Is it fair to 'make work pay'?*

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Abstract

The design of the income transfer program for the lower incomes is a hot issue in current public policy debate. Should we stick to a generous welfare state with a sizeable basic income, but high marginal tax rates for the lower incomes and thus little incentives to work? Or should we 'make work pay' by subsidizing the work of low earners, but possibly at the cost of a smaller safety net? We think it is difficult to answer this question without making clear what individuals are (held) responsible for and what not. First, we present a new fair allocation, coined a Pareto Efficient and Shared resources Equivalent (PESE) allocation, which compensates for different productive skills, but not for different tastes for working. We also characterize a fair social ordering, which rationalizes the PESE allocation. Second, we illustrate the optimal second-best allocation in a discrete Stiglitz (1982, 1987) economy. The question whether we should have subsidies for the low earners or not crucially depends on whether the low-skilled have a strictly positive or zero skill. Third, we simulate fair taxes for a sample of Belgian singles. Our simulation results suggest that 'making work pay' policies can be optimal, according to our fairness criterion, but only in the unreasonable case in which none of the unemployed are ever willing to work.

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1 Motivation

Focussing on the tax-benefit system as a whole, many European countries combine a sizeable basic income with high (empirical) marginal taxes for the low income earners. These programs are praised for their redistributional appeal, directing large transfers towards the low incomes in society. But, at the same time, critics have held these schemes responsible for large unemployment traps, because they do not provide incentives to (start) work(ing). Therefore, some continental European countries —such as Belgium, Finland, France, Germany, Italy and the Netherlands— have proposed and/or introduced tax credit schemes recently; see Bernardi and Profeta (2004) for an overview. At the same time, the US and the UK, with a much longer tradition in tax credit schemes, have reinforced the role of their tax credits. The increased policy interest for such 'making work pay' schemes, i.e., policies aiming at subsidizing the low income earners, is its ability to tackle two problems at the same time. It has a positive effect on employment (the number of people working and, to a lesser extent, the aggregate labour hours), while it increases the income of poor households; see Pearson and Scarpetta (2000) for an overview.

While 'making work pay' schemes may attain desirable objectives, it is not clear whether it is also optimal to make work pay given the government's budget constraint. The 'welfarist' optimal income tax literature consists of three canonical models, depending on whether labour supply responses are modelled intensively and/or extensively (Heckman, 1993). *First*, in a Mirrlees (1971) economy, individuals respond via the intensive margin, i.e., by varying their labour hours or effort. Marginal taxes should be non-negative everywhere (Mirrlees, 1971), which excludes the possibility of subsidizing work. At the bottom, the marginal tax has to be zero, but only in case everybody works (Seade, 1977). Once there exists an atom of non-workers, the marginal tax rate has to be positive (Ebert, 1992) and, according to some numerical simulations (Tuomala, 1990), rather high. Using the empirical earnings distribution, (i) a U-shaped pattern of positive marginal tax rates and (ii) high marginal tax rates at the bottom turn out to be optimal in many cases (for (i) see Diamond, 1998, Saez, 2001 and Salanié, 1998 and for (ii) see Piketty, 1997, Bourguignon and Spadaro, 2000 and Choné and Laroque, 2005). *Second*, in Diamond's (1980) approach, individuals respond via the extensive margin, i.e., they choose to work or not. Marginal tax rates can be negative, suggesting at least the possibility of subsidizing the work of low earners. *Third*, Saez (2002) presents a unifying framework where individuals can respond via both margins. Support for one of the two income transfer schemes depends on the relative importance of both response margins and on the redistributive tastes of government. He proposes a sizeable basic income (around \$7300/year), but combined with a tax exemption at the bottom (for incomes up to \$5000/year).

In the same year of Mirrlees' (1971) seminal contribution, Rawls (1971) criticizes the welfarist approach, in which only utilities are allowed to judge the desirability of different social states; see also Kolm (1966) for an early statement of this critique. Rawls (1982) presents the following example of expensive preferences. Otherwise identical individuals differ in their preferences towards food: some are happy with a simple meal, some only with an expensive dinner. Rawls asks the question whether we really want to distribute resources in such a way that those with expensive tastes get more resources than others. If we do not like such a distribution, we have to use an argument to decide which tastes are expensive. Such an argument has to be based on non-utility factors. Hence, it implies a rejection of the welfarist approach. In the aftermath of Rawls' influential work, many alternative theories of distributive justice were proposed. Although very diverse in equalisandum, they almost all have Dworkin's (1981) cut in common. Dworkin claims that not all individual characteristics can (should) be considered as morally arbitrary. Therefore one has to make a clear cut between endowments (e.g., skills, talents, handicaps, etc.) and ambitions (e.g., preferences, effort, tastes, etc.). He introduces personal responsibility: individuals are responsible for their ambitions —as long as they identify with them— but not for their endowments. As a consequence, a fair distribution scheme should be ambition-sensitive, but endowment-insensitive.

In an optimal income tax setting, fairness could require to compensate for differences in productive skill (endowment), but not for differences in taste for working (ambition), as measured by the consumption-labour trade-off.[1] Schokkaert *et al.* (2004) introduce such fairness considerations in different ways and calculate the corresponding optimal linear income tax, which turns out to be positive. Allowing for non-linear tax schemes, results change drastically. Boadway *et al.* (2002) analyze the optimal non-linear income tax according to a weighted utilitarian or maximin social planner where different weights are chosen for different tastes. Negative marginal taxes (for the low income earners) are optimal in case sufficient weight is given to the hard-working individuals. Fleurbaey and Maniquet (2005a,b) characterize fair social orderings to analyze non-linear income taxes. In both theoretical studies, it is optimal to direct the largest subsidies to the hard-working poor (the agents having the lowest skill and choosing the largest labour time), as long as the lowest skill is strictly positive.

In the next section we present a new fair allocation, coined a Pareto Efficient and Shared resources Equivalent (PESE) allocation. As the name suggests, the optimal allocation is Pareto efficient and all individuals are indifferent between their bundle and what they would get if it were physically possible to divide or share all resources, including the productive skills. Section 3 characterizes a fair social ordering, which rationalizes the PESE allocation. In section 4, we introduce a 'discrete' Stiglitz (1982, 1987) economy with (i) four types of individuals (defined by a low or high productive skill and a low or high taste for working) and (ii) a government who wants to install fair taxes, but cannot observe individuals' type. We show that fairness recommends to subsidize the low earners, as long as the low-skilled individuals have a strictly positive skill. In case the low-skilled have a zero skill, subsidies

[1] Roemer *et al.* (2001) consider the education level of the parents as the compensating variable.

can never be optimal. These theoretical results are in line with those found in Fleurbaey and Maniquet (2005a,b). In section 5, we enhance realism by simulating fair taxes for Belgian singles, while carefully paying attention to the calibration of the compensation (hourly wages) and responsibility (taste for working) variable. Similar to the welfarist simulation results often found in Mirrlees economies, our non-welfarist fairness criterion also suggests a U-shaped pattern of positive marginal tax rates in almost all cases. Negative marginal tax rates, and thus 'making work pay' policies, can only be optimal in the unreasonable case in which all the unemployed are never willing to work.

2 Equality of resources revisited

When all resources in society are alienable and divisible, Dworkin (1981) proposes to divide resources equally (endowment insensitivity), followed by an auction to reallocate resources according to taste (ambition sensitivity). This leads to a Pareto efficient and envy-free allocation. To study fair income taxation, however, we have to introduce productive resources (skills), which are not alienable, and therefore a problem arises. In labour economies, Pareto efficient and envy-free allocations do not exist, in general.

A first class of solutions tries to extend the above Dworkinian auction by assigning property rights over leisure. Varian (1974) analyzes two, rather extreme, solutions. One may divide consumption goods equally and either (i) assign each individual his own leisure, or (ii) give each individual an equal share in each of the agents' (including his own) leisure time. After trade, the resulting competitive (and hence Pareto efficient) equilibria are called respectively (i) wealth-fair and (ii) income-fair. In the wealth-fair allocation, productive talents are a private good and the resulting allocation does not compensate at all for inabilities. In the income-fair allocation, productive talents are a collective good. The high-skilled has to buy back his expensive leisure and is therefore punished for being a high skill type, resulting in a slavery of the talented. Intermediate solutions exist where skills are neither purely private, nor purely collective (Fleurbaey and Maniquet, 1996, Kolm, 1996, Maniquet, 1998).

A second class of solutions starts from the concept of fair-equivalence; see, e.g., Kolm (2004) for an overview and a critique. Pazner and Schmeidler (1978) define an allocation to be fair-equivalent if everyone is indifferent between his bundle in this allocation and the bundle he would receive in a 'hypothetical' fair, i.e., envy-free, allocation. It then suffices to define an interesting 'hypothetical' fair allocation and to look whether there exist Pareto efficient ones, among all fair-equivalent allocations. The resulting allocation is called a Pareto efficient and fair-equivalent (PEFE) allocation. Pazner and Schmeidler (1978) propose an egalitarian allocation —an allocation where everybody consumes the same consumption-leisure bundle— as the fair one. We propose a different fair allocation, which we coin a 'shared resources' allocation. This is the allocation which would result, if it were (physically) possible to divide or share all resources, including the productive ones. To make this idea more precise, we introduce some notation.

A fixed number of individuals, denoted $i \in N = \{1, \ldots, n\}$, differ in skills and preferences. Skill $s \in \mathbb{R}_+$ defines production (called gross income in the sequel) in a linear way, or $y = s\ell$, with $\ell \in [0, 1]$ the amount of labour. We denote a skill profile by $\mathbf{s} = (s_1, \ldots, s_n) \in \mathbb{R}^n_+$. Preferences —containing the trade-off between consumption and labour— are represented by continuously differentiable utility functions

$$U : \mathbb{R} \times [0, 1] \to \mathbb{R} : (c, \ell) \mapsto U(c, \ell),$$

which are strictly increasing (resp. strictly decreasing) in consumption c (resp. labour ℓ) and strictly quasi-concave. We call $\mathcal{U}$ the corresponding set of utility functions and, normalizing the consumption price to one, we refer to c as the net income in the sequel. A utility profile is denoted by $\mathbf{U} = (U_1, \ldots, U_n) \in \mathcal{U}^n$. An economy $e = (\mathbf{s}, \mathbf{U})$ is completely defined by a skill and a utility profile; all economies are gathered in a set $\mathcal{E} = \mathbb{R}^n_+ \times \mathcal{U}^n$.

Individuals are (held) responsible for their tastes, but not for their skills. Therefore, we want to compensate individuals for different outcomes which are only due to different skills, but not for different outcomes which are only caused by different tastes for working. In case skills are alienable —think, e.g., of individuals as farmers who receive, as a matter of brute bad luck, either a blunt or a whetted scythe (the skill s) to harvest crops (the consumption c)— there is a particularly simple and attractive way to obtain a fair allocation:

(a). each individual pays (or receives) the same lump-sum amount of money $a \in \mathbb{R}$,

(b). each individual can use each skill (including his own) for a time equal to $\frac{1}{n}$ at most.

Individuals are assumed to be rational: given (b), the budget set which maximizes net incomes (for all possible labour choices) starts using the highest skill for the first $\frac{1}{n}$ time units, followed by the second highest skill for an additional $\frac{1}{n}$ time units, and so on. Given a, we call such a budget set the 'shared resources' budget set and the resulting allocation (which ultimately depends on the tastes in society) is called the 'shared resources' allocation in the sequel.

To illustrate these concepts, suppose (i) there are only two skill types possible in society, say low (L) and high (H), which are equally represented in the skill pool $\mathbf{s}$, and (ii) there are only two tastes for working possible, also called low (L) and high (H). An allocation $\mathbf{z} = (z_{LL}, z_{LH}, z_{HL}, z_{HH})$ contains one bundle $z_{st} = (c_{st}, \ell_{st})$ for each of the four types st, with s referring to the skill (low or high) and t referring to the taste (low or high). Figure 1 illustrates the budget sets and (resulting) allocations in case (a) each individual receives the same lump-sum amount of money a, but productive resources are not shared, and (a)+(b) each individual receives the same lump-sum amount a and also the productive resources are shared (each individual can work with each of the skills half-time at most).

