CERME 13

13TH CONGRESS OF THE EUROPEAN SOCIETY FOR RESEARCH IN MATHEMATICS EDUCATION

10-14 July 2023
Budapest
Hungary

PROCEEDINGS OF THE THIRTEENTH CONGRESS OF THE EUROPEAN SOCIETY FOR RESEARCH IN MATHEMATICS EDUCATION

Editors: Paul Drijvers, Csaba Csapodi, Hanna Palmér, Katalin Gosztonyi and Eszter Kónya

Organised by: Alfréd Rényi Institute of Mathematics and Eötvös Loránd University
Budapest, Hungary

2023

erme european society for research in mathematics education

Alfréd Rényi Institute of Mathematics

Eötvös Loránd University
Faculty of Science

Publisher

Alfréd Rényi Institute of Mathematics, Budapest, Hungary and ERME
ISBN 978-963-7031-04-5

Recommended citation for the proceedings

Drijvers, P., Csapodi, C., Palmér, H., Gosztonyi, K., & Kónya, E. (Eds.). (2023). *Proceedings of the Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13).* Alfréd Rényi Institute of Mathematics and ERME.

Recommended citation for single entries in the proceedings

Authors (2023). Title of paper/poster. In P. Drijvers, C. Csapodi, H. Palmér, K. Gosztonyi, & E. Kónya (Eds.), *Proceedings of the Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13)* (pp. xx–yy). Alfréd Rényi Institute of Mathematics and ERME.

A sketch of analysis of the teacher's moves towards the implementation of "scientific debates" in the mathematics classroom

Daniel Zimmer[1], Laure Ninove[1,2] and Vanessa Hanin[1]

[1]UCLouvain, Belgium; daniel.zimmer@uclouvain.be

[2]Haute École Vinci, Belgium

As part of an ongoing research on the practice of "scientific debate" in French-speaking Belgium, we observed some teachers practicing scientific debates in their mathematics courses. We notice they intervene more in the discussion than what we would naively expect. In this preliminary study, we analyse the teacher's discourse during one short discussion episode and try to classify his interventions to understand how he supports his pupils to voice their opinion about the discussed problem and how he pushes them to question their own opinions.

Keywords: Classroom discussions, critical thinking, interaction, mathematics education, inquiry-based teaching.

Introduction

So-called "scientific debate" in mathematical classrooms was developed as a way to foster critical thinking and allow pupils to develop deeper mathematical understanding. The implementation of such discussions in the classroom requires a radical change in the teacher's posture and attitude. In theory, as advised by the programmatic work of Legrand (1993, 2001) and his colleagues of the IREM de Grenoble (e.g. Leroux & Lecorre, 2007), inspired amongst others by the work of Brousseau (1992) and Lakatos (1976), the implementation of scientific debate in the mathematics classroom intends to let the students experience, to some degree, the activity of a scientific community and share with the teacher the responsibility of the validity of the discussed arguments. This approach can be linked to inquiry-based mathematics teaching (see for example Artigue & Blomhøj, 2013), with a particular emphasis on verbalisation by the pupils in whole-classroom discussions. Few works in the French-speaking tradition attempt to characterize explicitly the role of the teacher in such discussions.

We observed two experienced teachers practicing scientific debates on a regular basis in their classrooms – the first one in middle school, with pupils aged 11 to 13 years old, and the second one in mathematical teacher training. In this paper, we focus on a short debate that was recorded in a grade 8 classroom of the first aforementioned teacher. When reviewing debates recorded in classrooms, we found that the teacher intervenes in the discussion more that we would expect: almost half of the speaking turns in some instances. This came as a surprise as one goal of the practice of scientific debate in the classroom is to let pupils express themselves and reclaim speech from the teacher.

