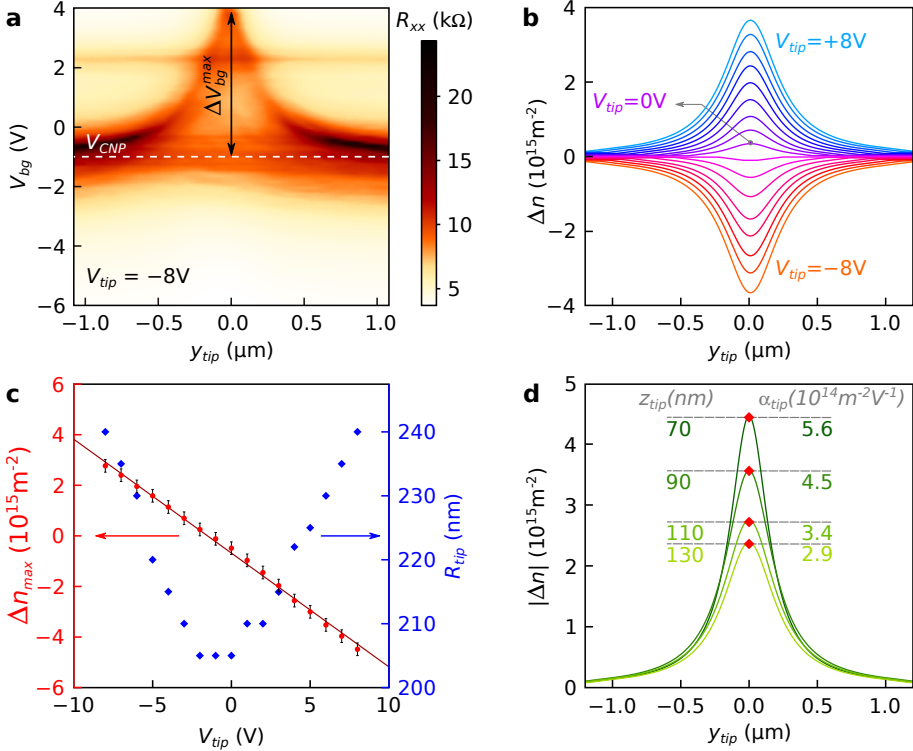


Figure 4.2: Evaluation of the tip-induced potential. **a**, Resistance map as a function of V_{bg} and of the tip position along a line passing by the centre of the constriction perpendicularly to the transport axis with $z_{tip} = 90$ nm and $V_{tip} = -8$ V. **b**, Fits of the tip-induced resistance maxima as a function of tip position extracted for different values of V_{tip} , from -8 V to $+8$ V, and $z_{tip} = 90$ nm, converted into local carrier density. **c**, Evaluation of maximum charge density change around the center of the tip-affected region as a function of tip voltage (red points), and linear fit giving a tip lever-arm $C_{tip} = 4,5 \times 10^{14} \text{ m}^{-2} \text{ V}^{-1}$ (red line). R_{tip} , corresponding to half-width at half maximum of the Lorentzian fits in **b**, as a function of V_{tip} (blue dots). **d**, Tip lever-arm parameter evolution with z_{tip} , for $V_{tip} = -8$ V.

ΔV_{bg}^{max} in a density shift thanks to the lever arm α introduced in Eq. (3.1) as

$$\Delta n_{max} = \alpha \Delta V_{bg}^{max}. \quad (4.2)$$

Curves of Δn vs y_{tip} for different V_{tip} are shown in Fig. 4.2b. They have been obtained by adjusting the parameters Δn_{max} and R_{tip} in Eq. (4.1) to fit the tip-induced resistance maxima of maps similar to that shown in Fig. 4.2a. The values of these parameters for different V_{tip} are shown in Fig. 4.2c. As expected, Δn_{max} is proportional to V_{tip} and we can define the tip lever arm as

$$\alpha_{tip} = \frac{\Delta n_{max}}{V_{tip} - V_{WF}}, \quad (4.3)$$

where V_{WF} is the voltage offset caused by the difference of wavefunction between graphene and the tip, leading to a tip-induced potential at $V_{tip} = 0$ (Fig. 4.2b). The tip lever arm can be tuned by changing z_{tip} as shown in Fig. 4.2d. Finally, the HWHM R_{tip} depends also on V_{tip} and z_{tip} and varies between 200 and 300 nm. This is an important parameter since it allows to tune the pn junction steepness.

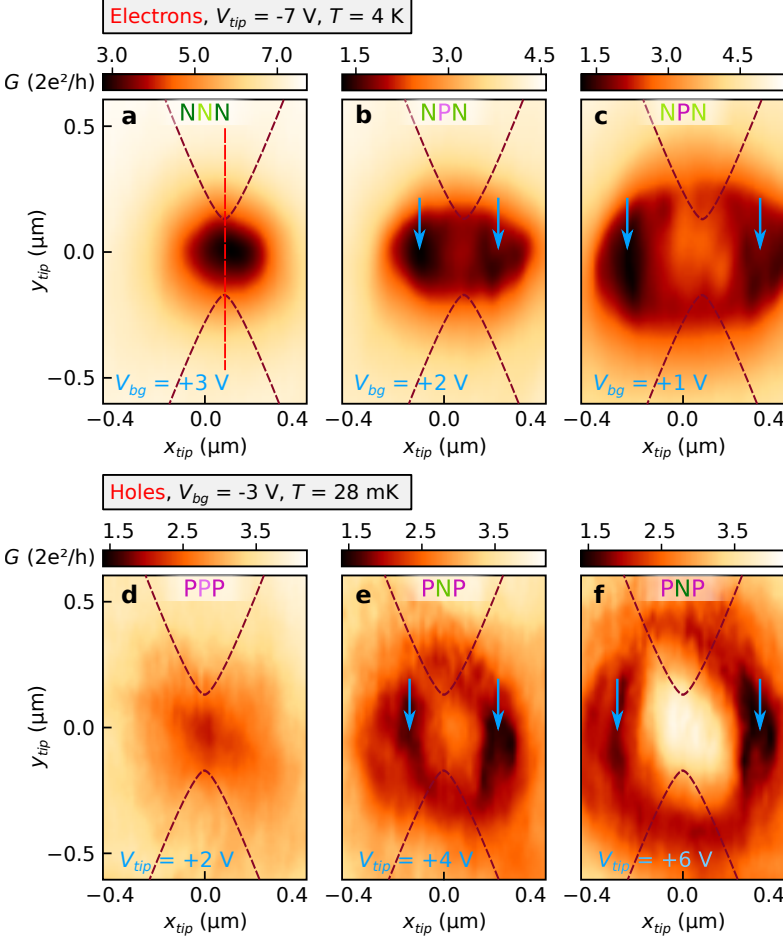


Figure 4.3: SGM images evolution with carriers density and temperature. The dark red dashed lines indicate the constriction edges. The blue arrows indicate the position of the low G spots. **a-c**, SGM images on electron side at 4K, for $V_{tip} = -7$ V and $z_{tip} = 120$ nm, inducing a density change $\Delta n_{max} \sim -2.3 \times 10^{15} \text{ m}^{-2}$ for a bulk density $n = 3 \times 10^{15} \text{ m}^{-2}$ (a), $n = 2.2 \times 10^{15} \text{ m}^{-2}$ (b) and $n = 1.5 \times 10^{15} \text{ m}^{-2}$ (c). **d-f**, SGM images at 28 mK on holes side for $n = -1.5 \times 10^{15} \text{ m}^{-2}$ and $z_{tip} = 120$ nm for different V_{tip} corresponding to $\Delta n_{max} = 1.2 \times 10^{15} \text{ m}^{-2}$ (d), $\Delta n_{max} = 2.5 \times 10^{15} \text{ m}^{-2}$ (e) and $\Delta n_{max} = 3.7 \times 10^{15} \text{ m}^{-2}$ (f).

4.1.2 Transport through the lens and a constriction

With this fully characterized tip influence, we analyze SGM images obtained in different conditions of bulk carrier densities n , tip voltages V_{tip} , and temperature T (Figs. 4.3 and 4.4). Figures 4.3a-c show the evolution of the SGM contrast with carrier density. V_{bg} is chosen such that bulk transport is governed by electrons. In Fig. 4.3a, the highest tip-induced density change ($\Delta n_{max} \sim -2.3 \cdot 10^{15} \text{ m}^{-2}$) is lower than the bulk electron density ($n = 3 \cdot 10^{15} \text{ m}^{-2}$). It therefore yields an n^+nn^+ configuration, inducing a quasi-isotropic decrease of G as the negatively biased tip moves close to the constriction center. This means that G is governed by the constriction region, and that reducing the carriers density in the center of the constriction reduces the total transmission, as naturally expected. This prominent influence of the constriction on transport justifies the validity of the method to characterize the tip-induced potential described in section 4.1.1. For lower bulk densities (Figs. 4.3b,c), the tip bias creates a circular n^+pn junction. In this case, an important qualitative change is observed. Contrary to Fig. 4.3a, we observe a local maximum of G when the tip is placed at the very center of the constriction, surrounded by an elliptic region of reduced conductance. Interestingly, the conductance drop is not isotropic anymore, and two spots of lower G are clearly visible on both sides of the

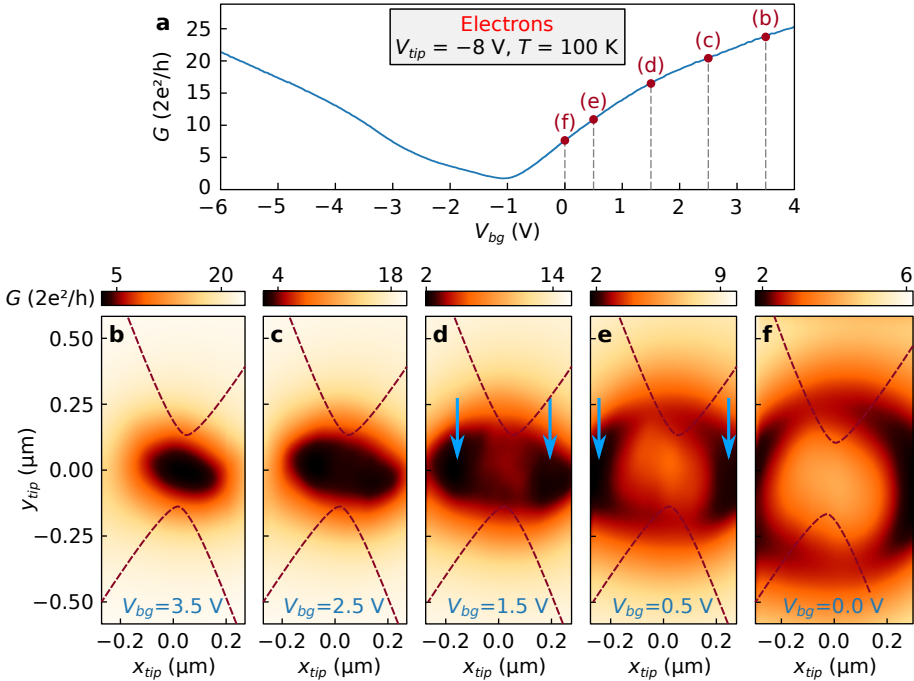


Figure 4.4: Signatures of Veselago lensing at high temperature. **a**, G vs V_{bg} across the QPC, measured in four-contacts configuration, at a temperature around 100 K. **b-f**, SGM maps for a tip voltage -8 V and tip-to-graphene distance of ~ 100 nm, for the gate voltages indicated on **a**.

constriction (blue arrows in Figs. 4.3b-c).

A similar behavior is observed for hole-type carriers, as shown in Figs. 4.3d-f. Here, the bulk hole density is fixed ($V_{bg} = -3$ V) and V_{tip} is varied. Once again, reducing the hole density below the tip leads to a quasi-isotropic single spot of reduced G (Fig. 4.3d). But when V_{tip} is chosen such that it locally changes the charge carriers sign (i.e., a circular pnp region is created), a local maximum of G is observed when placing the tip at the very center of the constriction, and two regions of lower G are visible when placing the tip on both sides of the constriction along the transport axis (Figs. 4.3e,f). Note that data in Figs. 4.3d-f are recorded at $T = 28$ mK, which does not qualitatively change the main observations compared to Figs. 4.3a-c, recorded at 4 K. The main features are robust to temperature changes, and still observable up to $T \sim 100$ K (Figs. 4.4b-f) with SGM maps obtained with $V_{tip} = -8$ V and $z_{tip} \sim 100$ nm. In these maps, the vertical and horizontal length scales are only roughly estimated because we have no calibration for our piezoelectric scanner at 100 K. These images therefore just bring a qualitative insight on the temperature robustness of the observed features.

4.2 Simulations

In the following, we discuss the origin of the observed features in the SGM maps of Fig. 4.3 with tight-binding simulations (performed with the KWANT package (section 3.3.2) and a recursive Green's function code [187]).

I still want to thank Viet-Hung Nguyen for his help with the simulations presented below, as he performed them with his home-made tight-binding code.

4.2.1 SGM maps image current density

The charge carrier flow around a tip-induced potential is first investigated in an infinite graphene sheet without the constriction (Figs. 4.5a,c). The tip-induced perturbation is included by imposing an on-site potential

$$U(\mathbf{r}_i) = \frac{U_{max}}{1 + \left(\frac{|\mathbf{r}_i - \mathbf{r}_{tip}|}{R_{tip}} \right)^d}, \quad (4.4)$$

where $\mathbf{r}_i$ is the position of the site i , U_{max} is the maximal value of the tip-induced potential, and d is a decay exponent used to tune the potential steepness. The current density $|\mathbf{J}|$ is computed at a given Fermi energy E_F . Note that all distances, energies, and potential are scaled to match the experimental parameters. Figure 4.5a presents the current density around a Lorentzian potential ($d = 2$ in Eq. (4.4)) fixed at $\mathbf{r}_{tip} = (0, 0)$, for $R_{tip} = 300$ nm and $V_{max} = 2E_F$, therefore inducing a circular pn junction. In this configuration,

$|\mathbf{J}|$ is maximal at the center of the lens. Figure 4.5b shows the computed SGM map when the perturbation is scanned around a 250-nm-wide constriction (represented by blue dashed lines), i.e., the system transmission as a function of the Lorentzian potential position.

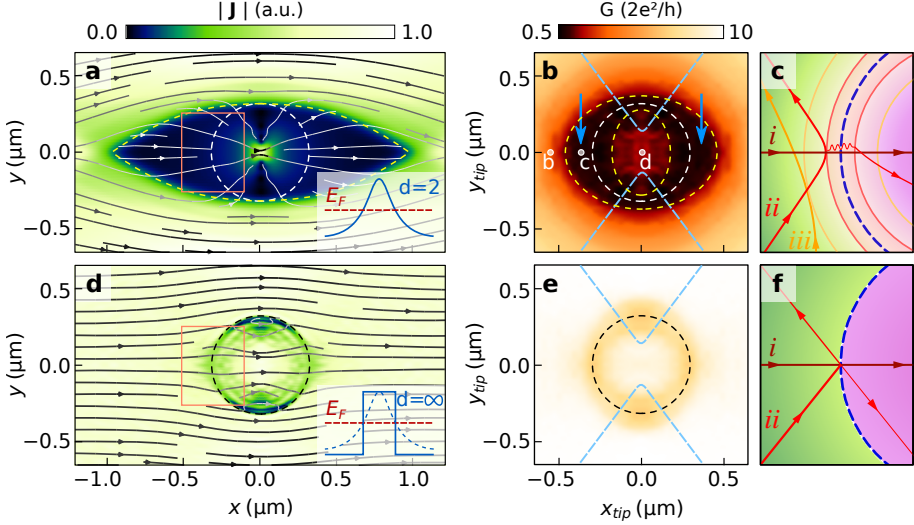


Figure 4.5: Tight-binding simulations comparing SGM and current density maps. **a**, Simulations of current density $\mathbf{J}$ in a graphene sheet around the fixed tip potential of Eq. 4.4 characterized by the parameters $V_{max} = 2E_F$ and $d = 2$. The dashed line corresponds to the intersection between the tip potential and E_F . **b**, Simulated SGM map obtained by displacing the same potential as **a** around a 250 nm-wide constriction. The edges of the constriction are depicted with blue dashed lines. **c**, Qualitative sketch illustrating the electron trajectories (i, ii and iii) impinging the circular smooth pn junction in an area corresponding to the orange square in **a**. A *forbidden zone* is associated to each trajectory (red boundaries for trajectory ii and orange for iii). **d**, Same as **a** for an abrupt pn junction characterized by $d = \infty$ and $V_{max} = 2E_F$. **e**, simulated SGM map corresponding to **d**. Insets: schematics of potential profile vs position, Lorentzian profile is indicated as dashed blue line for comparison. **f**, Same as **c** for an abrupt pn junction (orange square in **d**). In that case, there is no forbidden zone and the trajectory ii is refracted through the junction with a negative index.

These tight-binding simulations capture the essential features observed in the experimental SGM images (Fig. 4.3), and the comparison with current density maps shines light on the mechanisms at play in the experiment. Indeed, scanning the tip in the vicinity of a constriction yields SGM images that are qualitatively similar to the current density pattern around and through the tip-induced circular pn junction. This result contrasts with SGM images obtained on GaAs quantum point contacts, which take advantage of the tip-induced backscattering to image the unperturbed electron wave functions [160], or trajectories through the bulk disordered potential [2, 151]. In graphene, backscattering is prevented by pseudospin conservation, leading to Klein tunneling

(section 1.2.3.1), and this reverses the paradigm of SGM experiment. In turn, this allows one to study the Dirac fermion flow through the tip potential as detailed below.

Klein tunneling is characterized by a perfect transmission for trajectories impinging perpendicularly to a pn junction (trajectory *i* in Fig. 4.5c). Besides this effect, charge carriers tunnel through the junction at finite angle trajectories and are refracted with a negative index (trajectory *ii* in Fig. 4.5c). Given the circular geometry in our experiment, which offers a large collection of incidence angles, these two manifestations of the same physical phenomenon do contribute to the high $|J|$ spot at the pn junction center. The smoothness of the pn interface strongly impacts the transmission probability at finite incident angle [25, 33, 35, 188]. For equal carrier densities in the p and n regions, this probability for an incidence angle ϕ is given by $T(\phi) = \exp(-\pi k_F W \sin(\phi))$ (section 1.2.3.2), where k_F is the Fermi wave vector, and W the junction width. This indicates that trajectories impinging the pn junction with small incident angles are refracted with a negative index and contribute to the total transmission in our experimental conditions while trajectories with a larger angle do not go through the pn junction (trajectory *iii* in Fig. 4.5c) and are associated to backscattering.

The SGM images reveal two main features of the charge flow through the pn junction:

- The local maximum of transmission, observed when the tip is placed at the center of the constriction, reflects the high current density at the lens core in the $n pn$ or $p np$ configuration.
- The reduced conductance spots (low G spots) observed when the tip is placed slightly away from the constriction, indicated by arrows in Fig. 4.5b and very similar to the experimentally observed spots in Fig. 4.3, are the loci of all the tip positions for which the transmission of Dirac fermions through the constriction is strongly reduced. This is reminiscent of the effect visible in Fig. 4.5a where the tip also induces low current density regions (low $|J|$ spots in dark shades) on the sides of the tip-induced pn junction.

Interestingly, the smoothness of the pn junctions plays a crucial role in the emergence of the low G and low $|J|$ spots. They are due to the smooth electrostatic potential evolution that bends electron trajectories away from the lens, as would photons in a gradient index medium. They almost vanish and match the loci of zero charge density in the case of an infinitely sharp circular potential, as shown in Fig. 4.5d for the current density, and in Fig. 4.5e for the corresponding simulated SGM map.

4.2.2 Anti-focusing with a constriction

Nevertheless, the patterns observed in the SGM maps in the case of a smooth junction are significantly distorted with respect to the corresponding current density patterns. For example, the distance from the low G spots to the constriction center in Fig. 4.5b is different than the distance from the low $|J|$ spots to the lens center in Fig. 4.5a. The constriction shape plays a central role in this distortion, as it alters Dirac fermions trajectories. The finite opening angle of the etched region causes a focusing of current density lines toward the constriction, as depicted in Figs. 4.6b-d where we plot the current density $\mathbf{J}$ through the lens for different constriction opening widths. Whereas one of the low $|J|$ spots induced by the lens in Fig. 4.6a is located in the constriction (red star) in these simulations, the total transmission is still large because of trajectories refocused by the constriction edges.

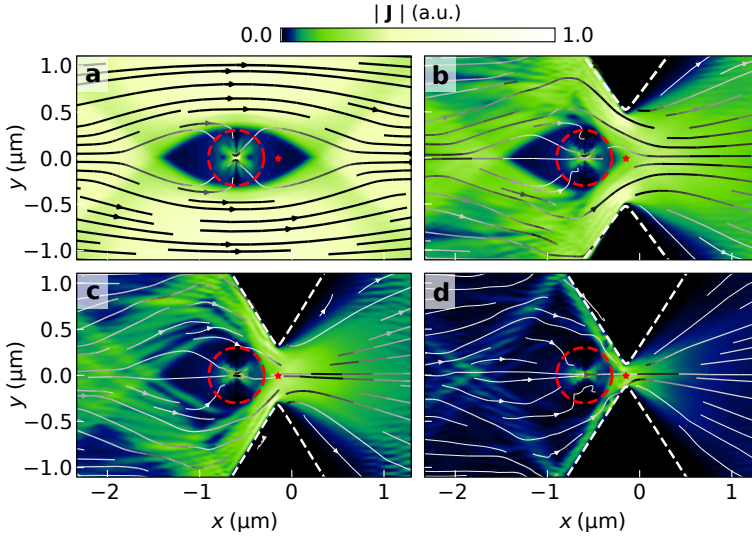


Figure 4.6: Simulation of current density when varying the constriction width. A Lorentzian potential ($V_m = 2E_F$, same as Figs. 4.5a,b) creates a circular pn junction (red dashed line, $R_{tip} = 300$ nm). **a**, Current density computed without constriction. Two spots of low current density appear, similarly to figure 4.5a. **b-d**, Current density computed for constriction widths of respectively 1050 nm (**b**), 650 nm (**c**) and 250 nm (**d**). The distance between the tip and the constriction is 600 nm.

