STABLE HETEROGENEOUS CARTELS *

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ABSTRACT

We define a notion of cartel stability, which allows firms to perceive the impact of their actions on overall competitiveness in the market. We demonstrate that in an industry with linear demand and marginal cost conditions and possibly diverse firms a stable cartel always exists. Furthermore, we determine the characteristics of stable cartels such as uniqueness, size and possible heterogeneity.

0 INTRODUCTION

The intrinsic instability of cartels was pointed out by Stigler when he observed :

> "the major difficulty in forming a merger is that it is more profitable to be outside a merger than to be a participant. The outsider sells at the same price but at the much larger output at which marginal cost equals price. Hence the promoter of a merger is likely to receive much encouragement from each firm - almost every encouragement, in fact - except participation." (Stigler [1950]).

At the same time however, some cartels do prevail. Even though the chiseling effect has often been advanced, the oil and other export cartels appear to last for considerable periods of time. Despite the risk for prosecution, price fixing arrangements are still common practice.[1] As a possible explanation of the stability of cartels one can suggest that Stigler's point is well taken only insofar as firms breaking away from the cartel are myopic and ignore the effects of their actions on the structure of the market; for it is conceivable that the fall in price due to increased overall competitiveness in the market leads to a fall in profit for the individual contemplating a move out of a cartel which goes beyond the advantage of joining the competitive fringe. It is the purpose of this paper to analyse this argument.

Using a simple price leadership model d'Aspremont, Jacquemin, Jaskold Gabszewicz, and Weymark [1981] established that a stable cartel would always exist whenever the number of firms is finite and that its size is a decreasing function of the number of firms in the industry;[2] furthermore, no non-negligible group of firms stays in the cartel if its choice has no impact on market structure, as is the case when the number of firms tends to infinity. Donsimoni, Economides and Polemarchakis [1981] examined alternative market conduct arrangements and extended the argument for the existence of a stable cartel. The characteristics of stable cartels, in particular their uniqueness, were left as an open question.

In this paper we maintain the assumption of price leadership and examine the impact of variations in cost and demand conditions on the structure of stable cartels. In the first section all firms face identical cost conditions. We demonstrate that the size of a stable cartel increases with the efficiency of firms; in addition, it is shown that the stable cartel is unique below a certain level of relative cost efficiency. Beyond that level, two situations may occur : if the number of firms is higher than a fixed number uniqueness prevails; otherwise there are two stable cartels one of which comprises all the firms in the industry. In the second section, firms face different cost conditions. We demonstrate that a stable cartel still always exists. As intuition suggests we find that the proportion of firms of a given cost type in a stable cartel is an increasing function of their efficiency. In fact a stronger result obtains : no less efficient firm stays in a stable cartel if at least one more efficient firm does not find it profitable to enter. In addition, under homogeneous as well as heterogeneous cost conditions, the size of a stable cartel is an increasing function of consumers' willingness to pay and a decreasing function of the elasticity of demand as well as of the number of firms in the industry.

Observe that the question of uniqueness raised here has nothing to do with the question whether multiple (stable or not) cartels exist simultaneously; this latter question can not even be raised within a framework of price leadership.

Before proceding with a formal model, we should discuss the notion of equilibrium we shall adopt. Each firm takes a two-fold decision : it decides whether or not to join the cartel and it decides on the amount of output to produce. We suppose that the decision concerning membership in the cartel is taken prior to the output decision and we solve for an equilibrium sequentially : once formed a cartel acts as a price leader without considering the possibility of defection among its members; this determines the equilibrium level of price, output and (up to side-payments) profits for firms inside as well as outside the cartel. The stability or not of a cartel arrangement can then be analyzed by considering whether a firm inside (outside) the cartel can attain higher profits by defecting from (entering into) the cartel, assuming that no other firm alters its cartel membership decision as a result. That is, unlike the second stage, output game, any conjectural variation is eliminated from the first stage, cartel formation game. The formulation clearly allows firms to recognize that their choice affects the overall competitiveness of the market (size of the cartel) and this is the key element in the argument for the existence of stable cartels.

In Section 1 we consider the simpler case of identical firms; heterogeneity among firms is introduced in Section 2; Section 3 concludes.

1. IDENTICAL FIRMS

There are N firms in the industry, indexed by $n = 1, \ldots, N$, producing a single output with quadratic cost functions $C(q_n) = q_n^2 / 2\beta$, where q_n is the output for firm n and $\beta > 0$ is a cost efficiency parameter. K firms, indexed by $c = 1, \ldots, K$, form a cartel and act as price leaders so as to maximize joint profits. The remaining $(N - K)$ firms, indexed by $f = 1, \ldots, (N - K)$, constitute the competitive fringe and act as price takers. Market demand for the homogeneous good is linear, $Q = N(a - bp)$, where p is the price of the output and $a, b > 0$ are demand parameters, which measure the willingness to pay and the elasticity of demand, respectively. Profits per firm are equal to total revenue minus total cost of the output supplied by the firm. It follows that all firms in the cartel (in the fringe) produce the same level of output, which equates the marginal cost to the marginal revenue associated with the residual demand function faced by the cartel (to the prevailing price); all firms in the cartel (in the fringe) make the same profits. The assumption concerning the distribution of revenue among cartel members is innocuous in this section, since all firms face identical cost conditions. The multiplicative term N in the demand function is introduced in order to examine the impact of an increase in the number of firms (size of the industry) without reducing into a situation of infinitesimally small output per firm.

Let $\omega = (a, b, \beta, N)$ be a vector of industry cost and demand characteristics, and let the (relative) size of a cartel be α; clearly α is a point in the set $\{0, \frac{1}{N}, \ldots, 1\}$, yet for computational simplicity we consider α in the closed interval $[0,1]$. For fixed ω and $\alpha \in [0,1]$, the industry equilibrium is described by the triple (p, q_c, q_f) of price for the output and production levels for firms inside and outside the cartel :[3]

$$
\begin{aligned}
p(\omega, \alpha) &= \frac{a(b+\beta)}{(b+\beta)^2 - \alpha^2\beta^2}\,; && \text{(a)} \\
q_c(\omega, \alpha) &= \frac{a\beta}{b+\beta+\alpha\beta}\,; && \text{(b)} \qquad (1) \\
q_f(\omega, \alpha) &= \frac{a(b+\beta)\,\beta}{(b+\beta)^2 - \alpha^2\beta^2}\,. && \text{(c)}
\end{aligned}
$$

As intuition suggests, the equilibrium price increases with the size of the cartel and consumers' willingness to pay; it decreases with demand elasticity and cost efficiency. Furthermore, output per firm in the competitive fringe (cartel) increases (decreases) with the size of the cartel, while they both increase with consumers' willingness to pay and firms' efficiency. Finally, firms inside the cartel always produce less than firms outside.