If we consider the usual role of the teacher – Hattie (2012), for example, reports that 89% of the classroom talk is teacher talking and this is likely the case in most "traditional" classrooms around the globe – it is actually not so surprising that implementing fruitful classroom discussions would require the teacher to actively model mathematical argumentation and stimulate the pupils' participation. This concurs with the remark of Carreyre and Salin (1982) about the role of the teacher

in the devolution of a problem, as cited by Margolinas (1993) in the context of Brousseau's theory of didactical situations:

> The second condition [concerning the teacher's role] concerns the way the teacher invests him/herself in the activity. A wrong interpretation of this type of work might induce to think the teacher has to stay neutral in order not to influence the kids. *This neutrality should be limited to judgements emitted about their propositions*. On the contrary, the kids must feel that the teacher expects a lot from them, that s/he is engaged in their tasks, that s/he shares the joy of success and the disappointment of a temporary failure; his/her behaviour must be constantly supportive. (as cited in Margolinas, 1993, p. 45. The translation is ours, but the emphasis is Margolinas.)

A natural question is thus to try and characterize the content and purpose of these interventions. Do they address the content of the discussion or the mere organisation of it? The research question we address in this paper is thus the following: "What is the content of a teacher's interventions and how does he support his pupils during the conduct of a scientific debate in his classroom?"

Context of the observed discussion

The observed classroom is a grade 8 half-class[1] with pupils from multicultural, unprivileged background. Only ten pupils were present. The teacher is a member of the Groupe d'Enseignement Mathématique (GEM)[2], who practiced scientific debate in his classes for a few years and is committed to teach his pupils critical thinking through discussion and mathematical arguments.

In the presented extract, the class discussed a problem that was posed to the pupils as part of a test about exponentiation of integer numbers. Although this discussion happened in the beginning of the schoolyear (end of September), the pupils had the same teacher in grade 7 and have already practiced such debates previously. The problem is the following: "You are playing a game of double or nothing where you are asked twenty questions. A correct answer on the first question yields one euro. For each subsequent correct answer, your gain doubles, so that the second correct answer yields two euros, the third yields four euros, and so on. How many correct answers do you need to give to obtain 256 euros? And 1024?" After the test, one pupil asked the teacher to discuss the problem with the class. The (recorded) discussion happens the next time the class got together. It is worth noting that the demand for discussing this problem with the whole class stemmed from one of the pupils. Three pupils proceeded to explain their reasoning, two of them being correct and one of them, failing to acknowledge the fact that the first question is worth one euro, hence 2^0, concluded she needed to answer correctly to only eight questions to gain 256 euros, because $256 = 2^8$. In total, seven pupils actually intervened in the discussion .

This episode is consistent with the definition of "scientific debate" (Legrand, 2001): in a whole-classroom discussion mediated by the teacher, pupils got to publicly defend their ideas and give explanations and arguments. During this discussion, the teacher did not evaluate the truth of proposed

[1] To better mathematical learning, this school chose to split grade 8 classrooms in two for math lessons.

[2] Founded in 1977 by Nicolas Rouche, the GEM is a group consisting of math teachers and math teacher trainers who gather regularly to work together on building collaborative teaching resources.

arguments but rather ensured that everyone had an opportunity to understand others' arguments and that the discussion stayed focused.

Results

Preliminary quantitative remarks

In about thirty minutes, we counted 202 speaking turns, 90 of which are the teacher talking. The teacher's interventions are in average about 27 words/speaking turn, while the pupils' interventions, although shorter, are approximately 15.5 words/speaking turn. This is significantly higher than what is observed in traditional "Initiation-Response-Evaluation" (I-R-E) patterns as reported for example by Veyrunes and Saury, who measured average pupils' responses of approximately 3 words/answer (2009, p. 10). In terms of number of words, the teacher's interventions amount for about 58% of the transcript, while the pupils' share amounts to about 42% of the words.

