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Optimal feedback control of rhythmic movements - the bouncing ball revisited

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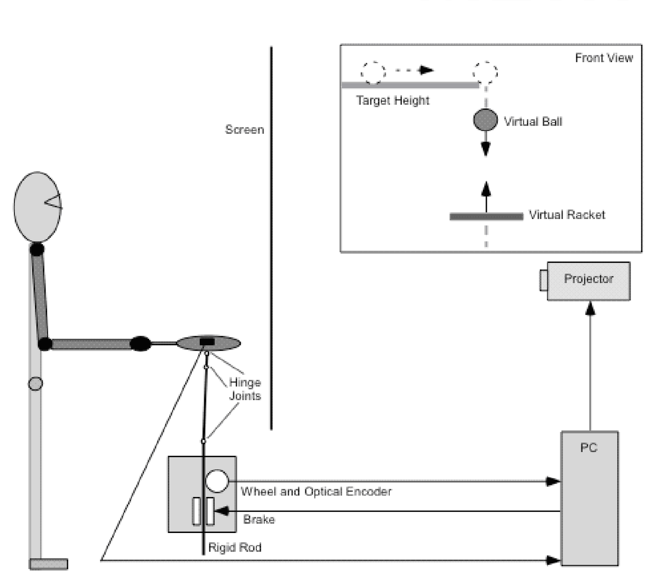
INTRODUCTION

- Optimal Feedback Control (OFC) has been introduced to motor control as a potential mechanism for the functioning of the nervous system (Todorov, 2005). This framework has been primarily tested in model tasks involving reaching (Liu & Todorov, 2007; Diedrichsen 2007). Can this framework be generalized beyond discrete, i.e. to rhythmic movements?
- Although rhythmic movements have been discussed as a different class of movements with different control principles (Schaal et al., 2004; Hogan & Sternad, 2007), they may also involve active "discrete" control elements, e.g., in the context of a visually specified task.
- Rhythmically bouncing a ball constitutes such a hybrid movement. While a series of studies have revealed that passive stability is central, recent studies also demonstrated that active control is present especially to keep the ball bouncing to a visually specified target height (Wei et al, 2007, 2008). How are these control elements blended into the rhythmic task?
- Can OFC be adapted to also account for rhythmic movements? Can the continuous dynamics be blended with intermittent control?
- To provide a test bed for OFC the experiment created conditions that would present additional challenges for the controller: the task was executed under different gravity conditions that slowed down the movements and modified the racket trajectory significantly.

SUMMARY

The control of rhythmic intermittent interactions between effector and object is accounted for by a hybrid Two-Layer architecture: Layer-1, the discrete-time dynamics, computes the state-to-reach of the effector at the next impact in order to bring the object to its target state. Layer-2, the continuous-time dynamics, designs the effector trajectory between impacts with the objective to reach the target state at the end of the cycle. Several optimality principles guided these their designs: Layer-1 minimized the correlation between successive apex errors, such that any apex error was on average corrected in one impact, i.e. the fastest possible; Layer-2 exploited classical OFC principles as applied for the execution of discrete movements. Passive stability mechanisms might be embedded in the controller design, e.g. if the racket trajectory decelerates around impacts (Wei et al., 2008).

METHODS



- 8 subjects practiced the task of bouncing a ball with a racket during a single session of 1 hour;
- the ball was only virtual;
- the session was divided into 7 blocks, each with different gravity acting on the ball (from g_1=0.61m/s² to g_7=9.81 m/s²).

$$T=\sqrt{\frac{8}{g}\left(b^\star_{apex}-b^\star\right)}$$

When the subject bounced the ball to the specified target height, different gravities introduced different bounce-to-bounce periods.

