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Professor Buchanan's article on the Theory of Clubs [1] provided a most interesting extension of the theory of public goods. So interesting in fact that the problem seemed worth reexamination. After recalling Buchanan's solution, this note will discuss under what conditions it is valid, and what are the solutions, if any, for other cases. It concludes with a discussion of the significance of the theory of clubs for the more general case of public goods.

Buchanan talks of a swimming pool as the facility to be provided by the club. We shall use the example of a golf course, as well as others, drawn from a wider range of semi-public goods with different caracteristics. When it comes to algebra, we shall, rather inconsistently, assume continuity.

I

It is easiest to start by reproducing Buchanan's first three diagrams. Fig. I has two pairs of curves. One pair shows total costs and benefits to the individual member of a golf club for a nine-hole facility, when club membership is varied. All members are assumed at this stage to share equally the total costs and to have identical preference scales. The other pair of curves relates to an 18-hole facility. Optimum club membership is reached for each type when the slopes of the benefit and of the

1) "An Economic Theory of Clubs", Economica, February 1965

cost curves are parallel. This ensures that the consumer surplus of our typical member is maximized. By adding other pairs of curves for all possible facility sizes, one can find the locus of optimum membership for given size : this Buchanan calls the N_{opt} line.

Figure I

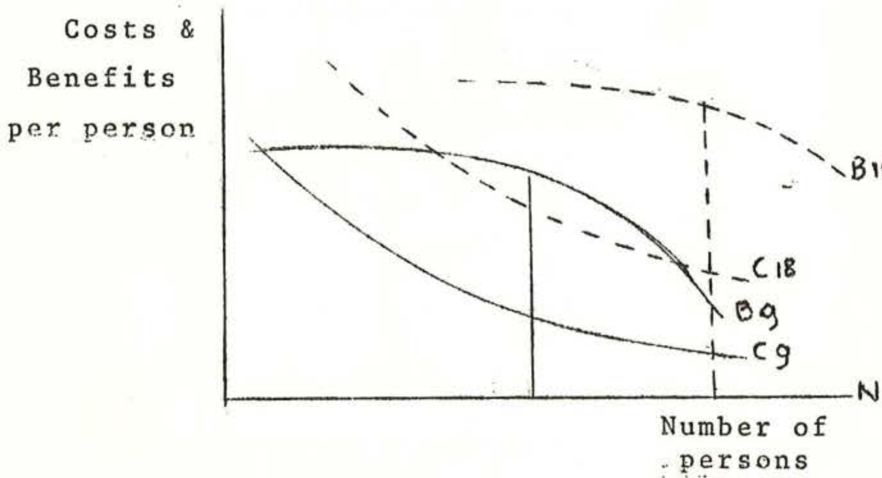


Figure II

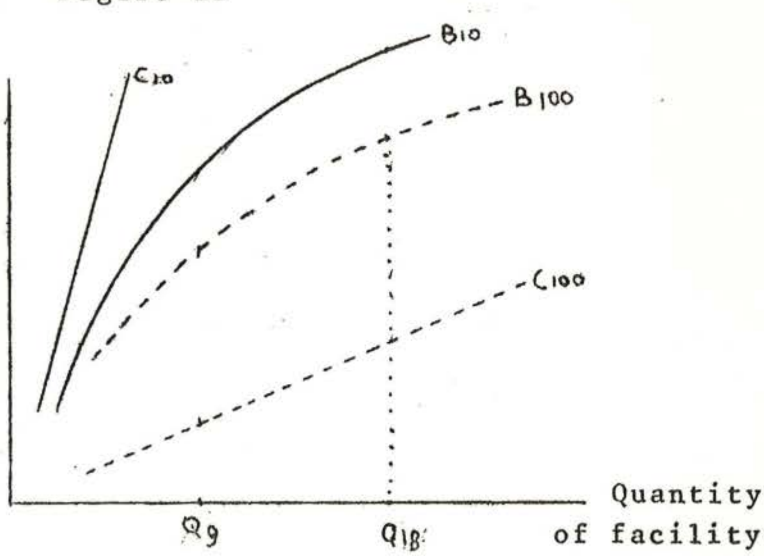
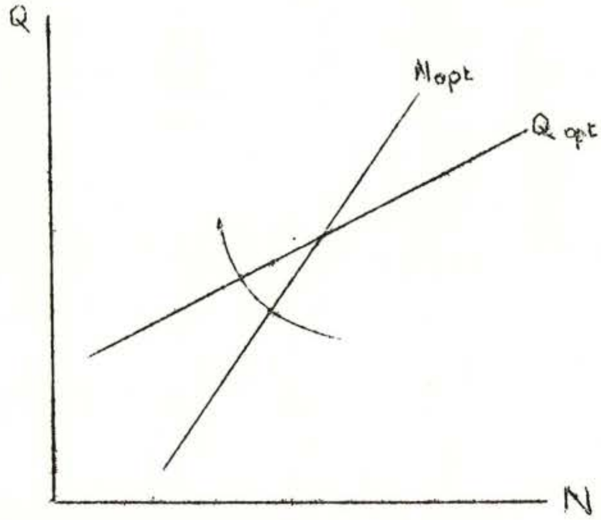