The impact of the constriction on the conductance is better understood by simulating the current density (Figs. 4.7b-d) for the tip positions indicated in the SGM map of Fig. 4.5b (along the transport axis). Figure 4.7b is similar to 4.6d and the conductance remains large in Fig. 4.7b due to the refocused trajectories along the constriction edges, as discussed above. The lowest total transmission is obtained when the forbidden zone (Fig. 4.5c) associated to the lens also blocks these trajectories (Fig. 4.7c). Finally, when placing the tip at the center of the constriction, current density lines are perpendicular to the pn

interface (Fig. 4.7d) and the efficient Klein tunneling in this configuration leads to a large conductance.

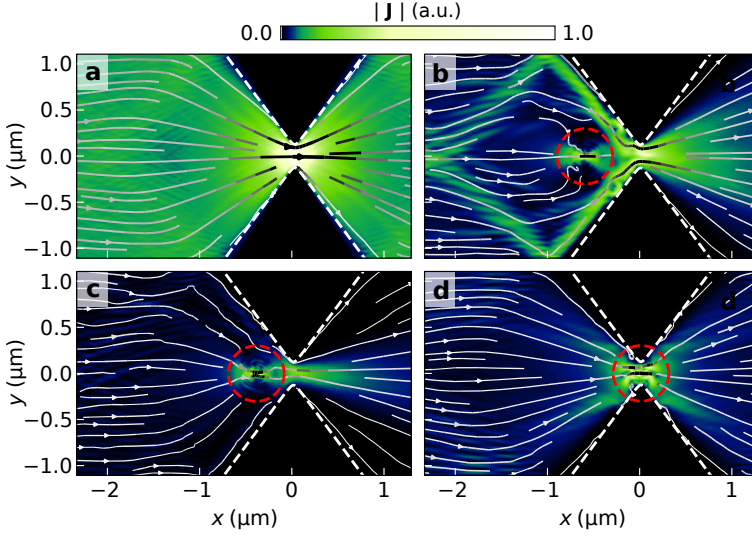


Figure 4.7: Distortion of the flow by the constriction. **a**, Current density through a 250 nm constriction without tip potential. **b-d**, A Lorentzian potential ($V_m = 2E_F$, same as Figs. 4.5a,b) creates a circular pn junction (red dashed line, $R_{tip} = 300$ nm). The tip position is moved along x and is located at a distance from the constriction of 600 nm (**b**) 400 nm, (**c**) and 0 nm (**d**).

4.2.3 The constriction as a current detector

In section 4.2.2, the low G spots origin and shape in the SGM maps of Fig. 4.5 have been discussed with respect to the electron flow around and through the movable lens. However, due to the bending of the electron trajectories along the constriction edges, the low G spots do not have exactly the same shape as the low J spots. In this section, we detail how the design of the constriction can be modified to obtain a direct correspondence between the SGM map and the current density map without constriction (Fig. 4.5a). To do so, a constriction with vertical boundaries is studied (Fig. 4.8a).

The SGM map obtained in the new constriction geometry is shown in Fig. 4.8b. It can be compared to the SGM map of Fig. 4.5a obtained with the triangular-shaped constriction. The conductance curves in Fig. 4.8 highlights the key differences between the two geometries. Three zones can be distinguished, indicated by arrows in Fig. 4.8b and c:

- The yellow arrow indicates the zone of increased conductance caused by Klein tunneling.
- The light blue arrows indicate the low G spots. The position and the shape of these spots are rather similar for both constriction geometries,

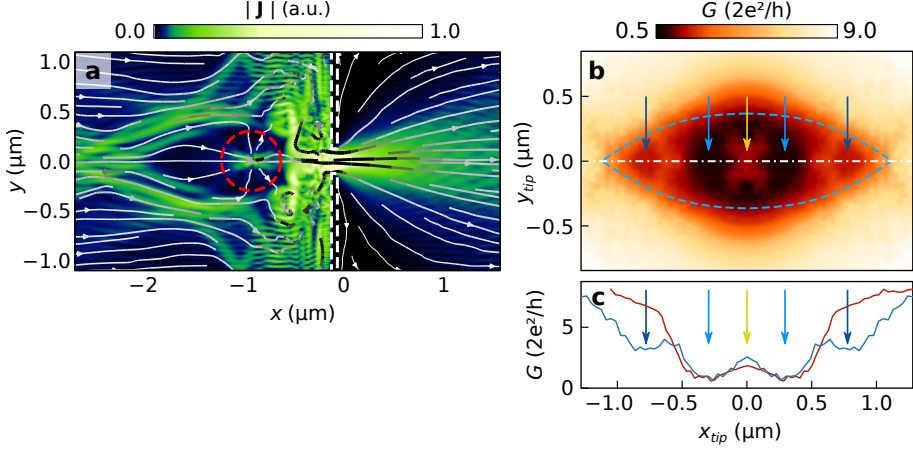


Figure 4.8: Effect of the constriction shape on the SGM maps. **a**, Current density map in a constriction defined by vertical boundaries in presence of a Lorentzian potential ($V_m = 2E_F$) that creates a circular pn junction (red dashed line, $R_{tip} = 300$ nm). **b**, SGM map obtained by moving the same Lorentzian potential in the same constriction as **a**. The blue dashed line shows the limits of the low J zone shown in Fig. 4.5a. **c**, Conductance curve obtained along the white dotted line of **b** ($y_{tip} = 0$) for vertical boundaries (red curve) and triangular boundaries (blue curve). The arrows show three characteristic zones of the SGM map.

as highlighted in Fig. 4.8c. The shape of the low G spots therefore mainly depend on the constriction width and the effect of the pn interface.

- The dark blue arrows indicate plateaus of reduced conductance only visible for the vertical boundaries (blue curve in Fig. 4.8c). By comparing Fig. 4.8b and d, it appears that these zones of reduced conductance are directly related to the low J . To emphasise this correspondence, the delimitation of the low J zone (yellow dotted line in Fig. 4.8d) is reported on Fig. 4.8b.

The appearance of the latter zone, for the vertical boundaries, can be explained by a less efficient bending of electron trajectories toward the center of the constriction, compared to the oblique edge (see section 4.2.2). Indeed, the vertical edges will mostly back-scatter electrons instead of bending their trajectories as illustrated by Fig. 4.8a (in comparison to Fig. 4.6). The low J spots have therefore a real influence when the potential is in front of the constriction. This explains the good correspondence between Fig. 4.8b and Fig. 4.5a.

4.3 Conclusion and perspectives

In this chapter, we have demonstrated that an electrically biased SGM tip can be used to create a circular pn junction-based Veselago lens in graphene. In that framework, the current density around and through this lens have been imaged

thanks to conductance maps recorded by scanning the tip in the vicinity of a constriction. Indeed, this is equivalent to keeping the Veselago lens fixed while moving the constriction, so that only the current density at the center of the constriction contributes to the total conductance, resulting in an equivalence between the two quantities. However, we have shown in Fig. 4.6 that the V-shaped constriction used in the experiment refocuses charge carriers and impairs the correspondence between current density and conductance. As discussed in section 4.2.3, it can be improved by changing the constriction geometry. Hence, future devices featuring narrower constrictions with vertical boundaries could be studied to test the theoretical results of Fig. 4.8.

In another work, the pn junction smoothness has been increased by changing the tip height (Fig. 4.2), leading to charge carriers confinement in the lens [186], a task usually made difficult because of Klein tunneling (section 1.2.3.1). With such an enhanced confinement, FP interference signatures inside the lens have been evidenced. These results highlight the importance of the pn junction smoothness, and hence the graphene-to-gate distance, when designing electron-optics devices.

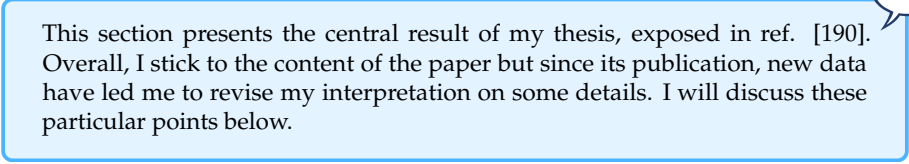
Finally, the experiment reported in this chapter could be performed in bilayer graphene (BLG). This material exhibits what is called anti-Klein tunneling, corresponding to a perfect reflection of charge carriers at a sharp pn junction [7, 189], that would therefore result in no transmission through the QPC when the tip-induced circular pn junction is located in the constriction. However, applying a perpendicular electric field to the BLG plane opens a gap between the conduction and the valence bands and Klein tunneling should be recovered in this situation, due to a continuous change of the Berry phase .

chapter 5

Quantum Hall breakdown in graphene

In section 2.2.1, we have seen that the topological breakdown of a QH state occurs when a conducting path forms between counterpropagating QHECs, leading to the backscattering of charge carriers towards the injection contact. In semiconductor-based 2DESs, counterpropagating QHECs flow along opposite edges of the sample and their breakdown is caused by percolation through the bulk. In graphene, the presence of both down- and upstream QHECs along the same edges lead to a completely different picture. This chapter presents experimental SGM data revealing the microscopic mechanism causing the topological breakdown of graphene QH states. From these data, we also discuss the spatial configuration of QHECs in the sample both for electrons and holes. The influence of contacts is highlighted and allows to explain the electron-hole asymmetry often observed in the graphene QH regime.

5.1 Antidots poisoning



This section presents the central result of my thesis, exposed in ref. [190]. Overall, I stick to the content of the paper but since its publication, new data have led me to revise my interpretation on some details. I will discuss these particular points below.

5.1.1 Imaging the topological breakdown in graphene

The location of topological breakdown can be pinpointed in real space thanks to SGM, provided that the QHECs are sufficiently close to each others. Indeed, if the percolation paths are spread through a bulk region much larger than the tip perturbation, no contrast is expected (section 3.1.3.1). QPCs are therefore the ideal platform given that QHECs are brought in close proximity in the constriction. In such devices, built from semiconductor-based 2DESs, sharp fluctuations have been evidenced in the longitudinal resistance measurements (Fig. 2.11a) [4, 191]. SGM maps, obtained on the same samples, feature circular fringes centered in the vicinity of the constriction (Figs. 2.11b-d). These signatures are explained by the presence of an antidot (section 2.2.2) coupled to the counterpropagating QHECs in the constriction. The potential perturbation of the tip changes the resonance conditions when it is moved in its vicinity, turning on and off QHECs backscattering [4, 191].

Since we had a graphene constriction in our SGM setup, at ultra-low temperature, we logically went into the QH regime to look after topological breakdown. At the time, the SGM data of Marguerite *et al.* [10] were not yet published and we expected the same signatures as those described above. What we observed quite surprised us...

First, we present transport and SGM data obtained in our two graphene QPCs (samples G1 and G2 - section 3.2.3) at the onset of the $\nu = -6$ plateau where R_{xx} , while close to zero, exhibit fluctuations (Figs. 5.1b and 5.2a) as observed in semiconductor-based 2DESs. However, in the corresponding SGM images, the contrast is not located in the vicinity of the constriction, as expected. Here, non-zero R_{xx} regions draw sets of concentric rings centered all along the edges and whose number and position evolve with the tip polarization V_{tip} (Figs. 5.1c-e for sample G1 and Figs. 5.2b-f for sample G2).

The observation of SGM contrast at large distance from the constriction (~ 500 nm in Fig. 5.1, and a few μm in Fig. 5.2) demonstrates that the constriction does not play a significant role here, which is counter-intuitive in the textbook framework of QH effect in conventional semiconductor-based 2DESs. In this picture, the counterpropagating QHECs are separated by an insulating bulk region much larger than the tip-induced perturbation. Away from the constriction, the edge states should only circumvent the perturbation and no tip-effect should be expected in principle.

5.1.2 Antidots and upstream modes

The key missing ingredient in the picture, allowing to solve the puzzling SGM signatures along the devices edges, is the presence of both up- and downstream QHECs along the same edge in graphene, as discussed in details in section

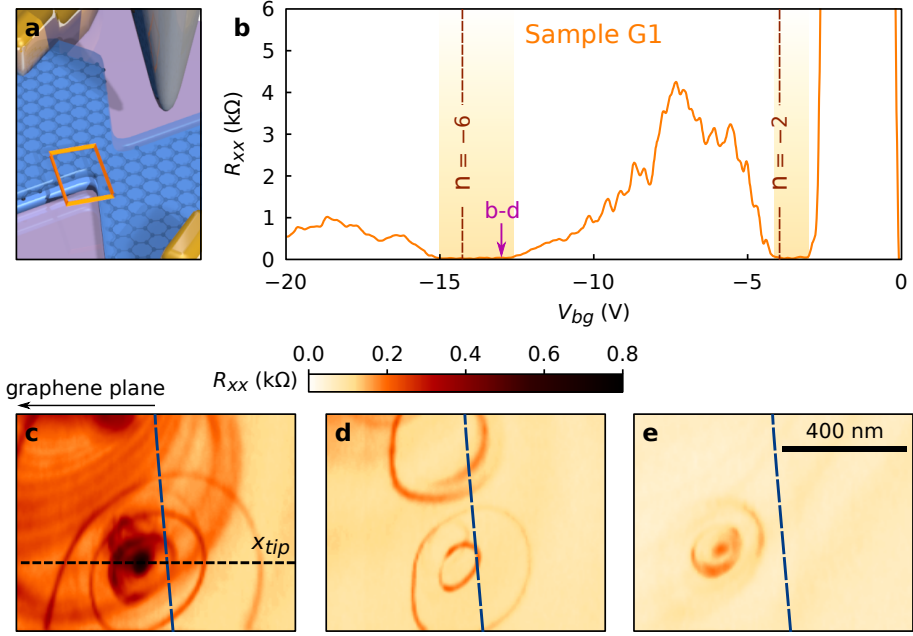


Figure 5.1: Imaging the topological protection break-down. **a**, Schematic of the scan areas (see Fig. 3.8). **b**, Longitudinal resistance R_{xx} as a function of V_{bg} , at $B = 10$ T, measured in sample G1. **c-e**, SGM maps of R_{xx} as a function of tip position. The graphene edge is shown with a dark blue dashed line. The scanning area is sketched by the red rectangle in **a**, located ~ 500 nm away from the constriction. The data are recorded with $V_{bg} = -13$ V - arrow in **b** - and $V_{tip} = +3$ V (**c**), 0 V (**d**) and -6 V (**e**).

2.3. Tunneling between counterpropagating QHECs can then be mediated by the presence of localized states associated with antidots, which pin circular QHECs loops in-between the QH channels (Fig. 5.3a). These antidots are at the origin of the characteristic concentric rings of non-zero R_{xx} in Figs. 5.1c-e and Figs. 5.2b-f. Note that these SGM signatures do not originate from a direct coupling of the counterpropagating QHECs induced by the tip potential alone: this would yield iso-resistance stripes following the edge topography [10, 171]. The absence of such stripes in SGM maps (Figs. 5.1) testifies that the tip perturbation is small enough to avoid inducing direct backscattering. This suggests that antidots are not created by the tip perturbation but are preexisting in the device.

Next we detail how the tip influences tunneling through such an antidot. As detailed in section 3.1.3.2, antidots are expected to host discrete energy levels in the QH regime, corresponding to the resonance condition reached when the charging energy is equal for N and $N + 1$ charges in the localized state. These energy levels are shifted under the tip-induced local modification of potential landscape, as sketched in Figs. 5.3b-c. The high R_{xx} rings in Figs. 5.1c-e are the loci of tip positions leading to an alignment between one of the antidot's

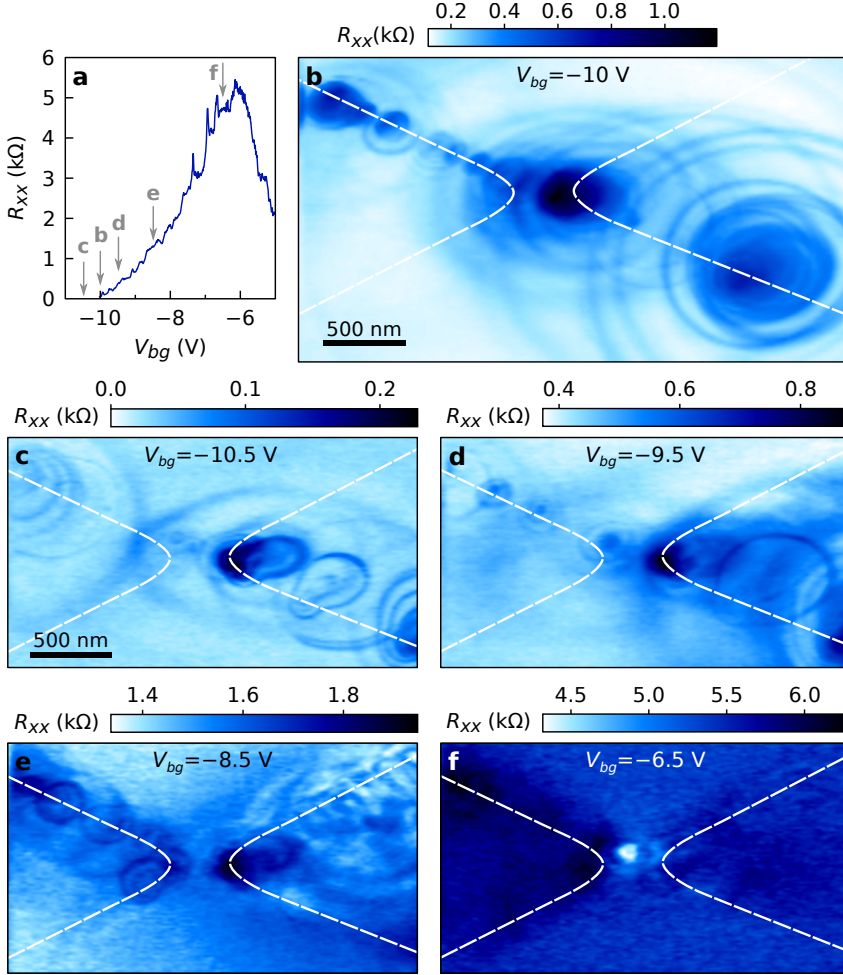


Figure 5.2: Imaging the break-down of topological protection in sample G2. **a**, Zoom on Fig. 3.11b between $\nu = -6$ and $\nu = -2$ corresponding to the loss of topological protection (non-zero R_{xx}). **b-f**, SGM maps obtained at 14 T and $V_{tip} = +3$ V (**b**) and $V_{tip} = 0$ V (**c-f**). The V_{bg} values used for each map are indicated with the arrows in **a**. The rings of higher R_{xx} are the loci of tip positions leading to a resonance between an antidot localized state and the counterpropagating QHECs and hence the breakdown of topological protection of these channels.

discrete energy levels and the QHECs potential (Fig. 5.3c) whereas low R_{xx} between the rings corresponds to Coulomb blockade [176, 177] (Fig. 5.3b).

This picture is supported by the emergence of Coulomb diamonds in scanning gate spectroscopy (Fig. 5.4c) [100]: applying a DC bias between source and drain allows to overcome Coulomb blockade as soon as the source-drain energy windows overlaps a localized state energy (section 1.1.4). In this framework, the position of the antidot corresponds to the center of the high R_{xx} rings

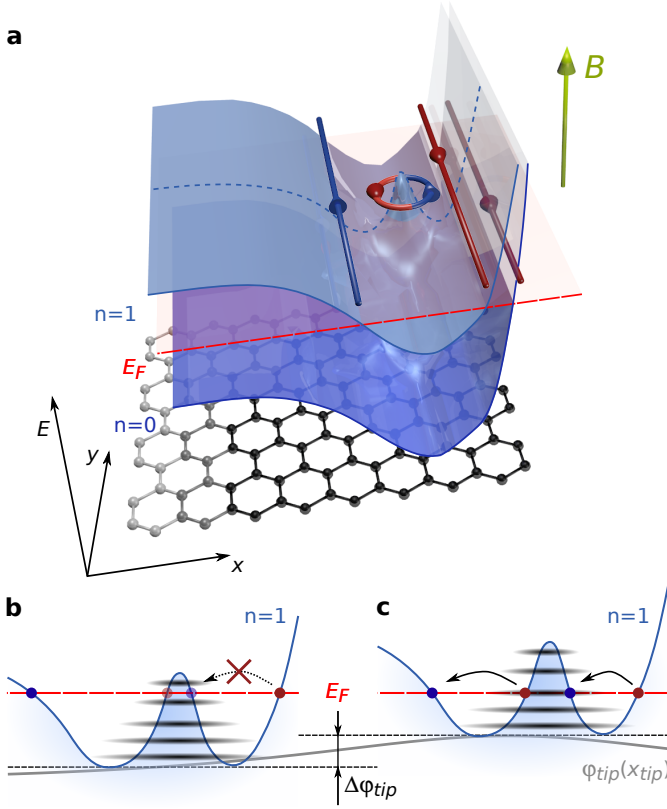


Figure 5.3: Artist' view of QHECs at graphene edge. **a**, The two lowest LLs arising due to the perpendicular magnetic field (green arrow) are represented as blue semi-transparent surfaces. Because of electrostatics at the graphene edge (on the right side), they are bent and the $n = 1$ LL crosses twice the Fermi energy E_F (red plane) yielding two downstream QHECs (in red) and to an upstream QHEC (in blue). An antidot is located between the counterpropagating QHECs and pins a QHEC island. **b,c**, Line profile across the QHEC island (blue dotted line in A) for the $n = 1$ LL (electron charge carriers). Discrete energy levels are represented in black. The tip-induced potential $\phi_{tip}(x_{tip})$ (gray line) tunes discrete energy levels positions with respect to E_F when varying the tip position x_{tip} . When E_F lies between two discrete energy levels, transport is not allowed via the QHEC island (**b**) whereas when a discrete energy level is aligned with E_F , charge carriers can tunnel between the counterpropagating QHECs (red and blue dots) through the QHEC island (**c**).

