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**Bearing capacity of shallow and deep foundations
on rock mass: Analytical and Experimental
investigations**

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Abstract

It is well known that, except for potential installation problems, most rock masses are excellent and trouble-free foundation materials. A significant economy can be therefore realized in design if the foundation can be constructed on or in the surface or near-surface rock. Nevertheless, the construction of challenging structures with usually high loads and special requirements on rock foundations, have made it necessary to determine the bearing capacity of rock media accurately. It is within this framework that the present research was carried out. This dissertation provided analytical solutions for bearing capacity problems of both shallow and deep foundations resting on or in a rock mass that obeys the Hoek and Brown failure criterion.

Considering the strip footing problem, improvements were brought by incorporating a non-associated flow rule in the plane strain upper bound limit analysis technique and analysing its effects on failure mechanisms and bearing capacity values. By assuming a rigid strip footing resting on a rock mass obeying the generalized Hoek and Brown failure criterion, the upper bound estimate for the bearing capacity was derived by preserving exactly the original nonlinear criterion. The multi tangential method was used to handle non-linear strength criterion issues. Furthermore, a sufficiently significant but conveniently simple formulation of a non-constant dilatancy angle model was derived. Closed-form expressions of the maximum resistance work rate were developed in any virtual velocity field. For practical purposes in rock engineering, tables and charts allowing easy and accurate estimates of the bearing capacity factor N_{σ_0} were provided. The effect of the flow rule on N_{σ_0} was carefully examined using different dilatancy parameters. To investigate the efficiency of the proposed solution, a series of static penetration tests of small-scale model footings (strip and circular) laying on a highly porous calcarenite rock mass was conducted. A reduction factor F_r defined as the ratio of the measured acceptable bearing capacity to ultimate theoretical one was introduced. This factor reflects by how much should the theoretical bearing factor N_{σ} reduced to get an acceptable value of bearing capacity. It was suggested to estimate F_r values as a function of the rigidity index and the friction angle of the rock mass.

As for the bearing capacity of the pile foundation, an analytical calculation procedure was proposed to predict the shaft resistance for piles drilled to some depth into a rock layer. The developed solution requires a numerical integration over the embedment depth. Thus, for practical purposes, design charts, as well as an approximate solution, were provided allowing easy and accurate estimates of the average shear resistance. Results obtained from developed solutions were in excellent agreement when compared to published analytical results and very close to finite element analysis

results. Moreover, this analysis procedure offered reasonable results when compared to field test measurements.

Furthermore, and always within the framework of pile foundation, an analytical solution based on the cavity expansion theory was proposed for computing the bearing capacity at the tip of a pile embedded in rock mass obeying the H-B failure criterion. The pile end bearing capacity was evaluated by assuming that the pressure exerted at the boundaries of a pressure bulb immediately beneath the pile tip is equal to the limit pressure required to expand a spherical cavity. Besides, a relationship was derived to predict the pile load-settlement response. To demonstrate the applicability of the presented solution, the results of the present work were compared to those of 91 field tests from technical literature. Despite some limitations, it was found that the computed end bearing resistance was in good agreement with the measured field test results.

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List of Abbreviations and Symbols

Unless stated otherwise, abbreviations and symbols employed in the manuscript are defined as follows:

Abbreviations

AFR	Associated Flow Rule
BTS	Brazilian Tensile Strength
CPT	Cone Penetration Test
DI	Displacement Index
EPB	Elastic-Plastic Boundary
FEM	Finite Element Modelling
GSI	Geological Strength Index
H-B	Hoek and Brown
Jcond89	Joint condition rating
LST	Large Strain Theory
MR	Modulus Ratio
N-AFR	Non-Associated Flow Rule
NCR	Non-Compliant Region
PMT	Pressuremeter Test
RF	Roughness Factor
RMR	Rock Mass Rating
RQD	Rock Quality Designation
SD	Standard Deviation
SG	Strain Gauge
SR	Spacing Ratio
SRC	Shaft Resistance Coefficient
SST	Small Strain Theory
UCS	Uniaxial Compressive Strength
ULS	Ultimate Limit State
UTS	Uniaxial Tensile Strength

Symbols (English)

		<i>Unit</i>
a	Exponent of the H–B criterion expression	[–]
B	Foundation width	[m]
c	Cohesion	[Pa]
C_1, C_2, C_3	Integration constant	[–]
D	Disturbance factor	[–]
D_f	Embedment Depth	[m]
E_i	Deformation modulus of the intact rock	[Pa]
E_{rm}	Deformation modulus of the rock mass	[Pa]

E_{50}^t	Young's modulus obtained from the stress-strain curve	[Pa]
$f_1, f_2, \dots, f_5$	Normalized bearing capacity function	[-]
F_c	Compressibility factor related to cohesion	[-]
F_q	Compressibility factor related to surcharge load	[-]
F_γ	Compressibility factor related to unit weight	[-]
F_R	Reduction factor	[-]
ϕ	Plastic potential function	[-]
G	Shear modulus	[Pa]
I_r	Rigidity index	[-]
k_0	Ratio of horizontal to vertical stress	[-]
k_{eq}	Equivalent stiffness	[N/m]
k_R	Rock foundation stiffness	[N/m]
k_F	Model footing stiffness	[N/m]
k_∞	Vertical stiffness of circular foundation on half-space	[N/m]
k_ψ	Dilatancy factor	[-]
k_ϕ	Friction factor	[-]
$k_{\psi,\phi}$	Friction and dilatancy factor	[-]
m_i	Strength parameter of the intact rock mass	[-]
m_b	Strength parameter of the rock mass	[-]
$m_{b,dil}$	Dilatancy parameter of the plastic potential function	[-]
m_b^{peak}	Peak strength parameter of the rock mass	[-]
m_b^{res}	Residual strength parameter of the rock mass	[-]
N_c	Bearing capacity factor related to cohesion	[-]
N_q	Bearing capacity factor related to surcharge load	[-]
N_γ	Bearing capacity factor related to unit weight	[-]
N_{σ_0}	Bearing capacity factor of weightless rock mass without surcharge load	[-]
N_σ	Bearing capacity factor of self-weight rock mass with a surcharge load	[-]
n_q	Exponent for use in the empirical expression for pile base resistance embedded in a rock mass.	[-]
n_τ	Exponent for use in the empirical expression for unit shaft resistance of a pile embedded in a rock mass.	[-]
P_0	Far-field pressure	[Pa]
P_i	Internal cavity pressure	[Pa]
P_y	Yield pressure of an expanding cavity	[Pa]
P_{lim}	Limit pressure of an expanding cavity	[Pa]
q_0	Surcharge load	[Pa]
q_b	End bearing resistance	[Pa]
$q_{b,ult}$	Ultimate end bearing capacity	[Pa]
Q_b	Base load capacity of a pile	[N]
Q_s	Shaft load capacity of a pile	[N]
Q_{ult}	Total load capacity of a pile	[N]
r	Radial coordinate	[m]
r_e	External cavity radius	[m]
r_i	Internal cavity radius	[m]

r_p	Plastic cavity radius	[m]
R	Change of variable for r	[–]
R_i	Change of variable for r_i	[–]
R_p	Change of variable for r_p	[–]
s	H-B derived parameter	[–]
s^{peak}	Peak H-B derived parameter	[–]
s^{res}	Residual H-B derived parameter	[–]
s_c, s_q, s_γ	Shape factors	[–]
s_{m_i}	Joint spacing of the i^{th} discontinuity family	[–]
u_{EPB}	Radial displacement at the EPB	[m]
W_0	0 th branch of the Lambert W-function (Omega function)	[–]

Symbols (Greek)

		<i>Unit</i>
α	Dilation parameter of the plastic potential function	[–]
α_q	Empirical constant for use in the empirical expression for base resistance.	[–]
α_τ	Empirical constant for use in the empirical expression for unit shaft resistance.	[–]
β	Constant solely dependent on the exponent a	[–]
γ_s	Soil unit weight	[kN/m ³]
γ_R	Rock unit weight	[kN/m ³]
γ^p	Plastic shear strain	[–]
δ	Normalized displacement at the EPB	[–]
ε_{circ}	Circumferential strain	[–]
ε_{long}	Longitudinal strain	[–]
ε_r	Total radial strain	[–]
ε_r^e	Elastic radial strain	[–]
ε_r^p	Plastic radial strain	[–]
ε_θ	Total circumferential strain	[–]
ε_θ^e	Elastic circumferential strain	[–]
ε_θ^p	Plastic circumferential strain	[–]
ε_v^p	Volumetric plastic strain increment	[–]
ε_1^p	Axial plastic strain increment	[–]
ξ	Change of variable: Ratio between cavity radius and plastic radius	[–]
η	Constant issued from LST	[–]
λ_i	Frequency of the i^{th} discontinuity family	[–]
ν	Poisson's ratio	[–]
σ_1	Major principal stress	[Pa]
σ_2	Intermediate principal stress	[Pa]
σ_3	Minor principal stress	[Pa]
σ_{ci}	Uniaxial compressive stress of the intact rock	[Pa]
σ_c	Uniaxial compressive strength	[Pa]
σ_h	Horizontal stress	[Pa]

σ_n	Normal stress	[Pa]
σ_{peak}	Peak stress	[Pa]
σ_r	Radial stress	[Pa]
σ_t	Uniaxial tensile stress	[Pa]
σ_v	Vertical stress	[Pa]
σ_θ	Circumferential stress	[Pa]
τ	Shear stress	[Pa]
τ_{avg}	Average ultimate unit shear stress	[Pa]
ϕ	Friction angle	[°]
ψ	Dilatancy angle	[°]
ω	Dilatancy coefficient	[—]
Ω	Inverse of the normalized plastic radius	[—]

Chapter I. Introduction

1. Background

For a geotechnical engineer, the foundation design of structures generally requires knowledge of different factors such as superstructure loads, subsoil conditions, and settlement specifications. Based on these factors, the foundation type may be defined among one of two conventional categories:

- (1) Shallow foundation such as spread and wall footings, mat foundations, etc. transfer structural loads to “near-surface” soils. What is meant by near-surface is an embedment depth D_f that is equal to or less than three to four times the foundation width B (Braja M.Das, 2002), as illustrated in Figure I-1. Shallow foundations are typically used for small to medium-sized buildings when near-surface soil conditions are adequate. Due to their relatively low cost and ease of construction, shallow foundations are far more prevalent than the second foundation type described below.
- (2) Deep foundation such as piles, diaphragm walls, etc. are typically used when adverse near-surface soil conditions are encountered (i.e., soft clays, collapsible soils, and/or expansive soils) and structural load requirements are sufficiently large to warrant the additional cost of deep foundations, or when a high lateral load capacity is needed.

Furthermore, a significant economy can be achieved in design if the foundation can be constructed on or in the surface or near-surface rock. It is well known that most rock masses are excellent and trouble-free foundation materials. Nevertheless, the construction of challenging structures such as skyscrapers, large bridges, nuclear power plants, arch-dams, onshore and offshore wind turbines (cf. Figure I-2 – Figure I-3), etc. with usually high loads and special requirements on rock foundations, have made it necessary to determine the allowable bearing capacity of rock media accurately. Additionally, designers cannot blindly follow the old aphorism, “*when in doubt, put it on a rock*,” because natural rock masses are among the most variable of natural materials. Rock masses have intrinsic characteristics as a result of discontinuities, anisotropy, etc. Furthermore, rock mechanics experts have concentrated on these characteristics, creating a difference with soil mechanics approaches. That means that a simple theoretical treatment of the ultimate bearing capacity with a simple failure criterion would be unrealistic. It is mostly for this reason

that the design approach of foundations on or in a rock mass has been mostly empirical until recent years.

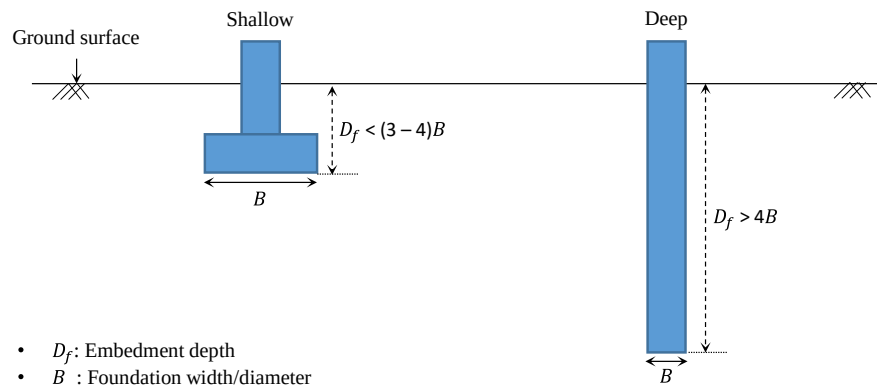


Figure I-1: Shallow and deep foundations domain in terms of foundation width/diameter and depth.



Figure I-2: The Mike O'Callaghan-Pat Tillman Memorial Bridge over the Hoover dam – Lake Mead (<http://wt.kimiq.com/water-level-lake-powell/>)



Figure I-3: Wind turbine farm founded in a rock formation – Norway River
(<https://windeurope.org/newsroom/news/alcoa-signs-another-ppa-deal-in-norway/>)

Fortunately, there have been many recent studies that have vastly improved our understanding of the mechanical behaviour of rock masses, paving the way towards more rational analytical treatments of rock-foundation systems, as described by Hoek et al (2002) among others. In such cases, rock mass could be regarded as a homogenous, isotropic, and continuous medium, as stated by some authors (Serrano and Olalla, 2002a; Girgin *et al.*, 2006; Saada, Maghous and Garnier, 2011; Keawsawasvong, Thongchom and Likitlersuang, 2020) among others. That will be legitimate when (1) the rock mass is so weak that its behaviour exerts a greater influence than its discontinuities or (2) when the rock mass is extremely fractured, both homogeneously and isotopically. A distinction should also be made between large and small foundations (Merifield, Lyamin and Sloan, 2006). Large foundations affect a considerable zone of the rock mass. In such a case, the stress effect upon the potential flow surfaces arising from gravity is comparable to the one arising from the uniaxial compressive strength. In such conditions, the rock behaves in a plastic manner and can be analysed using the theory of plasticity. That is not the case for small foundations since rock behaviour may be brittle. Indeed, only a small volume of rock is affected by small foundations, and that results in negligible gravitational stress compared to the rock strength.

Given the above, there are two modelling approaches in rock. In the first approach, the rock mass may be modelled as a discontinuum composed of individual blocks that interact with each other via stiffness and plasticity, introduced by the intervening fractures. The second approach is to model the rock mass as a continuum where classic plasticity theory combined with various solution techniques (upper and lower bound limit analysis, limit equilibrium method, etc.) could be applied. In other words, the former approach allows yielding to occur along with specific structures within the

rock mass (sliding of a wedge into an opening ...). The latter approach may allow complex non-linear material models but might not adequately represent discontinuum response such as slip and separation of blocks of material. Which failure modes are possible, and how the rock mass should be considered (continuum or discontinuum) depends on the scale of the problem i.e. discontinuities spacing compared to the overall size of the construction element studied, as described by Hoek and Brown (1980), Merifield et al. (2006) and Marinos (2019) among others. These effects are illustrated in Figure I-4.

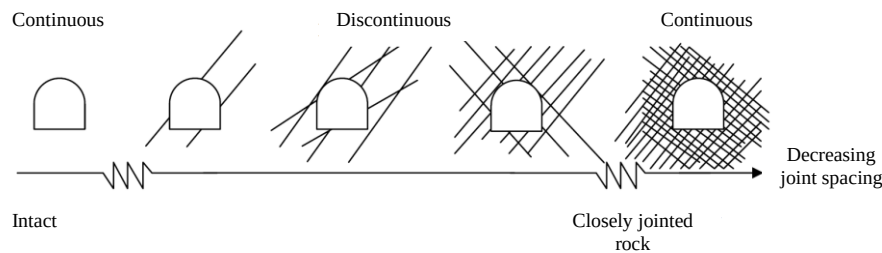


Figure I-4: Example of continuous and discontinuous rock medium – Tunnel size is kept constant, while the joint spacing is decreased (Edelbro, 2004).

Another feature of the rock mass is its nonlinear strength character that heavily affects the mechanical behaviour and cannot be simply ignored. Experimental data show that the nonlinear H-B criterion (Hoek, Carranza and Corkum, 2002) describes the rock mass behaviour more rigorously. This criterion is an empirically derived relationship used to describe the nonlinear increase in peak strength of isotropic rock with increasing confining stress. Its parabolic form distinguishes it from the classical linear failure criteria such as the M-C criterion (C.A. Coulomb, 1773; Mohr, 1900). Since introducing this criterion, it has been widely accepted to describe rock mass behaviour, and it has been applied to various projects and applications such as tunnelling and underground excavations, where it is crucial to estimate the tunnel convergence (Brown *et al.*, 1983; Wang, 1996; Carranza-Torres, 2004; Sharan, 2008; Rojat, Labiouse and Mestat, 2015). Furthermore, the choice of the nonlinear H-B failure criterion stems from its ability to be fitted to a wide range of nonlinear strength envelopes while respecting the convexity of the yield surface in the principal stress space. A better example is sand, which is the lower spectrum of geomaterial. Sand has indeed a nonlinear behaviour at low confining stress as described by Ud-Din et al (2011), Krabbenhoft et al. (2012), and Ziccarelli et al. (2017) among others.

2. Knowledge gaps

Bearing capacity for both shallow and deep foundations represents a long-standing geotechnical problem toward which many works have been devoted. For the shallow case, most solutions for bearing capacity on rock mass are based on the general bearing capacity equation developed for the linear M-C failure criterion (Brinch Hansen, 1970; Meyerhof, 1978), with modifications to account for rock mass characteristics by introducing an “appropriate” cohesion and friction angle values (c, ϕ). One of such classical bearing capacity equation is written in the following form:

$$q_{ult} = cN_c + q_0N_q + \frac{1}{2}\gamma BN_\gamma \quad (\text{I-1})$$

It is, however, well-known that the failure envelope of most rock masses is nonlinear, as it has been emphasized by several experimental works (Franklin and Hoek, 1970; Hobbs, 1970; Wawersik and Fairhurst, 1970; Bieniawski, 1974; Ramamurthy and Arora, 1994; Hoek and Brown, 1997). Therefore, the determination of an appropriate (c, ϕ) values is not straightforward. As an alternative, the H-B failure criterion, combined with the continuous medium assumption, has been widely used by practicing engineers to study the bearing capacity problem of strip and circular foundations (Kulhawy and Carter, 1992; Serrano, Olalla and González, 2000; Yang and Yin, 2005; Merifield, Lyamin and Sloan, 2006; Saada, Maghous and Garnier, 2008, 2011; Clausen, 2013; Keshavarz, Fazeli and Sadeghi, 2016; Chihi and Saada, 2020; Keawsawasvong, Thongchom and Likitlersuang, 2020). Despite the reliability of the aforementioned scientific works, flow plasticity has been vastly ignored and only the associated flow rule case (AFR) was considered (probably due, at least in part, to the simplicity of this hypothesis). Nevertheless, it is known that rock masses, like classical soils, do not always obey an AFR, and most researchers who have studied dilatancy in rock have acknowledged the need to use an N-AFR (Vermeer and de Borst, 1984; Detournay, 1986; Hoek and Brown, 1997; Sterpi, 1999; Alejano and Alonso, 2005). It is within this framework that improvements could be brought by incorporating an N-AFR and analysing its effects on failure mechanisms and bearing capacity values.

On the other hand, piles are a commonly used form of deep foundations usually used to support structures with heavy and concentrated loads by transferring the load at the required depth. Their load capacity derives from their end bearing and skin friction resistance. Nevertheless, the assessment of these two components for a pile embedded in a rock layer is a common problem in civil engineering towards which many empirical approaches have been devoted, as described by Vipulanandan et al. (2007) among others. These empirical relations have limitations related to using only

the uniaxial compressive strength of the intact rock σ_{ci} . σ_{ci} is only one of many other parameters that affect the strength of the rock mass. To tackle these limitations, rock mass behaviour can be better described by the H-B failure criterion making the analytical treatment of the problem more accessible. It is within this framework that improvements could be brought by developing an analytical solution to predict pile load capacity in rock layers.

3. Aims and objectives

This dissertation aims to explore the most promising approaches to enhance the prediction of the bearing capacity of shallow/deep foundations in rock mass behaving as an elastoplastic continuum and obeying the nonlinear H-B shearing failure criterion. In other words, the main objective of this dissertation is to develop general analytical closed-form solutions for classical geotechnical problems (bearing capacity of shallow/deep foundation) in materials with nonlinear strength envelope. The applicability/efficiency of these solutions to geomaterials, in particular, rock masses under certain limits featured later, will be investigated. Furthermore, conclusions will be drawn for practical purposes in geotechnical engineering.

Concretely, the author aims at addressing some of the shortcomings that are still open in the literature as follows:

For shallow foundations

- Propose a sufficiently significant but conveniently simple formulation of a “non-constant” dilatancy angle model for a material obeying the H-B criterion that can be readily implemented in numerical and analytical codes.
- Incorporate an N-AFR for the problem of a strip footing resting on a material obeying the H-B criterion. Analyse its effects on failure mechanisms as well as bearing capacity values.
- Provide bearing graphs and/or tables and/or explicit equations for the estimation of bearing capacity.
- Conduct static penetration tests on reduced scale model footing and compare experimental results to the analytical ones.

For pile foundations

- Develop an analytical calculation procedure to predict the shaft resistance for a pile drilled to some depth into a rock layer obeying the H-B criterion.
- Develop complete rigorous and closed-form solutions for stress and displacement fields within the framework of cavity expansion problem in a material obeying the H-B criterion by implementing both small and large strain theories (SST and LST).
- Provide graphs and/or tables and/or explicit equations for the estimation of the limit pressure of both cylindrical and spherical cavities expanding in such nonlinear material.
- Correlate pile end bearing resistance to the limit pressure.
- Demonstrate the applicability of the developed solutions using experimental data from numerous case studies.

4. Outline of the dissertation

The dissertation is divided into six main chapters, including this introduction part. The content of each chapter is summarized below.

- Chapter II

This chapter is composed of three main sections. The first one, entitled “Rock behaviour,” is devoted to a critical literature review of failure criteria for intact rock and rock mass. The presented criteria are selected based on the facts that they are published, well known, and deemed suitable for different applications. Furthermore, the elastic parameter, as well as the post-failure behaviour of rock mass, are also reviewed.

The other two sections present an extensive literature survey on the bearing capacity of both shallow and deep foundations on or in rock masses. Solutions for shallow foundation problems are classified according to the rock category (intact, layered, and jointed). In contrast, solutions for deep foundations problems are classified following one of the two approaches categories: (1) empirical and (2) semi-analytical approaches. Furthermore, a comparative study between published solutions is carried out.

- Chapter III

This chapter analyses the plastification phenomenon from the theoretical point of view of an ideal, homogenous, isotropic, and continuous rock mass behaving nonlinearly and obeying the H-B failure criterion under the penetration of a rigid strip footing. An elastic–perfectly plastic medium with both associated and non-associated flow rules is considered.

To obtain simple results that can be applied immediately with closed-form analytical solutions, two techniques are used. The first one that gives “exact” results relies on the upper bound limit analysis technique, preserving exactly the H-B failure criterion expression. The second technique employs equivalent M-C parameters and resorts to the classical bearing capacity equation, giving approximate results.

The proposed analytical solutions are finally validated using numerical modelling and experimental results obtained from simple model-scale footing tests on three different rock masses and nonlinear sand.

- Chapter IV

This chapter complements chapter III. It presents the experimental methodology undertaken to assess the bearing capacity of both strip and circular footings resting on an intact rock mass. This chapter is composed of three main sections. The first one is devoted to present the materials used for the reduced scale experimental setup, including the geometrical and mechanical properties of an intact calcarenite rock mass. Secondly, test equipment and procedure are itemized. Thereafter, experimental results are presented, discussed, and then compared to the analytical results obtained from the developed theoretical solutions.

- Chapter V

This chapter provides an analytical method for calculating the ultimate bearing capacity of a pile embedded in rock mass that obeys the H-B failure criterion. An analytical calculation procedure is proposed to predict the shaft resistance for piles drilled to some depth into such nonlinear material. The developed solution is compared to the published ones and validated using FEM. Furthermore, the efficiency/applicability of the shaft resistance solution is examined using experimental data of numerous case studies in rock layers.

On the other hand, the cavity expansion theory is used to predict the pile end bearing capacity. This can be achieved by assuming that the pressure exerted at the boundaries of the plastic bulb immediately beneath the pile tip is equal to the pressure required to expand a spherical cavity. Since the cavity expansion problem in a material with a nonlinear strength envelope has not been addressed in the literature, the development of complete rigorous solutions for stress and displacement fields is the main objective of this chapter. From a practical point of view, pile end bearing capacity is correlated to the limit pressure. The efficiency of this correlation is thereafter examined using experimental data of numerous case studies.

- Chapter VI

This chapter contains a summary of the main conclusions and presents perspectives for future research.

5. List of publications

Some papers published or submitted while writing this dissertation are cited below in chronological order:

- **Journal articles**

- [1] **Gharsallaoui, H.**, Jafari, M. and Holeyman, A. (2019) “Cavity Expansion in Rock Masses Obeying the ‘Hoek–Brown’ Failure Criterion,” *Rock Mechanics and Rock Engineering*. Springer Science and Business Media LLC, pp. 1–15. [doi: 10.1007/s00603-019-01920-7](https://doi.org/10.1007/s00603-019-01920-7).
- [2] Jafari, M., **Gharsallaoui, H.**, Victor, KH. and Holeyman, A. (2019) “End bearing response of open-ended pipe piles embedded in rock,” *International Journal of Rock Mechanics and Mining Sciences*. Pergamon, 119, pp. 46–57. [doi: 10.1016/j.ijrmms.2019.04.008](https://doi.org/10.1016/j.ijrmms.2019.04.008).
- [3] Jafari, M., **Gharsallaoui, H.** and Holeyman, A. (2020) “Effect of soil rigidity and tensile strength on bearing capacity of embedded foundations,” *Géotechnique Letters*. Thomas Telford Ltd., 10(1), pp. 1–6. [doi: 10.1680/jgele.19.00070](https://doi.org/10.1680/jgele.19.00070).
- [4] **Gharsallaoui, H.**, Jafari, M. and Holeyman, A. (2020) “Pile end bearing capacity in rock mass using cavity expansion theory,” *Journal of rock mechanics and geotechnical engineering*. doi.org/10.1016/j.jrmge.2020.03.004
- [5] **Gharsallaoui, H.**, Jafari, M. and Holeyman, A. (2020) “Bearing capacity of a strip footing on a rock mass by incorporating a non-associated flow rule in upper bound limit analysis,” *Géotechnique*. **Submitted**
- [6] **Gharsallaoui, H.**, Jafari, M. and Holeyman, A. (2020) “Shaft resistance of rock-socketed piles.” *International Journal of Geotechnical Engineering*. **Submitted**

- **Peer-reviewed conference articles**

- [1] **Gharsallaoui, H.**, Jafari, M. and Holeyman, A. (2019) “Bearing capacity of surface foundations resting on Hoek – Brown materials using equivalent Mohr-Coulomb parameters.” in *Proceedings of the XVII ECSMGE-2019 Geotechnical Engineering foundation of the future*. [doi: 10.32075/17ECSMGE-2019-0808](https://doi.org/10.32075/17ECSMGE-2019-0808).
- [2] **Gharsallaoui, H.**, Jafari, M. and Holeyman, A. (2019) “Cavity expansion analysis in elastic – brittle plastic rock mass.,” in *27th European Young Geotechnical Engineers Conference*. [doi: 10.13140/RG.2.2.30537.57448](https://doi.org/10.13140/RG.2.2.30537.57448)

Chapter II. Literature review

A reliable estimate of the strength and deformation characteristics of rock masses is needed for any rock engineering design. Considering this, a critical literature review of failure criteria for intact rock and rock masses is firstly presented. Furthermore, the elastic parameters, as well as the post-failure behaviour of rock mass, are also reviewed.

This chapter attempts subsequently to review the current state of the art concerning analytical and design methods for strip footings and pile foundations problems in rock masses. We must state that the subject of the strip footing problem has been relatively comprehensively covered in recent years compared to the pile foundation design for which the empirical solutions dominate.

1. Rock behaviour

The term ‘rock’ as used by geotechnical engineers encompasses a wide range of materials, from hard polyminerallic igneous and metamorphic rocks, such as granite and dolerite, on the one hand, to soft lithified argillaceous rocks, such as mudstone and shale, on the other side. We do not know exactly how a rock fails, either in terms of the precise details of each micro-crack initiation and propagation or terms of the total structural breakdown as many micro-cracks propagate and coalesce. In both cases, describing and prescribing rock failure and post-failure behaviour have been two significant difficulties of rock mechanics specialists.

Nevertheless, as engineers, we should like some measure of the failure properties and the ability to predict when failure will occur. That had led to a variety of rock failure criteria and rock constitutive models based on experimental and theoretical efforts and concepts, such as Plane Griffith Crack Theory (Griffith, 1921), Mohr-Coulomb (M-C) criterion (C.A. Coulomb, 1773; Mohr, 1900), and Hoek and Brown (H-B) failure criterion (Hoek, E. and Brown, 1980; Hoek, 1983; Hoek and Brown, 1997, 2019; Hoek, Carranza and Corkum, 2002).

Because of the complexities in rock masses behaviour, which are suffering different conditions, such as joint effects, disturbance, and anisotropic nature of rocks, finding an efficient failure criterion that can widely interpret rock failure initiation cannot be easily achieved.

In what follows, a critical review of the literature on failure criteria for intact rock and rock masses is presented. These criteria are selected based on the fact that they are

published, well known, and deemed suitable for different applications. Furthermore, the elastic parameters as well as the post-failure behaviour of the rock mass, are also reviewed.

1.1. Failure criteria

The published failure criteria can be classified following one of the two main failure criteria categories: (1) Classical failure criteria and (2) Empirical failure criteria for both intact and rock mass.

1.1.1. Classical failure criteria

- Coulomb failure criterion

The Coulomb failure criteria (C.A. Coulomb, 1773) is considered the oldest one. Coulomb proposed for masonry and soil a shear resistance S expression of the form:

$$S = ca + \frac{1}{n} N \quad (\text{II-1})$$

where c , a , N and $1/n$ denote, respectively, the cohesion per unit area, the area of the shear plane, the normal force on the shear plane, and the coefficient of internal friction.

In modern terms, $n = \cot(\phi)$ and Eq. (II-1) is usually written in the form:

$$\tau = c + \sigma_n \tan(\phi) \quad (\text{II-2})$$

where τ the shear strength per unit area, c is the unit cohesion, σ_n is the normal stress on the shear plane, and ϕ is the friction angle. It should be noted that Eq. (II-2) can be applied in rock mechanics for shear along joints and discontinuities and, in some cases, of the intact rock itself.

- Mohr-Coulomb failure criterion

This failure criterion was proposed by Mohr (1900) following the earlier work of Coulomb (1776). It considers failure by both yielding and fracture of the material under consideration. Otto Mohr presented a detailed graphical method to describe the state of stress at one point (cf. Figure II-1).

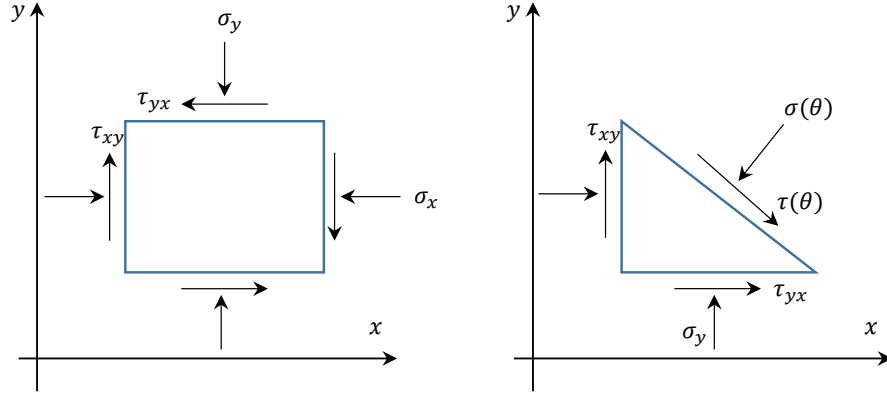


Figure II-1: State of stress at a point.

As shown, the state of stress in a point can be expressed as:

$$\begin{cases} \sigma(\theta) = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos(2\theta) + \tau_{xy} \sin(2\theta) \\ \tau(\theta) = -\frac{\sigma_x - \sigma_y}{2} \sin(2\theta) + \tau_{xy} \cos(2\theta) \end{cases} \quad (\text{II-3})$$

In the above equation, θ denotes the inclination of the plane at which σ_y and τ_y act. The elimination of angle θ from the above equation gives the following equation of a circle of radius R :

$$\left(\sigma(\theta) - \frac{1}{2}(\sigma_x + \sigma_y) \right)^2 + \tau(\theta)^2 = \left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2 = R^2 \quad (\text{II-4})$$

As shown in Figure I-3, the functional relationship between normal and shear stresses (cf. Eq. (II-3)) can be plotted on the $\tau - \sigma$ plane and they are expressed as:

$$\begin{cases} \sigma_n = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos(2\theta) \\ \tau = \frac{\sigma_1 - \sigma_3}{2} \sin(2\theta) \end{cases} \quad (\text{II-5})$$

Here σ_1 and σ_3 are, respectively, the major and the minor principal stresses. θ is the inclination of the plane.

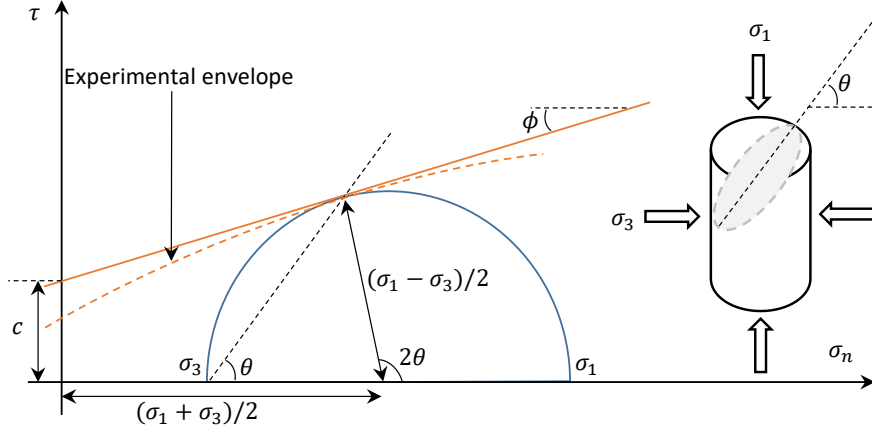


Figure II-2: Mohr failure theory plotted in the τ - σ plane.

Identifying the radius of the Mohr's circle as R , the following relation is obtained:

$$R = \frac{\sigma_1 - \sigma_3}{2} = c \cos(\phi) + \frac{\sigma_1 - \sigma_3}{2} \sin(\phi) \quad (\text{II-6})$$

From the above equation, the expression of the major principal stress as a function of the minor principal stress can be obtained:

$$\sigma_1 = \frac{2c \cos(\phi)}{1 - \sin(\phi)} + \sigma_3 \frac{1 + \sin(\phi)}{1 - \sin(\phi)} \quad (\text{II-7})$$

which can be further written as:

$$\sigma_1 = \sigma_c + k\sigma_3 \quad (\text{II-8})$$

Here $\sigma_c = \frac{2c \cos(\phi)}{1 - \sin(\phi)}$ and $k = \frac{1 + \sin(\phi)}{1 - \sin(\phi)}$ denote, respectively, the uniaxial compressive strength and the slope of the line relating σ_1 and σ_3 (cf. Figure II-3).

Furthermore, the M-C parameters can be expressed as follows:

$$\begin{cases} \phi = \sin^{-1} \left(\frac{k - 1}{k + 1} \right) \\ c = \sigma_c \frac{1 - \sin(\phi)}{2 \cos(\phi)} \end{cases} \quad (\text{II-9})$$

Note that rock cannot support significant tensile stresses. Thus, a tension cut-off is often included, as shown in Figure II-3.

Among the reasons why it is interesting to use the M-C criterion is that it is described by a simple mathematical expression that makes it easy to understand.

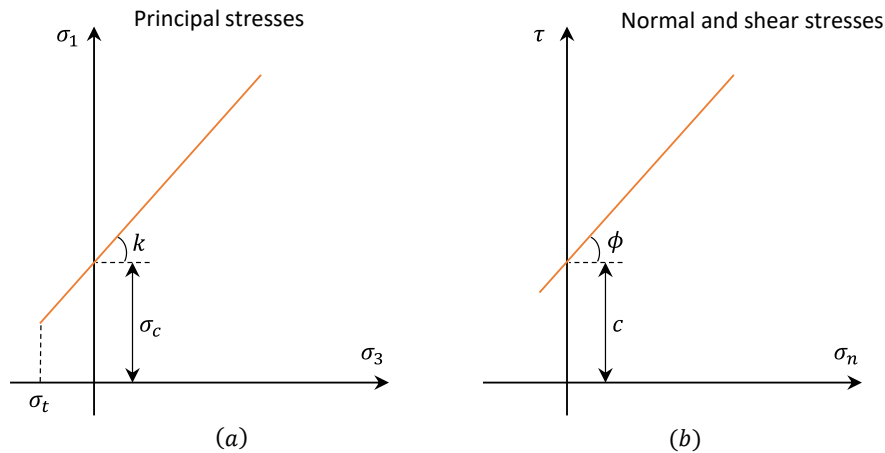


Figure II-3: Mohr-Coulomb criterion in terms of (a) principal stresses and (b) normal and shear stresses.

- Plane Griffith crack theory

Griffith (1921) postulated that the fracture of brittle materials, such as steel and glass, is initiated at tensile stress concentrations. Thin cracks (now referred to as Griffith cracks) are distributed throughout an isotropic, elastic material. Griffith (1921) based his determination of the conditions under which a crack would extend on his energy instability concept:

“A crack will extend only when the total potential energy of the system of applied forces and material decreases or remains constant with an increase in crack length.”

These microscopic defects in intact rock could be small cracks, fissures, or grain boundaries. As discussed by Hoek and Bieniawski (1984), the theory predicts a parabolic failure envelope and thus yields an envelope of the same general form as that observed for intact rocks.

Griffith (1921) extended his theory to the case of applied compressive stresses. Neglecting the influence of friction on the cracks, which will close under compression, and assuming that the elliptical crack will propagate from the points of maximum tensile stress concentration (point “P” in Figure II-4), Griffith obtained the following criterion for crack extension in-plane compression:

$$\begin{cases} (\sigma_1 - \sigma_3)^2 - 8T_0(\sigma_1 + \sigma_3) = 0, & \sigma_1 + 3\sigma_3 < 0 \\ \sigma_3 + T_0 = 0, & \sigma_1 + 3\sigma_3 > 0 \end{cases} \quad (\text{II-10})$$

where T_0 is the uniaxial tensile strength of the material and σ_1 and σ_3 are, respectively, the major and the minor principal stresses.

Note that the uniaxial compressive strength is obtained by setting $\sigma_3 = 0$ in Eq. (II-10) giving:

$$\sigma_c = 8.T_0 \quad (\text{II-11})$$

The Griffith criterion can also be expressed in terms of shear stress, τ , and normal stress, σ_n that acts on the plane containing the major axial of the crack. This resulted in a parabolic relation at failure:

$$\tau^2 = 4T_0(\sigma_n + T_0) \quad (\text{II-12})$$

Even though the strength envelope, which is given by Griffith criterion strongly reassembles the general form observed for tests on rock specimens, it does not provide a good model. Largely, the influence of friction on the cracks is neglected.

Many researchers like Wawersik and Fairhurst (1970) and Jaeger (1971) tried to modify the Griffith criterion (Griffith, 1921) by taking into account the influence of friction on the cracks. Their modifications have not led to a satisfactory agreement with experimental results.

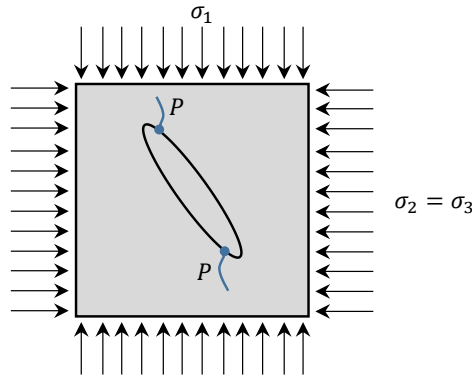


Figure II-4: Griffith crack model for plane compression (Griffith, 1921).

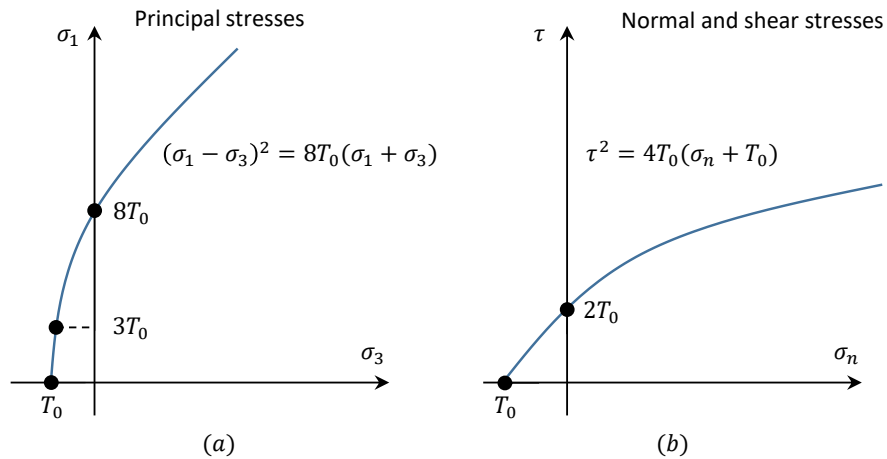


Figure II-5: Griffith strength envelopes for plane compression in terms of (a) principal stresses σ_1, σ_3 and (b) shear and normal stresses τ, σ_n .

1.1.2. Empirical failure criteria

Empirical failure criteria for rock masses are mainly based on triaxial test data of small “intact” rock samples. The term ‘intact rock’ is defined in engineering terms as the smallest element of rock block not cut by any fracture. There are always some microfractures in the intact rock, but these should not be treated as fractures. The physical and mechanical characteristics of the intact rock are shown in Figure II-6.

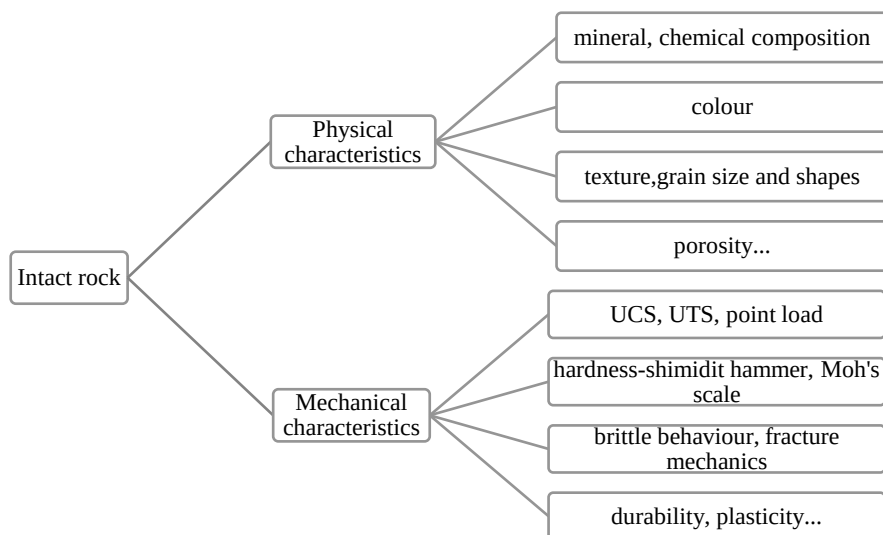


Figure II-6: Intact rock characteristics.

On the other hand, 'Rock mass' differs from 'intact rock.' Rock mass refers to in-situ rock together with its discontinuities and weathering profile. Considering this, some of the empirical failure criteria of intact rock have been verified for rock mass.

The most well-known failure criterion for intact rock followed by those for rock mass are listed below in chronological order.

a. Failure criteria for intact rock

- Hobbs criterion (1960)

Hobbs's strength criterion (Hobbs, 1960) was initially developed for intact rock, followed by a suggestion for broken rock (Hobbs, 1970). The first criterion was based on triaxial tests on nine different coals. The strength criterion for the nine coals was expressed as:

$$\sigma_1 = \sigma_c + \sigma_3 + F\sigma_3^f \quad (\text{II-13})$$

where

- σ_c : uniaxial compressive strength of the rock,
- σ_1 : major principal stress,
- σ_3 : minor principal stress
- F and f : empirical constants.
- Fairhurst criterion (1965)

Fairhurst's criterion (Fairhurst, 1965) can be considered as a generalization to Griffith's theory (Griffith, 1921). He concluded that the uniaxial tensile strength of materials with a low compression/tension ratio is underestimated by Brazilian tests in which radial loading is applied to the disc-shaped specimen. This criterion is formulated as:

$$\begin{cases} \sigma_1 - \sigma_t = 0; \text{ if } m(m-2)\sigma_1 + \sigma_3 > 0 \\ \text{else} \\ (\sigma_1 - \sigma_3)^2 + 2(m-1)^2\sigma_t(\sigma_1 + \sigma_3) - 4(m-1)^2\sigma_t^2 \left(\frac{1}{4}(m-1)^2 - 1 \right) = 0 \end{cases} \quad (\text{II-14})$$

with m taken as:

$$m = \sqrt{1+n}; n = \left| \frac{\sigma_c}{\sigma_t} \right| > 3 \quad (\text{II-15})$$

In the particular case where $n = 8$, Eq. (II-14) gives the condition derived by Griffith (1921). This criterion can also be expressed in terms of normal and shear stresses as:

$$\tau^2 = (m - 1)^2 \sigma_t (\sigma_t - \sigma_n) \quad (\text{II-16})$$

The simplified version of Eq. (II-14), which is easier to compare with other criteria for intact rock, is:

$$(\sigma_1 - \sigma_3)^2 = a + b(\sigma_1 - \sigma_3) \quad (\text{II-17})$$

- Franklin criterion (1970)

The Franklin criterion (Franklin and Hoek, 1970) was developed based on some comparatives between previous linear and nonlinear criteria. He suggested the following equation:

$$(\sigma_1 - \sigma_3) = A(\sigma_1 + \sigma_3)^B \quad (\text{II-18})$$

The suggested criterion reduced the strength prediction error compared to the linear criterion. By putting the minor principal stress $\sigma_3 = 0$ in the above equation, the major principal stress is $\sigma_1 = \sigma_c$ and $A = \sigma_c^{1-B}$ where σ_c is the uniaxial compressive strength, and B is a dimensionless parameter ranging between 0.6 and 0.9. Eq. (II-18) can be rewritten as follows:

$$(\sigma_1 - \sigma_3) = \sigma_c^{1-B} (\sigma_1 + \sigma_3)^B \quad (\text{II-19})$$

When comparing to Griffith criterion (Griffith, 1921), it was found that both criteria are identical when $B = 0,5$ but since $0,6 \leq B \leq 0,9$, it does not fit Franklin's data.

- Bieniawski criterion (1974)

The Bieniawski criterion (Bieniawski, 1974) was only proposed for intact rock. Bieniawski declared that the triaxial peak strength of a wide range of different rocks could be obtained by the following equations:

$$\left\{ \begin{array}{l} \frac{\sigma_1}{\sigma_c} = 1 + A \left(\frac{\sigma_3}{\sigma_c} \right)^\alpha \\ \frac{\tau_m}{\sigma_c} = 0,1 + B \left(\frac{\sigma_m}{\sigma_c} \right)^\beta \end{array} \right. ; \quad \left\{ \begin{array}{l} \tau_m = \frac{\sigma_1 - \sigma_3}{2} \\ \sigma_m = \frac{\sigma_1 + \sigma_3}{2} \end{array} \right. \quad (\text{II-20})$$

In the above equation, σ_1 and σ_3 denote, respectively, the major and the minor principal stresses. A , B , α , and β are constants:

- α and β : constants equal to 0,75 and 0,9, respectively.
- A and B : constants dependent on rock type (cf. Table II-1).

Table II-1: *A and B values for Bieniawski criterion (Bieniawski, 1974)*

Rock type	A	B
Norite	5,0	0,8
Quartzite	4,5	0,78
Sandstone	4,0	0,75
Silestone	3,0	0,7
Mudstone	3,0	0,7

- Ramamurthy criterion (1985)

Ramamurthy criterion (Ramamurthy and Arora, 1994) is based on uniaxial compression and triaxial tests for more than 100 different rock types. It is expressed as:

$$\sigma_1 = \sigma_3 + B\sigma_3 \left(\frac{\sigma_c}{\sigma_3} \right)^a \quad (\text{II-21})$$

where a and B are constants gained by tests on the intact rock. σ_1 , σ_3 and σ_c are, respectively, the major principal stress, the minor principal stress, and the uniaxial compressive strength.

Ramamurthy criterion was later updated (Ramamurthy, 2004), and the new version was expressed as:

$$\sigma'_1 = \sigma'_3 + (\sigma'_3 + \sigma_t)B_i \left(\frac{\sigma_c}{\sigma'_3 + \sigma_t} \right)^a \quad (\text{II-22})$$

The new intact rock Ramamurthy criterion contains tensile strength σ_t . Moreover, it is written as a function of effective principal stresses. With the tensile strength considered, the values of the exponent a and B_i may be estimated as:

$$\begin{cases} a = 2/3 \\ B_i = 1.3 \left(\frac{\sigma_{ci}}{\sigma_t} \right)^{\frac{1}{3}} \end{cases} \quad (\text{II-23})$$

Since B_i is very sensitive to the tensile strength when evaluated experimentally, Ramamurthy suggested that a Brazilian test should be used to determine the tensile strength.

- Johnston criterion (1985)

An alternative expression for intact rock has been presented by Johnston (1985):

$$\sigma'_{1N} = \left(\frac{M}{B} \sigma'_{3N} + 1 \right)^B \quad (\text{II-24})$$

where:

- $\sigma'_{1N} = (\sigma'_1/\sigma_c)$ and $\sigma'_{3N} = (\sigma'_3/\sigma_c)$: normalized effective principal stresses at failure (σ'_c is the effective uniaxial compressive strength of the intact rock).
- M and B : rock constants.

Note that, by putting $\sigma'_{3N} = 0$, we have the uniaxial compressive strength $\sigma_c = \sigma'_1$, which is correct. Otherwise, the ratio of uniaxial compressive strength σ_c to uniaxial tensile strength σ_t is found by putting $\sigma'_1 = 0$ and $\sigma'_3 = \sigma_t$, which yields:

$$\frac{\sigma_c}{\sigma_t} = -\frac{M}{B} \quad (\text{II-25})$$

An advantage of Eq. (II-24) is that it can be fitted to a wide range of strength envelope shapes by changing the value of B . For example, putting $B = 1$ gives a linear envelope:

$$\sigma'_{1N} = M\sigma'_{3N} + 1 \quad (\text{II-26})$$

which for

$$M = \frac{1 + \sin(\phi)}{1 - \sin(\phi)} \quad (\text{II-27})$$

corresponds precisely to the normalized linear M-C criterion (cf. Eq. (II-7)).

By fitting Eq. (II-24) to effective strength data from a wide range of clays soils and rocks, Johnston (1985) found the data to be reasonably represented by:

$$\begin{cases} B = 1 - 0,0172(\log(\sigma_c))^2 \\ M = 2.065 + 0.276(\log(\sigma_c))^2 \end{cases} ; [\sigma_c] = kPa \quad (\text{II-28})$$

The general trends are decreasing B with increasing σ_c and an increasing M with increasing σ_c .

- Yoshida criterion (1990)

The Yoshida failure criterion (Yoshida, Morgenstern and Chan, 1990) is a sample nonlinear criterion expressed in terms of major principal stress σ_1 and minor principal stress σ_3 , the uniaxial compressive strength σ_{ci} and three strength parameters : A , B and S . It is expressed as:

$$\sigma_1 = \sigma_3 + A\sigma_{ci} \left(\frac{\sigma_3}{\sigma_{ci}} - S \right)^{1/B} \quad (\text{II-29})$$

To assess the parameters A, B, and S, the least square method on results from triaxial compression tests or direct shear tests is recommended.

When $\sigma_3 = 0$, the following condition should be satisfied:

$$A = (-S)^{-1/B} \quad (\text{II-30})$$

Note that the Mohr-Coulomb criterion (cf. Eq. (II-7) – (II-9)) can be derived from this criterion if parameter B is equal to one. In such a case, we have:

$$\begin{cases} A = \frac{2 \sin(\phi)}{1 - \sin(\phi)} \\ S = -\frac{c}{\sigma_{ci} \tan(\phi)} \end{cases} \leftrightarrow \begin{cases} \phi = \sin^{-1} \left(\frac{A}{2 + A} \right) \\ c = -\frac{AS\sigma_{ci}}{\sqrt{4 + 2.A}} \end{cases} \quad (\text{II-31})$$

b. Failure criteria of rock mass

- Bieniawski-Yudhbir criterion (1983)

The Bieniawski-Yudhbir criterion (Yudhbir, Lemanza and Prinzl, 1983) was developed with the idea of covering all conditions of rocks varying from intact rock to highly jointed rock, which covers the brittle to ductile behaviour range. Yudhbir et al. (1983) used the original Bieniawski criterion (Bieniawski, 1974) and changed it to suit rock masses, mainly for soft rocks by testing a mixture of jointed gypsum-celite specimens. Therefore, he took the original formula of Bieniawski (cf. Eq. (II-20)) and updated it to the following general expression:

$$\frac{\sigma_1}{\sigma_{ci}} = A + B \left(\frac{\sigma_3}{\sigma_{ci}} \right)^\alpha \quad (\text{II-32})$$

in which A and B are dimensionless values referring to the rock mass quality and the rock type, respectively. The exponent α is independent of the rock mass quality, and it is equal to 0.65.

The uniaxial compressive strength of the rock mass is expressed as a function of σ_{ci} and the rock mass rating (RMR) as:

$$\frac{\sigma_c}{\sigma_{ci}} = A = \exp \left(7.65 \frac{RMR - 100}{100} \right) \quad (\text{II-33})$$

- Hoek and Brown (H-B) criterion (2018)

The H-B failure criterion is an empirical criterion developed through curve fitting of triaxial test data. It was first introduced in 1980 and has been subsequently updated in 1983, 1997, 2002, and 2019. The H-B failure criterion, which assumes isotropic rock, should only be applied to rock masses in which there is a sufficient number of closely spaced discontinuities. In other words, the H-B failure criterion is valid either for intact rocks or for jointed rock masses with sufficiently dense and randomly distributed joints. The latest version of the H-B criterion (Hoek and Brown, 2019) is defined by (positive compressive stresses):

$$\sigma_1' = \sigma_3' + \sigma_{ci} \left(m_b \frac{\sigma_3'}{\sigma_{ci}} + s \right)^a \quad (\text{II-34})$$

in which σ_1' and σ_3' denote, respectively, the major and minor principal stresses at failure under triaxial loading conditions. σ_{ci} denotes the uniaxial compressive strength of the intact rock ranging from 1 (weak rock) to 250 [MPa] (very strong rock). The strength parameters m_b , s and the exponent a describe the rock mass strength characteristics and depend on H-B core parameters GSI , D and m_i :

- GSI is the Geotechnical Strength Index that characterizes the quality of rock masses (structure and surface condition of the joints). GSI varies from 10 for very poor surface quality to 100 for intact or massive rock (cf. Figure II-7)
- D is a disturbance factor that varies from 0, for undisturbed in-situ rock masses, to 1, for very disturbed rock masses.
- m_i is the intact frictional strength parameter gained from a triaxial test performed under different ambient pressures. It varies from 2 to 33. Some approximate values of m_i estimated by Hoek, for five types of rocks are listed below:
 - ❖ $m_i \cong 7$ for carbonate rocks with well-developed crystal cleavage (dolomite, limestone, and marble), as adopted for the impoverished rock masses,
 - ❖ $m_i \cong 10$ for lithified argillaceous rocks (mudstone, siltstone shale, and slate),
 - ❖ $m_i \cong 15$ for arenaceous rocks with strong crystals and poorly developed crystal cleavage (sandstone, and quartzite), as adopted for the average rock masses,

- ❖ $m_i \cong 17$ for fine-grained polyminerallic igneous crystalline rocks (andesite, dolerite, diabase, and rhyolite) and
- ❖ $m_i \cong 25$ for coarse-grained polyminerallic igneous and metamorphic rocks (amphibolite, gabbro, gneiss, granite, and quartz-diorite), as adopted for the very good rock masses.

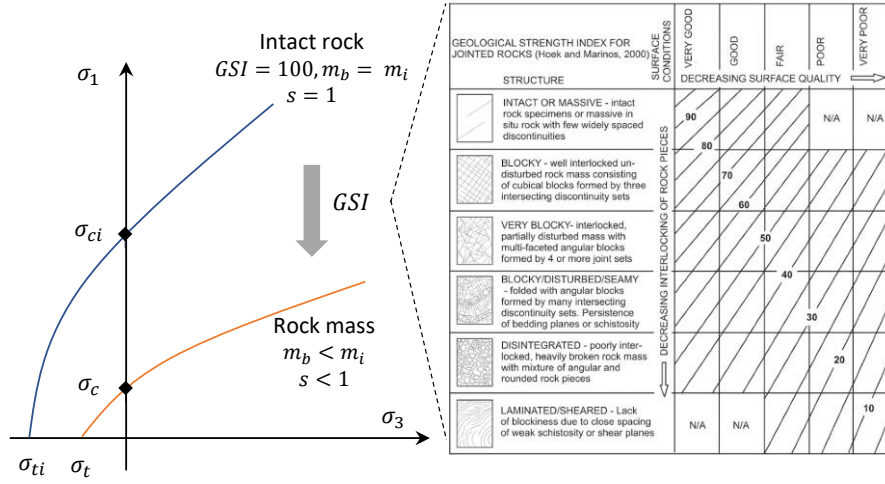


Figure II-7: H-B failure envelope: Intact rock vs. rock Mass.

The strength parameters are defined by:

$$m_b = m_i \exp\left(\frac{GSI - 100}{28 - 14D}\right), \quad (\text{II-35})$$

$$s = \exp\left(\frac{GSI - 100}{9 - 3D}\right), \quad (\text{II-36})$$

$$a = \frac{1}{2} + \frac{1}{6} \left(\exp\left(-\frac{GSI}{15}\right) - \exp\left(-\frac{20}{3}\right) \right), \quad (\text{II-37})$$

As can be seen in Figure II-8, the frictional strength parameter m_b increases gradually with increasing GSI values until reaching the value of m_i when GSI equals 100. For $GSI < 40$, the exponent a is slightly larger than 0.5. For $GSI > 40$, a is approximately 0.5. s increases exponentially with increasing GSI value until reaching 1 when GSI equals 100.

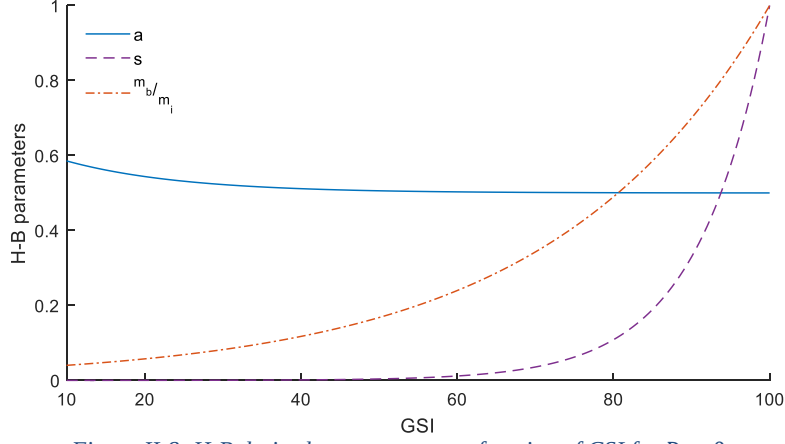


Figure II-8: H-B derived parameters as a function of GSI for $D = 0$.

On the other side, the Uniaxial Compressive Strength (UCS) of the rock mass is obtained by setting $\sigma'_3 = 0$ in Eq. (II-34), giving:

$$UCS = \sigma_c = \sigma_{ci} s^a \quad (II-38)$$

While the uniaxial tensile strength of the rock mass σ_t is obtained by setting $\sigma'_1 = 0$ in Eq. (II-34), giving:

$$\frac{\sigma_t}{\sigma_{ci}} + \left(m_b \frac{\sigma_t}{\sigma_{ci}} + s \right)^a = 0 \quad (II-39)$$

If $a = 0.5$, the tensile strength can be calculated explicitly as:

$$\frac{\sigma_t}{\sigma_{ci}} = \frac{1}{2} \left(m_b - \sqrt{m_b^2 + 4s} \right) < 0 \quad (II-40)$$

Otherwise, σ_t should be evaluated by a numerical tool.

It should be emphasized that if a tension cut off is desired, the following approximate relationship between the compressive to tensile strength ratio, $\sigma_{ci}/|\sigma_t|$ and m_i can be used:

$$\frac{\sigma_{ci}}{|\sigma_t|} = 0.81m_i + 7 \quad (II-41)$$

On the other hand, normal and shear stresses are related to principal stresses by the equation published by Balmer (1952) as follows:

$$\sigma'_n = \frac{\sigma'_1 + \sigma'_3}{2} - \frac{\sigma'_1 - \sigma'_3}{2} \frac{d\sigma'_1/d\sigma'_3 - 1}{d\sigma'_1/d\sigma'_3 + 1} \quad (\text{II-42})$$

$$\tau = (\sigma'_1 - \sigma'_3) \frac{\sqrt{d\sigma'_1/d\sigma'_3}}{d\sigma'_1/d\sigma'_3 + 1} \quad (\text{II-43})$$

in which

$$d\sigma'_1/d\sigma'_3 = 1 + am_b \left(m_b \frac{\sigma'_3}{\sigma_{ci}} + s \right)^{a-1} \quad (\text{II-44})$$

is obtained by direct derivation of Eq. (II-34).

1.1.3. Discussion

- **General remarks**

The published failure criteria are classified following one of the two main failure criteria categories: (a) Classical failure criteria, (b) Empirical failure criteria for both intact and rock mass.

Empirical failure criteria are mainly based on triaxial testing of small intact rock samples, and a few of them have been verified against test data for rock masses.

The criteria are formulated in terms of major and minor principal stresses σ'_1 and σ'_3 without any consideration of the intermediate principal stress σ'_2 .

It is found that the nonlinear H-B failure criterion more rigorously represents rock mass behaviour. This criterion is therefore selected to conduct the analytical studies. The advantages/limitations and the applicability of the H-B criterion are discussed below.

The main advantages of the H-B criterion are:

- (1) It is expressed in a nonlinear form, which agrees with experimental data over a range of confining stresses.
- (2) It was developed through an extensive evaluation of laboratory test data covering a wide range of intact rock types.
- (3) It provides a straightforward empirical means to estimate rock mass properties.
- (4) It can be fitted to a wide range of strength envelope shapes while respecting the convexity of the yield surface in the principal stress space. For example, putting $a = 1$ or $m_i = 0$ gives a linear envelope.

Nevertheless one of the most important limitations of the H-B failure criterion is the independence of the intermediate principal stress σ_2 . That can be justified by pointing to triaxial extension and compression tests by (Brace and Walsh, 1964) that showed no significant variation between results when $\sigma_2 = \sigma_3$ and $\sigma_2 = \sigma_1$. It was concluded that σ_2 had a negligible influence on failure. However, more pronounced influence of σ_2 was discounted as involving brittle / ductile transition in the failure process (Mogi, 1971). The following empirical equation was proposed by Mogi (1971):

$$(\sigma_1 - \sigma_3) = f(\sigma_1 + \beta\sigma_2 + \sigma_3) \quad (\text{II-45})$$

in which, β is a constant that varies from 0 and 1 and f is an empirical function that depends on the rock type.

One should also quote the Zhang–Zhu (2008) criterion which is a 3D version of the original H-B criterion. It is expressed as follows:

$$\frac{1}{\sigma_{ci}^{\frac{1}{a}-1}} \left(\frac{3}{\sqrt{2}} \tau_{oct} \right)^{\frac{1}{a}} + \frac{m_b}{2} \left(\frac{3}{\sqrt{2}} \tau_{oct} \right) - m_b \sigma'_{m,2} = s \sigma_{ci} \quad (\text{II-46})$$

Here τ_{oct} and $\sigma'_{m,2}$ are, respectively, the octahedral shear stress and the effective mean stress defined by:

$$\tau_{oct} = \frac{1}{3} \sqrt{(\sigma'_1 - \sigma'_2)^2 + (\sigma'_2 - \sigma'_3)^2 + (\sigma'_3 - \sigma'_1)^2} \quad (\text{II-47})$$

$$\sigma'_{m,2} = \frac{\sigma'_1 + \sigma'_3}{2} \quad (\text{II-48})$$

Zhang–Zhu (2008) not only inherits the advantages of the Hoek–Brown strength criterion but takes into account the influence of the intermediate principal stress. At a 2D stress state (triaxial or biaxial), Zhang–Zhu's (2008) criterion will simply reduce to the form of the original H-B strength criterion.

- **Terzaghi's principle of effective stress – rock mass**

The effective stress concept ($\sigma' = \sigma - u$) in geomechanics advocated by Terzaghi (1923, 1936) is useful for understanding the behaviour of water-saturated geomaterials. It has been widely accepted for soil mechanics, but its validity for other porous media, such as rock, has been questioned by many researchers (Lanyon, Kingdon and Herbert, 1992; Cornet and Jianmin, 1995). For porous rock, Terzaghi's equation, in its well-known form, is not adequate and the following modification was proposed (Skempton, 1960, 1984):

$$\sigma' = \sigma - \alpha u \quad (\text{II-49})$$

Here α is a reduction factor for pore pressure that varies between 0 and 1. The expression of α according to various authors is summarized in Table II-2.

Table II-2: Expression of α according to various authors

References	α
(Terzaghi, 1936; King Hubbert and Rubey, 1959)	1
(Fillunger, 1930; Terzaghi, 1945; Lubinski, 1954; Biot, 1955)	n^*
(Schiffman, 1970)	$n \leq \& \leq 1$
(Biot and Willis, 1957; Skempton, 1960; Nur and Byerlee, 1971; Bishop, 1973)	$1 - \left(\frac{k}{k_s}\right)^{**}$
* n: porosity of the rock ** k and k_s: bulk modulus of the dry porous rock and the rock without pores (solid or grain bulk modulus), respectively.	

- **Scaled H-B failure criterion**

The H-B failure criterion defined by Eq. (II-34) can be ‘scaled’ to obtain a unique relationship independent of the H-B parameters σ_{ci} , m_b and s . The transformation suggested by the author involves dividing Eq. (II-34) by the term $(\sigma_{ci} m_b^\beta)$ and adding the term $(s/m_b^{\beta/a})$ to both sides. With these manipulations, the H-B failure criterion expression becomes:

$$\frac{\sigma'_1}{\sigma_{ci} m_b^\beta} + \frac{s}{m_b^{\beta/a}} = \frac{\sigma'_3}{\sigma_{ci} m_b^\beta} + \frac{s}{m_b^{\beta/a}} + \left(\frac{\sigma'_3}{\sigma_{ci} m_b^\beta} + \frac{s}{m_b^{\beta/a}} \right)^a \quad (\text{II-50})$$

in which $\beta = \frac{a}{1-a}$ is a constant solely depending on the exponent $a \neq 1$.

Thus, the scaled (non-dimensional) minor and major principal stresses are defined naturally as:

$$\sigma_j^* = \frac{\sigma'_j}{\sigma_{ci} m_b^\beta} + \frac{s}{m_b^{\beta/a}}; j = \{1,3\} \quad (\text{II-51})$$

When expressed in terms of σ_1^* and σ_3^* , the H-B failure criterion permits a simplified and normalized treatment of the rock mass failure condition.

Thanks to such scaling, the H-B failure criterion is formally simplified as follows:

$$\sigma_1^* = \sigma_3^* + (\sigma_3^*)^a \quad (\text{II-52})$$

Londe (1988) made a similar transformation for a rock obeying the 1983 H-B version

with the assumption that the exponent a is equal to 0.5, i.e., valid for $GSI > 25$. Contrary to Londe's transformation, the author's 'scaled' H-B failure criterion, given per Eq. (II-52) applies to any rock obeying the generalized H-B failure criterion – last version (Hoek and Brown, 2019). Figure II-9 depicts the H-B criterion under its scaled form for three GSI values.

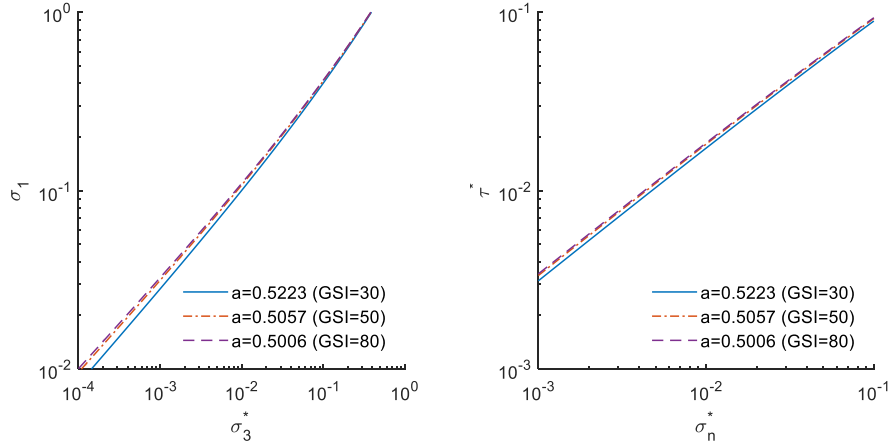


Figure II-9: H-B failure criterion yield surface (scaled form) for three values of GSI

In the plane $\tau - \sigma_n$, with the same manipulations, normal and shear stresses are scaled, and their expressions are related to each other using a parametric form depending exclusively on the scaled minor principal stress σ_3^* as follows:

$$\sigma_n^* = \frac{\sigma_n'}{\sigma_{ci} m_b^\beta} + \frac{s}{m_b^{\beta/a}} = \sigma_3^* + \frac{(\sigma_3^*)^a}{d\sigma_1^*/d\sigma_3^* + 1} \quad (II-53)$$

$$\tau^* = \frac{\tau}{\sigma_{ci} m_b^\beta} = (\sigma_n^* - \sigma_3^*) \sqrt{d\sigma_1^*/d\sigma_3^*} \quad (II-54)$$

By substituting σ_n^* by its expression in the above equation, the following scaled shear stress is obtained:

$$\tau^* = \frac{\tau}{\sigma_{ci} m_b^\beta} = (\sigma_3^*)^a \frac{\sqrt{d\sigma_1^*/d\sigma_3^*}}{d\sigma_1^*/d\sigma_3^* + 1} \quad (II-55)$$

where

$$d\sigma_1^*/d\sigma_3^* = 1 + a(\sigma_3^*)^{a-1} \quad (II-56)$$

is obtained by direct derivation of Eq. (II-52).

Using the scaled form of the H-B failure criterion will lead to significant simplification in the analysis of stresses in rock masses. This will be detailed later in Chapter III and Chapter V when considering the problem of bearing capacity of shallow and deep foundations, respectively.

- **Applicability of the H-B failure criterion**

- Scale effects

A special account should be taken when modelling the rock mass with the H-B criterion since this latter is valid only in a continuous, isotropic medium. Indeed, a rock mass can be considered as a continuous medium in two cases: (1) purely intact rock mass, (2) jointed rock mass with sufficiently dense and randomly distributed joints, where joint spacing is small enough compared to the overall size of the construction element studied.

In other words, when the structure being analysed is large and the block size is relatively small, the rock mass can be modelled with the H-B criterion. Furthermore, when the block size is of the same order as that of the structure being analysed or when one of the discontinuity sets is significantly weaker than the others, the H-B failure criterion should not be used. Figure II-10 shows the transition from an isotropic intact rock specimen, through highly anisotropic rock mass in which failure is controlled by one or two discontinuities, to an isotropic heavily jointed rock mass.

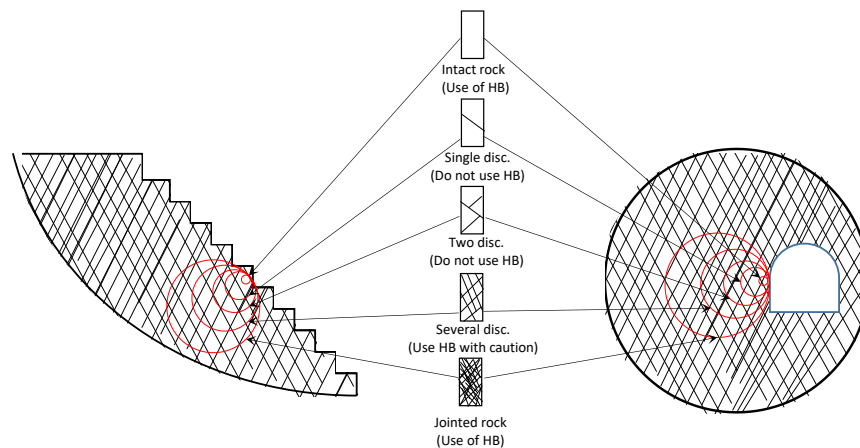


Figure II-10: Idealized diagram showing the transition from intact to a heavily jointed rock mass with an increasing sample size (Hoek, E. and Brown, 1980; Marinos, Marinos and Hoek, 2007; Marinos, 2019)

In line with the idealized diagram depicted in Figure II-10, “small spacing” is a relative concept in the sense that it depends on the foundation width B . This concept

may be quantified by the “Spacing Ratio (SR)” parameter (Serrano and Olalla, 1996). SR is defined as:

$$SR = B \sum_{i=1}^n \frac{1}{S_{m_i}} = B \sum_{i=1}^n \lambda_i \quad (\text{II-57})$$

where B is the foundation width, S_{m_i} is the joint spacing of the i^{th} discontinuity family, λ_i is the frequency of the i^{th} discontinuity family, and n is the number of discontinuity families.