Figure II approaches the same problem from a different angle. Here membership of the club is fixed, cost and benefit curves are drawn for a variation of facility size, and the conditions for optimum are derived in the same way as above. As Fig. II is drawn, with ten members only, even the smallest size golf course is not worth its cost.

Figure III



With Figure III we come to the heart of the matter. Here it is best to quote in full : "The results derived from Fig. I and II are combined in Figure III. Here the two variables to be chosen, goods quantity and club size (Note : they are called here facility size and club membership) are measured on the ordinate and the absissa respectively. The values for optimal size for each goods quantity, derived from Fig. I, allow us to plot the curve N_{opt} in Fig. III. Similarily, the values for optimal goods quantity for each club size, derived from Fig. II, allow us to plot the curve Q_{opt}."..." Fig. III may be interpreted as a standard preference map depicting the tastes of the individual for the two components, goods quantity and club size for the sharing of that good. The curves N_{opt} and Q_{opt} are lines of optima and G is the highest attainable utility level, for the individual, the top of his ordinal utility mountain."

Here there is a bit of a puzzle : why should there be two distinct lines ? And if there are, what of their respective positions ? Will they intersect at a maximum, as Buchanan assumes, or at a minimum, or not at all ? The next part will show that all these solutions do indeed occur, under appropriate assumptions for the benefit and cost curves. But first, a point of clarification will prove useful.

Buchanan places both costs and benefits on the ordinate and compares them to derive maximum consumer surplus. If benefits are measured in units of utility, money costs, to be made comparable, must be multiplied by the marginal utility of money. Costs then express the utility of the marginal unit of consumption foregone in order to pay club dues. The position of the cost curve thus reflects the level of income as well as the amount of club dues. Should income increase, and the marginal utility of money decline, the cost curve will shift downward and to the left in relation to the benefit curve. In his algebra, Buchanan handles this in terms of marginal rates of substitution, but the variation of consumer surplus and the summit of the utility mountain are inevitably cardinal concepts. The exposition will be simpler if we allow ourselves to use them.

II

The theory of clubs can be applied to all public or semi-public goods that are subject to the phenomenon of "crowding" or "congestion". It cannot apply to the "pure" type, such as national defence, TV broadcasts or street lighting, since here the cost of accomodating an additional consumer or user is identically zero.

Excluding then these pure cases, the benefit derived by a single consumer will always depend on the relation between the size of the facility provided and the number of users. For instance a four-lane highway will provide the same conditions of speed and safety with, say, twice as much traffic as a two-lane highway. The ratio $\frac{Q}{N}$ will always appear as an argument in the benefit function : it gives a measure of the "relative size" of facilities.