(at low V_{tip} , screening effects can however distort and shift high R_{xx} rings, as discussed in section 5.1.3). Based on Figs. 5.1c-e, we pinpoint antidots positions at a distance between 50 and 150 nm from sample G1 boundaries. This is in agreement with the estimated upstream QHEC position extracted from other local probe results [10, 106] (section 2.3).

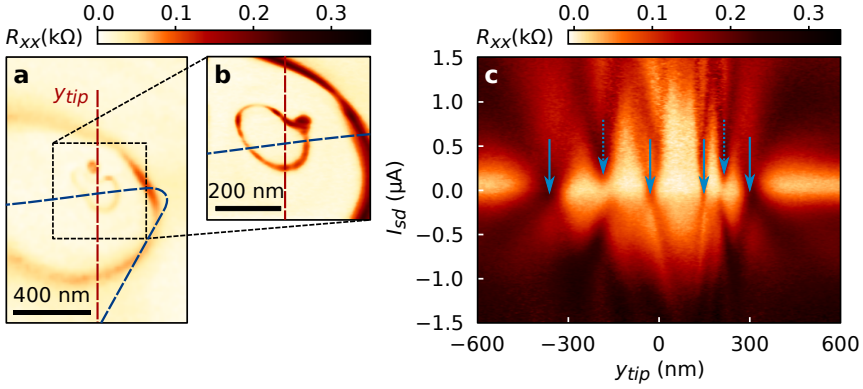


Figure 5.4: Coulomb diamonds related to a single antidot a, SGM map obtained at 12 T with $V_{bg} = -17.75$ V and $V_{tip} = 0$ V in sample G1. The blue dashed line indicate sample edges. **b**, Zoom in the square area indicated with a dotted line in **a**. High R_{xx} rings are more contrasted due to slower scan rate. **c**, R_{xx} map as a function of the tip position (along the red dashed line y_{tip} in **a**) and the source-drain DC current I_{sd} . Blue arrow indicate the resonance signatures (high R_{xx} rings in **a** and **b**). These resonances widen by increasing the absolute value of I_{sd} so that diamond-shaped regions of zero R_{xx} appear in the graph. The dotted arrows indicate weaker resonances not visible in **a** and **b**.

Based on the information we had at our disposal when interpreting the SGM data in Figs. 5.1c-e and 5.2b-f, it is logical that we invoked Coulomb blockade in the above discussion (also exposed in ref. [190]). However, other data we exploited later suggest that this phenomenon is not at play here and that the SGM signatures can be explained in a full coherent transport framework, which was counter-intuitive given the current state of the literature back then. This is the subject of chapter 6.

A fundamental question emerging at this point concerns the origin of the observed antidots. Atomic defects at the edges of graphene have often been invoked as source of perturbation for charge transport (section 1.2.4). However, if they were involved in the present case, it would remain to explain how they could yield potential landscapes similar to the one presented in Fig. 5.3a, with a potential extremum located 50 to 150 nm from the edge. More realistically, such potential landscape could originate from two known possible sources: (i) nanoscale random strain fluctuations, known to induce charge density inhomogeneities in graphene [27] (ii) remote charged impurities in the dielectric hBN layer [192]. Both sources lead to local variations of Dirac point energies (typically $\sim 50 - 100$ meV at $B = 0$ T, over typical distances $\sim 50 - 100$ nm [29]), probably ubiquitous in all hBN/graphene/hBN heterostructures. While our experiment does not allow to discriminate between strain- or impurity-induced potential fluctuations, it provides data on antidots distance from device borders, as well as on their spatial distribution along the borders of

graphene devices: the typical distance between neighboring antidots is in the range 100-500 nm, from data in Figs. 5.1c-e and 5.2b-f, i.e. compatible with data from ref. [29] (Fig. 1.5). Since the tip-induced potential perturbation extends beyond 500 nm, high R_{xx} rings originating from remote antidots can superimpose, as shown on Figs. 5.1c-e and 5.2b-f.

5.1.3 Tip potential and screening

The spatial locations of the antidots being unveiled, we now examine how their signatures emerge and evolve as a function of V_{bg} . For this purpose, we scan the tip across one of the antidots as indicated in Fig. 5.5a (the scan area in Fig. 5.5a corresponds to the red rectangle in Fig. 5.5b) and plot in Fig. 5.5c the SGM line profile as a function of V_{bg} in the vicinity of $\nu = -6$ for a constant V_{tip} . It is well known from earlier SGM experiments that such a plot allows to infer the tip potential perturbation from the V_{bg} -shift of the localized state resonances [144, 176]. Here, the antidot resonances undergo a Lorentzian evolution (Fig. 5.5c), as expected for a tip-induced potential perturbation (section Table 4.1.1). Examining Figs. 5.5a, c and d together, one can get the full picture of the fate of the resonances associated with an antidot: Figs. 5.5c,d evidence that peaks identified by the red and blue dashed lines undergo a parallel evolution with the approaching tip perturbation, and are therefore associated with the same antidot, whose location is clearly identified in the SGM map in Fig. 5.5a. Importantly, the resonances are also observed when the tip is far away from the device edges which means that the tunneling through the antidot is not necessarily triggered by the tip potential. Indeed, the resonance signatures can be tuned by V_{bg} as shown in Fig. 5.5d. It confirms that antidots are preexisting in the device and are not created by the tip-induced potential.

Some resonances are also visible for $V_{bg} < -21.5$ V in Fig. 5.5c. These resonances evolve slightly with the tip position. We ascribe them to signatures of the topological breakdown occurring between the $\nu = -6$ and $\nu = -10$ plateaus, far from the region scanned by the tip in Figs. 5.1c-e. It explains the small influence of the tip on these signatures.

In Fig. 5.5c, we image the tip-induced potential while $V_{tip} = 0$, which is contradictory at first sight. Nevertheless, the workfunction difference ΔV_{tip}^{WF} between the tip and the graphene plane should be added to V_{tip} to find the effective tip voltage [193]. For that purpose, we have recorded different maps, similar to Fig. 5.5c, at different values of V_{tip} (Fig. 5.6) and we expect the antidot resonances to follow a Lorentzian evolution along y_{tip} with an amplitude proportional to $V_{tip} + \Delta V_{tip}^{WF}$ (section 4.1.1). However, A striking feature appears in Figs. 5.6a,b. When following a resonance line, it first reaches a maximum then a minimum value and the curvatures changes between them. In other words, the tip-induced potential exhibits a positive biased feature when the tip is outside the graphene plane and a negative biased feature when the tip is above graphene, far from the expected Lorentzian evolution. Schnez

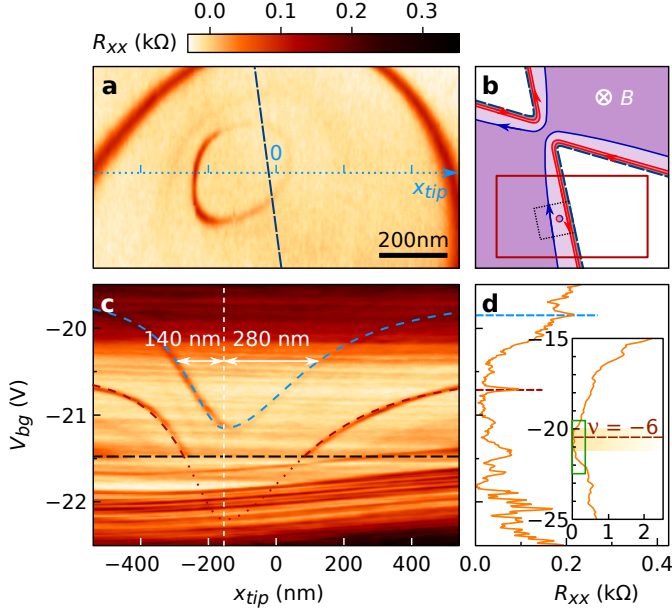


Figure 5.5: Coupling counterpropagating QHECs via an antidot. The data are obtained in sample G1 for $V_{tip} = 0$ V and $B = 14$ T. **a**, SGM map obtained at $V_{bg} = -20.85$ V, by scanning the tip inside the red rectangle indicated in the schematic picture of the device sketched in **b**. **b**, The QHECs are represented by red (downstream) and blue (upstream) continuous lines, and dashed line delineate the constriction. **c**, R_{xx} recorded as a function of V_{bg} and the tip position x_{tip} , along the light blue dotted line in **a**. The resonances signatures (highlighted with the red and blue dashed lines) allow to measure the tip-induced potential variation at the QHEC island location as a function of x_{tip} . Blue and red dashed lines are fits obtained with two merged half-Lorentzian functions. Above graphene, the half width at half maximum is 140 nm whereas it is 280 nm when the tip is above the etched area. The black dashed line indicates the V_{bg} limit beyond which one of the resonances disappears. **d**, Longitudinal resistance R_{xx} as a function of V_{bg} around $\nu = -6$ - zoom on the green rectangle of the inset.

et al. observed the same kind of signature and attributed it to two different origins (Fig. 3.4) [172]. First, the coupling between the tip and the studied antidot is not symmetric on both sides of the antidot because of the difference of screening between the back gate and the graphene plane. Second, the coupling between the back gate and the antidot also changes with the tip position due to screening effects. The combination of both effects explain how the features of Figs. 5.6a,b emerge. Nevertheless, by assuming that the averaged tip-induced potential is the weakest in Fig. 5.6a, we can estimate $\Delta V_{tip}^{WF} \approx 3$ V.

Screening effects are also visible in Figs. 5.2c, where the resonance signatures in the SGM maps are not circular but exhibit pear shapes. Furthermore, some of these signatures are centered outside the graphene plane. This comes from the fact that the position of device G2 edges were approximated from electrostatic

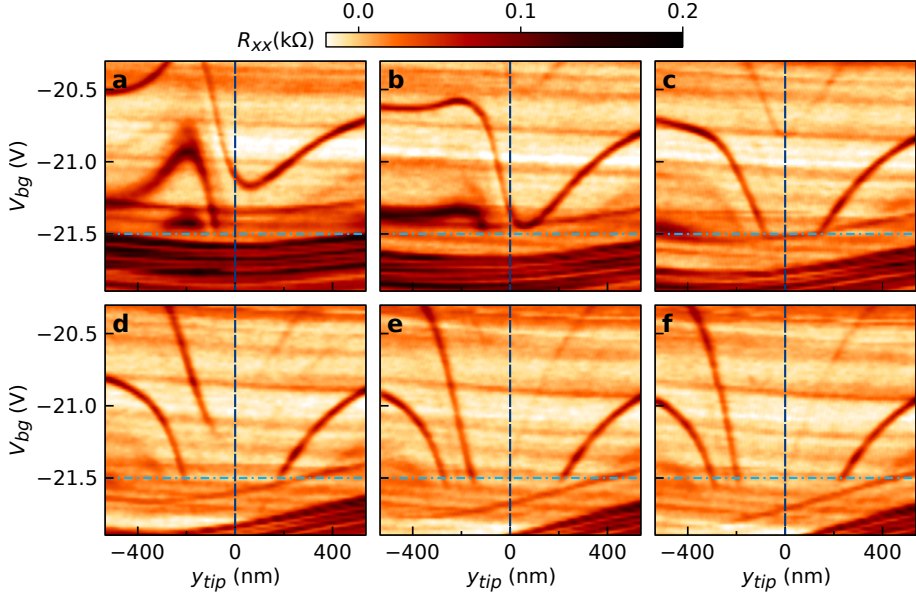


Figure 5.6: Effect of the screening on the tip-induced potential a-f, R_{xx} maps when varying y_{tip} and V_{bg} obtained in the same conditions as Fig. 3 of the manuscript with $V_{tip} = -3$ V (a), -2 V (b), -1 V (c), 0 V (d), $+1$ V e and $+2$ V (f). Resonance signatures corresponding to the studied antidot all disappear when $V_{bg} < -21.5$ V (light blue dash-dotted lines), as discussed in the manuscript. The dark blue dashed lines indicate the device edge position.

force microscopy maps (Fig. 3.9c). The transition between different electrostatic environments when crossing the edge results in a smooth signal when the tip is 55 nm above the graphene plane and hence an uncertainty on the edges position of about 100 nm in our case. In sample G1, the borders have been mapped more conventionally by atomic force microscopy (AFM) and the sharp transition of the measured height at the edges decreases the uncertainty on their position.

5.1.4 Simulations

Another way to tune the position and configuration of QHECs, but at the local scale, consists in varying both tip voltage and position. This is realized in Fig. 5.7a showing the evolution of R_{xx} when scanning the tip along the dashed line in Fig. 5.1e and varying V_{tip} . The different visible resonances corresponding to the same antidot undergo parabolic evolution with V_{tip} as expected for localized states [144, 177]. At low V_{tip} , these resonances are separated by $R_{xx} \sim 0$ regions while a finite R_{xx} region (in dark in Fig. 5.7a) is reached at larger positive V_{tip} . This evolution is also clearly visible in Fig. 5.7b showing R_{xx} versus the maximum tip-induced decrease in hole-density $|n_{max}|$ deduced from V_{tip} (section 4.1.1), for a fixed x_{tip} (with the tip on top of the antidot - black dotted line in Fig. 5.7a). At lower tip perturbation, transport is determined by

tunneling through the antidot as discussed above (left inset of Fig. 5.7b). As the tip-induced perturbation increases, the antidot grows and merges with up- or downstream QHECs. The confinement of charge carriers in the antidot is then suppressed and the backscattering is only induced by the coupling between the counterpropagating QHECs, as depicted in the right inset of Fig. 5.7b.

Small anecdote by the way: when writing this paragraph in ref. [190], Benoit and I had the idea to call the paper "Put the tip on top of antidots to spot where tip-top topology stops" but we were afraid that it would be a bit too intrepid. I therefore thank Boris for having kept a word game in the title with his proposal of *antidots poisoning*.

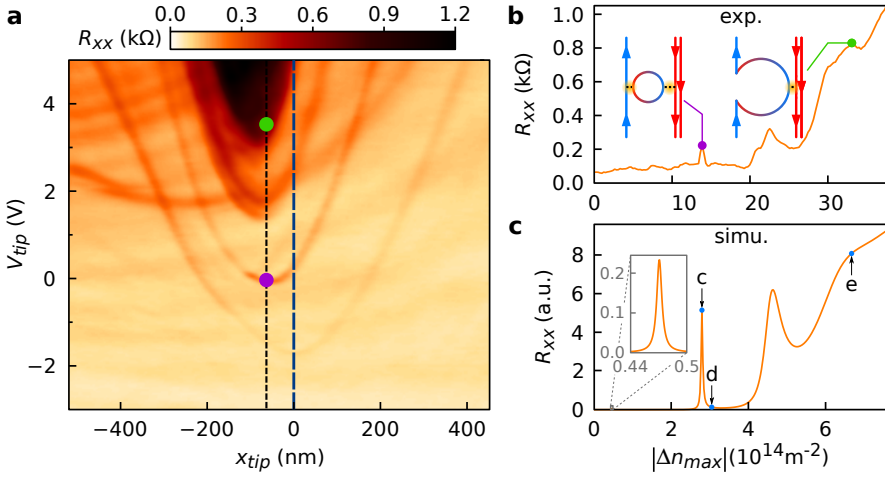


Figure 5.7: Tip-controlled tuning of transport through a QHEC island. **a**, Evolution of R_{xx} as a function of V_{tip} (measured along the black dotted line in Fig. 5.1d at $V_{bg} = -13$ V and $B = 12$ T). **b**, R_{xx} profile for $x_{tip} \sim -60$ nm (tip on top of the antidot *i.e.* along the black dotted line in **a**). V_{tip} has been converted in the maximum tip-induced hole density decrease $|\Delta n_{max}|$. **c**, Simulations of R_{xx} as a function of $|\Delta n_{max}|$ at the lower edge of $\nu = -6$ plateau, at $B = 12$ T.

5.1.4.1 Tight-binding simulations of an antidot

Tight-binding simulations reproduce the observed phenomenology and provide further insights in the underlying physics through real space images of the local current density (JDOS) in the different backscattering regimes. Using the KWANT package [184] (section 3.3.2), we model one edge of the device as a 150 nm-wide graphene ribbon represented in Fig. 5.8a where the colors correspond to the onsite potential landscape (term $U(\mathbf{r}_i)$ in Eq. 3.5). In our simulations, we focus on a single side of the device, and neglect the bulk region contribution (more details in section 5.1.4.2). The antidot potential is positioned close to the center of Fig. 5.8a. In this geometrical configuration, counterpropagating

QHECs (straight arrows in Fig. 5.8a,b) encompass the QHEC island (curved arrows in Fig. 5.8a,b) for the Fermi energy corresponding to the red dashed line of Fig. 5.8b. The tip potential shifts the relative position of the LLs with respect to the Fermi energy, thereby tuning the distance and coupling between the QHECs and the antidot.

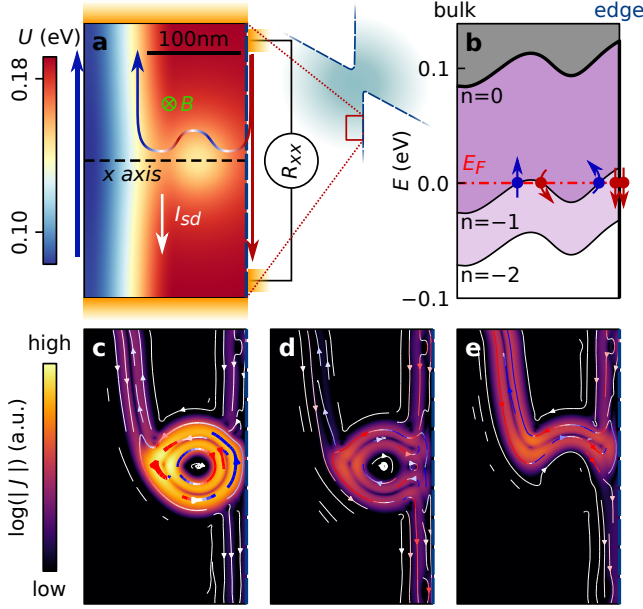


Figure 5.8: Tip-controlled tuning of transport through a QHEC island. **a**, Scheme of the simulated system chosen to capture the physics along one of the edges of the whole sample used in the experiment. The colors corresponding to the onsite potential landscape. The antidot corresponds to the circular region where the potential is lower, centered at 45 nm from the graphene edge. The 4 leads required to compute R_{xx} are represented in yellow. The arrows illustrate the charge carriers flow in QHECs (forward in red and backward in blue). QHECs appear at the natural graphene edges (left blue and right red arrows). By considering only downstream transport, the backward left channel does not contribute. **b**, Profile of the three lowest LLs ($n = 0, -1, -2$) along the black dashed line in **a**. This graph features an infinitely sharp confinement potential at the edge (right side of the figure), which yields two downstream QHECs (red straight arrows). **c-e**, Simulated maps of current density (JDOS) obtained for the three Δn_{max} values indicated with arrows in Fig. 5.7c. On top of the R_{xx} peak (**c**), the JDOS around the antidot is maximal compared to the situation of zero R_{xx} (**d**). The high JDOS in the antidot highlights that the resonance condition is reached. **e**, The region of finite R_{xx} in Fig. 5.7c corresponds to direct backscattering of QHECs. The colored arrows indicate the direction of the local current density.

Interestingly, we observe a striking qualitative correspondence between the measured (Fig. 5.7b) and simulated (Fig. 5.7c) longitudinal resistance as a function of $|\Delta n_{max}|$: at low $|\Delta n_{max}|$, finite R_{xx} peaks are separated by $R_{xx} \sim 0$ states and at larger $|\Delta n_{max}|$, R_{xx} remains finite. The $|\Delta n_{max}|$ scale (distance

between the peaks) depends mainly on the size of the considered antidot as well as on eventual Coulomb interactions, not captured in our simulations. Furthermore, $|\Delta n_{max}|$ is not defined exactly in the same way in the simulation and in the experiment, as discussed at the end of section 5.1.4.2. Therefore, the comparison between experimental and simulated typical $|\Delta n_{max}|$ scales will remain qualitative.

The sequence of JDOS maps shown in Figs. 5.8c-e provides a real space illustration of the peaks' origin. Comparing Figs. 5.8c and d, corresponding respectively to finite and zero R_{xx} (Fig. 5.7c), we observe that, while in both cases the antidot is coupled to downstream QH channel (right of the figures), current through the antidot is significantly larger in the case of Fig. 5.8c (as indicated by the brighter contrast in log scale at the antidot position). Coupling between up- and downstream QH channels is therefore much more efficient, yielding finite R_{xx} . At much higher $|\Delta n_{max}|$ (Fig. 5.8e), the JDOS map reveals that the raised antidot potential results in the merging of the antidot with the upstream QHEC, confirming the schematic picture sketched in the right inset of Fig. 5.7b.

5.1.4.2 More details on the simulated system

In the graphene QH regime, charge carriers flow without dissipation in the downstream QHECs except when an antidot induces their coupling with the upstream channels. In that framework, the bulk of the sample does not contribute to transport. Therefore, simulations can capture the coupling mechanisms by considering only a small region along one of the device edges in which an antidot is located between the counterpropagating QHECs (Fig. 5.8a). In this configuration, the downstream QHECs (red arrows at the right of Fig. 5.8c) appear along the device border due to edge confinement (hard wall potential). In the same way, "parasitic" upstream QHECs (blue arrow at the left of Fig. 5.8a) exists where the bulk is supposed to be insulating. We avoid the contribution from these QHECs by only computing the downwards transport (white arrow in Fig. 5.8a).

