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**Stabilized hotspots and vortex patterns
in superconducting NbN and Nb
nanostructured strips and loops**

Dissertation submitted in preparation for the degree of
Docteur en Sciences de l'Ingénieur
by

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- (1) **Microwave power dependence of the retrapping current of superconducting nanostrips**
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- (2) **Switching current modulations induced by vortices rearrangement in mesoscopic superconducting loops**
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- (3) **Discontinuous hotspot growth related to the thermal healing length in superconducting NbN microstrips**
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- (4) **Stabilization of non-self-spreading hotspots in current- and voltage-biased superconducting NbN microstrips**
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- (5) **Non-equilibrium resistive states of superconducting NbN microstrips in a transverse magnetic field**
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- (6) **Vortex detection by electrical transport measurements on a single lead nanowire under axial magnetic field**
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- (7) **Superconducting properties of perforated NbN films using ordered arrays of ferromagnetic nanowires**

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Introduction

The year 2011 celebrates the centennial of the superconductivity that was discovered by Kamerlingh Onnes in Leiden in The Netherlands [1]. This breakthrough in material sciences has closely followed the first liquefaction of the helium-4, a major advance that has opened prospects for the study of the low-temperature behavior of the matter. In particular, the ability for the electrons to form a superconducting condensate of Cooper pairs below the critical temperature has established itself as one of the leading topic in condensed matter and material physics. The craze around this discovery remains intact, especially due to the wide variety of present and potential applications that take advantage of the striking properties of the superconductivity, i.e., the cancellation of the electrical resistivity and the complete expulsion of the magnetic field lines from the interior of the superconductor. During the last 100 years, the superconductivity community was confronted to many technological and fundamental challenging issues. Among them, one finds the comprehension of the physical processes that occur when the samples sizes are reduced below the superconducting characteristic lengths, namely the coherence length ξ and the magnetic field penetration depth λ . This thesis aims to explore more deeply this concern. Indeed, **the two basic properties of the superconductivity are investigated by electrical transport measurements performed on multi-probe niobium nitride [NbN] and niobium [Nb] nanostructured strips and loops** that have been fabricated by electron beam lithography at the Laboratoire de Photonique et de Nanostructures (LPN-CNRS) of Marcoussis.

The main property of the superconductivity is the ability for the paired electrons to move without dissipation under the application of a direct current. This zero electrical resistance is preserved insofar as the applied current does not exceed the critical current above which the superconductor switches into the (dissipative) normal state. The current-carrying capabilities of narrow thin-film strips are crucial for many applications. In particular, whether a current-driven superconductor presents intermediate resistance levels between the perfect super-

conducting state and the fully normal state remains the subject of intensive researches. In the seventies, the stepwise evolution of the sample voltage under an increasing current was associated to the development of regular dissipative units that involve the localized slippage of the superconducting phase, hence the name *phase-slip center* [2]. It was first stated that the nucleation of these phase-slip centers requires small lateral sizes compared to the coherence length ξ of the material. However, it was progressively accepted that similar dissipative structures may develop in samples much wider and thicker than ξ [3]. From this time, rare are the experimental works that do not explain the regular steps observed in the current-driven evolution of the resistance by the concept of phase-slippage phenomena. A major point, however, seems missing - at least often neglected - in this interpretation of the current-driven behavior. Indeed, a resistive area submitted to a direct transport current is the site of an intense dissipation by Joule effect. As long as the heating locally exceeds the heat removal capabilities, the stabilization of resistive *hotspot* domains is conceivable [4]. **Investigating the distinction between the current-induced stabilization of normal hotspot domains and of phase-slippage phenomena in superconducting nanostructures is the first aim of this thesis.** Although the nucleation of the latter mechanisms was originally considered, the heating effects turn out to be responsible for the numerous intermediate states that are probed under a direct electrical bias [5].

The second aim of the electrical transport measurements is to better understand the high-frequency response of the NbN and Nb microstrips. The interest is twofold. From a fundamental viewpoint, one feature of the phase-slippage phenomena model is that the superconducting wavefunction oscillates in time at a characteristic frequency that ranges well above the gigahertz. The ability for a phase-slippage phenomenon to interact with another dissipative unit or with an external electromagnetic radiation can thus be considered as a potential tool to discriminate the latter mechanisms from the nucleation of purely thermal domains. On the other hand, many practical optoelectronic detectors that are based on similar superconducting nanowires involve heating-induced phenomena, as it is the case for the superconducting nanowire single-photon detectors [6]. Contrary to the latter devices that are based on the destruction of the perfect superconducting state due to the photon-induced formation of a dissipative hotspot, the present study interests in the evolution of the current-driven normal-to-superconducting transition. This thesis succeeds in proposing an experimental procedure that allows one to determine the tiny fraction of the total input power which is effectively absorbed by the normal domains in the continuous

irradiation mode [7]. By contrast, the usual optoelectronic systems are efficient to detect events that involve a few photons but are not used to quantify the power which is locally available in the steady-state regime.

While the two previous topics are intimately related to the thermal bistability under Joule heating of a current-carrying superconductor, **the third subject of this thesis interests in probing the other remarkable property of the superconductivity, i.e., the expulsion of the magnetic field from the interior of a (narrow) superconductor.** Contrary to bulk type-II superconductors for which the perfect Meissner state is destroyed at the first critical field B_{c1} , reducing the width of the strips delays the penetration of magnetic vortices up to the superheating field $B_{SH} > B_{c1}$. In the last few years, the influence of the width on this critical field below which the vortices are completely expelled was studied by scanning probe microscopy techniques on various materials [8,9]. The main limitation is the setup resolution that restricts the study to micrometer-wide strips. Here, the magnetic field evolution of the critical current is used to investigate the vortices entrance into field-cooled NbN nanostrips and Nb loops. The overall shape and the periodic modulations above B_{SH} are used to characterize the penetration of the first vortices and the rearrangement of the vortices patterns, respectively. The present electrical transport measurements appear to be efficient tools to extract the superheating field of submicrometer-wide strips [10] whose width is as small as a few tens of nanometers, in excellent agreement with the predicted theoretical width-dependence.

The manuscript is organized as follows. The chapter 1 introduces the properties of the superconductivity that are required to discuss the experimental results. The fabrication and the characterization of the NbN and Nb thin films and nanostructures are presented in the chapter 2. The chapter 3 is devoted to the description of the electrical transport properties of the NbN and Nb strips as a function of the bias and of the coolant temperature. The focus is successively put on the thermal bistability under Joule heating (3.1), on the stabilization of normal hotspot domains (3.2) induced by the nanoscale cross-section modulations (3.3), on the impact of a continuous microwave irradiation (3.4) and, finally, on the distinction between normal hotspot domains and phase-slippage phenomena (3.5). The chapter 4 discusses the magnetic field dependence of the critical current for both the NbN strips and the Nb loops. The entrance of the first vortices (4.2) and the rearrangement of the vortices patterns (4.3) are described, as well as the superheating fields that are extracted from these electrical transport measurements (4.4). In close relation with the chapter 4, the appendix A discusses the impact of the vortices in the observation of the fluxoid quantization effect.

Chapter 1

Key concepts of superconductivity

This chapter aims to look over the main theoretical and experimental concepts required to discuss the findings presented throughout this manuscript. The section 1.1 briefly introduces the macroscopic superconductivity, i.e., the electrical and magnetic behavior of samples whose dimensions are much larger than the superconducting characteristic lengths. The two next sections interest in the electrical behavior of superconducting samples submitted to an external transport current, especially on the ability to stabilize intermediate resistive domains between the perfect superconducting state and the fully normal state. The section 1.2 focuses on the utmost impact of the thermal (Joule) dissipation in the stabilization of localized normal domains while the section 1.3 is devoted to the development of resistive states in the charge-mode disequilibrium. Other kinds of non-equilibrium dissipative processes are discussed in the section 1.4 which deals with the response of granular superconductors submitted to an electromagnetic irradiation. Once again, the thermal dissipation has an undeniable influence. The section 1.5 is slightly dissociated from the previous ones in the sense that it focuses on the magnetic behavior of narrow superconducting strips in a perpendicular applied field. In particular, it describes the conditions of entrance and arrangement of the vortices when the transverse dimensions are reduced.

1.1 Overview on the macroscopic superconductivity

Three years after having succeeded to liquefy helium, Heike Kamerlingh Onnes and its group [1] discovered in 1911 that the electrical resistivity of mercury drops when the temperature is reduced below ~ 4 K. The unmeasurable resistivity of a bulk material below the so-called critical temperature T_c was the first - and spectacular - manifestation of the superconductivity. Such a transition between the normal state (above T_c) and the superconducting state (below T_c) was measured soon after in many other metallic elements (tin, lead, niobium, ...), alloys (NbN, NbTi, ...) and inter-metallic compounds (Nb₃Sn, Nb₃Ge, ...). The critical temperature was found to depend on the material, with values of the order of a few Kelvins for pure metals (see the first line of table 1.1) up to a few tenths of Kelvins for alloys and inter-metallic compounds (see the second line of table 1.1). In the race for the achievement of the highest critical temperature, a major advance was done in 1986 by Georg Bednorz and Karl Müller [11]. They discovered that the semiconducting oxygen-deficient compounds Ba-La-Cu-O were superconducting near 30 K, opening the way to the observation of superconductivity above the boiling point of liquid nitrogen (77 K). These oxide superconductors made of copper-oxide planes (whose most famous representative is the yttrium-based YBa₂Cu₃O₇) are often called *high temperature superconductors*. Another interesting recent finding concerns the ionic binary compound MgB₂, whose superconducting behavior was highlighted in 2001 by Nagamatsu et al. [12]. Its discovery still causes a great interest. This is also the case for the iron-based superconductors discovered in 2006 with the LaFeOP compound ($T_c \sim 5$ K) [13]. The lively interest for this new superconductors actually leads to a critical temperature that exceeds 50 K, e.g., $T_c = 53.5$ K for the GdFeAsO_{1- δ} compound [14].

Table 1.1: Critical temperature of representative bulk superconductors (pure elements, alloys and inter-metallic compounds, high temperature superconductors). The two superconductors of interest in the present study are the niobium (Nb) and the niobium nitride (NbN).

	Al	Sn	Pb	Nb
T_c [K]	1.2	3.72	7.19	9.2
	NbTi	NbN	Nb ₃ Sn	Nb ₃ Ge
T_c [K]	9.5	~ 16	18.3	23.2
	MgB ₂	YBa ₂ Cu ₃ O ₇	Bi ₂ Sr ₂ Ca ₂ Cu ₃ O ₁₀	HgBa ₂ Ca ₂ Cu ₃ O _x
T_c [K]	39	~ 90	~ 110	~ 130

1.1.1 Fundamental properties of the superconductivity

Scientists first considered that a superconductor comes down to a simple perfect conductor. From the Maxwell - Faraday equation $\vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$ (where $\vec{E}$ is the electric field and $\vec{B}$ is the magnetic induction), one knows that a perfect electrical conductor goes against any variation of the magnetic induction within it so that the induction has to be constant. The latter depends on the thermo-magnetic history of the conductor, i.e., on whether an external magnetic field is applied when the sample is cooled through its characteristic temperature. However, a *superconductor* is more than a simple ideal conductor. In 1933, Walther Meissner and Robert Ochsenfeld [15] discovered that the magnetic field inside a superconductor is always zero, independently of its thermo-magnetic history. This so-called Meissner effect results from the development of persistent screening currents in a thin penetration layer near the surface of the material which annihilate the applied field deep inside the bulk. A magnetization $\vec{M}$ appears inside the superconductor and cancels the magnetic excitation $\vec{H}$ so that the magnetic induction $\vec{B} = \mu_0(\vec{H} + \vec{M})$ is zero (μ_0 is the vacuum permeability). Superconductors thus exhibit a perfect diamagnetism characterized by a magnetic susceptibility $\chi \equiv \frac{M}{H} = -1$. Together with the cancellation of the electrical resistivity below T_c , the diamagnetic behavior is the real signature of the superconductivity.

The Meissner effect implies the existence of a critical magnetic field above which it becomes energetically more favorable for the material to lie in the normal resistive state. The thermodynamic critical field H_c is related to the difference in Gibbs free-energy density ΔG between the superconducting and the normal states when no magnetic field is applied. Indeed, the latter is given by $\Delta G = G_n(0) - G_s(0) = \frac{\mu_0 H_c^2}{2}$. It is found empirically that H_c varies with the temperature following the parabolic law $H_c(T) \approx H_c(0) [1 - (T/T_c)^2]$. The concept of critical field is intimately related to the maximum current density that can flow through a superconductor without dissipation. Indeed, it corresponds to the applied current density that produces a magnetic field H_c at the surface of the conductor. According to the Silsbee's rule for a bulk superconducting wire, the critical current density in the absence of an applied magnetic field is given by $j_c = 2H_c/r$, where r is the wire radius. The figure 1.1 illustrates the three-dimensional critical surface phase-diagram of representative superconductors. It is defined by the three important superconducting properties, i.e., the critical temperature $T_c(H, j)$, the critical field $H_c(T, j)$ and the critical current density $j_c(T, H)$. Each of them is maximum when the other two are zero.

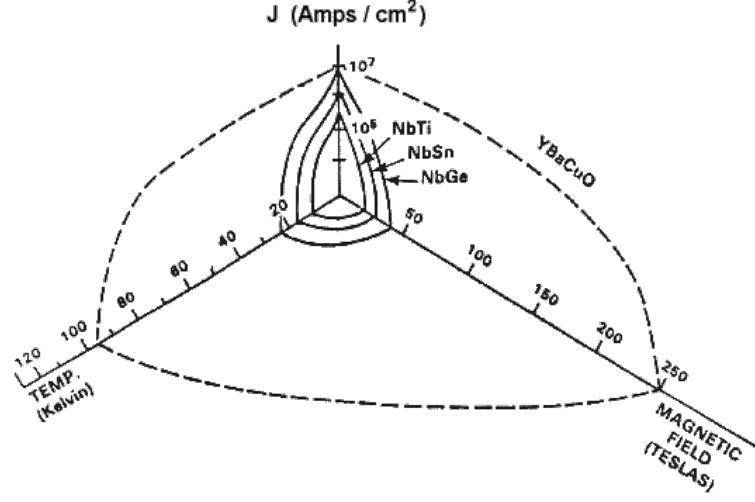


Figure 1.1: Three-dimensional critical surface phase-diagram of representative superconductors that involves the temperature, the magnetic field and the applied current density. All the points within the surface defined by $T_c(H, j)$, $H_c(T, j)$ and $j_c(T, H)$ lie in the superconducting state.

1.1.2 Successive advances in the understanding of the superconductivity

1.1.2.1 A classical phenomenological theory : the London equations

In 1935, Fritz and Heinz London [16] proposed the first phenomenological theory that explains the electromagnetic behavior of superconductors. Based on an original idea of Gorter and Casimir [17], they considered the two-fluid model according to which the free electrons of total density n are either condensed into a superfluid (responsible for the superconducting properties) or remain normal. The density of the superconducting electrons n_s decreases from n to zero when the temperature is increased from 0 K to T_c . Contrary to the classical electronic model of Drude [18], which is based on the competition between the scattering of the electrons on the positively-charged ions of the lattice and their acceleration by the electric field $\vec{E}$, the two brothers stated that the motion of the superconducting electrons in an electric field is described with no scattering (or frictional) term so that they are freely accelerated by $\vec{E}$. The average velocity of the superconducting electrons

$\langle \vec{v}_s \rangle$ is thus related to the field $\vec{E}$ by the relation $n_s m \frac{d\langle \vec{v}_s \rangle}{dt} = e \vec{E}$ (where m is the mass of an electron and e is the electronic charge). Together with the relation of the supercurrent density $\vec{j}_s = n_s e \langle \vec{v}_s \rangle$, this leads to the first equation of the London theory,

$$\frac{d\vec{j}_s}{dt} = \frac{n_s e^2}{m_s} \vec{E}, \quad (1.1)$$

which describes the perfect conductivity of the superconducting state : the flow of superconducting electrons meets no resistance and the electric field is zero in the stationary regime. After some mathematical treatment of Eq. 1.1, one obtains the equation $\vec{\nabla} \times \vec{j}_s = - \frac{n_s e^2}{m_s} \vec{B}$ which relates the supercurrent density and the magnetic field. This second London equation allows one to describe the spatial variation of a static magnetic field into a bulk superconductor. This is expressed by Eq. 1.2

$$\left(\frac{\mu_0 n_s e^2}{m_s} - \nabla^2 \right) \vec{B} = \left(\frac{1}{\lambda_L^2} - \nabla^2 \right) \vec{B} = 0, \quad (1.2)$$

where λ_L is called the London magnetic field penetration depth and corresponds to the length over which the field varies at the surface of the superconductor. Indeed, applying Eq. 1.2 to an infinite plane located in $x = 0$ gives the following dependence $B(x) = B(0) \exp^{-x/\lambda_L}$. The field is exponentially screened from the interior of the superconductor and decreases from a value equal to the applied field at the surface down to zero deep inside the material. A similar equation holds for the supercurrent density so that the latter also falls off over the same length near the surface of the superconductor. The parameter λ_L involves the density of superconducting electrons which tends to zero as the temperature approaches the critical one. As a consequence, λ_L diverges at $T \rightarrow T_c$ following the frequently used empirical temperature dependence $\lambda_L(T) \approx \lambda_{L0} / [1 - (T/T_c)^4]^{1/2}$ which is valid in the entire temperature interval [19]. In the local electrodynamics, it is usual to denote the theoretical limiting value (at $T = 0$ K) of the penetration depth of a pure superconductor by $\lambda_{L0} = \sqrt{\frac{m_s}{\mu_0 n e^2}}$, where the superconducting electrons density is replaced by the total electronic density n .

The London equations fall in the local electrodynamics limit as they relate the supercurrent density (equivalently, the average velocity of the superconducting electrons) to the magnetic field at the same location. This means that the spatial extension of the current carriers has to be smaller than the characteristic length λ_L over which the magnetic field varies. In 1953, Alfred Brian Pippard [20] introduced the notion of coherence length and proposed a non-local generalization of the London

equations. He proposed that any perturbation at a particular point of the superconductor influences the superfluid within a distance ξ_0 from that point. Indeed, considering that only the electrons within T_c from the Fermi surface play a role in the superconducting transition, the uncertainty principle gives $\Delta x \gtrsim \frac{\hbar}{\Delta_p} \approx \frac{\hbar v_F}{k_B T_c}$, where v_F is the Fermi velocity, $\hbar$ the reduced Planck constant and k_B the Boltzmann constant. This defines the (intrinsic) Pippard's coherence length

$$\xi_0 = a \frac{\hbar v_F}{k_B T_c}, \quad (1.3)$$

which is a material-dependent parameter that gives the typical size of the superconducting electron pairs (a is a numerical constant of order of unity). It can be much larger than λ_L in pure metals. Therefore, the local London equations can not be used for pure superconductors as the magnetic field presents noticeable changes over ξ_0 . The London formalism, however, is well-suited to describe the behavior of *dirty* materials that contain numerous impurities so that the electronic mean free path ℓ is much smaller than ξ_0 . In very dirty materials, the coherence length is even replaced by the mean free path. In the generalization proposed by Pippard, the effective coherence length ξ , defined by

$$\frac{1}{\xi} = \frac{1}{\xi_0} + \frac{1}{\ell}, \quad (1.4)$$

is used to include the effect of the electronic scattering over the impurities. The table 1.2 summarizes the intrinsic coherence length ξ_0 and the London magnetic field penetration depth at zero temperature λ_{L0} for a few representative superconductors.

Table 1.2: Intrinsic coherence length ξ_0 and London magnetic field penetration depth λ_{L0} at zero temperature for representative bulk superconductors. For the high temperature superconductor $\text{YBa}_2\text{Cu}_3\text{O}_7$, the values are given in the direction parallel and perpendicular to the CuO_2 planes, respectively.

	Sn	Pb	Nb	NbN	MgB ₂	YBa ₂ Cu ₃ O ₇
ξ_0 [nm]	230	83	38	3	3.6	1.5/0.25
λ_{L0} [nm]	34	37	39	200	130	27/180

1.1.2.2 A quantum phenomenological theory : the Ginzburg-Landau formalism

In 1950, Vitaly L. Ginzburg and Lev D. Landau [21] proposed the first quantum - but still phenomenological - theory of the superconducti-

vity (GL theory). Their explanation is based on the second-order phase transition theory developed by Landau to explain the transition of a system from a disordered state to an ordered state when the temperature is reduced below a particular critical temperature. In the Landau formalism, the system is characterized by an order parameter which is zero above the critical temperature and continuously increases without discontinuity at $T < T_c$. The GL theory identifies this order parameter with the effective condensate complex wavefunction of the superconducting electrons $\Psi(\vec{r}) = |\Psi(\vec{r})| \exp^{i\theta(\vec{r})}$, where θ is the phase of the superconducting electrons. The density of superconducting electrons is related to the squared amplitude of the wavefunction following the expression $|\Psi(\vec{r})|^2 = n_s/2$, where the factor $\frac{1}{2}$ takes into account that the superconducting electrons are paired. A relevant feature of the GL theory is that the superconducting electrons are described with a single spatial coordinate $\vec{r}$, so that the coherent behavior of all the superconducting electrons is suggested. Generally speaking, the Landau theory of second-order phase transitions considers the expansion of the free-energy in powers of the order parameter. It is only strictly valid near the critical temperature where the order parameter is small. In the GL theory, the Gibbs free-energy of the superconducting state is expressed in an applied magnetic field by the following equation

$$G_s = G_n + \alpha|\Psi|^2 + \frac{\beta}{2}|\Psi|^4 + \frac{1}{2m}|\frac{\hbar}{i}\vec{\nabla}\Psi - 2e\vec{A}\Psi|^2 + \frac{\vec{B}^2}{2\mu_0}, \quad (1.5)$$

where G_n is the Gibbs free-energy in the normal state, $\alpha(T)$ and $\beta(T)$ are phenomenological parameters that depend on the temperature, $\vec{A}$ is the magnetic vector potential such that $\vec{B} = \vec{\nabla} \times \vec{A}$ and $\hbar$ is the reduced Planck constant. The last term but one is related to the kinetic energy¹ while the last term stems from the magnetic contribution. At a given temperature $T < T_c$, the order parameter takes the value that minimizes the free-energy with respect to fluctuations in $\Psi(\vec{r})$ and $\vec{A}$. This leads to the Ginzburg-Landau equations

$$\alpha\Psi + \beta\Psi|\Psi|^2 + \frac{1}{2m} \left[\frac{\hbar}{i}\vec{\nabla} - 2e\vec{A} \right]^2 \Psi = 0, \quad (1.6)$$

$$\vec{j}_s = \frac{i\hbar e}{m} [\Psi\nabla\Psi^* - \Psi^*\nabla\Psi] - \frac{(2e)^2\vec{A}}{m}|\Psi|^2, \quad (1.7)$$

1. In quantum mechanics, the effective momentum operator under a magnetic field is expressed by $\vec{p} = \frac{\hbar}{i}\vec{\nabla} - 2e\vec{A}$.

that require a numerical resolution except if strong assumptions are done. Here, the equations 1.6 and 1.7 are presented because they lead to an expression of the second characteristic length of the superconductivity. The latter is given in the GL formalism by

$$\xi_{GL} = \sqrt{\frac{\hbar^2}{2m|\alpha|}}, \quad (1.8)$$

where $\alpha(T)$ is the temperature-dependent parameter previously introduced. The GL coherence length represents the characteristic scale over which the order parameter may vary. It is related but not the same that the temperature-independent intrinsic coherence length ξ_0 introduced by Pippard. As $|\alpha| \propto (T_c - T)$, the coherence length diverges near the critical temperature, as it is the case for the magnetic field penetration depth which is expressed in the GL formalism by

$$\lambda_{GL} = \sqrt{\frac{m}{4\mu_0 e^2 \Psi_0^2}}, \quad (1.9)$$

where $\Psi_0 \equiv \sqrt{\frac{-\alpha}{\beta}}$ is related to the paired electrons density deep inside the superconducting material, i.e., far away from the material surfaces.

1.1.2.3 A few words on the microscopic theory of Bardeen, Cooper and Schrieffer

In order to explain the isotopic effect [22, 23], Frölich suggested in 1950 that superconductivity is due to an electron-electron interaction which is transmitted through the vibrations of the crystal lattice, i.e., by the phonons. This concept is at the heart of the first microscopic theory of the superconductivity that was proposed in 1957 by John Bardeen, Leon Cooper and John Robert Schrieffer [24]. The BCS theory is based on a fundamental theorem of Cooper [25] that states that, for a system of many electrons at small temperature, any weak attractive interaction can bind two electrons close to the Fermi surface into a so-called Cooper pair. This happens when the lattice vibrations are in phase with the oscillations of the electron density, insofar as its frequency ω is smaller than the maximum possible frequency of the ion system, i.e., the Debye frequency ω_D . It is equivalent to say that electrons are attracted to each other if their energies differ from the Fermi level by no more than $\hbar\omega_D$. These electrons are paired over a finite distance (the coherence length) which can be three orders of magnitude larger than the lattice spacing. A

Cooper pair occupies the states $(\vec{k} \uparrow)$ and $(-\vec{k} \downarrow)$ in the momentum space so that the total spin is zero. All the Cooper pairs thus behave as Bose particles and present the noticeable property to condense in the lowest energy level (the ground state) when the temperature is reduced below the critical one. The Cooper pairs condensate carries the dissipation-free electric current so that the charge of an elementary superconducting current-carrier is $2e$.

A relevant novelty of the BCS theory is the introduction of an energy gap Δ which separates the spectrum of the elementary excitations (the quasiparticles) from the ground-state energy level. The energy required to break a Cooper pair corresponds to twice the gap as both electrons have to reach an excited level. In the framework of the BCS theory, the critical temperature and the superconducting energy gap at zero temperature are described by the expressions

$$k_B T_c = 1.14 \hbar \omega_D \exp \left(-\frac{1}{N_0 V} \right), \quad (1.10)$$

$$\Delta_0 = 2 \hbar \omega_D \exp \left(-\frac{1}{N_0 V} \right), \quad (1.11)$$

where N_0 is the density of states at the Fermi level and V is the energy of interaction responsible for the Cooper pairs formation. From Eqs. 1.10 and 1.11, one finds that $2\Delta_0 = 3.52 k_B T_c$, a relation which is in good agreement with several experiments. As a final comment, let's notice that the highest theoretical value that can take the critical current (known as the depairing current) is intimately related to the energy gap. Indeed, the superconducting state is destroyed when the kinetic energy of the Cooper pairs exceeds 2Δ .

1.1.3 Vortices in bulk type-II superconductors

Using the characteristic lengths of the superconductivity, Ginzburg and Landau have defined the so-called GL parameter $\kappa \equiv \lambda_{GL} / \xi_{GL}$ which is finite for $T \rightarrow T_c$ and approximately independent of the temperature. The latter allows one to distinguish between the two types of bulk superconductors, i.e., those of type-I (with $0 < \kappa < 1/\sqrt{2}$) and those of type-II (with $\kappa > 1/\sqrt{2}$). The distinction is related to the sign of the surface energy at the interface between a superconducting and a normal region. For type-I superconductors, the positive contribution that results from the reduction of the condensation energy (on a length-scale ξ) overcomes the benefit of the attenuation of the magnetic energy (on a length-scale λ). As a consequence, it becomes energetically unfavorable for the

material to be subdivided into a regular array of alternating normal and superconducting areas that maximizes the normal/superconductor interfaces. The magnetic response of a type-I superconductor is characterized by the perfect Meissner effect up to the thermodynamic critical field H_c [see the figure 1.2(a), top]. Above this field, the superconductor abruptly becomes normal following a first-order transition (the latent heat of transformation is non-zero). The situation is rather different for a type-II superconductor. Indeed, the negative surface energy makes favorable the penetration of the magnetic field on the form of normal filaments embedded in the superconductor, i.e., the so-called *vortices*. This is illustrated in Fig. 1.2(b). The first vortex enters the bulk superconductor at the lower critical field H_{c1} ($< H_c$) which is given in the Ginzburg-Landau formalism (for an ideally long, thin sample of zero demagnetization factor) by the equation 1.12,

$$H_{c1} = \frac{\Phi_0}{\mu_0 \pi \lambda^2} \ln \left(\frac{\lambda}{\xi} \right), \quad (1.12)$$

where Φ_0 is the superconducting flux quantum ($\equiv \frac{h}{2e}$). Starting from H_{c1} , the magnetization continuously decreases when the external magnetic field is increased [see the figure 1.2(a), bottom]. It vanishes at the upper critical field $H_{c2} = \frac{\Phi_0}{2\pi\xi^2}$ ($> H_c$) at which the vortices become so close that their cores overlap. The order parameter Ψ (and thus the density of Cooper pairs) is zero in the center of the vortex core. At sufficiently low fields (i.e., when the vortices are not so close from each other), the order parameter reaches its bulk value on a length-scale given by the coherence length. As a consequence, the vortex core size is often approximated by 2ξ . On the other hand, each core is surrounded by swirling supercurrents that extend on the magnetic field penetration depth and generate a field with the same orientation that the external one (parallel to the normal core). For a type-II superconductor, the density of superconducting electrons varies much more rapidly than the magnetic field so that a region $\sim \lambda$ exists where $\Psi \sim 1$ while the magnetic field still remains large. An important property of the vortices is that each of them carries a magnetic flux which is quantized and equals to the flux quantum $\Phi_0 \equiv \frac{h}{2e}$.

The mixed state of a type-II superconductor is thus characterized by an increasing number of vortices as the magnetic field is increased from H_{c1} to H_{c2} . Vortices do not interact with each other when they are distant from more than the magnetic penetration depth λ because their core does not *feel* the supercurrents that circulate around their neighbors. When the interdistance is reduced below λ , each pair of vortices (with the

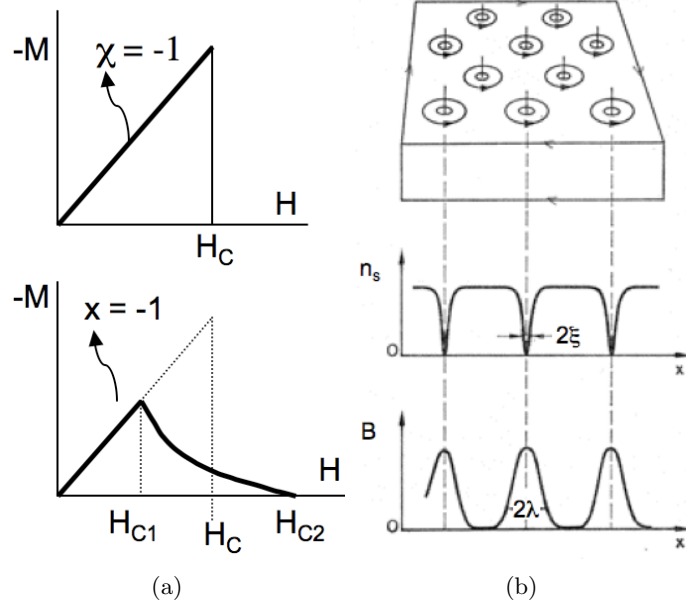


Figure 1.2: (a) Comparison of the magnetic response of a type-I (top) and a type-II (bottom) superconductor submitted to an external magnetic field (H_c is the thermodynamic critical field, H_{c1} is the lower critical field and H_{c2} is the upper critical field). (b) Mixed state of a type-II superconductor at a field $H \gtrsim H_{c1}$ [26]. The variations of the Cooper pairs density (on a length-scale $\sim 2\xi$) as well as of the local magnetic field (on a length-scale $\sim 2\lambda$) are depicted.

same orientation) interact repulsively. In the high- κ approximation [19], one finds that the force exerted on a vortex (per unit length of the vortex) is given by Eq. 1.13

$$\vec{f} = \vec{j}_s \times \vec{\Phi}_0, \quad (1.13)$$

where j_s represents the total supercurrent density due to all the other vortices at the location of the vortex core². For a bulk material, only a symmetrical array of vortices (with $\vec{j}_s = \vec{0}$) ensures a stable position of the vortices. By applying the GL theory to superconducting alloys, Alexei A. Abrikosov [27] was the first to discuss the formation of regular array of vortices. A few years later, it was shown that the triangular configuration is the more stable, i.e., it has a slightly lower free-energy than the square one. Following the work of Essmann and Träuble [28], many works have experimentally demonstrated the formation of the so-called Abrikosov vortices lattices. This is illustrated in Fig. 1.3 with (a)

2. The equation 1.13 remains valid if one includes an external transport current.

a magnetic-force microscopy image of a thin niobium film [29] and (b) a magneto-optical image of a NbSe₂ single crystal [30]. The formation of perfectly regular vortices arrays further requires that the intrinsic pinning of the materials is reduced. Indeed, if the samples contain a large amount of defects (i.e., grain boundaries, dislocation walls, voids, ...), the vortices are pinned and are not able to probe the lowest energy states.

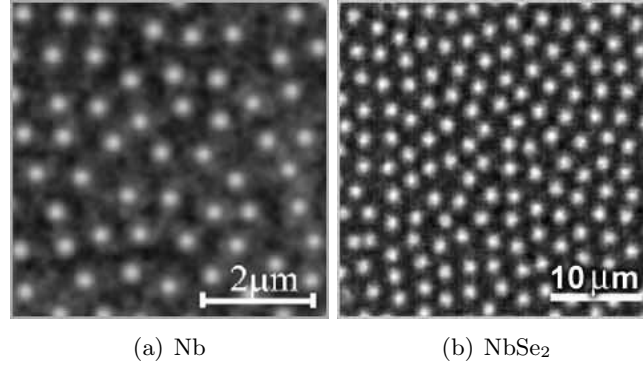


Figure 1.3: Formation of isotropic triangular vortices lattices in type-II superconductors. (a) Magnetic-force microscopy image obtained on a niobium film at 40 G and at 4.3 K [29]. (b) Magneto-optical image of a NbSe₂ crystal obtained at 3 G and at 4.3 K [30].

The pinning of the vortices due to structural defects has a huge impact on the current-carrying capabilities of type-II superconductors lying in the mixed-state. Indeed, it requires the application of a sufficiently strong transport current in order for the associated Lorentz force to exceed the pinning force and then to tear the vortices off the defects³. Because of the Lorentz force, the vortices tend to move in a direction transverse to the current with a velocity $\vec{v}$. This leads to the development of an electric field $\vec{E} = \vec{B} \times \vec{v}$, which is parallel to the applied current density. The flow of vortices under the action of a transport current (the flux-flow regime) is a resistive process that results in the dissipation of energy. The appearance of a measurable longitudinal voltage (typically a few μV) is often used to define the critical current density j_c of a type-II superconductor. The latter has to be distinguished from the instability current density j^* introduced in the section 1.2 to characterize the current-induced transition to the fully normal state.

3. For a homogeneous and defect-free sample, the application of an infinitesimally small transport current is sufficient to move the vortices.

1.2 Heating effects in superconducting strips

As discussed in the section 1.1, the first particularity of the superconductivity is the abrupt drop of the electrical resistivity below the critical temperature. The main consequence is the absence (resp. the existence) of a dissipation by Joule effect when an external transport current smaller (resp. larger) than the critical one is applied. It is essential to keep in mind this trivial - at least basic - statement in order to understand the current-induced (heating-related) behavior of macroscopic and mesoscopic superconducting samples.

1.2.1 Thermal bistability

The thermal steady-state of a current-carrying conductor is defined by the power balance condition

$$Q(T) = W(T), \quad (1.14)$$

where $Q(T)$ and $W(T)$ represent the heat source and the heat evacuation capabilities at a given sample temperature. The dissipation power $Q(T)$ stems from the Joule self-heating. It depends on the current density j averaged over the cross-section area through the relation $Q(T) = jE(T) = j^2\rho(T)$ [Wm^{-3}], where $E(T)$ is the electric field strength and $\rho(T)$ is the electrical resistivity. The thermal bistability under a sufficient Joule heating is observed in many systems like in the phase transition of metals from a high-conductivity to a low-conductivity state [31,32], in superconducting [33–37], magnetic [38,39] or structural transitions, ... The thermal bistability arises when Eq. 1.14 is fulfilled for more than one temperature so that a transition may happen from a *cold* state at a temperature T_{cold} to a *hot* state at a temperature T_{hot} . The bistability condition is verified if the dissipation varies sharply in a narrow temperature interval [4]. In the particular case of superconducting materials, the thermal bistability results from the stepwise evolution of the electrical resistivity which strongly varies near its critical temperature from zero up to the normal state value.

The thermal bistability arises when the current density lies in the interval $[j_r, j^*]$. Only the homogeneous state $T = T_1$ is allowed for $j < j_r$, while above j^* the sample lies in the homogeneous state $T = T_{\text{hot}}$. This leads to a hysteresis in the interval $j_r < j < j^*$ with a forward transition from T_{cold} to T_{hot} at $j = j^*$ and a reverse transition from T_{hot} to T_{cold} that occurs when the current is reduced below j_r . All the bistable conductors (either superconductors or normal metals) exhibit this significant hysteresis in their voltage-current characteristics. This

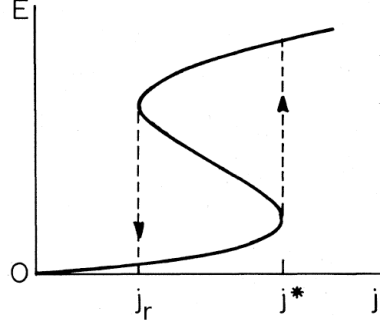


Figure 1.4: From [4] : Hysteretic electrical behavior of a bistable conductor (E is the electric field and j is the applied current density). The vertical dashed arrows indicate the current-driven transitions that arise at the instability current density j^* and at the retrapping current density j_r . The solid S-shaped curve illustrates the voltage-driven transition between the cold state and the hot state.

is schematically depicted in Fig. 1.4. For superconducting materials, a convenient way to characterize the width of the hysteresis is to define the dimensionless Stekly parameter α which corresponds to the ratio of the heat generated in the normal state at the instability current and of the heat transfert capabilities. Assuming that the transverse dimensions are sufficiently small, the heat evacuation capabilities are mainly determined by the heat transfer from the film to the substrate⁴. The function $W(T)$ is given by Eq. 1.15

$$W(T) = \frac{h(T)}{d}(T_c - T_0), \quad (1.15)$$

where $h(T)$ is the heat-transfer coefficient per unit area from the film to the substrate, d is the sample thickness, T_c is the temperature inside the normal domain (assumed to be the critical temperature), and T_0 is the substrate temperature (assumed to be the coolant temperature). Accordingly, the Stekly parameter is expressed by

$$\alpha = \frac{\rho_n j^{*2} d}{h_{T_c} (T_c - T_0)}, \quad (1.16)$$

where ρ_n is the normal state resistivity, j^* is the instability current density and h_{T_c} is estimated close to the critical temperature. The parameter α describes the relative role of the Joule self-heating in superconductors, the self-heating being dominant if $\alpha > 1$ and negligible if $\alpha \ll 1$ [4]. A

4. This is expected when the temperature slowly varies through the cross-section, i.e., when $d \ll \frac{\kappa}{h}$, where κ is the thermal conductivity of the conductor.

thermal instability of the resistive state only occurs if $\alpha > 1$. Such condition results in a negative differential conductivity and is often found in thin superconducting films whose critical current density is high (especially at low coolant temperatures) and whose electrical conductivity in the normal state is low. Furthermore, following the above-described assumption on the heat transfer capabilities, it is convenient to define the minimum normal-zone existence current I_{m0}

$$I_{m0} = \left[\frac{h_{T_c} W^2 d (T_c - T_0)}{\rho_n} \right]^{1/2}, \quad (1.17)$$

which corresponds to the minimal current required to heat up a sample above its critical temperature (W is the sample width). The associated current density $j_{m0} = \frac{I_{m0}}{Wd}$ is exactly equivalent to the current density j_r , so that the current density interval over which the thermal bistability appears is given by $j_{m0} < j < j^*$. From Eq. 1.17, the latter appears to noticeably depend on the characteristics of the sample as well as on the cooling conditions.

1.2.2 Heating models

Bremer et al. [40] were among the first to discuss the thermal effects in thin superconducting films. They have pointed out that some anomalous features in the current-induced superconducting-to-normal transition found an explanation in the propagation of normal regions that appear at the weakest parts of the samples. Since then, a considerable amount of work has been performed in order to better understand the impact of the self-heating on the electrical transport properties of superconducting samples (for detailed reviews on this subject, see Gurevich et al. [4] and Dharmadurai et al. [41]). The evacuation of the heat dissipated by the Joule effect was found to be influenced by many factors, among which the material quality, the sample homogeneity, the coupling between the film and the substrate [42, 43], the cooling conditions [44], the presence of an additional dissipation source [45, 46], ... Several heating models have been proposed to describe the behavior of superconductors submitted to a significant heating. At first, the aim was to explain the large hysteresis of the current-voltage curves. The electron heating model of Shklovskii et al. [47] considers an almost perfect heat removal characterized by a temperature of the conduction electrons larger than the lattice which remains at the coolant temperature T_0 . By contrast, Decker and Palmer [48] consider that a limited heat removal leads to the thermalization of the conduction electrons at the lattice temperature which is larger than the coolant one. In both cases, the

underlying idea is that the electric field which develops along a resistive normal domain increases the temperature of the conduction electrons, this elevated temperature being responsible for the hysteresis of the current-voltage curves. Indeed, the superconducting recovery (or re-trapping) current occurs when the bias current is equal to the depressed critical current at the elevated electronic temperature (this statement is valid insofar as the current-induced transition from the superconducting state to the normal state finds its origin in the self-heating nucleation of a normal domain) [49]. Following the Decker-Palmer heating model, the electronic temperature T_e may be much higher than T_c if the heat transfer conditions are poor and/or the power levels involved are large. In this case, the film returns in the superconducting state only when the applied current is sufficiently reduced so that the electron temperature falls just below T_c . This approach has been followed by Skocpol, Beasley and Tinkham in their hotspot model [44], which describes the stabilization of a localized normal domain maintained by Joule heating in superconducting thin-film microbridges. They have considered that the heat generated in a localized dissipative region is transferred in two ways. The first one is the surface heat transfer through the temperature discontinuity that arises at the boundary of the thin film and the substrate. For sufficiently small transverse dimensions, the substrate is assumed to be at the coolant temperature T_0 and the heat transfer per unit surface is given by Eq. 1.15. The second contribution to the heat evacuation comes from the thermal conduction within the film. This latter term depends on the thermal conductivity κ through the expression $\kappa \frac{d^2T}{dx^2}$. Taking into account both heat transfer mechanisms leads to the definition of a characteristic thermal length called the thermal healing length which is defined by Eq. 1.18,

$$\eta = \left(\frac{\kappa d}{h} \right)^{1/2}, \quad (1.18)$$

where d is the sample thickness and h is the heat-transfer coefficient from the film to the substrate. A major feature of the hotspot model is that the electrical behavior depends on the ratio between the sample length L and the thermal healing length η . This is intimately related to the dominant heat evacuation mechanism. Indeed, the surface heat-transfer dominates if $L > \eta$, while the conduction within the film becomes the dominant mechanism for $L < \eta$. As all the samples studied in the present manuscript fall in the former limit, the model appropriate for strips much longer than the thermal healing length will now be described in more details.

1.2.2.1 Model of the localized normal hotspot (SBT)

As mentioned by the authors, this model allows one to « *determine the current required to generate sufficient heat in order to sustain the temperature distribution associated with a normal region of a given size* » [44]. Skocpol et al. have considered the one-dimensional model of a bridge containing a normal domain whose both extremities (i.e., the interfaces between the normal domain and the superconducting regions) are symmetrically located at a distance x_0 from the center of the strip. Furthermore, they have considered a location-dependent electronic temperature $T_e(x)$, whose distribution satisfies the heat-flow equations

$$-\kappa_n \frac{d^2 T_e}{dx^2} + \frac{h}{d}(T_e - T_0) = \left(\frac{I}{Wd} \right)^2 \rho_n \quad (|x| < x_0), \quad (1.19)$$

$$-\kappa_s \frac{d^2 T_e}{dx^2} + \frac{h}{d}(T_e - T_0) = 0 \quad (|x| > x_0), \quad (1.20)$$

that correspond to the normal (N) and superconducting (S) areas, respectively. I is the current that flows through the bridge cross-section, W is the width, κ_n and κ_s are the thermal conductivities in the normal and superconducting state, respectively. In this model, the temperature at the ends of the bridge $T_e(\pm L/2)$ is fixed to the coolant temperature while the temperature at the N/S interfaces is set to $T_e(x_0) = T_c$. The heat-flow equations are resolved to determine the temperature distribution across the whole sample considering the continuity of the temperature T_e and of the heat flux $\kappa \frac{dT_e}{dx}$ at the N/S interfaces. The figure 1.5(a) shows the typical temperature profile obtained for a long superconducting bridge. This temperature distribution has been obtained by assuming $\eta_n = \eta_s$ ($= \eta$). One sees that the electronic temperature at the center of the normal hotspot domain (i.e., at the center of the strip) is higher than the critical temperature and increases when the N/S interfaces become closer to the extremities of the sample.

These self-consistent temperature profiles are valid insofar as a sufficient dissipation by Joule heating is generated in order to sustain a normal domain inside the bridge. The SBT model further expresses the associated minimum current which takes the form

$$I_0(x_0) = \left[\frac{hW^2d(T_c - T_0)}{\rho_n} \right]^{1/2} \times \left[1 + \left(\frac{\kappa_s}{\kappa_n} \right)^{1/2} \coth \left(\frac{x_0}{\eta_n} \right) \coth \left(\frac{\frac{1}{2}L}{\eta_s} - \frac{x_0}{\eta_s} \right) \right]^{1/2}, \quad (1.21)$$

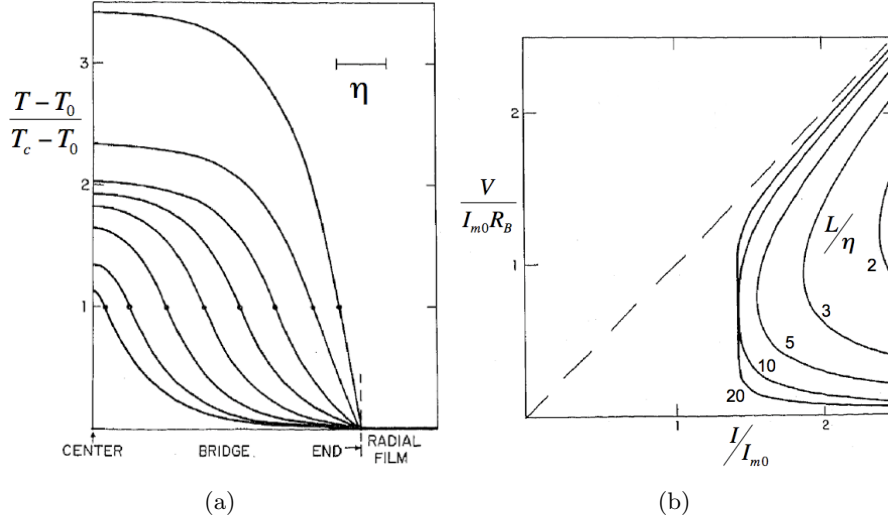


Figure 1.5: From [44] : (a) Electronic temperature in a microbridge longer than the thermal healing length η for different lengths of the normal hotspot domain. The temperature is assumed to be the coolant temperature T_0 at the extremities of the films. The normal/superconducting interfaces are located at $T=T_c$ (see the $\bullet$ symbols). (b) Theoretical current-voltage curves of a long superconducting bridge containing a normal hotspot domain. The bridge length L is normalized with respect to the thermal healing length η (L/η varies from 2 to 20). The scaling current I_{m0} is the minimum normal-zone existence current and R_B is the resistance of the bridge.

where the scaling current $\left[\frac{hW^2 d(T_c - T_0)}{\rho_n} \right]^{1/2}$ is the current that generates just enough heat in order to balance the surface heat transfer between the normal film and the substrate (this scaling current is exactly equivalent to the minimum normal-zone existence current I_{m0} of Eq. 1.17, page 19). By contrast, it is of the highest importance to stress that the current $I_0(x_0)$ is the current that *generates sufficient heat to balance (the) surface heat transfer at the temperature which is required well inside the normal region to maintain a stable N/S interface when the conduction in the vicinity of the interface is also considered* » [44]. As a consequence, the current $I_0(x_0)$ corresponds to the so-called minimum normal-zone propagating current I_{p0} which defines the current level above which a stable normal domain will propagate across the whole specimen [41, 44]. In addition, the model allows one to determine the shape of the associated current-voltage characteristics. For each hotspot length, the corresponding voltage is calculated using the Ohm's law,

$$V(x_0) = I_0(x_0) \rho_n \frac{2x_0}{Wd}, \quad (1.22)$$

where the normal state resistivity is assumed temperature-independent. Typical results are depicted in Fig. 1.5(b) where the theoretical current-voltage characteristics are given for different normalized bridge lengths L/η . For $L \gg x_0 \gg \eta$, the current-voltage characteristics approach to the constant current given by $I_{m0} \times \left(1 + \frac{\kappa_s}{\kappa_n}\right)^{1/2}$. Further assuming that $\kappa_n = \kappa_s$, Skocpol et al. have fixed the prefactor to $\sqrt{2}$, an inaccurate value in practice since the electronic thermal conductivity significantly decreases in the superconducting state due to the decreasing number of electrons which are able to carry the heat, i.e., the quasiparticles [50]⁵. Furthermore, the model is based on temperature-independent thermal parameters h , κ and ρ_n so that its validity appears questionable at very low temperatures. Several modifications of the original SBT model have been proposed to better describe the heat-transfer environment of the localized normal area⁶. In all cases, the proportionality of I_{m0} and I_{p0} with $(T_c - T_0)^{1/2}$ remains a relevant conclusion.

In conclusion, the heating models highlight that the hysteresis of the current-voltage characteristics is related to the increase of the temperature of the conduction electrons with respect to the coolant temperature T_0 . The main concept which underlies the stabilization of a dissipative hotspot domain inside a superconducting sample is that the total heat dissipated inside the normal region has to exceed the heat evacuation capabilities. In their hotspot model, Skocpol, Beasley and Tinkham have determined the minimum current required to sustain a normal domain by Joule heating. This leads to the expression of the characteristic thermal currents I_{m0} and I_{p0} , i.e., the minimum normal-zone existence current and the minimum normal-zone propagating current, respectively. Both currents are proportional to $(T_c - T_0)^{1/2}$, where T_c and T_0 are the temperature inside the normal domain and the temperature of the coolant, respectively.

1.2.2.2 Dynamics of the electron energy transfer

In the previous section, the thermal effects in superconducting materials have been related to an increase of the temperature of the conduction electrons with respect to the coolant temperature T_0 . Such heating

5. The reduction of the thermal conductivity in the superconducting state is discussed in section 3.1.2 (page 109).

6. Yamasaki et al. suggested to replace the usual temperature dependence of the surface heat transfer by a more appropriate $(T_c^4 - T_0^4)$ term [51]. This proves to be valid on a wider temperature range although some deviations have been observed for $T_0 < 0.7T_c$. Dharmadurai et al. [52] further considered temperature-dependent thermal conductivities, with no impact on the temperature dependence of I_{m0} , I_{p0} .

mainly depends on the way by which the electrons transfer their energy to their surroundings. A conducting film deposited on a dielectric substrate is basically composed of two interacting subsystems, namely the electrons and the phonons. Considering an insulating substrate, the latter subsystem is further divided into the phonons in the film and the phonons in the substrate. Furthermore, the situation slightly differs in a superconducting thin film given that the electron system also has to be divided into two parts : the *superconducting* Cooper pairs and the *normal* quasiparticles. All these four subsystems are involved in the thermal dynamics because they are able to exchange heat : the electrons and the phonons are coupled by the electron-phonon interactions while the phonons are coupled together through the interface between the film and the substrate. A thermal equilibrium is reached if all subsystems are described by equilibrium distribution functions using the same temperature. However, the electrons are often submitted to a heat source which causes their temperature to increase. The excess energy is then transferred to the phonons of the film until a steady-state regime is reached, with different temperatures for both the electrons and the phonons [53]. In practice, the electrons are driven far out the thermal equilibrium with the phonons, especially when the electron-phonon interaction is weak.

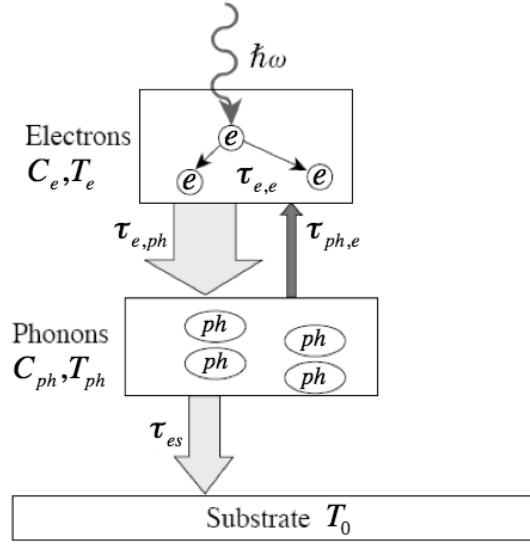


Figure 1.6: From [54] : Schematic description of the different ways by which the electrons and phonons subsystems transfer their energy ($\tau_{e,e}$ is the intrinsic electron-electron thermalization time, $\tau_{e,ph}$ and $\tau_{ph,e}$ are the electron-phonon and phonon-electron interactions times and τ_{es} is the phonon escape time through the substrate).

The subsequent non-equilibrium situation is often called the hot-electron regime. The latter is described by the hot-electron model valid at a coolant temperature T_0 close to the critical one, and, by extension, inside and in the vicinity of a normal domain where the quasiparticles have a temperature T_e close to or slightly higher than the critical one. In the vicinity of T_c , the fraction of Cooper pairs is significantly reduced (the superconducting gap is suppressed) and the unpaired electrons behave like normal ones. The hot-electron model [54, 55] considers that both normal electrons and phonons are treated by normal-state distribution functions with electron and phonon effective temperatures T_e and T_{ph} , respectively. Both temperatures are assumed to be reached rapidly and uniformly throughout the whole sample. This latter condition requires that the intrinsic electron-electron thermalization time $\tau_{e,e}$ is small compared to the other energy exchange times, i.e., the electron-phonon $\tau_{e,ph}$ and the phonon-phonon interactions times. The latter interaction is often related to the time τ_{es} required for the phonons in the film to escape into the substrate. This thermalization scheme is depicted in Fig. 1.6, where C_e and C_{ph} are the electron and phonon specific heats, respectively. The back-flow energy transfer results from the reabsorption of non-equilibrium phonons by the electrons. It is described by the interaction time $\tau_{ph,e} = \tau_{e,ph} \times \frac{C_{ph}}{C_e}$, which is derived from the energy-flow balance in equilibrium. Following this two-temperature approach, the hot-electron effect is described by a pair of coupled linear equations,

$$\frac{dT_e}{dt} = -\frac{(T_e - T_{ph})}{\tau_{e,ph}} + \frac{1}{C_e}W(t), \quad (1.23)$$

$$\frac{dT_{ph}}{dt} = \frac{C_e}{C_{ph}} \frac{(T_e - T_{ph})}{\tau_{e,ph}} - \frac{(T_{ph} - T_0)}{\tau_{es}}, \quad (1.24)$$

which express the time evolution of the temperatures T_e and T_{ph} when a time-dependent dissipation source $W(t)$ is present, either the Joule heating power or an external power density absorbed by the electron subsystem. In Fig. 1.6, the electron subsystem is heated by an external electromagnetic irradiation of energy $\hbar\omega$. The equation 1.23 has strong similarities with Eq. 1.19 (page 21) which describes the steady-state heat-flow inside a normal domain. Here, the main differences are (i) that the diffusion of the electrons inside the film is not taken into account and (ii) that the electron temperature may vary in time⁷. This

7. The non-zero time-derivative of the electrons and phonons temperatures is required to characterize the behavior of photonic devices where the electron system is heated by a pulsed irradiation.

electron-heating two-temperature model is valid insofar as the effective temperatures only slightly deviate from their equilibrium values. This model is introduced here to highlight the various channels by which the hot-electrons relax their energy with the other subsystems. Such heat transfer mechanisms are relevant in order to determine the time-scale as well as the length-scale required for an electron located inside a normal domain to return in the superconducting condensate. Both these parameters are intimately related through the concept of the electronic diffusivity D_e (also called the diffusion constant), which is of particular interest because it allows one to translate all the electronic characteristic times into a characteristic length L by using the expression 1.25,

$$L = \sqrt{D_e \tau}, \quad (1.25)$$

where τ is a particular relaxation time⁸. Following Eq. 1.25, the characteristic thermal length which corresponds to the length required for the hot-electrons to cool down from a hot normal domain to the neighboring superconducting regions is given by the relation

$$\eta = \sqrt{D_e \tau_e}, \quad (1.26)$$

where τ_e is the electron cooling time (or the electron temperature relaxation time). This definition of the thermal healing length is equivalent to Eq. 1.18 (page 20) as $D_e = \kappa_e / C_e$ and $\tau_e = C_e d / h$, where C_e is the electron specific heat, κ_e is the electronic thermal conductivity, d is the thickness and h is the heat-transfer coefficient from the film to the substrate. In particular, it was previously stated that the solution of the hotspot model depends on the ratio between the sample length L and the thermal healing length η , in agreement with the dominant heat evacuation mechanism. In long microbridges ($L > \eta$), the electron-phonon interactions are known to dominate the cooling mechanisms (phonon-cooled regime), which is equivalent to consider that the heat-transfer across the interface dominates. The phonons remain at a temperature close to the bath temperature and quickly go through the substrate. By contrast, if $L < \eta$, the conduction within the film becomes the dominant mechanism, i.e., the heat transfer is mainly impacted by the diffusion of the hot-electrons through the contacts. This diffusion-cooled regime is controlled by the electron out-diffusion time $\tau_d = (L^2 / \pi^2 D_e)$. Here, the samples are in the phonon-cooled regime because the thermal healing

8. For example, if one considers the elastic scattering time τ_{elas} involved in the conductivity process, the resulting length-scale is the electron mean free path $\ell = \sqrt{D_e \tau_{elas}}$; if one considers the inelastic electron-electron scattering time $\tau_{e,e}$, the characteristic length-scale for the electrons to thermalize is given by $L_{e,e} = \sqrt{D_e \tau_{e,e}}$.

length is smaller than the length of the microbridge. As a consequence, the average electron cooling time essentially depends on the electron-phonon interaction time $\tau_{e,ph}$, on the phonon-electron interaction time $\tau_{ph,e}$ and on the phonon escape time to the substrate τ_{es} [54, 55]. All these interaction times noticeably differ from one sample to the other because they depend on both the structural quality and the sample thickness. The *average* electron cooling time is usually expressed by the relation $\tau_e^{-1} = \tau_{e,ph}^{-1} + \tau_{ph,e}^{-1} + \tau_{es}^{-1}$, which takes into account all the contributions [56].

1.2.3 Localization of normal hotspot domains

The hotspot model proposed by Skocpol, Beasley and Tinkham considers that the superconducting bridge as well as the heat transfer conditions are perfectly homogeneous along the whole sample length. This is a rough assumption which is hardly verified in practice. Indeed, the samples are systematically related to a certain level of inhomogeneities associated with spatial variations of the sample cross-section, or of the electrical properties, or of the heat removal capabilities⁹. All these inhomogeneities alter the local thermal currents $I_{m0}(x)$ and $I_{p0}(x)$ that differ from the currents I_{m0} and I_{p0} of the homogeneous sample. The functions $I_{m0}(x)$ and $I_{p0}(x)$ are of the greatest importance in order to describe the stabilization of localized normal domains inside a superconductor. The inhomogeneities may be classified according to their size : an inhomogeneity whose characteristic size L_{inh} is larger than the thermal healing length η is called a smooth (or extended) one while, in the opposite case, it is referred as a point (or local) inhomogeneity [4, 57]. Bezuglyj et al. have shown that both types of inhomogeneities have similar qualitative effects on the localization and the propagation of normal domains [57].

1.2.3.1 Stepwise current-voltage curves in inhomogeneous superconductors

The roughness of the sample cross-section proves to have a significant impact - certainly the dominant one - on the stabilization and the propagation of a normal domain. Indeed, a metastable N/S interface located

9. On one hand, either the normal or the superconducting electrical properties may locally vary along the whole length of a sample. As an example, the local heat production $Q(x)$ may be non-uniform if the electrical conductivity slightly varies throughout the sample. On the other hand, non-uniform heat removal capabilities may result from a varying thermal resistance between the film and the substrate. For samples immersed in liquid helium, a non-uniform heat transfer due to the formation of vapor bubbles at the sample surface was reported [44].

at a position x_0 remains at equilibrium insofar as the applied current I lies in the range $[I_{m0}(x_0), I_{p0}(x_0)]$, where $I_{m0}(x_0)$ and $I_{p0}(x_0)$ denote the minimum normal - zone existence current and the minimum normal - zone propagating current at the interface. The equilibrium condition leads to a zero velocity of the N/S interface. Now, if the applied current exceeds the current $I_{p0}(x_0)$, the normal domain will expand until the N/S interface reaches an adjacent part of the superconducting sample located at x'_0 for which $I_{p0}(x'_0) > I$. As a consequence, even small modulations of the cross-section may pin a N/S boundary if the latter inequality is locally fulfilled. At increasing I , the jump of the N/S interface into the neighboring inhomogeneities may reoccur until the normal domain expands onto the whole sample length. The localization of stable resistive hotspots that do not occupy the whole sample thus produce voltage steps in the voltage-current characteristics. In the current-biased regime, the steps usually result from the extension of an existing resistive domain which starts at the weakest location of the sample and proceeds by successive jumps of the N/S interfaces. However, the appearance of several dissipative regions is also conceivable, especially in the voltage-biased regime. In this case, the resistive domains appear within the inhomogeneities and then grow and merge until the whole sample becomes normal. Such situation is favored if several local inhomogeneities ($L_{inh} < \eta$) are present, leading to a metastable dissipative structure characterized by localized domains whose length is fixed by the spacings between the inhomogeneities [4]. Furthermore, the appearance of a multiplicity of hotspots is particularly favored if the thermal healing length is short [58].

The latter description highlights that the variation of the local minimum normal-zone propagating current $I_{p0}(x)$ noticeably determines the shape of the forward part of the current-voltage characteristics, i.e., when the direct current is increased. From the discussion of the section 1.2.2.1, the minimum normal-zone existence current $I_{m0}(x)$ only differs from $I_{p0}(x)$ by a term which may be considered constant when $L \gg x_0 \gg \eta$. As the current $I_{m0}(x)$ is exactly equivalent to the superconducting recovery (or retrapping) current $I_r(x)$, similar voltage jumps arise in the backward part of the current-voltage characteristic. In this case, each voltage step corresponds to the disappearance or the contraction of an existing resistive domain.

