

Low Complexity Iterative LMMSE Receiver for Continuous Phase Modulation Based on a Multiple Access–Multipath Channel Analogy

Ulaş Güntürkün, *Member, IEEE*, Luc Vandendorpe, *Fellow, IEEE*

Abstract—Low complexity iterative reception of Continuous Phase Modulation (CPM) is addressed using Linear Minimum Mean–Square Error (LMMSE) filtering empowered by Soft Interference Cancellation (SIC). The transmitter emits a convolutionally encoded, true CPM signal with a constant envelope. At the receiver end, Laurent–Mengali–Morelli’s decomposition is exploited to reduce the detection complexity, where the exact transmitted CPM signal is approximated as a superposition of a small number of parallel Pulse Amplitude Modulated (PAM) and phase-encoded waveforms. Decomposition pulses are treated as parallel Inter–Symbol Interference (ISI) channels and the phase-encoded pseudo-symbols are interpreted as Multiple Access Interference (MAI). Hence the transmitted signal is regarded to inherently contain controlled ISI and MAI, which are equalized for at the receiver front-end using LMMSE–SIC filters. The receiver back-end comprises a serially concatenated turbo scheme encompassing a Soft Input–Soft Output (SISO) CPM detector and a SISO channel decoder. Bit error rate simulations performed on various CPM schemes show that the proposed receiver performs well for long modulation pulses, which have traditionally been deemed infeasible for implementation due to the prohibitive complexity. Consecutively, the proposed receiver enables to exercise ubiquitous CPM waveforms with arbitrarily long pulses at low computational costs, leading to substantial savings in spectral occupancy as well as receiver complexity.

Index Terms—Continuous Phase Modulation, CPM, Constant envelope, Laurent’s decomposition, Mengali and Morelli’s decomposition, CDMA, MIMO, Turbo, Iterative detection

I. INTRODUCTION

THE constant envelope property of Continuous Phase Modulation (CPM) makes it a suitable modulation format for nonlinear and bandwidth–limited channels. Such channels typically arise in satellite communications due to the use of stable and saturated class–C power amplifiers. In addition to the constant envelope property, CPM possesses some other desirable assets, including the inherent phase encoder structure that can be coupled with an outer code to provide a powerful error correction mechanism [1–5]. Fast roll–off of spectral side-lobes is another great merit of CPM, which enables to minimize adjacent channel interference in Frequency–Division Multiplexing (FDM) schemes [6–9].

Ulaş Güntürkün was with the Inverse Problems and Cognitive Systems Laboratory, Teknopark Istanbul, Istanbul 34906, Turkey by the time this article was prepared for submission. He is currently with Marecomms Inc., Halifax, NS B3N 0E7, Canada. Luc Vandendorpe is with the Institute of Information and Communication Technologies, Electronics and Applied Mathematics (ICTEAM) at Université catholique de Louvain, Avenue Georges Lemaitre 4, 1348 Ottignies-Louvain-la-Neuve, Belgium. Corresponding author’s e-mail: ulas.gunturkun.ca@ieee.org

Complex baseband equivalent of a CPM waveform is defined by three parameters, the modulation index h , cardinality M , and a phase–smoothing pulse¹ with length L [10, 11]. Modulation index is given as the ratio between two relatively prime integers κ and p such that $h \triangleq \kappa/p$, $\kappa < p$. h determines how closely the internal phase states are spaced and therefore has a direct impact on the probability of error. Cardinality dictates how many bits are packed in one CPM symbol, and influences both the error probability and spectral occupancy. The pulse length L , on the other hand, acts as the modulation memory and large values of L leads to significantly faster roll–off of spectral side-lobes than lower values of L . Given these modulation parameters, the number of trellis states required for the optimum Maximum Likelihood (ML) CPM demodulator² is given as pM^{L-1} .

A. Problem Statement

Due to the exponential dependence of the number of trellis states on pulse length L , CPM has traditionally been exercised with short pulses in practice (e.g., $L \leq 3$). It is, however, desirable to use CPM waveforms with long pulses to take advantage of fast spectral roll–offs. In order to illustrate this point, let us inspect Fig. 1(a), where Power Spectral Densities (PSD) for three quaternary CPM waveforms are shown. Clearly, for a fixed modulation index and cardinality, larger values of L lead to substantial savings in spectral occupancy. In Fig. 1(b), we plot the PSDs of three other CPM waveforms for a binary constellation, and $L \geq 5$. Inspecting the roll–offs of spectral side-lobes in Fig. 1(a) and Fig. 1(b), the advantage of using CPM waveforms with long pulses is evident.

As we noted, however, this advantage comes at a high computational cost. For example, the optimum demodulation of CPM schemes in Fig. 1(b) would require trellises with $pM^{L-1} \in \{64, 80, 48\}$ states, respectively, for $\{h = 1/2, \text{B6RC}\}$, $\{h = 2/5, \text{B5RC}\}$, and $\{h = 1/3, \text{B5RC}\}$. Unfortunately such high number of states is generally undesirable for practical implementations. Therefore there is a clear need to reduce the CPM demodulation complexity for large L and arbitrary M , h with an acceptable performance degradation.

¹Typical CPM pulse shapes and the corresponding abbreviations are: Raised Cosine–“RC”, Rectangular–“REC” and Gaussian–“G”.

²The number of matched filters required for optimum ML demodulation is M^{L-1} . However, once the matched filters are implemented, their coefficients can be fixed. Therefore, the receiver complexity is heavily dominated by the number of trellis states, which have to be re-visited for each sample interval.

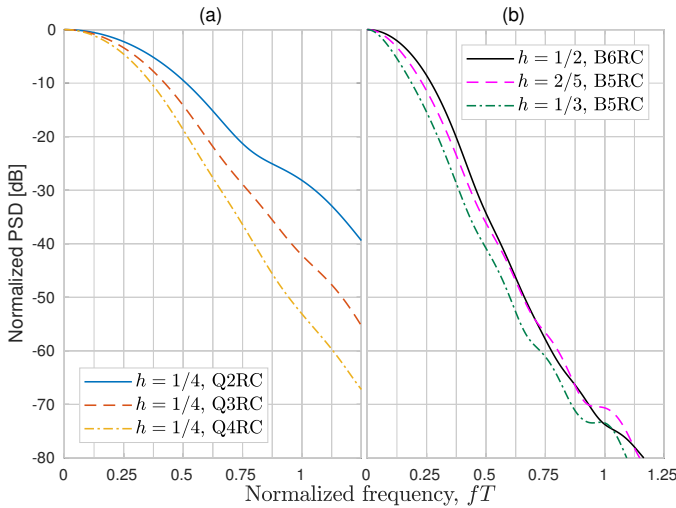


Fig. 1. Normalized Power Spectral Density (PSD) for (a) quaternary, (b) binary CPM schemes studied in this paper. h is the CPM modulation index and the abbreviation “QLRC” represents Quaternary CPM with a Raised Cosine pulse occupying L symbol periods. T denotes the duration of one symbol.

B. Background and Related Work

The contributions in [12–19] offer various complexity reduction techniques for CPM, based on Laurent’s [20] and Mengali&Morelli’s [21] decomposition methods. All of these studies exploit the constant envelope property of CPM in order to simplify the optimum ML demodulator. In [14], a method is presented to significantly reduce the number of front-end matched filters required for demodulation, provided by Laurent’s decomposition. The work in [14], however, does not lend itself to a reduction in the number of trellis states.

Kaleh, on the other hand, expands on Laurent’s decomposition further, and proposes to use a Linear Minimum Mean Square Error (LMMSE) filter prior to the phase detector in [12, Section V]. However, his technique is demonstrated only on a binary Gaussian Minimum Shift Keying (GMSK) system with no obvious strategy or a hint to generalize to higher-order cardinalities. As such, Kaleh’s work does not utilize interference cancellation either. In [13], the authors build on Kaleh’s work and concatenate an LMMSE equalizer with a *a posteriori* probability decoding, which is shown to form a powerful receiver architecture applied to GMSK signals with long pulses, $L = 5$ and $L = 6$. However, as in the case of [12], the work in [13] is also limited to binary CPM with static LMMSE filters that do not offer a solution for $M > 2$ and do not incorporate interference cancellation.

Colavolpe et al. [15–19] noticed that the trellis states can be defined in a much simpler way than that for an optimum CPM demodulator by exploiting the recursive phase encoding property of the pseudo-symbols that stem from Laurent–Mengali–Morelli decomposition (to be referred to as *LMM decomposition* in short hereinafter). They note that as long as the self interference in a CPM signal is not prominent, which generally holds true for short pulses ($L \leq 3$), demodulation complexity can be drastically reduced to $M - 1$ matched filters followed by a p -state trellis for CPM phase detection. While the work developed in [15–19] indeed lends itself to an elegant

and efficient low complexity CPM reception scheme for short pulses, its performance degrades significantly for longer pulses (i.e., $L \geq 4$) as will be demonstrated in this paper.