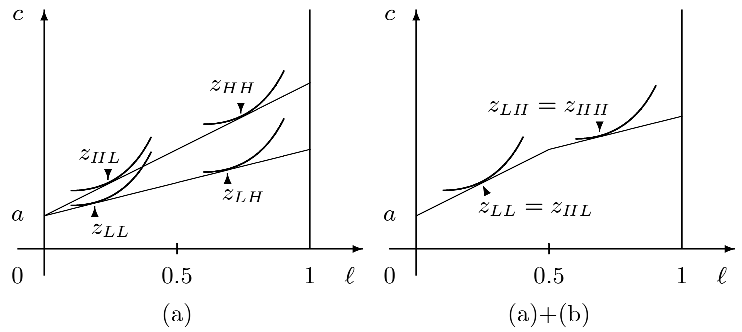

Figure 1: Allocation change when sharing productive resources.

Sharing productive resources is not technically feasible in many cases. Labour market productivities, due to inborn characteristics such as intelligence, talents, handicaps and so on, are typically always inalienable. Still, we could consider the allocation, which would arise if it were possible to divide and share all resources equally, as an interesting 'hypothetical' case. However, the resulting hypothetical 'shared resources' allocation is not necessarily Pareto efficient in the *actual* economy.[2] Therefore, we propose to focus on the Pareto Efficient and Shared resources Equivalent (PESE) allocation. More precisely, given the skill and preference technology, it is possible to construct a unique allocation $\mathbf{z}$ such that for each individual the marginal rate of substitution equals his skill level (Pareto efficient) and each individual is indifferent between his actual bundle and his bundle in a 'shared resources' allocation (shared resources equivalent, for a given lump-sum amount of money a in the *hypothetical* economy).

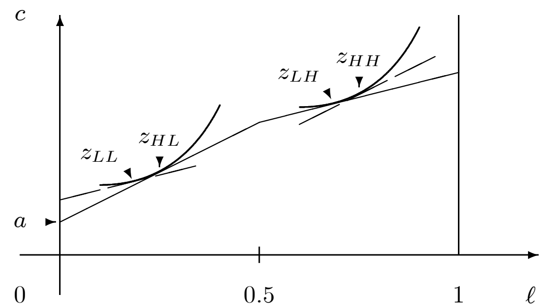

Figure 2: A Pareto efficient and shared resources equivalent allocation.

[2] Although fairness is the only thing that matters for a hypothetical allocation, notice that our 'shared resources' allocation is neither efficient in the *hypothetical* economy. In figure 1, case (a)+(b), individuals with types LL and HL have some time left to use the highest skill, while type LH and HH individuals would love to use it, and so there is room for Pareto improvements in the hypothetical economy. Therefore another plausible fair allocation —but more difficult to compute— would be obtained if we allow for trading time slots in the hypothetical economy.

Figure 2 illustrates this PESE allocation for the same economy as in figure 1.[3]

We are now ready to define a PESE allocation in general. Let $\mathcal{Z} = (\mathbb{R} \times [0,1])^n$ be the set of allocations $\mathbf{z} = (z_i)_{i \in N}$, containing one bundle $z_i = (c_i, \ell_i)$ for each individual i in N. We define a well-being concept closely linked to the PESE allocation.

WELL-BEING: For each allocation $\mathbf{z} \in \mathcal{Z}$, the vector of well-being levels $\mathbf{w} = (w_i)_{i \in N} \in \mathbb{R}^n$ is defined by the amounts of money w_i which would make individual i indifferent between (i) receiving (or paying) this amount of money w_i and sharing all productive resources equally, and (ii) his actual bundle z_i. Because the well-being vector $\mathbf{w}$ depends on the allocation $\mathbf{z}$ and the economy $e = (\mathbf{s}, \mathbf{U})$, we write $\mathbf{w} = \mathbf{W}(\mathbf{z}, e)$, with $w_i = \mathbf{W}_i(\mathbf{z}, e)$.

Notice that in the PESE allocation presented in figure 2, the well-being levels are the same for all individuals and equal to a. This leads to the following general definition of PESE allocations, which belong to Pazner and Schmeidler's class of PEFE allocations (cfr. infra).

PESE ALLOCATION: An allocation $\mathbf{z} \in \mathcal{Z}$ is a PESE allocation if and only if $\mathbf{z}$ is Pareto efficient and all individuals have the same well-being.

3 A 'shared resources' social ordering

In case it is not possible to recognize the less from the more productive and the lazy from the hard-working individuals, it is more convenient to characterize a 'shared resources' social ordering for such a second-best setting.

We start with two observations. *First*, a bundle that puts an individual on a higher indifference curve leads to a higher well-being level (and vice-versa). As such, our definition of well-being corresponds with one specific, but, according to us, interesting cardinalization of preferences. *Second*, well-being has to be interpreted as a 'relative' measure of fair treatment, in the spirit of the PESE allocation. If two individuals had the same well-being, they would have been treated equally fairly, because both individuals would have been indifferent between their actual bundle and the bundle they would choose if (a) they received the same lump-sum amount of money and (b) all productive resources were shared equally. If one individual had a strictly lower well-being compared to another, he would have been treated unfairly with respect to the other, because both individuals would have been indifferent between their actual bundle and the bundle they would choose if (b) all productive resources were shared equally, but (a) the former individual received a strictly lower lump-sum amount of money.

[3] Notice that, in the presence of a budget constraint of the government, the PESE allocation in figure 2 is not necessarily feasible in the *actual* economy. It should be clear, however, that given the skill and preference technology, all bundles and indifference curves can be translated downwards (by reducing a) leading to a unique feasible PESE allocation; see also Fleurbaey and Maniquet (2005c).

Because for each individual, well-being is ordinally equivalent to utility (point one in the previous discussion), we want social welfare to increase when all well-beings increase: this guarantees that the weak Pareto principle —strictly higher utilities should lead to strictly higher social welfare— is satisfied. At the same time, well-being differences are due to unfair treatment (point two in the previous discussion), which we want to eliminate as much as possible. Therefore, a fair social ordering should combine Pareto efficiency with a maximal priority for the worst-off in terms of well-being. More precisely the ordering should satisfy:

> FAIRNESS: For each economy $e \in \mathcal{E}$ and for all allocations $\mathbf{z}, \mathbf{z}' \in \mathcal{Z}$: $\min \mathbf{W}(\mathbf{z}, e) > \min \mathbf{W}(\mathbf{z}', e)$ implies that $\mathbf{z}$ should be strictly socially preferred to $\mathbf{z}'$.

First, possible candidates are the maximin and the leximin rule applied to well-beings, but in the sequel we do not bother about ranking allocations with equal minimal well-being levels, basically because the above definition is sufficient for optimization purposes. Second, the 'shared resources' social ordering has some formal similarity with Fleurbaey and Maniquet's (2005c) $\tilde{s}$-implicit budget leximin function, where $\tilde{s}$ is a reference skill level. In our case, the reference skill $\tilde{s}$ is piece-wise linear and endogenously defined by the skill pool in society. As such, laissez-faire allocations are selected in case all individuals have the same skill. Third, some readers might find the chosen priority too extreme. However, this extreme viewpoint can be easily derived from weaker assumptions, our next topic.

We discuss the issue here in an informal way, while we provide a characterization result in appendix A. First, we restrict ourself to a 'consequentialist' social ordering, which here means that only individual well-being levels matter for social evaluation. We can interpret consequentialism here as a responsibility axiom: individual utility levels are irrelevant to rank allocations. Furthermore, we add Fleurbaey and Maniquet's (2005a) principle to compensate for differences in outcomes which are only due to differences in skills. More precisely, their compensation principle is a weak one: it only requires that a Pigou-Dalton transfer (in terms of net income) from a rich to a poor individual —with both the same preferences and the same labour— should be welfare improving. Combining consequentialism with this weak compensation principle leads to a much stronger principle, called Hammond's (1976) equity: if one individual is worse-off compared to another (in terms of well-being), then any increase in the former's well-being at the cost of any decrease in the latter's well-being should be approved of. A similar result occurs in Fleurbaey and Maniquet (2005a). Once Hammond equity is obtained, it is only a small step towards the above fairness definition. More precisely, it suffices to add the Pareto principle —higher utilities and thus higher well-beings improve social welfare— and anonymity —the names of the individuals do not matter.

4 Fair taxes: theory

In the previous section, we characterize a fair social ordering, inspired by the PESE allocation. In this section, we analyze what happens when the government uses this fair social ordering to calculate optimal taxes in a discrete Stiglitz economy (1982, 1987) with four types, which are not observable to the government.

All individuals in $N = \{1, \ldots, n\}$ can have four types, denoted $(s,t) \in S \times T$, where s is the skill level and t the taste for working; we abbreviate types as $st \in ST$. Each type st is represented by $n_{st} > 0$ individuals, with $n = \sum_{st \in ST} n_{st}$. Skills can be low or high, or $s \in S = \{L, H\}$, with $0 < L < H$; later on, we come back to the issue of zero skills. Tastes for working can also be low or high, or $t \in \{L, H\}$, which correspond with a utility function U_t.[4]

As before, utility functions belong to $\mathcal{U}$, but we impose some additional properties. Let V_{st} represent the preferences in the consumption-income space for type st, more precisely $V_{st} : \mathbb{R} \times [0, s] \rightarrow \mathbb{R} : (c, y) \mapsto V_{st}(c, y) \equiv U_t\left(c, \frac{y}{s}\right)$.[5] We impose two additional properties on the utility functions U_t; see Stiglitz (1982, 1987) for the first and Boadway *et al.* (2002) for the second property:

Single-crossingness: A higher taste for working t corresponds with a lower marginal rate of substitution (denoted $MRS_t = -\frac{\partial U_t / \partial \ell}{\partial U_t / \partial c}$), expressing the view that individuals with a higher taste for working require less compensation (in terms of net income c) to work a little bit longer. Formally: $MRS_L > MRS_H$ in $\mathbb{R} \times [0, 1]$.

Indistinguishable middle type: The types LH and HL have the same preferences in the consumption-income space. Formally, there exists a continuous and strictly increasing function $\phi : \mathbb{R} \rightarrow \mathbb{R}$, such that $V_{LH} = \phi \circ V_{HL}$ in $\mathbb{R} \times [0, L]$.

Both assumptions together, the marginal rates of substitution in consumption-income space (denoted $MRSY_{st} = -\frac{\partial V_{st} / \partial y}{\partial V_{st} / \partial c}$) are also single-crossing, more precisely:

$$MRSY_{LL} > MRSY_{LH} = MRSY_{HL} > MRSY_{HH}, \text{ in } \mathbb{R} \times [0, L] \text{ and}$$

$$MRSY_{HL} > MRSY_{HH} \text{ in } \mathbb{R} \times [L, H].$$

We focus in the sequel on allocations $\mathbf{x} = (x_{LL}, x_{LH}, x_{HL}, x_{HH})$ in consumption-income space, thus $\mathbf{x} \in \mathcal{X} = (\mathbb{R} \times [0, L])^2 \times (\mathbb{R} \times [0, H])^2$, containing one bundle $x_{st} = (c_{st}, y_{st})$ for each type $st \in ST$. The program of the government is to find the best allocation(s) $\mathbf{x}$ —'best' according to the fair social ordering defined in section 3— subject to (i) incentive compatibility constraints (no type envies another type's bundle) and (ii) a feasibility constraint (the

[4] The exact scalars do not matter here, so we stick to the notation of L to denote low taste for working and/or low-skilled and $H > L$ to denote high taste for working and/or high-skilled.