Serrano and Olalla (1996) suggested that the H-B failure criterion applies for jointed rock mass when SR is greater than 60. One example is a jointed rock mass with 4 families of discontinuities each appearing at least 15 times across the foundation width. Otherwise ($SR < 60$), this criterion is not valid. An exception is when the value of SR is so low ($SR < 0.8 - 4$) that the rock mass can be considered as intact.

The concept of the small spacing, i.e. SR will be further specified in Chapter III when dealing with the bearing capacity problem of shallow foundation.

○ *s-values condition – Stress ratio condition*

The H-B failure criterion is only recommended to describe an intact rock mass or heavily jointed rock mass. This means that the H-B failure criterion given per. Eq. (II-34) is only valid if the term “ s ” is equal to 1 (intact rock) or around 0 (very fractured). However, ‘ s ’ is an exponential function of the Geological Strength Index GSI (cf. Eq. (II-36)), as depicted in Figure II-11. Accordingly, the limit of applicability in terms of s -values translates to $GSI < 80$ for $D = 0$. For intact rocks, $s = 1$ when $GSI = 100$ and $D = 0$.

On the other hand, in the recent article by Hoek and Brown (2019), it is emphasized that the H-B failure criterion was developed to deal with brittle shear failure in rock masses. The range of applicability of this criterion is therefore determined by the transition from brittle to ductile shear failures as exemplified in Figure II-12. This transition is defined by $\sigma_1 \geq 4\sigma_3$ based on triaxial compression tests on Indiana Limestone by Schwartz (1964). This limit was later alleviated by Mogi (1966) who investigated the average transition from brittle shear to ductile shear failures on a wide range of rock types. Mogi (1966) has found that the transition occurs at $\sigma_1 \geq 3.4\sigma_3$.

Let LSR be the Limit Stress Ratio beyond which the H-B failure criterion is not applicable. Therefore, the stress ratio condition is given by the following inequality (scaled form):

$$\sigma_1^* \geq \sigma_{1.\min}^* = (LSR)\sigma_3^* + \frac{s}{m_b^{\beta/a}}(1 - LSR) \quad (II-58)$$

Here, $\sigma_{1.\min}^*$ is the minimal major principal stress (scaled) beyond which the stress ratio condition is not satisfied.

To summarize the foregoing, the use of the H-B criterion to describe the behaviour of a rock mass required two fundamental conditions. Firstly, the “s-values condition”, ensures that the rock mass is either intact (i.e. $s = 1$) or very fractured (i.e. s close to 0). Secondly, the “stress ratio condition”, ensures that no ductile shear failure has occurred. During the investigation, both conditions will be always considered when producing and analysing results.

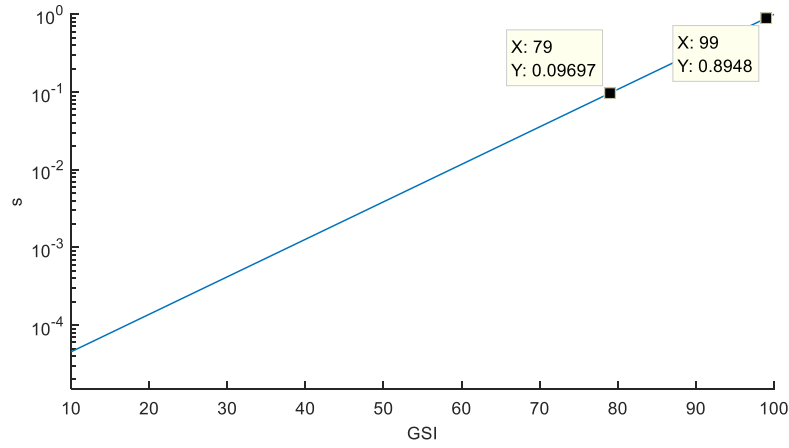


Figure II-11: s values as a function of GSI and for $D = 0$.

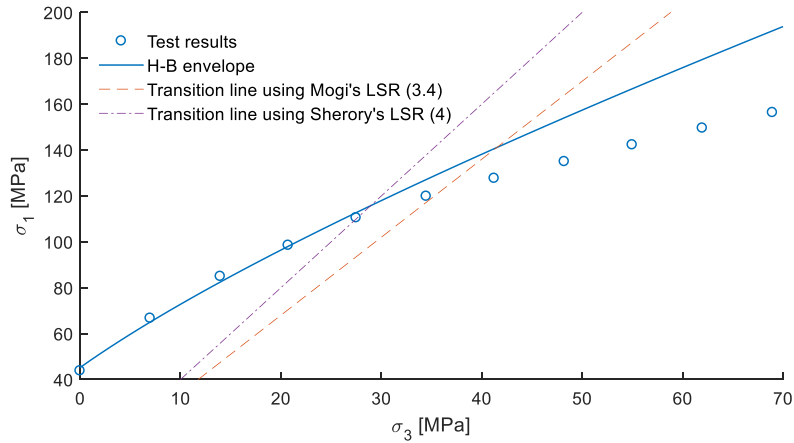


Figure II-12: Triaxial tests on Indiana limestone by Schwartz (1964) – H-B envelope and transition lines from brittle shear to ductile failures.

1.2. Elastic parameters

The deformations modulus expression was first introduced in 1997 (Hoek and Brown, 1997). It is expressed as a function of σ_{ci} and GSI as:

$$E_{rm}[GPa] = \begin{cases} \sqrt{\frac{\sigma_{ci}}{100}} 10^{\left(\frac{GSI-10}{40}\right)}, & \sigma_{ci} \leq 100 \text{ MPa} \\ 10^{\left(\frac{GSI-10}{40}\right)}, & \sigma_{ci} > 100 \text{ MPa} \end{cases} \quad (\text{II-59})$$

Later on, the above equation was updated and generalized to include the effect of the disturbance factor D (Hoek and Diederichs, 2006):

$$E_{rm} = E_i \left(0.02 + \frac{1 - D/2}{1 + \exp\left(\frac{60 + 15D - GSI}{11}\right)} \right) [MPa] \quad (\text{II-60})$$

If reliable values of intact rock modulus E_i are not available, one can resort to the modulus ratio MR to estimate the intact modulus E_i as:

$$E_i = MR \cdot \sigma_{ci} \quad (\text{II-61})$$

Furthermore, when no information on the intact rock deformation modulus neither the MR is available, the following alternative equation for estimating the rock mass modulus E_{rm} can be used:

$$E_{rm} = 10^5 \left(0.02 + \frac{1 - D/2}{1 + \exp\left(\frac{75 + 25D - GSI}{11}\right)} \right) [MPa] \quad (\text{II-62})$$

The evolution of the normalized deformation modulus as a function of GSI and for four different values of D are plotted in Figure II-13. As can be seen, the deformation modulus values are almost 2% E_i for GSI values lower than 30.

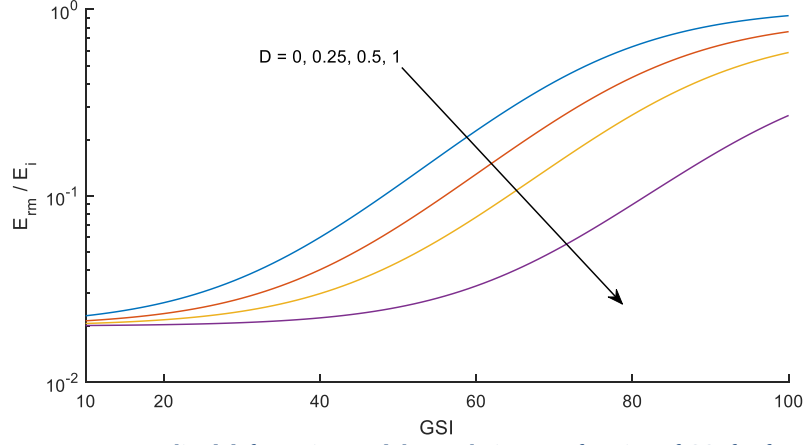


Figure II-13: Normalized deformation modulus evolution as a function of GSI for four values of D .

On the other hand, the Poisson's ratio can be estimated using the formula proposed by Vásárhelyi (2009), dependent on GSI and the Poisson's ratio of the intact rock ν_i as:

$$\nu = -0.002GSI + \nu_i + 0.2 \quad (II-63)$$

Note that if reliable values of ν_i are not available, a simplified equation proposed by Vásárhelyi (2009) provides an estimate of Poisson's ratio as a function of GSI and m_i :

$$\nu = -0.002GSI - 0.003m_i + 0.457 \quad (II-64)$$

The Poisson's ratio contours are depicted in Figure II-14. As can be seen, as the quality of the rock mass decreases, the Poisson's ratio increases.

1.3. Post-failure behaviour

- *Perfectly plastic – brittle plastic and softening post-failure behaviours*

When using numerical models to study the progressive failure of rock masses, estimates of the post-peak or post-failure characteristics of the rock mass are required. In most of these models, the H-B failure criterion is treated as a yield criterion, and the analysis is carried out using the plasticity theory.

In light of this, if we consider a rock mass that obeys the H-B failure criterion, as soon as the rock mass yields, the peak strength parameters (m_b^{peak} and s^{peak}) and the peak

deformation modulus (E^{peak}) drop to residual strength parameters (m_b^{res} and s^{res}) and residual deformation modulus E^{res} . Therefore, two cases can be distinguished:

- Case (1) Residual parameters are equal to the peak ones. Then we are defining an "ideally" elastic-plastic material (cf. Figure II-15).
- Case (2) Residual parameters are lower than the peak ones. In such cases, Hoek (2006) said that they, the rock engineering community, do not have good models to describe this post-failure behaviour. However, he suggested the brittle and the strain-softening behaviours as a starting point (cf. Figure II-15).

Note that the disturbance factor D can be used to achieve strength and modulus reduction after failure. It is found that $D = 0.7$ is appropriate in most cases (Hoek, 2006a).

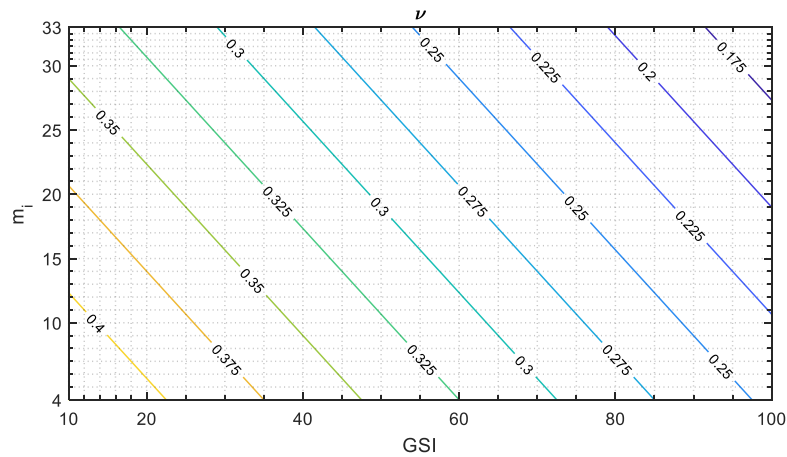


Figure II-14: Poisson's ratio contours as a function of GSI and m_i .

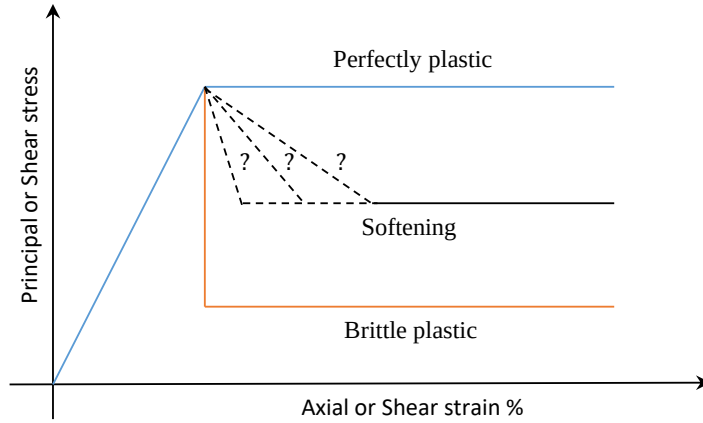


Figure II-15: Post-failure behaviour of rock mass.

- *Brittle – Ductile failure transition*

Although several definitions of the brittle-ductile transition have been proposed, one common definition depends primarily on macroscopic considerations. The distinction between the brittle and the ductile behaviour depends on whether a macroscopic fracture forms (strain localization occurs) after a substantial permanent strain (Paterson and Wong, 2005; Walton *et al.*, 2017; Yu *et al.*, 2019; Wang *et al.*, 2020). Such a categorization only describes the macroscopic behaviour of rocks without considering the deformation mechanism of rocks involving micro cracking and frictional sliding along grain boundaries in the brittle field (Rutter, 1986; Evans, Fredrich and Wong, 2011). In contrast, ductile deformation transitions to delocalized cataclasis (or “cataclastic flow”) as stated by Paterson and Wong (2005) among others. A schematic illustrating changes in failure patterns about internal factors, such as pore collapse, and external factors such as confining stress and temperature, and ductility is plotted in Figure II-16.

In light of this, Walton *et al.*, (2017) performed an extensive uniaxial and triaxial compression testing program on Indiana Limestone to assess its behaviour across the brittle-ductile transition. In addition to 20 uniaxial compression tests from which a UCS value of 53 MPa was deduced, 34 triaxial compressive tests were performed at confining stress σ_3 of 2, 4, 5, 8, 10, 15, 20, 30, 40, 50 and 60 MPa. They found out that specimens tested at $\sigma_3 = 30 \text{ MPa}$ (0.6 UCS) displayed a fully ductile failure mechanism whereas specimen tested at $\sigma_3 = 15 \text{ MPa}$ (0.3 UCS) and $\sigma_3 = 20 \text{ MPa}$ (0.4 UCS) displayed transitional mechanisms, which were neither fully brittle nor fully ductile. Interestingly, it was also found out that brittle-ductile transition approximately corresponds to the line with zero intercept $\sigma_1/\sigma_3 \approx 5$ which is in

accordance with the transition line defined by $\sigma_1 = 4\sigma_3$ based on triaxial compression tests on a similar Indiana Limestone by Schwartz (1964).

Note that the applicability of the H-B criterion taking into account the brittle-ductile failure transition is further reviewed in the upcoming subsection 1.4.1 of this chapter.

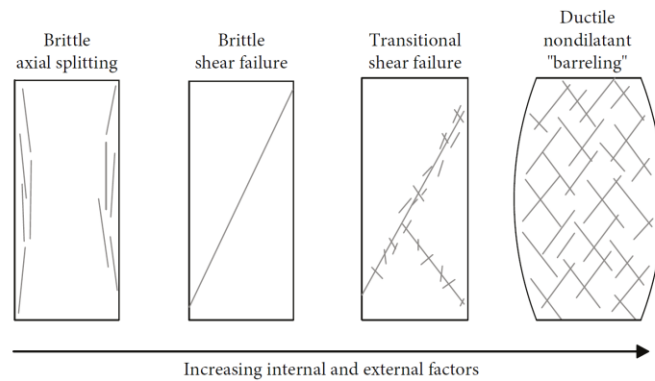


Figure II-16: Schematic illustrating changes in the failure in relation to internal and external factors and ductility – after Walton et al., (2017).

2. Shallow foundation on rock mass

2.1. Overview

Bearing capacity of shallow foundation is an old geotechnical problem toward which many works have been devoted. Most solutions for bearing capacity on the rock are based on the general bearing capacity equation developed for soil, with appropriate modifications to account for rock mass characteristics such as spacing and orientation of discontinuities, condition of the discontinuities, and strength of the rock mass. The available solutions are classified following one of the three rock categories: (1) Intact, (2) Layered, and (3) Jointed rock mass.

2.1.1. Intact rock mass

In this case, the ultimate bearing capacity is limited to the bearing stress that causes fracturing in the rock. Practically, the rock is considered as intact if joints spacing (S) is more than four to five times the foundation width (B), i.e., $S/B > 4 \sim 5$. In this case, based on field load tests, Rowe and Armitage (1987) stated that the rock fracturing could be expected to occur when the bearing capacity is approximately 2.7 times the rock uniaxial compressive strength. For design, the following is recommended:

$$q_{ult} = 2.5\sigma_{ci} \quad (\text{II-65})$$

2.1.2. Layered rock mass

For layered rock mass, failure can occur either by flexure if the rigid layer is relatively thick or by punching shear if the rigid layer is thin. Both modes of failure are controlled fundamentally by the tensile strength of the intact rock (Sowers and Sowers, 1979). To the author's knowledge, neither case has been studied adequately nor has an analytical solution been proposed.

2.1.3. Jointed rock mass

Here, we distinguish between 4 distinct cases depending on discontinuous verticality α and joints spacing S as detailed below.

When discontinuities are vertical or nearly vertical ($\alpha > 70^\circ$) and open joints are present with spacing less than the foundation width, i.e., $S/B \leq 1$, failure can occur (theoretically) by unconfined compression strength according to Sowers (1979). Bearing capacity can be estimated as:

$$q_{ult} = \sigma_{ci} = 2c \tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right) \quad (\text{II-66})$$

Here, (c, ϕ) are M-C strength properties of the rock mass.

Otherwise, if the nearly vertical joints are closed, a general wedge failure may develop, and the bearing capacity can be expressed using the classical Bell's solution for plane strain conditions (Bell, 1915):

$$q_{ult} = cN_c s_c + \frac{B}{2} \gamma N_\gamma s_\gamma + \gamma D_f N_q s_q \quad (\text{II-67})$$

in which B, γ and D_f are respectively the foundation width, the effective unit weight of the rock mass, and the foundation depth. N_c, N_γ and N_q are bearing capacity factors given as a function of $N_\phi = \tan^2(\frac{\pi}{4} + \frac{\phi}{2})$ by:

$$\begin{cases} N_c = 2 \sqrt{N_\phi} (N_\phi + 1) \\ N_\gamma = \sqrt{N_\phi} (N_\phi^2 - 1) \\ N_q = N_\phi^2 \end{cases} \quad (\text{II-68})$$

and s_c, s_γ and s_q are shape factor given by:

$$s_c = 1 + \frac{N_q}{N_c}; \quad s_\gamma = 0.6; \quad s_q = 1 + \tan(\phi) \quad (\text{II-69})$$

Furthermore, if the spacing is greater than the foundation width, i.e., $S/B > 1$, failure occurs by splitting, leading eventually to general shear failure. This problem has been evaluated by Bishnoi (1968) and developed further by Goodman (1981) and Kulhawy and Carter (1992). The solution is expressed as:

$$q_{ult} = JcN_{cr} \quad (\text{II-70})$$

where J is a correction factor (cf. Figure II-17), c is the cohesion of the rock mass (estimated as 10% σ_{ci}) and N_{cr} is the bearing capacity factor expressed as a function of $N_\phi = \tan^2(\frac{\pi}{4} + \frac{\phi}{2})$ as:

$$N_{cr} = \frac{2 \cdot N_\phi^2}{1 + N_\phi} \cot(\phi) \frac{S}{B} \left(1 - \frac{1}{N_\phi} \right) - N_\phi \cot(\phi) + 2 \sqrt{N_\phi} \quad (\text{II-71})$$

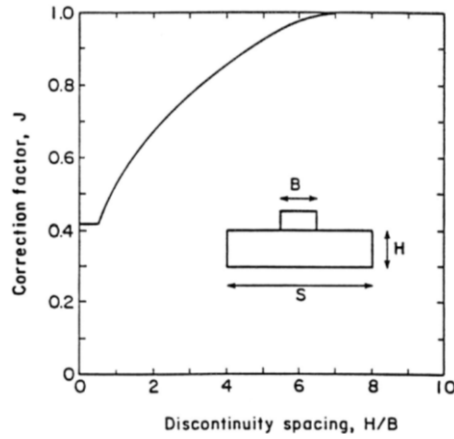


Figure II-17: Correction factor J as a function of the ratio of horizontal discontinuity spacing H to foundation width (H/B) (Bishnoi, 1968).

Finally, in case of highly jointed rock mass ($S/B \ll 1$), a rational approach for estimating the ultimate bearing capacity is to apply the classical bearing capacity equation with appropriate cohesion and friction angle values (c, ϕ). It is, however, well-known that the failure envelope of most rock masses is nonlinear as it has been emphasized by several experimental works. Therefore, the determination of (c, ϕ) values for highly jointed rock mass is not straightforward.

Among the nonlinear failure criteria proposed previously in Subsection 1.1 of this chapter, the H-B failure criterion is considered to model the strength properties of isotropic rock mass reasonably well. Several authors investigated the bearing capacity of foundation resting on rock masses modelled by this last strength criterion. The employed methods combined with perfect plasticity and associated flow rule, mainly fall within one of the four categories (a) limit equilibrium method (b) slip line method (c) limit analysis method, and (d) finite element or finite difference numerical techniques.

Against this background, Carter and Kulhawy (1992) proposed a simple lower bound solution for the bearing capacity of a strip footing resting on a weightless rock mass that obeys the H-B failure criterion. The details of the lower bound stress field are as shown in Figure II-18. The resolution refers to the theorem of the lower limit in which the stresses state must satisfy the equilibrium and must not violate the plasticity conditions. The ultimate bearing capacity of a strip footing may be evaluated as follows:

$$q_{ult} = N_{\sigma_0} \sigma_{ci} ; N_{\sigma_0} = [s^a + (m_b s^a + s)^a] \quad (\text{II-72})$$

Here, N_{σ_0} is the bearing capacity factor (weightless rock).

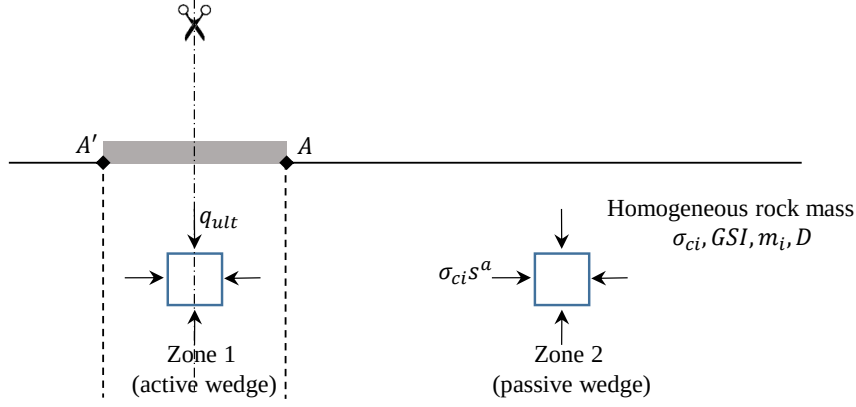


Figure II-18: Bearing capacity analysis (Kulhawy and Carter, 1992)

On the other hand, one should quote the pioneer contributions of Serrano et al. (2000). Disregarding the rock self-weight, they solved the differential equations system governing the stress field by implementing the characteristics method (slip line method). Their solution is expressed as follows:

$$q_{ult} = \beta_n (N_\beta - \zeta_n) \quad (\text{II-73})$$

where ζ_n and β_n are constants that depend on H-B parameters σ_{ci} , m_b , a and s according to the following expressions:

$$\begin{cases} \beta_n = A_n \sigma_{ci} \\ \zeta_n = \frac{s}{m_b A_n} \end{cases} ; A_n = \left[\frac{m_b (1 - a)}{2^{\frac{1}{a}}} \right]^{\frac{a}{1-a}} \quad (\text{II-74})$$

N_β is a factor which can be determined graphically from (Serrano, Olalla and González, 2000)

Later on, Yang and Yin (2005) developed an upper bound solution to estimate the bearing capacity of a strip footing resting on rock mass modelled with the H-B criterion. The idea was to replace the original H-B criterion with an ‘optimal’ tangential Mohr-Coulomb domain. It was found that this procedure amounts to overestimating the overall strength capacities of the rock material, which in turn leads to overestimating the ultimate bearing capacity.

A finite element approach was also developed by Merifield et al. (2006) to assess the bearing capacity of strip footing on rock mass that obeys the generalized H-B failure

criterion. The computed upper and lower bound estimates for the bearing capacity factor were found to be within 5% of each other. Consequently, the average values of upper and lower bound bearing capacity factor, published by these authors in the form of tables, can be considered as highly accurate estimates for the actual collapse load. Merifield et al. (2006) have also studied the effect of the rock self-weight on bearing capacity values. The effect of the rock weight γ was found to decrease with increasing GSI .

Later on, Saada et al. (2008) implemented the kinematic approach for the problem of ultimate bearing capacity making use of two classes of failure mechanisms: a generalized Prandtl-type (Prandtl, 1920) failure mechanism and a multi-wedge translation mechanism (Michalowski, 1997). Unlike the approach developed by Yang and Yin (2005), an upper bound estimate for bearing capacity preserving approximatively the original modified H-B failure criterion was derived.

We can also mention the pioneer contribution of Clausen (2013). Disregarding the rock self-weight, he numerically solved the bearing capacity problem of circular surface footing on rock mass obeying the generalized H-B failure criterion.

Studies that generalize the existing solutions have been published very recently. Indeed, the effect of eccentricity and load inclination (Chihi and Saada, 2020; Keawsawasvong, Thongchom and Likitlersuang, 2020), as well as seepage forces (AlKhafaji, Imani and Fahimifar, 2020) and seismic conditions (Keshavarz, Fazeli and Sadeghi, 2016), have been incorporated within the framework of bearing capacity of a strip footing resting rock mass obeying the H-B criterion.

On the other hand, an interesting study was made on the effect of joint spacing on bearing capacity (Imani, Fahimifar and Sharifzadeh, 2012b, 2012a). The upper bound theorem of limit analysis (to prescribe a general shear failure) as well as a discreet element method (to prescribe failure due to excessive deformations i.e. bearing capacity corresponding to a settlement equals $10\%B$) were used to investigate the bearing capacity of the jointed rock foundation. It was assumed that the rock mass contains two orthogonal tight joint sets and obeys the M-C failure criterion. To account for the joint spacing, the “spacing ratio of a foundation (SR)” defined per Eq. (II-1) was adopted (cf. Figure II-19). The proposed general shear failure mechanism and the corresponding hodograph are depicted in Figure II-20.

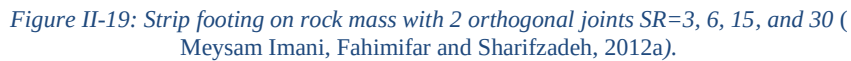


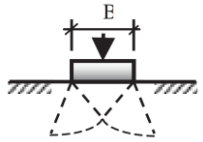
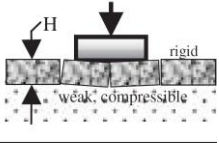
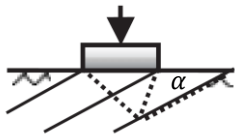
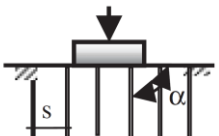
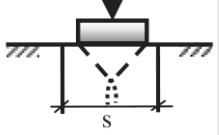
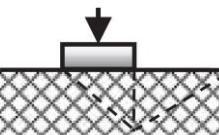
Figure II-20: Failure mechanism and the corresponding hodograph (Imani, Fahimifar and Sharifzadeh, 2012)

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2.2. Discussion and conclusion

The bearing capacity solution and its corresponding failure mode depending on the rock category (intact, jointed...) and joints characteristics are summarized below in Table II-3. It has been proven (Imani, Fahimifar and Sharifzadeh, 2012b, 2012a) that joint spacing affects the bearing capacity of rock foundation when $SR < 30$. Otherwise, there is no influence. This conclusion supports the conservative assumption that the H-B failure criterion is applicable for jointed rock mass when SR is greater than 60 (Serrano and Olalla, 1996).

Table II-3: Typical bearing capacity failure modes associated with various rock categories.

Rock category	Joint inclination α and joint spacing S .	SR	Failure mode	Bearing capacity solution
Intact	$\frac{S}{B} \gg 1$	$< 0.8 - 4$		(II-65) (II-72) (II-73)
Layered	$0^\circ < \alpha < 20^\circ$	—		—
Jointed	$20^\circ < \alpha < 70^\circ$ $\frac{S}{B} < 1$ or $\frac{S}{B} > 1$	> 4 < 60		(II-70)
	$70^\circ < \alpha < 90^\circ$ $\frac{S}{B} < 1$			(II-66)
	$70^\circ < \alpha < 90^\circ$ $\frac{S}{B} > 1$			(II-67)
	$\frac{S}{B} \ll 1$			(II-72) (II-73)

On the other hand, the published solutions that are based on the H-B failure criterion were given for GSI range between 10 and 100 (with an increment of 10) and a certain strength parameter value ($m_i \in [5, 10, 20, 30, 35]$). For a rock mass of which GSI and/or m_i differ from the offered range, a graphic reading and/or interpolation from published charts and/or tables is required, which is not always practical and can lead to uncertain results.

To compare those solutions, three different quality rock masses that obey the H-B failure criterion were considered, namely poor, average, and good (cf. Table II-4).

The results are presented in Table II-5. It has been found that published solutions, which are made for weightless rock and absence of surcharge load and presented in the form of charts and tables are in excellent agreement with each other, as published by Serrano et al. (2000), Merifield et al. (2006), and Saada et al. (2008) among others. Nevertheless, the procedure proposed by Yang and Yin (2005) amounts to overestimating the overall strength capacities of the rock material, which in turn leads to overestimating the ultimate bearing capacity. Furthermore, the lower bound results obtained from the method proposed by Kulhawy and Carter (1992) are on the conservative side due to the simple lower bound stress field that was assumed. Their estimates of the bearing capacity factor N_{σ_0} are typically 30% – 80% below compared to the average finite element limit analysis results (Merifield, Lyamin and Sloan, 2006).

Table II-4: H-B core parameters for three quality rock masses

Rock quality	H-B core parameters			
	σ_{ci} [MPa]	m_i [/]	GSI [/]	D [/]
Poor	10	7	30	0
Average	80	15	50	0
Good	150	25	80	0

Table II-5: Ultimate bearing capacity of a strip footing resting on weightless rock mass according to different published works.

Rock quality	q_{ult}/σ_{ci} [/]				
	(Kulhawy and Carter, 1992)	(Serrano, Olalla and González, 2000)	(Yang and Yin, 2005)	(Merifield, Lyamin and Sloan, 2006)	(Saada, Maghous and Garnier, 2008)
Poor	0.108	0.293	0.533	0.300	0.256
Average	0.457	1.395	2.732	1.400	1.368
Good	2.353	7.118	13.499	7.118	7.149

3. Pile foundation in rock mass

Piles are a commonly used form of deep foundations that provide support for structures, transferring their load to layers of soil or rock that have enough bearing capacity and suitable settlement characteristics (cf. Figure II-21). The bearing capacity of the pile is roughly composed of two components: (1) shaft friction resistance and (2) base resistance. Nevertheless, the assessment of these two components for a pile embedded in a rock mass is a common problem in civil engineering towards which many empirical approaches have been devoted for both shaft resistance (Kaderabek, T. J., Reynolds, 1981; Williams and Pells, 1981; Gupton, C. and Logan, 1984; R. K. Rowe and Armitage, 1987; Toh *et al.*, 1989; Reese, L.C. and O'Neil, 1990; Zhang, 1997) and base resistance (Coates, 1965; R. K. K. Rowe and Armitage, 1987; Le Tirant, Marshall and Argema, 1992; Zhang and Einstein, 1998; Vipulanandan, Hussain and Usluogulari, 2007). One should also quote the pioneering contribution of Holeyman (2017). He investigated experimentally the pipe pile driving problem in three artificial rock mass using a reduced scale pipe pile model. Results from the heavily instrumented driving tests indicate that pipe pile driving is easy in a cellular mortar with an UCS not exceeding 6 MPa but virtually (numerically) impossible in a mortar with UCS approaching 28 MPa. It was also proved that for a mortar with UCS of 11 MPa, driving is still feasible under a reasonable drop height (to avoid pile toe plastification / damage). Regarding the end bearing resistance, it was reported that the unit toe resistance increases from approximately $2\sigma_{ci}$ at the rock surface to reach 6 to 8 σ_{ci} at a penetration depth equivalent to 15 – 20 times the pile thickness.

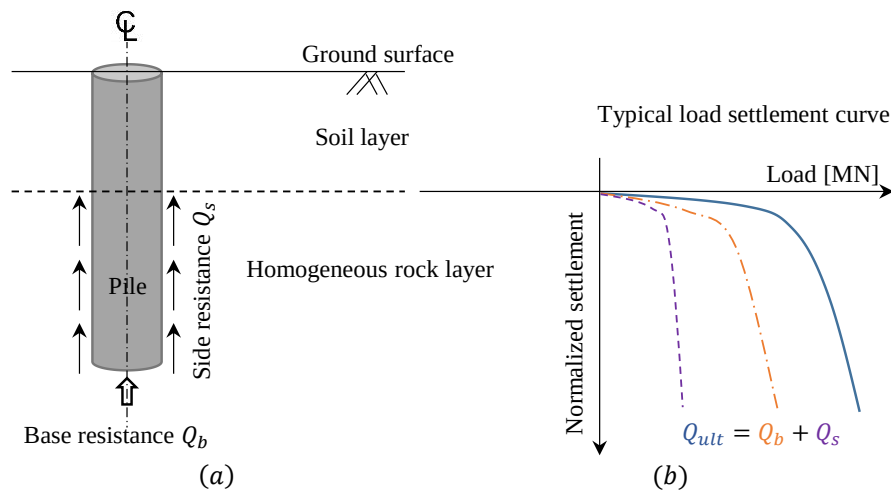


Figure II-21: (a) Pile foundation in homogenous rock layer – (b) Typical load settlement curve.

Nevertheless, there is not only empirical approaches. There are some semi-analytical approaches to estimate the bearing capacity of a pile embedded in a rock mass. So, the available solutions are classified following one of the two approaches categories: (1) empirical and (2) semi-analytical approaches.

To examine the efficiency of the empirical approaches, a database of conducted experimental case studies, originating from an extensive bibliographic study, is summarized in Table 0-21 and Table 0-22 of Appendix 4. This database is composed of 113 and 91 case studies, respectively, for shaft/base resistance problems of a pile embedded in a rock layer. For each case, the rock type, geometry details of the rock layer as well as the soil above it, the measured unit shear resistance, as well as the base resistance and their corresponding settlements, were extracted from original papers. The uniaxial compressive strength of the intact rock σ_{ci} , the RQD of the rock mass and a deformation modulus E_{rm} are also retrieved if they were available. This database has been also completed for use in Chapter V to validate the developed solutions for the bearing capacity problem of a pile embedded in the rock layer. Further details will be given thereafter.

3.1. Empirical approaches

3.1.1. Empirical approaches based on UCS of intact rock

The empirical approaches are based on either full-scaled in situ tests or laboratory tests to predict both unit shear resistance τ_{ult} and the ultimate end bearing capacity $q_{b,ult}$. According to these references, there are two kinds of approaches. The first kind relies on a linear function of the uniaxial compressive strength σ_{ci} whereas the second one relies on a linear function of the square root of σ_{ci} . Thus, the expression of the ultimate bearing capacity of a pile foundation in terms of τ_{ult} and $q_{b,ult}$ can be generalized as follows:

$$\tau_{ult} = \alpha_{\tau}(\sigma_{ci})^{n_{\tau}} \quad (\text{II-75})$$

$$q_{b,ult} = \alpha_q(\sigma_{ci})^{n_q} \quad (\text{II-76})$$

In the above equation, α_{τ} and α_q denote empirical constants, respectively, for unit shaft resistance and base resistance that varies between authors (cf. Table II-6 and Table II-7). n_{τ} and n_q are exponents equal to one for the first kind of approach and approaching 0.5 for the second kind.

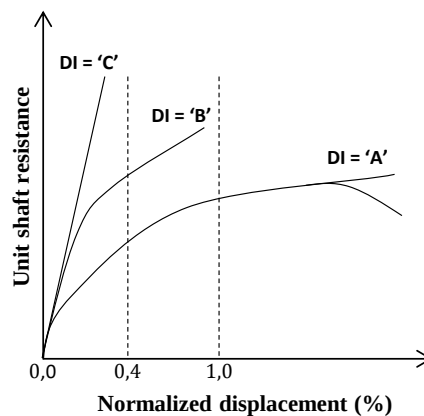
Table II-6: Values of α_τ and n_τ for use in Eq. (II-75).

α_τ	n_τ
0.1 – 0.4	1
0.2 – 0.6	0.5

Table II-7: Values of α_q and n_q for use in Eq. (II-76)

References	α_q	n_q
(Coates, 1965)	3	1
(R. K. K. Rowe and Armitage, 1987)	2.7	1
(Le Tirant, Marshall and Argema, 1992)	4.5	1
(Zhang and Einstein, 1998)	4.83	0.51
(Vipulanandan, Hussain and Usluogulari, 2007)	4.66	0.56

It is essential to mention that the values of the so-called ‘ultimate’ shaft resistance measured during the test are not necessarily the “true” ultimate ones in most cases. Indeed, a Displacement Index (DI), which is an approximate measure of the pile displacement and degree of mobilization of side resistance for a given maximum side resistance value, should be assigned for each test case (Ng *et al.*, 2001). There are three different “DI”: (A, B, or C). DI is assigned by observation of the field side resistance-displacement relationships. As shown in Figure II-22, an index “A” is assigned, firstly, to test results, which are observed to be close to the fully mobilized value. It is defined by the displacement reaching a minimum of 1% of the socket diameter and the gradient of the resistance-displacement relationship becoming $< 30 \text{ [kPa/mm]}$ at the end of the curve. Secondly, an Index “B” is assigned to test results not achieving an index “A” but recorded at a displacement of at least 0.4% of the pile diameter and for which the gradient of the resistance displacement relationship is $< 200 \text{ [kPa/mm]}$ at the end of the curve. Finally, an index “C” is assigned to the remaining results.

Figure II-22: Displacement Index (Ng *et al.*, 2001).

The same applies to the empirical relations for the base resistance. Different interpretations of the load test data are used by original authors to develop the empirical approaches. According to those interpretations, the ultimate end bearing capacity is defined by way of conventions as:

- The bearing resistance at a certain pile head displacement, say 10% of the pile diameter,
- The bearing resistance at a certain pile base displacement, say 5% of the pile diameter,
- The maximum applied test load,

The ultimate end bearing capacity can also be deduced by using a finite element method or the hyperbolic load transfer function approach to analyse the load-displacement response of the rock socketed piles. Indeed, the measured load-displacement curve is matched, and then the ultimate end bearing capacity is determined at a particular pile base/head settlement of 5/10 %. Because of these different interpretations, it is hard to figure out which one gives accurate estimations of the Ultimate Limit State (ULS). Therefore, the so-called ‘ultimate’ end bearing capacity determined from Eq. (II-76) is not necessarily the ‘true’ ultimate one.

By referring to the collected database and more precisely the UCS column (cf. Table 0-21 and Table 0-22 of Appendix 4), a simple comparison between the measured shaft resistance and the measured base resistance and those predicted from empirical relations (cf. Eqs. (II-75) and (II-76)) is made and it is depicted in Figure II-23 and Figure II-24, respectively. As can be seen, empirical approaches did not yield entirely satisfactory results. This is mainly because these empirical relations rely only on the uniaxial compressive strength of the intact rock mass σ_{ci} and do not take into account the partially disturbed state of the rock mass represented by GSI (or RQD).

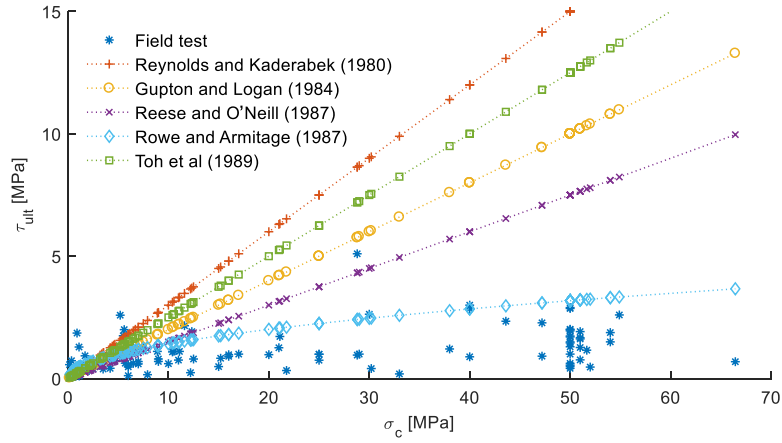


Figure II-23: Ultimate unit shaft resistance as a function of the uniaxial compressive strength (UCS) σ_c : Field test vs empirical relations.

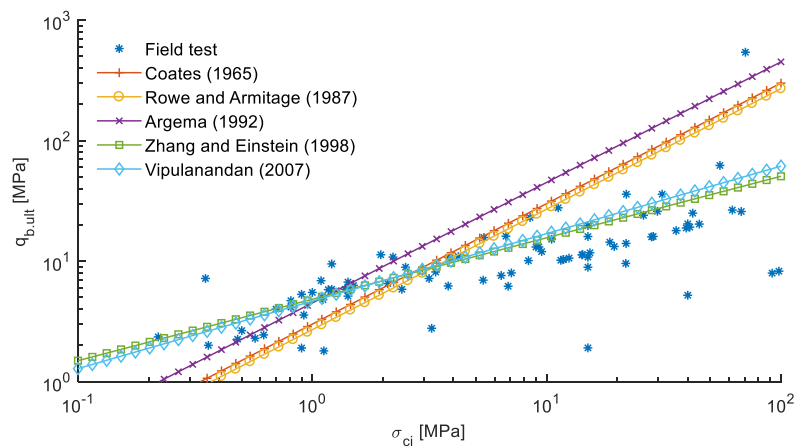


Figure II-24: Ultimate end bearing capacity as a function of the unconfined compressive strength σ_{ci} : Field test vs. empirical relations.

3.1.2. Empirical approaches considering other factors

In general, many other factors affect the bearing capacity of piles embedded in a rock mass. Major factors are listed as follows (Seol et al, 2008):

- (1) Strength of the rock materials,
- (2) Rock mass structure (discontinuities),
- (3) Modulus of the concrete relative to the modulus of the rock mass,
- (4) Confining stress, and
- (5) Construction-related factors, including roughness of the shaft-rock interface.

Therefore, instead of using a single parameter .i.e., UCS, all relevant parameters need to be considered. However, the interaction between all these elements is complex and cannot be implemented reliably using empirical methods.

With that in mind, Zhang (2010) developed a new relation for determining $q_{b,ult}$ by considering the effect of discontinuities, represented by the Rock Quality Designation (RQD), on σ_{ci} . Still, his empirical relation has limitations relating to using only RQD to represent the effect of discontinuities. Many other factors of the Rock Mass Rating (RMR) system, such as discontinuities spacing, condition, and orientation, can have a substantial effect on the strength of the rock mass.

Regarding the side resistance, one of the physical factors that has a significant influence is the shaft roughness. In light of this, Horvath and Kenney (1979) and Horvath et al., (1983) recommended the use of a roughness factor RF to describe how rough the rock-concrete interface of a rock socket is:

$$RF = \frac{\Delta r_h}{r} \frac{L_r}{L_s} \quad (II-77)$$

where Δr_h is the average roughness (asperity) height, L_r is the total length along the sidewall profile, r is the nominal radius of the socket, and L_s is the nominal socket length. Besides they proposed an empirical correlation to calculate the ultimate shaft resistance based on RF as:

$$\tau_{ult} = 0.8\sigma_c(RF)^{0.45} \quad (II-78)$$

Where σ_c is the unconfined compressive strength of the rock or the concrete shaft, whichever is smaller.

Meanwhile, Seidel and Collingwood (2001) suggested a dimensionless parameter defined as the shaft resistance coefficient (SRC) to consider the influencing factors as follows:

$$SRC = \eta_c \frac{MR}{1 + \nu} \frac{\Delta r_h}{B} \quad (II-79)$$

in which, Δr_h is the average roughness (asperity) height. η_c is the reduction factor for the construction method that ranges between 0.8 to 1. MR is the modulus ratio, ν is the Poisson's ratio and B is the pile diameter. The ultimate side resistance is thus computed as:

$$\tau_{ult} = SRC \cdot \sigma_c \quad (II-80)$$

Note that the above relationship does not apply for side resistance coefficient values exceeding 2.1., as stated by Seidel and Collingwood (2001).

To evaluate the efficiency and accuracy of the above equations incorporating the roughness factor, one can resort to the experimental results performed by Dai et al. (2017). They investigated experimentally the effect of sidewall roughness on the shaft resistance of rock-socketed piles by performing tests on 10 model piles socketed in a weak, artificial rock made of sand, cement, gypsum powder, and water. The mix is characterized by a uniaxial compressive strength of 6 MPa (after 20 days), an elastic modulus of 608 MPa, i.e. $MR = 101$ and a unit weight of 22 kN/m³. The 10 model piles, with 50 mm diameter and 500 mm length, were made of solid plexiglass characterized by a compressive strength of 100 MPa and an elastic modulus of 2.8 GPA. Two model piles were smooth i.e. $RF = 0$ and the remaining model piles were rough with a RF ranging from 0.04 to 0.23. More details can be found in (Dai et al., 2017). It was found that the maximum side resistance increase with increasing wall roughness. However, there is an upper limit of socket roughness of almost 0.07 beyond which very little additional side resistance was developed. On the other hand, a comparison between the measured peak side resistance values and those obtained from the empirical approaches (cf. (II-78) and (II-80)) was performed as summarized in Table II-8. It can be noted that both Horvath et al., (1983) and Seidel and Collingwood (2001) methods underpredicted the peak side resistance for piles with rough sidewall.

Table II-8: Comparison of the measured side resistance and the estimated one (MPa) for 10 model piles installed in rock with UCS equal to 6 MPa – after Dai et al. (2017)

Pile (RF)	P1 (0)	P2 (0)	P3 (0.4)	P4 (0.4)	P5 (0.082)	P6 (0.082)	P7 (0.127)	P8 (0.127)	P9 (0.232)	P10 (0.232)
Measured	1.14	1.30	2.23	2.67	2.82	2.29	2.70	2.69	2.93	2.98
Eq.(II-78) ⁽¹⁾	—	—	1.29	1.29	1.56	1.56	1.9	1.9	2.50	2.50
Eq.(II-80) ⁽²⁾	—	—	1.80	1.80	—	—	—	—	—	—
(1) Estimated ultimate side resistance according to Horvath et al., (1983)										
(1) Estimated ultimate side resistance according to Seidel and Collingwood (2001). Note that the mean roughness height Δr_h is 1, 2, 3, 5 mm, respectively for piles P3, P5, P7 and P9 and P4, P6, P8 and P10.										

Despite the efforts of researchers to incorporate shaft roughness into the design, the majority of pile design is still done in the absence of any consideration of this factor. This is possible for two reasons: (i) the reliable measurement of the roughness factor is not trivial, and hence is not undertaken in routine design; and (ii) the incorporation of the roughness factor is, of itself, insufficient to significantly improve the prediction of shaft resistance.

3.2. Semi-analytical approaches

To tackle the limitations related to the empirical approaches, rock masses can be better characterized by the H-B failure criterion, which allows for rock structure and surface conditions to be considered by using the Geological Strength Index (GSI). Kulhawy and Prakoso (1999) stated that practising engineers should use GSI as an alternative to the RQD to prescribe the rock surface state. As a result, this failure criterion has often been used in the literature to provide analytical solutions.

In this context, one can cite the work of Serrano and Olalla (2002, 2004). They proposed a semi-analytical solution to predict both shaft and base resistance of a pile embedded in a rock mass that obeys the 1997 H-B failure criterion. Their solutions were later generalized for the 2002 H-B failure criterion (Serrano, Olalla and Galindo, 2015). To conduct their analyses, they propose to reformulate the H-B failure criterion expression by involving the Lambe parameters p and q which led to consider simplification of equations governing the mechanical problem.

For the shaft problem, Serrano and Olalla (2004) and Serrano et al. (2015) proposed a rational calculation procedure. It was assumed that the pile strength material is greater than the rock strength. That means that the pile-rock interface is equal to the rock strength. It was also assumed that the horizontal stress σ_h (normal to the pile shaft) is proportional to the vertical one represented by the coefficient of the proportionality $k_0 = \sigma_h/\sigma_v$. At failure, stresses on the shaft are exerted according to Mohr's envelope that is consistent with the H-B failure criterion. Published solutions should be numerically solved. Thus, charts of the average unit shear stress (dimensionless) as a function of the normalized vertical stress at the pile tip were published for some practical cases of H-B core parameters. Using these charts is nevertheless not practical and does not ensure a precise reading of accurate values.

On the other hand, for the base resistance problem (Serrano and Olalla, 2002a; Serrano, Olalla and Galindo, 2014), the derivation is based on the analysis of a wall in plane strain conditions. By adopting an associated flow rule, they use the characteristic line method generalized for the non-linear failure criterion to analyse the plastic region. Nevertheless, their mathematical development was quite involved and cumbersome. De Beer's (1970) shape factor is adopted to account for the three-dimensional effect of the pile. Depending on the socket length, four different situations are distinguished: deep or semi-deep piles with overburden pressure less than or greater than the uniaxial compressive strength of the intact rock σ_{ci} .

In general, results gained from their theory, when compared to in situ test results (Serrano and Olalla, 2002b, 2006), show a high scatter and do not yield a satisfactory result which put in doubt their method and its applicability to such a problem.

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Chapter III. Bearing capacity of shallow foundation

1. Applicability

The H-B failure criterion, combined with the continuous medium assumption, has been widely used by practising engineering to study the bearing capacity problem of shallow foundation (cf. Chapter II for topic review). Considering this, plasticity theories combined with various solution techniques (upper and lower bound limit analysis, limit equilibrium method, characteristic lines method, and finite element limit analysis, etc.) were employed to solve such problems. In all published works, the effect of spacing and orientations of discontinuities was neglected, i.e., the rock mass was considered as a continuous medium, while not forgetting limitations discussed in subsection 1.1.3 of Chapter II.

Attention should indeed be paid to the scale effects when considering foundations on rock masses. Regarding the bearing capacity problem considered herein, the applicability of the H-B failure criterion is best described by referring to Figure III-1. As shown, three main groups can be differentiated for rock masses:

- Group I: The rock is intact and isotropic. The H-B failure criterion can be applied. In this case, the theory of plasticity is valid with some restrictions.
- Group II: The rock mass is affected by only a few families of discontinuities (less than four). Its behaviour is anisotropic, and the H-B failure criterion is not applicable.
- Group III: The rock mass is affected by several discontinuities giving rise to “small spacing” between discontinuities (four families or more). That is the case of an isotropic heavily jointed rock mass. The H-B failure criterion can be therefore applied, and the theory of plasticity is valid without restrictions.

It should be noted that, with intact rock mass (Group I), failure may be brittle, so the theories of plasticity may not be appropriate, and the solutions proposed thereafter are not reliable.

In line with the above discussion, Merifield et al. (2006) stated that there would be a distinction in the ultimate behaviour of “large” and “small” foundations. For “large” foundations, it seems reasonable to assume that such a problem can be studied using plasticity theories since in such case, a considerable volume of the rock mass is affected and the stresses caused by the gravity on the potential flow surfaces are comparable to that part of the strength that is due to the rock cohesion. However, with “small” foundations, a small volume of the rock mass is influenced, and that induces negligible stress caused by the rock self-weight compared to the rock strength. In this case, a brittle failure may dominate, and theories of plasticity may not be appropriate.

Finally, and as a synthesis, it will be assumed that the rock mass is either (1) isotropic intact (group I) or; (2) isotropic heavily jointed with “small spacing S ” between discontinuities compared to the foundation width B (group III) so that the H-B failure criterion, combined with the theory of plasticity, could be applied, while not forgetting s -values and stress ratio conditions discussed in Chapter II-□

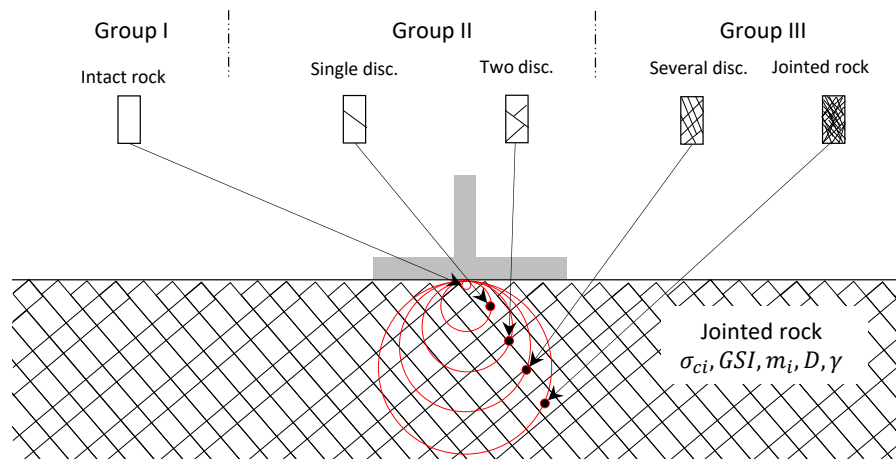


Figure III-1: Applicability of the H-B yield criterion for shallow foundations (Hoek, E. and Brown, 1980; Serrano and Olalla, 1996; Merifield, Lyamin and Sloan, 2006).

2. Bearing capacity using upper bound limit analysis

Nota:

The content of this section is submitted to *Géotechnique*

Despite the reliability of existing scientific works, the author notes that further improvements could still be suggested. An upper bound estimate for the bearing capacity preserving exactly the original generalized H-B failure criterion will be derived. Besides, solutions will be presented for both associated and non-associated flow rules.

A closed-form expression of the maximum resistant work rate developed in any virtual strain field will be derived by employing a multi tangential method to handle the nonlinear strength criterion issues. It should be emphasized that the adopted method fully considers the actual unevenly distributed stress, and it is a nonlinear limit analysis method in the strict sense.

2.1. Problem statement and basic hypothesis

The plane strain bearing capacity to be analysed is as shown in Figure III-2. A rigid strip footing of width B rests on a homogeneous and isotropic rock mass of unit weight γ located at a depth D_f from the rock mass surface. The weight of the rock mass above the footing is replaced with a surcharge load $q_0 = \gamma D_f$.

The homogenous and isotropic rock mass is idealized as a perfectly plastic material and follows the associated flow rule. The failure of the rock mass is governed by the generalized H-B failure criterion with bearing in mind the limit of applicability of this criterion as detailed previously in subsection 1.1.3 of Chapter II.

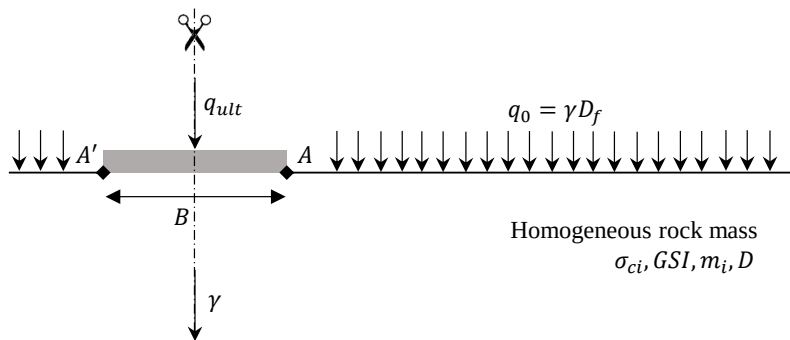


Figure III-2: Strip footing problem idealization

The system is subjected to the action exerted through the footing and surface surcharge, as well as the gravity in the rock mass. Therefore, the ultimate bearing capacity can be written as:

$$q_{ult} = \sigma_{ci}N_{\sigma_0} + q_0N_q + 0.5\gamma BN_\gamma \quad (\text{III-1})$$

where σ_{ci} is the uniaxial compressive strength of the intact rock. N_{σ_0} , N_q and N_γ are bearing capacity factors related to the uniaxial compressive strength σ_{ci} , the surcharge load q_0 and the unit weight γ of the rock mass, respectively. Note that the above equation is similar to that proposed for soil obeying the linear M-C failure criterion. The difference is that $\sigma_{ci}N_{\sigma_0}$ is used in the above equation for rock masses following the H-B failure criterion rather than that cN_c for a soil following the M-C failure criterion.

On the other hand, in the $\sigma_n - \tau$ stress plane, where σ_n and τ are the normal and shear stresses, the H-B failure criterion is expressed in a scaled form per Eqs. (II-53) – (II-56). If we consider the tangential line to the H-B curve at the location of tangency point M as shown in Figure III-3, the following expression holds:

$$\tau = c_t + \sigma_n \tan(\phi_t) \quad (\text{III-2})$$

where c_t and $\phi_t = \arctan(d\tau/d\sigma_n)$ are respectively, the intercept of the straight line to τ axes and the tangential friction angle. The normalized cohesion c_t^* and the friction angle ϕ_t are related to each other using a parametric form depending exclusively on the scaled minor principal stress σ_3^* :

$$\tan(\phi_t) \left(= \frac{d\tau^*}{d\sigma_n^*} \right) = \frac{a(\sigma_3^*)^{a-1}}{2\sqrt{d\sigma_1^*/d\sigma_3^*}} \quad (\text{III-3})$$

$$c_t^* = \frac{c_t}{\sigma_{ci}m_b^\beta} = \frac{(1-a)(\sigma_3^*)^a}{2\sqrt{d\sigma_1^*/d\sigma_3^*}} + \frac{s}{m_b^{\beta/a}} \tan(\phi) \quad (\text{III-4})$$

From the above equations, the following explicit relationship between c_t^* , ϕ_t and the H-B core parameters holds:

$$c_t^* = \frac{1-a}{2\sqrt{k_{\phi_t}}} \left(\frac{k_{\phi_t} - 1}{a} \right)^{-\beta} + \frac{s}{m_b^{\beta/a}} \tan(\phi_t); \quad k_{\phi_t} = \tan^2 \left(\frac{\pi}{4} + \frac{\phi_t}{2} \right) \quad (\text{III-5})$$

This equation is fundamental for assessing the ultimate bearing capacity. That will be detailed hereinafter.

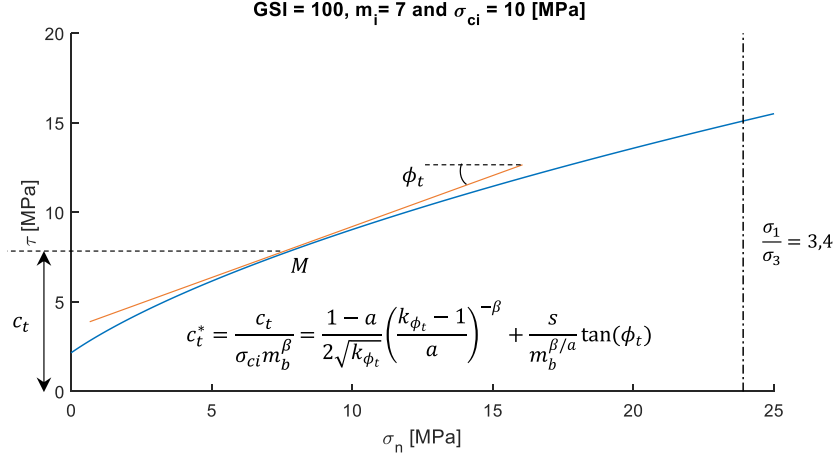


Figure III-3: H-B failure envelope in the Mohr plane – Tangential line.

2.2. Upper bound limit theorem – Governing equations

The upper bound limit theorem (Salencon, 1990) states that in a compatible mechanism of plastic deformation satisfying the velocity at the boundary of the geomaterial masses, the rate of work W_{ext} by external forces is less than or equal to the rate of maximum resisting work W_{int} in any kinematically admissible velocity fields denoted $\underline{V}$ under applied loading (γ, q_0, q_{ult}):

$$W_{ext}(\gamma, q_0, q_{ult}, \underline{V}) \leq W_{int}(\underline{V}) \quad (\text{III-6})$$

Details of how W_{ext} and W_{int} are computed will be given thereafter in subsection 2.2.2 of this chapter.

One should also quote that the above inequality expresses in dual form the compatibility between the strength capacities of the materials and the equilibrium of the structure.

To obtain a better solution (lower upper bounds), the rate of work must be done for as many trial kinematical admissible velocity fields as possible. The lowest possible upper bound solution is therefore found with an optimization scheme by trying various possible kinematically admissible failure mechanisms.

2.2.1. Multi wedge translation failure mechanism

When the limit analysis method is adopted, the chosen failure mechanism is of great importance for adequately estimating the collapse load. Therefore, the effective multi-wedge discretization model proposed by Michalowski (1997) for an M-C material will

be used for the calculation of the ultimate bearing capacity with the nonlinear H-B criterion. Since the failure mechanism is symmetric, only half of the problem domain is presented (cf. Figure III-4).

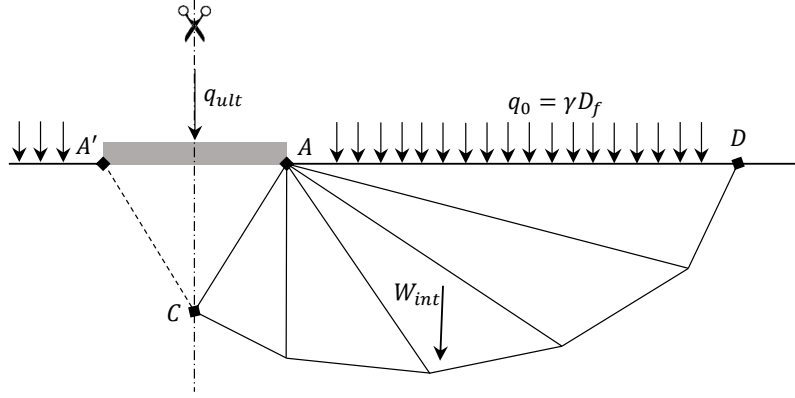


Figure III-4: Failure mechanism for bearing capacity of a strip footing

The plastic region under the strip footing is defined by $k + 1$ triangular wedges (cf. Figure III-5). The wedge $AA'B$ beneath the strip footing is an isosceles triangle characterized by an angle $\theta \in]0, \pi/2[$ and a side (interface) length $L_1 = \frac{B}{2\cos(\theta)}$. The geometry of the remaining k triangles are listed below ($i = 1, \dots, k$):

- b_i : length of the base,
- L_i : length of the interface,
- α_i : angle formed by the straight lines L_i and L_{i+1} and
- β_i : angle formed by the straight lines L_i and d_i .

The following geometric relations can be established for a triangle wedge ($i = 1, \dots, k$):

$$L_i = L_1 \prod_{j=1}^{i-1} \frac{\sin(\beta_j)}{\sin(\alpha_j + \beta_j)}; \quad L_1 = \frac{B}{2\cos(\theta)} \quad (\text{III-7})$$

$$b_i = L_i \frac{\sin(\alpha_i)}{\sin(\alpha_i + \beta_i)} \quad (\text{III-8})$$

On the other hand, the wedge $AA'C$ moves vertically as a rigid body with the same initial downwards velocity V_0 as the footing. Each wedge (i) moves with a velocity V_i inclined at ϕ_i with respect to the base b_i . Moreover, the relevant velocity $V_{i,i+1}$ along the interface L_i is inclined at $\phi_{i,i+1}$ with respect to the interface length L_i . To ensure that these velocities are kinematically admissible, it will be further assumed

that the movement of two adjoining wedges does not cause overlapping or separation. It is further assumed that the wedges are rigid (and thus incompressible) bodies. Since the movement is symmetrical, only the velocity field on the right-hand side is considered (cf. Figure III-6).

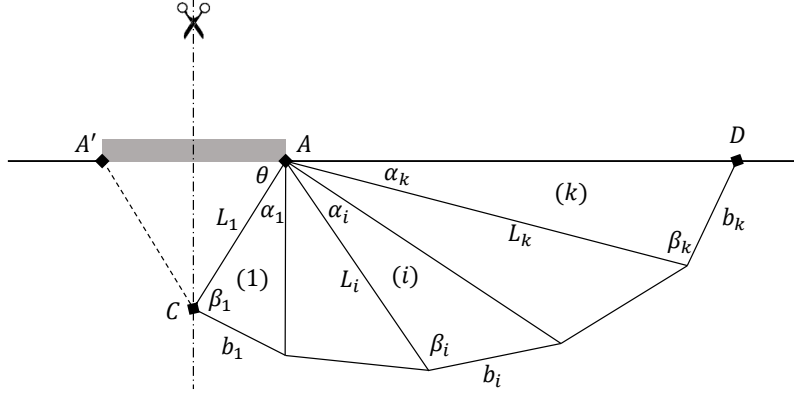


Figure III-5: Geometry details of the failure mechanism

On the other hand, the wedge velocity hodograph of the 1st and the i^{th} wedge are depicted in Figure III-7. The velocities of each wedge V_i and $V_{i,i+1}$ can be expressed as a function of initial downwards velocity V_0 and the set of angular parameters $\theta, \alpha_i, \beta_i, \phi_i$ and $\phi_{i,i+1}$ ($i = 1, \dots, k$). The following expressions hold (law of sines):

- 1st wedge

$$V_1 = V_0 \frac{\cos(\theta - \phi_{0,1})}{\sin(\beta_1 - \phi_1 - \phi_{0,1})} \quad (\text{III-9})$$

$$V_{0,1} = V_0 \frac{\cos(\beta_1 - \phi_1 - \theta)}{\sin(\beta_1 - \phi_{0,1} - \phi_1)} \quad (\text{III-10})$$

- i^{th} wedge ($i = 1, \dots, k$)

$$V_i = V_1 \prod_{j=1}^{i-1} \frac{\sin(\alpha_j + \beta_j - \phi_j - \phi_{j,j+1})}{\sin(\beta_{j+1} - \phi_{j+1} - \phi_{j,j+1})} \quad (\text{III-11})$$

$$V_{i,i+1} = V_i \frac{\sin(\alpha_i + \beta_i - \beta_{i+1} + \phi_{i+1} - \phi_i)}{\sin(\beta_{i+1} - \phi_{i+1} - \phi_{i,i+1})} \quad (\text{III-12})$$

It should be emphasized that, unlike what was assumed in previous works (Yang and Yin, 2005; Saada, Maghous and Garnier, 2008), the friction angle varies from slip surface to another, i.e., $\phi_i \neq \phi_{i,i+1}$. Similarly, the cohesion given per Eq. (III-5)

differs from a slip surface to another, i.e., $c_i = c_t(\phi_i) \neq c_{i,i+1} = c_t(\phi_{i,i+1})$. In other words, a higher degree of freedom is added to the current failure mechanism compared to the presently published ones.

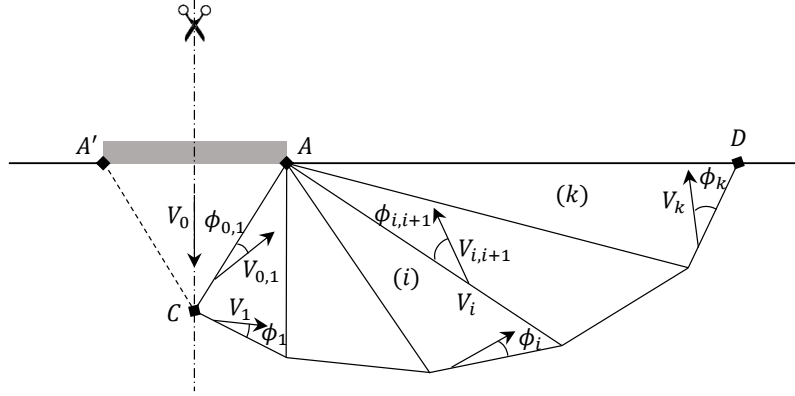


Figure III-6: Velocity field of the failure mechanism.

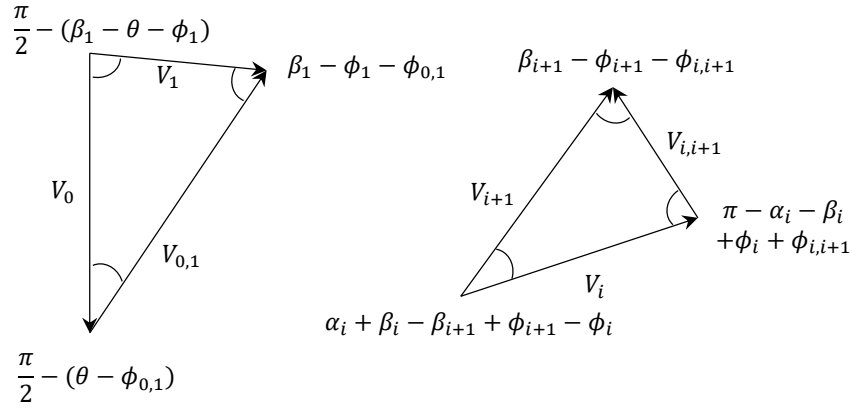


Figure III-7: Velocity hodograph for the 1st and the i^{th} wedge elements

2.2.2. Rate of work by resisting and external forces

The rate of maximum resisting work W_{int} is calculated in the first place along the wedge base b_i between the rock mass in motion and the rock mass at rest just below (cf. Figure III-5). In the second place, W_{int} is computed along the interface L_i between adjoining two wedges. In mathematical terms, W_{int} is expressed as:

$$W_{int} = \frac{B}{2} V_0 \sigma_{ci} (f_1 + f_2 + f_3) \quad (\text{III-13})$$

On the other hand, external forces include the pressure acting on the strip footing q_{ult} , the surcharge load q and the rock self-weight γ , which implies:

$$W_{ext} = \frac{B}{2} q_{ult} V_0 + \frac{B}{2} q_0 V_0 f_4 + \frac{B^2}{4} V_0 \gamma f_5 \quad (\text{III-14})$$

Equating rate of work done by internal (resisting) and external forces results in the following equation of ultimate bearing capacity to be minimized with respect to the angular parameters $\theta, \alpha_i, \beta_i, \phi_i$ and $\phi_{i,i+1}$ ($i = \{1 \dots k\}$):

$$q_{ult} = \sigma_{ci} N_\sigma = \sigma_{ci} \min_{\theta, \alpha_i, \beta_i, \phi_i, \phi_{i,i+1}} \left\{ (f_1 + f_2 + f_3) - \frac{q_0}{\sigma_{ci}} f_4 - 0.5 \frac{\gamma B}{\sigma_{ci}} f_5 \right\} \quad (\text{III-15})$$

Here N_σ is the bearing capacity factor. $f_{j=\{1 \dots 5\}}$ are non-dimensional functions that depend on angular parameters $(\theta, \alpha_i, \beta_i)$ and the instantaneous friction angles $(\phi_i, \phi_{i,i+1})$. Their mathematical expressions are reported in Appendix 2.1 (p.243). Note that the angular parameters should comply with the following constraints:

$$\left. \begin{aligned} \theta, \alpha_i, \phi_i, \phi_{i,i+1} &\in]0, \pi/2[; \beta_i \in]0, \pi[; \alpha_i + \beta_i \in]0, \pi[\\ \beta_1 - \phi_1 - \theta &\in]0, \pi/2[; \alpha_i + \beta_i - \phi_i - \phi_{i,i+1} \in]0, \pi[; \alpha_i + \beta_i \geq \beta_{i+1} \\ \alpha_{i-1} + \beta_{i-1} - \beta_i + \phi_i - \phi_{i-1} &\in]0, \pi[\end{aligned} \right\} \quad (\text{III-16})$$

The above equation of the ultimate bearing capacity expressed as a fraction of the uniaxial compressive strength is more convenient than that given per Eq. (III-1). Besides, this equation is more consistent with previous bearing capacity representations (Merifield, Lyamin and Sloan, 2006; Saada, Maghous and Garnier, 2008).

2.3. Plastic flow rule

2.3.1. Overview

Based on the theory of plasticity (Bishop, 1953; Lubliner and Moran, 1992), a material is characterized by a failure criterion $f = 0$ and a plastic potential ϕ . In the broadest sense, if the model includes hardening or softening, the failure criterion and plastic potential depend not only on the stress tensor $[\sigma_{ij}]$, but also on a plastic or softening parameter. However, there is no need to take the plastic parameter into consideration if a perfectly plastic behaviour is assumed.

One can also quote that if the plastic potential coincides with the failure criterion ($f \equiv \phi$), the material is said to obey the normality rule (i.e. AFR). In such a case, the plastic strain increment vector is normal to the failure curve and points outwards from it.

Although normality can be reasonably assumed for clayey soils (Graham, Noonan and Lew, 1983; Wood, 2014), it is clear that the failure criterion and the plastic potential cannot be considered to be identical for many materials such as granular soils and rock masses, where an N-AFR prevails. That entails the introduction of an independent angle of dilatancy ψ that controls the inelastic volume changes. This angle is not only a suitable parameter for the description of soils, but also appears to be useful for rocks, as described by Rudnicki and Rice (1975) and Vermeer and de Borst (1984) among others. A critical review of existing empirical and theoretical models of determining dilation is undertaken in Subsection 2.3.2 of this chapter.

