Concentration Control in a Tubular Reactor by a Late Lumping Approach



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Tubular chemical reactors are often modelled by using parabolic partial differential equations (PDE). Usually control strategies based on these equations make use of lumping and approximation of the model prior to controller design. This paper presents an approach to design a control strategy directly based on PDE for concentration control via the inlet boundary condition. The strategy uses a two-level controller. The inner loop is a Lyapunov-based adaptive controller that also estimates on-line the process kinetic parameters. The outer loop combines feedforward and feedback actions to track the set point. The performance of the controller is illustrated by numerical simulation of a bleaching tower example.

INTRODUCTION

The modelling of tubular reactors is usually performed by using mass and energy balances on a thin slice of the reactor. This approach leads to partial differential equations models. Those models are not typically used in process control where the use of lumped model such as first order plus delay transfer functions is the common practice. However, PDE models offer a structured framework in which process information such as kinetics is directly available for identification or adaptation purposes.

Various approaches have been considered to directly use PDE phenomenological models. Ray [1] proposed to divide control approaches on PDEs in two groups. The first group is composed of early lumping methods where a preliminary discretization of the PDE model is used to obtain a set of ODEs. This lumping is often realized by numerical techniques such as finite differences, orthogonal collocation or finite elements. Early lumping techniques include the use of global differentiation proposed by Dochain [2] as an approximation of partial derivatives. Christofides [3] has used also early lumping techniques on parabolic systems in a two-time scale framework in which only dominant modes are considered. The second group of techniques is based on late lumping methods where the controller design problem is solved directly with the PDE model. When necessary, lumping may be applied for controller implementation. The control of parabolic PDE have been previously addressed by Hong and Bentsman [4]. They provide a design solution for systems in which the control action explicitly appears in the PDE system.

In this paper, we consider the problem of controlling the outlet concentration of a tubular reactor with using the inlet concentration as the manipulated variable. The design is done using directly a parabolic PDE model. The kinetic parameters are the main source of uncertainty in the dynamical model of reactors, they are assumed here to be unknown, and an adaptive controller is proposed in order to handle this uncertainty. The main advantage of considering a PDE model in the control design is to use the available process dynamics knowledge such as hydrodynamics and kinetics. The tuning of the controller is then easier since it is based on physical knowledge about the process dynamics. Furthermore, the proposed adaptive control algorithm makes use of feedforward and feedback approach based on the model. This approach is then efficient for a large spectrum of values for the Peclet number.

The first part of this paper presents the type of model considered and the different assumptions. The second part of the paper shows the proposed late lumping controller design. This approach is then applied to a bleaching reactor in the third section followed by some simulations results.

PDE MODELLING

The dynamics of an isothermal tubular reactor are obtained from mass balances. If some mixing occurs in the reactor, the well-known convection-dispersion-reaction model is obtained. For a reactor involving *m* reactions on *n* chemical species, the following parabolic model is obtained:

$$\frac{\partial A(z,t)}{\partial t} = -\boldsymbol{v}\frac{\partial A(z,t)}{\partial z} + \boldsymbol{D}\frac{\partial^2 A(z,t)}{\partial z^2} - KR(A(z,t)) \tag{1}$$

$$\boldsymbol{D}\frac{\partial A(z,t)}{\partial z} = v(A(0,t) - A_{in}) \tag{2}$$

$$\boldsymbol{D}\frac{\partial A(z,t)}{\partial z} = 0 \tag{3}$$

In these equations, $A(z,t)$ (mol/l) is a $(n,1)$ vector of the species concentrations profiles. The reactor hydrodynamics is characterized by two (n,n) diagonal matrices: $\boldsymbol{v}$ (m/s), the superficial velocity for each component and $\boldsymbol{D}$ (m^2/s), the axial dispersion coefficient for all components. Kinetics are described by the $-KR(A(z,t))$ term where *K* is an (n,m) matrix containing all kinetic parameters and which $R(A(z,t))$ is a $(m,1)$ vector field of the remaining portion of the kinetics. It is assumed that $\boldsymbol{v}$, $\boldsymbol{D}$, and $R(A(z,t))$ terms are supposed known and the *K* matrix is unknown. In this model, the manipulated variable is the concentration of one reactant at the inlet and it is noted A_{in}^*. The controlled variable is the concentration of one species at the outlet and it is noted A_{out}^*. Thus, a single input -single output problem is considered.

CONTROLLER DESIGN

An internal model approach is proposed that takes advantage of the whole information of the PDE model. The error between the model and the system is used in a direct adaptation scheme. The role of this part of the algorithm is to insure that the model will track the system, in particular when the operating conditions or the raw materials properties have a significant influence on the kinetics. The information acquired on the model is then used by a feedforward algorithm to schedule the needed inlet concentration. Finally, a feedback external loop will insure a proper reference adaptation in case of external perturbation or model mismatch. Those two external loops will then insure set-point tracking. The proposed controller structure is shown in figure 1.