[5] Whenever $s = 0$, we define $V_{st}(c, y) = c$ for all $(c, y) \in \mathbb{R} \times \{0\}$.

sum of all taxes is larger than the government requirement $g \in \mathbb{R}$). Recall our definition of well-being in section 2; with a slight abuse of notation, we write the well-being of type st in allocation $\mathbf{x}$ as $w_{st} = \mathbf{W}_{st}(\mathbf{x}, e)$. We get:

$$\underset{\mathbf{x}\in\mathcal{X}}{\text{MAX}} \ \underset{st\in ST}{\text{MIN}} \ \left(\mathbf{W}_{st}(\mathbf{x}, e)\right)_{st\in ST} \text{ subject to} \tag{$*$}$$

INCENTIVE COMPATIBILITY CONSTRAINTS $IC_{st,(st)'}$:

$$\begin{aligned} V_{st}(x_{st}) &\geq V_{st}\left(x_{(st)'}\right), \forall st \in \{H\} \times T, \forall (st)' \in ST, \\ V_{st}(x_{st}) &\geq V_{st}\left(x_{(st)'}\right), \forall st \in \{L\} \times T, \forall (st)' \in ST \text{ with } y_{(st)'} \leq L. \end{aligned}$$

FEASIBILITY CONSTRAINT:

$$\sum_{st\in ST} n_{st}(y_{st} - c_{st}) \geq g.$$

Since the program defined by ($*$) focuses on the minimal well-being, the equity-efficiency trade-off is extreme: priority is given to the worst-off (in terms of well-being), irrespective of the change in the average well-being level. The traditional welfarist optimal tax literature captures the equity-efficiency trade-off by the social marginal value of consumption as a function of income, expressed in terms of the value of public funds; see, e.g., Saez (2001). In the present non-welfarist approach this trade-off is more complex: it is captured by a function of all well-beings —putting all weight on the worst-off— where each well-being is in turn a function of the received consumption-income bundle as well as the individual's taste for working and the skill profile in society. As a result, the social marginal value of consumption can differ for taxpayers with the same income. For example, when the LH-type has the lowest well-being, the LH- and HL-types might have the same income (due to the 'indistinguishable middle type'-property), but different social marginal values of consumption.

Consider now an economy with four types $\{LL, LH, HL, HH\}$, with strictly positive skills and tastes represented by utility functions satisfying single-crossingness and indistinguishable middle type. Figure 3 presents the laisser-faire outcome for such a particular economy (with $g = 0$). Notice that the dashed lines —with the kink at $\ell = \frac{n_{HL}+n_{HH}}{n}$, the proportion of high skilled— in the right hand panel allow us to measure individual well-being.

In lemma 1 of appendix B we show that in any implementable allocation, type LL cannot have a higher well-being than type HL and type LH cannot have a higher well-being than type HH. Hence, program ($*$) can focus on the low skilled types LL and LH. In the laisser-faire equilibrium of figure 3, type LH has a lower well-being than type LL.

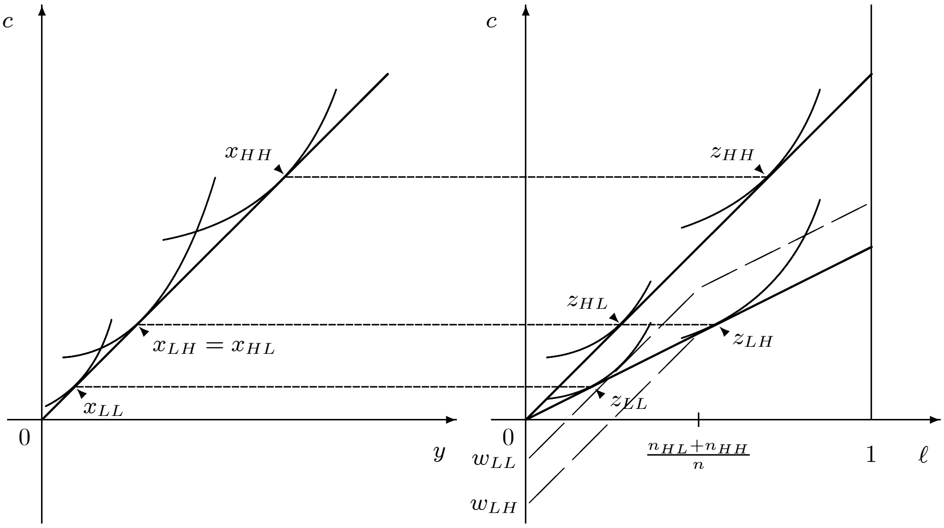

Figure 3: Laisser-faire equilibrium for a particular economy.

Therefore program $(*)$ departs from this laisser-faire situation by increasing type LH's (and type HL's) well-being at the cost of decreasing some other types' well-being. The resulting allocation 'makes work pay': contrary to the low income type LL, the middle income types LH and HL receive a subsidy. Such a 'making work pay'-feature still holds at the optimum. Proposition 1 tells us that the lowest income type, the 'undeserving poor' with type LL, must always receive lower subsidies (or pay higher taxes) than the second lowest income type, the 'hard-working poor' with type LH; this result is in line with Fleurbaey and Maniquet (2005a,b).[6]

Proposition 1. *Consider a four type economy with skills $0 < L < H$ and tastes represented by utility functions $U_L, U_H \in \mathcal{U}$ which satisfy single-crossingness and indistinguishable middle type. Consider a government who optimizes the program defined by $(*)$. In an optimal allocation $\mathbf{x}^* \in \mathcal{X}$ we must have $y^*_{LL} - c^*_{LL} \geq y^*_{LH} - c^*_{LH}$.*

We have to put this result in perspective, however. Although it is reasonable to assume that all individuals (with a capacity for work) have strictly positive productive skills, individuals might be constrained in their choice due to labour market frictions. Minimum wage laws, rationing and so on, may prevent individuals, in particular the low-skilled, from working. Suppose, in our four type economy, that the low-skilled individuals are willing, but cannot work due to such constraints, which are beyond their responsibility. In such a case, their skills are nullified and, as shown in appendix B, the LH-type individuals always have the

[6] All proofs can be found in appendix B.

lowest level of well-being. Because these individuals will never work, maximizing the minimal well-being boils down to maximizing the basic income in society; see Fleurbaey and Maniquet (2005a,b) for a similar result. This turns proposition 1 round, or, the LL- and LH-type individuals (with $y^*_{LL} = y^*_{LH} = 0$) must always receive higher subsidies (or pay lower taxes) than the second lowest income type (here the HL-type individuals). Notice that the 'indistinguishable middle type'-property is not needed for this result.

Proposition 2. *Consider a four type economy with skills $L = 0 < H$ and tastes represented by utility functions $U_L, U_H \in \mathcal{U}$ which satisfy single-crossingness. Consider a government who optimizes the program defined by $(*)$. In an optimal allocation $\mathbf{x}^* \in \mathcal{X}$ we must have $y^*_{LL} - c^*_{LL} = y^*_{LH} - c^*_{LH} \leq y^*_{HL} - c^*_{HL}$.*

Both proposition 1 and 2 are based on simple fictitious economies. In proposition 1 everyone is able to work whereas in proposition 2 the low skilled types LL and LH cannot work, even if they want to. In a more realistic setting, unemployment is the result of a low ability and/or a low willingness to work. Consider now the following economy $\{0K, L0, LK', HK'\}$, where the first two types never work but for different reasons: type $0K$ is never able to work as his skill equals zero and type $L0$ is not willing to work, where $t = 0$ denotes an infinite work aversion. Figure 4 presents the laisser-faire allocation for such an economy.

As before, type LK' can never have a larger well-being than type HK'. Also type $0K$ can never have a larger well-being than type $L0$. So program $(*)$ can focus on types $0K$ and LK'.

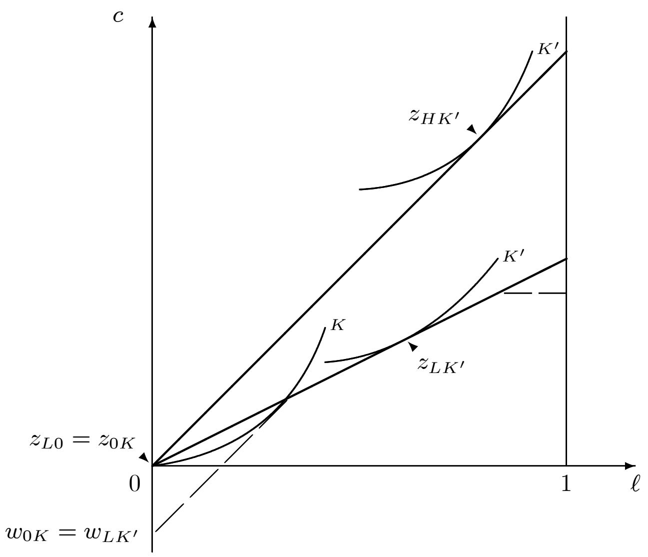

Figure 4: A more realistic scenario.

In this economy, it is not clear whether $0K$ or LK' has the lowest well-being. We have constructed figure 4 in such a way that both types have the same well-being. However, choosing a lower (resp. higher) taste for working for type $0K$ would lead to a higher (resp. lower) well-being for type $0K$. This indeterminacy leaves open whether it is optimal to 'make work pay' in a more realistic setting. At least one could expect that the higher the taste for working for type $0K$, the higher the optimal basic income will be. This intuition will be confirmed in the next section where we simulate fair taxes for a sample of Belgian singles. Simulation allows us to focus on (i) more realistic economies with many different types and, more importantly, on (ii) different and more realistic scenarios concerning the willingness of the unemployed to work. The different scenarios in (ii) have a crucial impact on the tax-benefit scheme for the low earners.

5 Fair taxes: simulation results

5.1 Calibration

We use a sample of singles from the 1997 wave of the Panel Study for Belgian Households.[7] We only include singles with a capacity for work (students, pensioners, sick, or handicapped singles are excluded). We observe (i) the pre-tax yearly labour income y, (ii) the amount of labour ℓ, normalized such that $0 \leq \ell \leq 1$, where $\ell = 1$ corresponds with 2925 hours, i.e., 45 weeks times 65 hours, (iii) the gross hourly wage rate σ (only observed for those who worked, i.e., both $y, \ell \neq 0$) which leads to a gross yearly wage rate $s = 2925\sigma$ and (iv) the total net unemployment benefit β (only observed for those who were partly or completely unemployed in 1997) from which we derive the net yearly unemployment benefit $b = \frac{\beta}{1-\ell}$, i.e., the net unemployment benefit one would obtain if full-time unemployed ($\ell = 0$).

First, we consider all individuals for which we possess all of the above information. We consider quasi-linear preferences (which excludes income effects) represented by utility functions:[8]

$$U_t : \mathbb{R} \times [0,1] \to \mathbb{R} : (c, \ell) \mapsto U_t(c, \ell) = c - \frac{1}{t}\frac{\varepsilon}{1+\varepsilon}\ell^{\frac{1+\varepsilon}{\varepsilon}},$$

with t the taste parameter (possibly different for different individuals) and ε the labour supply elasticity (the same for all individuals). Preferences in consumption-income space become:

$$V_{st} : \mathbb{R} \times [0,s] \to \mathbb{R} : (c, y) \mapsto V_{st}(c, y) = c - \frac{1}{s^{\frac{1+\varepsilon}{\varepsilon}}t}\frac{\varepsilon}{1+\varepsilon}y^{\frac{1+\varepsilon}{\varepsilon}}.$$

The net income of a Belgian single equals $y - \tau_{97}(y) + b(1 - \frac{y}{s})$, with $\tau_{97}(\cdot)$ the actual tax system for singles in Belgium in 1997 (reported in appendix C) and $b(1 - \frac{y}{s})$ the benefit when working $\ell = \frac{y}{s}$ units of time. Both tax and benefit parts separately, as well as the resulting budget set (the solid line), are illustrated in figure 5.

[7] Mortelmans, D., Casman, M.-T. (2002) PSBH. Panel Study on Belgian Households (1992-2002) Universiteit Antwerpen, Université de Liège.

[8] Due to quasi-linearity, other non-labour income (e.g., due to rents, gifts, alimony, child allowances) does not matter for the labour choice of an individual. Furthermore, we keep individuals responsible for the other non-labour income and it is therefore excluded from our analysis.

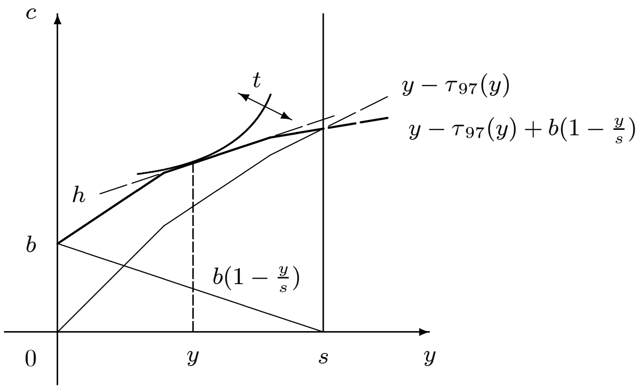

Figure 5: Calibration of the taste parameter t.