Beyond this, a classification of possible types of benefit functions is needed :

1.- The simplest case occurs when there is no interaction in consumption, apart from potential crowding ; and there is no additional benefit from larger facilities, once relative size is adequate (saturation immediately occurs). Thus, as long as the drains don't overflow, there is always water on the tap, or power on the switch, you don't care about the sewerage or water supply system or power grid, nor do you care about the number of people who share these facilities with you.

The benefit function will then be homogeneous with respect to relative size and can be written :

$$B = B(\frac{Q}{N})$$

There will however be a discontinuity in the curve :

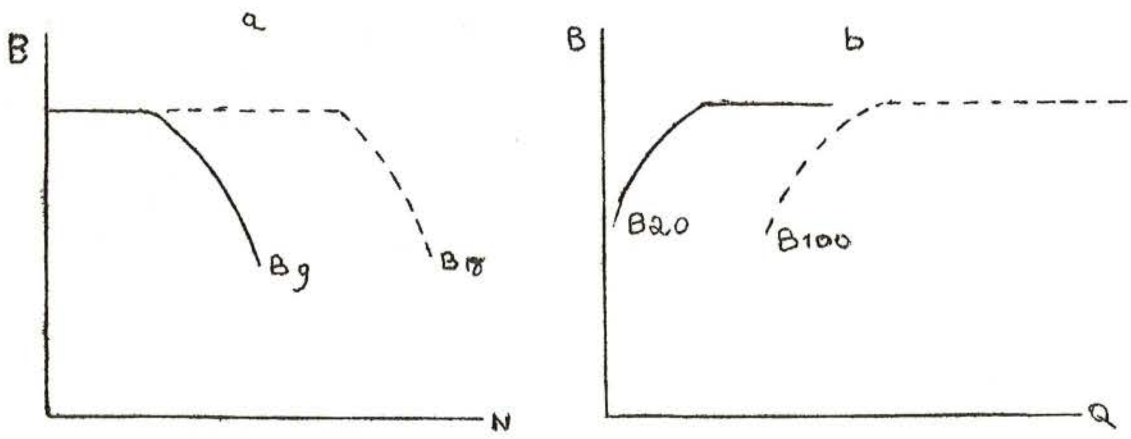
$$\frac{\partial B}{\partial(\frac{Q}{N})} = 0 \qquad \text{for } \frac{Q}{N} > (\frac{Q}{N})^*$$

$$\frac{\partial B}{\partial(\frac{Q}{B})} > 0 \qquad \text{for } \frac{Q}{N} < (\frac{Q}{N})^*$$

where $(\frac{Q}{N})^*$ denotes the point of incipient congestion (where an overflow or load shedding will occur exceptionnally).

Figures IV illustrate these benefit functions. It will be noticed that the discontinuity will raise no problem. Because of the slope of the cost functions, the optimal points we are concerned with, will always occur to the right of the point of discontinuity in Fig. IVa and to its left in Fig. IVb.

Figures IV



2.- A second type of benefit function would arise from services where increased facility size has positive value, but membership matters only in terms of relative size. This could be the case of Buchanan's swimming pool : the bigger the pool, the better, but you are indifferent to the number of people in or around it as long as they don't get in your way. Alternatively, it may be more pleasant to have some girls around admiring your high diving tricks. In both cases, the benefit function can be written :

$$B = B(\frac{Q}{N}, Q)$$

In the first example, the behaviour of the function with respect to $\frac{Q}{N}$ would be the same as in the homogeneous case ; in the second example, the terms in $\frac{Q}{N}$ could be quadratic, exhibiting a maximum for the relative membership optimal for each facility size. In both cases, with $\frac{Q}{N}$ held constant, (same relative size or degree of crowding), the partial derivative with respect to Q would be positive, though in most cases one would expect the second order partial to be negative (decreasing returns to absolute size, relative size constant).

The second example could thus be represented for instance by a function quadratic in $\frac{Q}{N}$ and logarithmic in Q. This would correspond pretty well to Fig. I, except that the summit of the benefit curve should move to the right more or less proportionately to the increase in facility size.