The dimensions of the simulated system are $150 \times 250 \text{ nm}^2$. The increase of potential at the origin of the upstream QHEC is obtained by varying smoothly the onsite potential $U(\mathbf{r}_i)$ from U_{bulk} in the bulk to $U_{bulk} + \Delta U_{edge}$ along the edges (inset of Figs. 5.9 and 5.8a,b), where $\Delta U_{edge} = 0.6 \cdot U_{bulk}$. The antidot is modeled with a Gaussian function centered 45 nm away from the edge and having a FWHM of 72 nm (Fig. 5.8a,b). A scaling factor $s = 4$ (Eq. (3.8)) has been used to decrease the computation time but has no influence on the output results (a convergence study has been performed by varying this parameter). R_{xx} as a function of U_{bulk} is shown in Fig. 5.9. As in the experiment, it features $R_{xx} = 0$ regions, when QHECs are topologically protected, separated by large R_{xx} regions when the breakdown through an antidot occurs. Interestingly, the effective integer filling factors ν_{eff} are shifted compared to the integer filling factors in the bulk ν_{bulk} (Fig. 5.9), exactly as in the fan diagram of Fig. 3.10.

These results are perfectly explained with the presence of a charge accumulation at the edges. Indeed, the upstream mode at the origin of the backscattering (and hence at the transition to the plateau $\nu + 4$) appears whereas the bulk is still characterized by the integer filling factor ν . Since the charge carrier density in the bulk is expected to coincide with the theoretical value obtained from an infinite plates capacitor model, the transition appears at lower V_{bg} . The effective lever-arm is therefore larger than with the capacitive model.

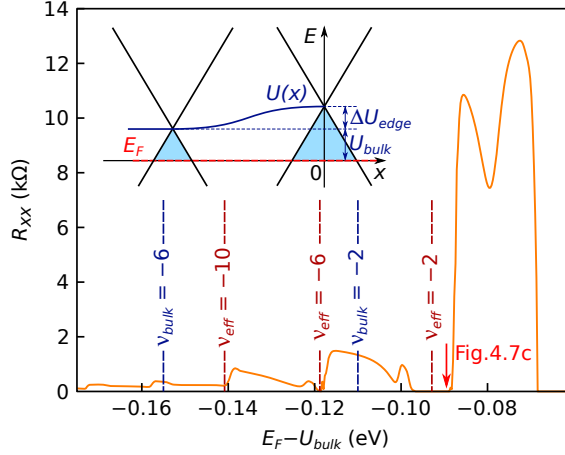


Figure 5.9: Transport measurement with a charge accumulation along the edges. R_{xx} as a function of U_{bulk} . The blue dotted lines indicate the integer filling factors in the bulk ν_{bulk} while the red dotted line indicate the effective integer filling factors deduced from the R_{xx} simulation with an increase of charge carriers along the edges. Inset: Evolution of the potential at a distance x from the graphene edge ($x = 0$). Hole-type charge carriers are obtained by defining a positive onsite parameter with a zero Fermi energy $E_F = 0$ eV. In the bulk, $U(x) = U_{bulk}$, and it increases to $U_{bulk} + \Delta U_{edge}$ along the edges.

The R_{xx} curve as a function of Δn_{max} presented in Fig. 5.7c is obtained for $U_{bulk} = 0.0895$ eV, at the onset of the $\nu_{eff} = -6$ state (red arrow in Fig. 5.9), as in the experiment (Fig. 5.1b). In this simulation, the tip-induced potential is modeled with a Lorentzian function added to the onsite potential (section 4.2.1) and writes

$$U_{tip}(\mathbf{r}_i) = \frac{U_{max}}{1 + \left(\frac{\mathbf{r}_i - \mathbf{r}_{tip}}{R_{tip}} \right)^2},$$

with $\mathbf{r}_i$ the position of site i , U_{max} the tip potential maximum, $\mathbf{r}_{tip}$ the tip position in the $x - y$ plane and R_{tip} the HWHM. From the experimental characterization of the tip potential (Fig. 5.5), we have chosen $R_{tip} = 200$ nm and the potential deformation due to screening has been ignored. The tip-induced change of charge carriers density Δn_{max} defined in Eq. (4.2) can be directly related to U_{max} with Eq. (3.7) to give

$$\Delta n_{max} = \frac{4}{9\pi} \left(\frac{U_{bulk} + U_{max}}{at} \right)^2 - \frac{4}{9\pi} \left(\frac{U_{bulk}}{at} \right)^2, \quad (5.1)$$

where U_{bulk} is the uniform onsite potential in the graphene bulk (Fig. 5.9). From this definition, it appears that Δn_{max} is overestimated in the experiment (Fig. 5.5b) compared to the simulation (Fig. 5.5c). Indeed, the experimental Δn_{max} is proportional to the effective lever arm v_{eff} (Eq. (3.2)) which is larger than the theoretical lever arm in the bulk due to fringing fields (section 3.2.3.2). On the contrary, Δn_{max} in the simulation depends only on the potential U_{bulk} in the bulk and is therefore expected to be smaller than the experimental value, explaining the difference of scale between Figs. 5.7b and c.

5.1.5 Loss of the antidot signature

In the simulations presented in section 5.1.4, charge carriers flowing in up- and downstream QHECs are perfectly transmitted to the leads as they are the ideal extension of the small region of interest in Fig. 5.8a. However, we can wonder if this picture is realistic in the actual device. Indeed, by focusing on one of the contacts (Figs. 5.10a-d), it appears that a plateau at $G_{xy} = -10 e^2/h$ should be observed between the $\nu = -2$ and $\nu = -6$ states as the extra upstream QHEC can also contribute to transport (Fig. 5.10m). Nevertheless, we have seen in section 2.2.3 that QHECs certainly do not enter the contacts and that the transfer of charge carriers is impaired when QHECs are remote from the contacts. Given that the upstream QHEC is estimated to be separated by a ~ 200 nm-wide incompressible region from the downstream QHECs by (section 5.1.2) [10, 11, 106], it is quite unlikely that the upstream QHEC is coupled to the contacts (Figs. 5.10i-l). It therefore remains to explain how the upstream QHEC can contribute to backscattering in this framework.

A clue to solve this puzzling question can be found in the evolution of the antidot resonances with the tip position in Fig. 5.5. An intriguing behavior is revealed for the resonance highlighted by the red dashed lines in Fig. 5.5c: below a threshold of $V_{bg} < -21.5$ V (black dashed line), signatures of this resonance vanish. Given that resonances are visible provided that (i) a discrete state associated with an antidot is tunnel-coupled to up- and downstream QHECs and (ii) the upstream QHEC allows charge carriers to be sent back to the injection contact, it looks like this threshold is caused by a sudden decoupling between the upstream QHEC and the injection contact below $V_{bg} < -21.5$ V. Finally, The origin of the decoupling is expected to take place far from the antidot as the threshold is independent of V_{tip} as demonstrated in Fig. 5.6.

The above discussion suggests that an accurate understanding of the backscattering mechanism requires to draw a picture of the QHECs configuration in the whole sample, that consider their coupling with the contacts. This is the subject of section 5.2.

5.1.6 Summary and open questions

Put together, our data shed a new light on the combined role of electrostatics (fringing fields or charged impurities) and antidots at graphene edges in QH

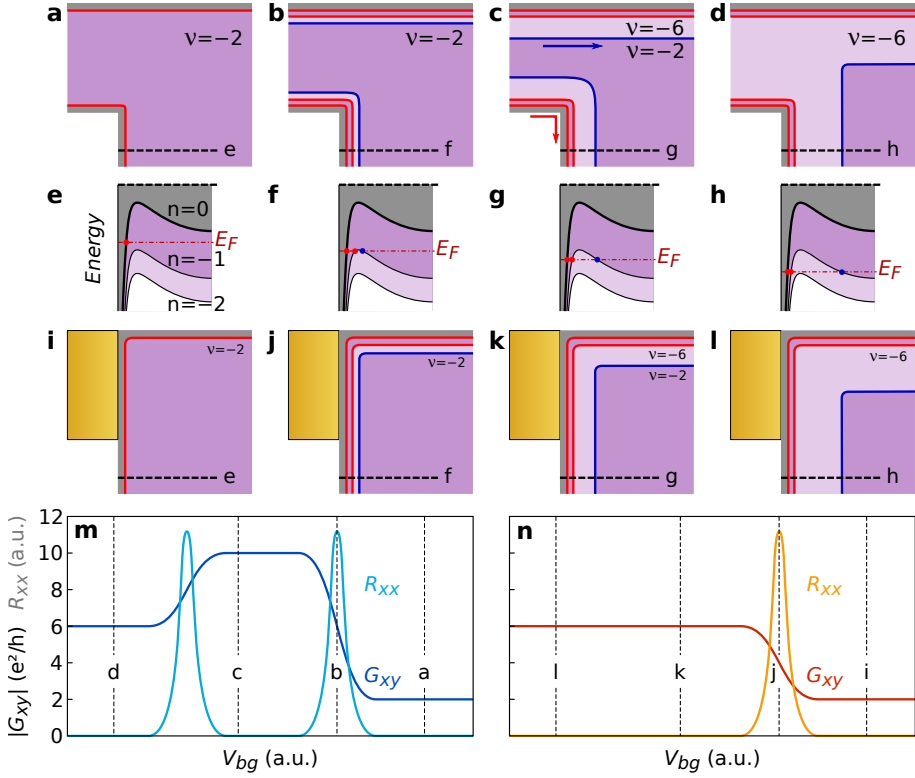


Figure 5.10: Illustration of the QHECs spatial structure for two models of contacts. The different schematics are shown at the transition between $\nu = -2$ and $\nu = -6$. **a-d**, Downstream (red) and upstream (blue) QHECs in the case of ideal contacts, smoothly connected with the sample. **e-h**, Potential profile (thick line) along the edges and the three first Landau levels ($n = 0$ to $n = -2$, in thin lines). QHECs appear where the Fermi energy (red dashed line) cross the Landau levels. **i-l**, Similar schematics as **a-d** for realistic contacts. The downstream QHECs are equilibrated with the contacts due to their close proximity while the upstream QHEC is separated from the contact by a large incompressible region. **m, n**, Schematic evolution of longitudinal resistance R_{xx} and Hall conductance G_{xy} as a function of V_{bg} in the case of the ideal contacts (**m**) and in the case of the line contacts (**n**).

breakdown. Both ingredients are likely ubiquitous in most graphene-based heterostructures studied up to now, but with variations in the importance of the different contributions. Indeed, fringing fields become much weaker when the gate is placed closer to graphene, for example when a graphite backgate is used below hBN. Furthermore, charged impurities at hBN etched edges depend on the etching recipe, and Dirac point inhomogeneities may be more or less pronounced depending on strain accumulated in the layers or on the quality of hBN.

In the previous sections, the SGM signatures along the graphene edges have been explained with the formation of up- and downstream QHECs. However, other SGM experiments performed on semiconductor-based 2DEs have also highlighted R_{xx} contrast along the edges without the presence of an upstream QHEC [171, 194]. In these studies, the SGM signatures have been ascribed to tunneling through the incompressible region separating co-propagating QHECs. This incompressible strip appears due to the smooth confinement potential at the borders of the GaAs/AlGaAs device. In graphene, however, an absence of upstream QHEC would imply that the QHECs form at the hard walls of the device, with no incompressible strip between the QHECs (Fig. 2.4). No SGM contrast should therefore be visible except in the vicinity of the constriction as mentioned before. It is also noteworthy that upstream QHECs have also been observed in InAs-based 2DEs [195, 196] and are not exclusive to graphene.

5.2 Influence of the contacts

Whereas the previous section relies on strong experimental evidences, the interpretation given below and exposed in ref. [197] is more speculative. Nevertheless, it is my believe that it offers an interesting theoretical framework to interpret data in the future.

5.2.1 Measurements in the electron side

In section 5.1, all the experimental data were obtained for holes. The SGM maps performed close to $\nu = -6$ in Figs. 5.1c-e and 5.2b-f all exhibit contrast along the edges, testifying to the presence of antidots coupling up- and downstream QHECs. The question emerging naturally at this point is whether the same kind of signatures is also visible for electrons. The few SGM data reported with electrons by Marguerite *et al.* suggest that holes also accumulate along the edges instead of electrons [10]. However, these data were obtained at low magnetic field ($B \sim 1$ T) and at low electron density in the bulk, meaning that the contribution of fringing fields to the CA@GE is weak. In the following, we present data obtained in device G1 at $B = 4$ T, to stand at the verge of the regime where R_{xx} exhibits large and irregular fluctuations (Fig. 3.10).

5.2.1.1 Topological protection at low and high magnetic fields

First, we look at the transport measurement of the device at $B = 4$ T (orange curve in Fig. 5.11b). Noticeably, R_{xx} does not fall to zero for the filling factors $\nu = \pm 4(n + 1/2)$, contrarily to R_{xx} measured at $B = 12$ T (yellow curve in Fig. 5.11b). It highlights a weaker topological protection of QHECs at lower magnetic field. This observation can be understood by remembering that the width of the wavefunction associated to a QHEC is given by the magnetic length $l_B = \sqrt{\hbar/eB}$ (Eq. (2.5)). Since both up- and downstream QHECs flow along the same edge in graphene, they can overlap at low magnetic field while

they are kept away from each other at higher field, as illustrated in Fig. 5.12a. However, an overlap of the counterpropagating QHECs does not mean they are coupled. The inter-channel equilibration requires the presence of impurities, as antidots, allowing the transfer of charge carriers from down- to upstream modes [169].

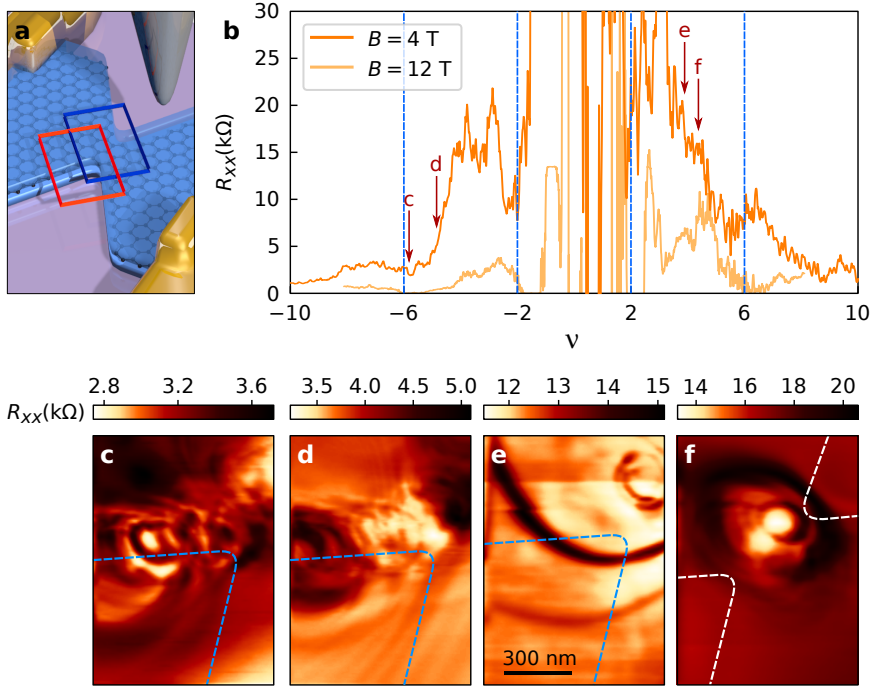


Figure 5.11: Electron-hole asymmetry in transport and SGM data. **a**, Schematic of the scan areas (see Fig. 3.8). **b**, R_{xx} as a function of the filling factor ν for a magnetic field of 4 T (orange) and 12 T (yellow) measured at $T = 100$ mK. The values of $\nu = \pm 4(n + 1/2)$ at integer n are indicated with blue dashed lines. **c-f**, SGM maps obtained in the vicinity of the constriction for holes (**c,d**) and electrons (**e,f**). The scan area is depicted in **a** with a red rectangle for **c-e** and with a dark blue one for **f**. The V_{bg} values are indicated with arrows in **b** and $V_{tip} = 0.5$ V in **c-e** and 1.5 V in **f**. For holes, SGM contrast appears centered on the sample edge whereas it is located exclusively at the center of the constriction for electrons.

5.2.1.2 SGM data in the electron side

Next, we compare the SGM maps recorded for holes (Figs. 5.11c,d) and electrons (Figs. 5.11e,f). For holes, the SGM contrast draws rings-like features along the edges, as observed at larger magnetic field (section 5.1.1). For electrons, however, SGM contrast is centered only in the constriction region, as indicated with the SGM fringes of larger R_{xx} in Fig. 5.11c and with spots of smaller R_{xx} in Fig. 5.11d. These signatures are similar to those found in semiconductor-based 2DEs (Figs. 2.11c-e), where there is no upstream QHEC. In these systems,

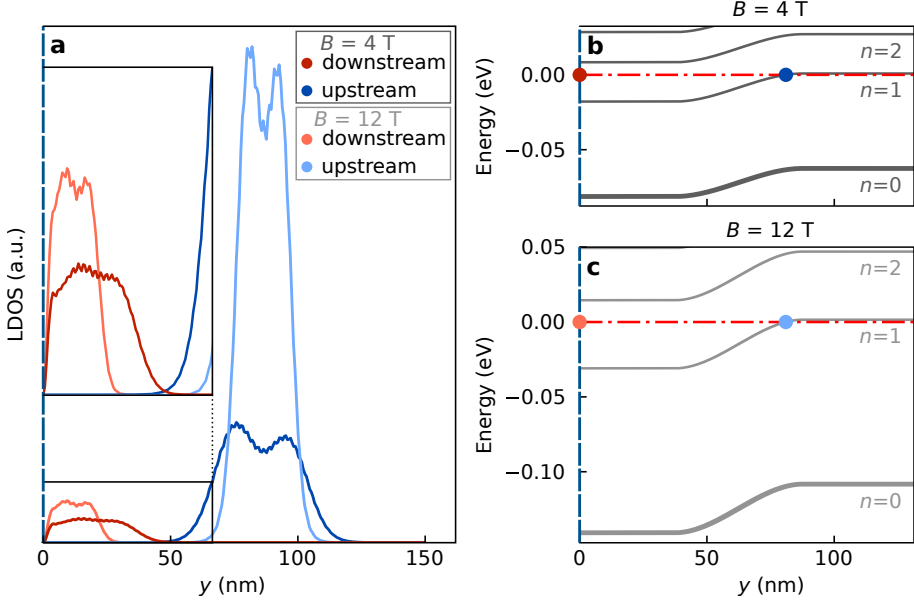


Figure 5.12: Width of up- and downstream QHECs at $B = 4$ and 12 T. **a**, KWANT simulation of the LDOS as a function of the distance y from the graphene edge. The LDOS associated to downstream (upstream) QHECs is presented in dark red (blue) for $B = 4$ T and in light red (blue) for $B = 12$ T. **b,c**, Potential as a function of the distance y from the graphene edge (thick line) used to simulate the LDOS of **a** and the three first Landau levels ($n = 0, 1, 2$) for $B = 4$ T (**b**) and $B = 12$ T (**c**). The potential has been qualitatively designed to induce an increase of the charge carriers density at the edges proportional to the bulk density by a factor $\eta = 0.69$ (a factor 0.3 between edge and bulk onsite parameters, as detailed in section 5.1.4.2). Upstream modes (blue dots) arise when the Fermi energy (red dashed-dotted line) crosses one of the bent Landau level. Downstream modes (red dots) appear due the infinitely sharp confinement potential at the graphene edge.

topological breakdown was assigned to the presence of an antidot located in the vicinity of the constriction and coupling the QHECs running at the opposite device edges.

As a first hypothesis to account for electron-hole asymmetry, one could speculate that upstream QHECs only exist for holes. Impurities and defects along the borders could in this case favor holes accumulation and induce such charge asymmetry, as proposed in refs. [10, 11] (referred to hereafter as the "impurity model"). While our data can not undoubtedly discard this scenario, the impurity model alone fails to yield a full picture of QHECs in graphene especially when considering other experimental results reviewed in section 2.3.4. On the other hand the mainstream theory, namely the inhomogeneous screening of the backgate potential (the fringing field model), predicts the emergence of upstream QHECs both for holes and electrons, and consequently an apparent charge symmetry refuted in the SGM maps of Fig. 5.11. In the

following, we show that electron-hole asymmetry can also be explained in the fringing field model framework.

5.2.2 Edge channels configuration around the contacts

In the model that we develop hereafter, we assume that charge carriers accumulate along the edges in the same way for electrons and holes. The bending of energy bands leads in both cases to the presence of upstream QHECs.

5.2.2.1 Doping induced by the line contacts

In this scenario, electron-hole asymmetry originates from the contacts. In particular, the difference of work-function between graphene and chromium, forming the adhesion layer of contacts (section 3.2.2), yields a hole-type doping below the metallic regions overlapping the stack illustrated in Fig. 5.13a. The work-function-induced difference of the potential can be approximated by [198, 199]

$$\Delta\varphi = \frac{\sqrt{1 + 2\alpha e|W_{\text{Cr}} - W_{\text{G}}|} - 1}{\alpha}, \quad (5.2)$$

where $W_{\text{Cr}} = 4.5$ eV [200] and $W_{\text{G}} = 4.48$ eV [198] are the work-functions of chromium and graphene respectively and $\alpha = 2e^2 t_{\text{hBN}} / (\epsilon_{\text{hBN}} \pi \hbar^2 v_{\text{F}}^2)$ with $t_{\text{hBN}} \sim 20$ nm the vertical distance between graphene and the overlapping metal, $2 < \epsilon_{\text{hBN}} < 4$ the hBN permittivity and $v_{\text{F}} \sim 10^6$ m/s the Fermi velocity of graphene. With these data, we find $6 < \Delta\varphi < 8$ meV. As a result, charge carrier density is larger below the overlapping metal, with respect to the bulk, when the bulk is doped with holes, whereas it is smaller for an electron-doped bulk.