1.2.3.2 Observation by low-temperature scanning electron microscopy

The formation of stable hotspots configurations in superconducting samples has been nicely observed by Eichele et al. using a low - temperature scanning electron microscope (LTSEM) [58, 59]. They have studied the evolution of the stationary hotspots configuration for different values of the applied bias. In practical terms, the power of an electron beam was used to slightly perturb the local heat-balance equation. This results in a modification of the sample voltage, which was found - both theoretically and experimentally - to be maximum at both ends of the hotspot where the electronic temperature is close to the critical one, i.e., when a small temperature increment noticeably modifies the electrical resistance¹⁰. Doing this, the precise location of the hotspots boundaries has been clearly identified. The figure 1.7 illustrates a typical result obtained by LTSEM in the case of a tin sample (length ~ 2 mm, width ~ 100 μm , thickness ~ 0.5 μm) [58, 59]. The two-dimensional image of the voltage signal is represented at increasing and decreasing voltage-bias. The bright areas correspond to the boundaries of the hotspot. The expansion and the shortening of the normal domain is observed at increasing and decreasing voltage-bias, respectively. At increasing bias, the right-side N/S boundary remains pinned at a fixed location while the hotspot expansion is realized by successive jumps of the left-side extremity (see the sequences 1 - 4). By contrast, one sees at decreasing bias (sequences 4 - 7) that the left-side boundary remains motionless and the hotspot shrinks from the right. In addition, Eichele et al. have shown that further increasing the bias when the hotspot expands on the whole sample length leads to the penetration of the normal domain into the neighboring contacts. This expansion was shown particularly relevant when the contacts are made with the same material.

Furthermore, the thermal healing length η is known to determine the extension of the temperature gradient which is sustained at a N/S interface. The hotspot boundary thus extends on a length close to η . This was also highlighted by Eichele et al. that have observed a sample-dependent width of the voltage signal [58]. In particular, the signal width (around twice the thermal healing length) becomes sharper with decreasing purity for samples made of pure Sn ($\eta \approx 50 - 150$ μm), O₂-doped Sn ($\eta \approx 20 - 30$ μm) and granular Al ($\eta \approx 1.5$ μm). This is expected given that

¹⁰. The voltage signal finds its origin in the small expansion and shift of the hotspot due to the heat dissipation of the electron beam. The irradiation power being lower than the Joule dissipation due to the direct current, the former is considered as a small perturbation to the system.

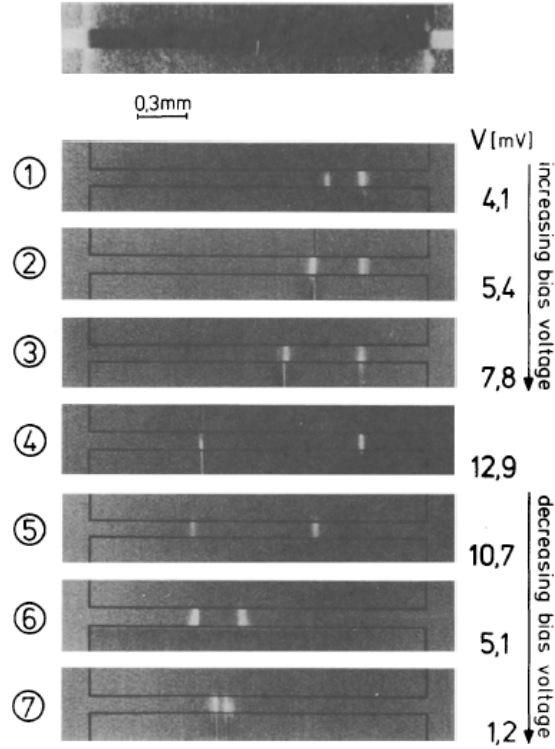


Figure 1.7: From [58,59] : Formation and propagation of localized hotspots in a tin microbridge ($T_0 = 2.1$ K) observed by low-temperature scanning electron microscopy. The figure represents the two-dimensional image of the voltage change that results from the perturbation induced by the electron beam dissipation.

the thermal healing length depends on the purity of the superconducting film (notably through its thermal conductivity). The most striking result is that the voltage-biased normal domains configurations of the Al samples were characterized by many individual hotspots that extend across the whole sample width. As a consequence, it appears that a sufficiently small thermal healing length favors the formation of a multiplicity of hotspots whose longitudinal extension can be smaller than the sample width. This is a strong deviation from the behavior treated by Skocpol, Beasley and Tinkham which consider the self-heating propagation of a single hotspot domain across a whole (and perfectly homogeneous) sample.

1.3 Non-equilibrium superconductivity

The dissipation of energy has always been a subject of concern in the investigation of the electrical transport behavior of superconducting structures. Since the beginning of the 1970's, it was well accepted that the heating dominates at coolant temperatures well below the critical one. This originates from the different temperature dependence of the Joule heat dissipated at the critical current [i.e. $RI_c^2 \propto (T_c - T_0)^3$] and the Joule dissipation required to sustain a normal hotspot domain [i.e. $RI_{m0}^2 \propto (T_c - T_0)$], the former one increasing more rapidly than the latter when the coolant temperature is decreased¹¹. In section 1.2, the deviations from the equilibrium has been considered from the *purely thermal* viewpoint, i.e., by considering that the steady-state non-equilibrium thermal effects manifest themselves by a global increase of the temperature of the conduction electrons above the critical one. In particular, it was shown that the unavoidable inhomogeneities that are present along the sample length may prevent the motion of the normal hotspot boundaries, leading to the appearance of voltage steps in both parts of the current-driven electrical characteristics. However, another kind of dissipative mechanisms was proposed to explain the voltage steps reported in the I-V curves at low voltages and very close to the critical temperature¹². In these conditions, the heating effects are small and allow one to probe the quantum properties of the superconducting state.

This section is devoted to the non-equilibrium processes that were first introduced to describe the disequilibrium of the electronic system very close to the critical temperature and were extended on a wider temperature range. First, the stationary non-equilibrium processes are discussed in section 1.3.1. A special attention is given on the conversion of an applied current through the boundary between a normal and a superconducting region. This is of great importance in order to understand the time-dependent charge-mode disequilibrium depicted in section 1.3.2. The temporal evolution of such dissipative mechanisms will be discussed in section 1.3.3. The focus is put on the challenging (and essential) distinction between the phase-slippage phenomena and the localization of normal hotspot domains introduced in Sec. 1.2.3.

11. The Ginzburg-Landau theory predicts that the critical current varies with the coolant temperature as $I_c \propto (T_c - T_0)^{3/2}$ while, from Eq. 1.15, the heat transfer through the substrate varies linearly with the coolant temperature.

12. For the conventional low- T_c materials, the heating effects are prevented above a temperature crossover which lies roughly at 0.1 K below T_c .

1.3.1 Stationary non-equilibrium processes

In a superconductor, the electronic population is decomposed into the superconducting *ground state* condensate of Cooper pairs and the elementary (or single) excited *quasiparticles* of electron-type or hole-type. The excited states of a superconductor are separated from the ground state by the BCS energy gap Δ . In thermal equilibrium, all the excited states are occupied following the Fermi distribution function $f_0(E_k/k_B T) = [1 + \exp^{E_k/k_B T}]^{-1}$, where E_k is the energy of the excitation that occupies a state $\mathbf{k}$ in the momentum space and k_B is the Boltzmann constant. The two branches of the energy spectrum are symmetrically occupied with quasiparticles so that the total charge is zero. This equilibrium situation may be disturbed under an external perturbation which can be described by considering a set of deviations $\{\delta f_k\} \equiv f_k - f_0(E_k/k_B T)$, where f_k is the perturbed occupation function. This set of departures can be decomposed into two orthogonal parts, which are even or odd with respect to inversion through the Fermi surface. The former situation corresponds to a variation of the temperature. It does not perturb the symmetry of the energy spectrum as the number of quasiparticles that move up to higher energy levels varies by the same amount on both branches. This *energy-mode disequilibrium*, which is also referred as the *longitudinal* or *temperature* mode, has been described in section 1.2. It is mainly excited by the phonons and the photons (i.e., neutral perturbations) although charged perturbations also impact on it. In Ref. [19], this energy-mode disequilibrium is characterized by a non-equilibrium temperature T^* which is not a thermodynamical temperature but rather defines an effective quasiparticle temperature. In contrast to the energy-mode disequilibrium, the symmetry of the electronlike and holelike branches of the energy spectrum can be broken by charged perturbations with energies $E_k > \Delta(T)$ that occupy some states $\mathbf{k}$ on the electron branch spectrum. The injection of electrons into the superconductor or the conversion of a normal current near a normal/superconductor interface fall in this category. This odd *charge-mode* disequilibrium, which is also referred as the *transverse* or *imbalance* mode, is associated to a net quasiparticle charge density Q^* whose expression is given at the page 407 of Ref. [19]. As detailed in the following paragraphs, the charge imbalance process is well suited to explain the conversion of a direct current which flows through a normal/superconductor boundary.

The overpopulation of the electron branch, which manifests itself by the existence of a non-zero Q^* at a particular location of the superconductor, requires a decrease by the same quantity of the number of electrons

in the ground state in order to ensure the overall electrical neutrality¹³. As the chemical potential μ_s of the Cooper pairs depends on the number of superconducting electrons, μ_s is shifted from its equilibrium value by a quantity which is proportional to the charge imbalance Q^* following the expression

$$Q^* = 2N(0)(E_F - \mu_s), \quad (1.27)$$

where $N(0)$ is the density of states in the normal metal at the Fermi level, E_F is the Fermi level and the factor two takes into account that two electrons with opposite spins occupy each energy level. As the charge imbalance Q^* depends on the location inside the superconductor, the nonequilibrium chemical potential of the Cooper pairs becomes a function of the coordinate x (assuming that the x -direction is normal to the N/S boundary). In addition, the electrochemical potential of the superconducting electrons is given by $\phi_s = e\phi + \mu_s$, where ϕ is the electrostatic potential and e is the electron charge. As ϕ_s must be constant under stationary conditions, the existence of a charge imbalance requires the presence of an electric field E which ensures that the Cooper pairs are not accelerated at all. This electric field is expressed by

$$E = -\nabla\phi = \frac{1}{e}\nabla\mu_s = -\frac{1}{2eN(0)}\nabla Q^*, \quad (1.28)$$

where the last equality stems from Eq. 1.27. The equation 1.28 means that a flow of quasiparticles is continuously entering through the location of charge imbalance, carrying a charge which is transferred to the condensate by relaxation processes following the relation

$$\vec{\nabla} \cdot \vec{j}_n = -eQ^*/\tau_{Q^*}, \quad (1.29)$$

where j_n is the normal component of the total current density and τ_{Q^*} is the relaxation time of the charge imbalance. This latter relation is only valid when the gap is small ($\Delta \ll k_B T_c$), i.e., when the temperature is close to T_c . Using the Ohm's law and the equations 1.28 and 1.29, one finds that

$$\nabla^2 Q^* = \frac{1}{\Lambda_{Q^*}^2} Q^*, \quad (1.30)$$

where $\Lambda_{Q^*} = \sigma_n \tau_{Q^*} / 2e^2 N(0)$ represents the depth over which the electric field penetrates into the superconductor [σ_n is the normal conductivity at low temperatures which is written for a normal metal as σ_n

13. The electroneutrality of the material requires that the charge of all electrons at every point is identical and equal in absolute value to the local charge of the material.

$= \frac{2}{3}e^2N(0)\ell v_F$, where ℓ is the electron mean free path and v_F is the electron velocity at the Fermi surface]. It follows that the penetration depth of the longitudinal electric field takes the form

$$\Lambda_{Q^*} = \left(\frac{\ell v_F \tau_{Q^*}}{3} \right)^{1/2}. \quad (1.31)$$

Given that the electronic diffusivity $D_e = (1/3)v_F\ell$, one finds that $\Lambda_{Q^*} = (D_e\tau_{Q^*})^{1/2}$, which is exactly equivalent to the relation 1.25 (page 26) obtained by using τ_{Q^*} as the particular relaxation time.

In the charge-disequilibrium mode, a finite time is thus required for the quasiparticles system in order to relax its charge Q^* . This also holds in the energy-mode for the relaxation time of the temperature T^* . The relaxation times τ_{Q^*} and τ_{T^*} indeed correspond to the time needed for the perturbations δf_k to relax to zero (equivalently for the actual distribution function f_k to approach the equilibrium Fermi function f_0). The rate of relaxation is dominated in both cases by the inelastic electron-phonon scattering processes although the elastic scattering from impurities also plays a role in the relaxation of the charge imbalance. In addition, only a fraction of the scattering events are efficient, with the most relevant ones for the elementary excitations whose energies E_k lie in the range $\Delta \leq E_k \leq 2\Delta$. Near the critical temperature, the fraction of occupied states which are fully effective in the relaxation processes is given by $\sim \Delta/k_B T_c$ so that the relaxation times τ_{T^*} and τ_{Q^*} will differ from the inelastic scattering time at the Fermi surface (τ_E) by the inverse factor. A detailed calculation at $T \approx T_c$ leads to the following expressions for the relaxation times of the energy- and charge-mode disequilibrium [60]

$$\tau_{T^*} = \tau_R^{(L)} \approx 3.7 \tau_E \frac{k_B T_c}{\Delta} (= \tau_\Delta), \quad (1.32)$$

$$\tau_{Q^*} = \tau_R^{(T)} \approx \frac{4}{\pi} \tau_E \frac{k_B T_c}{\Delta}, \quad (1.33)$$

where the superscripts (L) and (T) mean longitudinal and transversal, respectively. It is striking to observe that τ_{T^*} and τ_{Q^*} only differ from a factor which is close to 2.9 so that the characteristic length for both disequilibrium regimes (the thermal healing length η and the quasiparticle diffusion length Λ_{Q^*} , respectively) are very close to each other.

1.3.2 Time-dependent charge-mode disequilibrium

In section 1.3.1, it was shown that the charge imbalance that results from the passage of a direct current through the interface between a

superconducting and a normal region leads to the existence of an electric field in the superconductor. This electric field is attenuated over a length-scale given by the quasiparticle diffusion length Λ_{Q^*} over which the normal current converts to a supercurrent. This is a key concept in order to understand what happens in non-stationary non-equilibrium processes, especially when a superconductor is brought above its critical current under the action of an increasing external bias.

1.3.2.1 Phenomenological model for the quasi-one dimensional case : the phase-slip centers

Webb and Warburton [61] and, a few years later, Meyer and Minnigerode [62] were among the first to observe the stepwise increase (resp. decrease) of the voltage when a direct current is swept forward (resp. backward) in long superconducting whiskers and microbridges. Indeed, as illustrated in Fig. 1.8 with the current-driven characteristics of a tin whisker (section $\sim 2 \mu\text{m}^2$), a regular step structure arises in the resistive transition.

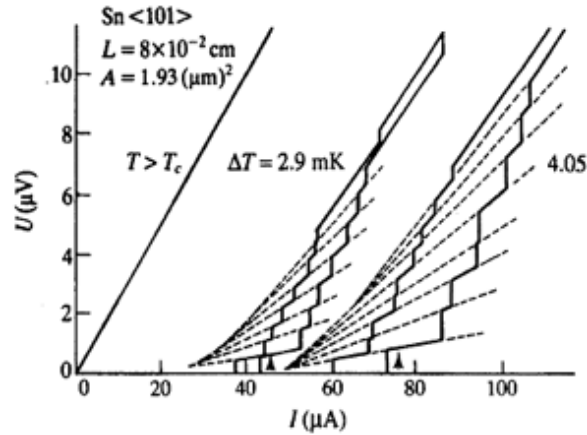


Figure 1.8: From [62] : Current-voltage characteristics of a tin whisker crystal showing a regular variation of the voltage due to the nucleation of successive phase-slip centers. The curves were obtained very close to the critical temperature.

The three major observations are that (i) each voltage step leads to a similar increase of the differential resistance dV/dI , (ii) the intersection of the intermediate parts of the transition with the current axis appears at a single point and (iii) the forward and the backward parts of the curves nicely match together so that the same dissipative structure is probed in both sides of the hysteresis [63]. In addition, the observation

of a stepwise structure in the forward part of the current-driven curves requires to work very close to the critical temperature ($T_c - T_0 \sim 5 - 30$ mK). At lower coolant temperatures, the heating dominates and the transition from the superconducting state to the normal state is abrupt with no intermediate dissipative state. For this reason, it was stated that this kind of resistive structure requires a quasi-one dimensional sample with lateral sizes smaller than the superconducting coherence length $\xi(T)$ [the latter diverges as one comes closer to T_c so that the appearance of a stepwise structure is made easier close to T_c].

To explain the stepwise current-driven curves observed on quasi-one dimensional superconductors, Skocpol, Beasley and Tinkham have developed the phase-slip center (or SBT) model [2]. The main idea is that the superconductor is not perfectly homogeneous along its whole length so that the applied current first reaches its critical *depairing* value at a particular weak spot. Further increasing the current creates a flow of normal electrons which, accordingly with the charge-imbalance process, leads to the establishment of an electric field. The Cooper pairs are then locally accelerated above their critical velocity and the order parameter $|\Psi|$ vanishes. At this time, the applied current as a whole is carried as a normal one. Since the creation of Cooper pairs remains locally possible, a supercurrent reappears and the process repeats itself. The term phase-slip center (PSC) stems from the fact that the phase of the superconducting electron wave-functions slips by 2π each time $|\Psi|$ cancels in the middle of the PSC. In this model, the so-called *beating heart* of the PSC carries an alternative supercurrent that oscillates in time with a period given by the Josephson relation $\omega = 2eV/\hbar$, where V is the voltage developed by a single PSC. The core of the PSC is assumed to have a size of the order of the coherence length since $\xi(T)$ governs the spatial variation of the superconducting gap. In the region of length $\xi(T)$, the time-averaged chemical potential of the Cooper pairs varies abruptly while it remains constant outside. As the time-averaged supercurrent is depressed at the center of the PSC, a dissipative normal current $j_n = -(\sigma_n/e) \nabla \bar{\mu}_q$ is required to ensure that the total current remains constant. The change of the electrochemical potential of the quasiparticles $\bar{\mu}_q$ therefore occurs on a much longer length-scale than the coherence length and defines the longitudinal extension of a PSC. Indeed, the normal current that flows through the PSC converts back to a supercurrent on the length $\Lambda_{Q^*} > \xi$ required for the quasiparticles to join the superconducting condensate by inelastic scattering processes.

The spatial evolution of the chemical potentials involved in the SBT model was interestingly evidenced by the work of Dolan and Jackel [64] and, a few years later, by Aponte and Tinkham [65]. The former made

independent measurements of both $\bar{\mu}_s$ and $\bar{\mu}_q$ by using regularly and closely spaced superconducting and normal probes along tin and indium thin-film microbridges [see Fig. 1.9(a)] while the latter studied the exponential decay of the quasiparticle current in long tin microbridges. The main results of Dolan and Jackel are given in Fig. 1.9(b) where the voltage detected by each probe at a fixed current is shown with respect to the probe location.

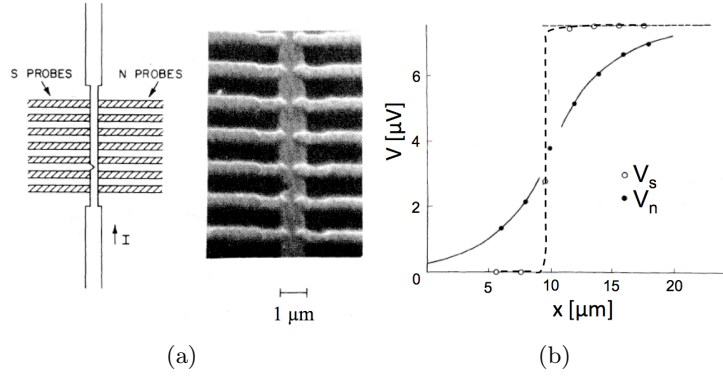


Figure 1.9: From [64] : (a) Multi-probe configuration of a tin sample attached to regularly-spaced normal (N) and superconducting (S) probes. (b) Spatial evolution of the potentials $V_s = \bar{\mu}_s/e$ and $V_n = \bar{\mu}_q/e$ near a phase-slip center.

The abrupt change of the voltage $V_s = \bar{\mu}_s/e$ was shown consistent with the (presumably discontinuous) variation of the chemical potential of the Cooper pairs over a distance of the order of ξ while $V_n = \bar{\mu}_q/e$ exponentially decays on both sides of the dissipative region on a length-scale given by Λ_{Q^*} . Dolan et al. have further shown that injecting quasiparticles through one of the normal probe gives rise to a similar charge imbalance, i.e., with the same decay length Λ_{Q^*} . In addition, the voltage drop associated with the formation of a phase-slip center is intimately related to the quasiparticle diffusion length. By considering that the average value (over time) of the supercurrent is around half the critical one ($I_s = \beta I_c$ with $\beta \sim 0.5$), one finds that the total voltage difference across a single PSC is given by

$$\bar{V} = \frac{2\Lambda_{Q^*}\rho_n}{S}I_n, \quad (1.34)$$

where $I_n = (I - \beta I_c)$ is the normal component of the current, ρ_n the normal state resistivity and S the cross-section area. Although its admitted simplicity, the equation 1.34 agrees with many experimental works

(see e.g. [65–68] and [69] for a review). In particular, the quasiparticle diffusion length has been estimated from the differential resistance of the stepped I-V curves of several tin microbridges and whiskers. The estimations agree with both the magnitude and the mean free path dependence of Λ_{Q^*} (see Eq. 1.31, page 34). Indeed, $\Lambda_{Q^*} \propto \ell^{0.5}$ with a relaxation time of the charge imbalance $\tau_{Q^*} \sim 800$ ps, a typical value for tin. In these conditions, the quasiparticle diffusion length varies between a few microns ($\ell \sim 20$ nm) up to several tens of microns ($\ell \sim 1$ μ m). One has to notice that the agreement between Eq. 1.31 and the experimental estimations of Λ_{Q^*} allows one to validate the charge imbalance model but does not validate by itself the phase-slip center theory.

In the context of the present work, the all-important feature of the SBT model is related to the explanation of the successive voltage steps observed under current-biased conditions, which is in turn intimately linked with the interactions between neighboring PSCs. Indeed, as the non-equilibrium currents generated at both sides of a PSC extend on a length Λ_{Q^*} , two PSCs interact if they are distant from less than $\sim 2\Lambda_{Q^*}$. The main assumption of the SBT model is that the occurrence of a PSC requires that the superconducting component of the current (and not the total current!) locally exceeds the critical one at some other point of the sample. Given that the normal current associated to a PSC reduces the local supercurrent $I_s(x) = I - I_n(x)$, the formation of additional PSCs in the neighborhood of existing ones is prevented and the successive phase-slip centers tend to avoid those already formed. This approach explains the succession of steps observed at increasing current by taking into account that the localized dissipative units interact through their time-averaged diffusive normal currents. However, as it was mentioned by Tinkham [70], *« the general spacing of the predicted steps is in reasonable agreement with the experimental results, but clearly the positions of the first steps are separated by the inevitable inhomogeneities, which overwhelm the exponentially weak interactions in the ideal model »*. This latter assertion will be discussed in section 3.5. In particular, the description of the formation of successive phase-slip centers question about the real impact of the sample inhomogeneities in the appearance of the voltage steps in the current-driven characteristics.

1.3.2.2 Kinematic vortices and phase-slip lines

The previous discussion highlights that the onset and the development of successive phase-slip centers cause a stepwise transition to the normal state for narrow and thin superconductors. This transition occurs in a current interval given by $I_c^{GL} < I < I_{c2}$, where I_c^{GL} is the

Ginzburg-Landau depairing current and I_{c2} corresponds to the current at which the film goes into the fully normal state [71]. At first, it was stated that such dissipative structure only manifests itself for samples whose transverse dimensions are smaller than both the coherence length $\xi(T)$ and the effective penetration depth $\lambda_{\perp}(T) \approx \lambda^2(T)/d$ (where λ is the London magnetic field penetration depth and d is the thickness). Indeed, the reduced dimensions hinder the penetration of Abrikosov vortices so that these *vortex-free* samples are able to carry a maximum uniformly distributed current equal to the depairing one¹⁴. Furthermore, the current-driven behavior of films much wider than the effective penetration depth $\lambda_{\perp}$ noticeably differs from this one-dimensional *narrow channel* limit. In this latter case, Abrikosov vortices may penetrate due to either the external field or the self-magnetic field that results from the applied current. The presence of vortices significantly perturbs the current-carrying behavior of the films. On one hand, the current is no more uniformly distributed through the whole cross-section. On the other hand, the departure from the purely superconducting state is significantly impacted by the pinning capabilities of the films, which in turn depend on the structural quality of the bulk and of the edges. The limit $W \gg \lambda_{\perp}$ was extensively discussed in the review of Dmitrenko et al. [72].

The intermediate case $\xi(T) \ll W \ll \lambda_{\perp}(T)$ is of particular interest. Weber and Kramer [3] were among the first to treat theoretically the behavior of thin quasi-two-dimensional current-carrying films using the generalized time-dependent Ginzburg-Landau equations. For $d \ll \xi \ll W$, they came to the conclusion that two solutions exist for the dissipative non-equilibrium state. The first one is the *vortex street* (VS) solution which is defined by the motion of vortices perpendicularly to the direction of the current. There is a strong topological analogy between the motion of a vortex and a phase-slip event since in both cases the superconducting phase varies by 2π between both extremities of the sample¹⁵. In this framework, the vortices of different polarities nucleate in pairs, move in opposite directions across the section and annihilate themselves. Let's notice that the VS solution is exactly equivalent to the *kinematic vortices*, a concept introduced by Andronov et al. [73] in order to better distinguish the vortices involved here with the usual (slow) Abrikosov

14. As there is not enough space for the normal core of a vortex to accommodate the sample dimensions, the destruction of the purely superconducting state is not masked by the flux-flow motion usually observed in type-II superconductors.

15. A vortex may be considered as an isolated cancellation of the order parameter so that turning around it gives a phase variation of $\pm 2\pi$ depending on the vortex polarity.

vortices. The second dissipative mechanism is referred as the *phase-slip line* (PSL) solution, which is the two-dimensional analog of the phase-slip center. In such a case, the order parameter $|\Psi|$ vanishes periodically in time over the full width of the sample, i.e., along a line perpendicular to the direction of the current flow. As a consequence, the formation of localized phenomena involving the slippage of the superconducting phase is not restricted to the strips that strictly satisfy the one-dimensional limit but are also theoretically possible in the two-dimensional geometry. In addition, the vortex street motion at high velocity significantly and permanently reduces the order parameter, so that a PSL may be viewed as a vortex street (or a kinematic vortex) with an infinite velocity.

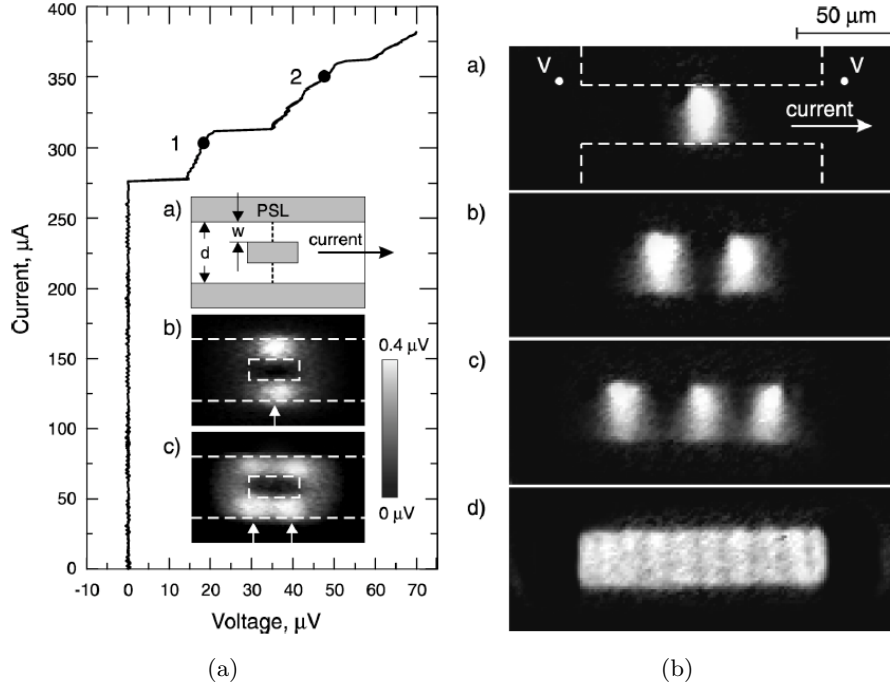


Figure 1.10: From [74] : Observation of phase-slip lines in wide superconducting tin strips. (a) Current-voltage characteristic of a strip with a window in the middle. The insets (b) and (c) are laser scanning microscopy images that correspond to the point 1 and 2 of the current-driven curve. (b) Formation and rearrangement of successive PSLs under an increasing applied current.

Both solutions of the dynamics of the resistive state in the two-dimensional case lead to the appearance of voltage steps in the current-voltage characteristics. Many experimental works have been performed in order to distinguish between these mechanisms, either in the absence [72, 74–77] or in the presence [74, 78] of an external magnetic field,

or in an alternating electromagnetic field [69,74,79,80]. Let's focus on two works that evidence the development of phase-slip lines in wide superconducting strips. Sivakov et al. [74] have studied the current-carrying capabilities of pure Sn and of InSn alloy strips whose width varies between 6 and 50 μm (the thickness is 100 nm and some samples have a rectangular window in the middle). The measurements were performed at a coolant temperature of 10 - 30 mK below the critical one. As illustrated in Fig. 1.10(a), the experimental current-voltage characteristics present the characteristics of the phase-slip lines : the transition is step-wise, the resistive branches have approximately the same excess current and the differential resistance is proportional to the number of branches. Each step was interpreted as the nucleation of an additional PSL.

In addition, low temperature scanning laser microscopy images were obtained on a 30 μm -wide strip [see the figure 1.10(b)]. They reveal the rearrangement in space of the PSLs that arises each time a new dissipative unit is created under an increasing bias. The authors mentioned that these measurements allow one to distinguish between the characteristic distribution of the phase-slippage phenomena and the « *possible local inhomogeneities in the films* ». Furthermore, Dmitriev et al. [75] have extensively discussed the current-driven behavior of tin thin films whose width varies from 7 to 50 μm (the thickness is around 100 - 300 nm). By studying the temperature-dependence of the characteristic currents (namely the critical current, the maximum current for the existence of the Abrikosov vortex state, and the current above which the current-voltage characteristic becomes non-linear), they came to the conclusion that phase-slip centers and phase-slip lines may be observed on the same sample depending on the coolant temperature. Close to the critical temperature, the first PSC arises at the Ginzburg-Landau depairing current which is uniformly distributed over the whole cross-section while well below T_c the first PSL nucleates at the vortex-state instability current. As a consequence, they considered that the physical meaning of both dissipative structures is the same, although it originates from either the destruction of the uniform superconducting state, or from the departure from the steady viscous flow of Abrikosov vortices. In addition, they considered that the formation of *phase-slip planes* is conceivable to explain the current-driven transition of films whose thickness is much larger than the coherence length.

1.3.3 Phase-diagram for phase-slip centers and hotspots

The main issue regarding the current-induced breakdown of the superconductivity is that either the nucleation of successive phase-slippage

phenomena (see Sec. 1.3.2) or the stabilization of normal hotspot domains (Sec. 1.2.3) manifest themselves by voltage discontinuities in the current-voltage characteristics. The required clear distinction between these dissipative mechanisms is all the more a challenging question since phase-slippage phenomena are also reported in microbridges whose transverse dimensions are much larger than the coherence length of the superconducting material.

Recently, it was proposed that applying nanoseconds current steps allows one to more easily distinguish between these two types of dissipative structures [81–83]. Two relevant informations are extracted from these pulsed measurements : the delay time for the appearance of a voltage signal after the application of a critical current step, and the time domain evolution of the measurable signal. The former feature is not essential in the context of this work, although it is instructive to briefly discuss it. The delay (or latency) time τ_d corresponds to the time required to suppress the energy gap after the application of a current bias [19]. It was considered for a long time to be a special feature of the nucleation of the phase-slippage phenomena [81, 84, 85] although it is now well established that the onset of a thermal hotspot domain may also be delayed [86]. In particular, Ladan et al. [82] have explained the time delay of the hotspot formation by the required passage through a PSC before the hotspot can nucleate, i.e., a hotspot never nucleates unless a PSC is first created. In their analysis, both types of dissipative mechanisms *« derive from the same original process and their delay times are not expected to behave differently »*. Let's mention that τ_d noticeably depends on the applied current with values that amount typically to a few hundred nanoseconds just above the critical current to a few nanoseconds for $I/I_c \sim 1.5$ (see [19] and references therein for a quantitative treatment of the delay time). The time domain evolution of the measurable signal voltage has to be discussed in more details because it is considered as an unambiguous evidence for the distinction between hotspots and phase-slippage phenomena. Indeed, the phase-slippage phenomena are considered as stable-in-time processes that give rise to a constant voltage signal (this was observed far below the critical temperature and for a current intensity just above the critical one). On the other hand, hotspots are assumed to grow or collapse in time depending on the current value so that a hotspot manifests itself in the constant-current bias mode by voltage signals that linearly vary in time. Three situations are usually considered [87–89] : (i) no hotspot exists if the dissipation does not exceed the thermal threshold, i.e., if the applied current is smaller than the minimum normal-zone existence current I_{m0} (see Eq. 1.17, page 19) ; (ii) the hotspot expands with no other limit that the complete tran-

sition (for a perfectly homogeneous sample) if the current is larger than the minimum normal-zone propagating current I_{p0} (see Eq. 1.21, page 21) which is larger than I_{m0} due to the heat conduction at the hotspot boundaries; (iii) the hotspot regresses in the intermediate region $I_{m0} < I < I_{p0}$ (decreasing voltage signal). As a consequence, the main assumption concerning the time domain evolution of normal hotspots is that the latter have a time-dependent size, i.e., a hotspot never keeps a constant length. This assumption will be discussed in section 3.5.

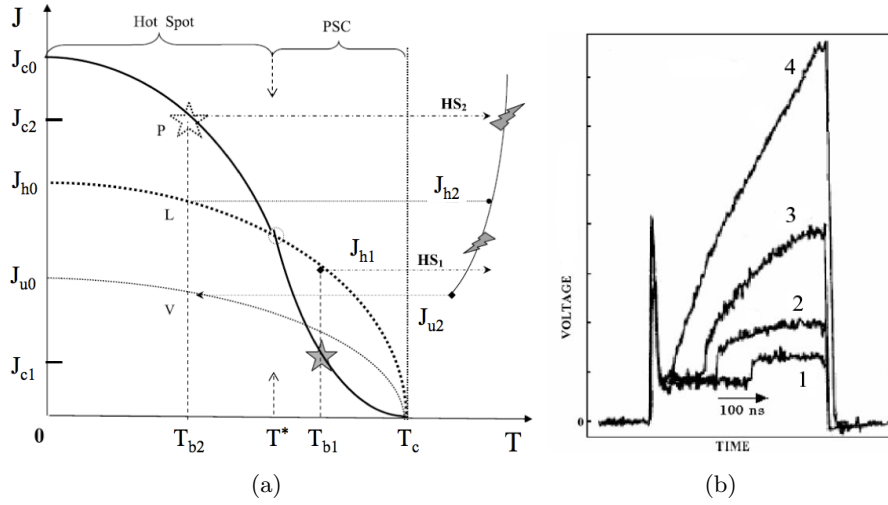


Figure 1.11: From [82] : (a) Current density vs. temperature diagram of a superconducting strip (see the text for an explanation of the various notations used by the authors). (b) Time-evolution of the voltage after a rectangular current pulse is applied to a niobium strip ($W = 10 \mu\text{m}$, $L = 3 \text{ mm}$, $d = 80 \text{ nm}$). The constant signal of the curve 1 ($T_0 = 6 \text{ K}$, $I = 16.5 \text{ mA}$) corresponds to a phase-slip center, the curve 2 ($T_0 = 5.8 \text{ K}$, $I = 21.3 \text{ mA}$) is interpreted as a PSC with strong heating, the curve 3 ($T_0 = 5.2 \text{ K}$, $I = 25.2 \text{ mA}$) is a hotspot through which a slowly decreasing current is flowing and the curve 4 ($T_0 = 4.3 \text{ K}$, $I = 33.6 \text{ mA}$) is a linearly rising hotspot.

The main advance of the pulsed measurements is that the discrimination between stable-in-time PSCs and expanding (or collapsing) hotspots may be performed in a quite large temperature interval. In addition, the different temperature-dependence of the characteristic currents for the nucleation of a phase-slippage phenomenon and for the feeding of hotspot domains¹⁶ allows one to predict the formation of one or the other of the dissipative mechanisms. This is illustrated in Fig. 1.11(a) with the schematic current density - temperature ($j - T$) diagram of a typical

16. The critical current I_c for the nucleation of a phase-slippage phenomenon is known to vary as $(1 - T_0/T_c)^{3/2}$ close to the critical temperature, while the minimum existence (I_{m0}) and propagation (I_{p0}) currents vary as $(T_c - T_0)^{1/2}$.

superconducting strip. In this figure, the critical current density is denoted by j_c while both current threshold densities related to the normal hotspot domains are referred as j_u ($\equiv j_{m0}$) and j_h ($\equiv j_{p0}$). The $j - T$ diagram is constructed by considering that the temperature for j_c is the *film* temperature, while for j_u and j_h the abscissa means the *substrate* temperature T_0 . In addition, from the intersection between the temperature evolution of j_c and j_h , it was demonstrated on niobium [82] and on YBCO [83] strips that a well-defined crossing temperature T^* separates the domain near T_c (where the first response to an increasing current is a phase-slip center) from the low-temperature domain (where the first response to an increasing current is a hotspot). The figure 1.11(b) illustrates the transition from the nucleation of a stable PSC at sufficiently large temperature to the spreading of a hotspot domain when the coolant temperature is reduced below the crossing temperature. Using the same notations that in Fig. 1.11(a), the appearance of a dissipative domain can be summarized as follows. Above the crossing temperature (for $T_{b1} > T^*$), the critical current density j_c is first reached if one increases the current amplitude. This results on the nucleation of a phase-slippage phenomenon (stable-in-time voltage signal). Further increasing the current leads to the transformation of this phase-slip center into a normal hotspot domain which expands through the whole sample at $j > j_h$. On the other hand, increasing the current at a temperature smaller than the crossing one ($T_{b2} < T^*$) is such that the curves $j_u(T_{b2})$ and $j_h(T_{b2})$ are first crossed. A relevant point is that the critical current density $j_c(T_{b2})$ has to be exceeded in order for the bias current to destroy the superconducting state. In other words, a current density $j_h(T_{b2})$ is sufficient to maintain a localized hotspot domain insofar as the latter has been previously formed. As a consequence, a voltage signal only appears for $j > j_c(T_{b2})$. This leads to a spreading hotspot since $j \geq j_c > j_h$. The authors consider that the point L in Fig. 1.11(a) is a *latent* hotspot which is not accessible directly by a single current step, but can be reached *from above* after a dissipative domain is created at a larger current density. This is a relevant feature that subtends the whole forthcoming discussion of chapter 3.

1.4 Microwave-induced heating in thin films

The electrical response of a superconducting thin film submitted to an electromagnetic irradiation is described in the framework of the two-fluid model (see section 1.1.2.1, page 8) by a complex conductivity whose real and imaginary components reflect the interactions of the normal electrons and of the superconducting electrons, respectively [19]. For alternating currents whose frequency is below the energy-gap (frequency), the latter model allows one to characterize the finite electrical losses of the normal electrons. The main benefit of using thin-film superconductors for microwave applications lies in their extremely low dissipation, which is characterized by the real part of their surface impedance [90]. Although limited at low irradiation power, this surface resistance significantly increases with the power and non-linearities may occur. The main challenge in studying the microwave-induced nonlinear dynamics of thin-film superconductors is that there is no universal mechanism, neither for perfect crystals nor for granular materials. Indeed, numerous nonlinear sources can coexist depending on the experimental conditions. Among the numerous possible origins reported in the literature, one finds the increase of the quasiparticles density due to the breaking of the Cooper pairs [91, 92], the penetration and the viscous motion of vortices which are more efficient at the weak-links or at the material defects [93, 94], the interaction between the microwaves and the network of superconducting grains that are coupled via the grain boundaries Josephson junctions [45, 95–98] ...

Furthermore, in the phase-slip center model of Skocpol, Beasley and Tinkham, the supercurrent oscillates in time with a period given by the Josephson relation $\omega = 2eV/\hbar$, where V is the voltage developed by a single phase-slippage phenomenon. Such behavior is reminiscent of the Shapiro steps for which current steps are observed at voltage levels that depend on the radiation frequency [99]. This explains the considerable work that has been done to better describe the impact of an electromagnetic irradiation on the nucleation and the evolution of phase-slippage processes. In particular, Sivakov et al. [74] have observed Shapiro steps on their wide tin strips that were prone to the appearance of phase-slip lines. Other kinds of microwave-induced non-equilibrium effects have been observed : enhancement of the supercurrent or appearance of the first phase-slip center [79, 100], enhancement of the stabilization of successive phase-slip centers [101], ac properties of the phase-slip lines [80] ... In this context, Dmitriev et al. [69] have highlighted that ac phase-slip centers may develop in the absence of a direct current insofar as the irradiation power exceeds a threshold value P_c , which is defined as the

minimum power at which the critical current of the channel is zero. As the resistance of an ac PSC - which is equivalent to the resistance of a dc PSC - remains constant whatever is the power $P > P_c$, the authors concluded that this feature could not be related to a normal domain with a length of the order of the quasiparticle diffusion length. In addition, it was shown that the resistance of the strips increases by discrete values when the microwave power exceeds P_c at a fixed and low transport current.

The existence of the latter power threshold is experimentally observed not only in narrow superconducting films but also on bulk or thin-film samples. Indeed, many practical applications that involve the microwave response of a superconducting film are limited by deviations from the linear regime when the microwave input power is increased. This is especially the case for polycrystalline samples for which the granular character significantly alters the response to a microwave irradiation. The nonlinear electrodynamics effects manifest themselves in the evolution of the frequency-dependent properties by weak (at low power) or strong (above a particular power threshold) nonlinearities. The former category brings together (i) the smooth increase of the surface impedance, (ii) the shift of the resonance frequency, (iii) the distortion of the resonance line shape, ... while the latter manifests itself by (i) sharp variations of the surface impedance, (ii) multiple discontinuities in the resonance curves, (iii) metastable response of the scattering or reflection coefficients, (iv) thermal breakdown, ...

1.4.1 Overheating of the intergranular regions

A relevant source of nonlinearities is the microwave dissipation which results from both the finite surface impedance and the generated ac-currents that flow through the films. If the heat removal capabilities are not sufficient to balance the microwave power, the films temperature may increase uniformly throughout the whole sample. In this case, the nonlinear behavior comes from the temperature dependence of the superconducting properties. Such global heating mainly occurs in the continuous mode, although it is also present in the pulsed-mode depending on the duration of the pulse and the sample dimensions [46]. The heating may also be concentrated within localized defects, which results from their larger resistance compared to the grains. In this case, the heat is preferentially transferred away along the defects [102], which may lead to the transition to the normal state of the weakest parts if the temperature locally exceeds the critical temperature. Nearly all the physical properties of polycrystalline superconducting thin films reflect a signifi-

cant intergranular coupling through the network of internal Josephson junctions that form at the grain boundaries. For this reason, the heating effects at the intergranular regions remain a matter of concern.

The overheating of the Josephson-junction system has been interestingly discussed by Stankowski et al. [45, 103] who studied the microwave absorption of Y-Ba-Cu-O thin films using the electron paramagnetic resonance technique. Their model considers four thermodynamical subsystems [the microwaves, the Josephson junctions system (JJS), the bulk sample and the coolant] which interact through the microwave absorption and through the heat transport processes. They introduced the concept of a local temperature T_{JJS} of the Josephson junction system. The latter may differ from the bulk temperature T_{bulk} due to the Joule heating that results from the absorption of a fraction of the incident power. In their experiments, the recorded signal is directly related to the microwave absorption and takes zero values when the JJS has a temperature $T_{JJS} \geq T_c$. At low incident power level, the microwave absorption signal vanishes at the critical temperature of the bulk sample, showing that $T_{JJS} = T_{bulk}$. By contrast, the microwave absorption cancels at an effective critical temperature which linearly decreases as the input microwave power is increased. Together with the thermal hysteresis that appears at sufficiently moderate power levels between the heating and the cooling sweeps, the shift of the effective critical temperature indicates that the Josephson junctions system is kept in the normal state even if, at the same time, the bulk sample remains superconducting. From their measurements, Stankowski et al. have pointed out that the heating of the internal boundaries Josephson junctions may have a significant impact on the intergranular coupling of polycrystalline superconductors. The effect of the local Joule heating that manifests itself by the formation of hotspot domains at the weak-links has been reported for many granular thin films, e.g., in high temperature superconductor [104–106], in NbN [107] and recently in MgB₂ [108, 109].

1.4.2 Microwave-induced thermal breakdown

The microwave heating-induced breakdown of the superconductivity has a threshold character in the sense that the microwave dissipation has to exceed a certain input power level in order to overcome the heat transfer capabilities and to lead to a thermal instability. The investigation of this microwave-induced thermal breakdown is intimately related to the issue of the superconducting power handling capabilities. The thermal instability under a sufficient power level takes several forms depending on the superconducting property of interest. Given that an exhaustive

description is beyond the scope of this section, the latter is dedicated to relevant experimental results that highlight the link between the thermal bistability and the strong microwave-induced nonlinearities observed on granular superconductors.

By studying the frequency-driven resonance response of NbN stripline microwave resonators, Abdo et al. [107, 110] have reported a very wide variety of nonlinear features that arise above a particular input microwave power : multiple and sharp variations of the reflection parameter S_{11} , hysteretic behavior in the vicinity of the jumps with clockwise or counterclockwise evolution depending on the power level, ... All these abrupt variations of the resistive losses have been attributed to the local heating of the weak-links that form at the boundaries of the NbN columnar structure. An interesting feature is the metastable nature of the jumps which was demonstrated by sweeping the microwave frequency at some fixed input power levels. Doing so, the reflection parameter S_{11} switches many times between two well-defined values in a very small frequency interval. All these nonlinear features, especially the transitions between two bistable states, have also been reported by Ghigo et al. [108] in their investigation of the switching response of MgB_2 resonators. From their analysis of the scattering parameter S_{21} , they have highlighted that, at sufficiently low coolant temperatures and high input microwave power, the system *« oscillates between two states that are well defined and stable with respect to time »*¹⁷. Their model is based on the formation of hotspots at the intergranular weak-links. The all-important feature is that the abrupt transitions are directly related to the thermal bistability discussed in section 1.2.1. Indeed, both states in the resonance curves were shown to be equivalent to the dissipative states that correspond to the intersections between the Joule dissipation power $Q(T)$ and the heat evacuation capabilities $W(T)$. In this sense, the bistable states in the resonances curves are directly linked to the current densities j_r and j^* that define the interval of the thermal bistability in the current-biased conditions [109]. The interplay between the microwave dissipation and the heat removal capabilities was also highlighted by Pukhov et al. [111, 112] who have theoretically studied the power-induced switching of thin high temperature superconductors deposited onto a thermally stabilized substrate. The heat transfer conditions between the sample and its surroundings appear to be the main factor that determines the threshold microwave power required to switch the film into the normal state. The hotspot N/S interfaces were shown to be at

17. Let's notice that the transition between the bistable states of the resonance curve becomes smooth at coolant temperatures close to the critical one.

rest when the absorbed power equals the propagation one (P_p) and to expand into the superconducting regions when $P > P_p$, i.e., when their velocity becomes positive. The authors further notice that P_p is theoretically proportional to the normal state electrical conductivity, although the presence of defects results in a reduction of the microwave power threshold required to initiate the breakdown phenomena. For typical Y-Ba-Cu-O thin films deposited onto MgO substrates, P_p amounts to 10^2 Wcm^{-2} . The existence of a power threshold was also revealed by Wosik et al. [46] who have studied the response of Y-Ba-Cu-O thin films deposited on LaAlO_3 substrates up to the superconductivity-quenching microwave power levels. Using sufficiently long pulsed microwave signals to charge the cavity of their resonators, they discussed the relation between the input microwave power and the transmitted power. At low power levels, the transmitted microwave power was shown to behave linearly with the input one. Their main observation is that a drop of the transmitted power arises above a certain input power value, which indicates that the film has switched into a higher resistive state. Their model considers that the heating takes place within one or more localized defects. A subsequent increase of the input power leads to a global heating of the superconductor when the heat is not removed efficiently, leading to the so-called rf-induced quenching of the superconducting state. Their conclusion was that the heat locally produced can be sufficient to keep the weak-link in the normal state, even if, at the same time, the microwave currents drop due to the limited duration of the microwave pulse. The distinction between local and global heating has also been examined by Cohen et al. [113] who have studied the evolution of the resonance curve shape of high temperature superconducting resonators as a function of the delay time between each frequency step. At a sufficiently high input power, the resonance curves remain symmetric for a delay time of 0.3 s, while a pronounced non-linear behavior arises by increasing the delay time up to 12 s. The authors consider that the non-linear effects arise in the films at a lower frequency threshold due to the longer heating (or delay) time. The reduction of the frequency resonance was then directly related to a global temperature rise. Even though their theoretical heating model reproduces only qualitatively the experimental results, the global heating of their films was interestingly shown to be generated if the delay time is sufficiently long, i.e., in the second time-scale.

1.5 Vortices in narrow type-II superconductors

A major property of the superconductivity is the Meissner effect, i.e., the complete expulsion of an applied magnetic field when a sample is cooled through its critical temperature. In the section 1.1, the magnetic behavior of bulk superconductors was shown to depend on the Ginzburg-Landau parameter $\kappa \equiv \lambda_{GL}/\xi_{GL}$ which is smaller (type-I superconductivity) or larger (type-II superconductivity) than the critical parameter $\kappa_c = 1/\sqrt{2}$. The former category goes normal at the thermodynamical field H_c while the latter is characterized by the penetration of vortices at a field H_{c1} and by the conservation of the superconductivity up to the second critical field H_{c2} . For bulk and isotropic materials, the vortex-vortex repulsive interactions lead to the formation of the so-called Abrikosov lattice, i.e., a triangular and periodic array [19] that minimizes the free-energy of the system. However, reducing the sizes of the samples below the in-plane penetration depth may significantly impact the entrance and the arrangement of vortices in type-II superconductors. This will be the subject of the present discussion.

First, the ability to maintain a metastable Meissner phase well above the bulk field H_{c1} is discussed in the section 1.5.1. The main theoretical models as well as some recent experimental observations in favor of the delay in the vortex entrance are presented in Sec. 1.5.2. Finally, the ability for the vortices to rearrange themselves into anisotropic lattices as the magnetic field is increased is highlighted in the section 1.5.3. The associated modulations of the critical current as well as the impact of a direct transport current on the rearrangement of the flux-line lattice are also presented.

1.5.1 Surface barrier for vortex penetration in macroscopic superconductors

Bean and Livingston [114] were among the first to highlight the influence of the surfaces in the magnetic behavior of superconducting samples. They have evidenced that the vortices must overcome an energy barrier at the surface in order to penetrate the interior of a superconductor. Studying the evolution of the Gibbs free-energy of an isolated vortex as a function of its distance from an ideally perfect semi-infinite surface has allowed them to determine the minimum external magnetic field that cancels the barrier surface. The Gibbs free-energy G is obtained by integrating the total force exerted on the vortex over the distance vortex-surface [115]. The total force is composed of an attractive and a repulsive term. The former contribution results from the condi-

tion of zero field at the surface, or equivalently from the required boundary condition of zero current in the direction normal to the surface. This condition is fulfilled by applying the superposition principle i.e. by considering an image-vortex located outside the superconductor. Both the real vortex and the image-vortex interact attractively because they have opposite signs. The latter contribution arises when an external magnetic field is applied parallel to the surface with the same sign as the real vortex. Indeed, the Meissner currents that circulate near the surface on a distance given by the penetration depth tend to repel the vortex from the surface.

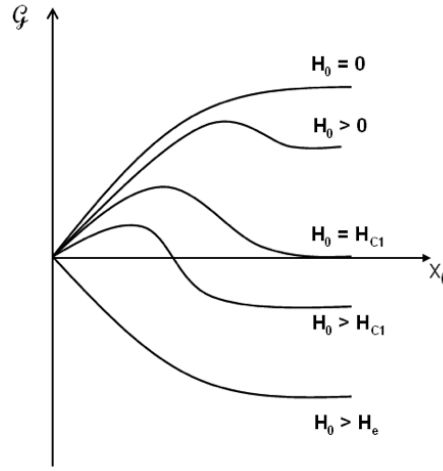


Figure 1.12: From [115] : Gibbs free-energy profile of an isolated vortex as a function of its distance from the surface. The curves are depicted at various applied magnetic fields and the free-energy is measured with respect to its value with no vortex.

The resulting profile of the Gibbs free-energy is depicted in Fig. 1.12 for different values of the external magnetic field. First, one sees that an isolated vortex is unstable inside the superconductor when the external magnetic field is zero ($H_0 = 0$). Sufficiently far from the surface, G is simply given by the free-energy of an isolated vortex. Increasing the magnetic field below H_{c1} leads to a metastable solution which is not favored thermodynamically. It becomes favorable for a vortex to penetrate the superconductor when $H_0 = H_{c1}$. However, the vortex entrance is still hindered by the so-called Bean-Livingston barrier. Further increasing H_0 leads to a gradual reduction of the energy barrier. The latter finally vanishes when the applied field equals the penetration field, which is often referred as the superheating field for the Meissner state (H_{SH}). As a consequence, the Meissner state exists as a metastable state up

to the superheating field $H_{SH} > H_{c1}$. For $H_{c1} < H_0 < H_{SH}$, the Bean-Livingston barrier is such that the perfectly superconducting state is locally stable, even though the global energy of the system is not the lowest one¹⁸. In practice, the Bean-Livingston barrier may be significantly suppressed. Indeed, the presence of surface defects or irregularities may locally increase the field above H_{SH} although the average value remains much smaller [114]. As a consequence, vortices may penetrate at $H_0 \ll H_{SH}$. The limit of the superheating state is determined by the sample boundaries and geometry only if the effects of these extrinsic factors are minimized.

The Bean-Livingston barrier also impacts the vortex escape. Indeed, a vortex has to overcome an energy barrier in order to leave the superconductor at decreasing field. By analogy with the superheating field, one may consider a supercooling field H_{sc} such that vortices remain trapped inside the superconductor for fields $H_{sc} < H_0 < H_{c1}$. This geometrical barrier is able to trap the flux into the sample even though a bulk pinning is not present.

1.5.2 Critical induction models

After Bean and Livingston [114] have introduced the concept of surface barrier for vortex entrance, many works have been devoted to the determination of the superheating field, both theoretically and experimentally. For a thin strip geometry with the external field applied perpendicularly, all these studies came to the conclusion that the critical induction above which the first vortices penetrate varies as

$$B_{SH} \sim k \frac{\Phi_0}{W^2}, \quad (1.35)$$

where Φ_0 is the superconducting flux quantum ($\equiv \frac{h}{2e}$) and W is the strip width. The parameter k differs only slightly from one model to the other depending on the assumptions regarding the size of the vortex core. In the expression 1.35, the relevant information is that B_{SH} is roughly proportional to W^{-2} so that the superheating field noticeably increases with a reduction of the sample width. The superheating field B_{SH} is also called the critical field for complete vortex expulsion as the vortices are expelled from the material for $B < B_{SH}$.

¹⁸. Here, the existence of a superheated metastable Meissner state is described for a type-II superconductor. However, it is relevant to notice that the field H_{SH} is a property of both type-I and type-II superconductors as a superheated Meissner state is also observed in type-I superconductors up to a field $H_{SH} > H_c$.

1.5.2.1 Critical field for vortex entrance in the London limit

Many theoretical models consider the free-energy of a vortex in a superconducting strip as a function of the distance x of the vortex from the center of the strip (i.e., the transverse position). The free-energy is expressed in the London limit by the equation 1.36

$$G(x) = \frac{4\pi}{\mu_0} \left\{ \frac{\Phi_0^2}{4\pi^2 \lambda_\perp} \ln \left[\frac{2W}{\pi\xi} \sin \left(\frac{\pi x}{W} \right) \right] - \frac{\Phi_0 B}{2\pi \lambda_\perp} x(W-x) \right\}, \quad (1.36)$$

where μ_0 is the vacuum permittivity, Φ_0 is the flux quantum, B is the applied field, W is the sample width and $\lambda_\perp \approx \lambda^2/d$ is the effective penetration depth (λ is the London magnetic field penetration depth and d is the thickness). The first term is independent of the applied magnetic induction and corresponds to the nucleation energy of the vortex. The second term corresponds to the interaction between the vortex and the screening currents induced by the external magnetic induction. This latter term is responsible for the minimum of $G(x)$ that appears when the applied induction exceeds a particular threshold. The equation 1.36 is not valid at a distance $\sim \xi$ from the edges. This results from the London approximation which neglects the vortex core. From Eq. 1.36, there are two ways to define a critical field depending on whether the vortices are in a stable state or in a metastable state in the center of the strip. The former model was introduced by Likharev et al. [116] and predicts that the vortices are trapped inside a strip insofar as they are absolutely stable, i.e., the Gibbs free-energy equals zero in the middle of the strip. This requires that the external induction exceeds the threshold B_s given by

$$B_s = \frac{2}{\pi} \frac{\Phi_0}{W^2} \ln \left(\frac{\alpha W}{\xi} \right), \quad (1.37)$$

where the constant α is related to the calculation of the vortex energy when it is located at a distance $\sim \xi$ from the edges¹⁹. Above B_s , the free-energy with a vortex is lower than in the absence of vortices. The other model, proposed by Clem et al. [117], considers that the relevant field is defined by the metastable equilibrium, i.e., the Gibbs free-energy is minimum at the center of the strip. This is characterized by the field B_0 given by

$$B_0 = \frac{\pi}{4} \frac{\Phi_0}{W^2}. \quad (1.38)$$

19. The constant α was found to be $1/4$ in [116], $2/\pi$ in [117] and $1/\pi$ in [118].

The equations 1.37 and 1.38 for B_s and B_0 are two possible expressions of the critical field for vortex entrance. Both have been corroborated by experimental observations. In particular, the work of Stan et al. [8] is really noticeable. Indeed, by imaging their samples with a scanning Hall-probe microscopy, the authors were able to determine the maximum field below which the vortices are completely expelled by field-cooled niobium strips. Their main experimental results are depicted in Fig. 1.13. The figure 1.13(a) shows an image obtained on a $10\text{ }\mu\text{m}$ -wide strip after the sample was cooled through T_c to 7 K under an applied field of $85\text{ }\mu\text{T}$. A consequence of the Bean-Livingston barrier for $\lambda_\perp > (W, d)$ is that the vortices strongly interact with the Meissner currents so that they tend to align along the center of the strip where their energy is the lowest. In addition, Stan et al. have determined the critical field of strips of various widths by counting the number of vortices that were present at different applied fields. They have found that no vortex is present in a field range centered around zero. To give an order of magnitude, the critical field amounts to less than $1\text{ }\mu\text{T}$ for $W = 100\text{ }\mu\text{m}$ and to around 2 mT for $W = 1.6\text{ }\mu\text{m}$.

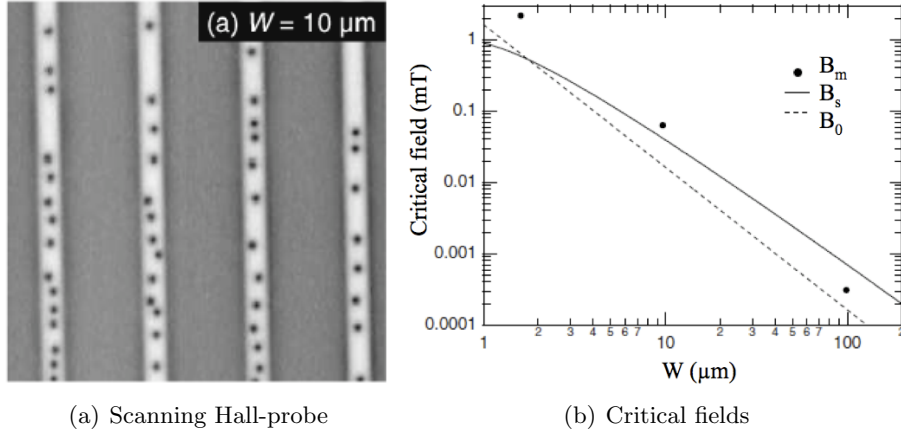


Figure 1.13: From [8] : (a) Scanning Hall-probe microscopy of a $10\text{ }\mu\text{m}$ -wide niobium strip obtained after the sample has been cooled through its critical temperature under an applied magnetic induction of $85\text{ }\mu\text{T}$. The darker spots indicate the location of the vortices. (b) Experimental ($\bullet$) and theoretical critical fields as a function of the strip width. The curves B_s and B_0 correspond to the stable and metastable solutions for the possible critical field for complete vortex expulsion, respectively.

An important feature of their quantitative analysis is that the equation 1.37 is applied with the coherence length at the temperature T_f at which the vortices become pinned. The physical arguments for such an assumption are the following. In a field-cooled experiment, the vortices

begin to nucleate near T_c where they are able to move inside the strip. Further cooling the sample increases the pinning capabilities so that the vortices freeze out at a temperature close to T_c below which they remain pinned. As a consequence, the vortices patterns that are imaged at low temperatures are considered to be properly accounted for in the $T_f \sim T_c$ regime. The temperature T_f was determined by imaging the samples at different substrate temperatures and appears to be very close to T_c , i.e., $1 - \frac{T_f}{T_c} = 0.0015$. In Fig. 1.13(b), the stable solution B_s is depicted by considering $\xi(T_f) = 320$ nm and $\alpha = 2/\pi$ (in its hand, the metastable solution B_0 does not depend on ξ). Although the narrowest strip can not be described in the London limit²⁰, the authors argued that the stable solution B_s is the relevant one to explain their observations.

A similar study was performed by Kuit et al. [9] on thin-film high temperature $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ strips whose width varies between 2 and 50 μm . The figure 1.14(a) illustrates typical images obtained with a scanning superconducting quantum interference device (SQUID) microscope on a 35 μm -wide strip cooled at different magnetic inductions. The location of the vortices is given by the bright dots. Once again, the vortices are positioned at the middle of the strip when the applied magnetic induction is sufficiently low.