It is also in order to notice the contributions in [22, 23], where the authors employ an LMMSE–SIC filter to equalize for an actual multipath channel and use Rimoldi’s decomposition of binary CPM [24] that results in a similar doubly iterative receiver architecture as the one proposed herein. Likewise, the authors of [25, 26] also develop an LMMSE–SIC filter to equalize for Intersymbol Interference (ISI) channels. We stress, however, that the contributions in [22, 23, 25, 26] focus on mitigating multipath channels using binary CPM signals rather than achieving a generalizable low complexity demodulation of CPM for all h , M , and L , which is our objective herein.

C. Statement of Contributions

Building and expanding on the previous work outlined above, we propose a combination of LMMSE filters empowered by Soft Interference Cancellation (SIC) and a p -state Soft Input–Soft Output (SISO) CPM symbol detector for reduced complexity demodulation of CPM for arbitrary h , M and L . Hence the proposed receiver architecture can be considered to generalize the LMMSE receiver in [12, 13] to encompass higher-order constellations using Mengali and Morelli’s approach [21] and bind it together with the detector proposed by Colavolpe et al. [15–19] to build a doubly iterative scheme.

Our inspiration in embodying the proposed receiver lies in the similarity between the LMM-decomposed CPM signal structure and a synchronous Code Division Multiple Access (CDMA) signal. More specifically, pulses arising from LMM decomposition can be treated as controlled ISI channels when $L > 1$, whereas the pseudo-symbols that drive each pulse can be regarded as user symbols that cause controlled Multiple Access Interference (MAI) for $M > 2$, analogous to a synchronous CDMA scheme. We also notice the similarity between the proposed receiver and a Multiple Input–Multiple Output (MIMO) co-channel demodulation scheme [27, Section 7.7].

Motivated by the impressive performance of LMMSE–SIC filtering technique on CDMA systems [28, 29] then, we apply LMMSE–SIC filters to equalize for the ISI and MAI that are regarded to be contained inherently in CPM waveforms for $L > 1$ and $M > 2$, respectively. Following the equalization stage at the front-end, the receiver back-end is governed by a serially concatenated turbo scheme, where a p -state SISO CPM detector and a SISO channel decoder jointly demodulate/decode the original message content. The *a posteriori* probabilities generated by the CPM SISO detector are used for calculating the average values of interfering pseudo-symbols which are subtracted from the received signal vectors to be processed by LMMSE filters. Therefore the overall receiver encompasses a nested iterative loop, for equalization using LMMSE–SIC filters, and for turbo demodulation/decoding.

Consequently, the main contribution of this work is a generalizable low complexity CPM receiver that offers the

capability to receive all CPM waveforms with no limitations on the pulse length, modulation index, or cardinality.

In what follows, we formally describe the original CPM waveform and LMM decompositions, on which we build the generalized low complexity receiver, and present simulated Bit Error Rate (BER) analysis.

II. LAURENT, MENGALI AND MORELLI'S DECOMPOSITION OF CPM

Let us begin the formal treatment describing the CPM complex baseband envelope signal $z(t)$ [10, 11],

$$z(t, \alpha) = \sqrt{\frac{2E_s}{T}} \exp \left\{ j2\pi h \sum_{n=0}^{\infty} \alpha[n] q(t - nT) \right\} \quad (1)$$

where $\alpha[n] \in \{\pm 1, \pm 3, \dots, \pm M-1\}$ denotes a modulation symbol belonging to an M -ary alphabet at discrete-time n , and E_s [Joule] denotes the average symbol energy. The vector $\alpha \triangleq [\alpha[0], \alpha[1], \dots, \alpha[n], \dots]^T$ denotes the history of modulation symbols. T [s] is the symbol duration, and $q(t)$ stands for the CPM phase-smoothing response, which is produced by integrating an underlying frequency pulse with a duration of LT [s], taking on a raised cosine (RC), rectangular (REC) or Gaussian (G) shape.

Laurent, Mengali and Morelli's decomposition of CPM allows us to express $z(t)$ in (1) as a superposition of a finite number of Pulse Amplitude Modulated (PAM) and phase-encoded waveforms as given in (2)

$$z(t, \alpha) = \sum_{i=0}^{F-1} \sum_n a_i[n] c_i(t - nT) \quad (2)$$

where $a_i[n]$ are the pseudo-symbols, and $c_i(t)$ are the decomposition pulses. Calculation of these terms can be achieved by following a sequel of simple algebraic operations in [20] for binary CPM, and in [21] for higher-order CPM schemes. In order to specify the number of decomposition terms, let us define some intermediate parameters as $Q \triangleq 2^{L-1}$, $P \triangleq \lceil \log_2 M \rceil$. Then we have $F = Q^P (2^P - 1)$ terms in the decomposition, which may be impractically large for higher-order CPM schemes and large values of L .

As noted in [20, 21], however, most of the signal energy is concentrated in the first $M-1$ components in (2). Hence a very simple approach to make a practical use of LMM decomposition is to retain the first $M-1$ terms, and discard the rest in (2). We will term this approach as the *principal pulse approximation* (PPA). While this simplification is shown to work well for $L \leq 2$ in [15–19], our results indicate that non-principal terms carry a non-negligible amount of signal energy for $L > 2$.

A. Optimized Approximate Decomposition

In order to improve the approximation quality for $L > 2$, we follow the ideas developed in [20, Section IV] and [21, Section V] and use Mean-Square Error (MSE) optimization to improve the quality of approximate decomposition. To proceed, let us introduce another parameter,

$$J \triangleq 2^P - 1 \quad (3)$$

The idea behind the MSE optimization is to calculate the weights that minimize the MSE between the original CPM baseband equivalent signal in (1) and a weighted combination of $F-J$ non-principal components added to the J principal components. The MSE-optimized pulses, denoted by $g_i(t)$, are related to the original decomposition pulses $c_i(t)$ as

$$g_i(t) = c_i(t) + \sum_{\delta=J}^{F-1} \sum_{\lambda} \xi_{i,\delta}[\lambda] g_{\delta}(t - \lambda T), \quad 0 \leq i < J \quad (4)$$

where $\xi_{i,\delta}[\lambda]$ are the MSE optimization weights. As seen in (4), the philosophy of the MSE optimization is to transfer as much relevant information from the non-principal pulses ($\delta \geq J$) into the principal pulses ($i < J$) as possible, exploiting the statistical similarity between non-principal and principal pseudo-symbols. In other words, optimization weights are proportional to the cross-correlations between the principal pseudo-symbols $a_i[n]$, $i \leq J-1$ and non-principal pseudo-symbols $a_{\delta}[n]$, $\delta \geq J$ as described in [21, eq. (82)]. Pseudo-symbol autocorrelations and cross-correlations are also nicely detailed in [21, eq. (14,15)].

We skip the further details in calculation of the optimization weights and refer the interested reader to [21, eq. (78–84)] to proceed directly to the final expression for the optimized approximate decomposition, which is given as

$$z(t, \alpha) \approx \sum_{i=0}^{J-1} \sum_n a_i[n] g_i(t - nT) \quad (5)$$

In Fig. 2(a) and Fig. 2(b), pulses obtained with principal component approximation and those obtained with MSE optimization are shown for binary and quaternary CPM schemes, respectively. The variance of discrepancy between the original CPM waveform and the MSE-optimized decomposed waveform is denoted by σ_{opt}^2 in Fig. 2. The variance of error made by the principal pulse approximation is denoted by σ_{ppa}^2 .

As seen in the top panel in In Fig. 2(a), the MSE-optimized pulse and the principal pulse are identical. This is due to the fact that the principal and non-principal pseudo-symbols are orthogonal for binary CPM with $h = 1/2$. Therefore the optimization weights are zero (i.e., $\xi_{i,\delta}[\tau] = 0, \forall i, \delta$), leading to $g_0(t) = c_0(t)$ in (4). For smaller modulation indices, however, cross-correlations among principal and non-principal pseudo-symbols become spread-out in the lag domain. Hence the MSE-optimized approximate decomposition leads to a more accurate representation of the true CPM waveform than the simple principal pulse approximation as seen in the middle and bottom panels in Fig. 2(a). For $h = 1/3$, in particular, MSE optimization leads to an approximation error that is smaller than half of that obtained by PPA.

In Fig. 2(b), we compare the MSE-optimized and PPA pulses for quaternary CPM, where the modulation index is fixed at $h = 1/4$ and the underlying frequency pulse varies from 2RC to 4RC. While the MSE optimization and PPA produce similar accuracies for $L = 2$, MSE optimization clearly outperforms PPA for $L = 3$ and $L = 4$, as seen in the middle and the bottom parts of Fig. 2(b), respectively. Thus, we will use the MSE-optimized pulses in the receiver design.