However, Eq. (5.2) certainly fails to give a precise value of $\Delta\varphi$. Indeed, the values of the workfunctions taken from the literature (especially that of chromium, less studied) depend on the experimental method. For instance, the chromium workfunction has been measured in ref. [200] by evaporating the metal on a clean doped GaAs surface. However, there is no guarantee that it would remain the same if measured with a graphene device. Performing ab-initio simulations is another way to assess chromium workfunction [198, 199]. For chromium, nevertheless, the task is far from being trivial. Indeed, chromium presents different crystal orientations whose workfunctions are different, as studied more extensively for other metals [201, 202]. It is therefore difficult to know the relative weight of each crystalline phase in the final workfunction value. A last consideration concerns the fact that impurities can be trapped between hBN and the metallic layer of the contacts during the fabrication process. It can also influence the accumulation of charge carriers density (either holes or electrons) below the contacts.

5.2.2.2 Electron-hole asymmetry in the QHECs configuration

In the QH regime, the modulation of charge carrier density captured by Eq. (5.2) has an important influence on the QHECs spatial configuration in the

vicinity of the contacts, which differs for holes and electrons, as illustrated in Figs. 5.13b,c. In this schematic, down- and upstream QHECs flow along the edges where one of the bent Landau levels crosses twice the Fermi energy. This is illustrated in Fig. 5.13d for holes and in Fig. 5.13f for electrons, using a schematic drawing of the potential profile close to device edges. However, this picture is no longer valid around and below the contacts pads, i.e., in the doped orange-shaded region in Fig. 5.13b-c. For holes, the upstream (clockwise) QHEC circumvent the contact zone, due to the increase of holes density (Fig. 5.13e). For electrons, the lower charge density close to the contacts (Fig. 5.13g) leads to the merging of the upstream (counterclockwise) with the closest downstream (clockwise) QHECs (Fig. 5.13c). These QHECs are therefore perfectly equilibrated.

To go beyond the above-described qualitative model, we performed tight-binding simulations, using the KWANT package [184]. Maps of the absolute value of the on-site potential around the contacts are shown in Fig. 5.13h for holes and in Fig. 5.13i for electrons. They feature an increase of charge carrier density along the edges, proportional to the bulk density (section 5.1.4.2), in addition to a constant positive offset energy $\Delta E = 7$ meV (Eq. (5.2)) around the contacts. The resulting current density maps are presented in Figs. 5.13j,k. The QHECs are visible in lighter tones and match the qualitative picture of Figs. 5.13b,c.

5.2.3 Edge channels configuration in the whole sample

We now consider the entire sample to examine the consequences of these different QHECs spatial configurations in the vicinity the contacts for holes and electrons, and the relationship with the experimental data of Fig. 5.11.

5.2.3.1 For holes

For holes (left side of Fig. 5.14), it appears that the upstream QHEC (blue) forms a closed loop, as illustrated in Fig. 5.14a. In the absence of coupling with the downstream QHECs (red), this loop is topologically equivalent to a localized state in QH theory (section 2.2.1.2) since it is not directly coupled with contacts and should not influence transport. The effective filling factor of the system is therefore given by the region outside the loop (here $\nu = -6$, in light purple).

To observe the topological breakdown of QHECs in the above picture, holes running along one of the sample edges should be backscattered to the opposite edge. This can only be achieved by coupling the upstream QHEC loop both on the upper and lower sides of the sample (trajectory shown with a black dotted line in Fig. 5.14a), for example at locations pinpointed by concentric black circles in Fig. 5.14a. The SGM contrast highlighted along the sample edge in Fig. 5.11b is therefore the signature of one of the two coupling spots between

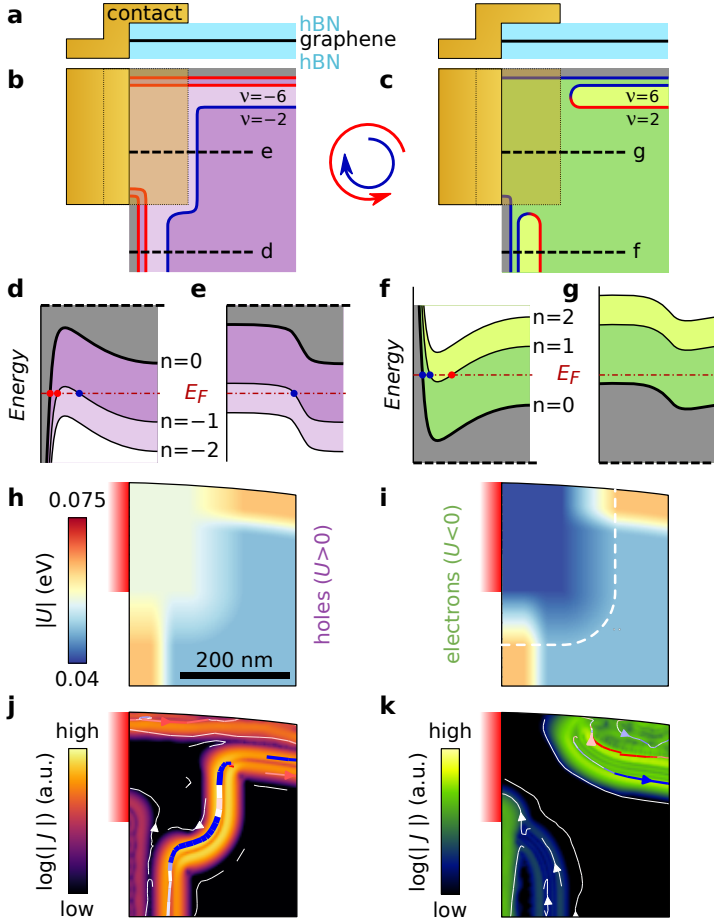


Figure 5.13: QHECs spatial configuration around the contacts. **a**, Side view of the device (schematic) in the vicinity of one of the contacts. **b,c**, Top view of one of the sample corner, around the contact. Due to the increase of charge carrier density at the device borders, counterpropagating QHECs run along the edges (clockwise in blue and counterclockwise in red). If the contacts dope graphene with holes, the upstream (clockwise) QHEC for holes circumvent the contact area (**b**) whereas the upstream (counterclockwise) QHEC for electrons is merged with the nearest downstream (clockwise) QHEC (**c**). **d-g**, Evolution of the potential (thick line) and the three first Landau levels (LLs) along the black dashed lines in **b** and **c**. QHECs appear where LLs cross the Fermi energy (red dashed line). **h,i**, Absolute value of the onsite potential landscape $|U|$ used for the tight-binding simulations for holes with $V_{bg} = -2.4$ V (**h**) and electrons with $V_{bg} = 2.4$ V (**i**). The semi-infinite leads in the simulated system are depicted in red. **j,k**, Current density maps obtained from tight-binding calculations for holes (**j**) and electrons (**k**).

upstream QHECs. In particular, the coupling is achieved through an antidot, as depicted in Fig. 5.14c.

We correctly capture the mechanism leading to holes backscattering in simulations by introducing these antidots in the simulated potential landscape, with Gaussian functions centered close to the edges (see the potential map in Fig. 5.14e). Fig. 5.14g shows a simulated current density map where propagating QHECs are not coupled through the antidots. This is in stark contrast with Fig. 5.14i where coupling is achieved at a slightly different value of V_{bg} (calculated from E_F), so that holes flow through the inner QHEC loop, connecting both edges.

5.2.3.2 For electrons

For electrons (right side of Fig. 5.14), the equilibrated QHECs form loops running along the edges, as illustrated in Fig. 5.14b. Contrary to holes, the effective filling factor of the system is given by the bulk (here $\nu = 2$, in dark green), as for conventional semiconductor-based 2DESSs. The topological breakdown of QHECs is then achieved by coupling the upstream QHECs of the upper and lower sides of the sample (trajectory shown with a black dotted line in Fig. 5.14b), which is much more likely to happen in the vicinity of the constriction, as illustrated with concentric black circles in Fig. 5.14b. Electrons running along one of the edges (in a QHEC loop) can then be backscattered to the opposite side of the device. The SGM contrast highlighted at the center of the constriction in Figs. 5.11c-d is therefore the consequence of the coupling between the opposite QHECs loops through an antidot located at the center of the constriction, as depicted in Fig. 5.14d [4].

Simulations support this picture, when introducing an antidot at the center of the constriction (Fig. 5.14f). Fig. 5.14h depicts a current density map where QHECs loops are visible along the edges but the absence of coupling with the opposite edge prevents backscattering. When coupling is active (at different V_{bg} , as in Fig. 5.14j), electrons flow from one edge to the other through the central antidot and can be backscattered.

5.2.3.3 A small simulation trick

In Figs. 5.14e-j, the *measurement* leads used to compute R_{xx} and G_{xy} are much thinner than the *injection* leads located at the left and right bottom corners of the system, used to inject and collect current (modeled by plane waves in Kwant). The reason is that one must prevent current to leave the sample through the measurement leads when computing the local current density maps presented in Figs. 5.14g-j to correctly capture the trajectories of charge carriers. These leads are therefore chosen thin enough to prevent plane waves to enter or leave them. However, the width of the measurement leads does not affect the calculation of R_{xx} (Fig. 5.15a). Indeed, it is possible to compute R_{xx} by imposing a zero current in the three measurement leads, while all the current enters and leaves the system by the two injection leads (section 3.3.2). We have confirmed this by computing R_{xx} for different widths of the measurement leads.

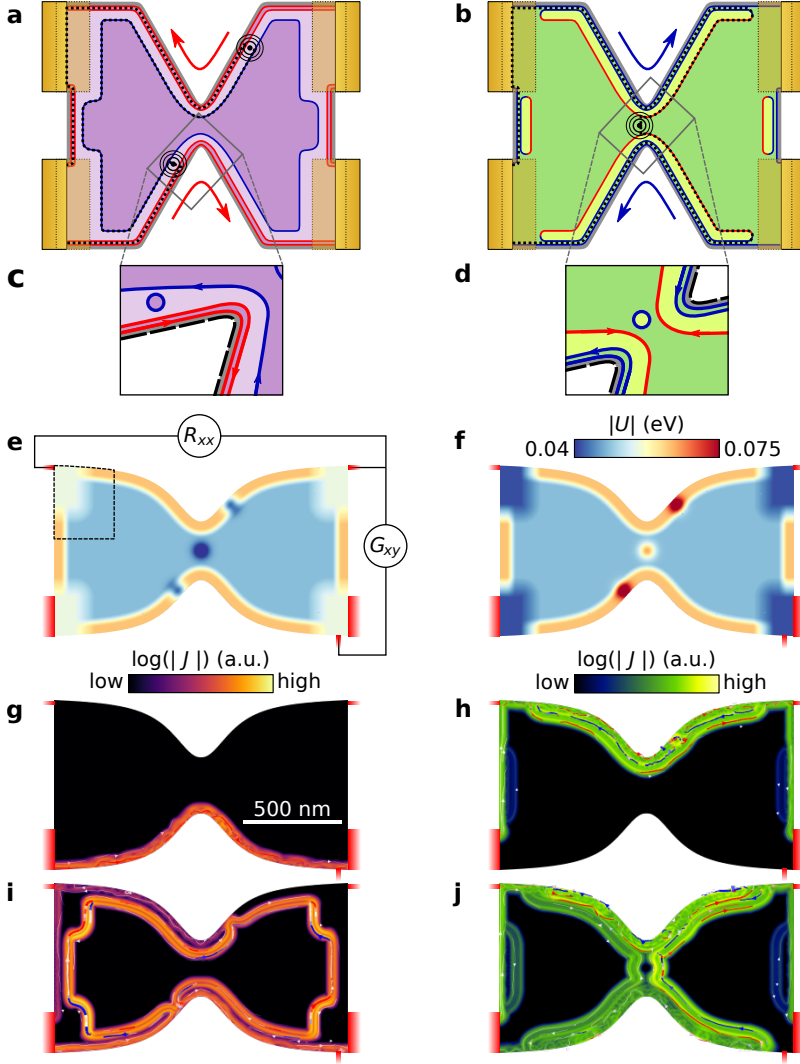


Figure 5.14: QHECs spatial configuration in the whole device. **a**, For holes, the upstream QHEC describes a closed loop so that the effective filling factor is given by the light purple area's order (here $\nu = -6$). **b**, For electrons, the upstream QHEC forms a closed loop as it connects with the nearest downstream QHEC so that the filling factor is given by the dark green area's order (here $\nu = 2$). **c**, For holes, SGM signatures along the edges in Fig. 5.11b are caused by the coupling between upstream QHECs occurs via an antidot located close to the constriction. **d**, For electrons, coupling between opposite border QHECs via an antidot. **e,f**, Potential landscapes of the simulated systems, with colors corresponding to the absolute value of the onsite potential landscape $|U|$, with the same parameters as in Figs. 5.13h,i. **g-j**, Current density maps obtained for holes with $V_{bg} = -4.8$ V (**g**), -3.8 V (**i**) and electrons with $V_{bg} = 6.4$ V (**h**), 7.8 V (**j**) as indicated with arrows in Fig. 5.15a.

5.2.4 Electron-hole asymmetry in transport

The simulated R_{xx} and G_{xy} curves as a function of V_{bg} (derived from the Fermi energy E_F), obtained in this system, are presented in Fig. 5.15a for holes (purple) and electrons (green). When R_{xx} is zero, the Hall conductivity exhibits the expected plateaus for graphene at $G_{xy} = 4(n + 1/2)e^2/h$, with n an integer. Non-zero R_{xx} , accompanied by the transition between two Hall plateaus, indicates the coupling between the opposite device edges, due to the mechanisms discussed in Fig. 5.14, and the non-zero probability for charge carriers to be back-scattered. The width of the transition between plateaus is governed by the latter coupling, so that in the experiment, with a larger amount of potential fluctuations, the non-zero-resistance transition region may obscure the signatures of QH effect at integer-filling factor in the low B range. Nevertheless, the edge state picture is still valid in this regime, as shown in Fig. 5.14i-j, corresponding to finite R_{xx} .

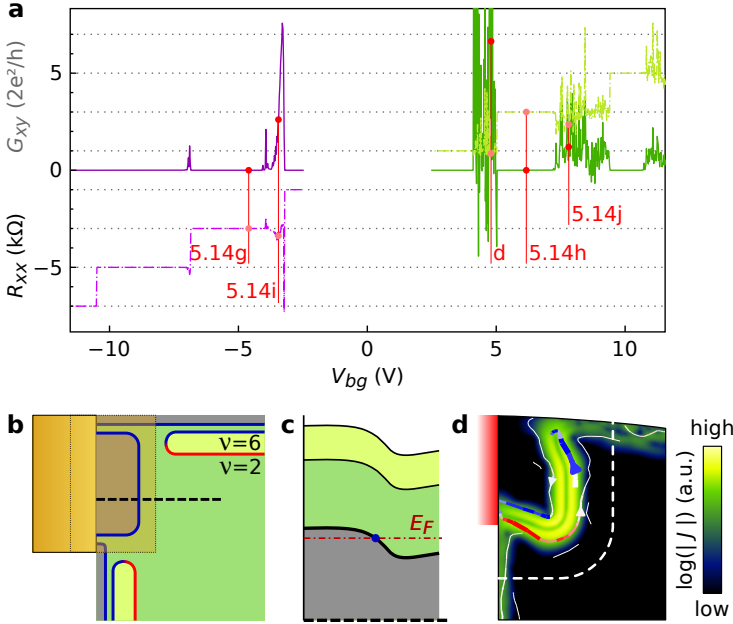


Figure 5.15: Transport properties through the device. **a**, Simulated R_{xx} (plain line) and G_{xy} (dotted line) as a function of V_{bg} for a magnetic field $B = 4$ T. **b**, Spatial configuration of the QHECs at the origin of the fluctuations in **a**. **c**, Spatial evolution of the potential (thick line) and of the three first Landau levels (LLs) along the black dashed lines in **b**. **d**, Current density maps obtained from tight-binding calculations for $V_{bg} = 4.8$ V, as indicated with a red line in **a**.

Another striking feature of the curve is the presence of large R_{xx} fluctuations on the electron side, similarly to the experimental data presented in Fig. 5.11a. These fluctuations originate from extra QHECs running below the contacts, inducing a direct backscattering of charge carriers in the contact area (Figs.

5.15b,c). This mechanism prevents a proper equilibration between the propagating QHECs and the contacts. In addition, current injection is strongly hampered in this situation, preventing proper determination of R_{xx} . Parasitic doping below the contacts therefore explains measurements artifacts such as the large variation of resistance even towards negative values reported here (Fig. 5.11a) as well as in previous experiments [183].

5.2.5 Summary and open questions

In summary, we have highlighted that line contacts, used for state-of-the-art encapsulated graphene samples, can lead to different spatial configurations of the upstream QHECs on the electron and hole sides. This yields distinct mechanisms at the origin of the QH topological breakdown for both charge carriers types. For holes, the breakdown occurs by coupling the upstream QHECs through antidots located at the device edges whereas for electrons, all the QHECs running along the same edge are equilibrated and the breakdown occurs through the bulk, similarly to conventional semi-conductor 2DESs. This interpretation provides a convincing explanation for microwave impedance microscopy results, where the QH topological breakdown was shown to coincide either with a conducting bulk, either with conducting edges, depending on the charge carriers type [106].

As already precised at the beginning of section 5.2, though, the link between the experimental SGM maps and the theoretical discussion that follows is quite speculative due to the limited amount of data, obtained on a single device. More extensive investigations are then required, involving, e.g., different types of contact materials and a statistically significant number of studied devices.

Concerning the model itself, the QHECs spatial configuration in the vicinity of the constriction deserves more discussion. Indeed, the distance between up- and downstream QHECs has been estimated around 200 nm in section 5.1.2 but the constriction itself is 250 nm wide. Two scenarios are then possible:

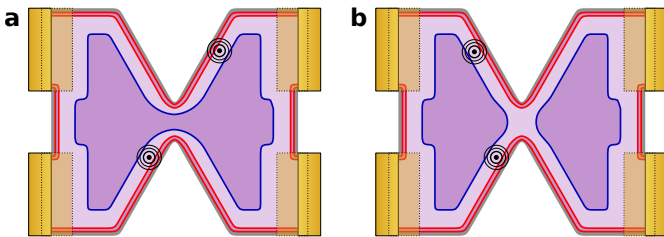


Figure 5.16: QHECs in the constriction. **a**, The distance between the up- and downstream QHECs is reduced in the constriction due to the smaller graphene width. **b**, The upstream QHEC does not enter the constriction and form two loops on both sides of the QPC.

- The distance between the up- and downstream QHECs is reduced in the constriction due to the smaller width of the graphene plane in this region [105] (Fig. 5.16a).
- No upstream QHECs exists in the constriction because of an overall increase of density in its vicinity (Fig. 5.16b). Two QHEC loops are formed on both side of the QPC. In this configuration, the coupling between up- and downstream QHECs is achieved in the same loop, on both opposite edges.

chapter 6

Quantum Hall nano-interferometers

In section 2.4.2.3, we have seen that source-drain bias spectroscopies allows to determine whether a QHI is in the AB or in the CD regime. In this chapter, such measurements, combined with SGM, have been used to characterize the quantum Hall breakdown through (anti)dots in graphene (section 6.1) and in a semiconductor-based 2DES (section 6.2).



The source-drain bias spectroscopies that I had measured on a graphene antidot exhibited very different signatures depending on the (V_{bg}, B) couple at which was done the measurement, associated either to the AB or to the CD regime. However I didn't see any reason for such an abrupt change of regime and I wondered if the AB regime could not explain of its own both signatures, but it was only an intuition. In a beautiful day of May, I was reading the very well written article of Deprez *et al.* [12], just before the evening meal, and I discovered that they explain spectroscopy results with a simple model involving the approximation of Eq. (2.23). I decided to quickly test if the same toy model, without this approximation, could also yield signatures usually associated to the CD regime. While I was under the pressure of joining the table on which stood the ready meal, something wonderful happened (the kind of event that happens only once or twice during a thesis): my Python code worked at the first attempt... and its output confirmed my intuition. Beside explaining the data in graphene, this model also allowed to resuscitate old data obtained in the group more than ten years ago, but that is the subject of section 6.2.

6.1 In graphene

In the previous chapter, we have seen that the QH topological breakdown in graphene is caused by the presence of antidots located between the up- and downstream QHECs. In particular, the data presented in section 5.1.2 led us to consider Coulomb blockade as the main mechanism affecting the electrical transport through the antidots, and hence the backscattering from the down- to the upstream QHECs. In this section, we revise this picture under the light of source-drain spectroscopies obtained at lower magnetic field. With the help of a simple analytical model, we show that the transport through the antidot can be explained in a pure coherent framework. The data presented in this section have been obtained in sample G1 (section 5.1) with the couples (V_{bg}, B) shown in Fig. 6.1.

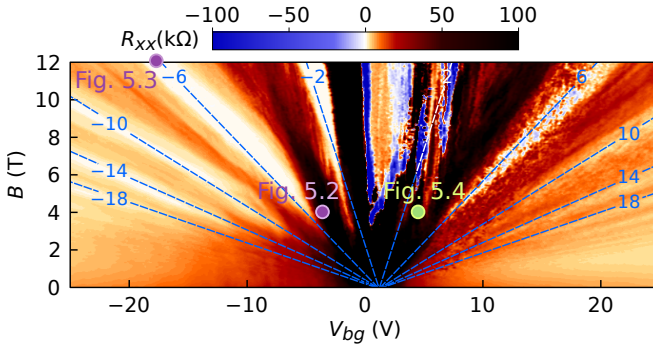


Figure 6.1: Fan diagrams for sample G1. Same as Fig. 3.10 with the coupled (V_{bg}, B) for each figures in this section.