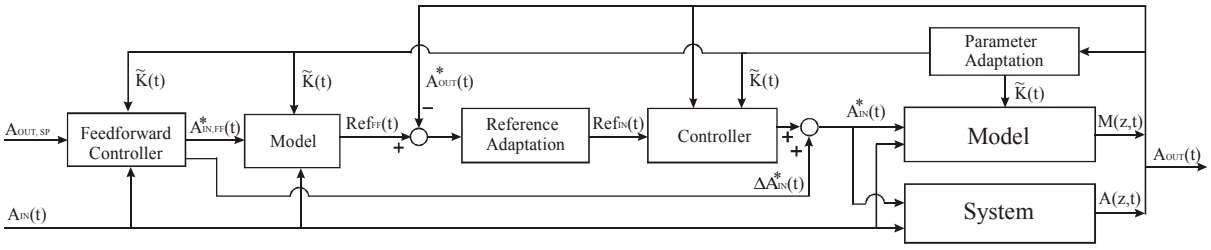


Figure 1: Controller structure

Adaptive controller design

The direct adaptation controller is designed by using a Lyapunov-based argument. In this scheme, the kinetic parameters of the model are adapted to minimize the error between the model and the system. The controller design is summarised in the following proposition.

Proposition:

Let us consider the following PDE system where the *K* matrix is unknown:

$$\frac{\partial A(z,t)}{\partial t} = -\boldsymbol{v}\frac{\partial A(z,t)}{\partial z} + \boldsymbol{D}\frac{\partial^2 A(z,t)}{\partial z^2} - KR(A(z,t)) \tag{4}$$

$$\boldsymbol{D}\frac{\partial A(0,t)}{\partial z} = v(A(0,t) - A_{in}) \tag{5}$$

$$\boldsymbol{D}\frac{\partial A(l,t)}{\partial z}=0 \tag{6}$$

Let us consider the following corresponding internal PDE model:

$$\frac{\partial M(z,t)}{\partial t}=-\boldsymbol{v}\frac{\partial M(z,t)}{\partial z}+\boldsymbol{D}\frac{\partial^2 M(z,t)}{\partial z^2}-\widetilde{K}(t)R(M(z,t)) \tag{7}$$

$$\boldsymbol{D}\frac{\partial M(0,t)}{\partial z}=v(M(0,t)-A_{in}) \tag{8}$$

$$\boldsymbol{D}\frac{\partial M(l,t)}{\partial z}=0 \tag{9}$$

The error between the model solution $M(z,t)$ and the solution of the system $A(z,t)$ tends to zero when t tends to infinity when the following control law and adaptation law are used:

$$\begin{aligned}\left(A_{in}(t)-Ref_{in}(t)\dot{A}_{in}(t)\right)=-\varepsilon\Bigg(&\sum_{i=1}^{n}\left\langle e_i(z,t),-v_{ii}\frac{\partial e_i(z,t)}{\partial z}\right\rangle\\&+\sum_{i=1}^{n}\left\langle e_i(z,t),\sum_{j=1}^{m}\widetilde{K}_{ij}(t)Q_j(z,t)\right\rangle+\frac{1}{\varepsilon}\left(A_{in}^{*}(t)-Ref_{in}(t)\right)\dot{Ref}_{in}(t)\Bigg)\end{aligned} \tag{10}$$

$$\dot{\psi}_{ij}(t)=-a_{ij}\left\langle e_i(z,t),R_j(A(z,t))\right\rangle \tag{11}$$

Proof: This proof is based on Lyapunov's second method. To build a Lyapunov function the following error equations are considered:

$$e(z,t)=A(z,t)-M(z,t) \tag{12}$$

$$\psi(t)=\widetilde{K}(t)-K \tag{13}$$

$$Q(z,t)=R(A(z,t)-R(M(z,t)) \tag{14}$$

These equations are used to build the following error function:

$$V(z,t)=\frac{1}{2}\sum_{i=1}^{n}\left\langle e_i(z,t),e_i(z,t)\right\rangle+\frac{1}{2\varepsilon}(A_{in}^{*}(t)-Ref(t))^2+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{1}{a_{ij}}\psi_{ij}^2(t) \tag{15}$$

The time derivative of this function can be calculated by using the expressions of the profile time derivatives and the kinetic parameter time derivative:

$$\begin{aligned}\dot{V}(z,t)=&\sum_{i=1}^{n}\left\langle e_i(z,t),-v_{ii}\frac{\partial e_i(z,t)}{\partial z}\right\rangle+\sum_{i=1}^{n}\left\langle e_i(z,t),-D_{ii}\frac{\partial^2 e_i(z,t)}{\partial z^2}\right\rangle\\&+\sum_{i=1}^{n}\left\langle e_i(z,t),\sum_{j=1}^{m}\psi_{ij}(t)R_j\left(A(z,t)\right)\right\rangle+\sum_{i=1}^{n}\left\langle e_i(z,t),\sum_{j=1}^{m}\widetilde{K}_{ij}(t)Q_j(z,t)\right\rangle\\&+\frac{1}{\varepsilon}(A_{in}^{*}(t)-Ref(t)_{in})\dot{A}_{in}(t)+\frac{1}{\varepsilon}(A_{in}^{*}(t)-Ref(t)_{in})\dot{Ref}_{in}(t)\\&+\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{1}{a_{ij}}\psi_{ij}(t)\dot{\psi}_{ij}(t)\end{aligned} \tag{16}$$

