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Bohr's Theory of the Hydrogen Atom: A Selective Realist Interpretation

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Abstract This paper proposes a reconstruction of 1913 Bohr's theory of the hydrogen atom in the framework of the model-theoretic approach of theories. On the basis of this reconstruction, it is argued that Bohr's theory is not internally inconsistent and can't be qualified as fictitious. Then, a selective realist interpretation of Bohr's theory can be defended according to which electrons occupy energy levels. An agnostic attitude however is recommended as far as the electron's trajectories are concerned.

Bohr's theory of the hydrogen atom has been the subject of extensive discussion in philosophy and history of science. Yet, no detailed reconstruction of Bohr's theory and way of proceeding in the framework of the model-theoretic approach of theories has been provided so far.[1] Here, I propose such a reconstruction, but I also give its due to the role of propositions as important ingredient of scientific theories. In fact, theories consist in mathematical structures, namely models, and also in propositions made true or satisfied by those models. Furthermore, a scientific theory provides useful information about entities in the world only if the targeted entities do possess the properties referred to in the theory.

On the basis of my proposed reconstruction, I will attempt to substantiate the following two claims. First, contrary to common opinion, Bohr's theory is *not* internally inconsistent and cannot then readily dubbed a fiction. Second, a

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[1] But see my recent paper [1].

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selective realist interpretation of Bohr's theory, which remains non-committal about the existence of electron's trajectories, can still be defended.

1 Models and Theories in Physics

In the following discussion, I will rely on a *synthetic* view of theories according to which a theory is made up of models and propositions, such as laws, that are satisfied or made true by those models. If what I call *ontic propositions* or judgements [2], that is, propositions[2] that affirm that some entities in the world possess the properties involved in the theory, are true, then the theory is applicable to those entities. And the theory can be said to be (approximately) true of those entities if their properties are related in the way described by the theory. Let us briefly explicate these notions.

According to Tarski [3] and Suppes [4], a model is, in the first place, a mathematical *structure*, that is, a set of individuals on which some relations hold. A structure *M* thus consists in two ingredients, namely a *domain* D of individuals and a set or relations $r_1, r_2, \ldots r_n$ on these individuals:

$$M = <\mathrm{D},\ r_1,\ r_2,\ r_3 \ldots > \ \text{or}\ M = <\mathrm{D}, r_i > \ (1 \le i \le n)$$

For a structure to become a mathematical model it must satisfy or make true some propositions. Mathematically, a model always is a model *of* a set of propositions. The propositions and the models that satisfy them constitute a *theory.*

In physics, models are used to *represent.* If representation is to be successful, the user of the model must establish a specific kind of representational relation between the representing structure and what it represents. A *sine qua non* condition for the success of mathematical representation in physics (and, in general, in the empirical sciences) is to look at the entities (things, processes or whatever) in the world as *systems*, that is, as domains of individual *properties* structured by specific relations. To take a familiar example, the systems of cartesian mechanics are the structures composed of the properties of "figure and motion" organized by geometrical relations.

Success in scientific representation presupposes an initial abstracting attitude in which the scientist looks at the entities in the world as *structures of properties*. This I called the *primary* or *original* abstraction [5]. The following abstracting step, or *secondary* abstraction, consists in paying attention only to the properties that are judged to be relevant according to some interests (such as positions and velocities) and their numerical values.

Together with Patrick Suppes [4], Newton da Costa and Steven French [6], among others, I contend that the construction of a *homomorphism* is a *necessary*

[2] I make the customary distinction between statements and propositions: propositions are the semantic content of statements.

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(*albeit* certainly *not* sufficient) condition for successful representation in science [2, 7]. A homomorphism is a function between two domains that leaves invariant the relations holding on these domains. In this sense, a homomorphism preserves the form. Structures that are homomorphic are said to be *structurally similar*. An *isomorphism* is a one–one homomorphism.[3] A mathematical structure acquires the status of model of a real (or possible) system only if we have constructed a homomorphism or an isomorphism between the model and the (possible) targeted system. Such a requirement is an extremely weak constraint on representation. We can construct a homomorphism between a mathematical structure and any entity in the world. We only have to select some properties of the entity and some relations among those properties. Then, we construct a homomorphism between the mathematical structure and the structure of the entity's properties. In science we of course strive to construct informative representations. To achieve this goal some more restrictive conditions will be required.