We calibrate t such that the choice of y is rationalized for each single. More precisely, the slope of the individual's budget set at y, denoted $h\,(y)$, should be equal to the marginal rate of substitution between consumption and gross income for the quasi-linear preferences V_{st}; we get[9]

$$t = \frac{1}{s} \frac{1}{h\,(y)} \left(\frac{y}{s}\right)^{\frac{1}{\varepsilon}}, \text{ with } h\,(y) = 1 - \tau_{97}'(y) - \frac{b}{s}.$$

Second, since we could only observe gross yearly wages s (resp. net yearly unemployment benefits b) for individuals who worked in 1997 (resp. individuals who received unemployment benefits in 1997), we complete our dataset by imputing values for s and b, whenever unobserved, via a Heckman selection model; see Greene (2003) for a definition. Thus, in estimating s and b, we correct for a possible sample selection bias, due to the fact that we only observe wages s for those who worked and benefits b for those who were (permanently or temporarily) unemployed. The variables used for the imputation as well as the estimation results are described in appendix D.[10]

We end up with a heterogeneous sample of 621 singles who differ in skills s and tastes t, which drive their labour market behaviour; appendix E contains some descriptive statistics for our dataset. Two points are worth mentioning here. The non-responsibility parameter s and the responsibility parameter t in our dataset are barely correlated: using a low labour supply elasticity $\varepsilon = 0.1$ for singles, the correlation between s and t equals -0.071, suggesting independently distributed skills and tastes. With a strong correlation, compensating for skills

[9] In order to have preferences that are strictly decreasing in labour, t must be strictly positive. Therefore, we drop all individuals with $h(y) \leq 0$ out of the sample (17 observations).

[10] Alternatively, one could choose not to impute s and b, e.g., by setting b equal to zero and drop all unemployed from our dataset. However, imputing s and b enhances the realism of our economy and hence also the validity of our simulation results. Moreover, we believe that the ability and/or willingness to work of the unemployed plays a central role in the policy debate concerning the design of the optimal income transfer program.

only (and not for tastes) would be a dubious exercise. In addition, given the nature of our quasi-linear preferences, all unemployed receive a taste for working $t = 0$. To put it differently, all unemployed are considered unwilling to work. We relax this crucial assumption later on.

5.2 Results

Rather than using allocations as in the government program $(*)$, we use a piece-wise linear tax-benefit scheme as our instrument to approximate a non-linear tax scheme. As we are mainly concerned with the bottom incomes, we consider a piecewise linear tax-benefit scheme up to yearly gross earnings of€ 20000 in steps of€ 500 and we use a constant marginal tax rate afterwards. Using either a wider range of piecewise linear taxes (up to€ 80000 in steps of€ 500) or a finer grid (up to€ 20000 in steps of€ 250) does not change our results for the bottom incomes drastically. Remarkably, using a wider range leads to approximately constant marginal tax rates for incomes above€ 20000 which, except for the very high incomes, approximate $\frac{1}{1+\varepsilon}$, the optimal linear tax rate when maximizing basic income (see Atkinson, 1995). Given such a tax-benefit scheme, individuals choose their best bundle (according to their tastes and skills) and, therefore, incentive constraints are automatically satisfied. For the feasibility constraint, we use the total government requirement (g) in the actual system, which is (in per capita terms) equal to€ 3851. Finally, to obtain realistic proposals, we add participation constraints to the government program $(*)$, such that no one prefers the bundle $(0, 0)$ to the allocated bundle in the optimum.

5.2.1 Three simulations

In this subsection we report and discuss simulation results for three cases; a sensitivity analysis with respect to the main parameters is postponed to the next subsection.

No income effects and a low labour supply elasticity do not seem unrealistic for singles; see, e.g., Blundell and MaCurdy (1999) for an empirical assessment. Using a labour supply elasticity $\varepsilon = 0.1$, figure 6 depicts the chosen bundles in the consumption-gross income space for the following cases.[11]

First, we present the Rawlsian optimal allocation (denoted RAWLS in figure 6), i.e., the one which maximizes the basic income, as a benchmark case. It installs a high basic income equal to€ 9363 and high (and almost constant) positive marginal tax rates for the bottom incomes.

Second, we show the optimal allocation according to our fair social ordering in the extreme case that all unemployed are never willing to work (denoted 100% in figure 6). This gives a low (yearly) basic income equal to€ 518, moderate subsidies fading in around € 3000 and fading out around€ 10500, followed by progressive taxes. Thus, it seems fair to 'make work pay', even if it brings about a very low basic income.

[11] Notice that dotted lines do not represent an optimal tax-benefit schedule, but are only connecting the chosen consumption-gross income bundles.

Third, the 100%-case is clearly an extreme viewpoint. For example, minimum wage laws in Belgium could keep some individuals (especially those with low skills s) from working and, therefore, our calibration might underestimate their true taste parameter t, thus overestimating their well-being level w. More reasonably, at least some of the observed unemployment must be involuntary, especially in the case of singles. Suppose unemployment is voluntary for $p\%$ of the unemployed: their taste levels remain equal to zero. The other $(1-p)\,\%$ are constrained, i.e., although they would like to work, they are and will always remain constrained at $y = 0$, but we want to use their 'true' taste parameter to calculate well-being levels. Taste levels are unfortunately unknown (and difficult to infer from our data), therefore we make the following assumption: we assign all the constrained unemployed the same taste level in a way to be explained later on. Although simplifying, giving them the same taste level allows us to disregard the exact value of p, as long as it differs from 100%. The reason is that all the constrained unemployed end up with the same well-being and our maximin-type criterion is not sensitive to population size. Thus, for example, only one (or all but one) constrained unemployed would lead to the same result. For the moment, we suppose that their tastes equal the average taste of those currently working. Although one might be inclined to choose a lower taste level for the constrained unemployed, we refer to the fact that skills and tastes in our dataset are approximately uncorrelated. The optimal allocation for this case is denoted <100% in figure 6.[12]

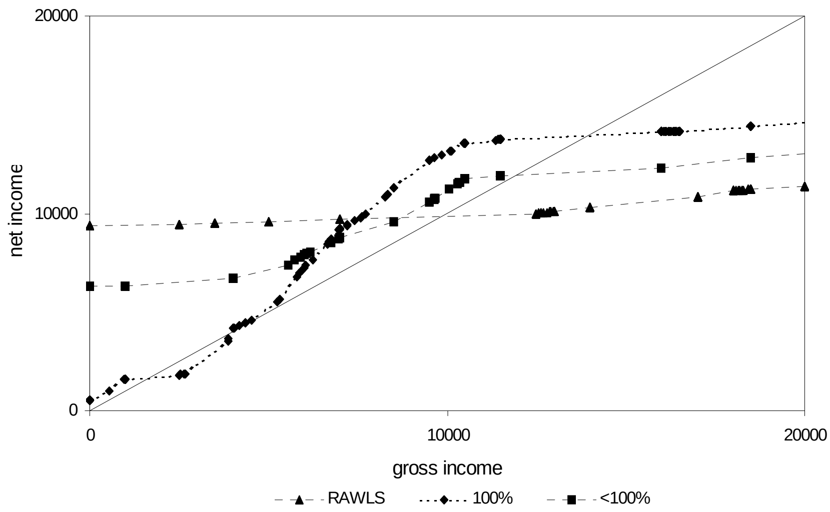

Figure 6: Optimal allocation for the Rawlsian case, the 100%- and <100%-case.

We get a sizeable basic income of€ 6318. In addition, it displays a U-shaped pattern of positive marginal tax rates: high marginal taxes for the bottom incomes, followed by an

[12]For reasons of calculation, we choose p equal to 90% and assign the average taste level to the unemployed with the lowest productivities.

almost neutral region (for incomes between € 4000-€ 10500) and again high marginal taxes afterwards. Although all individuals with zero skill are not able to work (as in proposition 2 of section 4), the <100%-case does not maximize the basic income.

Recall from the discussion at the end of section 4 that individuals with zero skill do not necessarily have the lowest well-being in more realistic economies. Table 1 summarizes for all three cases (in columns) and for different gross income groups G (in rows) (i) the proportion of individuals $\frac{|G|}{n}$ and (ii) the average tax rate $(\frac{1}{|G|}\sum_{i\in G}(y_i - c_i))$. Average tax rates are monotonically increasing in gross income, except in the 100%-case, which is in line with our previous observation that 'making work pay' can only be optimal in the unreasonable case where none of the unemployed is ever willing to work.

	RAWLS		100%		<100%	
income group	prop.	avg. tax	prop.	avg. tax	prop.	avg. tax
0	0.21	-9363	0.19	-518	0.19	-6318
$0 < y \leq 5000$	0.07	-5595	0.05	-98	0.05	-4294
$5000 < y \leq 10000$	0.06	-2679	0.06	-2222	0.05	-1610
$10000 < y \leq 15000$	0.14	3370	0.17	-2424	0.18	-615
$15000 < y \leq 20000$	0.26	7205	0.23	2785	0.24	4972
$20000 < y$	0.26	15856	0.30	13104	0.29	14714

Table 1: Some characteristics for the Rawlsian case, the 100%- and <100%-case.

5.2.2 Sensitivity analysis

As we believe that no one would be willing to defend the 100%-case, we focus on the <100%-case for our sensitivity analysis. We investigate the impact of changing (i) the labour supply elasticity and (ii) the taste level assigned to the constrained unemployed. Although we arbitrarily choose to half and double the original parameter values, other choices lead to the same qualitative results.

First, the labour supply elasticity ε equaled 0.1 in our previous simulations. Figure 7 shows the impact of halving and doubling ε on our optimal allocation. For comparison, we still present the three cases depicted in figure 6, here denoting the <100%-case by 'eps=0.1' for obvious reasons. Changing ε has a large impact on both marginal tax rates and installed basic income but the U-shaped pattern of marginal tax rates remains intact; see Saez (2001) for similar results in welfarist Mirrlees economies. As expected, choosing a higher labour supply elasticity leads to lower marginal tax rates everywhere and, for a given budget constraint, it installs a (substantially) lower basic income. Furthermore, it introduces subsidies for gross income earners between € 6000 and € 10000, but still the largest subsidies are directed towards the unemployed. For lower values of ε, marginal tax rates are everywhere positive, excluding 'making work pay'-type policies.

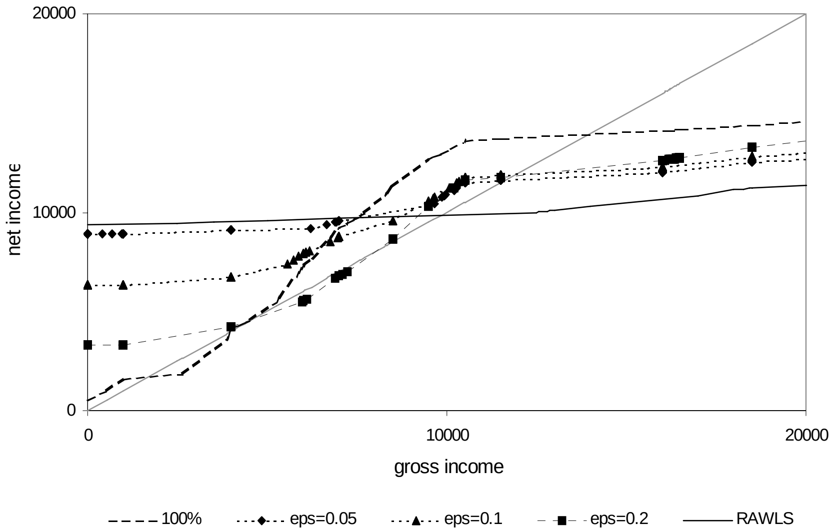

Figure 7: Measuring the impact of varying ε.

Second, the taste level assigned to the constrained unemployed equaled the average taste of the working class in the previous section. Here we consider two additional cases: half of the average taste of those currently working (denoted HALF in figure 8) and the double of the average taste (denoted DOUBLE in figure 8).