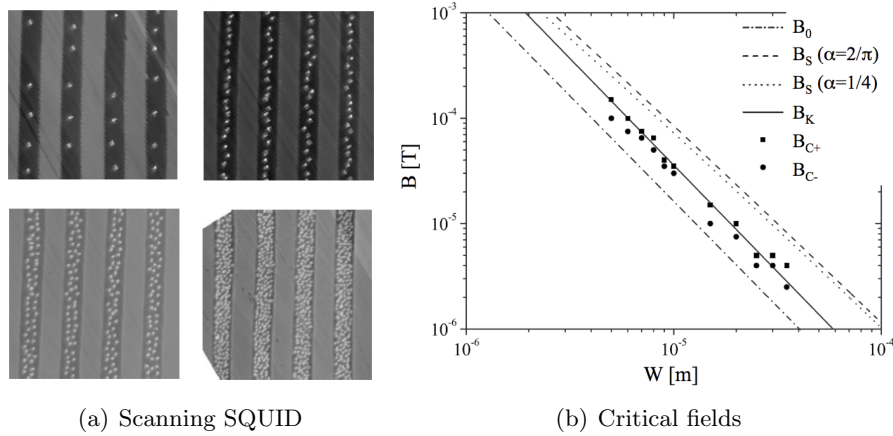


Figure 1.14: From [9] : (a) Scanning superconducting quantum interference device microscope images obtained on a 35 μm -wide YBCO strip field-cooled at 5 μT , 10 μT , 20 μT and 50 μT . (b) Experimental (markers) and theoretical (lines) critical inductions for vortex trapping depicted with respect to the strip width. The B_K model takes into account the energy required to produce a vortex-antivortex pair.

20. That will be discussed in section 1.5.2.2.

From their experiments, the authors have extracted an upper and a lower limit for the critical induction, i.e., they have determined the lowest induction B_{c-} at which vortices were present (at decreasing field) and the highest induction B_{c+} at which vortices were not observed (at increasing field). The results are summarized in Fig. 1.14(b) together with the metastable (B_0) and the stable (B_s) solutions for the critical field for vortex entrance. The temperature at which the vortices become pinned is assumed to be $T_f = 0.98 T_c$, which gives a coherence length $\xi(T_f) \sim 10.4$ nm [with $\xi(0) \sim 3$ nm]. In order to better account for their results, the authors proposed an intermediate model whose main idea is to integrate the energy required to produce a vortex-antivortex pair. This term adds to the nucleation energy of the vortex and to the interaction energy between a vortex and the screening currents which are involved in Eq. 1.36. By considering the vortex-antivortex generation rate [$\sim \exp(-E_{pair}/k_B T)$, where E_{pair} is the energy required to form a vortex-antivortex pair] and the vortex escape rate [$\sim \exp(-E_B/k_B T)$, where E_B is the difference in Gibbs energy between the local maximum and minimum at the center of the strip], the authors obtained numerically that the critical field is better expressed by the equation

$$B_K = 1.65 \frac{\Phi_0}{W^2}, \quad (1.39)$$

that lies between the metastable and stable solutions. The B_K solution is depicted by the solid line in Fig. 1.14(b) and satisfactorily reproduces the critical fields determined on the YBCO samples.

1.5.2.2 Critical field for vortex entrance in narrow strips

The theoretical models discussed in section 1.5.2.1 fall in the London limit. This simplified formalism is justified by the possibility to obtain analytical expressions when the vortex core is neglected. Strictly speaking, the condition $W \gg \xi$ is not fulfilled for the narrowest strip studied by Stan et al. [8] as $W/\xi \sim 5$. For similar or even smaller width-to-coherence length ratios, the theory predicts that $B_0 > B_s$, which is of course not conceivable. In this case, the finite size of the vortex core has to be considered, which motivates the use of the Ginzburg-Landau theory.

The critical fields for vortex entrance (or equivalently for complete vortex expulsion) of infinitely long and narrow strips ($W/\xi \sim 3-5$) have been treated using the Ginzburg-Landau formalism by Sánchez-Lotero et al. [119]. The stable critical field is determined by equating the Gibbs free-energy of the system with one vortex row and the system in the

Meissner state. On the other hand, the metastable solution is defined by the field at which a plateau in the free-energy arises in the center of the strip. As illustrated in Fig. 1.15 with the two top curves, the stable (B_s) and the metastable (B_0) solutions are larger than the solutions obtained in the London formalism. A major result is that the dependence of the critical field with respect to the sample width is similar (in this width range) to the one predicted in the London limit. Indeed, the critical fields are found to vary as $W^{-2.216}$ with a proportionality constant $\sim \Phi_0$, which is close to the general London form given by Eq. 1.35. In addition, the authors found that the best agreement with the experimental critical field obtained by Stan et al. is given by the stable solution in the Ginzburg-Landau theory. This is highlighted in Fig. 1.15 by the solid square which corresponds to the experimental value. Contrary to the London formalism, the free-energy takes a finite value when the vortices are placed on the edge of the strip at B_s or B_0 . This more correct description confirms that a vortex located near an edge is unfavored in comparison to the Meissner state.

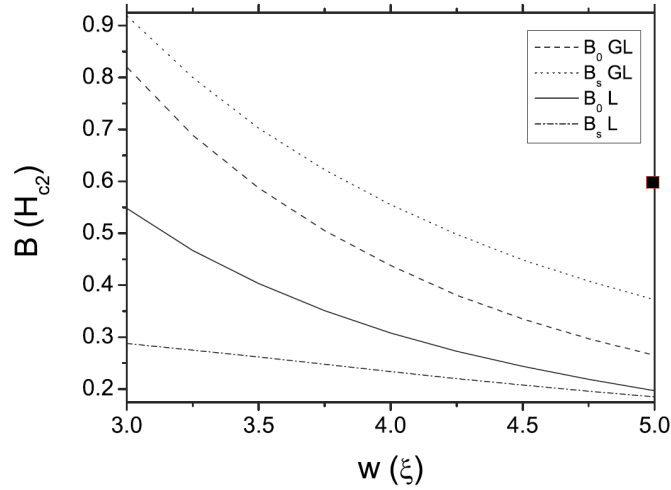


Figure 1.15: From [119] : Evolution of the stable (B_s) and metastable (B_0) critical fields determined in the Ginzburg-Landau and in the London formalisms for width ranging between 3ξ and 5ξ . The solid square indicates the experimental value obtained by Stan et al. [8] on a narrow superconducting strip.

1.5.3 Formation and reorganization of anisotropic lattices

The critical induction models allow one to estimate the maximum applied field below which no vortices are present inside a thin superconducting strip. Insofar as the applied magnetic field exceeds the superheating field, a typical field-cooling procedure results in the freezing of the vortices at a temperature which is very close to the critical one [8, 120]. In the previous sections, only a few number of vortices has been considered. At low magnetic fields, their strong interactions with the Meissner circulating currents result in their alignment along the center of the strip. This leads to the formation of an anisotropic one-dimensional vortices chain parallel to the longitudinal direction [see the figures 1.13(a) and 1.14(a)]. However, further increasing the applied field reduces the vortex-vortex interdistance, which in turn increases the vortex interaction energy. At some field, the anisotropic lattice becomes unstable and splits into two chains. This section is devoted to the (similar) rearrangements that occur at regularly spaced *matching fields* B_N , i.e., to the transition from N to $N+1$ vortices rows.

1.5.3.1 Dimensional phase transition

One of the first experimental evidence of the dimensional transition of the vortex lattice was highlighted by Guimpel et al. [121] who have studied the low-field magnetization of niobium/copper superlattices. The field was applied parallel to the surface so that the relevant geometrical parameter is the thickness. The figure 1.16(a) illustrates their main results, i.e., the flux expulsion with respect to the applied field for a Nb(16 Å)/Cu(16 Å) (top) and a Nb(54 Å)/Cu(54 Å) (bottom) multilayer. The linear regime observed at low field reflects the perfect Meissner state. The expulsion becomes not complete at higher fields and the magnetization presents two well-defined maxima which corresponds to the transition from a one-dimensional to a two-dimensional array of vortices. The vortices in the one-dimensional configuration are thought to be centered along the sample because the thickness ($d \sim 1 \mu\text{m}$) is comparable to the penetration depth [$\lambda(0) \sim 0.5 \mu\text{m}$]. Brongersma et al. [122] also observed a series of maxima in the magnetic moment of Nb/Cu multilayers. The main difference is that the total thickness was smaller ($d \sim 100 - 400 \text{ nm}$) which leads to a larger number of peaks (up to six). Illustrative results are depicted in Fig. 1.16(b). Once again, the maxima were related to particular configurations of the vortices which first form a single chain in the middle of the film before splitting into multiple-chain configurations that separate the sample in

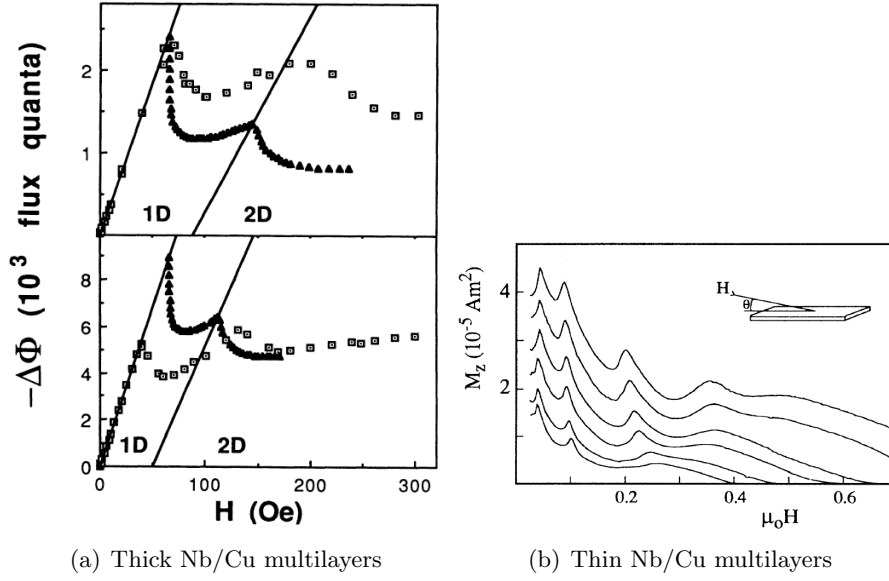


Figure 1.16: (a) Field dependence of the flux expelled by a Nb(16 Å)/Cu(16 Å) multilayer structure (total thickness $\sim 0.8 \mu\text{m}$, top) and by a Nb(54 Å)/Cu(54 Å) multilayer structure (total thickness $\sim 1 \mu\text{m}$, bottom). The coolant temperature is 1.7 K (from [121]). (b) Field dependence of the magnetic moment of thinner Nb/Cu multilayers (total thickness $\sim 300 \text{ nm}$). From top to bottom, the coolant temperature varies between 1.8 K and 4.1 K (from [122]).

equally thick parts. The authors mentioned that the position of the peaks is temperature-independent while their number is reduced when the temperature increases due to the temperature dependence of the upper critical field.

The dimensional phase transitions reported in [121] and in [122] were both explained in the framework of the London theory by considering the long-ranged vortex-vortex interactions. A few years later, Bolech et al. [123] studied the behavior of thin (homogeneous and isotropic) type-II superconducting films submitted to a parallel field. In the framework of the time-dependent Ginzburg-Landau theory, they have confirmed the existence of a series of maxima in the magnetization curves. However, the latter discontinuities were related to the penetration of vortices, and not necessarily to some structural modifications of the flux-line lattice. In other words, they have found theoretically that two peaks in the magnetization curves may be related to the same vortex structure, i.e., with the same number of vortices rows.

1.5.3.2 Matching effects in the critical current

From the beginning of the section 1.5, the entrance and the rearrangement of the vortices in type-II superconductors have been discussed *without an applied transport current*. The samples were most of the time field-cooled with no current and their current-carrying capabilities were not discussed. However, the rearrangement of an unstable vortices pattern into a more stable one interestingly goes hand in hand with a minimum of the critical current.

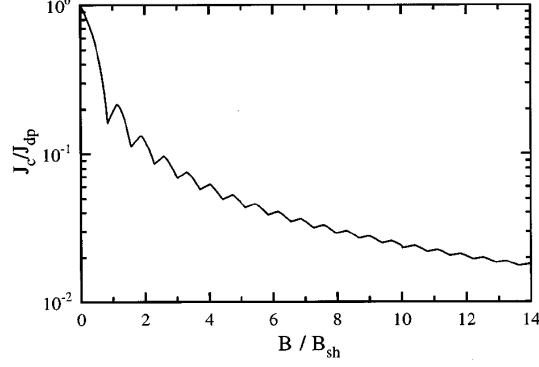
Mawatari and Yamafuji [124] investigated this issue by considering a superconducting film submitted to a transport current parallel to the surface and perpendicular to the magnetic field (the latter is also oriented parallel to the surface). Within their model, the critical current density is minimum each time the vortices lattice rearranges from a N-1 to a N rows configuration. The critical current density then increases above each matching field until the next rearrangement occurs. Mawatari et al. explicitly took into account the existence of a surface barrier for the entry of the vortices and showed that the characteristic scaling-field of interest is the superheating field. Ziese et al. [125] have computed the model of Mawatari et al. under the assumption²¹ that $\xi/\gamma d \ll 1$ (γ is an anisotropy factor defined for high temperature superconductors as the ratio between the penetration depth along the c-axis and along the CuO₂ planes). The calculated ratio between the critical current density (j_c) and the depairing critical current density (j_{dp}) is depicted in Fig. 1.17(a) as a function of the applied parallel field. A well-defined minimum occurs each time the flux-line lattice rearranges. An interesting observation is that the matching fields B_N varies linearly with the number N of vortices rows according to the relation

$$B_N = B_{SH} (0.19 + 0.71N), \quad (1.40)$$

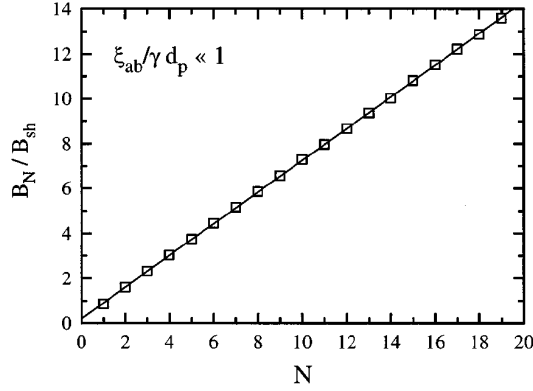
which is illustrated in Fig. 1.17(b). Although the equation 1.40 corresponds to a situation *with an applied transport current*, it brings into play the field B_{SH} which is by definition the maximal field at which the surface barrier vanishes *without an applied current*. The authors further mentioned that the presence of a transport current favors the penetration of vortices, even for field below the superheating one. The magnetic behavior of Bi₂Sr₂CaCu₂O₈ thin films ($\gamma \sim 150$) has been successfully described. The linear evolution $B_N \propto N$ was also observed in the low-field magnetization of Nb/Cu multilayer, although the matching fields

²¹. In practical terms, this is realized by considering the equations of the flux-line lattice in the first order of $\xi/\gamma d$.

were significantly smaller than expected from the model of Mawatari et al. This was interpreted by the ineffectiveness of the surface barrier for the entry of the vortices.



(a) Critical current density



(b) Matching fields

Figure 1.17: From [125] : (a) Computed evolution of the critical current density j_c of a superconducting film submitted to a parallel magnetic field. j_c is normalized with respect to the depairing critical current density j_{dp} and the calculations are performed in the limit $\xi/\gamma d \ll 1$. (b) Linear evolution between the matching field B_N and the number of vortices rows N . The matching fields are defined by the minima in the critical current density.

In the model of Mawatari et al. [124], a defect-free superconductor is assumed. However, the intrinsic *bulk* pinning (as opposed to the surface pinning) may hinder the formation of an ordered vortices lattice, predominantly at low temperatures where the bulk pinning is stronger²². The

²². Close to the critical temperature, the thermal fluctuations are mostly responsible for the destruction of the flux-line lattice.

possible deviations from the model due to the existence of defects are however not very clear. As an example, Pan et al. [126] have noticed no inconsistency between the simple model without bulk pinning and the magnetic behavior of their niobium thin films ($d \sim 120$ nm) for which the pinning is not negligible.

As a final comment, let's notice that the formalism described in [124] and [125] can be used with a magnetic field applied perpendicularly to the film surface. In this case, the sample width determines the periodicity of the matching fields [127] and the thickness d has to be replaced by W in the expression of the superheating field.

1.5.3.3 Rearrangement of the flux-line lattice under an increasing transport current

The theoretical works described in the section 1.5.3.2 were done to explain the magnetic behavior of thin anisotropic superconductors submitted to a parallel external field. Strictly speaking, the proposed model is valid only for three-dimensional anisotropic samples. However, the current-carrying capabilities of isotropic thin films are also characterized by a non-monotonic evolution of the critical current as a function of the field.

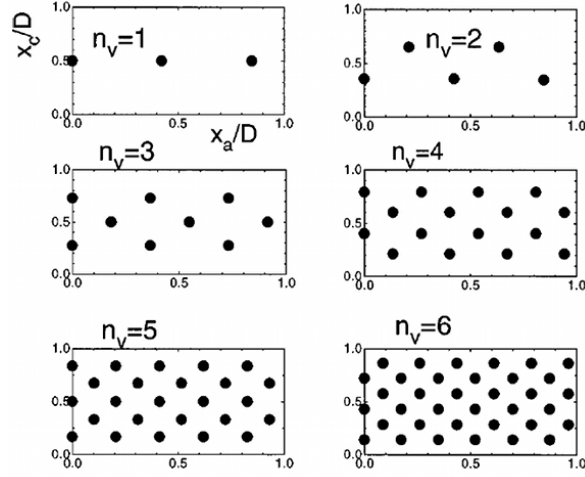


Figure 1.18: From [128] : Equilibrium vortices configurations without an applied transport current density ($j_t = 0$). The vortices locations are defined by x_a and x_c in the longitudinal and transverse directions, respectively (D is the film thickness). The number of vortices rows (n_v) increases with the parallel applied magnetic field.

This was studied by Carneiro et al. [128] who calculated the equilibrium configurations of vortices as well as the critical current at zero temperature of isotropic films submitted to a parallel magnetic field. The film is assumed free of imperfections (no intrinsic pinning) with a thickness smaller than the penetration depth. In the absence of a transport current, the equilibrium configurations consist in the formation of N (here, referred as n_v) chains of vortices. Increasing the field in a magnetic region of unchanged N leads to a slight increase of the vortices number in the film which manifests itself by a reduction of the distance between adjacent vortices in a row. As illustrated in Fig. 1.18, the transitions go hand in hand with an increase by one of the number of the vortices rows which are always separated by the same intrachain spacing²³. The novelty in the work of Carneiro et al. [128] is that the transitions that involve a change by one of the number of rows may also occur if the transport current is swept ($j_t > 0$) at a constant applied field.

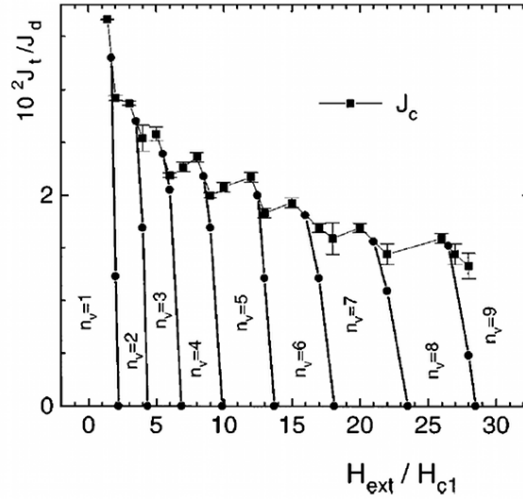


Figure 1.19: From [128] : Equilibrium phase-diagram of the vortices configuration expressed as a function of the transport current density and of the applied magnetic field ($T_0 = 0$ K, thickness $\sim$ penetration depth). j_t is normalized with respect to the depairing critical current density j_d .

Two situations have to be distinguished depending on the value of the applied field. On one hand, if the field is not too close to the one at which a transition takes place at $j_t = 0$, the equilibrium configuration at $j_t > 0$ has the same number of rows as without a transport

²³. Let's notice that the authors normalize the applied field with respect to the lower critical field H_{c1} , which is smaller than the superheating field.

current. In this case, the chains are slightly shifted to the film surface (the ones that are the closest to the surface being more shifted) and the distance between the vortices in the same row decreases (revealing a larger number of vortices in the film). On the other hand, if the field is close to the one at which a transition takes place at $j_t = 0$, a transition is observed in the number of vortices rows when the transport current is increased. As it is the case without a transport current, the transition is associated to a noticeable change of the vortices position and by a small change in their number. The equilibrium vortices configurations that are favored under a non-zero transport current are summarized in the zero-temperature phase diagram of Fig. 1.19. At a constant field, one sees that the number of the vortices chains can differ by one when j_t is increased from zero. This leads to a slight reduction of the matching field at which a minimum is observed in the critical current compared to the field at which the transition arises at $j_t = 0$. In the context of the present work, the similarities that exist between the vortices rearrangements at constant field when a transport current is applied (or not) is of the greatest relevance. Indeed, the evolution of the critical current with respect to the magnetic field is the main experimental tool that will be used in the chapter 4 to characterize the rearrangements of the vortices patterns in the studied field-cooled nanostructured strips and loops.

Chapter 2

Samples fabrication and characterization

This chapter is devoted to the fabrication and the basic characterization of the niobium nitride (NbN) and niobium (Nb) samples. Firstly, the section 2.1 deals with the properties of the as-grown thin films which are obtained by sputtering deposition. The impact of the sputtering conditions on the evolution of the electrical properties (critical temperature, residual resistivity, ...) is emphasized. Details on the nanofabrication process and on the experimental setup (cooling conditions, electrical measurements) are given in section 2.2. Finally, the section 2.3 discusses the main electrical properties that are extracted from the resistance vs. temperature and the resistance vs. magnetic field curves obtained at low applied current density.

2.1 Thin film deposition

2.1.1 Sputtering principle

Both Nb and NbN have a very high melting point which amounts to 2468°C and to 2573°C, respectively. The deposition of these materials using an evaporator is anything but an easy task. For this reason, an alternative process - the sputtering - proves to be the most widely used technique for the deposition of thin films [129]. It consists in the ejection of the material source from a target, and its following deposition onto a substrate. In this process, a plasma of an inert gas (usually argon) is created in a closed chamber, which is beforehand pumped down to the vacuum base pressure. A negative potential of several hundred volts is then applied on the material free target, which allows the positive gas

ions of the plasma (Ar^+) to be accelerated towards the target. Sputtered (or detached) particles are then ejected if the energy transferred to a lattice site is greater than the binding energy. The sputtered particles, among which only a small percentage is ionized, then have a probability to deposit onto the substrate.

The sputtering process requires that a sufficient amount of ions reaches the target. In order to increase the ionization rate inside the plasma, the secondary electrons that result from the collisions between the gas ions and the target can be trapped close to the surface by a ring magnet positioned below the target. This is called the magnetron sputtering. Such confinement of the secondary electrons leads to higher deposition rates and allows one to sustain a stable plasma using lower inert gas pressures. This increases the kinetic energy of the sputtered particles that impact the substrate. The above-described process is commonly used for the fabrication of conducting materials (metals or doped semiconductors) whose chemical composition is given by the target material. The deposition of nitridic (NbN , NbTiN , ...) or of oxidic (Al_2O_3 , TiO_2 , ZnO , ...) films further requires to inject nitrogen or oxygen as a reactive sputtering gas, in addition to the Ar gas. Here, all the studied superconducting thin films have been obtained using the dc magnetron sputtering process, either in an Ar atmosphere (for Nb) or in an Ar + N_2 atmosphere (for reactively sputtered NbN). The target is nominally made of 99.9% pure Nb.

2.1.2 Optimization of the deposition conditions

Most of the films have been deposited at room temperature, either at the LPN/CNRS laboratory for Nb (by C. Ulysse and L. Couraud) or at the UCL/IMCN laboratory for NbN. Some NbN films have also been deposited at the LPN/CNRS laboratory with a substrate holder heated up to several hundred of Celsius degrees (up to 600°C). The deposition conditions are known to noticeably alter the structural, electrical, superconducting, optical ... properties of Nb [130, 131] and NbN [132–137] thin films. The film properties are very sensitive to the total pressure and, for NbN, to the relative amount of nitrogen in the gas atmosphere. The temperature deposition and the substrate are also relevant parameters.

The Nb thin films have been deposited in an Ar gas atmosphere with a total pressure of 9.75 mTorr (1000 W dc power, base pressure around 6×10^{-8} Torr). As discussed in section 2.1.3, these conditions insure satisfactory superconducting properties, so that no optimization was required. The situation appears completely different for the NbN films

for which the sputtering conditions were optimized in order to achieve the largest critical temperature. Among the various sputter parameters (the sputter current and voltage, the partial and total pressures, the temperature deposition, ...), only the influence of the nitrogen partial pressure was studied in details. The relative amounts of argon and nitrogen were controlled by gas flowmeters. For the room temperature process, the argon pressure was fixed at 7.9 mTorr (38.5 Sccm¹) while the nitrogen pressure was varied between 0.7 mTorr (1.7 Sccm) and 2.2 mTorr (5.2 Sccm). Consequently, the total pressure was not constant but varied from 8.6 mTorr to 10.1 mTorr, leading to nitrogen partial pressures between 8 % and 22 %. Given that the working pressure has a moderate impact on the critical temperature², the total pressure variation is here considered small enough to compare the films between them. A summary of the main sputter parameters used for the thin film deposition is given in the table 2.1.

Table 2.1: Sputtering conditions of the Nb and NbN thin films : working pressure WP, nitrogen partial pressure p_{N_2} , sputter current I_{SP} and target power (density) P_{SP} .

	WP	p_{N_2}	I_{SP}	P_{SP}	
	[mTorr]	[%]	[A]	[W]	[Wcm ⁻²]
Nb ($\sim 20^\circ\text{C}$)	9.75	0	1	1000	5.5
NbN ($\sim 20^\circ\text{C}$)	8.9	11	1	360	4.4
NbN (600°C)	3.75	9.6	0.55	200	1.1

2.1.2.1 Influence of the nitrogen partial pressure

The figure 2.1(a) summarizes the evolution of the critical temperature (■, left axis) and of the normal state resistivity (●, right axis) of 100 nm-thick NbN films (width ~ 0.5 mm, length ~ 35 mm) as a function of the nitrogen partial pressure. The normal state resistivity ρ_n is measured just above the superconducting-to-normal transition (i.e., at 15 K) while the critical temperature T_c is determined at half the normal state value of resistance. The lower and upper bars represent the width of the transition measured at 5 % onset and 90 % endpoint, respectively.

1. Sccm denotes cubic centimeter per minute.

2. For example, Wang et al. [133] have shown that the maximum critical temperature of their films deposited at ambient substrate temperature only differs by 1.5 K (from 16 K to 14.5 K) when the total pressure varies from 2 to 10 mTorr. Nevertheless, the normal state resistivity was found to rapidly increase with the total pressure of the gas mixture [133–135, 137].

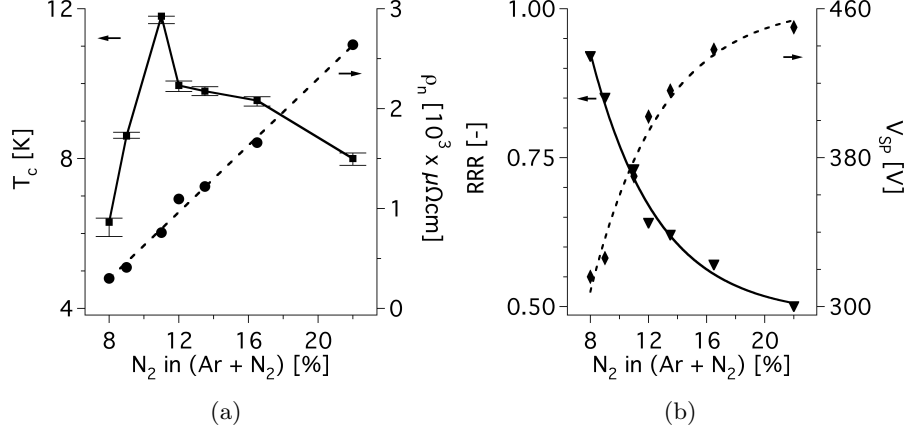


Figure 2.1: Properties of 100 nm-thick sputtered NbN films as a function of the relative amount of nitrogen in the gas atmosphere. (a) Critical temperature T_c (left axis, $\blacksquare$) and normal state resistivity $\rho_n = \rho(15K)$ (right axis, $\bullet$). (b) Residual resistivity ratio RRR (left axis, $\blacktriangledown$) and target voltage V_{SP} (right axis, $\blacklozenge$). T_c is determined at mid-transition, while the errors bars describe the transition width at 5 % and 90 % of the normal state resistance. The thin lines are guides for the eyes.

On one hand, one sees that ρ_n increases quasi-linearly with the injected nitrogen pressure. The influence of the pressure on ρ_n is usually attributed to intergrain effects [137]. As p_{N_2} increases from the optimum value, the growth of the superconducting grains is affected by the presence of non-superconducting phases that nucleate at the grain boundaries. The resistivity of the film thus increases due to the large number of barriers between the grains. On the other hand, the critical temperature rapidly increases, reaches a maximum around $p_{N_2} \sim 11$ % and slightly drops at higher partial pressures. This is the usual evolution although the peak position depends on the deposition setup. Indeed, for room temperature processes and for similar total pressures, the peak was observed between $p_{N_2} \sim 9$ % [133] and $p_{N_2} \sim 22$ % [135]. It was suggested that the largest T_c values correspond to film whose chemical composition is close to the stoichiometric one.

The target voltage increases monotonically with the nitrogen partial pressure, as illustrated in the figure 2.1(b) [$\blacklozenge$, right axis]. This is the expected evolution for a constant deposition current (here fixed at 1 A). The latter mainly determines the rate of the deposition onto the substrate by influencing the time available for the incoming sputtered particles to diffuse at the surface and to agglomerate with existing growth clusters. The deposition rate amounts to 0.73 nms^{-1} at the optimum

nitrogen partial pressure $p_{N_2} \sim 11\%$. The evolution of the residual resistivity ratio RRR ($\equiv \frac{\rho(300K)}{\rho(15K)}$) is shown in the figure 2.1(b) [▼, left axis]. As explained in more details in the section 2.1.3, the RRR is intimately related to the polycrystalline structure of the thin film : a RRR less than one reveals a granular or columnar structure with a pronounced electron scattering at the grain boundaries. This is consistent with the observed evolution of the RRR as a function of the normal state resistivity because the higher the intergrain resistance (and thus the normal state resistivity), the smaller the RRR.

2.1.2.2 Influence of the substrate

The substrate is known to play a relevant role in the achievement of high-quality thin films. In particular, this is crucial for ultra-thin films (for recent improvements, see e.g. [138–140]) because the bottom layers in contact to the substrate often present a poor crystallinity. For NbN deposition, the MgO [133] and the M-plane oriented (1010) sapphire [141] are currently the best substrates, especially due to their low lattice mismatch and their defect free surface. Many works have revealed a clear degradation of the critical temperature (≈ 2 K) for films deposited on silicon, sapphire [including R-plane (1012) orientation] and glass wafers [141, 142]. The reduction of the critical temperature often goes together with an increase of the electrical resistivity.

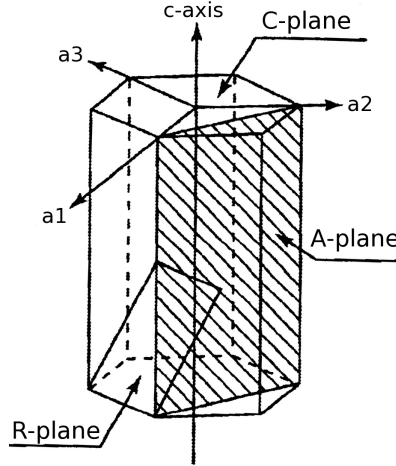


Figure 2.2: Hexagonal/rhombohedral structure of the sapphire crystal Al₂O₃. The typical orientations are indicated : R-plane (1012), A-plane (1120) and C-plane (0001). The C-plane orientation has been used to deposit the NbN thin films.

Two types of substrates have been considered here to deposit the NbN thin films : silicon wafers covered by a thermally grown amorphous silicon oxide layer and sapphire wafers (thickness : $330 \pm 20 \mu\text{m}$, roughness $< 0.5 \text{ nm}$). Sapphire is a highly anisotropic aluminum oxide (Al_2O_3) which belongs to the hexagonal/rhombohedral crystal system. Among the three orientations that are illustrated in Fig. 2.2, the C-plane (0001) orientation has been used. No noticeable variations of the electrical properties have been observed between the different substrates. In addition, all the niobium thin films have been deposited on Si/SiO₂ substrates.

2.1.3 Thin film characterization

2.1.3.1 Morphology

The thin film growth is a non-equilibrium process which is mainly determined by the ability for the sputtered atoms to move at the substrate surface. The surface mobility noticeably impacts the grain size and depends on the substrate temperature T_s , the surface and bulk diffusion, the energy of the atoms, ...

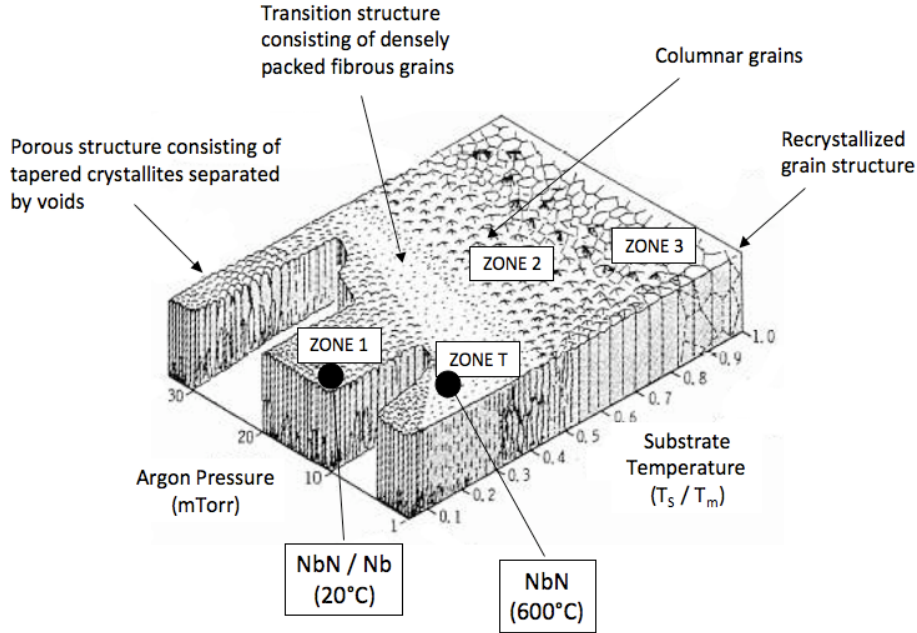


Figure 2.3: From [143] : Microstructure zone diagram for metal films deposited by magnetron sputtering. The structural zones are pointed out. The conditions of deposition of the NbN and Nb thin films are indicated by the black circles (T_s is the substrate temperature and T_m is the material melting point).

The impact of the substrate temperature and of the working pressure on the thin-film morphology is summarized in the structural zone diagram of the figure 2.3 on which the conditions of deposition of the NbN and Nb films are indicated by the black circles. The NbN films deposited at room temperature are characterized by a very small normalized temperature $T_s/T_m \approx 0.01$, where T_m is the melting point temperature. Such a low T_s/T_m ratio leads to a negligible diffusion at the substrate surface, so that the film structure should correspond to the structural zone 1 (Z1) which is characterized by the formation of columns with a diameter $\sim 10\text{-}20$ nm separated by voids of a few nm [143]. The situation differs for the films sputtered at $0.1 < T_s/T_m < 0.5$. In this case, the structure should correspond to the zone structure T (ZT) which consists in fibrous grains with dense grain boundaries, i.e., with less voids. A tilted and a sectional view of a NbN film (58 nm-thick) deposited at 600°C ($T_s/T_m \approx 0.23$) is given for illustration in the figure 2.4. At these deposition temperatures, a change of the crystallinity may arise when the film thickness increases [142]. Indeed, the growth structure tends to evolve from a ZT mode to the structural zone 2 (Z2), which is characterized by columns with tight grain boundaries between them, i.e., a better crystal structure.

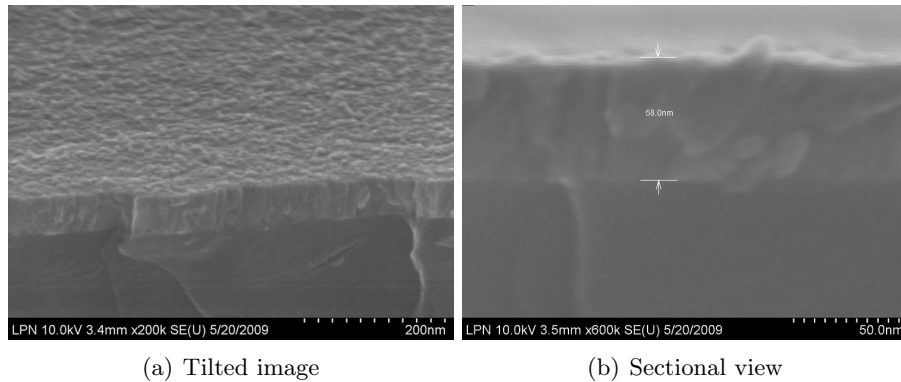


Figure 2.4: SEM images of a 58 nm-thick NbN film deposited on a substrate heated at 600°C (C. Ulysse, LPN/CNRS).

The granular structure of the NbN thin films deposited at ambient temperature is thus thought to contain a much higher density of voids than the films deposited at higher substrate temperatures. One way to corroborate these expected structural morphologies is to study the mass density (ρ_m) of the films, which is strongly dependent on the density of voids at the grain boundaries (to a lesser extent, it may also be affected by the presence of vacancies). The mass density has been determined by

X-ray reflectometry measurements amounts to 8 gcm^{-3} for the NbN films deposited at 600°C . This value differs by only 5 % from the bulk value ($\rho_m^{\text{bulk}} = 8.43 \text{ gcm}^{-3}$ [144]). At the opposite, the room temperature process leads to a much smaller $\rho_m = 7 \text{ gcm}^{-3}$ (around 17 % smaller than the bulk one). This highlights that a much denser structure is obtained for higher T_s/T_m ratios. In addition, the sputter conditions of course impact the granular morphology (lattice parameter, crystallite disorder, mean grain size, ...). In particular, the nitrogen partial pressure p_{N_2} induces significant modifications of the mean grain size, which was found to vary from 50 to 8 nm when p_{N_2} lies in the range between 2 % and 20 % [137]. At $p_{N_2} \sim 11$ % (which is the optimum value use here), a mean grain size of around 10 nm is reported. As illustrated by the atomic force microscopy image of Fig. 2.5, this value is very close to the one observed for the present thin films.

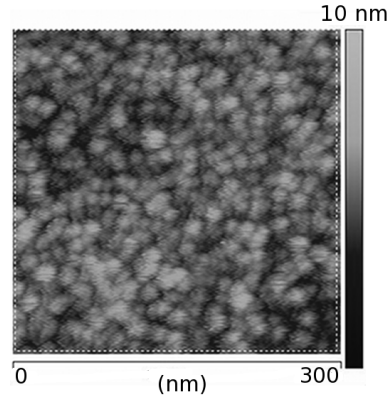


Figure 2.5: AFM image (300 nm x 300 nm) of the top surface of a NbN thin film that reveals a pronounced granular structure with a mean grain size of 10 - 15 nm (L. Couraud, LPN/CNRS).

The exact morphology of the Nb films is less clear. Okolo et al. [131] have reported densely packed fibrous grains structure for their sputtered Nb thin films (structural zone T). However, their films have been obtained with a substrate heated at 100°C and at a much lower working-pressure (3.45 mTorr)³. Here, the low deposition temperature ($T_s / T_m \approx 0.01$) as well as the quite high working-pressure (9.75 mTorr) are such that a structural zone Z1 is expected. The pronounced electrical metallic behavior of these films, however, suggests that the conducting grain boundaries are quite dense.

3. A denser microstructure is obtained at low pressures because the atoms that deposit onto the substrate have an higher energy.

2.1.3.2 Temperature dependence of the resistivity

All the studied Nb thin films have a metallic behavior with a sharp transition to the superconducting state at low temperatures. The table 2.2 summarizes the main parameters extracted from the resistance vs. temperature evolutions of three representative samples (thickness $d = 200, 20$ and 10 nm). As expected, the critical temperature decreases with a reduction of the sample thickness, from a value very close to the bulk one for the thickest film down to 6 K at $d = 10$ nm. The transition width amounts to 0.15 K for all the films. The decrease of T_c is accompanied by an increase of the residual resistivity (measured just above the normal-to-superconducting transition) and by a decrease of the residual resistivity ratio $RRR (\equiv \frac{\rho(300K)}{\rho(9.5K)})$. The obtained values for ρ_n are consistent with other published data [145]. Due to the pronounced granular structure of the films, the normal state resistivity is of course much higher than for bulk samples. Indeed, values as low as $7.25 \times 10^{-4} \mu\Omega\text{cm}$ have been reported for macroscopic samples ($RRR \sim 16000$) [146]. In addition, the evolution of ρ_n with respect to the thickness reflects the degradation of the film structure which is more and more affected by the quality of the substrate surface when the thickness is reduced. In particular, the affinity of the niobium for the oxygen atoms at the oxidized substrate surface may prevent the formation of a perfect crystalline order on the first deposited layers. In addition, the effective thickness of the thinnest Nb films is probably reduced due to the formation of a Nb_2O_5 amorphous oxide layer (~ 2 nm) on the top surface exposed to ambient conditions [131, 147].

Table 2.2: Main electrical transport properties of the as-grown Nb thin films extracted from their resistance vs. temperature curves : critical temperature (T_c), normal state resistivity (ρ_n) and residual resistivity ratio ($RRR \equiv \frac{\rho(300K)}{\rho(9.5K)}$).

d [nm]	T_c [K]	ρ_n [$\mu\Omega\text{cm}$]	RRR [-]
~ 200	9.16	2.4	5.7
20	7.4	7	3.2
10	6	14	2.1

The temperature dependence of the resistivity of the NbN thin films is rather different and depends on the crystalline structure that forms during the sputtering process. Indeed, the niobium-nitrogen system can crystallize into several phases among which the most frequent are the hexagonal δ' -NbN and β -Nb₂N phases and the cubic δ -NbN phase [148]. Only the cubic phase presents a transition to the superconducting state

at low temperatures. The impact of the sputtering conditions on the crystalline structure is illustrated at the figure 2.6 with two NbN films (30 nm-thick) deposited at 600°C (200 W dc power) but using different total pressures. The gray curve corresponds to the film obtained at the lowest pressure (0.75 mTorr). This gives an hexagonal δ' -NbN structure which exhibits a metallic behavior : the resistivity increases in a monotonous way with the temperature. At the opposite, a pressure of 3.75 mTorr gives rise to a cubic δ -NbN characterized by a non-metallic conduction and a superconducting transition, here at 11.4 K (black curve). The non-metallic electrical conduction, which is characterized by a residual resistivity ratio $RRR \equiv \frac{\rho(300K)}{\rho(15K)}$ smaller than unity, is known to be strongly influenced by the grain-boundary scattering, i.e., by the nonmetallic barrier present at the grain boundaries [132,135,137]. This behavior was explained by considering the transmission of the electrons between the low-resistivity grains which are separated by very disordered intergranular regions. Because of the small and temperature-independent probability for the electrons to go through the grain boundary, Nigro et al. [132] have concluded that the tunnelling mechanism is the dominant one (by opposition with the thermally activated hopping between grains). Small RRR values thus imply that the electrons are more and more localized between the grains [137].

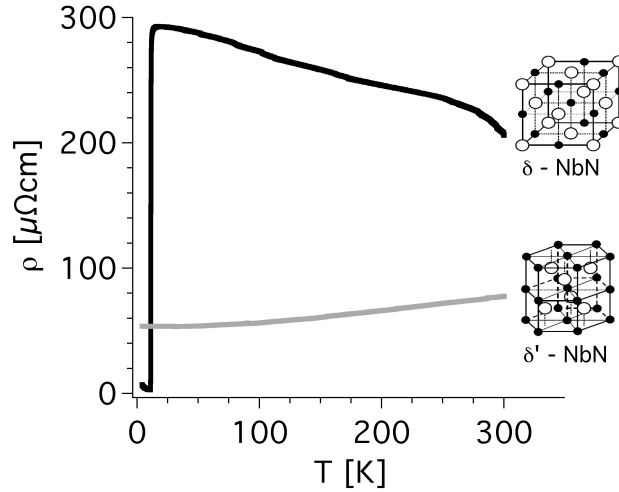


Figure 2.6: Electrical resistivity as a function of the temperature for two NbN films deposited at different working-pressures. The black curve corresponds to a cubic phase δ -NbN (3.75 mTorr, non-metallic behavior) while the gray curve corresponds to an hexagonal δ' -NbN structure (0.75 mTorr, metallic behavior).

2.1.3.3 Homogeneity through the film thickness (NbN)

The comparison of the NbN films deposited at high and at room substrate temperatures is worth considering, especially because they behave differently as a function of the film thickness. The figure 2.7 summarizes the evolution of the critical temperature at mid-resistance T_c (a), the normal state resistivity ρ_n (b) and the residual resistivity ratio RRR (c) as a function of the film thickness. The open and full symbols correspond to the films deposited at room temperature and at 600°C, respectively. The full and dashed curves are guides for the eyes. The boundaries of the normal-to-superconductor transition (at 10 % onset and 90 % offset of the transition) are marked by the lower and upper bars in the part (a). All the films deposited in the room-temperature process have been obtained with the optimum nitrogen partial pressure $p_{N_2} = 11$ %.

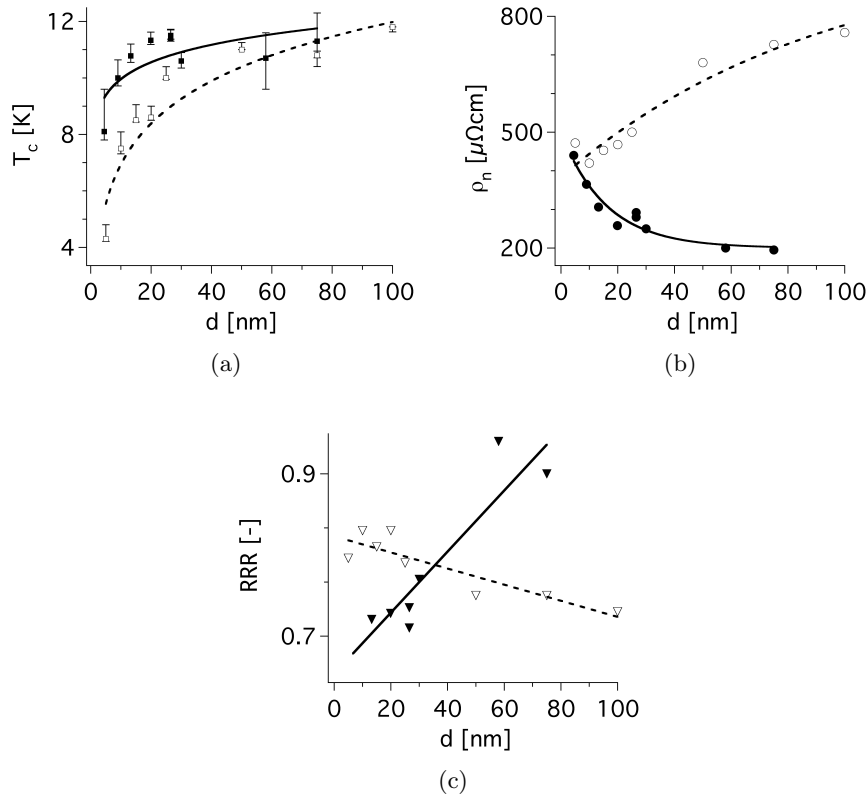


Figure 2.7: Evolution of the critical temperature T_c (a), the normal state resistivity ρ_n (b) and the residual resistivity ratio RRR (c) as a function of the NbN film thickness. The open and full symbols correspond to the films deposited at room temperature and at 600°C, respectively. The curves are guides for the eyes.

The usually reported evolutions of T_c , ρ_n and RRR are observed for the films deposited at 600°C [132, 133, 142]. One sees that the critical temperature is almost constant down to 20 nm and gradually decreases with a further reduction of the thickness. This reduction is accompanied by an increase of the normal state resistivity which appears to be, in the whole thickness range, around two times larger than for other reported films (e.g. in [142]). The associated reduction of the RRR indicates that the grain boundaries are probably more and more disordered. A relevant parameter has to be considered to explain the degradation of T_c and ρ_n : the homogeneity of the chemical composition through the film thickness. Indeed, it is well-known that the achievement of stoichiometric NbN thin films is far from being easy. The main explanation for a slight variation of the niobium-to-nitrogen ratio is the formation of a compound layer on the target surface, a phenomenon which is called the target poisoning [149]. As the properties of the compound layer significantly differ from the properties of the original material, the target poisoning may noticeably impact the thin film composition as well as the sputtering rate. Another possible origin for the varying composition is a change of the reactive sputtering conditions once the shutter is opened above the target⁴. The varying composition through the thickness has been highlighted by high-quality images obtained with a scanning transmission electron microscope (L. Largeau, LPN/CNRS). They have revealed that the first five nanometers contain less nitrogen atoms or, more precisely, a larger amount of nitrogen vacancies. This is illustrated in the figure 2.8. The left image was obtained in the High - Angle Annular Dark Field mode (HAADF-STEM), which is highly sensitive to the variations of the atomic number (Z-contrast images). The brighter region of the NbN film close to the thin oxide layer that covers the Si substrate indicates an impoverishment of the lighter atom ($Z_N = 7$) with respect to the heaviest one ($Z_{Nb} = 41$). Such contrast does not appear in the Bright - Field mode (BF-STEM), as illustrates with the right-side image. This non-homogeneous composition of the NbN film on the first layers of course plays a significant role in the degradation of the critical temperature.

The impact of the film structure and of the niobium content on the electrical properties is more apparent if one looks at the properties of the films deposited at room temperature. Indeed, for $d \geq 50$ nm, one sees that ρ_n is 3-4 times larger than for the films deposited at high temperatures, i.e., around 10 times higher than for the least resistive NbN films

4. Once the shutter is opened during a room-temperature process, the nitrogen flow has to be slightly adjust (a few percents) in order to maintain a constant sputtering voltage.

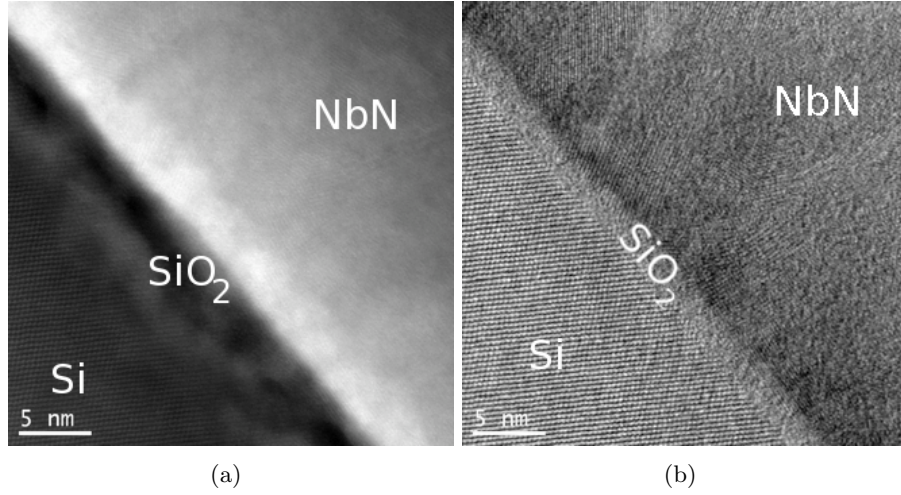


Figure 2.8: Scanning transmission electron microscopy images of a NbN thin film deposited at 600°C. (a) High - Angle Annular Dark Field mode image. The silicon substrate is covered by a thin amorphous oxide layer (SiO_2) on top of which was deposited the NbN film. The brighter area of the NbN film reveals a larger density of nitrogen vacancies. (b) Bright - Field mode image (L. Largeau, LPN/CNRS).

reported in the literature. This illustrates that, in the ambient temperature process, there is a large amount of voids at the grains boundaries which strongly impact the electrical coupling between the grains. But the most surprising features of the room temperature deposition are the reduction (resp. increase) of the normal state resistivity (resp. residual resistivity ratio) observed when the thickness is reduced. These evolutions are unusual and find no explanation based on a sudden evolution of the film structure. Once more, the most convincing explanation is that the chemical composition of the film is non-homogeneous, i.e., the niobium-to-nitrogen ratio is larger at least on the first tens of nm. Even though the film structure still consists of grains separated by voids, the intragrain resistivity is lowered due to the presence of a larger fraction of Nb atoms. This is consistent with the abrupt decrease of the critical temperature which arises below $d \approx 30$ nm, whereas the decrease arises for $d \leq 10$ nm for the films deposited at 600°C.

2.1.3.4 Summary

The thin films have been obtained by dc-magnetron sputtering deposition. All the films have a pronounced granular morphology which results from the low temperature of the substrate during the film growth.

The Nb thin films have an electrical metallic behavior with a sharp normal-to-superconducting transition at low temperatures. By contrast, the NbN films present a non-metallic behavior ($RRR < 1$) due to the disordered and non-metallic barriers that form between the grains. The quality of these intergrain regions is noticeably impacted by the substrate temperature and by the working pressure. Here, the room temperature process results in a porous structure which is characterized by a large voids density at the grain boundaries, a small mass density and a huge normal state resistivity. In addition, an inhomogeneous density of nitrogen vacancies through the film thickness is thought to explain the unusual thickness dependence of the normal state resistivity observed on the NbN films deposited at ambient temperature.

2.2 Experimental details

2.2.1 Nanofabrication

The samples have been fabricated by standard electron beam lithography at the LPN/CNRS laboratory (Damien Lucot). The main stage of the process lies in the definition of the desired pattern onto the thin film. This is realized with the deposition of a temporary mask on top of the regions of the film which have to be protected during the subsequent etching step. A schematic view of this first stage of the process is depicted in Fig. 2.9. The as-grown film (i) is covered by a thin polymethyl methacrylate (PMMA) layer (ii). Some regions of this electro-sensitive resist are exposed to an electron beam (iii) using Gaussian Beam Lithography Systems (JEOL JBX5D2U or VISTEC EBPG 5000 plus). The minimum diameter of the beam spot is less than 10 nm, which corresponds to ultimate designs in the 20 - 30 nm range⁵. The resist is then developed by a mixture of isopropyl alcohol and of methyl isobutyl ketone (MIBK). Only the non-exposed regions of the PMMA remain after the developing step, i.e., the PMMA is a positive resist (iv). An Al_2O_3 layer (thickness $\sim 30 - 40$ nm) is then deposited on top of the resist in order to fill the regions where the resist has been removed (v). The metallization is followed by the so-called lift-off during which the non-exposed resist is taken out. Doing so, the desired pattern is transferred onto the film through the remaining alumina layer (vi). After the lift-off process, the non-required parts of the film are withdrawn by a reactive ion etching (RIE) process. The high-energy ions of the reactive

5. For additional information, see the website of the Laboratoire de Photonique et de Nanostructures - <http://www.lpn.cnrs.fr/en/Commun/presentation.php>

SF_6 plasma attack the unprotected film surface and react with it. This process allows one to obtain very steep edges. After the etching process, the Al_2O_3 mask can be removed using caustic soda.

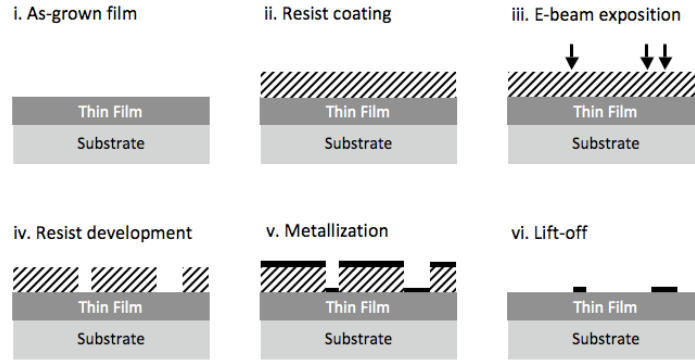


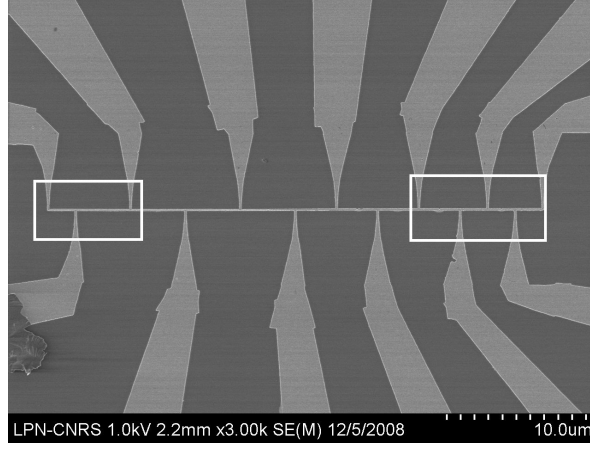
Figure 2.9: Schematic description of the lift-off process used to define the desired pattern on top of the as-grown thin film. The pattern is defined by a thin Al_2O_3 layer used to protect the film during the subsequent etching stage of the process.

The resolution of the nanostructures may be impacted by the etching process. Indeed, the weakest regions of the films (the grain boundaries) are probably more easily damaged during the chemical etching, which may result in small modulations at the edges of the nanostructures with a typical size given by the mean grain size of the thin film. However, the main limitation is the roughness of the alumina mask that depends on the resolution of the PMMA resist. Indeed, the latter has a resolution of 10 nm with the room-temperature development technique used here (the mask roughness is not significantly impacted by the granularity of the alumina layer which is composed of very small grains, i.e., less than 5 nm in diameter). As discussed in the section 3.3.1 (page 125), the resolution of the PMMA has a clear impact on the electrical behavior of the measured nanostructures. One way to improve the ultimate resolution would be to use a negative resist, e.g., the hydrogen silsesquioxane (or HSQ) whose resolution amounts to ~ 5 nm.

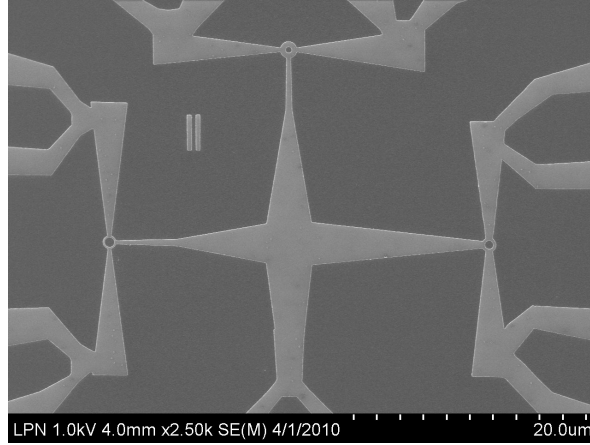
2.2.2 Multi-probe configuration

The figure 2.10 shows two multi-probe configurations that have been fabricated by standard e-beam lithography. The structures of interest are the parallelepiped strips horizontally positioned at middle-height of the image (a), and the loops on the image (b). Each structure (strip

or loop) is connected to much larger contacts that are made with the same superconducting material. The contacts are much wider than the studied structure in order to avoid their undesired current-biased transition to the normal state. The contacts finish by macroscopic pads used for the wirebonding process (not shown). In addition to the ability to perform standard four-probe measurements, these multi-probe configurations allow one to apply the bias and to measure the resulting voltage simultaneously at different locations along the sample.



(a) Strip



(b) Loop

Figure 2.10: SEM images of the nanofabricated multi-probe configurations. (a) A series of parallelepiped segments of different lengths. The framed areas are enlarged in Fig. 2.11. (b) Three loops with the same mean radius and a varying branch width.

2.2.2.1 Submicrometer-wide strip

The lateral dimensions of the parallelepiped strips ranging well below the micrometer, the term submicrometer(-wide) strip is preferred throughout the manuscript. Different segment lengths were investigated from $1\text{ }\mu\text{m}$ to $12\text{ }\mu\text{m}$ but their properties are found to depend slightly on this parameter. As extensively described in the chapters 3 and 4, the relevant geometrical dimensions are the width and the thickness. For this reason, a constriction was intentionally placed at the middle of some segments to study the impact of a strong cross-section reduction (or weak link) on the electrical transport properties of the strips. The typical shape of a weak-link is shown in Fig. 2.11(c). The constriction width is around half the full width of the strip.

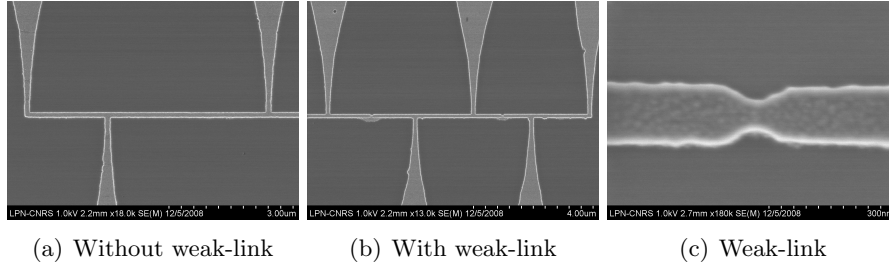


Figure 2.11: Enlargements of the rectangular parts of Fig. 2.10(a) illustrating the shape of the strips with (a) and without (b) a weak-link. A further enlargement of an intentionally placed half-wide constriction is shown (c).

The table 2.3 summarizes the main geometrical characteristics of the submicrometer strips whose electrical transport behavior will be discussed in chapters 3 and 4. The prefix **S** indicates that the samples belong to a strip multi-probe configuration. One has to distinguish between two series of samples. All the samples of the first series have been fabricated with films sputtered at room temperature so that their normal state resistivity noticeably depends on the thickness. As the normal state resistivity has a significant influence on the properties discussed in chapter 3, the sample thickness is indicated by the number that follows the type of material (the smaller is the thickness, the smaller is the normal state resistivity). For a fixed thickness, the samples are then ordered with respect to their increasing width and/or length. The strips of the second series deserve to be described in more details. They have been specifically designed for the study of the chapter 4 and belong to the same multi-probe configuration. The main advantage is that they have the same structural and geometrical characteristics, except for their width which

Table 2.3: Characteristics of the NbN and Nb submicrometer-wide strips : substrate, length L, thickness d and width W. The width of the intentionally placed constriction W_{WL} is specified (if present). Al_2O_3 refers to C-plane oriented sapphire wafers and SiO_2/Si means silicon wafers covered by a thermally grown amorphous silicon oxide.