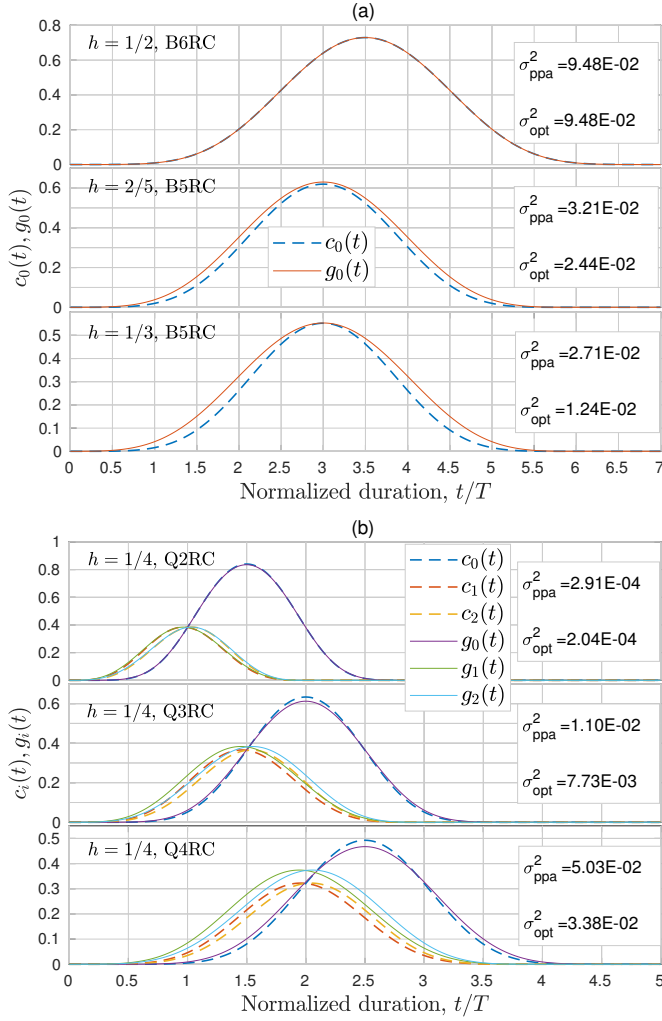


Fig. 2. CPM Pulse optimization for (a) binary, (b) quaternary CPM schemes studied in this paper. Legend in the middle panel in (a) applies to the upper and the lower panels in (a) as well. Likewise legend in the upper panel in (b) applies to the rest of (b). h is the CPM modulation index and the abbreviations "BLRC" and "QLRC" represent Binary and Quaternary CPM, respectively, with a Raised Cosine pulse occupying L symbol periods. T denotes the duration of one symbol. σ_{ppa}^2 and σ_{opt}^2 denote, respectively, the variance of approximation error for principal pulse approximation and that for MSE optimization. $c_i(t)$ and $g_i(t)$ refer to the principal pulses, and the MSE-optimized pulses, respectively, for the i 'th decomposition component.

B. Recursive Phase Encoding Property

A careful analysis of the derivation of CPM pseudo-symbols in [21, eq.(10,41)] reveals that $a_i[n]$ obeys Markovian property, and can be expressed as a function of the modulation symbol $\alpha[n]$ and $a_0[n-1]$ [17]. This property can be exploited to form a unit-rate CPM recursive phase encoder with a p -state trellis, which commonly governs the evolution of $a_i[n]$ for $i \in [0, J-1]$. The CPM recursive phase encoder state $x[n]$ is then described as [17]

$$x[n] = [x[n-1] + \alpha'[n]]_p \quad (6)$$

where $[\cdot]_p$ denotes the modulo- p operator and $x[0] = 0$. The non-negative constellation points $\alpha'[n] \in \{0, 1, \dots, M-1\}$ are obtained as

$$\alpha'[n] \triangleq \frac{\alpha[n] + M - 1}{2} \quad (7)$$

TABLE I
RADIX-2 REPRESENTATION AND THE CORRESPONDING BPSK DECOMPOSITIONS OF MODULATION SYMBOLS.

(a) $M = 2$

$\alpha[n]$	-1	1
$\alpha'[n]$	0	1
$\zeta^{(0)}[n]$	0	1
$\theta^{(0)}[n]$	-1	1

(b) $M = 4$

$\alpha[n]$	-3	-1	1	3
$\alpha'[n]$	0	1	2	3
$\zeta^{(0)}[n]$	0	1	0	1
$\zeta^{(1)}[n]$	0	0	1	1
$\theta^{(0)}[n]$	-1	1	-1	1
$\theta^{(1)}[n]$	-1	-1	1	1

TABLE II
CPM PSEUDO-SYMBOL EXPRESSIONS AS A FUNCTION OF RECURSIVE PHASE ENCODER STATE AND MODULATION SYMBOL REPRESENTATIONS.

Cardinality	Closed Form Expressions for CPM Pseudo-symbols
$M = 2$	$a_0[n] = \exp \left\{ \sqrt{-1} \pi h (2x[n] - n) \right\}$
$M = 4$	$a_0[n] = \exp \left\{ \sqrt{-1} \pi h (2x[n] - 3n) \right\}$ $a_1[n] = \exp \left\{ \sqrt{-1} \pi h (2x[n-1] + 4\zeta^{(1)}[n] - 3n + 1) \right\}$ $a_2[n] = \exp \left\{ \sqrt{-1} \pi h (2x[n-1] + 2\zeta^{(0)}[n] - 3n + 2) \right\}$

Clearly the CPM recursive phase encoder state takes on p different values, which are mapped on to phase values of pseudo-symbols, in a similar fashion to trellis-coded modulation.

For the development of SISO CPM detector in the next section, radix-2 representation of $\alpha'[n]$ are required, along with the corresponding Binary Phase Shift Keying (BPSK) values, which are given in Table I. In Table II, we present the final pseudo-symbol expressions for $M = 2$ and $M = 4$, which are the practical alphabet sizes considered in this paper.

III. DEVELOPMENT OF THE REDUCED COMPLEXITY RECEIVER

We stress that the MSE-optimized approximate decomposition given in (5) is utilized only at the receiver side, whereas the transmitter emits a true CPM waveform. In Fig. 3, the transmit/receive block diagram is presented where $d[n]$ denotes a message bit at n whereas $u[n]$ is a convolutionally encoded bit. Encoded bit blocks are interleaved and mapped onto the CPM modulation symbols, $\alpha[n]$, which are assumed uncorrelated as per previous work.

A. Front-End Matched Filtering

Following from (5), the receiver front-end employs J parallel correlators, which are implemented as matched filters with impulse responses $g_j^*(-t)$, respectively, for $j =$

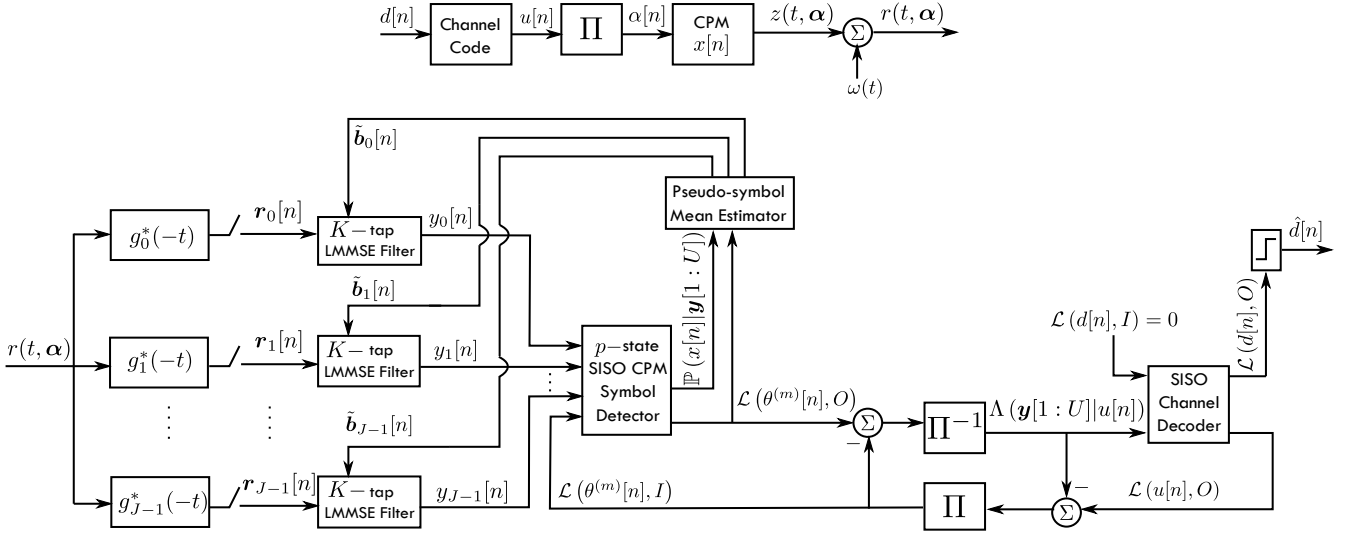


Fig. 3. Block diagram of the proposed generalized low complexity SCCPM transceiver. $x[n]$ denotes the state of CPM modulator. $\mathcal{L}(\cdot, I)$ and $\mathcal{L}(\cdot, O)$ represent, respectively, the *a priori* and *a posteriori* $\mathcal{L}$ -values of their first argument. $\Lambda(y[1:U] | u[n])$ denotes the log likelihood ratio (LLR) of coded bits and U is the number of coded bits in a block. $\omega(t)$ is the Additive White Gaussian Noise (AWGN) with a spectral density of N_0 [Watts/Hz] affecting the complex baseband equivalent transmit waveform, $z(t, \alpha)$. Symbols Π and Π^{-1} denote, respectively, interleaving and de-interleaving operations. For binary CPM $J = 1$, and hence is only one matched filter/MMSE filter cascade in the receiver. For quaternary CPM $J = 3$, and there are 3 parallel filter cascades.