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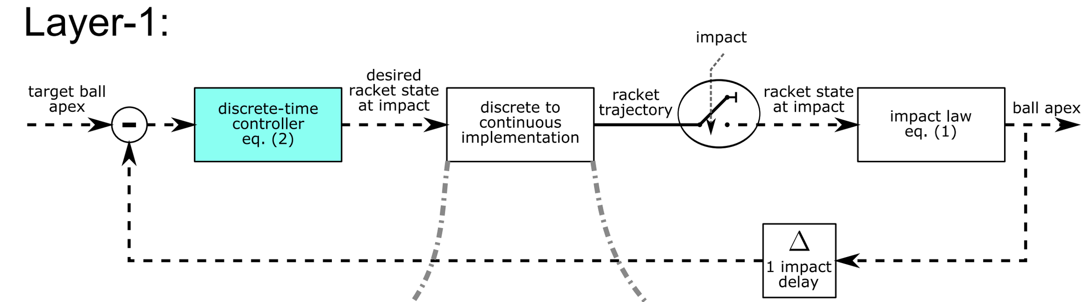
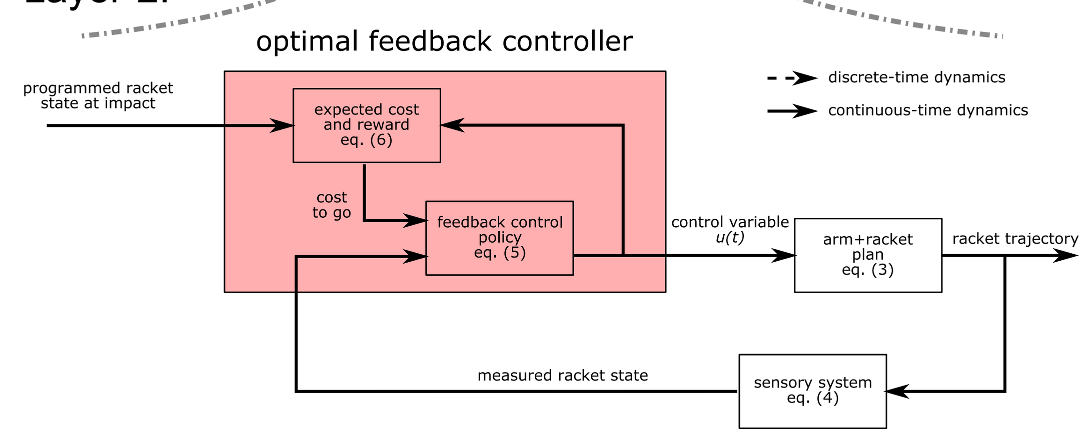
TWO-LAYER MODEL

Layer-1:



Layer-2:



Impact law:

$$\left(\dot b_k-\dot r_k\right) = -\alpha\left(\dot b_k^- - \dot r_k\right)$$
$$\Leftrightarrow \dot b_k = -\alpha \dot b_k^- + (1+\alpha)\dot r_k \qquad (1)$$

Discrete-time controller:

$$\dot r_{des,k+1}=\dot r_{des,k}+\kappa_D\left(b^\star_{apex}-b_{apex,k}\right) \qquad (2)$$

Arm+racket plan:

$$I\ddot\theta(t)+\gamma\dot\theta(t)=f(t)$$
$$\tau\dot f(t)+f(t)=u(t)+cu(t)\dot\sigma(t)$$

discretization:

$$\mathbf{x}_{t+1}=A\mathbf{x}_t+Bu_t+cu_t\epsilon_t \qquad (3)$$

Sensory system:

$$\hat{\mathbf{x}}_t=\mathbf{x}_t+N\eta_t \qquad (4)$$

Control policy:

$$u_t=-L_t\hat{\mathbf{x}}_t \qquad (5)$$

Cost function:

$$Q=w_{pos}\int_{t_k}^{t_{k+1}}\|r(t)-r_0\|^2\,dt+\|\dot r_{k+1}-\dot r_{des,k+1}\|^2+w_{energy}\int_{t_k}^{t_{k+1}}\|u(t)\|^2\,dt \qquad (6)$$