	Sample	Substrate	L [μm]	d [nm]	W [nm]	W_{WL} [nm]
Chap. 3-4	SNbN-20-1	Al_2O_3	3	20	70	35
	SNbN-20-2	Al_2O_3	4	20	70	-
	SNbN-20-3	Al_2O_3	2	20	110	60
	SNbN-20-4	Al_2O_3	2	20	120	-
	SNbN-20-5	Al_2O_3	3	20	110	70
	SNbN-20-6	Al_2O_3	3	20	100	-
	SNbN-25-1	Al_2O_3	2	25	120	70
	SNbN-25-2	Al_2O_3	3	25	130	-
	SNbN-75-1	SiO_2/Si	3	75	115	-
	SNbN-75-2	SiO_2/Si	3	75	115	70
	SNbN-75-3	SiO_2/Si	10	75	125	75
	SNbN-75-4	SiO_2/Si	1	75	150	-
	SNbN-75-5	SiO_2/Si	4	75	160	85
	SNbN-75-6	SiO_2/Si	12	75	160	85
	SNb-75-1	SiO_2/Si	3	75	130	-
	SNb-75-2	SiO_2/Si	3	75	130	-
	SNb-75-3	SiO_2/Si	3	75	240	-
Chap. 4	SNbN-W1	SiO_2/Si	4	26	40 (20)	-
	SNbN-W2	SiO_2/Si	4	26	55 (45)	-
	SNbN-W3	SiO_2/Si	4	26	85 (80)	-
	SNbN-W4	SiO_2/Si	4	26	100	-

increases from left to right on the SEM image of Fig. 2.12(a). The nominal width of the narrowest strip (SNbN-W1) is very small and amounts to ~ 40 nm. As detailed in chapter 4, the width determined by SEM measurements (see the figures 2.12(b), 2.12(c) and 2.12(d) that correspond to the strips SNbN-W1, SNbN-W2 and SNbN-W3, respectively) overestimates the (smallest) sample width which impacts the departure from the perfect superconducting state. The latter is indicated between brackets. As a final comment, let's notice that the strips of the second series have been fabricated based on a thin film sputtered at a substrate temperature of $\sim 600^\circ\text{C}$ so that their normal state resistivity is smaller than the least resistive sample of the first series.

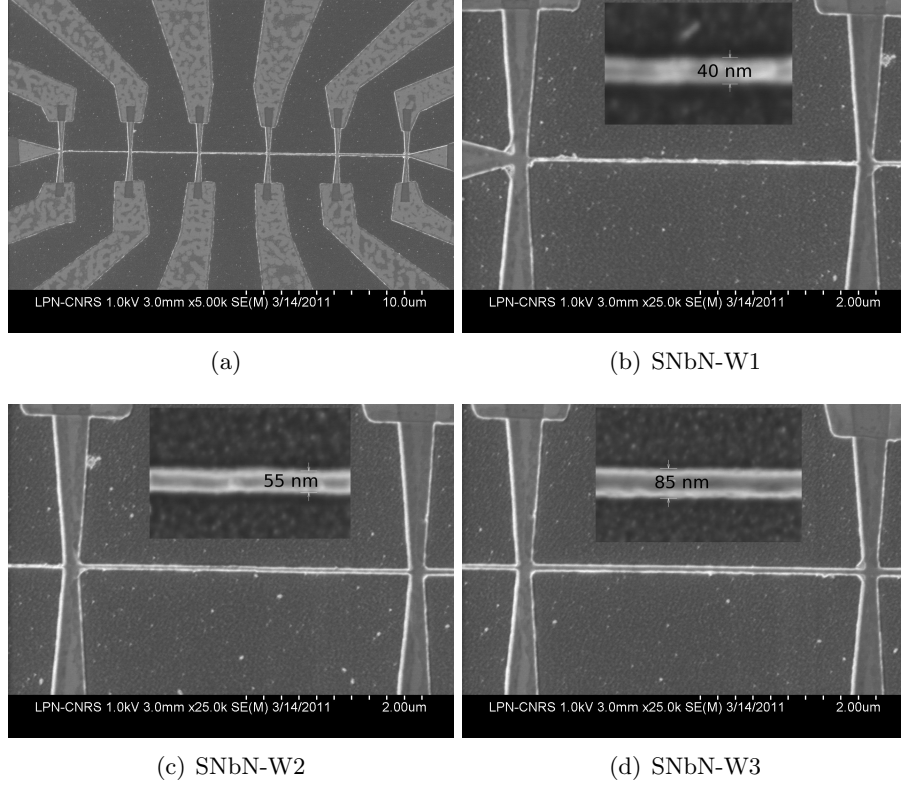


Figure 2.12: SEM images of the strips SNbN-W1-2-3-4 (a) whose width increases from left to right. This is illustrated by the other SEM images that correspond to the strips SNbN-W1 (b), SNbN-W2 (c) and SNbN-W3 (d). The nominal width of the narrowest strip (SNbN-W1) amounts to ~ 40 nm.

2.2.2.2 Loops

Several multi-probe loops configurations were designed, each of them containing three loops. The loops of a single configuration have in common one geometrical parameter, which makes the comparison of the electrical properties easier. Among all the fabricated loops, only two thirds were measurable or were perfectly superconducting at low temperatures. The figure 2.13 displays six Nb loops whose properties will be extensively discussed in the chapter 4. The samples whose name contains the same number belong to the same multi-probe configuration, i.e., they have in common a geometrical parameter. The first configuration (LNb-1) has a constant outer radius $r_o = 950$ nm, the second one (LNb-2) has a constant mean radius $r_M = r_o - r_i \approx 500$ nm (where r_i is the inner radius) and the third one (LNb-3) has a constant branch width $W = 150$

nm. The main geometrical characteristics are summarized in table 2.4. The prefix **L** indicates that the samples belong to a loop multi-probe configuration. In the current-biased measurements, it is worth noting that the transport current density is maximum either in a branch of the loop or in the area close to the contact, depending on their relative size. The loops LNb-1a and LNb-3a have a contact width larger than twice the branch width so that the weak region (i.e., the region where the critical current density is first reached) is located in one of the two branches. On the contrary, the other loops have a contact width smaller than twice the branch width so that the critical area is close to the contact. The critical width is of course a relevant parameter. The values that have to be taken into account are underlined in table 2.4.

Table 2.4: Geometrical characteristics of the Nb loops : thickness d , inner radius r_i , outer radius r_o , mean radius $r_m = \frac{r_o+r_i}{2}$, branch width $r_o - r_i$ and contact width. The samples whose name contains the same number belong to the same multi-probe configuration. The critical width, i.e., the width of the cross-section where the critical current density should be first reached according to the SEM images, is underlined.

	Sample	d [nm]	r_i [nm]	r_o [nm]	r_M [nm]	$r_o - r_i$ [nm]	Contact width [nm]
Chap. 4	LNb-1a	75	850	950	900	<u>100</u>	280
	LNb-1b	75	700	950	825	250	<u>420</u>
	LNb-1c	75	500	950	725	450	<u>650</u>
	LNb-2a	20	420	580	500	160	<u>300</u>
	LNb-2b	20	255	715	485	460	<u>400</u>
	LNb-3a	20	215	365	290	<u>150</u>	380

Each loop is connected to three right-angled contacts. This asymmetric configuration was initially designed to study the impact of the contacts position on the fluxoid quantization effect. However, most of the measurements reported in this manuscript focus on different properties and only the symmetrical contacts have been used. Furthermore, the contacts split in two at a certain distance from the loop, so that only pseudo four-probe measurements can be performed : the voltage that may develop along the triangular parts of the contacts is always integrated in the total measured one. Nevertheless, as most of the measurements concern the departure from the perfect superconducting state under an increasing transport current (see the chapter 4), the contacts remain in the zero resistance state far from the loop and do not affect the experience. This pseudo four-probe configuration has thus a negligible impact on the experimental results.

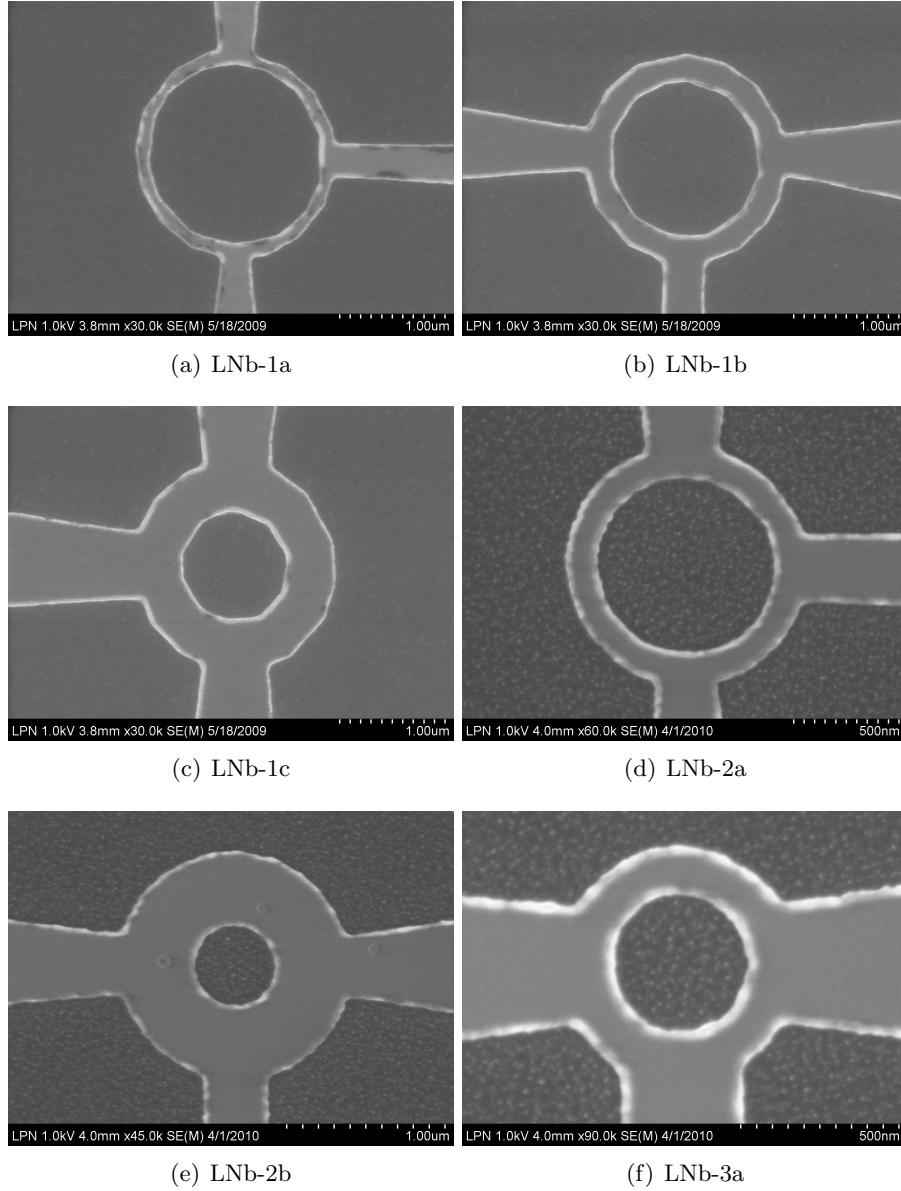


Figure 2.13: SEM images of the studied niobium loops. All the loops that contain the same digit in their name belong to the same multi-probe configuration.

2.2.3 Cooling conditions

A large part of this manuscript being devoted to the heat exchange between the superconducting structures and their surroundings, the cooling conditions of the substrate/sample configuration deserve to be described in details. The cryocooler system is divided into two parts : the cryogen-free (or dry) system and the variable temperature insert (VTI). The former allows one to cool the superconducting magnet down to a base temperature of less than 3 K, while the latter is used to control the sample temperature in the range 1.5 K - 300 K.

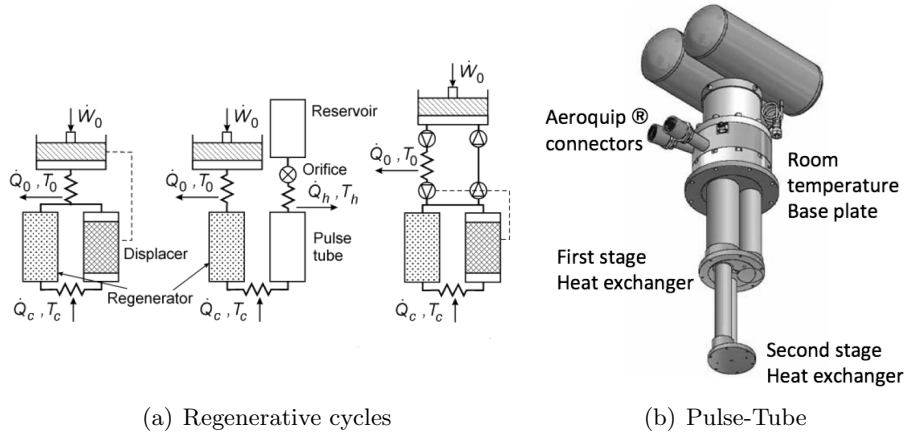


Figure 2.14: (a) From [150] : Schematics of three common types of regenerative cryocoolers. From left to right : Stirling, pulse-tube and Gifford-Mc Mahon. The cryocooler used for the measurements belongs to the pulse-tube type. (b) From Cryomech, Inc. : Representative schematic of the PT415 cold head.

Cryogen-Free Magnet The cooling of the magnet is achieved using a Gifford-Mac Mahon-type pulse-tube refrigerator (GM-PTR). Like the Stirling or Gifford-Mac Mahon refrigerators, the GM-PTR is a regenerative cryocooler that operates with oscillating flows and oscillating pressures using a high-pressure helium as the working fluid (see [150] for a review of recent improvements in the field). The figure 2.14(a) illustrates the main components of the three more common types of regenerative cryocoolers, namely the Stirling, the pulse-tube and the Gifford-McMahon (from left to right). In the pulse-tube cryocooler used here, the oscillating pressure (1 Hz) is achieved by a compressor connected to the cold end side of the pulse-tube by two flexible stainless steel lines. This is realized thank to the Aeroquip ® connectors depicted at the figure 2.14(b). The flexible lines carry the compressed helium from the compressor to the cold head and carry the low-pressure helium back

(the helium flex lines are filled with 99.999% pure helium at a pressure of ~ 16 bar at rest). Contrary to the Stirling and the Gifford-Mac Mahon refrigerators that use a displacer to separate the cooling effect (expansion) from the heating one (compression), the in-phase motion of the helium gas with the oscillating pressure is ensured in the PTR by the use of a warm-end orifice connected to a reservoir [151]. The latter has to be sufficiently voluminous in order to maintain a constant pressure inside it during the oscillating flow. This unique feature of the pulse tubes results in a cold head that has no moving part at cryogenic temperatures, so that the reliability of the cryocooler is greatly enhanced and the vibrations and the cost are lowered. One has to note that the pressure varies gradually into the whole system so that the helium velocities are quite low. The name *pulse-tube* appears quite inadequate because there is no pulse in the system.

Once the compressor is switched on, the pulse-tube cold head progressively cools down. The main components of the Cryogen-Free Magnet system are the heat exchangers of the first and second stages [see the figure 2.14(b)], the magnet and the helium pot. The nominal base temperatures of these four components are summarized in the table 2.5.

1 st stage	34.5 K
2 nd stage	3 K
Magnet	3.1 K
He Pot	2.9 K

Table 2.5: Nominal base temperatures of the Cryogen-Free Magnet components, namely the first and second stages, the magnet and the He Pot.

Variable Temperature Insert The high-pressure working-fluid of the Cryogen-Free Magnet system is uncoupled from the pure helium gas used to cool down the VTI. The figure 2.15 illustrates the associated helium flow circuit. A scroll pump ensures the circulation of the helium, whose temperature is reduced by conduction with the cold parts of the Cryogen-Free Magnet (1st stage, 2nd stage, He Pot). The helium reaches the base part of the VTI where is located the sample, allowing to benefit from variable temperature and magnetic field environments. The surface side of the sample/substrate configuration is cooled in a gaseous atmosphere with a pressure of a few mbar. The base temperature of the VTI is 1.5 K and the temperature is regulated by a Temperature Controller (LakeShore - Model 340) using a calibrated Cernox resistor. The temperature at the extremity of the measurement probe slowly oscillates with a long-term peak-to-peak temperature stability of 5 - 10 mK.

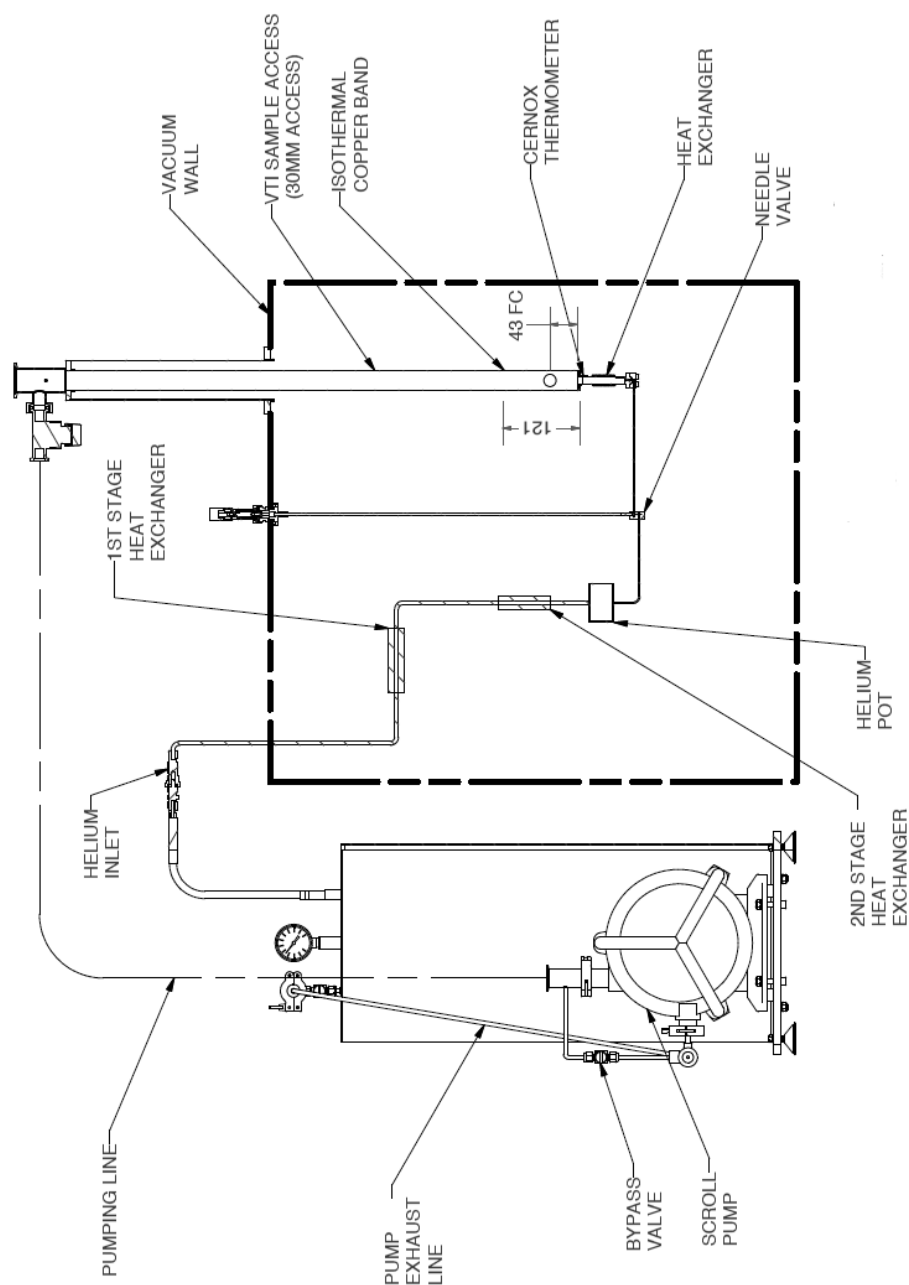


Figure 2.15: From Cryogenic Limited : Schematic of the helium flow circuit (VTI) which allows one to control the sample temperature in the range 1.5 K - 300 K. The dimensions are indicated in millimeters.

2.2.4 Measurements setup

After the lithographic process, the sample substrate is glued onto a ceramic Leadless Chip Carrier (LCC). The sample pads are then electrically contacted to the LCC holder through a wire-bonding process using wires made of aluminum. As illustrated in the figure 2.16, the sample holder is mounted on a dedicated 32-pin socket positioned at one extremity of the measurement probe.

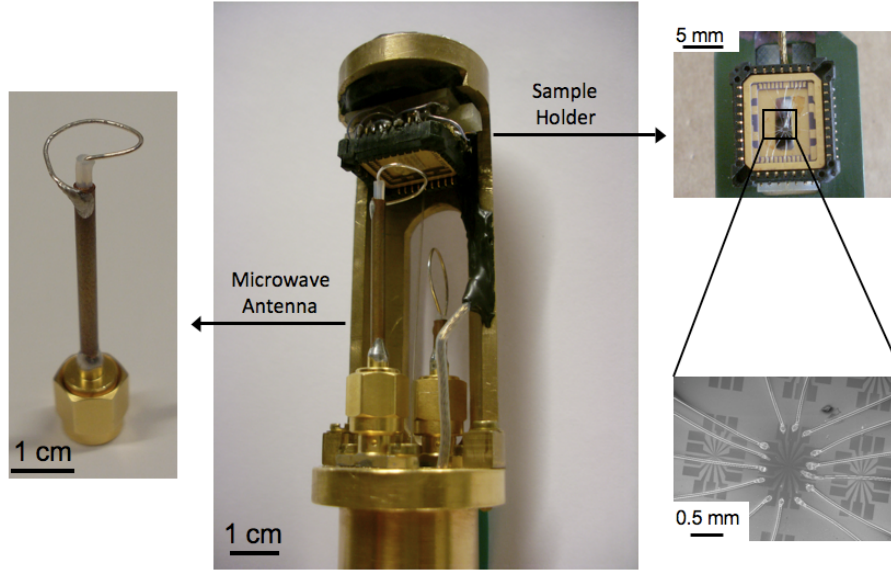


Figure 2.16: Picture of the extremity of the measurement probe. The sample (holder) lies at the top of the central image and is located at approximately 5 mm from the home-made copper-loop antenna used to emit microwave radiations inside the measurement chamber.

The electrical transport measurements are performed in direct conditions, either by applying a current-bias or a voltage-bias. In both modes, the bias is applied directly to the measured sample and the resulting voltage is probed using the nearest contacts on each side. This prevents any spurious transition of adjacent segment whose critical current should be reduced. Studying the evolution of the sample resistance with respect to the temperature or the magnetic field requires that a small dc current-bias is applied (compared to the critical one). By contrast, the voltage-current characteristics are obtained by sweeping the current - for a V-I curve - or the voltage - for a I-V curve - in order to exceed the critical current. Most of the time, the dc bias is swept down to zero after the sample resistance has reached its normal state value so that the overall

electrical characteristic is obtained. The current bias is applied using programmable Keithley devices, either a K2400 sourcemeter or a K224 current source. The maximum current needed to measure the samples varies between $100\ \mu\text{A}$ (NbN) and a few mA (Nb). The associated programming resolution amounts to 5 nA and 500 nA for the K2400, and to 50 nA and $5\ \mu\text{A}$ for the K224. The voltage at the extremities of the sample is measured using digital multimeters (Keithley 2000 or Keithley 196), with a resolution of $1\ \mu\text{V}$ and $0.1\ \mu\text{V}$, respectively. No special attention has been required to eliminate the parasitic voltages generated by the varying temperature of the measurement lines. Indeed, these thermoelectric voltages are much smaller than the measured signals. The samples are connected to the direct current sources and to the voltmeters through measurement lines that are filtered to remove the undesired frequency noise.

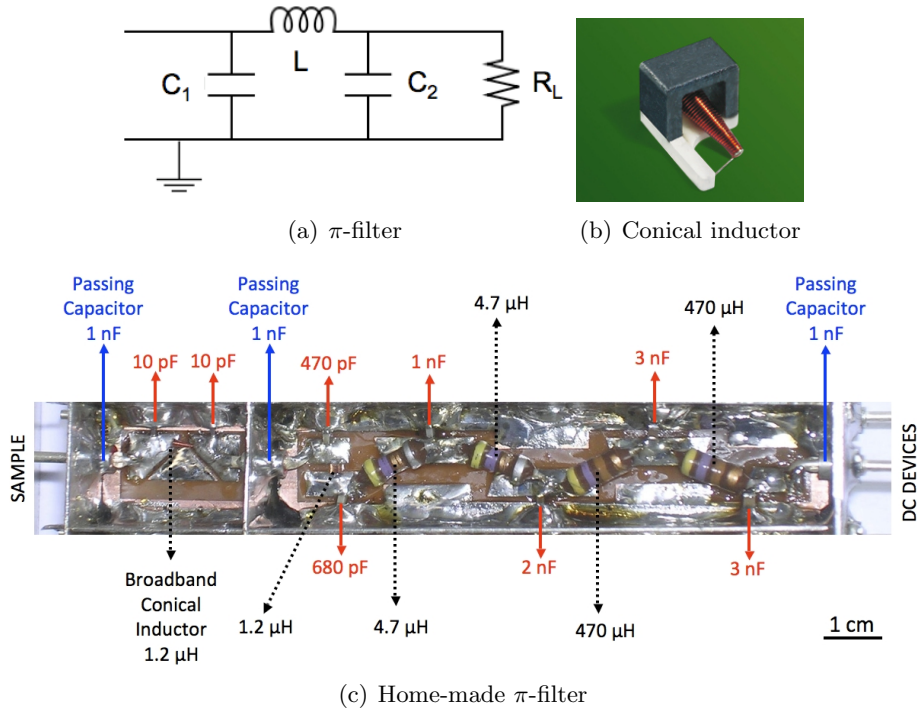


Figure 2.17: (a) Schematic view of a π -filter. C_1 and C_2 are the capacitors, L is the inductor and R_L is the load resistance. (b) Broadband Conical Inductor by Coilcraft ® operating up to 40 GHz. The length of the conical central part is around 3 mm. (c) Picture of the home-made « high-frequency multi-stage π -filter » which is composed of a succession of capacitors and inductors, including some *Surface Mount Technology* components.

A typical π -filter is schematized in Fig. 2.17(a), where C_1 and C_2 are the capacitors, L the inductor and R_L the load resistance (here, the sample). The basic principle is that the capacitors offer low reactance to the frequency component and blocks the direct signal, while the inductor offers a high reactance to the frequency component and allows the direct signal to flow through it. As a result, only the direct bias flows across the resistance load which is significantly less influenced by the frequency signal. The components (inductors and capacitors) of the home-made multi-stage π -filters have been optimized by Sébastien Michotte (IMMC, UCL) to filter out the unwanted signals on a wide frequency-range. Two series of filters have been used. The « low-frequency filter » has a cutoff at ~ 1 kHz ($C_1 = C_2 = 1 \mu\text{F}$, $L = 1$ mH). The second series is illustrated by the picture 2.17(c) which depicts the succession of capacitors and inductors used, including many *Surface Mount Technology* components. Especially due to the Broadband Conical Inductor⁶ [see Fig. 2.17(b)], this « high-frequency filter » is operative up to 40 GHz.

The measurement probe is fitted with a semirigid coaxial cable that allows one to emit radiations inside the experimental chamber (see the left part of Fig. 2.16). The electromagnetic radiation was applied through a home-made copper-loop antenna connected to a microwave generator (Anritsu 68377C Microwave Synthesizer or Wiltron 6769B Sweep Generator). The antenna - around 9 mm in diameter - was located at approximately 5 mm from the sample holder. The frequency was swept between 10 MHz and 20 GHz. One way to characterize the effectiveness of power delivery through the line transmission is to measure the power which is reflected back to the source. This allows one to calculate the return loss (RL) which is defined by

$$RL = 10 \log_{10} \left(\frac{P_{IN}}{P_R} \right), \quad (2.1)$$

where P_{IN} is the incident power and P_R is the power reflected back to the source. The return loss is a measure (in dB) of the mismatch between the transmission line and the antenna : the line and the antenna are well matched if the return loss is high. The figure 2.18 illustrates a typical evolution of the return loss of a home-made antenna as a function of the applied frequency. The power reflected to the source has been measured at the input of the coaxial cable connected to the antenna with an incident power of -10 dBm (equivalently of 0.1 mW). In order to benefit from the largest microwave powers inside the experimental

6. The Broadband Conical Inductor (BCS Series) by Coilcraft ® operates as a series of narrow-band inductors over a wide frequency range, i.e., from 10 MHz to 40 GHz.

chamber, the measurements of the section 3.4 (page 133) have been usually (not always) performed at the well-defined frequencies at which the return loss is high.

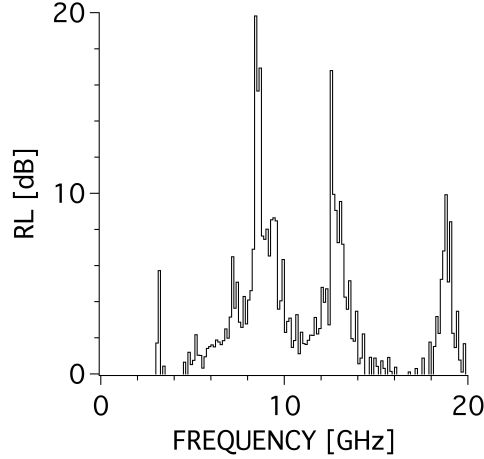


Figure 2.18: Evolution of the return loss of a home-made antenna as a function of the microwave frequency. The incident power is -10 dBm. The mismatch between the transmission line and the antenna is the lowest when the return loss is high.

The experimental setup used to apply the voltage-bias is depicted at the figure 2.19. It consists in a room temperature resistor $R_V = 1 \, \Omega$ in parallel with a dc current source (I_V). The voltage that develops at the extremities of the sample is measured with a voltmeter (V_0) and the current that flows through the sample is determined by measuring the voltage across a small resistor R_I in series with the sample (R_I varies between $10 \, \Omega$ and $100 \, \Omega$, depending on the desired accuracy for the determination of the current intensity). The load line of the voltage source is affected by all the resistances in series with the measured sample : the π -filter resistances ($\sim 2 \times 46 \, \Omega$), the resistance of the probe leads ($\sim 2 \times 60 \, \Omega$) and the resistance used to measure the current that flows through the sample (R_I). The resulting load line is flat enough to obtain detailed voltage-current characteristics of samples with relatively high normal-state resistance, which is the case for the NbN nanostructures.

The high-field solenoid superconducting magnet is oriented parallel with respect to the VTI sample access so that the magnetic field is perpendicular to the measured samples. Magnetic field as high as $\pm 8.2 \, \text{T}$ can be reached, although the measurements were performed up to maximum $7 \, \text{T}$. The smallest magnetic field increment used in the measurements amounts to $10 \, \mu\text{T}$.

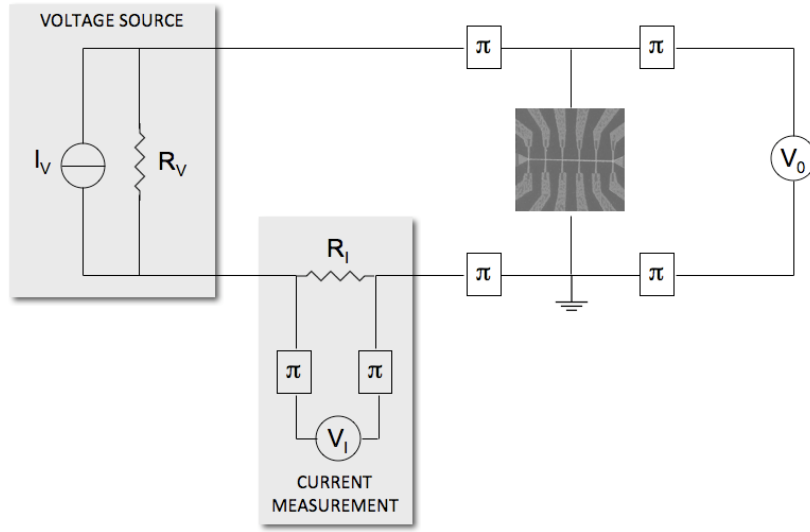


Figure 2.19: Schematic of the experimental setup used to apply the voltage-bias. The dc current source I_V and the room temperature resistor R_V in parallel constitute the voltage source. The voltage that develops across the resistance R_I allows one to determine the current that flows throughout the sample. The voltmeter V_0 is used to measure the voltage that develops at the extremities of the sample. The measurement lines are filtered by multi-stage π -filters that are referred by the symbol π .

2.3 Nanostructures characterization

This section is devoted to the main electrical properties that can be extracted from the evolution of the resistance with respect to both the temperature (R-T curves) and the magnetic field (R-B curves). The superconductor-to-normal transition has been studied in each case by applying a low dc current compared to the critical one⁷. This allows one to avoid any undesired variation of the resistance that may result from, e.g., the dissipation by Joule heating. The temperature and the magnetic field have been slowly varied. In the former case, this limits the possible temperature shift between the Cernox resistance and the studied sample. All the parameters discussed in this section are summarized in the tables 2.6 (page 103) and 2.7 (page 104).

⁷. Depending on the sample cross-section area, the applied current varies between $1 \mu\text{A}$ and $10 \mu\text{A}$.

Critical temperature

The critical temperature T_c is determined at half the normal state value of resistance. The largest critical temperature of the NbN nanostructured samples was measured on the thickest strips (≈ 12 K for $d = 75$ nm). As the thickness is reduced, the critical temperature gradually decreases down to around 9 K for a thickness of 20 nm⁸. The transition width is defined at 10% and 90% of the normal state value of resistance and amounts to around 1 K for all the studied samples. Although the critical temperature of the NbN submicrometer strips is close to the value measured on the as-grown films [see Fig. 2.7(a), page 75], T_c appears sensitive to the cross-section area. The reduction of the critical temperature when the width is reduced is illustrated in Fig. 2.20(a) with the normalized resistance R/R_{15K} vs. temperature curve of three NbN samples that have the same thickness but a different width [from left to right, the curves correspond to the samples SNbN-W1 ($W \sim 40$ nm), SNbN-W2 ($W \sim 55$ nm) and SNbN-W3 ($W \sim 85$ nm)].

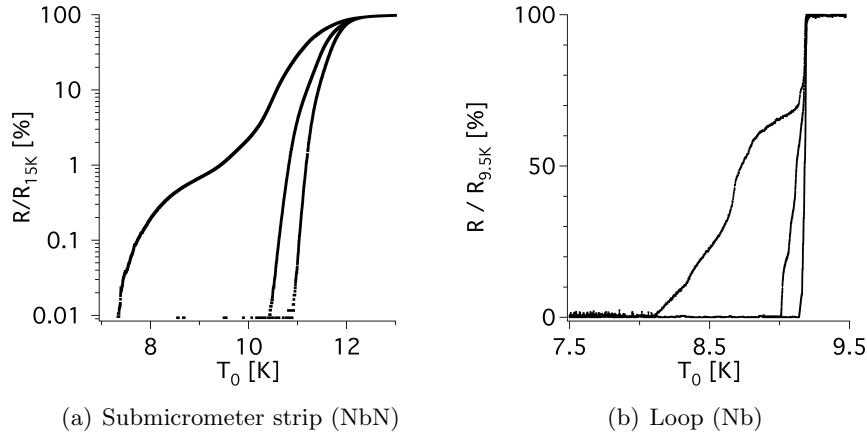


Figure 2.20: Normalized resistance vs. temperature curves of superconducting nanostructures that reveal a decrease of the critical temperature when the sample width is reduced. (a) NbN submicrometer strips SNbN-W1, SNbN-W2 and SNbN-W3 (from left to right). (b) Niobium loops LNb-1a, LNb-1b and LNb-1c (from left to right).

A similar behavior is observed on the Nb nanostructured samples. The critical temperature for the wider parts ($W \gtrsim 300$ nm) amounts to 9.17 K and 8 K for the 75 nm-thick and the 20 nm-thick samples, respectively. T_c is also found to decrease with a further reduction of the

8. As discussed at the table 1.1 (page 6), the critical temperature of a bulk NbN amounts to 16 K.

sample width. Such variation is illustrated in Fig. 2.20(b) for the loops LNb-1a-b-c (normalized resistance $R/R_{9.5K}$). Apart from the contacts that behave similarly for these three samples, multi-stages transitions arise when the sample width is continuously varied. One observes that the narrowest part of LNb-1c (branch width : 450 nm) goes to the superconducting state at the same temperature as the contacts. By contrast, the spreading of the transition for LNb-1a and LNb-1b is consistent with the reduced width of their branches with respect to the contacts. These multi-stage transitions of the temperature-driven transition from the normal state to the superconducting state are well known [152]. They reflect the width dependence of the critical temperature of samples whose contacts are made from the same material than the studied sample.

Residual resistivity

The residual resistivity of the nanostructured samples increases compared to the as-grown films. For the NbN submicrometer strips, the normal state resistivity ρ_{15K} decreases monotonically between 1300 $\mu\Omega\text{cm}$ and 650 $\mu\Omega\text{cm}$ as the thickness is reduced from 75 nm down to 20 nm. By contrast, the residual resistivity of their corresponding thin films amounts to $\sim 700 \mu\Omega\text{cm}$ and $\sim 500 \mu\Omega\text{cm}$. Similarly, the normal state resistivity $\rho_{9.5K}$ of the Nb samples (either strips or loops) is around 10 - 12 $\mu\Omega\text{cm}$ while values around 3 - 7 $\mu\Omega\text{cm}$ are measured for the films. Such variations are thought to result from the granular structure whose impact becomes more and more significant as the sample width is reduced down to a few times the mean grain size.

Furthermore, the residual resistivity ratio RRR slightly varies from 0.82 for the thinnest NbN submicrometer strips down to 0.73 for the thickest ones. These values are very close to the one obtained on the thin films. This implies that the nonmetallic barrier presents at the grain boundary is not affected by the e-beam process. In addition, the residual resistivity ratio of the niobium samples is reduced to $\text{RRR} \sim 4$ for $d = 75 \text{ nm}$ and to $\text{RRR} \sim 2.6$ for $d = 20 \text{ nm}$.

Upper critical field

The upper critical field B_{c2} of dirty type-II superconductors is known to be directly related to the density of states at the Fermi level N_0 and to the electronic mean free path ℓ . As both N_0 and ℓ decrease with the grain size, the film morphology impacts B_{c2} which is reduced (resp. enhanced) by a decrease of N_0 (resp. of ℓ) [153]. For a one-gap

dirty superconductor⁹, the Werthamer-Helfand-Hohenberg expression [154] relates the zero-temperature critical field $B_{c2}(0)$, the slope near the critical temperature $B'_{c2} = |\frac{dB_{c2}}{dT}|_{T_c}$, and the normal state resistivity ρ_n . This relation is given by Eq. 2.2,

$$\begin{aligned} B_{c2}(0) &= 0.69 T_c B'_{c2}, \\ &= 0.69 T_c \frac{4ek_B}{\pi} N_0 \rho_n, \end{aligned} \quad (2.2)$$

where N_0 is the density of states at the Fermi level, e is the electron charge and k_B is the Boltzmann constant. Given that B_{c2} depends on ρ_n , one expects noticeable variations (i) between the NbN and the Nb samples and (ii) between the NbN samples of different average resistivities.

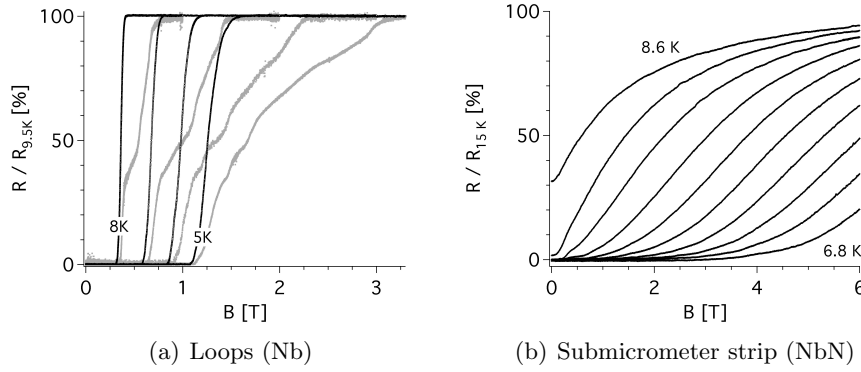


Figure 2.21: Normalized resistance vs. magnetic field curves of superconducting nanostructures. (a) Niobium loops LNb-1a (gray) and LNb-1c (black). From left to right, the temperature varies from 8 K down to 5 K. (b) NbN strip SNbN-20-6. From left to right, the temperature varies from 8.6 K down to 6.8 K.

The field-induced transition of the Nb nanostructures is illustrated in Fig. 2.21(a) for the loops LNb-1a (gray) and LNb-1c (black). Significant differences are observed between both samples, which differ by the width of their narrowest parts¹⁰. The superconductor-to-normal transition of LNb-1c is quite abrupt which reveals that the critical field of the contacts and of the loop branches ($r_o - r_i \approx 450$ nm) are similar. By contrast, the resistance vs. magnetic field curve of LNb-1a ($r_o - r_i \approx 100$ nm) shows a clear multi-stage transition, which reflects the increase

9. The NbN and Nb samples fall in the dirty limit, as discussed at page 100.

10. Let's remind that the loops are measured in a pseudo-four contacts configuration so that the contacts contribute to the measured resistance.

of B_{c2} as the width is reduced. In particular, this field-induced behavior nicely corresponds to the previously reported temperature-induced transition of Fig. 2.20(b). Indeed, it appears that the parts of the contacts that lie far away from the loop (larger critical temperature, smaller upper critical field) contribute to around 30% of the normal state value while the remaining contribution to the total resistance comes from the reduced parts of the contacts and from the loop itself (smaller critical temperature, higher critical field). As both loops have been fabricated with the same Nb thin film (identical mean grain size, similar resistivity), the increase of the upper critical field when the sample width is reduced below $\sim 350 - 400$ nm mainly results from the incomplete Meissner effect which arises when one of the sample dimensions becomes smaller than twice the magnetic penetration depth. This is corroborated by the estimated values of the magnetic penetration depth $\lambda_{\perp} \approx \frac{\lambda^2}{d}$ which amounts to 180 nm, i.e., around half the width below which an increase of B_{c2} is observed¹¹. Furthermore, in order to accurately determine the upper critical field of this pseudo-four contacts structures, each portion of the transition needs to be associated to the appropriate part of the sample. The upper critical field is then defined by the magnetic field value at which the resistance amounts to half the normal state value of the loop resistance (for the loop LNb-1a, this resistance criteria corresponds to 95% of the total normal state resistance).

The resistance vs. magnetic field behavior of the NbN submicrometer strips is also characterized by a significant spreading [see Fig. 2.21(b) - sample SNbN-20-6]. However, as the strips are measured in a four-contact configuration, this broadening is not thought to result from the small width modulations. Among all the parameters that may impact B_{c2} , the variation of the normal state resistivity through the film thickness is probably the major explanation : the first layers may have a reduced upper critical field than the more resistive ones on the top surface. As a result, the determination of B_{c2} is quite tough. In order to associate the upper critical field to the measured (and average) value of ρ_n , B_{c2} is defined for the NbN strips by the magnetic field at which the resistance amounts to 50% of the normal state value.

In addition, for a type-II superconductor, the upper critical field B_{c2} is inversely proportional to $[\xi_{GL}(T)]^2$, where ξ_{GL} is the Ginzburg-Landau coherence length [19]. In the dirty limit, ξ_{GL} has the following temperature dependence near T_c ,

11. See the page 100 for a detailed estimation of $\lambda_{\perp}$.

$$\xi_{GL} = 0.85 \frac{(\xi_0 \ell)^{0.5}}{(1 - T/T_c)^{0.5}}, \quad (2.3)$$

where ξ_0 is the intrinsic coherence length and ℓ is the electron mean free path. Therefore, the upper critical field of granular superconductors in the dirty limit is expected to linearly depend on the temperature near the critical one. This is illustrated in Fig. 2.22 which shows the temperature evolution of B_{c2} of NbN and Nb nanostructures as a function of the reduced temperature T/T_c . The slope of these dependences in the vicinity of T_c allows one to evaluate the upper critical field at zero temperature (see the relation 2.2). The obtained values are summarized in the table 2.6 (page 103). For the Nb samples, B_{c2} ranges for the thickest films between 2 T ($W \sim 450$ nm) and 4.35 T ($W \sim 100$ nm). The thinnest Nb samples have an upper critical field around 2.5 - 2.8 T. These values are of course much larger than for a bulk material ($B_{c2}^{bulk} \approx 0.45$ T [155]) and are consistent with other published data [153]. The upper critical field of the NbN nanostructures is significantly larger. Indeed, it varies between 22 T and 76 T when ρ_n increases from $700 \mu\Omega\text{cm}$ to $1000 \mu\Omega\text{cm}$. These values are much higher than those usually obtained on less resistive NbN thin films which rarely exceed 20 T [156, 157]. In the next section, these values will be discussed in the framework of Eq. 2.2.

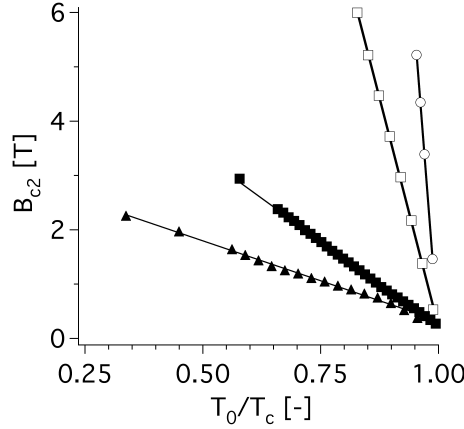


Figure 2.22: Evolution of the upper critical field of NbN and Nb nanostructures with respect to the reduced temperature T_0/T_c . The full and the open symbols correspond to the Nb and the NbN samples, respectively (▲ : SNb-75-3, ■ : LNb-1a, □ : SNbN-20-6, ○ : SNbN-75-4).

Density of states at the Fermi level

The density of states at the Fermi level N_0 is an important parameter because it determines the density of the free-energy difference between the superconducting state and the normal state through the relation $\Delta F = 0.5N_0\Delta^2$, where Δ is the superconducting energy gap. Recent experimental and theoretical works have shown that the density of states of NbN is strongly influenced (among others) by the film thickness [157], the structural phase of the thin films [158] and the nitrogen deficiency [159]. For optimized sputtering conditions, values around 1.6×10^{28} and 2.6×10^{28} states/(m³ eV) are found (the largest values go together with the largest critical temperatures) [156, 157].

From the knowledge of the upper critical field and of the normal state resistivity, N_0 can be determined using Eq. 2.2. N_0 is found to be around 2.5×10^{28} states/(m³ eV) for all the NbN samples. These values being very close to other reported data [156, 157], one can consider that the very large values of the upper critical field of the NbN samples come from their huge normal state resistivities, and not to a noticeable variation of N_0 . The density of states of the Nb samples is found to be larger, with values around 10^{29} states/(m³ eV). However this estimation overestimates the real value which is expected to be similar than for the NbN samples, i.e., $N_0 \approx 2 \times 10^{28}$ [18]. Such disagreement may result from an underestimated residual resistivity and this value has to be considered as a rough estimation. Fortunately, the following discussion only requires to know the product $(N_0\rho_n)$, which is directly - and accurately - given by the slope $B'_{c2} = |\frac{dB_{c2}}{dT}|_{T_c}$ (see Eq. 2.2).

Electronic diffusivity

The electronic diffusivity D_e is of particular interest¹² because it allows one to determine the length associated to any electronic relaxation time τ . The electronic diffusivity of the NbN and the Nb samples is determined with the free-electron relation (or Einstein's relation),

$$D_e = (e^2\rho_n N_0)^{-1}, \quad (2.4)$$

that relates N_0 , ρ_n and D_e . The electronic diffusivity of the NbN samples varies between $0.12 \text{ cm}^2\text{s}^{-1}$ and $0.36 \text{ cm}^2\text{s}^{-1}$ when ρ_n is reduced from 1000 to 500 $\mu\Omega\text{cm}$. These estimations are consistent with the values reported for less resistive NbN thin films ($\rho_n \sim 300 \mu\Omega\text{cm}$) for which D_e amounts to 0.5 - 0.6 cm^2s^{-1} [157]. The electronic diffusivities of the NbN samples are shown in Fig. 2.23 as a function of ρ_n^{-1} . As expected,

12. The concept of electronic diffusivity is introduced at the page 26.

the overall trend is linear with the slope directly related to the almost constant N_0 . The Nb samples have a normal state resistivity around two orders of magnitude smaller than the NbN samples. This leads to a much higher electron diffusivity of $\sim 3 \text{ cm}^2\text{s}^{-1}$, in agreement with other reported values [160].

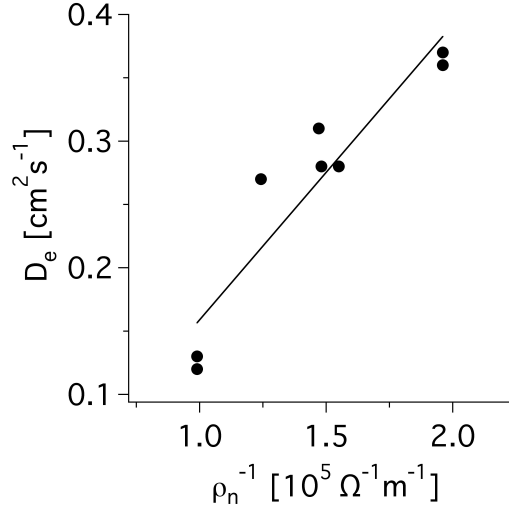


Figure 2.23: Evolution of the electronic diffusivity of the NbN samples with respect to the inverse of the normal state resistivity. The electronic diffusivity is determined using the relation 2.4 and the product $N_0\rho_n$ which is experimentally given by the slope of the temperature dependence of the upper critical field B_{c2} near the critical temperature (see the equation 2.2, page 96).

Characteristic lengths in the dirty limit

Up to now, the electronic transport properties of the NbN and Nb nanostructures have been discussed in the superconducting dirty limit, i.e., under the assumption that $\ell < \xi < \lambda$ (with ℓ the electron mean free path, ξ the coherence length and λ the magnetic penetration depth). These three characteristic length-scales are intimately interrelated because the theory predicts that $\xi \propto \ell^{0.5}$ and $\lambda \propto \ell^{-0.5}$ [19]. The purpose of this section is to justify *a posteriori* that the dirty limit condition is here verified.

Electron mean free path

The Einstein relation 2.4 together with the definition of the electronic diffusivity $D_e = \frac{1}{3}v_F\ell$ allows one to express the electron mean free path

with respect to both the residual resistivity ρ_n and the density of states at the Fermi level N_0 following the expression,

$$\ell = \frac{3}{e^2 v_F N_0 \rho_n}, \quad (2.5)$$

where v_F is the Fermi velocity which is given in the free-electron model by $v_F \equiv \frac{\hbar k_F}{m} = \frac{N_0 \hbar^3 \pi^2}{m^2}$ (where k_F is the radius of the Fermi sphere). The electron mean free path can thus be determined with the estimated values of N_0 and ρ_n . For the NbN samples, ℓ is found to be ~ 1 Å, i.e., below the values reported for samples with a lower residual resistivity [156,157]. In addition, as the estimation of N_0 was shown to overestimate the real value for the Nb samples, ℓ is determined in this case by taking $v_F \sim 8 \times 10^5 \text{ ms}^{-1}$ (see [18,145]) and the known product ($N_0 \rho_n$). Doing so, one obtains a mean free path of a few nm.

Coherence length

For a type-II superconductor, the Ginzburg-Landau coherence length at zero temperature is related to the upper critical field through the relation 2.6,

$$\xi_{GL}(0) = \left[\frac{\Phi_0}{2\pi B_{c2}(0)} \right]^{0.5}, \quad (2.6)$$

where $\Phi_0 \equiv \frac{h}{2e} = 2.07 \times 10^{-15} \text{ Tm}^{-2}$ is the flux quantum. The coherence length noticeably varies from one sample to the other, depending mainly on their normal state resistivity. The most resistive NbN strips have a coherence length of 2 nm, around half the value calculated for the least resistive samples (3.8 nm). For the Nb nanostructures, $\xi_{GL}(0)$ ranges between 9.5 nm and 13 nm. The condition $\ell < \xi$ is thus verified for all the samples.

Magnetic penetration depth

The magnetic penetration depth at zero temperature is expressed in the dirty limit by the Bardeen-Cooper-Schrieffer (BCS) expression,

$$\lambda_{BCS}(0) = \sqrt{\frac{\hbar \rho_n}{\pi \mu_0 \Delta(0)}}, \quad (2.7)$$

where $\hbar$ is the reduced Planck constant, μ_0 is the vacuum permeability and $\Delta(0)$ is the superconducting energy gap at zero temperature. On one hand, NbN is known to be a strong-coupling superconductor whose

energy gap is experimentally given by $2\Delta_{NbN}(0) = 4.15k_B T_c$ [133, 161, 162]. Here, this gives a superconducting energy gap between 1.6 meV and 2.1 meV. On the other hand, Nb is in the intermediate-coupling limit with the superconducting energy gap given by $2\Delta_{Nb}(0) \approx 3.6k_B T_c$ [153, 163, 164]. $\Delta_{Nb}(0)$ amounts to 1.2 meV and 1.4 meV for the 20 nm-thick and for the 75 nm-thick structures, respectively. Based on these estimations of $\Delta(0)$, the magnetic penetration depth at zero temperature is calculated using Eq. 2.7. For NbN, $\lambda_{BSC}(0)$ takes values as high as 0.8 - 1 μm , in agreement with the expected and observed trend [162, 165]. For the Nb structures, $\lambda_{BSC}(0)$ amounts to 120 - 130 nm, in agreement with [145]. As a consequence, the dirty limit condition $\ell < \xi < \lambda$ is clearly fulfilled for all the samples, all the more so in sufficiently thin films (for $d \ll \lambda$) the effective penetration depth $\lambda_{\perp} \approx \frac{\lambda^2}{d}$ is even much larger¹³. The estimations of $\lambda_{\perp}$ are summarized in the table 2.6.

Summary : main electrical transport properties

The electrical transport properties discussed in the section 2.3 are summarized here for representative NbN and Nb nanostructures (either strips or loops). The table 2.6 refers to the geometrical characteristics as well as to the parameters extracted from the resistance vs. temperature curves. The table 2.7 refers to the parameters extracted from the resistance vs. magnetic field curves.

13. Strictly speaking, the condition $d \ll \lambda$ is not fully verified for the 75 nm-thick samples as their thickness is not significantly smaller than λ .

Table 2.6: Geometrical characteristics of the NbN and Nb nanostructures and their electrical transport properties extracted from the resistance vs. temperature curves : length L (strip) or mean radius r_M (loop), width W (strip) or branch width $r_o - r_i$ (loop), width of the weakest area W_{WL} , thickness d , normal state resistivity ρ_n , residual resistivity ratio RRR, critical temperature T_c and critical temperature of the strip constriction (if present).

Strip		L [μm]	W [nm]	W_{WL} [nm]	d [nm]	ρ_n [$\mu\Omega\text{cm}$]	RRR [-]	T_c [K]	$T_{c,WL}$ [K]
NbN	SNbN-20-1	3	70	35	20	680	0.82	9.15	8.65
	SNbN-20-2	4	70	-	20	660	0.82	9.2	-
	SNbN-20-3	2	110	60	20	680	0.82	8.7	8.5
	SNbN-20-4	2	120	-	20	690	0.82	8.7	-
	SNbN-20-5	3	110	70	20	675	0.82	8.7	8.5
	SNbN-20-6	3	110	-	20	645	0.82	8.7	-
	SNbN-25-1	2	120	70	25	805	0.79	9.65	9.55
	SNbN-25-2	3	130	-	25	785	0.79	9.65	-
	SNbN-75-1	3	115	-	75	1220	0.73	11.4	-
	SNbN-75-2	3	115	70	75	1230	0.73	11.4	11
	SNbN-75-3	10	125	75	75	1200	0.77	12.5	-
	SNbN-75-4	1	150	-	75	1000	0.75	11.75	-
	SNbN-75-5	4	160	85	75	1285	0.75	11.75	11.65
	SNbN-75-6	12	160	85	75	1160	0.73	11.75	11.65
	SNbN-W1	4	40 (20)	-	26	250	0.62	11.3 (9.1)	-
	SNbN-W2	4	55 (45)	-	26	250	0.62	11.7	-
	SNbN-W3	4	85 (80)	-	26	250	0.62	11.8	-
	SNbN-W4	4	100	-	26	250	0.62	11.9	-
Nb	SNb-75-1	3	130	-	75	10	-	8.6	-
	SNb-75-2	3	130	-	75	8.8	2.9	9	-
	SNb-75-3	3	240	-	75	9.2	3.2	8.9	-
Loop		r_M [nm]	$r_o - r_i$ [nm]	W_{WL} [nm]	d [nm]	ρ_n [$\mu\Omega\text{cm}$]	RRR [-]	T_c [K]	$T_{c,WL}$ [K]
Nb	LNb-1a	900	100	100	75	11.6	3.9	8.65	-
	LNb-1b	825	250	420	75	11.6	-	9.05	-
	LNb-1c	700	450	650	75	11.7	4.4	9.17	-
	LNb-2a	500	160	300	20	12.2	2.6	7.75	-
	LNb-2b	480	455	400	20	11.9	-	7.65	-
	LNb-3a	290	150	150	20	12.1	2.6	7.75	-

Table 2.7: Electrical transport properties of the NbN and Nb nanostructures extracted from their resistance vs. magnetic field curves : upper critical field at zero temperature $B_{c2}(0)$, density of states at the Fermi level N_0 , electronic diffusivity D_e , electron mean free path ℓ , coherence length $\xi(0)$ (at $T_0 = 0$ K), superconducting energy gap $\Delta(0)$ (at $T_0 = 0$ K), magnetic penetration depth $\lambda(0)$ (at $T_0 = 0$ K) and effective magnetic penetration depth $\lambda_\perp \equiv \frac{\lambda^2}{d}$.

		$B_{c2}(0)$									
		[T]	N_0	D_e	ℓ	$\xi(0)$	$\Delta(0)$	$\lambda(0)$	$\lambda_{\perp}$		
			$[10^{28}/(\text{m}^3\text{eV})]$	$[\text{cm}^2\text{s}^{-1}]$	[Å]	[nm]	[meV]	[nm]	[μm]		
Strip	NbN	SNbN-20-1	22.3	1.9	0.31	1	3.8	1.6	830	69	
		SNbN-20-2	-	-	-	-	-	1.7	815	66	
		SNbN-20-3	-	-	-	-	-	1.6	850	72	
		SNbN-20-4	-	-	-	-	-	1.6	860	73	
		SNbN-20-5	23.4	2	0.28	0.8	3.8	1.6	850	72	
		SNbN-20-6	23.3	2.1	0.28	0.7	3.8	1.6	830	69	
		SNbN-25-1	27.3	1.8	0.27	0.8	3.5	1.7	880	62	
		SNbN-25-2	-	-	-	-	-	1.7	870	60	
		SNbN-75-1	-	-	-	-	-	2	1000	26	
		SNbN-75-2	-	-	-	-	-	2	1000	27	
		SNbN-75-3	-	-	-	-	-	2	1000	20	
		SNbN-75-4	76.5	3.3	0.12	0.2	2.1	2.1	890	21	
		SNbN-75-5	-	-	-	-	-	2.1	1000	27	
		SNbN-75-6	-	-	-	-	-	2.1	960	24	
Nb	SNb-75-1	2.24	21.5	2.3	11	12.1	1.3	112	0.17		
	SNb-75-2	-	-	-	-	-	1.4	100	0.14		
	SNb-75-3	2	20	3.4	13	12.9	1.4	102	0.14		
Loop	Nb	LNb-1a	3.7	19	1.8	11	9.4	1.3	120	0.19	
		LNb-1b	-	-	-	-	-	1.4	118	0.18	
		LNb-1c	2	9.5	3.5	21	12.9	1.4	118	0.18	
		LNb-2a	2.8	15.3	2.1	13	10.8	1.2	130	0.85	
		LNb-2b	2.7	15.3	2.1	13	11	1.2	129	0.83	
		LNb-3a	2.5	13.7	2.4	14	11.5	1.2	129	0.83	

Chapter 3

Stabilization of normal domains by electrical transport measurements

This chapter is devoted to the description of the electrical behavior of the NbN and Nb submicrometer-wide strips. All the results have been obtained without an applied (static) magnetic field, i.e., the only parameters involved in the experiments are the direct bias (current or voltage), the coolant temperature ($T_0 < T_c$) and an external microwave dissipation. The chapter is divided into five main parts. In section 3.1, the overall current-biased (V-I) characteristics are described in the framework of the thermal bistability, in agreement with the model of a localized hotspot in a long microbridge. The ability to stabilize a large number of intermediate dissipative states between the perfect superconducting and the fully normal states is discussed in Sec. 3.2. The origin of the localization and the propagation of these normal domains is discussed in section 3.3. In particular, the stepwise evolution of the voltage in both parts of the current-voltage characteristics is shown to predominantly result from the nanoscale modulations of the cross-section. In section 3.4, the electrical behavior of the submicrometer-wide strips is studied under a continuous microwave irradiation. All the experimental observations reinforce the major role played by the heat dissipation in the stabilization of the intermediate states between the perfect superconducting state and the fully normal state. Finally, the section 3.5 discusses the main (dis)similarities that exist between the stabilization of normal hotspot domains (that are observed here) and the phase-slippage phenomena (that are often considered to explain the current-driven transition of superconducting strips and whiskers).

3.1 Thermal bistability and hotspot regime

In section 1.2.1 (page 17), it was stated that the bistability under Joule heating of a current-carrying conductor leads to an hysteresis of the V-I curves in the interval $[I_{m0}, I^*]$, where I_{m0} is the minimum normal-zone existence current and I^* is the instability current¹. The figure 3.1 illustrates this feature with the overall electrical behavior of representative NbN [SNbN-75-1 (a)] and Nb [SNb-75-2 (b)] samples at different substrate temperatures T_0 . The results are expressed as a resistance ($R \equiv \frac{V}{I}$) vs. current (I) curve.