3.- The third type of benefit function would reverse the second : facilities matter only in relative terms but absolute membership is significant. The telephone service provides an example, where benefits increase with the number of subscribers, as long as the system does not get overloaded. Here the benefit curve can be written :

$$B = B(\frac{Q}{N}, N)$$

In the telephone case, with $\frac{Q}{N}$ constant, the partial derivative with respect to N would be positive, in other cases it could be negative.

4.- Finally there is the case where absolute facility size as well as absolute membership affect the benefit curve, in addition to relative size. For instance, doubling the size of a school or university as well as its population, may permit a better library or more specialized instructors (or a better football team), but may also leave the student feeling lost in a crowd. This case will also cover the most typical "club" situation. Even without crowding, too large an absolute membership will entail declining influence in decision making for the typical member. Besides, the club will lose what may be its most attractive characteristic, its exclusiveness (Buchanan's paper rules out this type of consideration as non economic).

To preserve symetry, this benefit function will be written :

$$B = B(Q, \frac{Q}{N}, N)$$

under the assumption that with $\frac{Q}{N}$ constant, $\frac{\partial B}{\partial Q}$ will be positive but declining, and $\frac{\partial B}{\partial N}$ negative in the relevant range.

Turning now to the cost side of the story, the assumptions underlying the cost function are as follows :

First, cost per member depend only on membership and facility size. Thus club dues or taxation are the only charges, and direct user cost, based on the use made of the facilities, are not considered. The analysis is restricted to the cost of subscibing to the telephone service and the benefit of being able to call up any other subscriber, and leaves out the cost and benefit of any individual call. An extension of the analysis in that direction would be useful, but is not attempted here.

Second, costs are shared equally between members. This is an allowable simplification as long as we are dealing with "typical" members, enjoying the same taxable income. This assumption implies that the cost curves of Fig. I will be rectangular hyperbolae (1).

Third, total costs are a function of Q only, and costs per member will thus depend on $\frac{Q}{N}$. It is only in the case, however,

(1) This is strictly accurate only if the cost curves are expressed in money terms, but not on our scale which measures subjective costs taking into account the marginal utility of money, which itself rises as costs increase. However, we shall be concerned only with points approximately on the same $\frac{Q}{N}$ line and these will lie in the neighbourhood of the same budget line, except where returns to scale are strongly increasing or decreasing.

of constant returns to scale in the total cost function that individual cost functions will be straight lines, as in Fig. II, or that these cost functions will be homogeneous in $\frac{Q}{N}$. For the more general case, the cost function can be written indifferently as :

$$C = C(Q,\frac{Q}{N}) \text{ or as : } C = C(\frac{Q}{N},N)$$

since with relative size constant, absolute size can be measured in terms of either Q or N.

Of course an assumption of constant returns is a little odd when dealing with public goods characterized by indivisibilities. Beyond a certain scale, however, the assumption may yield a fair approximation. The minimum size golf course may be 9 holes, but the 18-Hole course may cost about twice as much to lay down. If unit costs continue decreasing with scale, the cost curves in Fig. II must bend down.

Combining now the benefit and cost curves, we may start with the simplest case where both functions are homogeneous in $\frac{Q}{N}$ (independance and saturation on the consumption side, constant returns on the cost side). The surplus function can then be written :

$$S = B(\frac{Q}{N}) - C(\frac{Q}{N}) = S(\frac{Q}{N})$$

In this case, Buchanan's Q_{opt} and N_{opt} lines are one and the same : any point lying on one of them will also lie on the other. The Q_{opt} line will be the locus of points such that $\frac{\partial S}{\partial Q} = 0$ for different preassigned values of N. In the same way, the N_{opt} line is defined by : $\frac{\partial S}{\partial N} = 0$ for different values of Q. Now :

$$\frac{\partial S}{\partial Q} = \frac{\partial S}{\partial (\frac{Q}{N})} \frac{1}{N} = 0 \quad \text{and :} \quad \frac{\partial S}{\partial N} = \frac{\partial S}{\partial (\frac{Q}{N})} (- \frac{Q}{N^2}) = 0$$

and both expressions reduce to :

$$\frac{\partial S}{\partial (\frac{Q}{N})} = 0$$

Further, since the surplus curve is a function of relative size, only, the level of surplus will be constant all along the single $Q_{opt} - N_{opt}$ line. If continuity is assumed, there are an infinite number of equilibrium points. Here is a first, though admittedly not very interesting, case where the Buchanan solution breaks down.