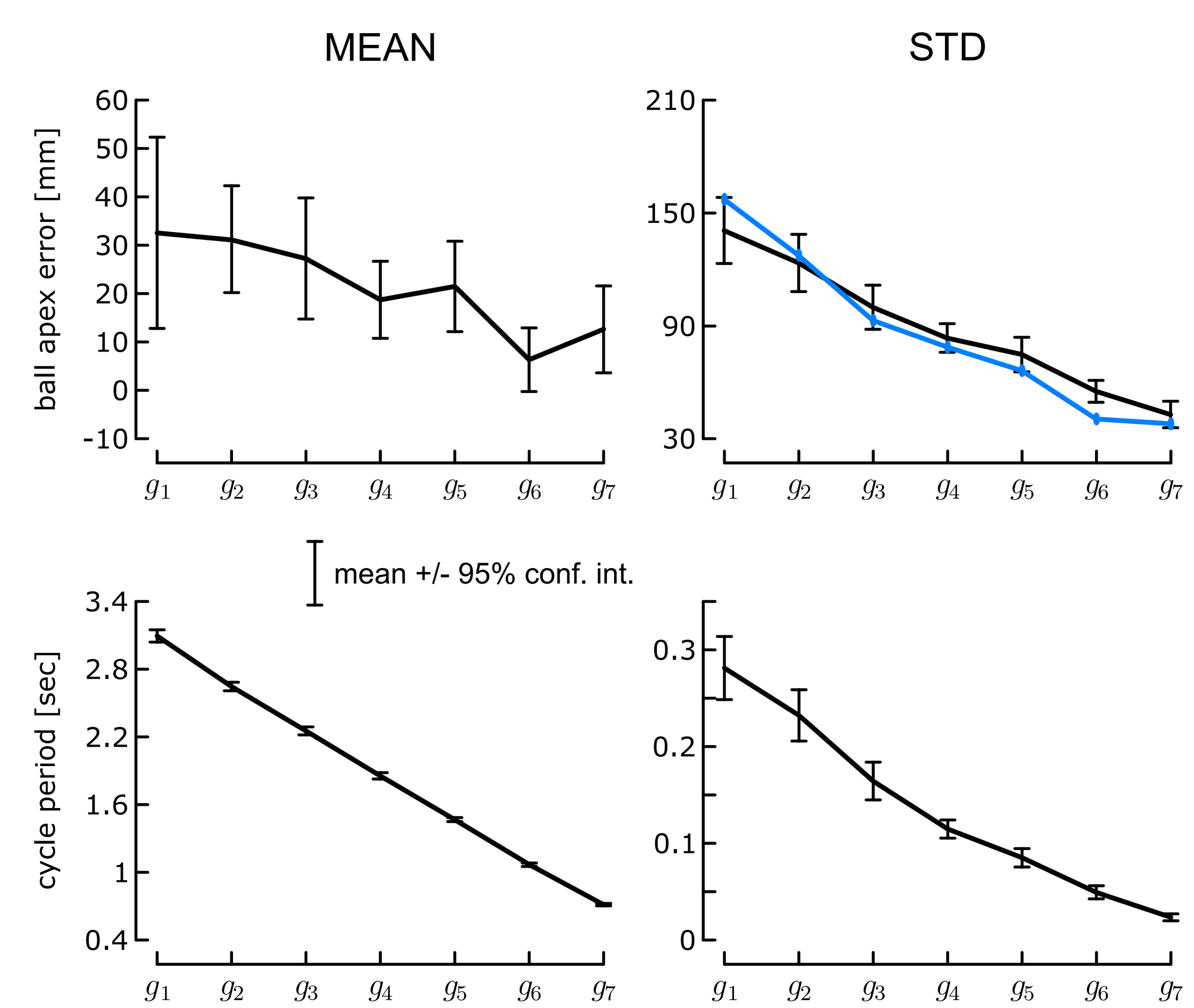
The ball state is only modified at impact. Layer-1 computes the racket position and velocity at the next impact time

$$t_{k+1}=t_k+\frac{\dot b_k+\sqrt{\dot b_k^2-2g(b^\star-b_k)}}{g}$$

that then bounces the ball up to the desired apex.
Since the racket trajectory is time-continuous, Layer-2 applies optimal feedback that brings the racket to this target state/time, while minimizing the control cost.

The state vector x includes racket position, velocity, the torque produced by the muscle, and the desired target velocity.

APEX ERROR AND MOVEMENT PERIOD

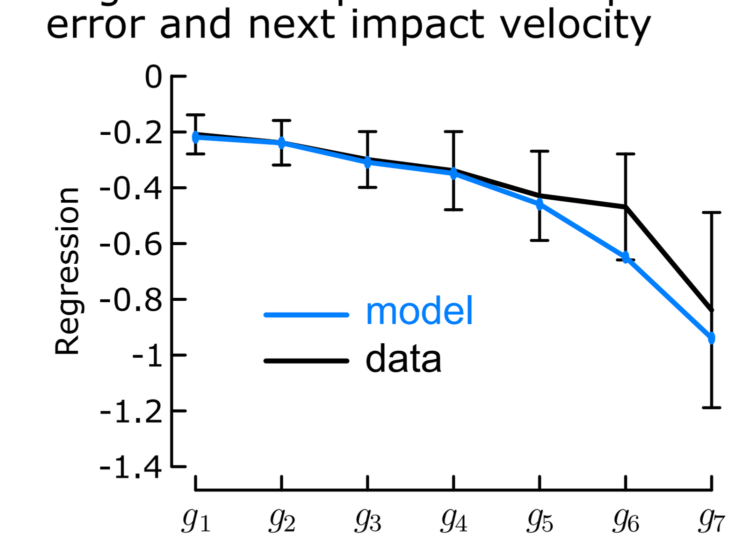


Subjects slightly overshot the target in all gravity conditions. Cycle period scaled linearly with the gravity as expected.