$0, 1, \dots, J-1$, as seen in Fig. 3. Since all LMM decomposition pulses are real-valued, we proceed with $g_j^*(-t) = g_j(-t)$. Then let $r_j[n]$ denote a sample measured at the output of j 'th matched filter at instant n which is given as

$$r_j[n] \triangleq \int_{nT}^{(n+L_j)T} r(t, \alpha) g_j(t - nT) dt, \quad j = 0, \dots, J-1 \quad (8)$$

where $r(t, \alpha)$ is the received complex baseband waveform defined as

$$r(t, \alpha) = z(t, \alpha) + \omega(t) \quad (9)$$

In (9), $z(t, \alpha)$ denotes the true CPM transmit waveform, corrupted by circularly symmetric AWGN $\omega(t)$ with a PSD of N_0 [Watts/Hz]. The receiver uses the expression in (5) to approximate to the true CPM waveform. Then substituting (5) in (9) and that in (8), we derive the discrete-time observation model sampled at symbol duration, which is given as

$$r_j[n] = \sum_{i=0}^{J-1} \sum_{l=-L_i+1}^{L_j-1} a_i[n+l] \rho_{i,j}[l] + v_j[n], \quad j = 0, \dots, J-1 \quad (10)$$

where $\rho_{i,j}[l]$ is described in (11) and it represents the l 'th sample of the correlation function between the i 'th transmit and j 'th receive pulses, which, respectively, are L_i and L_j symbol durations-long.

$$\rho_{i,j}[l] \triangleq \int_0^{L_j T} g_i(\tau - lT) g_j(\tau) d\tau \quad (11)$$

$v_j[n]$ in (10) is a zero-mean noise sample observed at the output of the j 'th matched filter as described in (12), the second-order statistics of which are derived in Appendix A,

$$v_j[n] \triangleq \int_{nT}^{(n+L_j)T} \omega(t) g_j(t - nT) dt \quad (12)$$

Samples observed at the matched filter outputs will be presented into the LMMSE filters on a sliding-window basis,

with the window length denoted by K , which is also the number of taps for each Wiener filter. We split K filter taps into W prediction and $V+1$ smoothing taps so that $K = W + V + 1$. In order to set the stage for the derivation of LMMSE filters, we first build a matrix representation for the matched filter outputs. To that end, let us first describe the vector of incoming samples for the j 'th LMMSE filter as

$$\mathbf{r}_j[n] \triangleq [r_j[n+W], \dots, r_j[n], \dots, r_j[n-V]]^T \in \mathbb{C}^{K \times 1} \quad (13)$$

where $\mathbb{C}$ denotes the complex space. Our objective is now to relate the filter input vectors $\mathbf{r}_j[n]$ to the CPM pseudo-symbols $a_i[n]$, which play the role of filter desired responses for $i = j$. For this, we construct pseudo-symbol vectors $\mathbf{a}_i^{(L_j)}[n] \in \mathbb{C}^{(L_i+L_j+K-2) \times 1}$ described as

$$\mathbf{a}_i[n] \triangleq [a_i[n+W+L_j-1], \dots, a_i[n], \dots, a_i[n-V-L_i+1]]^T \quad (14)$$

Each of the J matched filters receive contributions from each of the $i = 0, 1, \dots, J-1$ pseudo-symbols that are considered to be contained in the transmitted signal. Therefore we stack the vectors $\mathbf{a}_i[n]$ in a taller vector for each of the J receive filters. Then let us describe the overall pseudo-symbol vector as

$$\mathbf{b}_j[n] \triangleq [\mathbf{a}_0^T[n], \mathbf{a}_1^T[n], \dots, \mathbf{a}_{J-1}^T[n]]^T \in \mathbb{C}^{(L_\Sigma+J(L_j+K-2)) \times 1} \quad (15)$$

where L_Σ is defined as the sum of all transmit pulse lengths, i.e., $L_\Sigma = \sum_{i=0}^{J-1} L_i$.

Next, let us build the vectors made up of the composite pulse (pulse correlation) samples, $\rho_{i,j}[l]$ given as

$$\boldsymbol{\rho}_{i,j} \triangleq [\rho_{i,j}[L_j-1], \dots, \rho_{i,j}[0], \dots, \rho_{i,j}[-L_i+1]]^T \in \mathbb{R}^{(L_i+L_j-1) \times 1} \quad (16)$$

where $\mathbb{R}$ denotes the real space. Using the vectors $\boldsymbol{\rho}_{i,j}$, we construct the matrices $\mathbf{G}_j \in \mathbb{R}^{K \times (L_\Sigma+J(L_j+K-2))}$ as given in

(17), which play the role of channel convolution matrices. In particular, $\mathbf{G}_j$ pertains to the j 'th multiple access-ISI channel, which is imagined to be formed between the j 'th receive pulse and all J transmit pulses,

$$\mathbf{G}_j \triangleq \begin{bmatrix} \rho_{0,j}^T, \mathbf{0} & \rho_{1,j}^T, \mathbf{0} & \cdots & \cdots & \rho_{J-1,j}^T, \mathbf{0} \\ 0 & \rho_{0,j}^T, \mathbf{0} & \rho_{1,j}^T, \mathbf{0} & \cdots & \cdots & \rho_{J-1,j}^T, \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \rho_{0,j}^T, \mathbf{0} & \rho_{1,j}^T, \mathbf{0} & \cdots & \rho_{J-1,j}^T, \mathbf{0} \end{bmatrix} \quad (17)$$

Note that the vectors $\mathbf{0}$ in (17) represent the zero-padding vectors with appropriate dimensions, so that the final matrix expression in (18) holds,

$$\mathbf{r}_j[n] = \mathbf{G}_j \mathbf{b}_j[n] + \mathbf{v}_j[n], \quad j = 0, 1, \dots, J-1 \quad (18)$$

where the noise vector is defined as

$$\mathbf{v}_j[n] \triangleq [v_j[n+W], \dots, v_j[n], \dots, v_j[n-V]]^T \in \mathbb{C}^{K \times 1} \quad (19)$$

B. LMMSE-SIC Equalizer Bank

We can now proceed to the details of LMMSE-SIC filter design. As indicated in (10), $\rho_{i,j}[l]$ can be considered analogous to a tapped delay line with $L_i + L_j - 1$ symbol-spaced taps, acting like a discrete ISI channel. Likewise, contributions from those pseudo-symbols with indices $i \neq j$ in (10) can be regarded as multiple access interference. This fact is captured in the matrix representation in (18) in a compact form, where the matrix $\mathbf{G}_j$ can be considered to encode how ISI and MAI are contained in the received vectors at matched filter outputs.

Based on this insight, we treat $a_i[n]$ in (10) as the desired response for the j 'th equalization filter for $i = j$, and try to equalize for it. This leads us to develop J parallel equalization schemes, the objective of which is two-fold:

- Mitigate the controlled ISI by suppressing all terms for which $l \neq 0$ in (10),
- Cancel the controlled MAI by subtracting all terms from the equalizer inputs for which $i \neq j$ in (10).

We remark that the nature of our equalizer bank closely resembles those developed in [28, 29], which were originally proposed for CDMA systems. Similar to a CDMA scheme, noise and interference inflict comparable amounts of impairment on the received signal in a CPM scheme for $L > 3$ and/or $M > 2$. Thus, MMSE criterion presents itself to be an adequate design choice in its ability to provide a balance between noise reduction and interference suppression.

The LMMSE equalizers operate in two different modes: feed-forward mode and the feedback mode. For the former, the lack of *a priori* information in the first equalization iteration results in a feed-forward equalization scheme. We abbreviate this mode as “nai” denoting “no *a priori* information”. After several demodulation/decoding iterations at the back-end, *a priori* information on the interfering symbols becomes available, which is fed back into the equalizer bank through the feedback links as seen in Fig. 3. We treat such *a priori* information to be perfect [28], and abbreviate this mode as “fai”, which is short for “full *a priori* information”. Hence we separately derive filter tap-weight vectors for the feed-forward (“nai”) and feedback (“fai”) cases, respectively.