By using integration by parts of the second order derivative and applying the boundary conditions, the following expression is obtained :

$$\begin{aligned}\dot{V}(z,t)=&\sum_{i=1}^{n}\left\langle e_i(z,t),-v_{ii}\frac{\partial e_i(z,t)}{\partial z}\right\rangle+\sum_{i=1}^{n}\left\langle \frac{\partial e_i(z,t)}{\partial z},-D_{ii}\frac{\partial e_i(z,t)}{\partial z}\right\rangle-\sum_{i=1}^{n}e_i(0,t)D_{ii}e_i(0,t)\\&+\sum_{i=1}^{n}\left\langle e_i(z,t),\sum_{j=1}^{m}\psi_{ij}(t)R_j\left(A(z,t)\right)\right\rangle+\sum_{i=1}^{n}\left\langle e_i(z,t),\sum_{j=1}^{m}\widetilde{K}_{ij}(t)Q_j(z,t)\right\rangle\\&+\frac{1}{\varepsilon}(A_{in}^{*}(t)-Ref(t)_{in})\dot{A}_{in}+\frac{1}{\varepsilon}(A_{in}^{*}(t)-Ref(t)_{in})\dot{Ref}_{in}(t)\\&+\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{1}{a_{ij}}\psi_{ij}(t)\dot{\psi}_{ij}(t)\end{aligned} \tag{17}$$

Considering the control and adaptation law of the proposition leads to :

$$\dot{V}(z,t) = -\sum_{i=1}^{n} \left\langle \frac{\partial e_i(z,t)}{\partial z}, D_{ii} \frac{\partial e_i(z,t)}{\partial z} \right\rangle - \sum_{i=1}^{n} e_i(0,t) D_{ii} e_i(0,t) \tag{18}$$

Since this expression is negative definite, the asymptotic stability of the system is guaranteed. Furthermore, the system will converge to the model since at the equilibrium point the Lyapunov function guarantees that the following equations are satisfied for all the components.

$$\frac{\partial e_i(z,t)}{\partial z} = 0 \qquad e_i(0,t) = 0 \tag{19}$$

Then by the definition of the integral and application of the triangle inequality, it can be shown that:

$$\left\| e_i(z,t) \right\| = 0 \qquad \blacksquare \tag{20}$$

In practice, the proposed control law must be modified to avoid division by zero. The following control law then becomes :

$$\dot{\tilde{A}}_{in} = \frac{(A_{in}^* - Ref_{in})^2}{w + (A_{in}^* - Ref_{in})^2} \tilde{A}_{in}^2 + \frac{(A_{in}^* - Ref_{in})}{w + (A_{in}^* - Ref_{in})^2} f_c \tag{21}$$

$$\dot{A}_{in} = \dot{\tilde{A}}_{in}^* \tag{22}$$

$$f_c = \sum_{i=1}^{n} \left\langle e_i(z,t), -v_{ii} \frac{\partial e_i(z,t)}{\partial z} \right\rangle + \sum_{i=1}^{n} \left\langle e_i(z,t), \sum_{j=1}^{m} \tilde{K}_{ij}(t) Q_j(z,t) \right\rangle + \frac{1}{\varepsilon} \left(A_{in}^*(t) - Ref_{in}(t) \right) \dot{Ref}_{in}(t) \tag{23}$$

In this expression, *w* is a small positive tuning parameter. This control law is equivalent to the original one if $\dot{\tilde{A}}_{in} = 0$. Furthermore, when the error between the model and the system is zero, the control law reduces to the internal reference expression which is $\dot{A}_{in}(t) = \dot{Ref}_{in}(t)$.

Design of the external loops

The external part of the controller is composed of a feedforward controller and a feedback loop. The feedforward algorithm is composed of two parts, a controller and a PDE model. The controller part calculates the concentration of the manipulated species required in order to reach the set point at steady state. This calculation is based on the measurement of all other species at the inlet and on the kinetic parameters identified by the model. This problem is solved using a typical relaxation algorithm. The model part of the feedforward algorithm is used to filter the required inlet concentration through a model of the reactor. The output of this model could than be compared to the actual output of the reactor. The error between these two signals can then be used to feed the reference adaptation algorithm which is indeed the feedback regulator. The regulator used for this purpose is a simple integrator such as:

$$\dot{Ref}_{in}(t) = -\theta (A_{out}^*(t) - A_{out,SP}^*) \tag{24}$$

APPLICATION TO A BLEACHING REACTOR

In order to illustrate the performance of the late lumping controller presented in the preceding section, the algorithm is applied to a bleaching reactor example. The PDE modeling of this system as well as early lumping control algorithms have been presented in [5,6]. In this model, *L* represents lignin, which is the colored component of pulp, and *C* represents Chlorine Dioxide, the bleaching agent. The following PDE model is then considered:

$$\frac{\partial}{\partial t}\begin{bmatrix} C(z,t) \\ L(z,t) \end{bmatrix} = -\begin{bmatrix} v & 0 \\ 0 & v \end{bmatrix} \frac{\partial}{\partial z}\begin{bmatrix} C(z,t) \\ L(z,t) \end{bmatrix} + \begin{bmatrix} D & 0 \\ 0 & D \end{bmatrix} \frac{\partial^2}{\partial z^2}\begin{bmatrix} C(z,t) \\ L(z,t) \end{bmatrix} - \begin{bmatrix} k_C & 0 \\ 0 & k_L \end{bmatrix} L^2(z,t) C^2(z,t) \tag{25}$$