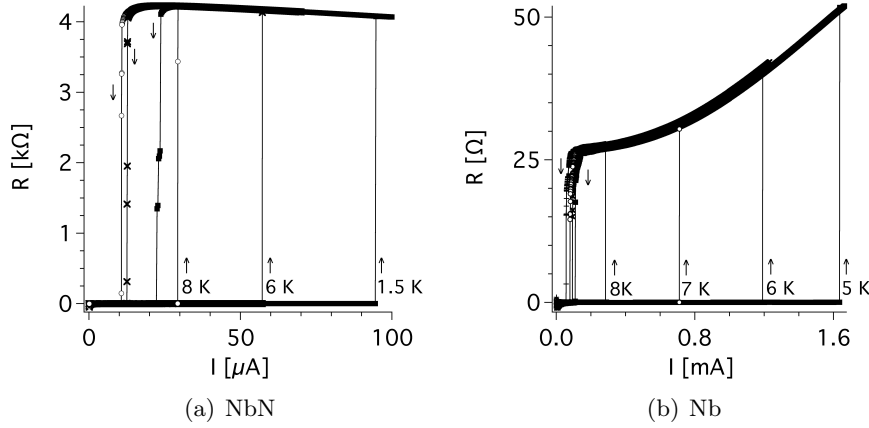


Figure 3.1: Overall current-biased characteristics of representative NbN (SNbN-75-1) and Nb (SNb-75-2) submicrometer-wide strips obtained at different coolant temperatures : 1.5 K, 6 K and 8 K for NbN and 5 K, 6 K, 7 K and 8 K for Nb.

In this temperature range (i.e., $\frac{T_0}{T_c} < 0.8$), both samples abruptly switch in the fully normal state when the applied current exceeds the instability current I^* . After the jump to the normal state, the resistance evolves differently for both samples when the applied current is reduced. This is directly related to the increase of the electronic temperature T_e which takes its maximum value when the applied current (and thus the dissipation) is maximum², and decreases when the current is reduced. Due to their different temperature coefficient above T_c (see the section

1. Given that a non-zero voltage may appear at a current I_c smaller than the instability current I^* , the latter is used throughout the manuscript to denote the formation of a highly dissipative domain.

2. Based on the resistance vs. temperature curves obtained at low applied current, the electronic temperature T_e can be estimated just after the transition to the normal state. For the largest instability currents of Fig. 3.1, T_e amounts to 60 K and 110 K for the NbN and Nb samples, respectively.

2.1.3.2, page 73), an increase of T_e reduces (resp. increases) the resistance of the NbN (resp. Nb) samples. The observed evolution of the resistance in the normal state thus agrees with the underlying idea of the heating models (see Sec. 1.2.2, page 19) : the electronic temperature has to be reduced below the critical temperature T_c in order to recover the perfect superconducting state. This arises below the retrapping current $I_r \equiv I_{m0} \lll I^*$.

3.1.1 Temperature dependence of the retrapping current

From Fig. 3.1, one sees that the width of the hysteresis $[I_r, I^*]$ gradually narrows when the substrate temperature increases. This results in a monotonic reduction of the Stekly parameter $\alpha \equiv \left(\frac{I^*}{I_r}\right)^2 = \left(\frac{j^*}{j_r}\right)^2$ (see Eq. 1.16, page 18), which takes values well above one at low T_0 and gradually decreases down to $\alpha \sim 1$ close to T_c . This is illustrated at Fig. 3.2 with the temperature dependence of the Stekly parameter expressed with respect to the reduced temperature T_0/T_c for representative Nb and NbN strips.

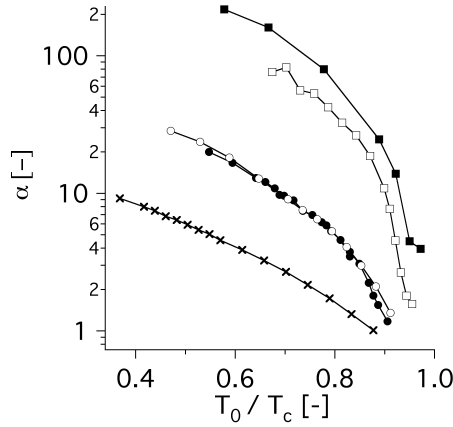


Figure 3.2: Evolution of the Stekly parameter α with respect to the reduced temperature T_0/T_c of NbN [SNbN-20-6 ($\circ$), SNbN-75-1 ($\bullet$), SNbN-75-5 ($\times$)] and Nb [SNb-75-2 ($\blacksquare$), SNb-75-3 ($\square$)] microstrips.

The reduction of the parameter α at increasing coolant temperatures is directly related to the temperature dependence of the characteristic current densities j^* and j_r . On one hand, j^* is experimentally found to vary linearly with the coolant temperature T_0 with an instability current density extrapolated at zero temperature of $j_{Nb}^*(T_0 = 0) \sim 20 \text{ MAcm}^{-2}$

and $j_{NbN}^*(T_0 = 0) \sim 1\text{-}2 \text{ MAcm}^{-2}$ (the values are summarized in the table 3.1, page 114). On the other hand, the retrapping current density varies as $(T_c - T_0)^{1/2}$, which agrees with Eq. 1.17 (page 19). The figure 3.3 shows the temperature evolution of j_r for three representative NbN and Nb samples with different normal state resistivities. The slope of the curves at $T_0 \approx T_c$ in this plot of the logarithm of the retrapping current density j_r as a function of $\left(1 - \frac{T_0}{T_c}\right)$ is 0.5 for all the microstrips. The slight deviation observed at substrate temperatures well below T_c (see the dashed lines) is well-known and results from the temperature dependence of the heat transfer coefficient [51] and/or temperature-dependent thermal conductivities [52].

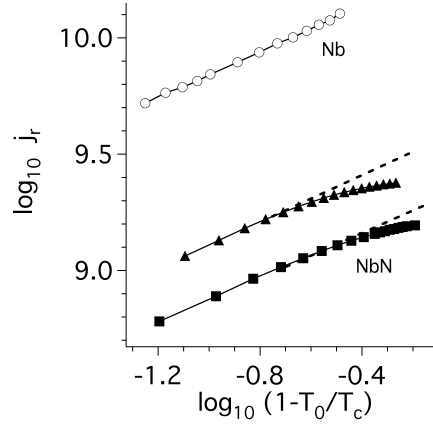


Figure 3.3: Log-log plot of the retrapping density current j_r as a function of $\left(1 - \frac{T_0}{T_c}\right)$ for three samples of different normal state resistivity : SNbN-75-5 (■, $\rho_n = 1285 \mu\Omega\text{cm}$), SNbN-20-5 (▲, $\rho_n = 675 \mu\Omega\text{cm}$) and SNb-75-3 (○, $\rho_n = 9.2 \mu\Omega\text{cm}$). The slope of the curves at temperature closed to the critical temperature is 0.5.

In addition, one sees at Fig. 3.3 that the retrapping current density increases when the normal state resistivity is reduced. Indeed, j_r is around 10 times larger for the Nb samples than for the (more resistive) NbN ones. Similar variations are also observed between the NbN samples of different ρ_n . The equation 1.17 expressed in units of a density power,

$$\rho_n j_r^2 = \frac{h_{T_c} (T_c - T_0)}{d}, \quad (3.1)$$

indicates that similar heat transfer conditions from the film to the substrate correspond to a similar Joule dissipation at the normal - to -

superconductor transition. The temperature dependence of the retrapping current is a well-known tool to quantitatively characterize the thermal transport across an interface [41], which is usually expressed by the heat-transfer coefficient h (equivalently by the thermal boundary resistance defined as the inverse of the heat transfer coefficient) [42]. Based on the y-axis intercept of the curves in Fig. 3.3, the heat-transfer coefficient close to the critical temperature h_{T_c} is found to vary only slightly between all the samples, with values between 1.5 and $4 \times 10^5 \text{ Wm}^{-2}\text{K}^{-1}$. The latter estimation is one order of magnitude smaller than reported values of various bulk metal/dielectric interfaces ($h \approx 10^7 \text{ Wm}^{-2}\text{K}^{-1}$ at low temperatures) [166, 167]. Accordingly, the heat removal appears to be mainly limited by the interface between the film and the substrate. However, some samples present a slight increase of the heat-transfer coefficient when the coolant temperature is reduced to its lowest value, i.e., $T_0 = 1.5 \text{ K}$. Since the measurements were not performed in vacuum, this may result from an enhanced heat removal at the interfaces between the sample and the condensed helium (or even the superfluid helium as the latter is below its lambda temperature $T_\lambda = 2.17 \text{ K}$). In any case, the observed evolution of the retrapping current density with respect to ρ_n (the larger is ρ_n , the smaller is j_r) is the direct consequence of the similar heat removal capabilities, and thus of the similar density power dissipated at the retrapping. The product $\rho_n j_r^2$ falls in the $\mu\text{W}\mu\text{m}^{-3}$ range for all the samples, from one to a few tens of $\mu\text{W}\mu\text{m}^{-3}$ depending on the coolant temperature.

3.1.2 Consistency with the model of the localized hotspot

Together with the heat transfer coefficient h_{T_c} , the thermal conductivity κ is required to estimate the thermal healing length of the substrate/thin film configuration (defined at Eq. 1.18, page 20). In normal metals, the heat currents have components from both the conduction electrons and the quantized lattice vibrations, i.e., the phonons. The thermal conductivity is splitted into two contributions,

$$\kappa = \kappa_e + \kappa_{ph}, \quad (3.2)$$

where κ_e and κ_{ph} are the electronic and lattice contributions, respectively [50]. At low temperatures, the electron scattering due to the impurities and the lattice defects dominates. As a consequence, κ_e can be estimated using the simple model of an electron gas in metals (i.e., the Wiedeman-Franz law [168]) which states that the ratio of the electronic thermal conductivity κ_e and of the electrical conductivity σ ($\equiv \rho^{-1}$) is

proportional to the temperature. The electronic thermal conductivity in the normal state³ is given by Eq. 3.3,

$$\frac{\kappa_{e,n}}{\sigma_n} = L_0 T_c, \quad (3.3)$$

which is expressed at the critical temperature T_c [σ_n is the electrical conductivity in the normal state and L_0 is the Lorentz number given by $\frac{\pi^2}{3} \left(\frac{k_B}{e}\right)^2 = 2.44 \times 10^{-8} \text{ W}\Omega\text{K}^{-2}$ with k_B is the Boltzman constant and e is the electron charge]. Due to their high residual resistivity, the NbN samples have small $\kappa_{e,n}$ that range between 20 to 35 $\text{mWm}^{-1}\text{K}^{-1}$. This contrasts with the values found for Nb, i.e., around 2.1 and 2.5 $\text{Wm}^{-1}\text{K}^{-1}$ (all the values are summarized in the table 3.1 at page 114). This latter expression is not valid in the superconducting state because the electrons that condense into Cooper pairs do not contribute to the heat transport. The remaining fraction of electrons (the quasiparticles) contributes, but their number rapidly decreases when the temperature is lowered below T_c [50]. In the coolant temperature range $[0.3T_c, 0.8T_c]$, the ratio between the electronic thermal conductivity in the superconducting state ($\kappa_{e,s}$) and in the normal state ($\kappa_{e,n}$) is known to be quasi-linear [170, 171]. As a consequence, it is often assumed that the thermal conductivity in the superconducting state is given by $\kappa_{e,s} = \frac{T_0}{T_c} \times \kappa_{e,n}$ [35] (T_0 is the coolant temperature), although a more pronounced empirical decrease is also reported with $\kappa_{e,s} = \left(\frac{T_0}{T_c}\right)^3 \times \kappa_{e,n}$ [169]. In addition, the dominant term in a metallic material is usually the electronic contribution [170, 172], especially in nanoscale devices where the lattice contribution is significantly reduced compared to the bulk value of the same material [167, 173–175]. Indeed, the phonons are efficiently scattered by both the surfaces and by the nanometer-scale voids located between the grains [166] and their mean free path ℓ_{ph} becomes comparable to the nanoscale devices sizes. As κ_{ph} is proportional to the phonon mean free path ℓ_{ph} through the Debye relation

$$\kappa_{ph} = \frac{1}{3} (Cv\ell)_{ph}, \quad (3.4)$$

one sees that the reduction of ℓ_{ph} strongly affects the lattice contribution κ_{ph} (C_{ph} is the phonon heat capacity and v_{ph} is the phonon velocity). As a consequence, the phonons contribution to the thermal conductivity is most of the time considered negligible, and only the electronic term

3. The localized hotspot model used here considers a temperature-independent electronic thermal conductivity in the normal state. More detailed descriptions often assume that $\kappa_{e,n}$ varies linearly with the temperature inside the hotspot region [169].

is considered. This assumption appears to be reasonably accurate for the niobium samples whose electronic conductivity is large. However, the reduced $\kappa_{e,n}$ of the NbN samples questions about its validity. The underestimation will be discussed in section 3.2.2.

From the above-described determination of the thermal conductivity, the thermal healing length in the normal state becomes accessible through the relation

$$\eta = \left(\frac{\kappa d}{h T_c} \right)^{1/2}, \quad (3.5)$$

which is derived from the equation 1.18 (page 20) at a temperature close to the critical one. The obtained values are summarized in the table 3.1 (page 114). The thermal healing length of the NbN samples appears to be small, between 40 nm and 70 nm ($L/\eta \sim 20 - 80$). All the Nb samples also fall in the limit of a long microbridge, but with smaller ratios $L/\eta \sim 3$ (their thermal healing length is close to one micron). As a consequence, the model of Skocpol, Beasley and Tinkham described in the section 1.2.2.1 (page 21) is applicable and the resulting V-I characteristics may be compared with the experimental results. This is done in Fig. 3.4 for representative NbN (a, b, c) and Nb (d) samples that differ by their transverse dimensions. The full black curve corresponds to the experimental current-biased characteristic. Although only the current-biased recovery of the superconducting state is described by the hotspot model, the overall curves (including the current-biased instability to the fully normal state) are depicted for clarity. The red curve (■ symbols) corresponds to the theoretical voltage vs. current evolution determined with only one adjustable parameter : the heat transfer coefficient between the thin film and the substrate h . All the others coefficients involved in the model are determined for each sample based on their measured transport properties (T_c , ρ_n , κ_n , η) or are deduced at the coolant temperature T_0 from the assumed temperature dependence (κ_s). One sees that the theoretical hotspot model is well suited to describe quantitatively the current range over which the retrapping transition takes place. All the theoretical curves of Fig. 3.4 have been obtained with the heat transfer coefficient adjusted between 1 and $2 \times 10^5 \text{ Wm}^{-2}\text{K}^{-1}$, i.e., close to the values determined from the temperature dependence of the retrapping current near T_c .

For the Nb sample [Fig. 3.4(d)], the theoretical V-I characteristic significantly deviates at high voltages. One has to remember that (i) the instability current density is more than one order of magnitude higher for Nb compared to NbN and (ii) that the hotspot model of Skocpol, Beasley

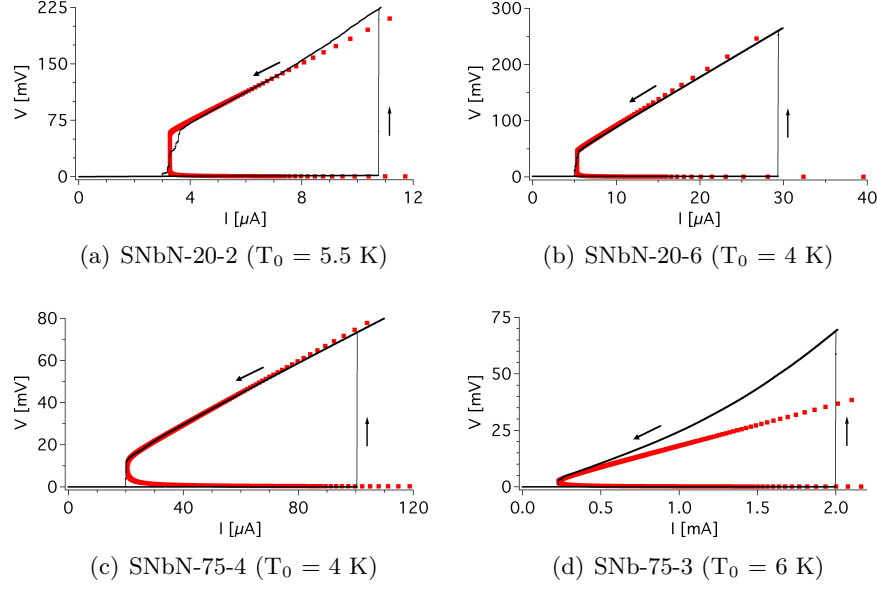


Figure 3.4: Comparison between the experimental (full black curve) and the theoretical (red ■ curve) V-I characteristics of representative NbN and Nb samples which differ by their transverse dimensions. The theoretical model of the localized hotspot in a long microbridge ($L > \eta$) is used with only one adjustable parameter, i.e., the heat transfer coefficient between the thin film and the substrate (h). The coolant temperature T_0 is indicated for each subfigure.

and Tinkham considers a temperature-independent residual resistivity above T_c . As a consequence, the model underestimates the pronounced increase of the electronic resistivity that results from the dissipation by Joule heating at high currents. The observed discrepancy clearly highlights that a strong electron overheating ($T_e \gg T_c$) takes place in practice at high currents. The gap between the theoretical and the experimental V-I curves tends to diminish when the current is reduced which is expected as the electronic temperature comes closer to the critical one. Such deviations are not observed for the NbN samples, even at the lowest coolant temperatures. This comes from the small variation of their electronic resistivity above T_c .

The table 3.1 summarizes the main electrical transport properties discussed in the section 3.1. In addition, the last column contains the average electron cooling time calculated using Eq. 1.26 (page 26) and the estimated thermal healing length (see the table 3.1) and electronic diffusivity (see the table 2.7, page 104). The average electron cooling time τ_e is slightly less than 100 ps for the thinnest NbN samples and increases up to

350 ps for the thickest ones, while values of a few nanoseconds are found for the Nb samples. From the discussion of the section 1.2.2.2, one knows that the average cooling time depends on both the electron-phonon interaction time $\tau_{e,ph}$ and on the phonon escape time to the substrate τ_{es} . On one hand, the direct energy flow from the electrons to the phonons usually dominates over the reabsorption of the non-equilibrium phonons by the electrons because $\tau_{ph,e} \gg \tau_{e,ph}$ (in other words, the electronic specific heat C_e is smaller than the phonon specific heat C_{ph}). However, this energy back-flow can not be neglected in NbN because the ratio C_{ph}/C_e is small (≈ 6.5) [176]. For Nb, although the ratio C_{ph}/C_e is still much lower, the effective escape of the phonons to the substrate is often considered fast enough ($\tau_{ph,e} > \tau_{es}$) to prevent the energy back-flow to the electrons [54]. For NbN, the electron-phonon interaction time was found to depend on the electronic temperature following the relation $\tau_{e,ph} \sim 500 T_e^{-1.6}$ ps [177]. Several values were reported between 10 ps and 60 ps [54, 55, 176, 178, 179], more than one order of magnitude smaller than for Nb [180]. On the other hand, the phonon escape time to the substrate mainly depends on the sample thickness [181]. For microstrips whose phonon mean free path exceeds the film thickness, τ_{es} is proportional to the thickness and inversely proportional to the film-substrate phonon transmission α following the relation $\tau_{es} \approx \frac{4d}{\alpha u}$, where u is the speed of sound and d the thickness [182]. For NbN films deposited on silicon or sapphire, the latter expression gives a phonon escape time of ~ 130 ps for $d = 20$ nm and to ~ 500 ps for $d = 75$ nm ($u \sim 3000$ ms $^{-1}$, $\alpha \sim 0.2$) [179]. In addition, noticeable variations have been observed in the literature, from a few tens of picoseconds for ultra-thin films ($d \approx 3.5$ nm) up to 160 ps for $d = 20$ nm [54, 55, 176, 183]. Therefore, one finds a satisfactory agreement between the values extracted here and the expected values of a few hundreds of ps for NbN and around a few ns for Nb. A more precise determination of τ_e is beyond the scope of this manuscript.

3.2 Stabilization of normal hotspot domains

For a non-ohmic conductor, an up-and-down sweep in the current bias regime generally gives the envelope of the hysteretic transition. It was found in section 3.1 that the overall V-I characteristics of the NbN and Nb submicrometer-wide strips reflect a pronounced thermal bistability, in agreement with the model of a normal hotspot domain that expands in a superconducting bridge much longer than the thermal healing length. In the forward part, the abrupt transition to the fully nor-

Table 3.1: Electrical and thermal transport properties extracted from the resistance vs. current curves of the NbN and Nb submicrometer-wide structures : normal state resistivity ρ_n , retrapping current density j_r at $T_0/T_c = 0.8$, instability current density j^* at $T_0 \rightarrow 0$, heat transfer coefficient close to the critical temperature h_{T_c} , electronic thermal conductivity in the normal state κ_n ($= \kappa_{e,n}$) and thermal healing length in the normal state η .

	ρ_n [$\mu\Omega\text{cm}$]	$j_r(0.8T_c)$ [MAcm $^{-2}$]	$j^*(0)$ [MAcm $^{-2}$]	h_{T_c} [10 5 Wm $^{-2}$ K $^{-1}$]	κ_n [mWm $^{-1}$ K $^{-1}$]	η [nm]	τ_e [ps]	
NbN	SNbN-20-1	680	0.2	1.2	4	33	40	55
	SNbN-20-2	660	0.2	1.7	3.1	34	40	70
	SNbN-20-3	680	0.2	1	2.7	31	50	75
	SNbN-20-4	690	0.2	1.6	2.3	31	50	85
	SNbN-20-5	675	0.2	1.9	2.3	31	50	100
	SNbN-20-6	645	0.2	1.8	2.7	31	50	90
	SNbN-25-1	805	0.2	1.7	4.5	29	40	60
	SNbN-25-2	785	0.2	1.3	5.3	30	40	50
	SNbN-75-1	1220	0.1	1	3.8	23	70	370
	SNbN-75-2	1230	0.1	1	4.2	23	65	340
	SNbN-75-3	1200	0.1	1	4.1	24	66	360
	SNbN-75-4	1000	0.1	1.9	5.2	28	60	340
	SNbN-75-5	1285	0.1	0.9	4.2	22	60	330
	SNbN-75-6	1160	0.1	0.7	3	25	80	500
Nb		ρ_n [$\mu\Omega\text{cm}$]	$j_r(0.8T_c)$ [MAcm $^{-2}$]	$j^*(0)$ [MAcm $^{-2}$]	h_{T_c} [10 5 Wm $^{-2}$ K $^{-1}$]	κ_n [Wm $^{-1}$ K $^{-1}$]	η [μm]	τ_e [ns]
	SNb-75-1	10	0.7	18	1.6	2.1	1	3.9
	SNb-75-2	9	1	27	2.1	2.5	0.95	4.3
	SNb-75-3	9.2	1	24	3.7	2.4	0.7	1.4

mal state results from the extremely fast expansion of a normal hotspot through the whole sample. By contrast, the fully superconducting state is recovered at the retrapping current at which the normal domains progressively shrink. As a consequence, the overall electrical behavior only deals with two states, either the perfect superconducting state ($T_e \sim T_0$) or the fully normal state ($T_e \gtrsim T_c$). However, the stabilization of intermediate dissipative states becomes possible if a dc voltage bias is applied instead of a current bias. Doing this, the abrupt transition at the instability current is avoided and one may focus on the parts of the current-voltage characteristics that cannot be easily probed in the current-biasing mode.

3.2.1 Hotspot plateau

The stabilization of intermediate dissipative states in NbN microstrips is illustrated in Fig. 3.5 which shows (i) the overall current-biased behavior of the sample SNbN-25-2 at two different coolant temperatures (a) and (ii) an enlargement of the current range where the resistive states are observed (b). In the voltage-biased regime (green curves), the current rapidly decreases at low voltage after the switching point is reached [see the left-directed arrow in the bottom of Fig. 3.5(b)] and several resistive states appear by sweeping the bias up and down. All the results are very reproducible from one run to another, and each dissipative state can be stabilized without time evolution (on the timescale accessible here which is much larger than 0.1 s). From these results, one sees that the investigation of the intermediate region between the perfect superconducting state and the fully normal state requires that the load line of the dc voltage source intersects the hotspot plateau. As detailed in the section 2.2.4 (page 89), the voltage source consists in a room-temperature resistor in parallel with a dc current source with a resulting load line (~ 4 mV / 10 μ A) which is flat enough to measure the hotspot plateau of the NbN microstrips but is too sharp for the Nb samples. Indeed, their small normal state resistance and, to a lesser extent, their large characteristic current densities are such that the load line intersects the I-V curve of the Nb sample when the latter is in the fully normal state. Even at coolant temperatures close to T_c , the voltage-biased experiments do not allow the hotspot plateau of the Nb strips to be probed. Its investigation is however feasible in the current-biased mode under the additional heating of a microwave irradiation. This is discussed in the section 3.4.3.

In addition, in the voltage-biased curve of Fig. 3.5(b), there are as many configurations of localized normal hotspot domain(s) as there are intermediate dissipative states. The ratio between the measured resis-

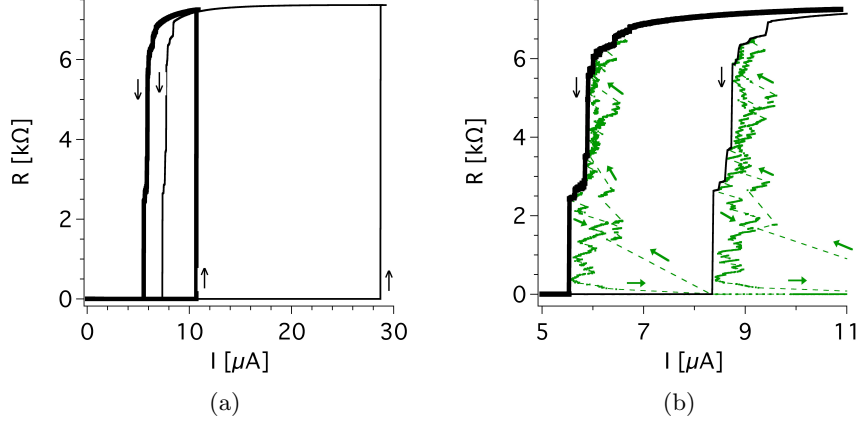


Figure 3.5: Hysteretic V-I and I-V curves of the sample SNbN-25-2 measured at $T_0 = 6$ K (thin curve) and 8 K (thick curve). (a) Overall current-biased curves characterized by an abrupt transition at the instability current. (b) Enlargement of the hotspot plateau (i.e., the current range left-bounded by the direct retrapping current) obtained by applying a voltage-bias (green dashed curves). The black curves correspond to the current-driven recovery of the superconducting state. The curves obtained at $T_0 = 6$ K are shifted by $+1 \mu\text{A}$ for clarity.

tance R_{HS} and the normal state resistance R_n reflects the total fraction of the sample length which is in the normal state, i.e., the normal domain(s) occupies(y) a total length given by $L_{HS} = L \times \frac{R_{HS}}{R_n}$, where L is the sample length. In addition, one sees that the hotspot plateau measured at different coolant temperatures T_0 present strong similarities, both in the forward and in the backward parts of the voltage-biased curves. This is directly related to the unchanged position of the inhomogeneities that pin the boundaries of the normal domains⁴. As the inhomogeneities profile along the sample length may significantly differ from one sample to the other, the number as well as the resistance level of the intermediate resistive states significantly vary. The figure 3.6 highlights the large variety of hotspot plateaus that have been observed on NbN submicrometer-wide strips that differ by their transverse dimensions. As it was the case in Fig. 3.5(b), several tens of dissipative states are observed between the perfect superconducting state and the fully normal state. This behavior is totally reproducible and has been observed on the whole set of NbN strips. The dissipative states are stable in a narrow current range (the so-called hotspot plateau), which is always

4. The origin of the inhomogeneities that prevent the motion of the normal-to-superconducting interfaces will be detailed in section 3.3.

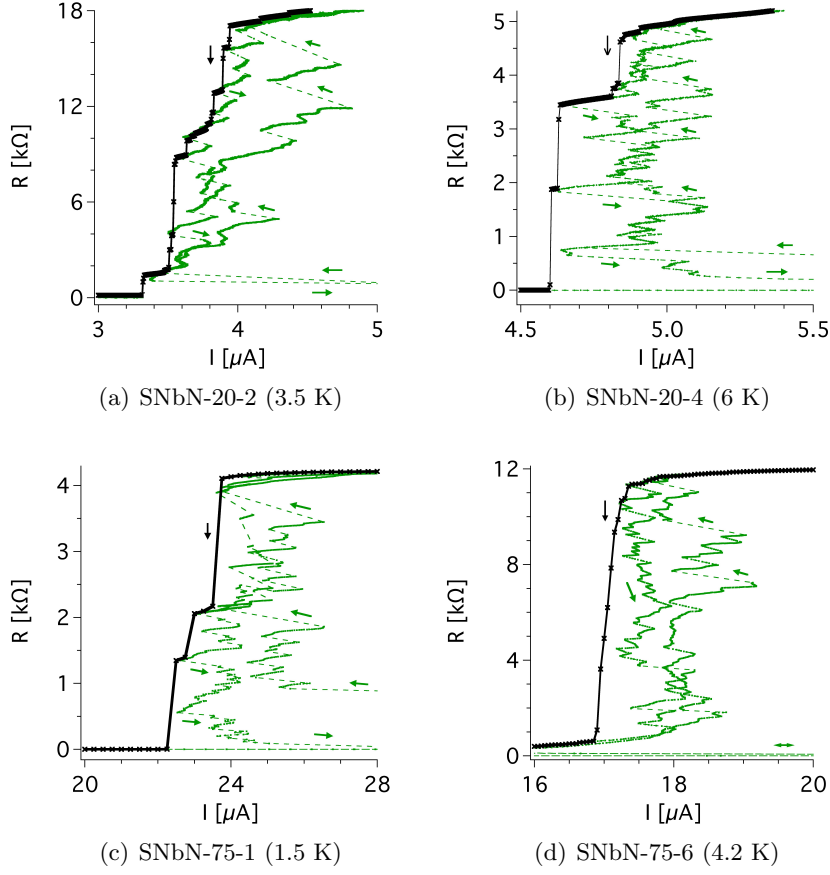


Figure 3.6: Voltage-biased curves (dashed green) of NbN submicrometer-wide strips that differ by their transverse dimensions. The backward part of the current-biased characteristic (black continuous curve) defines the left-side boundary of the stabilization current range (T_0 varies from 1.5 K up to 6 K).

left-bounded by the current-biased retrapping transition (black continuous curve). This is a relevant observation that makes sense because the retrapping current is by definition the current below which the heat removal through the substrate exceeds the Joule heating dissipation, so that a normal domain can not remain stable and collapses. The electronic temperature may be considered close to the critical one at the N/S interfaces and increases above T_c inside the normal domain [see the temperature profile of the figure 1.5(a), page 22].

3.2.2 Normal-zone propagating current

Through the concept of the retrapping current I_r , only the balance between the heat dissipated inside a normal domain and the heat evacuated through the substrate is usually considered. However, in section 1.2.2.1, the heat conduction in the vicinity of the normal / superconducting (N/S) interfaces was shown to be crucial in order to describe the current required to induce their motion [41, 44].

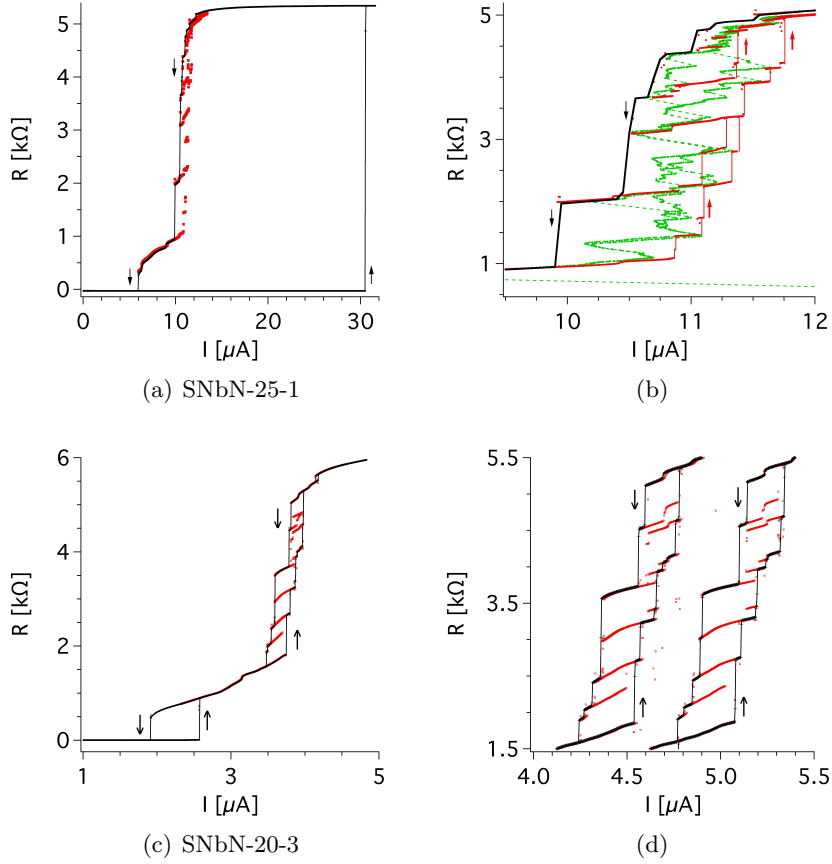


Figure 3.7: Electrical behavior of SNbN-25-1 (a) and SNbN-20-3 (c) measured well below ($T_0 = 1.65$ K) and close to ($T_0 = 7.25$ K) their respective critical temperature. The low-resistance part results from the intentionally placed constriction. (b) and (d) Enlargement of the hotspot plateau : envelope of the current-biased behavior (solid black), voltage-biased curve (dashed green) and current-biased curve obtained by sweeping the current back and forth after a resistance jump is detected in the overall characteristic (red curve) [$T_0 = 1.5$ K (b), $T_0 = 6.75$ K and 7 K (d)].

This is essential to notice that both contributions to the heat transfer are involved in the whole current range of stabilization of a normal domain. The main point is that a dissipation threshold⁵ has to be reached at the extremities of the normal domain in order to destabilize the N/S interfaces. This happens at the minimum normal-zone propagating current I_{p0} (see Eq. 1.21, page 21) above which the N/S interface tends to propagate until being pinned by another inhomogeneity. Nevertheless, the voltage-biased mode does not reveal the complete current interval of stabilization because each variation of the sample resistance leads to a change of the current which is imposed by the load line of the experimental setup. Monitoring the whole domain of stability thus requires to apply a dc current bias instead of a voltage bias. The intermediate dissipative states of the hotspot plateau are probed by sweeping the current back and forth at numerous points of the overall characteristic.

This is illustrated by the red curves of Fig. 3.7 obtained on two samples that contain a weak-link (SNbN-25-1 and SNbN-20-3). In the former case [curves (a) and (b)], the coolant temperature lies well below the critical one ($T_0 = 1.65$ K) so that the instability current ($I^* \approx 30$ μ A) is well above the hotspot plateau. Here, sweeping the current back and forth in the retrapping region is mandatory in order to probe the numerous intermediate dissipative states. One sees that the voltage-biased (green) and the current-biased (red) curves nicely match each other. In the latter case [curves (c) and (d)], the coolant temperature is sufficiently close to the critical temperature to prevent the transition of the whole sample when the instability current of the weak-link is reached. The normal domain length evolves by discrete jumps in both the forward and the backward parts of the curves, so that the stepwise envelope of the hotspot plateau is directly reachable. Once again, the whole stability range of the intermediate normal hotspot domains is highlighted. It is of the utmost importance to stress that (i) all the resistance steps in the retrapping transition have an equivalent resistance jump in the forward transition, and (ii) that the temperature dependence of the right-side boundary current is the same as for the retrapping current, i.e., $I \propto (T_c - T_0)^{1/2}$. This latter point is illustrated in Fig. 3.8 with the comparison of the retrapping (full symbols) and of the propagating (open symbols) current densities of the two NbN samples of Fig. 3.3. In addition, one sees in Fig. 3.7 that the right-side boundary of the stabilization current range follows the same trend as the retrapping current, which means that the ratio I_{p0}/I_r only slightly varies with the

5. The dissipation threshold corresponds to the maximum dissipation that can be sustained at a stable N/S interface when the heat removal capabilities through the substrate and by conduction within the film are considered.

resistance level. For these reasons, one can dependably consider that the right-side boundary of the stabilization current range is the current I_{p0} .

In the original work of Skocpol, Beasley and Tinkham [44], the minimum normal-zone propagating current is expressed by the equation

$$I_{p0} \approx I_r \times \left(1 + \frac{\kappa_s}{\kappa_n}\right)^{1/2}, \quad (3.6)$$

where κ_s and κ_n are the thermal conductivities in the superconducting state and in the normal state, respectively. The authors have considered that $\kappa_s = \kappa_n$, which is equivalent to consider identical contributions for the heat transfer through the substrate and for the heat conduction within the film. Indeed, the latter assumption leads to a dissipation ratio $(I_{p0}/I_r)^2 = 2$, with the numerator that includes both conduction channels while the denominator only contains the heat transfer through the substrate. As discussed in the section 1.2.2.1 (page 21), this leads to a rough estimation of the ratio $I_{p0}/I_r = \sqrt{2}$. This value, however, may significantly differ in practice. A ratio κ_s/κ_n smaller than unity [equivalently, a ratio $(I_{p0}/I_r)^2 < 2$] would correspond to a situation where the contribution of the heat transfer through the substrate dominates the whole heat removal capabilities. By contrast, a ratio κ_s/κ_n larger than unity [equivalently, a ratio $(I_{p0}/I_r)^2 > 2$] would correspond to a dominant contribution of the conduction within the film. As illustrated by the figures 3.6 and 3.7, the width of the stabilization current range of the NbN samples (at a fixed resistance level) is found to be around 10 % - 20 % of the retrapping current value. The ratio $I_{p0}/I_r < \sqrt{2}$ is quite constant (for a given sample) in the whole coolant temperature range. The heat removal capabilities are thus mainly defined by the transfer through the substrate with, at first approximation, a contribution of the heat conduction within the film that amounts to 20 % to 30 %. The electrical transport behavior observed on $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ strips [184] is enlightening to reinforce the latter interpretation. Indeed, the numerous dissipative states that are observed on these samples have a much wider current range of stabilization that leads to a ratio $I_{p0}/I_r > \sqrt{2}$ [or a ratio $(I_{p0}/I_r)^2 > 2$]. This is consistent with the experimentally observed temperature evolution of the thermal conductivity of metallic BiSrCaCuO crystals. Indeed, the latter materials present a significant increase of the thermal conductivity as the temperature is lowered below T_c , so that $\kappa_s > \kappa_n$ in a wide temperature interval⁶ [186, 187].

6. More generally, this so-called *bump anomaly* below the critical temperature is observed in the heat conduction of Cu-O plane superconductors, including the high temperature $\text{YBa}_2\text{Cu}_3\text{O}_7$ [185].

The previous discussion has shown that all the NbN samples are characterized by a ratio I_{p0}/I_r slightly above unity. For the sample SNbN-25-1 (see Fig. 3.7), it amounts to ~ 1.1 . At this low coolant temperature ($T_0/T_c \sim 0.17$), one can reasonably assumed that the electronic contribution to the thermal conductivity in the superconducting state is close to zero. Further assuming that the lattice contribution is mainly determined by the phonon-boundary scattering (so that κ_{ph} is considered unchanged between the superconducting and the normal states), the expression 3.6 is equivalent to

$$\left(\frac{I_{p0}}{I_r}\right)^2 \sim 1 + \frac{\kappa_s}{\kappa_n} \approx 1 + \frac{\kappa_{ph}}{\kappa_{e,n} + \kappa_{ph}}, \quad (3.7)$$

from which the lattice contribution of the thermal conductivity of the NbN microstrips appears to be around a fourth of the electronic contribution in the normal state, i.e., $\kappa_{ph} \sim 7.5 \text{ mWm}^{-1}\text{K}^{-1}$. This is of course a rough estimation which only gives an approximate idea of the relative contributions of both the conduction electrons and the phonons to the thermal conductivity of the NbN samples.

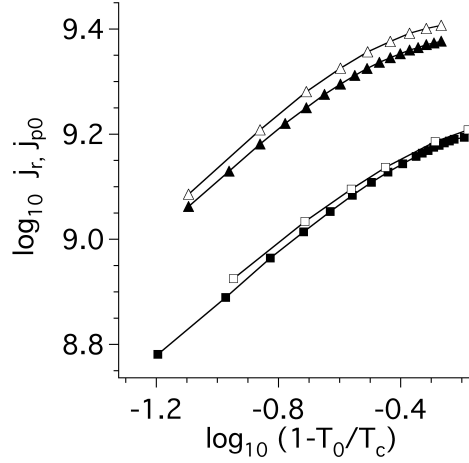


Figure 3.8: Log-log plot of the retrapping current density j_r (full symbols) and of the propagation current density j_{p0} (open symbols) as a function of $\left(1 - \frac{T_0}{T_c}\right)$ for two NbN samples of different normal state resistivity : SNbN-75-5 ($\blacksquare$, $\rho_n = 1285 \mu\Omega\text{cm}$) and SNbN-20-5 ($\blacktriangle$, $\rho_n = 675 \mu\Omega\text{cm}$). The slope of all the curves at temperature closed to the critical temperature is 0.5.

3.2.3 Simultaneous polarization of nanostructures in series

The obtention of a current-voltage characteristic usually consists in the application of the bias at the extremities of a sample and in the measurement of the resulting voltage using the nearest contacts on each side. Here, the multi-probe configuration also allows one to apply the bias at the extremities of several samples in series and to measure the resulting voltage simultaneously at different locations. The scanning electron microscopy image of Fig. 3.9(a) illustrates a typical 5-probe configuration which consists in applying the bias using the leads 2 and 4 and in measuring the voltages using the leads 1-3 and 3-5. This type of measurement gives complementary results, in both the current-biased and the voltage-biased regime. The latter mode is, however, the most interesting because the current that flows through the microstrips depends on the resistance levels of all the samples. This is illustrated in Fig. 3.9(b) and 3.9(d) with the comparison of the (forward) voltage-biased curves obtained in the 4-probe and in the 5-probe configurations for two microstrips : with a weak-link (SNbN-75-2, defined by the leads 3 and 4) and without a weak-link (SNbN-75-1, defined by the leads 2 and 3). The green dashed curves correspond to the 4-probe measurements while the red and blue curves correspond to the 5-probe measurements. The results obtained in the 5-probe configuration may be depicted in a different manner with the evolution of the sample resistance [Fig. 3.9(c)] and of the current [Fig. 3.9(e)] as a function of the total applied voltage, i.e., as a function of the sum of the voltages that develop at the extremities of the two microstrips. To understand these unintuitive results, one has to keep in mind that (i) the same current flows throughout the samples, (ii) when the applied voltage is increased, the current intensity is reduced each time the resistance of one of the samples increases and (iii) the samples areas close to the contact have a larger normal-zone propagating current than the microstrip. Having these three clues in mind, the curves of Fig. 3.9 are described as follows :

- (1) The first normal domain nucleates at the weak-link of the sample SNbN-75-2 when the current exceeds $29 \mu\text{A}$ (not shown). The current is reduced down to $12 \mu\text{A}$ due to the non-zero resistance level which develops in this segment. The sample SNbN-75-1 remains in the perfect superconducting state.
- (2) Increasing the voltage-bias leads to the extension of the first normal hotspot domain which propagates by successive jumps of its N/S interfaces. The correspondence in Fig. 3.9(b) is clear. At this stage, the 4-probe and the 5-probe configurations give similar results.

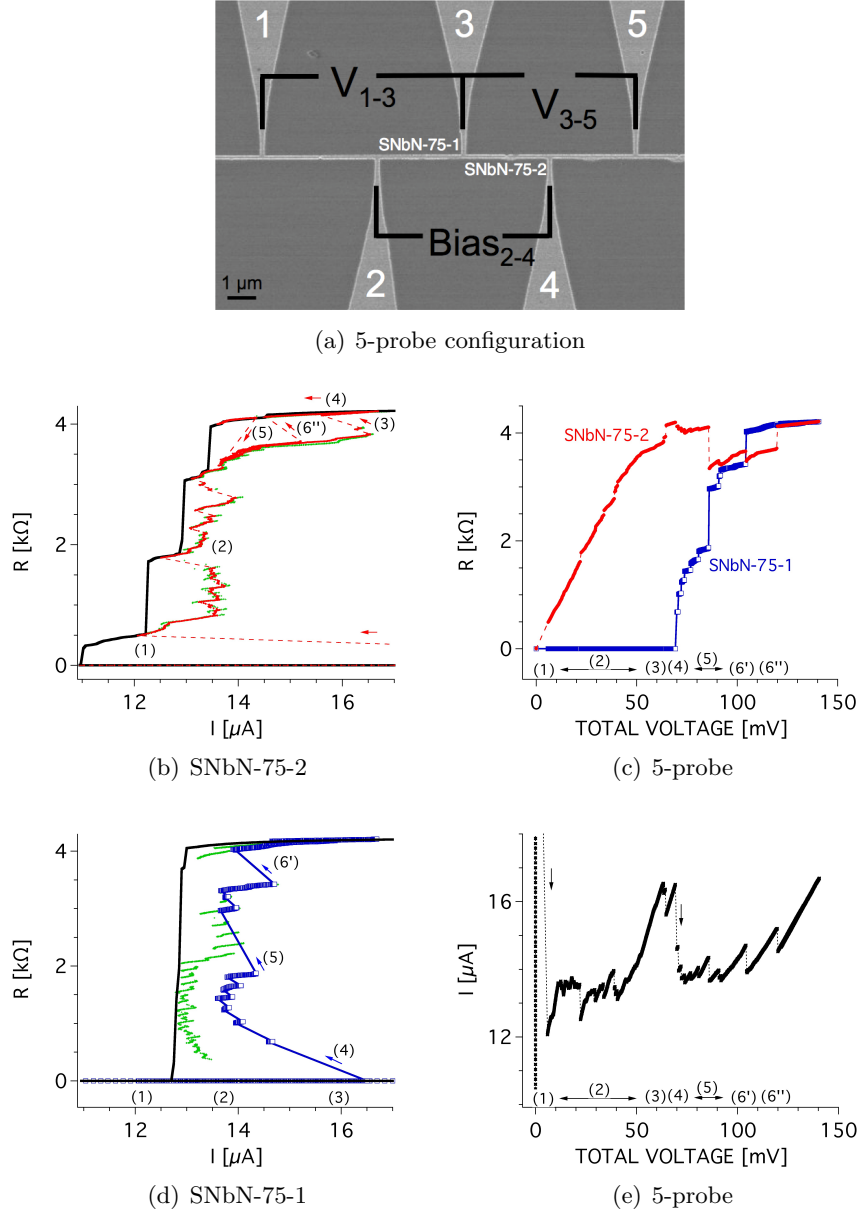


Figure 3.9: (a) SEM image of a 5-probe configuration which consists in applying the bias at the extremities of two strips in series so that the same current is flowing through them. In the 5-probe configuration, the leads 1 and 3 are used to measure the voltage at the extremities of SNbN-75-1, while the leads 3 and 5 are used for the sample SNbN-75-2. Forward voltage-biased curves of SNbN-75-2 (b) and SNbN-75-1 (d) obtained by applying the voltage in a 4-probe (green) and in a 5-probe (red or blue) configuration. Evolution of the resistance (c) and of the current (e) as a function of the total voltage that develops at the extremities of the two microstrips in series.

- (3) When the total applied voltage V_{bias} is around 60 mV, the microstrip SNbN-75-2 is in the normal state. The areas close to the contacts remain superconducting until the hotspot propagates through them due to a further increase of the bias. This is reflected by the small resistance increase at $V_{bias} \sim 65$ mV. The sample SNbN-75-1 is still in the zero-resistance state.
- (4) Given that the hotspot is close to the microstrip SNbN-75-1, a slight increase of the bias (and thus of the current) is sufficient to induce the propagation of the normal domain in this latter sample. This arises at $I \sim 16.5 \mu\text{A}$ ($V_{bias} \sim 70$ mV). The current is then reduced down to around $14 \mu\text{A}$ and the sample SNbN-75-2 remains in the same dissipative state than in point (3).
- (5) The subsequent increase of the voltage-bias leads to the stepwise propagation of the hotspot in the sample SNbN-75-1. It is particularly interesting to notice that the intermediate dissipative states are stabilized in the 5-probe configuration in a different range of the stabilization current interval than in the 4-probe measurement. This reflects once again that a voltage-biased measurement does not give access to the whole current range of stabilization. At $V_{bias} \sim 85$ mV, the large increase of the resistance of the sample SNbN-75-1 results in a reduction of the current below the minimum normal-zone existence current of the area close to the contact of the sample SNbN-75-2 (located at the opposite of the sample SNbN-75-1). As a consequence, the hotspot shrinks in sample SNbN-75-2 and its resistance is reduced.
- (6) A further increase of the bias leads to the expansion of the hotspot in the areas close to the extremities of the microstrip at $V_{bias} \sim 105$ mV [point (6'), sample SNbN-75-1] and $V_{bias} \sim 120$ mV [point (6''), sample SNbN-75-2]. From this point, increasing V_{bias} only results in an increase of the current that flows throughout both samples.

The above-described propagation of a normal domain that expands through two microstrips has been observed with at most four microstrips in series with no significant variations. The main features are always present, especially the nucleation and the propagation of a normal domain in the weakest segment and the impact of the *stronger* areas close to the contacts located between the strips which prevent the propagation of the hotspot onto the adjacent segment. This latter point also manifests itself in the current-biased regime insofar as the coolant temperature is sufficiently large in order to prevent the transition of both microstrips when the first normal domain nucleates. This is illustrated in

Fig. 3.10 with the forward part of the current-biased characteristic of the sample SNbN-75-1 and SNbN-75-2 obtained in the 5-probe configuration at $T_0 = 9.75$ K. At first, both samples lie in the perfect superconducting state. At $I \approx 6.25 \mu\text{A}$ (point 1), the instability current is reached at the weak-link of the segment SNbN-75-2. The instability current of the segment SNbN-75-1 is not exceeded so that it remains in the zero-resistance state. Further increasing the current bias leads to the propagation of the hotspot around the constriction (point 2') and through the main width (point 2''). At $I \approx 8.25 \mu\text{A}$, the area close to the contacts remains superconducting. Finally, the minimum propagating current of the area close to the contact is reached (point 3). This leads to the abrupt transition of the whole sample SNbN-75-1 as the applied current then exceeds its normal-domain propagating current.

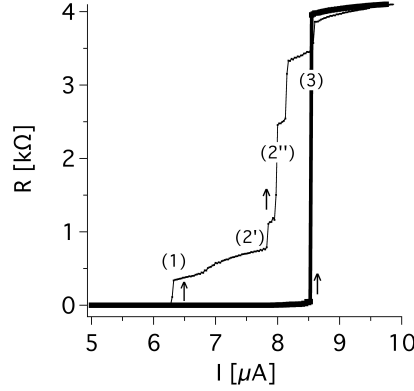


Figure 3.10: Forward part of the current-biased characteristic of the samples SNbN-75-1 (thick curve) and SNbN-75-2 (thin curve) in a 5-probe configuration, i.e., the current is swept simultaneously through the two microstrips in series ($T_0 = 9.75$ K).

3.3 Localization and propagation of the normal domains

3.3.1 Nanometric modulations of the sample cross-section

In the section 3.2, the boundaries of the current range of stabilization of the intermediate dissipative states have been related to the minimum current for the existence (I_r) and for the propagation (I_{p0}) of a normal domain. Another important feature of the hotspot plateau of the NbN submicrometer-wide strips is that these both characteristic thermal cur-

rents do not necessarily take a single value, i.e., the retrapping current as well as the propagating current spread over a well-defined interval.

3.3.1.1 Stepwise retrapping and propagating transitions

The spreading of the currents I_r and I_{p0} is once more illustrated in Fig. 3.11(a) with the enlargement of the hotspot plateau of the sample SNbN-20-6 (the green dashed curve and the continuous black curve correspond to the voltage- and the current-biased measurements, respectively).

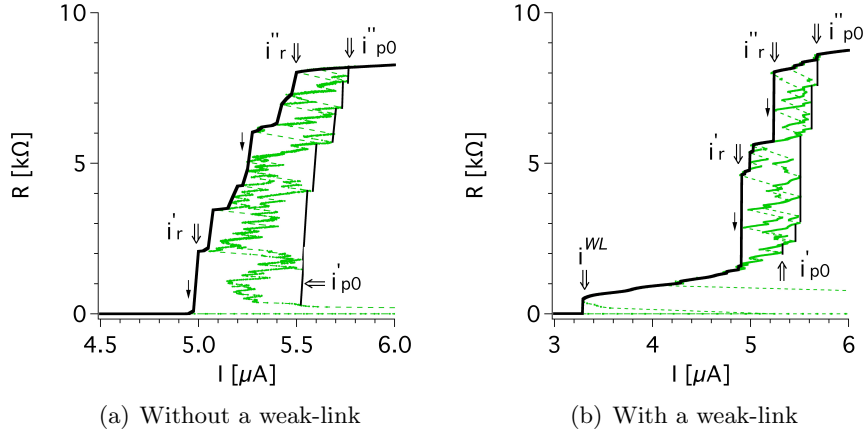


Figure 3.11: Hotspot plateau of the segment SNbN-20-6 (a) and SNbN-20-5 (b) at $T_0 = 4$ K. The voltage-biased characteristic (dashed green) and the current-biased retrapping transition (black continuous) are shown. The currents I_r'' and I_p0'' mark the beginning and the end of the current-driven normal-to-superconductor transition, while I_r' and I_p0' mark the beginning and the end of the current-driven superconductor-to-normal transition, while I_{WL} indicates the characteristic current of the weak-link. The thin solid lines are guides for the eyes used to point the spreading between the currents I_p0' and I_p0'' .

The spreading of the left and right boundaries of the stabilization interval are here denoted by $[I_r', I_r'']$ and by $[I_{p0}', I_{p0}'']$, respectively. With the reasonable assumption that the thermal properties are uniform on the whole sample length, the current densities j_r ($\equiv j_{m0}$) and j_{p0} are expected to be independent of the location. A normal domain in a sample with a perfectly uniform cross-section would thus be characterized by a *single* minimum existence current and by a *single* minimum propagation current. As a consequence, the most convincing explanation of the observed spreadings of I_r and I_{p0} is that the cross-section is not uniform along the whole sample length. This assumption is corroborated by the electrical behavior of a strip that contains a strong constriction. The figure 3.11(b) shows the hotspot plateau of the sample SNbN-20-5,

which only differs from the sample SNbN-20-6 by a weak-link intentionally placed at its middle. The current intervals $[I'_r, I'_{p0}]$ and $[I''_r, I''_{p0}]$ fall in the same range for both samples, i.e., between $5 \mu\text{A}$ and $5.5 \mu\text{A}$. They correspond to the part of the segments with a plain width. The main modification is observed in the low-resistance part of the V-I and I-V curves, which is broadened down to the current $I_{WL} \ll I'_r$ ⁷. The clear reduction of the weak-link retrapping current supports the assumption of a fixed current density j_r and the subsequent spreading of the thermal currents. Indeed, the ratio between the mean retrapping current $\frac{I''_r + I'_r}{2}$ and the current I_{WL} amounts to ~ 1.54 , which is very close to the ratio $W/W_{WL} \sim 1.57$ between the sample width and the weak-link width.

Table 3.2: Ratio between the mean retrapping current of the segment $[(I''_r + I'_r)/2]$ and the retrapping current of the weak-link (I_{WL} , if present) and ratio between the sample width (W) and the weak-link width (W_{WL}). Normalized spreading of the minimum normal-zone existence and propagation currents ($\Delta I_{r,p0}$) and the associated variation of the sample width (ΔW).

Sample	$\frac{(I''_r + I'_r)/2}{I_{WL}}$ [-]	$\frac{W}{W_{WL}}$ [-]	$\Delta I_{r,p0}$ [%]	ΔW [nm]
SNbN-20-1	1.8	2	16	11
SNbN-20-2	-	-	18	12
SNbN-20-3	1.87	1.83	9	10
SNbN-20-4	-	-	7	8
SNbN-20-5	1.55	1.57	6	7
SNbN-20-6	-	-	9	10
SNbN-25-1	1.75	1.71	12	14
SNbN-25-2	-	-	7	10
SNbN-75-1	-	-	7	8
SNbN-75-2	1.23	1.64	10	11
SNbN-75-3	1.27	1.67	7	9
SNbN-75-4	-	-	5	9
SNbN-75-5	1.16	1.88	5	8
SNbN-75-6	1.15	1.88	3	5

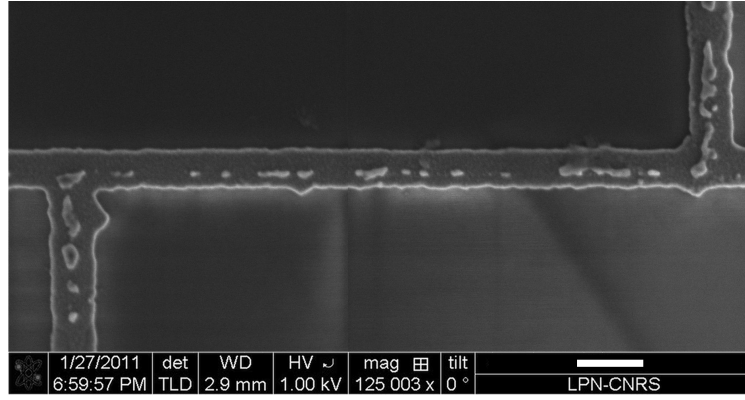
The table 3.2 summarizes the ratios $\frac{(I''_r + I'_r)/2}{I_{WL}}$ and $\frac{W}{W_{WL}}$ for the NbN microstrips that contain a weak-link. These ratios take similar values

7. Starting from the fully normal state, the last region which remains in the dissipative state is located where the applied current density (and thus the local dissipation) is maximum, i.e., around the weak-link.

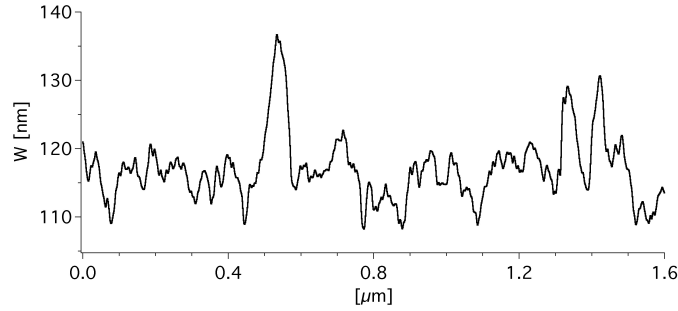
for the thinnest samples. However, one observes a large departure for the thickest ones. One explanation for such discrepancy lies in the assumption that the sample thickness is constant. Indeed, it is well known that the roughness of a columnar microstructure increases with the film thickness [131]. In addition, the estimation of the sample width by scanning electron microscopy was sometimes not performed on the measured strips, but on neighboring strips obtained during the same fabrication process. The widths used in the calculations may thus vary by several percents from the true ones. In addition, a convenient way to describe the spreading of the retrapping current is to normalize the difference $I_r'' - I_r'$ with respect to the mean value $(I_r'' + I_r')/2$ [the same procedure applied for I_{p0} gives identical results]. As summarized in Tab. 3.2, all the obtained values ($\Delta I_r \approx \Delta I_{p0}$) range between 3 % and 18 %, depending on the samples. If one assumes a uniform thickness, the spreading mainly results from the modulations of the sample width. As a consequence, it is more suitable to normalize the spreading of the thermal current using their corresponding width modulations $\Delta W = \Delta I_{r,p0} \times W$ which take similar values for all the samples with an average value of 10 nm.

These estimations of the width modulations obtained by electrical transport measurements are corroborated by a careful study of the scanning electron microscope images. Indeed, the latter reveal that some width modulations are present along the sample edges. This is illustrated in Fig. 3.12 with a representative strip fabricated at the same time that the sample SNbN-25-1 (the scalebar represents 200 nm). In order to quantify the width fluctuations more easily, the image is first transformed into a white and black image using a grayscale threshold. The evolution of the width is then monitored at each position along the sample. The resulting profile is depicted on Fig. 3.12(b). Numerous width modulations are observed through the whole sample length. Their mean grain size amounts to around 10 - 15 nm⁸, which is very close to the values extracted from the spreading of the retrapping current. In addition, the estimation is close to the mean grain size of the NbN films - see the AFM image of Fig. 2.5 (page 72). These nanoscale width modulations probably result from the roughness of the alumina mask that defines the desired pattern on top of the thin film during the fabrication process. Indeed, the resolution of the resist that defines the mask is comparable to the width modulations that are observed here [see the section 2.2 (page 78) for additional details on the resolution of the lithographic process]. The modulations of the sample width may also result

8. Similar width variations of ~ 10 nm have been observed by Marsili et al. on their 100 nm-wide NbN strips [188].



(a)



(b)

Figure 3.12: (a) SEM image of a representative NbN submicrometer-wide strip (length : $2 \mu\text{m}$, width : 120 nm). The scalebar represents 200 nm . (b) Modulation of the width of the sample shown in the part (a). The SEM image was first transformed into a white and black image using a grayscale threshold. The width profile was then monitored at each position along the sample. The mean size of the width modulations amounts to $10 - 15 \text{ nm}$, which is close to the typical grain size of the thin film.

from the etching of the NbN grains during the subsequent step of the fabrication process. Indeed, especially due to the chemical nature of the etching, this is conceivable to consider that the high-energy ions from the reactive plasma mainly damage the weakest regions of the thin films, so that the etching is more efficient at the grain boundaries. The first explanation, however, appears to be the dominant one.

3.3.1.2 Similarities between the temperature- and the current-driven retrapping transitions

In the usual current-biased measurements, the direct current is swept while the coolant temperature T_0 is maintained constant during the

whole experiment. In these conditions, it was previously shown that the voltage (or resistance) steps of the current-driven transition from the normal state to the superconducting state are intimately related to the nanoscale cross-section modulations. From Eq. 3.1 (page 108), one sees that the heat balance equation may be altered by a modification of the applied current or by a modification of the coolant temperature T_0 . Indeed, varying the current modifies the heat dissipation term, while varying the coolant temperature modifies the heat removal term. This means that, at a constant applied dc current, the variations of the cross-section may also result in a spreading of the coolant temperature at which the power balance is locally fulfilled. As a consequence, sweeping the coolant temperature T_0 at a sufficiently large applied current is strictly equivalent to the sweep of the current at a fixed T_0 [33]. This is illustrated in Fig. 3.13 with the temperature-driven and the current-biased retrapping transition of the sample SNbN-20-2 [for clarity, the curves are shown separately in Fig. 3.13(a) and 3.13(b) and their superposition is given in Fig. 3.13(c)]. On one hand, the current-biased experiment was performed at a fixed coolant temperature $T_0 = 7.5$ K. The beginning and the end of the retrapping transition are denoted by I_r'' and I_r' , respectively. On the other hand, the temperature-driven retrapping transition was measured at a constant current $I = 2.5 \mu\text{A}$, which is close to the mean retrapping current $(I_r'' + I_r')/2$ at 7.5 K.

