Using the concept of partial age to diagnose sediment transport processes in a river

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Abstract. Steady-state sediment transport phenomena in a river (horizontal advection, deposition and resuspension) are diagnosed using the concept of partial age. The relevant partial ages are introduced for suspended and deposited sediment particles. They can be deduced from physical arguments alone, i.e. without explicitly solving the relevant partial age equations. In the process, a tentative explanation is provided as to why the suspended sediment age is independent of the vertical sediment flux at the bottom of the water column.

Introduction

Using tools of CART[1] (Constituent-oriented Age and Residence time Theory), Deleersnijder (2018) investigates some aspects of sediment transport in a highly-idealised, steady-state, one-dimensional river model. There are suspended sediment particles in the water column and, at the riverbed, lies a layer of deposited sediment. All geometrical, hydrodynamic and sediment-related parameters and variables are assumed to be constant. Seeking inspiration in Mercier and Delhez (2007), Gong and Shen (2010) as well as Ralston and Geyer (2017), the age of sediment in the water column and in the deposited layer is introduced as diagnostic variables measuring the time elapsed since entering the domain of interest through the upstream boundary.

Despite strong simplifying assumptions, Deleersnijder (2018) leaves a number of issues partly or completely unanswered. The most puzzling question probably is the following one:

Why is the suspended sediment age independent of the vertical sediment flux at the bottom of the water column?

The objective of the present working note is to make an attempt to answer this question under the key assumption that the deposited sediment layer is vertically well mixed. To do so, the concept of partial age (Deleersnijder 2014, Deleersnijder et al. 2014, Mouchet et al. 2016) is resorted to. It is seen that the relevant ages may be deduced from relatively simple physical arguments without having to solve explicitly the steady-state, one-dimensional partial age equations. Of course, these equations are also solved (Appendix C), leading to the same solutions as those inferred from physical reasoning alone.

Hereinafter classical age equations are solved to obtain suspended and deposited sediment ages. It is because a physical interpretation of these ages is rather difficult to build that another diagnostic strategy is resorting to, which relies on a generalisation of the classical age, namely the notion of partial age. This concept if briefly recalled in a general framework. Then, it is applied to the problem at hand, proving to be very successful.

It is noteworthy that the notations used in the present working note are different from those of Deleersnijder (2018), for it is convenient to adopt hereinafter notations well suited to the mathematical developments related to partial ages.

[1] www.climate.be/cart

Suspended and deposited sediment ages

The geometry of the domain is illustrated in Figure 1. The (horizontal) along-flow coordinate is denoted x, whilst z is the vertical coordinate (increasing upwards), with $z = -(H^d + H^w)$, $z = -H^w$ and $z = 0$ at the bottom of the deposited sediment layer, the river bed and the water-air interface, respectively. Positive constants U and φ represent the water velocity and the upward or downward sediment flux at the bottom of the water column, respectively. Along-flow diffusion is considered negligible. The superscript "*w*" refers to the water column, whilst "*d*" is related to the deposited sediment layer.

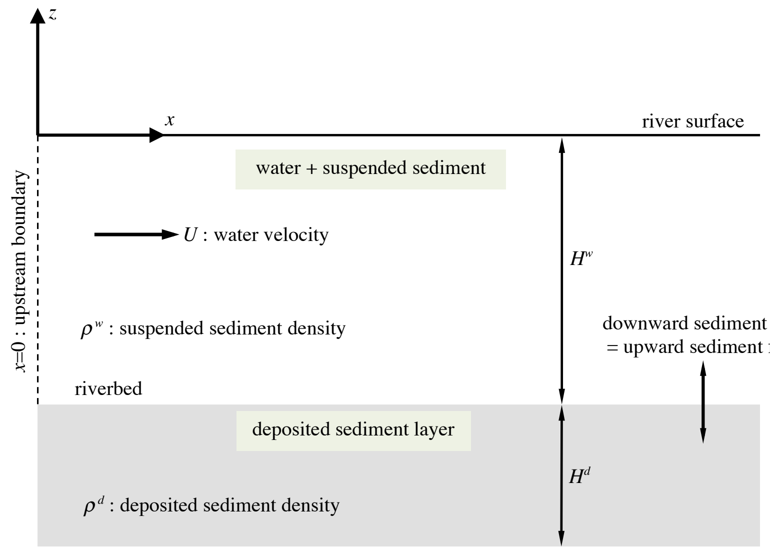

Figure 1. Illustration of the geometry and key parameters of the model used for studying sediment transport phenomena in a river. All of the parameters mentioned in the figure are positive constants.