To examine the issue of the stability of an industry equilibrium we must adopt a concept of stability for a collusive arrangement. As noted by d'Aspremont and Jaskold Gabszewicz [1982] there is "no unified theoretical framework for analyzing the effectiveness of collusive coordination"; one may emphasize either individual or unanimous stability. In this paper we focus on the schemes which are individually stable; that is schemes such that no individual move is desirable. To be precise let us introduce some notation. Denote by $\pi_i(\omega, \alpha) = \pi(q_i(\omega, \alpha), p(\omega, \alpha))$ the profits of a firm of type i, i = c,f, at the industry equilibrium associated with ω and α. Holding ω constant, the π_i's can be written as functions of α alone :

$$\pi_c(\alpha) = \frac{\beta}{2} \frac{\delta^2}{(1+\gamma)^2 - \alpha^2}, \qquad \text{(a)}$$

$$\pi_f(\alpha) = \frac{\beta}{2} \left[\frac{\delta(1+\gamma)}{(1+\gamma)^2 - \alpha^2} \right]^2, \qquad \text{(b)} \qquad (2)$$

where $\delta = \frac{a}{\beta}$ and $\gamma = \frac{b}{\beta}$.

We can now define partial and then total stability of a cartel, for a given ω.

<u>DEFINITION 1 a</u> : A cartel of size α, $\alpha \in [\frac{1}{N}, 1]$, is *internally stable* if it is not profitable for a firm in the cartel to join the competitive fringe; i.e., if profits in the cartel corresponding to the industry equilibrium at (ω,α) are higher than profits in the fringe corresponding to the industry equilibrium at (ω, α^-), $\alpha^- = \alpha - \frac{1}{N}$:

$$\pi_c(\alpha) \geqslant \pi_f(\alpha - \frac{1}{N}). \qquad (3.a)$$

<u>DEFINITION 1 b</u> : A cartel of size α, α $[0, 1 - \frac{1}{N}]$, is *externally stable* if it is not profitable for a firm in the competitive fringe to join the cartel; i.e., if profits in the fringe corresponding to the industry equilibrium at (ω,α) are higher than profits in the cartel corresponding to the industry equilibrium at (ω, α^+), $\alpha^+ = \alpha + \frac{1}{N}$:

$$\pi_f(\alpha) \geqslant \pi_c(\alpha + \frac{1}{N}). \qquad (3.b)$$

<u>DEFINITION 2</u> : A cartel of size α, $\alpha \in [0,1]$, is *stable* if it is externally stable for $\alpha \in [0, \frac{1}{N})$, it is both externally and internally stable for $\alpha \in [\frac{1}{N}, 1 - \frac{1}{N}]$ and it is internally stable for $\alpha \in (1 - \frac{1}{N}, 1]$.

We can now state the following proposition :

PROPOSITION 1 : *A stable cartel always exists. If firms are not too efficient relative to market demand, i.e.,* $\gamma = \frac{b}{\beta} > (k-1)$, $k = \frac{8}{(1+\sqrt{17})}$, *the stable cartel is unique. Otherwise, two situations may prevail : if the number of firms is greater than a fixed number* $\overline{N}$, *uniqueness prevails; if it is lower than* $\overline{N}$ *two stable cartels exist, one of which necessarily includes all the firms in the industry.*

To establish the above proposition we introduce two lemmas which characterize the profit functions inside and outside the cartel and are interesting in their own right. These lemmas specify the shape of the the profit functions (convexity and horizontal distance). This information will be useful since the notions of internal and external stability have geometric analogues. Figure 1 illustrates the relationship between the horizontal distance between the profit functions and the concepts of internal and external stability, respectively.

LEMMA 1 : *Profits in the cartel and in the fringe are equal to each other and to the perfectly competitive profits at* $\alpha = 0$. *They are both increasing convex functions of* α. *Profits in the fringe are higher than in the cartel for* $\alpha \in (0, 1]$. *Profits in the cartel are equal to the monopoly profits at* $\alpha = 1$.

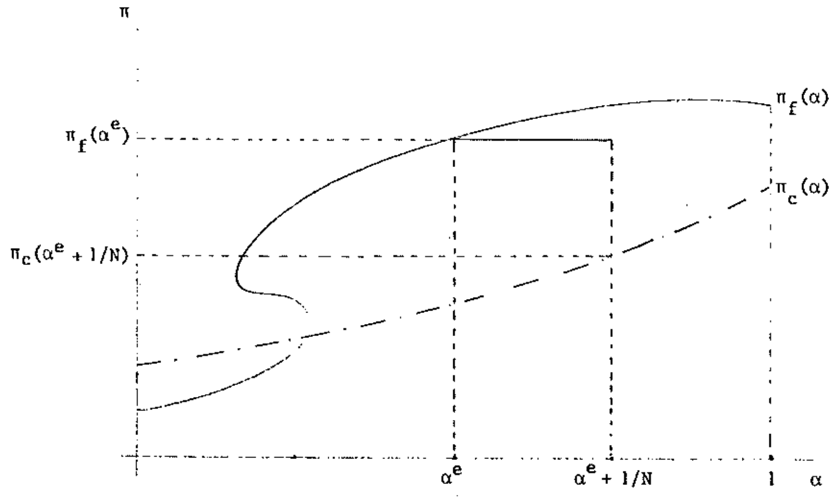
$$\pi_f(0) = \pi_c(0) = \pi^0 = \frac{\beta}{2}\,\frac{\delta^2}{(1+\gamma)^2}\,; \qquad \text{(a)}$$

$$\pi_f'(\alpha) > \pi_c'(\alpha) > 0, \quad \alpha \in [0, 1]; \qquad \text{(b)}$$

$$\pi_f''(\alpha) > \pi_c''(\alpha) > 0, \quad \alpha \in [0, 1]; \qquad \text{(c)} \qquad (4)$$

