



Research paper

Finite-volume coupled surface-subsurface flow modelling in earth dikes

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ABSTRACT

Earthen embankments are subjected to increasing threats because of climate change inducing sequences of severe drought periods followed by floods, possibly leading to overtopping of the structures. Consequently, the water saturation of the dike can vary significantly both in space and time, and the resulting groundwater flow can affect the free-surface flow in case of overtopping. Conversely, the free-surface flow can modify the pore water content, which controls erosion and slope instabilities. In this paper, a combined approach to such situations is presented, in which the degree of saturation and the flow through the embankment are simulated by solving the two-dimensional Richards equation on an unstructured mesh with an implicit finite volume scheme that is coupled to the system of shallow-water equations solved in one dimension using an explicit finite-volume scheme. The coupled model is validated on several situations of flows through and over earthen embankments with different constitutive materials.

Keywords: Richards equation; shallow-water equations; overtopping flows; finite volume; embankment; numerical simulation

1 Introduction

In the current context of climate change and ageing infrastructures, breaching of earthen embankments becomes a key issue. Dikes have an essential protective role in flood defence, coastal protection or for the storage of waste from the mining industry. Sea-level rise, increased extreme precipitations and drought events (IPCC, 2021) have put additional stress on water control structures such as dikes. Failure of these structures may present a high risk for stakeholders and neighbouring communities, and devastating consequences such as human life losses, material or environmental damages. Recent examples include the environmental disaster in Brumadinho (Brazil, 2019) where a tailings dam failed, causing 259 casualties, the levee failures in New Orleans (2005) leaving the city completely flooded for several days or the recent intense rainfall of July 2021 causing dike failures in the Netherlands and Belgium (Meerssen in South Limburg, Herk-de-Stad in Limburg). Analyses of previous events have shown that overtopping is the main cause

of failure (Foster et al., 2000). This phenomenon is complex, combining fast erosive flow over the dike with seepage through the dike. Flow through the dike can lead to internal erosion, but also modifies the water saturation level inside the embankment, affecting the breaching initiation and the pace towards complete failure when combined to overtopping. This internal flow is highly dependent on the initial saturation of the dike material, whose effects on the failure have been largely overlooked so far. However, at full saturation, cohesion of the dike material is greatly reduced, and thus the dike strength. To avoid such situations, drains are often placed in the dike at its downstream part to control the saturation profile by lowering the level of the water table in the dike and therefore increasing the time required for the full saturation. Therefore, a better understanding of the combination of the initial dike saturation with the overtopping flow and the internal flow will fill a knowledge gap with the ambition to provide a physically based model that can be used to reduce the risks associated with these phenomena.

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Many numerical models have been developed to design or assess the risk of failure of existing embankments when they are submitted to overtopping or internal flows. Most of these models allow to consider only the overtopping flow, by solving the shallow water equations (Shao et al., 2006; Shiach et al., 2004; Van Emelen et al., 2016), or the internal flow inside the pores of the embankment by solving the Richards equation (Chávez-Negrete et al., 2018 Tsai et al., 2008). For this latter equation, in most cases, the numerical resolution is performed either by finite elements, finite volumes or finite differences, implying different types of spatial discretization (Farthing & Ogden, 2017), using generally structured meshes. From the temporal point of view, most models use an implicit time discretization (Farthing & Ogden, 2017), because of the inability of the explicit formulations to solve the problem when the soil is saturated (Caviedes-Voullième et al., 2013).

Coupled models considering the interaction between subsurface and surface water flows exist in the literature but they are mostly used to understand the processes of irrigation or watershed replenishment (Furman, 2008; Weill et al., 2009). Moreover, these models tend to focus on variables such as water content and infiltration, rather than pore pressure/suction which is of interest in case of embankment stability issues. Therefore, coupled models are still not widespread in the engineering community for solving practical field cases, and decoupled models are often preferred where the focus is set on one of the models, i.e. overtopping flow or infiltration, while the second one is solved with empirical parameters or functions (Furman, 2008). Indeed, solving the coupled problem accurately in a comprehensive way poses several challenges, including changing boundary conditions as the soil can be submerged or not and these conditions may change over time, resulting in possible mass conservation errors linked to the numerical schemes used.

Different numerical methods have been proposed to address these challenges and simulate hydrostatic surface-subsurface flows. Casulli (2017) presented a method that uses a vertically integrated continuity equation, which is based on the principles of the Richards and Navier-Stokes equations on an orthogonal unstructured grid. An algorithm based on the Darcy law was used to solve the groundwater flow in the saturated case considering purely hydrostatic flow. Similarly, De Maet and Hanert (2011) proposed an explicit solver for the Richards equation in the unsaturated zone, but uses a method called the “false transient method” to handle the case when soil becomes saturated because of the explicit solver being unable to handle the saturated case. Additionally, Caviedes-Voullième et al. (2012) presented an algorithm that employs a non-iterative external coupling algorithm, using the 2D shallow-water equation for surface flows and the 3D Richards equation for subsurface flows on structured meshes. Finally, Mizutani et al. (2013) proposed a coupled solver that was used to study the influence of soil saturation on a dike’s overtopping flow using a bi-dimensional structured mesh.