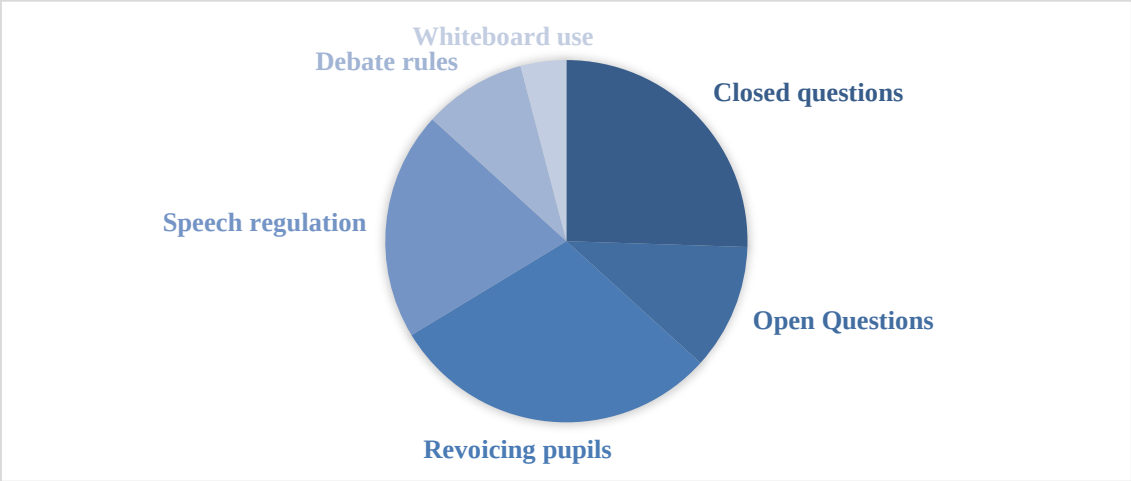

Figure 1: Representation of the repartition of teacher interventions during the debate

To analyse the teacher's interventions, the first author read the transcript and openly coded the teacher's utterances. We then proceeded to assemble the generated codes into meaningful categories. The categories which emerged and the share they represent in the teacher's discourse (in terms of number of speaking turns) are shown on Figure 1 above. We detail below the different chosen categories.

Questioning the pupils

More than a third of the teacher's interventions during the observed discussion are dedicated to questioning the students. Ten interventions out of the ninety are open questions, asking pupils to react and present their thinking to the classroom, as for example in the following excerpt.

33[3] Teacher: […] Qu'est-ce que tu penses que tu pourrais répondre,
[…] What do you think you can answer,
et quelle méthode tu peux utiliser ?
and which method can you use?

[3] Numbers refer to speaking turns during the same lesson. In about 45 minutes, there were in this case 372 speaking turns. Only the first 202 speaking turns (about 30 minutes) were addressing the presented problem.

34	Amina[4]:	Moi j'avais fait un cube de cent carrés. *I did a cube of a hundred squares.*
35	Teacher:	Pardon ? *Excuse me?*
36	Amina:	Un cube de cent carrés, enfin je prends un deux fois deux. *A cube of a hundred squares, well I take one two times two.*
37	Teacher:	Tu veux venir au tableau ? *Do you want to use the board?*

Here, the first open question, turn 33, prompted the pupil to explain her reasoning to the classroom.

During the discussion, the teacher also asked a fairly large number of closed questions, for example asking a pupil for more precision on a specific point, or asking the class whether they understood or were convinced by some pupil's argument.

Most of these closed questions happened as part of traditional I-R-E patterns, towards the beginning and the end of the discussion. In the beginning of the lesson, the teacher proceeded to recall details of the discussed problem by using pupils' answers, and towards the end of the discussion, he leaned on a pupil's answers to recapitulate a reasoning used to assert that one should answer nine questions correctly to gain 256 euros:

166	Amina:	Du coup, si on a neuf réponses, on a 256 euros, c'est pour ça que, pour moi *Well, if we have nine answers, we get 256 euros, that's why, for me* c'est neuf réponses. *it's nine answers.*
167	Teacher:	Donc une réponse c'est un euro. *So, one answer is one euro.*
168 – 180		[I-R-E sequence where the pupil and the teacher advance through each case: one answer, one euro, two answers, two euros and so on]
181	Teacher:	Neuf réponses ? *Nine answers?*
182	Amina:	Deux cents cinquante-six. *Two hundred fifty-six.*
183	Teacher:	OK. Est-ce que, lorsque je fais ça il y a quelque chose qui ne va pas ? *OK. Is there anything wrong when I do this?* Est-ce qu'on peut trouver une faille dans mon raisonnement ? *Can we find a breach in my reasoning?* C'est possible, je ne sais pas. *It's possible, I don't know.*