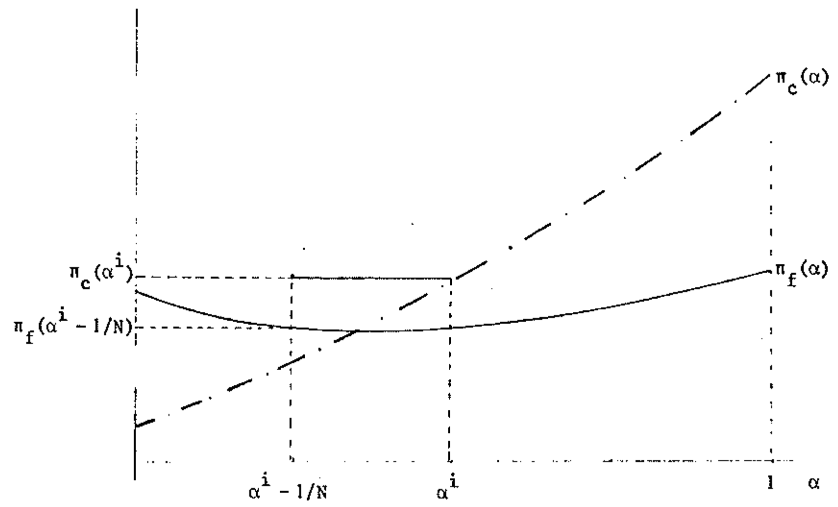
$$\pi_f(\alpha) > \pi_c(\alpha), \quad \alpha \in (0, 1]; \qquad \text{(d)}$$

$$\pi_c(1) = \pi^1 = \frac{\beta}{2}\,\frac{\delta^2}{(1+\gamma)^2 - 1}\,. \qquad \text{(e)}$$



A cartel of size α^e is externally stable : $\pi_f(\alpha^e) > \pi_c(\alpha^e + 1/N)$

Figure 1.a



A cartel of size α^i is internally stable : $\pi_c(\alpha^i) > \pi_f(\alpha^i - 1/N)$

Figure 1.b

π^0 and π^1 are computed by assuming that all firms in the industry are price takers (π^0) or that they act as a single monopolist (π^1). Equation (4-a) ((4-e)) obtains by setting α equal to zero (one) in equations (2). Taking the first and second order derivatives with respect to α in (2), equations (4-b) and (4-c) follow. Comparing (2-a) and (2-b), (4-d) follows.

Q.E.D.

From Lemma 1, we know that the π_i's, $i = c, f$, have well defined inverse functions for $\pi \in [\pi^0, \pi^1]$. Denote by $\alpha_i(\pi)$, $i = c, f$, these inverses. Taking into account the definition of π^0 we obtain that :

$$\alpha_c(\pi) = (1+\gamma)\left(1 - \left(\frac{\pi^0}{\pi}\right)\right)^{\frac{1}{2}} ; \qquad \text{(a)}$$

$$\alpha_f(\pi) = (1+\gamma)\left(1 - \left(\frac{\pi^0}{\pi}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}} . \qquad \text{(b)} \qquad (5)$$

<u>LEMMA 2</u> : *The horizontal distance between the profit functions of cartel members and of competitive firms is equal to zero at* $\pi = \pi^0$. *If firms are not too efficient relative to market demand, i.e.,* $\gamma > (k-1)$, *it increases monotonically as* π *varies from* π^0 *to* π^1. *Otherwise, it increases monotonically as* π *varies from* π^0 *to* $\overline{\pi} = \pi^0 k^2$ *and decreases monotonically as* π *varies from* $\overline{\pi}$ *to* π^1 :

$$\alpha_c(\pi^0) = \alpha_f(\pi^0) = 0 ; \qquad \text{(a)}$$

$$\alpha_c'(\pi) \gtreqless \alpha_f'(\pi), \quad \text{for} \quad \pi \lesseqgtr \overline{\pi} ; \qquad \text{(b)} \qquad (6)$$

$$\overline{\pi} \gtreqless \pi^1, \quad \text{for} \quad \gamma \gtreqless (k-1). \qquad \text{(c)}$$

(6-a) follows from the definition of the α_i functions. Differentiate $\alpha_c(\pi) - \alpha_f(\pi)$: a monotonically decreasing function of π obtains which is strictly positive at $\pi = \pi^0$ and reaches zero at $\pi = \overline{\pi}$. Hence (6-b) follows. To obtain (6-c), note that both $\overline{\pi}$ and π^1 can be written as multiples of π^0. Comparing these multiples (6-c) obtains.

Q.E.D.

Lemmas 1 and 2 establish that there are two basic configurations of the profit functions in term of α as illustrated by Figures (2-a) and (2-b). In both cases the profit functions are monotonically increasing functions of α. Whereas, however, in Figure (2-a) the horizontal distance is monotonically increasing, in Figure (2-b) it first increases until $\pi = \overline{\pi}$ and then decreases.

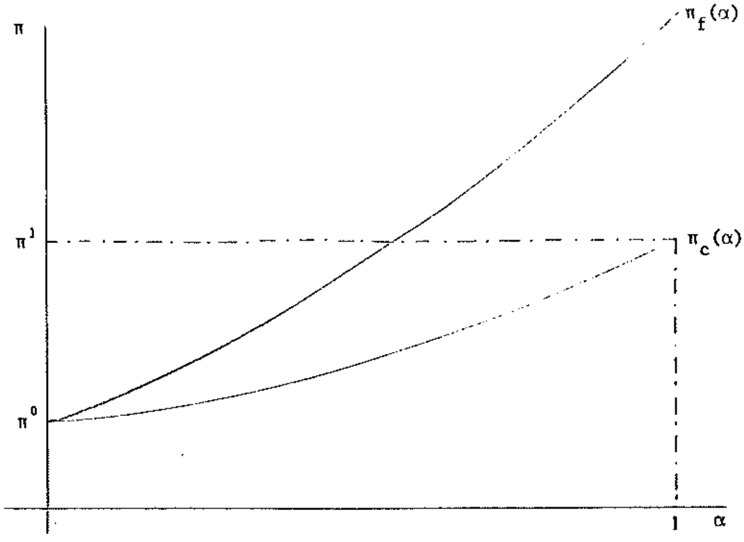
Before proceding with the proof of Proposition 1, let us give an intuitive argument. Suppose we are in the case in which the horizontal distance between the profit functions is monotonically increasing and let π^* be a profit level at which this distance is equal to $\frac{1}{N}$ (Figure 3). Let α_f^* and α_c^* be the points at which the functions $\pi_f(\alpha)$ and $\pi_c(\alpha)$ attain the value π^*, respectively; observe that, by the definition of π^*, $\alpha_c^* - \alpha_f^* = \frac{1}{N}$. It follows that, barring pathology, there exists a unique element α^* of the set $\left\{\frac{1}{N}, \dots, \frac{N-1}{N}\right\}$ in the closed interval $[\alpha_f^*, \alpha_c^*]$. It is now clear that α^* constitutes a stable cartel. Let π^i be the profit inside the cartel at α^*; the horizontal distance between the profit functions is inferior to $\frac{1}{N}$ for all π's below π^i and it follows that α^* is internally stable; similarly, let π^e be the profit outside the cartel at α^*; the horizontal distance between the profit functions is greater than $\frac{1}{N}$ for all π's greater than π^e and it follows that α^* is externally stable.