The aim of the present article is therefore to present a coupled finite-volume numerical model of both flows over and through earthen dikes using bi-dimensional unstructured meshes for groundwater flow and ensuring mass conservation, featuring a robust coupling between the two types of flows. By using unstructured meshes, the model provides a flexible and adaptable framework for analysing complex geometries of earthen dikes.

The model’s emphasis on pore pressure analysis is of particular significance in the context of dikes. Pore pressure, which refers to the pressure exerted by water within the void spaces of soil, plays a critical role in the stability and integrity of earthen dikes. By accurately capturing and analysing pore pressure distribution, the model provides valuable insights into potential weak zones, seepage patterns, and areas prone to failure. This information is essential for designing and maintaining robust dikes that can withstand hydraulic pressures and mitigate the risk of catastrophic failures when subject to extreme weather events as expected to happen in the coming future.

The study is essentially divided into three parts: (i) the development and validation of an implicit Richards equation solver using finite volumes on unstructured meshes for simulating the movement of water in unsaturated and saturated soils, (ii) the coupling of both models and a first validation on academic cases, and (iii) the application of the coupled model to dikes (considering a 2D cross-section) to highlight possible outcomes obtained with the presented scheme.

2 Finite-volume resolution of the Richards equations

2.1 Richards equation and model closure

The Richards equation (Richards, 1931) is a nonlinear partial differential equation describing the transfer of water in unsaturated soils under non-permanent conditions. It is based on the combination of the continuity equation and the Darcy equation.

Let Ω be the bidimensional domain, $\partial\Omega$ its boundary and $\mathbf{n}$ the normal vector at the boundary pointing outwards. The Richards’s equation can be written, in its mixed form, as:

$$\frac{\partial \theta(h)}{\partial t} - \nabla[K(h) \nabla(h+z)] = 0 \quad (1)$$

where θ is the volumetric water content, $h(\mathbf{x}, t)$ the pressure head and K the hydraulic conductivity. One of the main difficulties of the Richards equation lies in its nonlinearity caused by the relationships between the volumetric water content, the pressure head and the hydraulic conductivity. Multiple models exist in order to describe the relations between those variables, but the most frequent one is the Mualem-van Genuchten model (1980), used in this work. The latter was developed in order to facilitate the representation of the so-called soil–water retention curve (SWRC) that links volumetric water content and pressure head.

The development of this model follows the difficulty of determining the hydraulic conductivity in the case of unsaturated soils. While the experimental determination of the hydraulic conductivity is relatively simple in the case of a saturated soil, the hydraulic conductivity in an unsaturated soil is more difficult to determine because of its dependency on many factors. Moreover, its extensive measurement is expensive and time consuming as it usually requires the use of different methodologies to determine its evolution for various ranges of pressure.

The Mualem-van Genuchten model used in this work describes the transitioning between the unsaturated and the saturated state of the soil with the following relationships:

$$\theta = \begin{cases} \frac{\theta_s - \theta_r}{(1 + (\alpha|h|)^\eta)^\mu} + \theta_r & \text{if } h \leq 0 \\ \theta_s & \text{if } h > 0 \end{cases} \quad (2)$$

$$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r} \quad (3)$$

$$K = \begin{cases} K_s S_e^L [1 - (1 - S_e^{1/\mu})^\mu]^2 & \text{if } h \leq 0 \\ K_s & \text{if } h > 0 \end{cases} \quad (4)$$

$$C = \begin{cases} -\mu \eta \alpha^\eta \frac{\theta_s - \theta_r}{(1 + \alpha^\eta |h|^\eta)^{\mu+1}} |h|^\eta h^{-1} & \text{if } h \leq 0 \\ 0 & \text{if } h > 0 \end{cases} \quad (5)$$

$$\mu = 1 - \frac{1}{\eta} \quad (6)$$

where θ_s is the saturated volumetric water content, θ_r the residual volumetric water content, S_e the equivalent volumetric water content, K_s the saturated hydraulic conductivity and C the hydraulic capacity which represents the volumetric water content variation due to a head pressure variation ($\partial\theta/\partial h$).

The other parameters in Eqs (2–6) need to be calibrated using laboratory measurements.

Still, some of those have a physical meaning and can be obtained using closure relations. For example, η is linked to the pore pressure distribution and describes the slope of the curve (Mualem, 1976). Some authors (e.g. Caviedes-Voullième et al., 2013; Soltani et al., 2021) suggest that the α parameter is related to the air-entry pressure h_e . This pressure is defined (Brooks & Corey, 1964) as the pressure state where the entry of air (and thus the evacuation of water) into the largest pore becomes possible during the desaturation process of a saturated soil (Fig. 1). This pressure can be found graphically by crossing the horizontal asymptote at $\theta = \theta_s$ with the tangent to the SWRC (Benson et al., 2014).

Soltani et al. (2021) highlight the fact that an incorrect evaluation of this parameter leads to a very poor evaluation of the water content, which is directly related to the relative permeability of the soil but also to its weight and shear strength.