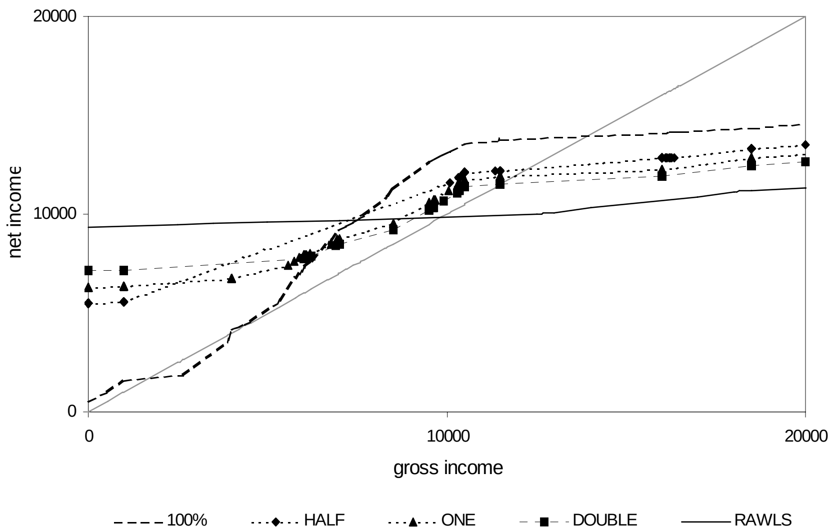

Figure 8: Measuring the impact of varying tastes.

These variants have a clear interpretation. The HALF-case (resp. DOUBLE-case) corresponds with the assumption that each constrained unemployed needs two times more (resp. half of the) additional consumption for a unit of additional labour compared to the 'average' worker. Figure 8 presents the HALF- and DOUBLE-cases, as well as all cases displayed in figure 6, while here we denote the <100%-case by 'ONE'.

Choosing less or more conservative estimates for the tastes of the constrained unemployed plays a moderate role. As expected from our discussion of figure 4 in the previous section, the higher their taste for working, the lower their well-being, which puts upward pressure on the optimal basic income. Furthermore, halving or doubling the assigned taste level does not alter the U-shaped pattern of positive marginal tax rates.

To conclude, our sensitivity analysis suggests that a U-shaped pattern of marginal tax rates is optimal according to our fairness criterion. Furthermore, negative marginal tax rates and thus 'making work pay'-policies never occur except for higher labour supply elasticities.

6 Conclusion

Given the increased importance many governments attach to 'making work pay' policies, we examine whether subsidizing low earners is optimal according to a specific 'fair' social ordering. Fairness considerations are kept simple in this paper: we want to compensate individuals for differences in productive skills, but we keep them responsible for their tastes for working.

We consider a discrete Stiglitz (1982, 1987) economy with four types, defined by a low or high productive skill and a low or high taste for working, and a government that wants to install fair taxes, but cannot observe individuals' type. We show that fairness recommends to subsidize the low earners, as long as the low-skilled individuals have a strictly positive skill. In case the low-skilled have a zero skill, subsidies can never be optimal.

As these theoretical results only hold for simple fictitious economies, we simulate fair taxes for a sample of Belgian singles. Our fairness criterion suggests a U-shaped pattern of positive marginal tax rates in almost all cases. This strongly suggests that negative marginal tax rates, and thus 'making work pay' policies, cannot be optimal in the reasonable case in which at least some unemployed are willing to work, but cannot due to exogenous labour market constraints.

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Appendix A: A characterization

We provide a characterization for the rule presented in section 3. A rule f maps economies into orderings, or $f : \mathcal{E} \to \mathcal{R} : e \mapsto R_e = f(e)$, with $\mathcal{R}$ the set of all orderings (complete and transitive binary relations) defined over allocations $\mathbf{z}$ in $\mathcal{Z}$; call P_e and I_e the corresponding asymmetric and symmetric relation. We define some properties for f.

Our Pareto principle is equal to Pareto Indifference and the Weak Pareto principle together, i.e., if everyone is indifferent between allocations $\mathbf{z}$ and $\mathbf{z}'$, then $\mathbf{z}$ should also be socially indifferent to $\mathbf{z}'$ and if everyone strictly prefers allocation $\mathbf{z}$ to $\mathbf{z}'$, then $\mathbf{z}$ should also be socially strictly preferred to $\mathbf{z}'$. Anonymity requires that the names of the individuals do not matter. Formally:

Pareto: For each economy $e \in \mathcal{E}$ and for all allocations $\mathbf{z}, \mathbf{z}' \in \mathcal{Z}$: If $U_i(z_i) = U_i(z_i')$ for all $i \in N$, then $\mathbf{z} I_e \mathbf{z}'$. If, $U_i(z_i) > U_i(z_i')$ for all $i \in N$, then $\mathbf{z} P_e \mathbf{z}'$.

Anonymity: For each economy $e \in \mathcal{E}$, for each allocation $\mathbf{z} \in \mathcal{Z}$ and for each permutation $\pi : N \to N$ over individuals: If $U_i = U_j$ for all $i, j \in N$, then $\mathbf{z} I_e \pi(\mathbf{z})$, with $\pi(\mathbf{z}) = \left(z_{\pi(1)}, \ldots, z_{\pi(n)}\right)$.

In line with the idea to compensate for differences in outcomes which are only due to differences in skills, compensation (Fleurbaey and Maniquet, 2005a) requires that a Pigou-Dalton transfer (in terms of net income) from a rich to a poor individual with the same preferences and the same labour should be welfare improving:

Compensation: For each economy $e \in \mathcal{E}$, for all allocations $\mathbf{z}, \mathbf{z}' \in \mathcal{Z}$ and for all individuals $i, j \in N$: If (i) $\ell_i = \ell_j = \ell_i' = \ell_j'$ and $U_i = U_j$, (ii) $\exists \delta > 0$ such that $c_i = c_i' + \delta < c_j' - \delta = c_j$ and (iii) $z_k = z_k'$ for all $k \neq i, j$, then $\mathbf{z} R \mathbf{z}'$.

Finally, in line with (i) our well-being definition and (ii) the idea that individuals are responsible for their tastes, Utility Independence requires the ranking of two allocations to be the same (i) whenever they give rise to the same well-being vector, (ii) irrespective of the utility profile:

Utility Independence: For all economies $e = (\mathbf{s}, \mathbf{U}), e' = \left(\mathbf{s}, \mathbf{U}'\right) \in \mathcal{E}$ and for all allocations $\mathbf{z}, \mathbf{z}' \in \mathcal{Z}$: If $\mathbf{W}(\mathbf{z}, e) = \mathbf{W}(\mathbf{z}, e')$ and $\mathbf{W}(\mathbf{z}', e) = \mathbf{W}(\mathbf{z}', e')$, then $\mathbf{z} R_e \mathbf{z}' \Leftrightarrow \mathbf{z} R_{e'} \mathbf{z}'$.

Given these axioms, we should focus on the minimal well-being in society, or:

Proposition. *If a rule $f : \mathcal{E} \to \mathcal{R}$ satisfies Pareto, Anonymity, Compensation and Utility Independence, then, for each economy $e \in \mathcal{E}$ and for all allocations $\mathbf{z}, \mathbf{z}' \in \mathcal{Z}$:* $\min \mathbf{W}(\mathbf{z}, e) > \min \mathbf{W}(\mathbf{z}', e)$ *implies* $\mathbf{z} P_e \mathbf{z}'$.

Proof.

First, we show, in three steps, that Pareto Indifference (the first part of the Pareto axiom) and Utility Independence for f are equivalent with Neutrality for f:

NEUTRALITY: For all economies $e = (\mathbf{s}, \mathbf{U})$, $e' = (\mathbf{s}, \mathbf{U}') \in \mathcal{E}$ and for all allocations $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d} \in \mathcal{Z}$: If $\mathbf{W}(\mathbf{a}, e) = \mathbf{W}(\mathbf{b}, e')$ and $\mathbf{W}(\mathbf{c}, e) = \mathbf{W}(\mathbf{d}, e')$, then $\mathbf{a} R_e \mathbf{c} \Leftrightarrow \mathbf{b} R_{e'} \mathbf{d}$.

1. If f satisfies Neutrality then f also satisfies Pareto Indifference. Suppose the antecedent of Pareto Indifference is true for a certain economy $e \in \mathcal{E}$ and two allocations $\mathbf{z}, \mathbf{z}' \in \mathcal{Z}$, i.e., $U_i(z_i) = U_i(z_i')$ for all $i \in N$. As such, z_i lies on the same indifference curve as z_i' for all individuals and, by definition of our well-being concept, $\mathbf{W}(\mathbf{z}, e) = \mathbf{W}(\mathbf{z}', e)$. Let $e' = e$ and define allocations $\mathbf{a} = \mathbf{d} = \mathbf{z}$ and $\mathbf{b} = \mathbf{c} = \mathbf{z}'$. As a consequence, $\mathbf{W}(\mathbf{a}, e) = \mathbf{W}(\mathbf{b}, e')$ and $\mathbf{W}(\mathbf{c}, e) = \mathbf{W}(\mathbf{d}, e')$ are true by construction. Using Neutrality, we get $\mathbf{a} R_e \mathbf{c} \Leftrightarrow \mathbf{b} R_{e'} \mathbf{d}$, or equivalently, $\mathbf{z} R_e \mathbf{z}' \Leftrightarrow \mathbf{z}' R_e \mathbf{z}$ ($\bullet$). Because of completeness of R_e, we must have either $\mathbf{z} R_e \mathbf{z}'$ (and also $\mathbf{z}' R_e \mathbf{z}$ via ($\bullet$)) or $\mathbf{z}' R_e \mathbf{z}$ (and also $\mathbf{z} R_e \mathbf{z}'$ via ($\bullet$)). Both cases, lead to $\mathbf{z} I_e \mathbf{z}'$ establishing Pareto Indifference.

2. If f satisfies Neutrality then f also satisfies Utility Independence. Suppose the antecedent of Utility Independence is true, i.e., there exist two economies $e = (\mathbf{s}, \mathbf{U})$,$e' = (\mathbf{s}, \mathbf{U}') \in \mathcal{E}$ and two allocations $\mathbf{z}, \mathbf{z}' \in \mathcal{Z}$ such that $\mathbf{W}(\mathbf{z}, e) = \mathbf{W}(\mathbf{z}, e')$ and $\mathbf{W}(\mathbf{z}', e) = \mathbf{W}(\mathbf{z}', e')$. Simply choose $\mathbf{a} = \mathbf{b} = \mathbf{z}$ and $\mathbf{c} = \mathbf{d} = \mathbf{z}'$ such that $\mathbf{W}(\mathbf{a}, e) = \mathbf{W}(\mathbf{b}, e')$ and $\mathbf{W}(\mathbf{c}, e) = \mathbf{W}(\mathbf{d}, e')$ holds. Using Neutrality, we get $\mathbf{a} R_e \mathbf{c} \Leftrightarrow \mathbf{b} R_{e'} \mathbf{d}$, or equivalently, $\mathbf{z} R_e \mathbf{z}' \Leftrightarrow \mathbf{z} R_{e'} \mathbf{z}'$ establishing Utility Independence.

