Proceedings of the IASS Annual Symposium 2016
"Spatial Structures in the 21st Century"
26–30 September, 2016, Tokyo, Japan
K. Kawaguchi, M. Ohsaki, T. Takeuchi (eds.)

Vector-Based 3D Graphic Statics (Part II): Construction of Force Diagrams

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Abstract

2D graphic statics is based on two interdependent diagrams, namely the form and the force diagrams. To each edge of the first one corresponds a pair of vectors in the second one that can be generally overlapped as a single element. In the case of vector-based 3D graphic statics, the reciprocity between the two diagrams is normally not attained due to the impossibility of overlapping all pairs of vectors in the force diagram. This paper explains how to construct the force diagram in three dimensions while proposing a solution to deal with the non-overlapping pairs of vectors. This procedure applies to any class of pin-jointed spatial frameworks. Alternative ways of assembling the force diagram are suggested and a specific double-layered configuration is described. The 3D force diagram defined here can be regarded as a direct extension of the 2D force diagram and can be effectively used for early design explorations.

Keywords: structural design, graphic statics, three-dimensional static equilibrium, force diagram, topology.

1. Introduction

Exploring non-conventional spatial structures might prove a difficult design problem and designers usually lack adequate interactive tools. Graphic statics has demonstrated its relevance to support the design of two-dimensional structures in an interactive and informed way. One of its main benefits is the possibility to represent the form and the inner forces in two interdependent diagrams. In this context, extending graphic statics to the third dimension would allow new possibilities for both structural design and analysis in three dimensions. There are different ways to embark on this topic (Jasienski et al. [8]), such as the vector-based and the polyhedral-based approaches. This contribution is focused on the vector-based approach to define a design framework where both diagrams are visualized and handled fully in three-dimensions. In fact, vector-based 3D graphic statics can be regarded as a direct extension of 2D graphic statics, which is itself vector-based. On the one hand, as shown by Mackinlay [9], the use of vectors (defined in position, lengths and angles), in place of polyhedra (defined by areas, volumes and densities) seems to increase the accuracy in quantitative perceptual tasks. This aspect is directly related to the legibility of the diagrams, which plays a key role in graphic statics. On the other hand, working with vectors introduces some disadvantages in comparison to the polyhedral-based approach, especially in the construction and transformation of the diagrams.

Graphic statics was originally theorized by Rankine [18], Maxwell [11], Culmann [2] and then Cremona [3], based on the reciprocity between the form and force diagrams. Maxwell and Culmann already pointed out the existence of mechanically possible 2D equilibrium configurations for which a force diagram cannot be drawn as a reciprocal of the form diagram. In graph theory, as demonstrated by Whitney [21], the lack of reciprocity between two figures is related to their topological properties. Two figures cannot be reciprocal of each other if their topological graph is not planar — i.e. a graph that can be drawn on a plane without crossing members. Crapo and Whiteley [5], Crapo [4], and Micheletti [14] have extended the notion of reciprocity in different ways to some cases of structures with non-planar graphs.

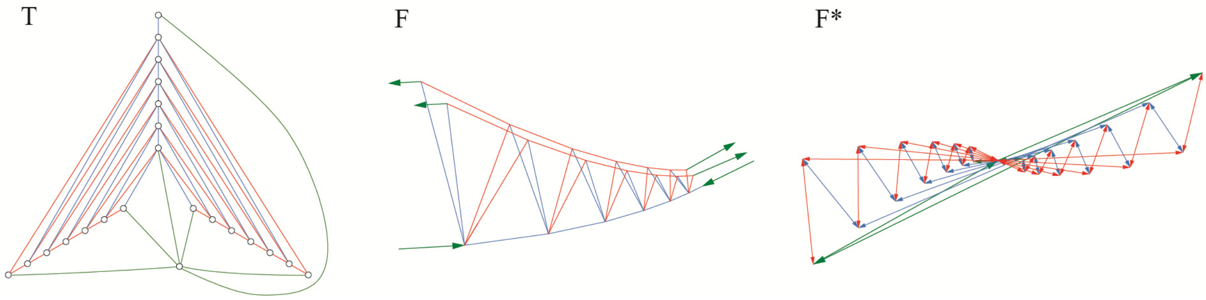

Fig. 1 Spatial truss with corresponding diagrams: Topological **T** – Form **F** – Force **F***

The aim of the present work is to support the solution of any class of pin-jointed spatial frameworks, especially focusing on structures that don't have an underlying planar graph and therefore don't have reciprocal force diagrams. Specifically, the presented approach suggests a series of assembly rules how to construct 3D force diagrams out of individual vector-based force polygons, representing the equilibrium of the single nodes of the structure. Hints to deal with transformations on these vector-based force diagrams that preserve their interdependency to the corresponding form diagrams are also introduced.