According to a selective scientific realist, some propositions of the theory are true, or at least approximately so, and some entities in the world possess the properties referred to in the theory. Entity realism and proposition realism go hand in hand. A realist interpretation of a theory rests on the existence of properties, that is, on the truth of ontic statements which assert that entities which possess those properties exist. Thus to assert that electrons exist is tantamount to saying that there are entities in the world which have a charge, a spin, a mass, an energy and may be properties of velocity and position at all times that make up what is called a "trajectory". Furthermore, those properties—more accurately, the values for those properties (which also are properties…)—are organized by means of relations that constitute a model. Such a model correctly represents a real system of properties if it is true that some entities in the world possess those properties and that the propositions describing the relations between those properties are true. For example, if it is true that there are entities in the world, such as planets, that are located at some distance to a centre and have an orbital frequency and if some mathematical relation holds between the values of orbital frequencies and distances, then a model in which these quantitative properties are structured by means of this relation correctly represents a real system.

2 Modelling an Observable Entity: The Sky

In order to model the observable entity we call the sky, we must first look at the sky as a system and not as a celestial vault of stupendous beauty. This is the primary or original abstracting move. We call some moving bright spots "planets"

[3] Two structures $M = \langle D, r_i \rangle$ and $M^* = \langle D^*, r^*_i \rangle$ are isomorphic if and only if there exists a one–one function *f* such that for all r_i and for all n-uple $(a_1, \ldots a_n)$ of elements of D which stand in relation r_i, there exists a n-uple $(a^*_1, \ldots a^*_n)$ of elements of D* which stand in a r^*_i such that $a^*_1 = f(a_1), \ldots, a^*_n = f(a_n)$ ([4], pp. 54–57).

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and other bright spots that don't seem to move one with respect to another "fixed stars". In the secondary abstracting move, we pay attention to the changing spatial relations of the five visible planets with respect to the fixed stars. More specifically we are interested in the distances of the planets from the sun and also in their orbital frequencies, that is the number of their complete revolutions in some unit of time. We see thus the sky as a *system* the elements of which are the *observed* distances to the sun and orbital frequencies of the five planets. Let me emphasize that the elements of the system considered are *not* the planets, but some of their *properties*, namely their distances to the sun and their orbital frequencies, while other properties such as their brightness and colour are left aside.

The distances to the sun and orbital frequencies of the five visible planets are observable with the naked eye. These properties can be approximately ordered, and thus structured, according to their sizes observed from the earth. (In remote antiquity, astronomers have already observed that the orbital frequency of a planet is inversely proportional to its distance to the sun). By doing so, we have constructed an *observable* or *phenomenal* structure *O*.

Aiming at precision, we use measuring apparatuses to determine the distances *a'* and the orbital frequencies ω' of the planets (I'll use a " ′ " to refer to data, *i.e. measured* values, as opposed to theoretically *calculated* values). Now, we are confronted with the perennial astronomical problem of finding a mathematical function which fits the measurement outcomes, *i.e.* the *data*. Assuming that planets move on circles[4] and placing a hypothetical observer at the centre (the sun) of those circles, we can construct a data structure *K* the domain of which consists in the distances and frequencies measured at the centre. (These data can be obtained from measurements performed on earth by means of a simple mathematical formula). The measured distances and frequencies approximately verify the following equation:

$$\omega' = 2\pi/\left(a'^3 k\right)^{1/2} \tag{1}$$

This is Kepler's third law (*k* is approximately a constant). The *data structure K* consists in the measured distances and frequencies organized by Eq. (1). Then, we establish a homomorphism between the phenomenal structure *O* and the data structure *K* and claim that the latter represents the latter. In other words, the data structure *K* is a data *model* that represents the phenomenal system or structure *O*.