$$\begin{bmatrix} D & 0 \\ 0 & D \end{bmatrix} \frac{\partial}{\partial z}\begin{bmatrix} C(0,t) \\ L(0,t) \end{bmatrix} = \begin{bmatrix} v & 0 \\ 0 & v \end{bmatrix} \frac{\partial^2}{\partial z^2} \left(\begin{bmatrix} C(0,t) \\ L(0,t) \end{bmatrix} - \begin{bmatrix} C_{in}^* \\ L_{in} \end{bmatrix} \right) \tag{26}$$

$$\begin{bmatrix} D & 0 \\ 0 & D \end{bmatrix} \frac{\partial}{\partial z}\begin{bmatrix} C(l,t) \\ L(l,t) \end{bmatrix} = 0 \tag{27}$$

In this model, the controlled variable is L_{out} and the control variable is C_{in}^*. The following internal model is used:

$$\frac{\partial}{\partial t}\begin{bmatrix} N(z,t) \\ M(z,t) \end{bmatrix} = -\begin{bmatrix} v & 0 \\ 0 & v \end{bmatrix}\frac{\partial}{\partial z}\begin{bmatrix} N(z,t) \\ M(z,t) \end{bmatrix} + \begin{bmatrix} D & 0 \\ 0 & D \end{bmatrix}\frac{\partial^2}{\partial z^2}\begin{bmatrix} N(z,t) \\ M(z,t) \end{bmatrix} - \begin{bmatrix} \tilde{k}_C & 0 \\ 0 & \tilde{k}_L \end{bmatrix} N^2(z,t)M^2(z,t) \tag{28}$$

$$\begin{bmatrix} D & 0 \\ 0 & D \end{bmatrix}\frac{\partial}{\partial z}\begin{bmatrix} M(0,t) \\ N(0,t) \end{bmatrix} = \begin{bmatrix} v & 0 \\ 0 & v \end{bmatrix}\frac{\partial^2}{\partial z^2}\left(\begin{bmatrix} M(0,t) \\ N(0,t) \end{bmatrix} - \begin{bmatrix} M_{in}^* \\ N_{in} \end{bmatrix}\right) \tag{29}$$

$$\begin{bmatrix} D & 0 \\ 0 & D \end{bmatrix}\frac{\partial}{\partial z}\begin{bmatrix} M(l,t) \\ n(l,t) \end{bmatrix} = 0 \tag{30}$$

The direct adaptive control law is then:

$$\dot{\tilde{C}}_{in} = \frac{(C_{in}^* - Ref_{in})^2}{w + (C_{in}^* - Ref_{in})^2}\tilde{C}_{in} + \frac{(C_{in}^* - Ref_{in})}{w + (C_{in}^* - Ref_{in})^2} f_c \tag{31}$$

$$\dot{C}_{in} = \tilde{C}_{in}^* \tag{32}$$

$$\begin{aligned} f_c = &\left\langle e_L(z,t), -v\frac{\partial e_L(z,t)}{\partial z}\right\rangle + \left\langle e_C(z,t), -v\frac{\partial e_C(z,t)}{\partial z}\right\rangle \\ &+ \left\langle e_L(z,t), \tilde{k}_L(t)(L(z,t)^2 C(z,t)^2 - M(z,t)^2 N(z,t)^2)\right\rangle \\ &+ \left\langle e_C(z,t), \tilde{k}_C(t)(L(z,t)^2 C(z,t)^2 - M(z,t)^2 N(z,t)^2)\right\rangle + \frac{1}{\varepsilon}\left(C_{in}^*(t) - Ref_{in}(t)\right)\dot{Ref}_{in}(t) \end{aligned} \tag{33}$$

The adaptation laws are given by the following equations:

$$\dot{\tilde{k}}_L(t) = -a\left\langle e_L(z,t), L(z,t)^2 C(z,t)^2\right\rangle \tag{34}$$

$$\dot{\tilde{k}}_C(t) = -b\left\langle e_C(z,t), L(z,t)^2 C(z,t)^2\right\rangle \tag{35}$$

Finally, the feedback regulator is:

$$\dot{Ref}_{in}(t) = -\theta(C_{out}^*(t) - C_{out,SP}^*) \tag{36}$$

In order to apply the above controller on a reactor with a limited number of sensors, these equations have to be modified. For the purpose of this study, only two sensors are considered. In order to obtain a maximal efficiency of the algorithm the sensor must be located at the reactor inlet and outlet since the kinetics are monotonic decreasing functions. The resulting control and adaptation laws for the bleaching reactor application are then:

$$\dot{\tilde{C}}_{in} = \frac{(C_{in}^* - Ref_{in})^2}{w + (C_{in}^* - Ref_{in})^2}\tilde{C}_{in} + \frac{(C_{in}^* - Ref_{in})}{w + (C_{in}^* - Ref_{in})^2} f_c \tag{37}$$