The paper is organized as follows. Section 2 explains the graphical solution of the internal equilibrium of a pin-jointed spatial framework on a single node basis. Section 3 deals with the construction of the force diagram based on the assembly of the nodal force polygons in self-stressed structures. The issue of non-overlapping pairs of vectors in the force diagram is discussed following a graph-theoretic approach. A procedure to deal also with external forces is here presented and various ways to assemble the force diagrams are shown. Section 4 briefly suggests further developments regarding transformations of the force diagram, while the last section summarizes the current results of the research.

2. Solving internal equilibrium

2.1. Node-by-node procedure

In this section, a procedure to solve the internal equilibrium of a pin-jointed spatial framework is presented. In this process, a 3D skew force polygon is built for each node of the structure. The solution of the equilibrium of a pin-jointed framework in two dimensions using a node-by-node procedure is known since the introduction of graphic statics, as recalled by Cremona [4]. What follows is an extension to three dimensions of this well-known procedure.

As a preliminary step, the determinacy of the structure has to be evaluated. The degree of static indeterminacy *n* of a space structure is (Calladine [1]):

$$n = b - 3v + s + m \tag{1}$$

where b is the number of bars, v is the number of vertices, s are the support force variables — i.e. the number of restraints imposed by the support conditions, 6 if the structure is self-stressed —, and m is the number of inextensible mechanisms. The result n represents the number of independent states of self-stress in the structure. The value n therefore represents the numbers of indeterminacies in the structure. By varying the value of the force parameters all the possible solutions of the problem can be found. The solution of the internal equilibrium in statically determinate structures is carried out after having solved the external global equilibrium — i.e. the equilibrium between applied loads and support forces – see D'Acunto et al. [6]).

The internal equilibrium of the structure can be solved node by node as explained in the next section, ranking at every step of the process the nodes in ascending order according to the number of unknown forces. Before starting with the resolution of the nodes, the bars with null force can be identified and removed by applying some basic rules (Pirard [17]).

2.2. Equilibrium of a spatial node

Given a pin-jointed framework, any node can be isolated together with the external forces applied on it and the internal forces generated by the bars connected to it. The result, obtained by means of an imaginary cut, generates what is known as the free body diagram (Marti [10], p.44). In this section, the geometrical construction to solve the equilibrium of a node regarded as a free body diagram is described. The procedure will be exemplified using a node P_0 connected to three other nodes $\{P_1, P_2, P_3\}$ by means of three bars $\{f_{10}, f_{20}, f_{30}\}$ and a given external force $\mathbf{f_{00}}$ (Figure 2). In order to represent the general case in 3D, the three bars and the external force are chosen to be non-coplanar and a coplanar triplet out of these four elements does not exist.

Based on this set-up, in the form diagram F three vectors $\{\mathbf{f_{10}}, \mathbf{f_{20}}, \mathbf{f_{30}}\}$ can be defined in such a way that $\mathbf{f_{i0}} = \mathbf{p_0} - \mathbf{p_i}$, being $\mathbf{p_0}$ the position vector of P_0 and $\mathbf{p_i}$ the position vectors of P_i , for i =1, 2, 3. Additionally, three unit vectors $\{\mathbf{d_{10}}, \mathbf{d_{20}}, \mathbf{d_{30}}\}$ can be obtained: $\mathbf{d_{i0}} = \frac{\mathbf{f_{i0}}}{||\mathbf{f_{i0}}||}$.

In the force diagram F* the force $\mathbf{f_{00}^*}$, which is a free vector equivalent to $\mathbf{f_{00}}$, defines with its tail and head two points O_0^* and O_1^* respectively. A plane Π_1 is set parallel to $\{\mathbf{d_{10}}, \mathbf{d_{20}}\}$ and includes O_1^*. A second plane Π_2 is set parallel to $\{\mathbf{d_{20}}, \mathbf{d_{30}}\}$ and includes O_0^*. Note that the order of the vectors in the force diagram can be freely chosen and consequently the equilibrium state would not be affected in case Π_1 is set to include O_0^* and Π_2 is set to include O_1^* therefore the order of the vectors in the force diagram can be freely chosen. The intersection of the planes Π_1 and Π_2 is a line l_{20}^* parallel to $\mathbf{d_{20}}$. Furthermore, a line l_{10}^* parallel to $\mathbf{d_{10}}$ is defined passing through the point O_1^* and being consequently included in the plane Π_1. A second line l_{30}^* parallel to $\mathbf{d_{30}}$ is defined passing through the point O_0^* and being consequently included in the plane Π_2. From these geometrical constructions the points O_2^* and O_3^* are obtained: $O_2^* = l_{10}^* \cap l_{20}^*$ and $O_3^* = l_{20}^* \cap l_{30}^*$.