A model is useful if it conveys interesting information about some entities in the world. Statements such as "The orbital frequency of Mercury ω'_1 is equal to $2\pi/(a'^3_1 k)^{1/2}$" are (approximately) satisfied by this data model.[5] To get information

[4] Remember that we have Bohr's model (which assumes that electrons move on circular orbits) in mind. Later on, Bohr's model was refined by Sommerfeld who introduced elliptical orbits.

[5] This appears to be redundant, even trivial. But see Suppes' very simple example of a domain D of two natural numbers 1, 2 organized by the order relation ≥, namely $D = <\{1, 2\}, \geq>$ which satisfies propositions such as $2 \geq 1$ ([4], p. 26).

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on the planetary motions we must know the meanings of the numerical symbols: they refer to the orbital frequencies and distances of the planets. More generally, we must know the *code* used in representing. Besides, the measurement data must be sufficiently *accurate*. The data model conveys information about what is going on in the world only if some propositions are true, otherwise the data model represents only a simply *possible* phenomenal system. If predicative propositions such as "The orbital frequency of Mercury is ω'_1" and "The orbital frequency of Mercury is a'_1" are true, the data model correctly represents the phenomenal system. Such predicative propositions are *not* made true by the data model but they provide the grounds for its accuracy. Their truth warrants that the data model possesses some informative content. I call predicative propositions of this type *ontic propositions* since they attribute properties to some concrete real entities, such as planets. If these ontic propositions are true, then it is also true that the real properties denoted by ω'_1 and a'_1 stand in relation (1).

So far, we have constructed structures that are merely kinematical. Isaac Newton introduced dynamical quantities, such as mass and force, in order to provide an explanation of Kepler's laws. The gravitational force is a central force proportional to the masses involved, which decreases with the square of the distance between them. Using Newton's laws, supplemented with initial conditions, we are in a position to calculate the properties, namely the radiuses a and orbital frequencies ω, for all possible stationary orbits of a two-mass system. To each orbit is associated a specific energy level W. We then construct a *theoretical model* H the domain of which contains the *calculated* energy levels of the five planets, their masses m, their orbital frequencies ω and distances to the centre a, which (exactly) verify the following equation:

$$W = 2\pi^2 m\omega^2 a^2 \tag{2}$$

In the next step, we construct a homomorphism from H into K that maps the theoretical orbital frequencies and distances in H into approximately equal measured orbital frequencies and distances in K. The theoretical model H can be merged in a larger structure N, which contains all possible trajectories and energy levels for the two-mass system and also Newton's laws. In van Fraassen's terms, H is an *empirical substructure* of N, but it is also a *theoretical* structure since it is included in a larger theoretical structure. The wider model N is said to be *empirically adequate*. In this way, the phenomenal structure O has been successfully *embedded* in the wider Newtonian model N.[6]

[6] I added the notion of phenomenal or observational model as the intermediary link between phenomena and data models to van Fraassen's model-theoretical approach, which, as the reader noticed, is one of my main sources of inspiration (see [7]).

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My construction can be summarized in the following way:

Sky (phenomenon)
↓ abstraction
Phenomenal structure *O* of *observed* planetary orbital frequencies and distances
$\sim$ isomorphism
Data model *K* of *measured* frequencies and distances
$\sim$ homomorphism
Empirical structure *H* of *calculated* frequencies and distances
$\cap$ set-theoretical inclusion
Newtonian theoretical model *N*

Here, the theoretical Newtonian model corresponds to a two-mass system in which one mass is much bigger than the other. The domain of this Newtonian model contains all the continuous possible energy levels of the smaller mass. The *differences* between two possible energy levels are obtained by simple calculation. This will be relevant when we get to the Bohr model.