3. If f satisfies Pareto Indifference and Utility Independence then f also satisfies Neutrality. Suppose the antecedent of Neutrality holds, i.e., there exist two economies $e = (\mathbf{s}, \mathbf{U})$ and $e' = (\mathbf{s}, \mathbf{U}') \in \mathcal{E}$ and four allocations $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d} \in \mathcal{Z}$ such that $\mathbf{W}(\mathbf{a}, e) = \mathbf{W}(\mathbf{b}, e')$ and $\mathbf{W}(\mathbf{c}, e) = \mathbf{W}(\mathbf{d}, e')$. Let us focus on an arbitrary individual $i \in N$. Because $\mathbf{W}_i(\mathbf{a}, e) = \mathbf{W}_i(\mathbf{b}, e')$, the indifference curve of U_i through a_i and U_i' through b_i are tangent to the same 'shared resources' opportunity set defined by $\mathbf{s}$. Given $U_i, U_i' \in \mathcal{U}$, both indifference curves must cross at least once in $\mathbb{R} \times [0, 1]$. Choose a bundle α_i where both cross. Repeating this construction of α_i for all individuals, we get an allocation $\boldsymbol{\alpha} \in \mathcal{Z}$ such that $\mathbf{W}(\boldsymbol{\alpha}, e) = \mathbf{W}(\mathbf{a}, e) = \mathbf{W}(\mathbf{b}, e') = \mathbf{W}(\boldsymbol{\alpha}, e')$. In the same way, define an allocation $\boldsymbol{\beta} \in \mathcal{Z}$ such that $\mathbf{W}(\boldsymbol{\beta}, e) = \mathbf{W}(\mathbf{c}, e) = \mathbf{W}(\mathbf{d}, e') = \mathbf{W}(\boldsymbol{\beta}, e')$. Using Pareto Indifference and transitivity of R_e and $R_{e'}$, we get:

$$(\bullet) \;\; \mathbf{a} R_e \mathbf{c} \Leftrightarrow \boldsymbol{\alpha} R_e \boldsymbol{\beta} \text{ and } \boldsymbol{\alpha} R_{e'} \boldsymbol{\beta} \Leftrightarrow \mathbf{b} R_{e'} \mathbf{d}.$$

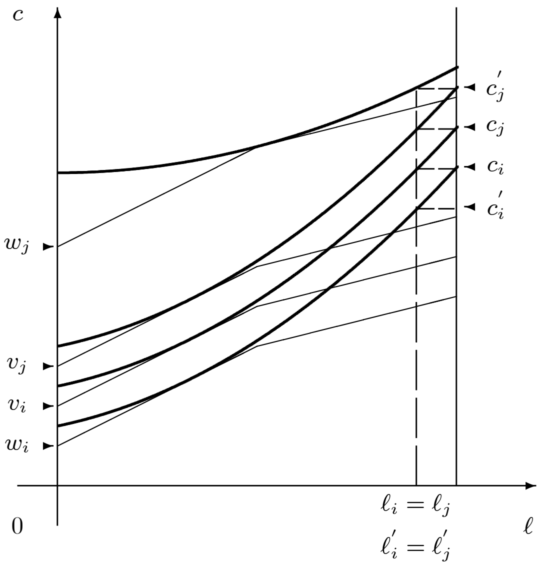
Using Utility Independence, we get $\boldsymbol{\alpha} R_e \boldsymbol{\beta} \Leftrightarrow \boldsymbol{\alpha} R_{e'} \boldsymbol{\beta}$. Using ($\bullet$), we get $\mathbf{a} R_e \mathbf{c} \Leftrightarrow \mathbf{b} R_{e'} \mathbf{d}$, establishing Neutrality.

Second, a rule f satisfies Neutrality if and only if there exists a unique ordering R^* defined over $\mathbb{R}^n$, such that, for each economy $e \in \mathcal{E}$ and for all allocations $\mathbf{z}, \mathbf{z}' \in \mathcal{Z}$ we have $\mathbf{z} R_e \mathbf{z}'$ if and only if $\mathbf{W}(\mathbf{z}, e) \;\; R^* \;\; \mathbf{W}(\mathbf{z}', e)$; see Bossert and Weymark, 2004, theorem 2 for a proof.[1] This has to be interpreted as follows: only well-being levels matter to rank two allocations (given a fixed size n of the population and a fixed skill vector $\mathbf{s}$).

Third, the unique ordering R^* inherits certain properties from f: R^* must satisfy weak Pareto* (if $v_i > w_i$ for all $i \in N$, then $\mathbf{v} P^* \mathbf{w}$) and Anonymity* ($\mathbf{v} I^* \pi(\mathbf{v})$ with $\pi : N \rightarrow N$ a permutation of individuals in N). This follows from Pareto and Anonymity for f straightforwardly. We show that, given Compensation for f, R^* must also satisfy

[1] It suffices to notice that our set-up is sufficiently rich to obtain welfarism: for any two well-being vectors $\mathbf{v}, \mathbf{w} \in \mathbb{R}^n$, there exist two allocations $\mathbf{z}, \mathbf{z}' \in \mathcal{Z}$ and an economy $e \in \mathcal{E}$ such that $\mathbf{W}(\mathbf{z}, e) = \mathbf{v}$ and $\mathbf{W}(\mathbf{z}', e) = \mathbf{w}$.

Hammond Equity*: For all well-being vectors $\mathbf{v}, \mathbf{w} \in \mathbb{R}^n$ and for all individuals $i, j \in N$: If (i) $w_i < v_i < v_j < w_j$ and (ii) $v_k = w_k$ for all $k \neq i, j$, then $\mathbf{v}R^*\mathbf{w}$.



Suppose the antecedent of Hammond Equity* holds, or there exist two well-being vectors $\mathbf{v}, \mathbf{w} \in \mathbb{R}^n$ and two individuals $i, j \in N$ such that $w_i < v_i < v_j < w_j$ and $v_k = w_k$ for all $k \neq i, j$ hold. The figure illustrates how it is possible to construct bundles z_i, z_j and z_i', z_j' and a utility function $U_i = U_j \in \mathcal{U}$ such that $\mathbf{W}_i(\mathbf{z}, e) = v_i$, $\mathbf{W}_j(\mathbf{z}, e) = v_j$ and $\mathbf{W}_i(\mathbf{z}', e) = w_i$, $\mathbf{W}_j(\mathbf{z}', e) = w_j$ and the antecedents of the Compensation principle are satisfied for i, j, i.e., (i) $\ell_i = \ell_j = \ell_i' = \ell_j'$ and $U_i = U_j$ (ii) $\exists \delta > 0$ such that $c_i = c_i' + \delta < c_j' - \delta = c_j$. The bundles z_i, z_j and z_i', z_j' can be extended with bundles $z_k = z_k'$ for the other individuals $k \neq i, j$ to obtain allocations $\mathbf{z}$ and $\mathbf{z}'$, such that $\mathbf{W}_k(\mathbf{z}, e) = v_k = w_k = \mathbf{W}_k(\mathbf{z}', e)$ holds for all $k \neq i, j$. Using Compensation, we must have $\mathbf{z}R_e\mathbf{z}'$ and thus, via neutrality, also $\mathbf{v}R^*\mathbf{w}$ must hold.

Finally, Given the axioms for R^*, Tungodden (2000, theorem 1) shows that $\min \mathbf{v} > \min \mathbf{w}$ implies $\mathbf{v}P^*\mathbf{w}$, for any vectors $\mathbf{v}, \mathbf{w} \in \mathbb{R}^n$, which, given neutrality, completes our proof.

Appendix B: Proofs of propositions 1 and 2

To prove propositions 1 and 2, we need two 'tricks' and two lemmas. We start with the tricks.

Consider an implementable and feasible allocation $\mathbf{x} \in \mathcal{X}$ as in figure B1. The bundles x_{LL} and x_{HH} lie somewhere in the left and right shaded zone, respectively, to satisfy the incentive constraints. The bundle x° is constructed to satisfy $V_{HL}(x^\circ) = V_{HL}(x_{HL})$ and $MRSY_{HL}(x^\circ) = 1$. Now, consider the allocation $\mathbf{x}^+ \in \mathcal{X}$ with $x_{st}^+ = x_{st}$ for all types $st \neq HL$ and x_{HL}^+ is constructed by moving x_{HL} on his indifference curve towards the bundle x°.

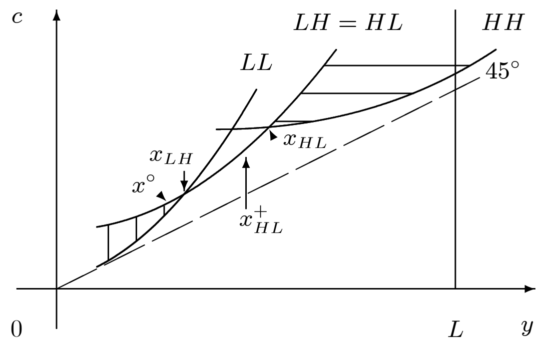


Figure B1: The allocations $\mathbf{x}$ and $\mathbf{x}^{+}$ illustrating trick 1.

It is clear that the allocation $\mathbf{x}^{+}$ is implementable. Furthermore, given the preference technology defined by $\mathcal{U}$, we have $y^{+}_{HL} - c^{+}_{HL} > y_{HL} - c_{HL}$. Thus, the allocation $\mathbf{x}^{+}$ is also feasible, with $\sum_{st \in ST} n_{st} \left(y^{+}_{st} - c^{+}_{st}\right) - \sum_{st \in ST} n_{st} \left(y_{st} - c_{st}\right) = m > 0$. The amount of money m can now be freely redistributed to the net income of all types (while still satisfying all incentive constraints) resulting in a weak Pareto improvement and thus also an improvement according to the government's program $(*)$. More generally, we obtain:

TRICK 1: Consider an implementable and feasible allocation $\mathbf{x} \in \mathcal{X}$ and a type st whose bundle x_{st} can be moved along his indifference curve (i) without violating incentive constraints and (ii) making an amount of money m free for redistribution. The allocation $\mathbf{x}$ cannot be optimal according to program $(*)$, because everyone can be made strictly better-off (by redistributing the amount of money m to the net incomes of all types), without violating incentive constraints.

To illustrate the second trick, consider an implementable and feasible allocation $\mathbf{x} \in \mathcal{X}$ as in figure B2.

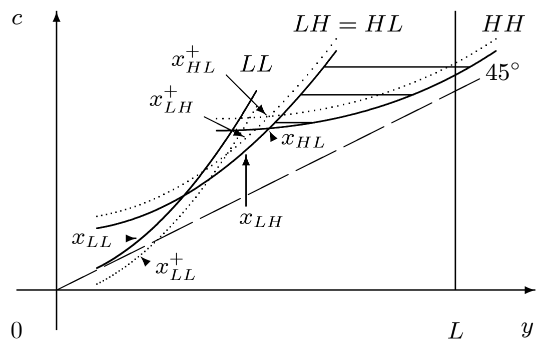


Figure B2: The allocation $\mathbf{x}$ and $\mathbf{x}^{+}$ illustrating trick 2.

Again, the bundle x_{HH} lies somewhere in the right shaded zone to satisfy the incentive constraints. Now it is possible to construct a feasible and implementable allocation $\mathbf{x}^{+} \in \mathcal{X}$, transferring in $\mathbf{x}$ some net income from type LL to the other types LH, HL and HH. Whether or not the resulting allocation is better according to program $(*)$, ultimately depends on the well-being levels in society: if LL is strictly better off compared to one of the other types, it is always possible to find an allocation $\mathbf{x}^{+}$ which is better according to program $(*)$. We summarize

TRICK 2: Consider an implementable and feasible allocation $\mathbf{x} \in \mathcal{X}$ and one or more types st whose bundle(s) x_{st} can be moved downwards (i) without violating incentive constraints and (ii) making an amount of money $m > 0$ free for redistribution to the other types. The allocation $\mathbf{x}$ cannot be optimal according to program $(*)$, if all donor type(s) st were strictly better off in $\mathbf{x}$ compared to (one of) the other types.