C. Feed-Forward (Static) LMMSE Filtering (nai)

Let $\mathbf{w}_j^{\text{nai}} \in \mathbb{R}^{K \times 1}$ denote the tap-weight vector for the j 'th feed-forward LMMSE filter. In the absence of *a priori* information, the vectors $\mathbf{r}_j[n]$ observed at the matched filter outputs are directly presented to the LMMSE filters (without interference subtraction). Let then $y_j^{\text{nai}}[n]$ denote the response of the j 'th LMMSE filter to $\mathbf{r}[n]$, which is given as

$$y_j^{\text{nai}}[n] = \left(\mathbf{w}_j^{\text{nai}} \right)^{\mathcal{H}} \mathbf{r}_j[n] \quad (20)$$

where $\mathcal{H}$ denotes the Hermitian (conjugate transpose) operator. Applying MMSE criterion to the linear filtering operation described in (20) results in the classical Wiener (LMMSE) solution:

$$\mathbf{w}_j^{\text{nai}} = \underset{\mathbf{w}_j^{\text{nai}}}{\operatorname{argmin}} \mathbb{E} \left[\left(a_j[n] - \left(\mathbf{w}_j^{\text{nai}} \right)^{\mathcal{H}} \mathbf{r}_j[n] \right)^2 \right] = \left(\mathbf{R}_{jj}^{\text{nai}} \right)^{-1} \mathbf{k}_j^{\text{nai}} \quad (21)$$

where $\mathbf{R}_{jj}^{\text{nai}} \in \mathbb{R}^{K \times K}$ is the autocorrelation matrix for the filter input vectors, as given in (22), which exhibit wide-sense stationarity property

$$\mathbf{R}_{jj}^{\text{nai}} \triangleq \mathbb{E} \left[\mathbf{r}_j[n] \mathbf{r}_j^{\mathcal{H}}[n] \right] = \mathbf{G}_j \mathbf{B}_j \mathbf{G}_j^{\mathcal{H}} + \mathbf{\Psi}_j \quad (22)$$

Note that we have defined the pseudo-symbol autocorrelation matrix as $\mathbf{B}_j \triangleq \mathbb{E} [\mathbf{b}_j[n] \mathbf{b}_j^{\mathcal{H}}[n]] \in \mathbb{R}^{(L_{\Sigma} + J(L_j + K - 2)) \times (L_{\Sigma} + J(L_j + K - 2))}$. Entries of $\mathbf{B}_j$ can be derived following from [21, eq (14)].

$\mathbf{\Psi}_j$ is the covariance matrix for the colored noise vector $\mathbf{v}_j[n]$ observed at the j 'th matched filter output as described in (59) in Appendix Section A.

$\mathbf{k}_j^{\text{nai}} \in \mathbb{R}^{K \times 1}$ is the cross-correlation vector between the filter input vector and the desired response pertaining to the j 'th equalizer and is given as

$$\mathbf{k}_j^{\text{nai}} \triangleq \mathbb{E} \left[\mathbf{r}_j[n] a_j^*[n] \right] = \mathbf{G}_j \mathbf{B}_j \mathbf{e}_j \quad (23)$$

The role of vector $\mathbf{e}_j \in \mathbb{R}^{1 \times (L_{\Sigma} + J(L_j + K - 2))}$ is to maintain that column of $\mathbf{B}_j$ which corresponds to $\mathbb{E} [\mathbf{b}_j[n] a_j^*[n]]$ and nullify all other columns as defined in (24)

$$\mathbf{e}_j \triangleq \begin{bmatrix} \mathbf{0}_{1 \times (W + (j+1)(L_j + K - J + 1) + \sum_{s=1}^{J-1} L_{j-s} + J - K - 2)}, 1, \\ \mathbf{0}_{1 \times (V + (J-j)(L_j + K - J + 1) + \sum_{s=1}^{J-1} L_{j+s} + J - K - 2)} \end{bmatrix} \quad (24)$$

In (24), $\mathbf{0}_{\dots}$ are all-zero row vectors with the dimensions given in the subscripts in (24) and $L_j = 0, \forall j < 0, j \geq J$.

Substituting (22) and (23) in (21), the tap-weight vector for the j 'th feed-forward LMMSE filter is obtained as

$$\mathbf{w}_j^{\text{nai}} = \left(\mathbf{G}_j \mathbf{B}_j \mathbf{G}_j^{\mathcal{H}} + \mathbf{\Psi}_j \right)^{-1} \mathbf{G}_j \mathbf{B}_j \mathbf{e}_j \quad (25)$$

1) *Statistics of the Feed-Forward LMMSE Equalizer Output:* It is shown in [29] that an output sample of the LMMSE filter can be represented as a scaled and noisy version of the filter desired response. In this representation, the noise process accounts for the residual noise and interference after

LMMSE filtering, and can be well approximated to be zero-mean, Gaussian-distributed. Then the LMMSE filter output sample observed at n can be described as

$$y_j^{\text{nai}}[n] = \mu_j^{\text{nai}} a_j[n] + v_j^{\text{nai}}[n] \quad (26)$$

where $\mu_j^{\text{nai}} \leq 1$ is the attenuation factor (i.e., gain) of the j 'th LMMSE filter and we have $v_j^{\text{nai}}[n] \sim \mathcal{N}(0, \sigma_{v_j^{\text{nai}}}^2)$ for the noise sample.

Note that by virtue of the LMMSE filtering, multiple access-ISI channel model in (10) is reduced into a memoryless signal+noise equivalent observation model in (26). Multiplying both sides of (26) by $a_j^*[n]$ and exploiting the fact that $\mathbb{E}[a_j[n]a_j^*[n]] = 1$, we obtain the filter gain as

$$\mu_j^{\text{nai}} = \mathbb{E}[y_j^{\text{nai}}[n]a_j^*[n]] = (\mathbf{w}_j^{\text{nai}})^{\mathcal{H}} \mathbf{k}_j^{\text{nai}} \quad (27)$$

Multiplying both sides of (26) by $y_j^{\text{nai}}[n]$, taking the statistical expectation of that multiplication, and skipping the necessary algebraic manipulations for brevity, we obtain the variance of the equivalent memoryless observation model noise as

$$\sigma_{v_j^{\text{nai}}}^2 = \mu_j^{\text{nai}} - (\mu_j^{\text{nai}})^2 \quad (28)$$

D. LMMSE Filtering with SIC (fai)

After several iterations in the inner iteration loop, estimates of instantaneous realizations of pseudo-symbols, denoted by $\tilde{a}_j[n]$, $\forall j < J$ are produced by the SISO CPM detector as seen in Fig. 3. We assume that these estimates provide the outer equalization loop with full *a priori* information in the sense that $\tilde{a}_j[n] = a_j[n]$ [28]. Then let us describe the interference estimate vector given as

$$\tilde{\mathbf{a}}_i[n] \triangleq [\tilde{a}_i[n+W+L_j-1], \dots, \tilde{a}_i[n], \dots, \tilde{a}_i[n-V-L_i+1]]^T \quad (29)$$

where we use a special font for the symbol at instant n to distinguish this symbol from the rest in the vector. In particular, $\tilde{a}_i[n]$ is set to 0 when the vector $\tilde{\mathbf{a}}_i[n]$ is processed by the $j = i$ 'th LMMSE filter, and set to $\tilde{a}_i[n] = \tilde{a}_i[n]$ otherwise. This is due to the fact that $\tilde{a}_i[n]$ plays the role of interference for those LMMSE filters for which $j \neq i$, whereas it is a desired quantity for the LMMSE filter for which $j = i$. With this note in mind, let us stack the vectors $\forall i \leq J-1$ in (29) in a column vector to obtain

$$\tilde{\mathbf{b}}_j[n] \triangleq [\tilde{a}_0^T[n], \tilde{a}_1^T[n], \dots, \tilde{a}_{J-1}^T[n]]^T \in \mathbb{C}^{(L_2+J(L_j+K-2)) \times 1} \quad (30)$$

As per the idea behind interference cancellation, $\tilde{\mathbf{b}}_j[n]$ in (30) is remodulated by $\mathbf{G}_j$, and subtracted from the vector of received samples, $\mathbf{r}_j[n]$. Then the output sample of the j 'th LMMSE-SIC filter observed at n is given as

$$y_j^{\text{fai}}[n] = (\mathbf{w}_j^{\text{fai}})^{\mathcal{H}} (\mathbf{r}_j[n] - \mathbf{G}_j \tilde{\mathbf{b}}_j[n]) \quad (31)$$

where $\mathbf{w}_j^{\text{fai}}$ denotes the LMMSE-SIC tap-weight vector in the presence of full *a priori* information. Similar to (21), the optimum tap-weight vector for the "fai" case is obtained as

$$\mathbf{w}_j^{\text{fai}} = (\mathbf{R}_{jj}^{\text{fai}})^{-1} \mathbf{k}_j^{\text{fai}} \quad (32)$$

where $\mathbf{R}_{jj}^{\text{fai}}$ denotes the autocorrelation matrix for the input of the j 'th LMMSE-SIC equalizer after perfect interference cancellation. Likewise, the cross-correlation vector between the LMMSE-SIC equalizer input and the desired response is denoted by $\mathbf{k}_j^{\text{fai}}$ with perfect interference cancellation. $\mathbf{R}_{jj}^{\text{fai}}$ and $\mathbf{k}_j^{\text{fai}}$ are given, respectively, in (33) and (34)

$$\mathbf{R}_{jj}^{\text{fai}} \triangleq \mathbf{G}_j \mathbf{e}_j \mathbf{e}_j^{\mathcal{H}} \mathbf{G}_j^{\mathcal{H}} + \mathbf{\Psi}_j \quad (33)$$

$$\mathbf{k}_j^{\text{fai}} \triangleq \mathbf{G}_j \mathbf{e}_j \quad (34)$$

The final expression for the equalizer tap-weight vectors with perfect interference cancellation is then obtained as

$$\mathbf{w}_j^{\text{fai}} = (\mathbf{G}_j \mathbf{e}_j \mathbf{e}_j^{\mathcal{H}} \mathbf{G}_j^{\mathcal{H}} + \mathbf{\Psi}_j)^{-1} \mathbf{G}_j \mathbf{e}_j \quad (35)$$