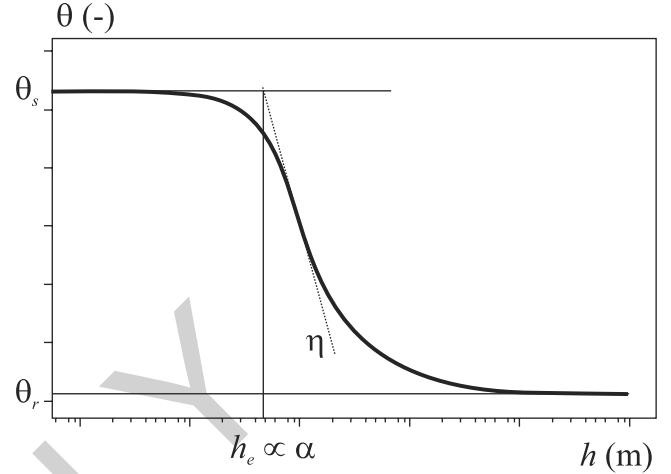


Figure 1 Schematic representation of the soil–water retention curve

2.2 Unstructured finite volume resolution

An implicit Eulerian solver is employed in this work. The primary reason behind employing implicit solvers is that the Richards equation, as explained in Caviedes-Voullième et al. (2013), cannot be effectively solved using an explicit scheme in the saturated zone. The development of a solver with an explicit scheme necessitates the value of the pore pressure to be estimated knowing the volumetric water content using the Mualem-van Genuchten relationship. However, it becomes inherently impossible to differentiate between two pore pressures solely based on the information that the soil is saturated. By using an implicit scheme, it becomes possible to directly express the scheme in terms of pore pressure. This allows differentiation between two pore pressures for a saturated soil.

Therefore, a system of equations is solved until convergence for each time step:

$$\mathbf{A}_{ij} \mathbf{h}_j^{n+1,m+1} = \mathbf{c}_i \quad (7)$$

where $\mathbf{A}$ is a square matrix of n by n elements containing terms calculated with the spatial and temporal integrations at time $n + 1$, but at iteration step m ; $\mathbf{h}$ is the vector of unknowns pressure heads and $\mathbf{c}$ the vector of external source terms. The terms detailed in Eqs (8–10) are obtained by applying the divergence theorem, consisting of applying the local mass conservation property, to Eq. (1). Mathematical approach and discussion of such schemes can be found in Bertolazzi and Manzini (2007);

$$\mathbf{A}_{i,i} = \frac{\mathbf{c}_i^{n+1,m}}{\Delta t} \Omega_i + \sum_k K_{i,k} L_{i,k} d_{i,k}^{-1} \quad (8)$$

$$\mathbf{A}_{i,j} = -K_{i,j} L_{i,j} d_{i,j}^{-1} \quad (9)$$

$$\mathbf{c}_i = \sum_k K_{i,k} L_{i,k} (n_z)_{i,k} + \frac{\Omega_i}{\Delta t} (\theta_i^{n,m} - \theta_i^{n+1,m} + C_i^{n+1,m} h_i^{n+1,m}) \quad (10)$$

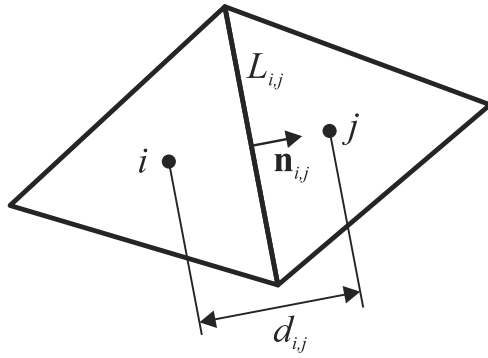


Figure 2 Computational cells for the Richards equation, definition of variables

Other terms than those presented previously include: Δt the time step, Ω_i the area of the computational cell, K_{ij} the hydraulic conductivity at the interface between two cells, L_{ij} the length of the edge and d_{ij} the distance between the centres of the two adjacent cells, $(n_z)_{ij}$ the outgoing z -axis pointing normal to the interface between two computational cells (Fig. 2).

2.3 Validation

Two classical benchmarks are used for the validation of the Richards equation solver, consisting in flows through a column of unsaturated soil: first the drainage of a soil column, and then the propagation of a sharp infiltration front.

The soil characteristics used for both tests are presented in Table 1. These soil data were presented first in a work by Celia et al. (1990) and the soil samples were collected in New Mexico. Since then, these soil data have been used in several studies to validate various Richard's solvers (Caviedes-Voullième et al., 2013; Kavetski et al., 2001; Su et al., 2022).

Drainage of soil column

Caviedes-Voullième et al. (2013) proposed an idealized academic test case for their one-dimensional solver. It consists of a nearly saturated 1 m high soil column with the parameters presented in Table 1. The initial pore pressure is $h = -20$ cm, meaning that the initial volumetric water content is $\theta = 0.323$. As the solver is bi-dimensional, the calculation was made on a 1 m depth and 0.5 m width plane. The average size for the mesh's triangles was 1 cm length edges, the integration time step is set at 10 s, while a constant numerical threshold of 0.001 cm is employed for convergence.