We can now establish Proposition 1. Let π^* be the profit level(s) in the closed interval $[\pi^0, \pi^1]$ at which the horizontal distance between the profit functions is equal to $\frac{1}{N}$:

$$\alpha_c(\pi^*) - \alpha_f(\pi^*) = \frac{1}{N}. \tag{7}$$

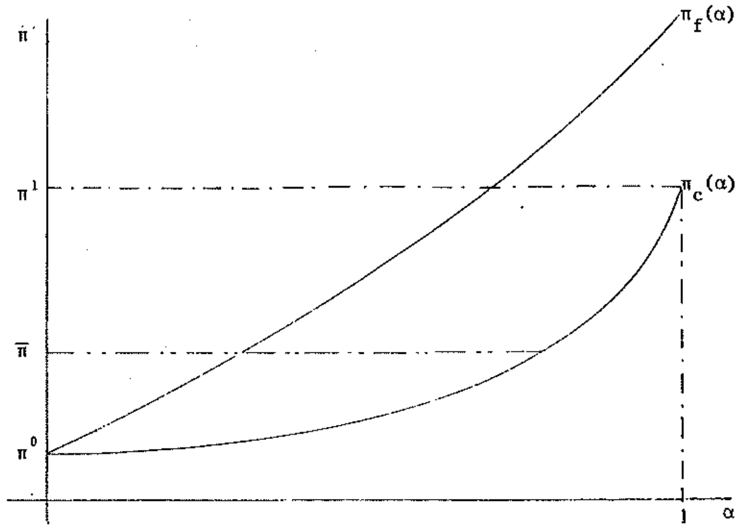
From Lemmas 1 and 2 it follows that π^* can never be equal to π^0,[4] and that three cases may occur : there may be zero, one, or two values of π^* such that (7) holds.

Consider the case where there is no value of π^* such that (7) holds. This implies, from Lemma 2, that the horizontal distance between the two profit functions is always less than $\frac{1}{N}$. It follows that $\alpha^* = 1$,



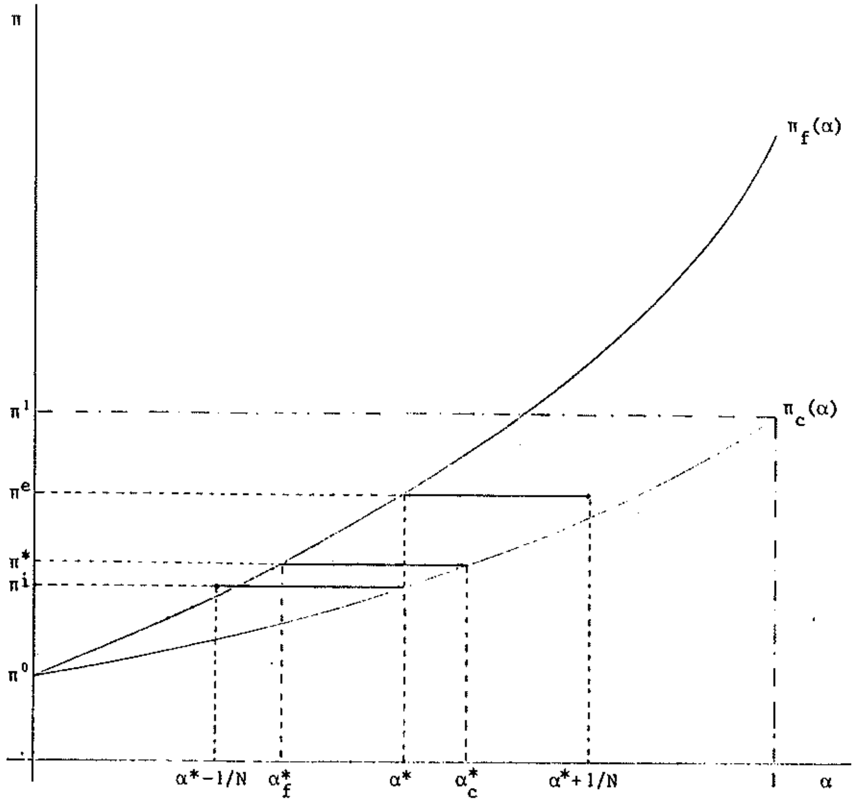
The horizontal distance between the profit functions increases monotonically as π varies from π^0 to π^1.

Figure 2.a



The horizontal distance between the profit functions increases (decreases) monotonically as π varies from π^0 to $\overline{\pi}$ (from $\overline{\pi}$ to π^1)

Figure 2.b



At π^* the horizontal distance between the profit functions is equal to $1/N$. π^* defines the interval (α_f^*, α_c^*). α^* is a multiple of $1/N$ in that interval, it is both internally and externally stable.

An Example of a Stable Cartel

Figure 3

i.e., all the firms in the industry form a stable cartel and this stable cartel is unique : From Definition 2, $\alpha^* = 1$ is a stable cartel if it is internally stable. This is clearly the case since at $\pi = \pi^1$, in particular, the horizontal distance is less than $\frac{1}{N}$ and therefore $\pi_c(1) > \pi_f(1 - \frac{1}{N})$. Uniqueness follows since for all α's less than 1 external stability fails, i.e., firms always wish to enter the cartel since the horizontal distance between the profit functions is less than $\frac{1}{N}$.