On the other hand, the use of an AFR is standard in rigorous limit analysis, since only for the normality rule can the upper and lower bound theorem be proven true. However, if an N-AFR is assumed, one reasonable approach is to find an energetically equivalent yield surface that provides the same rate of energy dissipation as the original one but with an AFR. This approach was firstly proposed by Davis (1968) and Rowe (1969) for a coaxial material (the coincidence of principal directions of stresses and deformations) and used later by Drescher and Detournay (1993) and Michalowski (1997) to investigate the effect of N-AFR on ultimate load for a linear M-C material. They concluded that the ultimate bearing capacity could be evaluated using the same AFR technique, but with equivalent cohesion c_{eq} and equivalent friction angle ϕ_{eq} as given below:

$$\tan(\phi_{eq}) = k_{\psi,\phi} \tan(\phi) \quad (\text{III-17})$$

$$c_{eq} = k_{\psi,\phi} c \quad (\text{III-18})$$

Here $k_{\psi,\phi}$, which is illustrated in Figure III-8, is a function of the friction and dilatancy angles expressed as:

$$k_{\psi,\phi} = \frac{\cos(\psi) \cos(\phi)}{1 - \sin(\psi) \sin(\phi)} \quad (\text{III-19})$$

In the case of a material obeying the H-B criterion, cohesion, friction angle, and dilatancy angle appearing in the above equations should be treated as instantaneous parameters because of their dependency on the confining stress. Once these latter are known, the bearing capacity could be calculated per Eq. (III-15), with c_t replaced by $(c_t)_{eq}$ and ϕ_t replaced by $(\phi_t)_{eq}$. Details on how these parameters are determined, based on the proposed dilatancy angle model, are provided below.

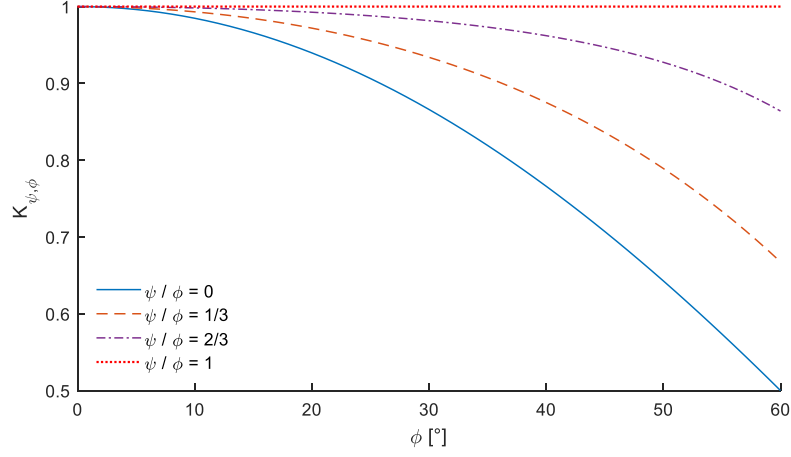


Figure III-8: Friction and dilatancy factor for use to calculate the equivalent cohesion c_{eq} and equivalent friction angle ϕ_{eq} to assess the ultimate bearing capacity (Drescher and Detournay, 1993; Michalowski, 1997)

2.3.2. Dilatancy angle model

To provide some background to the dilatancy angle model developed in this subsection, observations and proposals made by a range of researchers are firstly reviewed.

a. Background

One of the simplest and most commonly used plastic potential function, which originally emanates from the linear M-C failure criterion, is expressed as a function of a constant dilatancy angle ψ as:

$$\varphi : \sigma'_1 - k_\psi \sigma'_3 \leq 0 ; \quad k_\psi = \frac{1 + \sin \psi}{1 - \sin \psi} \quad (\text{III-20})$$

Early proposals concerning dilatancy generally assumed as either one of the following two constants: (1) $\psi = 0^\circ$ in an N-AFR or (2) $\psi = \phi$ in an AFR. Later on, the common practice is to define a constant dilatancy angle equals to $\psi = \phi - 20^\circ$. This rule of thumb originates from the recommendation of Vermeer and Borst (1984) stating that the dilatancy angle is at least 20° less than the friction angle. They also reported some typical values for ψ as listed in Table III-1. These values were estimated using the following formula:

$$\sin \psi = \frac{\dot{\varepsilon}_v^p}{-2\dot{\varepsilon}_v^p + \dot{\varepsilon}_1^p} \quad (\text{III-21})$$

where $\dot{\epsilon}_v^p$ and $\dot{\epsilon}_1^p$ are, respectively, the volumetric and axial plastic strain increments. An irreversible increase of volume is manifested by a positive dilatancy angle ($\psi > 0^\circ$) while a volume decrease (Plastic contraction) is manifested by a negative one ($\psi < 0^\circ$). $\psi = 0^\circ$ is the special case of incompressible flow.

Table III-1: Typical values of dilation angle for some materials as reported by Vermeer and Borst (1984)

Dense sand	Loose sand	Normally consolidated clay	Intact marble	Concrete
15°	$< 10^\circ$	0°	$12^\circ - 20^\circ$	12°

Hoek and Brown's (1997), and based on wide engineering experience, suggested the use of a constant dilatancy angle value related to the friction angle and rock mass quality. They proposed a value of $\psi = \phi/4$ for excellent and good quality rock mass, $\psi = \phi/8$ for average quality rock mass and $\psi = 0$ for poor quality rock mass.

However, the assumption of a “constant” dilatancy angle is unrealistic, as highlighted by Detournay (1986). He claimed that dilation must necessarily be a function of plastic strain and confining stress. He proposed the following dilatancy factor $k_\psi = \frac{1+\sin\psi}{1-\sin\psi}$ that decays exponentially from an initial frictional factor $k_\phi = \frac{1+\sin\phi}{1-\sin\phi}$ as a function of plastic shear strain γ^p as:

$$k_\psi = 1 + (k_\phi - 1)e^{-\gamma^p/\gamma^{p*}} \quad (\text{III-22})$$

where γ^{p*} can most usefully be related to the maximum inelastic volume increase Δ^* as:

$$\gamma^{p*} = \Delta^* / \ln\left(\frac{k_\phi - 1}{2}\right) \quad (\text{III-23})$$

Nevertheless, this interesting flow rule does not seem to be widely used in practice.

Medhurst (1997) conducted a highly reliable series of triaxial compressive tests on multi-scale coal samples (diameter of 61, 101, 146, and 300 mm) submitted to different confining stresses in the range of 0.2 MPa and 4 MPa. They pointed to the decrease in dilation as the confining stress increases. Medhurst's test results (1997) were later reinterpreted by Alejano and Alonso (2005), with the aim to develop a consistent model for estimating rock mass dilatancy, with potential applications in the tunnel and mine design. Based on plasticity dependency in the flow rule of Eq. (III-22), they proposed a dilation angle decay model using Detournay's definition of γ^{p*} (cf. Eq. (III-23)):

$$k_{\psi} = 1 + (k_{\psi^{peak}} - 1)e^{-\gamma^p/\gamma^{p*}} \quad (\text{III-24})$$

where ψ^{peak} is calculated as a function of UCS and the confining stress σ_3 as:

$$\psi^{peak} = \frac{\phi}{1 + \log_{10} \sigma_{ci}} \log_{10} \left(\frac{\sigma_{ci}}{\sigma_3 + 0.1} \right); [\sigma_{ci}] = [0.1] = \text{MPa} \quad (\text{III-25})$$

Here ϕ is estimated from the slope of the H-B failure criterion as per Eq. (III-3). As shown in Figure III-9, the above equation gives $\psi^{peak} = \phi$ for null confining stress ($\sigma_3 = 0 \text{ MPa}$) and as the value of σ_3 increases, ψ^{peak} value decreases nonlinearly until the value of zero is reached (for $(\sigma_3)_{\psi=0^\circ} [\text{MPa}] = \sigma_{ci} - 1/10$). Beyond this stress value, a negative dilatancy angle is assessed which reflects a plastic contraction. Special attention should therefore be paid if this flow rule is adopted, especially for problems with high stress levels. The accuracy of this model will be examined later in this subsection when validating the proposed dilatancy angle model.

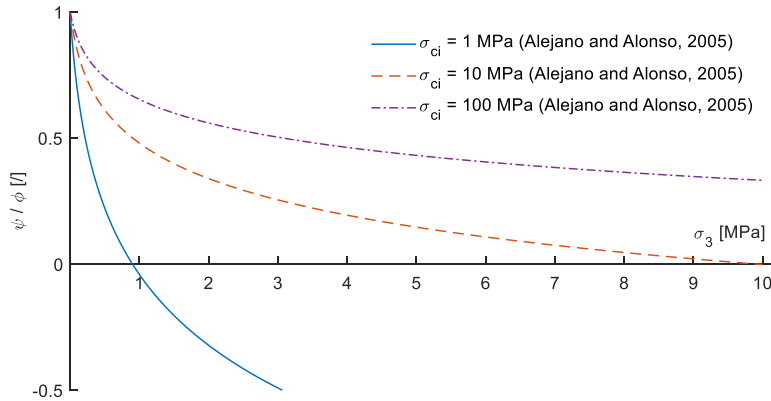


Figure III-9: Dilatancy angle model – (Alejano and Alonso, 2005).

Based on published data acquired from triaxial compression tests conducted on seven rock types – Witwatersrand quartzite (Crouch, 1970) / medium-grained sandstone (strong), silty sandstone and mudstone (Farmer, 1983) / medium-grained sandstone (weak), and seatearth (Hassani, White and Branch, 1984) / Moura coal (Medhurst, 1997) – at different confining stresses with volumetric strain measurement, Zhao and Cai (2010) suggested the following expression:

$$\psi = c_1 c_2 [\exp(-c_2 \gamma^p) - \exp(-c_3 \gamma^p)] / (c_3 - c_2) \quad (\text{III-26})$$

where γ^p is the plastic shear strain in %. c_1 , c_2 and c_3 are fitting coefficients that vary with σ_3 according to the following equations:

$$c_i = c_{i,1} + c_{i,2} \exp\left(-\frac{\sigma_3}{c_3}\right); i = \{1,2\} \quad (\text{III-27})$$

$$c_3 = c_{3,1} + c_{3,2}(\sigma_3)^{c_{3,3}}$$

They found Eq. (III-26) to be in good agreement with experimental results.

Arzúa and Alejano (2013) performed a series of unconfined and confined compressive tests on three different intact granitic rock samples locally known as Amarelo País, Blanco Mera, and Vilachán. The three granitic rocks were characterized with the H-B criterion and their H-B core parameters were reported as summarized in Table III-2. Furthermore, special attention was paid to the dilatancy angle. Experimental results once again reveal the dilatancy angle dependencies on confining stress and plastic shear strain. It was found that the proposed by Alejano and Alonso (2005) (cf. Eq. (III-25)) did not fit the laboratory data in terms of peak dilatancy angle. However, the process of dilation decay was reasonably well represented by Zhao and Cai's (2010) model (cf. Eq. (III-26)).

Table III-2: H-B core parameters of three granitic rocks as reported by Arzúa and Alejano (2013).

Granitic rock	σ_{ci} [MPa]	m_i	s	a
Amarelo País	76.6	41.0	1	0.5
Blanco Mera	135.5	37.8	1	0.5
Vilachán	116.3	28.2	1	0.5

Walton *et al.* (2015) performed a highly reliable series of uniaxial and triaxial compressive tests on Indiana limestone, Carrara marble, and Toral de Los Vados (TLV) Limestone. They noted that the experimental strength envelope shape is adequately represented by an H-B fit. H-B core parameters were reported as listed in Table III-3. In general, their results are consistent with those reported in the literature for the same rock type – Carrara marble: $\sigma_{ci} = 93.6 \text{ MPa}$ (Howarth and Rowlands, 1987) / Indiana limestone: $\sigma_{ci} = 56.9 \text{ MPa}$ (Cargill and Shakoor, 1990) and $m_i = 5.5$ (Ramamurthy, 2001). As for dilatancy angle, data were found to show a trend similar to that of the model proposed by Zhao and Cai (2010) .i.e., as per Eq. (III-26). The peak dilatancy angle was found to decrease with increasing confinement whereas the rate of dilatancy angle decay with continued strain was found to increase with increasing confinement.

Table III-3: H-B core parameters of three rock types as reported by Walton et al. (2015).

Rock type	σ_{ci} [MPa]	m_i	s	a
Indiana limestone	52.6	7.1	1	0.5
Carrara marble	94.3	5.5	1	0.5
Toral de Los Vados Limestone	116.8	21.8	1	0.5

Arzúa and Alejano's results (2013), as well as those of Walton et al. (2015), among others, are reinterpreted in the following subsection to validate the proposed dilatancy angle model.

b. Dilatancy angle model – Governing equations

We aim to propose a sufficient significant but conveniently simple formulation of a dilatancy angle model of a perfectly plastic material that obeys the H-B criterion. This model should be readily implemented in numerical and analytical codes, and it should notably be employed for the bearing capacity problem investigated in this chapter.

The simplest and most efficient method is to select a plastic potential function similar to that of failure. Dilation is then defined using a dimensionless parameter denoted as $m_{b.dil}$, different from the H-B strength parameter m_b that allows us to consider the non-linear character of the H-B failure criterion. The parameter $m_{b.dil}$ varies between zero (N-AFR with no volume change) and the H-B derived parameter m_b (AFR). Thus, an N-AFR occurs when the plastic potential function given by:

$$\mathcal{G} : \sigma'_1 - \sigma'_3 - \sigma_{ci} \left(m_{b.dil} \frac{\sigma'_3}{\sigma_{ci}} + s \right)^a \leq 0 \quad (\text{III-28})$$

is different from the yield condition, i.e. $m_{b.dil} \neq m_b$. Otherwise, an AFR prevails. In other words, let α be the ratio between $m_{b.dil}$ and m_b , i.e.:

$$\alpha = \frac{m_{b.dil}}{m_b} \begin{cases} = 1 & \text{AFR} \\ < 1 & \text{N - AFR} \end{cases} \quad (\text{III-29})$$

Note that the potential function is expressed in a scaled form as:

$$\mathcal{G} : \sigma_1^{*dil} - \sigma_3^{*dil} - (\sigma_3^{*dil})^a \leq 0 \quad (\text{III-30})$$

The superscript '*dil*' in the above equation means that stresses are scaled using dilatancy strength parameters $m_{b.dil}$. Figure III-10 represents graphically both yield and plastic potential functions for a given H-B parameter. However, it would be more appropriate to introduce the dilatancy angle ψ since, for most practitioners, there is an intuitive feel for the physical meaning of ψ , as for ϕ , and that it is not the case for $m_{b.dil}$. Given that ψ depends on the confining stress, it will be thus treated as an instantaneous angle referred to by a subscript "t". If we consider the tangential line to the plastic potential curve at the location of the tangency point M as illustrated in Figure III-10, the following expression holds:

$$k_{\psi_t} = \frac{1 + \sin \psi_t}{1 - \sin \psi_t} = \tan^2 \left(\frac{\pi}{4} + \frac{\psi_t}{2} \right) = \frac{d\sigma_1^{*dil}}{d\sigma_3^{*dil}} = 1 + a(\sigma_3^{*dil})^{a-1} \quad (\text{III-31})$$

ψ_t can be further expressed as a function of $m_{b.dil}$ (i.e. $\alpha = \frac{m_{b.dil}}{m_b}$) and σ_3^* by substituting the term σ_3^{*dil} by:

$$\sigma_3^{*dil} = \frac{\sigma_3^*}{\alpha^\beta} + \frac{s}{m_b^{\beta/a}} \left(\frac{1}{\alpha^{\beta/a}} - \frac{1}{\alpha^\beta} \right) \quad (III-32)$$

which results in the following fundamental expression:

$$k_{\psi_t} = 1 + a \left[\frac{\sigma_3^*}{\alpha^\beta} + \frac{s}{m_b^a} \left(\frac{1}{\alpha^a} - \frac{1}{\alpha^\beta} \right) \right]^{a-1} \quad (III-33)$$

For the bearing capacity problem studied in this chapter, it is essential to relate both dilatancy and friction angles for use in Eq. (III-15) combined with Eqs. (III-17) to (III-19).

Considering this, the instantaneous friction angle can be calculated from the slope of the tangent to the H-B yield function, i.e.

$$k_{\phi_t} = \frac{1 + \sin \phi_t}{1 - \sin \phi_t} = \tan^2 \left(\frac{\pi}{4} + \frac{\phi_t}{2} \right) = \frac{d\sigma_1^*}{d\sigma_3^*} = 1 + a(\sigma_3^*)^{a-1} \quad (III-34)$$

The above equation can be further expressed as:

$$\sigma_3^* = \left(\frac{k_{\phi_t} - 1}{a} \right)^{-\beta/a} \quad (III-35)$$

Substituting Eq. (III-35) into Eq. (III-32) and thereafter into Eq. (III-31) results in a fundamental explicit relationship between ψ_t and ϕ_t for use in Eq. (III-15) combined with Eqs. (III-17) to (III-19) to evaluate the ultimate bearing capacity. The following holds:

$$k_{\psi_t} = 1 + a \left[\frac{\left(\frac{k_{\phi_t} - 1}{a} \right)^{-\frac{\beta}{a}}}{\alpha^\beta} + \frac{s}{m_b^a} \left(\frac{1}{\alpha^a} - \frac{1}{\alpha^\beta} \right) \right]^{\frac{a}{\beta}} \quad (III-36)$$

From the above equations, one can note that this model is marked by its conveniently simple formulation thanks to its dependency on one unknown parameter $m_{b.dil}$ (i.e. $\alpha = \frac{m_{b.dil}}{m_b}$) that can be easily obtained from simple triaxial compressive tests (This will be detailed in the next subsection “c.”). Furthermore, one can easily

switch from an AFR ($\alpha = 1$) to an N-AFR ($\alpha < 1$). ψ curves are plotted in Figure III-11 for α equal to 1 (AFR), 0.25 and 0.1. As can be seen, by further increasing the confining stress, and unlike Alejano and Alonso's model (2005) (cf. Figure III-9), the dilatancy angle value tends to zero (critical state), which is in accordance with Vermeer and Borst's finding (1984).

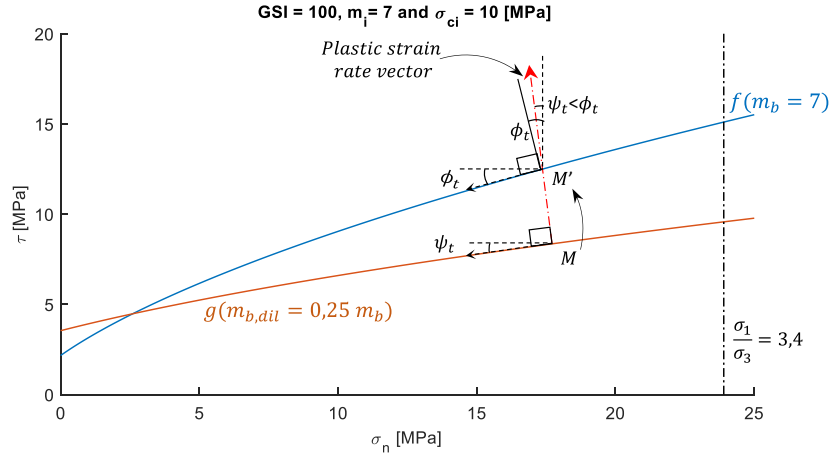


Figure III-10: Failure envelope and potential plastic function of the H-B failure criterion.

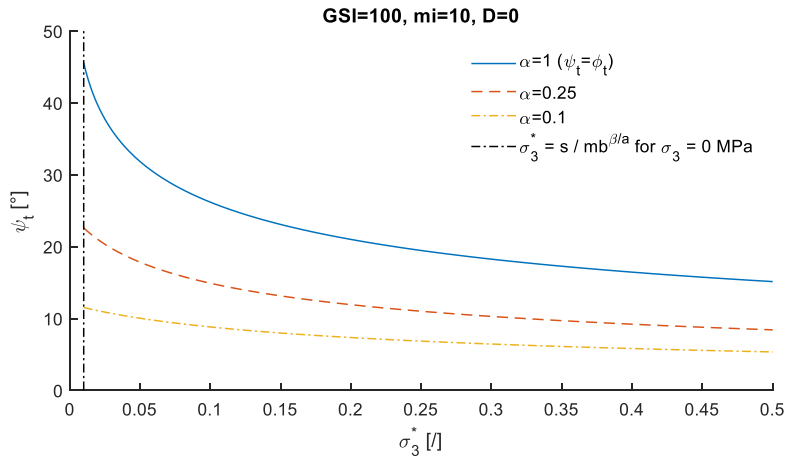


Figure III-11: Evolution of the dilatancy angle as a function of the scaled confining stress for different cases of non-associated flow rule (cf. Eq. (III-33)).

It can be noted that this model incorporates the dependence of the dilatancy angle only on the confining stress. At first sight, this sounds very reasonable in particular for perfectly plastic post-failure behaviour. However, and considering the efficiency of existing dilatancy angle models (as reviewed at the beginning of this subsection) that incorporate both confining stress and plastic shear strain γ^p , it would be very valuable

to further incorporate γ^p to enhance the proposed model in particular when strain-softening post-failure behaviour is encountered. One can follow Detournay (1986) and Alejano and Alonso (2005) in adopting an exponential decay function for k_{ψ} that gradually drops from its initial value, which corresponds to the previously calculated peak value (cf. Eq. (III-36)), to a value of one, which corresponds to a zero plastic volume increase. This null value is proposed in the light of the fact that a rock cannot dilate infinitely, as described by Vermeer and Borst (1984) and Detournay (1986) among others, although negative values (representing contractions) have been reported for highly porous rocks submitted to very high confinement stress, which is not a very common occurrence. Baud et al.'s (2009) experiments on Saint-Maximin Limestone are part of this scarcity. One can thus propose:

$$k_{\psi_t} = 1 + (k_{\psi_t^{peak}} - 1) e^{-\gamma^p / \gamma^{p*}} \quad (\text{III-37})$$

Here $k_{\psi_t^{peak}}$ is given per Eq. (III-36). γ^{p*} is a plastic constant that should be estimated for each rock type. Its value can be calculated according to:

$$\gamma^{p*} = -\gamma^p \left[\ln \left(\frac{k_{\psi_t} - 1}{k_{\psi_t^{peak}} - 1} \right) \right]^{-1} \quad (\text{III-38})$$

Medhurst's test results on coal samples (1997) as reinterpreted by Alejano and Alonso (2005) reveal an average γ^{p*} value of around 20 mstrain with moderate variability.

c. Dilatancy angle model – Validation

In this subsection, Arzúa and Alejano's results (2013), as well as those of Walton et al. (2015) are employed to validate the proposed dilatancy angle model. These results are derived from a highly reliable series of uniaxial and triaxial compressive tests on two different intact granitic rock samples locally known as Blanco Mera and Vilachán by Arzúa and Alejano (2013) and on Indiana limestone, Carrara marble, and Toral de Los Vados (TLV) Limestone by Walton et al. (2015). It was noted that the experimental strength envelope shape of the aforementioned rock samples is adequately represented by an H-B fit and the H-B core parameters are as listed in Table III-2 and Table III-3. These H-B parameters, and by resorting to Eq. (III-34), translate to a decreasing instantaneous friction angle for increasing confining stress as plotted in Figure III-12 and Figure III-13. Furthermore, experimental results from Arzúa and Alejano (2013) and Walton et al. (2015) include measurements of the peak dilatancy angle that decreases with increasing confining stress (cf. Figure III-12 and Figure III-13). Therefore, the proposed dilatancy angle model can be easily calibrated, i.e. find out the value of $\alpha = m_{b,dil}/m_b$, by resorting to Eq. (III-36). As can be seen in Figure III-12, for the granitic rock mass, the fit quality is quite good, with R^2 values

falling above 0.9. The same finding goes for the Carrara Marble (cf. Figure III-13). However, for the TLV rock mass and the Indiana limestone (cf. Figure III-13), it is rather the dilatancy model of Alejano and Alonso (2015) that offers a fairly fit quality compared to the proposed one.

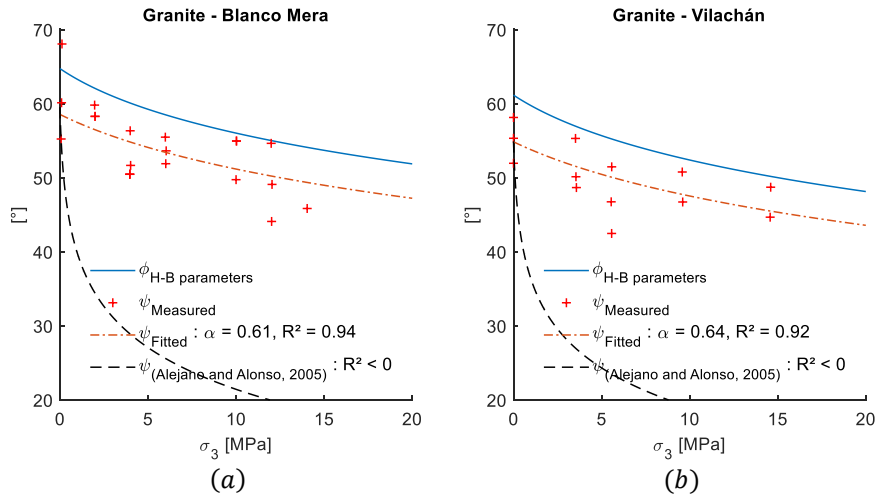


Figure III-12: Peak dilatancy angle vs. confining stress – Comparison between measured values and predicted ones according to the proposed peak dilatancy angle model (cf. Eq. (III-33)) and Alejano and Alonso's model (2005) (cf. Eq. (III-25)) : (a) Granite Blanco Mera / (b) Granite – Vilachán (Arzúa and Alejano, 2013)

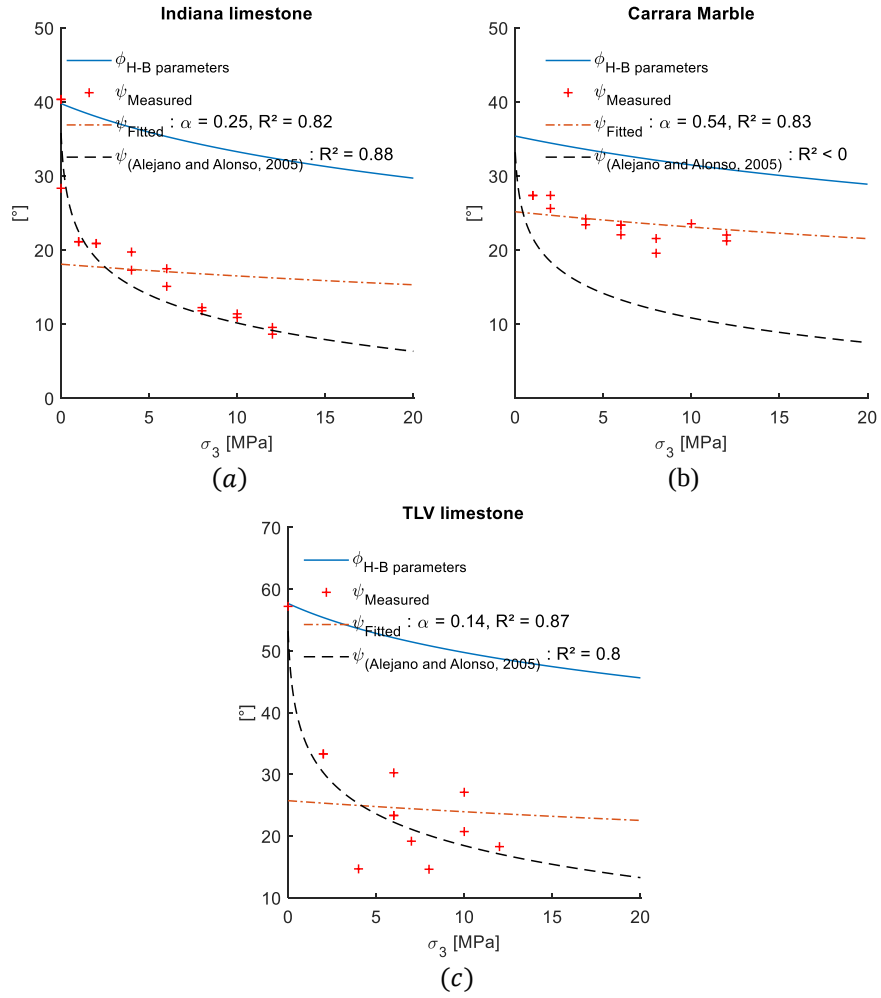


Figure III-13: Peak dilatancy angle vs. confining stress – Comparison between measured values and predicted ones according to the proposed peak dilatancy angle model (cf. Eq. (III-33)) and Alejano and Alonso's model (2005) (cf. Eq. (III-25)) : (a) Indiana Limestone / (b) Carrara marble and (c) TLV limestone (Walton et al., 2015).

2.4. Results and discussion

Using the upper bound analysis technique, the bearing capacity of a strip footing resting in rock mass obeying the H-B criterion is investigated. As shown previously (cf. Figure III-5), this technique is based on the discretization of the rock domain beneath the footing into k triangular wedges. Theoretically, increasing the wedge number will improve the lower upper bound estimate significantly. In this light, Soubra (1999) and Saada et al. (2008) showed that selecting $k = 14$ is sufficient in the context of M-C and H-B failure criteria, respectively. This value is therefore adopted throughout the present analysis.

On the other hand, it should be recalled that the upper bound solution given per Eq. (III-15) is obtained by minimization as a function of the angular parameters $\theta, \alpha_i, \beta_i, \phi_i$ and $\phi_{i,i+1}$ ($i = \{1 \dots k\}$). That corresponds to $4k + 1$ variables, subjected to the constraints given per Eq. (III-16).

In the following, we present and discuss: (1) the bearing capacity of weightless rock mass using both AFR and N-AFR, (2) the effect of the surcharge load, and (3) the effect of rock self-weight on bearing capacity values.

2.4.1. Weightless rock mass without surcharge load

The bearing capacity is firstly investigated for a strip footing resting on weightless rock mass without surcharge load q_0 . In this case, the bearing capacity solution (Eq. (III-15)) reduces to:

$$q_{ult} = \sigma_{ci} N_{\sigma_0} = \sigma_{ci} \min_{\theta, \alpha_i, \beta_i, \phi_i, \phi_{i,i+1}} \{f_1 + f_2 + f_3\} \quad (\text{III-39})$$

To evaluate the validity of the proposed method, results are firstly compared to the finite element solutions (average of upper and lower bound estimates) and the upper bound solutions obtained, respectively, by Merifield et al. (2006) and Saada et al. (2008). Since these latter were obtained by assuming an AFR, the dilatation parameter $m_{b,dil}$ will be set equal to the strength parameter m_b , i.e. $\alpha = 1$.

Results gained from the present analysis are on the conservative side compared to published solutions (Merifield, Lyamin and Sloan, 2006; Saada, Maghous and Garnier, 2008). Table III-4 summarizes results gained from the present work solution and the published ones for the case of $m_i = 10$. It can be noted that the estimates derived in this work improve those obtained in the aforementioned works. This improvement is attributed to the fact that the proposed methodology preserves exactly the original generalized H-B failure criterion.

For practical purposes in rock engineering, several computations of the bearing capacity factor have been performed considering the following ranges for the strength parameters: $GSI \in [10,100]$, $m_i \in [5, 7, 10, 15, 17, 20, 25, 30]$ and $D \in [0, 0.1, 0.2, 0.3, 0.6]$. It should be emphasized that the GSI range $GSI \in [80,100[$ for which the s-values condition is not satisfied is provided for the sake of completeness.

Furthermore, the effect of the flow rule on N_{σ_0} is carefully incorporated using different dilatancy parameter $\alpha = m_{b,dil}/m_b$ equals to $[0, 0.1, 0.25, 1]$. A value of $\alpha = 1$ (i.e. $\psi_t = \phi_t$) means that the rock obeys the AFR and the value of $\alpha = 0$ (i.e. $\psi_t = 0^\circ$) means that the rock obeys the N-AFR with no volume change. The obtained numerical results are summarized in Table 1 – Table 20 of Appendix 2.2 (p. 245 – 256).

Table III-4: Bearing capacity factor $N_{\sigma_{ci}}$: Present work (cf. Eq.(III-39)) vs Published results ($\gamma = 0 \text{ kN/m}^3, q_0 = 0 \text{ kPa}, D = 0, m_i = 10$ and $\alpha = 1$ (AFR)).

GSI	$N_{\sigma_0} = q_{ult}/\sigma_{ci} \text{ []}$		
	Present work	Merifield et al. (2006)	Saada et al. (2008)
100	8.245	8.896	–
90	5.297	5.758	–
80	3.419	3.745	3.753
70	2.215	2.444	2.451
60	1.436	1.597	1.602
50	0.927	1.037	1.041
40	0.586	0.659	0.663
30	0.351	0.397	0.399
20	0.184	0.209	0.210
10	0.068	0.077	0.078

Figure III-14 provides the variation of N_{σ_0} with m_i for different values of GSI and α . It can be noted that N_{σ_0} increases with an increase in the value of GSI and m_i . Furthermore, for a given GSI and m_i , as dilatancy parameter $\alpha = m_{b,dil}/m_b$ decreases, the bearing capacity factor naturally decreases too.

An approximate explicit solution of the bearing capacity factor N_{σ_0} can be provided. The approximate solution is expressed as:

$$N_{\sigma_0} = (a_1 m_i^{b_1} + c_1) \exp((a_2 m_i^{b_2} + c_2) GSI) \quad (\text{III-40})$$

Here a_j, b_j and c_j ; $j=1,2$ are parameters depending on the dilatancy ratio α . These latter are summarized in Table III-5. Note that values of the bearing capacity factor gained from the above expression agree within 5% the exact one.

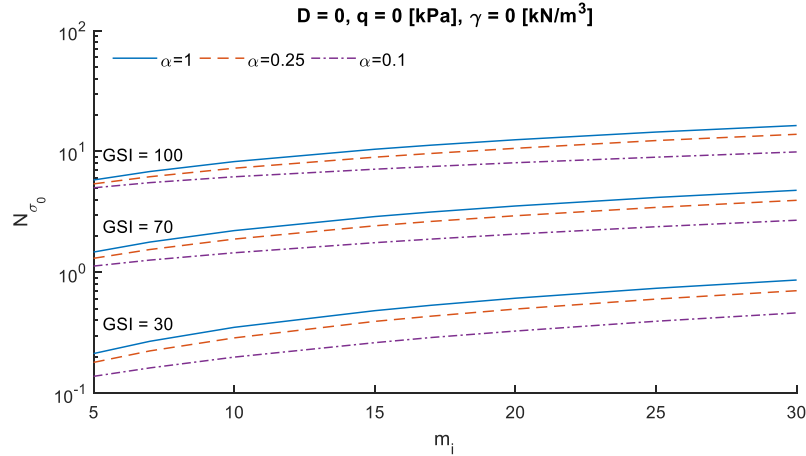


Figure III-14: Bearing capacity factor for weightless rock without surcharge load.

Table III-5: Parameters of the approximate solution of the bearing capacity factor.

		$\alpha = 1$	$\alpha = 0.25$	$\alpha = 0.1$	$\alpha = 0$
a_1	$\times 10^3$	11.060	6.419	1.295	73.400
a_2		48.100	54.280	-0.9500	23.240
b_1		979.900	1070.000	1359.000	-313.500
b_2		-64.890	-73.490	689.100	219.500
c_1		13.150	17.750	27.330	-22.380
c_2		1.792	-01.510	52.150	20.8600

For illustration purposes, the critical slip surface obtained from the numerical optimization corresponding to $k = 14$, $\gamma = 0$ kN/m³, $q = 0$ kPa, $D = 0$, $GSI \in \{30, 50, 70, 100\}$, $m_i \in \{5, 7, 10\}$ and $\alpha = m_{b,dil}/m_b \in \{0, 0.25, 1\}$ are depicted in Figure III-15 – Figure III-16. The footing width B is set equal to 1 m. Since the failure mechanism is symmetrical, only half of the failure surfaces are illustrated. It can be observed that the critical failure mechanism is composed of radial shear regions fanning between the active triangular wedge immediately beneath the footing and a Rankine passive wedge. Besides, for a given GSI , as m_i increases, so do the extent of the shear regions as well as the Rankine passive wedge. This effect is expected since the increase of m_i results in an increase in the strength of the rock mass, which subsequently induces the increase of the instantaneous M-C parameters. However, for a given m_i and an increasing GSI , this effect is reversed, and a reduction of the extent of both shear regions and Rankine passive wedge is observed. Moreover, for a given GSI and m_i , as dilatancy parameter $\alpha = m_{b,dil}/m_b \leq 1$ decreases, an enlargement of the extent of shear regions, and the Rankine passive wedges are noticed. These observations are consistent with Veiskarami et al.'s finding (2014) for sandy soil.

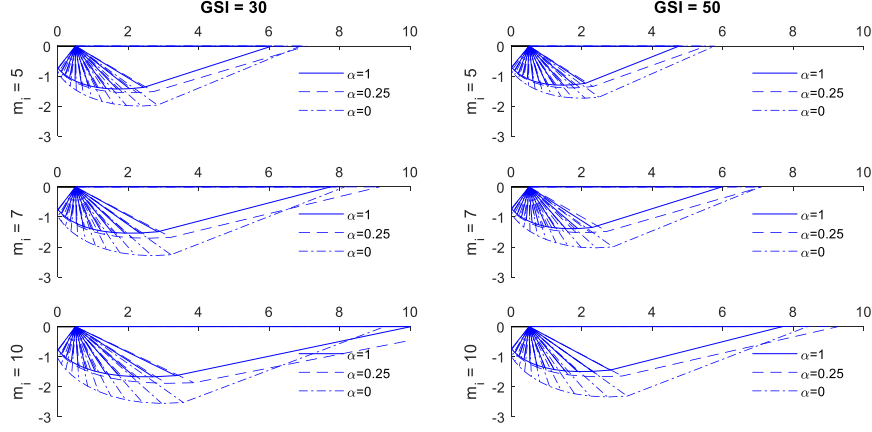


Figure III-15: Critical slip surface of a strip footing resting on weightless rock mass for different H-B core parameters and different flow rules – GSI = {30, 50}.

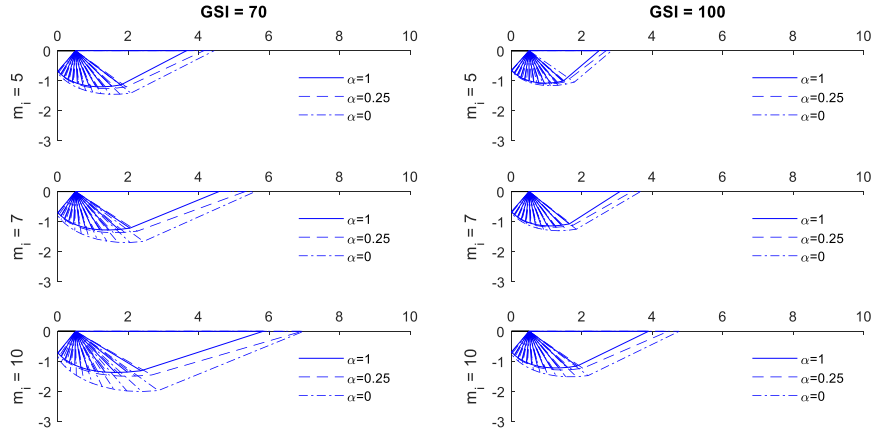


Figure III-16: Critical slip surface of a strip footing resting on weightless rock mass for different H-B core parameters and different flow rules – GSI = {70, 100}.

2.4.2. Effect of surcharge

In the presence of a surcharge load q_0 , the bearing capacity solution for a weightless rock mass writes:

$$q_{ult} = \sigma_{ci} N_\sigma = \sigma_{ci} \min_{\theta, \alpha_i, \beta_i, \phi_i, \phi_{i,i+1}} \left\{ (f_1 + f_2 + f_3) - \frac{q_0}{\sigma_{ci}} f_4 \right\} \quad (\text{III-41})$$

To study the effect of the surcharge load, the nondimensional term q_0/σ_{ci} belonging to the following range $\{0, 10^{-2}, 10^{-1}\}$ is adopted. Figure III-17 and Figure III-18

illustrates this effect for an $m_i \in \{5,15\}$ and $GSI \in \{30,50,80,100\}$. The AFR and a N-AFR with $\alpha = 0.1$ are assumed.

The bearing capacity factor increases with increasing surcharge load. Furthermore, for a given surcharge load, the higher the GSI, the greater the value of N_σ compared to N_{σ_0} .

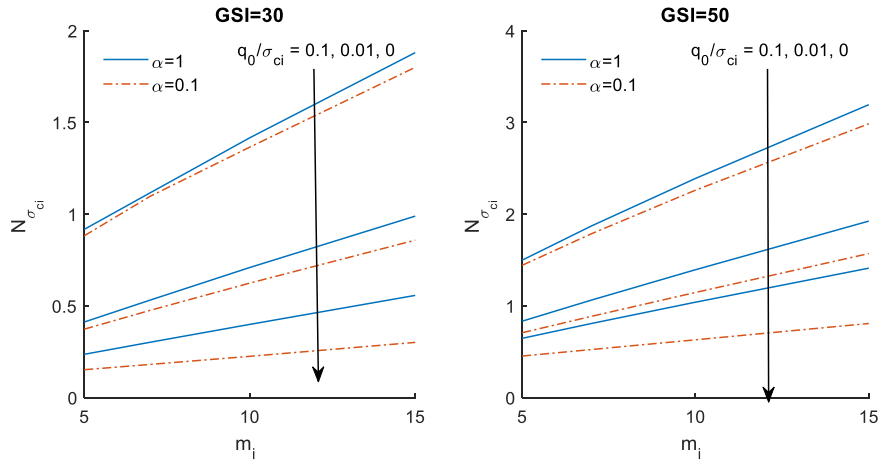


Figure III-17: Effect of the surcharge load q_0 on bearing capacity factor for $GSI = \{30,50\}$.

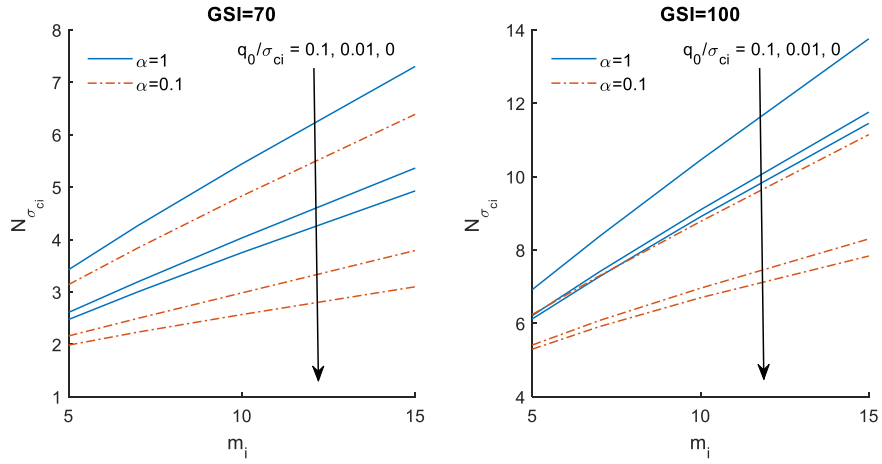


Figure III-18: Effect of the surcharge load q_0 on bearing capacity factor for $GSI = \{80,100\}$.

2.4.3. Effect of rock self-weight

We deal with the situation of a self-weight rock mass ($\gamma \neq 0 [kN/m^3]$) and absence of surcharge load ($q_0 = 0$). Thus, the bearing capacity solution reduces to:

$$q_{ult} = \sigma_{ci} N_{\sigma_{ci}} = \sigma_{ci} \min_{\theta, \alpha_i, \beta_i, \phi_i, \phi_{i,i+1}} \{ (f_1 + f_2 + f_3) - 0.5 \frac{\gamma B}{\sigma_{ci}} f_5 \} \quad (\text{III-42})$$

Analogously to Merrifield et al. (2006), the non-dimensional factor $\frac{\sigma_{ci}}{\gamma B}$ which belongs to the following range $\{125, 250, 500, 1000, 2000, 5000, 10000, \infty\}$ is used to study the effect of the rock weight γ on bearing capacity values. A value of $\frac{\sigma_{ci}}{\gamma B} = \infty$ means that the rock is weightless ($\gamma = 0$).

For $GSI \in \{30, 70, 100\}$, the bearing capacity factor gained from the above equation as a function of strength parameter m_i and for the different values of the fraction $\frac{\sigma_{ci}}{\gamma B}$ are depicted in Figure III-19. A general observation is that the bearing capacities evolutions are almost linearly dependent on m_i and become more and more independent of the rock self-weight when GSI increases. Indeed, for $GSI > 60$, the ratio of $\frac{\sigma_{ci}}{\gamma B}$ has almost no effect on the bearing capacity values.

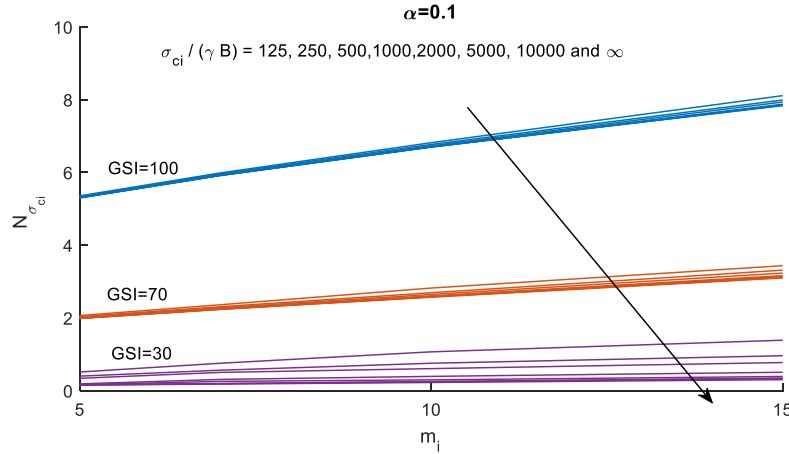


Figure III-19: Effect of rock self-weight on bearing capacity factor assuming a N-AFR with $\alpha = 0.1$.

2.5. Applicability of the H-B criterion – Stress ratio condition

The bearing capacity of a strip footing resting on a material obeying the H-B failure criterion was investigated. Using the upper bound limit analysis theorem, and by employing a multi tangential method to handle the nonlinear strength criterion issues, a closed-form expression of the maximum resistant work rate developed in any virtual strain field was developed. Nevertheless, it is necessary to ensure that a brittle shear failure has occurred below and around the strip footing i.e., the stress ratio condition was respected. This verification is carried out by conducting a parametric study for an $m_i = [3, 5, 7, 10]$, $GSI = [50, 70, 100]$ and by considering two dilatancy parameter

values: $\alpha = 1$ (AFR) and $\alpha = 0.1$ (N-AFR). The stress field generated below and around the strip footing is examined. The verification procedure is described as follows. For each value pair m_i and GSI :

- The values of the instantaneous friction angles $\phi_i, \phi_{i,i+1}$, for each wedge side (cf. Figure III-6), are obtained from the numerical minimization.
- The values of the scaled minor principal stress σ_3^* are computed using the following relationship $\sigma_3^* = \left(\frac{k_{\phi_t}-1}{a}\right)^{-\beta/a}$; $k_{\phi_t} = \frac{1+\sin \phi_t}{1-\sin \phi_t}$
- The scaled major principal stress is finally calculated as $\sigma_1^* = \sigma_3^* + \sigma_3^{*a}$.

The (σ_1^*, σ_3^*) for each wedge side, forming the H-B failure envelope, as well as the transition lines from brittle shear to ductile shear failure, are plotted in Figure III-20 to Figure III-25. Furthermore, the failure surfaces of each triplet (GSI, m_i and α) are illustrated. The wedge sides for which the stress ratio does not satisfy Mogi's LSR (i.e., $\sigma_1/\sigma_3 < 3.4$) are shown in red dashed lines forming a Non-Compliant Region (NCR). The area of this region (A_{NCR}), compared to the overall failure surface area (A_{total}), are also displayed. Firstly, it can be noted from the computed σ_3^* that the minor principal stress field beneath the strip footing is characterized by high σ_3^* values just below the footing base (wedges 1 and 2 as illustrated in Figure III-6) and decreases rapidly as one moves laterally from the footing base (remaining triangular wedges).

On the other hand, the NCR, if it exists, is a small part of the radial shear regions fanning between the active triangular wedge immediately beneath the footing and the passive Rankine wedge. For the cases with AFR ($\alpha = 1$), the NCR is more pronounced in comparison to the N-AFR cases. This observation is expected since the AFR, which is a purely theoretical case, results in a maximum irreversible increase of the inelastic volume. Furthermore, for a given GSI , increasing m_i results in a decrease in the extent of the NCR.

For a global view, a contour plot displaying the NCR area compared to the overall failure surface area as a function of $m_i \in [3,30]$ and $GSI \in [10,100]$ is depicted in Figure III-26. It should be emphasized that the interval $GSI \in [80,100]$, i.e., $s \in [0.1,0.9]$, is added for the sake of completeness. One can already note that the extent of the NCR is maximum ($A_{NCR} = 20\% A_{total}$) when the s-values condition is not fulfilled especially when $m_i < 7$. This somehow confirms why the H-B criterion is not applicable when $80 \leq GSI < 100$.

For the remaining GSI range, i.e., the s-values condition is satisfied, one can conclude that, for the strip footing problem being studied herein, the H-B failure criterion can

be applied without restriction for a $GSI \leq 70$ and $m_i \geq 5$ whatever the value of the dilatancy parameter α is. However, special attention must be paid if $GSI = 100$ and $m_i \leq 10$. It should be emphasized that for cases where the NCR area is at most 10% of the overall failure surface area, the H-B criterion may still be applicable. This can be explained by the fact that the (σ_1^*, σ_3^*) points defining the NCR are barely below the transition line as depicted in Figure III-23, i.e., neither fully brittle nor fully ductile.

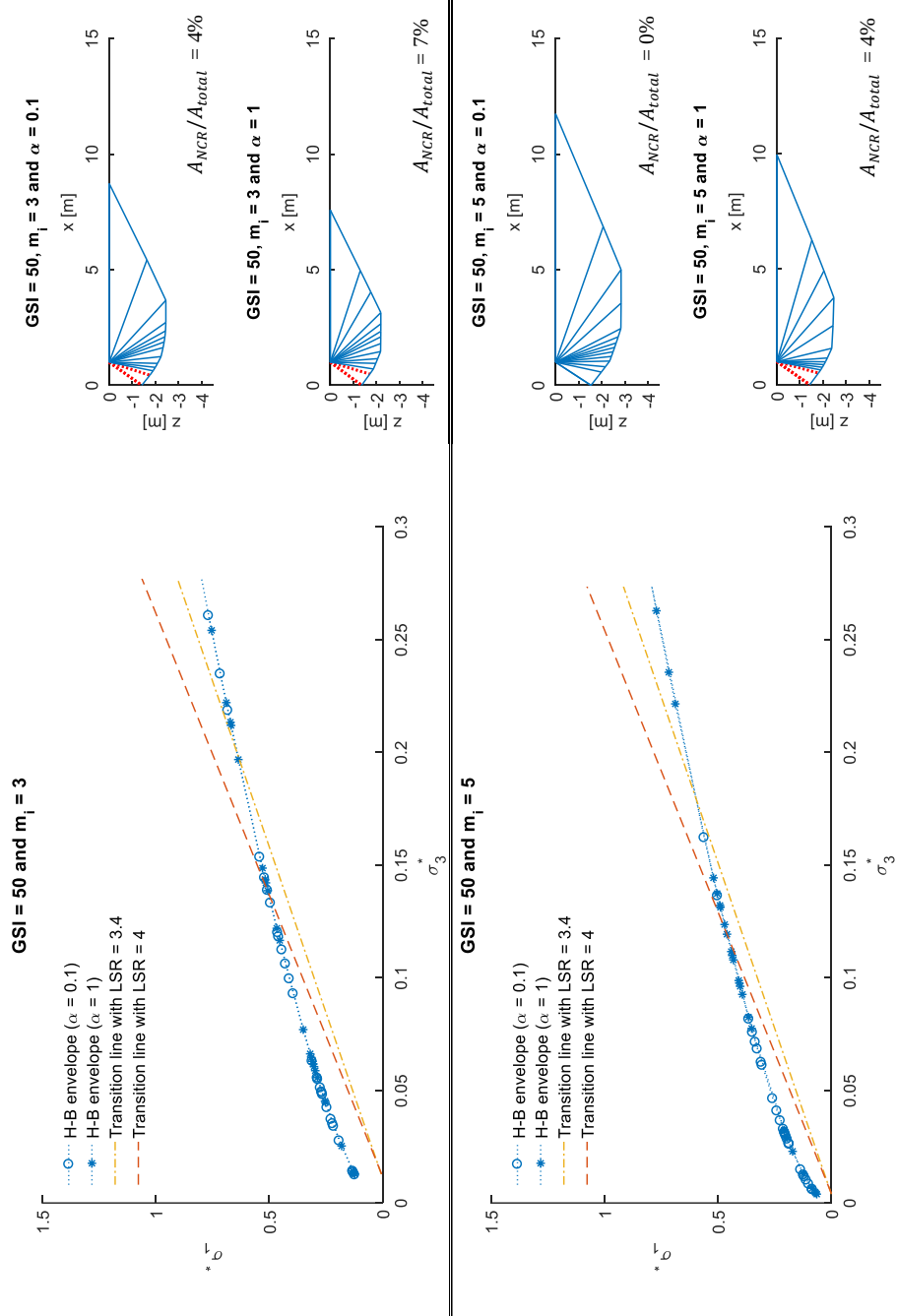


Figure III-20: (Left) H-B failure envelope obtained from the stress field generated below and around the footing vs. Transition lines from brittle to ductile shear failures using two limits state ratio LSR (3.4 and 4) – (Right) Failure surface: the red dashed wedge interfaces define the Non-Compliant Region NCR that does not satisfy Mogi's LSR (3.4) – $GSI=50, m_i = \{3,5\}$ and $\alpha = \{0.1, 1\}$

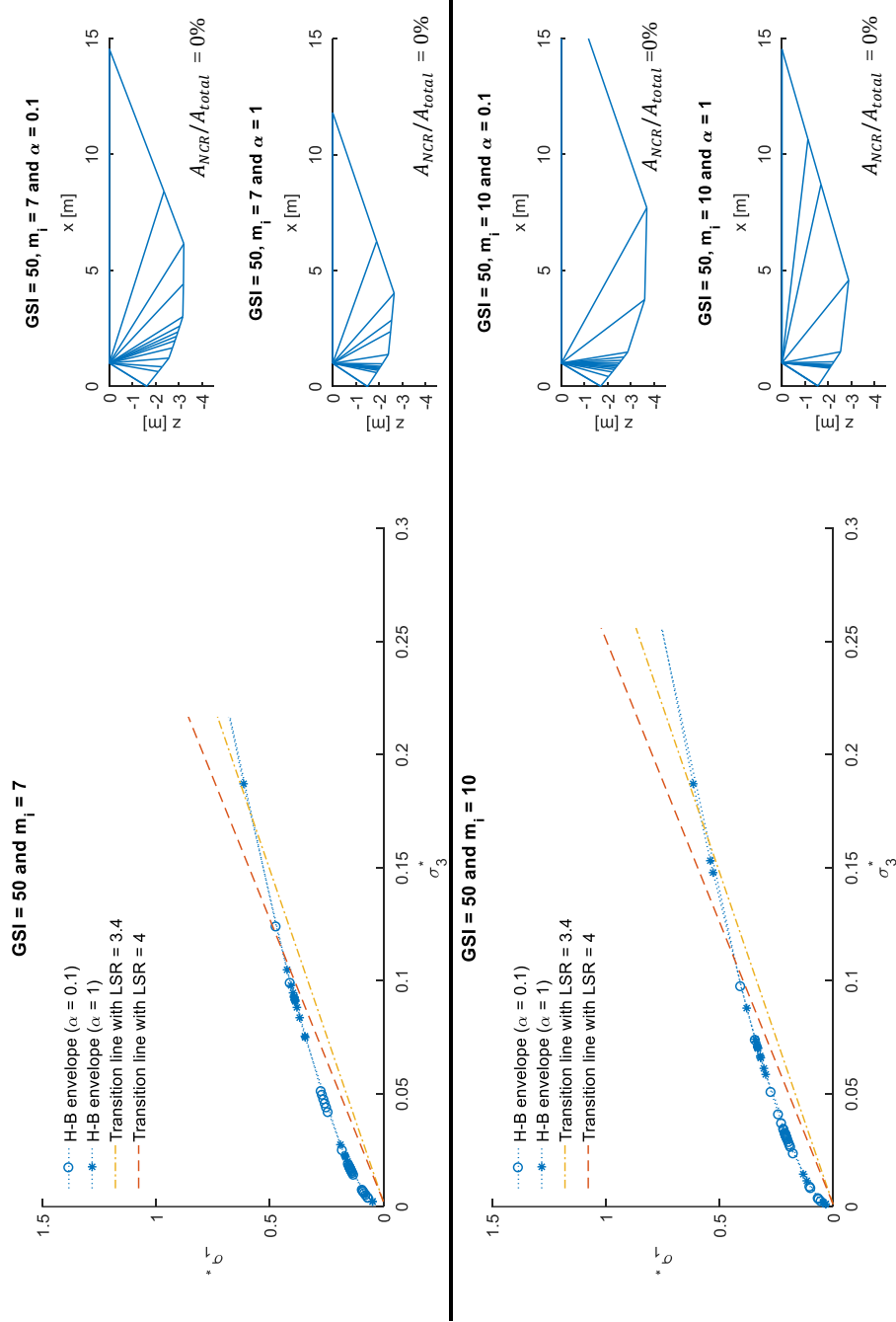


Figure III-21: (Left) H-B failure envelope obtained from the stress field generated below and around the footing vs. Transition lines from brittle to ductile shear failures using two limits state ratio LSR (3.4 and 4) – (Right) Failure surface: the red dashed wedge interfaces define the Non-Compliant Region NCR that does not satisfy Mogi's LSR (3.4) – $GSI=50$, $m_i = \{7,10\}$ and $\alpha = \{0.1, 1\}$.

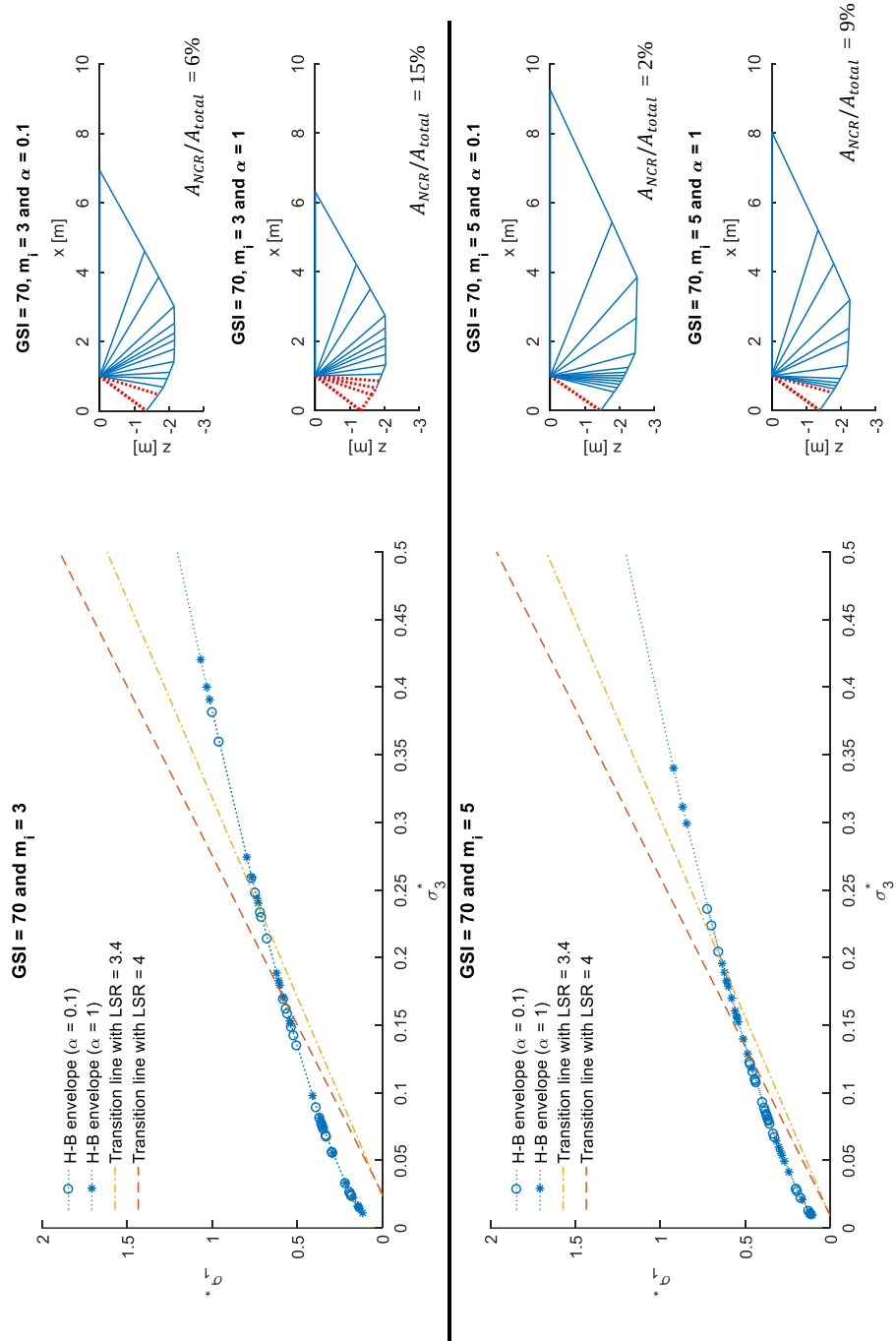


Figure III-22: (Left) H-B failure envelope obtained from the stress field generated below and around the footing vs. Transition lines from brittle to ductile shear failures using two limits state ratio LSR (3.4 and 4) – (Right) Failure surface: the red dashed wedge interfaces define the Non-Compliant Region NCR that does not satisfy Mogi's LSR (3.4) – $GSI=70$, $m_i = \{3,5\}$ and $\alpha = \{0.1, 1\}$.

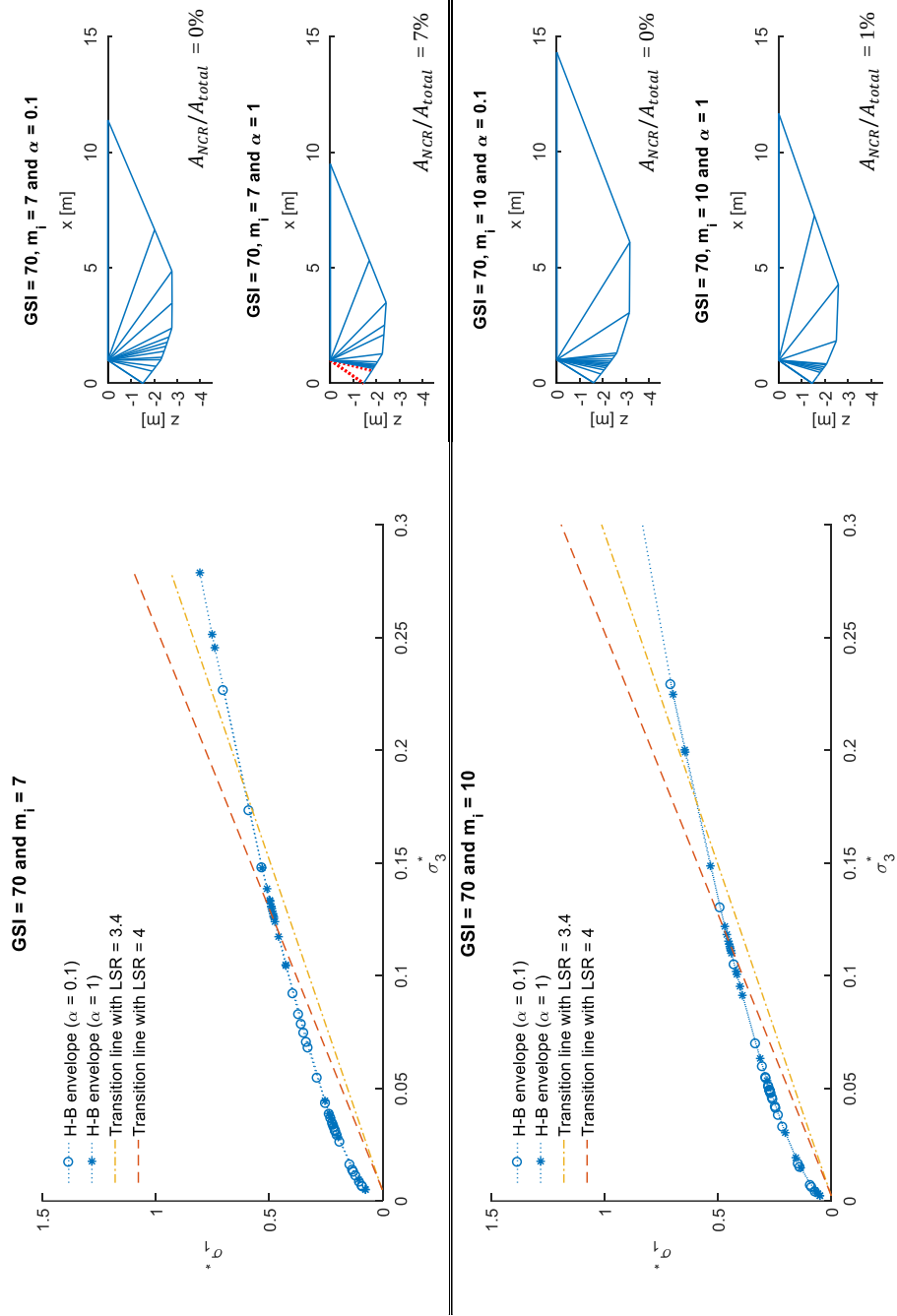


Figure III-23: (Left) H-B failure envelope obtained from the stress field generated below and around the footing vs. Transition lines from brittle to ductile shear failures using two limits state ratio LSR (3.4 and 4) – (Right) Failure surface: the red dashed wedge interfaces define the Non-Compliant Region NCR that does not satisfy Mogi's LSR (3.4) – $GSI=70, m_i = \{7,10\}$ and $\alpha = \{0.1, 1\}$.

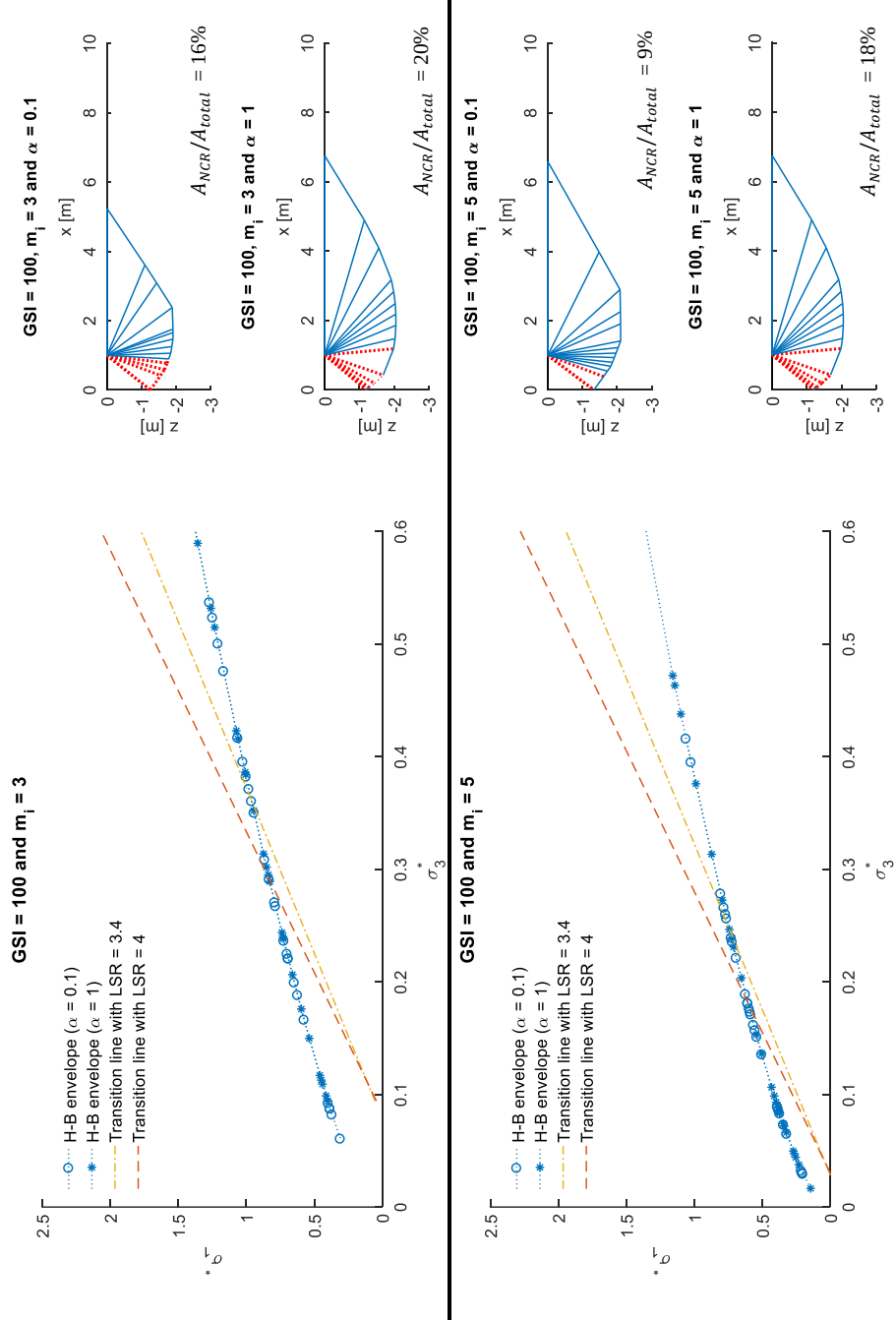


Figure III-24: (Left) H-B failure envelope obtained from the stress field generated below and around the footing vs. Transition lines from brittle to ductile shear failures using two limits state ratio LSR (3.4 and 4) – (Right) Failure surface: the red dashed wedge interfaces define the Non-Compliant Region NCR that does not satisfy Mogi's LSR (3.4) – $GSI=100$, $m_i = \{3,5\}$ and $\alpha = \{0.1, 1\}$.

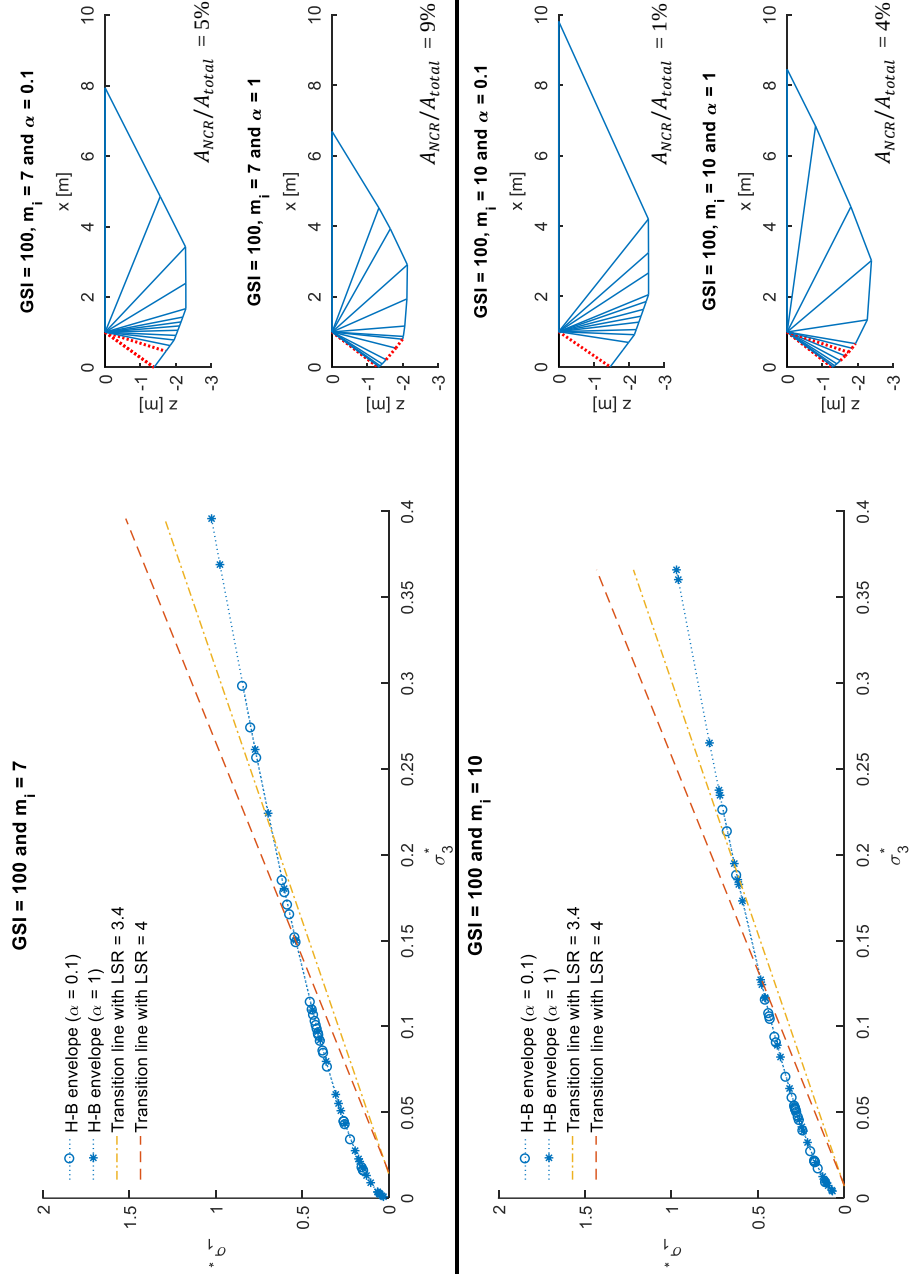


Figure III-25: (Left) H-B failure envelope obtained from the stress field generated below and around the footing vs. Transition lines from brittle to ductile shear failures using two limits state ratio LSR (3.4 and 4) – (Right) Failure surface: the red dashed wedge interfaces define the Non-Compliant Region NCR that does not satisfy Mogi's LSR (3.4) – $GSI=100$, $m_i = \{7,10\}$ and $\alpha = \{0.1, 1\}$.