Consider now the second, or third, type of benefit function in combination with the general cost function :

$$S = B(Q,\frac{Q}{N}) - C(Q,\frac{Q}{N}) = S(Q,\frac{Q}{N})$$

$$\text{or :} \quad S = B(N,\frac{Q}{N}) - C(N,\frac{Q}{N}) = S(N,\frac{Q}{N})$$

Although they are formally identical, the two cases will be treated separately because of different economic implications.

The general method will consist of studying the behaviour of these functions for $\frac{Q}{N}$ optimal. A little care is however needed. Since in this case there are distinct Q_{opt} and N_{opt} lines, there will be two optimal values for the $\frac{Q}{N}$ ratio. The hill of consumer surplus will have two ridges. It makes however no difference which ridge we chose to climb. We are only interested in establishing whether there is a summit, and it is clear that, if there is, both ridges must lead to it or merge before getting there. Formally, at the optimum :

$$\left[\frac{\partial S}{\partial(\frac{Q}{N})}\right]_{dN=0} = \left[\frac{\partial S}{\partial(\frac{Q}{N})}\right]_{dQ=0}$$

In the first case above, we separate the variables of the S-function by taking the partial derivative with respect to $\frac{Q}{N}$ for $dQ = 0$ (that is, we proceed along the N_{opt} line) and then the partial derivative with respect to Q.

Along the N_{opt} line, we have :

$$\left[\frac{\partial S}{\partial(\frac{Q}{N})}\right]_{dQ=0} = 0, \text{ written for short : } dX = 0$$

Along that line :

$$\left[\frac{\partial S}{\partial Q}\right]_{dX=0} = \left[\frac{\partial B}{\partial Q} - \frac{\partial C}{\partial Q}\right]_{dX=0}$$

Now for $dX = 0$, $\frac{\partial C}{\partial Q}$ will be approximately 0 for constant returns and negative for increasing returns (it will be exactly zero for constant returns only if N_{opt} is a straight line, implying that the degree of crowding remains the same as facilities and membership are increased proportionately). Then if $\frac{\partial B}{\partial Q}$ is positive or zero, it is clear that this expression cannot have a zero value (except when both terms on the right are zero : this is the previous homogeneous case, where all points satisfying $dX = 0$ are equally good).

To see intuitevely that there is in general no optimum, assume that with constant returns on the cost side, we double simultaneously both Q and N. Cost per member will be unchanged as well as that part of the benefits which depend on $\frac{Q}{N}$ (optimal degree of crowding). However, benefits also depend on the absolute value of Q, and therefore benefits will have risen for costs unchanged. The resulting surplus function will increase indefinitely, with absolute size.