Three forces $\{\mathbf{f_{10}}^*, \mathbf{f_{20}}^*, \mathbf{f_{30}}^*\}$ related to $\{\mathbf{f_{10}}, \mathbf{f_{20}}, \mathbf{f_{30}}\}$ are then defined as follows:

$$\mathbf{f_{10}}^* = (\mathbf{o_2^*} - \mathbf{o_1^*}) = \mu_{10}\mathbf{d_{10}} \qquad \mathbf{f_{20}}^* = (\mathbf{o_3^*} - \mathbf{o_2^*}) = \mu_{20}\mathbf{d_{20}} \qquad \mathbf{f_{30}}^* = (\mathbf{o_0^*} - \mathbf{o_3^*}) = \mu_{30}\mathbf{d_{30}}$$

where $\mathbf{o_i^*}$ represent the position vectors of O_i^* and the absolute values of μ_{i0} are the magnitudes of the forces $\mathbf{f_{i0}}^*$. The force $\mathbf{f_{00}^*}$ and the three forces $\{\mathbf{f_{10}}^*, \mathbf{f_{20}}^*, \mathbf{f_{30}}^*\}$ thus generate the 3D skew force polygon $\mathbf{f_{00}^*} | \mathbf{f_{10}}^* | \mathbf{f_{20}}^* | \mathbf{f_{30}}^*$.

In order to determine if the bars $\{\mathbf{f_{10}}, \mathbf{f_{20}}, \mathbf{f_{30}}\}$ connected to the node P_0 are in tension or compression, it is possible to evaluate the sign of μ_{i0}. Since the unit vectors $\mathbf{d_{i0}}$ are defined to be directed towards P_0, if $\mu_{i0} < 0$, then the corresponding bar is pulling the node and it is in tension. If $\mu_{i0} > 0$, then the bar is pushing the node and it is in compression. The presented method is also valid if more than one acting force on the node are already determined. In this case, in the force diagram F* the known forces are assembled in sequence, being O_0^* at the tail of the first known force and O_1^* at the head of the last known force. A construction equivalent to the one here explained was already described by Saviotti ([18], p. 48) for the decomposition of a force in space into three skew forces.

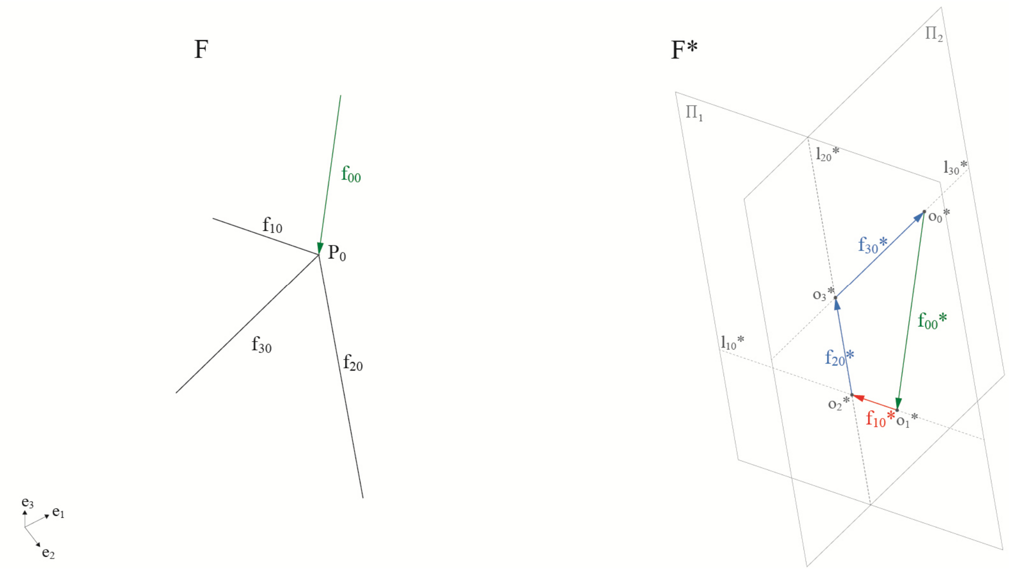

Figure 2: Equilibrium of a node in space (3 unknown forces: {$\mathbf{f_{10}}$*, $\mathbf{f_{20}}$*, $\mathbf{f_{30}}$*})

As an example, the presented procedure is applied to solve the equilibrium of the nodes of a self-stressed tetrahedron. Five 3D force polygons are built for the five nodes of the structure (Figure 3).