6.1.1 Checkerboard pattern

6.1.1.1 Antidots acting as Fabry-Perot interferometers

First, we use SGM to study hole transmission from down- to upstream QHECs at $B = 4$ T (Fig. 6.1). Figure 6.2a shows a R_{xx} map obtained by scanning the tip above one of the device edges. The concentric circular fringes are the signatures of transport through an antidot coupling the up- and downstream QHECs (chapter 5), as illustrated in Fig. 6.2b. An in-depth characterization of transport in this system is achieved via a local (tip-controlled) source-drain spectroscopy, shown in Fig. 6.2c. It has been obtained by varying the DC source-drain bias voltage V_{sd} while moving the tip along the white dashed line in Figs. 6.2a, i.e., crossing the location of the antidot, at constant V_{tip} . The derivative of the map in Fig. 6.2c with respect to x_{tip} (Fig. 6.2d) exhibits the typical checkerboard-like pattern observed in QHI in the AB regime (section 2.4.2.3). Figure 6.2d features AB oscillations vs x_{tip} , caused by a variation of the magnetic flux $\Phi = BA$ enclosed in the antidot of area A , as well as oscillations vs V_{sd} , linked to a bias-driven phase shift of $L\epsilon/\hbar v$ (Eq. (2.21)). Here, $L = \pi r$ where r is the radius of the QHI (Fig. 6.2b).

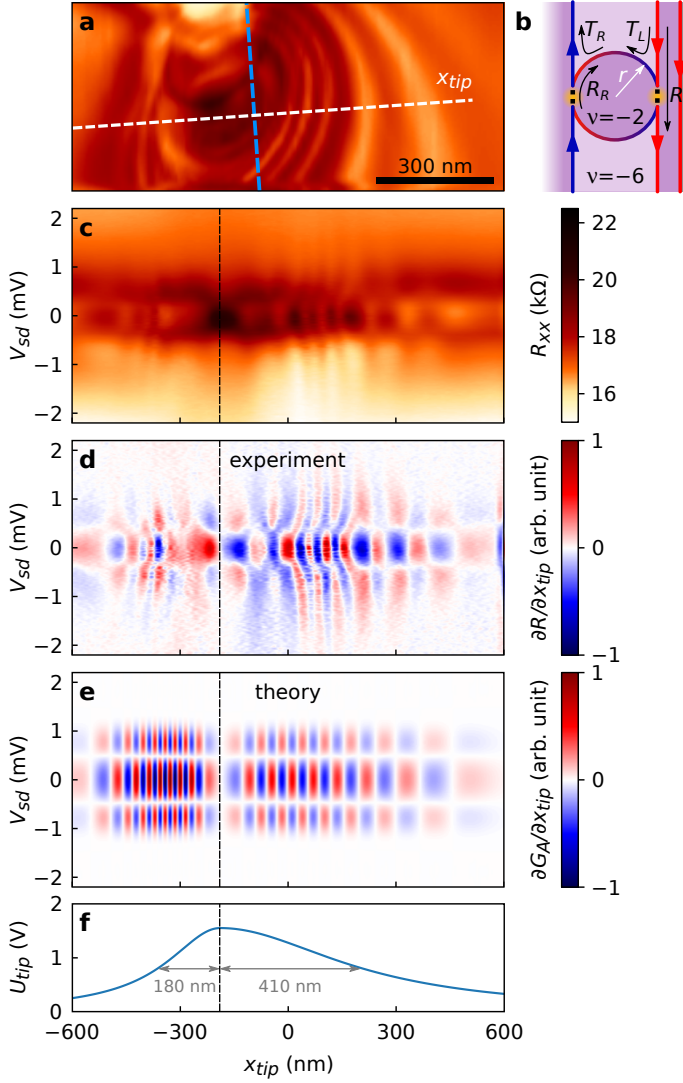


Figure 6.2: Checkerboard pattern in a source-drain spectroscopy. **a**, SGM maps of R_{xx} as a function of tip position, with $B = 4$ T, $V_{bg} = -3.6$ V (Fig. 6.1 and Fig. 5.11b) and $V_{tip} = 0$ V. The scan area is sketched by a red rectangle in Fig. 5.11b. **b**, Schematic of the antidot located between the upstream (blue) and downstream (red) QHECs. It forms a FP QHI strongly coupled to the QHECs, i.e., with low reflection coefficients at the two beam splitter depicted with black dotted lines highlighted in yellow. **c**, R_{xx} as a function of V_{sd} and tip position x_{tip} along the white dashed line in **a**. **d**, Derivative of R_{xx} in **c** according to x_{tip} . **e**, Theoretical map of $\partial G_A / \partial x_{tip}$ as a function of V_{sd} and x_{tip} . **f**, Tip potential (U_{tip} as a function of x_{tip}) used in **e**.

6.1.1.2 Numerical model

Now, we demonstrate that numerical simulations of the conductance G_A through the antidot (Fig. 6.2e) accurately reproduce the experimental data. The simulated spectroscopy map has been obtained by following the simple FP model introduced in section 2.4.1. However, we have not simplified the transmission expression (Eq. 2.19) as in Eq. (2.23). The conductance through the antidot then writes

$$G_A = \frac{e}{h} \frac{d}{dV_{sd}} \int_{-eV_{sd}/2}^{eV_{sd}/2} \frac{(1 - R_L)(1 - R_R)}{1 + R_L R_R - 2\sqrt{R_L R_R} \cos\left(\pi \frac{\Phi}{\Phi_0} + \frac{L\epsilon}{\hbar v} + \varphi\right)} d\epsilon, \quad (6.1)$$

where $\Phi = BA$ (Eq. (2.20)) with $A = \pi r^2$. The radius r is expected to change with B and U_{tip} (the tip-induced potential) as both parameters change the relative position of the Fermi level compared to the potential landscape at the origin of the antidot. We can then write

$$dr = \frac{\partial r}{\partial U_{tip}} dU_{tip} + \frac{\partial r}{\partial B} dB. \quad (6.2)$$

However, the parameters $\alpha_{tip} = \partial r / \partial U_{tip}$ and $\alpha_B = \partial r / \partial B$ are extremely difficult to assess and we will consider them as constant. Therefore, the radius evolves as

$$r = r_0 + \alpha_{tip} \Delta U_{tip} + \alpha_B \Delta B, \quad (6.3)$$

where r_0 is the radius without tip-induced perturbation at the initial value of B .

Next, it remains to describe the evolution of U_{tip} in Eq. (6.3) with the tip position x_{tip} . It is known that it should follow a Lorentzian evolution (section 4.1.1). In the case of a line scan such as in Fig. 6.2a, we consider different half-width at half-maximum (HWHM) to capture the change of screening depending if the tip is above or away from the graphene plane (as shown in Fig. 6.2f and discussed in section 5.1.3) which yields the following tip-induced potential

$$U_{tip}^*(x_{tip}) = \begin{cases} \frac{U_{max}}{1 + ((x_{tip} - x_A)/R_{tip,1})^2}, & \text{for } x_{tip} < x_c, \\ \frac{U_{max,2}}{1 + ((x_{tip} - x_A)/R_{tip,2})^2}, & \text{for } x_{tip} > x_c, \end{cases} \quad (6.4)$$

where U_{max} is the maximal tip-induced potential, x_A the antidot position, x_c the position of the graphene edge, $R_{tip,1}$ the HWHM above the graphene plane, $R_{tip,2}$ the HWHM outside the graphene plane and $U_{max,2}$ the amplitude of the Lorentzian function outside the graphene plane that guarantees the continuity of the function at $x = x_c$. However, Eq. 6.4 does not guarantee the continuity of $\partial U_{tip}^* / \partial x_{tip}$, since it considers an abrupt transition between the graphene plane where screening is at play and away from the graphene plane. To overcome

this issue, we perform a convolution of $U_{tip}(x_{tip})$ with a Gaussian, yielding

$$U_{tip}(x_{tip}) = \frac{\int_{-\infty}^{\infty} U_{tip}^*(x) \exp\left(-(x - x_{tip})^2 / \Delta x^2\right) dx}{\int_{-\infty}^{\infty} \exp\left(-x^2 / \Delta x^2\right) dx} \quad (6.5)$$

where Δx is the width of the smoothing function. Here, we found that $\Delta x = 45$ nm provided the best fit to the experimental data for all the simulations.

6.1.1.3 Simulated spectroscopies

The map in Fig. 6.2e is obtained with small reflection coefficients at the left and right beam splitters ($R_L = R_R = 0.01$ - see Fig. 6.2b). It is therefore logical to recover a checkerboard pattern as described by the analytical expression of Eq. (2.25), obtained with an approximation. Here, the antidot radius is $r_0 = 100$ nm, compatible with the expected distance between up- and downstream QHECs estimated between 200 and 300 nm (section 5.1.2). The oscillation period vs x_{tip} has been fitted by taking into account a change of the antidot radius (and hence its area A) caused by the tip-induced electrostatic potential U_{tip} (r changes from 100 nm when $U_{tip} = 0$ to 108 nm when $U_{tip} = U_{max}$). The relationship between x_{tip} and U_{tip} is shown in Fig. 6.2f and is obtained with $U_{max} = -1.55$ V, $x_A = -230$ nm, $x_c = x_A$, $R_{tip,1} = 125$ nm and $R_{tip,2} = 350$ nm (close to the values extracted in Fig. (5.5)).

Finally, the oscillations vs V_{sd} in Eq. 6.1 have been fitted with a QHEC velocity $v = 1.5 \cdot 10^5$ m/s, in good agreement with former experimental results in semiconductor-based heterostructures [99, 127, 128, 203] and in graphene [12, 13]. Another interesting feature of Fig. 6.2d is the decrease of the oscillations visibility when $|V_{sd}|$ is raised. The decoherence mechanism leading to this behavior (section 2.4.2.3) is usually captured in numerical calculations by multiplying the pattern obtained with the FP model by $\exp(-\gamma|V_{sd}|^n)$, with $n = 1$ or 2 [99, 127, 204]. Here, we have chosen another approach, inspired by ref. [126], consisting in introducing a gaussian dephasing whose variance is proportional to V_{sd}^2 . Starting from $G_A^*(V_{sd}, x_{tip})$ in Eq. 6.1, it yields

$$G_A(V_{sd}, x_{tip}) = \frac{\int_{-\infty}^{\infty} G_A^*(\epsilon, x_{tip}) \exp\left(-\left(\frac{\epsilon - V_{sd}}{\gamma V_{sd}^2}\right)^2\right) d\epsilon}{\int_{-\infty}^{\infty} \exp\left(-\left(\frac{\epsilon - V_{sd}}{\gamma V_{sd}^2}\right)^2\right) d\epsilon} \quad (6.6)$$

where $\gamma = 500$ is a fitting parameter. This approach has provided better results with the simulations presented below.

At this point, we should stress that experiments yield maps of R_{xx} , the longitudinal resistance through the sample, whereas the model output, obtained

from Eq. 6.1, expresses the conductance G_A through the antidot. The two quantities are directly linked since a large value of G_A causes a strong backscattering from the down- to the upstream QHEC and, hence, a large R_{xx} .

6.1.2 Diamond pattern

In the following, we discuss the influence of the reflection coefficients R_L and R_R of the beam splitters (Fig. 6.2b), considering the experimental data obtained at different values of B and V_{bg} . First, Fig. 6.3 is obtained in similar conditions as Fig. 6.2 but at larger magnetic field ($B = 12$ T). The SGM maps in Figs. 6.3a,b feature a $R_{xx} = 0$ background with sharp concentric non-zero R_{xx} fringes centered on the antidot position, similarly to Fig. 5.1. The local spectroscopy map shown in Fig. 6.3c exhibits a clear diamonds pattern, in sharp contrast with the checkerboard pattern of Fig. 6.2. This kind of pattern is usually associated to Coulomb charging effects (section 1.1.4).

Interestingly, the same FP model used to generate Fig. 6.2e can also reproduce the results shown in Fig. 6.3c, by considering larger reflection probabilities R_L and R_R . The simulated map in Fig. 6.3e, obtained with $R_L = R_R = 0.7$ in Eq. 6.1, also displays a diamond pattern in good agreement with the experiment. In this model, the antidot radius increases from $r = 57.7$ nm to 59.5 nm with the change of tip-induced potential following the same evolution in x_{tip} as depicted in Fig. 6.2f with $x_c = x_A = -150$ nm in Eq. (6.4). Note that for the purpose of the simulation, the vertical scale of Fig. 6.3e should be expressed in V_{sd} whereas the map in Fig. 6.3c has been obtained by varying I_{sd} due to the presence of $R_{xx} = 0$ regions. Therefore, the QHECs velocity found in this case ($v = 1.4 \cdot 10^5$ m/s) suffers from a large uncertainty.

At this stage, it remains to explain what is the rationale behind the change of reflection coefficients R_L and R_R between Fig. 6.2 and 6.3. It can originate from two factors at least. First, the size of the antidot and the distance between the up- and downstream QHECs depend on the position of the Landau levels compared to the Fermi energy (section 5.1.4). The proximity between the antidot and the QHECs, and hence the value of R_L and R_R , is consequently different for each couple of B and V_{bg} . Second, the width of the QHECs wavefunction decreases at larger magnetic field, since it depends on the magnetic length $l_B \propto B^{-1/2}$. Therefore, the overlap between the antidot and the QHECs wavefunctions also decreases with B (Fig. 5.12), leading to an increase of the coefficients R_L and R_R (the transmission from the QHEC to the antidot drops), in agreement with the strong versus low coupling regimes observed between Fig. 6.2 and 6.3 respectively.

6.1.3 Intermediate pattern in the electron side

Next, we examine the electron side in Fig. 6.4 at $B = 4$ T. The SGM map in Fig. 6.4a, obtained at positive V_{bg} , shows fringes centered in the vicinity of the constriction, in opposition to the contrast located along the edges in the hole

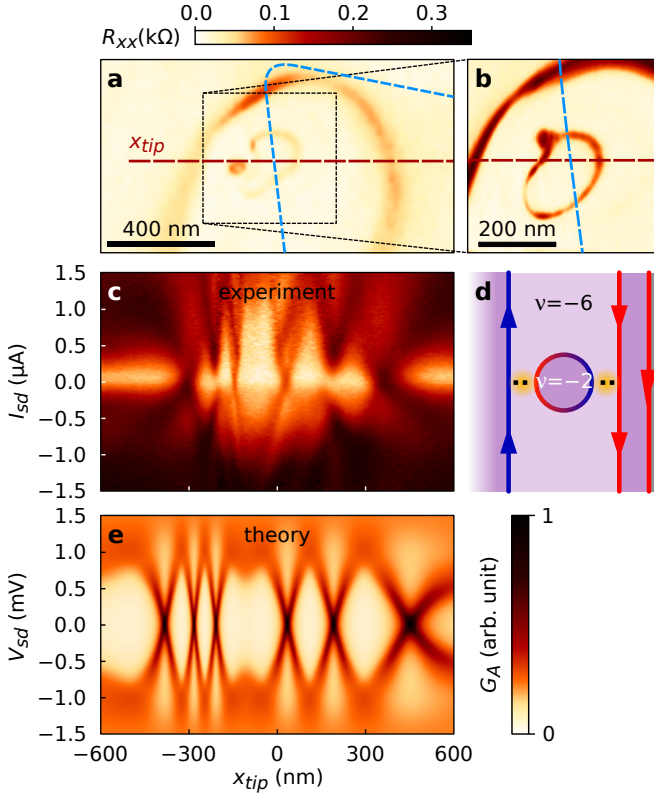


Figure 6.3: Diamond pattern in a source-drain spectroscopy. **a**, SGM maps of R_{xx} as a function of tip position, with $B = 12$ T, $V_{bg} = -17.75$ V (Fig. 6.1 and Fig. 5.11b) and $V_{tip} = 0$ V. The scan area is sketched by a red rectangle in Fig. 5.11a. **b**, zoom in **a**. **c**, R_{xx} as a function of I_{sd} and tip position x_{tip} along the red dashed line in **a**. Note that, due to the low R_{xx} , the spectroscopy is performed by applying a DC current I_{sd} instead of a voltage bias. **d**, Schematic of the FP QHI weakly coupled to the QHECs, i.e., with large reflection coefficients at the two beam splitter depicted with black dotted lines highlighted in yellow. **e**, Theoretical map of G_A as a function of V_{sd} and x_{tip} .

side (Figs. 6.2a and 6.3a,b). In section 5.2.3, we ascribe this difference to an asymmetry in the QHEC structure between the electron and hole sides caused by the electrical contacts. In that framework, the electron backscattering is mediated by an antidot located in the vicinity of the constriction (Fig. 6.4b) that couples the upstream QHECs transmitted along the etched opposite edges of the QPCs. The spectroscopy map measured on this antidot is shown in Fig. 6.4c and its derivative vs x_{tip} in Fig. 6.4d. The latter map features a strained checkerboard pattern well captured by the numerical simulation of Fig. 6.4e. It has been obtained with the intermediate reflection coefficients $R_L = R_R = 0.15$ and with an antidot radius going from $r = 70$ nm to 80.45 nm under the action of the tip-induced potential depicted in Fig. 6.4g. This curve has been obtained from Eqs. (6.4) and (6.5) with $U_{max} = -1.55$ V, $x_A = -180$ nm, $x_c = 20$ nm, $R_{tip,1} = 125$ nm and $R_{tip,2} = 250$ nm. The latter value is smaller than in Fig.

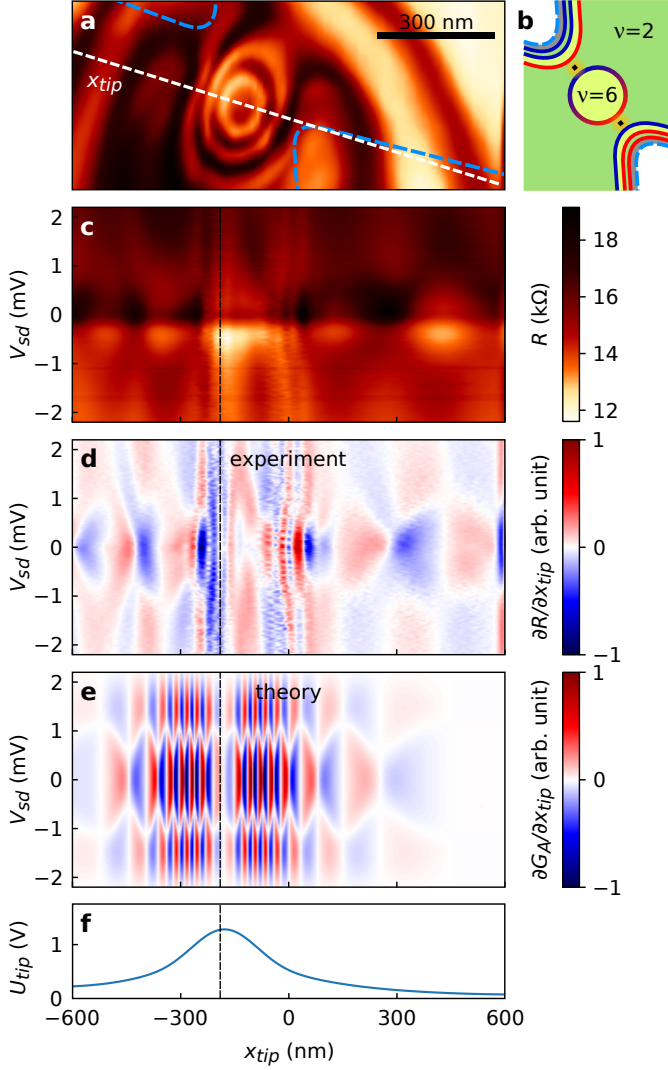


Figure 6.4: Intermediate regime in a source-drain spectroscopy. **a**, SGM maps of R_{xx} as a function of tip position, with $B = 4$ T, $V_{bg} = 4.5$ V (Fig. 6.1 and Fig. 5.11b) and $V_{tip} = 0$ V. The scan area is sketched by a blue rectangle in Fig. 5.11a. **b**, Schematic of the antidot located in the center of the constriction, between the upstream QHECs of both edges. It forms a FP QHI with intermediate reflection coefficients at the two beam splitter. **c**, R_{xx} as a function of V_{sd} and tip position x_{tip} along the white dashed line in (a). **(d)** Derivative of R_{xx} in **c** according to x_{tip} . **e**, Theoretical map of $\partial G_A / \partial x_{tip}$ as a function of V_{sd} and x_{tip} . **f**, Tip potential (U_{tip} as a function of x_{tip}) used in **e**.

6.2g because the tip is moved along the edge of the device (Fig. 6.4a), resulting in a larger screening. Finally, the QHECs velocity, extracted from a fit to the experimental oscillations vs V_{sd} , is $v = 2 \cdot 10^5$ m/s, i.e., similar to values found on the hole side.

The data presented in Fig. 6.4 is interesting for two reasons. First, it highlights the pattern obtained in an intermediate regime between checkerboard and diamonds. Second, it confirms the picture exposed in section 5.2.3, assigning the topological breakdown in the electron side to the presence of an antidot located in the vicinity of the constriction, that trigger the coupling between the QHECs on the opposite sides of the constriction.