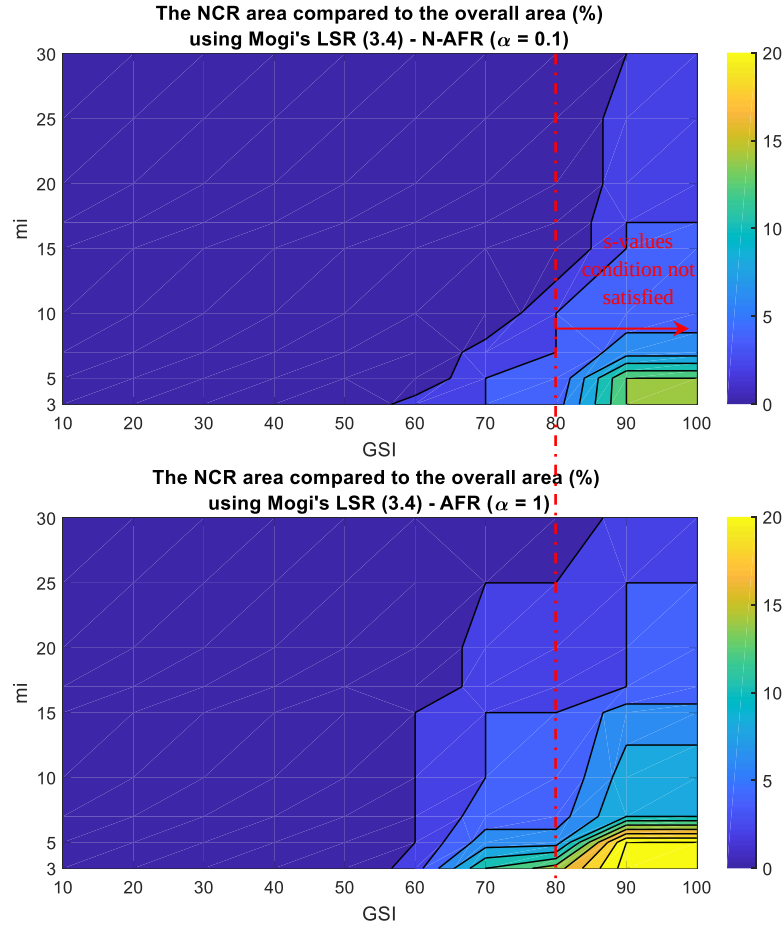


Figure III-26: Contour plot displaying the Non-compliant region (NCR) area compared to the overall area as a function of GSI and m_i using Mogi's limit state ratio – (Top) case of AFR (Bottom) Case of N-AFR with $\alpha = 0.1$.

2.6. Validation using OptumeG2 software

Currently, few softwares implement the kinematic approaches of limit analysis. The geotechnical analysis software OptumG2 (<https://optumce.com/products/optumg2>) belongs to this category. Regarding the optimization procedure, OptumeG2 is based on recent advances in the field of second-order cone programming. Besides, there is an optimized adaptive mesh procedure to improve the accuracy of the upper bounds of the calculated load. Furthermore, the generalized H-B failure criterion is available in the software. Numerical simulations are performed considering the following H-B parameters: $\sigma_{ci} = 1 [MPa]$, $m_i \in \{1,3,5,7,10\}$, $GSI \in \{30,50,70,100\}$ and $D = 0$. Note that the rock mass is assumed to be weightless without a surcharge load. It should be emphasized that only the associated flow rule case can be used in this analysis.

Figure III-27 plots the bearing capacity factor values obtained from OptumeG2 as a function of the results obtained from the developed solutions. As can be seen, the analytical results fall slightly below the line 1:1 with a relative error bracketed between 1% and 9%.

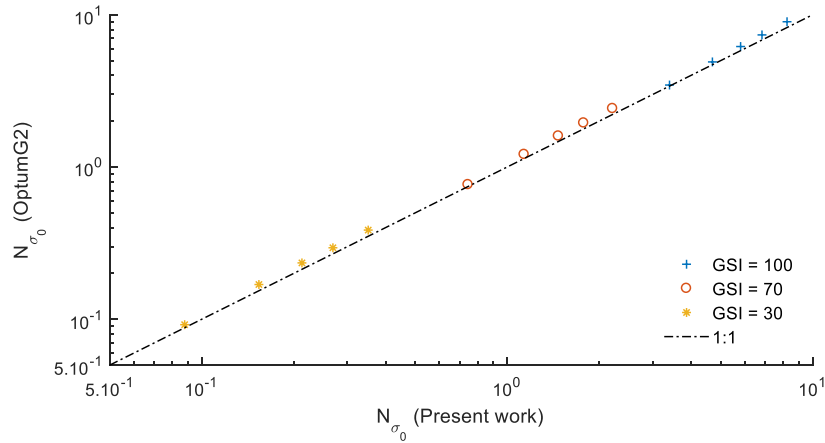


Figure III-27: Bearing capacity factor: Present work vs. OptumeG2 software - $m_i \in \{3,5,7,10\}$.

Figure III-28 to Figure III-31 depict elastic and plastic points over the model domain. The non-compliant region of plastic points for which Mogi's limit stress ratio are not respected ($\sigma_1/\sigma_3 < 3.4$) is highlighted. One can already note that failure paths from OptumeG2 are similar to the assumed multi-wedge translation failure mechanism (cf. Figure III-6) i.e., the failure mechanism is composed of radial shear regions fanning between the active triangular wedge immediately beneath the footing and a Rankine passive wedge. Furthermore, if we compare/examine the extent of non-compliant regions from the OptumeG2 model with those obtained from the developed solutions

(cf. Figure III-20, Figure III-21, Figure III-24 and Figure III-25), one can confirm the finding regarding the applicability of the H-B criterion to the bearing capacity problem of a strip footing on a rock mass (cf. Subsection 2.5 of this chapter).

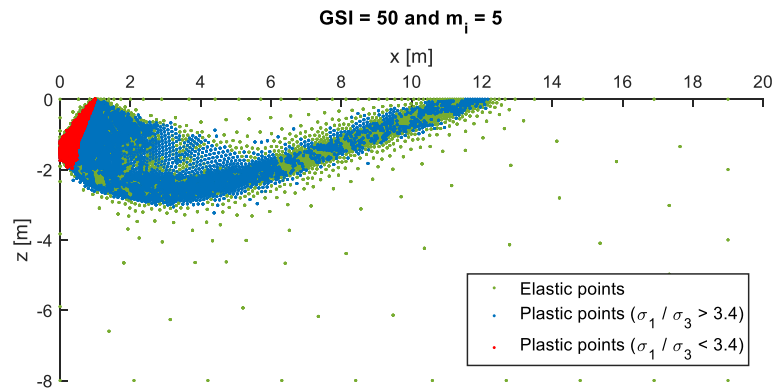


Figure III-28: Elastic and plastic points over the model domain – GSI = 50 and $m_i = 5$.

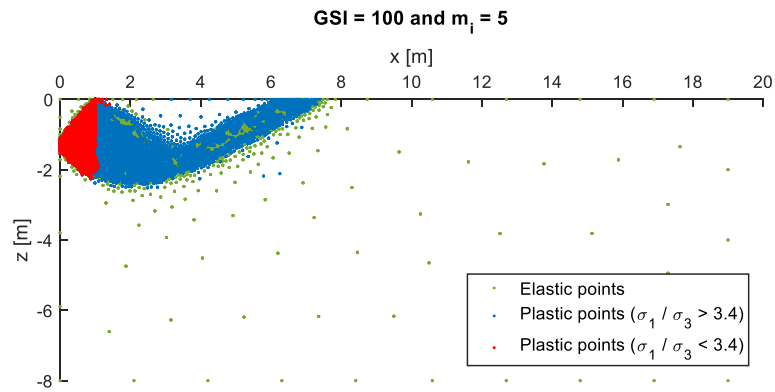


Figure III-29: Elastic and plastic points over the model domain – GSI = 100 and $m_i = 5$.

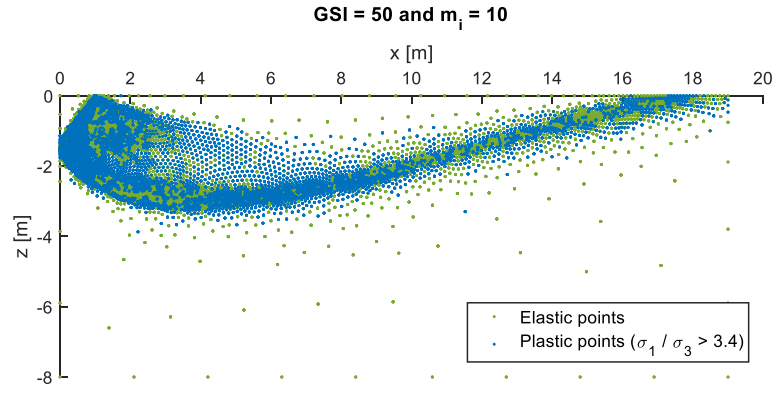


Figure III-30: Elastic and plastic points over the model domain – $GSI = 50$ and $m_i = 10$.

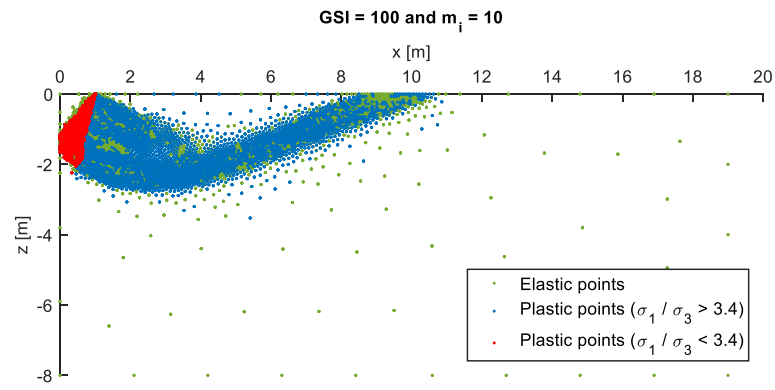


Figure III-31: Elastic and plastic points over the model domain – $GSI = 100$ and $m_i = 10$.

2.7. Comparisons with experimental results

2.7.1. Experimental results on rock material

To demonstrate the applicability of the proposed analytical solution, results of laboratory load tests carried out on three different rock materials (granite, limestone, and sandstone) are extracted from the literature (Rehman and Broms, 1971). The experimental tests consist of penetrating a 20 mm diameter steel dowel into the cubic rock specimens ($130 \times 130 \times 100$ mm), which were confined in rigid steel cylinders to prevent them from splitting during the load tests. For each case, the rock type, the measured bearing capacity, and its corresponding settlements are identified. The strength envelopes of these rocks can be characterized by the H-B criterion (Hoek, 2006b), and their H-B parameters can be calculated, as explained in Table III-6, based on the mechanical properties retrieved from the original paper.

Table III-6: Mechanical properties and deduced H-B parameters of the three different rock materials.

Rock type	$UCS^{(1)}$ [MPa]	$UTS^{(2)}$ [MPa]	$UCS/ UTS $	H-B parameters ($D = 0$)		
				$m_i^{(3)}$	$\sigma_{ci}^{(4)}$ [MPa]	GSI
Granite	230	-14.4	16	11.1	230	100
Sandstone	70.5	-5.9	12	6.2	70.5	100
Limestone	124	-12.4	10	3.7	124	100

(1) Measured unconfined compressive strength
 (2) Measured tensile strength
 (3) Strength parameter calculated per Eq. (II-41) as $m_i = (UCS/|UTS| - 7)/0.81$
 (4) Uniaxial compressive strength equal to UCS.

The load-displacement curves for the three rock materials are depicted in Figure III-32. The penetration of the model footing increased linearly with increasing load up to failure that occurred suddenly. Rehman and Broms (1971) stated that a cracking sound was heard in the granite specimen when the failure load was approached, and the intensity of the cracks increased rapidly before failure. However, no such sound was heard in the sandstone or limestone cases. Furthermore, for the test conducted on granite, local crushing failure occurred, and some granite chipping was observed around the footing perimeter.

On the other hand, and based on the plasticity theory, the ultimate bearing capacity can be calculated from the developed solution (cf. Eq. (III-15)). A factor s_f equals to 1.2 (De Beer, 1970) is assumed to consider the circular shape of the model-scale footing. The bearing capacities for granite, sandstone, and limestone are calculated by assuming an AFR and an N-AFR with $\alpha = 1/4$. The choice of this flow rule follows Hoek and Brown's (1997) recommendations i.e., of $\alpha = 1/4$ for excellent and good

quality rock mass. Both experimental and analytical results are summarized in Table III-7. It can be noted that calculated values by assuming an AFR are significantly overestimated by about 30%. Assuming N-AFR results in a relatively good agreement between measured and calculated values in particular for the sedimentary rock (13% for sandstone and 6 % for limestone).

It can be concluded that the ultimate bearing capacity of sedimentary rock can be predicted relatively satisfactorily by the theory of plasticity and by incorporating the mobilized dilation angle of the rock mass.

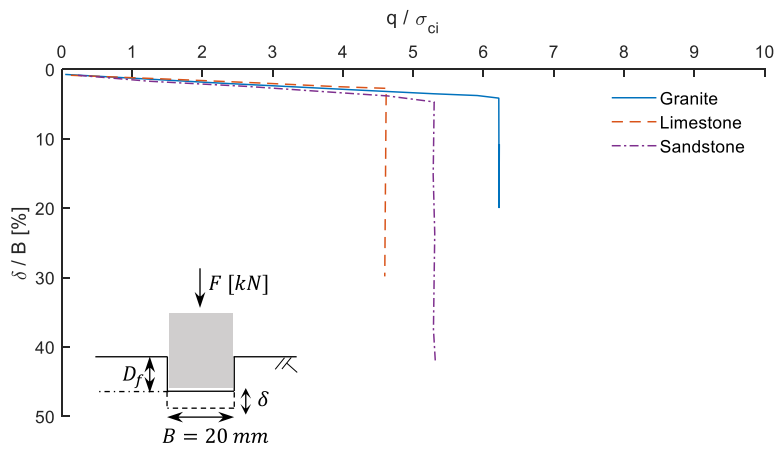


Figure III-32: Load – displacement curves of the three different rock materials – after (Rehman and Broms, 1971).

Table III-7: Bearing capacity of three different rock materials – Experimental vs. theoretical results.

Rock type	$\frac{\delta^{(1)}}{B}$	$\frac{D_f}{B}$	$\frac{q_{test}^{(2)}}{\sigma_{ci}}$	$\left(\frac{q_{ult}}{\sigma_{ci}}\right)_{\alpha=1}^{(3)}$	$\left(\frac{q_{ult}}{\sigma_{ci}}\right)_{\alpha=1/4}^{(4)}$	Error ⁽⁶⁾ [%]
Granite	4.1	0	6.2	9.0	7.9	31 / 21
Sandstone	4.7	0	5.3	6.7	6.1	20 / 13
Limestone	2.8	0	4.6	5.3	4.9	13 / 6

(1) Normalized measured displacement [%]. (Rehman and Broms, 1971).

(2) Normalized measured bearing capacity. (Rehman and Broms, 1971).

(3) Normalized theoretical ultimate bearing capacity assuming an AFR as per Eq. (III-15).

(4) Normalized theoretical ultimate bearing capacity assuming a NAFR with $\alpha = 0.25$ as per Eq. (III-15).

(5) Error between (2) & (3) and (2) & (4) [%].

2.7.2. Experimental results on nonlinear sand

One advantage of the H-B failure criterion is the ability to be fitted to a wide range of nonlinear strength envelopes while respecting the convexity of the yield surface in the principal stress space. Therefore, this feature may validate extreme points bracketing the developed analytical solutions by comparing analytical results using experimental results conducted, especially on nonlinear geomaterials such as sand.

In light of this, Krabbenhoft et al. (2012) presented some experimental results obtained from simple model-scale circular footing tests on dense Esbjerg sand (same denomination as in the original paper) with dry unit weight and a relative density of 17.5 kN.m^{-3} and 0.84, respectively. Triaxial tests carried out on the sand show that the friction angle ϕ depends strongly on the stress level. As can be seen from Figure III-33, the friction angle decreases nonlinearly with increasing confining pressure σ_3 with the $[0 - 100]$ kPa range. Another relevant piece of information that can be retrieved from their work is the nonlinear evolution of the dilatancy angle ψ as a function of σ_3 (cf. Figure III-33). It can be noted that both friction and dilatancy angles have the same nonlinear tendency, but ψ values remain considerably below ϕ values.

Centrifuge tests were also conducted on a reduced scale strip footing models at 1g, 25g, and 40g (Valore, Ziccarelli and Muscolino, 2017; Ziccarelli et al., 2017). The footing was resting on a dense silica sand B (same denomination as in the original papers) with dry unit weight and a relative density of 15.8 kN.m^{-3} and 0.95, respectively. The peak strength failure envelope of the sand obtained from direct shear tests is strongly curvilinear in the range of low normal stresses ($\sigma_n < 100 \text{ kPa}$) resulting in a nonlinear decrease of the peak friction angle with increasing normal stresses (cf. Figure III-34). The same tendency was observed for the peak dilation angle (cf. Figure III-34).

It is a well-known fact that dilation plays an important role in the study of the strength of soil/rock. As described in Chapter III-2.4, bearing capacity for dilatant materials with a non-associated flow rule ($\psi < \phi$) are smaller than those obtained when an associated flow rule ($\psi = \phi$) is assumed. It is, therefore, essential to properly quantify the flow rule and the appropriate dilatancy parameter.

In light of the foregoing, it becomes clear that the classical linear M-C criterion is ill-suited, and it is recommended to use the nonlinear H-B failure criterion combined with the proposed nonlinear flow rule to describe the sand behaviour more rigorously. Therefore, based on triaxial and direct shear test results, the H-B failure criterion can be calibrated using the friction angle expression (III-34). Then, the dilatancy parameter $m_{b,dil}$ can be deduced from the dilatancy angle expression (III-31). The values are summarized in Table III-8, and fitted curves are depicted in Figure III-33 and Figure III-34. Note that the H-B parameters values reported in Table III-8 should

be considered as constants of the fitting process i.e., no physical meaning. It should be emphasized that for the silica sand B tested under direct shear test, the gap between measured and predicted friction angle values may be notable at low normal stress because of the sensitivity of the test. Indeed, the continual decrease in the contact surface of the two halves during the test produces an error on the shear and normal stress measurements and that affects the Mohr-Coulomb failure envelope. This error is generally ignored. It should be emphasized that the scaled cohesion proper to each sand can be explicitly computed per Eq. (III-5) as a function of the friction angle and the H-B core parameters. For instance, the Silica sand B cohesion varies from 0 kPa to 10 kPa, respectively for normal stress between 10 kPa to 480 kPa.

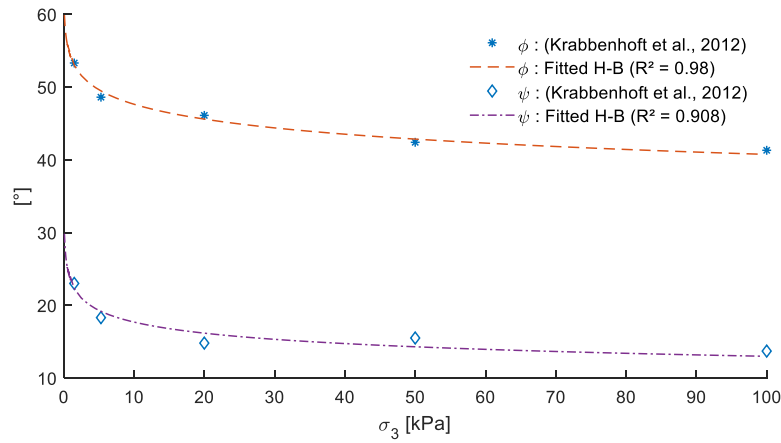


Figure III-33: Esbjerg sand - Friction and dilatancy angles from triaxial tests – Fitted HB curves.

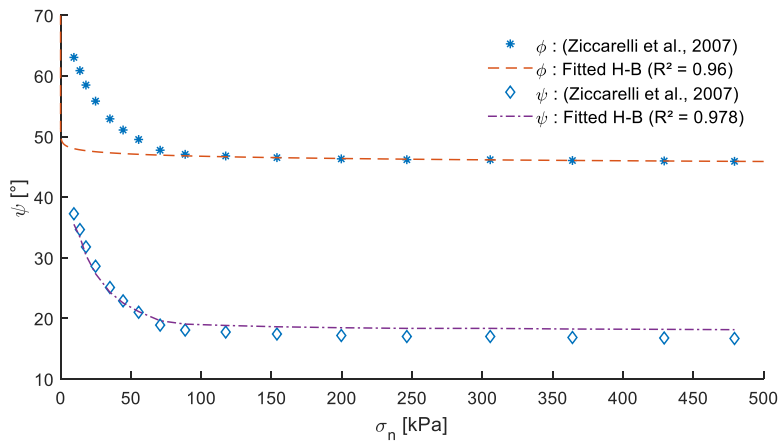


Figure III-34: Silica sand B - Friction and dilatancy angles from direct shear tests – Fitted H-B curves.

III.2. Bearing capacity using upper bound limit analysis

Table III-8: Tested Sand – Estimated H-B parameters.

Material ⁽¹⁾	σ_{ci} [kPa]	m_b	s	a	$\alpha^{(2)}$	Ref.
Esbjerg sand	3.09	13.46	3.01E-14	0.822	0.103	(Krabbenhoft, Clausen and Damkilde, 2012)
Silica sand B	10.17	6.22	2.26E-14	0.968	0.168	(Ziccarelli et al., 2017)

(1) Material denomination in the original paper (2) Dilation ratio equal to $m_{b,dil}/m_b$

The measured failure loads and the corresponding normalized settlements retrieved from the published papers (Krabbenhoft, Clausen and Damkilde, 2012; Ziccarelli *et al.*, 2017) are summarized in Table III-9. Tests B3, B4, B6 are conducted using 100 mm diameter circular footing whereas tests B59, B60 B61, and C01, C04, C09 are conducted using a 40 mm width strip footing and different lengths L equal to 100 mm and 160 mm (cf. Table III-9). For all tests, the circular/rectangular footing was initially resting on the sand surface. However, as load increased, the footing sinks into the ground to a depth δ , introducing a vertical effective stress at the foundation base level.

On the other hand, the ultimate bearing capacity of a strip foundation is given per Eq. (III-15). Since the uniaxial compressive strength σ_{ci} is too low, the sand self-weight as well as the overburden pressure $q = \delta\gamma$ cannot be ignored as they account for a significant part of the bearing capacity of the foundation. Besides, a factor s_f should be introduced to consider the circular or rectangular shape of the model-scale footing. s_f is computed as (Zhu and Michalowski, 2005):

$$s_f = \begin{cases} 1.2, & B = L \\ 1 + (1.3 \tan^2(\phi_{mean}) - 0.5) \left(\frac{L}{B}\right) e^{-L/B}, & B < L \end{cases} \quad (III-43)$$

in which ϕ_{mean} is the average friction angle of the silica sand equal to 47.5°. A shape factor of 1.32 and 1.15 is found for $L = 100$ and 160 mm, respectively.

For a matter of comparison, the theoretical ultimate bearing capacity assuming an associated flow rule is also reported (cf. Table II-5). One can note that values gained by assuming an AFR, when compared to the measured values, are overestimated with at least 120 % in case of circular footing (tests B3, B4, B5) and at least 60% in case of rectangular footing. However, introducing the nonlinear flow rule reduced the AFR values by a factor of 2.2 and 1.6 for circular and rectangular footings, respectively, and that agrees better with experimental results (cf. Figure III-35).

But, let's not forget that 1g tests on reduced-scale models (cases B3, B4, B6, B59, B60, and B61) suffer from severe limitations due to the scale effect associated foremost with the very low stresses generated in the soil beneath the model footing. However, employing a centrifuge machine enhances the geotechnical models by producing identical self-weight stresses in the model and prototype, which makes it

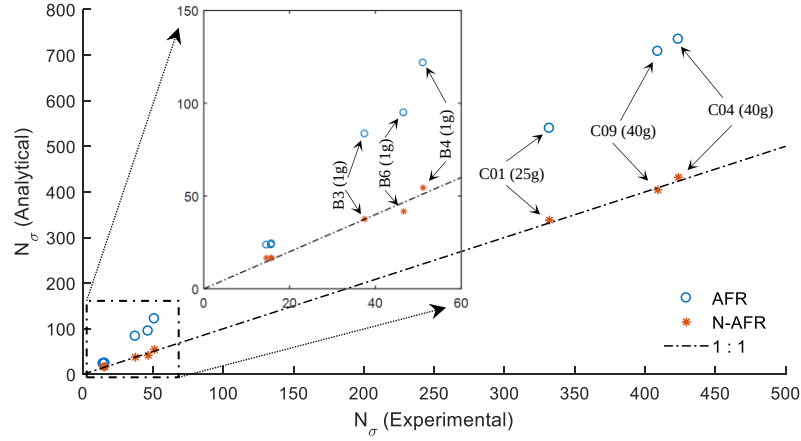


Figure III-35: Model-scale strip and circular footing tests on nonlinear sand – Analytical vs. experimental results.

Table III-9: Measured and theoretical bearing capacity of nonlinear sand.

Test ⁽¹⁾	$\frac{\delta^{(2)}}{B}$	$\frac{q_{test}^{(3)}}{\sigma_{ci}}$	$\left(\frac{q_{ult}^{(4)}}{\sigma_{ci}}\right)_{AFR}$	$\left(\frac{q_{ult}^{(5)}}{\sigma_{ci}}\right)_{NAFR}$	Error ⁽⁶⁾	Ref.
B3	6.3	37.5	83.5	37.6	122 / 0.2	Circular footing Ø100 mm (Krabbenhoft, Clausen and Damkilde, 2012)
B4	7.5	51.1	121.8	54.5	138 / 6.5	
B6	6.7	46.6	94.9	41.7	103 / -10.5	
B59	10.9	15.7	23.7	16.6	51 / 5.6	Strip footing B40 × L100 mm (Valore, Ziccarelli and Muscolino, 2017)
B60	12.7	15.8	24.3	16.7	53 / 6.0	
B61	10.5	14.6	23.6	16.6	61 / 13.6	
C01 (25g)	13.5	332.0	539.5	338.1	62 / 1.8	Strip footing B40 × L160 mm (Ziccarelli et al., 2017)
C04 (40g)	16.6	423.7	734.7	432.0	73 / 1.9	
C09 (40g)	16.2	409.1	708.2	404.5	73 / -1.1	
(1) Test denomination as in the original paper				(4) Normalized theoretical ultimate bearing capacity assuming an AFR as per Eq. (III-15)		
(2) Normalized displacement [%] retrieved from the original paper				(5) Normalized theoretical ultimate bearing capacity assuming a NAFR as per Eq. (III-15)		
(3) Normalized measured bearing capacity retrieved from the original paper (cf. Ref. column)				(6) Error between (3) & (4) / (3) & (5) [%]		

possible to obtain accurate data. This could explain why the errors between the measured and the calculated values are minimal, especially for tests C01 (25g), C04 (40g), and C09 (40g) (Error < 2%).

3. Bearing capacity using equivalent M-C parameters

Nota:

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Estimating the bearing capacity of surface foundations resting on rock masses is a necessary first step in the design of critical infrastructures. Although it is known that the rock masses behave nonlinearly according to the H-B failure criterion, there has long been a tendency to model rock failure as a linear M-C law for which extensive literature is available in terms of the foundation ultimate bearing response. That has provided an impetus for practising engineers to find equivalent friction angle and cohesion so that the classical bearing capacity relationships can be used.

In light of this, the main feature of the nonlinear H-B criterion is that the friction angle and cohesion vary as a function of the confining stresses. More precisely, the H-B criterion allows for higher friction angles at low confining stress and vice versa.

Nevertheless, the industry tends to use constant M-C parameters $\phi - c$ to characterize the rock masses. This is, indeed, because the physical interpretation of these parameters is much easier, and classical bearing capacity equations can be used. One such equation is written in the following form:

$$q_{ult} = cN_c + qN_q + 0.5\gamma BN_\gamma \quad (\text{III-44})$$

Here the non-dimensional parameters N_c , N_q and N_γ are bearing capacity factors, which are a function of the friction angle and are related, respectively, to the cohesion c , the surcharge load q and the unit weight γ of the soil.

3.1. Problem statement and basic hypothesis

To estimate equivalent M-C parameters, a direct comparison was made by fitting an average linear relationship to the curve generated by solving the equation for a range of minor principal stress σ'_3 values defined by $\sigma_t < \sigma'_3 < \sigma'_{3\max}$ (Hoek, Carranza and Corkum, 2002). The fitting process involves balancing the area above and below the M-C plot in the $(\sigma'_1 - \sigma'_3)$ plane within the $[\sigma_t, \sigma'_{3\max}]$ range (cf. Figure III-36). This results in the following equations for the friction angle ϕ and cohesive strength c :

$$\sin(\phi) = \frac{6 \cdot a \cdot m_b \cdot (s + m_b \cdot \sigma'_{3n})^{a-1}}{2 \cdot (1 + a)(2 + a) + 6 \cdot a \cdot m_b \cdot (s + m_b \cdot \sigma'_{3n})^{a-1}} \quad (\text{III-45})$$

$$\frac{c}{\sigma_{ci}} = \frac{[(1 + 2a) \cdot s + (1 - a) \cdot m_b \cdot \sigma'_{3n}] \cdot (s + m_b \cdot \sigma'_{3n})^{a-1}}{(1 + a)(2 + a) \cdot \sqrt{1 + \frac{6 \cdot a \cdot m_b \cdot (s + m_b \cdot \sigma'_{3n})^{a-1}}{(1 + a) \cdot (2 + a)}}} \quad (\text{III-46})$$

where $\sigma'_{3n} = \sigma'_{3\max}/\sigma_{ci}$. The issue of determining the appropriate value of $\sigma'_{3\max}$ for use in Eqs. (III-45) and (III-46) depends upon the specific application. There are no theoretically established methods for choosing this range. From experience and by trial and error, Hoek (Hoek, 2002) suggests, in the general case, the following value:

$$\sigma'_{3\max} = 0.25\sigma_{ci} \quad (\text{III-47})$$

More specific guidance was provided (Hoek, Carranza and Corkum, 2002) for selecting appropriate values of $\sigma'_{3\max}$ for tunnels and slopes. However, no guidance is provided for the case of surface foundations.

Therefore, the primary focus of this subsection is to provide specific guidance for selecting appropriate equivalent M-C parameters, which results in acceptable bearing capacity values and failure mechanism when compared to the original H-B failure criterion. Note that only the AFR case will be considered.

The proposed method will be initially established for a strip footing resting on weightless rock mass ($\gamma = 0 [kN/m^3]$) without a surcharge load ($q_0 = 0 [kPa]$). Then, the method will be extended to include rock self-weight for strip footing and circular footings, respectively.

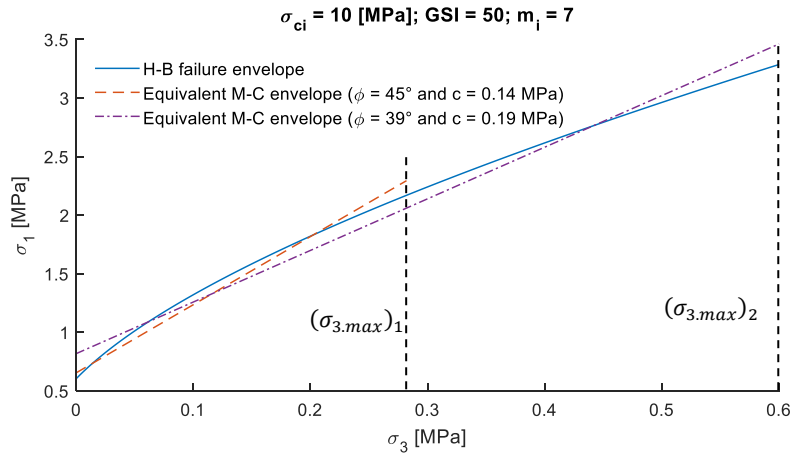


Figure III-36: M-C and H-B failure envelope

3.2. Strip footing resting on a weightless rock mass

In this case, Eq. (III-44) reduces to:

$$q_{ult}^{strip} = cN_c = c \frac{N_\phi e^{\pi \tan(\phi)} - 1}{\tan(\phi)} ; \quad N_\phi = \tan^2 \left(\frac{\pi}{4} + \frac{\phi}{2} \right) \quad (\text{III-48})$$

which can be further written in non-dimensional form, as:

$$\left(\frac{q_{ult}^{strip}}{\sigma_{ci}} \right)_{\gamma=0} = \frac{c}{\sigma_{ci}} \frac{N_\phi e^{\pi \tan(\phi)} - 1}{\tan(\phi)} \quad (\text{III-49})$$

here, the $\phi - c$ values are calculated per Eqs. (III-45) – (III-46) for a specific σ'_{3max} over which the equivalence between M-C and H-B criteria is considered.

Since the exact bearing capacity values for different H-B core parameters has been found previously in Chapter III-2.4 and results are summarized in Table 1 of Appendix 2.2, the above equation may be solved to determine the appropriate value of σ'_{3max} . Surprisingly, no such solution exists. The reason may be the fact that the minor principal stress field beneath the foundation has stress values starting from zero and the linear fit gained from Eqs. (III-45) – (III-46) overestimates the H-B failure envelope close to $\sigma_3 = 0$.

To tackle this problem, we proceeded as follows. Firstly, the σ'_{3max} has been chosen such that the difference is minimized between bearing capacity values evaluated per Eq. (III-49) and those gained from the upper bound limit analysis (cf. Table 1). It was found that σ'_{3max} varies as a function of UCS, GSI and m_i as follows:

$$\sigma'_{3max} = \frac{\sigma_c}{100} (1.83m_i + 34.02) \quad (\text{III-50})$$

The relationship between σ'_{3max}/σ_c and the geological strength index GSI is shown in Figure III-37 for different values of m_i . It is found that the normalized upper limit of confining stress suggested by Hoek ($\sigma'_{3max}/\sigma_c = 0.25 \cdot \sigma_{ci}/\sigma_c$) far exceeds the proposed one for $GSI < 80$ but it remains lower for GSI value close to 100.

Thereafter, the friction angle was downgraded, such that the bearing capacity would match upper bound solutions within 1%. Such a downgrade is on the conservative side since it implies a strength rock reduction. The factor adjusting the friction angle is defined as $f_\phi = \phi_{adj}/\phi$ (< 1). It is a function of GSI and m_i :

$$f_\phi = \exp \left[\frac{1}{100} \left\{ b_1 \exp \left(b_2 \frac{GSI}{100} \right) - b_3 \exp \left(b_4 \frac{GSI}{100} \right) \right\} \right] \quad (\text{III-51})$$

III.3. Bearing capacity using equivalent M-C parameters

here b_j ; $j=1,2,3,4$ are parameters depending on the strength parameter m_i . These four parameters can be correlated to m_i using a power relation of the form $b_j = p_j \cdot m_i^{\frac{q_j}{100}} + t_j$; $j=1,2,3,4$. Wherein p_j , q_j and t_j are constants given in Table III-10.

To sum up, the following steps should be followed to assess the bearing capacity of a strip footing resting on a weightless rock mass:

- (1) Choose the H-B input parameters.
- (2) Calculate σ'_{3max} per Eq. (III-50),
- (3) Calculate M-C parameters per Eq. (II-33) – (II-34),
- (4) Calculate the adjusted friction angle ϕ_{adj} per Eq. (II-39),
- (5) Evaluate the bearing capacity per Eq. (III-49)

The calculation process can be easily programmed in an Excel spreadsheet.

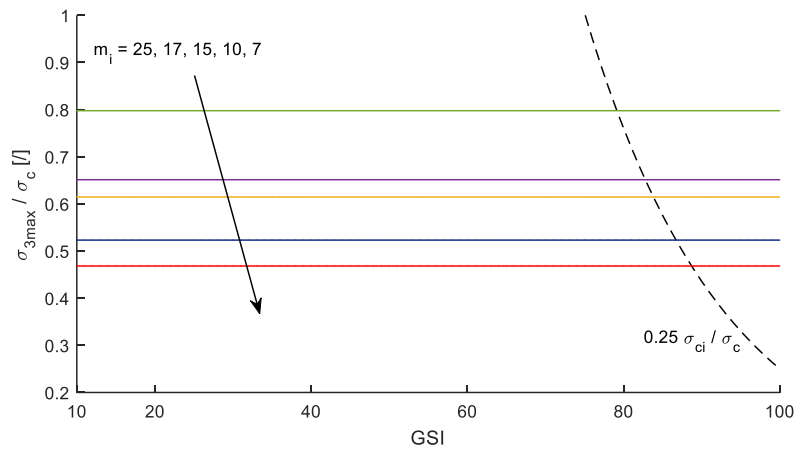
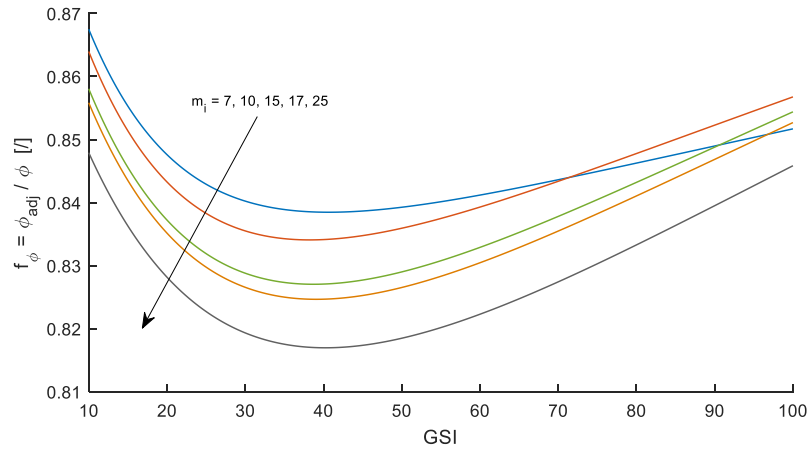


Figure III-37: Normalized upper limit of confining stress as a function of GSI for different values of m_i .

Table III-10: Power relation constants

	$j = 1$	$j = 2$	$j = 3$	$j = 4$
p_j	818.40	7.59	29.80	-17.50
q_j	-0.76	-162	-70.60	-53.90
t_j	-826	-0.52	-18.40	-1.59

Figure III-38: Adjustment factor for the friction angle f_{ϕ} .

The proposed method is applied to the three-quality rock type defined below.

Table III-11: H-B core parameters and M – C equivalent parameters for three different quality rock masses.

Rock quality	H-B core parameters				M-C equivalent parameters		
	σ_{ci} [MPa]	m_i [/]	GSI [/]	D [/]	c [MPa]	ϕ [°]	f_{ϕ} [/]
Very poor	10	7	30	0	0.038	48.22	0.840
Average	80	15	50	0	1.327	49.65	0.829
Very good	150	25	80	0	15.819	46.80	0.833

The normalized bearing capacity values calculated according to the proposed approach are summarized in Table III-12. It can be noted that these values are in excellent agreement compared to values gained from the proposed upper bound method.

Table III-12: Normalized bearing capacity of a strip foundation with an adjusted friction angle of three different quality rock masses (weightless case)

Rock quality	q_{ult}/σ_{ci} [/]		
	Equivalent M-C method	Upper bound method	Error (%)
Very poor	0.303	0.300	+1.0
Average	1.415	1.400	+1.0
Very good	7.142	7.150	-0.1

The original M-C, the H-B, and the adjusted failure criteria for the three different quality rock masses are depicted in Figure III-39. As can be seen, the failure envelope of the present work is a conservative estimate of the actual failure envelope.

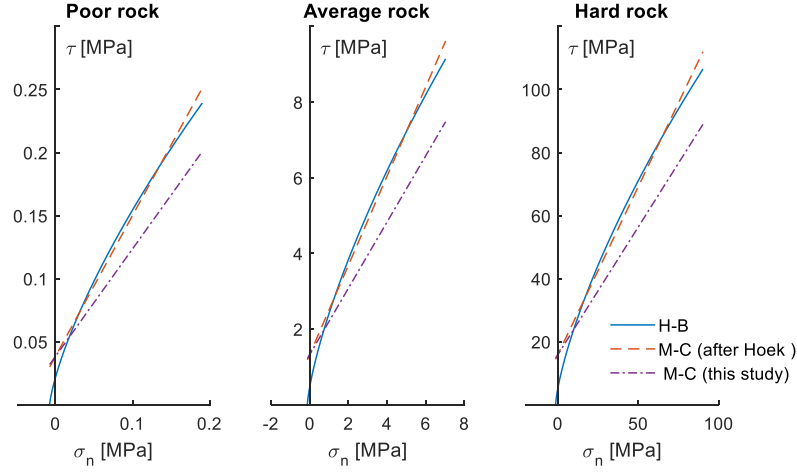


Figure III-39: H-B and equivalent M-C failure envelopes as given by Hoek et al. (2002) vs. M-C failure envelope of this study – Poor, average, and hard rock.

3.3. Applicability of the method for strip footing resting on a self-weight rock mass

In this section, the previous case is extended to include the effect of rock self-weight ($\gamma \neq 0$) on bearing capacity values. Thus, the following bearing capacity equation (normalized) should be used instead of Eq. (III-44):

$$\left(\frac{q_{ult}^{Strip}}{\sigma_{ci}} \right) = \left(\frac{q_{ult}^{Strip}}{\sigma_{ci}} \right)_{\gamma=0} + 0.5 \frac{\gamma B}{\sigma_{ci}} N_\gamma \quad (III-52)$$

Several investigators worked in this area to refine the bearing capacity solution. While the exact values for N_γ remain unknown, the values for N_c as given per Eq. (III-48) is generally agreed upon. It was found that the bearing capacity factor N_γ varies widely between authors (Brinch Hansen, 1970; Meyerhof, 1978; Loukidis, Chakraborty and Salgado, 2008; Lau and Bolton, 2011; Ahmad, 2013; Van Baars, 2014). The difference among these bearing capacity factors is shown in Figure III-40. As can be seen, the bearing capacity factor gained from Van Baars's solution (Van Baars, 2014), which is derived from FEM analysis for rough and smooth footing, gives more conservative values and will be adopted hereinafter. It is expressed as:

$$N_\gamma = 2e^{\pi \tan(\phi)} \tan(\phi) \quad (III-53)$$

To study the effect of rock self-weight on bearing capacity values, the non-dimensional factor $\frac{\sigma_{ci}}{\gamma B}$ belonging to the following range $\{125, 250, 500, 1000, 2000, 5000, 10000, \infty\}$ is used.

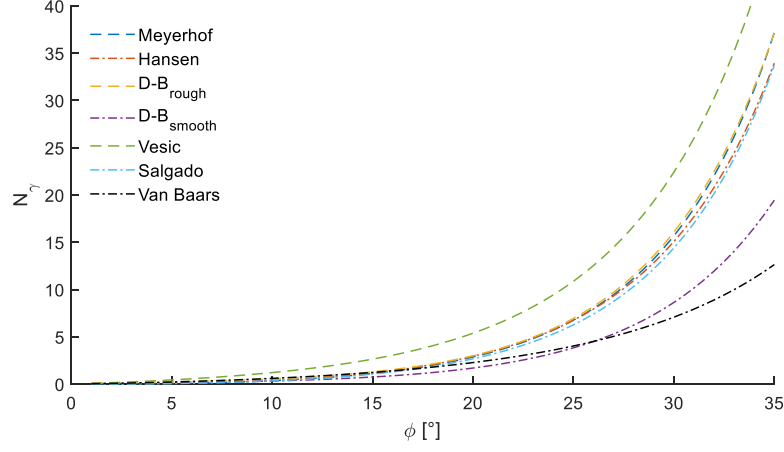


Figure III-40: Soil-weight bearing capacity factor found in the literature as a function of friction angle.

The bearing capacity evaluation procedure previously given is also followed, except that equation (III-52) is used instead of equation (III-49) to take into account the rock self-weight. For some GSI values, for example, $GSI \in \{10, 30, 60, 90\}$, the normalized bearing capacity $\frac{q_{ult}}{\sigma_{ci}}$ gained from Eq. (III-52) as a function of strength parameter m_i and for a constant value of the fraction $\frac{\sigma_{ci}}{\gamma B}$ are depicted in Figure III-41.

A general observation is that the evolution of $\left(\frac{q_{ult}^{Strip.f}}{\sigma_{ci}}\right)$ are almost linearly dependent on m_i and become more and more independent of the rock self-weight when GSI increases. Indeed, for $GSI > 60$, the ratio of $\frac{\sigma_{ci}}{\gamma B}$ has almost no effect on bearing capacity values.

By comparing charts of present work to published charts (Merifield, Lyamin and Sloan, 2006), it is concluded that curves presented the same tendency but bearing capacity values for $GSI < 30$, $\frac{\sigma_{ci}}{\gamma B} < 500$ and $m_i > 10$ remain 10% below the values graphically read from published charts, i.e.,

$$0.9q_{ult}^{Merifield} < q_{ult}^{This\ study} < q_{ult}^{Merifield}$$

This is essentially due to the conservative choice of bearing capacity factor N_γ .

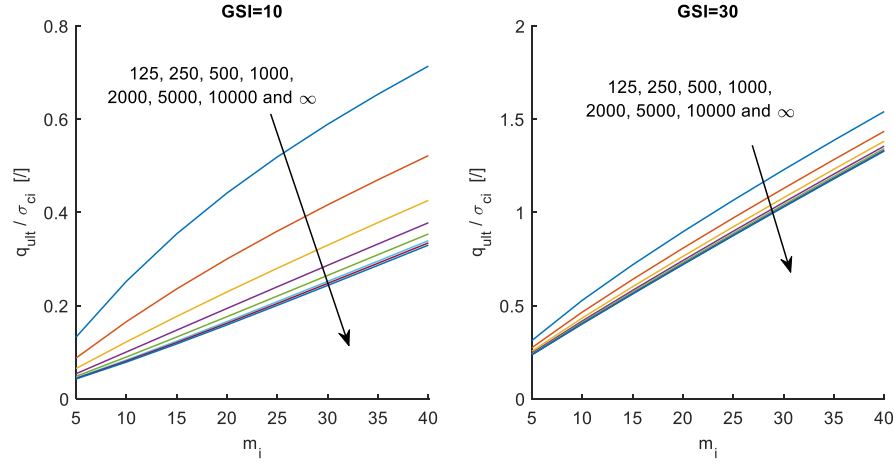


Figure III-41: Effect of the rock self-weight on the normalized bearing capacity for different m_i and $GSI = \{10,30\}$.

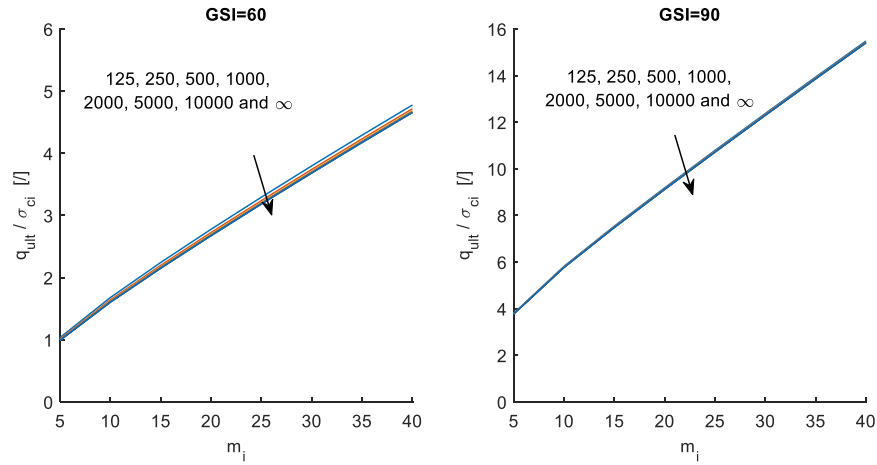


Figure III-42: Effect of the rock self-weight on the normalized bearing capacity for different m_i and $GSI = \{60,90\}$

3.4. Applicability of the method for circular footing

The proposed method described above is extended to include circular footing. A shape factor denoted s_c can be introduced to consider the circular shape of the footing. Thus, the bearing capacity expression is formulated, in non-dimensional form, as follows:

$$\left(\frac{q_{ult}^{circular}}{\sigma_{ci}} \right)_{\gamma=0} = s_c \left(\frac{q_{ult}^{strip}}{\sigma_{ci}} \right)_{\gamma=0} \quad (III-54)$$

A series of analyses were performed by comparing bearing capacity values evaluated using $\phi_{adj} - c$ pair and those published in the literature (Clausen, 2013). This analysis has led to the following formulation of the shape factor:

$$s_f = 1 + \left[0.22 \tan \left(\frac{\pi}{4} + \frac{\phi_{adj}}{2} \right)^{1.4} \right] \frac{B}{L} \quad (\text{III-55})$$

where B and L are, respectively, the foundation width and the foundation length. This factor goes to 1 for $L \rightarrow \infty$ (case of strip footing). In the case of circular footing, $B = L$ and the above equation is simplified to:

$$s_f = 1 + \left[0.22 \tan \left(\frac{\pi}{4} + \frac{\phi_{adj}}{2} \right)^{1.4} \right] \quad (\text{III-56})$$

Table III-13 summarizes the normalized bearing capacity of a circular footing calculated according to the proposed approach for the three different rock quality masses. As can be noted, the results are in excellent agreement compared to the published solutions. It can be concluded that the proposed approach is also valid in the case of a circular foundation resting on a weightless rock.

Table III-13: Normalized bearing capacity of a circular foundation with an adjusted friction angle of three different quality rock masses (weightless case).

Rock quality	q_{ult}/σ_{ci} [/]		
	This study	(Clausen, 2013)	Error (%)
Very poor	0.500	0.495	+0.9
Average	2.354	2.319	+1.5
Very good	11.568	11.612	+0.4

3.5. Validation using Finite Element Modelling (FEM)

In this section, results gained from the equivalent M-C method are validated using the FEM. A numerical model to calculate the bearing capacity of surface foundations is shown, using the geotechnical analysis software RS2.

Recalling that developed solutions are based on the assumptions of elastic perfectly plastic material behaviour and associated flow rule, the same assumptions are assumed in the numerical model.

The rock layer is first modelled using the H-B model. The model uses six parameters, which consist of elastic parameters (E, ν) and the H-B core parameters (σ_{ci}, m_i, GSI and $D = 0$). Then, the mechanical response of the rock is modelled according to the M-C model and the following peak parameters are defined: cohesion (c'), friction angle (ϕ') and tensile strength (σ_t).

i. Strip footing

In this case, the following aspects are considered:

- The 2D plane strain was used for modelling this problem,
- Due to symmetry, only half of the geometry is modelled,
- The lateral boundaries of the model are constrained in normal displacement. Since only half of the model is considered, the normal displacement is also constrained along the symmetry axis,
- A vertical downward uniform load is incrementally applied on the top and
- The load-settlement curve will be derived from the analysis results to assess the bearing capacity.

The geometry details of the model are as shown in Figure II-22. A graded mesh of 2380 triangular elements is used. It should be noted that the mesh was refined at the interface foundation.

The material properties for the three different quality rock masses are given in Table III-11 and the elastic parameters are summarized below in Table III-14.

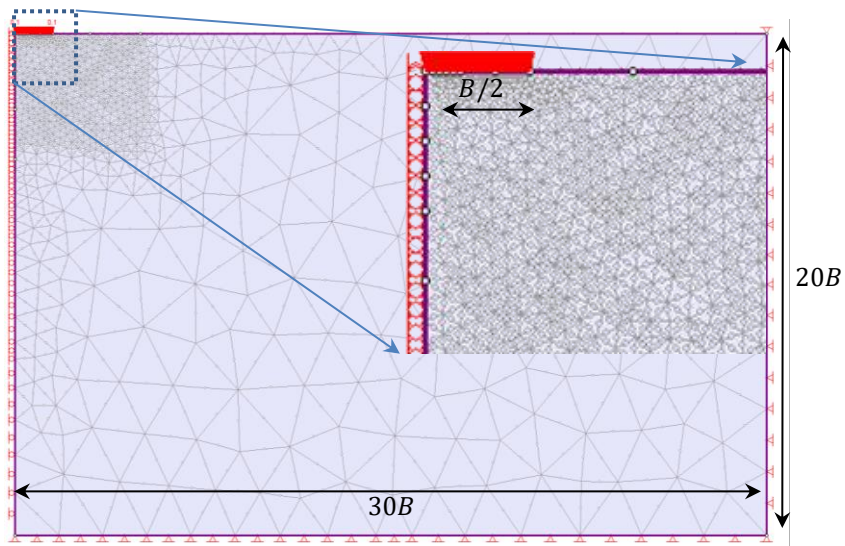


Figure III-43: Boundary conditions, mesh, and geometry details of the numerical model.

Table III-14: Elastic parameters of rocks

Rock quality	Elastic parameters	
	ν [/]	E [MPa]
Very poor	0.2	1000
Average	0.2	8944
Very good	0.2	56234

Load – displacement curves for poor, average, and hard rock are depicted in Figure III-44. A good agreement between the two model solutions is demonstrated and peak stresses equal to the proposed approach results are reached.

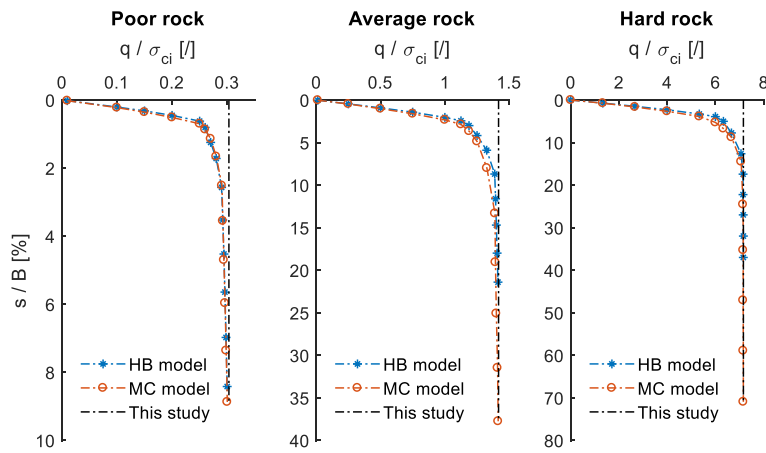


Figure III-44: Load – displacement curves – Strip footing resting on a poor, average, and hard weightless rock.

ii. Circular footing

In this case, the numerical model type is set as an axisymmetric. Load – displacement curves for poor, average, and hard rock are depicted in Figure III-45. The numerical results are close to the analytical ones, which confirm the validity of the proposed method.

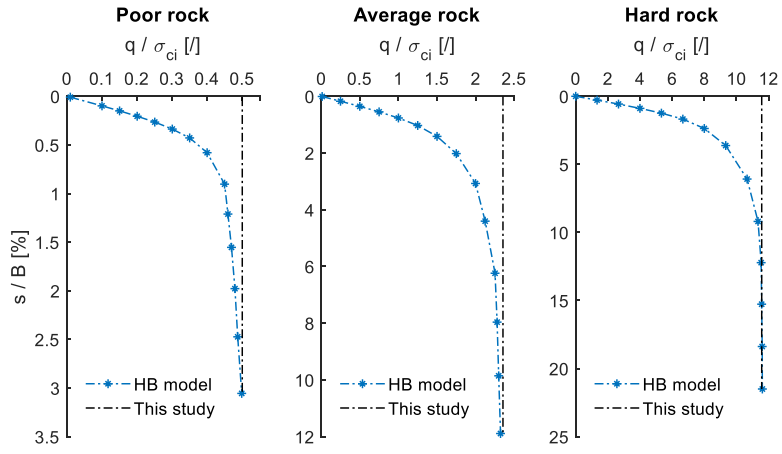


Figure III-45: Load – displacement curves – Circular footing resting on poor, average, and hard weightless rock.

4. Conclusion

The classical problem of assessing the ultimate bearing capacity of a strip footing resting on rock mass that obeys the H-B failure criterion has been investigated. Using the plane strain upper bound limit analysis technique, and by assuming a multi-wedge translation mechanism, it was focused on the effect of the non-associated flow rule on the bearing capacity factor. To handle the nonlinear strength criterion issues, the multi tangential method has been used.

The numerical results have been first compared to previous results published for the associated flow rule case emphasizing the high efficiency of the proposed approach.

For practical purposes in rock engineering, tables allowing easy and accurate estimates of the bearing capacity factor N_{σ_0} are provided. Effect of the flow rule on N_{σ_0} has been carefully implemented using different dilatancy parameter $\alpha = m_{b,dil}/m_b$ that ranges between $[0,0.1,0.25,1]$ (A value of $\alpha = 1$ means that the rock obeys the AFR and a value of $\alpha = 0$ means that the rock obeys the N-AFR with no volume change). It has been noted that for a given GSI and m_i , as the dilatancy parameter $m_{b,dil}$ (i.e. $\alpha = m_{b,dil}/m_b$) decreases, the bearing capacity factor naturally decreases too.

Finally, the effect of the surcharge load, as well as the rock self-weight, has been investigated. It has been found that the bearing capacity factor increases with increasing surcharge load. Furthermore, for a given surcharge load, the lower GSI , the greater the value of N_σ compared to N_σ (case without surcharge load). However, the rock self-weight has almost no effect on the bearing capacity factor if the fraction $\frac{\sigma_{ci}}{\gamma B} > 1000$ or if $GSI > 60$ more generally.

On the other side, and for a specific upper limit of confining stress over which the correspondence between the H-B and the M-C criteria can be considered, equivalent M-C parameters have been suggested using an adjustment friction angle factor. The following conclusions can be made:

(1) Results obtained from the proposed approach fit within 1% published solutions in case of a strip footing resting on weightless rock masses.

(2) Ignoring rock self-weight leads to a potentially very conservative estimate of the ultimate bearing capacity. This is especially the case for poor rock mass where the underestimation can reach 60%. This is particularly true when $GSI < 30$.

III.4. Conclusion

(3) The proposed method is also valid for estimating the bearing capacity of circular footing resting on weightless rock via shape factor. Bearing capacity values are in good agreement within 1.5 % compared to published results (Clausen, 2013).

(4) The comparison of the proposed approach with results from finite element limit analysis validates the accuracy of the proposed method.

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Chapter IV. Experimental program

1. Introduction

To investigate the efficiency and accuracy of the developed solutions in Chapter III, a series of static penetration tests of small-scale model footings (strip and circular) laying on a rock layer are performed. Hence, this chapter completes Chapter III and is divided into five main sections including this introduction part. The second section consists of defining the material used to emulate the rock layer including its geometrical and mechanical properties. Test equipment and procedure are itemized in the third section. Thereafter, experimental results are presented, discussed, and then compared to the analytical results in the fourth section. Finally, conclusions are drawn.

2. Calcarenite used as a rock layer – Characterization

To emulate the rock mass behaviour, calcarenite monoliths that came from the “Ouachee Et Corpechot” quarry (<https://www.ouachee-corpechot.com/>) located in Gouvieux, France are used. The rock is locally referred to as “de Saint-Leu” or “Roche Douce” (cf. Figure IV-1). It is provided in the form of 15 cm cylindrical prisms of an octagonal base for which the diameter of the inscribed cylinder is 45 cm. This “soft rock” (translation of the local French reference “Roche Douce”) is characterized by a porosity of 41 % and a compressive strength that ranges between 4.7 and 8.2 MPa (commercial values provided by the quarry as shown in Figure IV-1). For a detailed microscopic description of the rock mass, one can refer to Baud et al. (2009). They characterized a similar rock mass referred to as “Saint-Maximin Limestone” (SML) that came from a nearby quarry (<http://www.rocamat.fr/>). It consists of calcite (60%) and quartz (40%) embedded into a calcite matrix and frequent fossils. The average grain diameter is 150 μm and 100 μm for quartz and calcite, respectively. It is also reported that the unconfined compressive strength (UCS) of the SML is 12 MPa (Baud et al., 2017).

The mechanical properties of our calcarenite are investigated in the LEMSC laboratory (<https://uclouvain.be/fr/instituts-recherche/immc/lemsc>). The purpose of testing is first, to verify the commercial value of uniaxial compressive strength (UCS) and secondly, to determine the uniaxial tensile strength (UTS) as well as the elastic properties represented by Young’s modulus E and Poisson’s ratio ν . Twenty-two samples of 60 mm nominal diameter and 150 mm nominal length are thus cored. The denomination of each sample refers to the rock type and a number of inventory (1,2,3,...) designing the sample location as shown in the photo of Table V-1.

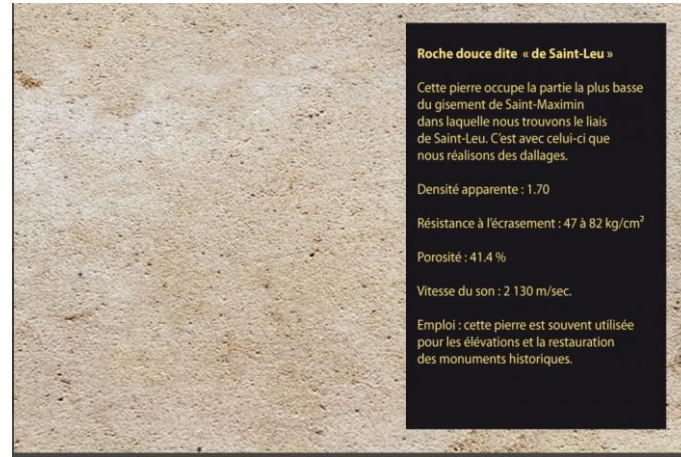
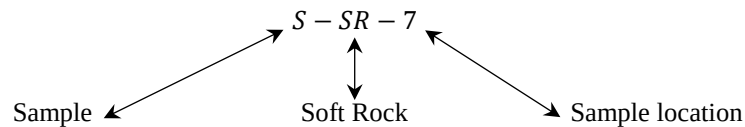


Figure IV-1: Texture and description of the “de Saint-Leu” rock mass (https://www.ouachee-corpechot.com/?page_id=15255#!)

An example of a denomination is given below:



Note that measurements of the sample diameter and length are collected using a digital sliding calliper (measurement accuracy of 0.01 mm). All samples are weighed to determine their density considering their exact volume. The physical properties of the samples are summarized in Table IV-1.

2.1. Uniaxial compressive strength (UCS)

Under uniaxial compression, the failure mode depends on the degree of the end constraints of the specimen offered by the platens of the testing machine and the surface quality of the parallel ends of the cylindrical sample. To mitigate such influence on UCS values, samples with an approximate length/diameter ratio of 2.5 were carefully prepared following the stipulation of the American Society for Testing and Materials (ASTM, 2010). A total of 7 “intact” samples located at locations 3, 9, 10, 12, 14, 15, and 17 are selected for the uniaxial compression test (cf. Table IV-1). Note that samples SSR-3, SSR-9, and SSR-10 are instrumented with three axial strain gauges placed on the lateral surface of the sample at equal angular intervals and mid-height, while samples SSR-12 and SSR-17 are instrumented with 4 90° 2-element Cross strain gauges for both longitudinal and circumferential deformations (cf. Figure IV-2). The instrumented samples serve, besides the determination of UCS, for

Table IV-1: Geometrical and physical properties of soft rock samples.

Physical properties	
Diameter [mm]	58.92 ± 0.12
Length [mm]	150.55 ± 0.15
Weight [g]	727.68 ± 19
Density [g/cm ³]	1.77 ± 0.04
Water content [%]	0.16 ± 0.03



assessing Young's modulus (case of SSR-3, SSR-9, and SSR-10) and Poisson's ratio (case of SSR-12 and SSR-17). Figure IV-3 shows the average stress evolution as a function of the average axial compression for the five instrumented samples. The peak value of the axial stress during the test provides the UCS value. It is found that these values are ranging between 5.1 MPa and 8.9 MPa with a mean value $\overline{UCS}$ equals to 7 MPa (SD=1.57 MPa), and that agrees well with quarry commercial values that range between 4.7 MPa and 8.2 MPa (except for SSR-3 where UCS = 8.9 MPa). UCS values are summarized in Table IV-2 and are represented in Figure IV-4 in the form of a histogram.

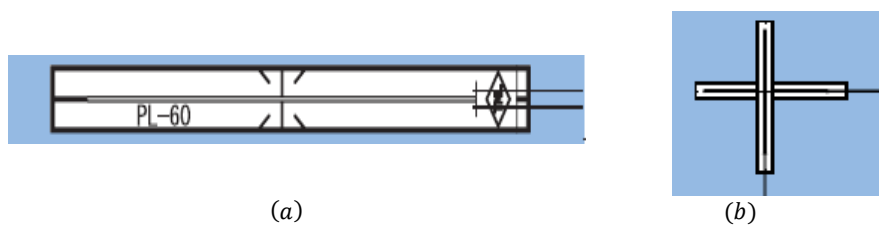


Figure IV-2: Strain gauges types :(a) Single element (b) 90° 2-element Cross.

Table IV-2: Uniaxial compressive strength of seven selected soft rock samples.

Sample ID →	SSR-3	SSR-9	SSR-10	SSR-12	SSR-14	SSR-15	SSR17
UCS [MPa]	8.9	5.5	7.7	8.1	5.1	8.2	5.5
$\overline{UCS}$ & SD [MPa]	7 & 1.57						

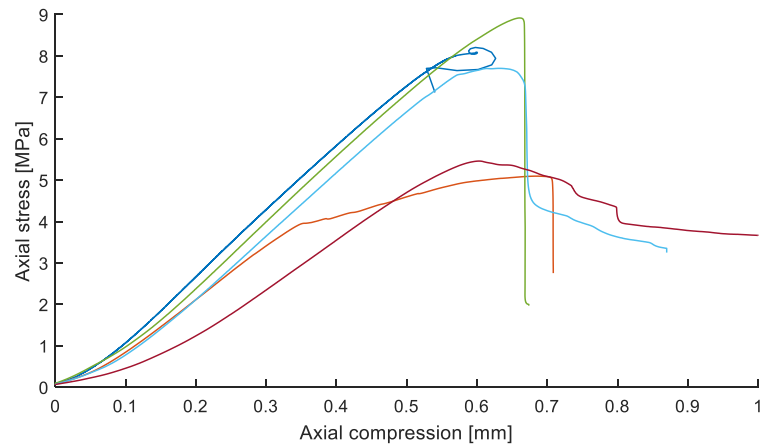


Figure IV-3: Stress evolution as a function of axial displacement during the compression test.

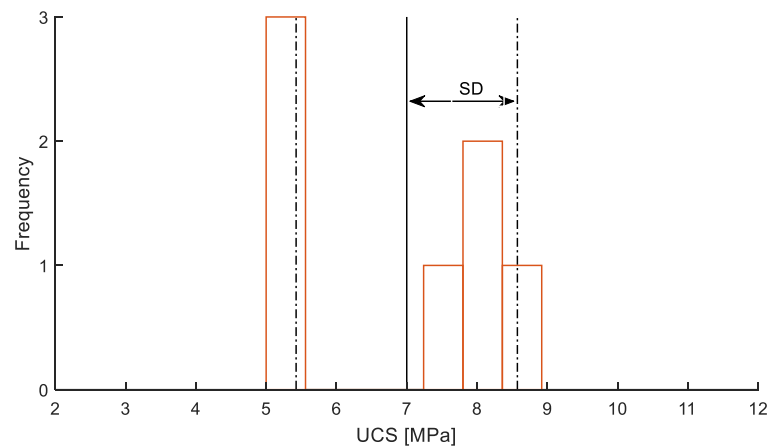
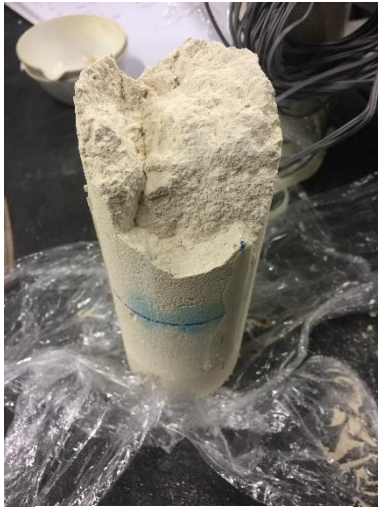


Figure IV-4: Histogram representation of the uniaxial compressive strength values.

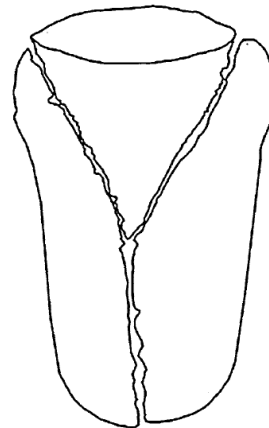
Various failure patterns were observed under the uniaxial compressive strength test. They can be categorized into four modes: axial splitting, shear along a single plane, multiple fracturing, and Y-shaped failure. Photographs of some samples were taken, and corresponding failure modes are schematically represented in Figure IV-5 and Figure IV-6.

SSR-3 / UCS = 8.9 MPa



Multiple Fracturing

SSR-9 / USC = 5.5 MPa

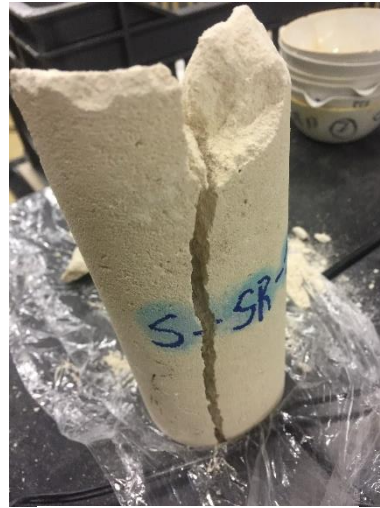
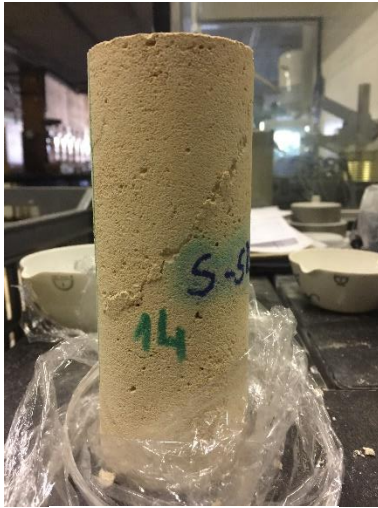


Y-Shaped Failure

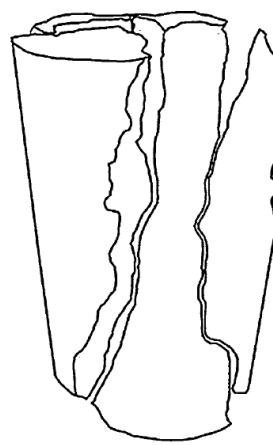
Figure IV-5: Failure modes observed on SSR-3 and SSR-9 and their corresponding schematic representations under uniaxial compressive test conditions.

SSR-14 / USC = 5.1 MPa

SSR-15 / USC = 8.2 MPa



Shearing along a single plane



Axial Splitting

Figure IV-6: Failure modes observed on SSR-14 and SSR-15 and their corresponding schematic representations under uniaxial compressive test conditions.

2.2. Elastic parameters

2.2.1. Young's modulus

The Young's modulus is calculated as tangent modulus about half of the peak strength. It is therefore evaluated as the slope of the stress-strain curves (longitudinal strain ε_{long}) determined between 40% and 60 % of the peak strength (σ_{peak}), i.e.:

$$E_{50}^t = \frac{\sigma(0.6\sigma_{peak}) - \sigma(0.4\sigma_{peak})}{\varepsilon_{long}(0.6\sigma_{peak}) - \varepsilon_{long}(0.4\sigma_{peak})} \quad (IV.1)$$

An example of stress-strain curves as measured by three single element strain gauges SG1, SG2, and SG3 (cf. Figure IV-2), during the compression test on sample SRR-10, are plotted in Figure IV-7. It is found that Young's modulus E_{50}^t of SRR-10 is comprised between 6.8 and 7.8 GPa with a mean value of 7.2 GPa .i.e., a modulus ratio $MR = E_{50}^t / \sigma_{ci}$ of 930. For SRR-3 and SRR-9, the mean E_{50}^t values are equal to 7.1 and 6.9 GPa, respectively (cf. Table IV-3). Considering the aforementioned values, our calcarenite rock mass is characterized by an average Young's modulus ($\overline{E_{50}^t}$) of 7.0 GPa and that corresponds to a MR of 1000. The latter MR value is consistent with Hoek and Diederichs (2006) guidelines for the selection of MR values for such type of rock mass (MR ranging between 800 – 1000).