$$\dot{C}_{in} = \tilde{C}_{in}^* \tag{38}$$

$$\begin{aligned} f_c = &\left\langle e_{L,in}(z,t), \tilde{k}_L(t)(L_{in}(z,t)^2 C_{in}(z,t)^2 - M_{in}(z,t)^2 N_{in}(z,t)^2)\right\rangle \\ &+ \left\langle e_{L,out}(z,t), \tilde{k}_L(t)(L_{out}(z,t)^2 C_{out}(z,t)^2 - M_{out}(z,t)^2 N_{out}(z,t)^2)\right\rangle \\ &+ \left\langle e_{C,in}(z,t), \tilde{k}_C(t)(L_{in}(z,t)^2 C_{in}(z,t)^2 - M_{in}(z,t)^2 N_{in}(z,t)^2)\right\rangle \\ &+ \left\langle e_{C,out}(z,t), \tilde{k}_C(t)(L_{out}(z,t)^2 C_{out}(z,t)^2 - M_{out}(z,t)^2 N_{out}(z,t)^2)\right\rangle \\ &+ \frac{1}{\varepsilon}\left(C_{in}^*(t) - Ref_{in}(t)\right)\dot{Ref}_{in}(t) \end{aligned} \tag{39}$$

$$\dot{\tilde{k}}_L(t) = -a\left(\left\langle e_{L,in}(z,t), L_{in}(z,t)^2 C_{in}(z,t)^2\right\rangle + \left\langle e_{L,out}(z,t), L_{out}(z,t)^2 C_{out}(z,t)^2\right\rangle\right) \tag{40}$$

$$\dot{\tilde{k}}_C(t) = -b\left(\left\langle e_{C,in}(z,t), L_{in}(z,t)^2 C_{in}(z,t)^2\right\rangle + \left\langle e_{C,out}(z,t), L_{out}(z,t)^2 C_{out}(z,t)^2\right\rangle\right) \tag{41}$$

SIMULATION RESULTS

Numerical simulations of the late lumping control approach applied the bleaching reactor example have been performed using a sequencing algorithm with a 100 node mesh [7]. In this algorithm, convection, dispersion and reaction phenomena are successively considered at each time step. The controller parameters

have been chosen to minimize overshoot and oscillations. Simulations are started at steady state with $L_{in} = 31\,Kappa$, $C_{in} = 2.35\,g/l$ and the M1 kinetic model of Table 1. The following controller parameters have been used:

$$\varepsilon = 0.0001 \qquad \theta = 0.002 \qquad a = 0.000005$$
$$b = 0.000005 \qquad w = 0.0000001$$

Figure 2 shows the controller response to flow variation. Since the model directly uses the measured flow, this results in a good disturbance compensation. Figure 3 shows the controller response to a set point change. In this case, the feed-forward algorithm gives an optimal response. The influence of the kinetic parameter variations is presented in Figure 4. In this simulation a variation of +20% on k_L is first applied, followed by a -20% variation of k_C. The simulation result shows an efficient adaptation of the model which then results in an adequate reference tracking.

In order to evaluate the robustness of the algorithm, three different models for process kinetics are considered as shown is Table 1. For this simulation, the sequence of perturbation shown in Figure 5 is applied.

Model 1	$k_c = 0.001L^2C^2$ $k_l = 0.001L^2C^2$
Model 2	$k_c = 0.00006L^3C^3$ $k_l = 0.0004L^3C^3$
Model 3	$k_c = 0.006LC$ $k_l = 0.035LC$ and constant drop at the inlet $L0 = -9$, $C0 = -1.3$

Table 1: Kinetic model for robustness analysis

Figure 6 shows the results of the robustness analysis and Figure 7 shows the corresponding kinetic parameter estimations. The results show that the late lumping approach have good robustness properties as long as the structure of the kinetics model in the controller is similar to one in the model. Otherwise, the internal model is not able to efficiently track the process model.

The purpose of this analysis was to assess the benefits of using a detailed nonlinear PDE model for a tubular reactor instead of a simple I/O model. This approach can be used as a benchmark to understand what performance improvement may be obtained by nonlinear methods. This approach has been compared to other design techniques in [6] for the particular bleaching reactor application.

CONCLUDING REMARKS

In this paper, we have presented a late lumping approach for the design of controllers of chemical processes whose dynamics are described by PDEs. The proposed control strategy is composed of two loops : an inner loop and an outer loop. The inner contains an adaptive controller which has been designed by considering Lyapunov theory based arguments and which allows to estimate on-line the (badly known or unknown) process kinetics parameters The outer loop combines feedforward and feedback actions; its objective is to allow the tracking of changing set-points. The proposed late lumping controller has been illustrated for the concentration control by the inlet of reactor of a bleaching process whose dynamics are described by parabolic PDE equations. The performance and robustness of the controller have been illustrated in numerical simulation for various operating conditions. The direct use of the PDE model in the control design makes the adaptive control really efficient when the kinetic structure is correctly estimated even if only a few sensors are actually available. The combination of the two (inner and outer) loops in the controller provides an adequate structure to integrate the advantages of a feedforward controller and adaptation mechanisms.