Table 1 Soil parameters for the test cases

Parameter	Value	Unit	Parameter	Value	Units
K_s	0.00922	cm s^{-1}	α	0.0335	cm^{-1}
θ_r	0.102	—	$\hat{\eta}$	2	—
θ_s	0.368	—	$\hat{L}$	0.5	—

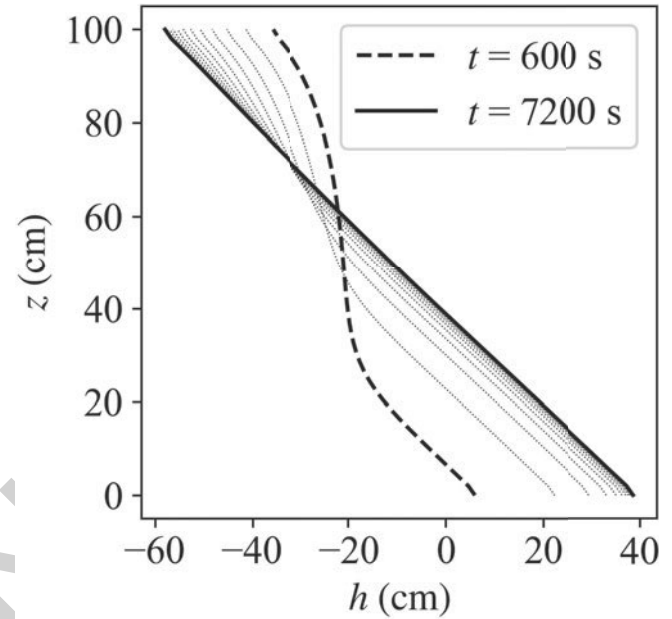


Figure 3 Results of test case 1: drainage of a soil column

The objective is to validate the solver by checking the mass-conservation as well as the ability of the solver to converge to a hydrostatic equilibrium. The waterflow is simply driven by gravity and the boundaries are waterproof, meaning that no water can flow out of the system.

The present solver provides convincing results as shown in Fig. 3, no water enters or flows out of the system and a perfect hydrostatic equilibrium is obtained.

A sharp infiltration front

In this test case, a sharp infiltration front is generated in a soil column by both the initial and the boundary conditions. The initial conditions are described by the following function, where values are expressed in cm:

$$h(x, y, z, t = 0) = \begin{cases} -1000 & \text{if } z > 0.6 \\ -75 - \frac{925}{0.6} & \text{if } 0 \leq z \leq 0.6 \end{cases} \quad (11)$$

where the z axis is pointing towards the bottom of the column. At the top of the column ($z = 0$ cm), $h = -75$ cm, as at the bottom of the column ($z = 60$ cm) a constant pressure of $h = 1000$ cm is imposed. These results have been used and compared as a benchmark in several articles (Horgue et al., 2015; Kavetski et al., 2001; Su et al., 2022). Results obtained by Horgue et al. (2015) are used as a reference solution and plotted in dashed lines to compare with the proposed solver. The results obtained with the present method match satisfactorily the reference ones (Fig. 4). In this case, the calculation was made on a 0.6 m depth and 0.25 m width plane. To accurately capture the gradient, triangular elements with edges measuring 0.2 cm in length were used at the top of the soil column, while triangular elements with edges measuring 1 cm in length were employed

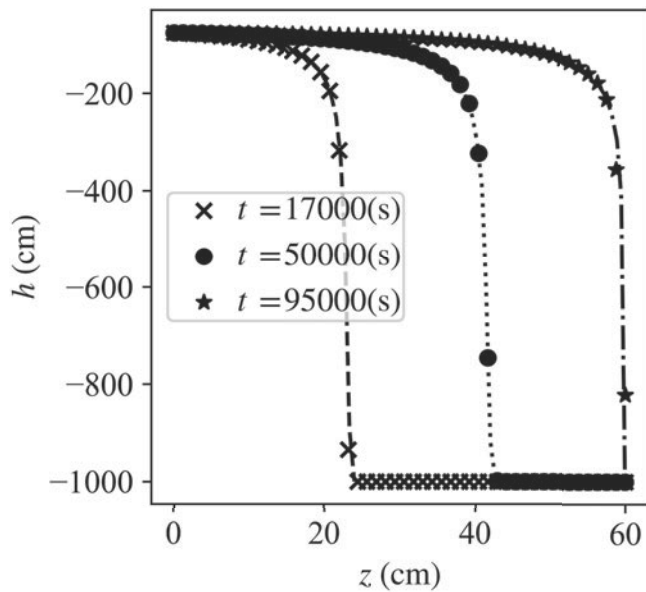


Figure 4 Results of test case 2: sharp infiltration front. Dashed lines represent

at the bottom. The integration time step was set at 60 s, and a constant numerical threshold of 0.001 cm was applied to ensure convergence.