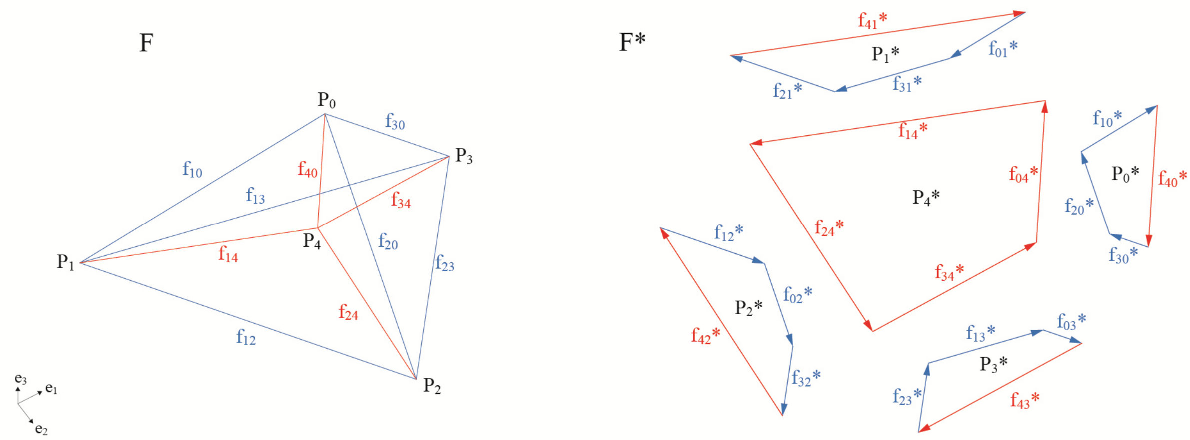

Figure 3: Self-stressed tetrahedron with the force polygons of all the individual nodes resolved

3. Construction of the force diagram

3.1. From the 3D force polygons to the 3D force diagram

After the 3D force polygons of the individual nodes are determined, a 3D force diagram of the entire structure can be assembled. For non-self-stressed structures, this includes also the 3D force polygon of the external forces representing the external global equilibrium (D'Acunto et al. [6]).

Every bar in the form diagram F will be represented in the force diagram F* by a pair of vectors, which are equivalent in magnitude and are opposite in direction. The two vectors belong to two different 3D force polygons. In order to reduce the number of elements in the force diagram F*, as many pairs of vectors as possible should be overlapped. As highlighted by Maxwell [13]"*We also represent every stress twice over, for it appears as a side of both the polygons corresponding to the two joints between which it acts. But if we can arrange the polygons in such a way that the sides of* *any two polygons which represent the same force coincide with each other, we may form a diagram in which every stress is represented in direction and magnitude, though not in position, by a single line, which is the common boundary of the two polygons which represent the points of concourse of the pieces of the frame".*

Considering the example of the self-stressed tetrahedron and trying to assemble the 3D force diagram of the entire structure, not all the pairs of vectors can be overlapped (**f_{20}*/ f_{02}*** and **f_{13}*/ f_{31}*** in Figure 3). As explained in the following section, this is a consequence of the topological properties of the system.

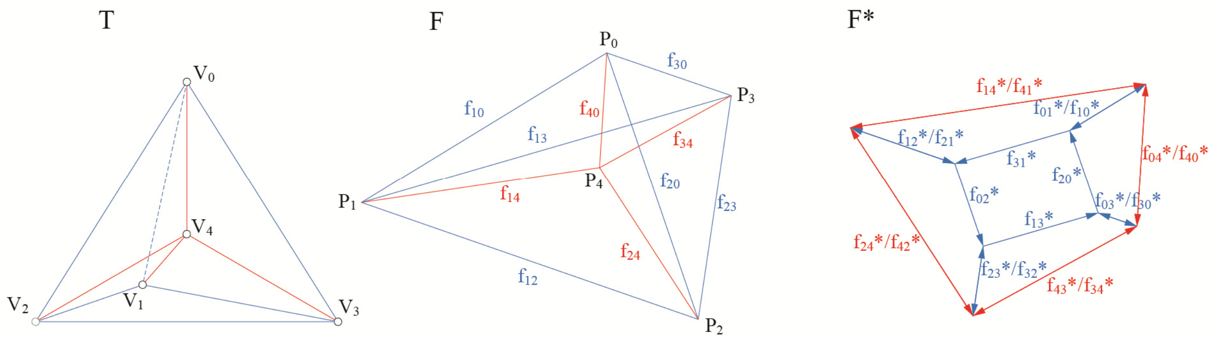

Figure 4: Self-stressed tetrahedron and the corresponding 3D force diagram

3.2. Non-overlapping pairs of vectors

The presence of non-overlapping pairs of vectors in the force diagram is not a peculiarity of spatial structures but can also occur in two-dimensional structures since it is only depending on the topological graph of the structure itself. This was already highlighted by Maxwell [11] for specific 2D cases. Some force diagrams developed by Culmann [2] were also built with non-overlapping pairs of vectors and as a result they were not reciprocal as highlighted by Cremona [3]. For reciprocal figures to be possible, Maxwell introduced the following conditions: "*every line belongs to two polygon only and e=s+f-2.*" [11] and also "*Since every polygon in one figure has three or more sides, every point in the other figure must have three or more lines converging to it; and since every line in the one figure has two and only two extremities to which lines converge, every line in the other figure must belong to two, and only two closed polygons*" [11]. Moreover, as explained by Maxwell [11] comparing the vector-based and the polyhedral-based approach to 3D graphic statics, "*Diagrams of forces in which the forces are represented by lines may be always constructed in space as well as in a plane, but in general some of the lines must be repeated (i.e. cannot be overlapped as explained). […] We have thus a complete but redundant diagram of forces […] This is a more convenient though less elegant construction of a diagram of forces, and it never becomes geometrically impossible as long as the problem is mechanically possible, however complicated the original figure may be".*