6.1.4 Absence of a Coulomb dominated regime

Nevertheless, finding the studied antidots in the AB regime in the different configurations discussed in Figs. 6.2, 6.3 and 6.4 is unexpected in regards to their small sizes ($A < 0.05 \mu\text{m}^2$). Indeed, up to our knowledge, no QHI operating in this regime has ever been reported with a size under $1 \mu\text{m}^2$, even with architectures optimized to reduce Coulomb interactions [12, 13, 128]. To better understand the transition between both regimes, one can consider the coefficient $\xi = C_{eb}/(C_{eb} + C_b)$ introduced in Eq. 2.26 [113]. We have seen that $C_{eb} \propto r$ is the capacitive coupling between the QHECs and the localized states enclosed in the QHI and $C_b \propto r^2$ is the localized states capacitance. By decreasing the QHI size, the coefficient C_b becomes negligible compared to C_{eb} and $\xi \sim 1$, meaning that the QHI should be in the CD regime (section 2.4.3). However, this reasoning is only valid provided that localized states are present inside the QHI. In the case of an antidot, this is mostly unlikely since its size is similar with the disorder potential fluctuations observed in graphene on hBN (Fig. 1.5) [29]. In other words, the QHI is actually made of a localized state. Hence, $C_{eb} = \xi = 0$, which supports the observation of the AB regime in our nano-size QHI.

6.1.5 Summary and open questions

In summary, we have used SGM to reveal the presence of preexisting nano-QHI, associated with antidots, causing the coupling between up- and downstream QHECs in graphene. From the local spectroscopies performed on these QHI, accurately reproduced with a simple FP numerical model, we find that even the smallest-radius (sub-100 nm) QHI operates in the AB regime. From the fit to experimental data, we have extracted both a value for the QHEC velocity, similar to previously extracted values in graphene QHIs, and for the antidot size, coherent with the expected distance between up- and downstream QHECs but much smaller than previously investigated QHI in the AB regime. According to leading theories in the field, QHI of such small size should lead to a CD regime. We explain this apparent contradiction by the absence of localized state inside the antidot, whose length scale is on the order of the disorder potential fluctuations.

Nevertheless, despite we have shown that transport through nano-QHIs associated to antidots can be modeled by a pure AB model, without taking into account the Coulomb interactions, we can not rule out completely the possibility that they play a role in the spectroscopy of Fig. 6.3c. Indeed the CD

regime should also yield a diamond pattern that could be mixed with the one originating from the AB oscillations. This should be characterized carefully when designing nano-QHI in the future.

6.2 In semiconductor-based electron gases

After exploring the model exposed in the previous section, we realized that it explains data obtained in our group, by Sébastien Faniel, more or less 10 years ago and never understood so far. This section presents those results and the associated discussion.

The device is built from an $\text{In}_{0.7}\text{Ga}_{0.3}\text{As}/\text{In}_{0.52}\text{Al}_{0.48}\text{As}$ quantum well grown by molecular beam epitaxy on an InP substrate (see [159, 205]). A 15 nm-thick quantum well buried 25 nm below the surface hosts a 2DEG with a bare electronic density $N_S = 5.7 \cdot 10^{15} \text{ m}^{-2}$ and a mobility $\mu = 5.3 \cdot 10^4 \text{ cm}^2/\text{Vs}$. The 2DEG was electrically contacted by means of Ge/Au ohmic contacts. The Hall bar and the QPC were patterned using electron beam lithography and wet etching. The QPC materializes thanks to two narrow trenches defining a $\sim 350 \text{ nm}$ -wide constriction (see Fig. 6.5a). The 2DEG areas beyond the etched trenches of the QPC were used as side gates (with an applied bias V_g) in order to control the effective width of the constriction.

6.2.1 Evolution of the magnetoresistance

Figure 6.5b compares the evolution of R_{xx} as a function of B with $V_g = 5 \text{ V}$ in two different regions of the sample: a macroscopic Hall bar (top panel) and the QPC region (bottom panel). Both curves show Shubnikov-de Haas oscillations and QH effect stemming from the magnetic depopulation of the LLs in the 2DEG. The bulk integer filling factor ν is deduced from the oscillations in the Hall bar. In the QPC trace, the more complex series of oscillations in the vicinity of the integer filling factor ν is highlighted in Fig. 6.5d. Similar resistance oscillations have been reported in QPCs and were attributed to the interaction between the counterpropagating QHECs through a small QHI (see Fig. 6.5c) located in the vicinity of the constriction [4, 87, 88, 90, 99, 206, 207]. The same mechanisms are at play in the present sample. We will indeed show below that successive types of QHIs, whose configuration depend on the magnetic field, are located in the constriction region.

We first focus on the R_{xx} evolution around $\nu = 4$ (Fig. 6.5d) and discover four different QHI configurations as B is raised (Figs. 6.6a-d). The whole picture, detailed hereafter, relies on a peculiar electrostatic potential landscape in the constriction region, with a central dot (larger electron density) surrounded by a region of lower electron density, fully determined by disorder-related potential fluctuations. This model is further justified in section 6.2.1.2, in particular with respect to another hypothesis involving a central antidot instead of a dot.

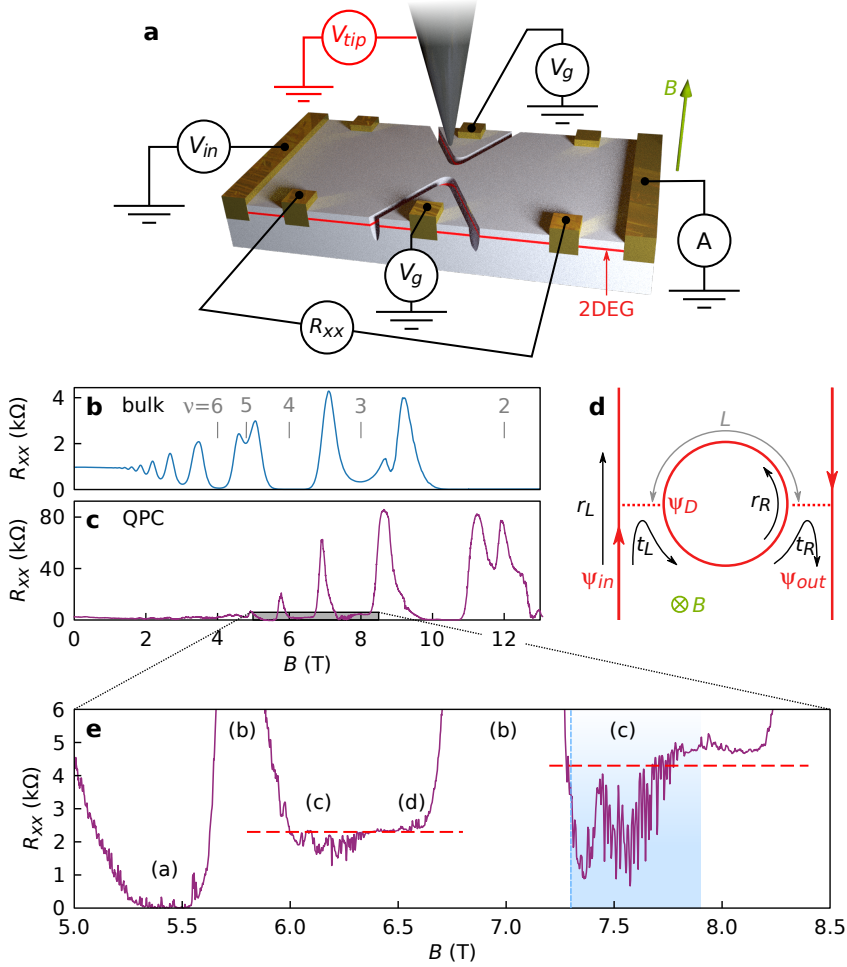


Figure 6.5: Magnetoresistance in a semiconductor-based QPC. **a**, Artist view of the experimental setup with the buried 2DEG depicted in red. The biased tip locally changes the electron density when applying the voltage V_{tip} and is moved at a distance $d_{tip} \sim 60$ nm above the 2DEG. A magnetic field B is applied perpendicularly to the 2DEG plane. **b**, R_{xx} as a function of B measured on a Hall bar patterned next to the QPC (top panel) and in the QPC (bottom panel) with $V_g = 5$ V. The vertical lines indicate integer filling factors. **c**, Schematic of a QHI formed by a QHEC loop coupling two counterpropagating QHECs (the dotted lines correspond to tunneling paths). **d**, Zoom of the gray shaded region in **b**. 2.2 kΩ and 4.3 kΩ plateaus are indicated with red dashed lines.

6.2.1.1 The different regimes of coupling through the dot

At the onset of the $\nu = 4$ QH state ($5.2 \text{ T} \lesssim B \lesssim 5.6 \text{ T}$), R_{xx} shows series of narrow peaks superimposed on a $0 \text{ } \Omega$ plateau. This indicates the presence of a QHI formed as the counterpropagating QHECs (red channels at the top and

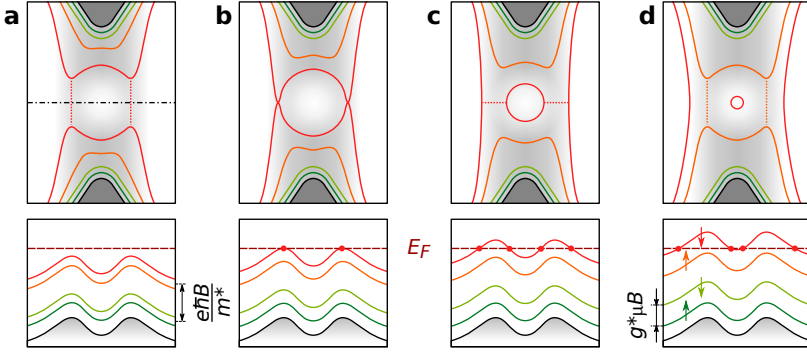


Figure 6.6: QHECs evolution with a dot in the constriction. **a-d**, Evolution of the QHECs spatial configuration with B in the vicinity of the constriction (top panels) and the corresponding Landau level positions compared to the Fermi energy E_F . Higher potential (lower electron density) is depicted with darker gray shades.

bottom of Fig. 6.6a) are coupled at both sides of the QPC (red dotted lines in Fig. 6.6a).

A large R_{xx} peak is then observed ($5.6 \text{ T} \lesssim B \lesssim 6 \text{ T}$). It is the signature of a large transmission probability between the counterpropagating QHECs, as sketched in Fig. 6.6b.

As B is further increased ($6 \text{ T} \lesssim B \lesssim 6.4 \text{ T}$), R_{xx} exhibits dips below a $\sim 2.2 \text{ k}\Omega$ plateau (red dashed line in Fig. 6.5d). Such a plateau appears when the inner QHEC is completely backscattered so that the region outside of the QPC exhibits a different filling factor (ν in the bulk) from the QPC region (filling factor ν^*), as illustrated in Fig. 6.6c. In such configuration, the resistance is given by [91]

$$R_{xx} = \frac{h}{e^2} \left(\frac{1}{\nu^*} - \frac{1}{\nu} \right), \quad (6.7)$$

where e is the electron charge and h is the Planck constant. The $2.2 \text{ k}\Omega$ plateau is consistent with $\nu = 4$ and $\nu^* = 3$. It indicates that the spin degeneracy of the LLs has been lifted due to Zeeman effect (section 2.1.1.3). The second LL (and its corresponding QHEC) is therefore split in two states: one spin down-polarized (in red in Figs. 6.6a-d) and one spin up-polarized (in orange). Based on this picture, the dips below the plateau are explained by the presence of a dot (QHEC loop) at the center of the constriction which acts as a QHI coupling the QHECs running at the left and right sides of the QPC in Fig. 6.6c. When the QHI formed by this dot (Fig. 6.5c) is active, these QHECs are not perfectly backscattered anymore and R_{xx} drops below the $2.2 \text{ k}\Omega$ plateau.

In the next B -range ($6.4 \text{ T} \lesssim B \lesssim 6.7 \text{ T}$), R_{xx} peaks are observed above the $2.2 \text{ k}\Omega$ plateau. This magnetoresistance sequence is close to the first regime (Fig. 6.6a) but the QHI is created by the spin up-polarized QHEC while the spin down-polarized QHEC is perfectly backscattered (Fig. 6.6d).

When B is further increased, the regimes highlighted in Fig. 6.5d are repeated with the spin up-polarized QHEC. R_{xx} then features fluctuations around a plateau close to $4.3 \text{ k}\Omega$ (right red dashed line in Fig. 6.5d), corresponding to the filling factors $\nu = 3$ and $\nu^* = 2$ in Eq. 6.7. In the remainder of this paper, we will further characterize the QHI formed by the spin-up polarized QHEC in the configuration of Fig. 6.6c.

6.2.1.2 Dot or Antidot?

In Figs. 6.6a-d, we have presented the QHECs configurations in the vicinity of the QPC that explains the transport curve of Figs. 6.5b,d. This relies on a global decrease of the electron density in the constriction region and a local increase of the density, due to potential inhomogeneities, forming a quantum dot in the center of the constriction. However similar transport data in the literature have been explained with the presence of an antidot near the QPC, associated with a local decrease of the electron density (section 2.2.2). In this section, we justify why the latter model (antidot), despite being simpler, is not consistent with the present data.

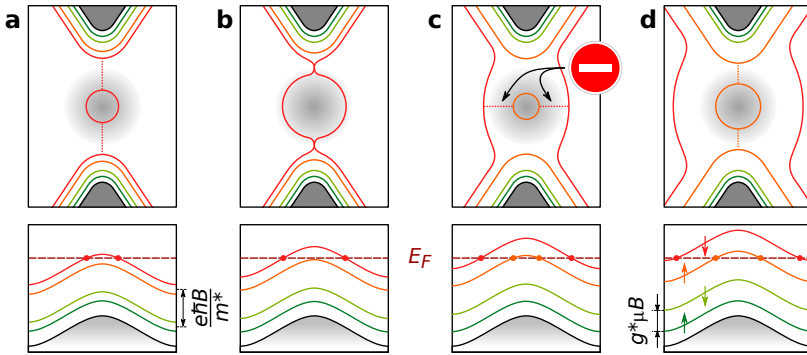


Figure 6.7: QHECs evolution with an antidot in the constriction. a-d, Evolution of the QHECs spatial configuration with B in the vicinity an antidot located in the center of the constriction (top panels) and the corresponding Landau levels position compared to the Fermi energy E_F . A high potential (low electron density) is depicted with darker gray shades.

Figure 6.7 presents the QHECs configurations in the situation where an antidot is located in the center of the constriction. At first sight, the four configurations are compatible with the different regimes in Fig. 6.5d. However, Fig. 6.7c presents a first inconsistency. Indeed, we have shown that the spin degeneracy is lifted and the third (orange) and fourth (red) LLs are polarized with opposite spins. Therefore, the tunneling between the counterpropagating QHECs via the antidot in Fig. 6.7c is prohibited.

A second argument against the antidot model (and hence in favor of the model proposed in Figs. 6.6a-d) is found in the R_{xx} evolution vs V_{tip} in Fig.

6.8b. As discussed in next section, large negative values of V_{tip} , leading to a decrease of the electron density, are associated with high reflection probabilities at the QHI beam splitters. It is inconsistent with the antidot model where a decrease of the electron density should yield a larger antidot, closer to the counterpropagating QHECs, leading to smaller reflection coefficient at the beam splitters (due to a higher transmission probability between the QHECs and the antidot). On the contrary, the model of Figs. 6.6a-d correctly captures the evolution of the transmission probability with V_{tip} .

6.2.2 DC bias spectroscopies

6.2.2.1 Transition from checkerboard to diamond patterns

Figure 6.8a displays a R_{xx} map as a function of the electrically-biased tip position, with the etched area defining the constriction indicated by the white shaded regions. This map features concentric fringes centered on the QPC, which clearly confirms that the QHI is located in the constriction. In the AB framework, these oscillations originate from a change of the magnetic flux $\Phi = BA$ enclosed in the QHI whose area A varies when the electrically-biased tip moves in its vicinity. As a result, interference conditions in the QHI alternate between constructive and destructive, which yields concentric fringes in the SGM image. Note that another explanation could be given for the same phenomenon in the framework of the CD regime (with the tip influencing energy levels of a tunnel-coupled quantum dot), but we will see in section 6.2.2.2 that we can discard this hypothesis.

Figure 6.8b shows R_{xx} as a function of V_{tip} for the tip position indicated with a green dot in Fig. 6.8a. When V_{tip} is tuned to large negative values (below ~ -3 V), R_{xx} reaches the 4.3 k Ω plateau, decorated with a set of narrow dips. This behavior is perfectly consistent with the picture presented in Fig. 6.5. Indeed, a negative V_{tip} decreases the electron density (i.e., the electrostatic potential landscape shifts upwards) which implies a smaller QHI size. Consequently, the transmission probability to the backscattered QHECs is decreased (Fig. 6.8c) compared to a less negative V_{tip} (Fig. 6.8d). With a smaller transmission probability, R_{xx} remains close to the plateau that corresponds to a perfect backscattering of the QHECs.

Next, we compare the DC bias spectroscopies in the blue (weak transmission) and orange (stronger transmission) ranges in Fig. 6.8b, to obtain further information on the different regimes of transmission between the dot and the QHECs. In this measurement, R_{xx} is recorded as a function of V_{tip} and V_{sd} . A high-pass filter has been applied to each map to get rid of the broad background evolution of R_{xx} and only keep the oscillations ΔR_{xx} originating from the QHI. As for the graphene QPC in section 6.1, we distinguish two regimes. First, Fig. 6.8e shows the spectroscopy obtained at large negative values of V_{tip} (weak transmission - blue in Fig. 6.8b). In this situation, ΔR_{xx} features a diamond pattern, usually associated with Coulomb charging effects. Second,

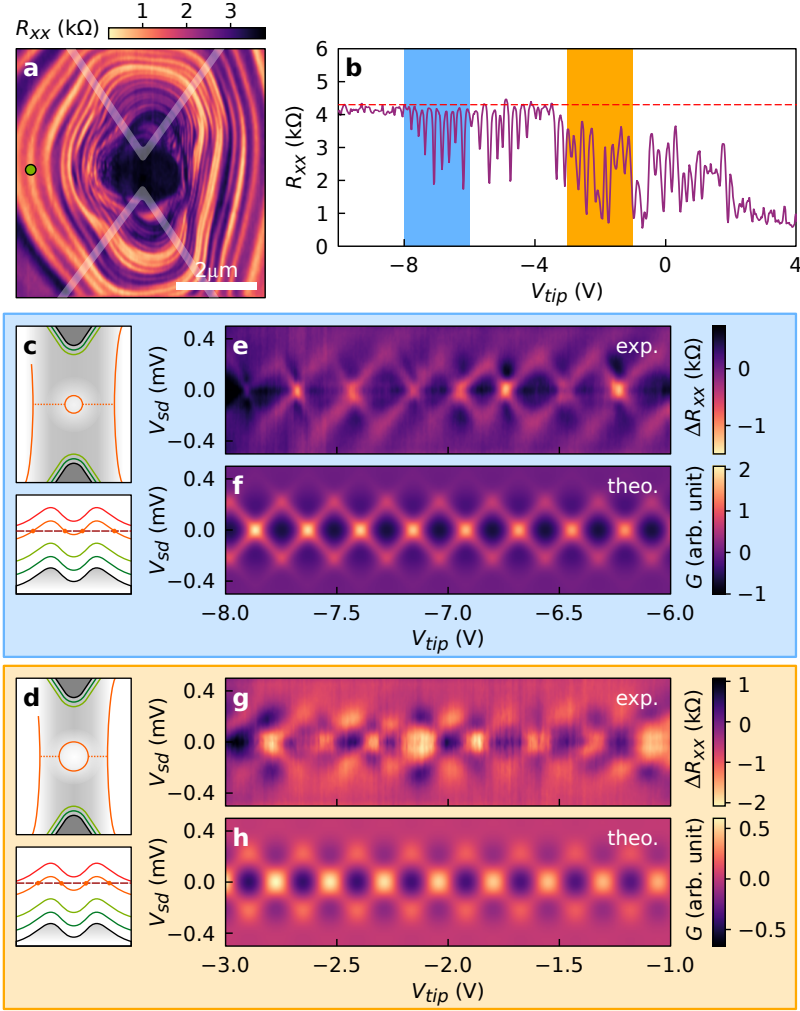


Figure 6.8: From diamond to checkerboard pattern when varying V_{tip} . **a**, SGM map obtained by recording R_{xx} as a function of the tip position at $B = 7.3$ T (blue dashed line in Fig. 6.5d) and with $V_{tip} = 0$ V. Note that due to the work function difference between tip and sample surface, $V_{tip} = 0$ V corresponds to a depleting tip. The borders of the sample are indicated by the white shaded area. **b**, Evolution of R_{xx} with V_{tip} for the tip position indicated with a green dot in **a**. The red dashed lines indicates the 4.3 kΩ plateau. **c,d**, Evolution of the QHEs spatial configuration with V_{tip} in the vicinity of the constriction (top panels) and the corresponding Landau level positions compared to the Fermi energy E_F (bottom panel) **c** and **d** correspond respectively to the blue and orange regions in **b**. **e,f**, Experimental map of ΔR_{xx} and simulated map of G , respectively, as a function of V_{sd} and V_{tip} (blue region in **b**). The maps feature diamond patterns. **g,h**, Experimental map of R_{xx} and simulated map of G , respectively, as a function of V_{sd} and V_{tip} (orange region in **b**). The maps feature checkerboard patterns.

when examining data at less negative values of V_{tip} (stronger transmission - orange in Fig. 6.8b), the obtained spectroscopy displays a checkerboard pattern (Fig. 6.8g). The latter pattern is the signature observed in QHIs in the AB regime.