3 Modelling the Unobservable: The Hydrogen Atom

Modelling *un*observable entities is of paramount importance in physics. On this, there is little disagreement between realists and antirealists. Scientists typically use well-established theories as heuristic guides for constructing novel models. As Mary Hesse [8] emphasized, this demarche assumes some kind of resemblance, or *positive analogy*, between the newly investigated entity and an already successfully modelled entity. Specifically, some properties and relations must be assumed to be shared by both entities. In attempting to model the hydrogen atom, Bohr [9] had good reasons to believe, on the basis of the scattering experiments with alpha particles performed by Geiger and Marsden, that the atom and the sky are analogous in some relevant respects. In accordance to what Perrin envisioned as soon as in 1901 and Rutherford later hypothesized, just as the sky can be partially seen as a system of planets revolving around the very massive sun, the atom can be considered as a system of electrons revolving around a very massive nucleus. Some properties are possessed by both the planets and the electrons, namely their distances to the centre and orbital frequencies. Moreover, since electrons and nucleus have opposite (equal) charges, they are bound to the nucleus by the Coulomb static force that has the same mathematical form as the gravitational force (in atoms, the gravitational force is much weaker than the electromagnetic force and can be neglected). Thus, the truth of ontic propositions attributing some relevant properties of the same kind to planets and electrons is assumed.

Treading in Bohr's footsteps, I model the hydrogen atom by constructing the structure of the properties of all possible stable circular orbits for a single electron revolving around the nucleus. These properties are the distances to the nucleus, the orbital frequencies and the energy levels. This purely theoretical construction deals with quantities that are *calculated* by means of classical laws. Then, it is easy to

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construct a homomorphism from a planetary structure H_K into an atomic structure H_R[7] which maps energies, distances and frequencies into energies, distances and frequencies and which leaves Eq. (2) invariant. Let me stress that H_K and H_R are not only structurally similar but they are also alike. They resemble each other in the sense of Mary Hesse's positive analogy.

Bohr now encounters a serious obstacle. Maxwell's laws of classical electrodynamics stipulate that an accelerated orbiting electron emits electromagnetic radiation. As a consequence, a revolving electron should continuously fall towards the nucleus while emitting a radiation the frequency of which is also continuous. On the contrary, the electromagnetic emission and absorption spectra provide ample evidence that stable atoms do not emit radiation and that the emission and absorption spectra of excited atoms are not continuous but discrete. Experimental data manifest a blatant contradiction between Rutherford's conception of the atom and classical electromagnetism. Taking a decisive clue from Balmer's formula,[8] Bohr boldly assumes first that an electron can revolve on closed stable orbits without emitting radiation and second that an electron emits or absorbs radiation only when jumping from one stable orbit to another. Moreover, since the atomic spectra exhibit sharp discontinuous lines, Bohr supposes that only some stable orbits are permitted. In the case of the hydrogen atom, Bohr assumes that its single electron moves on specific allowed *circular* orbits possessing discrete kinematical properties, namely their *radiuses* a_n, *orbital frequencies* ω_n and *energy levels* W_n (the allowed orbits are indexed by the natural number n).

Being aware of the importance of Planck's constant for the behaviour of atomic systems and guided by the necessity of recovering Balmer's formula, Bohr introduces the following non-classical condition for the quantization of energy as a function of the frequency $\omega_{n,}$, Planck's constant h and the quantum positive natural number n:

$$W_n = nh\frac{\omega_n}{2} \tag{3}$$

This quantization condition is absolutely central to Bohr's reasoning. Yet, this is his *sole* non-classical assumption. At this point, we have what Mary Hesse calls a *negative analogy*. Whereas planets can occupy a continuous array of stationary orbits, electrons are allowed to move on some selected orbits only. After stating the quantification condition, Bohr's reasoning is purely mathematical without resorting to other non-classical suppositions. An energy level W_n is a function of the mass m of the electron, its charge e, Planck's constant h and the quantum number n:

$$W_n = \frac{2\pi^2 me^4}{n^2h^2} \tag{4}$$

[7] The subscripts K and R are chosen in honour of Kepler and Rutherford.

[8] Bohr is quoted as having repeatedly said: "As soon as I saw Balmer's formula, the whole thing was immediately clear to me." (Rosenfeld's Introduction to reprinting of Bohr's papers [10], p. xxxix).