Assuming that the water column is vertically well mixed, the suspended sediment age equation reads:

$$\frac{\partial}{\partial t}(H^w \rho^w a^w) = H^w \rho^w - \frac{\partial}{\partial x}(H^w U \rho^w a^w) - \varphi a^w + \varphi a^d \quad , \tag{1}$$

where:

- $H^w \rho^w a^w$ is the vertical age inventory of the water column;
- $H^w \rho^w$ is the ageing term;
- $H^w U \rho^w a^w$ is the (horizontal) advective age flux;
- φa^w is the downward age flux from the water column into the deposited sediment layer;
- φa^d is the upward age flux from the deposited sediment layer into to the water column.

This equation is to be solved under the upstream boundary condition

$$\left[a^w\right]_{x=0} = 0 \quad . \tag{2}$$

The deposited sediment layer is regarded as vertically well mixed: all the variables are vertically homogeneous in this layer, though there are horizontal age contrasts, which are not caused by horizontal fluxes inside this layer. The deposited sediment age equation reads:

$$\frac{\partial}{\partial t}(H^d \rho^d a^d) = H^d \rho^d + \varphi a^w - \varphi a^d \tag{3}$$

where:

- $H^d \rho^d a^d$ is the vertical age inventory of the deposited sediment layer;
- $H^d \rho^d$ is the ageing term;
- φa^w and φa^d have been defined above.

Deleersnijder (2018) derived the steady-state solution of (1)-(3):

$$a^w(x) = \frac{H^w \rho^w + H^d \rho^d}{H^w \rho^w} \frac{x}{U} > \frac{x}{U} \tag{4}$$

and

$$a^d(x) = \underbrace{\frac{H^w \rho^w + H^d \rho^d}{H^w} \frac{x}{U}}_{=a^w(x)} + \frac{H^d \rho^d}{\varphi} \tag{5}$$

If sediment particles were neither settling toward the river bed nor being resuspended from the deposited layer, they would move in the downstream direction at the same speed as the water — and the sediment age in deposited layer would grow unboundedly. As a result, the age of suspended sediment would be equal to x/U, i.e. the time needed to travel over distance x at velocity U. However, this scenario is incorrect, since sediment particles lie motionless for a certain amount of time in the deposited sediment layer. Therefore, the speed at which they move in the downstream direction is smaller than the water velocity, which is the reason why their age is larger than x/U as may be seen in (4).

As mentioned above, the deposited sediment layer is assumed to be vertically well mixed. Therefore, on each vertical the key results of reservoir theory are applicable (Appendix B). Accordingly, the average time spent in each vertical is equal to the ratio of the vertical sediment inventory ($H^d \rho^d$) to the vertical sediment flux at the bottom of the water column (φ). As a consequence, the following relation holds valid

$$a^d(x) = a^w(x) + \frac{H^d \rho^d}{\varphi} \quad , \tag{6}$$

which is the counterpart of (C.4)[2]. This result is in accordance with (5).

The sediment age in the deposited layer cannot be prescribed to be zero at the upstream boundary of the domain (x=0). This is because there is no horizontal derivative in (3). In fact, at a steady state, (3) simplifies to an algebraic equation, thereby precluding the introduction of any type of boundary condition.

Using the classical age theory, it is hardly possible to gain further insight into the sediment transport processes taken into account in the present model. However, as will be seen below, the concept of partial age allows shedding more light on these phenomena.

[2] To obtain (6) from (C.4), the following substitutions are to be made: $a \to a^d$, $a^{in} \to a^w$, $m \to H^d \rho^d$ and $\phi \to \varphi$.

Partial ages

The Lagrangian view of the classical age is as follows: every particle of the constituent under consideration is "equipped" with a single watch. The age is then the time elapsed since the watch was (re-)set to zero. The partial age consists in a generalisation of this concept. The domain of interest is partitioned into N non-overlapping subdomains. Each particle has N watches and the n-th watch is ticking only in the n-th subdomain (i.e. it is at rest in all other subdomains). The time the n-th watch has been active is the n-th partial age. Clearly, the classical age is the sum of the all the partial ages. As was seen in Mouchet et al. (2016), partial ages provide additional pieces of information about the past evolution of the particles of the constituent under study.