Consider the case where there is a unique value of π^* such that (7) holds. By Lemma 2, it follows that the horizontal distance between the profit functions is monotonically increasing as π varies from π^0 to π^* and that it is less than $\frac{1}{N}$. Furthermore, as π varies from π^* to π^1, it is always greater than $\frac{1}{N}$; yet it may not monotonically increase (see Figure 2-b)). Barring pathological cases[5], there exists a unique element α^* of the set $\left\{\frac{1}{N}, \ldots, 1 - \frac{1}{N}\right\}$ in the closed interval $[\alpha_f(\pi^*), \alpha_c(\pi^*)]$. It can readily be shown that α^* constitutes a stable cartel and that it is unique : By Definition 2, α^* is stable if it is internally and externally stable. Let π^e be the profit attained in the fringe with a cartel of size α^*. Since the horizontal distance between the profit functions is greater than $\frac{1}{N}$ for all π's greater than π^*, it follows that π^e is greater than $\pi_c(\alpha^* + \frac{1}{N})$; this establishes external stability at α^*. Similarly, let π^i be the profit attained in the cartel with a cartel of size α^*. Since the horizontal distance between the profit functions is smaller than $\frac{1}{N}$ for all π's smaller than π^*, it follows that π^i is greater than $\pi_f(\alpha^* - \frac{1}{N})$; thus α^* is internally stable. Uniqueness follows trivially : For all values of α greater than α^* internal stability fails, i.e., firms in the cartel make lower profits than if they exited and, similarly, for all values of α smaller than α^* external stability fails, i.e., firms outside the cartel make smaller profits than if they joined the cartel.

Consider last the case where there are two values of π^* such that (7) holds.[6] From Lemma 2 this implies that we are in the configuration of Figure (2-b). In addition, one of the values of π^* is in the open interval $(\pi^0, \bar{\pi})$ and the other in the open interval $(\bar{\pi}, \pi^1)$[7]. Let

π_1^* be the smallest value of the two. This situation is identical to the previous situation where π^* was unique. Thus, the unique element α_1^* of the set $\left\{\frac{1}{N}, \ldots, 1 - \frac{1}{N}\right\}$ in the closed interval $[\alpha_f(\pi_1^*), \alpha_c(\pi_1^*)]$ is a stable cartel. However, the unique element α_2^* of the set $\left\{\frac{1}{N}, \ldots, 1 - \frac{1}{N}\right\}$ in the closed interval $[\alpha_f(\pi_2^*), \alpha_c(\pi_2^*)]$ is not stable since external stability fails. That external stability fails stems from the fact that the horizontal distance as π varies from π_2^* to π^1 is monotonically decreasing and less than $\frac{1}{N}$. Note, however, that in that case $\alpha^* = 1$ is itself a stable cartel since it is internally stable.

Q.E.D.

To illustrate the possibility of two stable cartels we introduce Table 1. Appropriate values of ω have been selected, namely, $a = b = 1$, $\beta = 50$, and $N = 10$. Comparing the profits attained inside and outside the cartel for different values of α it can readily be seen that $\alpha = \frac{3}{10}$ and $\alpha = 1$ are both stable cartels.

α	π_c	π_f
0	.00961	.00961
1/10	.00970	.00979
2/10	.00999	.01039
3/10	.01052	.01152
4/10	.01136	.01342
5/10	.01265	.01665
6/10	.01470	.02247
7/10	.01817	.03434
8/10	.02497	.06490
9/10	.04340	.19599
1	.24752	6.37438

TABLE 1 : An example of two stable cartels.

Proposition 1 establishes the existence of a stable cartel for arbitrary cost and demand conditions. The question follows whether the characteristics (size) of a stable cartel depend, and if so how, on cost and demand conditions. This is the object of the following Proposition which, in a way, formalizes the argument in Fellner [1949] and Stigler [1964] concerning the elements which facilitate collusion among firms.

PROPOSITION 2 : *The size of a stable cartel (not including all the firms in the industry) is an increasing function of consumer's willingness to pay and of cost efficiency. It is a decreasing function of the number of firms in the industry and of the elasticity of demand.*

To establish Proposition 2 it suffices to note that α^* and π^* are monotonically increasing functions of each other. The comparative static exercise is carried out for interior solutions only, i.e., $0 < \alpha^* < 1$, and hence we limit ourselves to the situation where the horizontal distance between the profit functions is increasing. It suffices to totally differentiate equation (7) which can be written as $F(\pi^*, \omega) = 0$. Since we restrict ourselves to the increasing horizontal distance we know that $F_{\pi^*} > 0$. That $F_N > 0$ follows immediately. The other components are more difficult to obtain. A simplifying computation stems from the recognition that a only enters through π^0, while b and β enter both through π^0 and $(1+\gamma)$ in equation (7). Yet, all results are unambiguous and the followings hold :

$$\frac{d\pi^*}{dN} = -\frac{F_N}{F_{\pi^*}} < 0, \quad \frac{d\pi^*}{da} = -\frac{F_a}{F_{\pi^*}} > 0 ;$$

$$\frac{d\pi^*}{db} = -\frac{F_b}{F_{\pi^*}} < 0, \quad \frac{d\pi^*}{d_\beta} = -\frac{F_\beta}{F_{\pi^*}} > 0.$$

Q.E.D.

2. HETEROGENEOUS FIRMS

In this section we introduce M types, indexed by $m = 1,\ldots,M$, of N identical firms each. Within each type, firms face the same quadratic cost function $C_m(q_{n,m}) = q_{n,m}^2 / 2\beta_m$, where $q_{n,m}$ is the output of the firm n within type m and $\beta_m > 0$ is a cost efficiency parameter characterizing type m firms. For convenience we rank types by decreasing efficiency : $\beta_1 \geqslant \ldots \geqslant \beta_m \geqslant \ldots \geqslant \beta_M$. Observe that since the β's are not necessarily all different the assumption that there is the same number of firms of each type is innocuous. A cartel is now characterized not only by its size but also by its composition; it comprises K_m firms of type m, $m = 1,\ldots,M$. The assumptions of the previous section still hold with minor changes to account for differentiated firms. In particular, firms in the cartel are assumed to receive the profits associated with their own production. Thus firms in the cartel have the same profit within each type but different across distinct types. Similarly, the fringe comprises $(N - K_m)$ firms of type m, $m = 1,\ldots,M$. Within each type firms in the fringe make the same profits while these profits differ across distinct types. Finally, market demand is assumed to be $Q = NM(a - bp)$ to account for the change in the number of firms compared with the first section.