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The kinematical quantities, that is, the orbital frequency ω_n and the radius a_n, have been eliminated: they don't appear in (4). We are now in a position to construct a theoretical model H_B the domain of which contains energies and natural numbers structured by Eq. (4). Then, we establish a homomorphic function f_B from H_B into H_R such that the couples (W_n, n) satisfying Eq. (4) are sent to triples (W, a, ω) satisfying Eq. (2) in such a way that the ordering between the energy levels is preserved. My reconstruction of Bohr's demarche can be summarized thus:

Two-mass system		Hydrogen atom
↓ abstraction		↓ abstraction
Keplerian model		Rutherford's model
H_K = <H$_k$, h$_k$>	∼	H_R = <H$_R$, h$_R$>
	Homomorphism (positive analogy)	
		∼ f_B
		H_B = <H$_B$, h$_B$>

In order to gain experimental access to the electron's energy levels, Bohr assumes that when an electron jumps from orbit n' to orbit n closer to the nucleus (with n smaller than n'), a single monochromatic quantum of radiation hv is emitted. The energy of this quantum of radiation is equal to the *difference* between the energies associated with these orbits:

$$W_n - W_{n'} = \frac{2\pi^2 me^4}{h^2}\left(\frac{1}{n^2} - \frac{1}{n'^2}\right) = hv \tag{5}$$

and

$$v = \frac{2\pi^2 me^4}{h^3}\left(\frac{1}{n^2} - \frac{1}{n'^2}\right) \tag{6}$$

Balmer's formula is a particular case of (6) for $n = 2$. The values of the emitted frequencies are not directly observable, but colours are and can be analysed with the aid of a spectrometer. From the series of lines on a photographic plate, we obtain a *data model* D_v of frequencies and natural numbers structured by formula (6) with the condition $n < n'$. Such a data model is homomorphic to a model of *differences* of energies which can immediately be constructed from H_B. For n' infinite, we can construct a data model D^*_v homomorphic to the model H_B. For an electron falling from infinity, that is, the binding of a free electron, the energy levels in H_B are mapped into electromagnetic frequencies such that pairs (W_n, n) satisfying (4) are mapped into corresponding pairs (v, n) verifying (6). If the measured frequencies are approximately equal to the calculated frequencies, we are entitled to claim that Bohr's theoretical model is empirically adequate.

The reconstruction of Bohr's demarche offered above, is based on his epoch-making 1913 papers. (For further discussion, see Ghins [1]).

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4 Some Philosophical Lessons

Two interesting philosophical lessons can be drawn from the reconstruction just proposed. First, Bohr's model is not *internally* inconsistent; thus, there is no need to resort to quasi-structures and partial homomorphisms as Newton da Costa and Steven French [6] propose. Second, given its internal consistency, Bohr's model remains open to a realistic interpretation and cannot be immediately interpreted as fictitious. This leaves open a selective realist interpretation of Bohr's model according to which energy levels are real whereas agnosticism with respect to the existence of electron's trajectories is recommended. Let us examine these two issues in turn.

4.1 Inconsistency

Most historians and philosophers contend that Bohr's model of the hydrogen atom is inconsistent. In a sense, they are quite right since Bohr's assumptions conflict with Maxwell's electromagnetism according to which accelerated electrons cannot occupy stable energy levels. Bohr's assumptions also conflict with classical mechanics since the latter allows a continuum of energy levels, in contradiction with the quantization condition (3). Several logicians and philosophers [6, 11, 12] even praise inconsistency as virtuous, since the empirical success of Bohr's theory allegedly proves that paraconsistent logic is useful in some instances of scientific modelling. However, together with Peter Vickers[9] and others, I will briefly argue that Bohr's model is *not internally* inconsistent.[10]

At the end of the day, in a physical theory, only the models and the equations satisfied by those models are relevant. The equations used by Bohr to deduce the electron's energy levels and Balmer's formula have not been proved to be inconsistent: it would behove to those who might wish to advocate their inconsistency to produce a proof of their contention. Granted, Bohr's crucial assumption (3) contradicts classical physics, but as Vickers and others (Bartelborth [14, 15]; Hendry [16, 17] aptly point out, *external* inconsistency must be sharply distinguished from *internal* consistency. If we take Bohr's mathematical theory of the hydrogen atom in itself, leaving aside its acknowledged conflict with classical physics, the logical consistency of the relevant equations can be safely assumed. The main charge of inconsistency is based, as we saw, on Bohr's assumption that the electron does *not* emit electromagnetic radiation while orbiting on an a stationary orbit n (especially on the lowest orbit for which n equals 1, *i.e.* the ground state).