To apply the concept of partial age to the model dealt with herein, it seems natural to provide each sediment particle with two watches. One of them is active in the water column and is identified by subscript "w", whilst the other one ticks only in the deposited sediment layer. Subscript "d" refers to the second watch. This leads to the introduction of four partial ages: a_w^w, a_d^w, a_w^d and a_d^d. Their precise definition is given in Table 1. Clearly, the ages referred to above must satisfy the following relations:

$$a^w = a_w^w + a_d^w \tag{7}$$

and

$$a^d = a_w^d + a_d^d \quad . \tag{8}$$

Table 1. Definition of the partial ages pertaining to the sediment particles located in the water column or in the deposited sediment layer.

partial age	**location of the sediment particle**	**elapsed time measured**
a_w^w	water column	time spent in the water column
a_d^w	water column	time spent in the deposited layer of sediment
a_w^d	deposited layer of sediment	time spent in the water column
a_d^d	deposited layer of sediment	time spent in the deposited layer of sediment

At a steady state, it is possible to derive the abovementioned partial ages on the basis of physical intuition and simple arguments alone. A sediment particle located at distance x to the upstream boundary, be it in the water column or in the deposited sediment layer, has spent a time equal to x/U in the water column. Therefore, the corresponding partial ages are

$$a_w^w(x) = \frac{x}{U} \tag{9}$$

and

$$a_w^d(x) = \frac{x}{U} \tag{10}$$

It is readily understood that a relation similar to (6) applies, in the deposited layer, to the partial ages related to the time spent in the deposited layer, yielding