Define $\omega = (a, b, \beta_1,\ldots,\beta_M, N, M)$ as the vector of cost and demand parameters, and $\alpha = (\alpha_1,\ldots,\alpha_M) \in [0,1]^M$ as the vector characterizing the cartel. The industry equilibrium for given ω and α is given by :

$$p(\omega,\alpha) = \frac{a(b+\overline{\beta})}{(b+\overline{\beta})^2 - \overline{\beta}_c^2}\,, \qquad \text{(a)}$$

$$q_{c,m}(\omega,\alpha) = \beta_m \frac{a}{(b+\overline{\beta})^2 - \overline{\beta}_c^2}\,, \quad m = 1,\ldots,M \qquad \text{(b)} \qquad (8)$$

$$q_{f,m}(\omega,\alpha) = \beta_m \frac{a(b+\overline{\beta})}{(b+\overline{\beta})^2 - \overline{\beta}_c^2}\,, \quad m = 1,\ldots,M, \qquad \text{(c)}$$

where $\overline{\beta} = \frac{1}{M}\sum_{m=1}^{M} \beta_m$, and $\overline{\beta}_c = \frac{1}{M}\sum_{m=1}^{M} \alpha_m \beta_m$.

The profit functions can easily be derived :

$$\pi_{c,m}(\alpha) = \frac{\beta_m}{2} \quad \frac{a^2}{(b+\overline{\beta})^2 - \overline{\beta}_c^2}, \qquad m = 1,\ldots, M ; \qquad (a)$$

$$\pi_{f,m}(\alpha) = \frac{\beta_m}{2} \left[\frac{a(b+\overline{\beta})}{(b+\overline{\beta})^2 - \overline{\beta}_c^2} \right]^2, \qquad m = 1,\ldots, M . \qquad (b) \tag{9}$$

It can readily be seen that by setting $M = 1$ in equations (8) and (9) the results of the first section obtain.

The profit schedule $\pi_{i,m}$, $i = c,f$, $m = 1,\ldots,M$, is a monotonically increasing function of α_m holding constant the other α's. Thus, it has a well defined inverse on $[\underline{\pi}_m , \overline{\pi}_m]$[9] :

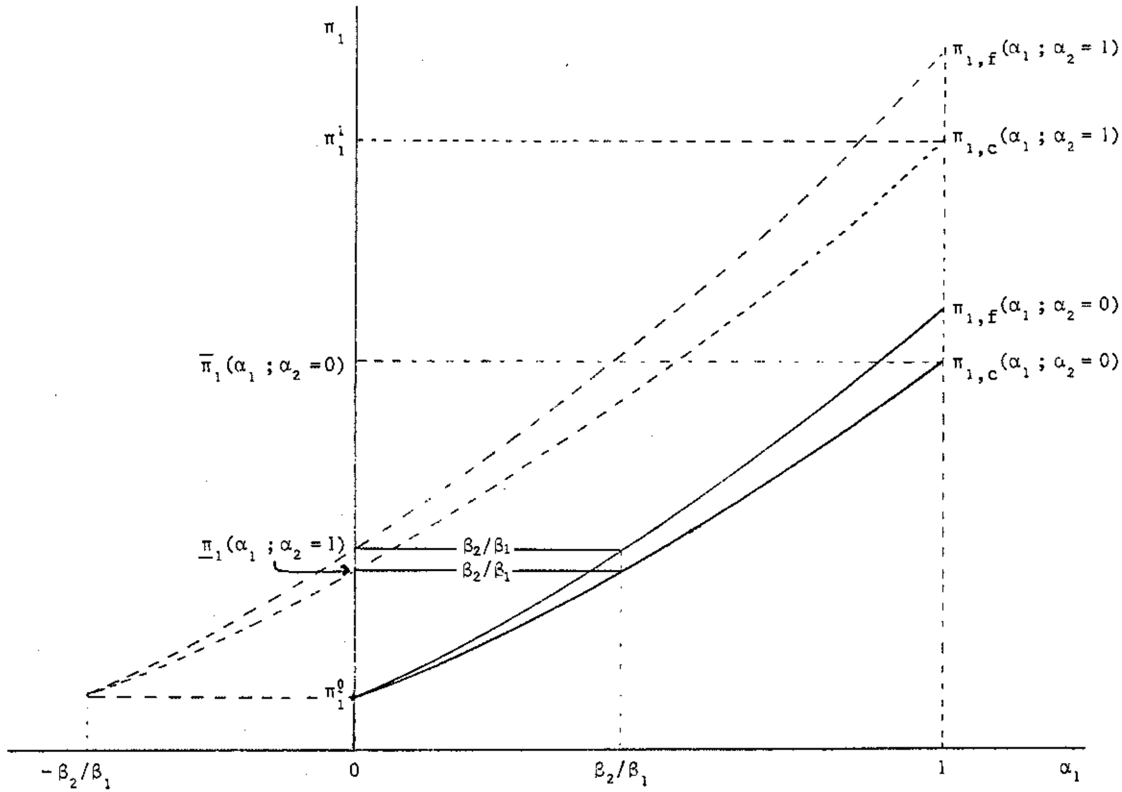
$$\alpha_{c,m}(\pi_m) = \frac{M(b+\overline{\beta})}{\beta_m} \left(1 - \left(\frac{\pi_m^0}{\pi_m}\right)\right)^{\frac{1}{2}} - \phi , \qquad (a)$$

$$\alpha_{f,m}(\pi_m) = \frac{M(b+\overline{\beta})}{\beta_m} \left(1 - \left(\frac{\pi_m^0}{\pi_m}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}} - \phi , \qquad (b) \tag{10}$$

where $\phi = \sum_{\substack{k=1 \\ k\neq m}}^{k=M} \alpha_k \frac{\beta_k}{\beta_m}$ and π_m^0 is the profit attained by type m firms under perfect competition.