Table IV-3: Young's modulus and corresponding modulus ratio of samples SSR-3, SSR-9, and SSR-10.

Sample ID	$(E_{50}^t)_{SG_1}^{(1)}$ [GPa]	$(E_{50}^t)_{SG_2}^{(1)}$ [GPa]	$(E_{50}^t)_{SG_3}^{(1)}$ [GPa]	$\overline{(E_{50}^t)}^{(2)}$ [GPa]	SD ⁽³⁾ [GPa]	MR ⁽⁴⁾ [/]
SSR-3	5.9	8.5	6.9	7.1	0.3	800
SSR-9	8.0	7.3	5.5	6.9	1.0	1250
SSR-10	7.8	7.1	6.8	7.2	0.4	930
				7.0		1000

(1) Young's modulus value deduced from the stress-strain curves as measured by the i^{th} strain gauge ($i = \{1,2,3\}$) during the compression test.

(2) Mean value of Young's modulus calculated as $\overline{(E_{50}^t)} = \frac{1}{3} \sum_{i=1}^3 (E_{50}^t)_{SG_i}$.

(3) Standard Deviation.

(4) Modulus Ratio defined as $\overline{(E_{50}^t)} / UCS$.

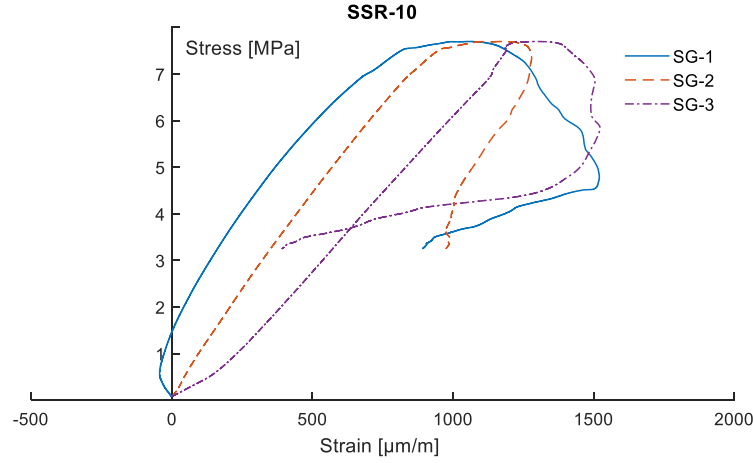


Figure IV-7: Stress-strain curves as measured by strain gauges SG1, SG2, and SG3, during compression test on sample SSR-10.

2.2.2. Poisson's ratio

The Poisson's ratio ν is defined as the ratio of circumferential strain ϵ_{circ} to the longitudinal strain ϵ_{long} within the elastic limit. It is therefore calculated from the stress-strain curves between 40% and 60 % of the peak strength σ_{peak} i.e.:

$$\nu = - \frac{\epsilon_{circ}(0.6\sigma_{peak}) - \epsilon_{circ}(0.4\sigma_{peak})}{\epsilon_{long}(0.6\sigma_{peak}) - \epsilon_{long}(0.4\sigma_{peak})} \quad (IV.2)$$

An example of stress-strain curves as measured by the 90° 2-element Cross strain gauges (cf. Figure IV-2) during the compression test on SSR-12 is depicted in Figure IV-8: Stress-strain curves as measured by the 90° 2-element Cross strain gauges for both longitudinal and circumferential strains during the compression test on SSR-12.. As mentioned previously, SSR-17 is also instrumented as SSR-12. Therefore, an average Poisson's ratio of almost 0.2 is found (SD=0.02).

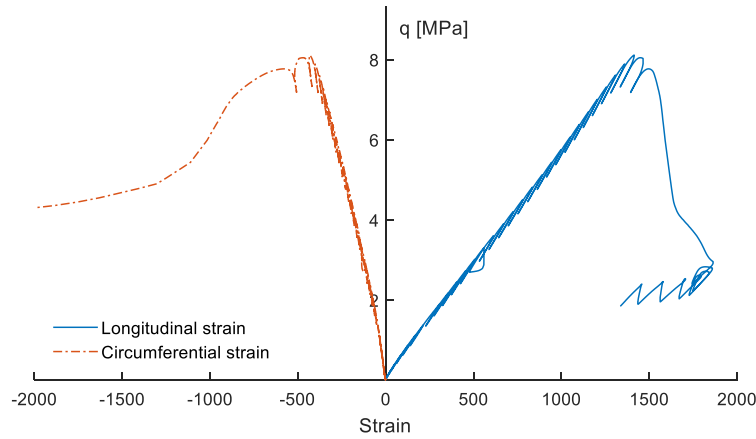


Figure IV-8: Stress-strain curves as measured by the 90° 2-element Cross strain gauges for both longitudinal and circumferential strains during the compression test on SSR-12.

2.3. Uniaxial tensile strength (UTS)

Three samples located at locations 2, 11, and 21 (cf. Table V-1) are selected for the Brazilian tensile strength (BTS) test. The samples are sliced using a circular saw equipped with a diamond blade to obtain circular discs samples. The cut surfaces of the samples were then grinded to achieve a high-quality surface that complies with the tolerances required by ASTM (2010). A total of 12 disk-like samples were tested under the Brazilian tensile test. They were compressed along a vertical diameter between two platens.

BTS results are depicted in Figure IV-9 in the form of a histogram. The BTS values are ranging between (-0.7) and (-1.48) MPa, with a mean value of -1.0 MPa and a standard deviation of 0.23 MPa (cf. Table IV-4).

Nevertheless, the Brazilian test, in which the tensile failure is induced as a centre of a diametrically loaded disc specimen is considered to overestimate the true tensile strength (Perras and Diederichs, 2014). Due to the complex stress distribution and the influence of the stress concentrations at the loading points, the calculation of the uniaxial tensile strength (UTS) requires significant correction. A general rule of thumb used in practice is that $UTS = 0.65 BTS$ which is in accordance with Perras and Diederichs' finding (2014) for sedimentary rocks ($UTS = 0.68 BTS$). Therefore, an UTS value of 0.68 MPa is adopted for our calcarenite rock mass.

Table IV-4: BTS values gained from 12 disk-like samples tested under the splitting tensile test.

BTS (average value) [MPa]	-1.0
SD [MPa]	0.23
UTS = 0.68 BTS [MPa]	0.68

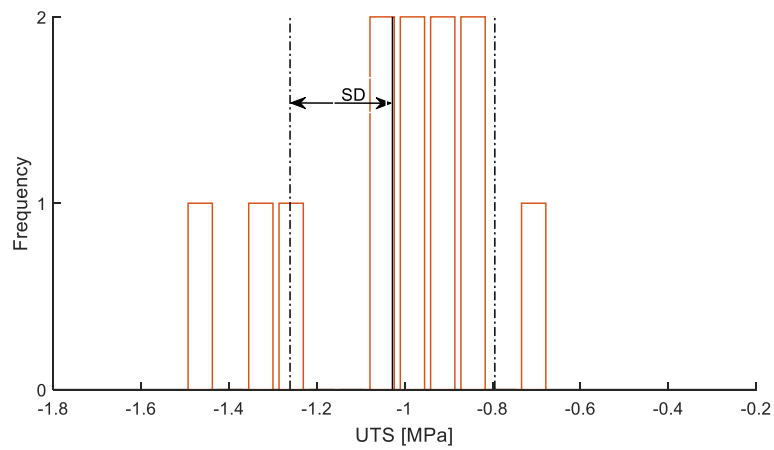


Figure IV-9: Histogram representation of the Brazilian tensile strength values.

Two failure patterns are observed under the splitting tensile strength test: central and non-central failures. Photographs document the two typical failure modes, as schematically represented in Figure IV-10.

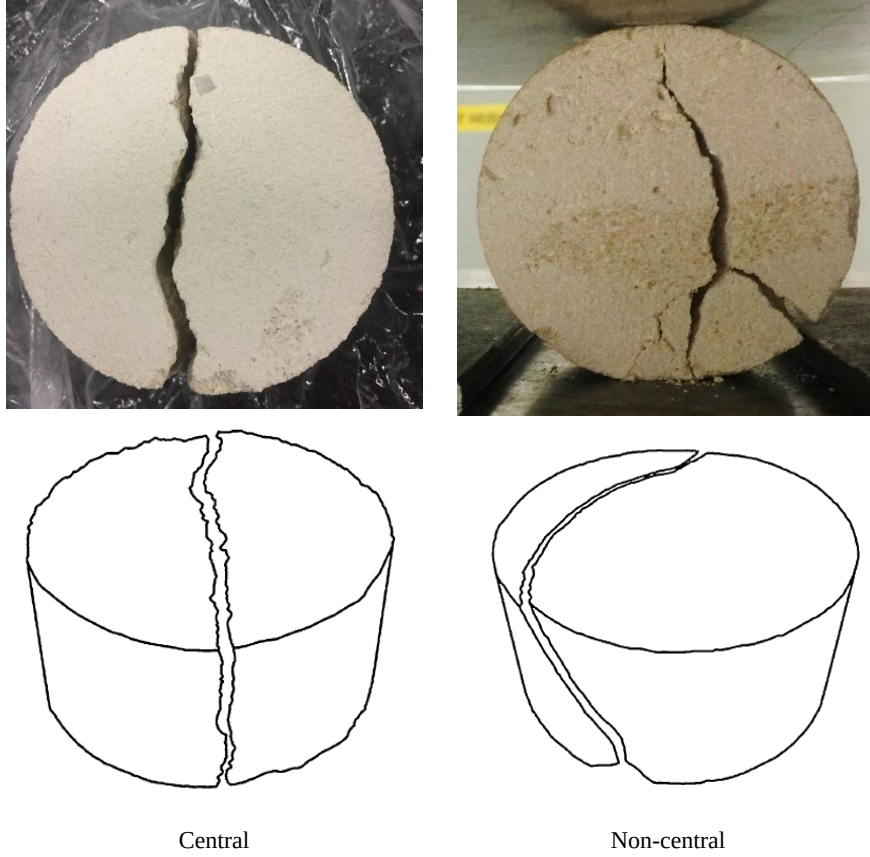


Figure IV-10: Failure modes observed on soft rock samples and corresponding schematic representations under Brazilian test conditions.

2.4. H-B parameters and dilatancy angle

To determine rock mass strength parameters based on the generalized H-B failure criterion, one can resort to UCS and UTS values already found previously and by following the guidelines of Hoek and Brown (2019). The average value for the UCS data set, which represents the value of the principal stress at zero confining stress, is considered as the unconfined compressive strength σ_{ci} , i.e., $\sigma_{ci} = \overline{\text{UCS}} = 7 \text{ MPa}$ (cf. Table IV-2). The strength parameter m_i is deduced from the following approximate relationship of the compressive to tensile strength ratio as:

$$\frac{\sigma_{ci}}{|\sigma_t|} = 0.81m_i + 7 \rightarrow m_i = 4.1 \quad (\text{IV-1})$$

Note that, according to Hoek and Brown (2019), the value of m_i should be approached with at least five triaxial tests, with confining stress ranging from 0 to one half of σ_{ci} ,

which is not the case here. However, for a similar rock mass, an $m_i = 4.2$ was reported from triaxial tests on Indiana limestone (Schwartz, 1964; Hoek and Brown, 2019) which agrees well with the estimated value within 2%.

Regarding the dilatancy angle, one should quote that the proposed dilatancy angle model as well as the published ones (cf. section 2.3.2 of Chapter III for topic review) defines a dilation function of the plastic strain and confining stress in such a way that the value of this parameter gradually drops from its initial value, which corresponds to the peak value, to a value of zero, which refers to a zero plastic volume increase. This null value was proposed in light of the fact that a rock cannot dilate infinitely, as described by Vermeer and Borst (1984) and Detournay (1986) among others, although negative values (representing contractions) have been reported for highly porous rocks submitted to very high confinement stress, which is not a very common occurrence. Baud et al.'s (2009) experiments on Saint-Maximin Limestone are part of this scarcity. They performed triaxial tests at high confining pressures ranging from $0.25\sigma_{ci}$ (3 MPa) to σ_{ci} (12 MPa) on a rock mass (SML) from a quarry nearby the quarry of our calcarenite. At the lowest confining pressures (3 – 5 MPa), Baud et al. (2009) stated that behaviour typical of the brittle faulting regime was observed. At confining pressures between 6 and 12 MPa, Baud et al. (2009) stated that stress-strain curves were mostly flat beyond the yield point and a volumetric failure emphasized by a cap surface has occurred (more details can be found in Baud et al. (2009)). Although the SML appears to be similar to our calcarenite because of its texture and its proximity to our calcarenite quarry, Baud's findings are not appropriate to describe our calcarenite rock mass for two main reasons. Firstly, mechanical properties of the SML (UCL=12 MPa and $m_i = 1.5$) are quite different of the calcarenite (UCL= 7 MPa and $m_i = 4.1$). Secondly, confinement stress generated below and around the footing are too low to reveal a volumetric rupture. It is therefore suggested to set a dilatancy parameter α equal to 0.25.

2.5. Monolith preparation

To prevent the splitting of the rock monolith during the loading tests, the octagonal block is confined using a 508 mm diameter steel tube of 8 mm wall thickness and 15 cm height. Non-shrinking grout is poured in the void left between the monolith and its confining tube (cf. Figure IV-11). The confined monolith is placed atop an unconfined calcarenite pedestal, acting as a base.

The denomination of each monolith refers to the rock type, its height expressed in 'cm,' and a letter of inventory (A, B ...), i.e.:

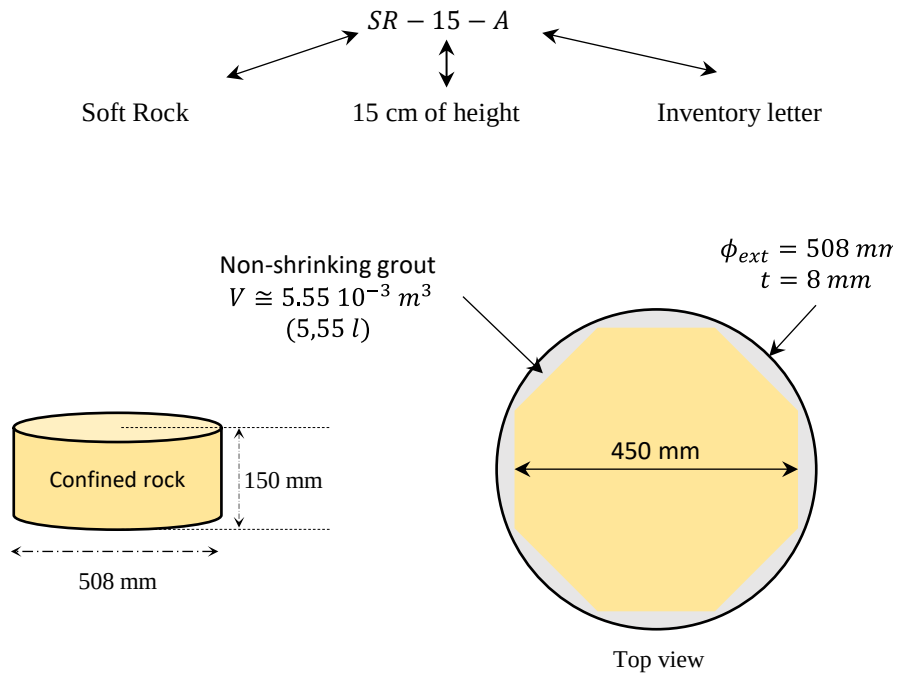


Figure IV-11: Schematic representation of the confined monolith – Geometric details.

3. Test equipment and test procedure

The experimental program is carried out on three confined monoliths denoted as SR15A, SR15B, and SR15C. For SR15A, a total of five static penetration tests of model strip footing are conducted at different positions P1, P2, P3, P4, and P5 located, respectively, at the Centre (C), East (E), West (W), South (S) and the North (N) as illustrated in Figure IV-12. A steel plate is used to model the reduced scale-tested footing. Its width B , length L , and height H are 8.27, 71.17, and 45.27 mm, respectively (cf. Figure IV-13). The selected width and height are sufficient to withstand bending stresses.

The confined monolith SR15B serves for static penetration tests using two different footing geometries. Firstly, three loading tests of model strip footing are conducted at positions P1 (C), P4 (S), and P5 (N). Secondly, six loading tests of model circular footing are conducted at positions P2 (E), P24 (S-E), P25 (N-E), P3 (W), P34 (S-W), and P35 (N-W), as illustrated in Figure IV-14. Note that a steel dowel of 11.85 mm diameter and 40 mm height is used as model circular footing (cf. Figure IV-13).

Finally, static load tests carried out on SR15C are identical to those carried out on SR15B, but the sequence of loadings tests of the model strip and circular footings is reversed.

A 106 mm spacing (minimum) between different positions, as shown in Figure IV-12 and Figure IV-14, is selected considering the influence of failure patterns arising from the theoretical critical slip surface obtained from the numerical optimization (cf. Section 2 of Chapter III). Figure IV-15 shows that the transverse extent of the Rankine passive wedge on both sides does not exceed 50 mm, which is below the adopted spacing.

A Zwick 250kN, an automatic compression test machine of 250kN load capacity, is used for the tests. The cyclic load test procedure laid out in Figure IV-16 is followed. As shown, a cycle is composed of three main phases. First, a preload phase of 0.5 kN is defined. Then, an incremental displacement of 1 mm with a displacement rate of 10 mm/h is imposed. Finally, an unloading phase prevails.

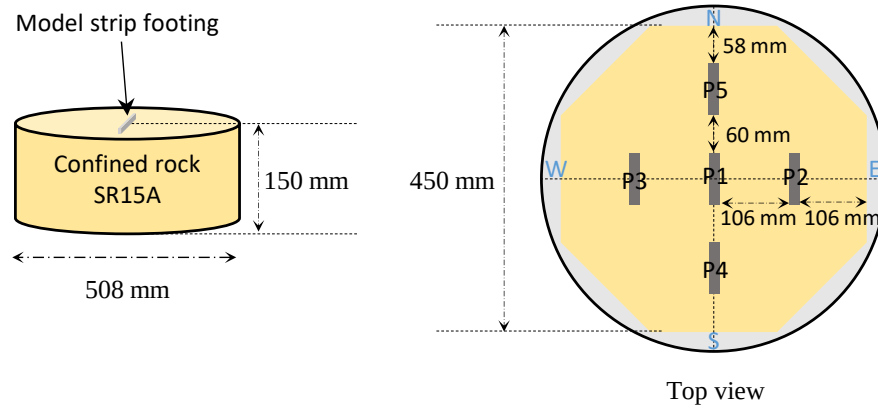


Figure IV-12: Positions of five static penetration tests conducted on SR15A.

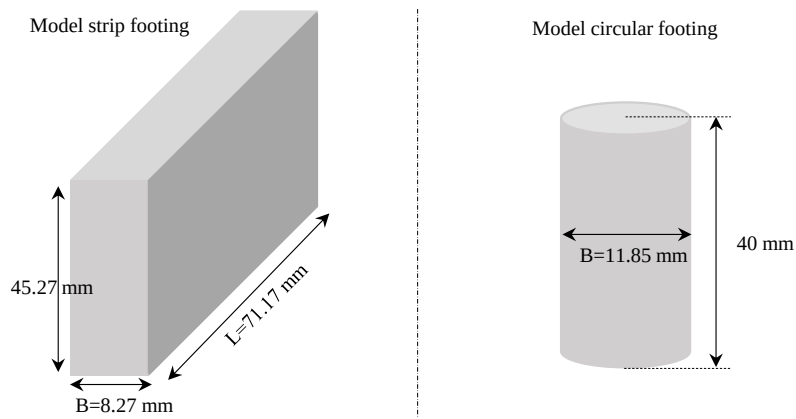


Figure IV-13: Model strip and circular footings – Geometric details.

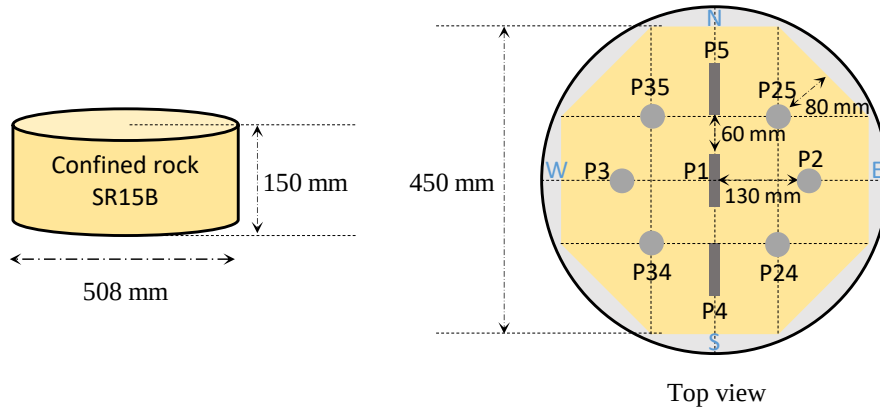


Figure IV-14: Positions of nine static penetration tests (3 strip + 6 circular) conducted on SR15B.

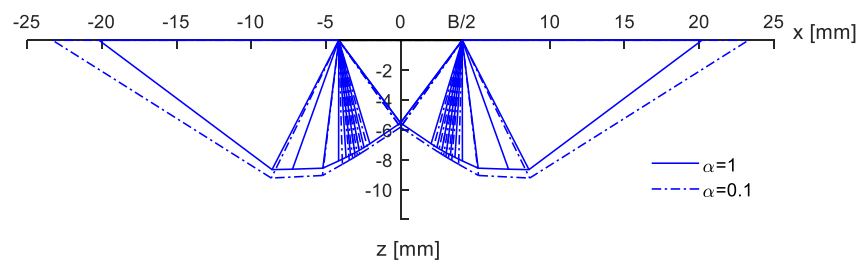


Figure IV-15: Theoretical critical slip surface of the model strip footing resting on Calcarene monolith obtained from the numerical optimization (cf. Section 2 of Chapter III).

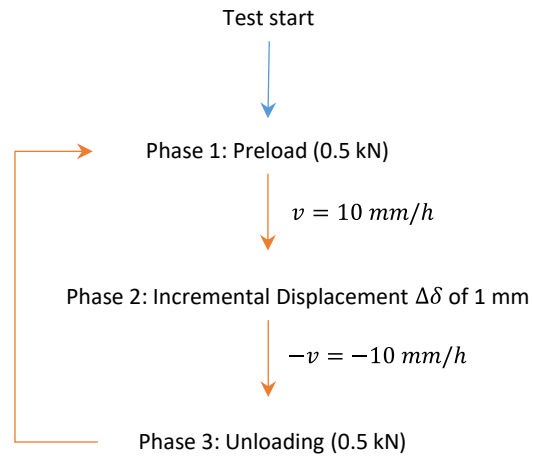


Figure IV-16: Load test procedure.

4. Experimental results and discussion

4.1. Test observations

During penetration tests, as load increases, the model footing sinks into the calcarenite layer to a significant depth. Nevertheless, bulging of calcarenite around the footing was absent. Besides, no cracking sound was heard and no calcarenite chipping around the footing perimeter was observed (cf. Figure IV-17). Furthermore, load-displacement curves do not have a dramatic break and the general trend of these curves is rather a bilinear type response as will be detailed below. Given the aforementioned observations, we might be in the presence of a punching shear failure rather than a local or general shear one. This type of failure occurs generally in a material of very high compressibility (Vesic, 1963). Yet, high compressibility implies high porosity which is the case of the calcarenite monoliths with a porosity $n = 40\%$.

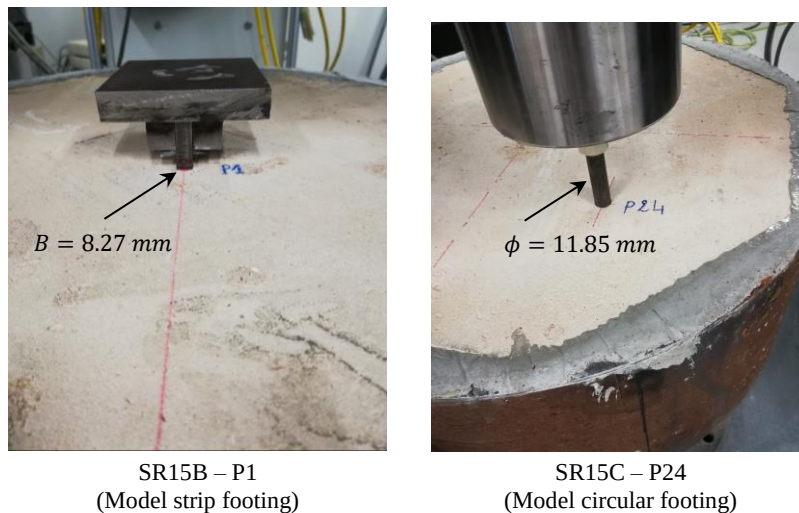


Figure IV-17: Model footings during static penetration tests.

Typical load-displacement curves of five static penetration tests of model strip footing conducted on SR15A are illustrated in Figure IV-18. Each curve depicts the load evolution (left y-axis) and the corresponding vertical stress (right y-axis) at a specific position. As can be seen, during the unloading phases, a slight rebound is noted. This phenomenon is mainly due to the elasticity of the test equipment/monolith. In what follows, and for the sake of clarity, the unloading/reloading part of the curves are removed, and only the part of load-displacement curves with increasing penetration is considered (cf. Figure IV-19 and Figure IV-20). The same applies to load-displacement curves gained from static penetration tests on model circular footing conducted on SR15B and SR15C (cf. Figure IV-21 and Figure IV-22).

The general trend of these curves can be categorized as a bilinear type response. As can be seen, a first branch characterized by a linear increase in load as the model footing sinks into the calcarenite layer is followed by a second ascending branch with lower stiffness. For punching shear failure, the bearing capacity is often defined as the first major non-linearity in the load-displacement curve. It then follows to consider the “kink point” between the first and second branches as the yield point. The coordinates of this point define the admissible settlement and the admissible stress and hence called admissible bearing capacity. Further details are provided in the upcoming subsection.

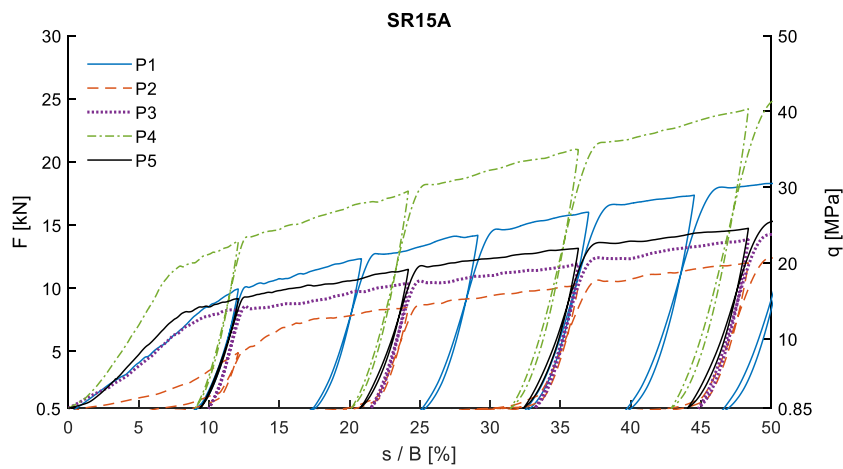


Figure IV-18: Typical load-displacement curves of five static penetration tests of model strip footing conducted on SR15A.

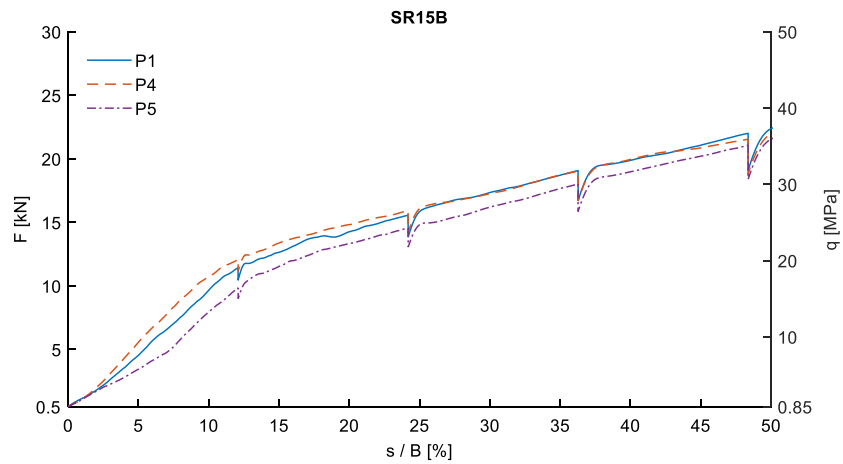


Figure IV-19: Load – displacement curves of three static penetration tests of model strip footing conducted on SR15B.

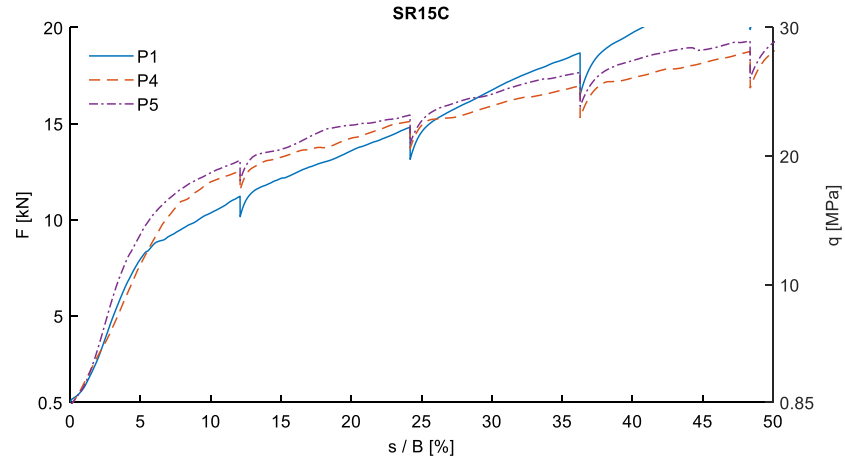


Figure IV-20: Load – displacement curves of three static penetration tests of model strip footing conducted on SR15C.

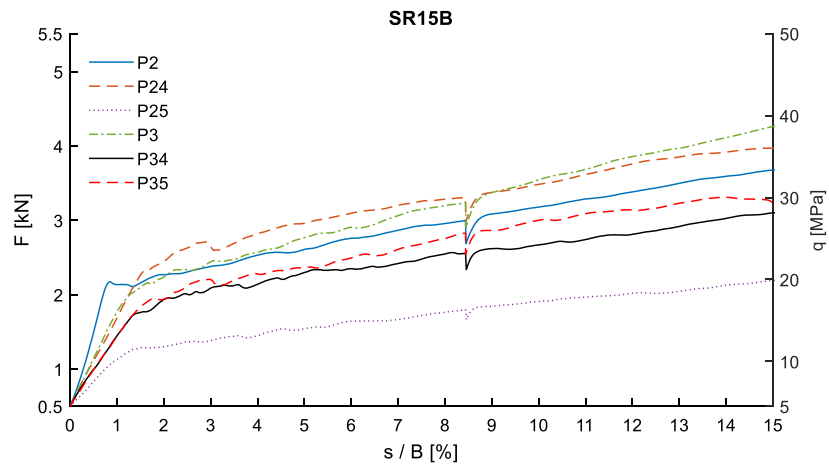


Figure IV-21: Load – displacement curves of six static penetration tests of model circular footing conducted on SR15B.

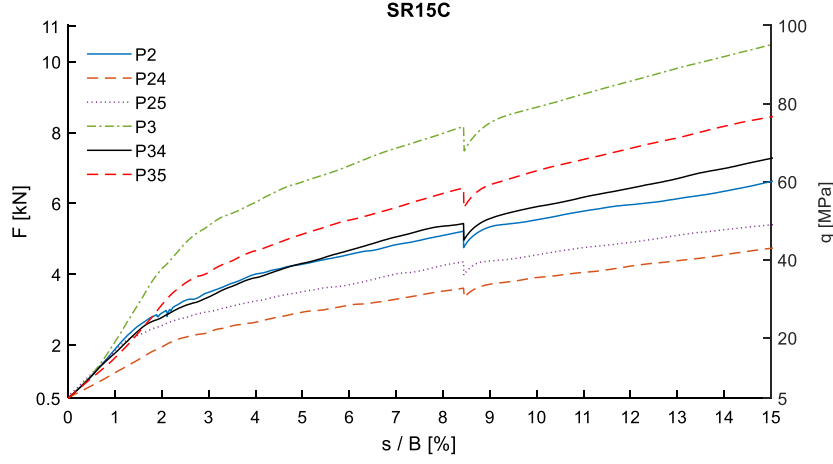


Figure IV-22: Load – displacement curves of six static penetration tests of model circular footing conducted on SR15C.

4.2. Interpretation of results and discussion

To analyse and interpret the experimental results, two main parameters are calculated based on the experimental load-displacement curves. The first parameter is a reduction factor $F_R \leq 1$ defined as the ratio of the measured acceptable bearing capacity to the ultimate theoretical one. This factor will reflect by how much should the theoretical bearing factor N_σ reduced to get an acceptable value of bearing capacity, i.e.,

$$q_{test} = F_R q_{ult} = F_R \sigma_{ci} N_\sigma; 0 < F_R \leq 1 \quad (IV-2)$$

The second parameter is the foundation stiffness k calculated as the peak slope of the unloading curves. k values will be compared to the published analytical solution of static vertical stiffness k_∞ of circular foundation on half-space (Johnson, Christiano and Epstein, 1975; Kausel and Roesset, 1975).

Furthermore, and to better qualify/quantify the monolith being tested, SR15B and SR15C are sampled ($\varnothing = 35 \text{ mm}$) as illustrated by the dashed circles in Figure IV-23, then tested under uniaxial compression. UCS values are summarized in Table IV-5. For sake of clarity, a UCS map specific to each monolith is shown in Figure IV-23. As expected, UCS values are quite dispersed, and this dispersion is more pronounced for SR15C than for SR15B. The lowest UCS values that range between 4 and 5 MPa are measured in the South-East side of the monolith SR15B, whereas the highest UCS values that range between 6 and 7 MPa are assessed on the South-West side. On the other hand, the UCS dispersion for SR15C is almost symmetrical concerning the N-S

axis. UCS values are minimal around the axis of symmetry and increase as one moves either East or West. To sum up, average UCS values of 5.4 and 6.6 MPa with a standard deviation (SD) of 0.9 and 1.4 MPa are, respectively, assessed for SR15B and SR15C. These values fall within the commercial values provided by the quarry (4.7 – 8.2 MPa). Note that for SR15C, UCS value differs by 6% less (22% for SR15B) than that evaluated previously (cf. Table IV-2).

Table IV-5: UCS values [MPa] gained from the uniaxial compression test on $\varnothing 35 \times 70$ h mm cylindrical samples.

Position ⁽¹⁾	SR15B	SR15C
P2-P24	4.3	8.0
P2-P25	N.A	12.0 ⁽²⁾
P3-P34	6.8	7.7
P3-P35	N.A	8.0
P4-P24	5.0	4.7
P4-P34	5.5	4.8
P5-P25	5.1	6.8
P5-P35	N.A	6.0
$\overline{UCS}$ / SD	5.4 / 0.9	6.6 ⁽²⁾ / 1.4

(1) The different positions can be visualized in Figure IV-23.

(2) The UCS value of P2-P25 is ignored for the calculation of $\overline{UCS}$.

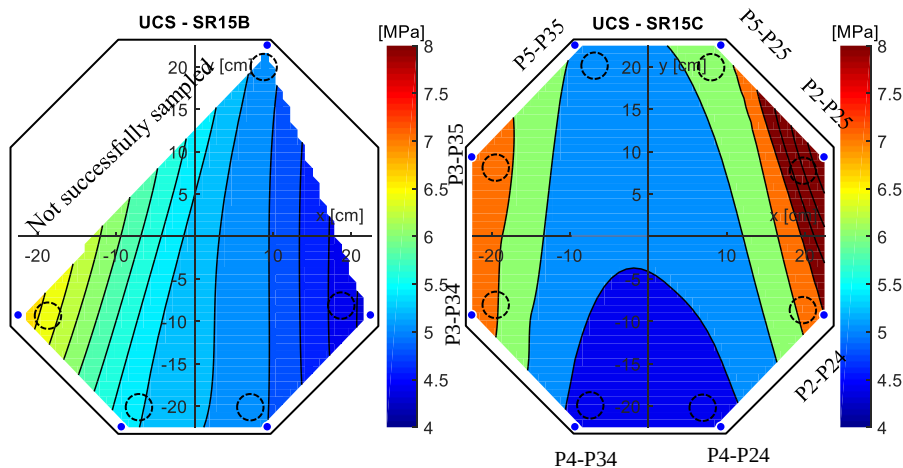


Figure IV-23: UCS [MPa] map specific to each confined monolith (SR15B & SR15C) – Dashed circles (proper scale) correspond to the positions of the samples ($\varnothing 35 \times 70$ h mm) tested under uniaxial compression.

- Reduction factor F_r

Values of the acceptable bearing pressure, as well as the corresponding settlement, are summarized in Table IV-6. For static penetration tests of model strip footing, the average values of 15 MPa, 17.9 MPa, and 14 MPa are respectively measured on

SR15A, SR15B, and SR15C. These values were reached for a normalized penetration depth s/B of 6.6%, 9% and 5.5%, respectively. For static penetration tests of model circular footing, acceptable bearing capacity values are slightly higher compared to the strip case. This is expected due to the shape effect of the circular footing. The measured acceptable bearing capacities are 18 MPa and 25 MPa (average values) for SR15B and SR15C, respectively. These values were reached for a quite similar penetration depth s/B less than 2%.

Table IV-6: Loading tests – Acceptable bearing capacity values.

Monolith	Model footing	Position	s/B [%]	q_{test} [MPa]
SR15A	Strip	P1	8.1	14.9
		P2	6.9	12.0
		P3	6.2	13.1
		P4	5.9	20.6
		P5	6.1	13.4
		Average	6.6	15
SR15B	Strip	P1	8.9	18.4
		P4	9.1	18.0
		P5	9.0	17.2
		Average	9.0	17.9
	Circular	P2	1.0	19.3
		P24	1.5	20.7
		P25	1.1	10.2
		P3	1.1	17.1
		P34	1.3	15.6
		P35	1.4	16.1
		Average	1.2	18
SR15C	Strip	P1	4.1	13.0
		P4	6.2	15.7
		P5	4.9	14.6
		Average	5.0	14
	Circular	P2	1.5	23.4
		P24	2.2	18.6
		P25	1.2	19.2
		P3	2.0	37.5
		P34	1.7	23.3
		P35	2.3	32.5
		Average	1.8	25

On the other hand, the theoretical ultimate bearing capacity from the exact solution is 34 MPa ($\sigma_{ci}N_{\sigma_{ci}}$) for the AFR ($\alpha = 1$) and it is 30 MPa assuming an N-AFR with $\alpha = 0.25$. It follows that the reduction factor F_R is ranging between 0.4 – 0.5 for the AFR case and 0.5 – 0.65 for the N-AFR case with $\alpha = 0.25$. It is worth mentioning that the introduction of such parameter (F_R) is a necessity for the practical use of the exact theoretical solution. Furthermore, it is noteworthy that F_R is comparable to the compressibility factors F_c , F_q and F_γ , introduced by Vesic (1973) to take into account

the effect of soil compressibility on soil bearing capacity. F_c, F_q and F_γ are related to cohesion c , surcharge load q and soil unit weight γ , respectively. Vesic (1973) developed the following equations for the compressibility factors as a function of the rigidity index and the friction angle of the soil:

$$F_q = F_\gamma = \exp \left\{ \left(-4.4 + 0.6 \frac{B}{L} \right) \tan(\phi) + \frac{3.07 \sin(\phi) \log(2I_r)}{1 + \sin(\phi)} \right\} \quad (IV-3)$$

$$F_c = F_q - \frac{1 - F_q}{N_q \tan \phi}$$

Our calcarenite is characterized by a rigidity index $I_r = \frac{G}{\sigma_{ci} m_b^\beta} = \frac{MR}{2(1+\nu) m_b^\beta} = 100$ and a peak friction angle values of 37° (AFR) and 33° (N-AFR with $\alpha = 0.25$) obtained from the numerical optimization corresponding to $k = 14$, $\gamma = 0$ kN/m³, $q = 0$ kPa, $D = 0$, $GSI = 100$ and $m_i = 4.1$. It then follows that the F_c values computed per Eq. (IV-3) are 0.4 for the AFR case and 0.61 for the N-AFR case with $\alpha = 0.25$. The latter values fall within the range of F_R values previously found. One may thus resort to Eq. (IV-3) to estimate an indicative value of the factor F_R to be used to predict the allowable bearing capacity for practical use.

- Foundation stiffness k

The foundation stiffness is the force required to move the body by a unit displacement. This parameter can be deduced experimentally from the slope of the loading/unloading – displacement curves. By assuming that rock foundation – model footing interaction can be idealized by a system of two springs in series as illustrated in Figure IV-24, the rock foundation stiffness k_R could be found as a function of the equivalent stiffness k_{eq} (calculated as the slope of the load-displacement curves) and the model footing stiffness k_F as:

$$k_R \left[\frac{kN}{mm} \right] = \frac{k_F k_{eq}}{k_F - k_{eq}}; \quad k_F = \frac{EA}{H} \quad (IV-4)$$

in which, H is the foundation height, $E = 210$ GPa is the model footing modulus of elasticity and A is the model footing surface area, leading to k_F of 2.73 and 0.58 MN/mm for the strip and circular models, respectively.

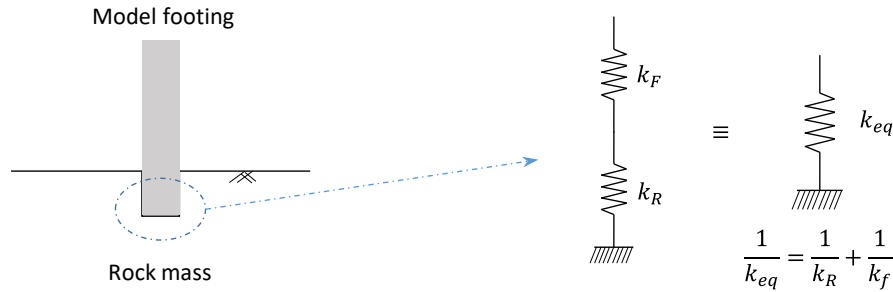


Figure IV-24: Rock foundation – model footing interaction idealized by a system of two springs in series.

Figure IV-25 and Figure IV-26 depict the evolution of k_R from both loading and unloading phases for all experimental test positions conducted on monolith SR15B and SR15C, respectively. It can be noted that stiffness curves gained from unloading phases differ from one position to the other within the same block and increase non-linearly as the model footing sinks into the calcarenite rock layer. This is not the case for stiffness curves deduced from loading phases where stiffness curves are very close to each other, within the same block, and stiffness values increase linearly and slightly with an increasing penetration depth. Furthermore, unloading stiffness values are higher compared to the reloading ones. These aforementioned observations can be explained by the fact that during the loading phase, both elastic and plastic deformations occur as the load increases. While in the unloading phase, only the elastic deformation recovers, leading to the load decreasing more rapidly as the deformations change.

For a matter of comparison between circular and strip model footings, the normalized stiffness values (obtained by dividing the stiffness values by the footing surface area) are reported in the right-y-axis as shown in Figure IV-25 and Figure IV-26. The normalized stiffness values bring to light that the foundation stiffness under circular footing is almost 2 – 3 times larger than that of the strip case. This can be explained by referring to Figure IV-27, in which stress bulbs based on Newmark's solution (Newmark, 1942) of Boussinesq's equation (J. Boussinesq, 1885) for circular and strip footings are illustrated. The spherical bulb formed under the circular footing is much more significant compared to the strip case, and that requires a significant force to move the footing downward i.e. a higher stiffness.

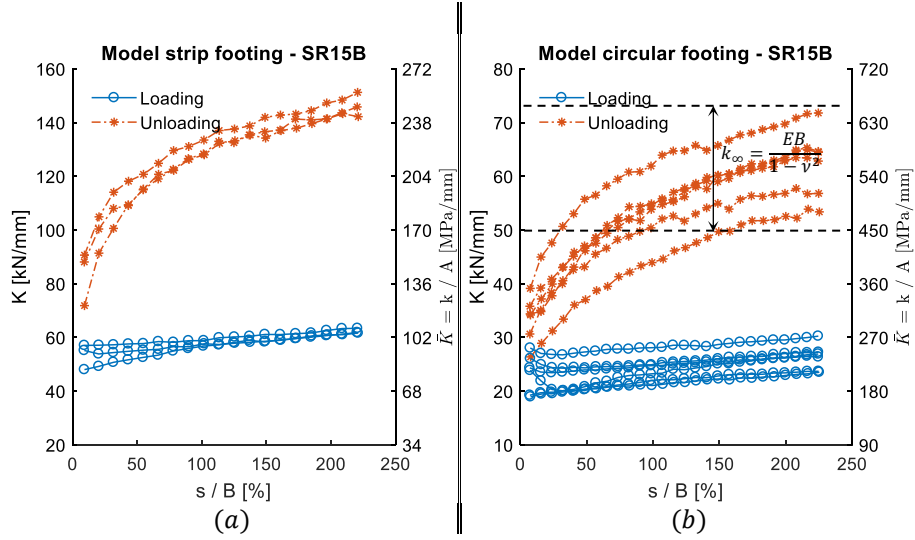


Figure IV-25: Foundation Stiffness based on static penetration tests conducted on the confined monolith SR15B – (a) model strip footing, (b) model circular footing.

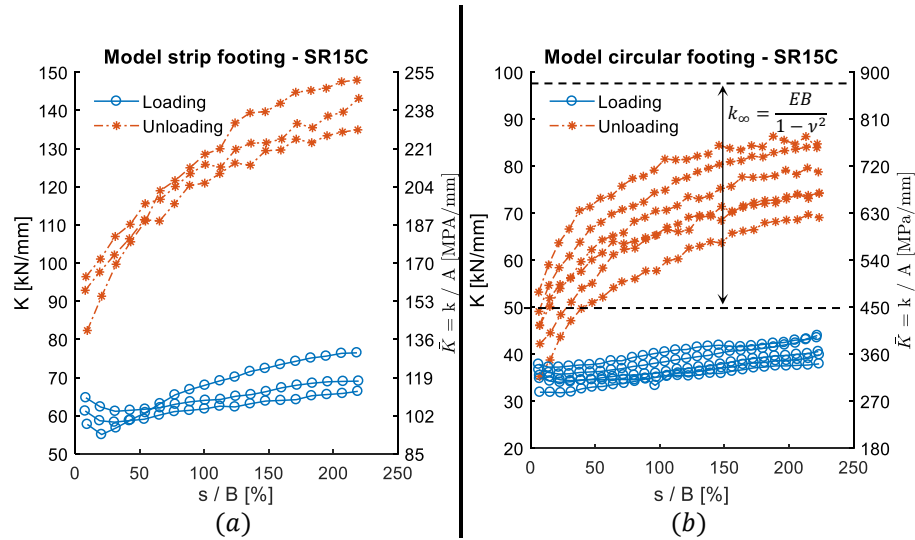


Figure IV-26: Foundation Stiffness based on experimental penetration tests conducted on the confined monolith SR15C – (a) model strip footing, (b) model circular footing.

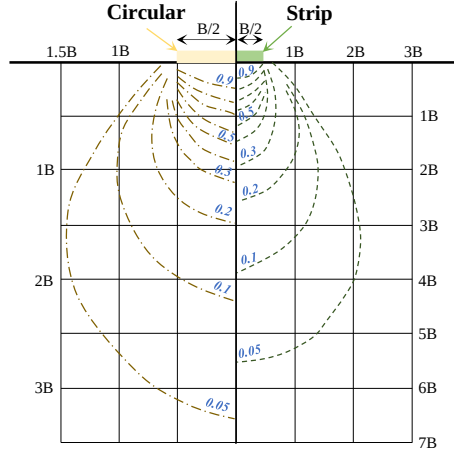


Figure IV-27: Stress bulbs based on Newmark's solution (Newmark, 1942) of Boussinesq's equation (J. Boussinesq, 1885) for circular and strip footings.

The measured foundation stiffness from unloading phases is compared to the published analytical solution (Johnson, Christiano and Epstein, 1975; Kausel and Roesset, 1975) of static vertical stiffness of circular foundation on half-space expressed as:

$$k_{\infty} = \frac{EB}{1 - \nu^2} \quad (\text{IV-5})$$

Here, $B = 11.85 \text{ mm}$ is the foundation diameter, E is the elastic modulus, and $\nu = 0.2$ is the Poisson's ratio. For the intact calcarenite, and by resorting to the UCS map specific to each monolith, E values, calculated as a product of $MR = 1000$ and UCS, ranges from 4 – 6 GPa for SR15B and 4 – 8 GPa for SR15C. It follows that the static vertical stiffness computed according to Eq. (IV-5) ranges from 50 – 74 kN/mm for SR15B and 50 – 98 kN/mm for SR15C. Within the same monolith, this remarkable gap between the minimum and maximum k_{∞} values is mainly due the UCS dispersion. It is found that the k_{∞} range agree with the measured ones.

5. Conclusion

A series of static penetration tests of small-scale model footings (strip and circular) laying on a rock layer is conducted. A Calcarenite rock mass is selected to emulate the rock layer. The mechanical properties of this latter are firstly determined from indirect tensile and uniaxial compression strength tests. The H-B core parameters are thereafter deduced.

The experimental campaign was performed with the hope to get the classical Buisman-type failure mechanism (Buisman, 1940). i.e. a general shear failure. Nevertheless, bulging of calcarenite around the footing was absent. Furthermore, load-displacement curves do not have a dramatic break and the general trend of these curves is rather a bilinear type response. Given the aforementioned observations, a punching shear failure rather than a local or general shear one took place. This type of failure occurs generally in a material of very high compressibility (Vesic, 1963). Yet, high compressibility implies high porosity which is the case of the calcarenite monoliths with a porosity $n = 40\%$. Furthermore, for such type of failure, the bearing capacity was defined as the first major non-linearity in the load-displacement curve reflected by a “kink point”. A reduction factor F_r defined as the ratio of the measured acceptable bearing capacity to ultimate theoretical one was introduced. This factor reflects by how much should the theoretical bearing factor N_o reduced to get an acceptable value of bearing capacity. It was found that F_r is comparable to the compressibility factor introduced in the literature to take into account the effect of soil compressibility on soil ultimate bearing capacity. It was therefore suggested to estimate F_r values by resorting to the published equation of the compressibility factor as a function of the rigidity index and the friction angle of the rock mass.

It should be emphasized that the performed 1g tests on reduced-scale models suffer from severe limitations due to the scale effect associated foremost with the very low stresses generated in the rock layer beneath the model footing. In particular, this means that calcarenite strength i.e. UCS, with respect to the confinement stress, is very high. Consequently, the above condition must be met at least in situ to extrapolate the results of the scale model experiments to the real size problem. It would be more interesting to employ a centrifuge machine that enhances the geotechnical models by producing identical self-weight stresses in the model and prototype, which makes it possible to obtain accurate data.

Chapter V. Bearing capacity of pile foundation

1. Introduction

Existing empirical relations to predict the bearing capacity of deep foundation in rock masses rely only on the unconfined compressive strength σ_{ci} of the intact rock (see Subsection 3.1 of Chapter II for topic review). However, as the rock mass supports the pile, it would be more realistic to consider rock mass properties instead of the intact one.

In this chapter, a semi-analytical method, not an empirical one, is proposed for calculating the ultimate bearing capacity of a pile embedded in rock mass that obeys the H-B failure criterion. With this in mind, Section 3 of this chapter is devoted to the development of an analytical calculation procedure to predict the shaft resistance for piles drilled to some depth into a rock layer. Section 4 presents the methodology to be followed to predict the pile end bearing capacity using the cavity expansion theory. Indeed, the cavity expansion theory was often used to model pile behaviour in two different ways:

- (1) The shaft resistance of the pile can be correlated to the cylindrical cavity limit pressure. Indeed, driving the pile into the rock mass can be modelled as the expansion of a cylindrical cavity from zero initial radius to the radius of the pile. That will give the stress change in the rock mass around the pile due to the pile installation, which can be used to estimate the shaft resistance.
- (2) The base resistance of the pile can be correlated to the spherical cavity limit pressure. Nevertheless, there is also a tendency to use cylindrical cavity limit pressure for the same purpose, especially for sands (Mitchell and Tseng, 1990). Salgado and Prezzi (2007) stated that cylindrical cavity expansion analysis is possible for a soil medium subjected to an anisotropic stress state. In contrast, spherical cavity expansion can be directly analysed for a medium in which isotropic stress field is established. Consequently, the choice of spherical cavity stems from the fact that the H-B criterion assumes isotropy.

To the author's knowledge, the cavity expansion problem in rock mass obeying the H-B failure criterion has not been addressed. The development of complete rigorous solutions for stress and displacement fields is the primary motivation of Section 2 of this chapter. From a practical point of view, these solutions are applied thereafter to analyse piles embedded in rock masses (Section 4).

2. Cavity expansion in a rock mass

Nota:

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2.1. Overview

The cavity expansion theory has been extensively developed and widely used to study a variety of practical geotechnical engineering problems. Among them, one can cite pile installation effects (Randolph and Wroth, 1979), prediction of the bearing capacity of deep foundations (Vesic, 1972; Yasufuku and Hyde, 1995), and interpretation of in-situ tests such as pressuremeter tests PMT (Gibson and Anderson, 1961) and cone penetration tests (Salgado, Mitchell and Jamiolkowski, 1997; Salgado and Prezzi, 2007). The main objective of this theory is to predict the stress and displacement fields around an expanding cavity. Most of the analytical solutions are based on either small or large strain theories and the solution procedure for acquiring a complete pressure-expansion relationship varies. The small strain solution predicts an increasing internal pressure as the cavity expands, whereas the large strain solution predicts approaching a limit pressure at vast expansion.

For cavities in rock masses, most practical applications still consider rock as soil and predict their behaviour by using soil-related failure criteria such as the linear M-C criterion for which extensive literature is available (Yu, 2000). Nevertheless, the non-linear H-B failure criterion represents the rock mass behaviour more rigorously. Since the introduction of this criterion, most analyses have focused on cavity contraction rather than cavity expansion problems. The reason lies in the abundant practical applications such as tunnelling and underground excavations, where it is crucial to estimate the tunnel convergence.

Earlier solutions for cavity contraction were developed by neglecting the contribution of elastic strains within the plastic region. These solutions for stress and displacement fields were provided for an elastic – brittle – plastic material obeying the 1980 H-B version (Brown *et al.*, 1983; Wang, 1996). Later on, Carranza-Torres and Fairhurst (1999) improved these solutions by considering the contribution of the elastic strains. They developed a rigorous solution for the cavity contraction problem for a material obeying the 1997 H-B version with both associated and non-associated flow rules but restricted to the elastic – perfectly plastic behaviour. After that, their solutions were updated by Carranza-Torres (2004) for the generalized H-B version. Despite the

usefulness of their solutions, this last does not apply to the brittle behaviour of rock mass. Shortly after that, Sharan (2003, 2005, 2008) overcame this shortcoming by developing closed-form solutions for stress and displacement fields for cavity contraction problems in an elastic – brittle – plastic rock mass. The perfectly plastic analysis is conducted as a limiting case. Sharan’s solutions (2003, 2005, 2008) are limited to the case of the non-associated flow rule. The alternative solving method was proposed later by Serrano et al. (2011) whose solutions yield comparable results. Finally, Rojat et al. (2015) suggested a complimentary analysis of the aforementioned work. They addressed the influence of the edges of the H-B failure envelope for an elastic perfectly plastic material. Besides, they presented a new closed-form solution for the case of an associated flow rule.

2.2. Problem statement and governing equations

Both cylindrical and spherical cavities in a finite rock mass are considered. The rock mass is treated as an isotropic dilatant material that obeys the H-B failure criterion.

The geometry of the problem and the boundary conditions are depicted in Figure V-1. Let r_i and r_e be, respectively, the internal and the external radii of the cavity. Let P_0 be the hydrostatic pressure that acts throughout the rock mass. Let P_i be the internal pressure applied on a cavity wall that increases monotonically from its initial value P_0 . As the internal pressure P_i increases, the rock mass will initially behave elastically, until reaching a yield pressure P_y . When the internal pressure P_i exceeds P_y , a plastic region will start spreading from r_i to the “plastic” radius r_p . The remainder of the domain ($r_p \leq r \leq r_e$) belongs to the elastic region.

Hereinafter, we deal firstly with the elastic perfectly plastic behaviour of the rock mass. The brittle behaviour will be treated after.

For such problems, the cylindrical coordinate system is adopted and the equation of equilibrium for the cavity problem is expressed in terms of radial and circumferential stresses as follows:

$$\frac{d\sigma_r}{dr} + k \frac{\sigma_r - \sigma_\theta}{r} = 0 \quad (\text{V-1})$$

which can be further written, in scaled form, as:

$$\frac{d\sigma_r^*}{dr} + k \frac{\sigma_r^* - \sigma_\theta^*}{r} = 0 \quad (\text{V-2})$$

Here k is a coefficient equal to 1 in the case of a cylindrical cavity and it is equal to 2 in the case of a spherical cavity.

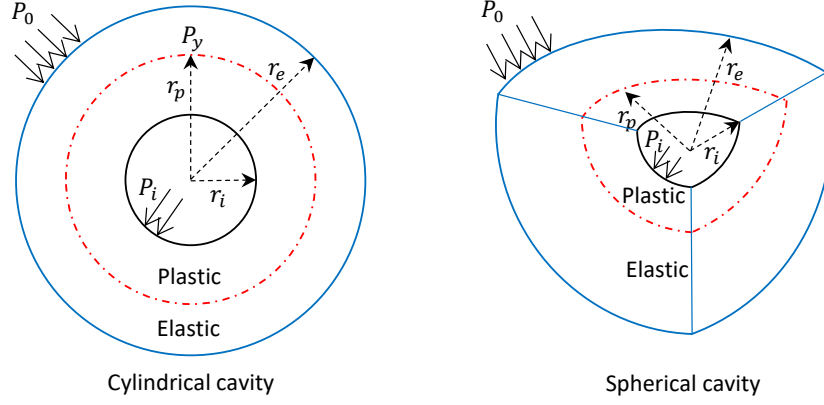


Figure V-1: Geometry of the problem and boundary conditions for both cylindrical and spherical cavities.

Note that major and minor principal stresses are assumed to be equal to radial and circumferential stresses, respectively, i.e.

$$\sigma_1 = \sigma_r \text{ and } \sigma_3 = \sigma_\theta \quad (V-3)$$

It should be emphasized that the analytical study described hereinafter is conducted based on the scaled form of the H-B failure criterion. That means that pressures are expressed, in scaled form, as:

$$P_c^* = \frac{P_c}{\sigma_{ci} m_b^\beta} + \frac{s}{m_b^{\beta/a}} \quad (V-4)$$

with the index $c = "0" \text{ or } "i" \text{ or } "y"$ denoting respectively the far-field condition, the cavity wall, and the yield condition. The normalized pressure will have an asterisk as a superscript. For sake of brevity, the term "scaled" will be mostly dropped when referring to a scaled variable unless stated otherwise.

To cover the range of values likely to be encountered in practice, it is assumed that the hydrostatic pressure P_0 is comprised between 0 and 1 [MPa]. Besides, H-B core parameters are ranging as follows: σ_{ci} [MPa] $\in$ [1,250], $m_i \in$ [4,33], $D = 0$ and $GSI \in$ [10,100]. Consequently, P_0^* evaluated by the above equation ranges from 10^{-4} to 10 (cf. Figure V-2).

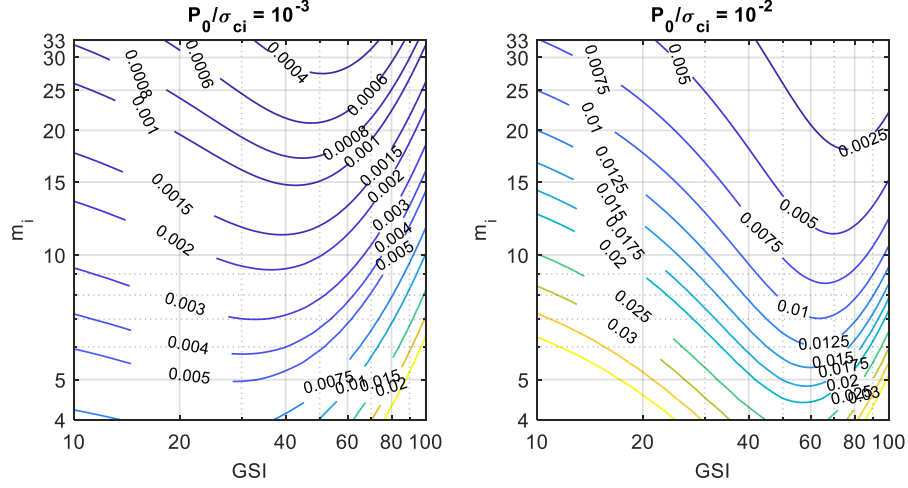


Figure V-2: Scaled far-field pressures P_0^* contour as a functions of $D = 0$, GSI , m_i and $P_0/\sigma_{ci} = \{10^{-3}, 10^{-2}\}$.

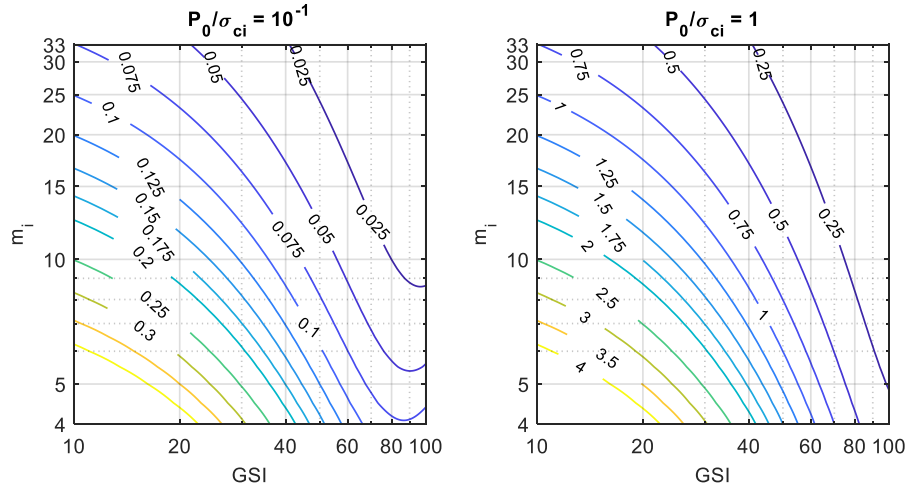


Figure V-3: Scaled far-field pressures P_0^* contour as a functions of $D = 0$, GSI , m_i and $P_0/\sigma_{ci} = \{10^{-1}, 1\}$.

2.3. Perfectly plastic finite medium

When the internal pressure P_i exceeds P_y , a plastic region starts spreading from r_i to the plastic radius r_p . The remainder of the domain ($r_p \leq r \leq r_e$) belongs to the elastic region. The latter is first studied before investigating the plastic region.

2.3.1. Elastic region ($r_p \leq r \leq r_e$)

Since solutions for this problem are well known, only the main results are presented below. Taking into account the relevant boundary conditions $\sigma_r^*(r = r_p) = P_y^*$ and $\sigma_r^*(r = r_e) = P_0^*$, the radial and circumferential stresses are expressed as:

$$\sigma_r^*(r) = \frac{P_0^* r_e^{k+1} - P_y^* r_p^{k+1}}{r_e^{k+1} - r_p^{k+1}} + \frac{P_y^* - P_0^*}{r_e^{k+1} - r_p^{k+1}} \left(\frac{r_p r_e}{r} \right)^{k+1} \quad (V-5)$$

$$\sigma_\theta^*(r) = \frac{P_0^* r_e^{k+1} - P_y^* r_p^{k+1}}{r_e^{k+1} - r_p^{k+1}} - \frac{P_y^* - P_0^*}{k(r_e^{k+1} - r_p^{k+1})} \left(\frac{r_p r_e}{r} \right)^{k+1} \quad (V-6)$$

On the other hand, total strains are written as functions of elastic and plastic strains as follows:

$$\varepsilon_r = \varepsilon_r^e + \varepsilon_r^p \quad (V-7)$$

$$\varepsilon_\theta = \varepsilon_\theta^e + \varepsilon_\theta^p \quad (V-8)$$

where ε_r^p and ε_θ^p denote, respectively, the radial and the circumferential plastic strains. They will be evaluated later when studying the plastic region.

ε_r^e and ε_θ^e are, respectively, the radial and the circumferential elastic strains expressed as a function of the Poisson's ratio ν and the rigidity index of the rock mass $I_r = \frac{G}{\sigma_{ci} m_b^\beta}$ as:

$$\varepsilon_r^e = \frac{1}{2I_r(1+\nu)^{k-1}} \left((1+\nu(k-2))(\sigma_r^* - P_0^*) - k\nu(\sigma_\theta^* - P_0^*) \right) \quad (V-9)$$

$$\varepsilon_\theta^e = \frac{1}{2I_r(1+\nu)^{k-1}} \left((1-\nu)(\sigma_\theta^* - P_0^*) - \nu(\sigma_r^* - P_0^*) \right) \quad (V-10)$$

Infinitesimal strain can be written in terms of radial displacement u as:

$$\varepsilon_r^e = -\frac{du}{dr} \quad (V-11)$$

$$\varepsilon_\theta^e = -\frac{u}{r} \quad (V-12)$$

Thus, the radial displacement u can be evaluated using one of the above equations as follows:

$$\begin{aligned} u(r) &= -r\varepsilon_\theta^e \\ &= \frac{P_y^* - P_0^*}{2I_r(1+\nu)^{k-1} \left(\frac{1}{r_p^{k+1}} - \frac{1}{r_e^{k+1}} \right)} \left(\frac{1-2\nu}{r_e^{k+1}} r + \frac{(1+\nu)^{k-1}}{kr^k} \right) \end{aligned} \quad (V-13)$$

2.3.2. Yield pressure P_y

When the internal pressure P_i applied on the cavity wall reaches the yield pressure P_y , the stress of the rock mass at the cavity face will satisfy the failure criterion. Thus, the scaled net yield pressure $\Delta P_y^* = (P_y^* - P_0^*)$ should satisfy the following equation:

$$\frac{\Delta P_y^*}{1 - \left(\frac{r_i}{r_e}\right)^{k+1}} = -\frac{\Delta P_y^*}{k \left(1 - \left(\frac{r_i}{r_e}\right)^{k+1}\right)} + \left[P_0^* - \frac{\Delta P_y^* \left(1 + k \left(\frac{r_i}{r_e}\right)^{k+1}\right)}{k \left(1 - \left(\frac{r_i}{r_e}\right)^{k+1}\right)} \right]^a \quad (\text{V-14})$$

It can be noted that the scaled yield pressure depends on P_0^* , the exponent a (i.e., GSI) and the ratio between the internal and the external cavity radius $\left(\frac{r_i}{r_e}\right)$.

The above equation provides an explicit solution when $a = 0.5$ and the following hold:

$$\Delta P_{y|a=0.5}^* = -k \left(1 - \left(\frac{r_i}{r_e}\right)^{k+1}\right) \times \frac{\sqrt{4P_0^*(k+1)^2 + \left(k \left(\frac{r_i}{r_e}\right)^{k+1} + 1\right)^2 + k \left(\frac{r_i}{r_e}\right)^{k+1} \left(k \left(\frac{r_i}{r_e}\right)^{k+1} + 1 - k\right) - k}}{2(k+1)^2} \quad (\text{V-15})$$

When $a \neq 0.5$, there is no explicit solution, and Eq. (V-14) should be solved numerically.

The evolution of P_y^* as a function of P_0^* is shown in Figure V-4 for both cylindrical and spherical cavities. As can be seen, the scaled yield pressure is relatively constant when $GSI > 30$ since the exponent $a \cong 0.5$. Furthermore, P_y^* is no longer influenced by the ratio r_e/r_i when this latter exceeds 10. In this case, we can assume that we are in the presence of an infinite medium.

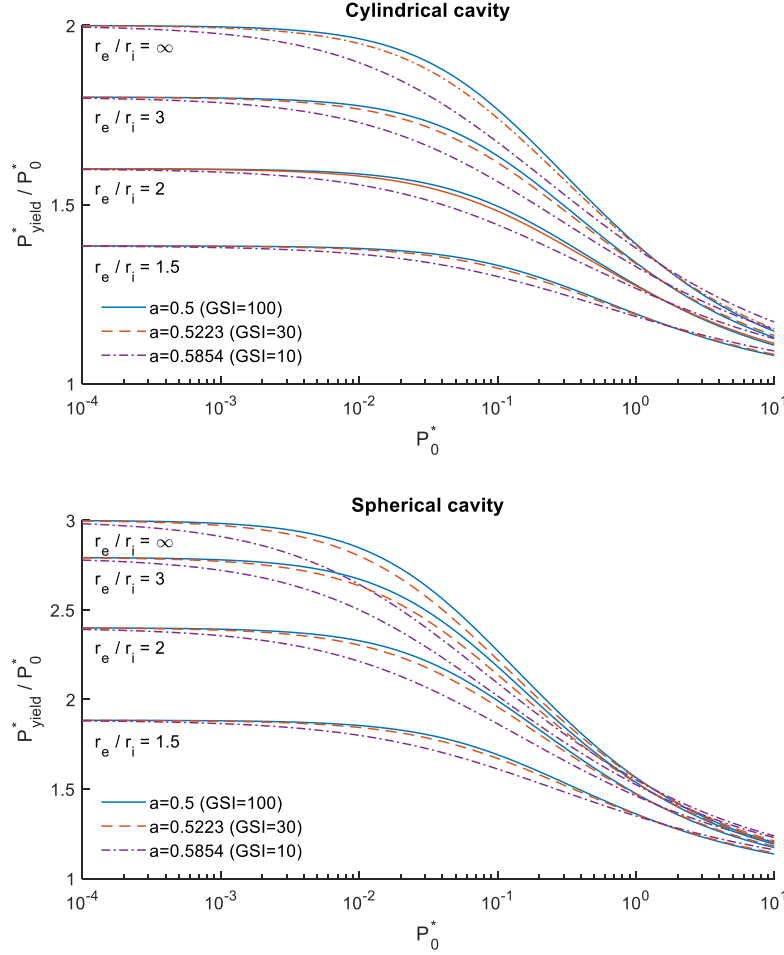


Figure V-4: Scaled yield pressure for finite H-B medium

2.3.3. Plastic region ($r_i \leq r \leq r_p$)

Determining the stress and displacement field in the plastic region for the nonlinear H-B criterion is quite involved, as detailed below.

i. Stress field

Substituting the H-B failure criterion expression given per Eq. (II-52) into the expression of equilibrium given per Eq. (V-2) results in the following differential equation of the circumferential stress:

$$\frac{d\sigma_\theta^*}{dr} + \frac{d(\sigma_\theta^*)^a}{dr} + k \frac{(\sigma_\theta^*)^a}{r} = 0 \quad (V-16)$$

The general solution of the above first-order nonlinear differential equation can be written as:

$$\sigma_{\theta}^*(r) = \left[aW_0 \left(\frac{C_1}{r^{\frac{k}{\beta}}} \right) \right]^{\beta/a} \quad (V-17)$$

and subsequently, the scaled radial stress is expressed per Eq. (II-52) as follows:

$$\sigma_r^*(r) = \left[aW_0 \left(\frac{C_1}{r^{\frac{k}{\beta}}} \right) \right]^{\beta/a} + \left[aW_0 \left(\frac{C_1}{r^{\frac{k}{\beta}}} \right) \right]^{\beta} \quad (V-18)$$

Here C_1 is a constant. W_0 is the 0th branch of the Lambert W -function (https://fr.wikipedia.org/wiki/Fonction_W_de_Lambert), also called the Omega function (cf. Appendix p.241).

To simplify the mathematical expressions, the following change of variable will be used:

$$R = C_1 r^{-\frac{k}{\beta}} \quad (V-19)$$

which allows Eqs. (V-17) – (V-18) to be reduced to

$$\sigma_{\theta}^*(R) = [aW_0(R)]^{\beta/a} \quad (V-20)$$

$$\sigma_r^*(R) = [aW_0(R)]^{\beta/a} + [aW_0(R)]^{\beta} \quad (V-21)$$

Based on the hypothesis of the continuous radial stress at the (EPB) and knowing that an internal scaled pressure P_i^* is exerted at the cavity wall, i.e.,

$$P_y^* = [aW_0(R_p)]^{\beta/a} + [aW_0(R_p)]^{\beta} \quad (V-22)$$

$$P_i^* = [aW_0(R_i)]^{\beta/a} + [aW_0(R_i)]^{\beta} ; \quad (V-23)$$

the plastic radius r_p can be calculated once the above two equations are solved for $R_p = C_1 r_p^{-\frac{k}{\beta}}$ and $R_i = C_1 r_i^{-\frac{k}{\beta}}$. Thus, the normalized plastic radius r_p/r_i is simply given by:

$$\frac{r_p}{r_i} = \left(\frac{R_i}{R_p} \right)^{\frac{\beta}{k}} \quad (V-24)$$

When $a = 0.5$, a closed-form solution of the plastic radius can be provided:

$$\left(\frac{r_p}{r_i}\right)_{a=0.5} = \left(\frac{\sqrt{4P_i^* + 1} - 1}{\sqrt{4P_y^*|_{a=0.5} + 1} - 1} \frac{e^{\sqrt{4P_i^* + 1} - 1}}{e^{\sqrt{4P_y^*|_{a=0.5} + 1} - 1}} \right)^{\frac{1}{k}} \quad (\text{V-25})$$

When $a \neq 0.5$, there is no closed-form solution for the plastic radius, and Eq. (V-24) should be solved numerically.

ii. Displacement field

A plastic flow rule is needed to determine the displacement field in the plastic region. Thus, a non-associated flow rule with a constant dilatancy angle ψ is adopted as:

$$\varepsilon_r^p + k\omega\varepsilon_\theta^p = 0 \quad (\text{V-26})$$

in which $\omega = \frac{1-\sin(\psi)}{1+\sin(\psi)}$

The above relationship between plastic strains can be further expressed per Eqs. (II-9) – (II-10) as :

$$\varepsilon_r + k\omega\varepsilon_\theta = \varepsilon_r^e + k\omega\varepsilon_\theta^e \quad (\text{V-27})$$

In the above equations, the strain-displacement relationship depends on whether a small deformation or large deformation theory is adopted. The SST solution predicts an ever-increasing internal pressure as the cavity expands, whereas the LST solution foresees the approach of a limit pressure prevailing at large expansion rates.