$$a_d^d(x) = a_d^w(x) + \frac{H^d \rho^d}{\varphi} \quad . \tag{11}$$

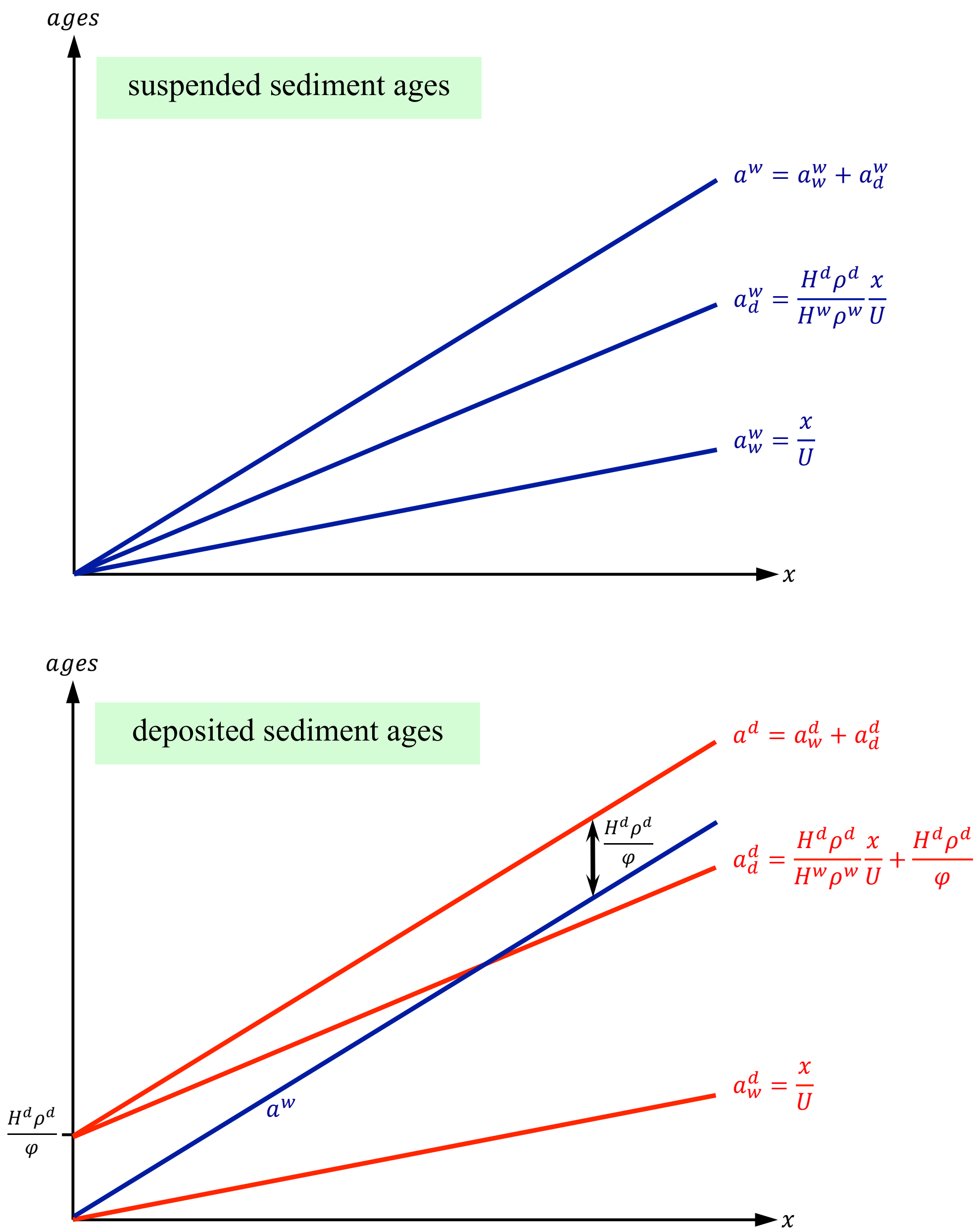

Figure 2. Graphs representing the steady-state sediment ages and partial ages derived in the present working note. They are linear function of x, the distance to the upstream boundary of the domain.

The water column and the deposited sediment layer are vertically well mixed and the exchange of sediment at the interface between these layers has no preferred direction. This is why the following relation

$$\frac{a_d^w(x)}{a_w^w(x)} = \frac{H^d \rho^d}{H^w \rho^w} \tag{12}$$

must hold valid. Then, combining (9) and (12) leads to

$$a_d^w(x) = \frac{H^d \rho^d}{H^w \rho^w} \frac{x}{U} \tag{13}$$

Finally, substituting (13) into (11) yields

$$a_d^d(x) = \frac{H^d \rho^d}{H^w \rho^w} \frac{x}{U} + \frac{H^d \rho^d}{\varphi} \tag{14}$$

Partial ages (9), (10), (13) and (14) are consistent with the classical ages derived above. Indeed, the sum of the partial ages related to the water column and those in the deposited sediment layer is equal to related classical ages (Figure 2):

$$a_w^w(x) + a_d^w(x) = \underbrace{\frac{x}{U}}_{=a_w^w} + \underbrace{\frac{H^d \rho^d}{H^w \rho^w} \frac{x}{U}}_{=a_d^w} = \frac{H^w \rho^w + H^d \rho^d}{H^w \rho^w} \frac{x}{U} = a^w(x) \tag{15}$$

$$\begin{aligned} a_w^d(x) + a_d^d(x) &= \overbrace{\frac{x}{U}}^{=a_w^d} + \overbrace{\frac{H^d \rho^d}{H^w \rho^w} \frac{x}{U} + \frac{H^d \rho^d}{\varphi}}^{=a_d^d} \\ &= \frac{H^w \rho^w + H^d \rho^d}{H^w \rho^w} \frac{x}{U} + \frac{H^d \rho^d}{\varphi} = a^d(x) \end{aligned} \tag{16}$$

Conclusion

Analysing sediment ages in the water column and in the deposited layer does not allow satisfactorily addressing the issue on which the present working note focuses, i.e. the question as to why the sediment age in the water column is independent of the vertical sediment flux at the river bed. Using the partial age concept, the answer is readily obtained. The suspended sediment age (i.e. the age of sediment particles located in the water column) is the sum of the time spent in the water column, which obviously is x/U, and the time spent in the deposited sediment layer, which is seen to be proportional to the aforementioned age because the sediment exchanges at the bottom of the water column have no preferred direction.

It must be realised, however, that all of the abovementioned explanations and interpretations are still of a tentative nature. This is because it will be necessary to find out why the sediment age in the water column remains independent of the vertical sediment flux at the river bed when the deposited sediment layer is no longer well-mixed and has layered structure (Deleersnijder 2018). Therefore, further research is needed.