3 Coupling between Richards and shallow-water equations

3.1 Spatial and temporal coupling

The shallow water equations are used to describe the flow over the dike. Considering a unit-width system, the equations can be written as:

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = q_{in} \quad (12a)$$

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x} \left(\frac{q^2}{h} + g \frac{h^2}{2} \right) = g h (S_0 - S_f) \quad (12b)$$

where h is the water depth, $q = uh$ the unit discharge with u the depth-averaged velocity, S_0 the bed slope, and S_f the friction slope calculated using Manning's friction formulation. Eqs (12) are solved using an explicit finite volume scheme with the fluxes at the interfaces evaluated using Roe's scheme (Soares Frazao & Zech, 2002).

Spatial coupling between the Richards and shallow-water equations is achieved at the interface between the two systems, i.e., the embankment surface, using appropriate boundary conditions. Although on site the interface between the subsurface environment and the surface flow domain is not so well defined (silt, sediments, etc.), it is generally considered that this interface is sharp to facilitate the mathematical approach (Furman, 2008). In the present model, as illustrated in Fig. 5, the subsurface and the surface cells are perfectly aligned. In

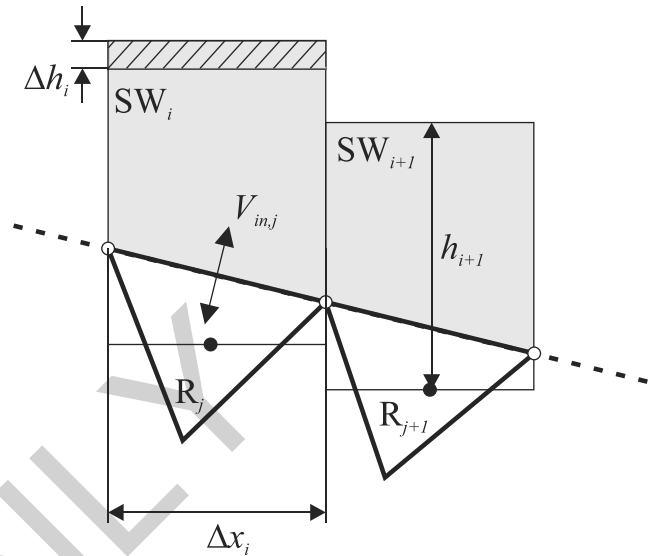


Figure 5 Schematic representation of the mass exchanged between the shallow-water equation cells (SW) and the Richards equation cells (R)

this way no interpolation or remapping is required to obtain the boundary information, which avoids error propagation. The algorithm consists in using the water depth of the surface flow as the boundary condition for the Richards' equation and using the calculated internal flows as source terms q_{in} in the mass conservation Eq. (12a).

From a temporal point of view, the resolution method is staggered in the sense that the resolution of the shallow-water equations requires small time steps due to the explicit nature of the resolution scheme, whereas the implicit resolution of the Richards' equation allows the use of large time steps without losing too much precision. Let us define t_1 as the most recent time at which the Richards' equation was solved, $\Delta t_{Richards}$, the Richards' equation time-step and Δt_{SW} the shallow-water's equations time-step. The global simulation time, denoted as t , is updated by adding Δt_{SW} until it reaches $t = t_1 + \Delta t_{Richards}$. At this juncture the Richards' equation is solved, the source terms are evaluated, and the same process resumes until the final simulation time is reached. Therefore, a careful selection of the Richards' equation time step must be done. Opting for an excessively large time step may lead to inaccurate evaluation of information corresponding to surface flow and source terms. Conversely, selecting an overly small time step significantly extends the computational time required. Generally speaking, the selection of the right time step is highly problem related. In cases involving soils with high hydraulic conductivities, it is advisable to use smaller time steps due to the rapid occurrence of significant changes. Conversely, situations involving soils with low conductivities can be solved with larger time steps, transfers of water take place at a slower pace. In addition, since the flow through the soil is much slower than the free-surface flow, no information is lost with this decoupling.

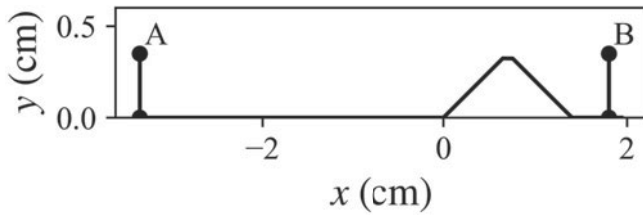


Figure 6 Schematic view of the experimental set-up

3.2 Source terms

Considering an unit-width dike as illustrated in Fig. 6, the mass-balance is formally established as follows for the source term representing the volume of water infiltrating in the dike $V_{in,i}$:

$$V_{in,i} = \sum_j v_{ij} L_{ij} \Delta t + (\theta_i^{n+1} - \theta_i^n) \Omega_i \quad (13)$$

where v_i denotes the flow velocity between the boundary cell and its adjacent cells evaluated using Darcy's law and L_i is the length of the corresponding interface. The second part of Eq. (13) takes into account the part of water absorbed by the soil of the cell itself. This volume tends very quickly towards 0 because the boundary cell is saturating almost instantaneously.