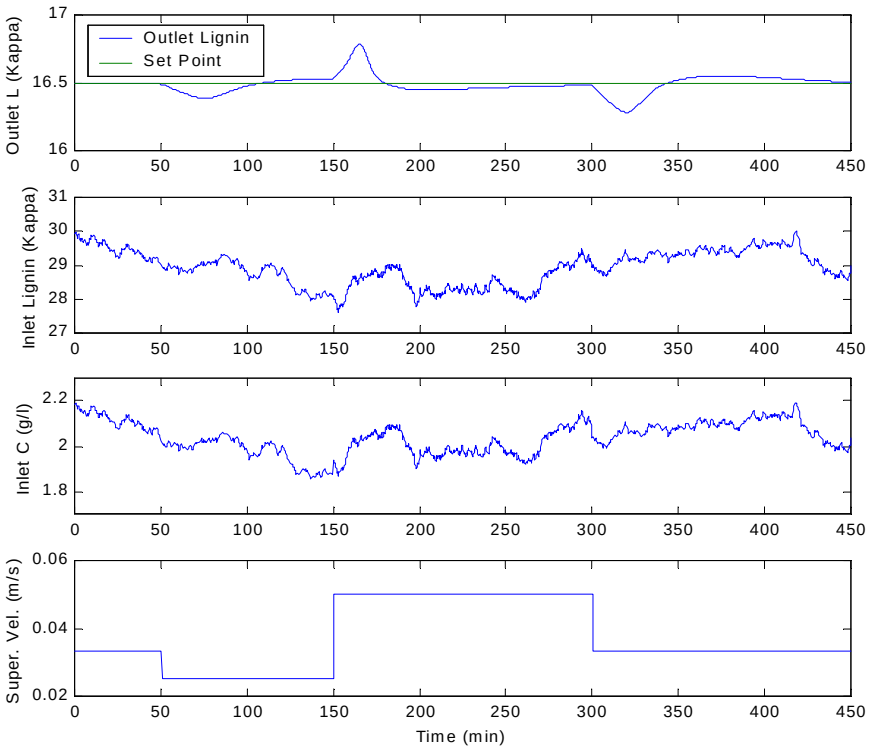


Figure 2: Flow variations

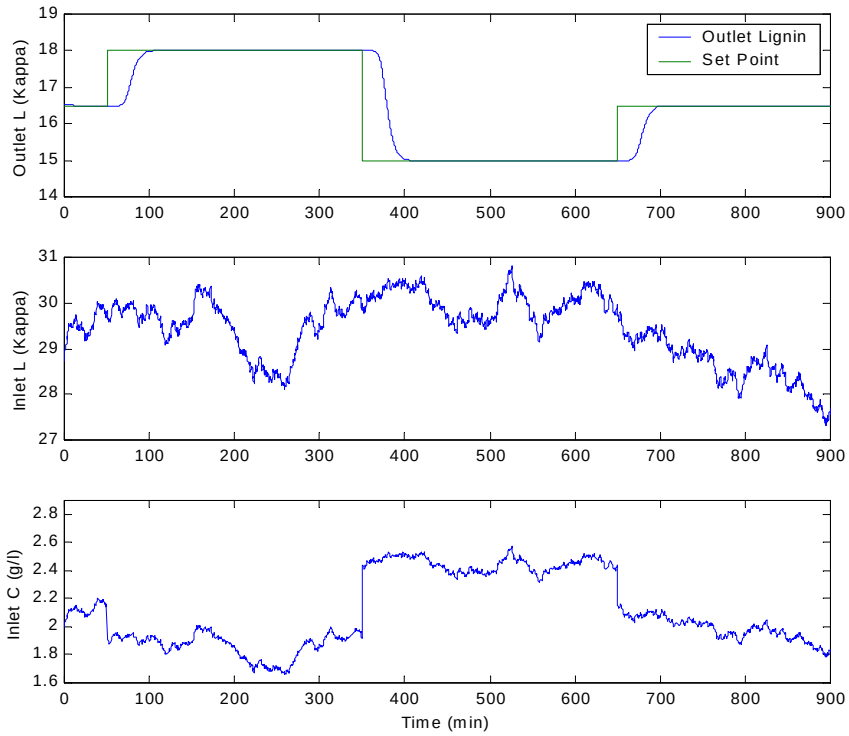


Figure 3: Set point variations

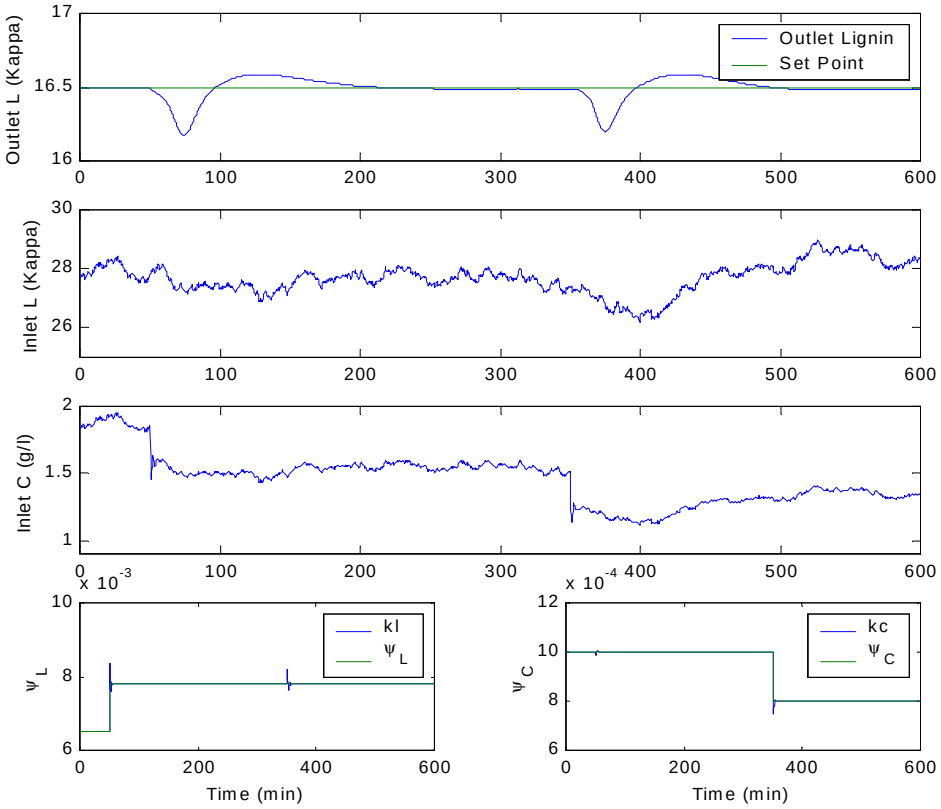


Figure 4: Kinetic parameters variations

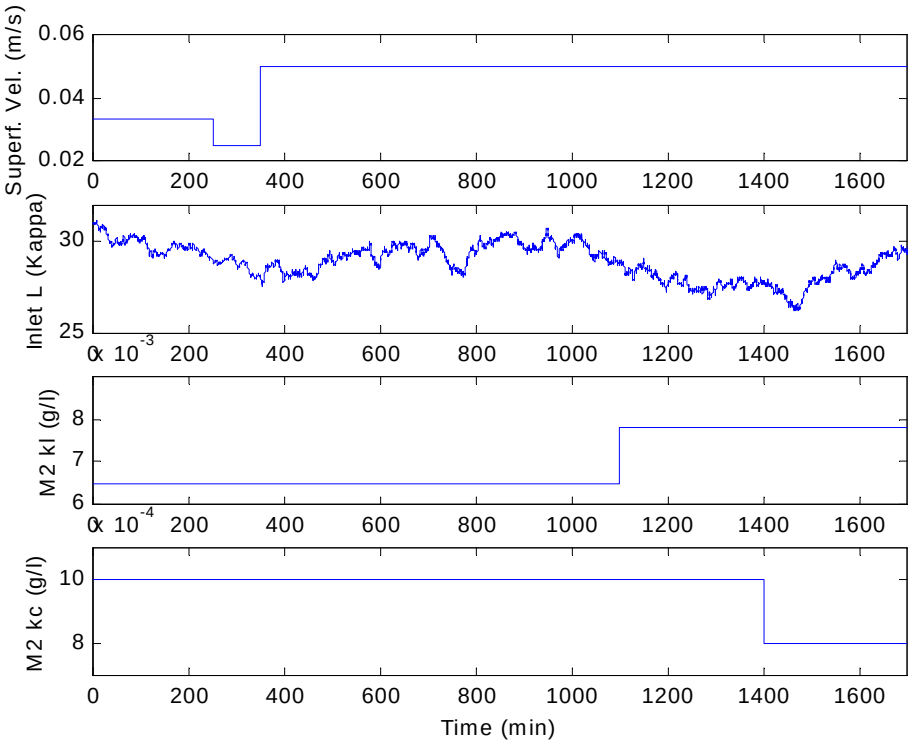


Figure 5: Parameter variations for robustness analysis

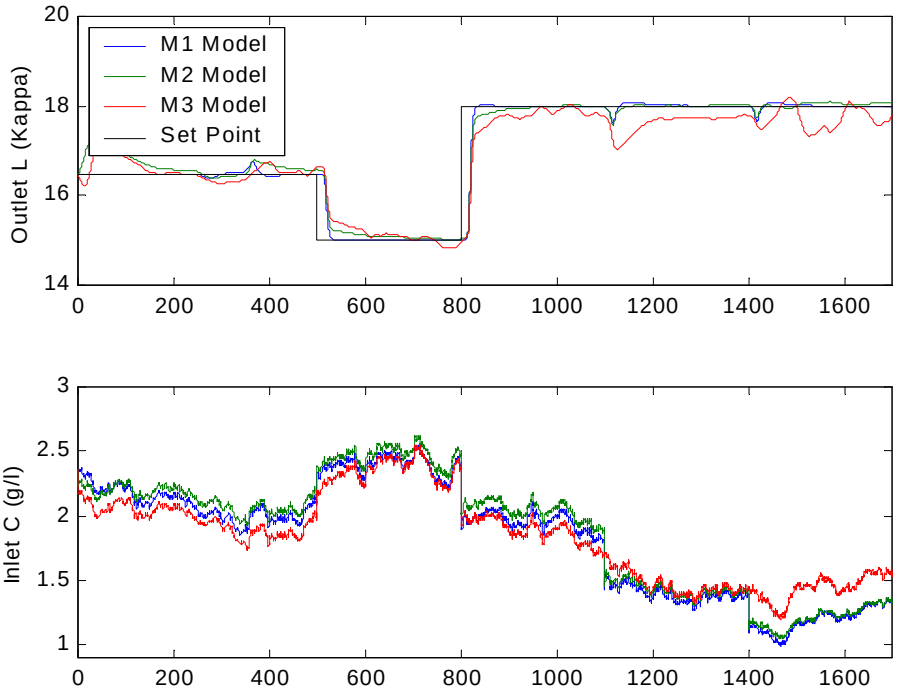


Figure 6: Robustness analysis

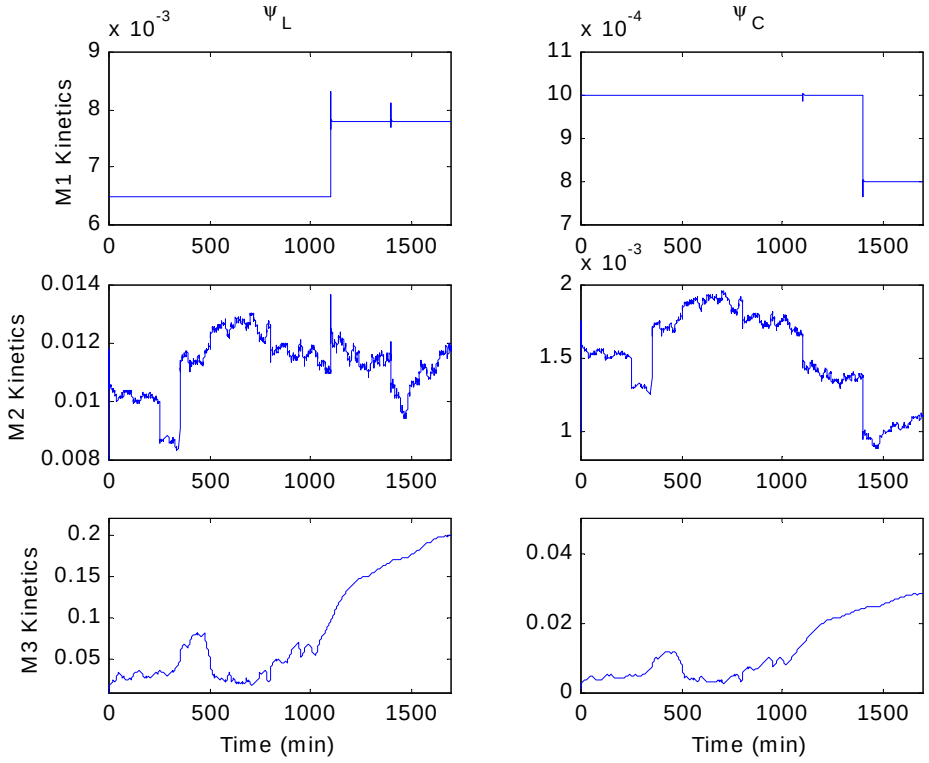


Figure 7: Kinetic parameters adaptation

NOMENCLATURE

$A(z,t)$	Vector of species concentration profile
v	Superficial velocity
D	Axial dispersion coefficient
A_{in}	Manipulated variable, concentration of specie at the inlet
A_{out}	Controlled variable, concentration of specie at the outlet
K	Kinetic parameters matrix
$KR(A(z,t))$	Kinetics