Although Vrakas (2016) provided a relationship between small and large strain solutions for general cavity expansion problems in elastoplastic material obeying the M-C failure criterion. The solution has limitations due to many several simplifying assumptions that facilitated its derivation. The main restrictive one was disregarding elastic strains within the plastic region. Such an assumption influences the results significantly. That is why two different resolution strategies are adopted separately hereinafter.

a. Small Strain Theory (SST)

In small deformations analyses, the fact that displacements modify the position of material points is ignored. Thus, the SST is only valid for small expansions. In this case, strains can be expressed in terms of radial displacement u as:

$$\varepsilon_r = -\frac{du}{dr} \quad (\text{V-28})$$

$$\varepsilon_\theta = -\frac{u}{r} \quad (\text{V-29})$$

It is found that the radial displacement should be computed as the solution of the following differential equation:

$$\frac{du}{dr} + k\omega \frac{u}{r} + D_1(\sigma_\theta^* - P_0^*) + D_2(\sigma_\theta^*)^a = 0 \quad (\text{V-30})$$

where D_1 and D_2 are two dimensionless coefficients defined as follows:

$$D_1 = \frac{(1 + k\omega)(1 - 2\nu)}{2I_r(1 + \nu)^{k-1}} \quad (\text{V-31})$$

$$D_2 = \frac{(1 + \nu(k - 2) - \nu k\omega)}{2I_r(1 + \nu)^{k-1}} \quad (\text{V-32})$$

The general solution of the above differential equation can be expressed as:

$$u(r) = r^{-k\omega} \left[C_2 - \int_1^r \rho^{k\omega} (D_1(\sigma_\theta^* - P_0^*) + D_2(\sigma_\theta^*)^a) d\rho \right] \quad (\text{V-33})$$

in which C_2 is a constant that can be determined using the known displacement at the (EPB), and ρ is an auxiliary integration variable.

Finally, the radial displacement is simply expressed in dimensionless form, as:

$$\begin{aligned} \bar{u}(r) = I_r \frac{u(r)}{r^{-k\omega}} = I_r \left[u_{EPB} r_p^{k\omega} \right. \\ \left. + \int_r^{r_p} \rho^{k\omega} (D_1(\sigma_\theta^* - P_0^*) + D_2(\sigma_\theta^*)^a) d\rho \right] \end{aligned} \quad (\text{V-34})$$

The integral of the right-hand side of the above equation can be numerically evaluated.

Otherwise, by using the variable change $R = C_1 r^{-\frac{k}{\beta}}$, and after some derivations, a closed-form solution can be provided as a function of the generalized exponential integral function $E_n(x) \equiv \int_1^\infty e^{-xt} t^{-n} dt$:

$$\bar{u}(r) = I_r (u_{EPB} r_p^{k\omega} + (A_1 + A_2 - A_3)) \quad (\text{V-35})$$

where

$$A_1 = \frac{D_1 a^{\frac{\beta}{a}} \left(R_p r_p^{\frac{k}{\beta}} \right)^{-(\alpha+1)}}{-\frac{k}{\beta}(\alpha+1)} \left[\frac{W_0(x) x^{\alpha+\frac{\beta}{a}}}{e^{(\alpha+\frac{\beta}{a})W(x)}} \left(e^{(\alpha+1)W(x)} + \frac{\beta}{a} E_{-(\alpha+\frac{\beta}{a})}(-(\alpha+1)W_0(x)) \right) \right]_R^{R_p} \quad (V-36)$$

$$A_2 = \frac{D_2 a^{\beta} \left(R_p r_p^{\frac{k}{\beta}} \right)^{-(\alpha+1)}}{-\frac{k}{\beta}(\alpha+1)} \left[\frac{W_0(x) x^{\alpha+\beta}}{e^{(\alpha+\beta)W(x)}} \left(e^{(\alpha+1)W(x)} + \beta E_{-(\alpha+\beta)}(-(\alpha+1)W_0(x)) \right) \right]_R^{R_p} \quad (V-37)$$

$$A_3 = \frac{D_1 P_0^*}{-\frac{k}{\beta}(\alpha+1)} (r_p^{k\omega+1} - r^{k\omega+1}) \quad (V-38)$$

in which $\alpha = -\frac{\beta}{k}(k\omega+1) - 1$

It should be emphasized that the above *SST* solutions are only valid for small expansions. Therefore, the prediction of the theoretical limit pressure is not possible when adopting this theory. Hence the interest of employing the large strain theory, which will be detailed hereinafter.

b. Large Strain Theory (LST)

To take into account the effect of large strain in the plastic region, one can follow Chadwick (1959) in adopting the logarithmic strains expressed as follows:

$$\varepsilon_r = -\ln\left(\frac{dr}{dr_0}\right) \quad (V-39)$$

$$\varepsilon_\theta = -\ln\left(\frac{r}{r_0}\right) \quad (V-40)$$

In the above equations, the index 0 is used to indicate the original length of a given radial distance r .

In this case, the following expression holds:

$$\ln\left(\left(\frac{r}{r_0}\right)^{k\omega} \left(\frac{dr}{dr_0}\right)\right) = D_1 P_0^* - D_1 \sigma_\theta^* - D_2 (\sigma_\theta^*)^a \quad (V-41)$$

Through the following change of variables

$$R = C_1 r^{\frac{k}{\beta}} \quad (V-42)$$

$$\xi = \left(\frac{r_0}{r_p} \right)^{k\omega+1} \quad (V-43)$$

The above equation is easily integrated over the interval $[r, r_p]$ leading to:

$$\eta \left[(1 - \delta)^{k\omega+1} - \left(\frac{r_0}{r_p} \right)^{k\omega+1} \right] = - \int_R^{R_p} \frac{e^{D_1 \sigma_\theta^* + D_2 (\sigma_\theta^*)^a}}{R^{\frac{\beta(k\omega+1)}{k} + 1}} dR \quad (V-44)$$

where

$$\delta = \frac{u_{(EPB)}}{r_p} = \frac{1}{2kI_r} (P_{yield}^* - P_0^*) \quad (V-45)$$

$$\eta = \frac{e^{D_1 P_0^*} \left(\beta \frac{(k\omega+1)}{k} \right)^{-1}}{R_p^{\beta \frac{(k\omega+1)}{k}}} \quad (V-46)$$

By assuming $r_0 = r_{i0}$ and $r = r_i$, Eq. (V-44) is rewritten as follows:

$$\eta \left[(1 - \delta)^{k\omega+1} - \Omega^{k\omega+1} \left(\frac{r_{i0}}{r_i} \right)^{k\omega+1} \right] = - \int_{R_i}^{R_p} \frac{e^{D_1 \sigma_\theta^* + D_2 (\sigma_\theta^*)^a}}{R^{\frac{\beta(k\omega+1)}{k} + 1}} dR \quad (V-47)$$

Here Ω is the inverse of the normalized plastic radius, i.e., Ω is as:

$$\Omega = \frac{r_i}{r_p} = \left(\frac{R_p}{R_i} \right)^{\beta/k} \quad (V-48)$$

Finally, the following implicit relationship between the cavity pressure and the cavity expansion is obtained:

$$\left(\frac{r_i}{r_{i0}} \right)^{k\omega+1} = \frac{\Omega^{k\omega+1}}{(1 - \delta)^{k\omega+1} - \frac{1}{\eta} \int_{R_p}^{R_i} \frac{e^{D_1 \sigma_\theta^* + D_2 (\sigma_\theta^*)^a}}{R^{\frac{\beta(k\omega+1)}{k} + 1}} dR} \quad (V-49)$$

Here, r_i is the instant cavity radius and r_{i0} is the original cavity radius before cavity expansion starts. The corresponding instant cavity volume V_i and original volume V_{i0} can be computed as:

$$\left(\frac{V_i}{V_{i0}} \right) = \left(\frac{r_i}{r_{i0}} \right)^{k+1} \quad (V-50)$$

Typical pressure – expansion curves of both cylindrical and spherical cavities are depicted for three GSI values (cf. Figure V-5). It is observed that the pressure applied at the cavity wall does not increase indefinitely, but that a limit pressure is reached.

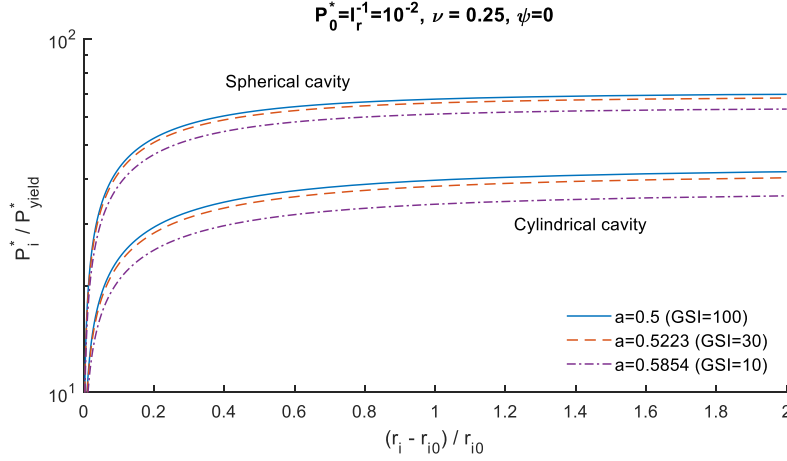


Figure V-5: Typical cavity pressure–expansion curves adopting the large strain theory

Furthermore, neglecting the contribution of elastic strains within the plastic region considerably simplifies the analysis of the cavity expansion problem. Indeed, the right-hand side of Eq. (V-47) vanishes. Hence, following the same procedure described above a relatively simple expression for the relationship between the cavity pressure and the cavity expansion is obtained:

$$\left(\frac{r_i}{r_{i0}}\right)^{k\omega+1} = \frac{\Omega^{k\omega+1}}{(1-\delta)^{k\omega+1} - \left(1 - \left(\frac{r_i}{r_p}\right)^{k\omega+1}\right)} \quad (V-51)$$

It should be noted that the above expression is exact for the particular case of incompressible ($\nu = 0.5$) material with no volume change ($\psi = 0^\circ$).

Considering the above development, one can conclude that neglecting the contribution of elastic strains within the plastic region may seem to be a reasonable assumption since the elastic strains are much smaller than the plastic strains. This, however, does have a significant effect on the predicted results. Figure V-6 depicts a comparison of the cavity pressure-displacement curves, which are computed according to the exact and the approximate solutions. It can be noted that for the same P_i^*/P_{yield}^* , the evaluated radial displacement is more significant when considering elastic strains.

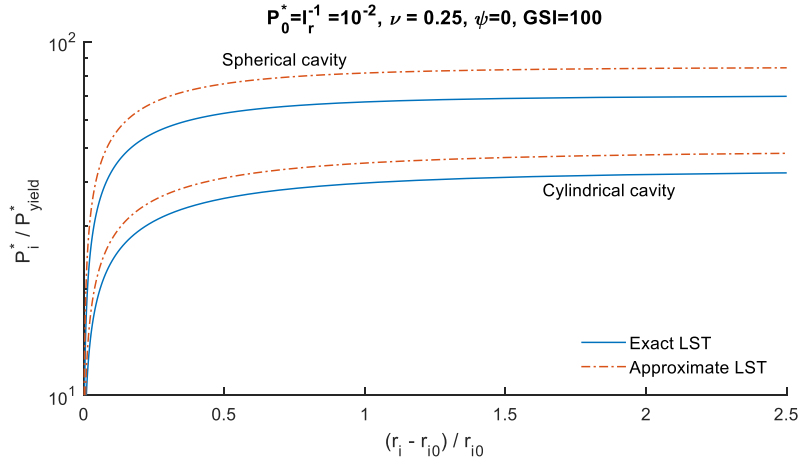


Figure V-6: Comparison of cavity pressure–expansion curves computed by employing the approximate and exact solution according to large strain theory.

c. SST vs. LST

Once the analytical expression of the radial deformation is found for both *SST* and *LST*, it would be interesting to visualize, for example, the cylindrical cavity wall deformation as a function of the applied internal pressure. As can be seen in Figure V-7, for small cavity wall deformations ($< 3\%$), the cavity pressure – cavity expansion curves derived from both *SST* and *LST* overlap. However, for higher deformations, the pressure applied at the cavity wall increases indefinitely in the case of *SST* while it reaches the theoretical limit pressure in the case of *LST*.

On the other hand, the cavity expansion theory has been extensively developed and widely used to study a variety of practical geotechnical engineering problems. Among them, prediction of the bearing capacity of pile foundation (Vesic, 1972; Yasufuku and Hyde, 1995). As demonstrated previously, the cavity expansion theory allows the prediction of stress and displacement fields around an expanding cavity, which are directly linked to the pile end bearing capacity. Indeed, an essential assumption in the use of cavity expansion theory in the context of pile end-bearing capacity is that the pressure exerted at the boundaries of the plastic bulb immediately beneath the pile tip is equal to the limit pressure P_{lim} required to expand a spherical cavity ((further details are provided in the upcoming section). However, it is demonstrated that the small strain solution predicts an increasing internal pressure as the cavity expands, whereas the large strain solution predicts approaching a limit pressure at vast expansion highlighting the advantages of *LST* for the tackled problem.

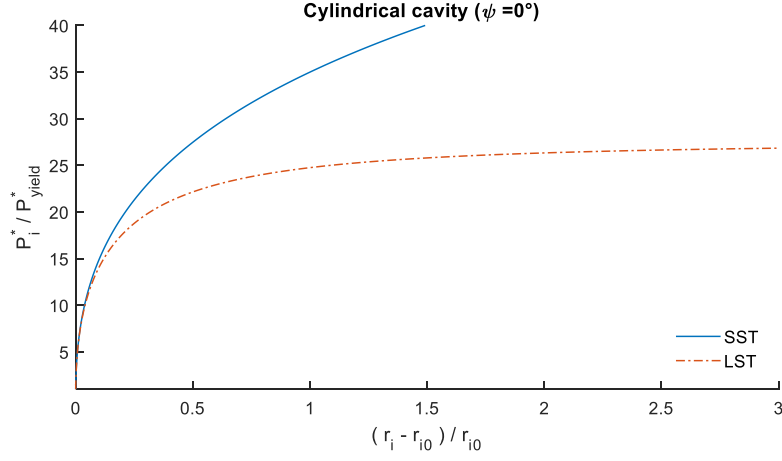


Figure V-7: Cavity wall deformation: SST vs. LST.

2.3.4. Infinite medium

It is of interest to consider the particular case of an infinite medium, as this case has many applications in geotechnical engineering. In this case, it merely needs to put $r_e \rightarrow \infty$ into the set of equations developed for the finite medium in its elastic region. For the plastic region, solutions remain the same. Therefore, the following simple expressions for stresses and displacement in the elastic region are obtained:

$$\sigma_r^*(r) = P_0^* + (P_y^* - P_0^*) \left(\frac{r_p}{r} \right)^{k+1} \quad (\text{V-52})$$

$$\sigma_\theta^*(r) = P_0^* + \frac{(P_y^* - P_0^*)}{k} \left(\frac{r_p}{r} \right)^{k+1} \quad (\text{V-53})$$

Besides, the net yield pressure $\Delta P_y^* = (P_y^* - P_0^*)$ should be computed by numerically solving the following equation:

$$\Delta P_y^* = -\frac{\Delta P_y^*}{k} + \left[P_0^* - \frac{\Delta P_y^*}{k} \right]^a \quad (\text{V-54})$$

which provides an explicit solution when $a = 0.5$ and the following holds:

$$\Delta P_y^*|_{a=0.5} = \frac{k(\sqrt{4P_0^*(k+1)^2 + 1} - 1)}{2(k+1)^2} \quad (\text{V-55})$$

On the other hand, the radial displacement is simply expressed as:

$$u(r) = -r\varepsilon_\theta^e = \frac{1}{2kI_r} (P_y^* - P_0^*) r_p \left(\frac{r_p}{r} \right)^k \quad (\text{V-56})$$

which can be further written as a function of the radial displacement at the Elastic-Plastic Boundary (EPB) as:

$$u(r) = u_{EPB} \left(\frac{r_p}{r} \right)^k ; u_{EPB} = \frac{(P_y^* - P_0^*)}{2kI_r} r_p \quad (V-57)$$

The radial distribution of radial and circumferential stresses for $a = 0.5854, 0.5223, 0.5057$, and 0.5 (i.e., $GSI = 10, 30, 50$, and 100) are depicted in Figure V-8 for both cylindrical and spherical cavities. It is observed that, unlike radial stress, there is a significant difference in circumferential stress values when $GSI < 30$. As shown, increasing a values (i.e. decreasing GSI values) results in a larger plastic region.

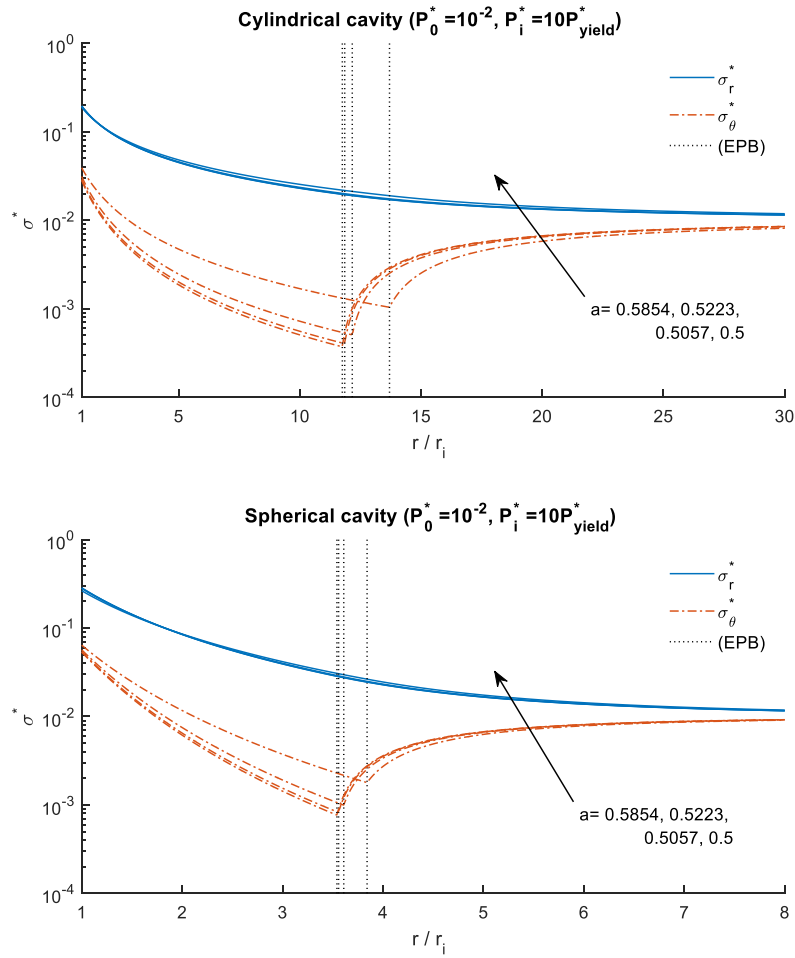


Figure V-8: Radial distribution of radial and circumferential stresses for different values of exponent a .

Typical pressure – displacement curves for both cylindrical and spherical cavities are depicted in Figure V-9. For a given internal pressure P_i^* , the dimensionless displacement at the cavity face increases with a lowering dilatancy angle. It is also observed that the normalized load-displacement curves are almost identical when $GSI > 30$.

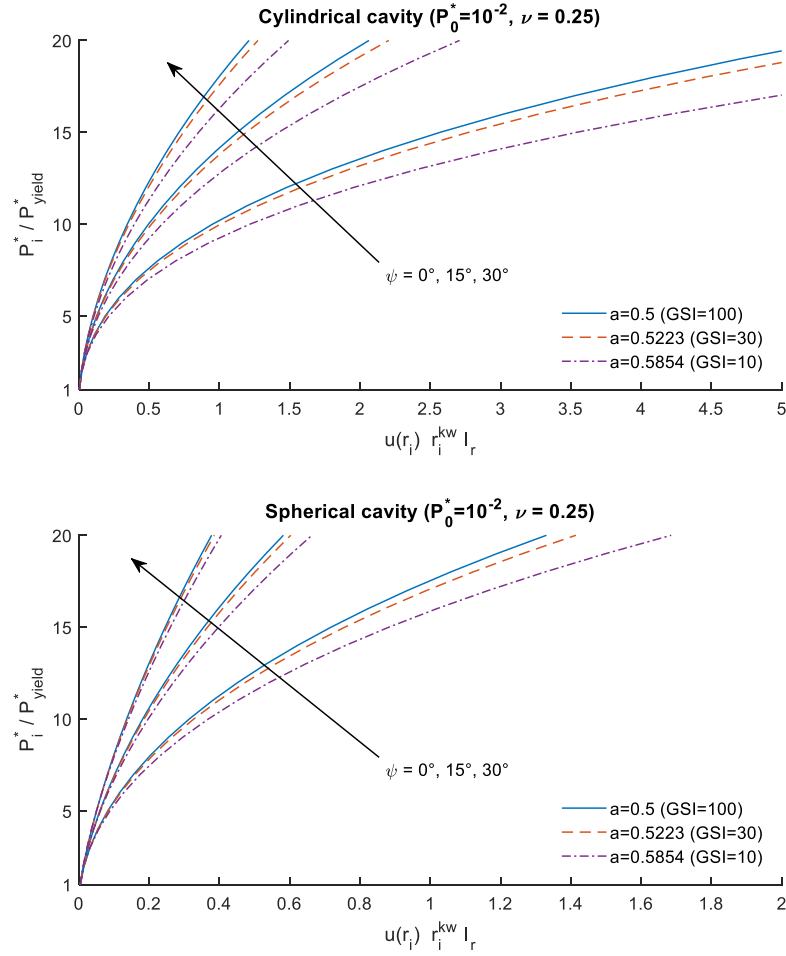


Figure V-9: Typical pressure – displacement curves employing the small deformation theory.

2.3.5. Limit pressure

The limit pressure denoted P_{lim}^* can be found by putting $r_i/r_{i0} \rightarrow \infty$ in Eq. (V-49) i.e. P_{lim}^* can be obtained implicitly from the following expression:

$$(1 - \delta)^{k\omega+1} = \frac{1}{\eta} \int_{R_p}^{R_i} \frac{e^{D_1 \sigma_\theta^* + D_2 (\sigma_\theta^*)^a}}{R^{\frac{\beta(k\omega+1)}{k} + 1}} dR \quad (V-58)$$

Note that P_{lim}^* depends on five main parameters, which are the far-field pressure P_0^* , the rigidity index $I_r = \frac{G}{\sigma_{ci} m_b^\beta}$, Poisson's ratio ν , the dilatancy angle ψ , and the exponent a (i.e., GSJ). When $a = 0.5$ and by considering an incompressible material ($\nu = 0.5$) with no plastic volume change ($\psi = 0$), a closed-form solution of the limit pressure can be provided and the following holds:

$$P_{lim}^* = \left(\frac{W_0(C_3)}{2} \right)^2 + \frac{W_0(C_3)}{2} \quad (V-59)$$

where:

$$C_3 = \frac{\left(\sqrt{4P_y^*|_{a=0.5}} + 1 - 1 \right) e^{\sqrt{4P_y^*|_{a=0.5}} + 1 - 1}}{(1 - (1 - \delta)^{k+1})^{\frac{k}{k+1}}} \quad (V-60)$$

Charts allowing easy and accurate estimation of the limit pressure for both cylindrical and spherical cavities are presented herein for $\psi = 0$ and $\nu = 0.25$. These charts provide a first aid to design engineers. To cover the range of values likely to be encountered in practice, the far-field pressure P_0^* ranging from 10^{-3} to 5.10^{-1} is adopted (cf. Figure V-2). The values of the inverse rigidity index $I_r^{-1} = [10^{-3}, 5.10^{-3}, 10^{-2}, 5.10^{-2}, 10^{-1}, 1]$ will be used to compute the limit pressure values. As shown in Figure V-10 – Figure V-11, an increase in limit pressure as the far-field stress and the shear modulus increase.

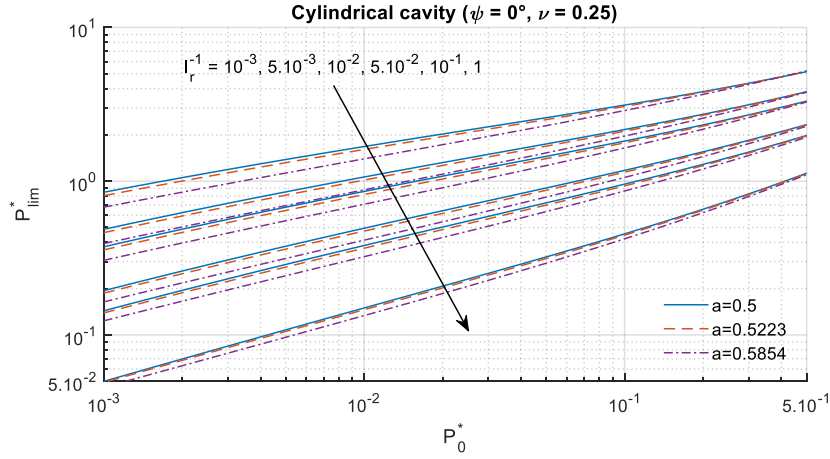


Figure V-10: Limit pressure chart for the cylindrical cavity

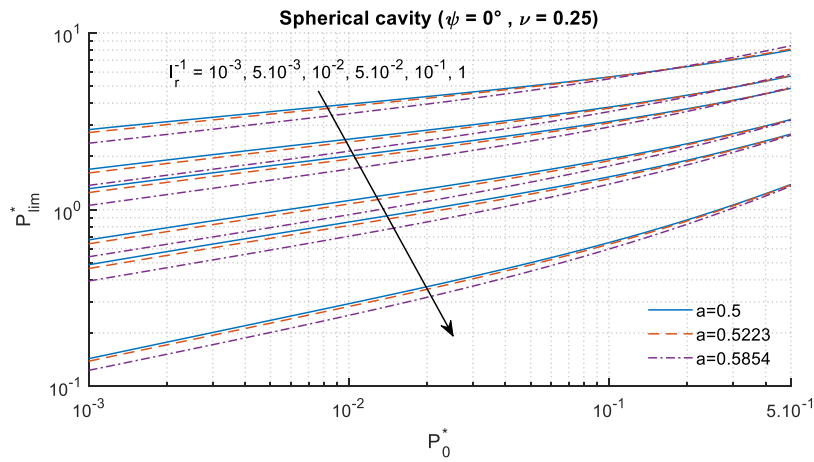


Figure V-11: Limit pressure chart for the spherical cavity

To illustrate the use of the provided charts, an example is given for computing the limit pressures of cylindrical and spherical cavities in a medium with a far-field pressure P_0 of 100 [kPa] consisting of an intact undisturbed rock mass ($D = 0$ and $GSI = 100$) with $m_i = 10$, $\sigma_{ci} = 10$ [MPa], $G/\sigma_{ci} = 200$, $\psi = 0^\circ$, and $\nu = 0.25$.

The H-B derived parameters are $a = 0.5$, $m_b = 10$ and $s = 1$. The scaling parameter, $\beta = \frac{a}{1-a} = 1$. Then, the scaled far-field pressure, $P_0^* = 1.1 \times 10^{-2}$ and $I_r^{-1} = 5 \times 10^{-2}$. Reading the scaled limit pressure from the above charts for these input values, $P_{lim}^* = 0.59$ and 1.42, respectively, for cylindrical and spherical cavities. The corresponding limit pressure values can be computed as $P_{lim} = \left(P_{lim}^* - \frac{s}{m_b^{\beta/a}} \right) \sigma_{ci} m_b^\beta$

resulting in $P_{lim} = 58 [MPa]$ and $141 [MPa]$, respectively, for cylindrical and spherical cavities.

2.3.6. Zero initial radius

In this subsection, the particular case of cavity expansion from zero initial radius is investigated. As stated by Hill (1998), this kind of problem has no characteristic length, and therefore the continuing deformation is geometrically self-similar. As a result, one can follow Hill (1998) in adopting the velocity approach to obtain a solution for the limit pressure of cavity expansion in an elastic-perfectly plastic rock mass.

It should be emphasized that developed stress solutions in both elastic and plastic regions in case of cavity expansion problem with the initial cavity radius still valid for this special case. The only significant change is the use of the velocity approach instead of the large strain theory to determine the limit pressure solution.

Hill (1998) stated that, in calculating the displacement of any individual particle, it is convenient to take the movement of the plastic boundary as the scale of 'time' or progress of the expansion, since the parameter r_p appears in the formulae for the stresses. We can speak of the velocity V of a particle, i.e., the particle is displaced by an amount Vdr_p when the plastic boundary moves outwards a further distance dr_p . The velocity V can be expressed directly in terms of the total displacement u which is a function of both the current radius r and the plastic radius r_p as follows:

$$du = \frac{\partial u}{\partial r_p} dr_p + \frac{\partial u}{\partial r} dr = \left(\frac{\partial u}{\partial r_p} + V \frac{\partial u}{\partial r} \right) dr_p \quad (V-61)$$

Equating the above equation to the amount Vdr_p , the following velocity expression is obtained:

$$V = \frac{\frac{\partial u}{\partial r_p}}{1 - \frac{\partial u}{\partial r}} \quad (V-62)$$

On the other hand, the increment of stress and strain are evaluated following a given element. Thus:

$$d\varepsilon_r = -\frac{\partial(du)}{\partial r} = -\frac{\partial V}{\partial r} \cdot dr_p \quad (V-63)$$

$$d\varepsilon_\theta = -\frac{du}{r} = -\frac{V}{r} \cdot dr_p \quad (V-64)$$

$$d\sigma_r^* = \left(\frac{\partial \sigma_r^*}{\partial r_p} + V \frac{\partial \sigma_r^*}{\partial r} \right) \cdot dr_p \quad (V-65)$$

$$d\sigma_{\theta}^* = \left(\frac{\partial \sigma_{\theta}^*}{\partial r_p} + V \frac{\partial \sigma_{\theta}^*}{\partial r} \right) \cdot dr_p \quad (\text{V-66})$$

Substituting the above equations into the plastic flow rule expression and taking into account the yield condition given by the following equation $d\sigma_r^* = (1 + a\sigma_{\theta}^{*a-1})d\sigma_{\theta}^*$, results in :

$$d\varepsilon_r + k\omega_1 d\varepsilon_{\theta} = (D_1 + aD_2\sigma_{\theta}^{*a-1})d\sigma_{\theta}^* \quad (\text{V-67})$$

Which can be further written in terms of velocity as:

$$\frac{\partial V}{\partial r} + k\omega_1 \frac{V}{r} + (D_1 + aD_2\sigma_{\theta}^*(r)^{a-1}) \left(\frac{\partial \sigma_{\theta}^*}{\partial r_p} + V \frac{\partial \sigma_{\theta}^*}{\partial r} \right) = 0 \quad (\text{V-68})$$

With the set of expression given above, the following differential equation for velocity holds:

$$\frac{\partial V}{\partial r} + k\omega_1 \frac{V}{r} + (D_1 + aD_2\sigma_{\theta}^*(r)^{a-1}) \left(V - \frac{r}{r_p} \right) \frac{\partial \sigma_{\theta}^*}{\partial r} = 0 \quad (\text{V-69})$$

in which $\frac{\partial \sigma_{\theta}^*}{\partial r}$ is obtained by direct derivation of Eq. (V-20) and the following holds:

$$\frac{\partial \sigma_{\theta}^*}{\partial r} = -\frac{k}{r} \frac{\sigma_{\theta}^*(r)}{a + \frac{a/\beta}{\sqrt{\sigma_{\theta}^*(r)}}} \quad (\text{V-70})$$

The general solution of the above differential equation is given as:

$$V(r) = \frac{e^{-(D_1\sigma_{\theta}^* + D_2\sqrt{\sigma_{\theta}^*})}}{r^{k\omega_1}} \left(C_4 - \int_1^r \frac{kC_1 \left(D_2 + D_1 W \left(\frac{C_1}{\xi^k} \right) \right) W' \left(\frac{C_1}{\xi^k} \right)}{2r_p \xi^{k(\omega_1-1)} e^{-(D_1\sigma_{\theta}^* + D_2\sqrt{\sigma_{\theta}^*})}} \cdot d\xi \right) \quad (\text{V-71})$$

in which C_4 is an arbitrary constant that can be determined using the known displacement at the (EP) boundary. Therefore, the velocity is expressed as:

$$\begin{aligned}
V(r) = & \frac{e^{2-(D_1\sigma_\theta^*(r)+D_2\sqrt{\sigma_\theta^*(r)})}}{r^{k.\omega_1}} \\
& \times \left(V_{(EP)} \frac{e^{(D_1\sigma_\theta^*(r_p)+D_2\sqrt{\sigma_\theta^*(r_p)})}}{r_p^{-k.\omega_1}} \right. \\
& \left. + \int_r^{r_p} \frac{kC_1 \left(D_2 + D_1 W \left(\frac{C_1}{\xi^k} \right) \right) W' \left(\frac{C_1}{\xi^k} \right)}{2r_p \xi^{k(\omega_1-1)} e^{-(D_1\sigma_\theta^*+D_2\sqrt{\sigma_\theta^*})}} . d\xi \right)
\end{aligned} \tag{V-72}$$

where $V_{(EP)} = V(r = r_p) = \frac{(k+1)\delta}{1+k\delta}$

At the cavity wall, $r = r_i$ and $V(r = r_i) = \frac{dr_i}{dr_p}$. Thus,

$$\begin{aligned}
\frac{dr_i}{dr_p} = & \frac{e^{-(D_1\sigma_\theta^*(r_i)+D_2\sqrt{\sigma_\theta^*(r_i)})}}{r_i^{k.\omega_1}} \\
& \times \left(V_{(EP)} \frac{e^{(D_1\sigma_\theta^*(r_p)+D_2\sqrt{\sigma_\theta^*(r_p)})}}{r_p^{-k.\omega_1}} \right. \\
& \left. + \int_{r_i}^{r_p} \frac{kC_1 \left(D_2 + D_1 W \left(\frac{C_1}{\xi^k} \right) \right) W' \left(\frac{C_1}{\xi^k} \right)}{2r_p \xi^{k(\omega_1-1)} e^{-(D_1\sigma_\theta^*+D_2\sqrt{\sigma_\theta^*})}} . d\xi \right)
\end{aligned} \tag{V-73}$$

Since we deal with the situation of cavity expansion from zero initial radius, Hill (1998) stated that deformation is assumed to be geometrically similar in the plastic region such that the ratio of the radius of the (EP) boundary to that of the cavity wall is constant, i.e. :

$$\frac{dr_i}{dr_p} = \frac{r_i}{r_p} \tag{V-74}$$

Equating (V-73) and (V-74) results in a non-linear equation of r_p/r_i which can be solved by numerical tool for the value of r_p/r_i . Once the value of r_p/r_i is obtained, Eq. (V-24) can be used to evaluate the limit pressure.

It should be emphasized that neglecting the contributions of elastic strains within the plastic region, considerably simplifies the analysis of the cavity expansion problem.

Indeed, the right-hand side of Eq. (V-67) vanishes. Hence, following the same procedure described above a relatively simple expression for the velocity is obtained:

$$V(r) = V_{(EP)} \left(\frac{r_p}{r} \right)^{k\omega_1} \quad (V-75)$$

Subsequently, the following equation of r_p/r_i is obtained by equating (V-74) and (V-75) for $r = r_i$:

$$\frac{dr_i}{dr_p} = \frac{r_i}{r_p} = V_{(EP)} \left(\frac{r_p}{r_i} \right)^{k\omega_1} \quad (V-76)$$

Thus,

$$\frac{r_p}{r_i} = (V_{(EP)})^{-\frac{1}{k\omega_1+1}} \quad (V-77)$$

The limit pressure can then be deduced by equating the above equation with Eq. (V-24). Note that When $a = 0.5$ and by considering an incompressible material ($\nu = 0.5$) with no plastic volume change ($\psi = 0$), a closed form solution of the limit pressure can be provided and the following holds:

$$P_{lim}^* = \left(\frac{W(C_5)}{2} \right)^2 + \frac{W(C_5)}{2} ; C_5 = \frac{(\sqrt{4P_y^* + 1} - 1)e^{\sqrt{4P_y^* + 1} - 1}}{v_{EP}^{\frac{k}{k\omega_1+1}}} \quad (V-78)$$

The above limit pressure expression is very similar to that developed in the case of cavity expansion with an initial cavity radius (Eqs. (V-59) – (V-60)). Indeed, the following term $(1 - (1 - \delta)^{k\omega_1+1})$ is just substituted with the velocity at the (EP) boundary $V_{(EP)} = \frac{(k+1)\delta}{1+k\delta}$.

The theoretical limit pressure corresponding to an expanding cavity from an initial non-null cavity radius (Eqs. (V-59) – (V-60)) is compared to the limit pressure corresponding to expanding cavity from zero initial radius. The ratio of these pressures is depicted in Figure V-12 for a range of P_0^* and $\frac{\sigma_{ci}m_b}{G}$ likely to be encountered in practice. As can be seen, limit pressure values evaluated using the velocity approach for zero cavity radius is within 0.04% of the limit pressure values evaluated using the LST for $\frac{\sigma_{ci}m_b}{G} > 10^{-1}$ and $P_0^* > 10^{-2}$. Otherwise, they are the same. It can be therefore concluded that cavity expansion from zero radius to any finite radius is mathematically equivalent to the expansion of an existing cavity to infinity.

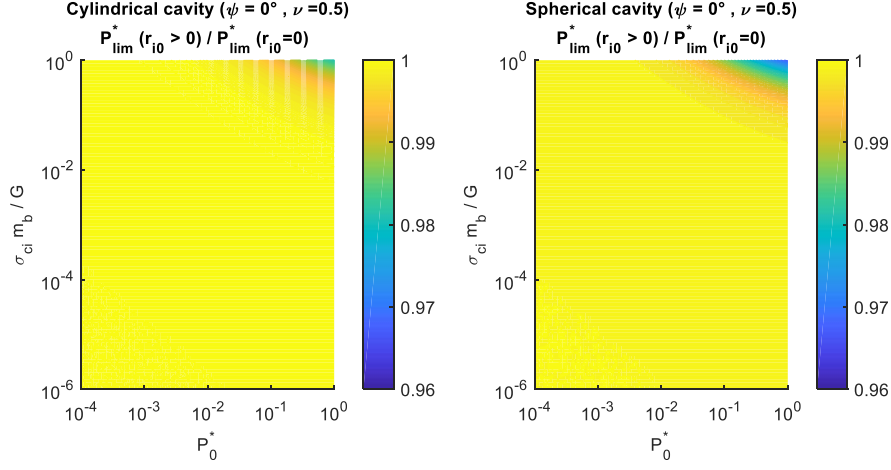


Figure V-12: Ratio of limit pressure evaluated for the special case of cavity expansion from an initial radius and zero initial radius case for both cylindrical and spherical cavities.

2.4. Brittle plastic finite medium

The brittle plastic post-failure is considered. In such a case, as the rock yields, the peak strength parameters m_b^{peak} and s^{peak} and the peak deformation modulus E^{peak} drop to residual strength parameters m_b^{res} and s^{res} and residual deformation modulus E^{res} . To ensure that a closed-form solution can be obtained, it is necessary to assume further that after yield, the strength of rock drops suddenly to its residual values (cf. Figure V-13).

On the other hand, the analytical study described hereinafter is almost the same compared to the perfectly plastic post-failure case with some minor changes. The changes consist mainly of scaling the governing equations of the problem using residual strength parameters reflected by 'res' as a superscript instead of the peak ones. It should be noted that the perfectly plastic model is simply a limiting case of the brittle one. Therefore, interpretations and conclusions made for the perfect plastic case remain valid in this case.

2.4.1. Yield pressure

When the internal pressure P_i applied on the cavity wall reaches the yield pressure P_y . The stress of the rock mass at the cavity face will satisfy the failure criterion expressed as a function of the peak strength parameters.

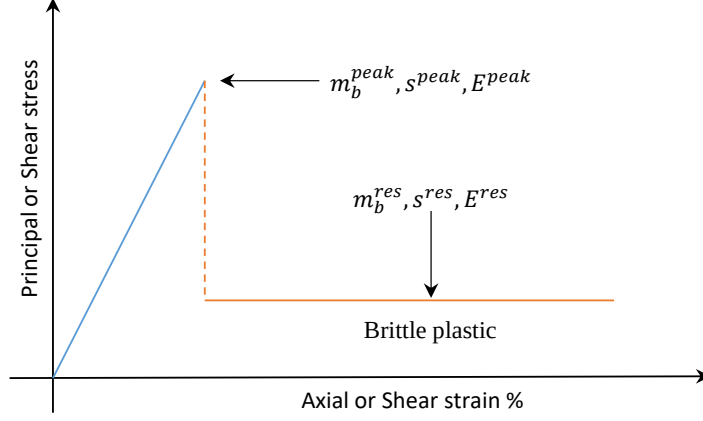


Figure V-13: Brittle plastic post-failure of the rock mass.

Thus, the net yield pressure $\Delta P_y^{*peak} = (P_y^{*peak} - P_0^{*peak})$ should satisfy the same equation as the perfectly plastic post-failure case, i.e.:

$$\frac{\Delta P_y^{*peak}}{1 - \left(\frac{r_i}{r_e}\right)^{k+1}} = -\frac{\Delta P_y^{*peak}}{k \left(1 - \left(\frac{r_i}{r_e}\right)^{k+1}\right)} + \left[P_0^{*peak} - \frac{\Delta P_y^{*peak} \left(1 + k \left(\frac{r_i}{r_e}\right)^{k+1}\right)}{k \left(1 - \left(\frac{r_i}{r_e}\right)^{k+1}\right)} \right]^a \quad (V-79)$$

The superscript ‘peak’ in the above equation means that pressures are scaled using peak strength parameters m_b^{peak} and s^{peak} .

Since the governing equations of the problem are scaled using residual strength parameters, the following relationship relating P_y^{*res} to P_y^{*peak} should be used from now on:

$$P_y^{*res} = \left(\frac{m_b^{peak}}{m_b^{res}}\right)^\beta P_y^{*peak} + \frac{1}{(m_b^{res})^\beta} \left(\frac{s^{res}}{m_b^{res}} - \frac{s^{peak}}{m_b^{peak}}\right) \quad (V-80)$$

2.4.2. Elastic region

Since the solution for this region is well known, only the main results are presented hereinafter. By considering the boundary conditions $\sigma_r^{*res}(r = r_p) = P_y^{*res}$ and $\sigma_r^{*res}(r = r_e) = P_0^{*res}$, radial and circumferential stresses are expressed as:

$$\sigma_r^{*res}(r) = \frac{P_0^{*res} r_e^{k+1} - P_y^{*res} r_p^{k+1}}{r_e^{k+1} - r_p^{k+1}} + \frac{P_y^{*res} - P_0^{*res}}{r_e^{k+1} - r_p^{k+1}} \left(\frac{r_p r_e}{r} \right)^{k+1} \quad (V-81)$$

$$\sigma_\theta^{*res}(r) = \frac{P_0^{*res} r_e^{k+1} - P_y^{*res} r_p^{k+1}}{r_e^{k+1} - r_p^{k+1}} - \frac{P_y^{*res} - P_0^{*res}}{k(r_e^{k+1} - r_p^{k+1})} \left(\frac{r_p r_e}{r} \right)^{k+1} \quad (V-82)$$

On the other hand, total strains are written as functions of elastic and plastic strains as follows:

$$\varepsilon_r = \varepsilon_r^e + \varepsilon_r^p \quad ; \quad \varepsilon_\theta = \varepsilon_\theta^e + \varepsilon_\theta^p \quad (V-83)$$

where ε_r^p and ε_θ^p denote, respectively, the radial and the circumferential plastic strains. They will be evaluated afterwards whereas ε_r^e and ε_θ^e are, respectively, the radial and the circumferential elastic strains given as:

$$\varepsilon_r^e = \frac{\left((1 + \nu(k-2))(\sigma_r^{*res} - P_0^{*res}) - k\nu(\sigma_\theta^{*res} - P_0^{*res}) \right)}{2I_r(1 + \nu)^{k-1}} \quad (V-84)$$

$$\varepsilon_\theta^e = \frac{\left((1 - \nu)(\sigma_\theta^{*res} - P_0^{*res}) - \nu(\sigma_r^{*res} - P_0^{*res}) \right)}{2I_r(1 + \nu)^{k-1}} \quad (V-85)$$

in which, ν is the Poisson's ratio. I_r is the rigidity index of the rock mass, and it is expressed according to the region state. When the elastic region prevails which is the case here, I_r is expressed as a function of the peak shear modulus as $I_r = \frac{G^{peak}}{\sigma_{cl}(m_b^{res})^\beta}$.

By substituting Eqs. (V-81) – (V-82) into Eqs. (V-84) or (V-85) and by taking into account that infinitesimal strain can be written in terms of radial displacement u as $\varepsilon_r = -\frac{du}{dr}$ and $\varepsilon_\theta = -\frac{u}{r}$, the radial displacement can be evaluated as:

$$u(r) = -r\varepsilon_\theta^e = \frac{P_y^{*res} - P_0^{*res}}{2I_r(1 + \nu)^{k-1} \left(\frac{1}{r_p^{k+1}} - \frac{1}{r_e^{k+1}} \right)} \times \left(\frac{1 - 2\nu}{r_e^{k+1}} r + \frac{(1 + \nu)^{k-1}}{kr^k} \right) \quad (V-86)$$

2.4.3. Plastic region

The analytical solutions for the extent of this plastic region and the related stresses and displacement fields remain the same as the perfectly plastic post-failure behaviour. The only minor change is that stresses should be scaled concerning residual parameters. Therefore, stresses are expressed as:

$$\sigma_\theta^{*res}(R) = [aW_0(R)]^{\beta/a} \quad (V-87)$$

$$\sigma_r^{*res}(R) = [aW_0(R)]^{\beta/a} + [aW_0(R)]^{\beta} \quad (V-88)$$

On the other hand, the normalized radial displacement can be computed as:

$$\begin{aligned} \bar{u}(r) = I_r \frac{u(r)}{r^{-k\omega}} = I_r \left[u_{EPB} r_p^{k\omega} \right. \\ \left. + \int_r^{r_p} \rho^{k\omega} (D_1(\sigma_{\theta}^* - P_0^*) + D_2(\sigma_{\theta}^*)^a) d\rho \right] \end{aligned} \quad (V-89)$$

It is important to mention that the rigidity index in the above equation is expressed this time as a function of the residual shear modulus as $I_r = \frac{G^{res}}{\sigma_{ci}(m_b^{res})^{\beta}}$.

2.5. Validation

2.5.1. Validation using Finite Element Modelling (FEM)

A finite element analysis is conducted to validate the analytical results. In light of this, the finite element softwares Plaxis.2018.0 and RS² 9 Modeller (Rocscience, 2019) are employed. Plaxis software is mainly used since it implements large deformations theory, also known as updated mesh analysis, which is essential to validate the limit pressure solution. On the other hand, RS² 9 Modeller is mainly used because it allows predicting deformations after a brittle failure. Details are given below for each case.

i. Case of perfectly plastic post-failure- Plaxis model

The geometry of the models, as well as the selected boundary conditions for both spherical and cylindrical cavities, are, respectively, shown in Figure V-14 and Figure V-15.

For the cylindrical cavity model, the initial cavity radius r_0 is set equal to 1 m. The width of the rock layer is selected to be 64m extended by a correcting layer with 64m width to simulate an infinite boundary (Burd and Houlsby, 1990). The elastic properties of the correcting layer are ν' and E' . They were specified such that (Burd and Houlsby, 1990):

$$\frac{1 + \nu}{E} = \frac{(n^2 - 1)(1 + \nu')(1 - 2\nu')}{E'(1 + n^2(1 - 2\nu'))}; n = 2 \quad (V-90)$$

For the spherical cavity, a soil domain with a size of 12m × 24m was created with the mesh shown in Figure V-15. The spherical cavity has an initial radius $r_{i0} = 0.1$ m located at a depth of 12m. It was found that the computational stability was enhanced by modelling the cavity as a linear elastic material; a nominal young modulus of

100 MPa and a Poisson's ratio of 0.2 were specified. The spherical cavity expansion is modelled by applying a positive volumetric strain to the cavity cluster allowing the cavity to expand. This modelling technique was employed successfully in previous studies employing Plaxis (XU, 2007; Xu and Lehane, 2008; Suryasentana and Lehane, 2014; Suzuki and Lehane, 2015). It should be noted that the spherical cavity expansion is a relatively hard boundary value problem that consumes a significant amount of computational power when solved using the finite element method.

The rock mass in the FEM model has properties similar to that of undisturbed claystone (Hoek, 2006b) ($D = 0$) with: $\sigma_{ci} = 25$ [MPa], $m_i = 2$ and $E_i/\sigma_{ci} = 150$, and $GSI = [30, 50, 75, 100]$. The deformation modulus and the Poisson's ratio, which depend on GSI and m_i , are calculated according to Eqs.(II-60) and (II-63), respectively. The dilatancy angle is set equal to zero. Note that the claystone is assumed to be weightless for the cylindrical cavity modelling while an initial isotropic stress P_0 of 240 kPa is set for the spherical cavity modelling.

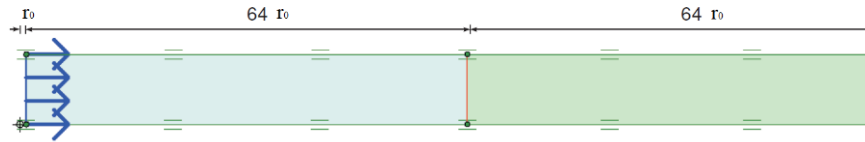


Figure V-14: Geometry and boundary conditions of the numerical model for cylindrical cavity expansion.

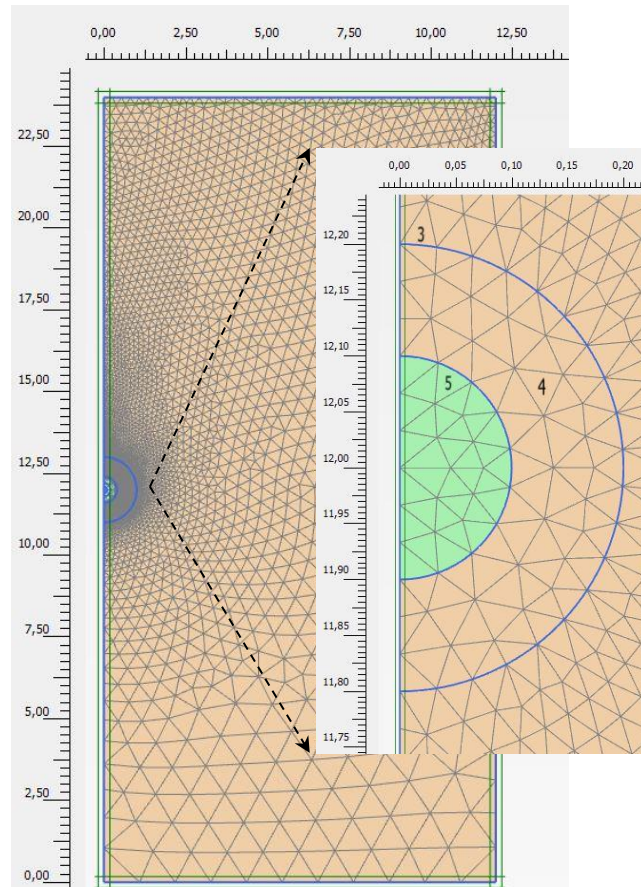


Figure V-15: Geometry and boundary condition of the numerical model for spherical cavity expansion.

As shown in Figure V-16, analytical solutions are in excellent agreement with the FEM predictions. Besides, the limit pressures values computed by FEM are briefly summarized in Table V-1 for both cylindrical and spherical cavities, with a graphic output (cf. Figure V-17) corresponding to one case ($GSI = 100$). As can be seen, the cavity pressure – cavity expansion curves derived from analytical solutions and finite element methods are in close agreement within 2%.

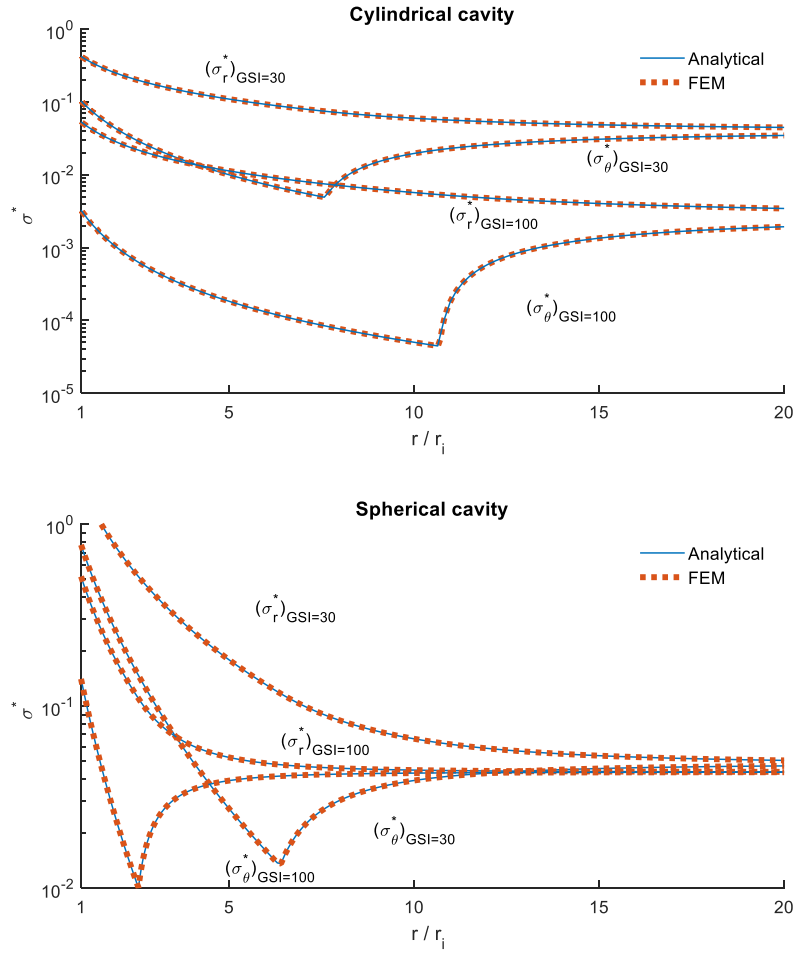


Figure V-16: Radial distribution of radial and circumferential stresses: Analytical vs. FEM

Table V-1: Comparison of limit pressure values Analytical vs. finite element method – Cylindrical cavity.

m_i	GSI	P_{lim}/UCS (Cylindrical)		Error (%)
		Eq. (V-58)	FEM	
2	30	5.86	5.87	0.17
	50	5.81	5.76	-0.86
	75	4.84	4.76	-1.65
	100	3.27	3.27	0.00

Table V-2: Comparison of limit pressure values: Analytical vs. finite element method – Spherical cavity.

m_i	GSI	P_{lim}/UCS (Spherical)		Error (%)
		Eq. (V-58)	FEM	
2	30	18.59	18.76	0.91
	50	13.38	13.48	0.74
	75	8.89	8.68	-2.36
	100	5.22	5.21	-0.19

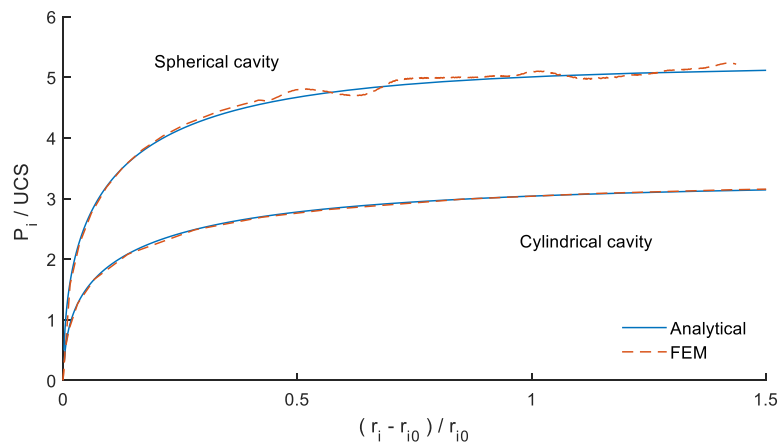


Figure V-17: Cavity pressure against cavity wall displacement according to the analytical (LST) and finite element method (Updated mesh)

ii. Case of brittle plastic post-failure - RS² 9 Modeller

Using the finite element software RS² 9 Modeller (Rocscience, 2019), a finite element analysis is conducted to validate the analytical results. The geometry of the models, as well as the selected boundary conditions for both cylindrical and spherical cavities, are as shown in Figure V-18. A plane strain conditions are assumed for the cylindrical cavity with an internal radius of 1m and default external boundary set as infinite. On the other hand, a spherical cavity with an internal radius $r_i = 0.1\text{ m}$ is modeled with a rock domain broad enough ($120r_i \times 240r_i$) to prevent boundary effect. The spherical cavity is located at a depth of 12m. A distributed load normal to the cylindrical and the spherical cavity cluster of 5 [MPa] and 10 [MPa], respectively is prescribed.

The rock mass in the FEM model has properties similar to that of undisturbed claystone ($D = 0$) with: $\sigma_{ci} = 35$ [MPa], $m_i = 4$ and $MR = E_i/\sigma_{ci} = 250$ and $GSI = 80$. The peak and residual deformation modulus are evaluated based on H-B core parameters as 7703 [MPa] and 4715 [MPa], respectively. These values can be

simply estimated from the software library (Rocscience, 2019). Since the Poisson's ratio ν does not usually affect rock behaviour in a significant manner, a standard value equal to 0.3 is assigned. Note that the dilatancy angle ψ as well as the far-field pressure P_0 are set equal to zero.

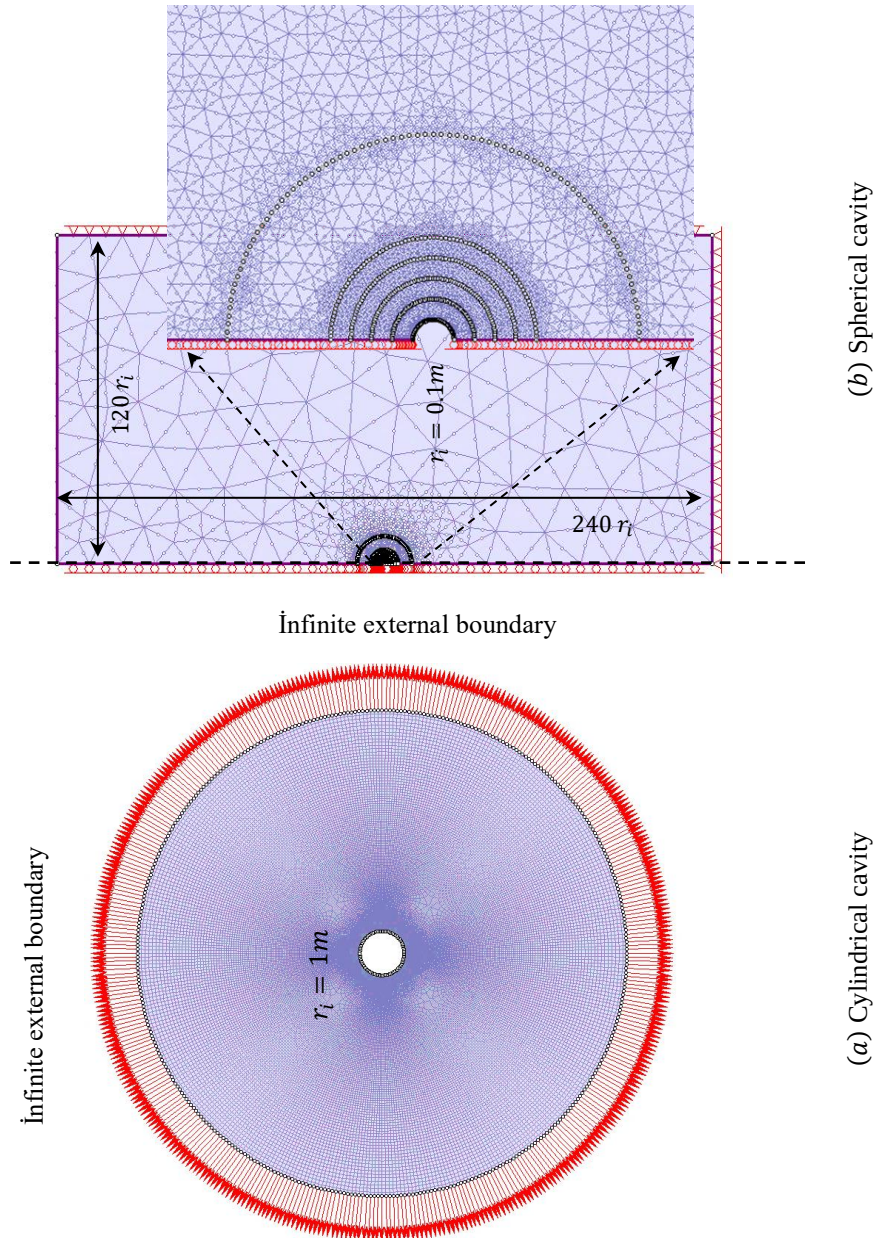


Figure V-18: Geometry and boundary conditions of the Finite element modelling

As shown in Figure V-19, analytical solutions are in excellent agreement with the FEM predictions emphasizing the accuracy of the developed solutions.

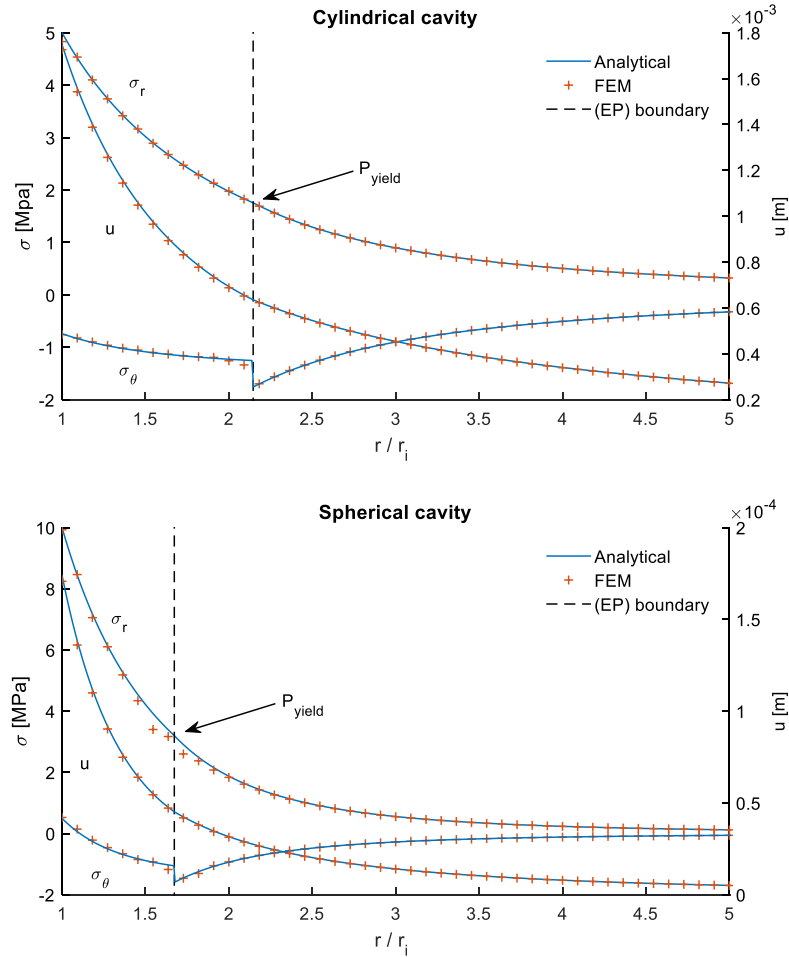


Figure V-19: Distribution of radial and circumferential stresses and displacement for both cylindrical and spherical cavities in an elastic brittle-plastic medium.

2.5.2. Limit pressure compared to published solutions

i. P_{lim} vs. analytical published solutions

In the absence of published large strain solutions for cavity expansion problem in an elastic perfectly plastic material obeying the H-B criterion, the limit pressure solutions are validated for a case of H-B criterion reduced to the Tresca criterion. Extensive published literature is available on cavity expansion analyses for a material obeying

the Tresca criterion (Yu, 2000). In this case, the H-B failure criterion simply reduces to:

$$\sigma'_1 = \sigma'_3 + \sigma_{ci} s^a, \quad m_b \rightarrow 0 \quad (V-91)$$

The above equation can be further written as:

$$\sigma'_1 - \sigma'_3 = \sigma_{ci} s^a = 2S_u \quad (V-92)$$

where $S_u = \frac{\sigma_{ci} s^a}{2}$ denotes the undrained shear strength of the material.

On the other hand, the limit pressure for a Tresca material can be computed as (Yu, 2000; Frikha and Bouassida, 2013):

$$P_{lim} = P_0 + \left(\frac{4}{3}\right)^{k-1} S_u \left[1 + \ln\left(\frac{G}{S_u}\right) + \ln\left(\frac{1+\nu}{3(1-\nu)}\right)^{k-1} \right] \quad (V-93)$$

For comparison matters, the following parameters are considered: $GSI = 100$ ($a = 0.5$), $m_i \rightarrow 0$, $\nu = 0.5$, $G/\sigma_{ci} = 300$ and $P_0/\sigma_{ci} = 0$. These rock mass parameters lead to an equivalent Tresca material with an undrained shear strength $S_u = \sigma_{ci}/2$.

The evolution of the scaled limit pressure as a function of a decreasing m_i ($m_i \rightarrow 0$) for both cylindrical and spherical cavities is plotted in (V-12). It is found that the limit pressure values gained from the developed solution are within 0.5% of those evaluated from the Tresca solutions (Yu, 2000) when the strength parameter $m_i = 10^{-3}$.

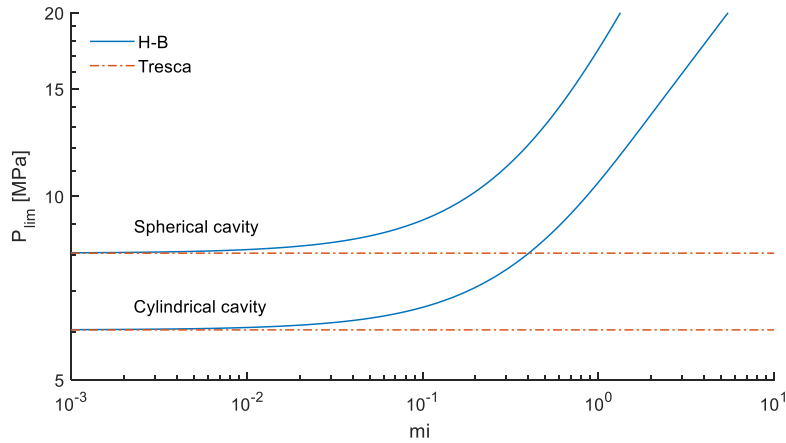


Figure V-20: Limit pressure approaching that of a cavity in a Tresca material when $m_i \rightarrow 0$.

ii. P_{lim} vs. Published FEM solutions

Hamdi (2016) investigated the limit pressure of a cylindrical cavity problem in four different rock masses using the finite element software Abaqus (<https://edu.3ds.com/fr/software/abaqus-edition-etudiante>). He implemented the SST numerically and, following the pressuremetric conventions, the limit pressure is defined as the pressure at which the cavity has doubled its volume i.e., $\frac{V_i}{V_0} = 2$ ($r_i/r_{i0} = \sqrt{2}$). Table V-3 summarizes the H-B core parameters, the elastic properties, the dilatancy angle, and the far field stress retrieved from the published thesis.

Table V-3: H-B core parameters for three quality rock masses

Rock mass ⁽¹⁾	H-B core parameters				Elastic parameters		$\psi^{(2)}$ [°]	$P_0^{(3)}$ [MPa]
	σ_{ci} [MPa]	m_i	GSI	D	E [MPa]	ν		
(CW)	48	28	10	0	700	0.2	0	0.25
(HW)	48	28	20	0	1200	0.2	0	0.25
(MW)	48	28	30	0	2000	0.2	0	0.25
(DL)	52	17	40	0	4000	0.33	5	1.5

(1) Same denomination as in the original work: St-Peter sandstone completely weathered gneiss (CW), St-Peter sandstone highly weathered gneiss (HW), St-Peter sandstone moderately weathered gneiss (MW) and St-Peter sandstone (DL) derived from Dittes *et al.* (2002)
(2) Constant dilatancy angle.
(3) Far-field pressure

Both yield and limit pressure values gained from the developed solutions as per Eqs. (V-14) and (V-58), respectively, and the published numerical results (Hamdi, 2016) are summarized in Table V-4. As can be noted, firstly, the computed yield pressure lies within 3% of the FEM values. On the other side, even if the analytical limit pressure is calculated for a cylindrical cavity expansion up to infinity compared to the published one where the cavity volume has been doubled, the present work results are on the conservative side except for (DL) rock mass where the limit pressure is within 12% the published one.

Table V-4: H-B core parameters for three quality rock masses

Rock mass ⁽¹⁾	$\frac{V_i}{V_0}$	(CW)	(HW)	(MW)	(DL)
P_{yield} [MPa] per Eq. (V-14)	—	0.486	0.497	0.505	2.94
P_{yield} [MPa] (Hamdi, 2016)	—	0.5	0.5	0.5	3.1
Error between P_{yield} [%]	—	2.8	0.6	-1	5.4
P_{lim} [MPa] per Eq. (V-58)	$+\infty$	9.86	14.25	19.40	65.21
P_{lim} [MPa] (Hamdi, 2016)	2	13	17	22.4	57.3
Error between P_{lim} [%]	—	31.8	19.3	15.4	-12.1

3. Shaft resistance

Nota:

The content of this section is submitted to the *International Journal of Geotechnical Engineering*.

An additional discussion covering the limit of applicability of the H-B failure criterion in terms of stress ratio condition is provided at the end of this section.

3.1. Introduction

Although the empirical approaches are useful for design, (cf. Section 3.1 of Chapter II for a topic review), they were developed from specific and limited data sets for which there were differences in geological conditions, drilling methods, and other conditions. Consequently, none of these approaches can be considered as a satisfactory predictor method. Because of the aforementioned limitations, analytical simple method is proposed to calculate the shaft resistance of a pile embedded in a rock mass. The method is based upon the H-B failure criterion, which represents the failure state of the in situ rock mass in terms of principal stresses, rock mass condition (surface condition) reflected by the Geological strength index GSI, rock type reflected by the strength parameter m_i as well as the uniaxial compressive strength of the intact rock. In addition, the horizontal stress term normal to the pile wall is directly considered to calculate the side resistance. In consequence, the proposed method constitutes an improvement over existing empirical methods, which mainly use the uniaxial compressive strength for estimating the side resistance, by considering the aforementioned extended parameters.

It should be emphasized that the proposed method provides estimates from the conditions comparable to existing empirical approaches. It means that progressive failure, time-dependent behaviour of concrete and multiple rock layer case, etc. are not considered.

3.2. Problem statement and basic hypothesis

The shaft resistance of a pile embedded in a rock mass that obeys the H-B failure criterion is investigated. The geometry of the problem and the stress diagram exerted on the pile wall are depicted in Figure V-21. It is assumed that the pile strength material is greater than the rock strength. That means that the pile–rock interface is equal to the rock strength. It is also assumed that this interface is relatively clean, not intentionally roughened and failure occurs through the rock mass rather through the concrete. The last assumption is not critical for most rocks (Sagong, Paik and Kim,

2007). Even though σ_{ci} may be much higher than that of concrete, this is not necessary the case for the in situ rock mass with discontinuities exhibiting much lower compressive strength. At failure, stresses on the shaft are exerted according to Mohr's envelope that is consistent with the H-B criterion as given per Eqs. (II-42) – (II-44).

It is also assumed that the horizontal stress σ_h (normal to the pile shaft) is proportional to the vertical one represented by the coefficient of the proportionality $k_0 = \sigma_h/\sigma_v$.

In this section, only lateral skin friction of the rock–pile interface is considered. In other words, the unit skin friction from the above layer, and the pile tip resistance are neglected.