The question of non-overlapping pairs of vectors in the force diagram has been recently treated using graph theory. Whitney [21] demonstrated that a graph has its reciprocal if and only if it contains none of the Kuratowski's graphs as a subgraph. According to Kuratowski's theorem, a planar graph can be mapped directly on the surface of a sphere. By extension, if the topological graph underlying a pin-jointed framework is not planar, then a reciprocal force diagram cannot be obtained. Van Mele and Block [20] presented a graph theoretical approach to 2D graphic statics. This is focused on the algebraic formulation of the form and force diagrams of structures with underlying planar graphs. Different ways to extend the notion of reciprocity to non-planar graphs can be found in literature. Micheletti [14] presented a method to extend the reciprocity to non-planar graphs representing 3D self-stressed networks.

Considering the example of the self-stressed tetrahedron, its underlying topological graph is not planar, being an isomorphic of K_5. This is one of the two elementary non-planar graphs that are presented in Kuratowski's definition of non-planarity (Harary [7]). Indeed, in the force diagram F* of the self-stressed tetrahedron non-overlapping pairs of vectors can be found (Fig.3). They constitute together a 3D close skew polygon ($\mathbf{f_{02}}$*| $\mathbf{f_{13}}$*| $\mathbf{f_{20}}$*| $\mathbf{f_{31}}$*).

3.3. Dealing with external forces

In literature, the application of graph theory to the definition of 3D force diagrams has been mainly studied in relation to self-stressed structures. In this section, an extension to structures with applied external forces is suggested.

Conventionally, the external forces are modeled as outer bars connected to the structure (Pirard [17]). In the underlying topological graph of a pin-jointed framework each of these elements is thus represented as a member with one of its extremities connected to a node of the structure and the other extremity as a leaf vertex. Contrary to the conventional approach, the present proposal is to bundle all the leaf vertices of the external forces into one unique vertex. This vertex would then correspond in the force diagram to the force polygon of the external global equilibrium. Each external force is thus represented twice in the force diagram F*, once in the force polygon of the external global equilibrium and once in the force polygon related to the node of the structure where the force is applied. In this way, it is possible to deal with the external forces without modifying the condition of planarity of the structure's underlying graph.

By applying the proposed rule, it is easy to detect a specific condition of non-planarity (hence of generation of non-overlapping pairs of vectors in the force diagram) that occurs in the case in the topological graph an external force is applied to an interior node of a pin-jointed framework. In this situation, the member representing the external force is connected with one of its extremities to the interior node of the structure and with the other extremity to the previously introduced vertex of the external global equilibrium. As such, this member will intersect at least another member of the graph, representing a bar of the pin-jointed framework. This condition was already described by Maxwell [12]*"if it [the external force] is in the interior of the frame it is a side of more than one other polygon, and it cannot be represented in the reciprocal diagram by a single line"*.

The self-stress tetrahedron of the previous example and a tetrahedron with three support forces and one applied load are used to illustrate how the external forces are dealt with in the proposed approach. As can be observed, based on the proposed convention, the topological graphs of the two tetrahedra are isomorphic and the resulting 3D force diagrams are equivalent in topological terms.

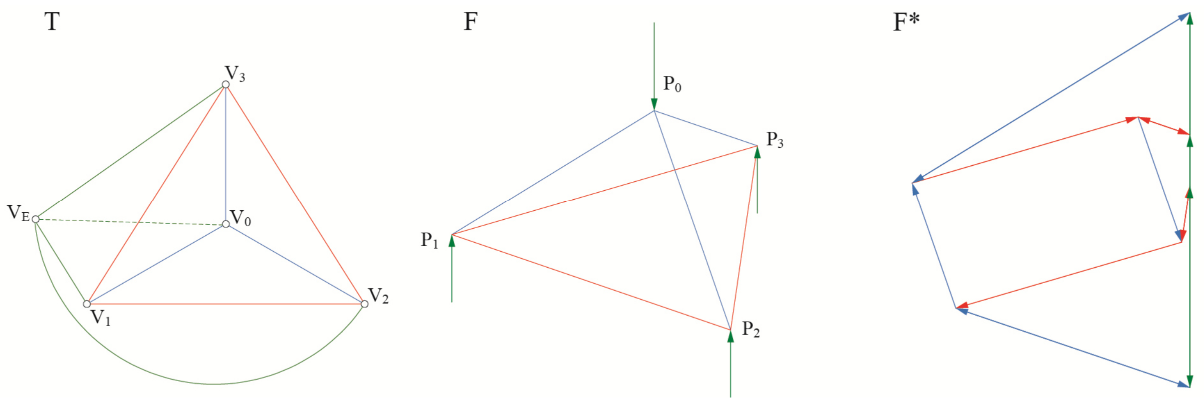

Figure 5: A tetrahedron with external loads. In the topological diagram **T**, V_E represents the vertex where all the external forces connect.