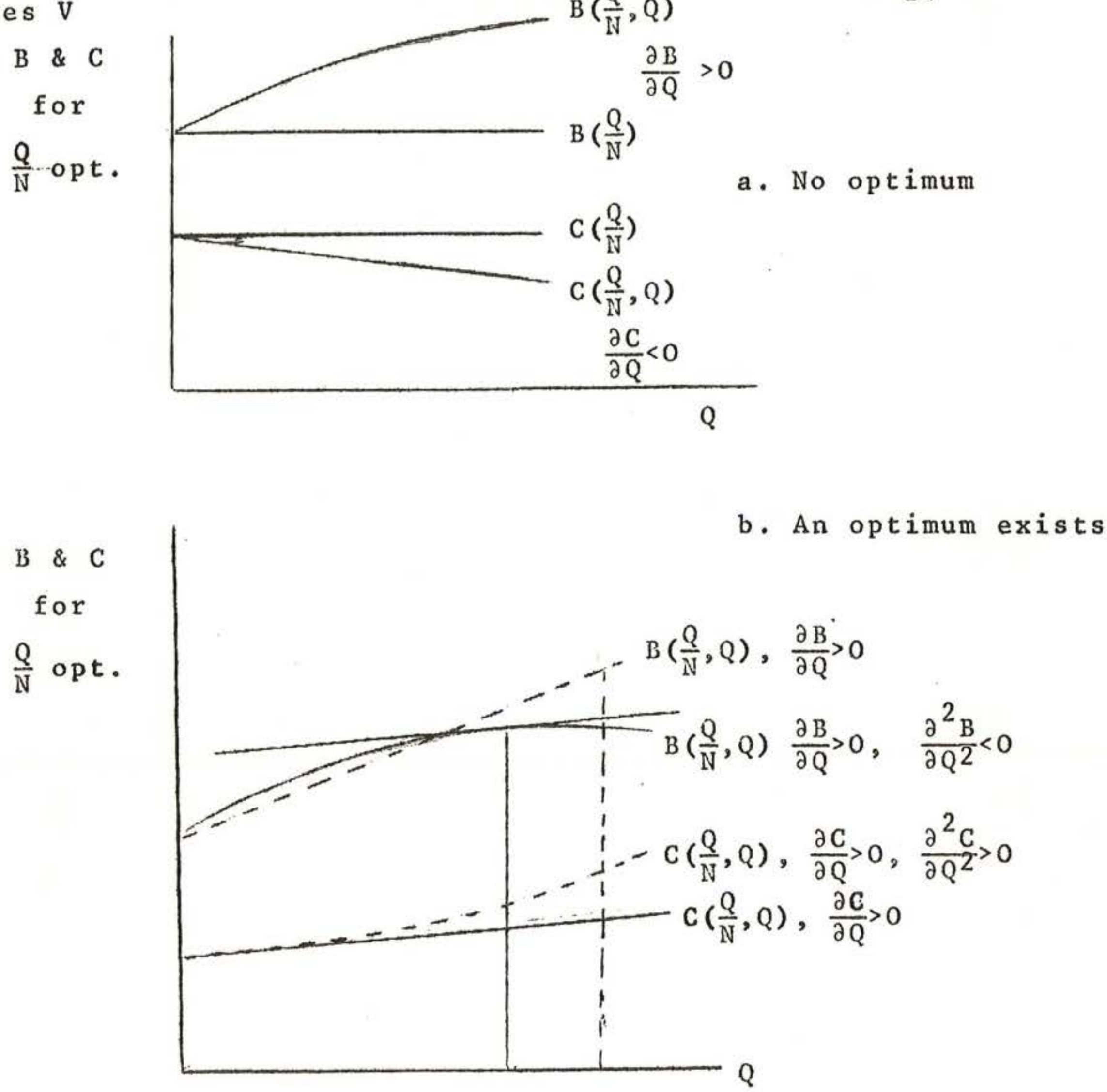
In other words, the Q_{opt} and N_{opt} lines, with different slopes, will increasingly diverge as Q and N are simultaneously increased. If continuity is assumed, they will intersect at a minimum or zero point (in terms of facilities and membership). The Buchanan solution thus also fails for unit costs constant (or declining) with scale.

There will be an optimal point in two cases :

1. If unit costs are increasing sufficiently to scale, and $\frac{\partial B}{\partial Q}$ is positive but declining (relative saturation) ;
2. If $\frac{\partial C}{\partial Q}$ is positive and rising (increasing unit costs to scale at an increasing rate).

These are fairly restrictive assumptions when dealing with public goods where indivisibilities loom large. Some of the possibilities are illustrated in Figures V, where unit costs and benefits are drawn for a movement along the N_{opt} line.