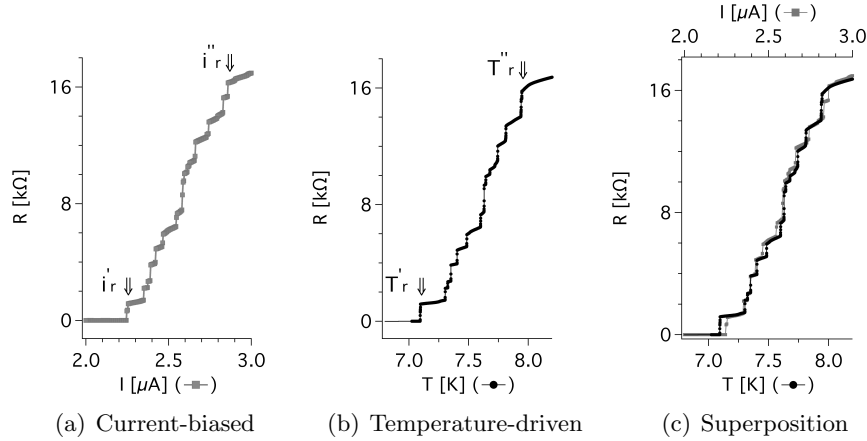


Figure 3.13: Current-biased (a) and temperature-driven (b) transition from the normal state to the superconducting state of the sample SNbN-20-2 obtained at a fixed coolant temperature ($T_0 = 7.5$ K) and at a constant applied current ($I = 2.5 \mu\text{A}$), respectively. The part (c) highlights that both curves nicely match each other.

One sees that the stepwise evolutions of the resistance are very similar. In particular, both curves appear to match together on the whole current or temperature range. This means that the heat dissipation and the heat removal are locally identical at the extremities of the shrinking normal domain hotspot. In addition, the major feature is that the spreading of the retrapping current $I_r'' - I_r'$ is quantitatively equivalent to the temperature spreading $T_r'' - T_r'$. Indeed, considering the difference between the heat balance (Eq. 3.1) at the beginning and at the end of the retrapping transitions gives the expression

$$\rho_n \left[\left(\frac{I_r''}{S''} \right)^2 - \left(\frac{I_r'}{S'} \right)^2 \right] = \frac{h}{d} (T_r'' - T_r'), \quad (3.8)$$

where S'' and S' denote the cross-sections that correspond to the current I_r'' and I_r' , respectively. From the measured values of $I_r'' = 2.86 \mu\text{A}$ and of $I_r' = 2.24 \mu\text{A}$ and using $S'' \approx S' = W \times d$, one obtains from Eq. 3.8 that $(T_r'' - T_r') = 0.73 \text{ K}$, in satisfactory agreement with the measured value that amounts to 0.84 K .

3.3.2 Hotspot propagation mechanisms

The stabilization of the normal domains is thus made easier by the nanoscale modulations of the cross-section which prevent the motion of the hotspot boundaries⁹. Each voltage step in the forward part of the V-I and of the I-V curves corresponds to the appearance of a newly localized resistive domain or to the extension of an existing one (the same features in the backward part correspond to the disappearance or the contraction of an existing resistive domain). However, an accurate description of the mechanism of the normal domains propagation further requires to distinguish between both biasing modes (i.e., applying the voltage or the current) because the bias noticeably impacts the way by which hotspots evolve.

In the current-bias regime, it is likely that a single hotspot nucleates at the weakest location and then expands by successive jumps of its N/S interfaces. Let's consider a hotspot whose length is smaller than the whole sample length, and which remains stable insofar as the propagation current of its boundaries are slightly larger than the applied current (i.e. $I < I_{p0}(x_0)$, with x_0 the location of the boundary whose propagation current is the smallest). An increase of the current above

9. The localization of normal hotspot domains due to the presence of inhomogeneities was explained in details in the section 1.2.3 (page 27).

$I_{p0}(x_0)$ leads to the propagation of the N/S interface until a neighboring inhomogeneity for which $I_{p0}(x'_0) > I$. The propagation of the hotspot by discrete jumps of these boundaries reoccurs each time the applied current exceeds the local propagation current. The situation is similar at decreasing current. In this case, the hotspot boundary remains stable insofar as the applied current is slightly larger than the local retrapping current. A reduction of the applied current below $I_r(x_0)$ makes the boundary unstable, so that the normal domain shrinks until a neighboring inhomogeneity for which $I_r(x'_0) < I$. In both cases, the successive hotspot lengths are defined by the profile of the nanoscale modulations (location as well as longitudinal extension). The latter play a relevant role in the definition of the voltage steps observed in the V-I curves.

As in the current-biased regime, the stabilization of localized normal domains in the voltage-biased mode is significantly affected by the cross-section modulations profile. However, applying the voltage-bias with a small resistor in parallel with a dc current source results in variations of the current that flows through the sample each time its resistance varies. The experimental setup has thus an impact on the details of the I-V curves. Once again, let's consider a hotspot whose length is smaller than the whole sample length. At increasing dc voltage, the hotspot becomes unstable if the propagation current at one of its boundaries is exceeded [$I > I_{p0}(x_0)$]. Its expansion results in an increase of the sample resistance and thus to a reduction of the current which is imposed by the load line of the experimental setup. This current reduction is such that the condition $I_{p0}(x'_0) > I$ may be fulfilled at a neighboring inhomogeneity which is located much closer than in the current-biased regime. For this reason, more states can be stabilized in the forward part of the voltage-biased regime than in the forward part of the current-biased regime. In the retrapping transition (i.e., at decreasing voltage-bias), the current that flows through the sample increases just after the hotspot contracts. The condition $I_r(x'_0) < I$ may thus be fulfilled at a location x'_0 closer to the initial position of the unstable hotspot boundary. As a consequence, there are potentially a much higher number of discrete hotspot lengths that can be probed in the voltage-biased regime than in the current-biased one. This is in nice agreement with Fig. 3.7(b) (page 118), where the number of intermediate states in the current-biased regime (red curve) is much smaller than in the voltage-biased mode (green curve). However, in the voltage-biased regime, the sample geometry is not sufficient by itself to address the issue of the hotspots configurations, especially because it requires to distinguish between a single or many localized hotspots. Indeed, applying the voltage is well-known to be the most favorable way to stabilize a large number of hotspots whose

length may be smaller than the sample width. In particular, Eichele et al. [58, 59] have shown that a multiplicity of hotspots is favored for samples with relatively small thermal healing length because the interactions between neighboring hotspots is weak and the coalescence is less probable¹⁰.

This condition is met in the NbN strips given that $\eta \approx 40 - 70$ nm. In addition, η strongly impacts the length of a newly created dissipative zone. On one hand, it roughly defines half the minimum size of a hotspot, in direct relation with the nucleation of a normal domain. On the other hand (i.e., the extension or the contraction of an existing hotspot), one has to remember that the temperature gradient at the N/S interfaces weakens the superconductivity on a length-scale given by the thermalization length. The localized inhomogeneity that pins the moving N/S boundary is thus expected to be at most at a distance η from the previous location of the boundary. Furthermore, the smallest difference of resistance between two nearest dissipative states is easily determined in the voltage-biased regime and appears to noticeably varies from one NbN sample to another, ranging from $\Delta R = 60 \Omega$ to 200Ω . Assuming that each normal domain expands across the whole width, it appears more convenient to express the gap between two nearest states in terms of an equivalent resistive sample length (here denoted $L_{\Delta R} = \Delta R \times \frac{Wd}{\rho_n}$), which amounts to around 50 nm, independently of the cross-section dimensions. The most consistent explanation is that $L_{\Delta R}$ reflects the spacing between two neighboring inhomogeneities, which is expected to be similar from one sample to the other. However, it is striking to realize that $L_{\Delta R}$ is very similar to the estimated thermalization length η so that the presence of a multiplicity of localized hotspots can not be neglected. Unfortunately, it is rather difficult to confirm a scenario at the expense of the other.

3.4 Continuous microwave irradiation

The two previous sections have been mainly devoted to the description of the stepwise electrical characteristics of the NbN submicrometer-wide strips. By means of an additional source of dissipation, the existence of intermediate dissipative states will now be discussed for both materials : NbN and Nb¹¹. The main advance of the present section is that the *purely thermal* explanation based on the stabilization of normal

10. See the section 1.2.3, page 27.

11. The reproducibility of the experiments performed under a continuous irradiation has been checked for both materials.

hotspot domains is totally corroborated, both qualitatively and quantitatively. In particular, it will be demonstrated that the evolution of the retrapping current with the input microwave power is a powerful tool to determine the density power effectively absorbed by the strips when they are in the normal state.

At first, the evolution of the hotspot plateau under a continuous microwave irradiation is described (Sec. 3.4.1). The theoretical background required to extract the power density absorbed by the strips as well as the quantitative treatment of the data are presented in Sec. 3.4.2. Finally, the ability to stabilize intermediate dissipative states in both parts of the current-driven characteristics is highlighted in Sec. 3.4.3.

3.4.1 Microwave dependence of the hotspot plateau

The figure 3.14 illustrates the impact of a microwave irradiation on the current-driven transition from the normal state to the superconducting state of the sample SNb-75-1. The curves were obtained at two microwave frequencies, i.e., at 8.9 GHz (a) and at 11.3 GHz (b). The coolant temperature is $T_0 = 6$ K. At this coolant temperature, the retrapping transition without a microwave irradiation arises at around $70 \mu\text{A}$ (see the farthest blue curve on the right). For both frequencies, the hotspot plateau is shifted to lower values when the input microwave power level increases, up to 1 mW at 8.9 GHz and up to 6.3 mW at 11.3 GHz¹². In other words, the current range of stabilization of a resistive domain is shifted to lower values when the input power is increased. Above an input power threshold (between 0.4 mW and 0.63 mW at 8.9 GHz and between 1 mW and 1.58 mW at 11.3 GHz), a normal domain remains stable down to zero applied current. This microwave-dependence of the hotspot plateau is similarly observed on the NbN microstrips insofar as the input power is sufficiently large. This is illustrated in Fig. 3.15 for the sample SNbN-75-1 ($T_0 = 6.8$ K, freq : 7 GHz) and SNbN-75-5 ($T_0 = 1.5$ K, freq : 7.5 GHz), the latter containing an intentionally placed constriction in its middle. The y-axis is displayed in a logarithmic-scale in order to better distinguish between all the intermediate dissipative states. At the studied coolant temperatures, the retrapping current without a microwave irradiation amounts to $11.7 \mu\text{A}$ and to $28.5 \mu\text{A}$ in

12. The total input power delivered by a microwave source is most of the time described using the dBm unit. Contrary to the decibel (dB), the dBm represents an absolute power which corresponds to the power ratio in dB referenced to one milliwatt, i.e., $P_{IN} [\text{dBm}] = 10 \times \log_{10} \left(\frac{P_{IN} [\text{mW}]}{1 \text{ mW}} \right)$. Although all the experiments have been performed by monitoring the input power in dBm, it is more convenient for the discussion to express the microwave input power in mW.

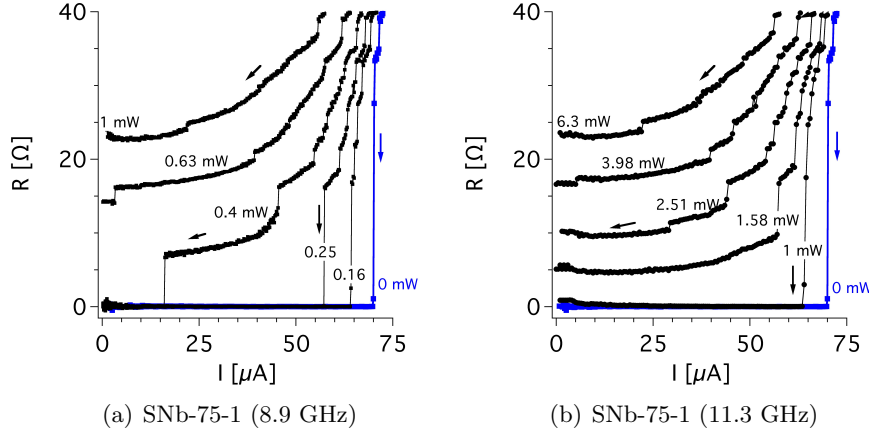


Figure 3.14: Evolution of the normal-to-superconducting transition of the nanostrip SNb-75-1 under an increasing microwave power ($T_0 = 6$ K). The microwave frequency is 8.9 GHz (a) and 11.3 GHz (b). The microwave input powers are indicated in mW.

Fig. 3.15(a) and 3.15(b), respectively (see the farthest blue curves on the right). Once again, a progressive shift of the hotspot plateau to lower currents is observed when the input microwave power increases. In addition, zero-crossing resistance steps (i.e., there is a finite resistance at bias current close to zero) are observed when the input power level exceeds a particular threshold.

For both materials, the main features of the microwave-dependence of the hotspot plateau (i.e., the shift to lower currents and the stabilization of normal domains at zero applied current) are qualitatively described by considering a local heating of the normal hotspot domains due to the microwave irradiation. First, one has to remember that a normal domain is stable insofar as the total dissipation (Joule dissipation and microwave dissipation) balances the heat removal capabilities. As a consequence, less Joule dissipation due to the direct-current is required to stabilize a normal domain inside the strip when the input microwave power increases (and thus the power density absorbed by the sample). Accordingly, the retrapping current is continuously shifted to lower currents when the microwave power increases. This is consistent with the decrease of the superconducting recovery current observed, e.g., in superconducting Al strips when the total microwave power increases [189]. Second, the microwave power threshold above which the zero-crossing resistance steps are observed basically corresponds to the point above which the local heating of the irradiation exceeds the heat removal capabilities, so that no applied current is required to stabilize a normal

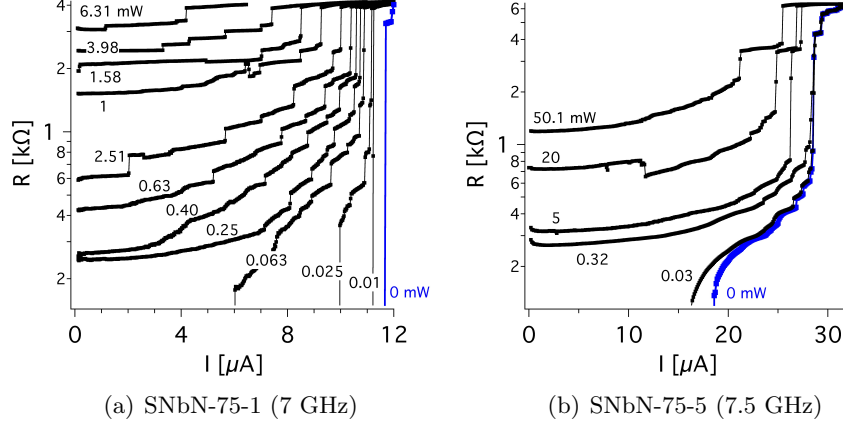


Figure 3.15: Normal-to-superconducting transition of the nanostrips SNbN-75-1 and SNbN-75-5 under an increasing microwave power [(a) : 7 GHz, $T_0 = 6.8$ K and (b) : 7.5 GHz, $T_0 = 1.5$ K]. The microwave input powers are indicated in mW.

domain. This was already mentioned by Skocpol et al. [44] that wrote that « *when I_{MW} (personal note : the average root mean square of the equivalent ac current generated by the microwave source) exceeds the minimum current required to sustain a hotspot, the microbridge latches into the normal state at all dc levels* ». In other words, if the microwave power absorbed by the conduction electrons exceeds the power required to stabilize a N/S interface (or if the temperature of the conduction electrons raises above T_c due to the microwave power alone), the dc current that has to be applied to stabilize a normal domain becomes zero.

From the figures 3.14 and 3.15, one sees that the input power levels involved in the measurements noticeably vary depending on both the coolant temperature and on the microwave frequency. The impact of the frequency is further highlighted in Fig. 3.16, which shows the superposition of the hotspot plateau of the sample SNb-75-1 [see Figs. 3.14(a) and 3.14(b)] obtained at two frequencies and at the same coolant temperature. The curves nicely match together, either in the whole current range (e.g., for $P_{IN}^{11.3GHz} = 6.3$ mW and $P_{IN}^{8.9GHz} = 1$ mW) or in a quite large current range (e.g., for $P_{IN}^{11.3GHz} = 2.51$ mW and $P_{IN}^{8.9GHz} = 0.4$ mW). Although the input microwave powers differ for both frequencies, one sees that the ratio between the input power levels at 11.3 GHz and at 8.9 GHz is close to 6.3 for each pair of superimposed curves. The correspondence between the curves means that the local microwave dissipation is similar for each pair of current-biased curves. The different input powers required to obtain a similar local dissipation is most proba-

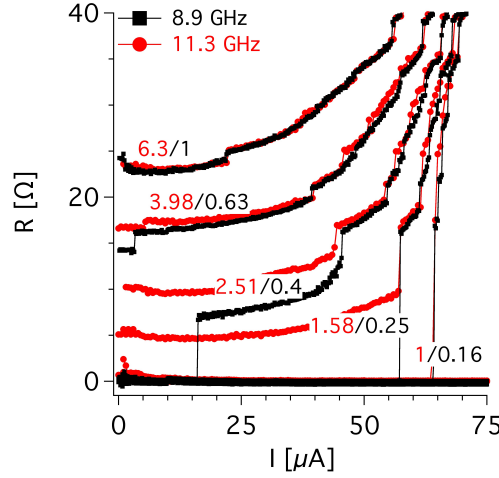


Figure 3.16: Superposition of the microwave dependence of the hotspot plateau of the sample SNb-75-1 at two microwave frequencies : at 8.9 GHz [black curves - see Fig. 3.14(a)] and 11.3 GHz [red curves - see Fig. 3.14(b)]. The coolant temperature is 6 K in both cases. The microwave input powers are indicated in mW.

bly related to the frequency-dependent portion of the input microwave power which is reflected back to the source. Indeed, the imperfect matching between the line transmission and the antenna is such that a finite portion of the total input power never reaches the experimental chamber¹³. As a consequence, one may consider that the fraction of the total microwave input power which is transmitted by the antenna is around six times smaller when the applied frequency is 11.3 GHz than when the frequency is 8.9 GHz. Larger input power levels are thus needed at 11.3 GHz to generate the same microwave heating. Such features highlight that (i) this is not the absolute value of the input microwave power which is relevant but the microwave power effectively absorbed by the samples and (ii) that the linear relation between the absorbed microwave power and the input level can noticeably differs from one applied frequency to another. This is of course a relevant point which emphasizes once more that comparing the microwave dependence of a particular property at several frequencies requires to determine the fraction of the input power which is effectively transmitted by the antenna and, then, which is absorbed by the samples. For this purpose, a quantitative analysis of the results is given in the next section.

¹³. See the figure 2.18 (page 92) which illustrates the evolution of the return loss of the home-made antenna as a function of the applied frequency. Let's remind that the transmission line and the antenna are well matched when the return loss is high.

3.4.2 Determination of the power density absorbed by the normal domains

3.4.2.1 Adaptation of the localized hotspot model

Skocpol et al. have extended their hotspot model in order to include the effect of the microwave irradiation on the direct current-voltage characteristics of superconducting microbridges [44]. The microwave field was considered equivalent to an ac current source with an average root mean square current I_{MW} . Assuming that the bridge responds only to an average heating of the microwaves, the term I_{MW}^2 was taken proportional to the microwave source input power. The impact of the microwaves is thus equivalent to an extra dissipation that adds to the Joule heating power that results from the direct current. The exact coupling between the total microwave power and the superconducting bridge is here unknown. As a consequence, it appears more convenient to reformulate the SBT model in order to express the minimum normal-zone propagating current as a minimum power density dissipation. The equation 1.19 (valid inside the normal domain) is rewritten in units of a density power [Wm^{-3}] on the form

$$\begin{aligned}\rho_n j_{p0}^2(x_0) &= \frac{h(T_c - T_0)}{d} \left[1 + \sqrt{\frac{\kappa_s}{\kappa_n}} \times F(x_0, \eta, L) \right], \\ &= K_{x_0}(T_0),\end{aligned}\tag{3.9}$$

where the function $F(x_0, \eta, L)$ brings together the terms related with the characteristic lengths of the model, namely the position x_0 of the N/S interface, the total length L and the thermal healing length η (assuming $\eta_n = \eta_s$). In the absence of microwaves, Eq. 3.9 describes the balance between the power dissipated by the direct-current Joule effect and the power removed outside the hot area, the latter being equivalent to the power required to stabilize a N/S interface. The notation $K_{x_0}(T_0)$ is introduced to emphasize that, for a given sample and a given hotspot length, the right-term of Eq. 3.9 only depends on the coolant temperature. A major feature is that the minimum dissipation $K_{x_0}(T_0)$ in the vicinity of the hotspot boundaries can be experimentally determined without any estimation of the thermal parameters because only ρ_n and j_{p0} are required. Furthermore, it can be considered constant at a given bath temperature T_0 . This is crucial in order to extend the above-described power balance by considering the alternating currents generated by the microwave irradiation inside the strip. The power balance may reflect this additional contribution by including the microwave power density

absorbed by the sample in the normal state (P_{MW}). This is done in the left-term of Eq. 3.10,

$$\rho_n j_{p,MW}^2(x_0) + P_{MW} = K_{x_0}(T_0), \quad (3.10)$$

where $j_{p,MW}(x_0)$ is the minimum normal-zone propagating current density at the hotspot boundaries when a microwave is applied. In practice, it is more convenient to only consider the heat transfer to the substrate for which a N/S interface remains stable. Indeed, the minimum normal-zone existence current I_{m0} (or $I_{m,MW}$ if a microwave is present) is more accurately - and easily - determined as it is directly given by the current-induced transition from the normal state to the superconducting state. This leads to a slightly modified expression of the Eq. 3.10 given by

$$\rho_n j_{m,MW}^2(x_0) + P_{MW} = K'_{x_0}(T_0), \quad (3.11)$$

where $j_{m,MW} (\equiv j_{r,MW})$ is now the minimum normal-zone existence current density at the hotspot boundaries when a microwave is applied and $K'_{x_0}(T_0) \equiv \rho_n j_{m0}^2$ corresponds to the experimentally determined minimum dissipation required to sustain a normal domain without a microwave irradiation. In equations 3.10 and 3.11, the density power P_{MW} is related to the electromagnetic energy that heats up the quasiparticles at the extremities of the normal domains configuration of total length $2x_0$ (by extension, it describes the microwave absorption inside the whole normal region). For a small radiation photon energy $\hbar\omega$ compared to (twice) the superconducting gap 2Δ , the paired electrons located far away from the N/S interfaces are not perturbed by the irradiation. No new quasiparticle is created in the superconducting regions at moderate levels of the incident power [19]. Only the existing quasiparticles move up to the higher energy levels of the excitation spectrum, leading to a nonequilibrium distribution of quasiparticles inside the normal domains.

The knowledge of the retrapping current densities j_{m0} (without irradiation) and $j_{m,MW}$ (with irradiation) is thus sufficient to determine the microwave power density P_{MW} absorbed at each input microwave power P_{IN} . Indeed, P_{MW} is given by the expression

$$P_{MW} = \rho_n [j_{m0}^2(x_0) - j_{m,MW}^2(x_0)], \quad (3.12)$$

which is derived from Eq. 3.11. In practical terms, one has to fix a resistance level between the perfect superconducting state and the normal

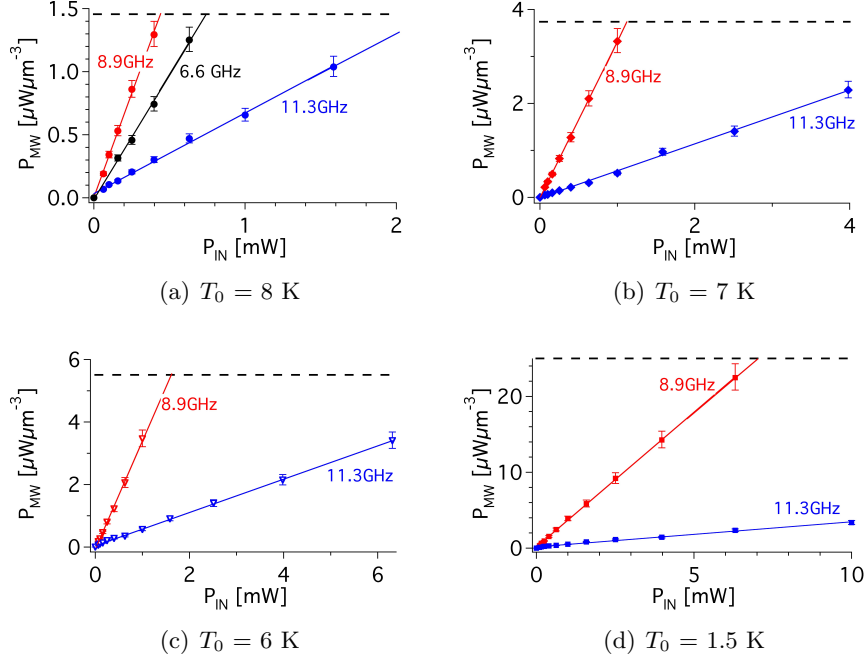


Figure 3.17: Evolution of the absorbed microwave power density with respect to the input power P_{IN} for the microstrip SNb-75-1. The data are depicted at four coolant temperatures : $T_0 = 8$ K (a), 7 K (b), 6 K (c) and 1.5 K (d). The microwave frequencies are 8.9 GHz and 11.3 GHz, except at $T_0 = 8$ K where the data are also displayed at 6.6 GHz. The dashed horizontal lines mark the heat evacuation capabilities level at the studied coolant temperature. The error bars take into account that the local sample width used in Eq. 3.12 may differ from the nominal one by ± 5 nm.

state in order to follow the evolution of the retrapping current of equivalent N/S interfaces (i.e., equivalent hotspot length). Moreover, each resistive state of the hotspot plateau has been previously related to the stabilization of the normal domain boundaries at particular locations along the sample length. Therefore, considering a resistance criterion R_{crit} implicitly means that one focuses on N/S interfaces located at a sample width which slightly differs from the nominal one. As the exact width profile is not known with precision, the absorbed microwave power densities are extracted from Eq. (3.12) using the nominal width as a reference. The nanoscale modulations thus lead to a small uncertainty in the obtained values. The same conclusion holds for the choice of R_{crit} (the results slightly depend on the resistance criterion but were checked to be very close for different criteria). Through the continuous shift to lower currents of the retrapping current, the use of electrical transport

measurements to determine the microwave power density absorbed by the normal domains represents the major result of this section.

The figure 3.17 shows the evolution of the absorbed microwave power density with respect to the power delivered by the microwave generator for the sample SNb-75-1. The data extracted from the experimental current-voltage curves are depicted at four coolant temperatures ($T_0 = 1.5$ K, 6 K, 7 K and 8 K) for microwave frequencies of 8.9 GHz and 11.3 GHz (apart from the curves at $T_0 = 8$ K for which the data are also available at a microwave frequency of 6.6 GHz). The resistance criterion is $R_{crit} = 30 \Omega$ and the error bars represent the deviations associated to the width modulations by considering that the width is ± 5 nm around the nominal one (let's remind that the nanoscale width modulations amount to ~ 10 nm). The relative error is around 15 % on the whole microwave power range. In each case, P_{MW} increases linearly with the input power, which is a major feature in order to validate the consistency of the procedure [190]. In addition, the fact that the slope depends on the microwave frequency [see e.g. the figure 3.17(a)] nicely indicates that the local heating of the normal domains does not depend on the absolute value of the input power, but on the fraction of the total power which is effectively transmitted by the antenna. A linear dependence is also observed for the NbN microstrip, as illustrated at the figure 3.18 for the sample SNbN-75-1 ($T_0 = 6.8$ K, freq : 7 GHz). The resistance criterion is $R_{crit} = 3.2$ k Ω .

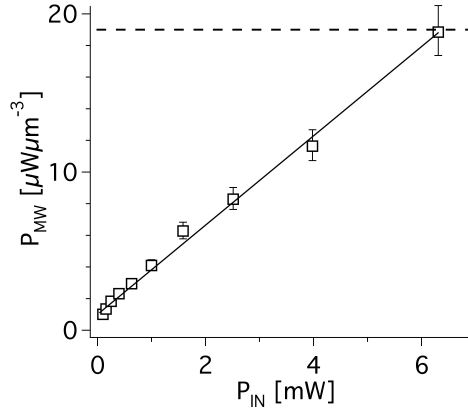


Figure 3.18: Evolution of the absorbed microwave power density with respect to the input power P_{IN} for the NbN sample SNbN-75-1 at $T_0 = 6.8$ K (freq : 7 GHz). The dashed horizontal line marks the heat evacuation capabilities level at the coolant temperature. The error bars take into account that the local sample width used in Eq. 3.12 may differ from the nominal one by ± 5 nm.

3.4.2.2 Relation to the heat removal capabilities

In Figs. 3.17 and 3.18, the horizontal dashed lines mark the value of $K'_{x_0}(T_0) \equiv \rho_n j_{m0}^2$, namely the heat removal capabilities at the studied coolant temperature. It describes the largest microwave power density that can be detected at these fixed coolant temperatures using the chosen resistance criterion. $K'_{x_0}(T_0)$ can thus be viewed as the upper bound of the detection range. Even though the extracted values of P_{MW} are of the same order of magnitude for both Nb and NbN (i.e., in the $\mu\text{W}\mu\text{m}^{-3}$ -range), one notices that $K'_{x_0}(T_0)$ differs between both materials. Indeed, $K'_{x_0}(T_0)$ increases from $1.5 \mu\text{W}\mu\text{m}^{-3}$ at 8 K to $25 \mu\text{W}\mu\text{m}^{-3}$ at 1.5 K for the Nb sample SNb-75-1, while, for the NbN sample SNbN-75-1, $K'_{x_0}(T_0)$ varies between $19 \mu\text{W}\mu\text{m}^{-3}$ at 7 K and around $70 \mu\text{W}\mu\text{m}^{-3}$ at 1.5 K. For these particular Nb and NbN microstrips, the heat removal capabilities $K'_{x_0}(T_0)$ thus increases by $3.6 \mu\text{W}\mu\text{m}^{-3}\text{K}^{-1}$ and $9.7 \mu\text{W}\mu\text{m}^{-3}\text{K}^{-1}$ for each coolant temperature rise of one degree. It is of the greatest importance to notice that such difference does not reveal any intrinsic properties of the material. Indeed, the microstrips are in the phonon-cooled regime and $K'_{x_0}(T_0)$ is mainly determined by the heat transfert through the substrate, which essentially depends on the thermal resistance at the interface between the substrate and the thin film [42]. In other words, $K'_{x_0}(T_0)$ is mainly determined by the heat transfer coefficient from the film to the substrate, which was shown (table 3.1, page 114) to vary at most by a factor of three between all the samples. For the Nb and NbN samples of Fig. 3.17 and 3.18, the heat transfer coefficient close to the critical temperature (h_{T_c}) amounts to $1.6 \times 10^5 \text{ Wm}^{-2}\text{K}^{-1}$ and $3.8 \times 10^5 \text{ Wm}^{-2}\text{K}^{-1}$, respectively. As expected, the ratio between the heat removal capabilities (≈ 2.6) is very close to the ratio of their respective heat transfer coefficients that amounts to 2.4.

3.4.2.3 Zero-crossing resistance steps

A relevant feature of the microwave-dependence of the hotspot plateau is that, above a particular power threshold, some zero-crossing resistance steps are observed. In Fig. 3.16, it was highlighted that the input power threshold noticeably depends on the microwave frequency. However, from the viewpoint of the density power effectively absorbed by the samples, there is only one power threshold which corresponds to the dissipation required to stabilize the boundaries of the normal hotspot domain. For a given frequency, the intersection between the linear power dependence of P_{MW} ($\propto P_{IN}$) and the heat removal threshold $K'_{x_0}(T_0)$ defines the minimum input power required so that the power density ab-

sorbed by the samples becomes sufficient to heat up the electrons above their critical temperature without any Joule heating due to the direct-current. In other terms, it defines from which microwave source power a particular resistance plateau is sustained down to zero bias current (see Fig. 3.14 for Nb and Fig. 3.15 for NbN). As already mentioned, these plateaus are observed whatever the microwave frequency and the coolant temperature are, provided that the power density absorbed by the electrons overcomes the heat evacuation capabilities. Moreover, let's notice that the resistance criterion R_{crit} can be set above any of the zero-crossing resistance step. Considering such criterion simply means that the N/S interfaces are stabilized at some locations where the width is slightly larger than the narrowest one. In this latter case, there is no additional uncertainty that the one associated to the nanoscale modulations of the cross-section.

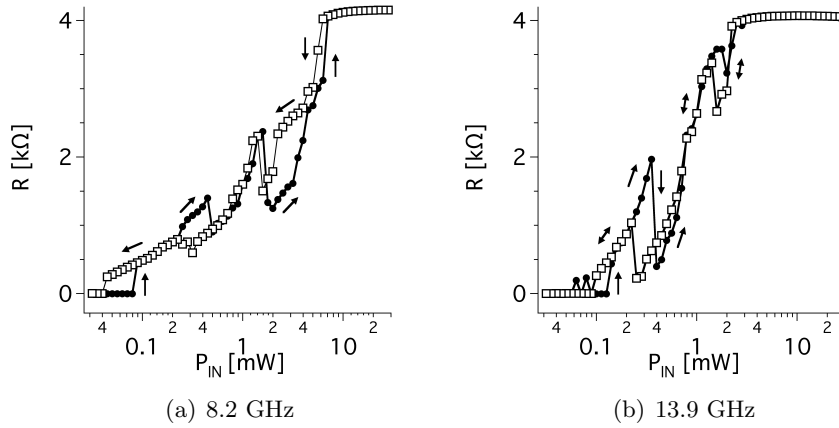


Figure 3.19: Superconducting $\leftrightarrow$ normal transitions of the sample SNbN-75-2 obtained by sweeping the input microwave power from 0.03 mW (-15 dBm) to 31 mW (+15 dBm). The coolant temperature is 7 K. The frequency is kept constant during each measurement and the applied current bias amounts to 1 μ A.

Another way to study the microwave heating effects that happen at low applied current density is to measure the evolution of the sample resistance under an increasing input power (with a low direct current kept fixed). This latter situation is illustrated in Fig. 3.19 which shows the evolution of the resistance of the sample SNbN-75-2 obtained by increasing ($\bullet$ curve) and by decreasing ($\square$ curve) the input power at some fixed microwave frequencies, i.e., at 8.2 GHz (a) and at 13.9 GHz (b). The applied dc current is fixed at 1 μ A for all the experiments. At increasing input power, the sample leaves the perfect superconducting

state to reach a resistance level of around 300Ω . The sample resistance then gradually increases with the input power. At each frequency, some abrupt (and counterintuitive) transitions arise at both sweeping directions, i.e., the resistance sometimes drops (resp. increases) when the microwave input power is increased (resp. is reduced). These transitions do not appear at the same input power so that hysteresis are observed between the upward and the downward sweeps. The origin of these transitions is unknown. They could result from a slight variation of the microwave emission each time the impedance of the strip is modified.

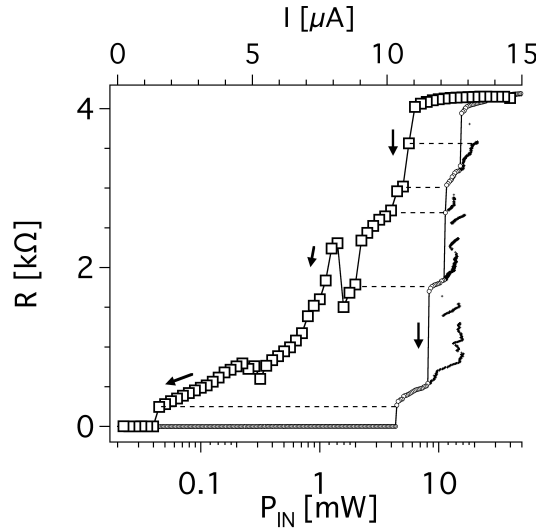


Figure 3.20: Comparison between the normal-to-superconductor transition of the sample SNbN-75-2 obtained (i) by decreasing the microwave input power at a fixed direct current of $1 \mu\text{A}$ [$\square$ curve - bottom axis] and (ii) by using a direct current (continuous) and voltage (dotted) bias without irradiation [top axis].

It is particularly interesting to relate this behavior with the direct biased measurements performed at the same coolant temperature without a microwave irradiation. This is done in Fig. 3.20 which compares the downward curve of Fig. 3.19(a) ($\square$ curve) with the hotspot plateau of the same sample obtained using a direct current (continuous curve) and voltage (dotted curve) bias. First, the largest resistance which is measured at $P_{IN} \approx 30 \text{ mW}$ corresponds to the resistance level measured when the areas close to the contacts have switched into the normal state [see Fig. 3.9(b) and the discussion of the 5-probe measurements in the section 3.2.3, page 122]. This means that, at decreasing input power, the first abrupt transition corresponds to the recovery of the su-

perconducting state in the areas which are more efficiently cooled thanks to the contacts. Second, the correspondence between the current-driven and the power-driven curves (see the horizontal dashed lines) indicate that the abrupt transitions observed when the input power is reduced are related to the variation of the hotspot length whose boundaries are pinned by different nanoscale modulations of the cross-section. In particular, one sees that the dissipative state which is stabilized at low input microwave power ($R \approx 300 \Omega$) is similar to the one that results from the formation of a normal domain around the weak-link of this sample.

3.4.2.4 Impact of the coolant temperature

The heat transfer from the normal domains and the substrate depends on the difference between the hotspot temperature and the coolant temperature. The electronic temperature at the boundaries of the normal domains being by definition close to the critical one at the re-trapping current, the hot electrons are similarly heated - whatever the coolant temperature is - by the microwaves at fixed microwave input power and frequency. The absorbed power is thus expected to vary very slightly with T_0 , on the contrary of the shift of the re-trapping current that, according to Eq. 3.12, depends on T_0 .

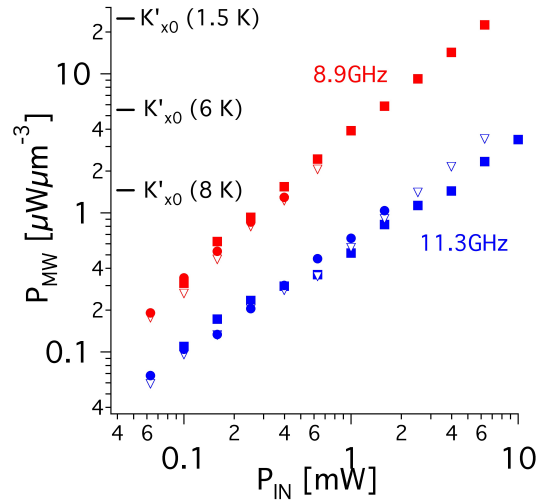


Figure 3.21: Temperature-independent relation between the absorbed power density and the total input power at $T_0 = 1.5 \text{ K}$ (■), 6 K (▽) and 8 K (●) [sample SNb-75-1, frequency : 8.9 GHz (red) and 11.3 GHz (blue)]. The heat evacuation capabilities levels are indicated for each coolant temperature on the top-left part of the figure.

This independence of P_{MW} with respect to T_0 is experimentally verified and illustrated for the sample SNb-75-1 in the figure 3.21, which shows that the relation $P_{MW} \propto P_{IN}$ is temperature-independent in the range 1.5 K - 8 K. This observation reinforces the validity of the procedure used to determine the microwave density power effectively absorbed by the microstrips. The horizontal lines indicate the value of the heat removal capabilities $K'_{x_0}(T_0)$ for each coolant temperature studied.

3.4.2.5 Comparison with photonic devices operating in the pulsed regime

All the measurements under a microwave irradiation have been performed in the continuous regime, i.e., the submicrometer-wide strips were continuously irradiated during the whole experiments. This steady-state behavior noticeably differs from the dynamic electron heating regime which is involved in most of the photonic devices. Indeed, the energy or the power of an incident radiation is often detected in the pulsed mode, in order to detect events that involve a few number of photons. Photonic devices may be classified either as bolometers or as photon (or quantum) detectors. Basically, the former category regroups all the devices for which an increase of the temperature results in a detectable change of the resistance (e.g., in the hot-electron bolometers and the transition edge sensors [183, 191]). In this case, the device temperature is maintained close to the critical one, in order to take advantage of the large temperature dependence of the resistance. The detectors of the second category are based on the photon-induced formation of a resistive domain in an ultra-thin meander. The hotspot disrupts the superconductivity across the wire, which results in a measurable voltage pulse (e.g., in the superconducting nanowire single-photon detector [6, 192]). In this case, the device is biased close to its critical current, although it remains in the perfect superconducting state between each photon detection. The improvement of the radiation sensors is still a subject of intensive research. In particular, there are plenty of recent papers which are related to the photonic devices, most of them being fabricated based on Nb or NbN superconducting ultrathin films [193–197].

The photons counting devices are currently operating well above the THz-range¹⁴, i.e., at frequencies much larger than the ones involved in the continuous-mode studied here. However, the basic mechanism is si-

14. At zero temperature, the superconducting gap amounts to ~ 2 meV for the NbN and Nb samples (see the table 2.7, page 104). In the THz-range, the energy $\hbar\omega$ of a single-photon is thus larger than twice the superconducting gap so that new quasiparticles may be created even at low incident power.

milar in both cases. Indeed, the subtended feature is the nucleation and the stabilization of a normal hotspot domain when the Joule heating dissipation exceeds the heat removal capabilities. In this sense, one may expect that the radiation power densities involved in both types of experiments take similar values, mainly defined by the manner by which the heat is removed away from the dissipative normal domains. From an applicative viewpoint, the evolution of the retrapping current in the continuous mode allows one to determine with accuracy the very small fraction of the input radiation power which is locally present inside an experimental chamber. As it was described above, the microwave power levels absorbed by the NbN and Nb microstrips range between a fraction of $\mu\text{W}\mu\text{m}^{-3}$ up to a few tens of $\mu\text{W}\mu\text{m}^{-3}$ at the lowest coolant temperatures. In addition to the intrinsic deviations that result from the nanoscale modulations of the sample cross-section, the resolution of the above-described procedure is given by the accuracy of the current source. Indeed, the smallest theoretical power density increment is given by $\frac{(\Delta I)^2}{I_{m0}^2} \times K'_{x_0}(T_0)$, where ΔI is the source resolution and I_{m0} is the retrapping current without irradiation. This gives a typical resolution of around $\frac{(5nA)^2}{(100\mu A)^2} \times 10 \mu\text{W}\mu\text{m}^{-3} \approx 0.1 \text{ pW}\mu\text{m}^{-3}$.

3.4.3 Normal domains stabilization at increasing current

Independently of the exact mechanism at the origin of the superconducting - to - normal transition, the instability current may be reduced in such a way that the minimum current for the propagation of a normal domain over the whole sample is not exceeded. As a consequence, intermediate dissipative states are observed in both the forward and the backward parts of the current-driven curves. The figure 3.22 illustrates this microwave-dependence of the overall current-driven characteristics for two NbN microstrips (the same samples as in Fig. 3.15, page 136). The curves were obtained at increasing input power, at fixed coolant temperature and frequency. The input power varies from $P_{IN} = 0 \text{ mW}$ (blue curves) up to $P_{IN} = 4 \text{ mW}$ [(a), SNbN-75-1, $T_0 = 6.8 \text{ K}$, frequency : 7 GHz] and up to $P_{IN} = 50 \text{ mW}$ [(b), SNbN-75-5, with a weak-link, $T_0 = 1.5 \text{ K}$, frequency : 7.5 GHz]. The forward and the backward parts of the current-driven behavior are shown, except for the curves without irradiation for which only the hotspot plateau is depicted¹⁵. In addition, some curves have been shifted along the x-axis for

15. Without irradiation, the abrupt transition from the superconducting state to the normal state arises at a current much larger than the retrapping one, i.e., at $43 \mu\text{A}$ for SNbN-75-1 [Fig. 3.22(a)] and at around $60 \mu\text{A}$ for SNbN-75-5 [Fig. 3.22(b)].

clarity (see the caption of Fig. 3.22). The following features are observed for both samples :

- (i) The overall current-driven behavior is shifted to lower currents when the microwave power level increases.
- (ii) At sufficiently high microwave powers, intermediate dissipative states are stabilized in both parts of the electrical characteristics.
- (iii) Zero-crossing resistance steps arise for both sweeping directions insofar as the microwave input power is sufficiently large.
- (iv) The non-zero resistance level close to zero may be larger in the forward part of the V-I characteristics than in the backward part.

All these features are consistent with the localization of normal hot-spot domains whose stability current range depends on the microwave absorption conditions, i.e., to the smaller Joule dissipation due to the direct-current which is required to stabilize a normal domain when the input microwave power increases. Starting from the perfect superconducting state, the instability current can be sufficiently reduced to range in the current interval of stabilization $[I_r, I_p]$, so that intermediate dissipative states are observed in the forward part of the curves (e.g. Fig. 3.22(a), $P_{IN} = 0.063$ mW). The behavior depicted by the curve $P_{IN} = 0.31$ mW of Fig. 3.22(b) is clarifying. At this quite low microwave input power, one sees that the region of reduced cross-section (i.e., the weak-link) is first impacted by the additional dissipation of the microwave irradiation. This reflects at low dc current by a non-zero resistance plateau at a value which corresponds to the normal state resistance of the constriction ($R \sim 300 \Omega$). In the forward part of this curve, the normal domain progressively expands when the dc current is increased, which reveals the right-side boundary of the hotspot plateau. The microwave power absorbed at this small input power is such that the retrapping transition of the main part of the sample is not perturbed. Indeed, one sees that the retrapping transition exactly coincides with the curve without a microwave. Furthermore, for both samples, there is a nice correspondence between some resistance plateaus observed at different microwave input powers (see the horizontal dashed lines). It is particularly noticeable that the resistance plateaus in Fig. 3.22(a) satisfactorily match the intermediate states of the voltage-biased characteristic obtained without a microwave (see the dotted blue curve left-bounded by the continuous current-biased retrapping transition). This supports the idea that each intermediate dissipative state corresponds to a normal domain whose extremities are pinned by fixed modulations of the sample cross-section. In addition, the profile of the cross-section modulations also

explains the different resistance levels of the zero-crossing steps that are observed in the forward and in the backward parts of some V-I curves [see the curves $P_{IN} = 1$ mW in Fig. 3.22(a) and the curves $P_{IN} = 5$ mW and 50 mW in Fig. 3.22(b)]. Indeed, Eichele et al. have shown that the inhomogeneities that determine the location of the boundaries of the normal domains may differ between the forward and the backward parts of the bias sweep. This was highlighted in section 1.2.3.2 (page 29) with the comparison of low-temperature scanning electron microscopy images of a tin microbridge and the corresponding overall voltage-driven characteristics. The inhomogeneities that lead to the localization of a normal domain at increasing applied current are not necessarily the same as those that prevent its shrinkage at decreasing current. For this reason, both parts of the V-I curves coincide at low applied current only when the same hotspot configuration is involved in both sweeping directions. This is observed, e.g., at $P_{IN} = 3.98$ mW in Fig. 3.22(a).

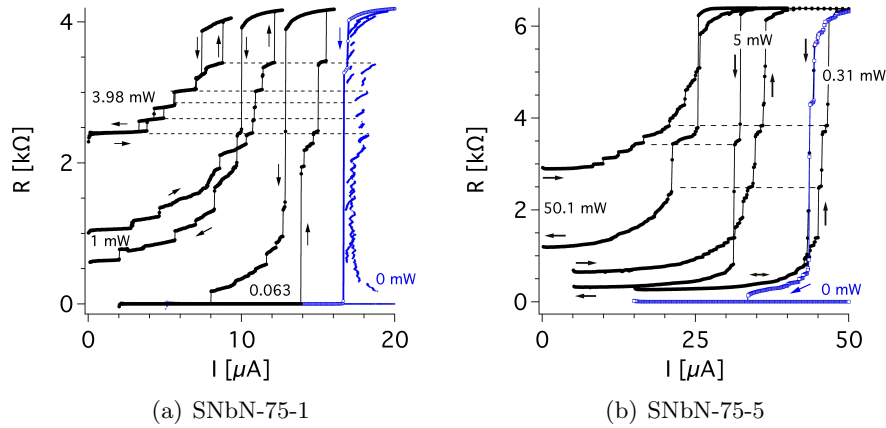


Figure 3.22: Current-biased behavior of NbN microstrips under an increasing input microwave power. (a) SNbN-75-1, $T_0 = 6.8$ K, 7 GHz. P_{IN} varies from 0 mW (in blue) up to 3.98 mW. The voltage-biased behavior without microwave is shown with the dotted blue curve. The curves at $P_{IN} = 0$ mW and $P_{IN} = 0.063$ mW are shifted by +5 μ A and +2 μ A for clarity. (b) SNbN-75-5, $T_0 = 1.5$ K, 7.5 GHz. P_{IN} varies from 0 mW (in blue) up to 50 mW. The curves at $P_{IN} = 0$ mW and 0.31 mW are shifted by +15 μ A and the curve at $P_{IN} = 5$ mW is shifted by +5 μ A for clarity. The horizontal dashed lines indicate the correspondence between some resistance levels.

Many intermediate dissipative states are also observed in the forward part of the current-biased characteristics of niobium microstrips. This is illustrated in Fig. 3.23, which shows the evolution of the V-I curves of the sample SNb-75-3 for four different increasing input microwave power ($T_0 = 8$ K, frequency : 6.7 GHz). At this temperature, the instability

current I^* without a microwave irradiation amounts to $410 \mu\text{A}$, while the retrapping transition takes place at $125 \mu\text{A}$. The main feature of the figure 3.23 is that successive transitions between well-defined resistance levels are observed in both current sweep directions (see the $\uparrow$ symbols). The exact explanation of these transitions is unknown. However, at all microwave input power levels, one sees that the transitions arise between intermediate dissipative states that have exactly the same resistance level. This is also the case at the other coolant temperatures and microwave frequencies. Once again, the localization of the extremities of a normal domain due to the presence of some modulations of the cross-section is highly probable.

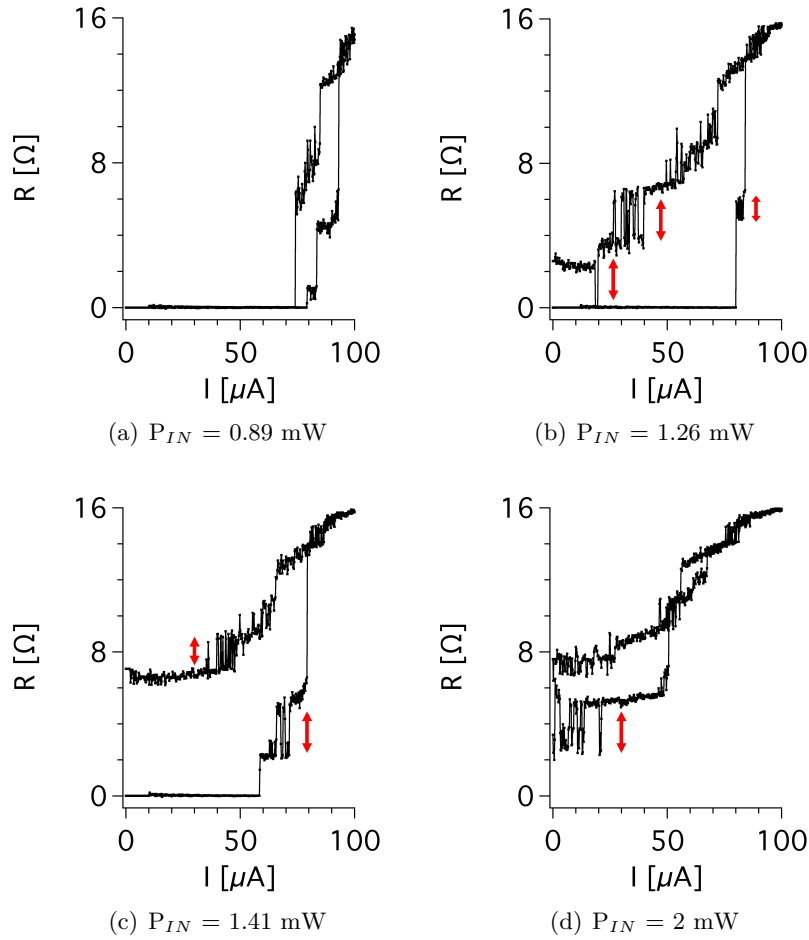


Figure 3.23: Current-biased behavior of SNb-75-3 under an increasing input microwave power. The coolant temperature is 8 K, the microwave frequency is 6.7 GHz.

3.5 Hotspots domains or phase-slippage phenomena ?

Up to now, the ability to stabilize dissipative normal domains in the hotspot plateau of superconducting microstrips has been discussed. It was shown that the voltage steps result from the localization of the N/S boundaries at some locations along the sample length. However, this current- and voltage-induced breakdown of the superconductivity in narrow microstrips remains a challenging issue, in particular due to the required clear distinction between the phase-slippage phenomena and the above-mentioned hotspot processes. The main experimental difficulty is that both dissipative mechanisms manifest themselves by voltage discontinuities in the V-I curves.

3.5.1 Stepwise current-biased characteristics

Let's remind that the stepwise growth of a stabilized normal hotspot domain requires that the applied current ranges in the stabilization interval $[I_r, I_p]$. The instability current I^* is included in this latter interval when the coolant temperature T_0 is in the range $[0.8T_c, 0.9T_c]$, so that a stepwise evolution of the voltage (equivalently of the resistance) is directly probed in the forward part of the current-biased characteristic. By contrast, I^* noticeably exceeds the propagating current of the widest part of the sample for $T_0 \in [0.2T_c, 0.8T_c]$. The first nucleated normal region thus rapidly converts into a spreading hotspot which expands through the whole sample. In this case, the forward part of the current-biased behavior is characterized by an abrupt variation of the voltage (equivalently of the resistance) up to the normal state value. At these low coolant temperatures, it is of the greatest importance to emphasize (i) that a large number of intermediate dissipative states can be stabilized insofar as the applied current is swept in the interval $[I_r, I_p]$ and (ii) that the intermediate dissipative states of the hotspot plateau find their origin in the same dissipative process that at higher T_0 , i.e., in the stabilization of a normal domain which is self-sustained by the Joule heating dissipation. As a consequence, all the regular voltage (or resistance) steps that appear in the current-biased curves (either in both parts at relatively high T_0 or only in the backward part at low T_0) are explained by purely thermal effects linked with the unavoidable modulations of the sample cross-section. This is summarized in Fig. 3.24 with the current-biased behavior of SNbN-75-3 at three coolant temperatures $T_0 = 9$ K, 9.7 K and 10.2 K (from right to left). The presence of a weak-link in the middle of the segment results in three different hotspot propagations :

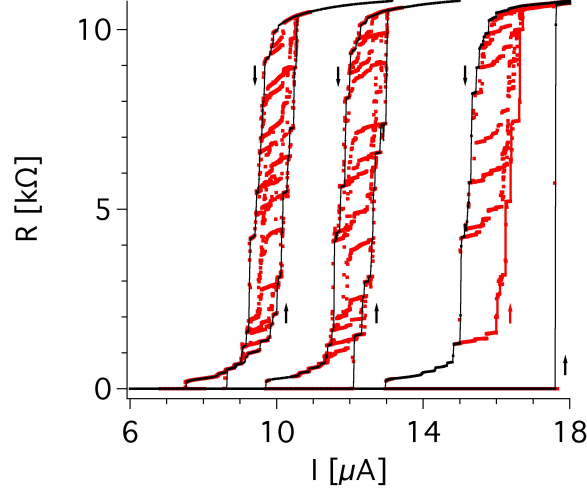


Figure 3.24: Current-biased curves of the sample SNbN-75-3 (from right to left, $T_0 = 9$ K, 9.7 K and 10.2 K). The overall (black) curve as well as the (red) curves obtained by sweeping the current back and forth at numerous points of the envelope are shown. The low-resistance part is due to the constriction at the middle of the sample. The curves at 9 K and 9.7 K are shifted by $+3 \mu\text{A}$ and $+1 \mu\text{A}$ for clarity.

$T_0 \leq 9$ K The instability current of the weak-link (I_{WL}^*) exceeds the minimum propagation current of the widest part of the sample so that the hotspot expands onto the whole sample. Each step that appears in the retrapping transition corresponds to a localized normal domain that can be stabilized in the hotspot plateau by sweeping the current back and forth.

$T_0 = 9.7$ K At this coolant temperature, I_{WL}^* exceeds the propagating current of the weak-link but is smaller than the propagating current of the other parts of the sample. As a consequence, the first normal domain expands around the constriction and grows by discrete resistance steps as the applied current is increased. In addition, the intermediate dissipative states that are stabilized by sweeping the current back and forth are similar than for $T_0 \leq 9$ K.

$T_0 \geq 9.7$ K The instability current of the weak-link lies below the retrapping current of the main sample width. The normal domain first propagates smoothly around the constriction and then expands by discrete jumps of the N/S interfaces as it is done at $T_0 = 9.7$ K. For this sample, the dissipative states are observed with exactly the same resistance levels up to $T_0 = 10.9$ K, i.e., up to $T_0/T_c \approx 0.87$.

The only requirement to observe a stepwise evolution of the voltage at increasing bias is that the cross-section is not uniform through the whole sample length. Of course, the presence of an intentionally placed constriction is not required, as illustrated in Fig. 3.25 with the current-biased characteristics of the sample SNbN-20-2 measured at reduced coolant temperature T_0/T_c of 0.82, 0.84, 0.87 and 0.9 (from right to left)¹⁶.

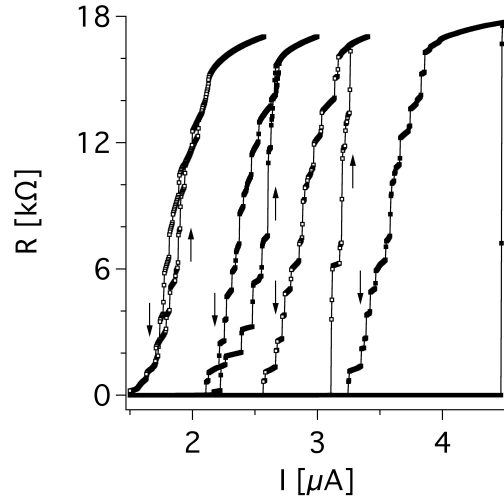


Figure 3.25: Stepwise current-biased characteristics of the sample SNbN-20-2 obtained at $T_0 = 7.5$ K, 7.75 K, 8 K and 8.25 K (from right to left). The curves obtained at 7.5 K, 7.75 K and 8 K are respectively shifted by $+1\mu\text{A}$, $+0.5\mu\text{A}$ and $+0.3\mu\text{A}$ for clarity.

3.5.2 (Dis)similarities

At first sight, the electrical behavior of the NbN and Nb submicrometer - wide strips present some similarities with the phase-slip centers (PSC) or with their two- (or three-) dimensional equivalent, i.e., the phase-slip lines (PSL) and the phase-slip planes. The main feature is that both parts of the current-driven characteristics are stepwise, which is most of the time considered as a sufficient signature for the presence of phase-slippage phenomena. However, a careful analysis reveals that the present resistive domains must be distinguished from the dissipative mechanisms that involve the slippage of the superconducting phase.

¹⁶. The electrical behavior of this sample far below the critical temperature is given in Fig. 3.6(a), page 117.

3.5.2.1 Scatter of the transition currents

In the phase-slip center model of Skocpol, Beasley and Tinkham, the scatter of the critical current was originally explained by the interactions between neighboring phase-slip centers (see the section 1.3.2.1, page 35). The latter were thought to interact if they are distant from less than twice the quasiparticle diffusion length Λ_{Q^*} . However, the nucleation of additional PSCs must require a certain amount of inhomogeneity along the sample length. This is essential in order to explain why other phase-slip centers do not nucleate far away from the existing ones (i.e., at a distance well longer than Λ_{Q^*}) when the successive critical currents are reached. The main challenge is thus to distinguish between both contributions.

Here, the scatter of the critical current results from the unavoidable inhomogeneities along the sample length. The main evidence is that the width of the transition is quantitatively explained by the nanoscale modulations of the sample cross-section (see the section 3.3, page 125). This highlights the necessity to minimize the defects in the bulk of the film but, before everything, on the edges. In the present nanofabrication process, the width modulations are close to the mean grain size of the polycrystalline thin films. As a consequence, it appears quite unlikely that the spreading of the current-driven transition may be significantly reduced. In the previous studies, the sample preparation technologies were such that inhomogeneities were most probably present with sometimes a substantial amount of defects along the edges of the film. Indeed, the films were either cut with a diamond - pyramid - stylus or were etched by photolithographic techniques. For this reason, it is likely that the real impact of the inhomogeneities is often underestimated in the explanation of the scatter of the critical current.

Furthermore, a striking correspondence is observed here between the critical current and the retrapping current (see the section 3.2.2). First, both currents present the same dependence with the coolant temperature. Second, the ratio between both currents is quite constant whatever the total sample resistance is. Third, each intermediate state in the forward transition can be stabilized down to the retrapping transition so that the states in the forward and the backward parts of the curves correspond to the same stabilized dissipative structures. As the retrapping transition undoubtedly results from purely thermal effects (i.e., the collapse of a normal hotspot domain when the applied current becomes too small to ensure a sufficient heat dissipation), the nucleation of successive phase-slippage phenomena is here rejected to explain the forward current-driven transition.

3.5.2.2 Temperature range for phase-slippage mechanisms

At first, the phase-slip center model was designed for samples whose transverse dimensions were small with respect to both the coherence length and the effective penetration depth. These conditions have been mainly established to explain the very small temperature range close to the critical temperature in which the stepwise characteristics were observed. The size restriction was challenged and it was considered both theoretically and experimentally that similar phase-slippage mechanisms could happen in the two-dimensional (phase-slip lines) and three-dimensional (phase-slip planes) limits (see the section 1.3.2.2, page 38). Here, both transverse dimensions are much larger than the coherence length of the material. The conditions of appearance of a phase-slip center or a phase-slip line are far from being satisfied in the coolant temperature range studied. Indeed, their development requires to perform the measurements very close to T_c , which is never the case here, even for the largest coolant temperatures. Using the temperature dependence of the coherence length in the dirty limit (see Eq. 2.3, page 98) and the values of $\xi(0)$ given in the table 2.7 [$\xi_{NbN}(0) \approx 3$ nm and $\xi_{Nb}(0) \approx 10$ nm], the conditions $W < \xi$ and $d < \xi$ are only fulfilled for $T_0 > 0.995T_c$. At these coolant temperatures, the current-driven characteristics are perfectly smooth without any voltage jumps. As a consequence, only the conditions of appearance of phase-slip planes are satisfied in the present work, although there is no evidence to corroborate the nucleation of such dissipative states.