[9] For a detailed discussion of the inconsistency charges that have been levelled against Bohr's model, see Vickers [13], Chap. 3).

[10] For more discussion on this, see (Ghins [1]).

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> (…) the assertion that the ground state was stable, so that an electron in such a state would not radiate energy (…). *This* is the central inconsistency. (Da Costa and French [6], p. 91)

What Bohr does is to separate Coulomb's law for electrostatics from the rest of classical electromagnetism. Having done this, he proceeds with his deductions (see Hendry [16] and Norton [18]). Then the contradiction with other laws of classical electromagnetism, *albeit* certainly problematic, becomes external to Bohr's theory of the hydrogen atom. Thus, given the internal consistency of Bohr's theory, there is no need to resort to quasi-structures as Da Costa and French propose ([6], p. 91). In a quasi-structure, three kinds of relations are distinguished. R_1 are the relations that we assume to hold between the elements of the domain; R_2 are the relations that we suppose not to hold between these elements; R_3 are the relations about which we are non-committal with respect to their holding or not between the elements of the domain (Da Costa and French [6], p. 19). It can readily be shown that propositions which contradict each other can be satisfied by an adequately constructed quasi-structure.[11] However, in my reconstruction of Bohr's theory, some parts of electrodynamics are simply ignored since they play no role in the deduction of the energy levels. This is a consequence of the abstraction procedure in play in any modelling activity. The mathematics of Bohr's theory just remains non-committal about the truth of some laws of electrodynamics when dealing with bound electrons. Obviously, Bohr doesn't need to take into account the classical law of electromagnetic emission by accelerated charges, since he postulates that the electron doesn't emit radiation when on a stationary orbit. Robert Millikan's contemporary reaction to Bohr's theory is worth quoting here:

> Bohr's first assumption (…) when mathematically stated takes the form [the fromula must be on the same line] $e^2/R^2 = (2\pi\omega)^2 mR$ in which e is the charge of the electron, ω the orbital frequency, and m the mass of the electron. This is merely the assumption that the electron rotates in a circular orbit… The radical element in it is that it permits the negative electron to maintain this orbit or to persist in this so-called "stationary state" without radiating energy even though this appears to conflict with ordinary electromagnetic theory. (Millikan 1917, quoted by vickers [19], p. 247)

Such reaction shows that some physicists were perfectly aware that Bohr constructed his theory by putting to work only some parts of classical theory while not taking some other parts into account. This doesn't imply that the external contradiction with Maxwell's electromagnetism was ignored. On the contrary, such contradiction was deemed to be a serious defect of the theory, including by Bohr himself who judged it "provisional" (Pais [20], p. 155). In a December 1913 talk, Bohr said:

> You understand, of course, that I am by no means trying to give what might ordinarily be described as an explanation… I hope that I have expressed myself sufficiently clearly so that you have appreciated the extent to which these considerations conflict with the admirably coherent group of conceptions which have been rightly termed the classical theory of electrodynamics. (Pais [20], p. 155)

[11] Some telling examples are discussed by Vickers [19].

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Now as then, physicists strive, and rightly so, to construct unified theoretical ensembles that are immune of contradiction. When a successful theory clashes with well-established background laws, scientists are confronted with a *conceptual problem* [21]. To retrieve consistency, scientists devote considerable efforts in revising parts of accepted physics. The history of science provides ample evidence for the fecundity of such efforts. In the present case, these efforts gave birth to standard quantum mechanics. Praising the virtues of inconsistency in some instances (if there are any) should not impair the pursuit of consistent theories, a pursuit which has been highly beneficial to scientific progress.