Besides two tricks, we need two lemmas. The first lemma tells us that the program $(*)$ can, loosely speaking, focus on the lower-skilled, because they are always worse-off in terms of well-being, more precisely:

Lemma 1*. Consider two types with the same taste for working $t \in T$ (and thus the same utility function $U_t \in \mathcal{U}$), but different skills $0 \leq L < H$. In an implementable allocation $\mathbf{x} \in \mathcal{X}$, with $V_{Ht}(x_{Ht}) \geq V_{Ht}(x_{Lt})$ (resp. $V_{Ht}(x_{Ht}) > V_{Ht}(x_{Lt})$) the lower-skilled type Lt is always worse off (resp. strictly worse off) compared to the higher-skilled type Ht, i.e., $\mathbf{W}_{Ht}(\mathbf{x}, e) \geq \mathbf{W}_{Lt}(\mathbf{x}, e)$ (resp. $\mathbf{W}_{Ht}(\mathbf{x}, e) > \mathbf{W}_{Lt}(\mathbf{x}, e)$).*

Proof. Consider two types with the same taste for working $t \in T$. We prove the case where skills satisfy $0 < L < T$ and $V_{Ht}(x_{Ht}) \geq V_{Ht}(x_{Lt})$; the other cases are analogous. Call (c_{Lt}, y_{Lt}) and (c_{Ht}, y_{Ht}) their bundles. Individuals with the same taste t have the same utility functions U_t and thus also the same indifference curves and therefore the same well-being level for bundles on the same indifference curve. Because our well-being measure is ordinally equivalent with utility, measured by U_t, it suffices to show that $U_t\left(c_{Lt}, \frac{y_{Lt}}{L}\right) \leq U_t\left(c_{Ht}, \frac{y_{Ht}}{H}\right)$. Suppose not, i.e., suppose (i) $U_t\left(c_{Lt}, \frac{y_{Lt}}{L}\right) > U_t\left(c_{Ht}, \frac{y_{Ht}}{H}\right)$. Because $V_{Ht}(x_{Ht}) \geq V_{Ht}(x_{Lt})$ we get, by definition of V_{Ht}, that (ii) $U_t\left(c_{Ht}, \frac{y_{Ht}}{H}\right) \geq U_t\left(c_{Lt}, \frac{y_{Lt}}{H}\right)$. Combining (i) and (ii), we obtain $U_t\left(c_{Lt}, \frac{y_{Lt}}{L}\right) > U_t\left(c_{Lt}, \frac{y_{Lt}}{H}\right)$, a contradiction given $U_t \in \mathcal{U}$ and $0 < L < H$. □

Lemma 2 tells us that it cannot be optimal —according to the government's program $(*)$— to treat the indistinguishable middle types LH and HL differently in case $y^*_{HL} \leq L$. Otherwise (if $y^*_{HL} > L$) it might be optimal to treat them differently, but only under certain conditions:

Lemma 2*. Consider a four type economy with skills $0 < L < H$ and tastes represented by utility functions $U_L, U_H \in \mathcal{U}$, which satisfy single-crossingness and indistinguishable middle type. Consider a government who optimizes the program defined by $(*)$. In an optimal allocation $\mathbf{x}^* \in \mathcal{X}$, we must have:*

(a). $x^*_{LH} = x^*_{HL}$, *if* $y^*_{HL} \leq L$, *or else,*

(b). $V_{HL}(x^*_{LH}) = V_{HL}(x^*_{HL})$, *with* $y^*_{LH} = L < y^*_{HL}$ *and* $MRSY_{HL}(x^*_{HL}) \leq 1$.

Proof of part (a). Suppose $y^*_{HL} \leq L$ and $x^*_{LH} \neq x^*_{HL}$. We show that it is always possible to construct another allocation $\mathbf{x} \in \mathcal{X}$, which is feasible, implementable and strictly better than $\mathbf{x}^*$ according to the government's program $(*)$. Because $y^*_{HL} \leq L$, the incentive compatibility constraints $IC_{LH,HL}$ and $IC_{HL,LH}$ require

$$\begin{aligned} V_{HL}(x^*_{HL}) &\geq V_{HL}(x^*_{LH}) \text{ and} \\ V_{LH}(x^*_{LH}) &\geq V_{LH}(x^*_{HL}) \Leftrightarrow V_{HL}(x^*_{LH}) \geq V_{HL}(x^*_{HL}), \end{aligned}$$

where the equivalence $\Leftrightarrow$ is due to indistinguishable middle types. We must have $V_{HL}(x^*_{HL}) = V_{HL}(x^*_{LH})$, or x^*_{LH} and x^*_{HL} must lie on the same indifference curve.

Given our preference technology $\mathcal{U}$, there are only two cases for $x^*_{LH} \neq x^*_{HL}$. Assume $x^*_{LH} < x^*_{HL}$, i.e., $c^*_{LH} < c^*_{HL}$ and $y^*_{LH} < y^*_{HL}$; for the other case $x^*_{LH} > x^*_{HL}$ simply switch subscripts HL and LH in the sequel. Define a bundle $x^\circ = (c^\circ, y^\circ)$ in $\mathbb{R} \times [0, L]$ such that x° also lies on the same indifference curve through x^*_{LH} and x^*_{HL}, i.e., $V_{HL}(x^\circ) = V_{HL}(x^*_{HL})$, and choose (i) $y^\circ = 0$, if $MRSY_{HL} \geq 1$ everywhere in $\mathbb{R} \times [0, L]$, (ii) $y^\circ = L$, if $MRSY_{HL} \leq 1$ everywhere in $\mathbb{R} \times [0, L]$, or else (iii) choose x° such that $MRSY_{HL}(x^\circ) = 1$. Each case leads to any of the following three cases: either (α) $y^\circ \leq y^*_{LH} < y^*_{HL}$, or ($\beta$) $y^*_{LH} < y^\circ < y^*_{HL}$ or (γ) $y^*_{LH} < y^*_{HL} \leq y^\circ$. In each of the three cases (α), (β) and (γ), it is possible to use TRICK 1, by moving either x^*_{HL} to the left on his indifference curve (in case (α) and (β)) or moving x^*_{LH} to the right on his indifference curve (in case (γ)), contradicting that $\mathbf{x}^*$ was optimal.

Proof of part (b). Suppose $y^*_{HL} > L$. We show that $x^*_{LH} < x^*_{HL}$, with $y^*_{LH} = L$ and $MRSY_{HL}(x^*_{HL}) \leq 1$, must hold. Recall that, in case $y^*_{HL} > L$, the incentive constraint $IC_{LH,HL}$ does not exist, because type HL's bundle is not attainable for LH.

We first show that the incentive constraint $IC_{HL,LH}$ must bind, i.e., $V_{HL}(x^*_{HL}) = V_{HL}(x^*_{LH})$. Suppose not, i.e., $V_{HL}(x^*_{HL}) > V_{HL}(x^*_{LH})$. Single-crossingness ensures that $V_{HL}(x^*_{HL}) > V_{HL}(x^*_{LL})$ and thus LL is strictly worse-off compared to HL (lemma 1); for the same reason, LH is strictly worse off than HH. Now, it is possible to use TRICK 2, transferring from type HL (and possibly HH as well, if $IC_{HL,HH}$ binds) to both other types LL and LH, which must improve the lowest well-being, contradicting that $\mathbf{x}^*$ was optimal according to program $(*)$.

Now, we are back in the same situation as in part (a), because both x^*_{LH} and x^*_{HL}, with $x^*_{LH} < x^*_{HL}$, lie on the same indifference curve (of type HL), i.e., $V_{HL}(x^*_{LH}) = V_{HL}(x^*_{HL})$, but here $y^*_{LH} \leq L < y^*_{HL}$. Now proceed as in part (a). Define the bundle $x^\circ = (c^\circ, y^\circ)$ in $\mathbb{R} \times [0, H]$ such that x° also lies on the same indifference curve through x^*_{LH} and x^*_{HL}, i.e., $V_{HL}(x^\circ) = V_{HL}(x^*_{HL})$, and choose (i) $y^\circ = 0$, if $MRSY_{HL} \geq 1$ everywhere in $\mathbb{R} \times [0, H]$, (ii) $y^\circ = H$, if $MRSY_{HL} \leq 1$ everywhere in $\mathbb{R} \times [0, H]$, or else (iii) choose x° such that $MRSY_{HL}(x^\circ) = 1$. Now, $y^\circ < y^*_{HL}$ is not possible (otherwise we can use TRICK 1, moving x^*_{HL} to the left on his indifference curve); thus $MRSY_{HL}(x^*_{HL}) \leq 1$. As a consequence $y^\circ \geq y^*_{HL}$ must hold. Now, $y^*_{LH} < L$ is not possible (because then $y^*_{LH} < L < y^\circ$ and using trick 1 again, we could move x^*_{LH} to the right on his indifference curve). Thus $y^*_{LH} = L$, which completes the proof. □

We are ready to prove propositions 1 and 2, on the basis of lemmas 1 and 2 and tricks 1 and 2.

Proof of proposition 1

Consider a four type economy with skills $0 < L < H$ and tastes represented by utility functions $U_L, U_H \in \mathcal{U}$, which satisfy single-crossingness and indistinguishable middle type. Consider a government who optimizes the program defined by $(*)$. In an optimal allocation $\mathbf{x}^* \in \mathcal{X}$, we must have $y^*_{LL} - c^*_{LL} \geq y^*_{LH} - c^*_{LH}$.

Our proof consists of two parts, depending on whether (a) $y^*_{HL} \leq L$ in the optimum $\mathbf{x}^*$, or (b) $y^*_{HL} > L$. Given the definition of $\mathcal{X}$, one of both cases must hold. We show, for both cases, that $y^*_{LL} - c^*_{LL} < y^*_{LH} - c^*_{LH}$ is not possible.

Proof of part (a). Suppose $y^*_{HL} \leq L$ (thus $x^*_{LH} = x^*_{HL}$ via lemma 2) and $y^*_{LL} - c^*_{LL} < y^*_{LH} - c^*_{LH}$. We consider four possible cases, depending on whether $IC_{LL,LH}$ and/or $IC_{LH,LL}$ bind, or not. In an optimum $\mathbf{x}^*$ of the program $(*)$, one of these four cases must hold. For all cases, we show that it is possible to construct a strictly better allocation according to the program $(*)$, which also satisfies the feasibility and incentive compatibility constraints.

1. $IC_{LL,LH}$ and $IC_{LH,LL}$ bind. This requires $x^*_{LL} = x^*_{LH}$ which contradicts $y^*_{LL} - c^*_{LL} < y^*_{LH} - c^*_{LH}$.

2. $IC_{LL,LH}$ binds, $IC_{LH,LL}$ does not bind. **2a.** If $MRSY_{LL}(x^*_{LL}) < 1$, we could use TRICK 1 moving x^*_{LL} somewhat to the right on his indifference curve. **2b.** We must have $MRSY_{LL}(x^*_{LL}) \geq 1$, from (2a). But, given the preference technology defined by $\mathcal{U}$, $y^*_{LL} - c^*_{LL} < y^*_{LH} - c^*_{LH}$ is not possible, a contradiction.

3. $IC_{LL,LH}$ does not bind, $IC_{LH,LL}$ binds: **3a.** Let us first focus on type LH. If $y^*_{LH} = 0$, then incentive constraints and single-crossingness require $x^*_{LL} = x^*_{LH}$, which violates $y^*_{LL} - c^*_{LL} < y^*_{LH} - c^*_{LH}$. So $y^*_{LH} > 0$. If $MRSY_{LH}(x^*_{LH}) > 1$, we can use TRICK 1 again, by moving both $x^*_{LH} = x^*_{HL}$ to the left on their (common) indifference curve. So $MRSY_{LH}(x^*_{LH}) \leq 1$ must hold. **3b.** We focus now on type LL. We must have either (i) $y^*_{LL} = 0$ or (ii) $y^*_{LL} > 0$. In case (ii), we have $MRSY_{LL}(x^*_{LL}) \leq 1$ (otherwise we can use TRICK 1, moving x^*_{LL} somewhat to the left on his indifference curve). **3c.** Due to lemma 1, either type LH or LL has the minimal well-being. Figure B3 illustrates (3a), (3b(ii)) and $y^*_{LL} - c^*_{LL} < y^*_{LH} - c^*_{LH}$; type HH's bundle is somewhere in the shaded zone. We measure well-being in (c, y)- rather than in (c, ℓ)-space. Therefore, one has to divide the slopes of the shared resources budget line by the skill level of the individual under consideration, here a low skilled individual and to multiply the proportion of high-skilled $\frac{n_{HL}+n_{HH}}{n}$ (which defines the kink in the budget set where the slope changes from $\frac{H}{L} > 1$ to 1) with the skill level of the individual under consideration. As a consequence, it is easy to verify that type LH is always strictly worse-off compared to type LL, irrespective of the proportion of high-skilled $\frac{n_{HL}+n_{HH}}{n}$ and irrespective of whether (3b(i)) or (3b(ii)) applies.