The teacher however concluded this last sequence by an open-ended question, asking the pupils whether they agree or can oppose something to this reasoning. This resort to I-R-E patterns inside a "debate" phase came as a surprise to us at first, as the goal of such phases is to let pupils gain more control over the discussion. It could be explained, following Veyrunes and Saury (2009), by the emergent character of such I-R-E dialogical sequences: as these authors argue, these patterns "emerge from interactions through processes of self-organisation" (p. 12) and many teachers use them almost without noticing. As Hersant (2004) remarks, this type of dialogic teaching tends to substitute itself

[4] To ensure anonymity, pupils' names have been changed to pseudonyms throughout.

to a "proper" institutionalization phase in some teacher's practice; here, it takes the place of monological sum-ups which could have been made by the teacher.

Revoicing and reformulating pupils' interventions

In about a third of his speaking turns, the teacher repeated in some way the pupils' answers. This kind of repetition can be very quick and literal, but sometimes the teacher went on to rephrase some pupil's argument to ensure it is being understood by the whole classroom and pupils can react to it properly. As we will see below, this was also used by the teacher to dodge the evaluation of pupils' contributions.

Distributing and regulating speech

A prominent role of the teacher is to regulate talk and give the floor to different pupils. Seventeen out of ninety speaking turns were dedicated to give voice to one of the pupils. Sometimes the teacher just gave permission to speak to pupils raising their hand and asking to participate, but sometimes he also selected pupils frowning or otherwise visibly disagreeing with the presented argument to react and add to the discussion, as exemplified below, turn 53. In a few instances, he also asked a pupil to repeat their argument if it was not loud enough to be widely heard, in particular when he himself couldn't hear it properly.

53	Teacher:	[…] Donc Amina propose… Siam, t'es pas d'accord, t'as pas l'air d'accord. *[…] So, Amina proposes… Siam, you disagree, you look like you disagree.*
54	Siam:	Elle a pas compté la – *She didn't count the –*
55	Teacher:	Tu peux te tourner. *You can turn around.*
56	Siam:	T'as pas compté de la bonne façon, parce qu'il faut faire… *You didn't count the right way, because you have to…* Il faut compter à partir du deux. […] *You have to begin counting at two. […]*

Setting and maintaining the discussion rules

One function of the teacher's interventions is to ensure that the talking happens smoothly and that pupils stay focused on mathematical arguments. He enjoined them in the beginning of the lesson to listen to others, to react respectfully, and to address directly their "opponent" instead of the teacher. This last rule is a difficult attitude to maintain for pupils in our observations of whole-classroom discussions. The teacher remains a privileged interlocutor for the pupils and it is difficult for them to address each other directly. In ninety interventions from the teacher in the observed period, nine were dedicated to recalling the rules, six of which were specifically enjoining them to address the other pupils instead of the teacher. See for example turn 55 above, where the teacher asked Siam to face towards Amina as she opposed her argument. One intervention was about respectfully listening to the speaker and the other two interventions were of a more "philosophical" nature, urging the pupils to take a step back from the heat of the debate and try to genuinely listen to other's arguments and question their point of view. This last intervention is reproduced below, turns 131 and 133.

131	Teacher:	[…] Alors, je vous rappelle, peut-être une petite règle par rapport au débat : *[…] So, I remind you, maybe a little rule about debates:* on essaye de se dire – plutôt, d'écouter l'argument qui est donné.

we try to think – instead, of listening to the argument given.
Plutôt que de se dire, ce que je pense est forcément la bonne solution,
Instead of thinking that what I think has to be the correct answer,
on essaye de s'ouvrir et de se dire –
we try to open up