Using the definition (13) of the infiltrated volume, let us define the equivalent infiltrated height extracted from the cell of the shallow water domain:

$$\Delta h_{in,i} = \frac{V_{in,i}}{\Delta x_i} = q_{in,i} \Delta t \quad (14)$$

This water height must be removed from the cell at the end of each Richards equation time step and is used to calculate the source term q_{in} of Eq. (12a).

4 Model validation

4.1 Flow through an embankment

Experiments by Ebrahimi et al. (2007) in which they collected an extensive set of data for the study of seepage flows are used to validate the proposed model. The test consists of applying a tidal wave on a sandy embankment. On both sides of the embankment an initial 274 mm water level is initially set. The sand is non-cohesive and has a mean diameter of 1 mm and a saturated hydraulic conductivity of 1 cm s^{-1} . No other soil parameters are provided by Ebrahimi et al. (2007); therefore, the general features of the test case are reproduced but the results cannot be straightforwardly compared.

A cyclic tidal wave was applied on one side of the embankment and the objective is to observe its influence on the other side. The tidal wave's equation is the following:

$$z_w(x=0) = 0.214 + 0.06 \cos\left(\frac{2\pi t}{355}\right) \quad (15)$$

The mean water level of the tidal wave is 0.214 m and its amplitude is 0.06 m. A 355 s period is imposed. The water level variation at up and downstream points of the embankment were monitored during the whole process. More technical details about the design of this experimental test case are extensively explained in Ebrahimi et al. (2007).

The computed results are illustrated in Fig. 7. Triangular elements with edges measuring 1 cm in length were used to describe the geometry of the dike and information between the two systems was exchanged every 5 s, corresponding to the Richards' equation time step. Again, a 0.001 cm numerical threshold was used to ensure the convergence of the Richards' equation solver. Elements describing the shallow-water domain have a width of 1 cm, except for the ones located on the dike.

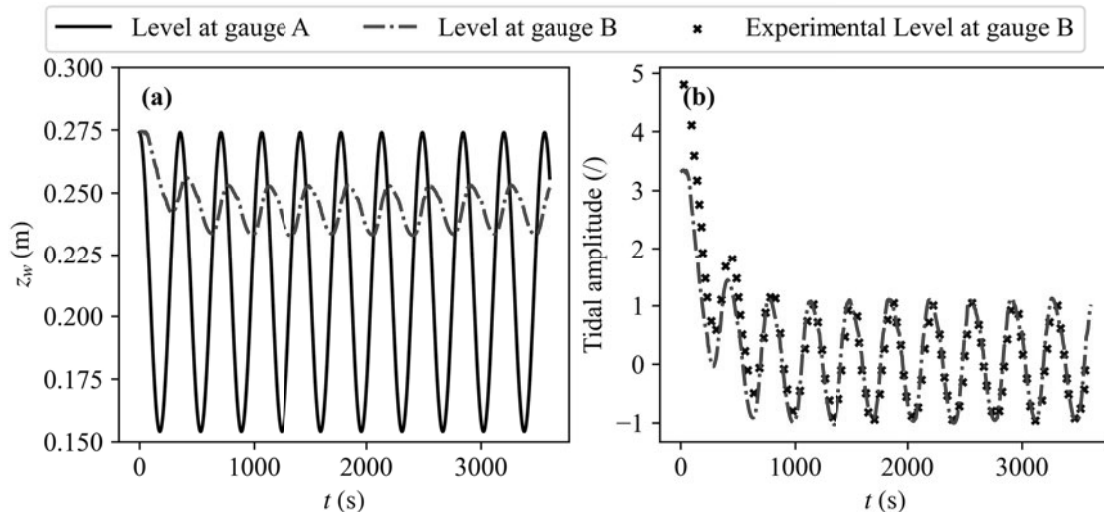


Figure 7 Left: numerical evolution of the water level at both sides of the dike. Right: comparison with adimensionalized experimental results