3.5.2.3 Temperature dependence of the quasiparticle diffusion length

In the framework of the phase-slippage phenomena, the length-scale of interest is the quasiparticle diffusion length over which the quasiparticles created at the core of the dissipative structure relax by inelastic scattering processes and join the superconducting condensate. According to the theory (page 34), the characteristic imbalance relaxation time τ_{Q^*} has the same order of magnitude as the inelastic scattering time, but diverges at the critical temperature [$\tau_{Q^*} \propto (1 - T/T_c)^{-1/2}$]. Following Eq. 1.31, Λ_{Q^*} should diverge weakly as $(1 - T/T_c)^{-1/4}$ although this latter evolution was not observed in the first experiments [2, 198]¹⁷. The experimental evidence of the temperature dependence of Λ_{Q^*} remains

17. Let's remark that some authors [67] have observed the temperature dependence of the differential resistance of their tin microbridges only after a *correction* of their results that took into account some heating effects.

a challenging and controversial issue. To his surprise, Tidecks has noticed that *« the differential resistance (of Sn and Sn-In whiskers) is usually found to be temperature independent throughout the investigated temperature range »* so that the associated time *« cannot be related to one of the steady-state charge imbalance relaxation times »* [199]. Here, the dissipative states present very similar resistance levels in the wide temperature range studied, so that the spatial extension of the normal domains appears temperature-independent. This is further highlighted in Fig. 3.26 with the voltage-biased characteristics of two NbN samples in the coolant temperature interval [1.5 K, 9 K] and [4 K, 7.5 K], respectively. At the lowest temperatures, the correspondence between the intermediate states is clear. As T_0 is increased above $\approx 0.8T_c$, the curves become smoother but the correspondence remains satisfactory. This reinforces that the number and the configuration of the resistive domains do not depend on a mechanism that involves a center of phase-slip events, but rather on the temperature-independent location of the nanoscale cross-section modulations.

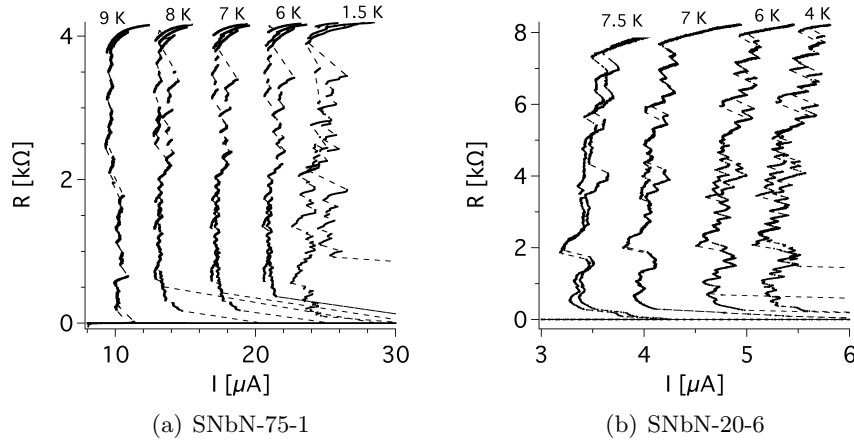


Figure 3.26: Temperature-dependence of the voltage-biased characteristics of two NbN strips. The coolant temperature varies from 1.5 K to 9 K [SNbN-75-1, (a)] and from 4 K to 7.5 K [SNbN-20-6, (b)]. For a sake of clarity, the curves $T_0 = 6$ K, 7 K and 8 K of the part (a) are shifted by $+8 \mu\text{A}$, $+5 \mu\text{A}$ and $+2 \mu\text{A}$, respectively.

3.5.2.4 Excess current

In the first experiments on phase-slip centers, it was observed that the various resistive states (when plotted as a voltage) intercept the current axis at around fifty percent of the critical current. This lead to

the conclusion that the average value (over time) of the supercurrent is around half the critical one (i.e., $I_s = \beta I_c$ with $\beta \sim 0.5$). In practice, β was found to significantly vary from around 0.4-0.6 [68, 70] up to 0.7-0.8 [74, 75, 80]. In addition, although this statement is often considered as a distinctive feature of the phase-slippage phenomena, the existence of a non-zero excess current was also observed in the framework of the normal hotspot domains stabilization [41].

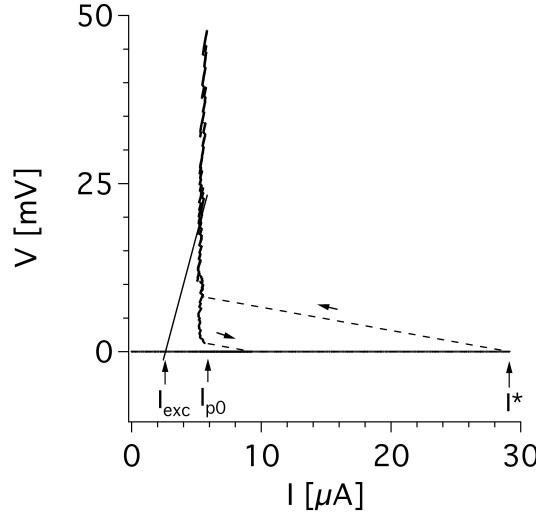


Figure 3.27: Voltage-biased characteristic of the sample SNbN-20-6 at $T_0 = 4$ K. The excess current of a dissipative state at half the normal state resistance is specified by the slanted line in the bottom-left.

Here, the slopes of the various resistive states (when plotted as a voltage) are roughly the same. At the lowest coolant temperatures at which the intermediate dissipative states have been observed, the excess current I_{exc} is less than 10 % of the instability current I^* , in a very poor agreement with the time-averaged supercurrent $I_s \approx 0.5I_c$. This is illustrated in Fig. 3.27 with the voltage-biased characteristic of the sample SNbN-20-6 at a coolant temperature $T_0 = 4$ K. The slanted line in the bottom-left of the figure is the extrapolation to the zero-voltage level of an intermediate dissipative state at a resistance level of half the normal state value. At this low coolant temperature, the excess current is around half the propagation current associated with this particular dissipative state. In addition, the instability current decreases more quickly than the excess current when the coolant temperature is increased. As a consequence, the ratio I_{exc}/I^* increases with T_0 , from around 10 %

at $T_0 = 4$ K to around 50 % at $T_0 = 8$ K, i.e., when the forward part of the current-driven characteristics becomes stepwise or, in other words, when $I^* \approx I_{p0}$. Interestingly, the ratio I_{exc}/I_{p0} remains quite unchanged at around 50 % in the whole temperature range. This is depicted in Fig. 3.28 with the ratios I_{exc}/I^* (■) and I_{exc}/I_{p0} (○) expressed as a function of the coolant temperature. The same temperature evolution is observed for all the NbN samples. This is a relevant feature which highlights that the stabilization of normal hotspot domains also manifests itself by a non-zero excess current. For a particular intermediate dissipative state, the latter amounts to a temperature-independent fraction of the minimum normal-zone propagating current.

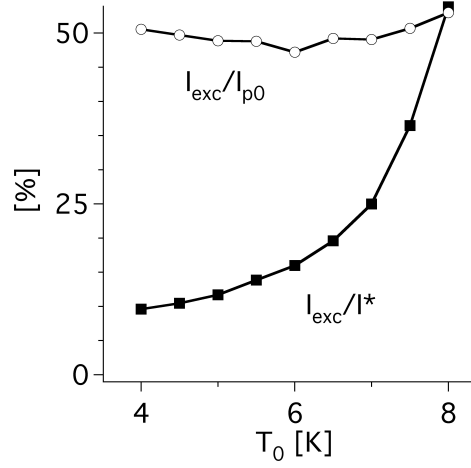
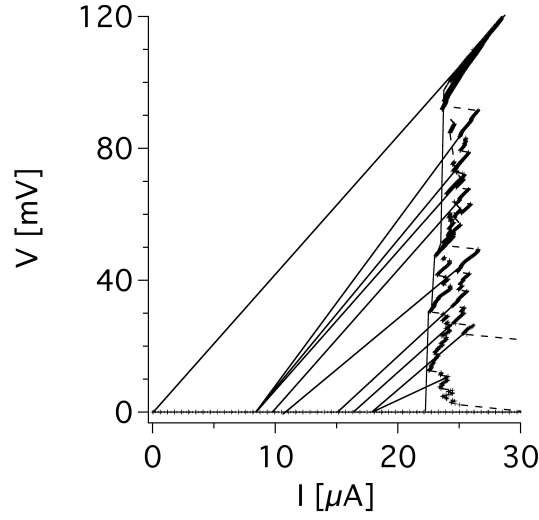
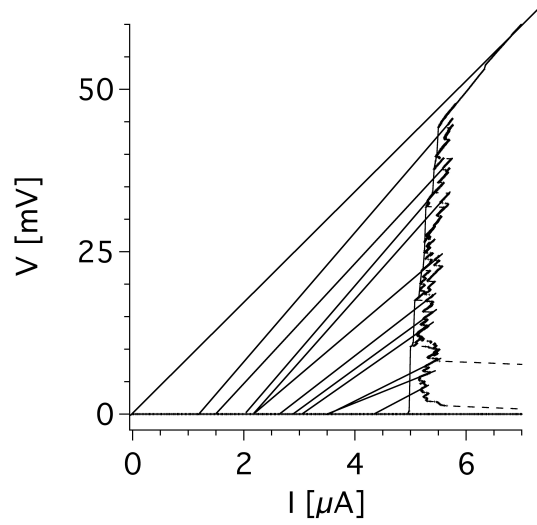


Figure 3.28: Temperature-dependence of the ratio between the excess and the instability currents I_{exc}/I^* (■) and between the excess and propagating currents I_{exc}/I_{p0} (○).

The resistance of the intermediate state has been specified in the previous paragraph because all the intermediate states do not give the same excess current. Indeed, the excess current is reduced as the resistance of the dissipative structure increases. This is illustrated in Fig. 3.29 with the hotspot plateau of two NbN submicrometer-wide strips that differ by their lateral dimensions. All the curves correspond to the smallest available coolant temperature. One sees that the excess current tends to zero when the sample resistance reaches the normal state value. Such behavior differs completely from published data on phase-slippage phenomena for which there is a single excess current throughout the sample resistance.



(a) SNbN-75-1 (1.5 K)



(b) SNbN-20-6 (4 K)

Figure 3.29: Hotspot plateau of two NbN submicrometer-wide strips that differ by their transverse dimensions. The reduction of the excess current as the resistance of the intermediate dissipative states increases is highlighted by the slanting lines.

3.5.2.5 An alternative explanation for non-equilibrium processes

The above-discussion highlights that numerous hotspots configurations can be stabilized in dirty superconducting submicrometer-wide strips whose transverse dimensions are much wider than their coherence length. In particular, the spotlight is turned on the utmost impact of the modulations of the sample cross-section in the stabilization of the latter dissipative states. A relevant point is that the formation of normal hotspot domains gives a stable signal below and above the crossing temperatures T^* insofar as the applied current density is swept in the interval $[j_r, j_{p0}]$ (see the discussion on the phase diagram $j - T$ at page 41). Having all these clues in mind, the conclusion is that much care is required when a regular stepwise behavior arises, especially when the coherence length of the material is very small and when the coolant temperature is much lower than the critical one.

Chapter 4

Determination of the critical field for vortex entrance

A powerful tool to determine the critical field for vortex entrance in the absence of a transport current consists in probing the presence of vortices by means of scanning microscopy techniques. As it was discussed in the section 1.5 (page 50), the latter techniques have allowed several authors to characterize the penetration of vortices in niobium as well as in yttrium-based high temperature superconductors. The main limitation turns out to be the experimental setup resolution as the samples width amounts to a few microns at best. This chapter highlights that electrical transport measurements are efficient tools to determine the critical field for vortex entrance (also called the superheating field) of much narrower samples, i.e., whose width amounts to a few tens of nanometers. In addition, it will be shown that studying the response of the frozen vortices configurations to a direct transport current is a way to probe their relative stability.

This chapter is divided into four parts. The section 4.1 depicts the experimental procedure followed to study the magnetic-field dependence of the critical currents. Although the superheating field is by definition associated to a situation with no applied transport current, two independent determinations of this latter field will be discussed. The penetration of the first vortices at the superheating field is described in Sec. 4.2 through the overall shape of the critical current vs magnetic field curves. Above this field, it will be shown that the arrangement of the frozen vortices patterns at regularly-spaced magnetic fields gives an indirect way to determine the critical field of interest. This is done in

the section 4.3. Finally, the section 4.4 highlights that all the extracted values are consistent together, as well as with recent values obtained by scanning probe microscopy on wider strips. The appendix A treats an additional feature of the critical current vs magnetic field curves, namely the observation of the fluxoid quantization effect. Indeed, one takes advantage of the loop configuration of the niobium samples to discuss the impact of the penetration of vortices on the observation of the periodic modulations of the circulating currents.

4.1 Introductory comments

The whole chapter is devoted to the magnetic-field dependence of two characteristic currents, namely the critical current I_c and the instability current I^* . The former current is associated to the destruction of the perfect superconducting state. It is defined by a voltage threshold V_{crit} which is fixed at a few μV (most of the time, $V_{crit} = 3 \mu\text{V}$). The latter current corresponds to the development of a highly dissipative domain. It is defined by a resistance criterion R_{crit} which is close to the normal state resistance value, and thus varies from one sample to the other. The measurement procedure is illustrated by the voltage-current characteristics of Fig. 4.1. The detection of I_c and I^* is performed by increasing the transport current from zero until the associated threshold is reached. The coolant temperature and the external magnetic field are maintained constant during the measurement. Insofar as the sample remains in the perfect superconducting state, the measured voltage is $\sim 1 \mu\text{V}$, i.e., the sensitivity of the voltmeter. As illustrated by the right curve of Fig. 4.1 ($B = 0 \text{ T}$), the departure from the perfect superconducting state manifests itself at low magnetic fields by an abrupt transition to the fully normal state. In this case, the critical current I_c and the instability current I^* take the same value that amounts here to $33 \mu\text{A}$. At higher magnetic fields (see the left curve in Fig. 4.1 for $B = 2 \text{ T}$), a non-zero voltage develops due to the vortices flux-flow motion. The critical current $I_c \approx 10 \mu\text{A}$ is reached when $V = 3 \mu\text{V}$. A further increase of the applied current is required to intensify the vortices motion and to lead to the instability, so that $I^* \neq I_c$ ¹. In addition, after each detection and whatever the criterion used is, the sample is brought to the normal state by the applied direct current. As a consequence, the critical electronic temperature is exceeded under the influence of the Joule heating before

1. The non-zero voltage which is measured just before the transition to the fully normal state is referred by V^* . Its field-dependence will be discussed in the section 4.3.2 (page 182).

the sample is field-cooled when the current source is switched off. This is not a temperature-driven field-cooling as the coolant temperature is maintained constant during the whole measurement.

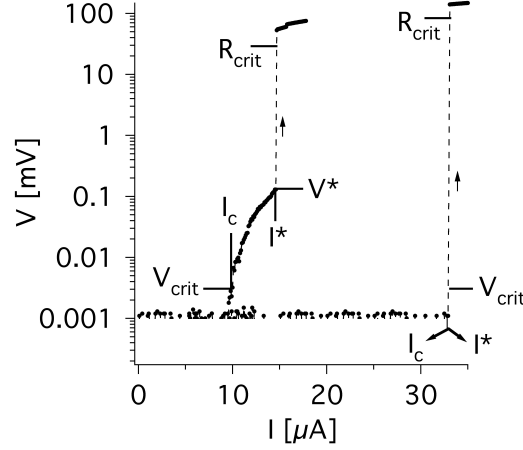


Figure 4.1: Forward part of representative current-driven characteristics of the sample SNbN-75-2 that illustrate the procedure used to detect the critical current I_c ($V_{crit} = 3 \mu\text{V}$) and the instability current I^* ($R_{crit} \approx R_n$). The coolant temperature is $T_0 = 6 \text{ K}$ and the applied magnetic field is $B = 0 \text{ T}$ (right) and $B = 2 \text{ T}$ (left). The resolution of the multimeter used to measure the voltage amounts to $1 \mu\text{V}$.

In anticipation of the following discussion, it is of the utmost importance to notice that the overall shape of the current vs magnetic field curves strongly depends on whether some vortices are present just after the current-driven field-cooling, and thus before the direct current is increased from zero. This is illustrated in Fig. 4.2(a) for the submicrometer-wide strip SNbN-20-6. This curve is representative of the low-temperature behavior of all the samples (here, $T_0 = 3 \text{ K}$, i.e., well below the critical temperature). In the magnetic field range centered around zero, the instability current noticeably decreases for both orientations of the field. This field range is called the *central peak* and is detailed in the section 4.2. Outside the central peak, the global decrease of the instability current is less pronounced. The appearance of modulations of the characteristic currents outside the central peak is made clearer in the section 4.3. It has to be mentioned that the evolutions of I_c and I^* are independent of the magnetic field sweeping direction (either in an increasing or in a decreasing magnetic field). As it was previously emphasized, the electrons are briefly heated above T_c each time I_c or I^* is reached, so that the sample undergoes a kind of thermal reset af-

ter each current detection. In addition, although the overall evolutions are nicely reproducible, a significant spreading of I_c and I^* is measured in the central peak region. This is illustrated in Fig. 4.2(b) which depicts the distributions of I_c and I^* obtained after having repeated several times the current detection at fixed magnetic fields between 0 T and 1.5 T [indicated by the arrows in Fig. 4.2(a)]. The large spreading in the peak region is thought to reflect the metastability of the superconducting state when no vortices are present inside the strip and when the applied current is close to the critical value [165, 200]. By contrast, the narrow spreading outside the central peak indicates that the formation of vortices patterns reduces the metastable nature of the superconducting state for $I \approx I^*$.

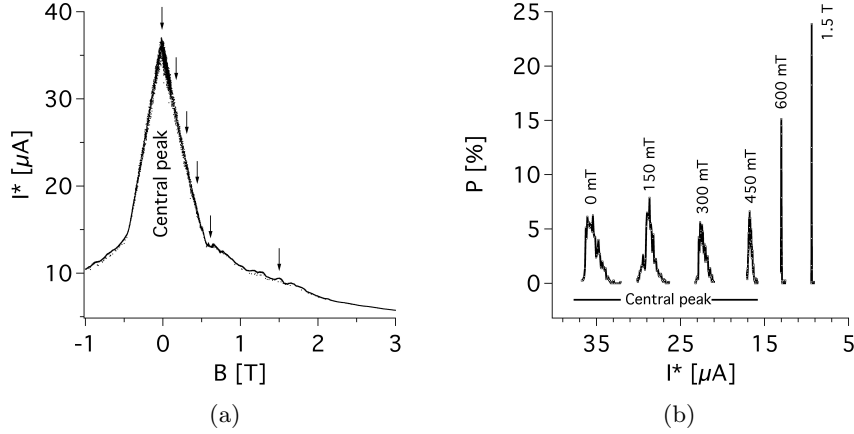


Figure 4.2: (a) Overall evolution of the instability current of SNbN-20-6 as a function of a perpendicular magnetic field. The curve is centered with respect to zero field. The coolant temperature lies far below the critical one ($T_0 = 3$ K). The instability current significantly decreases in the central peak region, while the trend is less pronounced at higher fields. A similar envelope is probed for the critical current I_c . (b) Instability current distributions at the magnetic field values referred by the arrows in the part (a). 800 measurements have been performed at each magnetic field.

Furthermore, studying the evolution of the instability current for both orientations of the applied transport current allows one to better understand the origin of the transition to the normal state. In particular, that reinforces that the instability current is intimately related to the penetration (in the central peak region) or to the motion (in the high-field regions) of vortices inside the sample². Indeed, from Eq. 1.13

2. As it will be made clearer in the next section, the same reasoning remains valid for the departure from the perfect superconducting state.

(page 15), one knows that the vortices are submitted to a Lorentz force when they are in presence of a non-zero net current. The latter force is given by the cross product between the net current density and the flux carried by the vortices (generally speaking, one flux quantum ϕ_0). As a consequence, the impact of a direct transport current in the positive magnetic field range would be the same in the negative field range if the direction of the current is reversed.

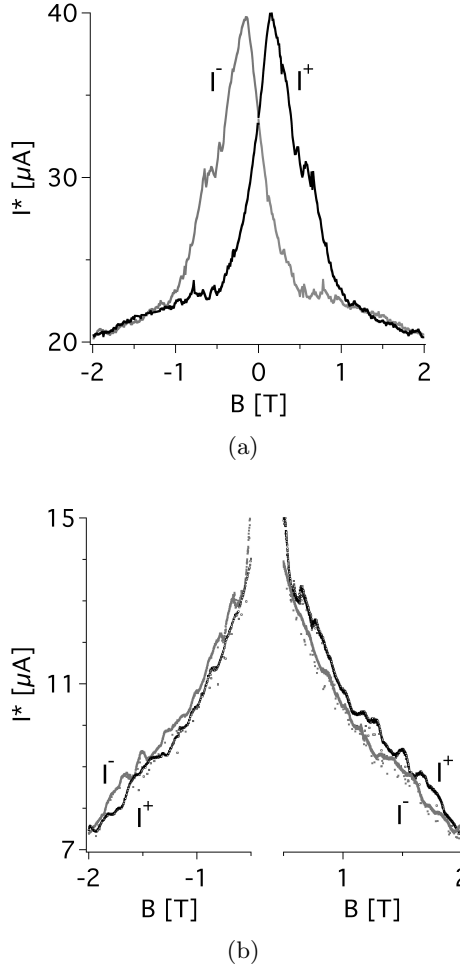


Figure 4.3: Magnetic field-dependence of the instability current obtained for two directions of the applied transport current (I^+ : black, I^- : gray). The curves obtained for one bias direction are the perfect mirror images of the curves obtained for the other direction. This is observed inside [(a), SNbN-75-6, $T_0 = 4.2$ K] and outside [(b), SNbN-20-6, $T_0 = 3$ K] the central peak region.

This is illustrated in the figure 4.3 which depicts the magnetic-field dependence of the instability current obtained for both directions of the applied transport current, arbitrarily denoted by I^+ (black curves) and I^- (gray curves). Whatever the field range of interest is, the curves obtained for one bias direction are the perfect mirror images of the curves obtained for the other direction. This is a clear illustration of the role played by the vortices in the determination of the characteristic critical currents, both in the central peak [Fig. 4.3(a)] and in the high-field regions [Fig. 4.3(b)].

In addition, one sees in Fig. 4.3(a) that the extremum of the central peak is not located at zero applied magnetic field. This shift of the instability current vs magnetic field curves is observed for all the NbN and Nb samples. This is illustrated in Fig. 4.4 which summarizes the position of the central peak extremum B_{cp} for a set of NbN submicrometer-wide strips ($\triangle$ symbols) and of Nb loops ($\blacktriangle$ symbols). The shift of the central peak does not result from the experimental setup. Although a residual (or remanent) magnet field may be present, the latter should take a much lower value than the experimentally determined B_{cp} . Above all, it should remain constant for all the samples. This is not the case here because B_{cp} varies from one sample to the other, i.e., the narrower is the sample, the larger is B_{cp} . The shift of the central peak is well-known for granular materials that contain strong pinning regions or holes that are able to trap a certain amount of residual magnetic flux [201]. A non-zero B_{cp} was experimentally observed on high temperature superconductors [202, 203] as well as on NbN and Nb bridges submitted to low perpendicular magnetic fields [204]. The shift was also studied by Shantsev et al. [205] who highlighted that a non-zero B_{cp} is a signature of the granular character of the superconductors. Indeed, in the framework of the critical-state model, they have patterned YBCO thin films and shown that the introduced granularity shifts the central peak of the magnetization curves. Here, the flux is thought to be preferentially trapped at the intergranular junctions between the superconducting grains. What is interesting here is the magnitude of the central peak shift. Indeed, B_{cp} varies over more than two orders of magnitude on the studied width-range. One way to compare the samples together is to calculate the product $B_{cp}W$ which corresponds to the equivalent flux trapped per unit length when the sample is cooled at zero field. Doing so, one obtains similar values for all the samples, i.e., a few times the flux quantum ϕ_0 per micron length. This gives an idea of the residual flux that may be trapped due to the polycrystalline structure of the samples. In order to properly compare the samples starting from an initial situation which is completely free of trapped magnetic flux, it is mandatory to monitor the

magnetic fields of interest with respect to the central peak. From now, all the currents vs magnetic field curves will be centered with respect to zero field.

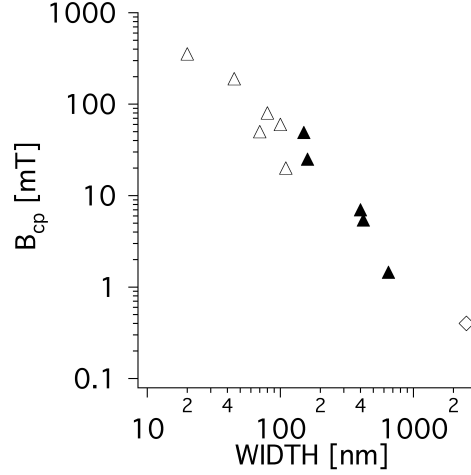


Figure 4.4: Position of the central peak extremum B_{cp} as a function of the width of the NbN submicrometer-wide strips ($\triangle$ symbols) and of the Nb loops ($\blacktriangle$ symbols). The $\diamond$ marker corresponds to the shift of the central peak of a NbN strip measured by Aomine et al. [204].

Finally, probing the vortices behavior through the magnetic-field dependence of the characteristic critical currents presents a significant drawback. Indeed, the location where a voltage arises first may vary along the sample length when the applied magnetic field is modified. The issue can be summarized as follows. When the sample is everywhere free of vortices, the critical (or instability) current is first reached at the location where the transport current density is the largest, i.e., at the narrowest parts. As the superheating field is reduced by an increase of the width, it is first exceeded at the widest parts of the sample. The presence of vortices may be such that the critical (or instability) current density is reached at this latter place. The situation may be reversed when the applied magnetic field is further increased above the superheating field of the narrowest parts, which become again the weakest parts of the samples. This matter is all the more pronounced when the cross-section presents variations throughout the sample length, as it is the case for the loop configurations where different sample widths are involved : the branches and the contacts. This drawback is also encountered in the microstrip configuration as the vortices that penetrate first into the (wider) contacts or into (wider) parts of the sample may im-

pact the capabilities of the strip to sustain a large supercurrent density. As a consequence, one can not be sure that the recorded magnetic-field evolution of the critical current corresponds to a single sample location on the whole magnetic-field range. It is essential to keep in mind this point for the following discussions.

4.2 Evidence of the first vortex entrance

4.2.1 Distinction between the critical current and the instability current

In Section 4.1, it was stated that the samples behave differently depending on whether some vortices remain stable after the current-driven field-cooling. Below the superheating field of the widest parts of the sample, the application of a bias current is mandatory in order for the vortices to penetrate the sample. The latter situation is encountered when the combination of the external and of the self-induced magnetic fields leads to the spontaneous nucleation of a single vortex that crosses the sample from one edge to the other somewhere along its length. As it was recently discussed by Bulaevskii [206], the energy (heat) released by the crossing of a vortex at high bias currents may be sufficient to create a normal-state region along the vortex trajectory. The latter rapidly expands through the whole sample if the applied current exceeds a particular threshold that was interpreted in the section 3.2.2 as the normal-zone propagating current. In this case, both criteria (namely the voltage criterion defining I_c and the resistance criterion defining I^*) are reached simultaneously because the voltage just before the instability is zero. Such approach remains valid insofar as the applied magnetic field is below the lowest superheating field of the sample. Above this latter field, some vortices penetrate during the current-driven field-cooling and may remain stable inside the sample. In this case, the characteristic currents are detected when the applied current is sufficiently large in order to tear the vortices off their pinning sites. Two situations have to be further distinguished. First, both detection criteria give equivalent characteristic currents if the vortices are efficiently pinned so that the local dissipation induced by their motion is large enough to create a normal domain that expands throughout the sample length. Second, the voltage criterion is first reached when the voltage at the extremities of the sample exceeds V_{crit} and a further increase of the direct current is required in order for the local dissipation to overpass the heat removal capabilities. The above-described distinction between I_c and I^* are illustrated in the figure 4.5 which compares the magnetic-field dependence of the critical

current (black curves) and of the instability current (red curves) for the strips SNbN-W1, SNbN-W2 and SNbN-W3 (from bottom to top).

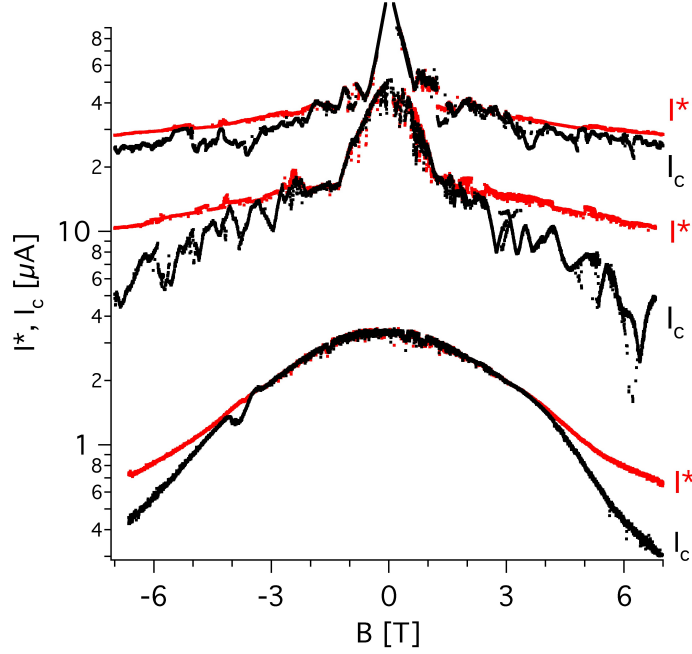


Figure 4.5: Low-temperature magnetic-field dependence of the critical current I_c (black curves) and of the instability current I^* (red curves). From bottom to top : SNbN-W1 ($W = 20$ nm), SNbN-W2 ($W = 45$ nm) and SNbN-W3 ($W = 80$ nm). The curves associated to the sample SNbN-W3 are shifted by $+15 \mu\text{A}$ for clarity. The coolant temperature is 3 K.

According to the previous description, the curves I_c and I^* are superimposed for each sample in the central peak magnetic-field region. The criteria are reached in both cases when a vortex enters the sample due to the application of the direct current. Outside the central peak, the curves get farther, which indicates the presence of vortices and their ability to move without leading to the switch to the normal state when the Lorentz force exceeds the pinning threshold. In particular, it is clarifying to discuss the evolution of the current I^* in the framework of the heating effects highlighted in the chapter 3. Indeed, there is a strong interplay between the instability current I^* and the current I_{p0} above which a normal hotspot domain expands. This is made clearer at Fig. 4.6(a) which shows the current-driven evolution of the resistance for the sample SNbN-W2 at zero magnetic field (■) and at $B = 7$ T (○).

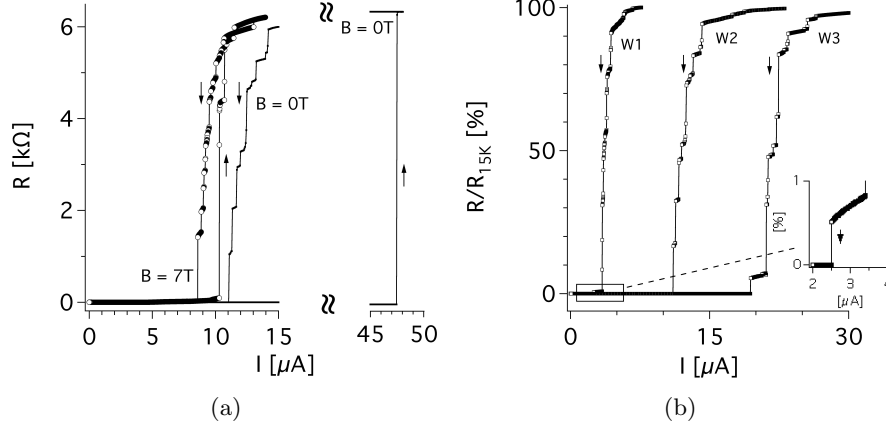


Figure 4.6: (a) Current-driven evolution of the resistance of SNbN-W2 at zero magnetic field (■ symbols) and at $B = 7 T$ (○ symbols). (b) Current-driven normal-to-superconducting transition for the samples SNbN-W1, SNbN-W2 and SNbN-W3 ($T_0 = 3 K$). The curves are normalized with respect to the normal state resistance value. The inset highlights that the narrowest strip contains a strong constriction.

Without magnetic field, both I_c and I^* are reached at $\sim 47.5 \mu A$. The abrupt superconducting-to-normal transition occurs well above the retrapping current I_r , which spreads between $11 \mu A$ and $14 \mu A$ ($I_c, I^* \gg I_{p0} > I_r$). The situation differs at $B = 7 T$. Indeed, the voltage threshold is reached at $I_c \sim 4.5 \mu A$ while the instability transition occurs at $I^* \sim 10 \mu A$. Interestingly, the instability current is sufficiently reduced to become smaller than the largest propagation current I_{p0} of the sample ($I^* < I_{p0}$). As a consequence, an intermediate dissipative state is stabilized in the forward part of the curve. Moreover, one sees that the ratio I_{p0}/I_r is close to 1.1, in perfect agreement with the width of the hotspot plateau which has been observed in Sec. 3.2.2 (page 118) on more resistive NbN samples³. This confirms that the instability current finds its origin in the propagation of a normal domain that nucleates at the location where the dissipation induced by the vortices motion exceeds the heat removal capabilities.

Furthermore, the magnetic field above which the curves I_c and I^* deviate indicates that the superheating field is exceeded at least in one part of the sample *and* that the vortices are able to move without leading to the complete transition to the normal state. For the samples SNbN-W2 and SNbN-W3 (top curves of Fig. 4.5), one sees that I_c and

3. Let's remind that the thin film used to fabricate the samples SNbN-W1-2-3-4 has been sputtered at $\sim 600^\circ C$. Its normal state resistivity is significantly reduced compared to the films deposited at room temperature.

I^* remain superimposed slightly above the magnetic field that corresponds to the end of the central peak. This probably comes from the enhanced stability of the first vortices that form a single chain in the middle of the film. Indeed, as the width is much smaller than the penetration depth, the vortices are efficiently pinned at the center of the strip due to their strong interactions with the surfaces (see the section 1.5.3, page 58). The situation is somewhat different for the sample SNbN-W1 (bottom curves). Indeed, the I_c and I^* curves deviate at $B \sim 3.5$ T while the central peak ends at $B \sim 5$ T⁴. This quite unexpected observation becomes clearer if one looks at the current-induced normal-to-superconducting transition. Indeed, in the section 3.3, the retrapping part of the voltage-current curves has been related to the unavoidable nanoscale modulations of the sample cross-section. One can take advantage of this feature to characterize the homogeneity of the samples cross-section. This is illustrated in Fig. 4.6(b) with the current-induced return to the superconducting state of the samples SNbN-W1, SNbN-W2 and SNbN-W3. The curves are normalized with respect to the normal-state resistance value. Interestingly, one sees in the enlargement of the framed low-resistance part that the sample SNbN-W1 contains a strong cross-section reduction, i.e., around 1.5 times smaller than the mean value. As the four samples SNbN-W1-2-3-4 have been fabricated during the same process, one can consider that the ratios of the retrapping currents properly reflect the ratios of the samples width. A detailed study of these ratios together with the estimations of the sample width obtained from scanning electron microscopy (see Fig. 2.12 at page 83) allows one to conclude that the most probable width to take into account are 15 nm and 20 nm (SNbN-W1), 45 nm (SNbN-W2), 80 nm (SNbN-W3) and 100 nm (SNbN-W4). The discussion of the chapter 3 on the heating effects in superconducting microstrips thus appears as an interesting tool to explain some particularities of the field-dependent evolution of the critical and instability currents.

4.2.2 Width-dependence of the central peak

The conclusion of the previous discussion is that much care is required when studying the central peak because the weakest area is not always located at the same place. Having this point in mind, one can study in more details the impact of the sample width on the magnetic-field range over which the central peak extends. Strictly speaking, the half-width of the central peak is of particular interest as one can directly relate it to the superheating field. The notation $B_{SH,peak}$ will be used to

4. The end of the central peak is more visible in the linear scale of Fig. 4.7(a).

indicate that this first estimation of the superheating field comes from the width of the central peak of the I_c and I^* curves [207].

The width-dependence is illustrated in the figure 4.7 with the central peak of representative NbN (a) and Nb (b) samples. The width of the central peak of the NbN samples clearly increases when the width is reduced, varying from $B_{SH,peak} \sim 0.6$ T for the sample SNbN-W3 ($W \sim 80$ nm) up to $B_{SH,peak} \sim 5$ T for the narrowest part of the sample SNbN-W1 ($W \sim 15$ nm). The same is true for the Nb loops of Fig. 4.7(b). Indeed, $B_{SH,peak}$ amounts to ~ 20 mT for LNb-2b ($W \sim 400$ nm) and to ~ 160 mT for LNb-2a ($W \sim 160$ nm). Interestingly, the central peak of the sample LNb-2a also presents a change of slope at around 50 mT. This is thought to correspond to the penetration of vortices in a (wider) contact of the loop whose width amounts to 300 nm. All these values are summarized in the table 4.1 where the samples are ordered with respect to the width W of the weakest area (equivalently, with respect to their decreasing central peak width). The main advance of the present electrical transport measurements is that the superheating field of very narrow samples is determined. It is exciting to probe samples whose width amounts to a few tens of nanometers, so that their characteristic fields $B_{SH,peak}$ become as high as several teslas.

Table 4.1: Half-width of the central peak ($B_{SH,peak}$) of the instability/critical current vs magnetic field curves for a set of NbN submicrometer-wide strips and of Nb loops. The samples are ordered with respect to the width W of the weakest area (equivalently, with respect to their decreasing central peak width).

	$B_{SH,peak}$ [mT]	W [nm]
SNbN-W1 (constriction)	5000	15
SNbN-W1	3500	20
SNbN-W2	1200	45
SNbN-W3	585	80
SNbN-W4	390	100
LNb-3a	175	150
LNb-2a (branch)	160	160
LNb-2a (contact)	54	300
LNb-2b	20	400
LNb-1c	11	650

As it was discussed in the section 1.5.2.1 (page 53), many theoretical and experimental works were dedicated to the impact of the sample size on the lowest critical field above which vortices are stable in the absence of a transport current [8,9,119]. Two theoretical models are based on the

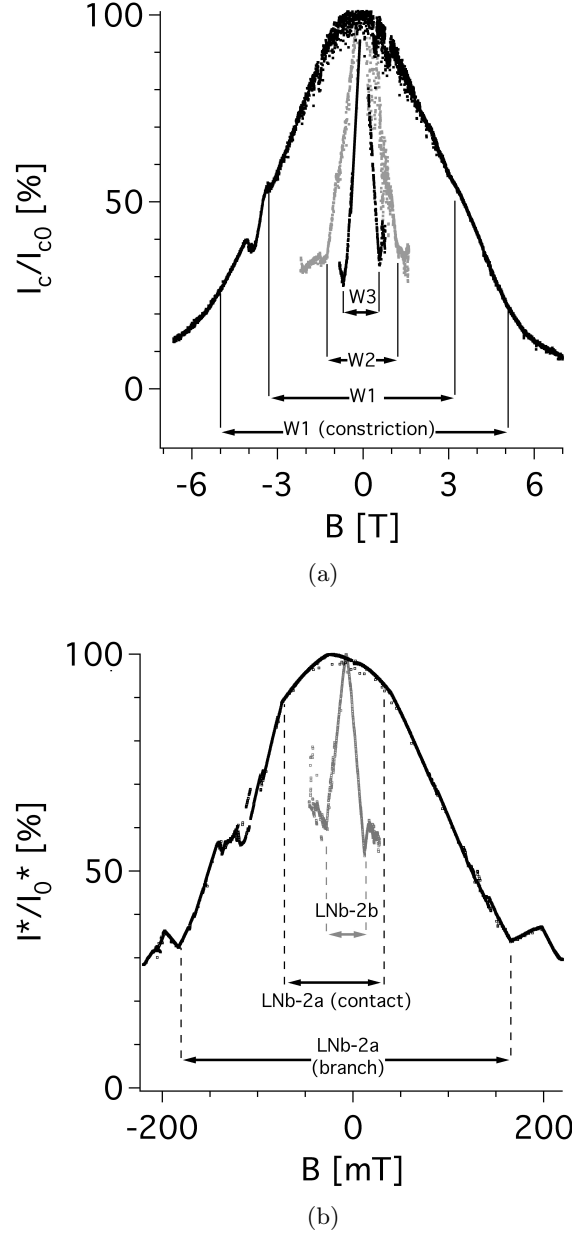


Figure 4.7: Central peak of the magnetic field-dependence of the critical/instability current obtained at $T_0 = 3$ K. The curves are normalized with respect to the extremum value. (a) SNbN-W1 ($W = 20$ nm), SNbN-W2 ($W = 45$ nm) and SNbN-W3 ($W = 80$ nm). (b) LNb-2a (branch : $W = 160$ nm) and LNb-2b ($W = 400$ nm). The central peak of the sample LNb-2a presents a change of slope at around 50 mT which corresponds to the first penetration of vortices in a contact of the loop ($W = 300$ nm).

Gibbs free-energy profile of an isolated vortex located inside a strip (i.e., the metastable B_0 and the stable B_s solutions), while a third one deals with the formation of vortex - anti-vortex pairs (i.e., the B_K solution). In summary, the metastable solution argues that a vortex can penetrate into a strip insofar as the magnetic field induction is large enough to induce a minimum of the Gibbs free-energy at the center of the strip. This occurs for fields $B > B_0$ with B_0 given by Eq. 1.38 (page 53). A further increase of the induction is required in order to induce an absolutely stable solution. This arises at the magnetic field B_s at which the Gibbs free-energy at the center of the strip presents an absolute minimum (see Eq. 1.37, page 53). Recently, Kuit et al. [9] proposed an intermediate model that takes into account the V-AV pair production energy. They show that the relevant critical induction is determined when the vortex generation rate is exactly balanced by the rate of escape, which occurs at a magnetic field B_K given by the expression 1.39 (page 56). A common feature of these three models is that the superheating field is roughly proportional to Φ_0/W^2 . This is illustrated in Fig. 4.8 which compares the experimental data extracted from the width of the central peak with the three theoretical models. The error bars take into account the uncertainty associated to the exact width of the samples. For the NbN samples, the uncertainty is estimated from the width of the current-driven retrapping transition of Fig. 4.6(b) through the relation $\Delta W = \Delta I_r \times W$ discussed in Sec. 3.3.1 (page 125).

The proportionality term does not depend on the sample width for the B_0 (metastable) and the B_K models, so that they are both depicted by straight lines in these log-log plots (see the dotted - for B_0 - and the solid - for B_K - curves). Besides, the stable solution B_s depends on the coherence length at the temperature T_f at which the vortices become pinned during the field-cooling. Many authors have indicated that the temperature T_f is very close to the critical one in a temperature-driven field-cooling ($T_f \approx 0.99 T_c$) [8, 208]. However, whether this statement remains valid here is not so clear. Indeed, the samples are field-cooled by switching off the applied current at a fixed coolant temperature so that the (unknown) temperature at which the vortices become pinned may be reduced or may even be close to the coolant one. For this reason, the stable solution B_s (with $\alpha = 2/\pi$) is depicted for different T_f/T_c ratios, namely 0.9, 0.6 and 0.3 (see the dashed curves labeled A, B and C, respectively)⁵. These latter curves correspond to different values of the coherence length $\xi(T_f)$ that are determined using the temperature

5. Let's remind that the $B_{SH,peak}$ values have been extracted from curves obtained at a coolant temperature $T_0 \sim 0.3T_c$.

dependence of the coherence length in the dirty limit (see Eq. 2.3, page 98). Taking $\xi(0) \sim 4$ nm and $\xi(0) \sim 11$ nm for the NbN and for the Nb samples respectively, one obtains that $\xi(T_f) \approx 4.8$ nm, 6.3 nm and 12.6 nm for the NbN samples [Fig. 4.8(a)] and 13.1 nm, 17.4 nm and 34.8 nm for the Nb samples [Fig. 4.8(b)]. One sees that the experimental data satisfactorily follow the theoretical width-dependence. For both types of samples, the stable model calculated for a temperature T_f close to the critical one significantly deviates from the experimental data. This seems to indicate that, in the present current-induced field-cooling, the vortices positions are frozen at a temperature which is closer to the coolant one. Besides, for the NbN samples, the agreement is impressive on the whole width-range if one compares the data with the stable solution calculated for $T_f = 0.3T_c$ (curve C). Concerning the Nb samples, the overall trend is verified although the theoretical models underestimate the experimental data for the narrowest studied loops. A definitive conclusion concerning the theoretical model that is better suited to describe the data is quite difficult as the B_s model (curves B and C) and the B_K model are very close to each other for the largest studied widths. In any case, the agreement between the experience and the theory supports that studying the width of the central peak gives interesting information with regard to the superheating field value.

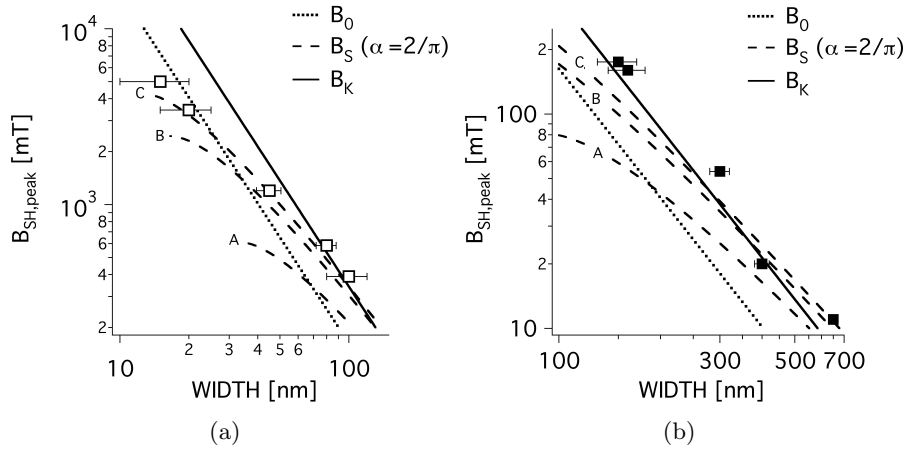


Figure 4.8: Evolution of the half-width of the central peak as a function of the width for the NbN strips (a) and the Nb loops (b). The values extracted from the central peak of the critical/instability current evolutions are compared with three theoretical critical induction models, namely the metastable solution B_0 (dotted curve), the stable solution B_s (dashed curves) and the B_K solution (solid curve). The stable solution is depicted for three values of the coherence length that correspond to different vortices pinning temperatures : $T_f = 0.9T_c$ (curve A), $0.6T_c$ (curve B) and $0.3T_c$ (curve C).

4.3 Vortices lattice rearrangement

The central peak of the magnetic-field dependence of the critical current (and equivalently of the instability current) gives a direct way to estimate the superheating field of the studied strips and loops. The essential feature is that whether vortices are present inside the samples just after the current-driven field-cooling impacts the overall trend of the curves (no vortices remain stable when $B < B_{SH,peak}$). The present section is devoted to the evolution of both characteristic currents *above* the superheating field, i.e., when vortices are present after the field-cooling and when their number varies due to an increase of the external field. It will be shown that the increasing number of vortices leads to their rearrangement into more or less stable patterns at regularly-spaced values of the external field. Interestingly, the position of the latter matching fields allows one to determine the superheating field of the samples.

4.3.1 Periodic modulations of the characteristic currents

4.3.1.1 NbN submicrometer-wide strips

The figure 4.9 depicts the wide variety of critical current modulations that appear above the superheating field for the submicrometer-wide strips SNbN-W2 (a) and SNbN-W3 (b). The magnetic field increment is 3 mT and the coolant temperature is $T_0 = 3$ K. Although this is not always obvious to track the local minima (especially for the strip SNbN-W3), a relevant feature is that the associated fields are regularly spaced, with a period that amounts to ~ 750 mT for SNbN-W2 and ~ 400 mT for SNbN-W3. The spacing thus increases with a decrease of the sample width. In accordance with the literature, the matching fields⁶ are referred by the notation B_N with $N = 1, 2, \dots$. The well-defined critical current minima reflect the rearrangement of the vortices from an unstable pattern into a more stable one. Indeed, as it was discussed in the section 1.5.3 (page 58), the vortices first form an anisotropic one-dimensional chain parallel to the longitudinal direction [for an illustration, see the figure 1.13(a) at page 54]. This occurs when the external

6. The term *matching* usually refers to an efficient pinning of the vortices by a periodic lattice of artificial pinning centers [209], leading to an enhanced critical current at specific values of the magnetic field. Here, the term *matching* has a different meaning : it indicates a commensurate relation between the vortices lattice and the strip width, which manifests itself by the rearrangement of the vortices into a more stable pattern (there is no artificial pinning centers). Here, the superconductivity is weakened at the matching fields and a further increase of the applied magnetic field is required in order for the vortices to form an optimal lattice whose critical current is locally maximum.

field is close to the superheating field, which is here denoted by B_1 as it corresponds to the transition from a situation with no vortex to a situation with one line of vortices ($N = 1$). A further increase of the applied field reduces the vortex-vortex inter-distance so that the anisotropic lattice becomes unstable at a particular field and splits into two chains. This occurs at the matching field denoted by B_2 . As the external magnetic field is increased, similar rearrangements occur at regularly-spaced matching fields B_N at which the number of vortices rows change from $N-1$ to N .

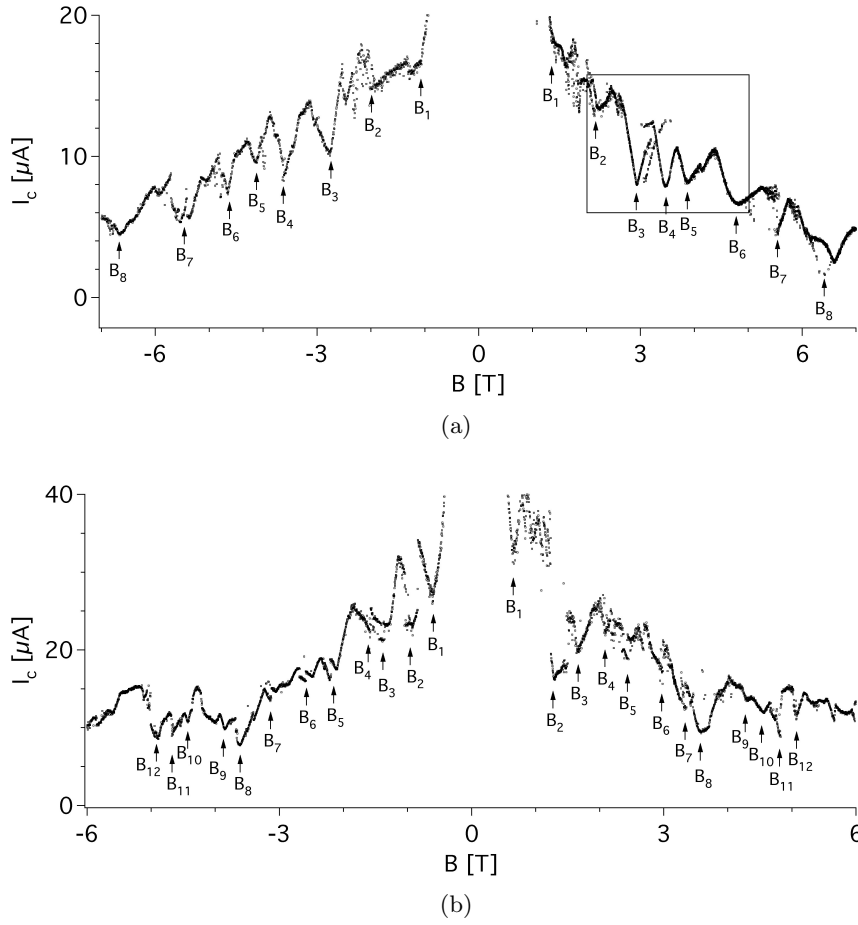


Figure 4.9: Evolution of the critical current as a function of a perpendicular magnetic field larger than the superheating field for the strips SNbN-W2 (a, $W = 45 \text{ nm}$) and SNbN-W3 (b, $W = 80 \text{ nm}$). The coolant temperature is 3 K. The fields B_N that correspond to local minima of the critical current are numbered for $N = 1, \dots$

Of course, this is a simplified scheme of the real configurations that take the vortices. Indeed, it is highly unlikely that the vortices form regular lines perfectly parallel to the longitudinal direction *along the whole length*. At best, one can consider that the vortices configurations tend *on average* to the ones illustrated in Fig. 1.18 (page 62). Finally, let's note that the measurement setup does not allow one to probe the rearrangement of the vortices patterns for the strip SNbN-W1. Indeed, this sample is so narrow that the first matching field $B_1 = B_{SH,peak}$ is close to the largest field that can be applied with the available high-field solenoid superconducting magnet (the field B_2 is not accessible for this sample).

Furthermore, the figure 4.10(a) shows an enlargement of the rectangular frame of Fig. 4.9(a). It is noteworthy to measure different values of the critical current at a given magnetic field (see, e.g., the field $B \sim 3.2$ T which is referred by the vertical line, and where I_c spreads from 10 to 12 μA). The measurements have been reproduced 500 times at this particular field [see Fig. 4.10(b), where only the 150 first are shown].

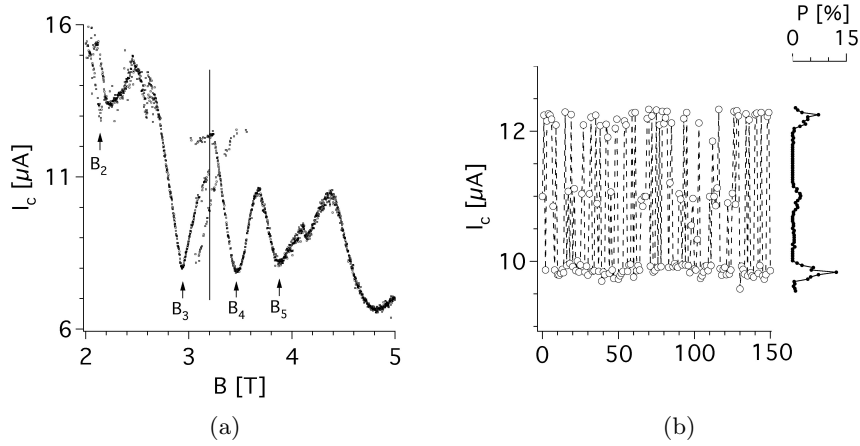


Figure 4.10: (a) Enlargement of the rectangular frame of the figure 4.9(a). (b) Numerous critical current measurements for the sample SNbN-W2 obtained at the fixed magnetic field referred by the vertical line in Fig. 4.10(a). 500 measurements have been performed and only the 150 first ones are shown. The distributions of the three well-defined values are depicted on the right side.

The three critical current levels that are pointed out nicely match the overall curve. The spreading of the distributions depicted on the right side of the figure is quite narrow, and the lowest value is the most frequently probed. Two hypothesis may be proposed to explain this behavior. First, a multimodal distribution at a fixed magnetic field might

reveal that the sample is randomly stabilized in different states during the current-driven field-cooling that follows each critical current detection (a larger critical current is related to a better stabilized starting state). The nanoscale modulations of the cross-section as well as some defects of the thin film may allow the vortices to be preferentially frozen at close but distinct locations. Second, the application of a direct transport current after the field-cooling might displace the vortices from their original positions (this possibility was introduced in the section 1.5.3.3, page 62). This current-induced reorganization of the vortices may also explain that different critical currents are probed at the same magnetic field. A relevant point is that all the vortices configurations that are involved here are motionless just after the current-driven field-cooling. The application of a bias current $I < I^*$ only results in the development of small electrical voltages that correspond to the slow motion of the vortices. In that sense, the present observations have nothing to do with the vortex street (or kinematic vortices) solution that has been introduced to explain the nucleation of resistive states in quasi-two-dimensional current-carrying films (see the section 1.3.2.2, page 38). Indeed, the latter dynamical regime is associated to the fast motion of vortices (perpendicularly to the direction of the current) that permanently sustain a highly dissipative state inside the sample.

4.3.1.2 Nb mesoscopic loops

The high-field modulations of the critical current are similarly observed on the Nb samples. This is illustrated in Fig. 4.11 for the loops LNb-3a, LNb-2a and LNb-2b (from top to bottom). For each sample, the overall evolution of the magnetic-field dependence of the instability current I^* is depicted on the left-side column while a detailed view is enlarged in the right-side column. The magnetic field increment is 100 μT and the coolant temperature lies far below the critical one ($T_0 = 3\text{ K}$, 4K and 3 K, respectively). Once more, the local minima are regularly spaced, with a period that amounts to around 65 mT (LNb-3a), 33 mT (LNb-2a) and 18 mT (LNb-2b)⁷. The spacing between the matching fields will be extensively described in the section 4.3.3. It is however interesting to make preliminary comments. First, the previous discussions have highlighted that the relevant widths to take into account are 150

7. For the loop LNb-3a, the period of interest is much larger than the one observed in the low-field part of the curve where a sawtooth-shaped evolution of the instability current occurs with a periodicity of 7.8 mT. As it is made clearer in the appendix A, this latter value is in excellent agreement with the theoretical periodicity of the fluxoid quantization which is calculated with the mean radius of the loop.

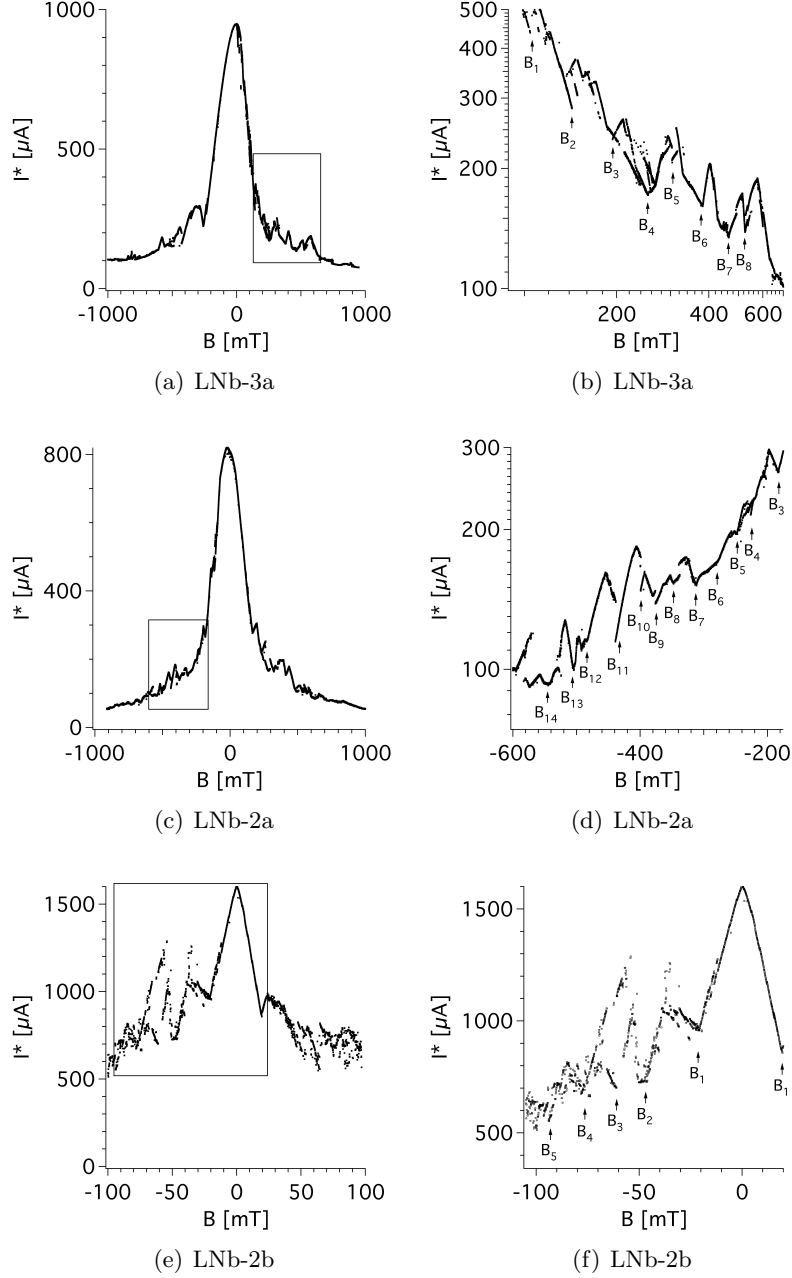


Figure 4.11: Overall and detailed views of the magnetic-field evolution of the instability current for the loops LNb-3a (a, b, $W = 150$ nm), LNb-2a (c, d, $W = 300$ nm) and LNb-2b (e, f, $W = 400$ nm). The bath temperature lies far below the critical one (3 K, 4 K and 3 K, respectively). The field values B_N that correspond to local minima of the instability current are numbered with respect to the number N of vortices rows.

nm for LNb-3a, 300 nm for LNb-2a and 400 nm for LNb-2b. As a consequence, the field periodicity increases with a reduction of the weakest part of the samples, in agreement with the trend observed on the NbN submicrometer-wide strips. Second, the field periodicity is here much smaller than the ones involved in Fig. 4.9. This is once more consistent because the associated NbN samples are much narrower (~ 45 nm for SNbN-W2 and ~ 80 nm for SNbN-W3).

In addition, let's notice that the matching fields B_N associated with the local instability current minima are numbered from $N = 1$ for the loop LNb-3a and LNb-2b while they are numbered from $N = 3$ for the loop LNb-2a. In the latter case, and in accordance with the discussion of the section 4.2.2 [in particular, see the figure 4.7(b)], the superheating field of the width of interest (i.e., a contact of the loop) is lower than the full spreading of the central peak which corresponds to a branch of the loop. The fields B_1 and B_2 are not shown in the enlargement of Fig. 4.11(d). As a final comment, let's notice that the current modulations depicted in Fig. 4.11 for the Nb samples correspond to the instability current I^* , while the modulations were only observable for the NbN samples on the magnetic-field evolution of the critical current I_c (see, e.g., the top curves of Fig. 4.5). This is thought to reflect distinct pinning thresholds. Indeed, the relative insensitivity of I^* outside the central peak of the NbN samples seems to indicate that the vortices are weakly pinned in these structures compared to the Nb ones. They are easily torn off their pinning sites and the critical current I_c is the most interesting tool to probe the stability of the vortices patterns. At the opposite, being able to detect clear variations of the instability current for the Nb samples might reflect an enhanced pinning of the vortices. Such assumption is corroborated by the higher instability current density of the Nb samples compared to the NbN ones, i.e., $j^*(0)_{Nb} \approx 10 \times j^*(0)_{NbN}$ [see the table 3.1 at page 114 which summarizes the zero-temperature instability current density of the studied NbN and Nb submicrometer-wide strips].