1) Statistics of the LMMSE-SIC Equalizer Output:

Following from the treatment presented previously for the "nai" case, the output samples generated by the LMMSE-SIC equalizer can be described given as

$$y_j^{\text{fai}}[n] = \mu_j^{\text{fai}} a_j[n] + v_j^{\text{fai}}[n] \quad (36)$$

where we have $v_j^{\text{fai}}[n] \sim \mathcal{N}(0, \sigma_{v_j^{\text{fai}}}^2)$. The LMMSE-SIC equalizer gain and $\sigma_{v_j^{\text{fai}}}^2$ are obtained as given in (37) and (38), respectively,

$$\mu_j^{\text{fai}} = (\mathbf{w}_j^{\text{fai}})^{\mathcal{H}} \mathbf{k}_j^{\text{fai}} \quad (37)$$

$$\sigma_{v_j^{\text{fai}}}^2 = \mu_j^{\text{fai}} - (\mu_j^{\text{fai}})^2 \quad (38)$$

E. SISO CPM Detector

At the output of LMMSE equalizers, we now imagine to have constructed J virtual parallel AWGN channels between the CPM pseudo-symbols $a_j[n]$ and the corresponding channel output samples, $y_j[n]$ ³. This significantly reduces the demodulation complexity, as the CPM memory content is eliminated from the demodulation/decoding problem. Therefore, regardless of the pulse length, L , we can now exploit classical iterative (turbo) decoding principles for joint demodulation and decoding, interpreting the CPM Phase Encoder (CPE) described in (6) as an inner code, and the convolutional channel code as an outer code of a serially concatenated scheme inside the inner iterative loop.

As depicted in Fig. 3, the SISO CPM detector processes LMMSE equalizer outputs to derive measurement metrics (i.e., log-likelihoods) for the MAP decoding method using the log-domain BJCR algorithm. Also fed into the SISO CPM unit are the *a priori* $\mathcal{L}$ -values on the coded and interleaved BPSK symbols, denoted by $\mathcal{L}(\theta^{(m)}[n], I) \triangleq \ln \left(\frac{\mathbb{P}(\theta^{(m)}[n] = +1)}{\mathbb{P}(\theta^{(m)}[n] = -1)} \right)$ that come from the SISO channel decoder. Using these two pieces of information, the SISO CPM unit outputs the *a posteriori* $\mathcal{L}$ -values on the modulation symbols (see eq.(50)), as well as discrete probabilities on the states of CPE, denoted

³For the nai case, $y_j[n] = y_j^{\text{nai}}[n]$ as described in (26), and for the fai case, $y_j[n] = y_j^{\text{fai}}[n]$ as given in (36).

by $\mathbb{P}(x[n] | \mathbf{y}[n])$, where $\mathbf{y}[n] \triangleq [y_0[n], \dots, y_{J-1}[n]]^T \in \mathbb{C}^{J \times 1}$ represents the vector containing all LMMSE equalizer output samples at n and $\ln(\cdot)$ denotes the natural logarithm. We elaborate on the application of BCJR algorithm [30] to SISO CPM detector in the subsequent parts of this section.

1) *Branch Metrics*: BCJR branch metrics [30] are derived from the probabilities

$$\mathbb{P}(x[n], \mathbf{y}[n] | x[n-1]) = \mathbb{P}(x[n] | x[n-1]) \times f(\mathbf{y}[n] | x[n], x[n-1]) \quad (39)$$

where $f(\cdot | \cdot)$ denotes the likelihood function (i.e., probability density function of $\mathbf{y}[n]$ conditioned on the current and previous CPE states, $x[n]$ and $x[n-1]$), and $\mathbb{P}(x[n] | x[n-1])$ denotes the CPE state transition probability from instant $n-1$ to n . As each transition is uniquely determined by a modulation symbol, the state transition probability can be written as the product of probabilities of individual bits composing the modulation symbol (see Table I) at instant n ,

$$\mathbb{P}(x[n] | x[n-1]) = \prod_{m=0}^{P-1} \mathbb{P}(\theta^{(m)}[n]) \quad (40)$$

The probabilities $\mathbb{P}(\theta^{(m)}[n])$ can be obtained *a priori* from the outer SISO channel decoder as depicted in Fig. 3. Since we will use log-domain MAP algorithm, we compute $\ln(\mathbb{P}(x[n] | x[n-1]))$ using $\mathcal{L}(\theta^{(m)}[n], O)$ as given in (41),

$$\ln(\mathbb{P}(x[n] | x[n-1])) = \sum_{m=0}^{P-1} \theta^{(m)}[n] \frac{\mathcal{L}(\theta^{(m)}[n], I)}{2} + \epsilon[n] \quad (41)$$

where $\epsilon[n]$ is a positive constant that does not depend on whether $\theta^{(m)}[n]$ takes on the value ± 1 (see Appendix B for the derivation of (41)).

Evaluation of the likelihood function in (39) is now in order. Inspecting the closed-form expressions of pseudo-symbols in Table II, and noting the state equation of CPE given in (6), it follows that the knowledge of $x[n]$ and $x[n-1]$ are sufficient to uniquely determine the values that $a_j[n]$ can take on at instant n . Therefore, the likelihood function $f(\mathbf{y}[n] | x[n], x[n-1])$ in (39) can be re-written as

$$f(\mathbf{y}[n] | x[n], x[n-1]) = f(\mathbf{y}[n] | a_0[n], \dots, a_{J-1}[n]) \quad (42)$$

This enables us to use the equivalent memoryless LMMSE channels given in (26) and (36) to evaluate the likelihood function in a straightforward manner. Taking into account that the white Gaussian noise samples observed at LMMSE outputs are statistically independent, we can write the likelihood function as

$$f(\mathbf{y}[n] | a_0[n], \dots, a_{J-1}[n]) = \prod_{j=0}^{J-1} \frac{1}{\sqrt{\pi\sigma_{v_j}^2}} e^{-\frac{|y_j[n] - \mu_j a_j[n]|^2}{\sigma_{v_j}^2}} \quad (43)$$

Taking the natural logarithms of both sides of (43) and manipulating the necessary algebraic operations, the log-likelihood function for the SISO CPM unit is obtained as

$$\Lambda(\mathbf{y}[n] | a_0[n], \dots, a_{J-1}[n]) \triangleq \ln(f(\mathbf{y}[n] | a_0[n], \dots, a_{J-1}[n])) \propto 2 \frac{\mu_j}{\sigma_{v_j}^2} \text{Re} \{y_j[n] a_j^*[n]\} \quad (44)$$

Following from (28) and (38), we can define a reliability factor for the j 'th LMMSE equalizer as⁴

$$\chi_j \triangleq 2 \frac{\mu_j}{\sigma_{v_j}^2} = \frac{2}{1 - \mu_j} \quad (45)$$

Then the final expression for the SISO CPM log-likelihood function becomes

$$\Lambda(\mathbf{y}[n] | a_0[n], \dots, a_{J-1}[n]) \propto \chi_j \text{Re} \{y_j[n] a_j^*[n]\} \quad (46)$$

where $\text{Re}\{\cdot\}$ denotes the real part of its argument. Combining (41) and (46), we describe the log-domain BCJR measurement metrics as

$$\gamma_{s',s}[n] = \sum_{m=0}^{P-1} \theta^{(m)}[n] \frac{\mathcal{L}(\theta^{(m)}[n], I)}{2} + \chi_j \text{Re} \{y_j[n] a_j^*[n]\} \quad (47)$$

where we omitted constant terms that cancel out during the iterative process. $(s', s) \in \mathcal{S}$ label the instantaneous values that the CPE state takes on at instants $n-1$ and n , respectively, such that $x[n-1] = s'$, $x[n] = s$, where $\mathcal{S} \triangleq \{0, \dots, p-1\}$.

2) *Forward/Backward Recursions and Decision Variables*: We use the “max-log-map” algorithm due to its simplicity. The log-domain forward and backward recursion variables are given, respectively, in (48) and (49) [31]

$$\eta_s[n] = \underset{s' \in \mathcal{S}}{\text{argmax}} (\eta_{s'}[n-1] + \gamma_{s',s}[n]) \quad (48)$$

$$\beta_{s'}[n-1] = \underset{s \in \mathcal{S}}{\text{argmax}} (\beta_s[n] + \gamma_{s',s}[n]) \quad (49)$$

with the boundary conditions $\eta_0[0] = 0, \eta_{s \neq 0}[0] = -\infty$ and $\beta_s[U] = \ln(1/p)$, $\forall s \in [0, p-1]$, where U denotes the number of coded bits in a block. This follows from the fact that the CPE starts from all zero state, and is left at a random state at $n = U$, because we do not append extra bits to the coded bit stream to terminate the CPE trellis.