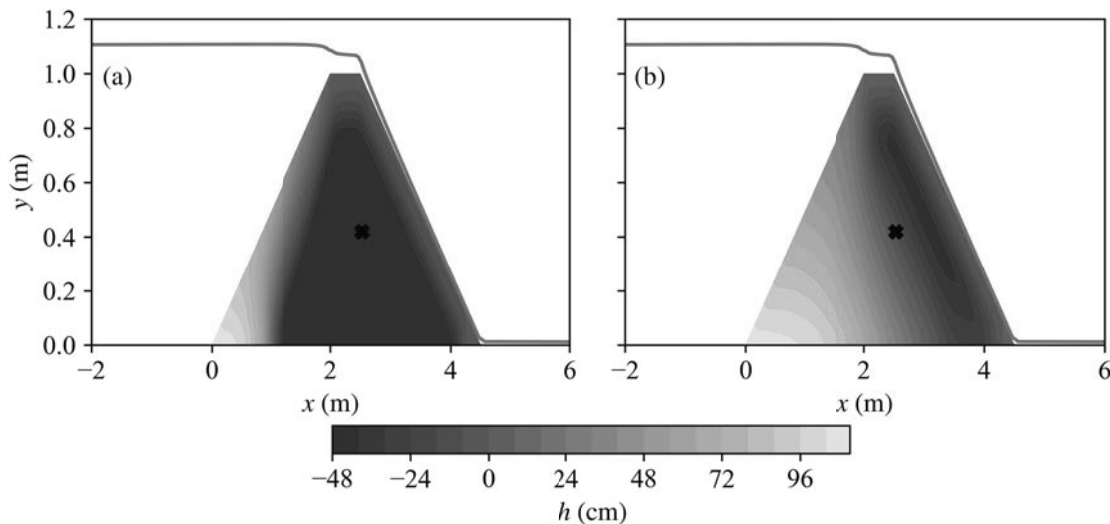


Figure 8 (a) Unsaturated dike overtopping, (b) saturated dike overtopping

On the dike, the elements are precisely aligned with the boundary cells of the Richards' domain, ensuring perfect alignment.

It can be observed that after two oscillations, a steady state takes place at the downstream side of the dike (gauge B, $x = 1.8$ m). The solver successfully replicates the system's steady state characteristics, which include a 90° phase lag between the oscillations of the two water surfaces and a wave amplitude of 0.02 m. These results are illustrated in Fig. 7, showing the amplitude normalized with the steady-state value. The good agreement in both amplitude and frequency suggests that the saturated hydraulic conductivity, which was the only soil parameter provided by Ebrahimi et al. (2007), predominantly governs the system's steady state behaviour.

4.2 Flow through and over embankment: influence of initial water saturation level

The objective of the current test case is to highlight the influence of the initial saturation on the internal pore pressure evolution when the dike is submitted to an overtopping flow.

The dike is 4.5 m wide and 1 m high and its soil has the same parameters as those presented in Table 1, excepted for the hydraulic conductivity set to $K_s = 0.05 \text{ cm s}^{-1}$ and a pore pressure initialized at $h = -40$ cm. Two cases are presented in which the initial water level at the upstream side is 0.75 m. In case (a) a $0.1 \text{ m}^3 \text{ s}^{-1}$ flow is directly released on the dike while in case (b) the same flow is released only after 500 s. As a result, it can be observed that during the first 500 s a saturation profile is developed from the upstream part to the downstream seepage face. Due to this profile, the steady state is obtained much faster as the unsaturated soil zone is reduced. In Fig. 8 we can observe the state of pore pressure in the soil 20 s after the release of the flow for case (a) and case (b). The initial water saturation of case (b) enables the water of the overtopping to penetrate much faster in the soil.

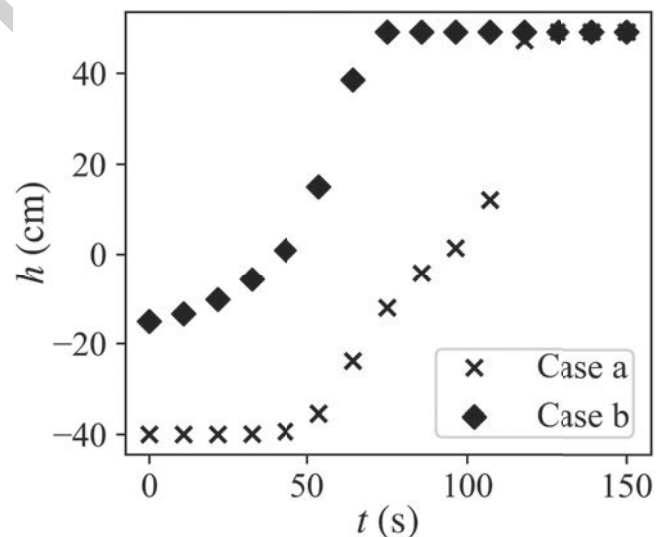


Figure 9 Comparison of the pore pressure evolution from the start of the overtopping

An observation gauge, represented with a black cross inside the dike in Fig. 8 ($x = 2.53$ m, $y = 0.42$ m) is used to represent the time evolution of the pore pressure, as illustrated in Fig. 9.

The comparison of the pore pressure values at the gauge emphasizes the importance of the initial soil saturation on the full saturation rate. Indeed, we can observe that the time needed to reach the steady state is two times faster in case of an initially almost saturated dike (Fig. 8).

As the pore pressure is a key factor for the strength of the dike, the time needed to reach full saturation also appears as a key factor.

4.3 Flow through and over an embankment

In this section, another test case is presented which examines the impact of different saturated hydraulic conductivities, without changing the other soil parameters, on the groundwater flow and