132 Siam: Mais monsieur, j'ai juste oublié de dire –
But Mister, I just forgot to say –

133 Teacher: Attends Siam, laisse-moi terminer s'il-te-plaît. Donc, j'ai juste dit :
Wait Siam, let me finish please. So, I just said:
on essaye d'écouter les réponses de tout le monde, ça va ?
we try to listen to everybody's answers, OK?
Et puis à la fin, même si on n'est pas d'accord – on a le droit,
And in the end, even if we disagree – we have the right,
on peut rester, et se dire à la fin : « Ben finalement, je suis pas d'accord ».
we can stay, and think in the end: "Well, actually, I disagree".
Mais quand même, on se dit, j'ouvre un peu mon esprit et se dire,
But still, we say, I open up my mind a little and we say,
peut-être que la solution est différente de celle que je pense. Peut-être
maybe that the answer is different than the one I think. Maybe.
Peut-être qu'elle est fausse, peut-être qu'elle est pas fausse.
Maybe it is wrong, maybe it is not wrong.
Mais dans ce cas-là, il faut donner un argument, plutôt mathématique.
But in that case, we have to give an argument, rather mathematical.
Ça va ? C'est vraiment l'argument mathématique. Oui, Siam.
OK? It's really about the mathematical argument. Yes, Siam.

Use of the whiteboard

In our observations, the pupils made use of the whiteboard freely to explain their reasoning. When noticing an argument was fairly long, the teacher asked the pupils whether they wanted to come forward and write on the board. One purpose of his interventions was thus to invite the pupils to use the board, exemplified above in speaking turn 37. Conversely, the pupils also asked to come to the board when they felt they needed it to explain their reasoning.

Refusal to explicitly evaluate pupils' contributions

It is interesting to observe that no "Evaluation" category emerged in our analysis of the teacher's talk. Indeed, during the whole discussion, the teacher refrained from explicitly evaluating the pupils' contributions. Even during the observed I-R-E sequences, he merely repeats the pupils' answers, without further validation. The question whether pupils interpret this repeating as (implicit) validation remains open.

Towards the end of the discussion, pupils requested explicit evaluation from the teacher, but he refused to give one and instead repeated the false arguments given by a pupil, asking the pupils to identify what's wrong or give reasons why they disagree.

153 Pupil: Mais du coup c'est quoi [la réponse] ?
But at the end of the day, what is it [the anwer]?

154 Teacher: Hmm…
Ehm…

155 Class: <laughs in the classroom>

156	Pupil[5]:	C'est neuf. *It's nine.*
157	Teacher:	Ben si tu… Est-ce que tu es convaincue ? *Well if you… Are you convinced?* Qu'est-ce que tu pourrais donner de plus pour être convaincue ? *What more could you say to be convinced?*
158	Pupil:	Ben que vous dites la réponse. *Well, that you give the answer.*
159	Teacher:	Pour convaincre… Bon alors, moi je suis comme Siam, je crois dans ma tête *To convince… Well then, I am like Siam, I think in my head* que la réponse c'est neuf, parce que j'ai vu dans le cours précédent… *that the answer is nine, because I saw in the previous course that…*
160	Siam:	Non, j'ai dit huit, Monsieur. *No, I said eight, Mister.*
161	Teacher:	Attends, pardon, oui t'as dit huit. Je pense que c'est huit, parce que *Wait, sorry, yes, you said eight. I think it is eight, because* j'ai vu dans le cours au début du chapitre une certaine légende de Sissa […] *I saw in the beginning of the chapter a certain legend of Sissa […]*

Discussion

We can identify three main roles fulfilled by the teacher's interventions in the discussion, each amounting here to about a third of interventions. The first one is the maintaining of the focus of the discussion and the practical organisation of it. This role is harnessed by the interventions addressing debate rules, whiteboard use and distribution of speech. The second one is to repeat and paraphrase the presented arguments, therefore ensuring that every pupil has a chance to properly understand it and react to it. The last role is questioning. We draw a distinction between closed and open questions, which serve different purposes. Open questioning is used to open the discussion and give the space to pupils to express themselves, while the closed questioning allows the teacher to locally help pupils elucidate their reasoning. Those three roles are of course interconnected, as for example revoicing and questioning can also concur to maintaining the focus on the discussed problem.