$M(z,t)$	Model vector of species concentration profile
$\widetilde{K}$	Model kinetic parameters matrix
$\varepsilon, a_{ij}, \theta, w$	Controller parameters
$V(t)$	Lyapunov function
ψ_{ij}	Kinetic parameters estimation error
Ref_{in}	Adaptive controller reference
$C(z,t)$	Chlorine Dioxide profile
$L(z,t)$	Lignin profile
k_C, k_L	kinetic parameters of the bleaching reactor

REFERENCES

1 Ray, W.H. (1981), *Advanced Process Control.* McGraw-Hill, New York.

2 Dochain, D. (1994), 'Contribution to the Analysis and Control of Distributed Parameter Systems with Application to (Bio)Chemical Processes and Robotics', Thèse d'agrégation de l'enseignement supérieur, Université Catholique de Louvain, Belgium.

3 Christofides, P.D. (1996), 'Nonlinear Control of Two-Time-Scale and Distributed Parameter Systems', Ph.D. Thesis, University of Minnesota, USA.

4 Hong, K.S., Bentsman, J. (1994), 'Direct Adaptive Control of Parabolic Systems: Algorithm Synthesis and Convergence and Stability Analysis', *IEEE Trans.Aut. Control*, **39(10)**, pp 2018-2033.

5 Renou, S., Perrier, M., Dochain, D., Gendron, S., (2000), 'Nonlinear Control Design for Pulp Bleaching', *Proc. Control Systems 2000*, PAPTAC, Victoria, Canada, pp 351-354.

6 Renou, S., Perrier, M., Dochain, D., Gendron, S., (2000), 'Partial Differential Equation Model Based Control: Application to a Bleaching Reactor', *CPC6 preprints*, Tucson (AZ), USA, pp 471-475.

7 Renou, S., Perrier, M., Dochain, D., Gendron, S., (2001), 'Solution of Convection-Dispersion-Reaction Equation by a Sequencing Method', *Submitted to Comp & Chem. Eng.*