The Nb samples also present multimodal distributions of the instability current at a given magnetic field. This is illustrated in Fig. 4.12 for the loop LNb-3a. The fields of interest are denoted [A] and [B] and are referred by the vertical lines in Fig. 4.12(a). The instability current I^* spreads from 280 to 360 μA and from 220 to 270 μA , respectively. The measurements have been reproduced 500 times at these particular fields [see Fig. 4.12(b), where only the 150 first are shown]. The three instability current levels pointed out at each field nicely match the overall curve. The spreading is very narrow. As for the NbN samples, the lowest values are more frequently probed. One can confidently consi-

der that these multimodal distributions at fixed magnetic field either reveal that the vortices are frozen in different configurations during the current-driven field-cooling that follows each instability current detection, or that the vortices rearrange from their original frozen pattern due to the application of the direct current.

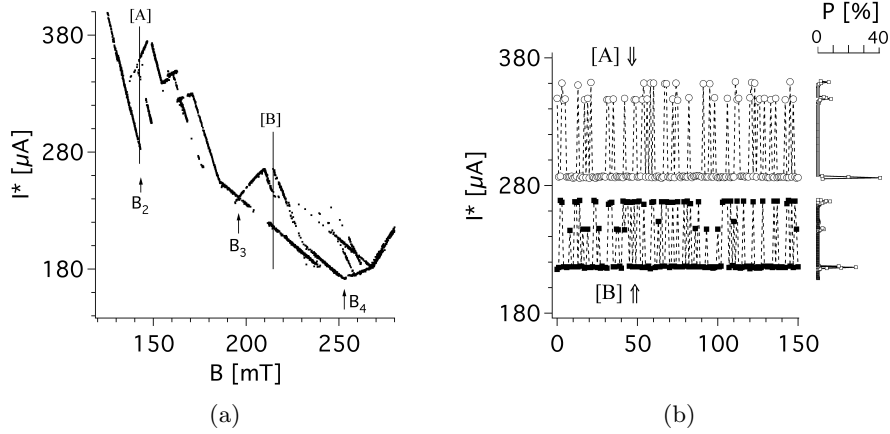


Figure 4.12: (a) More precise view of some matching fields of the figure 4.11(b). The symbols [A] and [B] refer to two particular magnetic fields studied in more details in the figure 4.12(b). (b) Numerous instability current measurements for the loop LNb-3a obtained at fixed magnetic fields ([A] : 193 mT, [B] : 265 mT). 500 measurements have been performed and only the 150 first ones are shown. Three well-defined values are observed at each field. The narrow distributions are depicted on the right side.

4.3.2 Evolution of the instability voltage

The evolution of the instability voltage V^* with respect to the magnetic field is instructive. This is illustrated (together with the instability current I^*) in Fig. 4.13 for the loop LNb-3a (a) and LNb-2a (b). The instability voltage, which is measured just before the transition to the fully normal state, takes either zero or non-zero values depending on the magnetic field range. The non-zero values are more often probed when the derivative of the instability current with respect to the applied field is negative, and V^* generally drops at the local I^* minima.

The ability to relate both I^* and V^* is the principal advantage of using a switching criterion rather than a criterion based on the appearance of a voltage. These experimental results are intimately related to the response of a type-II superconductor under a direct transport current, in particular to the stability of the vortices pattern that freezes during the field-cooling procedure. If the pinning threshold is small (i.e.,

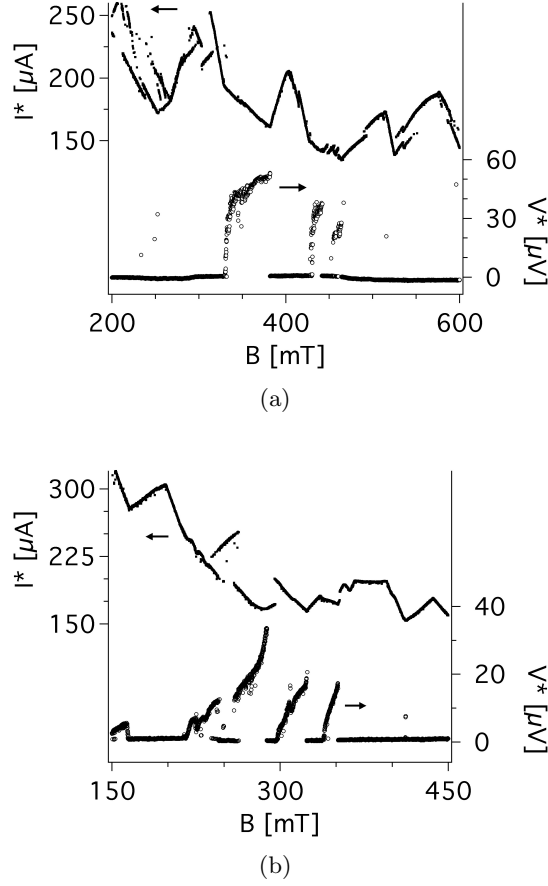


Figure 4.13: Magnetic-field dependence of the instability current I^* (left) and of the instability voltage V^* (right) for the loop LNb-3a [(a), $T = 3$ K] and for the loop LNb-2a [(b), $T = 4$ K]. V^* drops at the magnetic fields at which I^* is minimum.

the vortices configuration is not very stable), the vortices move at relatively low applied currents and a non-zero voltage appears at the extremities of the sample without leading to the complete transition to the normal state. In the other side, a large transport current must be applied when the pinning threshold is enhanced (i.e., for a more stable vortices configuration). In this case, the dissipation that results from the motion of a few vortices may be sufficient to induce the transition to the normal state, with a zero voltage measured just before the transition. The results of Fig. 4.13 reflect the above-described stability of the vortices arrangements that form in the arms and in the contacts of the loops. The modulations of the instability current highlight that more or less

stabilized initial states are formed during the field-cooling that precedes each measurement. This point is corroborated by the correspondence between I^* and V^* . Indeed, non-zero values of V^* are observed when the instability current decreases. In these field intervals, less stable vortices configurations develop and the vortices are easily displaced by the direct current. At the opposite, zero instability voltages mean that more stable configurations arise. As it was already mentioned, the instability current is the lowest when the unstable vortices system rearranges. In this sense, the sharp variations of V^* that occur in the vicinity of the I^* minima are of particular interest because they reinforce the occurrence of a transition from an unstable vortices configuration ($V^* \neq 0$) into a more stable one ($V^* = 0$) at the matching fields B_N . However, although V^* takes zero values when the vortices lattice rearranges, the associated minimum of the critical current indicates a non-optimal stability. The stability of the vortices lattice is further enhanced by an increase of the magnetic field until the critical current becomes locally maximum.

4.3.3 Matching fields position

4.3.3.1 Dependence on the coolant temperature

Most of the measurements have been performed well below the critical temperature, i.e., in the temperature-range where the current modulations are the more pronounced. The impact of the coolant temperature appears quite moderate, although no systematic study of the temperature evolution of the matching fields has been performed. Representative results are given in the figure 4.14(a) with the temperature evolution of the critical current vs magnetic field curve of the sample SNbN-W2. One sees that the position of the fields $B_{N \geq 2}$ remains almost unchanged in this temperature range (between $0.25T_c$ and $0.6T_c$). The matching fields that involve the transitions between a vortices pattern to another one only slightly decrease when the coolant temperature comes closer to the critical one. The situation slightly differs for the field $B_1 \sim B_{SH}$ which corresponds to the transition between a state with no vortex to a state with stable vortices. Indeed, as illustrated in Fig. 4.14(b), the field at which the first vortices line becomes stable appears more sensitive to the temperature as T_0 comes closer to T_c . This agrees with recent works on the temperature dependence of the superheating field [210, 211]. In particular, Valles et al. [210] have experimentally evidenced that the superheating field of their Nb samples behaves as $B_{SH} \propto 1-(T_0/T_c)^2$. Although there is a clear lack of data, the evolution observed here does not seem in contradiction with this latter trend.

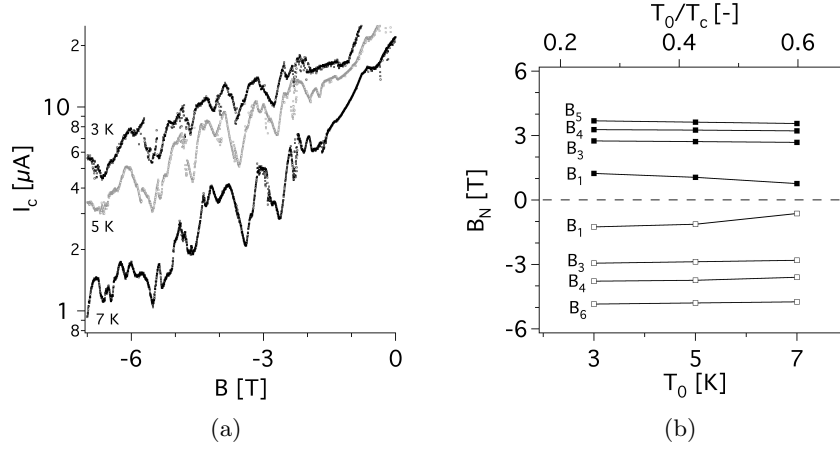


Figure 4.14: (a) Temperature dependence of the critical current vs magnetic field curves for the strip SNbN-W2. From top to bottom, the coolant temperature amounts to 3 K, 5 K and 7 K. (b) Evolution of the matching fields B_N with respect to the coolant temperature for the strip SNbN-W2. The values are summarized for both the positive and negative orientations of the magnetic field.

Furthermore, the present results do not depend on the direction of the field sweep⁸. They differ from many theoretical and experimental works (e.g., on mesoscopic Al disks [213, 214] and squares [215]) for which the field sweeping direction is relevant because the samples are not brought to the normal state at each magnetic field increment. In such measurements, the field at which the transition between a state with N vortices to a state with $N+1$ vortices is larger than the field at which the $N+1 \rightarrow N$ transition occurs. As a consequence, the authors distinguish between the penetration field (at increasing field) and the expulsion field (at decreasing field)⁹. Depending on the magnetic-field history, there is a field range over which two different vortices patterns are reachable. In the present experiment, there is only one possible scenario at each magnetic field : either some vortices remain stable inside the sample and form a more or less stabilized pattern, or there is no vortex after the

8. They have things in common with the electrical transport measurements performed in [212] on single Pb nanowires. In this latter case, the penetration of a vortex manifests itself by a drop of the forward critical current that occurs at lower fields when the coolant temperature increases.

9. The former was found to decrease when T_0 increases while the latter decreases very slightly (it is almost temperature-independent) when the system lies in a multi-vortex state [214, 215]. This comes from the unchanged currents that circulate near the sample edges and that result in almost T_0 -independent surface barrier for vortex expulsion.

current-driven field-cooling. It makes sense to draw a parallel between the transition that occurs here at B_1 and the penetration of the first vortex at increasing field observed in [214,215]. Indeed, in both cases, the initial pattern is the one with no vortex and the field of interest corresponds to the maximum delayed field below which the vortices are completely expelled.

4.3.3.2 Relation between the matching fields and the superheating field

The position of the matching fields for $N \geq 2$ is quite insensitive to the coolant-temperature. The following quantitative analysis is based on their low-temperature evolution. The dependence of B_N with respect to the number N of vortices rows is summarized in Fig. 4.15 for the NbN strips (a) and for the Nb loops (b).

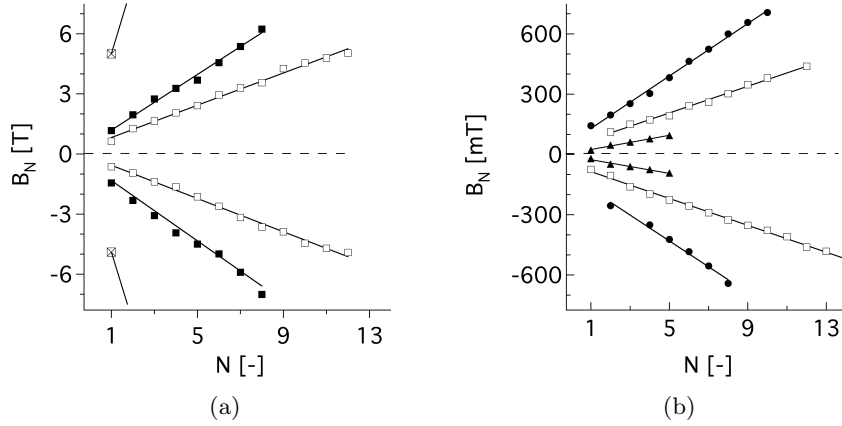


Figure 4.15: Evolution of the magnetic fields B_N that correspond to local minina of the critical or of the instability current as a function of the number N of vortices rows for (a) the strips SNbN-W1 ($\square$, $W = 20$ nm), SNbN-W2 ($\blacksquare$, $W = 45$ nm) and SNbN-W3 ($\square$, $W = 80$ nm), and for (b) the loops LNb-3a ($\bullet$, $W = 150$ nm), LNb-2a ($\square$, $W = 300$ nm) and LNb-2b ($\blacktriangle$, $W = 400$ nm). The B_N values are summarized for both the positive and negative orientations of the applied field.

The data are shown for both the positive and the negative orientations of the applied field, with no significant difference between them. The various slopes of course result from the varying spacing $\Delta B_N = (B_{N+1} - B_N)$ that are summarized in the table 4.2. In addition, the evolution of the fields B_N with the number of vortices lines has been discussed in details by Ziese et al. [125] (see the section 1.5.3.2, page 60). In the limit $\xi/\gamma W \ll 1$ (with γ an anisotropy factor which is here close to 1

and W the sample size perpendicular to the applied field), the fields B_N depend linearly on N according to the relation $B_N = B_{SH}(0.19 + 0.71N)$ [see Eq. 1.40]. The latter expression indicates that the formation of the first vortices lines is made easier under the action of a transport current ($B_1 < B_{SH}$). This remains valid for all the transitions between $N-1$ and N vortices rows. Indeed, as it was discussed in Sec. 1.5.3.3 (page 62), all the matching fields that are probed through the minima of the critical current are slightly reduced compared to the situation with no transport current. However, no information are available here concerning the shift of the B_N , which is assumed to be uniform on the whole field range. Fortunately, the exact positions of the fields B_N are not so relevant here as only the spacing between the fields is required to extract the superheating field through the relation $B_{SH,N} = \Delta B_N / 0.71$. The latter field is referred by $B_{SH,N}$ to indicate that it comes from the position of the matching fields B_N . The values that are obtained from linear fits of the experimental data are summarized in the table 4.2.

Table 4.2: Spacing $\Delta B_N = (B_{N+1} - B_N)$ for both directions of the applied field. The superheating field $B_{SH,N}$ is extracted using the relation 1.40 which gives that $B_{SH,N} = \Delta B_N / 0.71$. The NbN submicrometer-wide strips and the Nb loops are ordered with respect to their width W (equivalently, with respect to their decreasing superheating fields).

	ΔB_N [mT] ($B > 0$)	ΔB_N [mT] ($B < 0$)	$B_{SH,N}$ [mT]	W [nm]
SNbN-W2	690	750	1000	45
SNbN-W3	400	410	570	80
LNb-3a	65	64	91	150
LNb-2a (contact)	33	33	47	300
LNb-2b	18	17	25	400

The superheating fields $B_{SH,N}$ extracted from the evolution of the matching fields are compared in Fig. 4.16 with the theoretical models B_0 , B_s and B_K for the NbN strips (a) and for the Nb loops (b). As it was done for the characteristic field $B_{SH,peak}$, the stable solution B_s is depicted for three temperatures T_f at which the vortices become pinned during the field-cooling (T_f/T_c amounts to 0.9, 0.6 and 0.3 for the dashed curves labeled A, B and C, respectively). The error bars take into account the uncertainty associated to the exact width of the samples as well as the uncertainty related to the average spacing between the matching fields. For the NbN samples, the stable solution depicted at a coolant temperature close to the coolant one appears well-suited to explain the

data. Indeed, the stable solution at $T_f = 0.9T_c$ noticeably underestimates the experimental results while the B_0 and B_K solutions do not reflect the observed width-dependence. The same conclusion holds for the Nb samples although it appears really difficult to discriminate between the B_s and the B_K models as they are very close for the largest studied widths. The metastable solution B_0 also underestimates the results in the whole studied width range.

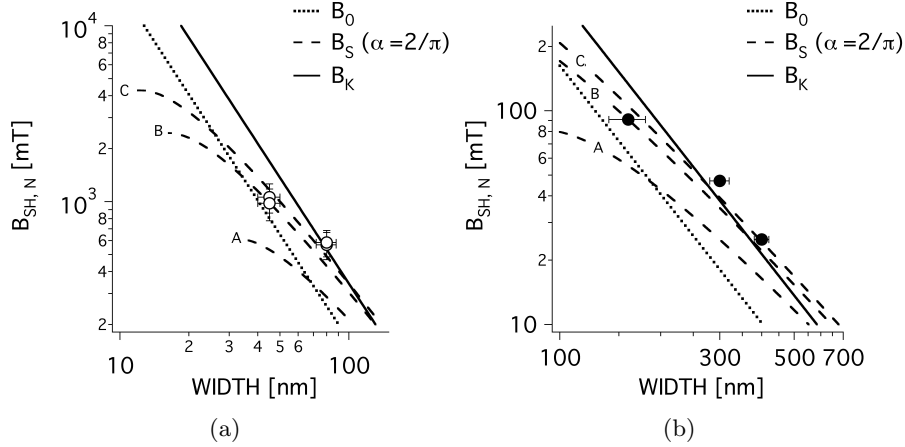


Figure 4.16: Width-dependence of the superheating field $B_{SH,N}$ extracted from the linear evolution of the fields B_N with respect to the number N of vortices rows for the NbN strips (a) and the Nb loops (b) [see Eq. 1.40]. The values extracted from the electrical transport measurements are compared with three theoretical critical induction models, namely the metastable solution B_0 (dotted curve), the stable solution B_s (dashed curves) and the B_K solution (solid curve). The stable solution is depicted for three values of the coherence length that correspond to different vortices pinning temperatures : T_f amounts to $0.9T_c$ (curve A), $0.6T_c$ (curve B) and $0.3T_c$ (curve C).

4.4 Superheating field - a summary

The previous sections have lead to two determinations of the superheating field, namely the field $B_{SH,peak}$ and the field $B_{SH,N}$. The former estimation is based on the entrance of the first vortices (see the section 4.2) and the superheating field is assimilated to the width of the central peak of the critical/instability currents vs magnetic field curves. The latter estimation is based on the arrangement of the vortices patterns under an increasing magnetic field (see the section 4.3) and the superheating field is determined thanks to the regular spacing between the matching fields at which the critical/instability currents are minimum.

All the experimental values of the field B_{SH} are summarized in a single frame in the figure 4.17. The open symbols correspond to the NbN samples while the closed symbols correspond to the Nb samples. The fields $B_{SH,peak}$ and $B_{SH,N}$ are depicted with squares and circles symbols, respectively.

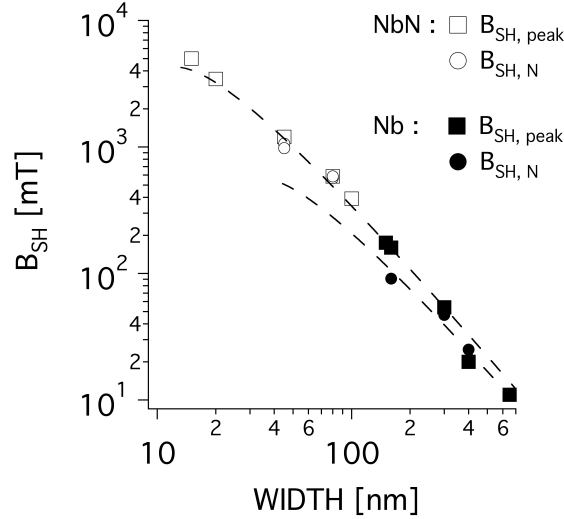


Figure 4.17: Summary of the superheating fields $B_{SH,peak}$ and $B_{SH,N}$ extracted from the electrical transport measurements of the sections 4.2 and 4.3. The data correspond to the width of the central peak ($\square$ for NbN and $\blacksquare$ for Nb) and to the values extracted from the positions of the matching fields B_N ($\circ$ for NbN and $\bullet$ for Nb). The solid curves are the stable solution B_s calculated with a reduced pinning temperature $T_f/T_c = 0.3$ [the top curve corresponds to the NbN samples with $\xi(0) \approx 4$ nm while the bottom curve corresponds to the Nb samples with $\xi(0) \approx 11$ nm].

From the conclusions of the sections 4.2.2 and 4.3.3, it appears that the model that compels an absolutely stable solution for the penetration of a vortex is better suited insofar as the vortices become frozen at a temperature close to the coolant one. This is illustrated in Fig. 4.17 with the dashed curves that correspond to the stable solution B_s at a reduced pinning temperature $T_f/T_c = 0.3$ [the top curve is for the NbN samples with $\xi(0) \approx 4$ nm while the bottom curve is for the Nb samples with $\xi(0) \approx 11$ nm]. One sees that the whole set of experimental data is consistent with this theoretical evolution although a definitive conclusion would require to know with precision the size of the vortices cores [$\sim 2\xi(T_f)$] at the temperature at which the vortices positions are frozen. This information, however, is not accessible in the present measurements. In any case, the overall agreement between the theory and the experimen-

tal data highlights that (i) studying the width of the central peak and (ii) studying the matching fields at which the vortices lattices rearrange themselves are interesting tools to determine the superheating field of narrow superconducting structures whose width is as small as a few tens of nanometers. In addition, the figure 4.18 compares the present estimations of the superheating field with the critical inductions experimentally determined by Stan et al. [8] with a scanning Hall-probe microscope (see the $\blacktriangle$ symbols). These values correspond to the maximum field below which the vortices are completely expelled by thin niobium strips whose width is in the μm -range (see the page 54 for further information). One notices that both the fields $B_{SH,peak}$ and $B_{SH,N}$ nicely follow the general trend given by these wider niobium strips. Here, the main advantage is that the superheating field is determined for much narrower samples, with no limitation due to the experimental setup resolution.

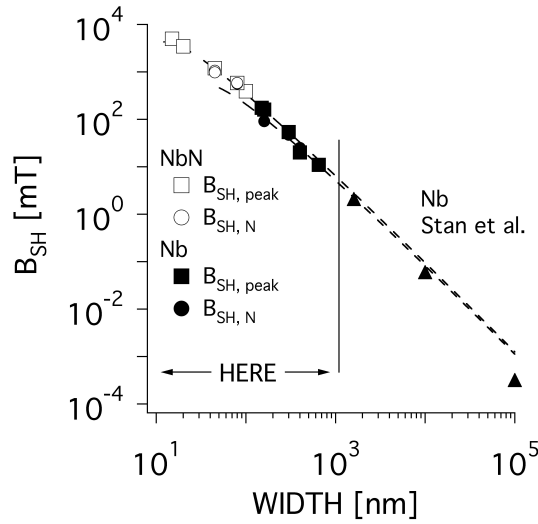


Figure 4.18: The same as in Fig. 4.17 with the values reported by Stan et al. [8] for wider niobium samples ($\blacktriangle$).

Conclusion

This Ph.D. thesis is devoted to the electrical transport properties of superconducting niobium nitride [NbN] and niobium [Nb] nanostructured strips and loops. The experiments have been performed on multi-probe structures lithographed by Damien Lucot at the Laboratoire de Photonique et de Nanostructures (LPN-CNRS) of Marcoussis. Unusual - and totally reproducible - properties have been underlined by applying a direct bias in various experimental conditions, i.e., under a continuous microwave irradiation, under a static external magnetic field, or simply as a function of the coolant temperature.

The current-carrying properties of the NbN and Nb submicrometer-wide strips turn out to be mainly dictated by the heating effects. First, this manifests itself by the strongly hysteretic current-voltage characteristics. The model of the localized self-heating hotspot in a strip much longer than the thermal healing length (η) is particularly well-suited to describe this thermal bistability under Joule dissipation. A relevant point is that both the magnitude and the temperature-dependence of the current-driven normal-to-superconducting transition current (succinctly referred as the retrapping current, I_r) are consistent with the evacuation of the Joule effect dissipation through the surroundings of the samples. Indeed, the retrapping current density j_r is given by the power balance between the Joule heating term $\rho_n j_r^2$ (where ρ_n is the normal state resistivity) and the removal capabilities through the strip/substrate interface that amount to a few $\mu\text{W}\mu\text{m}^{-3}$ for all the samples. This leads to a much lower retrapping current density for the NbN samples as their normal state resistivities are two orders of magnitude larger than for the Nb samples. The influence of the heating also manifests itself by the numerous dissipative states that are observed between the perfect superconducting state and the normal state of the NbN strips. These resistive domains are stabilized in the hotspot plateau, i.e., a current interval left-bounded by the retrapping current which is assimilated in this context as the minimum applied current that sustains a normal-zone. Interestingly, the right-side boundary of the stabilization range

has the same temperature dependence as I_r , so that it may be safely associated to the minimum applied current above which a normal-zone propagates. A relevant point is that the two boundaries of the stabilization current-range spread over a finite interval that quantitatively matches the nanoscale modulations of the sample width determined by scanning electron microscopy ($\Delta W \sim 10$ nm). As a consequence, each resistance step in the current-voltage characteristics is interpreted as a modification of the number or the length of localized normal hotspot domains whose boundaries are pinned at different locations along the sample length where the cross-section slightly differs from the nominal one. This is of the utmost relevance as one is often tempted to consider ideal samples whose cross-section modulations are negligible. The main message is that the nanoscale cross-section modulations are responsible for the stepwise evolution of the sample resistance under a decreasing as well as an increasing bias. In addition, the large number of dissipative states seems to reflect the small thermal healing length $\eta \sim 60$ nm of the NbN samples. This is especially the case in the voltage-driven mode because the variations of the applied current allow one to probe a much larger number of localized inhomogeneities along the sample length.

The above-described stepwise evolution of the resistance under both an increasing or a decreasing current reveals that the distinction between the current-induced stabilization of normal hotspot domains and of phase-slippage phenomena is far from easy. Indeed, most of the features of the current-voltage characteristics of the NbN submicrometer-wide strips could be explained in the framework of the phase-slippage phenomena. However, there are stronger arguments in favor of a purely heating-based scenario, especially to explain the scatter of the transition currents. The striking correspondence between the regular resistance steps in the forward and retrapping parts of the hotspot plateau indicates that the same stabilized dissipative structures are involved. As the retrapping transition undoubtedly results from purely thermal effects, the nucleation of successive phase-slippage phenomena is rejected to explain the forward current-driven transition. This is all the more verified that similar resistance levels are probed in a wide coolant temperature range, i.e., for $T_0 \in [0.2T_c, 0.8T_c]$, where T_c is the critical temperature. The only requirement to observe a stepwise evolution of the voltage for $T_0 < 0.8T_c$ is to sweep the applied current back and forth in the hotspot plateau, which is rarely performed in practice. Furthermore, the above-discussion highlights that numerous hotspots configurations are stabilized in dirty superconducting submicrometer-wide strips whose transverse dimensions are much wider than their coherence length. Much care is thus required when a regular stepwise behavior arises, especially

when the coherence length of the material is small (as for the high temperature superconductors) and when the coolant temperature is much lower than the critical one.

From a practical viewpoint, studying the spreading of the current-driven normal-to-superconducting transition proves oneself to be a powerful tool to characterize the evenness of the sample cross-section. This may provide useful information for many practical devices based on narrow thin-film superconducting strips. This is especially the case for the optoelectronic devices that involve heating-induced phenomena, e.g., for the NbN- and Nb-based superconducting nanowire single-photon detectors that operate in the pulsed-regime in order to detect a limited number of photons. This thesis further provides a complementary description of the response of similar structures in the steady-state regime, i.e., under a continuous microwave irradiation. The prevailing role played by the thermal dissipation effects is powerfully corroborated by the evolution of the hotspot plateau under an increasing microwave input power. Concretely, this manifests itself by a decrease of the normal-to-superconducting transition current whose shift is all the more pronounced when the microwave power density absorbed by the microstrips increases. This leads to the progressive enlargement of the current range over which the localized normal domains remain stable. Incorporating the microwave power density in the power balance between the normal domains and their surroundings indicates that the continuous irradiation comes down to a source of dissipation that adds up to the current-induced Joule heating. The main point is that a normal domain remains stable insofar as the total heat dissipation (Joule heating due to the direct current and microwave dissipation) balances the heat removal capabilities. A borderline case occurs when the microwave dissipation is sufficient by itself to maintain the electronic temperature above T_c . This leads to the appearance of zero-crossing resistance steps in both parts of the current-voltage characteristics. In addition, this thesis succeeds in determining the tiny fraction of the total input power which is effectively absorbed by the normal domains in the steady-state regime. The latter varies linearly with the total input power and falls in the $\mu\text{W}\mu\text{m}^{-3}$ range for both the NbN and Nb submicrometer-wide structures. The knowledge of the microwave power absorbed in the normal state may be of practical interest in the field of incident power detection. The upper limit of the detection range is given by the temperature-dependent heat removal capabilities through the substrate, while the relation between the input power and the absorbed power does not depend on the coolant temperature at a given microwave frequency.

In addition, the ultimate performances of electrically-biased superconducting nanostructured devices are often limited by the presence of vortices whose motion leads to an undesirable thermal dissipation. One way to weaken this downside is to reduce the samples width in order to increase the critical field below which the vortices are completely expelled in the absence of a transport current. This so-called superheating field (B_{SH}) has been determined for NbN and Nb structures whose width is as small as a few tens of nanometers. The electrical transport measurements are based on the magnetic-field dependence of their critical (I_c) and instability (I^*) currents that are defined by the appearance of a voltage and by the transition to the fully normal state, respectively. The first approach is based on the central peak region which corresponds to the field-range where no vortex remains stable after the current-driven field-cooling that precedes each current detection. The width of the central peak is directly related to the superheating field, which varies from ~ 10 mT up to ~ 5 T for the sample whose narrowest part has a width $W \sim 15$ nm. The second approach comes from the regularly-spaced minima of the critical/instability currents above B_{SH} . Interestingly, sharp drops of the instability voltage occur in the vicinity of the I^* minima, which reinforces the occurrence of a transition from an unstable vortices configuration into a more stable one at the associated matching fields. The spacing between the matching fields being related to the superheating field, their position provides an indirect way to extract the field of interest. Furthermore, all the experimental values of B_{SH} are consistent with the theoretical width-dependence, i.e., $B_{SH} \propto \Phi_0/W^2$, where Φ_0 is the flux quantum. A definitive discrimination between the various models is made difficult due to the unknown temperature at which the vortices become pinned in the current-driven field-cooled measurements. Nevertheless, the model that compels an absolutely stable solution for the penetration of a vortex seems better suited insofar as the vortices become frozen at a temperature close to the coolant one. The experimental data are in nice agreement with recent values obtained by scanning probe microscopy on wider niobium strips. Here, the main advantage is that the superheating field is determined for much narrower samples, with no limitation due to the experimental setup resolution. Finally, the weak-link-free superconducting Nb loops have revealed that the periodic oscillations of the critical current that result from the quantization of the fluxoid can be superimposed to the modulations induced by the rearrangement of the vortices patterns.

Outlook

This thesis originally aimed at investigating the basic properties of the superconductivity at the nanoscopic scale. From a fundamental viewpoint, it would be interesting to widen the experiments on commonly-used materials, especially the high temperature superconductors and the magnesium diboride compound. On the other hand, the electrical transport measurements have highlighted some features that are useful for applicative devices : (A) the cross-section roughness is characterized by the spreading of the retrapping current, (B) the power handling capabilities under a continuous irradiation are described by the microwave-dependence of the retrapping current, and (C) the critical field for vortex entrance of narrow samples is easily estimated. These findings are especially relevant for devices that consist in narrow thin-film strips. In this context, these outlooks focus on two additional concerns related to the performances of devices that usually consist in ultrathin NbN and Nb meanders, namely the superconducting nanowire single-photon detectors (SNSPDs)¹⁰. The latter are currently used in various domains of fundamental research as well as in characterization and measurement applications.

Thermal latching behavior The detection of single-photon events requires a very high counting rate that usually exceeds 1 GHz. The maximum count rate is mainly limited by the dead time, which is the minimum time between two consecutive detections. This dead time ($\sim$ nanosecond) is intimately related to the time required for the photon-induced normal domain to collapse after the bias current is redistributed through the shunt resistor. Many works have been recently performed in order to better understand the hotspot dynamics which are involved in the SNSPDs. In particular, it was recently shown that increasing the operation frequency leads to a stable electrothermal feedback : the device latches into a stable resistive state and can no longer detect photons [196, 216, 217]. With regard to the results presented in this manuscript, the latching occurs when the current that passes through the meander after the current redistribution through the shunt lies in the normal-domain stabilization current range. The hotspot plateau is thus of great importance because it impacts the working regime of the photonic devices. While some authors question about the opportunity to avoid the electrothermal latching by improving the thermal coupling between the strip and the substrate [217], others infer that the limited detection efficiency of their devices results from constrictions (either

10. The working principle of the single-photon detectors has been introduced in the section 3.4.2.5 (page 146).

thickness variations or material defects) [218]. These both concerns are rarely considered in a single explanation. By contrast, this manuscript emphasizes the utmost interdependence that exists between the cross-section roughness and the appearance of stable dissipative structures. As a consequence, it would be useful to perform a systematic study of the roughness impact on the SNSPDs count rate limitation.

Current-activated vortices One has to remember that a SNSPD is biased close to its critical current prior to the photon detection. A voltage pulse is measured each time a hotspot domain is created due to the interaction between the incident photon and the superconducting meander line. The dark counts correspond to similar voltage pulses that are recorded in the absence of photons (dark count rate ~ 10 kHz). Based on recent works on the current-assisted fluctuations in narrow thin-film strips [165, 200, 206], it is now well accepted that the dark counts originate from the current-activated motion of vortices [219, 220]. In direct relation, this manuscript highlights that the current-driven metastable nature of the superconducting state manifests itself by a broadened critical current distribution at low applied magnetic fields. In particular, the distribution spreading noticeably narrows above the critical field for vortex entrance, i.e., when stable vortices are present inside the superconducting strips in the absence of a transport current. As the formation of a stable vortices pattern makes more reproducible the current-biased departure from the superconducting state, it would be interesting to investigate the operating regime of the SNSPDs above the superheating field. In particular, a systematic study of the magnetic field dependence of the dark count rate would be clarifying.

Appendix A

Fluxoid quantization in mesoscopic loops

This appendix takes advantage of the loop configuration of the niobium samples to discuss in a few words the interplay between the entrance of vortices and the fluxoid quantization effect in a multiply connected superconductor¹. The fluxoid Φ' differs from the flux Φ that passes through the area inside a closed contour inside a superconductor by a line integral of the supercurrent around this latter contour. It is defined by the following equation [19],

$$\Phi' = \Phi + \left(\frac{m^*}{e^*}\right) \oint \vec{v}_s \cdot d\vec{s}, \quad (\text{A.1})$$

where $\Phi \equiv \oint \vec{A} \cdot d\vec{s}$ ($\vec{A}$ is the potential vector such that $\vec{B} = \vec{\nabla} \times \vec{A}$), m^* and e^* are the mass and the charge of a Cooper pair, respectively, and $\vec{v}_s$ is the supercurrent velocity. This year celebrates the half-centennial of the experimental observation of the quantization of Φ' [223, 224], a concept which was first introduced by Fritz London [225] that stated in 1950 that *there exists a universal unit for the fluxoid*. Indeed, by applying the Bohr-Sommerfeld quantum condition, the expression A.1 can be written as follows,

$$\Phi' = \frac{1}{e^*} \oint \left(m^* \vec{v}_s + e^* \vec{A}\right) \cdot d\vec{s} = \frac{\oint \vec{p} \cdot d\vec{s}}{e^*} = \frac{Lh}{e^*} = L\Phi_0, \quad (\text{A.2})$$

where $\vec{p}$ is the momentum of the Cooper pairs, h is the Planck constant, Φ_0 is the flux quantum and L is an integer called the fluxoid quantum number. If the flux Φ imposed at a given applied magnetic field differs

1. For reviews on this subject, see e.g. [221, 222].

from an integer multiple of Φ_0 , the quantization of Φ' requires the presence of circulating currents that ensure that the expression A.2 holds. The intensity of the latter currents is thus a periodic function of Φ/Φ_0 , which cancels when $\Phi/\Phi_0 = L$.

There is a relevant distinction between the multiply connected vortices lattice of a type-II superconductor and the above-discussed concept of fluxoid. In the former case, the vortices lattice is composed of quantized flux lines, each carrying a single (or many) flux quantum(a). However, this is conceivable, e.g., for a type-II superconducting loop, to have a non-zero fluxoid ($L \neq 0$) even in the absence of vortices in the material. The fluxoid can be viewed as a *coreless* vortex with the core replaced by the opening at the middle of the sample [222]. In this appendix, the fluxoid-related periodic modulations of the critical and instability currents are first discussed in A.1. The temperature evolution of the circulating currents are used to extract the kinetic inductance as well as the magnetic penetration depth of the samples. The impact of the vortices entrance on the observation of the fluxoid quantization is then briefly illustrated in A.2.

A.1 Modulations of the circulating currents

Consistency with the fluxoid quantization effect

A supercurrent flows inside a superconducting loop when the applied flux is a non-integer multiple of the flux quantum. For a perfectly symmetrical loop, the applied transport current splits equally in both branches of the loop. When $\Phi = L\Phi_0$, the circulating currents cancel and the measured critical current amounts to twice the critical current of each individual branch. For $\Phi \neq L\Phi_0$, the circulating currents flow clockwise or counter-clockwise depending on the exact value of the applied field. In this case, the critical current is reached in the branch where the circulating currents add up to the transport one. This leads to a saw-tooth evolution of the critical current with respect to the applied field, with a theoretical field periodicity $B_{\Phi_0}^{theo} = \frac{\Phi_0}{\pi r_M^2}$ given by the area enclosed by the mean radius r_M of the loop [226]. The figures A.1 and A.2 illustrate the temperature evolution of the periodic modulations of the instability current for the niobium loops LNb-1a and LNb-1b, respectively. Contrary to recent works that focus on the impact of the current leads location, either symmetrically or asymmetrically positioned around the loop [227–229], all the curves reported here have been obtained by applying the transport current using the symmetrical contacts.

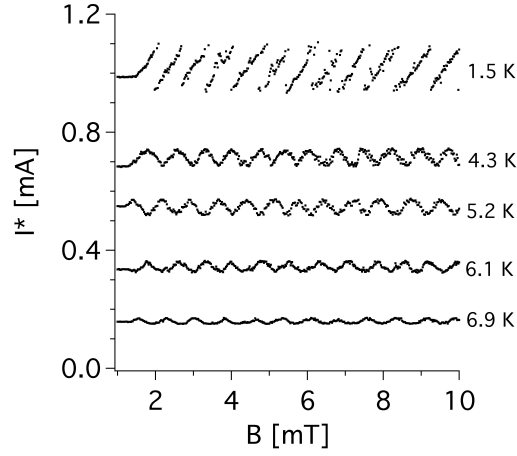


Figure A.1: Magnetic field-induced modulations of the instability current of the niobium loop LNb-1a that reflect the fluxoid quantization effect.

The curves of Figs. A.1 and A.2 are fully consistent with recent results obtained on niobium and aluminum loops that contain weak-links [230] or not [228,231]. The impact of the circulating currents is all the more pronounced when the coolant temperature is reduced with a full-height of the modulations that amounts to around 10 % of the instability current. As it was recently highlighted by Michotte et al. [228], studying the evolution of the instability current with respect to the applied field is an efficient tool to probe the fluxoid quantization effect at temperatures far below T_c . The modulations, however, remain observable at least until $T_0 = 0.8T_c$, which is the highest coolant temperature at which the measurements have been performed.

The table A.1 summarizes the experimental (B_{Φ_0}) and theoretical ($B_{\Phi_0}^{theo}$) field periodicities for the four loops for which the fluxoid quantization effect was observed. One sees a nice agreement between the experience and the theory, using the area enclosed by the mean radius r_M of the loop. The observation of the fluxoid quantization was expected for the loops LNb-1a and LNb-3a, as it was pointed out in the table 2.4 (page 84) that the critical width (i.e., the width of the cross-section where the critical current density is first reached) is located in a branch. Accordingly, it was also expected (and experimentally verified) that no effects are observed for the loops LNb-2a and LNb-2b as the critical width is located in one of the contacts, where the circulating currents do not flow. By contrast, this is quite surprising to probe the circulating currents for the loops LNb-1b and LNb-1c because the SEM images

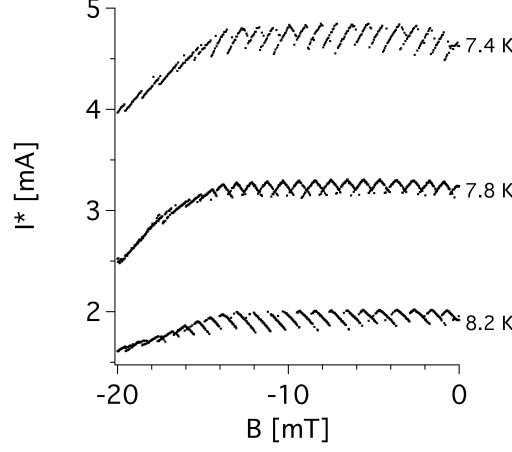


Figure A.2: Magnetic field-induced modulations of the instability current of the niobium loop LNb-1b that reflect the fluxoid quantization effect.

indicate that the critical width is located in one of the contacts. The electrical transport measurements, however, indicate that the critical width is also influenced by the presence of the circulating currents, and hence is most probably located in a contact but very close to the place where the two branches separate.

Table A.1: Experimental (B_{Φ_0}) and theoretical ($B_{\Phi_0}^{theo}$) field periodicities that are associated to the fluxoid quantization. The theoretical field scale $B_{\Phi_0}^{theo} = \frac{\Phi_0}{\pi r_M^2}$ is given by the area enclosed by the mean radius r_M of the loop (Φ_0 is the flux quantum).

	B_{Φ_0} [mT]	$B_{\Phi_0}^{theo}$ [mT]	r_M [nm]
LNb-1a	0.78	0.81	900
LNb-1b	0.97	0.97	825
LNb-1c	1.3	1.25	725
LNb-3a	7.8	7.8	290

The behavior of the loop LNb-3a deserves to be discussed in more details. Indeed, as it was illustrated in Fig. 4.11(b), this sample is prone to the observation of regularly-spaced minima of the instability current above the superheating field, with a period of around 65 mT. The figure A.3 focuses on the evolution of I^* in the low-field range. The observed periodicity of ~ 7.8 mT is in excellent agreement with the periodicity of the fluxoid quantization, which is much smaller than the one associated to the rearrangement of the vortices lattice above B_{SH} . As pre-

viously discussed, these two behaviors can be unambiguously discriminated thanks to the different size-dependence of the characteristic fields B_{Φ_0} and B_{SH} . In addition, one sees that the behavior reported in Fig. A.3(b) slightly differs from the one of Figs. A.1 and A.2. Indeed, a clear saw-tooth behavior is observed in the latter case, while only a few slanted decreases are visible for the loop LNb-3a. Interestingly, there exist strong similarities with the evolution of the instability current of niobium nanoSQUIDs measured before and after the nanofabricated weak-links were partially oxidized using atomic force microscopy techniques. Indeed, as illustrated by the figure 3 of the work of Faucher et al. [230], the modulations observed here in the low-field region of Fig. A.3 are close to the one observed on non-oxidized niobium nanoSQUIDs, while the behavior reported in Figs. A.1 and A.2 present strong similarities with oxidized samples. In Ref. [230], the oxidation of the weak-links goes together with a reduction of the instability current, which can be interpreted as a reduction of the superconducting cross-section. By contrast, the loops whose behavior is here the closest to the oxidized nanoSQUID have a larger nominal cross-section ($S \sim 75 \text{ nm} \times 100 \text{ nm}$ or 250 nm) than the one that present similarities with the non-oxidized nanoSQUID ($S \sim 20 \text{ nm} \times 150 \text{ nm}$). In the present case, both behaviors can not be directly related to a different superconducting cross-section and the explanation remains an open question.

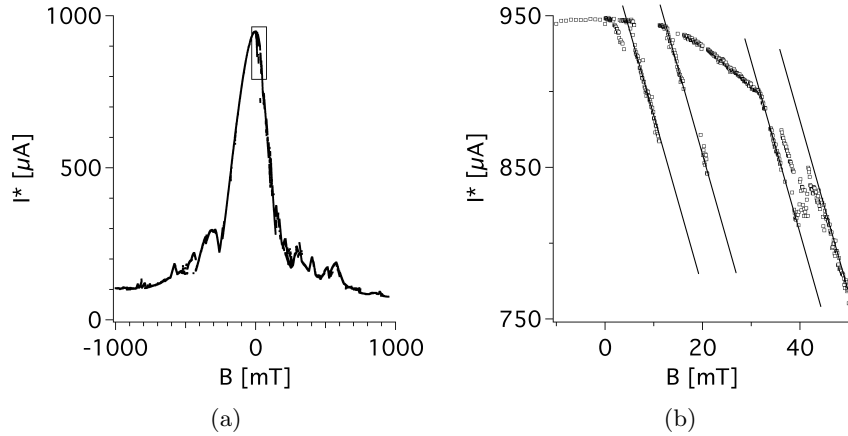


Figure A.3: (a) Overall evolution of the instability current of the loop LNb-3a as a function of a perpendicular magnetic field ($T_0 = 3 \text{ K}$). (b) Enlargement of the rectangular frame of the part (a). The behavior is reminiscent of the fluxoid quantization effect with a field periodicity $B_{\Phi_0} \sim 7.8 \text{ mT}$ given by the area enclosed by the mean radius of the loop.

Temperature dependence of the circulating currents

The periodic modulations of the critical current in a weak-link-free superconducting loop were first proposed by Fink et al. [232]. This is reminiscent of the behavior of conventional superconducting quantum interference devices (SQUID) that contain a weak-link in one or both branches of a superconducting loop. The weak-links can be either Josephson junctions or variable thickness bridges (see [233, 234] for examples based on niobium structures). In any case, the resolution of the device is related to the variation of the circulating currents with respect to a change of the applied magnetic flux. The sensitivity is thus defined by the variation ΔI^* which is associated to a flux variation $\Delta \Phi$. The evolution of the sensitivity with respect to the reduced coolant temperature T_0/T_c is given in μA per flux quantum in the figure A.4(a). The data are depicted for the loops LNb-1a (■), LNb-1b (●), LNb-1c (◇) and LNb-3a (△). At the lowest temperatures, these sensitivities range between 5×10^{10} and $2 \times 10^{11} \text{ AWb}^{-1}$. In addition, as previously discussed, the instability current of the loops evolves linearly with the applied field with a rather constant slope. This is consistent with the linear behavior observed for superconducting weak-links in the high inductance limit [235]. The global inductance L_G of the samples is given by the relation $\Delta \Phi = L_G \Delta I^*$. The temperature evolution of L_G is depicted in the figure A.4(b). At low temperatures, L_G amounts to $\sim 2 \text{ pH}$ for the loops LNb-1b and LNb-1c while slightly larger values are extracted for the loops LNb-1a ($L_G \sim 6 \text{ pH}$) and LNb-3a ($L_G \sim 10 \text{ pH}$). For a superconductor, the kinetic inductance, which originates from the kinetic energy of the superconducting condensate, often dominates in comparison to the geometrical one. From the Meservey relation [236]

$$L_K = \frac{m^*}{n_s e^* 2} \frac{L}{S} = \mu_0 \lambda_L^2 \frac{L}{S}, \quad (\text{A.3})$$

one sees that the kinetic inductance depends on the penetration depth λ_L , and hence on the coolant temperature (m^* , n_s and e^* are the mass, the density and the charge of the Cooper pairs, respectively, μ_0 is the vacuum permeability, L and S are the length and the cross-section area of a branch of the loop, respectively). If one neglects the geometrical inductance (so that $L_K \sim L_G$), the penetration depth of the samples can be determined using Eq. A.3. One sees in the figure A.4(c) that the low-temperature magnetic penetration depth λ_{L0} amounts to $\sim 120 \text{ nm}$ for all the samples, in agreement with the zero temperature $\lambda_{BSC}(0)$ values that were obtained in the section 2.3 in the dirty limit (see the table 2.7, page 104). In addition, the empirical temperature dependence of the

penetration depth is verified as the solid lines in Fig. A.4(c) correspond to the two-fluid model [19] relation $\lambda_L(T) \approx \lambda_{L0}/[1 - (T/T_c)^4]^{1/2}$. All these observations reinforce the validity of the interpretation based on the fluxoid quantization effect.

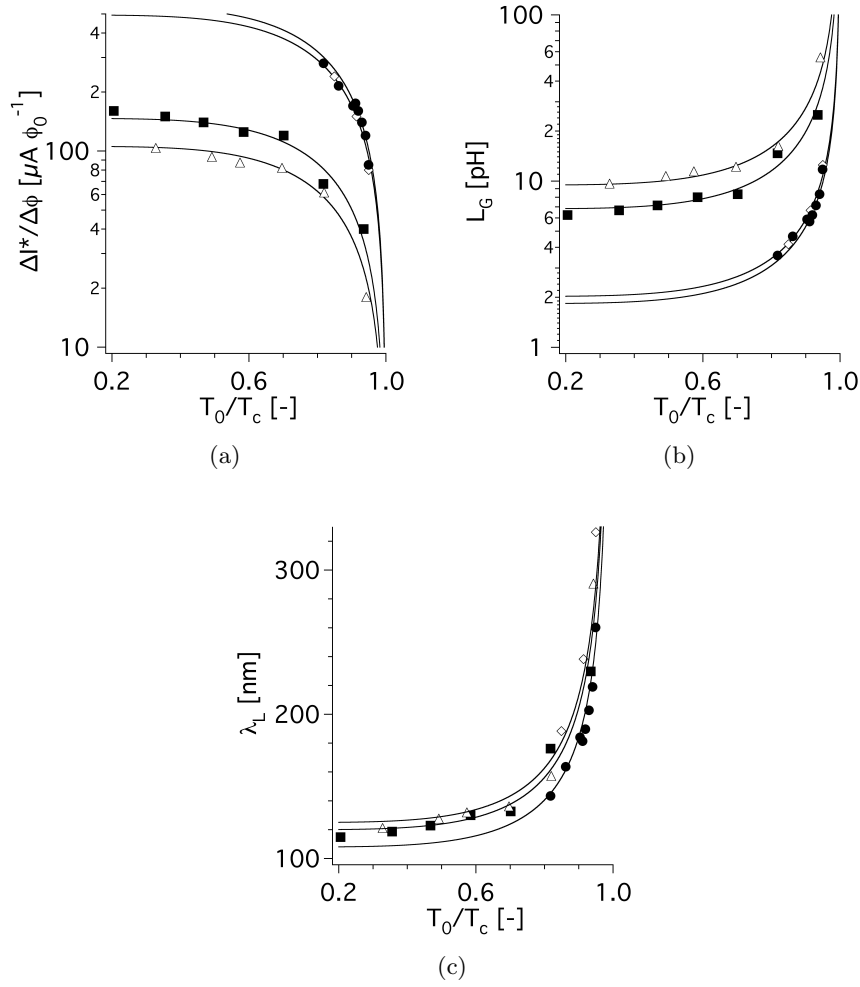


Figure A.4: Temperature dependence of the parameters extracted from the fluxoid quantization effect for the loops LNb-1a (■), LNb-1b (●), LNb-1c (◇) and LNb-3a (△) (a) Sensitivity $\Delta I^*/\Delta \Phi$ expressed in μA per flux quantum. (b) Global inductance $L_G = \Delta \Phi/\Delta I^*$. (c) Magnetic penetration depth λ_L .

Little-Parks effect

The oscillating circulating currents noticeably increase the kinetic energy of the superconducting condensate, while the free energy of the normal state is virtually independent of the flux. As a consequence, the critical temperature is also impacted by the quantization of the fluxoid. This results in oscillations of the resistance close to the critical temperature when the applied magnetic field is varied. This effect was first discussed by Little and Parks [237] who took advantage of the finite width of the resistive transition of tin thin-walled cylinders to highlight the periodic variations of T_c ($\Delta T_c \sim 3$ mK). From this time, many works were devoted to the observation of the so-called Little-Parks effect, e.g., on aluminum [238], niobium [152], and recently on high temperature YBCO loops [239].

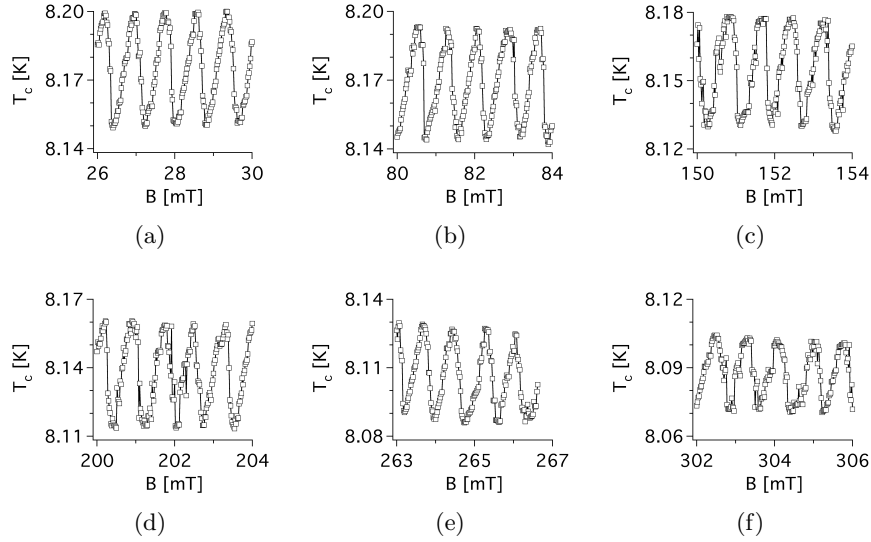


Figure A.5: Magnetic field-induced modulations of the critical temperature of the loop LNb-1a. The curves are obtained by recording the temperature at which the resistance of the sample remains at a constant value ($R_{crit} = 0.25 \Omega$, i.e., around 5 % of the normal state resistance of the loop). The direct transport current is $20 \mu\text{A}$.

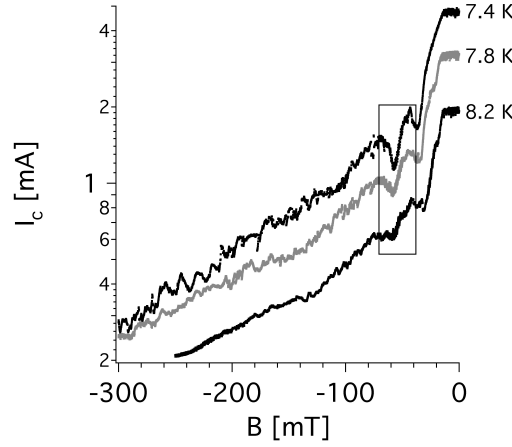
The Little-Parks effect was observed on the loop LNb-1a. The figure A.5 illustrates the periodic oscillations of the critical temperature that were obtained by applying a direct transport current of $20 \mu\text{A}$ and by keeping the sample resistance at around 5 % of the normal state value R_n . The field periodicity amounts to $B_{\Phi_0} \sim 0.8$ mT, in agreement with the value reported in the table A.1. The amplitude of the oscillations has been optimized by modifying the resistance level as well as the applied

current. By contrast with the latter parameter, the resistance level has a noticeable influence on the height of the oscillations. Indeed, as it was recently highlighted by Berdiyorov et al. [152], probing the Little-Parks effect on a loop whose normal-to-superconducting transition is multistage requires to adjust the resistance criterion [the multistage transition of the sample LNb-1a is given in the figure 2.20(b), page 94]. Here, the modulations of T_c continuously decrease as the resistance criterion is increased, from $\Delta T_c \sim 50$ mK for $R_{crit} = 0.05 R_n$ down to ~ 5 mK for $R_{crit} = 0.4 R_n$. In the conditions of Fig. A.5, ΔT_c progressively diminishes when the applied field is increased, and becomes indistinguishable at around 350 mT. As a final comment, let's notice that the largest ΔT_c noticeably exceeds the theoretical prediction which is given in the dirty limit by $0.18\xi_0\ell/r_M^2 \times T_c \approx 20 \mu\text{K}$ [19].

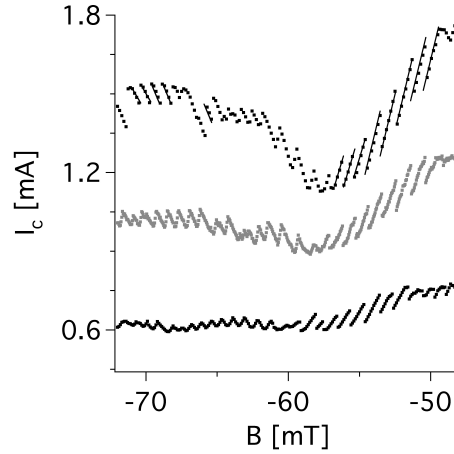
A.2 Impact of the presence of vortices

In the section A.1, it was shown that the fluxoid quantization effect manifests itself in the multi-probe mesoscopic Nb loops by periodic modulations of the critical (or instability) current, even if no intentional weak-links are present in the branches of the samples. Now, it will be shown (i) that the periodic oscillations of the critical current that result from the quantization of Φ' can be superimposed to the modulations induced by the rearrangement of the vortices patterns and (ii) that the entrance of vortices prevents a clear observation of the fluxoid quantization. The coexistence of modulations of the critical current due to the fluxoid quantization or the rearrangement of the vortices is illustrated in the figure A.6 with the evolution of the critical current I_c of the loop LNb-1b with respect to the applied magnetic field. The low-field part in Fig. A.6(a) [between -20 mT and 0 mT] has been previously enlarged in the figure A.2. Above the superheating field B_{SH} , one clearly sees some local minima of the critical current. A careful analysis indicates that the field periodicity ΔB_N is around 12 mT, which gives a superheating field $B_{SH,N} = \Delta B_N/0.71 \sim 17$ mT. This latter value agrees with the previous results obtained on other Nb loops that are summarized in the figure 4.16(b), page 188 [the width of the critical area here amounts to 420 nm]. In addition, the enlargement of the rectangular frame of the figure A.6(a) reveals additional periodic modulations of the critical current with a period $B_{\Phi_0} \sim 0.98$ mT, in agreement with the value reported in the table A.1 for the fluxoid quantization effect. Interestingly, the slope $\Delta I_c/\Delta \Phi$ is either positive or negative. As the direction of the applied current remains unchanged, this indicates that the voltage crite-

rion is reached at two different locations, i.e., in one branch of the loop, then in the other. As discussed by Michotte et al. [228], the presence of vortices inside the sample does not impact the flow of the circulating currents that ensure the quantization of Φ' . Indeed, the supercurrents that flow around each vortex do not contribute to the total circulating currents due to the symmetry of the current lines around the vortex core.



(a)



(b)

Figure A.6: (a) Field-dependence of critical current of the loop LNb-1b. (b) Enlargement of the rectangular frame of the part (a) that highlights the coexistence of the rearrangement of the vortices lattice and of the fluxoid quantization effect.

Finally, the figure A.7 illustrates the low-field evolution of I^* for three loops that belong to the same multi-probe configuration. From top to bottom, the width of the critical area decreases : 650 nm (LNb-1c), 420 nm (LNb-1b) and 100 nm (LNb-1a).

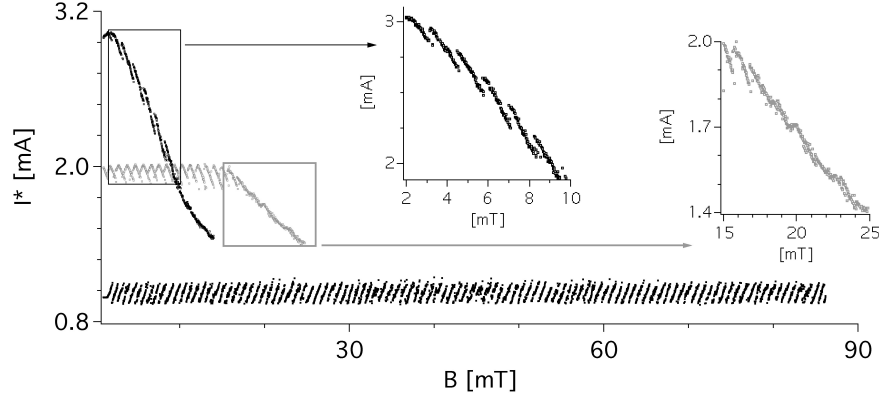


Figure A.7: Impact of the penetration of vortices in the observation of the fluxoid quantization effect. From bottom to top : LNb-1a ($W \sim 100$ nm, $T_0 = 1.5$ K), LNb-1b ($W \sim 420$ nm, $T_0 = 8.2$ K) and LNb-1c ($W \sim 650$ nm, $T_0 = 8.4$ K).

The overall envelopes significantly differ, which impact the observation of the saw-tooth evolutions. This is intimately related to the ability for a vortex to remain stable inside the sample just after the current-driven field-cooling or to the ability for a vortex to penetrate inside the sample under the action of the swept direct transport current. This is in turn related to the superheating field B_{SH} of the area which first goes to the normal state. The results can be summarized as follows. The sample LNb-1c has the lowest superheating field of the three samples, i.e., $B_{SH,peak} \sim 11$ mT. The overall envelope drops at a few mT due to the penetration of the first vortices that is favored at lower currents when the applied field increases. Concerning the sample LNb-1b, the modulations are clearly visible until the field exceeds ~ 16 mT. As indicated in the inset, less-defined modulations remain present above this field. Interestingly, the field value at which the envelope drops is close to the $B_{SH,N}$ value determined from Fig. A.6. As a consequence, one may consider that, above $B \sim 16$ mT, some vortices are present inside the sample before the current is increased from zero after the field-cooling. Lastly, the drop has not yet occurred at $B \sim 90$ mT for the sample LNb-1a. Unfortunately, this latter sample broke before the end of the measurements so that the exact field value at which the drop would have occurred is not known with precision. From the previous discussion [es-

pecially the figure 4.17, page 189], one sees that a superheating B_{SH} larger than 200 mT is expected for this sample ($W \sim 100$ nm), which is consistent with the unfavorable presence of vortices inside the sample in the magnetic field range that has been probed.

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