Then using the forward/backward recursion metrics and skipping the intermediate algebraic steps, the *a posteriori* $\mathcal{L}$ -values on the coded and interleaved bits are obtained as

$$\mathcal{L}(\theta^{(m)}[n], O) = \ln \left(\frac{\mathbb{P}(\theta^{(m)}[n] = +1 | \mathbf{y}[1:U])}{\mathbb{P}(\theta^{(m)}[n] = -1 | \mathbf{y}[1:U])} \right) \quad (50)$$

$$\approx \underset{\substack{(s',s) \Rightarrow \\ \theta^{(m)}[n]=+1}}{\text{argmax}} (\eta_{s'}[n-1] + \gamma_{s',s}[n] + \beta_s[n]) - \underset{\substack{(s',s) \Rightarrow \\ \theta^{(m)}[n]=-1}}{\text{argmax}} (\eta_{s'}[n-1] + \gamma_{s',s}[n] + \beta_s[n]) \quad (51)$$

where $\mathbf{y}[1:U]$ denotes the concatenation of all data measured at LMMSE outputs for the entire block of U coded bits.

⁴ μ_j in (45) equals μ_j^{nai} for the nai case, and $\mu_j = \mu_j^{\text{fai}}$ for the fai.

As seen in Fig. 3, another soft output that the SISO CPM detector is supposed to produce is the *a posteriori* probabilities $\mathbb{P}(x[n] | y[1 : U])$ to calculate the expected value of $a_0[n]$, and $\mathbb{P}(x[n-1], \zeta^{(m)}[n] | y[1 : U])$ to calculate the expected values of $a_j[n]$, $j > 0$ (see Table II(b)). Since there is a one-to-one mapping between $\zeta^{(m)}[n]$ and its BPSK correspondent $\theta^{(m)}[n]$ (see Table I), and since the modulation symbols are statistically independent from the CPE state, we have

$$\mathbb{P}(x[n-1], \zeta^{(m)}[n] | y[1 : U]) = \mathbb{P}(x[n-1] | y[1 : U]) \times \mathbb{P}(\theta^{(m)}[n] | y[1 : U]) \quad (52)$$

Following from [30, eq.(1,3)], the first factor in (52) is obtained as

$$\mathbb{P}(x[n-1] | y[1 : U]) \approx \frac{e^{\eta_s[n-1] + \beta_s[n-1]}}{\sum_{s \in S} e^{\eta_s[n-1] + \beta_s[n-1]}} \quad (53)$$

Likewise, $\mathbb{P}(x[n] | y[1 : U])$ can be obtained simply by replacing $n-1$ with n in (53). The probabilities, $\mathbb{P}(\theta^{(m)}[n] | y[1 : U])$, on the other hand, can be deduced by substituting $\mathcal{L}(\theta^{(m)}[n], O)$ from (50) in (60). Evaluating the pseudo-symbols with the probabilities $\mathbb{P}(x[n] | y[1 : U])$, $\mathbb{P}(x[n-1] | y[1 : U])$ and $\mathbb{P}(\theta^{(m)}[n] | y[1 : U])$, we can calculate the pseudo-symbol expectations $\tilde{a}_j[n] \triangleq \mathbb{E}[a_j[n] | y[1 : U]]$.

F. SISO Channel Decoder

SISO channel decoder is a standard convolutional SISO module [31]. The Log-Likelihood Ratios (LLR) on the coded bits required by the SISO channel decoder are obtained by de-interleaving the modulating bit LLRs delivered by the SISO CPM module, $\Lambda(y[1 : U] | \theta^{(m)}[n])$, defined as,

$$\begin{aligned} \Lambda(y[1 : U] | \theta^{(m)}[n]) &\triangleq \ln \left(\frac{f(y[1 : U] | \theta^{(m)}[n] = +1)}{f(y[1 : U] | \theta^{(m)}[n] = -1)} \right) \\ &= \mathcal{L}(\theta^{(m)}[n] | y[1 : U]) - \mathcal{L}(\theta^{(m)}[n]) \end{aligned} \quad (54)$$

which follows from Bayes' theorem. Since all message bits, $d[n]$, are assumed equi-probable *a priori*, $\mathcal{L}(d[n], I) = 0$, as seen in Fig. 3.

The *a posteriori* $\mathcal{L}$ -values, $\mathcal{L}(d[n], O)$, produced by the SISO channel decoder are thresholded, leading to the hard estimates of message bits. The *a posteriori* $\mathcal{L}$ -values, $\mathcal{L}(u[n], O)$, on the coded bits are used for constructing the extrinsic information on the coded bits given as

$$\mathcal{L}(u[n], I) = \mathcal{L}(u[n], O) - \Lambda(y[1 : U] | \theta^{(m)}[n]) \quad (55)$$

Interleaving $\mathcal{L}(u[n], I)$ results in the *a priori* $\mathcal{L}$ -values on the modulation bits which are fed into the SISO CPM detector as seen in Fig. 3, completing the inner iterative loop, as well as the entire receiver design.

IV. EXPERIMENTS

We demonstrate the performance of the proposed system on three binary, and three quaternary CPM schemes, all of which were chosen for their fast spectral roll-offs as seen in Fig. 1. In what follows, "Q(B)LRC" abbreviates Quaternary (Binary) CPM with a Raised Cosine pulse occupying L symbol periods.

For channel coding, we use a rate-1/2 non-recursive, non-systematic convolutional code with the octal generator representation (7, 5)₈ as in [16–18]. The number of message bits in a block is $N = 2048$ with the only exception in Fig. 4(c), as will be explained in the sequel. A pseudo-random bit interleaver is used in all experiments.

In the "nai" mode, 15 iterations are performed in the inner (SISO) loop without any interference feedback. Hence in this mode, LMMSE filter tap-weights are calculated only once and the equalization proceeds in a feed-forward fashion, which is marked as "Static LMMSE" in Fig. 4 and Fig. 5. In the "fai" mode, 5 and 3 iterations are performed, respectively, in the outer and the inner loops, adding up to 15 iterations in total. This is labeled as "LMMSE-SIC" in Fig. 4 and Fig. 5. All LMMSE filters are realized with $W = 2$ prediction, and $V + 1 = 3$ smoothing taps. Hence the total number of tap-weights for each LMMSE filter is $K = W + V + 1 = 5$.

A. Experiments on Quaternary CPM

In Fig. 4, BER simulation results are presented for the quaternary CPM schemes. In this case the receiver encompasses $J = 3$ matched filters, cascaded with $J = 3$ LMMSE filters and followed by the SISO CPM detector that has $p = 4$ states. The CPM scheme in Fig. 4(a) was shown to performs very close to the optimum CPM demodulator [16–18]. Hence, we reproduce the reduced complexity technique proposed in [16–18] and consider it as a frame of reference rather than simulating the optimum CPM receiver. We refer to the complexity reduction technique in [16–18] as "Colavolpe et al." in Fig. 4.

As seen in Fig. 4(a) and Fig. 4(b), the LMMSE-SIC receiver proposed herein is only marginally better than that in [16–18]. The interference-ignorant complexity reduction technique proposed by Colavolpe et al. is, however, simpler than our approach, since it requires only 3 matched filters, followed by a $p = 4$ -state CPM detector. Therefore, the method in [16–18] should be the obvious choice for implementation for $L \leq 3$.

For $L = 4$, on the other hand, as seen in Fig. 4(c), the receiver of Colavolpe et al. does not work any longer. This is because of the fact that for long pulses (e.g., $L > 3$), the controlled ISI in CPM transmit signal cannot be neglected anymore. Our proposed receiver, however, performs well for $L = 4$ as seen in Fig. 4(c), where, we also report results for $N = 4096$ to demonstrate that the performance of LMMSE-SIC receiver can be increased considerably for larger N .

It is also noted in Fig. 4 that the LMMSE-SIC receiver outperforms the static (feed-forward) LMMSE receiver markedly for all pulse lengths. This is due to the fact that for $M = 4$ and $h = 1/4$, the pseudo-symbol auto and cross correlations take on non-zero values only at the zero lag. Therefore, both the controlled ISI and the controlled MAI are highly uncorrelated with the desired response of each LMMSE filter. Hence interference suppression performs very successfully.

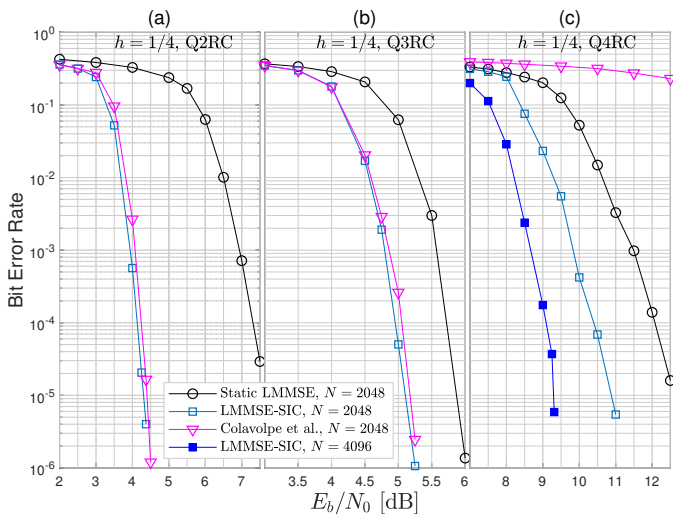


Fig. 4. Bit error rate analysis of serially concatenated Quaternary CPM with three low complexity demodulation/detection techniques. h is the CPM modulation index and the abbreviation “QLRC” represents Quaternary CPM with a Raised Cosine pulse occupying L symbol periods. T denotes the duration of one symbol. N is the number of message bits in a transmitted block. The legend printed in between (a) and (b) is applicable to (c) as well. “SIC” is short for “Soft Interference Cancellation”.