6.2.2.2 Ruling out the Coulomb dominated regime

At this point, a transition from AB to CD regime when varying V_{tip} can be excluded for at least two reasons. First, the characteristic energy scales in V_{sd} are expected to be different, because they are ascribed to different and uncorrelated mechanisms: a phase change explains the AB oscillations in V_{sd} whereas the Coulomb charging energy should determine the height of the Coulomb diamonds. Interestingly, the energy scale extracted from the spectroscopies in Figs. 6.8e,g remains the same for the checkerboard and the diamonds, with a value around 0.2 mV. Second, the Coulomb charging energy is determined by the size of the QHI and we will show in the next section that this size does not change significantly within the V_{tip} range of Fig. 6.8. Therefore, if the Coulomb charging energy does not dominate in Fig. 6.8g, nor should it in Fig. 6.8e. Figure 6.9c, obtained by varying B instead of V_{tip} , confirms that the checkerboard pattern (zoom shown in Fig. 6.9e) evolves continuously, and with a constant energy scale in V_{sd} , to a diamond pattern (zoom shown in Fig. 6.9f) with a small increase of the magnetic flux enclosed in the QHI.

Studying the evolution of R_{xx} oscillation amplitude ΔR_{xx} with the temperature yields a third reason to discard the hypothesis of a different origin for checkerboard and diamond patterns. The oscillation amplitude associated with both patterns (orange and blue regions in Fig. 6.9a) turns out to follow the same exponential dependence $\Delta R_{xx} = \Delta R_{xx,0} \exp(-T/T_0)$ with the temperature T (Fig. 6.9b), consistent with the AB effect [99]. The CD regime, on the contrary, would imply a T^{-1} dependence [4]. These observations suggest that the QHI remains in the pure AB regime, whatever the observed phenomenology (checkerboard- or diamond-like). In that framework, it remains to explain how a diamond pattern can be observed in the AB regime.

6.2.3 Fabry-Perot model

As for graphene in section 6.1, the transition between checkerboard and diamond patterns in Figs. 6.8c,f and 6.9c can be explained with a QHI in the AB regime whatever the V_{tip} or B range. Hereafter, we detail a model relying on the schematic shown in Fig. 6.5c, where the FP QHI is formed by a closed-loop QHEC (inside a dot) connected to the left (input) and the right (output) QHECs by two beam splitters (dotted lines). The conductance G_A through this QHI is then given by Eq. 6.1. From this expression, we extract the oscillation contribution of the conductance, expressed as

$$G_{osc} = G_A - \frac{e}{h} \frac{(1 - R_L)(1 - R_R)}{1 + R_L R_R}, \quad (6.8)$$

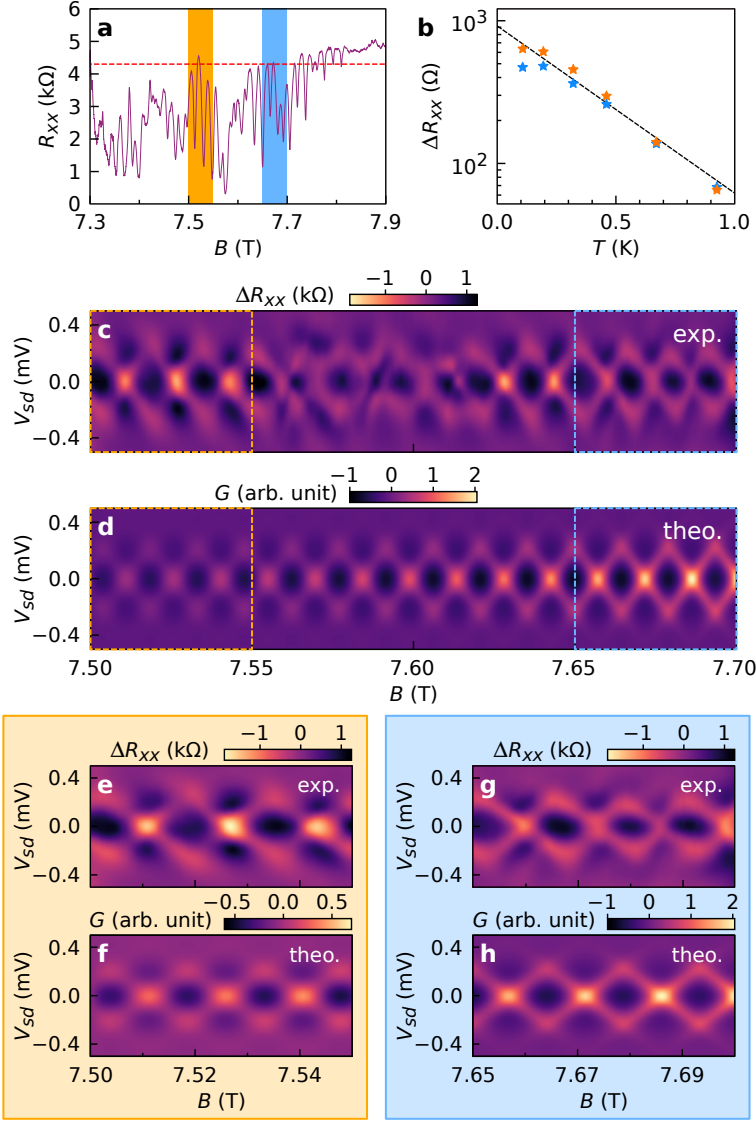


Figure 6.9: From diamond to checkerboard pattern when varying B . **a**, R_{xx} vs B (blue region in Fig. 6.5d) with $V_{tip} = 0$ V. The red dashed line indicates the 4.3 kΩ plateau. **b**, Oscillation amplitude ΔR_{xx} measured at different temperatures T for the two ranges of B (orange and blue regions in **a**). The temperature dependence, fitted with a continuous line, displays a similar exponential evolution for both B ranges. **c,d**, Experimental map of ΔR_{xx} and simulated map of G as a function of V_{sd} and B . **e,g**, Zooms on the low B range of **c,d** displaying a checkerboard pattern. **f,h**, Zooms on the high B range of **c,d** displaying a diamond pattern.

where $R_L = |r_L|^2$ and $R_R = |r_R|^2$ in Fig. 6.5c. Furthermore, the fading of visibility at large $|V_{sd}|$ is captured by multiplying Eq. 6.8 by a Gaussian

function, giving a conductance

$$G = G_{osc} \exp \left(-2\pi\gamma \left(\frac{V_{sd}}{\Delta V_{sd}} \right)^2 \right), \quad (6.9)$$

where γ and ΔV_{sd} are fitting parameters.

In the experimental spectroscopies of Figs. 6.8 and 6.9, the AB phase evolves with Φ (Eq. 2.20) through V_{tip} or B . Indeed, both parameters tune the dot area A as illustrated in Figs. 6.5f-h for a change of B , modifying the spacing between the LLs, and in Figs. 6.8e,h for a change of V_{tip} , shifting the potential and the LLs compared to the Fermi energy E_F . The QHI size, given by the arm length L , then evolves as

$$dL = \frac{\partial L}{\partial V_{tip}} dV_{tip} + \frac{\partial L}{\partial B} dB. \quad (6.10)$$

Since the terms $\partial L / \partial V_{tip}$ and $\partial L / \partial B$ are extremely difficult to assess, we assume that they are constant. The evolution of L with V_{tip} and B is therefore linear.

Furthermore, any change of the dot size also affects its proximity, and hence its transmission probability (dotted lines in Figs. 6.5e-h and Figs. 6.8e,h), with the backscattered QHECs. Since the coupling occurs through tunneling, the associated transmission probability evolves exponentially with the distance between the dot and the QHECs. In turn, since this distance evolves linearly with the dot size, one has

$$R_{L(R)} = \alpha \exp(-\beta L), \quad (6.11)$$

where α and β are positive coefficients to ensure that the reflection probability increases when the dot size decreases (see Fig. 6.5c). This expression is only valid when $R_{L(R)} \ll 1$.

6.2.4 Comparison between experiment and simulation

Next, we show that the model developed in the previous section accurately reproduces the experimental spectroscopy results. Before developing the comparison between experimental and simulation results, we should stress that experiment yields maps of ΔR_{xx} , the oscillating component of the resistance through the sample, whereas model results, obtained from Eq. 6.9, correspond to oscillating component of the conductance G through the dot. The two quantities are directly linked since a large value of G favors the transport through the QPC, lowering R_{xx} below the plateau value, and therefore increasing ΔR_{xx} .

The correspondence between the simulated G and experimental ΔR_{xx} spectroscopies is illustrated in Figs. 6.8e,f and Figs. 6.8g,h. Both simulated maps were obtained with the same parameters in Eq. 6.9: $B = 7.3$ T as in the experiment, the QHEC velocity $v = 0.5 \cdot 10^5$ m/s and the parameters associated to the visibility decay $\gamma = 1$ and $\Delta V_{sd} = 0.6$ mV. The following parameters were

the only ones adjusted when changing V_{tip} : $L = 404.6$ nm and $R_L = R_R = 0.05$ at $V_{tip} = -8$ V, and $L = 435.1$ nm and $R_L = R_R = 0.3$ at $V_{tip} = -1$ V (These values were used to find the free parameters in Eqs. 6.10 and 6.11). Three main outcomes can be highlighted from simulation results, and the good fit to the experimental data: (i) At large negative V_{tip} , a good correspondence has been obtained for a small dot size and a large reflection probability (weak transmission) and a diamond pattern is observed in this case, whereas at smaller negative V_{tip} (larger dot, stronger transmission), the model yields a checkerboard pattern. (ii) The dot size obtained from a fit to the AB oscillation is fully consistent with the constriction width of 350 nm. Indeed, the dot diameter is around 250 nm. (iii) The QHEC velocity v used to fit the V_{sd} oscillations is fully consistent with values obtained in previous studies [99, 127].

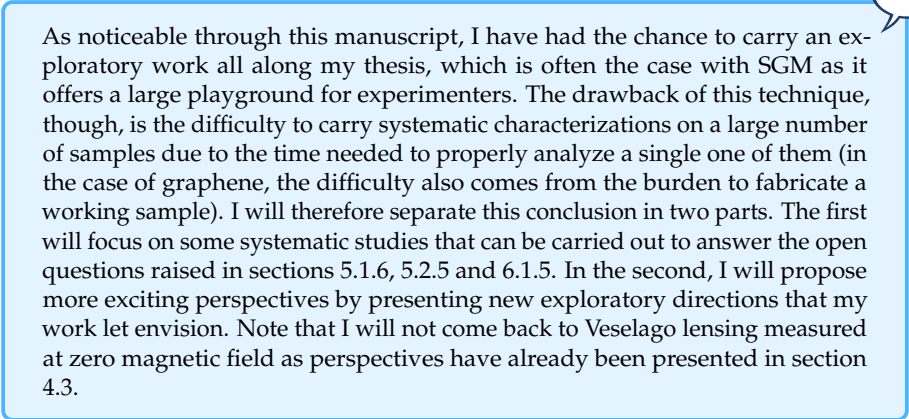
Figures 6.9d,g,h also show that the model can accurately reproduce the experimental data of Figs. 6.9c,e,f, respectively. The simulations have been performed with an evolution of the QHI parameters from $L = 427.3$ nm and $R_L = R_R = 0.3$ at $B = 7.5$ T to $L = 407.8$ nm and $R_L = R_R = 0.05$ at $B = 7.7$ T. The other constant parameters are the same as in Fig. 6.8. Again, the dot shape evolution and its coupling to QHECs follow the pictures proposed in Fig. 6.6c, with a size and a transmission probability decreasing with the magnetic field. This set of maps further draws a clear overview of the evolution from a checkerboard to a diamond pattern as the dot geometry and transmission are changed.

6.2.5 Summary and open questions

In summary, we have used SGM to reveal the presence of a quantum dot located in the vicinity of a QPC in a semiconductor-based 2DES. In the QH regime, it acts as a FP interferometer that modulates electrons backscattering between the two sides of the QPC. DC bias-spectroscopies reveal a continuous transition from checkerboard to diamonds patterns. With a simple FP model, we have shown that both patterns can be explained in a fully AB (coherent) regime, contrary to the usual practice that associates diamonds with the CD regime. We associate the observed transition with a change of transmission probability between the central QHEC loop and the counterpropagating QHECs flowing at each side of the QPC.

In section 6.1.5, we discuss the transport results of an antidot-based QHI in graphene and we raise the possibility that the diamond pattern can also partially be due to Coulomb interactions. Nevertheless, the data obtained in the $\text{In}_{0.7}\text{Ga}_{0.3}\text{As}/\text{In}_{0.52}\text{Al}_{0.48}\text{As}$ quantum well demonstrate that the CD regime can totally be ruled out to explain the diamond pattern observed in a QHI of the same length scale. The new proposed interpretation of diamond phenomenology will likely lead to revisit previous data, and opens the way towards engineering more complex interferometric devices with nanoscale dimensions.

Conclusion and perspectives



As noticeable through this manuscript, I have had the chance to carry an exploratory work all along my thesis, which is often the case with SGM as it offers a large playground for experimenters. The drawback of this technique, though, is the difficulty to carry systematic characterizations on a large number of samples due to the time needed to properly analyze a single one of them (in the case of graphene, the difficulty also comes from the burden to fabricate a working sample). I will therefore separate this conclusion in two parts. The first will focus on some systematic studies that can be carried out to answer the open questions raised in sections 5.1.6, 5.2.5 and 6.1.5. In the second, I will propose more exciting perspectives by presenting new exploratory directions that my work let envision. Note that I will not come back to Veselago lensing measured at zero magnetic field as perspectives have already been presented in section 4.3.

The need of systematic characterization

In the first part of chapter 5, the QH breakdown in graphene has been shown to originate from the presence of antidots triggering charge carriers backscattering between down- and upstream QHECs. This has been observed in two different samples, at different magnetic fields and charge carriers densities. The presence of an upstream QHEC involved in graphene QH breakdown has also been confirmed in other studies [10, 11]. Nevertheless, all these samples share something in common: the charge carriers density is varied by the mean of a silicon back-gate buried 300 nm underneath the graphene plane. If the upstream QHECs originate from a charge accumulation along the edges due to electrostatics effects (section 2.3.4), then a graphite back-gate used in state-of-the-art devices [208], located a few tens of nanometer below graphene, could change the QHECs configuration. Indeed, the distance between down- and upstream QHECs is expected to be similar to the graphene-to-gate distance [105]. A proper characterization of the back-gate influence could be done by performing SGM on two adjacent Hall bars, one above a silicon back gate and the other above a graphite back gate.

As further discussed in section 2.3.4, a large amount of edge impurities is suspected to attenuate the charge accumulation along the device borders

and hence, can prevent the formation of upstream QHECs. The effect of this disorder on charge accumulation has already been characterized from electrical transport measurements [109–111] but without data attesting the presence or not of upstream QHECs. It would therefore be interesting to join these kind of characterization with SGM or transport measurements designed to highlight the presence of upstream QHECs [196].

The subject of impurities along the edges leads us naturally to the question of the antidots origin, already raised in section 5.1.2. At the present time, we can only make suppositions (strain fluctuations for example) to explain their observation. A proper characterization of the potential landscape would therefore give precious information to precise their origin. To do so, scanning tunneling microscopy could be used, in the same way as Fig. 1.5 but along the borders. However, this technique requires that graphene is not encapsulated, which would possibly lead to a larger contamination along the edges. Another technique used to probe charge density is EFM (Fig. 3.9c). Since the tip is not in contact with the probed surface, graphene could be encapsulated. Nevertheless, the spatial definition depends both on the graphene-to-tip distance and on the radius of the tip apex [209]. Reaching a resolution below ~ 100 nm, the estimated size of antidots, therefore requires to use a very thin top hBN layer and an extremely sharp tip (with the constraint that it should be kept intact during the measurements). Nevertheless, overcoming these challenges would also offer the possibility to image the potential landscape at the origin of the upstream QHECs under the application of a back-gate voltage.

In the second part of chapter 5, we have seen that line contacts can dope graphene beneath the metallic region covering the stack, leading to a different QHECs configuration between the electron and hole sides and, hence, a different mechanism of QH breakdown between both types of charge carriers. A more in-depth study is therefore needed to clarify the contacts effect. It could involve the same SGM characterizations on similar devices, but with different metals used for the contacts, doping graphene either with electrons or holes. Furthermore, two scenarios of QHECs spatial configuration in the constriction region have been proposed in Fig. 5.16, with or without upstream QHECs going through the constriction. Identifying the right scenario could be achieved with side gates. If upstream QHECs exist inside the constriction, increasing the charge density in its vicinity would push upstream QHECs out of the constriction and backscattering would be prevented, with signatures similar to Figs. 5.5 and 5.6.

Finally, we have seen in chapter 6 that antidots are FP nano-QHIs in the AB regime in graphene. The absence of Coulomb charging effect, despite their small size, is explained by an absence of localized state enclosed in the QHIs. A similar conclusion has been drawn from the observation of a dot-based QHI of similar size in a semiconductor-based 2DEG. New designs of QHIs can then be envisioned, with unprecedented small sizes offering no compromise between high visibility and the absence of Coulomb effect. In graphene, the

up- and downstream modes appearing spontaneously at graphene edges can be replaced by counterpropagating QHECs controlled by top gates [210] and the antidot-based nano-QHI can be easily introduced by injecting charges into the hBN layer, as recently demonstrated in refs. [97, 98] (Fig. 2.10). With this technique, the size of the QHI can be tuned in-situ and localized states can even be added progressively inside the QHECs paths by changing the tip polarity, to better assess the role played by Coulomb interactions.

Towards exploratory directions

As a more ambitious project involving QHIs, one can imagine taking the advantage of the Corbino geometry shown in Fig. 2.5, free from the detrimental effects of graphene edges, to create an interferometer in the clean device bulk (Fig. 1.8). In this framework, the well defined fractional states observed in this geometry could be exploited to perform the non-Abelian braiding experiment proposed in section 2.4.4 with a double FP QHI [140]. This would require the use of nanosize patterned top gates. Furthermore, this kind of architecture opens the way to the realization of complex systems where several QHIs can interact with each other, while conserving the phase coherence thanks to the small size of such architecture.

Otherwise, the SGM experiments performed in this thesis could be transposed to other systems than graphene. In 2013, Geim and Grigorieva predicted that stacking different 2D materials on top of each other would lead to the emergence of new properties, shared by none of the layers taken alone [211]. This has been confirmed in 2018 with the discovery of superconductivity in magic angle ($\sim 1.1^\circ$) twisted bilayer graphene (MATBG) [212, 213], certainly the most important breakthrough in the field since the discovery of graphene itself. From then on, a zoo of exotic properties have been discovered in twisted materials. Among others, quantum anomalous Hall effect (QAHE) originating from a purely orbital ferromagnetism has been reported in MATBG aligned with hBN (section 2.1.4.3) [79, 80]. With a SQUID on tip (section 1.2.4.2), different magnetic domains have been evidenced in this material, each of them encircled by clock- or counterclockwise topologically protected edge channels [214].

In this framework, SGM could be used as in chapter 5 to pinpoint precisely where the topological breakdown of such exotic phases occurs and to precise what are its underlying mechanisms, as it would help the development of more robust topological states. Furthermore, Ribeiro-Palau *et al.* have shown that the top hBN layer can be twisted compared to graphene (or MATBG) with an AFM tip [215]. One can then imagine to tune the alignment angle between hBN and MATBG in-situ, and characterize its effect on the topological state robustness with SGM characterization.

Beside QAHE, quantum spin Hall effect has been revealed in WTe_2 , another 2D material (section 2.1.4.2) [75, 76]. The topological breakdown of the spin-

polarized counterpropagating edge channels has already been imaged with SGM in HgTe and has been attributed to preexisting localized states [216]. The same kind of experiment could therefore be conducted in WTe₂ to properly characterize the nature of these localized states, that can be magnetic or not [217].

APPENDICES

appendix

List of papers

Consequent contributions

- B. Brun, **N. Moreau**, S. Somanchi, V.-H. Nguyen, K. Watanabe, T. Taniguchi, J.-C. Charlier, C. Stampfer and B. Hackens, *Imaging Dirac fermions flow through a circular Veselago lens*, Phys. Rev. B **100**, 041401 (2019) [185]
- **N. Moreau**, B. Brun, S. Somanchi, K. Watanabe, T. Taniguchi, C. Stampfer and B. Hackens, *Upstream modes and antidots poison graphene quantum Hall effect*, Nature Communications **12**, 4265 (2021) [190]
- **N. Moreau**, B. Brun, S. Somanchi, K. Watanabe, T. Taniguchi, C. Stampfer and B. Hackens, *Contacts and upstream modes explain the electron-hole asymmetry in the graphene quantum Hall regime*, Phys. Rev. B **104**, L201406 (2021) [197]
- **N. Moreau**, B. Brun, S. Somanchi, K. Watanabe, T. Taniguchi, C. Stampfer and B. Hackens, *Upstream modes quantum Hall nano-interferometer in graphene*, in submission process [191]
- **N. Moreau**, S. Faniel 2, F. Martins, L. Desplanque, X. Wallart, S. Melinte, V. Bayot, and B. Hackens, *Revisiting Coulomb diamond signatures in quantum Hall interferometers*, in submission process [218]
- **N. Moreau**, T. Pirson, G. Le Brun, T. Delhaye, G. Sandu, A. Paris, D. Bol and J.-P. Raskin, *Could unsustainable electronics support sustainability?*, Sustainability **13**, 6541 (2021) [219]

Minor contributions

- B. Brun, **N. Moreau**, S. Somanchi, V.-H. Nguyen, K. Watanabe, T. Taniguchi, J.-C. Charlier, C. Stampfer and B. Hackens, *Optimizing Dirac fermions quasi-confinement by potential smoothness engineering*, 2D Materials **7**, 025037 (2020 [186])
- B. Brun, V.-H. Nguyen, **N. Moreau**, S. Somanchi, K. Watanabe, T. Taniguchi, J.-C. Charlier, C. Stampfer and B. Hackens, *Graphene whisperitronics: transducing whispering gallery modes into electronic transport*, in submission process

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