Figures V



When it is the argument N which appears separately in the benefit function, we proceed along the Q_{opt} line, then take the partial derivative with respect to N. Thus with the restriction :

$$\left[\frac{\partial S}{\partial (\frac{Q}{N})}\right]_{dN = 0} = 0 \text{ (noted : } dY = 0$$

we get the expression :

$$\left[\frac{\partial S}{\partial N}\right]_{dY = 0} = \left[\frac{\partial B}{\partial N} - \frac{\partial C}{\partial N}\right]_{dY = 0}$$

If $\frac{\partial B}{\partial N}$ is constantly positive along the Q_{opt} line (as in the telephone example), the analysis is the same as in the previous case, and only some rather special assumptions of increasing unit costs will yield an optimum position.

Here however it is quite reasonable to assume that cases exist where $\frac{\partial B}{\partial N}$ will change sign along the Q_{opt} line. In the genuine "club" case, for instance, keeping relative size optimal, benefits could well increase first with absolute membership (or size), and later decrease. Such cases will normally provide an optimum, unless there are strongly increasing returns.

The inclusion of absolute membership in the benefit function will thus offer better chances for the Buchanan solution, since it does not then depend on the existence of increasing costs to scale.

Finally, there is the more complex case, where the surplus function must be written :

$$S = B(Q,\frac{Q}{N},N) - C(Q,\frac{Q}{N}) = S(Q,\frac{Q}{N},N)$$

We may again proceed by restricting ourselves to one of the optimal lines, say N_{opt}. Then Q and N must vary together in some constrained proportion in order to stay on that line. (If N_{opt} were linear and homogeneous, they would vary in the fixed proportion $\frac{Q}{N}$). Along this line, we can express as before the effect of absolute size through the variable Q alone, and we define $dX = 0$ also as before.

Then when Q and N vary in their optimal ratio, we have :

$$\left[\frac{\partial S}{\partial Q}\right]_{dX=0} = \left[\frac{\partial B}{\partial N}\frac{dN}{dQ} + \frac{\partial B}{\partial Q} - \frac{\partial C}{\partial Q}\right]_{dX=0}$$

where $\frac{dN}{dQ}$ is the constrained variation that will keep us on the the N_{opt} line, and can be taken to be positive. $\frac{\partial B}{\partial Q}$ will also be positive.

Thus the possibility that this expression will go through a zero value will depend on $\frac{\partial C}{\partial Q}$ being strongly positive (decreasing returns) ; and/or on $\frac{\partial B}{\partial N}$ turning strongly negative (but remember : we are still on the N_{opt} line). There is also the possibility of multiple equilibria, but little more can be said without specifying the functions involved.

No hard and fast general conclusion can be drawn from this taxonomy. The range of application of the Buchanan optimum must remain a matter for individual judgement. It is clear however that the "club" analogy to public goods is somewhat misleading. Since, apart from cases of strongly increasing unit costs, the existence of an optimum depends on declining benefits to absolute membership (relative membership optimal), it implies interaction in consumption. Such interaction (benefits decline when the group becomes a crowd) will be more typical of true club situations, than of the majority of public or semi-public goods, subject to congestion, but supplied by some authority in more impersonal fashion.

As long as we are dealing with "typical" members, it is clear that there exists a wide spectrum of public goods where no position of unconstrained optimum can be defined, that is, where there exists no reason for applying a principle of voluntary exclusion. Outside constraints then become essential. Let us briefly look at them.

III

Among the constraints, the most obvious relates to potential membership. It applies first of all to public goods not subject to congestion. If national security were simply a matter of the size of the army, the larger the country (or the alliance), the greater security would be for any given level of individual taxation.

This will also be the typical problem of the local authorities. Given the size fo the community, is it worth the while putting in sewers or building a school ? Or, for that matter, building a golf course ?

The problem remains definite even if N is not fixed but is itself a function of Q. A better school will attract pupils from outside the town ; at the limit better public services will attract population to the town. One can still use Figure II to find optimum Q, by drawing the cost curve so as to take into account the fact that maximum N will change with increased Q.

Constraints on facility size will also arise. In the short run, when capital equipment is fixed. In the longer run, where there exist large-scale indivisibilities (the 18-hole golf course can be getting crowded or refuse applications long before it is worth doubling or duplicating its facilities). Permanently, if a specific facility or location is involved, or a scarce factor, such as "tradition" in a school or university. As with the membership constraint, this will always ensure that there is a second best optimum.