Observe that the horizontal distance between the profit schedules of a given type of firms is independent of the proportion of other firms in the cartel. Indeed, from (10) we obtain that $\alpha_{c,m}(\pi_m) - \alpha_{f,m}(\pi_m)$ does not depend on the other α_k's. However, profit schedules for type m firms are indexed by the proportion of other firms in the cartel. In particular, equation (10) indicates that the profit schedules are shifted to the left as the proportion of other firms in the cartel increases. Figure 4 illustrates this point in the simple case when $M = 2$ and where the horizontal distance between the profit schedules of type 1 firms is increasing monotonically as α_1 increases.

We can now define a stable cartel in this new context :



The solid (dotted) curves represent type 1 firms profit schedules given $\alpha_2 = 0$ $(\alpha_2 = 1)$
The dotted curves are obtained by shifting horizontally to the left (a shift of β_2/β_1) the solid curves.

Figure 4

DEFINITION 3 : A cartel of size $\alpha = (\alpha_1, \ldots, \alpha_M)$, is *stable* if for all types of firms m, $m = 1, \ldots, M$, it is externally stable when $\alpha_m \in [0, \frac{1}{N})$, it is both internally and externally stable for $\alpha_m \in [(\frac{1}{N}), 1 - (\frac{1}{N}))$ and it is internally stable for $\alpha_m \in (1 - (\frac{1}{N}), 1]$.

Before examining the existence of a stable cartel with heterogeneous firms we introduce two lemmas which characterize willingness to enter or to exit the cartel.

LEMMA 3 a : *Given* $\alpha^e = (\alpha_1^e, \ldots, \alpha_M^e) \in (0,1)^M$, *if no firm of type* m^e *wishes to join the cartel, then no less efficient firm of type* $m > m^e$ *wishes to join the cartel.*

LEMMA 3 b : *Given* $\alpha^i = (\alpha_1^i, \ldots, \alpha_M^i) \in (0,1)^M$, *if no firm of type* m^i *wishes to exit the cartel, then no more efficient firm of type* $m < m^i$ *wishes to exit the cartel.*

To establish these two lemmas we introduce two additional features of the profit schedules :

$$\frac{\pi_{i,m}}{\pi_{i,k}} = \frac{\beta_m}{\beta_k}, \; i = c,f ; \quad m, k = 1, \ldots, M. \tag{11}$$

Equation (11) obtains from equation (9). It says that the ratio of profits for two types of firms m and k is equal to the ratio of their efficiency parameters β, both for firms in the cartel and in the fringe. In addition, it can be shown differentiating (9) with respect to the share of a given type of firms in the cartel that the increase (decrease) in profit of type m firms associated with the entry (exit) of type k firms in the cartel is an increasing function of the efficiency of type k firms, both for firms in the cartel and in the fringe. This can be written as :

$$\frac{\partial}{\partial \beta_k} \left[\frac{\partial \pi_{i,m}}{\partial \alpha_k} \right] > 0, \quad i = c,f; \quad m, k = 1, \ldots, M. \tag{12}$$

We can now establish Lemma 3 . If no firm of type m^e wishes to exit the cartel it follows that α^e is externally stable for type m^e firms :

$$\pi_{f,m^e}(\alpha^e) \geqslant \pi_{c,m^e}(\alpha_1^e,\ldots,\alpha_{m^e}^e+\frac{1}{N},\ldots,\alpha_M^e). \tag{13}$$

Taking into account equation (11) we obtain for all $m = 1,\ldots,M$:

$$\pi_{f,m}(\alpha^e) \geqslant \pi_{c,m}(\alpha_1^e,\ldots,\alpha_{m^e}^e+\frac{1}{N},\ldots,\alpha_M^e). \tag{14}$$

In addition, taking equation (12) into account, we obtain that for all $m > m^e$:

$$\pi_{c,m}(\alpha_1^e,\ldots,\alpha_{m^e}^e+\frac{1}{N},\ldots,\alpha_m^e,\ldots,\alpha_M^e) \geqslant \\ \pi_{c,m}(\alpha_1^e,\ldots,\alpha_{m^e}^e,\ldots,\alpha_m^e+\frac{1}{N},\ldots,\alpha_M^e). \tag{15}$$

Combining (14) and (15), we observe that α^e is externally stable for type m firms, $m > m^e$:

$$\pi_{f,m}(\alpha^e) \geqslant \pi_{c,m}(\alpha_1^e,\ldots,\alpha_m^e+\frac{1}{N},\ldots,\alpha_M^e). \tag{16}$$

This completes the proof of Lemma 3 a. A symmetric argument would establish Lemma 3 b .

Q.E.D.

Having characterized profit schedules as well as entry and exit conditions across types of firms we can now demonstrate the following :

PROPOSITION 3 : *A stable cartel* α^* *exists. The proportion of firms of type* m*,* α_m^**, is an increasing function of the efficiency of the type,* β_m*. There is one type of firm* m^* *such that* $\alpha_{m^*}^* \in [0,1)$*. All firms of type* $m < m^*$ *are in the cartel and all firms of type* $m > m^*$ *are in the fringe.*

To establish Proposition 3 it suffices to procede step by step. Let

$\alpha_2^*, \ldots, \alpha_M^* = 0$. From Section 1, we know that there exists either one or two values of α_1 such that the cartel of size $(\alpha_1^*, 0, \ldots, 0)$ is stable from the point of view of type 1 firms. If α_1^* is unique and less than one the search for α^* is completed. By Lemma 3 a, if no firm of type 1 wishes to enter for $\alpha = (\alpha_1^*, 0, \ldots, 0)$, then no other firm wishes to enter, thus $\alpha^* = (\alpha_1^*, 0, \ldots, 0)$ is a stable cartel. If α_1^* is unique and equal to one we can set $\alpha_1 = \alpha_1^*$ and $\alpha_3^*, \ldots, \alpha_M^* = 0$, and search for α_2^* such that $\alpha = (\alpha_1^*, \alpha_2^*, 0, \ldots, 0)$ is stable from the point of view of type 2 firms. If α_2^* is less than one the search for α^* is over. By Lemma 3 a, if no firm of type 2 wishes to enter, then no firm of type $m > 2$ wishes to enter. Furthermore by Lemma 3 b, if no firm of type 2 wishes to exit, then no firm of type 1 wishes to exit. Thus $\alpha^* = (\alpha_1^*, \alpha_2^*, 0, \ldots, 0)$ is a stable cartel. This process can be repeated if $\alpha_2^* = 1$. If α_1^* is not unique, there are two values, one equal to 1 the other less than 1, which are stable cartels from the point of view of type 1 firms. If a proportion $\alpha^* < 1$ join the cartel, then no other firm will join by Lemma 3 a. Instead, if all firms of type 1 are in the cartel, then other firms of type 2 may wish to enter. The process of search described above can then be repeated.