Smaller gravity conditions (longer period) induced more variability in both apex error and cycle period.

IMPACT VELOCITY

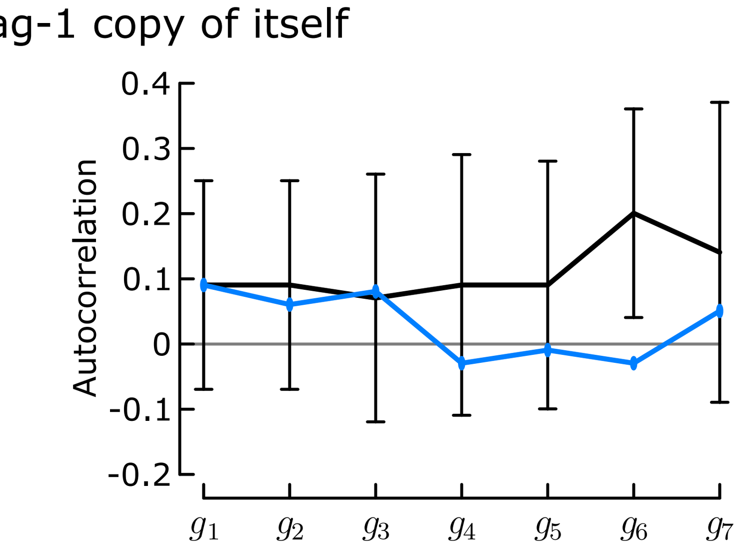
Regression slope between apex error and next impact velocity



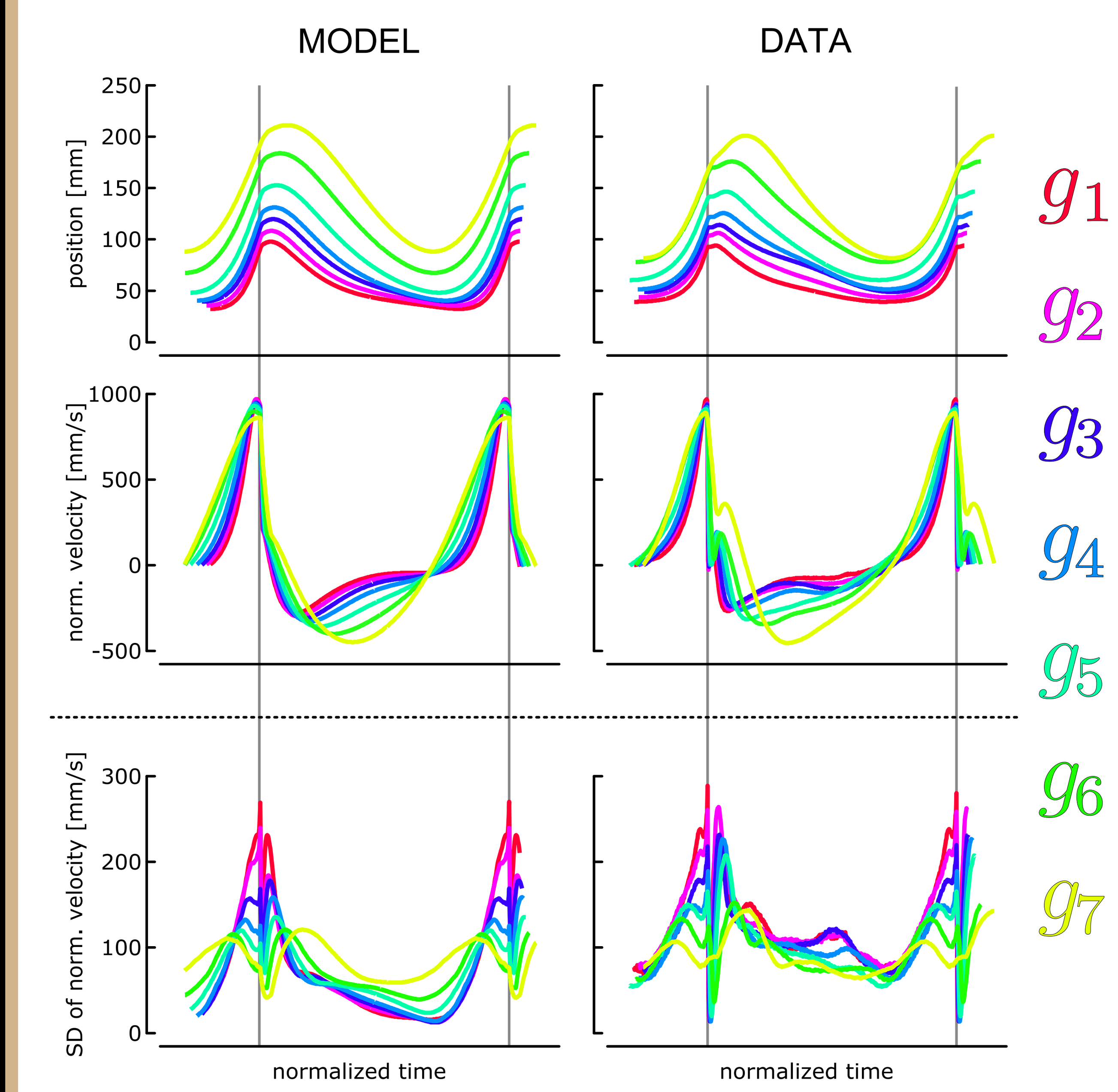
The impact gain tunes the correlation between apex errors at impact *k* (input) and the racket velocity at impact *k+1* (output); negative feedback.