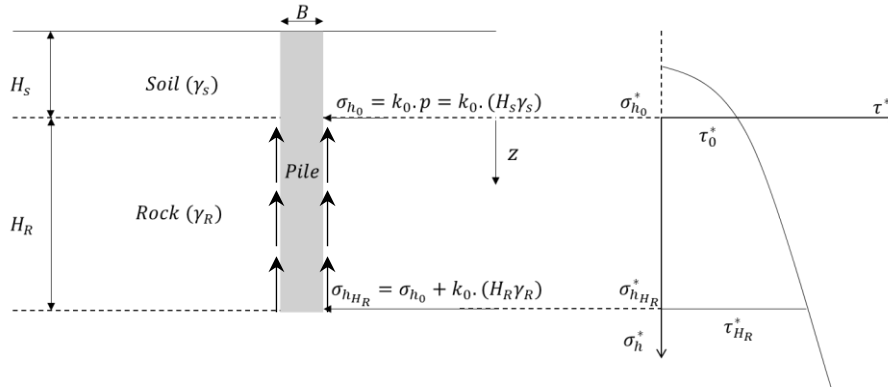


Figure V-21: Geometry of the problem and the stress diagram exerted on the pile wall.

3.3. Stresses on the shaft

As shown in the above figure, a vertical overburden pressure p applied on the rock surface is considered:

$$p = H_S \cdot \gamma_S \quad (V-94)$$

here H_S and γ_S denote, respectively, the thickness and the unit weight of the soil layer above the rock surface.

At any depth z below the rock surface, the vertical stress and subsequently the horizontal stress are expressed, as a function of the rock unit weight γ_R , as

$$\sigma_v = p + z \cdot \gamma_R \quad (V-95)$$

$$\sigma_h = k_0 \cdot \sigma_v = k_0(p + z \cdot \gamma_R) \quad (V-96)$$

Thus, the horizontal stress for the same depth z , in scaled form, writes:

$$\sigma_h^* = \frac{\sigma_h}{\sigma_{ci} \cdot m_b^\beta} + \frac{s}{m_b^{\beta/a}} = \frac{k_0(p + z \cdot \gamma_R)}{\sigma_{ci} \cdot m_b^\beta} + \frac{s}{m_b^{\beta/a}} \quad (V-97)$$

Therefore, referring to Eqs. (II-53) and (II-54), the scaled normal and shear stresses exerted on the pile wall at any depth z , are given as follows:

$$\sigma_n^* = \sigma_h^* = \frac{k_0(p + z \cdot \gamma_R)}{\sigma_{ci} \cdot m_b^\beta} + \frac{s}{m_b^{\beta/a}} = \sigma_3^* + \frac{(\sigma_3^*)^a}{2 + a(\sigma_3^*)^{a-1}} \quad (V-98)$$

$$\tau^* = \frac{\tau}{\sigma_{ci} \cdot m_b^\beta} = (\sigma_3^*)^a \frac{\sqrt{1 + a(\sigma_3^*)^{a-1}}}{2 + a(\sigma_3^*)^{a-1}} \quad (V-99)$$

The above two equations are fundamental for assessing the scaled average ultimate shear stress and, subsequently, the total shaft resistance. This will be detailed later.

a. Stresses at the rock surface ($z = 0$ m)

The horizontal stress at the rock surface is expressed as:

$$\sigma_{h_0}^* = \frac{k_0 p}{\sigma_{ci} \cdot m_b^\beta} + \frac{s}{m_b^{\beta/a}} = \sigma_{3_0}^* + \frac{(\sigma_{3_0}^*)^a}{2 + a(\sigma_{3_0}^*)^{a-1}} \quad (V-100)$$

Thus $\sigma_{3_0}^*$ can be evaluated by a numerical tool from the above equation and subsequently τ_0^* per Eq. (V-99) as follows:

$$\tau_0^* = (\sigma_{3_0}^*)^a \cdot \frac{\sqrt{1 + a(\sigma_{3_0}^*)^{a-1}}}{2 + a(\sigma_{3_0}^*)^{a-1}} \quad (V-101)$$

b. Stresses at pile toe ($z = H_R$)

Like the previous case, the scaled horizontal stress at pile toe is expressed as:

$$\sigma_{h_{H_R}}^* = \sigma_{h_0}^* + \frac{k_0 \gamma_R H_R}{\sigma_{ci} \cdot m_b^\beta} = \sigma_{3_{H_R}}^* + \frac{(\sigma_{3_{H_R}}^*)^a}{2 + a(\sigma_{3_{H_R}}^*)^{a-1}} \quad (V-102)$$

Thus $\sigma_{3_{H_R}}^*$ can be evaluated by a numerical tool from the above equation and subsequently the shear stress at the pile toe $\tau_{H_R}^*$ is calculated as follows:

$$\tau_{H_R}^* = (\sigma_{3_{H_R}}^*)^a \cdot \frac{\sqrt{1 + a(\sigma_{3_{H_R}}^*)^{a-1}}}{2 + a(\sigma_{3_{H_R}}^*)^{a-1}} \quad (V-103)$$

3.4. Total shaft resistance Q_s

The total shaft resistance Q_s of a pile with diameter B and embedded in rock at a depth H_R is defined naturally as:

$$Q_s = \pi B H_R \tau_{avg} \quad (V-104)$$

In which, τ_{avg} denotes the average ultimate unit shear stress throughout the pile embedment length into the rock layer H_R . τ_{avg} is expressed in scaled form as:

$$\tau_{avg}^* = \frac{\tau_{avg}}{\sigma_{ci} \cdot m_b^\beta} \quad (V-105)$$

On the other hand, τ_{avg}^* is defined naturally as:

$$\tau_{avg}^* = \frac{1}{H_R} \int \tau^* dz = \frac{\int \tau^* d\sigma_h^*}{\int d\sigma_h^*} = \frac{S}{\sigma_{h_{HR}}^* - \sigma_{h_0}^*} \quad (V-106)$$

Here, $\sigma_{h_0}^*$ and $\sigma_{h_{HR}}^*$ denote the scaled horizontal stresses, respectively, at depth $z = 0$ m and $z = H_R$ evaluated per Eq. (V-97).

Details of how the integral $S = \int_{\sigma_{h_0}^*}^{\sigma_{h_{HR}}^*} \tau^* d\sigma_h^*$ is evaluated are described below.

Firstly, referring to Eqs. (V-98) and (V-99), one can write:

$$d\sigma_h^* = \left(1 + \frac{2a(\sigma_3^*)^{a-1} + a(\sigma_3^*)^{2a-2}}{(2 + a(\sigma_3^*)^{a-1})^2} \right) d\sigma_3^* \quad (V-107)$$

$$\tau^* d\sigma_h^* = \tau^* \left(1 + \frac{2a(\sigma_3^*)^{a-1} + a(\sigma_3^*)^{2a-2}}{(2 + a(\sigma_3^*)^{a-1})^2} \right) d\sigma_3^* \quad (V-108)$$

Secondly, substituting the above equations into the integral expression $S = \int_{\sigma_{h_0}^*}^{\sigma_{h_{HR}}^*} \tau^* d\sigma_h^*$ results in the following key expression for evaluating S :

$$S = \int_{\sigma_{h_0}^*}^{\sigma_{h_{HR}}^*} \tau^* d\sigma_h^* = \int_{\sigma_{3_0}^*}^{\sigma_{3_{HR}}^*} (\tau_1^* + \tau_2^* + \tau_3^*) d\sigma_3^* \quad (V-109)$$

where

$$\tau_1^* = (\sigma_3^*)^a \frac{\sqrt{1 + a(\sigma_3^*)^{a-1}}}{2 + a(\sigma_3^*)^{a-1}} \quad (V-110)$$

$$\tau_2^* = 2a(\sigma_3^*)^{2a-1} \frac{\sqrt{1 + a(\sigma_3^*)^{a-1}}}{(2 + a(\sigma_3^*)^{a-1})^3} \quad (V-111)$$

$$\tau_3^* = a(\sigma_3^*)^{3a-2} \frac{\sqrt{1 + a(\sigma_3^*)^{a-1}}}{(2 + a(\sigma_3^*)^{a-1})^3} \quad (V-112)$$

Finally, the exact value of the integral S can be found numerically using, for example, MATLAB software (<https://nl.mathworks.com/products/matlab.html>). Otherwise, the Simpson rule (Matthews, 2004) can be used to calculate approximately the integral S . i.e.

$$S = \frac{\sigma_{3_{HR}}^* - \sigma_{3_0}^*}{6} \sum_{j=1}^3 \left[\tau_j^*(\sigma_{3_0}^*) + 4\tau_j^*\left(\frac{\sigma_{3_{HR}}^* + \sigma_{3_0}^*}{2}\right) + \tau_j^*(\sigma_{3_{HR}}^*) \right] \quad (V-113)$$

For practical purposes, charts allowing easy and accurate estimation of the average shear resistance as a function of the boundary horizontal stresses $\sigma_{h_0}^*$ and $\sigma_{h_{HR}}^*$ are proposed for some different values of GSI (cf. Figure V-22). These curves can be used approximately. Otherwise, a spreadsheet can be used to assess the average unit shaft resistance accurately. As can be seen, the general trend of these curves demonstrates that there is no significant influence of GSI on results.

The use of this chart is straightforward. Knowing the values of the boundary stresses $\sigma_{h_0}^*$ and $\sigma_{h_{HR}}^*$, the average shear resistance τ_{avg}^* which is equal to:

$$\tau_{avg}^* = \frac{S_{\sigma_{h_{HR}}^*} - S_{\sigma_{h_0}^*}}{\sigma_{h_{HR}}^* - \sigma_{h_0}^*} \quad (V-114)$$

which can be simply evaluated graphically. Otherwise, these curves can be fitted to an exponential function of the form:

$$S = A(\sigma_h^*)^B \quad (V-115)$$

where A and B are two constants equal to $2.77 \cdot 10^{-1}$ and 1.71 , respectively.

Substituting the above equation into Eq. (V-114) results in the following approximate average shear resistance expression:

$$\tau_{avg}^* = 0.277 \frac{(\sigma_{h_{HR}}^*)^{1.71} - (\sigma_{h_0}^*)^{1.71}}{\sigma_{h_{HR}}^* - \sigma_{h_0}^*} \quad (V-116)$$

To illustrate the use of a developed solution, an example is given below:

A pile with diameter $B = 1 \text{ m}$ embedded in weak rock R1 at a depth $H_R = 4 \text{ m}$ is considered. The unit weight of the rock layer is $\gamma_R = 20 \text{ kN/m}^3$. A vertical

overburden pressure $p = 100 \text{ kPa}$ applied on the rock surface is considered. The H-B core parameters of the rock layer are $\sigma_{ci} = 5 \text{ [MPa]}$, $m_i = 7$, $D = 0$, and $GSI = 30$. Thus, H-B derived parameters can be evaluated as:

$$a = \frac{1}{2} + \frac{1}{6} \left(\exp \left(-\frac{GSI}{15} \right) - \exp \left(-\frac{20}{3} \right) \right) = 0.522,$$

$$s = \exp \left(\frac{GSI-100}{9-3D} \right) = 4.19 \cdot 10^{-4} \text{ and}$$

$$m_b = m_i \exp \left(\frac{GSI-100}{28-14D} \right) = 0.574,$$

Then, boundary stresses are simply calculated as:

$$\sigma_{h_0}^* = \frac{k_0 p}{\sigma_{ci} m_b^\beta} + \frac{s}{m_b^{\beta/a}} = 0.038 \rightarrow \sigma_{3_0}^* = 0.050 \text{ per Eq.(V-100),}$$

$$\sigma_{h_{HR}}^* = \sigma_{h_0}^* + \frac{k_0 \gamma_R H_R}{\sigma_{ci} m_b^\beta} = 0.0673 \rightarrow \sigma_{3_{HR}}^* = 0.097 \text{ per Eq. (V-102),}$$

which results in the following normalized average unit shear stress.

$$\tau_{avg}^* = 5.70 \cdot 10^{-2} \text{ per Eq. (V-106) (or } \tau_{avg}^* \approx 5.84 \cdot 10^{-2} \text{ per Eq. (V-120))}$$

Finally, the total shaft resistance can be evaluated as

$$Q_s [MN] = \pi B H_R \tau_{avg} = \pi B H_R (\sigma_{ci} m_b^\beta) \tau_{avg}^* = 1.95 \text{ (or } Q_s [MN] \approx 2.00).$$

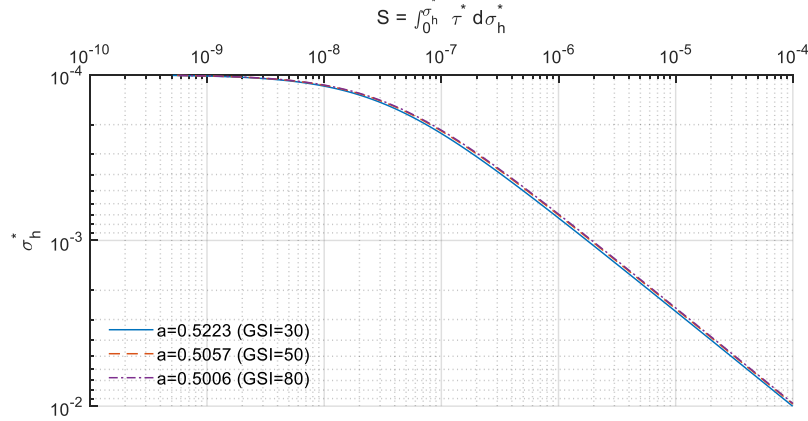


Figure V-22: Average shear resistance chart.

3.5. Validation using FEM

Using the geotechnical analysis software RS², a numerical model is implemented to compare the analytical results presented in Subsection 3.4 of this chapter to the numerical solutions obtained by the method of finite element. The elastic perfectly

plastic H-B model uses six parameters, which consist of elastic parameters (E, ν) and the H-B core parameters (σ_{ci}, m_i, GSI and $D = 0$).

Since the theoretical formulation was done in plane strain conditions, the following aspects were considered:

- A 2D plane strain is used for modelling this problem,
- Due to symmetry, only half of the geometry is modelled (cf. Figure V-23),
- The lateral boundaries of the model are constrained in normal displacement. Since only half of the model is considered, the normal displacement is also constrained along the symmetry axis (cf. Figure V-23),
- A vertical downward uniform displacement is incrementally applied on the top of the wall (elastic model)
- A joint between the wall and the rock is introduced. This contact is characterized by a “Material dependent” slip criterion.
- Loose soil is placed at the pile base to prevent the transmission of load at the tip of the wall.

The geometry details of the model are as shown in Figure V-23. A wall of thickness $B = 1\text{ m}$ embedded in rock layer at a depth $H_R = 4\text{ m}$ is considered. The unit weight of the rock layer is $\gamma_R = 20\text{ kN/m}^3$. A vertical overburden pressure $p = 100\text{ kPa}$ applied on the rock surface is considered. A graded mesh of 4000 triangular elements is used. The mesh is refined at the wall-rock interface.

To properly conduct the study, it was felt necessary to use for the numerical modelling, H-B core parameters range that cover practically all rocks. Therefore, we consider four different quality rock masses, namely, weak rock, medium-strong rock, strong rock, and a very strong rock (cf. Table V-5). These rocks will be used to compare shaft resistance values calculated by the proposed method against those gained from FEM.

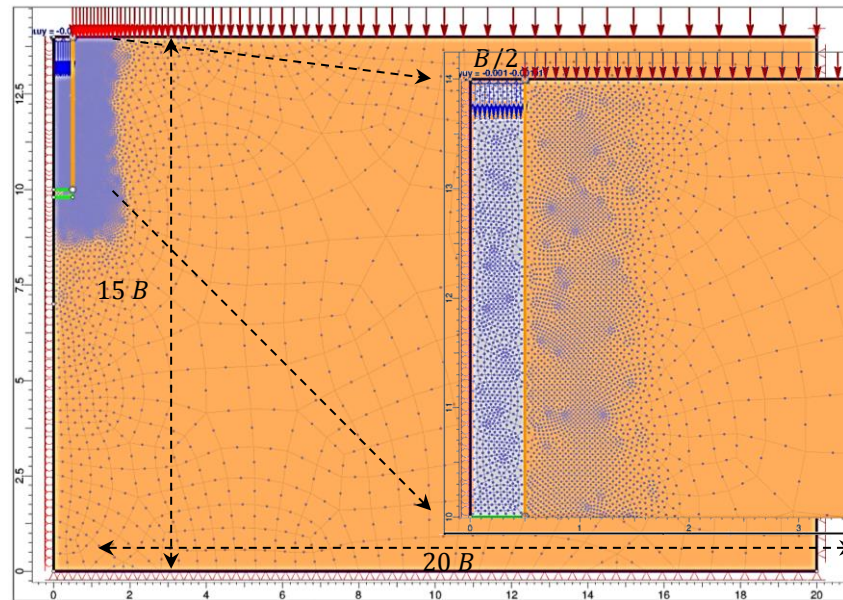


Figure V-23: geometry of the numerical model to assess the shaft resistance of a wall.

Table V-5: H-B core parameters for four different quality rock masses (Hoek, 2000).

Rock quality	H-B core parameters		
	σ_{ci} [MPa]	m_i	GSI
R1: weak	5	7	30, 50
R2: medium-strong	25	10	
R3: strong	50	15	
R4: very strong	100	25	

Numerical and analytical results of the total shaft resistance for the different rock masses defined above are summarized in Table V-6. As can be seen, results obtained by numerical integration are the same as those gained from published solutions (Serrano, Olalla and Galindo, 2015), emphasizing the efficiency of the proposed method. On the other hand, numerical results are in excellent agreement with analytical results.

Table V-6: Total shaft resistance [kN] obtained by analytical solution and numerical modelling for some different rock mass qualities.

GSI	Method	Rock quality			
		R1	R2	R3	R4
30	(1) : Analytical solution as per Eq. (V-104)	1.95	3.57	4.90	6.80
	(2) : (Serrano, Olalla and Galindo, 2015)	1.95	3.57	4.90	6.80
	(3) : Numerical solution	2.00	3.73	5.05	6.95
	Error between (1) and (3) (%)	2.5	4.4	3.0	2.2
50	(1) : Analytical solution as per Eq. (V-104)	2.71	5.55	7.96	11.44

(2) : (Serrano, Olalla and Galindo, 2015)	2.71	5.55	7.96	11.44
(3) : Numerical solution	2.79	5.66	8.05	11.64
Error between (1) and (3) (%)	2.9	2.0	1.13	1.74

3.6. Comparison with the state of the art

In this section, the 113 specific cases presented in Section 3 of Chapter II are considered to demonstrate the applicability of the proposed analytical solution. For each case, rock type, UCS, and geometry details of soil and rock layers were extracted from original papers. Still, these sets of values are not enough to make calculations. It is necessary to estimate the rest of the H-B input parameters such as GSI and m_i . To do that, one can resort to published guidelines and tables (Hoek and Brown, 1997). It is important to mention that the density of the soil overlying the rock layer is set equal to $10 [kN/m^3]$ if the soil is saturated and it is equal to $17.5 [kN/m^3]$ if not whereas the density of the rock layer is estimated from (Rocks.comprenature.com, 2015) based on the rock type. Table 0-21 summarizes for each case, the required input data to be able to make calculations and the corresponding unit shaft resistance gained from the developed solution by setting $k_0 = 1$ as well as the ‘measured’ one based on load tests. Note that setting $k_0 = 1$, i.e., the horizontal stress equal to the vertical stress stems from the fact that the collected data do not provide detailed roughness measurement and lateral stress ratio of the in situ rock mass (Sagong, Paik and Kim, 2007).

It is important to recall that the values of the so-called ‘measured’ shaft resistance are not necessarily the true ultimate one in most cases. Indeed, a Displacement Index (DI) is assigned for each case (see Subsection 3.1 of Chapter II for further details). As shown in Figure V-24, more than half of test results (51%) have a DI=“C,” 27% of test results have a DI=“B,” whereas the remaining results (22%) are characterized by a DI=“A.”

A comparison between the measured ultimate unit shaft resistances values against those evaluated by the proposed method is depicted in Figure V-25. Taking all the 113 cases, the scatter is high, and the results are not entirely satisfactory. However, neglecting test results with an index ‘B’ and ‘C’ where the unit shaft resistance is not fully mobilized, a good agreement between test results (with a DI ‘A’) and results gained from the proposed solution is noticed. It can be considered that this analysis procedure offers consistent results. It is essential to mention that results obtained from the approximate solution (cf. Eq. (V-116)) are in excellent agreement compared to those obtained from the exact solution (cf. Eq. (V-106)), which requires a numerical integration). The use of the approximate expression is efficient, and it allows easy and accurate estimation of the average shear resistance as a function of only boundary stresses $\sigma_{h_0}^*$ and $\sigma_{h_{HR}}^*$ without having to resort either to the parametric form depending

on the minor principal stresses σ_3^* or the numerical integration over the embedment depth.

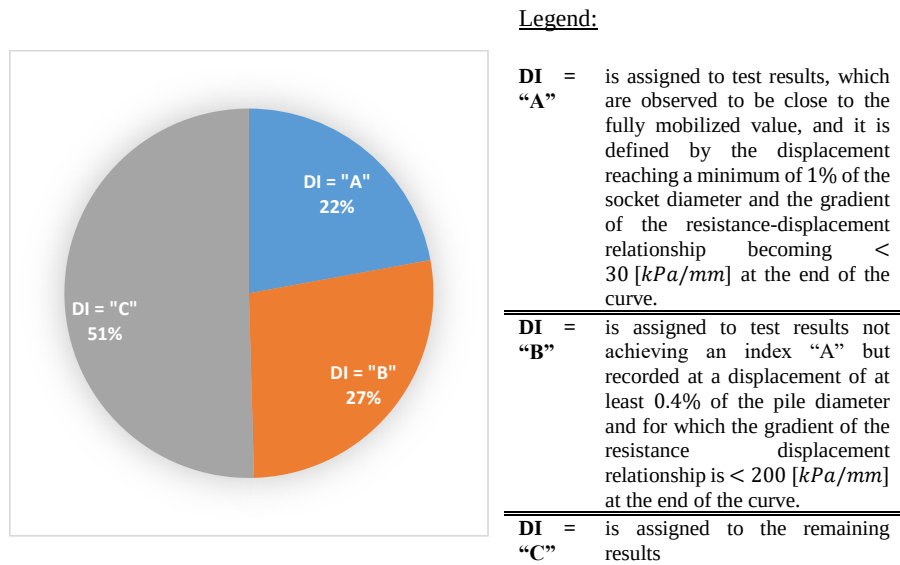


Figure V-24: Distribution of displacement index of test data results extracted from the literature.

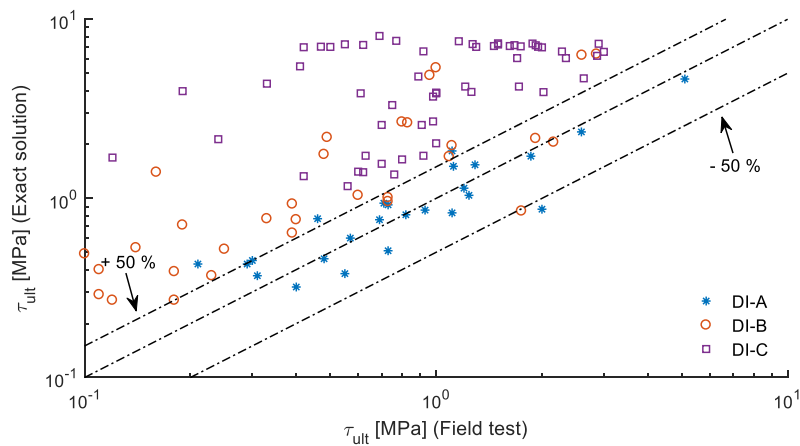


Figure V-25: Ultimate unit shaft resistance: analytical solution results vs. field test results.

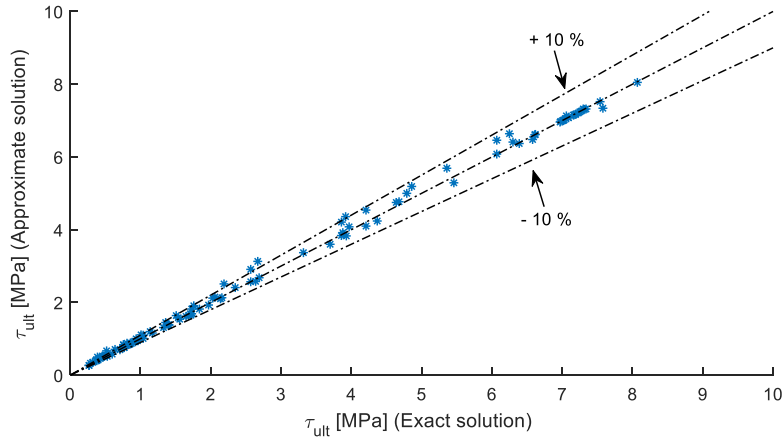


Figure V-26: Ultimate unit shaft resistance: Exact vs. approximate solution.

3.7. Conclusion

The H-B failure criterion is used in a scaled form to calculate the total shaft resistance of a pile embedded into the rock mass. The scaled form of the H-B failure criterion expression leads to considerable simplification in defining the shear strength problem at the pile–rock interface. The scaled shear stress τ^* and the scaled normal stress σ_n^* are related to each other using a parametric form depending exclusively on the scaled minor principal stress σ_3^* . These formulas were fundamental for assessing the normalized average shaft resistance τ_{avg}^* . For practical purposes, design charts, as well as an approximate solution, were provided allowing easy and accurate estimates of the normalized average shaft resistance τ_{avg}^* .

The analytical results were in excellent agreement with results obtained by numerical models.

On the other hand, a comparison has been made between results gained from the developed theory and those measured during field load tests of 113 specific cases. It is essential to mention that field test results were divided into three categories based on a displacement index. A good agreement between predicted results and test results belonging to the displacement index ‘A’ was noticed. However, the remaining test results are on the conservative side compared to the predicted ones, which are expected since the unit shear resistance was not fully mobilized during the load test.

The analytical results were calculated by assuming that the horizontal and vertical stresses acting at the pile shaft are the same, i.e. $k_0 = 1$. This assumption, which is on the conservative side due to the expansive effect of the concrete poured, was presumed since the collected data do not provide detailed roughness measurement and lateral

stress ratio of the in situ rock mass. Nevertheless, and to address the roughness influence, one can follow Sagong, Paik and Kim (2007) in adopting a horizontal stress expression that includes the roughness effect represented by the triangle asperity height Δr_h on the interface between pile and rock mass.

To make an excellent comparison with the theory developed here, it would be useful to conduct further field tests with all relevant rock medium data as well as the stress acting through the shaft of the pile.

3.8. Applicability of the H-B criterion to the shaft resistance problem – Stress ratio condition

The shaft resistance of a pile embedded in a material that obeys the nonlinear H-B failure criterion was investigated. To demonstrate the applicability/efficiency of the above analytical solution, a database of conducted experimental case studies of piles embedded in rock layers (113 cases), originating from an extensive bibliographic study was employed.

Therefore, the first necessary step consists of ascertaining the applicability of the H-B criterion according to the s -values condition. As shown in Figure V-27, only 12 case studies among 113 (11%) do not satisfy the s -values condition. Therefore, special attention is given to these cases when analyzing the results.

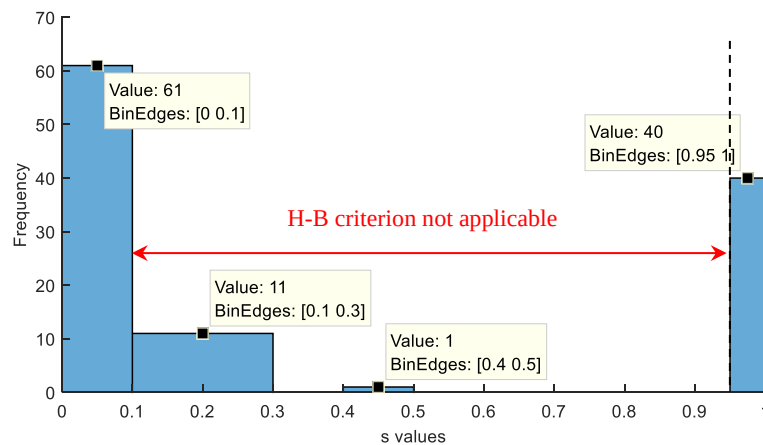


Figure V-27: Shaft resistance problem – Applicability of the H-B criterion according to the s -values conditions (113 case studies).

On the other hand, the second step is to check whether the stress ratio condition is respected. The verification procedure is given below. For each case study:

- The maximum horizontal stress σ_h^* (adverse one) is calculated at the pile embedment depth into the rock layer.
- The scaled minor principal stress σ_3^* value is deduced using the following relationship $\sigma_h^* = \sigma_3^* + \frac{(\sigma_3^*)^a}{2+a(\sigma_3^*)^{a-1}}$.
- The inequality $\sigma_1^* = \sigma_3^* + \sigma_3^{*a} \geq \sigma_{1.min}^* = (LSR)\sigma_3^* + \frac{s}{m_b^{B/a}}(1 - LSR)$ is finally examined.

Figure V-28 and Figure V-29 plot σ_1^* versus $\sigma_{1.min}^*$ using, respectively, Mogi's LSR and Schwartz's LSR. As can be seen, all results fall above the line $\sigma_1^* = \sigma_{1.min}^*$ i.e., the stress ratio condition is satisfied for all case studies. However, the 12 case studies among 113 that do not satisfy the s-values condition should be excluded from the analyses. Therefore, the predicted shaft resistance values for these cases (cf. Table X cases) are not reliable.

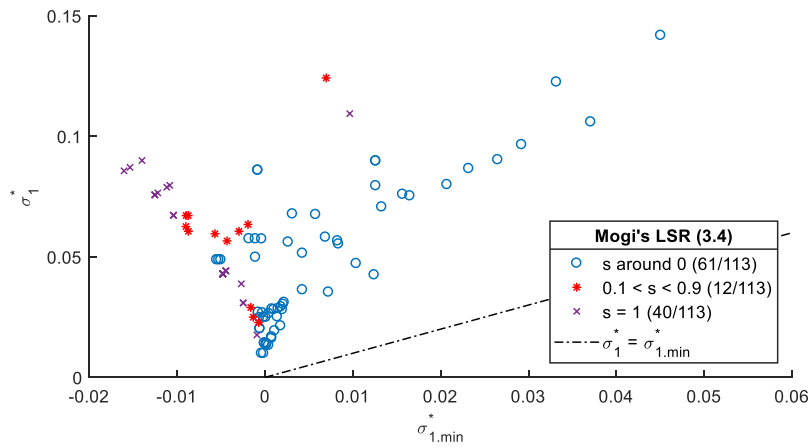


Figure V-28: Shaft resistance: σ_1^* vs. $\sigma_{1.min}^*$ using Mogi's LSR.

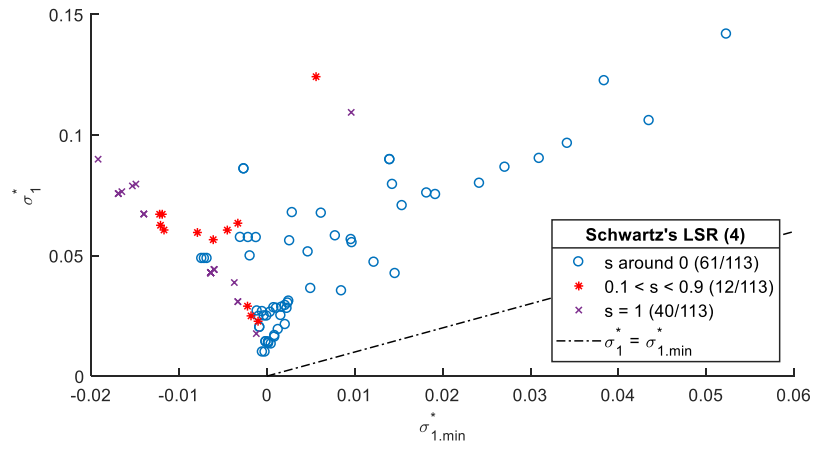


Figure V-29: Shaft resistance: σ_1^* vs. $\sigma_{1,min}^*$ using Schwartz's LSR.

4. Base resistance

Nota:

The content of this section was published in the *Journal of rock mechanics and geotechnical engineering*. doi.org/10.1016/j.jrmge.2020.03.004.

An additional discussion subsection (4.5) covering the limit of applicability of the H-B failure criterion in terms of stress ratio condition is provided at the end of this section.

4.1. Problem statement and basic hypothesis

An essential assumption in the use of cavity expansion theory in the context of pile end-bearing capacity is that the pressure exerted at the boundaries of the plastic bulb immediately beneath the pile tip is equal to the limit pressure P_{lim} required to expand a spherical cavity. Two methods, called mode 1 and mode 2, are proposed for the computation of the ultimate end bearing capacity $q_{b,ult}$. Therefore, the ultimate end bearing capacity will be denoted as $(q_{b,ult})_1$ for mode 1 and $(q_{b,ult})_2$ for mode 2.

The failure mechanism of mode 1, which was initially postulated by (Yasufuku and Hyde, 1995), is illustrated in Figure V-30. In this mode, active earth pressure conditions are considered to exist immediately beneath the pile tip and moment equilibrium is considered about point B, for the limit pressure P_{lim} , the ultimate end bearing capacity $(q_{b,ult})_1$ and the active pressure σ_A . The following equation relating P_{lim} to $(q_{b,ult})_1$ is established (Yasufuku and Hyde, 1995):

$$(q_{b,ult})_1 = \frac{P_{lim}}{1 - \sin(\varphi)} \quad (V-117)$$

Here φ is the friction angle.

The failure mechanism of mode 2 illustrated in Figure V-30 which was initially proposed by (Vesic, 1973), assumes a rigid cone of soil beneath the pile tip, with an angle $\alpha = \left(\frac{\pi}{4} + \frac{\phi}{2}\right)$. Outside the conical region, it is assumed that the normal stress acting on the cone face is equal to the limit pressure P_{lim} required to expand a spherical cavity.

A relationship between $(q_{b,ult})_2$ and P_{lim} can be obtained considering vertical equilibrium, i.e.:

$$\frac{(q_{b,ult})_2 \pi B^2}{4} = (P_{lim} \cos(\alpha) + \tau \sin(\alpha)) \frac{\pi B^2}{4 \cos(\alpha)} \quad (V-118)$$

which leads to:

$$(q_{b,ult})_2 = P_{lim} + \tau \tan(\alpha) \quad (V-119)$$

in which τ is the shear stress, and B is the pile base diameter.

On the other hand, Manandhar and Yasufuku (2013) stated that no plastic volumetric strain should be assumed when computing P_{lim} , i.e. the dilatancy angle will be considered to be zero for large strain analyses.

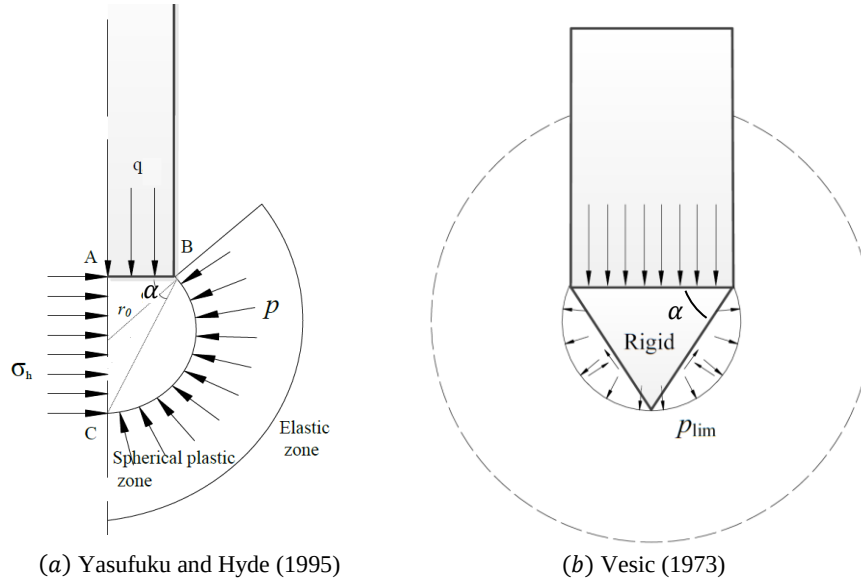


Figure V-30: Sketch of failure mechanism around the pile tip – Spherical cavity expansion.

4.2. End bearing capacity

In this section, details of how the ultimate end bearing capacity is calculated are given below firstly for mode 1 and then for mode 2.

For mode 1, the relationship between $(q_{b,ult})_1$ and P_{lim} is given per Eq. (V-112). Knowing that the friction angle ϕ is determined from the slope of the tangent to the H-B failure envelope, i.e.:

$$\frac{d\sigma_r^*}{d\sigma_\theta^*} = \tan^2 \left(\frac{\pi}{4} + \frac{\phi}{2} \right) = 1 + a(\sigma_\theta^*)^{a-1} \quad (V-120)$$

and by substituting the above equation into Eq. (V-112) and after some derivations, it is demonstrated that the scaled ultimate end bearing capacity $(q_{b,ult}^*)_1$ and the scaled

limit pressure P_{lim}^* are related to each other using a parametric form depending exclusively on the scaled circumferential stress σ_θ^* as follows:

$$\begin{cases} (q_{b,ult}^*)_1 = \sigma_\theta^* + \left(1 + \frac{a}{2}\right)(\sigma_\theta^*)^a + \frac{a}{2}(\sigma_\theta^*)^{2a-1} - \frac{s}{m_b^{\beta/a}} \frac{a}{2}(\sigma_\theta^*)^{a-1} \\ P_{lim}^* = \sigma_\theta^* + (\sigma_\theta^*)^a \end{cases} \quad (V-121)$$

For mode 2, the relationship between $(q_{b,ult})_2$ and P_{lim} given per Eq. (V-114) can be further expressed in a scaled form as:

$$(q_{b,ult})_2 = P_{lim}^* + \tau^* \tan(\alpha) ; \alpha = \left(\frac{\pi}{4} + \frac{\phi}{2}\right) \quad (V-122)$$

Substituting Eqs. (II-54) and (V-116) into the above equation result in the following parametric form relating $(q_{b,ult})_2$ and P_{lim}^* by means of σ_θ^* as:

$$\begin{cases} (q_{b,ult})_2 = \sigma_\theta^* + (\sigma_\theta^*)^a \\ P_{lim}^* = \sigma_\theta^* + \frac{(\sigma_\theta^*)^a}{2 + a(\sigma_\theta^*)^{a-1}} \end{cases} \quad (V-123)$$

The evolution of both $(q_{b,ult})_1$ and $(q_{b,ult})_2$ as a function of P_{lim}^* is depicted in Figure V-31 for different values of the exponent a (i.e. GSI) and by setting $m_i = 10$. As can be seen, the ultimate end bearing capacity values derived from both modes are close to each other's when $P_{lim}^* > 0.37$ whereas mode 2 offers conservative values when $P_{lim}^* < 0.37$. It is also observed that the ultimate end bearing capacity values are relatively constant when $GSI > 30$.

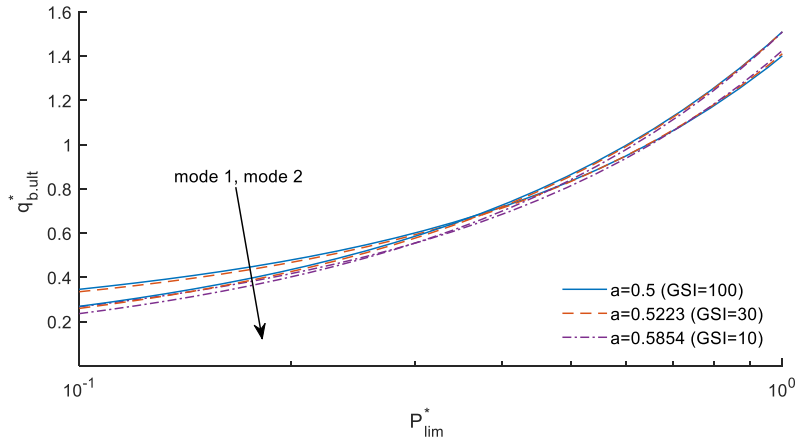


Figure V-31: 'True' ultimate bearing capacity for mode 1 and mode 2.

Due to the often-high capacity of piles in rock masses, it is rarely possible to mobilize the bearing capacity corresponding to the ultimate limit state. As a result, it is crucial to estimate the end bearing capacity of a pile at a certain settlement level at the pile base in the context of performance-based design. In this light, it will be necessary to compute not only the ultimate bearing capacity but also the load-settlement behaviour of piles. To achieve that, it will be further assumed that the volume of rock mass immediately beneath the pile tip which was displaced due to the pile settlement s_b , is equal to the increase of spherical cavity volume, i.e.,

$$s_b \frac{\pi B^2}{4} = \frac{4}{3} \pi (r_i^3 - r_{i0}^3) \quad (\text{V-124})$$

By taking into account that the instantaneous cavity radius r_i can be expressed as a function of the pile base diameter B (Yang, 2006) as:

$$r_i = \frac{B}{2 \cos \phi} \quad (\text{V-125})$$

Eqs. (V-124) can be further expressed as:

$$\frac{s_b}{B} = \frac{2}{3} \frac{\left(1 - \left(\frac{r_{i0}}{r_i}\right)^3\right)}{(\cos \phi)^3} \quad (\text{V-126})$$

in which $\left(\frac{r_{i0}}{r_i}\right)$ is given per Eq. (V-49). Referring to Eq. (V-116) the term $\cos \phi$ can be substituted by the following expression depending exclusively on the circumferential stress $\sigma_\theta^*(R_i)$ as:

$$\cos \phi = \frac{2 \sqrt{\left(1 + a(\sigma_\theta^*)^{a-1}\right)}}{2 + a(\sigma_\theta^*)^{a-1}}; \sigma_\theta^*(R_i) = [a W_0(R_i)]^{\frac{\beta}{a}} \quad (\text{V-127})$$

To sum up, the procedure described below should be followed to evaluate the ultimate end bearing capacity ($\frac{s_b}{B} = \infty$) as well as the end bearing resistance for a given settlement $\frac{s_b}{B}$ of a pile embedded in rock layer:

- (1) Choose input parameters: GSI , m_i , σ_{ci} , D , P_0 , G (or E), ν and $\psi = 0^\circ$
- (2) Evaluate H-B derived parameters: a , m_b and s .
- (3) Evaluate intermediate parameters: the scaled far-field pressure $P_0^* = \frac{P_0}{\sigma_{ci} m_b^\beta} + \frac{s}{m_b^{\beta/a}}$ and the rigidity index $I_r = \frac{G}{\sigma_{ci} m_b^\beta}$.
- (4) Evaluate the scaled limit pressure P_{lim}^* per Eq. (V-58). Alternatively, read it approximatively from the spherical cavity chart (cf. Figure V-11).

- (5) Evaluate, the scaled ultimate end bearing capacity $q_{b,ult}^*$ for both modes using Eqs. (V-121) and (V-123)
- (6) Evaluate the end bearing resistance for a given settlement $\frac{s_b}{B}$ using Eq. (V-49) combined with Eq. (V-126) for P_i^* . Then, q_b^* is evaluated per Eqs. (V-121) and (V-123), respectively, for mode 1 and mode 2.

An example is given below for computing the ultimate end bearing capacity as well as the end bearing resistance of a pile embedded in a rock mass with properties similar to that of an undisturbed ($D = 0$) claystone (Hoek, 2006b). Table V-7 summarizes the rock mass properties and the computed end bearing resistance q_b for different settlement values.

Table V-7: Rock mass properties – Limit pressure – End bearing resistance.

Input parameters:						
m_i	σ_{ci}	GSI	$\frac{E_i}{\sigma_{ci}}$	E_{rm} [GPa] Eq. (II-60)	ν Eq. (II-63)	P_0 [MPa]
2	25	100	150	3.73	0.25	0.25
H-B derived and intermediate parameters:						
a	m_b	s	β	P_0^*	I_r	
0.5	2	1	1	0.25	29.80	
Limit pressure and ultimate end bearing capacity for both modes:						
P_{lim}^*		$(q_{b,ult}^*)_1$		$(q_{b,ult}^*)_2$		
2.86		3.38		3.71		
Pile end bearing resistance for both modes:						
$(s/B) \%$	10	20	50	100	∞	
$(q_b)_1$ [MPa]	44.7	69.5	104.3	125.1	156.4	
$(q_b)_2$ [MPa]	49.5	77	115.5	138.6	173.2	

4.3. Comparison with the state of the art

In this section, the 91 field tests extracted from the literature are considered to demonstrate the applicability of the proposed analytical solution. For each case, the rock layer characteristics, the measured end bearing resistance, and settlements were identified. It is further assumed that all rocks obey the H-B failure criterion. Indeed, these types of rocks were characterized with the H-B criterion, and their parameters are published in (Hoek, 2006b). To compute the bearing capacity, the elastic and mechanical properties of the rock mass (deformation modulus E_{rm} , uniaxial compressive strength, etc.) were retrieved from the original papers. However, it was still necessary to estimate the parameters of the H-B criterion: GSI , and m_i and the elastic modulus for some cases. The parameter m_i was estimated based on published guidelines and tables for different rock types (Hoek, 2006b). Similarly, the elastic modulus of intact rock was also estimated based on rock type when necessary (Hoek

and Diederichs, 2006). Furthermore, and to profit from the large amount of test data accumulated using the traditional RMR system, the GSI values is calculated after Hoek and Carter (2013) as:

$$GSI = 1.5(JCond89) + (RQD/2) \quad (V-128)$$

in which, $JCond89$ denotes the joint condition rating that varies from 0 to 30. When RQD was not available, the GSI values were estimated based on the rock surface description in the original papers. The set of all these values are summarized in Table 0-22 (Appendix p. 265).

Finally, the analytical and measured end bearing resistances are also reported in Table 0-22. Note that the end bearing capacity obtained from the developed solutions is calculated as follows. Knowing the value of s_b/B , one can calculate the increase of the spherical cavity volume and subsequently the required pressure P_i^* to reach such expansion. The computed P_i^* values as a function of the normalized settlement s_b/B are plotted in Figure V-32. A hyperbolic law trend of the form $y = \frac{x}{x+constant}$ is observed. For the 6 case studies with a normalized settlement exceeding 40%, the pressure applied at the spherical cavity wall P_i^* reaches the limit pressure P_{lim}^* . For the remaining results, P_i^* varies from $3\%P_{lim}^*$ ($= 1.5P_y > P_y$ i.e. Plasticity was initiated) for an $s_b/B = 0.6\%$ to $72\%P_{lim}^*$ for an $s_b/B = 13.9\%$.

The measured end bearing resistance is plotted against the predicted bearing resistance computed as an average value considering both failure criteria (cf. Figure V-33). As can be seen, the scatter is low (within 20% of error) emphasizing that the present work solution provides better estimations than the empirical method (cf. Eq. (II-76). Furthermore, results indicate the crucial role of incorporating the allowable pile settlement in the bearing capacity computations.

On the other hand, and to better demonstrate the benefits of the proposed method, it will be interesting to visualize the measured load-displacement curve against the computed curve according to the present work. Figure V-34 to Figure V-38 display these curves for cases 2, 3, 4, 6, 7, 8, 74, 86, 87, and 89. As observed, the general trend of the curves agrees well.

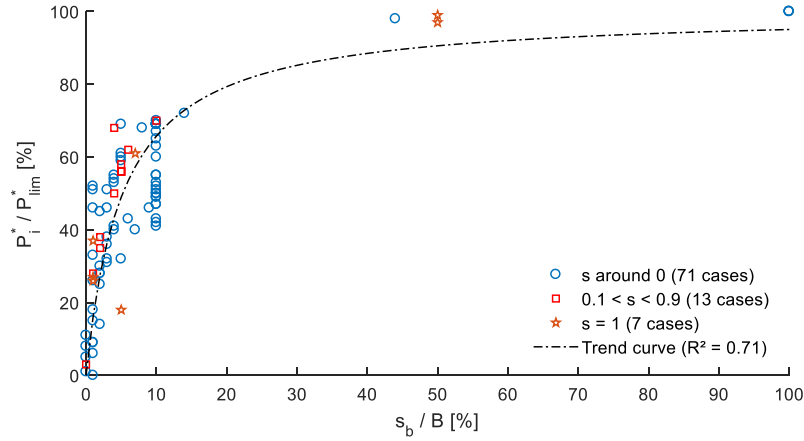


Figure V-32: Pressure applied at the cavity wall as a function of the normalized settlement – 91 case studies.

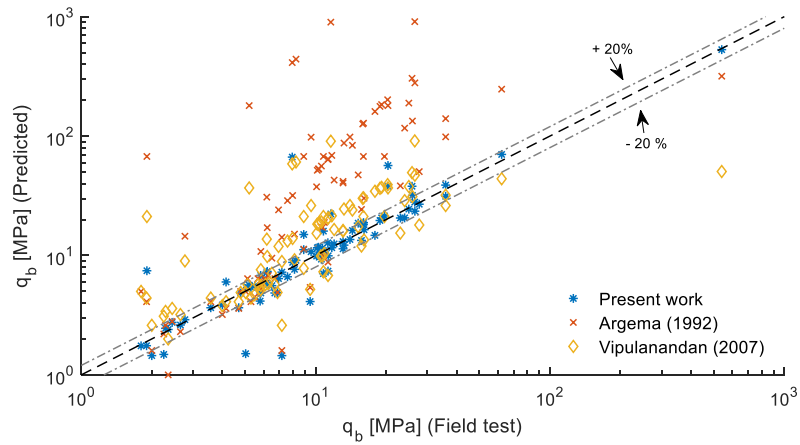


Figure V-33: Pile end bearing resistance: Field test vs. developed solution results.

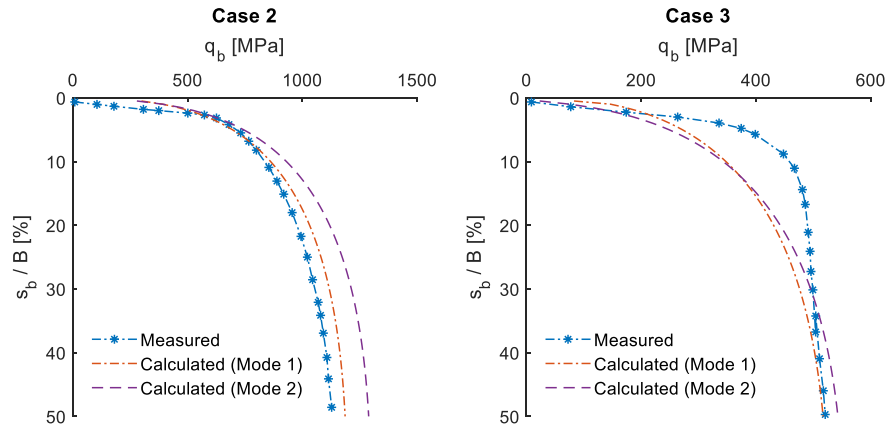


Figure V-34: Load – displacement curves: Predicted vs. Measured – Cases 2 & 3.

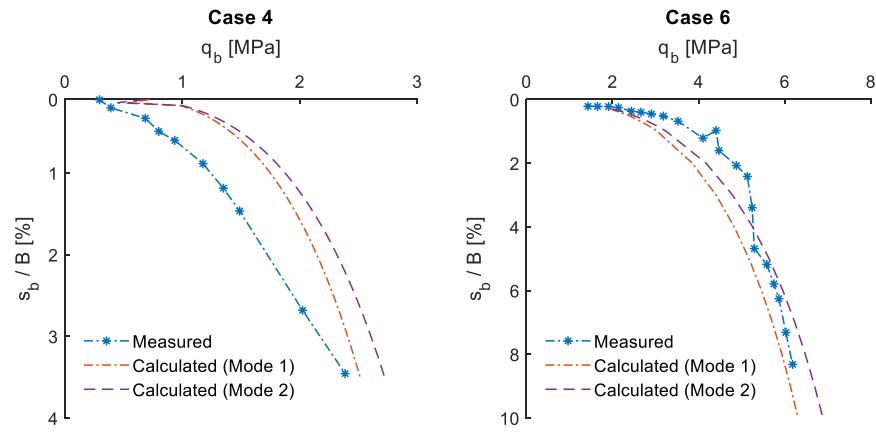


Figure V-35: Load – displacement curves: Predicted vs. Measured – Cases 4 & 6.

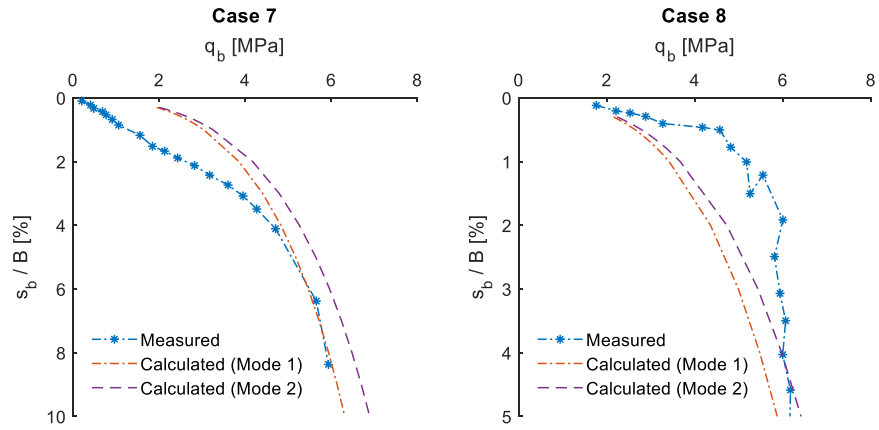


Figure V-36: Load – displacement curves: Predicted vs. Measured – Cases 7 & 8.

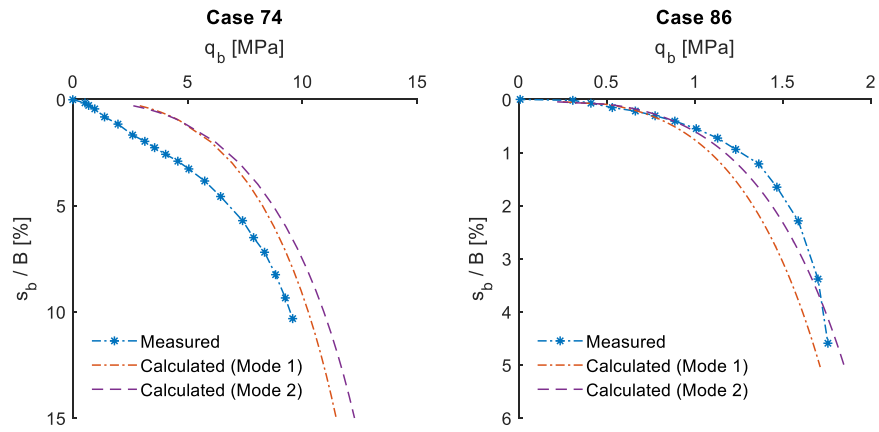


Figure V-37: Load – displacement curves: Predicted vs. Measured – Cases 74 & 86.

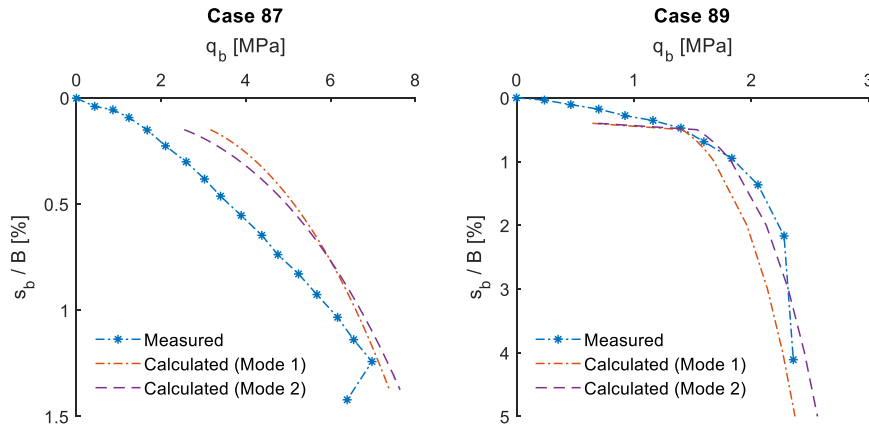


Figure V-38: Load – displacement curves: Predicted vs. Measured – Cases 87 & 89.

4.4. Conclusion

An analytical method for computing the ultimate end bearing capacity $q_{b,ult}$ of a pile embedded in a rock mass that obeys the H-B failure criterion was proposed. This method was developed based on the assumption that the pressure exerted at the boundaries of the bulb of rock immediately beneath the pile tip is equal to the limit pressure P_{lim} required to expand a spherical cavity. To compute the limit pressure, an analytical solution that requires a numerical integration over the plastic region is presented. Thereafter, $q_{b,ult}$ was evaluated as a function of P_{lim} by adopting two failure mechanisms around the pile tip. The limit pressure is computed by assuming a no dilatant material ($\psi = 0$). This assumption is on the conservative side since P_{lim} increases for an increasing ψ .

Results derived from the present work were compared to field load test results of 91 specific cases. It is found that the developed solution (cf. Eqs. (V-121) and (V-123)) agreed well with experimental results.

On the other hand, the existing empirical relations (cf. Eq. (II-76)) for estimating the ultimate end bearing capacity do not give reliable estimates. These empirical relations have limitations related to using only the uniaxial compressive strength of the intact rock σ_{ci} to predict the end bearing capacity, and this parameter is only one of many other parameters that affect the strength of the rock.

4.5. Applicability of the H-B criterion – Stress ratio condition

Using the cavity expansion theory, an analytical solution was developed to estimate the end bearing capacity of a pile embedded in a material obeying the nonlinear H-B failure criterion. To demonstrate the applicability/efficiency of the proposed solution, a database of conducted experimental case studies of piles embedded in rock layers (91 cases), originating from an extensive bibliographic study was employed. For these cases, the first necessary step consists of ascertaining the applicability of the H-B criterion according to the s -values condition. As shown in Figure V-39, only 13 case studies among 91 (14%) do not satisfy the s -values condition. Therefore, special attention is given to these cases when analyzing the results. Further details are provided below.

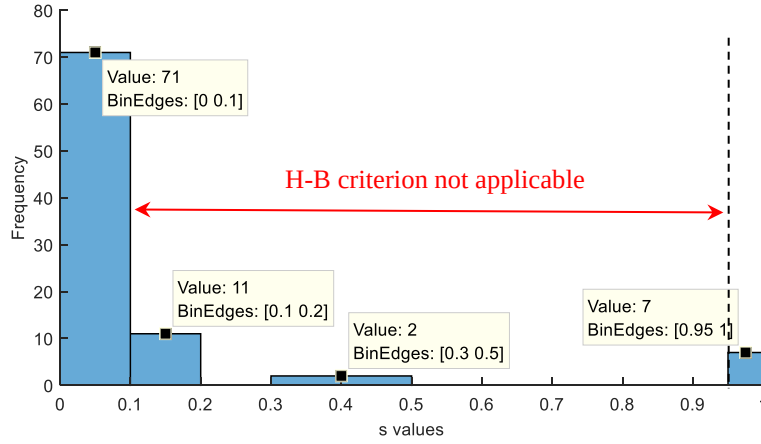


Figure V-39: Pile end bearing capacity – Applicability of the H-B criterion according to the s -values conditions (91 case studies)

On the other hand, the second step is to check whether the stress ratio condition has been respected. In light of this, the stress field generated below and around the pile tip should be examined. Since the proposed solution is based on the cavity expansion theory, it is then necessary to examine whether the pressure applied at the spherical cavity wall P_i respects the stress ratio condition. Details of how P_i is computed are previously given in subsection 4.2. To be in accordance with the nomenclatures already defined in the article and the manuscript, the stress ratio condition is simply re-expressed in a scaled form as:

$$P_i^* \geq (P_i^*)_{min} = (LSR)\sigma_\theta^* + \frac{s}{m_b^{\beta/a}} (1 - LSR) ; LSR = \{3.4, 4\} \quad (V-129)$$

Here $(P_i^*)_{min}$ is the minimal pressure beyond which the stress ratio condition is not satisfied. σ_θ^* is the circumferential stress.

If the stress ratio condition is not fulfilled i.e. $P_i^* < (P_i^*)_{min}$, one can assume the existence of a Non-Compliant Region (NCR) of radius r_{NCR} , within the plastic region of radius r_p , beyond which the stress ratio condition will be fulfilled again, as exemplified in Figure III-40. It will, therefore, be interesting to examine the extent of the NCR compared to the plastic region knowing that the ratio r_{NCR}/r_p is simply given by:

$$r_p/r_{NCR} = (R_{NCR}/R_p)^{\frac{\beta}{k}}; k = 2 \text{ for spherical cavity} \quad (V-130)$$

Here, R_p and R_{NCR} are found by solving the following equations,

$$\begin{cases} P_y^*(R_p) = \sigma_\theta^*(R_p) + \sigma_\theta^{*a}(R_p); \sigma_\theta^*(R) = [aW_0(R)]^{\beta/a} \\ (P_i^*)_{min}(R_{NCR}) = P_i^* \end{cases} \quad (V-131)$$

In the above equation, P_y^* denotes the scaled yield pressure.

In the following, one should first check for how many of the 91 case studies, the stress ratio condition is not satisfied (i.e. $P_i^* < (P_i^*)_{min}$). For these cases, the extent of the NCR is thereafter examined.

Figure III-41 plots P_i^* versus $(P_i^*)_{min}$ for the 91 case studies. Three groups of results are distinguished. The first group concerns the 71 cases with s-values less than 0.1, i.e., s-values condition fulfilled. As can be seen, 41/71 (58%) of the first group results fall above the line $P_i^* = (P_i^*)_{min}$ i.e., the stress ratio condition is satisfied. For the remaining results (30/71), the normalized NCR radius concerning the plastic region radius r_{NCR}/r_p , as a function of the far-field pressure P_0^* , is depicted in Figure III-42. It can be noted that the extent of the NCR increases exponentially as P_0^* increases. For $P_0^* < 10^{-2}$, r_{NCR} is about 10% r_p (average value of 5 cases). This ratio doubles for $10^{-2} < P_0^* < 10^{-1}$ i.e., $r_{NCR}/r_p = 20\%$ (average value of 18 cases). The remaining results (7/30) are characterized by $r_{NCR}/r_p = 40\%$ for $P_0^* > 10^{-1}$.

The second group of results concerns the 10 cases that do not satisfy the s-values condition and for which the NCR radius is almost 35% of the plastic radius.

The last group concerns the 4 cases with intact rock masses i.e., satisfies the s-values condition. The same observations as for the first group are noticed i.e., the extent of the NCR increases exponentially as P_0^* increases.

The above analyses lead to the following conclusions:

- 13 case studies among 91 that do not satisfy the s-values condition should be excluded from the analyses. Therefore, the predicted bearing capacity values for these cases, although they are close to the measured ones, are not reliable.

- 44 case studies among 91 satisfy both s-values and stress ratio conditions. Subsequently, the predicted bearing capacity values are reliable.
- For the remaining case studies that satisfy the s-values condition and do not fulfil the stress ratio condition, we can tolerate the 12 special cases for which the radius of the non-compliant region does not exceed 15% of the radius of the plastic region.

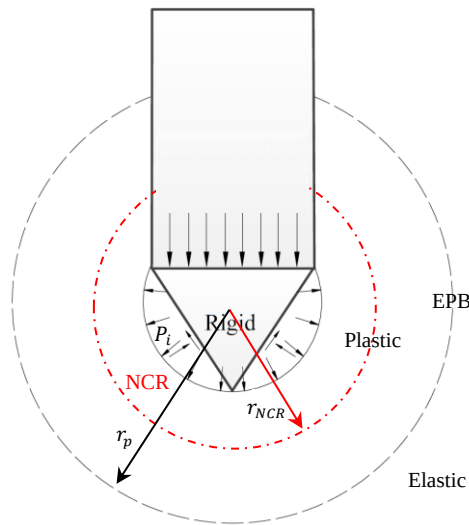


Figure V-40: Sketch of the failure mechanism around the pile tip – The extent of the Non-Compliant Region (NCR) within the plastic region is highlighted in a red dashed line.

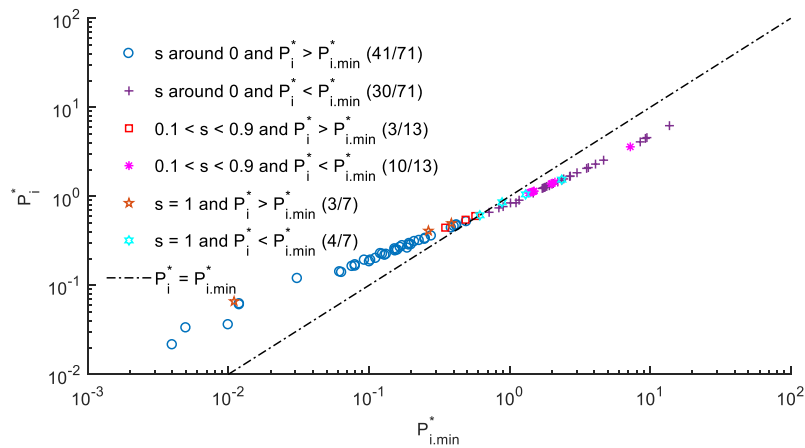


Figure V-41: Pile end bearing capacity problem: Pressure applied at the spherical cavity wall P_i^* vs. minimal pressure $P_{i,min}^*$ beyond which the stress ratio condition is not satisfied.

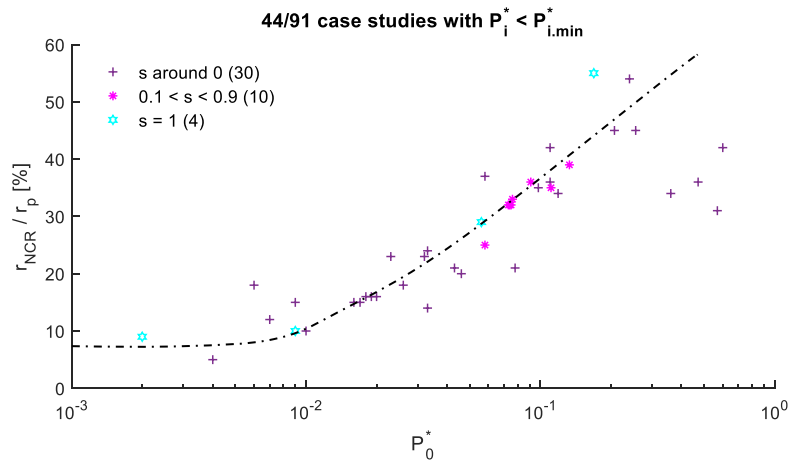


Figure V-42: The extent of the Non-Compliant region compared to the plastic region around the pile tip as a function of the far-field pressure.

Chapter VI. Conclusions and perspectives

1. Main conclusions

It is well known that most rock masses are excellent and trouble-free foundation materials. A significant economy can be therefore realized in design if the foundation can be constructed on or in the surface or near-surface rock. Nevertheless, the construction of challenging structures with usually high loads and special requirements on rock foundations, have made it necessary to determine the allowable bearing capacity of rock media accurately. It is within this framework that the present research was carried out. This dissertation provided analytical solutions for bearing capacity problems of both shallow and deep foundations resting on or in a rock mass that obeys the H-B failure criterion.

Considering this, a critical literature review of failure criteria for intact rock and rock masses was firstly conducted (Chapter II). It was found that the nonlinear H-B failure criterion more rigorously represents rock mass behaviour. This criterion is an empirically derived relationship developed through an extensive evaluation of laboratory test data covering a wide range of intact rock types. It describes the nonlinear increase in peak strength of isotropic rock with increasing confining stress. Besides, it provides a straightforward empirical means to estimate rock mass properties. However, one of the most important limitations of the H-B failure criterion is the independence of the intermediate principal stress σ_2 . It should be emphasized that a scaled form of the H-B failure criterion expression, which leads to a unique relationship independent of the H-B core parameters, was proposed by the author. Thanks to such scaling, the H-B criterion was described by a simple mathematical expression that makes it easy to understand. Furthermore, such scaling leads to a significant simplification in the analysis of stresses in rock mass when considering the problem of the bearing capacity of shallow and deep foundations.

On the other hand, an extensive literature survey on the bearing capacity of strip footing resting on rock mass was presented. Despite the reliability of existing scientific works, it was noted that further improvements could still be suggested. Using the plane strain upper bound limit analysis technique, and by assuming a multi-wedge translation mechanism, the effect of the non-associated flow rule on the bearing capacity factor was incorporated. Results gained from the developed solutions when compared to published results for the associated flow rule case are in good agreement which emphasizes the high efficiency of the proposed approach. Furthermore, and for practical purposes in rock engineering, tables allowing easy and accurate estimates of

the bearing capacity factor N_{σ_0} were provided. Effect of the flow rule on N_{σ_0} was carefully implemented using different dilatancy parameters. To demonstrate the applicability of the proposed method, results gained from the developed solutions were compared to those gained from published laboratory load tests carried out on three different rock materials (granite, limestone, and sandstone). Analysing the test data indicates that the ultimate bearing capacity of sedimentary rock can be predicted relatively satisfactorily by the theory of plasticity and by incorporating the mobilized dilation angle. To further investigate the efficiency of the theoretical solution, a series of static penetration tests of small-scale model footings (strip and circular) laying on a highly porous calcarenite rock mass was conducted. The experimental campaign was performed with the hope to get the classical Buisman-type failure mechanism (Buisman, 1940). i.e. a general shear failure. Nevertheless, and due to the high compressibility of the tested rock, a punching shear failure rather than a local or general shear one took place. As a result, a factor F_r reflecting by how much should the theoretical bearing factor N_σ reduced to get the measured value was introduced. It was found that F_r is comparable to the compressibility factor introduced in the literature to take into account the effect of soil compressibility on soil ultimate bearing capacity. It was therefore suggested to estimate F_r values by resorting to the published equation of the compressibility factor as a function of the rigidity index and the friction angle of the rock mass. On the other hand, one advantage of the H-B failure criterion is that it can be fitted to a wide range of strength envelope shapes. This feature allowed us to validate the developed solutions by comparing analytical results to the experimental ones conducted on nonlinear sand. On the other side, an approximate solution for estimating the bearing capacity of shallow foundations based on equivalent Mohr-Coulomb parameters was proposed. The method focused on providing specific guidance for selecting appropriate equivalent M-C parameters, which resulted in acceptable bearing capacity values and acceptable failure mechanism when compared to the original H-B case. Note that only the associated flow rule case was considered.

As for the bearing capacity problem of the pile foundation, an analytical calculation procedure was proposed to predict the shaft resistance for piles drilled to some depth into a rock layer. For practical purposes, design charts, as well as an approximate solution, were provided allowing easy and accurate estimates of the normalized average shaft resistance τ_{avg}^* . This analytical method was firstly validated using FEM. Thereafter, a comparison was made between results gained from the developed solutions and those measured during field load tests of 113 specific cases. It is essential to mention that field test results were divided into three categories based on a displacement index “DI”, which is an approximate measure of the pile displacement and degree of mobilization of side resistance for a given maximum side resistance value. A good agreement between predicted results and test results belonging to the displacement index ‘A’ (case of fully mobilized side resistance) was

noticed. However, the remaining test results ($DI = B$ and C) are on the conservative side compared to the predicted ones, which was expected since the unit shear resistance was not fully mobilized during the load test.

Furthermore, and always within the framework of deep foundation, the cavity expansion theory was used to predict the base resistance of the pile by correlating it to the spherical cavity limit pressure. To achieve that, complete rigorous and closed-form solutions for stress and displacement fields of both cylindrical and spherical cavities expanding in rock mass obeying the H-B criterion were developed. From a practical point of view, these solutions were applied thereafter to analyse driven and socketed piles in a rock layer. Pile end bearing capacity values predicted from the developed solutions when compared to field load test results of 91 specific cases were in excellent agreement within 20%.

2. Perspectives

The present research has allowed improving methods currently in use for assessing the bearing capacity of both shallow and deep foundations resting on rock mass. However, a margin for improvements is still open.

The complex behaviour of rock mass was only merely approached through the elastic perfectly plastic constitutive model. Improvements in rock mass behaviour can be made to enhance the model, in particular:

- The constitutive model assumes isotropy although rock mass, by nature and because of its depositional history, is anisotropic.
- The constitutive model is based on the H-B failure criterion that comes with a succinct expression in terms of the major and minor principal stresses (σ_1 and σ_3 , respectively) and a benefit of only requiring easily obtained material parameters. It would be interesting if the effect of the intermediate principal stress could be considered using the three-dimensional (3D) version of the H-B criterion.
- The effect of pore water pressure needs to be addressed.
- The H-B criterion expression needs a modification to include ductile shear failure. Such modification has already been proposed in the literature to model the nonlinear strength of frozen soil (Yang, Gao and Lai, 2013). The modified H-B criterion is formulated as follows:

$$\sigma_1 = A\sigma_3 + \sigma_{ci} \left(m_b \frac{\sigma_3}{\sigma_{ci}} + s \right)^a$$

In the special case $A = 1$, the above equation can be simplified as the original H-B criterion. Applying the modified H-B formula to the Indiana limestone (cf. Figure VI-1) results in the following H-B parameters: $\sigma_{ci} = 46.7 \text{ MPa}$ (45.2 MPa for the original H-B), $m_i = 7.2$ (4.2 for the original H-B), $s = 1$, $a = 0.5$ and $A = 0$ (1 for the original H-B) with $R^2 = 0.995$.