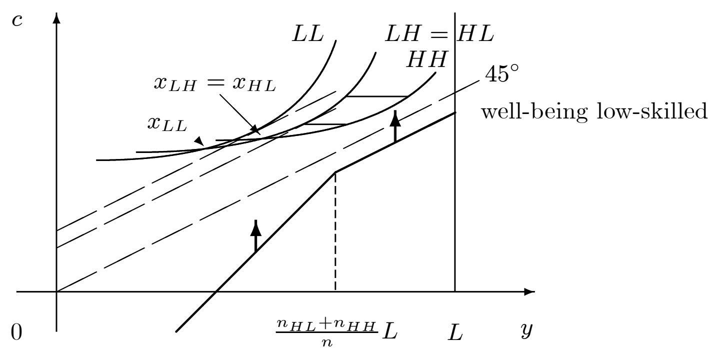


Figure B3: Type LL is strictly better off compared to type LH.

Since $IC_{LL,LH}$ does not bind, it is always possible to use TRICK 2 transferring a small amount of money from LL to the other types LH, HL and HH, improving the minimal well-being in society, a contradiction.

4. $IC_{LL,LH}$ and $IC_{LH,LL}$ do not bind. Using TRICK 1, it can be verified that only the following cases are possible: (i) $y^*_{LL} = 0 < y^*_{LH}$, $MRSY_{LL}(x^*_{LL}) \geq 1$ and $MRSY_{LH}(x^*_{LH}) \leq$

1, or (ii) $0 < y^*_{LL} < y^*_{LH}$, $MRSY_{LL}(x^*_{LL}) = 1$ and $MRSY_{LH}(x^*_{LH}) \leq 1$. We are back in the same situation as in (3). In both cases (i) and (ii) and, given $y^*_{LL} - c^*_{LL} < y^*_{LH} - c^*_{LH}$, type LH is strictly worse-off compared to LL, irrespective of the proportion of high-skilled. Here again, TRICK 2 can be used to obtain a contradiction.

Proof of part (b). Suppose $y^*_{HL} > L$ (thus $V_{HL}(x^*_{LH}) = V_{HL}(x^*_{HL})$, with $y^*_{LH} = L < y^*_{HL}$ and $MRSY_{HL}(x^*_{HL}) \leq 1$ via lemma 2) and $y^*_{LL} - c^*_{LL} < y^*_{LH} - c^*_{LH}$. It is again possible to consider four cases, depending on whether $IC_{LL,LH}$ and/or $IC_{LH,LL}$ bind or not, and to show, for each case, a contradiction. Actually, the proof is completely analogous as in steps 1-4 of part (a) and therefore omitted.

Proof of proposition 2

Consider a four type economy with skills $L = 0 < H$ and tastes represented by utility functions $U_L, U_H \in \mathcal{U}$, which satisfy single-crossingness. Consider a government who optimizes the program defined by $(*)$. In an optimal allocation $\mathbf{x}^* \in \mathcal{X}$ we must have $y^*_{LL} - c^*_{LL} = y^*_{LH} - c^*_{LH} \leq y^*_{HL} - c^*_{HL}$.

Proof. Suppose $y^*_{LL} - c^*_{LL} = y^*_{LH} - c^*_{LH} > y^*_{HL} - c^*_{HL}$ holds. **1.** Because $L = 0$ and $\mathbf{x}^* \in \mathcal{X}$, we must have $y^*_{LL} = y^*_{LH} = 0$ and, given the incentive constraints, also $c^*_{LL} = c^*_{LH}$ must hold. **2.** Due to lemma 1, the lowest well-being is either LH or LL, thus, given (1), we must maximize the basic income, i.e., maximize $c^*_{LL} = c^*_{LH}$. **3.** $y^*_{HL} = 0$ is excluded, otherwise we must have $c^*_{LH} = c^*_{HL}$ (due to incentive constraints) and $y^*_{LH} - c^*_{LH} > y^*_{HL} - c^*_{HL}$ would be violated. **4.** So, $y^*_{HL} > 0$ holds from (3). Now, x^*_{LH} and x^*_{HL} must lie on the same indifference curve, or $V_{HL}(x^*_{HL}) = V_{HL}(x^*_{LH})$. Otherwise (see the proof of lemma 2, part (b)) it would be possible to improve the situation of the worst-off types LL and LH, at the cost of the better-off types HL and HH (on the basis of TRICK 2). **5.** If $MRSY_{HL}(x^*_{HL}) > 1$ at $y^*_{HL} > 0$, we can use TRICK 1, moving x^*_{HL} to the left on his indifference curve. **6.** To summarize, we must have $MRSY_{HL}(x^*_{HL}) \leq 1$ and $y^*_{HL} > 0$ while $V_{HL}(x^*_{HL}) = V_{HL}(x^*_{LH})$ and $y^*_{LH} = 0$. But this contradicts $y^*_{LH} - c^*_{LH} > y^*_{HL} - c^*_{HL}$, given our preference technology defined by $\mathcal{U}$.

Appendix C: The Belgian tax system for singles

pre-tax income y	marginal tax rate (in %)
<€ 5032	0
€ 5033 –€ 6272	25
€ 6273 –€ 8304	30
€ 8305 –€ 11849	40
€ 11850 –€ 27268	45
€ 27269 –€ 40902	50
€ 40903 –€ 59990	52.5
>€ 59990	55

Appendix D: Imputation via a sample selection model

First, we present the variables used for imputing gross hourly wages σ and net hourly benefits b_h; afterwards, we show the estimates for both sample selection models.

Imputing gross hourly wages σ

In the wage equation, the independent variables are:

- age and its square (*age*, *agesq*)
- educational dummies indicating the highest achieved education level of the individual, starting from primary education (base case), lower secondary education (*dumeduc2*), higher secondary education (*dumeduc3*), higher education short type (*dumeduc4*), higher education long type (*dumeduc5*)
- a gender dummy (*sex*) taking the value of 1 for females

In the selection equation, the independent variables are:

- physical health dummies indicating the general health situation of the individual, ranging from very good (base case), good (*dumhealth2*), reasonable (*dumhealth3*), to bad (*dumhealth4*)
- mental health dummies indicating how often the individual feels depressed, ranging from never (base case), seldom (*dumdepri2*), at times (*dumdepri3*), regularly (*dumdepri4*), to frequently (*dumdepri5*); and how often the individual longs for death, ranging from never (base case), seldom (*dumdeath2*), at times (*dumdeath3*), regularly (*dumdeath4*), to frequently (*dumdeath5*)
- smoking dummies indicating smoking behaviour, ranging from never (base case), occasionally (*dumsmoke2*), to daily (*dumsmoke3*)
- care dummies indicating whether the individual has to take care for his children (*child*) taking the value of 1 if affirmative; and/or has to take care of others (*depperson*) taking the value of 1 if affirmative
- the independent variables of the wage equation

Imputing net hourly benefits b_h

The dependent variable is the marginal benefit per hour $b_h = \frac{b}{2925}$, with b the net yearly benefit. The independent variables are identical to those in the Heckman selection model imputing σ. In addition, we add in both the benefit and the selection equation civil status dummies, indicating whether the individual is divorced (*divorce*), taking the value of 1 if affirmative; widowed (*widow*), taking the value of 1 if affirmative; living together (*cohabit*), taking the value of 1 if affirmative.

Number of observations = 644, from which 136 censored and 508 uncensored. Wald's $\chi^2(7) = 265.68$ with $\Pr > \chi^2(7) = 0.00$ and likelihood ratio test of independent wage and selection equations results in $\chi^2(1) = 4.66$ with $\Pr > \chi^2(1) = 0.031$.

wage equation	Coefficient	Standard Error	$\Pr > \lvert z \rvert$
age	0.497	0.123	0.000
agesq	-0.003	0.002	0.075
dumeduc2	1.364	1.141	0.232
dumeduc3	2.834	1.103	0.010
dumeduc4	4.508	1.160	0.000
dumeduc5	5.617	1.149	0.000
sex	-0.917	0.383	0.017
cons	-2.680	2.568	0.297
selection equation	Coefficient	Standard Error	$\Pr > \lvert z \rvert$
dumhealth2	-0.576	0.176	0.001
dumhealth3	-0.686	0.227	0.002
dumhealth4	-1.221	0.476	0.010
dumdepri2	0.014	0.186	0.938
dumdepri3	-0.243	0.185	0.188
dumdepri4	-0.486	0.249	0.051
dumdepri5	-0.799	0.342	0.019
dumdeath2	0.374	0.173	0.031
dumdeath3	0.113	0.196	0.563
dumdeath4	0.160	0.297	0.591
dumdeath5	-0.657	0.352	0.062
dumsmoke2	-0.041	0.245	0.868
dumsmoke3	-0.219	0.138	0.114
child	-0.355	0.153	0.020
depperson	-0.232	0.210	0.269
age	0.179	0.040	0.000
agesq	-0.002	0.001	0.000
dumeduc2	0.587	0.254	0.021
dumeduc3	0.784	0.243	0.001
dumeduc4	1.584	0.305	0.000
dumeduc5	1.685	0.291	0.000
sex	-0.507	0.139	0.000
cons	-2.016	0.728	0.006

Number of observations = 638, from which 480 censored and 158 uncensored. Wald's $\chi^2(10) = 50.51$ with $\Pr > \chi^2(7) = 0.00$ and likelihood ratio test of independent benefit and selection equations results in $\chi^2(1) = 7.09$ with $\Pr > \chi^2(1) = 0.008$.

benefit equation	Coefficient	Standard Error	$\Pr > \lvert z \rvert$
age	0.115	0.051	0.025
agesq	-0.001	0.001	0.120
dumeduc2	0.361	0.259	0.163
dumeduc3	0.470	0.271	0.083
dumeduc4	0.181	0.358	0.613
dumeduc5	0.575	0.410	0.161
sex	-0.317	0.187	0.091
divorce	0.376	0.197	0.057
widow	-0.504	0.582	0.386
cohabit	-0.035	0.196	0.858
cons	-0.226	0.921	0.806
selection equation	Coefficient	Standard Error	$\Pr > \lvert z \rvert$
dumhealth2	0.304	0.154	0.048
dumhealth3	0.416	0.203	0.040
dumhealth4	0.384	0.415	0.355
dumdepri2	0.137	0.163	0.400
dumdepri3	0.244	0.166	0.142
dumdepri4	0.470	0.237	0.048
dumdepri5	0.334	0.329	0.311
dumdeath2	-0.464	0.154	0.003
dumdeath3	0.020	0.173	0.909
dumdeath4	0.369	0.254	0.145
dumdeath5	0.889	0.339	0.009
dumsmoke2	0.114	0.214	0.596
dumsmoke3	0.228	0.124	0.065
child	0.304	0.143	0.033
depperson	0.158	0.192	0.410
age	-0.108	0.040	0.007
agesq	0.001	0.001	0.006
dumeduc2	-0.376	0.242	0.120
dumeduc3	-0.707	0.232	0.002
dumeduc4	-1.049	0.265	0.000
dumeduc5	-1.428	0.270	0.000
sex	0.373	0.128	0.004
divorce	0.027	0.162	0.868
widow	0.036	0.490	0.941
cohabit	-0.329	0.144	0.022
cons	1.247	0.717	0.082

Appendix E: Some descriptive statistics

We report the number of observations (n), the minimum (min), the 25$^{\text{th}}$ percentile (p25), the median (p50), the 75$^{\text{th}}$ percentile (p75) and the maximum (max) for the following variables: age, normalized labour ℓ, observed gross hourly wages σ, imputed gross hourly wages $\hat{\sigma}$, observed and imputed gross hourly wages $(\sigma, \hat{\sigma})$, observed net hourly benefits b_h, imputed net hourly benefits $\hat{b}_h$ and observed and imputed net hourly benefits $(b_h, \hat{b}_h)$.

	n	min	p25	p50	p75	max
age	621	16	25	33	42	70
ℓ	621	0.000	0.231	0.554	0.615	1.000
σ	502	1.495	9.585	11.864	15.700	38.271
$\hat{\sigma}$	119	1.262	4.796	6.665	9.421	19.214
$\sigma, \hat{\sigma}$	621	1.262	8.594	11.065	14.404	38.271
b_h	143	0.166	0.915	1.849	3.051	3.661
$\hat{b}_h$	478	0.002	0.072	0.214	0.510	2.089
$b_h, \hat{b}_h$	621	0.002	0.115	0.345	0.928	3.661