Q.E.D.

As a final remark, note that the equilibrium condition similar to equation (7) can be written for each type of firm as :

$$\pi_m^* / \alpha_{c,m} (\pi_m^*) - \alpha_{f,m} (\pi_m^*) = \frac{1}{N}, \quad \pi_m^* \in (\underline{\pi}_m, \overline{\pi}_m), \tag{17}$$

where $\underline{\pi}_m$ $(\overline{\pi}_m)$ is the value of the profit per firm of type m in the cartel for $\alpha_m = 0$ $(\alpha_m = 1)$ given the other α_k's. In Figure 4, the range of variation of π_1 can be seen to vary as α_2 varies. Thus, the comparative static results established in Proposition 2 can easily be extended noting that the proportion of firms m in the cartel decreases with the number of types of firms M and increases with their average efficiency $\overline{\beta}$.

3. CONCLUSION

We have established the existence of possibly heterogeneous stable cartels and we have determined their characteristics such as uniqueness, size and composition. It should be the subject of further work to determine the extent to which alternative specifications might confirm or contradict the results derived above. Of particular interest are the possibility of side payments among cartel members, the adoption of a mode of market conduct other than price leadership and the possibility of differentiated output across firms.

FOOTNOTES

[1] Sherer [1981, p. 169] mentions that by 1967 several hundred cases were in front the EEC Commission, while every year a dozen or so cases are brought to court by the Federal Trade Commission in the United States. Insofar as export cartels are concerned, they are still legal in most developed countries and seem to last for substantial periods of time. A survey of international export cartels throughout the world can be found in UNCTAD [1971, chapter 3].

[2] The d'Aspremont et alii result was extende[illegible] 1981] who showed that under their assumptions, the number of firms in the cartel was equal to three so long as there was more than three firms in the industry. This result, however, depends crucially on the value of the parameters in the cost and demand functions as we show below.

[3] To obtain (1) set the supply of the competitive fringe equal to the sum of the inverses of each competitors marginal cost function; define the residual demand function faced by the cartel as market demand less the supply of the competitive fringe; equate the corresponding marginal revenue to marginal cost per cartel member in order to determine the output supplied by the cartel. The equilibrium price in (1-a) obtains by equating total supply and market demand. Finally the equilibrium quantities in the fringe and the cartel are obtained by equating marginal cost to the prevailing price and the corresponding marginal revenue.

[4] This case could occur, however, in the extreme case where $\frac{1}{N}$ equals zero, i.e., when there is an infinite number of firms. Observe that in this case the cartel of size α equals zero would be a stable cartel, as was already established by d'Aspremont et alii [1981].

[5] Observe that two pathologies may occur : $\alpha_f(\pi^*)$ and, by the definition of π^*, $\alpha_c(\pi^*)$ may be multiples of $\frac{1}{N}$ in which case there will be two values of α^* and, in turn, two stable cartels one of which would have one more firm than the other one; π^* may be equal to π^1 in which case both α^* equals one and $1 - \frac{1}{N}$ would be stable cartels.

[6]This case will occur when the two following conditions are satisfied : first, we must be in the situation corresponding to Figure (2-b), i.e., $\gamma > k$; second, it must be the case that the horizontal distance between the profit functions, computed at π equals $\overline{\pi}$ is greater than $\frac{1}{N}$. Hence, there is a maximum number of firms $\overline{N}$ such that below that number of firms there will be two stable cartels : $\overline{N} = (\alpha_c(\overline{\pi}) - \alpha_f(\overline{\pi}))^{-1}$.

[7]Observe that if π^* was equal to $\overline{\pi}$ there would be a unique stable cartel of size α equals one.

[8]Allowing for side payments might of course affect the results. Observe, however, that a full treatment of the possibility of side payments should involve the consideration of a third stage game, this time among the firms in the cartel. For an argument among these lines see Pitchik and Osborne [1982].

[9]Denote $\underline{\pi}_m$ ($\overline{\pi}_m$) as the minimum (maximum) profit attained by type m firms in the cartel given the other α_k's.

REFERENCES

d'Aspremont, C., and J. Jaskold Gabszewicz [1982], "On the Stability of Collusive Arrangements", mimeographed, C.O.R.E., Université Catholique de Louvain.

d'Aspremont, C., A. Jacquemin, J. Jaskold Gabszewicz, and J. Weymark [1981], "On the Stability of Dominant Cartels", Working Paper n° 8101, Institut des Sciences Economiques, Université Catholique de Louvain, forthcoming in *Canadian Journal of Economics*.

Chiappori, A. [1981], "Une note sur l'article de Messieurs d'Aspremont, Jacquemin, Jaskold Gabszewicz et Weymark", mimeographed, Université de Paris I.

Donsimoni, M.-P., N. Economides, and H. Polemarchakis [1981], "Stable Cartels", Document de Travail 8112, E.N.S.A.E., Institut National de la Statistique et des Etudes Economiques, Paris.

Fellner, W. [1949], *Competition Among the Few*, New York, Knopf.

Pitchik, C., and M. Osborne [1982], "Are Large Firms More Powerful than Small Ones ?", Discussion Paper Series No. 82-07, Department of Economics, New York University.

Scherer, F. [1980], *Industrial Market Structure and Economic Performance*, 2nd edition, Rand Mc Nally.

Stigler, G. [1950], "Monopoly and Oligopoly by Merger", *American Economic Review, 40*, 23-34.

Stigler, G. [1964], "A Theory of Oligopoly", *Journal of Political Economy, 72*, 57-71.

United Nations Conference on Trade and Development [1971], *Restrictive Business Practices*, New York, United Nations.