3.4. Assembly of the 3D force diagram

3.4.1. About the compactness of the force diagram

Given a complex spatial pin-jointed framework composed of several elements, its force diagram might present many non-overlapping pairs of vectors. As such, the force diagram itself can be assembled in many different ways, its compactness depending on the number of non-overlapping pairs of vectors.

Concerning the legibility of the force diagram, one strategy is to reduce the number of non-overlapping pairs of vectors to the minimum so that the force diagram is as compact as possible. The *degree of non-planarity of a graph* is defined as the minimum number of members of the graph that should be removed in order to make the graph planar. In the present context, this definition applies to the isomorphic graph of the structure that has the fewest amount of intersecting members. The hypothesis is here presented that the minimum amount of non-overlapping pairs of vectors in the force diagram can be deduced by the degree of non-planarity of the topological graph of the structure. Specifically, the hypothesis is that each degree of non-planarity implies the presence of two non-overlapping pairs of vectors in the force diagram. Hence, the number of non-overlapping pairs of vectors in a force diagram is always a multiple of 2. Moreover, it is a multiple of 4 the number of independent vectors that constitute the skew polygon defined by the non-overlapping pairs of vectors. Algorithms already exist to check the planarity of a graph. These might be also used to deduce the degree of non-planarity of the underlying graph of a given structure and as such to figure out the minimum number of non-overlapping pairs of vectors that have to be expected in the force diagram.

An opposite strategy to the one previously described is to assemble the force diagram in order to maximize the number of non-overlapping pairs of vectors. This can be achieved by constructing the force diagram in such a way that each nodal force polygon of the structure has at most one vector that is overlapped to one of another force polygon.

3.4.2. The double-layered force diagram configuration

A specific method is here proposed to construct vector-based 3D force diagrams in such a way that they are visually readable and have persistent geometrical properties that might be used to control their transformations. The force diagrams built according to the proposed method present a peculiar double-layered configuration. In particular, the two layers are represented respectively by the force polygon of the external forces (or the topological equivalent ones in self-stressed structures) and the force polygon of the non-overlapping pairs of vectors. These two layers are themselves connected through forks made of overlapping pairs of vectors. The following procedure explains how to build the double-layered force diagram configuration.

In the first step, all the external forces are assembled into a 3D skew force polygon representing the global equilibrium. This force polygon constitutes the first layer of the force diagram. In the second step, the nodal force polygons where the external forces are applied are introduced into the force diagram. This is done so that every external force belonging to each nodal polygon is overlapped to the corresponding force of the polygon of the global equilibrium. The sequential order of the external forces in the polygon of the global equilibrium is determined in such a way that each pair of adjacent nodal polygons defines an overlapping pair of vectors. In the form diagram, each overlapping pair of vectors represents the bar of the structure that connects the nodes corresponding to the adjacent nodal force polygons to which the vectors belong. These overlapping vectors constitute the previously introduced forks.

In the third step, the remaining nodal force polygons are assembled in the force diagram in order to maximize the number of overlapping pairs of vectors. The nodal polygons that only contain internal forces have to be placed in this step. Once all the force polygons are placed, a 3D skew polygon whose sides are composed by all the non-overlapping pairs of vectors is established, thus constituting the second layer of the force diagram. The most compact configuration of the force diagram is attained when the number of vectors composing the second layer is four times the degree of non-planarity of the underlying topological diagram of the structure, as defined in Section 3.2.

Eventually, the force diagram will be defined by two layers: one of the external forces, and one of the non-overlapping pairs of vectors, themselves connected by the forks of the overlapping pairs of vectors.