This will not be generally true of the familiar budget constraint. Fixing costs per head will simply mean fixing $\frac{Q}{N}$ at a level below optimum $\frac{Q}{N}$ (if the constraint is operetive). Further, since the costs and benefit curves obtained will be parallel to those resulting from $\frac{Q}{N}$ optimal, the previous analysis will apply and a constrained optimum will exist only under somewhat restrictive assumptions, unless additional constraints on potential membership or facility size are introduced.

To conclude, Buchanan's Theory of clubs was intended as "a step forward in closing the awesome Samuelson gap between the purely private and the purely public good". It certainly succeeds in this object, although Buchanan's assertion that "few, if any, goods satisfy the condition of extreme collectiveness" is open to question. Even where congestion occurs, that is, excluding the pure Samuelson case, there would remain a wide class of public goods where the largest potential membership is preferable to all arrangements involving the exclusion of potential participants, provided the quantity of the good to be shared can be varied at will.

IV

We may end pu by trying to relate Buchanan's analysis to the wider controversy on public goods.

Professor Samuelson has recently redefined public goods as those entering the utility function of more than one person (1) ; and reasserted the failure of the Wicksell-Lindahl theory of voluntary exchange tax. Thus leaving economists incapable of defining allocations for most goods.

Buchanan's paper can be read as an attempt to justify voluntary exchange, by showing that a private initiative system, applying the principle of exclusion, will indeed lead to a Pareto-optimal solution. The previous discussion has shown the limitations of that solution, even under the assumption that all members have the same indifference maps and income levels (or marginal utility of money).

What happens now if we drop this last assumption ? Assume you a keen and dedicated golfer, intent on always having a clear fairway ahead ; while I joined the club on my doctor's advice, do not really care for the game, but am interested in always finding convivial company around the bar. It is clear that the same $\frac{Q}{N}$ cannot be optimal for both of us, and your "best" club is not my "best" club.

Again, even if indifference maps are identical, income differences will modify optimum choices, since the cost curve will shift with the marginal utility of money. I may be as keen a golfer as you are, but, if poorer, will be more tolerant of incipient congestion in the interest of lower club dues. My optimal $\frac{Q}{N}$ will be lower than yours. And discrimination in dues between club members provides no answer. How is one to

(1) P.A. SAMUELSON : Pure Theory of Public Expenditure and Taxation, in : Public Economics (J; Margolis and H. Guitton, eds) London, Macmillan, 1969.

decide who pays what ? We are back against the Samuelson objection that real preferences will not be revealed.

The crucial distinction here may be in what may be called the degree of publicness of the good. In a small town, you and I will have to belong to the same club, since there is only one, and only by luck will it be optimal for both of us. In a metropolitan area, we could belong to different clubs and chose them more closely geared to our tastes and purses. The degree of publicness then depends on the size of the indivisibility of the public facility in relation to potential membership. Samuelson makes the point in terms of a cinema, which will be more of a public good in a village, where only one film is showing, and more of a private good in a large city where any consumer can find fare to his taste.

Thus the Theory of Clubs, where it offers a Pareto-optimal solution, will do so essentially for goods which are situated near the private end of the spectrum. The chances of any member attaining a position of full equilibrium falls off as the degree of publicness increases. Publicness could in fact be measured by the rising improbability of the Buchanan first best solution.

There still remains the welfare significance of Pareto-optimality. Take a case where it is attainable, say a community large enough to fill two schools of optimal size. Then, on the club principle, the rich men of the town would organize one, which would be expensive and provide highquality education. The poor men would organize the other, cheap and with low grade facilities. The solution is Pareto optimal : marginal conditions are met for both sets of citizens. Yet, who would invoke Pareto-optimality to defend this sytem against the ruling of the U.S. Supreme Court that it violates the constitution ? It is Pareto-optimal to provide the rich with better services than the poor, and using this criterion involves a value judgement.

Here also Samuelson was right in asserting that allocative efficiency could not be separated from distribution.