4.2 A Selective Realist Interpretation

In what follows, I will try to answer the following question: to what extent are we entitled to believe in a selective realist interpretation of Bohr's theory of the hydrogen atom? This is an epistemological issue. Thus, I will be concerned with the arguments in favour of the existence of some specific properties, the truth of some propositions and the accuracy of some representations or models.

As we saw, a scientific theory is a class of models together with the set of propositions that are satisfied by them. Taken in itself, a scientific theory is independent from the existence of any entity in the world. Classical mechanics can be exposed in textbooks irrespective of the actual existence of mechanical systems, just as Euclidean geometry can be developed independently of the existence of rigid rods. Very plausibly, Frederick Suppe [22] has defended a counterfactual account of theories according to which scientific laws are not committed to the existence of entities but solely assert that if entities of a specified kind existed, then their behaviour would be correctly described by some specific mathematical equations. For sure, a theory must at least leave open the possible existence of some systems in the world. For this to be the case, the theory must be logically consistent. Consistency has to do with the truth of propositions. Propositions that negate each other cannot both be true. An electron cannot emit and also not emit radiation at the same time. But, since there is no cogent reason to question the internal consistency of Bohr's theory, a serious obstacle in the way of its selective realist interpretation is removed. If Bohr's theory of the hydrogen atom was inconsistent, then it could be judged to be a fiction right away in the sense that a fiction must at least be something that we have good reasons to believe that it is false (Bokulich [23], p. 137 and [24], p. 92).

In the first place, we may ask if the entities of which Bohr's theory speak, namely electrons, exist. In other words, are there in the world entities which possess specific properties such as numerical values of mass and charge. The achievements of Bohr's theory, no matter how spectacular, do not provide sufficient grounds for believing that electrons exist. Bohr's deduction of the frequencies of spectral lines of the hydrogen atom (and the ionized helium) but also his prediction of the correct value for the Rydberg constant (a fact which strongly

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impressed Einstein) carry no obligation to commit ourselves to the existence of unobservable entities, such as electrons.[12]

Electrons are entities which are endowed with some properties by our theories only and not by direct observation. To be entitled to believe in the existence of electrons, diverse independent experimental methods of measuring their properties must deliver concordant quantitative results. This criterion is inspired from the manner in which we justify our beliefs in ordinarily observed entities. We feel confident about the existence of a specific observed entity, such as a sample of wine or water, when we are able to perform a diversity of observations from different perspectives that resort to different perceptual modalities (seeing, tasting etc.) and when these observations deliver approximately equal results (see Ghins [26] and [27], p. 108). In 1913 [28], Jean Perrin famously managed to convince the scientific community that atoms exist by showing that thirteen methods of measuring Avogadro's number give approximately equal values.[13] In the same manner, the quantitative results obtained by independent experimental methods provide good grounds to believe that entities having a mass *m* and a charge *e* equal to 9.11×10^{-31} kg and 1.6×10^{-19} Coul respectively, do exist. Such evidence in favour of the existence of electrons is *external* to Bohr's model. In other words, the truth of ontic propositions asserting that electrons possess mass, charge, and also energy and that their momentum and position can in some circumstances be measured, is established on the basis of evidence that comes outside Bohr's theory of the hydrogen atom.

We now come the following issue: is it reasonable, given the empirical adequacy of Bohr's theory of the hydrogen atom, to believe that an electron has a trajectory, that is, that it has a momentum (or velocity) and a position at all times and that its spacetime line is continuous? In Bohr's theory, the orbital frequencies ω and radiuses a of the trajectories of electrons are easily *calculable* from the experimental values of the emission and absorption frequencies. But these kinematical properties are not experimentally measurable by a variety of independent experimental methods, contrary to what is the case for planets.