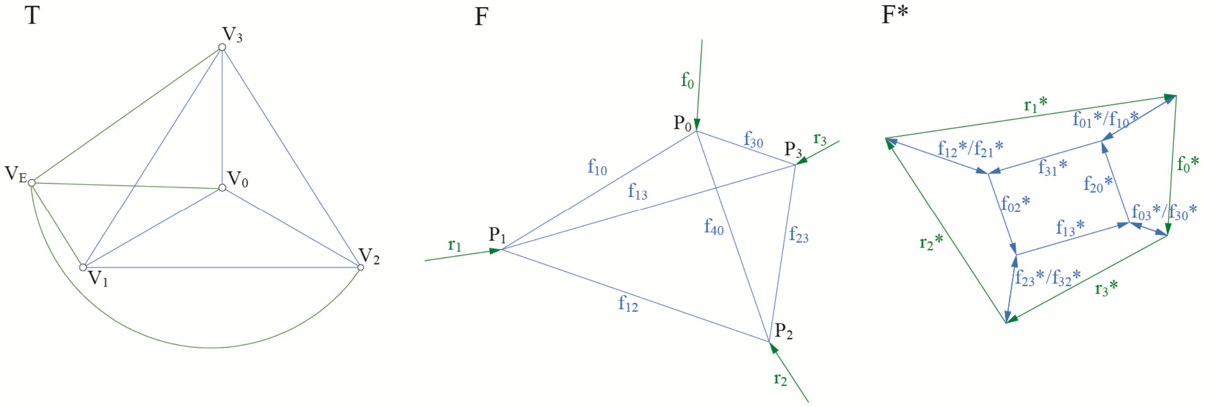

Figure 6: Assembling a tetrahedron with external loads in the double-layered configuration. The first layer is the skew force polygon composed of $\mathbf{f_0}$***|** $\mathbf{r_3}$***|** $\mathbf{r_2}$***| r1*,** the second layer is the skew force polygon composed of $\mathbf{f_{02}}$***|** $\mathbf{f_{13}}$***|** $\mathbf{f_{20}}$***|** $\mathbf{f_{31}}$***** and the forks are the following pairs of vectors $\mathbf{f_{12}}$***/**$\mathbf{f_{21}}$***,** $\mathbf{f_{23}}$***/**$\mathbf{f_{32}}$***,** $\mathbf{f_{03}}$***/**$\mathbf{f_{30}}$***,** $\mathbf{f_{01}}$***/**$\mathbf{f_{10}}$*****.

Other more complex examples are presented to show the visual readability of the diagrams. A spatial truss composed of 24 elements with 3 external forces and 2 supports is shown in Figure 7. Following the proposed arrangement rules, the most compact force diagram F* implies 16 (4x4) non-overlapping vectors. This can be deduced by considering that at least 4 members have to be removed in order to make the topological diagram T planar (represented in dashed line in T).

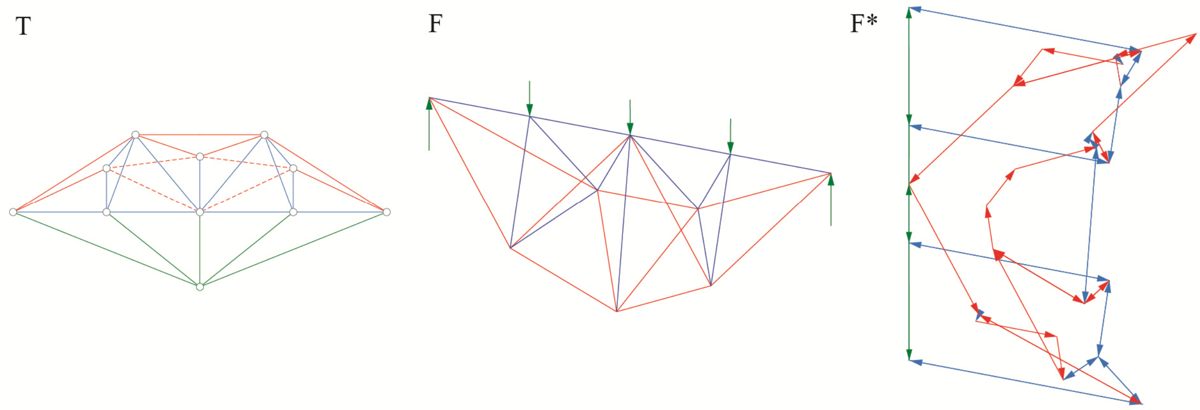

Figure 7: 3D truss composed of 24 bars – topological diagram **T** – form diagram **F** – force diagram **F***

In Figure 8 a double layered force diagram configuration for a compression-only surface structure with a degree of non-planarity of 12 is shown. In this case, 12 vectors of the external applied loads and 12 vectors of inner forces are represented twice in the force diagram F*.