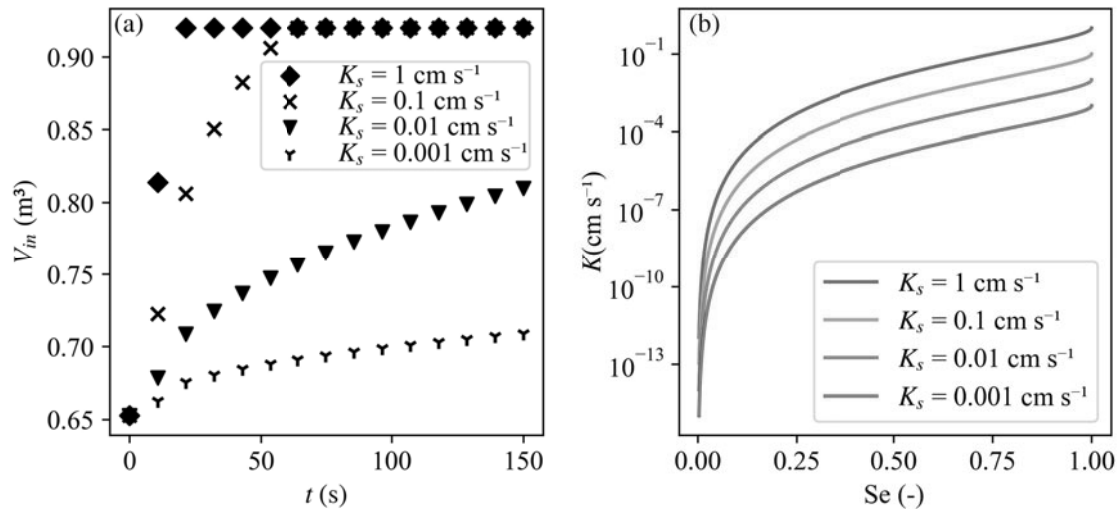


Figure 10 (a) Comparison of infiltrated water volume (b) comparison of the unsaturated hydraulic conductivities

internal pore pressure of an embankment dike when subjected to overtopping flow. In Fig. 10, the influence of the saturated hydraulic conductivity on the unsaturated hydraulic conductivity can clearly be identified. As the saturated conductivity is high ($K_s = 1 \text{ cm s}^{-1}$), the soil is likely to be formed of gravel. Values such as $K_s = 0.1 \text{ cm s}^{-1}$ or $K_s = 0.01 \text{ cm s}^{-1}$ can often be associated with sand and lower values ($K_s = 0.001 \text{ cm s}^{-1}$) represent silt.

The test case is conducted on the same geometry as the previous test case (illustrated in Fig. 8) and the same overtopping flow is released. The results, as shown in Fig. 10, clearly demonstrate the significant influence of hydraulic conductivity on the infiltration rate and internal pore pressure of the dike. The steady state is reached when all available volume in the pores of the soil is filled with water.

Given that the dike is fully submerged under water, there is no means for water to escape, making the infiltration rate and internal pore pressure particularly crucial for the stability of the structure. A higher hydraulic conductivity results in faster infiltration and a more significant impact on the internal pore pressure in the short-term. These findings are of great importance for the design and construction of dikes, particularly for clay core and clay surface dikes, as the hydraulic conductivity of the soil must be closely considered to ensure the stability and safety of the structure.

5 Conclusion

In this article a coupled solver for the numerical calculation of sub-surface and the surface flows on unstructured meshes was presented.

At first, we developed and validated a finite volume scheme for solving the Richards equation which appeared to be efficient and precise. The latter was validated against classical benchmarks, such as the drainage of a soil column or a sharp infiltration front. The simplicity of the formulation but also its

ability to solve the equation in the saturated as well as in the unsaturated zone renders the solver very interesting for further applications.

The subsurface flow was linked to the surface flow via a source term which allows the mass conservation to be respected. The resulting solver appears to be effective and reveals how the interaction between the surface and subsurface flows and the soil characteristics could affect the internal flow and the evolution of the degree of saturation of the dike, which is a key parameter in dike stability assessment. We emphasized the importance of the initial saturation level but also of the saturated hydraulic conductivity on the time needed to reach the steady state.

The solver demonstrated its ability to deliver good outcomes across a diverse set of test cases, encompassing complex combinations of flows. Its ability to simultaneously analyse multiple types of soils, focus on pore pressure, and ensure mass conservation through a robust coupling of flows makes it an asset for future numerical developments. Future work will be dedicated to the validation of the solver by comparison with experimental data.

Notation

θ	= volumetric water content (-)
θ_s	= saturated volumetric water content (-)
θ_r	= residual volumetric water content (-)
S_e	= equivalent volumetric water content (-)
K	= hydraulic conductivity (cm s ⁻¹)
K_s	= saturated hydraulic conductivity (cm s ⁻¹)
α	= parameter linked to the air-entry pressure (cm ⁻¹)
η	= parameter linked to the distribution of the pores in the soil (Mualem-van Genuchten model)
L	= parameter of the Mualem-van Genuchten model (-)

881	μ	= parameter of the Mualem-van Genuchten model (–)
	h	= pressure (cm)
	h_e	= pore entry pressure (cm)
	Δt	= time step (s)
886	$\Delta t_{Richards}$	= time step of the Richards' equation solver
	Δt_{SW}	= time step of the shallow water equations solver
	Ω_i	= cell area in the Richards domain's (m ²)
	$L_{i,j}$	= length of the specific edge between i and j (cm)
	n_z	= outgoing z-axis pointing normal (–)
891	h	= water depth (m)
	q	= unit discharge (m ² s ^{–1})
	g	= gravity (m s ^{–2})
	S_0	= bed slope (–)
896	S_f	= friction slope (–)

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