It might be surprising at first glance to notice the emergence of "Initiation-Response-Evaluation" patterns in the debate, because this practice implies a fairly strong control of the teacher, contrary to the openness to pupil's reasonings promoted by scientific debates. However, these are confined to the introduction and conclusion of the discussion and the teacher mostly does not interfere to validate or invalidate the pupils' arguments in the heart of the debate. As already mentioned, the emergence of such patterns is consistent with previous research by Veyrunes and Saury (2009), who show the recurrence of such patterns in teachers speak, even though teachers themselves might not realise how often they are using such patterns.

Our observations are also consistent with the quote cited from Margolinas (1993) above: the teacher refrains from evaluating pupils' contribution, but is far from passive and his neutrality is here limited to judgement on the validity of the pupils' arguments. We argue the conclusions of Leblanc and Sauvaire-Maltrana (2019) in the context of the analysis of the activity of teachers using "novel cooperative pedagogical forms" can be adapted in the case of "scientific debate" in mathematics

[5] Here, we couldn't identify which pupil was talking from the audio recording.

classrooms: the role of the teacher in this setting is more that of a “stage director” setting the stage to allow certain interactions and behaviours to emerge, rather than an “orchestra conductor” knowing beforehand exactly what sheet to play and on which pupil to rely to advance his agenda. His role is to moderate the discussion and to promote questioning among the pupils. He does not emit judgements on the pupil’s answers and arguments, but ensures they stay focused on the problem, that they interact respectfully and that every argument can be understood by the whole class.

References

Artigue, M., & Blomhøj, M. (2013). Conceptualizing inquiry-based education in mathematics. *ZDM, 45*(6), 797–810. https://doi.org/10.1007/s11858-013-0506-6

Brousseau, G. (1992). *Théorie des situations didactiques* [Theory of didactic situations.]. La pensée sauvage.

Carreyre, E., & Salin, M. H. (1982). *Compte rendu des recherches à l'école maternelle Jules Michelet - Classe 2 : année 1981-1982* [Report on research at Jules Michelet nursery school - Class 2: 1981-1982], IREM de Bordeaux.

Hattie, J. (2012). *Visible learning for teachers: maximizing impact on learners*. Routledge.

Hersant, M. (2004). Caractérisation d’une pratique d’enseignement, le cours dialogué [Characterisation of a teaching practice: the dialogic lesson]. *Revue canadienne de l’enseignement des sciences, des mathématiques et des technologies, 4*(2), 241–258.

Lakatos, I. (1976). *Proofs and Refutations: The Logic of Mathematical Discovery*. Cambridge University Press.

Leblanc, S., & Sauvaire-Maltrana, C. (2019). Comprendre l’activité d’enseignant(e)s utilisant des formes pédagogiques coopératives nouvelles dans le secondaire [Understanding the activity of teachers using new forms of cooperative teaching in secondary schools]. *Activités*, *16*(1). https://doi.org/10.4000/activites.3901

Legrand, M. (1993). Débat scientifique en cours de mathématiques et spécificité de l’analyse [Scientific debate in mathematics lessons and the specific nature of analysis], *Repères-IREM, 10*, 123–149.

Legrand, M. (2001). Scientific Debate in Mathematics Courses. In D. Holton, M. Artigue, U. Kirchgräber, J. Hillel, M. Niss, & A. Schoenfeld (Eds.), *The teaching and learning of mathematics at university level* (pp. 127–135). New ICMI Study Series, vol 7. Springer.

Leroux, L., & Lecorre, T. (2007). Le « Débat scientifique » en classe. Comment donner à l’élève une responsabilité scientifique réelle en cours de mathématique? [“Scientific debate" in the classroom. How to give students real scientific responsibility in maths lessons?]*APMEP-Plot, 19*, 2–15.

Margolinas, C. (1993). *De l’importance du vrai et du faux dans la classe de mathématiques* [The importance of true and false in the mathematics classroom]. La pensée sauvage.

Veyrunes, P., & Saury, J. (2009). Stabilité et auto-organisation de l’activité collective en classe: exemple d’un cours dialogué à l’école primaire [Stability and self-organisation of collective activity in the classroom: The example of a dialogic lesson in primary school]. *Revue Française de Pédagogie, 169*, 67–76.