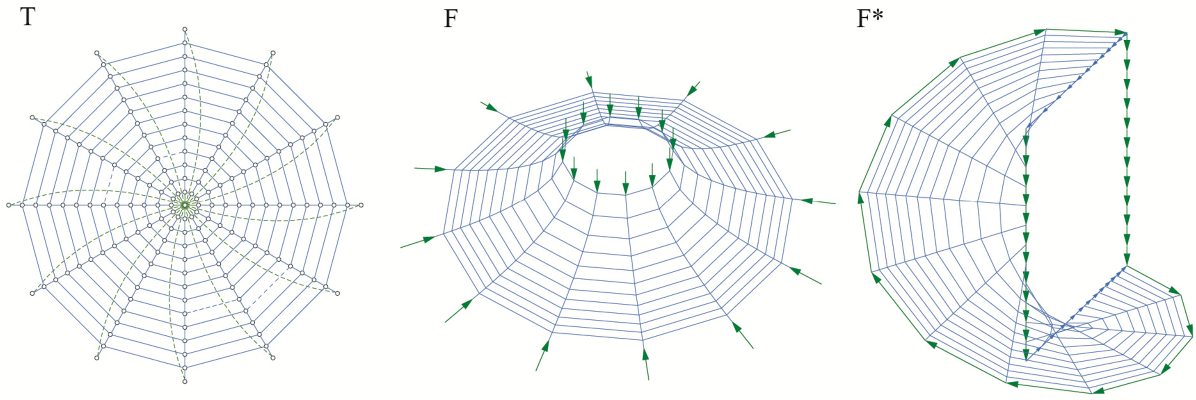

Figure 8: Compression only surface structure – topological diagram **T** – form diagram **F** – force diagram **F***

4. Further developments: transformations on the 3D force diagram

Given a pin-jointed framework, the goal of constructing its 3D force diagram is double: providing a visual understanding of the relationship between the forces within the structure and allowing the manipulation of these forces without affecting the topology of the structure. Regarding the second aspect, first tests conducted by the authors have shown that some transformations on the force diagram can be applied, even in the presence of non-overlapping pairs of vectors, without losing its interdependency to the form diagram, which can be transformed accordingly.

For example, if the position of the supports is not restrained, global scaling transformations (along 1, 2 or 3 directions) on the force diagram can be directly applied to the form diagram. Furthermore, local transformations like the displacement of individual points of the force diagram can be applied in such a way that the form diagram can be transformed accordingly without altering its topology. Future work will cover the families of transformations on the 3D force diagram and the necessary rules underlying these manipulations that preserve its interdependency to the 3D form diagram.

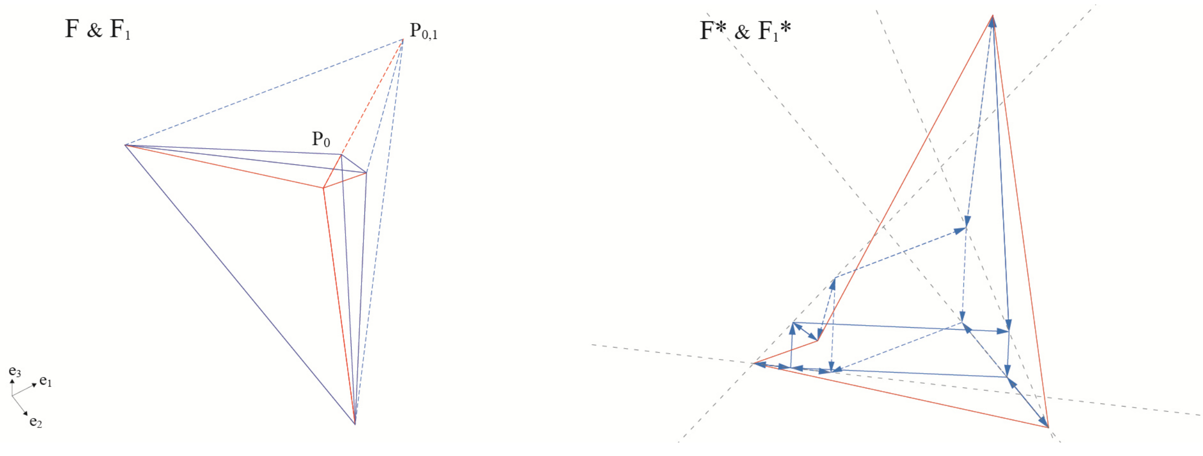

Figure 9: Self-stressed tetrahedrons **F** & $\mathbf{F_1}$ – Geometrical properties

5. Conclusion

The problem of assembling 3D force diagrams has been discussed in the present paper within the scope of vector-based graphic statics and graph theory. Based on existing methods, a procedure has been explained to find the equilibrium of the individual nodes of a given pin-jointed framework. Moreover, a rule that enables the treatment of external forces has been presented. The issue of non-overlapping vectors in the 3D force diagram has been examined with the aid of graph theory. Based on this, different approaches to construct the force diagram have been discussed and a specific procedure to build a double-layered configuration of the force diagram has been explained. Some complex case-studies have been shown to illustrate the visual convenience of the proposed double-layered configuration. The establishing of rules to construct the force diagrams represents a critical step forward in providing intuitive geometrical tools to visualize and handle the forces within any class of pin-jointed frameworks. Future research will focus on the geometrical transformations of the force diagrams that keep the topology of the form diagram invariant.

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