Anjan Chakravartty ([29], Chap. 1) proposes to divide unobservable properties into two distinct categories: those which are *detectable* by means of instruments and those which are not detectable but play an *auxiliary* role in the explanation of phenomena. Detectable properties are those, in the words of Chakravartty, "with which we have managed to forge significant causal contact" ([29], p. 60). Accordingly, the scientific realist should be committed to the existence of detectable properties only and remain agnostic about the existence of auxiliary properties. I contend that even more stringent conditions are required in order to justify the legitimacy of existence beliefs: only properties that are measurable by means of distinct and independent experimental procedures that lead to concordant

[12] Several serious objections have been addressed to the so-called no-miracles argument and the truth-tropic strength of inference to the best explanation [25].

[13] It is debatable however that all thirteen methods are independent from each other.

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quantitative values (up to an appropriate degree of approximation) can legitimately be attributed to some entities. In other words, a variety of concordant causal contacts must obtain.

Thus, it is reasonable to be committed to the truth of ontic propositions that assert that "There are entities that have a mass equal to 9.11×10^{-31} kg and a charge equal 1.6×10^{-19} Coul." as we saw. On the other hand, it is more prudent for the realist to refrain from claiming that electrons move on spatiotemporal trajectories, simply because we have not managed, at least so far, to measure their positions and velocities by several independent ant concordant methods and to experimentally determine the properties of their purported trajectories. The positions and velocities of electrons are not detectable. These are auxiliary properties, that may play some kind of explanatory role [24], but there is no convincing reason for believing that electrons possess them. An agnostic attitude *à la van Fraassen* ([30], p. 72) is recommended with respect to these properties. Such austere selective realism does *not* exclude the possibility that further evidence might be provided in the future in favour of electron trajectories.

One might object that the Heisenberg uncertainty inequalities prohibit the simultaneous measurement of positions and momenta. Those inequalities certainly are a consequence of the new quantum mechanics that was developed after 1913 and became standard later on. However, there is no totally convincing argument for interpreting Heisenberg's relations in a strong realist way and to conclude that electron trajectories do not exist.[14] Such a strong position, akin to atheism in theology, is not warranted. According to Bohm's theory, which is empirically equivalent to standard non-relativistic quantum mechanics, electrons move on trajectories, and can even be at rest. (According to Bohm's theory however, the purported electron trajectories in the hydrogen atom do not coincide with Bohr's orbits and cannot generally be identified with classical trajectories). My aim here is not to take sides in a sometimes (too) heated debate, but to point out that Bohm's theory remains at least a possibility and to recommend an agnostic, thus not atheistic, attitude with respect to the existence of electron's trajectories.

What about the existence of energy levels for bound electrons? According to Bohr's theory, differences in energy levels correspond to spectral lines in the emission spectra. The differences of energy levels are immediately *calculable* from the measured frequencies by means of Eq. (5) above. But the energy differences and the energies themselves are not measured in this way, only the electromagnetic frequencies or wavelengths are. Given the amount of experimental evidence and the variety of methods for ascertaining the emission (and absorption) of electromagnetic radiation by atoms, we can safely believe that atoms do emit and absorb electromagnetic energy and, given the principle of conservation of energy, that atoms occupy—quantified—states of energy. Yet, we need further experiments in order to be able to assert that atomic differences of

[14] I here diverge from Bokulich ([23], p. 137) and the standard interpretation of the Heisenberg principle.

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energies correspond to different states of the electron. Bohr's theory alone does not adduce sufficient grounds for believing that the nucleus doesn't play a role in accounting for the observed spectra. Such experimental evidence must come from outside. And it does. Collision experiments show that electrons can be expelled and captured again by atoms, and that the various energy states of atoms correspond to essentially different states of energy for electrons.

Summarizing, given its internal consistency, Bohr's theory cannot be judged to be a fiction, that is, something about which we have good reasons to believe that it does not correspond to some real entities in the world. On the contrary, there are experimental grounds in favour of the existence of electrons, namely entities that possess the real properties of mass, charge and energy. Given this, we also have reasons to believe that the mathematical relations verified by the quantitative values of those properties, such as Eq. (4), are true and that the structure H_B can be considered to be a reliable model and partially and approximately represents existing systems in the world.

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