B. Experiments on Binary CPM

In Fig. 5, we report the BER results on the binary CPM schemes that we consider. In this case, $J = 1$ and the controlled MAI is out of the picture, leaving the controlled ISI as the only source of self-interference. The receiver design takes an even simpler form with only one matched filter followed by a single LMMSE filter with $K = 5$ taps. For the binary CPM, we judge the performance of our receiver against the optimum CPM demodulator with full complexity, i.e., having pM^{L-1} states in its trellis. Since we consider $L \geq 5$ in Fig. 5, it is not necessary to reproduce the method of Colavolpe et al., which is not applicable to long pulses as demonstrated in the previous figure.

In Fig. 5(a), we present the BER for $\{h = 1/2, \text{B6RC}\}$. This CPM scheme leads to uncorrelated pseudo-symbols, and the controlled ISI is, therefore, uncorrelated to the desired LMMSE filter response. Hence, the LMMSE-SIC receiver outperforms the static LMMSE, and performs very close to the optimum. The SISO CPM detector for this scheme has $p = 2$ states in its trellis, leading to a very simple yet efficient receiver. The optimum CPM receiver, however, incorporates a $pM^{L-1} = 64$ -state trellis.

In Fig. 5(b), the BER results for $\{h = 2/5, \text{B5RC}\}$ are studied. For this scheme, the pseudo-symbol autocorrelations expand beyond lag zero, and the performance gap between the static LMMSE and LMMSE-SIC is much narrower than in Fig. 5(a). Both the static LMMSE and LMMSE-SIC perform very close to the optimum CPM demodulator that has $pM^{L-1} = 80$ states. The SISO CPM detector has $p = 5$ states for this scheme.

Finally, in Fig. 5(c), we report the BER results for $\{h = 1/3, \text{B5RC}\}$. Since the modulation index is even smaller than in Fig. 5(a) and Fig. 5(b), the pseudo-symbols become even more correlated, and the performances of static LMMSE and

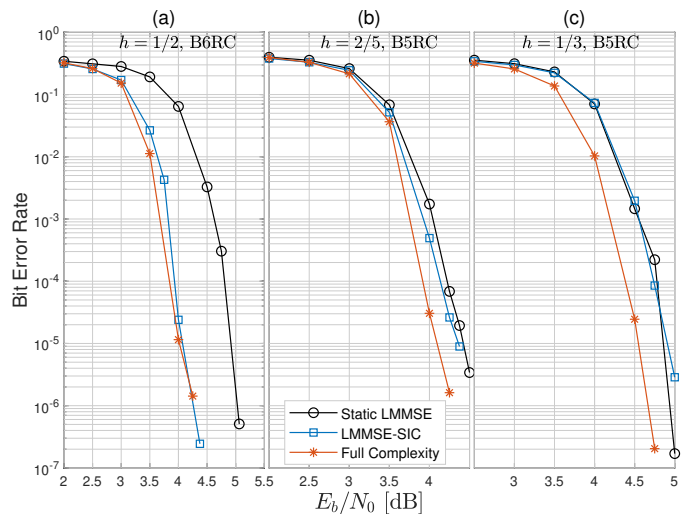


Fig. 5. Bit error rate analysis of serially concatenated Binary CPM with the proposed low-complexity demodulation/detection technique with contrast to the full complexity receiver. h is the CPM modulation index and the abbreviation “BLRC” represents Binary CPM with a Raised Cosine pulse having the length of L symbol durations. The legend printed on (b) is applicable to (a) and (c) as well. “SIC” is short for “Soft Interference Cancellation”.

LMMSE-SIC are nearly identical, and again, very close to the optimum receiver with $pM^{L-1} = 48$ states. The SISO CPM detector has $p = 3$ states in this final experiment.

V. CONCLUDING REMARKS

We proposed a reduced complexity Continuous Phase Modulation (CPM) receiver for serially concatenated CPM schemes (SCCPM). By virtue of treating the modulation memory as a source of controlled ISI when $L > 1$, and interpreting the modulation cardinality as a source of controlled MAI for $M > 2$, we presented a linear MMSE receiver empowered by soft interference cancellation to equalize for the self interference that is regarded to be contained inherently in a CPM waveform.

This leads to a reduction in the receiver complexity from a pM^{L-1} -state trellis decoder, required for an optimum CPM demodulator, to a p -state trellis decoder, preceded by $J = 2^{\lceil \log_2 M \rceil} - 1$ parallel matched filters and LMMSE equalizers, proposed by our approach. This provides a substantial drop in receiver complexity with an acceptable degradation in performance, as demonstrated in simulations, leading to a low complexity receiver design generalizable to all CPM formats.

We also nominate some CPM schemes as suitable candidates for frequency-division multiplexed constant envelope systems for future generations of satellite communications. These schemes, whose power spectral densities and performances were demonstrated, respectively, in Fig. 1(b) in Fig. 5, will be the subject of a “carrier packing” approach in a future communication. Our preliminary results using the elements of information theory suggest that significant bandwidth savings can be achieved in constant envelope communication using the spectrally advanced CPM waveforms with large L and using the LMMSE-SIC receiver architecture proposed herein.

APPENDIX A

NOISE AUTOCORRELATION AT MATCHED FILTER OUTPUTS

Let us describe the autocorrelation function of the discrete-time filtered noise sample $v_j[n]$ as given in (56) at lag τ

$$\psi_{v_j}[\tau] \triangleq \mathbb{E} \left[v_j[n] v_j^*[n + \tau] \right] \quad (56)$$

where $\mathbb{E}[\cdot]$ denotes the statistical expectation operator. Substituting (12) in (56), and noting that all pulses are real-valued functions, we obtain

$$\psi_{v_j}[\tau] = N_0 \int_0^{L_j T} g_j(t - \tau T) g_j(t) dt \quad (57)$$

$$= N_0 \rho_{j,j}[\tau] \quad (58)$$

where we made use of the definition of $\rho_{i,j}[l]$ in (11) to obtain (58) from (57).

The noise covariance matrix $\Psi_j \in \mathbb{R}^{K \times K}$ for the vector of coloured noise samples observed at the output of the j 'th filter (described in (19)) is given as

$$\Psi_j \triangleq \begin{bmatrix} \psi_{v_j}[0] & \psi_{v_j}[1] & \dots & \psi_{v_j}[K-1] \\ \psi_{v_j}[-1] & \psi_{v_j}[0] & \dots & \psi_{v_j}[K-2] \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{v_j}[-K+1] & \psi_{v_j}[-K+2] & \dots & \psi_{v_j}[0] \end{bmatrix} \quad (59)$$

APPENDIX B

RELATING $\mathcal{L}$ -VALUES TO PROBABILITIES

Probability of a binary symbol $\theta^{(m)}[n]$ is related to its *a priori* $\mathcal{L}$ -value [32, Ch. 5, eq(5.5)] given as

$$\mathbb{P} \left(\theta^{(m)}[n] = \pm 1 \right) = \left(\frac{e^{-\mathcal{L}(\theta^{(m)}[n])/2}}{1 + e^{-\mathcal{L}(\theta^{(m)}[n])/2}} \right) e^{\pm \mathcal{L}(\theta^{(m)}[n])/2} \quad (60)$$

Then taking natural logarithm of each side of (60), we have

$$\ln \left(\mathbb{P} \left(\theta^{(m)}[n] = \pm 1 \right) \right) = \pm \mathcal{L}(\theta^{(m)}[n])/2 + \epsilon[n] \quad (61)$$

where the term $\epsilon[n] \triangleq \left(\frac{e^{-\mathcal{L}(\theta^{(m)}[n])/2}}{1 + e^{-\mathcal{L}(\theta^{(m)}[n])/2}} \right) > 0 \forall \theta^{(m)}[n]$, and does not depend on whether $\theta^{(m)}[n]$ takes on the value ± 1 .

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Ulaş Güntürkün received his Ph.D. degree in Electrical Engineering at McMaster University, Hamilton, ON, Canada in 2007, where his dissertation addressed novel signal processing algorithms for the estimation and tracking of hidden inputs and states in nonlinear, nonstationary dynamic systems. For his post-doctoral work, he joined the Université catholique de Louvain in Belgium in 2008, where he worked on the development of low complexity Continuous Phase Modulation transmitter and receiver design, along with other aspects of contemporary communication systems until 2012. Thereafter, he received an independent research grant provided by the Turkish National Council of Technical and Scientific Research to continue his work on the information theoretic aspects of channels with memory. Between 2014 and 2015, he worked under a research contract granted by Aselsan to address the needs of Aselsan’s proprietary satellite communication systems. He was with Ultra Maritime Digital Communications Research Centre between 2015 and 2018, affiliated with Dalhousie University, Halifax, Canada, working on large-scale MIMO-OFDM techniques. In 2018, he founded Marecomms Inc. to design and implement a novel underwater acoustic communications system. His current research interests include 5G physical layer design and radar/sonar signal processing in addition to underwater acoustic and constant envelope communications.

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