Appendix A: main variables and parameters used herein

The main variables and parameters used in the present working note are listed below. The notation, meaning and units are provided.

notation	meaning of the variable or parameter	units
t	time	s
x	horizontal space coordinate	m
z	vertical space coordinate	m
U	(constant) water velocity	$\mathrm{m\,s^{-1}}$
H^w	(constant) water column height	m
ρ^w	(constant) suspended sediment density	$\mathrm{kg\,m^{-3}}$
a^w	suspended sediment age	s
a_w^w , a_d^w	suspended sediment partial ages	s
φ	(constant) upward or downward sediment flux at the river bed	$\mathrm{kg\,m^{-2}\,s}$
H^d	(constant) deposited sediment height	m
ρ^d	(constant) deposited sediment density	$\mathrm{kg\,m^{-3}}$
a^d	deposited sediment age	s
a_w^d , a_d^d	deposited sediment partial ages	s

Appendix B: age of a well-mixed reservoir

Consider an open domain containing constant mass m of a given constituent. The incoming and outgoing mass fluxes of this constituent are equal and are denoted ϕ ($\mathrm{kg\,s^{-1}}$). The age of incoming and outgoing particles are a^{in} and a^{out}, respectively. The age content of the domain is ma, where a denotes the mean age of the particles of the constituent under study that are present in the domain. The age content of the domain obeys differential equation

$$\frac{d}{dt}(ma) = \phi a^{in} - \phi a^{out} \quad . \tag{B.1}$$

If the domain is well mixed, then classical results of reservoir theory are applicable. In particular, the age of the outgoing particles is equal to the age the particles present in the reservoir (e.g. Bolin and Rodhe 1973):

$$a^{out}(t) = a(t) \quad . \tag{B.2}$$

Further assuming that the age of incoming particles is constant, the age is readily seen to be

$$a(t) = a^{in} + \frac{m}{\phi} + \left[a(0) - a^{in} - \frac{m}{\phi} \right] \exp\left(-\frac{t}{m/\phi} \right) \quad . \tag{B.3}$$

As time progresses, the age tends to a constant value that is independent of the initial condition:

$$a(\infty) = \lim_{t\to\infty} a(t) = a^{in} + \frac{m}{\phi} \quad . \tag{B.4}$$

The age of the outgoing particles is the sum of the age of the incoming particles and the ratio m/ϕ, which is the mean time spent in the domain.

Appendix C: solving the partial age equations

The age is an intensive property, but the age content (roughly speaking, the product of the mass and the age) is an extensive (i.e. additive) variable. For a given system, the budget of the age content can be made so as to ultimately derive its age. Accordingly, CART's three-dimensional age equations are derived from local age content budget considerations (Delhez et al. 1999, Deleersnijder et al. 2001). Similarly, the vertically-averaged age equations for the present model are derived from the budget of the relevant vertical age inventory. By the same token, the equations obeyed by the partial ages in the water column are obtained from the budget of the relevant vertical partial age inventory, which is influenced by ageing, if appropriate, horizontal advection and vertical fluxes at the river bed. A similar approach is resorted to in the deposited sediment layer, except that there is no horizontal transport in this subdomain. Accordingly, the partial age equations read:

$$\frac{\partial}{\partial t}(H^w \rho^w a_w^w) = \underbrace{H^w \rho^w}_{\substack{\text{ageing}\\ \text{term}}} \underbrace{- \frac{d}{dx}(H^w U \rho^w a_w^w)}_{\text{advection term}} \underbrace{- \varphi a_w^w + \varphi a_w^d}_{\substack{\text{vertical age fluxes}\\ \text{through the river bed}}} \quad , \tag{C.1}$$

$$\frac{\partial}{\partial t}(H^w \rho^w a_d^w) = \underbrace{- \frac{d}{dx}(H^w U \rho^w a_d^w)}_{\text{advection term}} \underbrace{- \varphi a_d^w + \varphi a_d^d}_{\substack{\text{vertical age fluxes}\\ \text{through the river bed}}} \quad , \tag{C.2}$$

$$\frac{\partial}{\partial t}(H^d \rho^d a_d^d) = \underbrace{H^d \rho^d}_{\substack{\text{ageing}\\ \text{term}}} \underbrace{+ \varphi a_d^w - \varphi a_d^d}_{\substack{\text{vertical age fluxes}\\ \text{through the river bed}}} \quad , \tag{C.3}$$

and

$$\frac{\partial}{\partial t}(H^d \rho^d a_w^d) = \underbrace{\varphi a_w^w - \varphi a_w^d}_{\substack{\text{vertical age fluxes}\\ \text{through the river bed}}} \quad . \tag{C.4}$$

Equations (C1)-(C.4) are to be solved under the following boundary conditions:

$$\left[a_w^w(x)\right]_{x=0} \;=\; 0 \;=\; \left[a_d^w(x)\right]_{x=0} \quad . \tag{C.5}$$

It is noteworthy that there is no ageing term in (C.2), i.e. the equation for a_d^w . This is because the watch associated with the deposited sediment layer, which is identified by subscript "*d*", is not ticking in the water column. For a similar reason, there is no ageing term in (C.4).