However, a non-negligible shortcoming is that the modified formula containing the “A” coefficient cannot be scaled as the original H-B criterion to give a simple relationship of the form $\sigma_1^* = \sigma_3^* + \sigma_3^{*a}$. As a result, the analytical developments lead to involved and cumbersome mathematical expressions without any closed-form solutions. It would be preferable to keep the same expression of the criterion so that the scaled form remains valid. Nevertheless, we propose to modify the values of the constants m_b , s and a or even the form of the constants equations. For

such a step, Hoek and Brown (2019) stated that practical engineers/designers intending to apply such modifications should ensure that they have done sufficient reading and research to enable them to define the range of applicability of these modifications and whether they apply to the problem under consideration. Applying the aforementioned modifications to the Indiana limestone (cf. Figure VI-1) results in the following H-B constants values : $\sigma_{ci} = 65.9 \text{ MPa}$, $m_i = 7.7$ (4.2 for the original H-B), $s = 0.0683$ (1 for the original H-B) and $a = 0.155$ (0.5 for the original H-B) with $R^2 = 0.994$. Note that the UCS value should be computed as $UCS = \sigma_{ci}s^a = 43.5 \text{ MPa}$ (45.2 MPa for the original H-B).

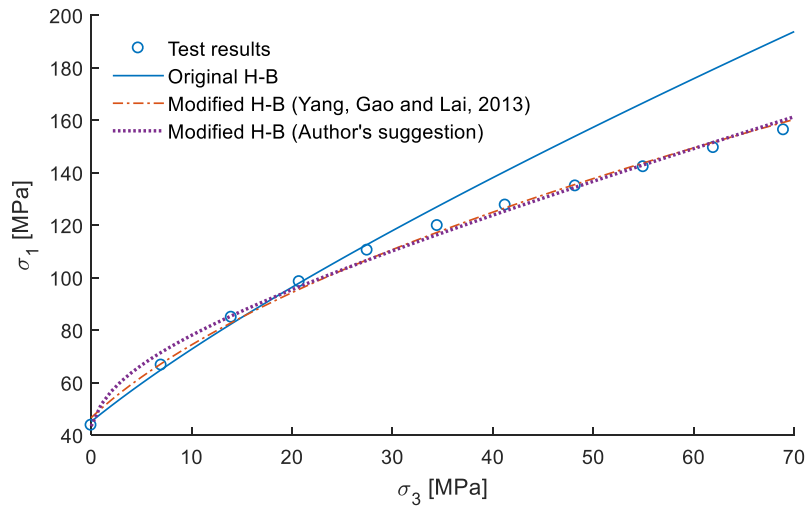


Figure VI-1: Triaxial test results of Indiana limestone – Original vs Modified H-B envelopes.

On the experimental front, future static penetration tests should focus on a well-characterized reference material .i.e. high reliable triaxial tests with high confining pressures should be performed to accurately estimate the H-B core parameters. The role of water which could reduce the compressive and tensile strength of the rock mass should be also investigated. Furthermore, bear in mind that the 1g tests on reduced-scale models suffer from severe limitations due to the scale effect associated foremost with the very low stresses generated in the rock layer beneath the model footing. It would be more interesting to employ a centrifuge machine that enhances the geotechnical models by producing identical self-weight stresses in the model and prototype, which makes it possible to obtain accurate data.

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Appendices

1. Lambert W- function

The Lambert W -function (https://fr.wikipedia.org/wiki/Fonction_W_de_Lambert), also called the Omega function or Product Logarithm, is a multivalued function, namely the branches of the inverse relation of the function:

$$f(w) = we^w \tag{1}$$

where w is any complex number and e^w is the exponential function.

For each integer $k \in \mathbb{Z}$, there is one branch, denoted by $W_k(z)$ which is a complex-valued function of one complex argument. These functions have the following property: if z and w are any complex numbers, then

$$we^w = z \leftrightarrow w = W_k(z); k \in \mathbb{Z}, z \in \mathbb{C}, w \in \mathbb{C} \tag{2}$$

When dealing with real numbers only, the two branches W_0 and W_{-1} suffice. In this case, the Lambert W -function is set as solutions of the equation:

$$x = ye^y, x \geq -1/e \tag{3}$$

and we get

$$y = \begin{cases} W_0(x), & x \geq 0 \\ W_{-1}(x), & -1/e \leq x < 0 \end{cases} \tag{4}$$

The two main branches W_0 and W_{-1} of the Lambert W -function which return real values are depicted in Figure 1.

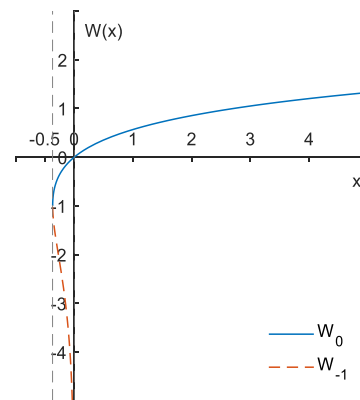


Figure 1: W_0 and W_{-1} branches of the Lambert W -function which return real values

Some special values of the principal branch are listed below:

- $W\left(-\frac{1}{e}\right) = -1$
- $W(0) = 0$
- $W(1) = \Omega = 0.56714329 \dots$ (the Omega constant)
- $W(e) = 1$
- $W(e^{1+e}) = e$

2. Strip footing problem

2.1. Energy dissipation and external work

The non-dimensional function for use in the bearing capacity solution (cf. Eqs. (III-15)) are expressed below:

$$f_1 = \frac{c_{0,1}}{\sigma_{ci}} \frac{\cos(\phi_{0,1}) \cos(\beta_1 - \phi_1 - \theta)}{\cos(\theta) \sin(\beta_1 - \phi_{0,1} - \phi_1)} \quad (5)$$

$$f_2 = \frac{\cos(\theta - \phi_{0,1})}{\cos(\theta) \sin(\beta_1 - \phi_{0,1} - \phi_1)} \times \sum_{i=2}^k \frac{c_{i,i+1}}{\sigma_{ci}} \cos(\phi_{i,i+1}) \frac{\sin(\alpha_{i-1} + \beta_{i-1} - \beta_i + \phi_i - \phi_{i-1})}{\sin(\alpha_{i-1} + \beta_{i-1} - \phi_{i-1} - \phi_{i-1,i})} \times \prod_{j=1}^{i-1} \frac{\sin(\beta_j)}{\sin(\alpha_j + \beta_j)} \times \prod_{j=1}^{i-2} \frac{\sin(\alpha_j + \beta_j - \phi_j - \phi_{j,j+1})}{\sin(\beta_{j+1} - \phi_{j+1} - \phi_{j,j+1})} \quad (6)$$

$$f_3 = \frac{\cos(\theta - \phi_{0,1})}{\cos(\theta) \sin(\beta_1 - \phi_{0,1} - \phi_1)} \times \sum_{i=1}^k \frac{c_i}{\sigma_{ci}} \cos(\phi_i) \frac{\sin(\alpha_i)}{\sin(\alpha_i + \beta_i)} \times \prod_{j=1}^{i-1} \frac{\sin(\beta_j)}{\sin(\alpha_j + \beta_j)} \frac{\sin(\alpha_j + \beta_j - \phi_j - \phi_{j,j+1})}{\sin(\beta_{j+1} - \phi_{j+1} - \phi_{j,j+1})} \quad (7)$$

$$f_4 = \frac{\cos(\theta - \phi_{0,1}) \sin(\beta_k - \theta - \sum_{i=1}^{k-1} \alpha_i - \phi_k)}{\cos(\theta) \sin(\beta_1 - \phi_{0,1} - \phi_1)} \frac{\sin(\beta_k)}{\sin(\alpha_k + \beta_k)} \times \prod_{i=1}^{k-1} \frac{\sin(\beta_i)}{\sin(\alpha_i + \beta_i)} \frac{\sin(\alpha_i + \beta_i - \phi_i - \phi_{i,i+1})}{\sin(\beta_{i+1} - \phi_{i+1} - \phi_{i,i+1})} \quad (8)$$

$$f_5 = \frac{\tan(\theta)}{2} + \frac{\cos(\theta - \phi_{0,1})}{2 \cos^2(\theta) \sin(\beta_1 - \phi_{0,1} - \phi_1)} \quad (9)$$

$$\times \sum_{i=1}^k \frac{\sin(\alpha_i) \sin(\beta_i)}{\sin(\alpha_j + \beta_j)} \sin\left(\beta_i - \theta - \sum_{j=1}^{i-1} \alpha_j - \phi_i\right) \times \prod_{j=1}^{i-1} \frac{\sin^2(\beta_j)}{\sin^2(\alpha_j + \beta_j)} \frac{\sin(\alpha_j + \beta_j - \phi_j - \phi_{j,j+1})}{\sin(\beta_{j+1} - \phi_{j+1} - \phi_{j,j+1})}$$

Note that the angular parameters should comply with the following constraints:

$$\left\{ \begin{array}{l} \theta, \alpha_i, \phi_i, \phi_{i,i+1} \in]0, \pi/2[; \beta_i \in]0, \pi[; \alpha_i + \beta_i \in]0, \pi[\\ \beta_1 - \phi_1 - \theta \in]0, \pi/2[; \alpha_i + \beta_i - \phi_i - \phi_{i,i+1} \in]0, \pi[; \alpha_i + \beta_i \geq \beta_{i+1} \\ \alpha_{i-1} + \beta_{i-1} - \beta_i + \phi_i - \phi_{i-1} \in]0, \pi[\end{array} \right. \quad (10)$$

2.2. Design tables (weightless rock mass without surcharge load)

- **Associated Flow Rule** ($\alpha = m_{b,dil}/m_b = 1$)

Table 1: Bearing capacity factor $N_{\sigma_{ci}}$ for weightless rock mass for $D = 0$ and AFR ($\alpha = 1$)

GSI	m_i									
	1	3	5	7	10	15	17	20	25	30
100	3.400	4.698	5.810	6.826	8.245	10.437	11.274	12.499	14.475	16.391
90	2.045	2.914	3.659	4.342	5.297	6.777	7.344	8.174	9.517	10.822
80	1.231	1.816	2.316	2.775	3.419	4.421	4.806	5.370	6.285	7.175
70	0.745	1.137	1.472	1.781	2.215	2.894	3.156	3.540	4.164	4.774
60	0.452	0.712	0.936	1.143	1.436	1.897	2.075	2.337	2.764	3.182
50	0.271	0.443	0.592	0.730	0.927	1.237	1.358	1.537	1.822	2.106
40	0.159	0.269	0.366	0.456	0.586	0.793	0.872	0.990	1.185	1.377
30	0.088	0.154	0.213	0.269	0.351	0.482	0.534	0.611	0.738	0.864
20	0.042	0.076	0.108	0.138	0.184	0.258	0.288	0.334	0.409	0.485
10	0.014	0.027	0.038	0.050	0.068	0.100	0.113	0.133	0.168	0.203

Table 2: Bearing capacity factor $N_{\sigma_{ci}}$ for weightless rock mass for $D = 0.1$ and AFR ($\alpha = 1$)

GSI	m_i									
	1	3	5	7	10	15	17	20	25	30
100	3.400	4.698	5.810	6.826	8.245	10.437	11.274	12.499	14.475	16.391
90	2.003	2.859	3.590	4.260	5.197	6.650	7.206	8.021	9.339	10.619
80	1.185	1.749	2.230	2.672	3.292	4.257	4.627	5.170	6.051	6.909
70	0.703	1.074	1.390	1.682	2.092	2.734	2.981	3.344	3.934	4.510
60	0.418	0.659	0.867	1.059	1.331	1.757	1.922	2.165	2.561	2.948
50	0.246	0.402	0.537	0.663	0.841	1.124	1.234	1.396	1.658	1.913
40	0.142	0.239	0.325	0.406	0.521	0.705	0.775	0.880	1.053	1.224

30	0.076	0.134	0.185	0.233	0.304	0.418	0.463	0.532	0.639	0.749
20	0.035	0.064	0.091	0.116	0.154	0.217	0.242	0.280	0.343	0.407
10	0.012	0.021	0.031	0.040	0.055	0.080	0.090	0.106	0.134	0.162

Table 3: Bearing capacity factor $N_{\sigma_{ci}}$ for weightless rock mass for $D = 0.2$ and AFR ($\alpha = 1$)

GSI	m_i									
	1	3	5	7	10	15	17	20	25	30
100	3.400	4.698	5.810	6.826	8.245	10.437	11.274	12.499	14.475	16.391
90	1.962	2.800	3.517	4.173	5.090	6.514	7.058	7.856	9.147	10.400
80	1.137	1.678	2.139	2.563	3.158	4.083	4.438	4.959	5.804	6.626
70	0.661	1.009	1.306	1.580	1.965	2.568	2.800	3.141	3.695	4.236
60	0.384	0.607	0.798	0.974	1.224	1.616	1.768	1.991	2.354	2.710
50	0.222	0.362	0.483	0.596	0.757	1.011	1.109	1.255	1.493	1.720
40	0.125	0.211	0.286	0.356	0.458	0.620	0.681	0.778	0.929	1.075
30	0.065	0.114	0.158	0.200	0.260	0.357	0.395	0.452	0.546	0.639
20	0.030	0.053	0.075	0.096	0.127	0.179	0.200	0.231	0.282	0.335
10	0.009	0.017	0.024	0.032	0.043	0.062	0.070	0.082	0.104	0.126

Table 4: Bearing capacity factor $N_{\sigma_{ci}}$ for weightless rock mass for $D = 0.3$ and AFR ($\alpha = 1$)

GSI	m_i									
	1	3	5	7	10	15	17	20	25	30
100	3.400	4.698	5.810	6.826	8.245	10.437	11.274	12.499	14.475	16.391
90	1.921	2.738	3.438	4.079	4.976	6.366	6.898	7.678	8.939	10.164
80	1.087	1.603	2.044	2.449	3.017	3.900	4.239	4.737	5.543	6.328
70	0.618	0.942	1.220	1.475	1.835	2.397	2.613	2.931	3.448	3.952
60	0.351	0.554	0.728	0.888	1.116	1.473	1.611	1.814	2.145	2.469
50	0.198	0.322	0.430	0.531	0.673	0.899	0.987	1.116	1.323	1.532

40	0.108	0.183	0.248	0.309	0.397	0.535	0.590	0.672	0.801	0.931
30	0.055	0.097	0.133	0.168	0.218	0.301	0.333	0.380	0.458	0.537
20	0.024	0.043	0.061	0.078	0.103	0.144	0.161	0.186	0.228	0.270
10	0.007	0.013	0.019	0.024	0.033	0.047	0.053	0.062	0.078	0.095

Table 5: Bearing capacity factor $N_{\sigma_{ci}}$ for weightless rock mass for $D = 0.6$ and AFR ($\alpha = 1$)

GSI	m_i									
	1	3	5	7	10	15	17	20	25	30
100	3.400	4.698	5.810	6.826	8.245	10.437	11.274	12.499	14.475	16.391
90	1.773	2.521	3.163	3.750	4.572	5.845	6.333	7.047	8.201	9.323
80	0.925	1.359	1.728	2.068	2.544	3.285	3.569	3.986	4.662	5.320
70	0.484	0.734	0.947	1.143	1.419	1.850	2.016	2.260	2.656	3.042
60	0.253	0.395	0.518	0.631	0.790	1.040	1.137	1.279	1.511	1.737
50	0.131	0.211	0.280	0.344	0.435	0.579	0.634	0.717	0.851	0.983
40	0.065	0.109	0.146	0.182	0.232	0.313	0.344	0.389	0.465	0.539
30	0.030	0.051	0.070	0.088	0.117	0.156	0.172	0.197	0.237	0.277
20	0.012	0.020	0.028	0.035	0.047	0.065	0.072	0.083	0.101	0.119
10	0.003	0.005	0.007	0.009	0.012	0.017	0.019	0.022	0.027	0.033

- **Non – Associated Flow Rule with $\alpha = m_{b,dil}/m_b = 0.25$**

Table 6: Bearing capacity factor $N_{\sigma_{ci}}$ for weightless rock mass for $D = 0$ and N-AFR with $\alpha = 0.25$

GSI	m_i									
	1	3	5	7	10	15	17	20	25	30
100	3.368	4.502	5.394	6.181	7.272	8.971	9.629	10.620	12.314	13.869
90	2.015	2.760	3.348	3.874	4.609	5.759	6.205	6.879	7.962	8.994
80	1.214	1.699	2.088	2.441	2.937	3.731	4.039	4.494	5.239	5.942
70	0.729	1.049	1.307	1.545	1.885	2.427	2.638	2.939	3.446	3.945
60	0.438	0.648	0.832	0.980	1.212	1.582	1.727	1.932	2.281	2.626
50	0.261	0.397	0.511	0.619	0.776	1.028	1.127	1.267	1.503	1.741
40	0.152	0.238	0.312	0.383	0.485	0.651	0.716	0.814	0.974	1.133
30	0.083	0.135	0.180	0.224	0.287	0.393	0.434	0.497	0.601	0.705
20	0.040	0.066	0.090	0.113	0.148	0.207	0.230	0.267	0.326	0.392
10	0.014	0.023	0.032	0.040	0.054	0.077	0.087	0.102	0.128	0.157

Table 7: Bearing capacity factor $N_{\sigma_{ci}}$ for weightless rock mass for $D = 0.1$ and N-AFR with $\alpha = 0.25$

GSI	m_i									
	1	3	5	7	10	15	17	20	25	30
100	3.368	4.501	5.394	6.181	7.272	8.971	9.619	10.631	12.314	13.869
90	1.981	2.708	3.284	3.801	4.522	5.651	6.099	6.749	7.813	8.946
80	1.165	1.636	2.010	2.350	2.830	3.592	3.888	4.314	5.025	5.724
70	0.688	0.994	1.235	1.459	1.780	2.292	2.487	2.777	3.256	3.727
60	0.406	0.600	0.802	0.908	1.122	1.466	1.593	1.790	2.113	2.431
50	0.237	0.360	0.464	0.562	0.707	0.935	1.017	1.149	1.365	1.579
40	0.136	0.212	0.278	0.340	0.431	0.578	0.637	0.723	0.865	1.007
30	0.072	0.117	0.156	0.194	0.250	0.340	0.376	0.431	0.521	0.610

20	0.033	0.055	0.076	0.095	0.124	0.173	0.193	0.224	0.274	0.325
10	0.011	0.019	0.026	0.032	0.043	0.062	0.069	0.081	0.101	0.122

Table 8: Bearing capacity factor $N_{\sigma_{ci}}$ for weightless rock mass for $D = 0.2$ and N-AFR with $\alpha = 0.25$

GSI	m_i									
	1	3	5	7	10	15	17	20	25	30
100	3.368	4.502	5.394	6.181	7.272	8.971	9.629	10.620	12.314	13.869
90	1.940	2.653	3.233	3.723	4.430	5.535	5.964	6.611	7.652	8.641
80	1.134	1.570	1.928	2.254	2.715	3.439	3.721	4.150	4.820	5.488
70	0.647	0.931	1.160	1.371	1.672	2.147	2.333	2.618	3.058	3.516
60	0.373	0.556	0.701	0.835	1.032	1.343	1.465	1.647	1.943	2.235
50	0.214	0.324	0.418	0.506	0.634	0.835	0.915	1.034	1.228	1.420
40	0.119	0.186	0.244	0.299	0.380	0.508	0.559	0.635	0.760	0.884
30	0.062	0.100	0.134	0.166	0.213	0.291	0.322	0.368	0.445	0.522
20	0.028	0.046	0.063	0.079	0.103	0.143	0.159	0.184	0.225	0.267
10	0.009	0.015	0.020	0.026	0.034	0.048	0.055	0.063	0.079	0.095

Table 9: Bearing capacity factor $N_{\sigma_{ci}}$ for weightless rock mass for $D = 0.3$ and N-AFR with $\alpha = 0.25$

GSI	m_i									
	1	3	5	7	10	15	17	20	25	30
100	3.368	4.501	5.394	6.181	7.272	8.971	9.619	10.631	12.314	13.869
90	1.894	2.594	3.146	3.639	4.330	5.410	5.839	6.461	7.456	8.529
80	1.069	1.500	1.843	2.154	2.594	3.291	3.555	3.964	4.603	5.525
70	0.604	0.870	1.084	1.333	1.561	2.009	2.177	2.443	2.856	3.265
60	0.341	0.504	0.637	0.762	0.941	1.229	1.336	1.510	1.770	2.296
50	0.191	0.289	0.372	0.451	0.564	0.748	0.814	0.919	1.092	1.262
40	0.104	0.162	0.212	0.259	0.329	0.441	0.485	0.550	0.658	0.765

30	0.053	0.084	0.113	0.139	0.179	0.244	0.270	0.309	0.373	0.438
20	0.023	0.037	0.051	0.064	0.083	0.115	0.129	0.148	0.182	0.216
10	0.007	0.012	0.015	0.020	0.026	0.037	0.041	0.048	0.060	0.072

Table 10: Bearing capacity factor $N_{\sigma_{ci}}$ for weightless rock mass for $D = 0.6$ and N-AFR with $\alpha = 0.25$

GSI	m_i									
	1	3	5	7	10	15	17	20	25	30
100	3.368	4.502	5.394	6.181	7.272	8.971	9.629	10.620	12.314	13.869
90	1.753	2.390	2.913	3.349	3.982	4.977	5.354	5.932	6.863	7.761
80	0.923	1.273	1.561	1.823	2.191	2.775	2.996	3.338	3.874	4.861
70	0.477	0.695	0.844	1.007	1.210	1.553	1.682	1.885	2.211	2.560
60	0.246	0.361	0.455	0.542	0.668	0.869	0.943	1.058	1.247	1.432
50	0.126	0.190	0.243	0.293	0.365	0.479	0.528	0.591	0.719	0.826
40	0.063	0.097	0.126	0.153	0.193	0.257	0.282	0.320	0.382	0.447
30	0.029	0.045	0.060	0.074	0.094	0.127	0.140	0.160	0.193	0.226
20	0.011	0.018	0.024	0.029	0.038	0.052	0.058	0.066	0.081	0.095
10	0.003	0.004	0.006	0.007	0.010	0.013	0.015	0.017	0.021	0.025

- **Non – Associated Flow Rule with $\alpha = m_{b,dil}/m_b = 0.1$**

Table 11: Bearing capacity factor $N_{\sigma_{ci}}$ for weightless rock mass for $D = 0$ and N-AFR with $\alpha = 0.1$

GSI	m_i									
	1	3	5	7	10	15	17	20	25	30
100	3.350	4.387	5.005	5.523	6.176	7.133	7.511	8.071	8.969	9.915
90	1.999	2.630	3.042	3.371	3.792	4.438	4.691	5.068	5.700	6.339
80	1.197	1.594	1.869	2.059	2.340	2.784	2.961	3.230	3.671	4.119
70	0.717	0.966	1.127	1.263	1.452	1.760	1.884	2.072	2.384	2.699
60	0.428	0.583	0.686	0.775	0.905	1.120	1.207	1.337	1.557	1.779
50	0.254	0.349	0.414	0.474	0.563	0.712	0.772	0.862	1.015	1.170
40	0.146	0.204	0.246	0.285	0.344	0.445	0.485	0.546	0.650	0.755
30	0.079	0.112	0.138	0.162	0.199	0.262	0.289	0.327	0.394	0.462
20	0.038	0.054	0.067	0.080	0.100	0.134	0.149	0.170	0.207	0.245
10	0.013	0.019	0.024	0.029	0.036	0.048	0.054	0.062	0.076	0.091

Table 12: Bearing capacity factor $N_{\sigma_{ci}}$ for weightless rock mass for $D = 0.1$ and N-AFR with $\alpha = 0.1$

GSI	m_i									
	1	3	5	7	10	15	17	20	25	30
100	3.350	4.387	5.005	5.523	6.176	7.133	7.511	8.071	8.969	9.915
90	1.963	2.580	2.984	3.305	3.720	4.355	4.603	4.973	5.593	6.217
80	1.152	1.534	1.782	1.982	2.253	2.680	2.850	3.110	3.535	3.965
70	0.677	0.912	1.064	1.193	1.371	1.663	1.782	1.956	2.254	2.550
60	0.397	0.540	0.648	0.718	0.838	1.038	1.132	1.240	1.443	1.648
50	0.231	0.317	0.376	0.431	0.511	0.646	0.702	0.783	0.922	1.062
40	0.130	0.181	0.218	0.253	0.306	0.395	0.431	0.486	0.578	0.671
30	0.069	0.098	0.120	0.141	0.173	0.228	0.250	0.284	0.342	0.400

20	0.032	0.046	0.057	0.068	0.084	0.113	0.124	0.143	0.173	0.205
10	0.011	0.016	0.020	0.023	0.029	0.039	0.043	0.049	0.061	0.072

Table 13: Bearing capacity factor $N_{\sigma_{ci}}$ for weightless rock mass for $D = 0.2$ and N-AFR with $\alpha = 0.1$

GSI	m_i									
	1	3	5	7	10	15	17	20	25	30
100	3.350	4.387	5.005	5.523	6.176	7.133	8.001	8.071	8.969	9.915
90	1.921	2.527	2.923	3.244	3.644	4.266	4.525	4.871	5.478	6.092
80	1.105	1.472	1.723	1.902	2.176	2.571	2.734	2.979	3.390	3.804
70	0.636	0.857	1.000	1.120	1.288	1.562	1.775	1.838	2.117	2.395
60	0.365	0.497	0.584	0.661	0.771	0.968	1.031	1.140	1.326	1.515
50	0.207	0.285	0.340	0.388	0.460	0.582	0.630	0.705	0.829	0.955
40	0.114	0.159	0.192	0.223	0.269	0.347	0.379	0.427	0.508	0.590
30	0.059	0.084	0.102	0.121	0.148	0.194	0.214	0.243	0.292	0.342
20	0.027	0.038	0.047	0.056	0.070	0.093	0.103	0.118	0.143	0.169
10	0.009	0.013	0.016	0.018	0.023	0.030	0.033	0.039	0.047	0.056

Table 14: Bearing capacity factor $N_{\sigma_{ci}}$ for weightless rock mass for $D = 0.3$ and N-AFR with $\alpha = 0.1$

GSI	m_i									
	1	3	5	7	10	15	17	20	25	30
100	3.350	4.387	5.005	5.523	6.176	7.133	7.511	8.071	8.969	9.915
90	1.893	2.472	2.858	3.166	3.563	4.170	4.407	4.762	5.354	5.952
80	1.056	1.408	1.634	1.818	2.066	2.457	2.612	2.851	3.239	3.633
70	0.595	0.801	0.934	1.047	1.204	1.459	1.561	1.716	1.976	2.235
60	0.334	0.468	0.534	0.603	0.704	0.870	0.938	1.039	1.209	1.381
50	0.185	0.255	0.302	0.346	0.410	0.518	0.561	0.627	0.738	0.849
40	0.100	0.139	0.167	0.193	0.233	0.301	0.328	0.370	0.440	0.510

30	0.050	0.071	0.087	0.102	0.125	0.164	0.180	0.204	0.245	0.287
20	0.022	0.031	0.039	0.046	0.057	0.076	0.083	0.095	0.115	0.136
10	0.007	0.010	0.012	0.014	0.018	0.023	0.026	0.029	0.036	0.043

Table 15: Bearing capacity factor $N_{\sigma_{ci}}$ for weightless rock mass for $D = 0.6$ and N-AFR with $\alpha = 0.1$

GSI	m_i									
	1	3	5	7	10	15	17	20	25	30
100	3.350	4.387	5.005	5.523	6.176	7.133	8.001	8.071	8.969	9.915
90	1.735	2.280	2.636	2.918	3.283	3.839	4.067	4.380	4.921	5.467
80	0.900	1.197	1.390	1.545	1.753	2.080	2.278	2.408	2.732	3.061
70	0.467	0.643	0.732	0.819	0.939	1.133	1.213	1.329	1.527	1.724
60	0.241	0.328	0.385	0.433	0.503	0.619	0.666	0.736	0.854	0.973
50	0.123	0.169	0.199	0.228	0.268	0.336	0.363	0.405	0.475	0.545
40	0.061	0.084	0.100	0.116	0.139	0.177	0.193	0.216	0.255	0.296
30	0.028	0.039	0.047	0.055	0.066	0.086	0.094	0.106	0.127	0.148
20	0.011	0.015	0.018	0.022	0.026	0.035	0.038	0.043	0.052	0.061
10	0.003	0.004	0.005	0.006	0.007	0.009	0.010	0.011	0.013	0.015

- **Non – Associated Flow Rule with $\alpha = m_{b,dil}/m_b = 0$**

Table 16: Bearing capacity factor $N_{\sigma_{ci}}$ for weightless rock mass for $D = 0$ and N-AFR with $\alpha = 0$

GSI	m_i									
	1	3	5	7	10	15	17	20	25	30
100	3.330	4.197	4.548	4.686	4.662	4.365	4.209	3.972	3.590	3.245
90	1.986	2.484	2.665	2.695	2.613	2.356	2.246	2.085	1.844	1.639
80	1.184	1.472	1.545	1.528	1.439	1.248	1.175	1.073	0.930	0.814
70	0.706	0.865	0.883	0.852	0.776	0.647	0.602	0.542	0.461	0.399
60	0.419	0.502	0.496	0.466	0.410	0.329	0.303	0.269	0.226	0.193
50	0.246	0.285	0.273	0.249	0.212	0.164	0.150	0.132	0.109	0.093
40	0.140	0.157	0.145	0.129	0.107	0.080	0.073	0.064	0.052	0.044
30	0.075	0.083	0.074	0.064	0.052	0.039	0.035	0.030	0.025	0.021
20	0.036	0.039	0.035	0.030	0.025	0.018	0.016	0.014	0.012	0.010
10	0.013	0.015	0.014	0.013	0.011	0.008	0.007	0.006	0.005	0.005

Table 17: Bearing capacity factor $N_{\sigma_{ci}}$ for weightless rock mass for $D = 0.1$ and N-AFR with $\alpha = 0$

GSI	m_i									
	1	3	5	7	10	15	17	20	25	30
100	3.330	4.197	4.548	4.686	4.662	4.365	4.209	3.972	3.590	3.245
90	1.946	2.437	2.614	2.644	2.563	2.311	2.203	2.045	1.809	1.608
80	1.139	1.417	1.487	1.471	1.385	1.201	1.130	1.033	0.894	0.783
70	0.666	0.816	0.834	0.805	0.733	0.611	0.568	0.511	0.435	0.376
60	0.388	0.465	0.459	0.432	0.380	0.305	0.280	0.249	0.209	0.179
50	0.223	0.259	0.248	0.226	0.192	0.149	0.136	0.119	0.099	0.084
40	0.125	0.140	0.129	0.115	0.095	0.072	0.065	0.057	0.046	0.039
30	0.065	0.072	0.064	0.056	0.046	0.034	0.030	0.026	0.022	0.018

20	0.030	0.033	0.030	0.026	0.021	0.016	0.014	0.012	0.010	0.008
10	0.010	0.012	0.012	0.010	0.009	0.007	0.006	0.005	0.005	0.004

Table 18: Bearing capacity factor $N_{\sigma_{ci}}$ for weightless rock mass for $D = 0.2$ and N-AFR with $\alpha = 0$

GSI	m_i									
	1	3	5	7	10	15	17	20	25	30
100	3.330	4.197	4.548	4.686	4.662	4.365	4.209	3.972	3.590	3.245
90	1.906	2.437	2.561	2.590	2.511	2.265	2.158	2.005	1.772	1.575
80	1.092	1.375	1.427	1.412	1.329	1.153	1.085	0.992	0.859	0.752
70	0.626	0.768	0.784	0.756	0.689	0.575	0.535	0.481	0.409	0.354
60	0.357	0.427	0.423	0.397	0.350	0.281	0.258	0.230	0.192	0.165
50	0.201	0.233	0.224	0.204	0.174	0.135	0.123	0.108	0.089	0.076
40	0.110	0.123	0.114	0.101	0.084	0.063	0.057	0.050	0.041	0.035
30	0.056	0.062	0.056	0.048	0.039	0.029	0.026	0.023	0.019	0.016
20	0.025	0.028	0.025	0.022	0.018	0.013	0.012	0.010	0.009	0.007
10	0.008	0.010	0.009	0.009	0.007	0.006	0.005	0.005	0.004	0.003

Table 19: Bearing capacity factor $N_{\sigma_{ci}}$ for weightless rock mass for $D = 0.3$ and N-AFR with $\alpha = 0$

GSI	m_i									
	1	3	5	7	10	15	17	20	25	30
100	3.330	4.197	4.548	4.686	4.662	4.365	4.209	3.972	3.590	3.245
90	1.867	2.335	2.505	2.534	2.457	2.216	2.113	1.962	1.735	1.542
80	1.060	1.300	1.365	1.351	1.273	1.104	1.039	0.950	0.823	0.721
70	0.586	0.718	0.734	0.708	0.646	0.539	0.501	0.452	0.384	0.332
60	0.326	0.392	0.403	0.365	0.321	0.258	0.237	0.211	0.177	0.152
50	0.180	0.209	0.200	0.183	0.156	0.121	0.110	0.097	0.080	0.068
40	0.096	0.108	0.100	0.089	0.074	0.056	0.051	0.044	0.036	0.031

30	0.048	0.053	0.048	0.042	0.034	0.025	0.023	0.020	0.016	0.014
20	0.021	0.023	0.021	0.018	0.015	0.011	0.010	0.009	0.007	0.006
10	0.007	0.008	0.008	0.007	0.006	0.005	0.004	0.004	0.003	0.003

Table 20: Bearing capacity factor $N_{\sigma_{ci}}$ for weightless rock mass for $D = 0.6$ and N-AFR with $\alpha = 0$

GSI	m_i									
	1	3	5	7	10	15	17	20	25	30
100	3.330	4.197	4.548	4.686	4.662	4.365	4.209	3.972	3.590	3.245
90	1.724	2.157	2.316	2.346	2.279	2.061	1.966	1.828	1.619	1.440
80	0.891	1.110	1.169	1.160	1.097	0.957	0.902	0.827	0.718	0.630
70	0.460	0.567	0.582	0.565	0.519	0.436	0.407	0.368	0.314	0.272
60	0.236	0.286	0.286	0.271	0.241	0.196	0.181	0.161	0.136	0.116
50	0.120	0.141	0.137	0.127	0.109	0.086	0.079	0.070	0.058	0.049
40	0.059	0.067	0.064	0.057	0.048	0.037	0.034	0.030	0.024	0.021
30	0.027	0.030	0.028	0.025	0.021	0.016	0.014	0.012	0.010	0.009
20	0.010	0.012	0.011	0.010	0.008	0.006	0.006	0.005	0.004	0.004
10	0.003	0.003	0.003	0.003	0.003	0.002	0.002	0.002	0.002	0.001

3. Experimental campaign

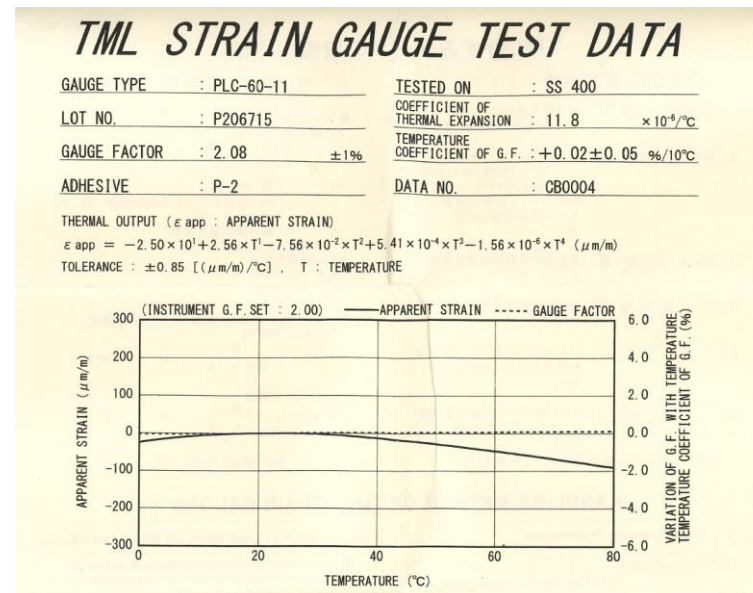


Figure 2: Manufacture's characteristics of strain gauges

4. Field test data extracted from the literature

3.1. Unit shaft resistance

Table 0-21: Database of measured unit shaft resistance of rock socketed piles – Estimated H-B parameters – Calculated ultimate unit shaft resistance.

Case	Rock type	H-B parameters			Characteristics of the soil and rock layers				Ultimate unit shaft resistance [MPa]		Field test	DI	Ref.
		$m_i^{(1)}$	$\sigma_c^{(2)}$ [MPa]	GSI	H_s [m]	γ_s [kN/m ³]	H_R [m]	$\gamma_R^{(3)}$ [kN/m ³]	Per Eq. (V-106)	per Eq. (V-120)			
1	Shale	8	12.20	75	2.40	17.5	3.35	26.0	2.14	2.10	0.24	C	(S. Thorburn, 1966)
2	Shale	8	0.48	25	7.60	17.5	6.00	26.0	0.37	0.41	0.31	C	(Matich and Kozicki, 1967)
3	Andesite	30	10.55	55	12.20	17.5	0.56	22.5	1.51	1.64	1.12	A	(Rosenberg and Journeaux, 1976)
4	Shale	8	21.10	85	16.70	17.5	0.91	26.0	4.21	4.10	1.72	C	(Rosenberg and Journeaux, 1976)
5	Chalk	9	1.00	60	19.30	17.5	8.00	25.0	0.71	0.70	0.19	B	(Buttling, 1976)
6	Mudstone	6	1.10	60	3.00	17.5	1.00	22.5	0.27	0.27	0.12	B	(Wilson, 1976)
7	Mudstone	6	1.10	60	3.00	17.5	1.00	22.5	0.27	0.27	0.18	B	(Wilson, 1976)
8	Shale	8	15.20	80	1.07	17.5	0.90	26.0	2.64	2.58	0.83	B	(Horvath and Kenney, 1979)
9	Shale	8	21.00	85	5.70	17.5	1.30	26.0	3.93	3.83	1.26	C	(Thorne, 1980)
10	Shale	8	5.40	65	0.57	17.5	1.40	26.0	0.83	0.84	1.11	A	(Horvath, Kenney and Kozicki, 1983)
11	Shale	8	11.10	75	0.57	17.5	1.40	26.0	1.84	1.82	1.11	A	(Horvath, Kenney and Kozicki, 1983)
12	Shale	8	5.60	65	0.57	17.5	1.40	26.0	0.87	0.87	2.00	A	(Horvath, Kenney and Kozicki, 1983)
13	Shale	8	5.50	65	0.57	17.5	1.40	26.0	0.85	0.85	1.75	B	(Horvath, Kenney and Kozicki, 1983)

Apx.4. Field test data extracted from the literature

14	Shale	8	10.40	75	0.57	17.5	1.40	26.0	1.71	1.69	1.09	B	(Horvath, Kenney and Kozicki, 1983)
15	Siltstone	9	7.00	75	7.20	17.5	6.80	26.5	1.41	1.38	0.60	C	(Radhakrishnan and Leung, 1989)
16	Siltstone	9	9.00	80	0.00	17.5	11.50	26.5	1.65	1.62	0.80	C	(Radhakrishnan and Leung, 1989)
17	Siltstone	9	9.00	80	0.00	17.5	7.30	26.5	1.56	1.54	0.70	C	(Radhakrishnan and Leung, 1989)
18	Limestone	15	2.30	35	8.25	17.5	2.45	24.5	0.64	0.71	0.39	B	(McVay, Townsend and Williams, 1992)
19	Limestone	15	6.71	55	10.00	17.5	1.00	24.5	1.14	1.20	1.20	A	(McVay, Townsend and Williams, 1992)
20	Limestone	15	4.41	45	11.00	17.5	1.20	24.5	0.94	1.00	0.71	A	(McVay, Townsend and Williams, 1992)
21	Limestone	15	1.72	30	6.10	17.5	1.10	24.5	0.46	0.53	0.48	A	(McVay, Townsend and Williams, 1992)
22	Limestone	15	3.55	45	7.60	17.5	3.10	24.5	0.76	0.83	0.69	A	(McVay, Townsend and Williams, 1992)
23	Limestone	15	4.55	45	5.80	17.5	3.70	24.5	0.81	0.88	0.82	A	(McVay, Townsend and Williams, 1992)
24	Granite	35	12.50	35	29.00	17.5	1.00	27.0	2.67	3.13	0.80	B	(CF, 1996)
25	Siltstone	9	6.00	70	2.40	17.5	10.00	26.5	1.17	1.16	0.56	C	(CF, 1996)
26	Siltstone	9	3.50	60	13.50	17.5	2.50	26.5	0.93	0.92	0.39	C	(CF, 1996)
27	Siltstone	9	6.50	70	13.00	17.5	3.00	26.5	1.40	1.38	0.62	C	(CF, 1996)
28	Marl	9	0.90	60	11.00	17.5	7.50	26.0	0.53	0.52	0.14	B	(Carrubba, 1997)
29	Breccia	24	15.00	40	16.50	17.5	2.50	30.0	2.19	2.51	0.49	B	(Carrubba, 1997)
30	Gypsum	10	6.00	90	26.00	17.5	11.00	22.5	1.69	1.64	0.12	C	(Carrubba, 1997)
31	Diabase	20	40.00	65	18.00	17.5	2.00	30.0	4.79	5.00	0.89	C	(Carrubba, 1997)
32	Limestone	15	2.50	35	11.00	17.5	2.50	24.5	0.76	0.84	0.40	B	(Carrubba, 1997)
33	Mudstone	7	3.20	80	21.00	17.5	1.00	26.0	1.04	1.01	0.60	B	(Walter, Burwash and Montgomery, 1997)
34	Siltstone	9	8.90	80	19.90	17.5	2.10	26.5	1.97	1.93	1.11	B	(Walter, Burwash and Montgomery, 1997)

Apx.4. Field test data extracted from the literature

35	Sandstone	21	11.60	60	19.80	17.5	2.20	22.5	2.06	2.15	2.16	B	(Walter, Burwash and Montgomery, 1997)
36	Limestone	15	43.60	85	4.12	17.5	5.00	24.5	6.07	6.08	2.34	C	(Excavation and Assembly, 1998)
37	Limestone	15	47.20	85	4.02	17.5	5.00	24.5	6.62	6.62	0.92	C	(Excavation and Assembly, 1998)
38	Limestone	15	47.20	85	3.77	17.5	5.00	24.5	6.61	6.61	2.28	C	(Excavation and Assembly, 1998)
39	Granite	35	50.00	60	5.63	17.5	1.50	22.5	3.92	4.36	2.02	C	(Long, 2018)
40	Limestone	15	25.00	75	5.50	17.5	2.80	24.5	3.32	3.37	0.75	C	(Long, 2018)
41	Limestone	15	54.00	90	5.50	17.5	2.80	24.5	7.34	7.32	1.50	C	(Long, 2018)
42	Limestone	15	29.00	80	5.50	17.5	2.30	24.5	3.88	3.92	1.00	C	(Long, 2018)
43	Limestone	15	54.00	90	5.50	17.5	2.30	24.5	7.32	7.31	1.88	C	(Long, 2018)
44	Limestone	15	51.00	90	6.00	17.5	3.30	24.5	7.16	7.15	1.92	C	(Long, 2018)
45	Limestone	15	50.00	90	7.00	17.5	2.00	24.5	7.09	7.08	1.46	C	(Long, 2018)
46	Limestone	15	50.00	90	11.00	17.5	3.00	24.5	7.25	7.24	0.55	C	(Long, 2018)
47	Siltstone	9	51.00	105	6.75	17.5	2.25	22.5	7.58	7.34	0.77	C	(Long, 2018)
48	Limestone	15	51.00	90	4.40	17.5	2.70	24.5	7.09	7.08	1.62	C	(Long, 2018)
49	Limestone	15	51.00	90	9.50	17.5	3.15	24.5	7.27	7.26	1.27	C	(Long, 2018)
50	Limestone	15	50.00	90	10.80	17.5	0.50	24.5	7.19	7.17	0.62	C	(Long, 2018)
51	Limestone	15	50.00	90	4.50	17.5	1.00	24.5	6.98	6.98	0.42	C	(Long, 2018)
52	Limestone	15	50.00	90	4.80	17.5	2.00	24.5	7.02	7.01	1.95	C	(Long, 2018)
53	Limestone	15	51.00	90	7.35	17.5	1.75	24.5	7.17	7.16	1.67	C	(Long, 2018)
54	Limestone	15	50.00	90	4.50	17.5	2.10	24.5	7.01	7.00	0.50	C	(Long, 2018)
55	Limestone	15	52.00	90	1.50	17.5	1.50	24.5	7.03	7.02	0.47	C	(Long, 2018)
56	Limestone	15	50.00	90	4.70	17.5	0.20	24.5	6.97	6.96	2.00	C	(Long, 2018)
57	Limestone	15	50.00	90	4.90	17.5	1.60	24.5	7.01	7.00	1.30	C	(Long, 2018)
58	Mudstone/limestone	9	16.00	70	6.50	17.5	3.00	22.5	2.57	2.57	0.91	C	(Long, 2018)
59	Mudstone/limestone	9	16.00	70	10.30	17.5	3.00	22.5	2.69	2.68	0.98	C	(Long, 2018)
60	Limestone/mudstone	10	25.00	75	4.00	17.5	3.00	24.5	3.86	3.84	1.00	C	(Long, 2018)

Apx.4. Field test data extracted from the literature

61	Limestone/mud stone	10	25.00	75	4.90	17.5	3.00	24.5	3.89	3.87	1.00	C	(Long, 2018)
62	Limestone/mud stone	10	40.00	85	8.20	17.5	0.50	24.5	6.58	6.48	3.00	C	(Long, 2018)
63	Limestone	15	50.00	90	12.00	17.5	0.50	24.5	7.23	7.21	1.50	C	(Long, 2018)
64	Tuff	18	30.00	75	33.60	17.5	2.00	22.5	4.68	4.77	2.63	C	(Zhan and Yin, 2000)
65	Fine ash tuff	18	50.00	90	27.20	17.5	2.00	25.0	7.30	7.32	2.90	C	(Ng et al., 2001)
66	Fine ash tuff	18	50.00	80	26.50	17.5	6.00	25.0	7.06	7.13	1.74	C	(Ng et al., 2001)
67	Coarse ash tuff	18	40.00	90	33.60	17.5	2.00	25.0	6.39	6.37	2.86	B	(Ng et al., 2001)
68	MetaSandstone	21	28.80	70	38.80	17.5	1.50	25.0	4.64	4.75	5.10	A	(Ng et al., 2001)
69	Granodiorite	32	6.00	45	23.00	17.5	2.10	27.0	1.76	1.91	0.48	B	(Ng et al., 2001)
70	Granite	35	50.00	70	37.40	17.5	1.20	27.0	6.07	6.46	1.70	C	(Ng et al., 2001)
71	Granite	35	10.00	40	56.60	17.5	3.60	27.0	3.86	4.21	0.61	C	(Ng et al., 2001)
72	Granite	35	50.00	70	40.80	17.5	0.90	27.0	6.25	6.64	2.87	C	(Ng et al., 2001)
73	Granite	35	28.80	60	45.80	17.5	2.50	27.0	4.86	5.19	0.96	B	(Ng et al., 2001)
74	Granite	35	50.00	75	22.70	17.5	1.50	27.0	5.36	5.69	1.00	B	(Ng et al., 2001)
75	Granite	35	38.00	65	20.90	17.5	3.00	27.0	4.21	4.54	1.21	C	(Ng et al., 2001)
76	Granite	35	7.00	35	37.20	17.5	0.80	27.0	2.57	2.91	0.70	C	(T.S.K. Lam, J.H.W. Yau, 1991)
77	Limestone	15	7.50	55	7.25	10.0	5.25	14.5	1.04	1.10	1.24	A	(Castelli and Fan, 2002)
78	Siltstone	9	20.00	90	8.53	10.0	2.41	15.0	3.70	3.60	0.98	C	(Gordon, Hawk and McConnell, 2004)
79	Sandstone	21	11.00	60	10.94	10.0	2.13	15.0	1.36	1.45	0.76	C	(Gordon, Hawk and McConnell, 2004)
80	Sandstone	21	17.00	70	11.98	10.0	2.35	15.0	2.03	2.12	1.00	C	(Gordon, Hawk and McConnell, 2004)
81	Sandstone	21	33.00	80	14.33	10.0	2.18	15.0	3.97	4.08	0.19	C	(Gordon, Hawk and McConnell, 2004)
82	Shale	8	0.40	25	10.00	10.0	8.20	16.0	0.29	0.33	0.11	B	
83	Shale	8	0.38	25	18.20	10.0	6.10	16.0	0.37	0.41	0.23	B	(Mackiewicz <i>et al</i> ., 1953)
84	Shale	8	0.45	25	24.30	10.0	6.10	16.0	0.45	0.50	0.30	A	(Mackiewicz <i>et al</i> ., 1953)

Apx.4. Field test data extracted from the literature

85	Claystone	6	2.10	50	14.00	17.5	6.10	24.5	0.77	0.76	0.46	A	(“Drilled shafts: Construction procedures and design methods,” 1990)
86	Claystone	6	2.10	50	14.00	17.5	6.10	24.5	0.77	0.76	0.33	B	(“Drilled shafts: Construction procedures and design methods,” 1990)
87	Siltstone	9	4.67	65	9.45	17.5	1.20	16.5	0.96	0.95	0.73	B	(“Drilled shafts: Construction procedures and design methods,” 1990)
88	Sandstone	21	3.36	40	10.65	17.5	11.00	16.5	1.01	1.11	0.73	B	(“Drilled shafts: Construction procedures and design methods,” 1990)
89	Cemented-Siltstone	9	30.20	100	10.60	17.5	7.00	16.5	5.46	5.29	0.41	C	(“Drilled shafts: Construction procedures and design methods,” 1990)
90	Hard siltstone	9	21.76	95	17.60	17.5	2.40	16.5	4.37	4.24	0.33	C	(“Drilled shafts: Construction procedures and design methods,” 1990)
91	Siltstone	9	4.20	65	42.00	10.0	6.10	16.5	1.33	1.31	0.42	C	(“Drilled shafts: Construction procedures and design methods,” 1990)
92	Sandstone	21	5.70	50	48.10	10.0	0.61	16.5	1.73	1.82	0.63	C	(“Drilled shafts: Construction procedures and design methods,” 1990)
93	Siltstone	9	6.16	70	48.71	10.0	4.80	16.5	1.73	1.69	0.92	C	(“Drilled shafts: Construction procedures and design methods,” 1990)
94	Limestone	15	66.45	90	4.50	10.0	1.20	14.5	8.07	8.05	0.69	B	(“Drilled shafts: Construction procedures and design methods,” 1990)
95	Sandstone	21	54.90	90	5.70	10.0	4.20	16.5	6.30	6.41	2.60	B	(“Drilled shafts: Construction procedures and design methods,” 1990)

Apx.4. Field test data extracted from the literature

96	Shale	8	12.33	75	5.20	10.0	3.96	16.0	2.16	2.13	1.92	B	("Drilled shafts: Construction procedures and design methods," 1990)
97	Limestone	15	0.15	10	7.00	17.5	6.10	14.5	0.60	0.62	0.57	A	("Drilled shafts: Construction procedures and design methods," 1990)
98	Shale	8	0.79	30	12.80	17.5	2.43	26.0	0.49	0.52	0.10	B	("Drilled shafts: Construction procedures and design methods," 1990)
99	Limestone	15	0.40	15	15.23	17.5	2.13	24.5	0.52	0.67	0.25	B	("Drilled shafts: Construction procedures and design methods," 1990)
100	Limestone	15	51.66	90	17.36	17.5	1.21	24.5	7.54	7.52	1.16	C	("Drilled shafts: Construction procedures and design methods," 1990)
101	Siltstone	9	0.35	25	18.57	17.5	3.00	26.5	0.51	0.55	0.73	A	("Drilled shafts: Construction procedures and design methods," 1990)
102	Shale	8	1.27	40	8.80	10.0	3.96	16.0	0.38	0.40	0.55	A	("Drilled shafts: Construction procedures and design methods," 1990)
103	Shale	8	1.13	35	6.00	17.5	3.35	26.0	0.40	0.42	0.11	B	("Drilled shafts: Construction procedures and design methods," 1990)
104	Shale	8	0.81	30	9.35	17.5	3.35	26.0	0.43	0.46	0.29	A	("Drilled shafts: Construction procedures and design methods," 1990)
105	Shale	8	7.92	70	6.70	10.0	5.48	16.0	1.40	1.39	0.16	B	("Drilled shafts: Construction procedures and design methods," 1990)
106	Sandstone	21	0.87	25	12.18	10.0	0.60	16.5	0.39	0.50	0.18	B	("Drilled shafts: Construction procedures and design methods," 1990)

Apx.4. Field test data extracted from the literature

107	Shale	8	0.61	30	3.00	17.5	7.62	26.0	0.32	0.35	0.40	A	(“Drilled shafts: Construction procedures and design methods,” 1990)
108	Sandstone	21	0.28	10	17.00	10.0	7.00	16.5	0.43	0.61	0.21	A	(“Drilled shafts: Construction procedures and design methods,” 1990)
109	Sandstone	21	5.20	50	24.00	17.5	5.48	16.5	2.35	2.41	2.59	A	(“Drilled shafts: Construction procedures and design methods,” 1990)
110	Phyllite	10	1.70	40	32.50	10.0	6.00	17.5	0.86	0.89	0.93	A	(Akgüner and Kirkit, 2012)
111	Schist	15	2.20	50	4.00	28.5	16.00	28.5	0.92	0.94	0.73	A	(Akgüner and Kirkit, 2012)
112	Greywacke	21	0.90	40	4.80	17.5	6.40	25.0	1.72	0.49	1.86	A	(Akgüner and Kirkit, 2012)
113	Claystone	6	1.10	40	11.50	10.0	8.50	14.5	1.54	1.58	1.29	A	(Akgüner and Kirkit, 2012)

(1) Estimated strength parameter of the intact rock by resorting to the Roclab free software program (<https://roclab.software.informer.com/1.0/>).

(2) Uniaxial compressive strength retrieved from the original paper.

(3) Estimated rock layer density according to the rock type ([Rocks.comparenature.com](https://rocks.comparenature.com/)).

3.2. End bearing resistance

Table 0-22: Database of measured end bearing resistance of rock socketed piles – Estimated H-B parameters – Calculated end bearing resistance.

Case	Rock type	H-B parameters ($D = 0$)				$MR^{(1)}$ (or $E_{meas}^{(2)}/E_{rm}^{(3)}$)	Soil and rock characteristics				$q_b^{(5)}$ [MPa]	$\frac{s_b^{(6)}}{B}$ (%)	Analytical solution [MPa]				References
		m_i	σ_{ci} [MPa]	RQD	GSI		H_s [m]	γ_s [kN/m ³]	H_R [m]	$\gamma_R^{(4)}$			$(q_b)_1$ at $\frac{s_b}{R}$ (%)	$(q_b)_2$ at $\frac{s_b}{R}$ (%)	$(q_b \cdot ult)_1$	$(q_b \cdot ult)_2$	
1	Granite	16	230.00	—	100	425	0.0	0.0	0.0	27.0	1432.4	5.0	1415.0	874.5	2967.8	2935.7	(Rehman and Brooms, 1971)
2	Limestone	10	124.00	—	100	700	0.0	0.0	0.0	15.0	1114.1	50.0	1186.0	1288.6	1196.6	1300.2	(Rehman and Brooms, 1971)
3	Sandstone	12	70.50	—	100	275	0.0	0.0	0.0	15.0	541.1	50.0	516.9	542.7	622.7	654.7	(Rehman and Brooms, 1971)
4	Diabase	15	0.50	—	50	87 ⁽²⁾	9.0	14.0	13.0	14.0	2.7	3.5	2.5	2.7	3.4	3.7	(Webb, 1976)
5	Mudstone	5	1.09	—	95	250	4.5	13.0	3.0	16.0	6.9	2.5	4.7	5.0	10.0	10.9	(Wilson, 1976)
6	Clay-Shale	4	1.42	—	85	200	6.1	17.5	1.1	22.5	5.7	5.0	5.2	5.6	8.5	9.3	(Aurora and Reese, 1978)
7	Clay-Shale	4	1.42	—	85	200	6.1	17.5	1.2	22.5	5.1	5.0	5.2	5.6	8.5	9.3	(Aurora and Reese, 1978)
8	Clay-Shale	4	1.42	—	90	200	5.7	17.5	1.6	22.5	6.1	5.0	5.9	6.4	8.6	9.3	(Aurora and Reese, 1978)
9	Clay-Shale	4	0.62	—	85	200	5.8	17.5	1.8	22.5	2.4	5.0	2.6	2.9	4.2	4.6	(Aurora and Reese, 1978)
10	Clay-Shale	4	0.81	—	85	200	5.5	13.0	3.3	22.5	4.7	6.2	3.5	3.8	5.3	5.8	(Goeke and Hustad, 1980)
11	Sandstone	15	9.26	88	55	275	0.0	0.0	16.9	15.0	13.1	1.0	12.4	10.4	40.0	43.0	(Glos and Briggs, 1983)
12	Sandstone	15	8.36	74	55	275	0.0	0.0	15.9	15.0	10.1	0.9	10.8	8.9	36.4	39.1	(Glos and Briggs, 1983)
13	Hardpan	20	1.38	—	35	(—) ⁽³⁾	1.5	17.5	0.0	22.0	5.8	0.7	5.3	5.7	11.8	12.7	(Baker, 1985)
14	Hardpan	20	0.57	—	30	(—) ⁽³⁾	1.2	17.5	0.5	22.0	2.3	0.5	2.4	2.5	5.9	6.3	(Baker, 1985)
15	Hardpan	20	1.11	—	35	(—) ⁽³⁾	0.4	7.5	0.5	10.0	4.8	2.6	4.7	5.0	8.8	9.5	(Baker, 1985)

Apx.4. Field test data extracted from the literature

16	Shale	5	3.82	—	65	200	11.0	15.0	2.0	15.5	10.7	15.3	11.1	12.0	13.5	14.6	(J.B. Hummert and T.L. Cooling, 1988)
17	Till	20	0.70	—	15	(—) ⁽³⁾	17.0	14.7	3.6	17.5	4.0	1.3	3.7	4.0	6.9	7.2	(Orpwood, Ahmad and Peaker, 1989)
18	Till	20	0.81	—	15	(—) ⁽³⁾	16.0	22.4	3.6	22.0	4.2	4.6	5.8	6.2	8.1	8.6	(Orpwood, Ahmad and Peaker, 1989)
19	Till	20	1.00	—	15	(—) ⁽³⁾	22.0	22.3	3.6	22.0	5.5	1.4	5.4	5.7	9.8	10.3	(Orpwood, Ahmad and Peaker, 1989)
20	Siltstone	9	9.00	55	50	375	5.8	7.5	1.5	16.5	13.1	4.9	12.8	13.4	23.0	24.9	(Radhakrishnan and Leung, 1989)
21	Gypsum	5	2.10	100	73	350	0.0	0.0	3.0	16.3	6.5	3.5	5.4	5.8	10.0	10.9	(Leung and Ko, 1993)
22	Gypsum	5	4.20	100	73	350	0.0	0.0	3.2	16.3	10.9	3.5	10.2	11.0	19.1	20.8	(Leung and Ko, 1993)
23	Gypsum	5	5.40	100	73	350	0.0	0.0	3.2	16.3	15.7	3.5	12.9	13.9	24.3	26.4	(Leung and Ko, 1993)
24	Gypsum	5	6.70	100	73	350	0.0	0.0	3.3	16.3	16.1	3.5	15.8	17.0	29.9	32.5	(Leung and Ko, 1993)
25	Gypsum	5	8.50	100	73	350	0.0	0.0	3.3	16.3	23.0	3.5	19.9	21.3	37.6	40.9	(Leung and Ko, 1993)
26	Gypsum	5	11.20	100	73	350	0.0	0.0	3.3	16.3	27.7	3.5	25.9	27.8	49.1	53.4	(Leung and Ko, 1993)
27	Shale	6	2.21	79	85	200	10.0	7.5	10.0	16.0	10.8	1.9	6.9	7.2	16.1	17.5	(William Robert Thompson, 1994)
28	Marl	5	0.90	100	60	200 ⁽²⁾	11.0	17.5	7.5	25.0	5.3	∞	5.2	5.6	5.2	5.6	(Carrubba, 1997)
29	Diabase - Breccia	20	15.00	10	10	200 ⁽²⁾	16.5	17.5	2.5	25.0	8.9	∞	14.6	15.4	14.6	15.4	(Carrubba, 1997)
30	Limestone	8	2.50	100	50	500 ⁽²⁾	11.0	17.5	2.5	25.0	8.9	∞	10.7	11.6	10.7	11.6	(Carrubba, 1997)
31	Claytill-Sandstone	15	0.90	—	70	275	4.7	7.5	4.3	15.0	1.9	2.1	1.9	1.6	4.8	5.1	(Rabeller, Bedenis and Thelen, 2000)

Apx.4. Field test data extracted from the literature

32	Limestone	10	1.21	100	60	700	8.0	7.5	19.5	14.5	5.8	5.1	6.5	7.1	10.1	11.0	(Miller and Loehr, 2003)
33	Claystone	4	1.96	90	85	358 ⁽²⁾	1.9	17.5	5.9	24.5	11.3	5.0	7.1	7.7	11.7	12.7	(Abu-Hejleh <i>et al.</i> , 2005)
34	Claystone	4	10.50	90	55	2387 ⁽²⁾	5.2	17.5	7.2	24.5	15.2	5.0	17.9	19.4	28.3	30.7	(Abu-Hejleh <i>et al.</i> , 2005)
35	Limestone	10	1.50	39	55	700	10.0	7.5	15.0	14.5	6.3	3.2	5.5	6.0	10.0	10.8	(Bullock, 2003)
36	Limestone	10	3.80	35	40	700	10.0	7.5	17.0	24.5	6.2	2.6	7.1	7.6	13.7	14.8	(Bullock, 2003)
37	Limestone	10	0.92	12	50	700	10.0	7.5	17.0	14.5	3.6	3.5	3.5	3.8	5.9	6.4	(Bullock, 2003)
38	Claystone	4	1.21	95.00	85	250	3.0	7.5	3.0	14.5	9.5	3.4	3.9	4.3	7.4	8.0	(Abu-hejleh and Attwooll, 2005)
39	Claystone	4	0.48	—	90	250	3.0	7.5	3.0	14.5	2.3	1.2	1.4	1.5	3.7	4.0	(Abu-hejleh and Attwooll, 2005)
40	Claystone	4	1.10	43	60	250	3.0	7.5	5.5	14.5	5.0	1.2	1.4	1.6	3.5	3.8	(Abu-hejleh and Attwooll, 2005)
41	Basalt	25	14.14	51	35	350	10.2	7.5	2.5	20.0	11.3	3.5	14.1	11.5	32.1	33.2	(Basarkar and Dewaikar, 2006)
42	Basalt	25	19.43	10	28	350	11.1	7.5	3.1	20.0	13.2	3.5	13.5	10.9	31.5	32.3	(Basarkar and Dewaikar, 2006)
43	Basalt	25	11.77	8	35	350	11.9	7.5	3.0	20.0	10.3	3.5	12.7	10.8	28.4	29.5	(Basarkar and Dewaikar, 2006)
44	Basalt	25	12.46	0	30	350	9.5	7.5	2.4	20.0	10.6	3.5	10.2	8.4	23.3	24.1	(Basarkar and Dewaikar, 2006)
45	Basalt	25	7.07	30	38	350	11.2	7.5	2.6	20.0	8.0	3.5	9.3	8.2	20.3	21.3	(Basarkar and Dewaikar, 2006)
46	Tuff	13	11.49	37	41	300	5.3	7.5	7.9	14.0	10.2	3.5	11.7	11.5	24.6	26.3	(Basarkar and Dewaikar, 2006)
47	Tuff	13	28.50	30	38	300	7.0	7.5	4.4	14.0	16.0	3.5	19.7	17.9	43.5	46.0	(Basarkar and Dewaikar, 2006)
48	Breccia	13	6.40	20	33	500	14.7	7.5	4.5	14.0	7.6	3.5	6.5	6.7	13.0	14.0	(Basarkar and Dewaikar, 2006)

Apx.4. Field test data extracted from the literature

49	Basalt	25	39.40	37	30	350	11.7	7.5	0.4	20.0	18.8	3.5	23.8	15.5	58.5	58.4	(Basarkar and Dewaikar, 2006)
50	Basalt	25	28.04	10	35	350	8.3	7.5	1.0	20.0	15.9	3.5	22.0	14.7	52.8	53.2	(Basarkar and Dewaikar, 2006)
51	Basalt	25	35.70	37	25	350	9.3	7.5	1.1	20.0	17.9	3.5	17.5	12.0	43.4	43.4	(Basarkar and Dewaikar, 2006)
52	Basalt	25	21.83	27	30	350	8.8	7.5	2.3	20.0	14.0	3.5	15.5	11.7	36.7	37.3	(Basarkar and Dewaikar, 2006)
53	Breccia	13	5.36	72	40	500	9.1	7.5	0.1	14.0	7.0	3.5	6.4	6.5	13.0	14.0	(Basarkar and Dewaikar, 2006)
54	Basalt	25	40.80	42	25	350	6.0	7.5	8.0	20.0	19.1	3.5	23.3	18.0	55.7	56.6	(Basarkar and Dewaikar, 2006)
55	Basalt	25	15.30	43	30	350	10.0	7.5	4.0	20.0	11.7	3.5	12.7	10.5	29.0	29.9	(Basarkar and Dewaikar, 2006)
56	Basalt	25	11.80	0	30	350	4.0	7.5	7.2	20.0	10.3	3.5	10.7	9.3	23.9	24.9	(Basarkar and Dewaikar, 2006)
57	Basalt	25	14.24	39	30	350	8.0	7.5	2.4	20.0	11.3	3.5	11.1	8.8	25.6	26.3	(Basarkar and Dewaikar, 2006)
58	Granite	32	98.00	95	10	425	42.2	17.5	0.9	27.0	8.3	0.1	12.0	6.2	96.9	98.9	(CEDD, 2011)
59	Granite	32	40.00	91	50	425	46.8	17.5	2.5	27.0	20.4	1.1	70.0	43.4	229.1	240.9	(CEDD, 2011)
60	Granite	32	91.70	96	50	425	37.5	17.5	1.1	27.0	8.0	0.2	113.3	20.1	438.9	452.4	(CEDD, 2011)
61	Granite	32	6.85	29	20	425	56.8	17.5	3.5	27.0	6.2	1.5	7.3	7.1	19.5	20.7	(CEDD, 2011)
62	Granite	32	200.00	—	10	425	23.2	17.5	1.5	27.0	11.6	0.7	28.4	16.2	154.2	153.8	(CEDD, 2011)
63	Granite	32	40.00	—	10	425	21.5	17.5	3.0	27.0	5.2	0.0	6.9	4.4	42.9	44.1	(CEDD, 2011)
64	Tuff	13	202.00	60	35	300	26.0	7.5	2.1	4.5	26.5	0.6	45.5	10.5	230.3	238.6	(CEDD, 2011)
65	Granodiorite	29	15.00	49	30	425	21.1	7.5	2.0	17.0	10.8	4.8	17.0	15.0	34.0	35.2	(CEDD, 2011)
66	Granodiorite	29	15.00	49	30	425	21.1	7.5	2.0	17.0	16.0	6.5	18.9	17.4	34.0	35.2	(CEDD, 2011)
67	Siltstone	9	62.00	50	35	375	38.4	7.5	1.5	16.5	26.5	1.1	24.8	22.2	79.2	85.0	(CEDD, 2011)
68	Siltstone	9	25.90	88	80	375	38.7	7.5	0.7	16.5	24.0	0.1	27.2	14.1	194.2	210.3	(CEDD, 2011)

Apx.4. Field test data extracted from the literature

69	Granite	32	15.00	49	30	425	47.4	7.5	2.0	17.0	1.9	0.6	9.9	5.0	40.7	42.4	(CEDD, 2011)
70	Granite	32	15.00	49	50	425	47.4	7.5	2.0	17.0	19.7	1.0	26.4	16.5	88.3	93.1	(CEDD, 2011)
71	Marble	9	42.00	83	45	850	39.9	17.5	0.7	24.5	25.0	0.3	25.8	23.1	126.3	136.9	(CEDD, 2011)
72	Marble	9	29.70	84	45	850	69.3	7.5	0.6	14.5	25.9	1.1	31.4	32.3	89.9	97.5	(CEDD, 2011)
73	Sandstone	17	3.36	—	60	275	10.7	10.0	5.7	16.5	8.1	1.7	7.9	7.4	20.8	22.4	(“Drilled shafts: Construction procedures and design methods,” 1990)
74	Siltstone	7	21.76	—	20	375	17.6	11.0	2.4	16.5	9.6	10.4	10.4	11.1	14.4	15.4	(“Drilled shafts: Construction procedures and design methods,” 1990)
75	Limestone	10	2.41	25	50	700	9.9	17.5	4.0	14.5	5.8	1.3	4.7	5.0	11.7	12.7	(“Drilled shafts: Construction procedures and design methods,” 1990)
76	Gneiss	28	3.87	12	50	525	21.0	7.5	2.9	17.0	10.5	2.3	10.9	10.4	26.0	27.9	(“Drilled shafts: Construction procedures and design methods,” 1990)
77	Sandstone	17	54.90	—	45	275	5.7	10.0	3.0	16.5	62.2	9.4	72.7	67.7	115.9	119.5	(“Drilled shafts: Construction procedures and design methods,” 1990)
78	Limestone	10	21.88	100	75	700	14.0	22.5	0.0	24.5	35.9	0.3	35.0	27.6	181.5	197.0	(“Drilled shafts: Construction procedures and design methods,” 1990)
79	Shale	6	18.60	92	50	200	17.2	10.0	0.0	16.0	14.4	3.1	15.9	16.5	33.1	35.8	(“Drilled shafts: Construction procedures and design methods,” 1990)

Apx.4. Field test data extracted from the literature

80	Clay-Shale	4	0.35	—	95	200	16.2	7.5	1.3	12.5	7.2	1.4	1.4	1.5	3.3	3.6	("Drilled shafts: Construction procedures and design methods," 1990)
81	Shale	6	1.42	100	85	200	8.8	10.0	3.7	16.0	6.7	2.4	4.7	5.0	10.3	11.2	("Drilled shafts: Construction procedures and design methods," 1990)
82	Sandstone	17	3.23	61	30	275	17.0	17.5	1.0	26.5	2.8	1.6	2.9	2.9	7.5	8.0	("Drilled shafts: Construction procedures and design methods," 1990)
83	Shale	6	0.36	—	95	200	13.5	12.0	0.2	16.0	2.0	0.9	1.4	1.5	4.2	4.5	("Drilled shafts: Construction procedures and design methods," 1990)
84	Shale	6	1.17	—	95	200	3.2	17.5	8.7	26.0	5.8	0.9	4.1	4.2	12.1	13.2	("Drilled shafts: Construction procedures and design methods," 1990)
85	Sandstone	17	9.52	36	40	275	25.0	12.0	4.6	16.5	12.0	3.3	12.6	12.4	26.5	28.4	("Drilled shafts: Construction procedures and design methods," 1990)
86	Chalk	7	1.12	—	30	1000	2.7	7.5	9.0	15.0	1.8	4.6	1.7	1.8	2.6	2.8	("Drilled shafts: Construction procedures and design methods," 1990)
87	Shale	6	3.15	—	80	200	12.4	17.5	3.7	7.2	7.2	1.3	7.1	7.4	19.4	21.1	("Drilled shafts: Construction procedures and

Apx.4. Field test data extracted from the literature

																	design methods," 1990)
88	Limestone	10	31.12	100	60	700	13.5	17.5	7.5	24.5	35.9	0.5	40.3	37.4	158.6	172.1	("Drilled shafts: Construction procedures and design methods," 1990)
89	Marl	5	0.22	—	80	175	17.4	17.5	18.4	25.0	2.4	4.2	2.3	2.5	3.3	3.6	("Drilled shafts: Construction procedures and design methods," 1990)
90	Phyllite- Shist	7	67.70	56	35	550	4.8	10.0	3.2	14.5	25.8	3.9	37.5	38.5	73.7	79.4	(Zhang, 2010)
91	Limestone	10	44.80	55	35	700	5.3	17.5	4.8	24.5	20.3	3.9	37.3	38.5	72.9	78.6	(Zhang, 2010)

(1) : $\mathbf{MR}$: Estimated modulus ratio. Then Young's modulus is computed as : $\mathbf{E_{rm}} = \mathbf{MR} \cdot \sigma_{ci} \left(0.02 + \frac{1-D/2}{1+\exp\left(\frac{60+15D-6SI}{11}\right)} \right)$ [MPa] (Hoek and Diederichs, 2006)

(2) : $\mathbf{E_{meas}}$ [MPa]: Measured Young's modulus retrieved from the original paper.

(3) : $\mathbf{E_{rm}}$ [MPa]: Estimated Young's modulus when $\mathbf{MR}$ cannot be determined: $\mathbf{E_{rm}} = 10^5 \left(0.02 + \frac{1-D/2}{1+\exp\left(\frac{75+25D-6SI}{11}\right)} \right)$ [MPa]. (Hoek and Diederichs, 2006).

(4) : $\mathbf{\gamma_R}$ [kN/m³]: Estimated rock layer density according to the rock type (Rocks.comparenature.com, 2015)

(5) : $\mathbf{q_b}$ [MPa]: Measured end bearing resistance retrieved from the original paper.

(6) : $\frac{s}{R}$ [%]: Maximum settlement measured during the test.