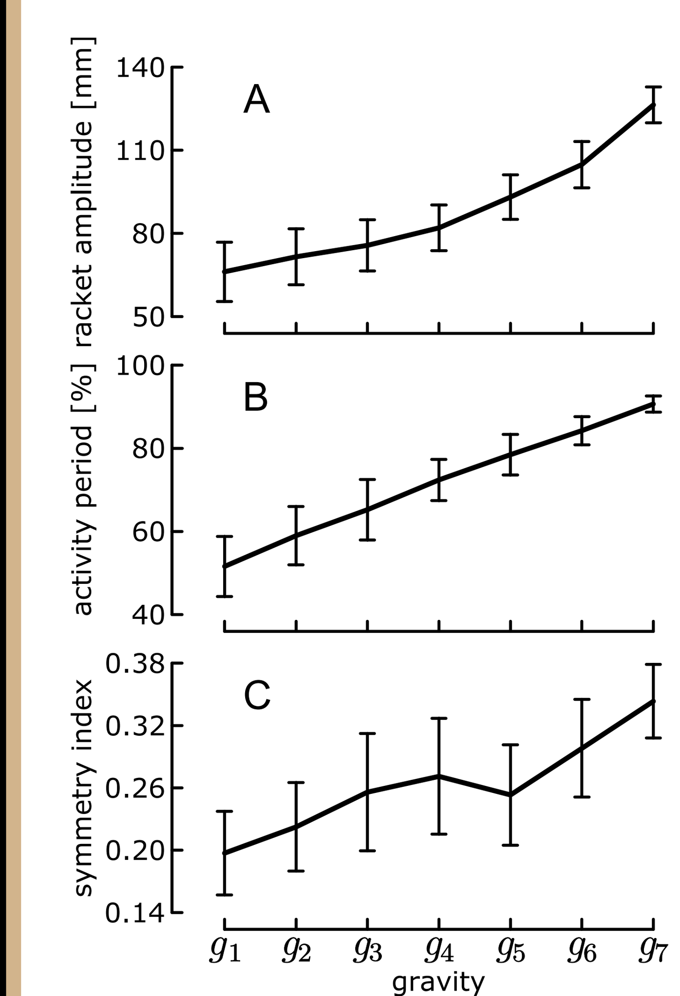
By fine-tuning the gain of Layer-1 ($\kappa_D = 0.5g_i$), the model predicts that the apex errors become uncorrelated.

Pearson correlation coefficien between apex error and lag-1 copy of itself



RACKET KINEMATICS





Manipulation of gravity slowed the cycle period and significantly changed the racket trajectories: The longer the period, the longer the dwell intervals between two successive impacts. Accordingly, racket amplitude (A), the percent of time corresponding to actual movement (B), and the symmetry index, i.e. the relative position of minimum velocity within the cycle (C), changed across gravity conditions.

The matrix $N=diag(0,0,0,n)$ in the model of the sensory system (4) models the inaccuracy to compute the actual velocity to reach at impact. A constant n for all gravity conditions predicts that (a) the variability in the racket trajectory should by larger for slower movements; (b) a slight twitch is velocity variability just before impact (fine control); and (c) the stabilization of the ball apex should be less accurate for slower periods.