As mentioned above, the steady-state solutions of (C1)-(C.4) should be

$$a_w^w(x) \;=\; \frac{x}{U} \quad , \tag{C.6}$$

$$a_d^w(x) \;=\; \frac{H^d\rho^d}{H^w\rho^w}\frac{x}{U} \quad , \tag{C.7}$$

$$a_w^d(x) \;=\; \frac{x}{U} \quad , \tag{C.8}$$

and

$$a_d^d(x) \;=\; \frac{H^d\rho^d}{H^w\rho^w}\frac{x}{U} \;+\; \frac{H^d\rho^d}{\varphi} \quad . \tag{C.9}$$

These solutions obviously satisfy boundary conditions (C.5). Below, it will be seen that they also satisfy the steady-state versions of (C.1)-(C.4). Substituting (C.6) and (C.8) into the right-hand side of (C.1) leads to

$$\begin{aligned} &H^w\rho^w \;-\; \frac{d}{dx}(H^wU\rho^wa_w^w) \;-\; \varphi a_w^w \;+\; \varphi a_w^d \\ &\quad= H^w\rho^w \;-\; \underbrace{\frac{d}{dx}\left(H^wU\rho^w\frac{x}{U}\right)}_{=H^w\rho^w} \underbrace{-\;\varphi\frac{x}{U} \;+\; \varphi\frac{x}{U}}_{=0} \;=\; H^w\rho^w \;-\; H^w\rho^w \;=\; 0 \end{aligned} \tag{C.10}$$

Substituting (C.7) and (C.9) into the right-hand side of (C.2) leads to

$$\begin{aligned} &-\;\frac{d}{dx}(H^wU\rho^wa_d^w) \;-\; \varphi a_d^w \;+\; \varphi a_d^d \\ &\quad= -\;\underbrace{\frac{d}{dx}\left(H^wU\rho^w\frac{H^d\rho^d}{H^w\rho^w}\frac{x}{U}\right)}_{=H^d\rho^d} \;-\; \varphi\frac{H^d\rho^d}{H^w\rho^w}\frac{x}{U} \;+\; \varphi\left(\frac{H^d\rho^d}{H^w\rho^w}\frac{x}{U}+\frac{H^d\rho^d}{\varphi}\right) \\ &\quad= -\,H^d\rho^d \;-\; \varphi\frac{H^d\rho^d}{H^w\rho^w}\frac{x}{U} \;+\; \varphi\frac{H^d\rho^d}{H^w\rho^w}\frac{x}{U} \;+\; H^d\rho^d \;=\; 0 \end{aligned} \tag{C.11}$$

Substituting (C.7) and (C.9) into the right-hand side of (C.3) leads to

$$\begin{aligned} &H^d\rho^d \;+\; \varphi a_d^w \;-\; \varphi a_d^d \\ &\quad= H^d\rho^d \;+\; \varphi\frac{H^d\rho^d}{H^w\rho^w}\frac{x}{U} \;-\; \varphi\left(\frac{H^d\rho^d}{H^w\rho^w}\frac{x}{U}+\frac{H^d\rho^d}{\varphi}\right) \\ &\quad= H^d\rho^d \;+\; \varphi\frac{H^d\rho^d}{H^w\rho^w}\frac{x}{U} \;-\; \varphi\frac{H^d\rho^d}{H^w\rho^w}\frac{x}{U} \;-\; H^d\rho^d \;=\; 0 \end{aligned} \tag{C.12}$$

Substituting (C.6) and (C.8) into the right-hand side of (C.4) leads to

$$\varphi a_w^w - \varphi a_w^d = \varphi \frac{x}{U} - \varphi \frac{x}{U} = 0 \qquad \text{(C.13)}$$

Q.E.D.

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