

Wear and erosion performance of metal-polymer hybrid materials facing multi-scale contact interactions

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Abstract

Abradable materials are used to improve the efficiency of turbomachines by reducing the gap between the tip of the blade and the casing. The abrasability property is the ability of a material to be worn out easily, in a controlled way, during a contact with the blade. In addition, abrasable materials must resist erosion caused by particles swallowed in the engine, including dust, sand, water droplets, ice and volcanic fly ashes. Thermally sprayed materials made of aluminum and polyester seem to comply with these apparently conflicting properties of combining good abrasability and erosion resistance. Yet, the origin of these good performances is not well understood. The purpose of the thesis is to identify the physical parameters that influence the abrasability and the erosion resistance in order to, either guide the design of more reliable systems based on these materials and to offer more innovative and more sustainable solutions.

To achieve this goal, hybrid model abrasable materials are manufactured, studied and characterized using microstructure analysis, mechanical and fracture testing as well as tribology measurements. Four families of model architected materials were processed and tested, including (i) metallic foams infiltrated with polyester, (ii) powder aggregates, (ii) polyester matrices with aluminum reinforcements and (iv) the classical thermally sprayed material. Experimental results on these materials confirm that, in terms of combining erosion and abrasability, some model materials exhibit much better performances than others.

In parallel with this experimental approach, physics-based models of abrasability and erosion are proposed, including elasticity, plasticity and fracture mechanics and adapted to three different wear mechanisms. These models are then refined and validated on the basis of experimental results and used to guide the selection and optimization of abrasable materials. Modelling results show that the key parameters are the quality of the interfaces between the aluminum and the polyester as well as the maximum elastic strain attained before entering plasticity. Our results also show that the dominant wear mechanism is affected by the penetration depth and by the geometry of the blade.

Finally, the calculation of the optimal set of properties required to improve both abrasability and erosion resistance points towards materials with low Young's modulus and large ability to deform elastically before entering plasticity, including high temperature elastomers, polymers and porous materials.

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List of symbols

Greek alphabets

α	Angle between and the tool frontal surface and a vertical plane	[radian]
β	Angle between the lateral surface of the tip and a horizontal plane	[radian]
β'	Angle between the lateral surface of the tip and the residual imprint	[radian]
δ	Complementary angle of the cutting angle $= \alpha + (\pi/2)$	[radian]
ε	Strain	[]
ε_{shear}	Shearing strain in the chip during cutting	[]
ε_r	Representative strain	[%]
ε_{r-ind}	Representative strain during indentation	[%]
ε_{r-plow}	Representative strain during plowing	[%]
ε_{r-cut}	Representative strain during cutting	[%]
ε_y	Tensile yield strain	[%]
ε_u	Ultimate tensile strain	[%]
η	Plasticity yield criterion proportionality factor	[]
λ	Friction angle $= \arctan(\mu)$	[radian]
μ	Friction coefficient	[]
$\mu_{Diamond}$	Friction coefficient between the diamond tip and the materials	[]
μ_{TA6V}	Friction coefficient between the TA6V tool and the materials	[]
ν	Poisson's ratio of the base material	[]
ν_1	Poisson's ratio of the tip	[]
σ	Stress	[N/m ²]

σ_{r-ind}	Representative stress during indentation	[N/m ²]
σ_{r-plow}	Representative stress during plowing	[N/m ²]
σ_{r-cut}	Representative stress during cutting	[N/m ²]
σ_{u-t}	Ultimate tensile stress	[N/m ²]
σ_{u-c}	Ultimate compression stress	[N/m ²]
σ_y	Initial yield stress	[N/m ²]
$\sigma_{y0.2}$	Yield stress measured at 0.2% of plastic strain	[N/m ²]
$\sigma_{y1\%}$	Yield stress measured at 1% of plastic strain	[N/m ²]
τ_y	Shear yield stress	[N/m ²]
ϕ	Primary shear plane angle	[radian]
ω	Erosion impact angle	[radian]

English alphabets

a	Radius of contact	[m]
a_{load}	Radius of contact during the first loading	[m]
a_{reload}	Radius of contact during the second loading	[m]
a_{unload}	Radius of contact during unloading	[m]
A	Vertical projection of the contact area	[m ²]
A_{el}	Vertical projection of the contact area in ideal elastic materials	[m ²]
A_{elpl}	Vertical projection of the contact area in elastoplastic materials	[m ²]
A_{cut}	Cross section of the cutted groove	[m ²]
A_{wear}	Cross section area of the residual groove after contact	[m ²]

A_{Unload}	Vertical projection of the contact area during unloading	[m ²]
b	Radius of the plastic zone	[m]
B_{OC}	Normalized orthogonal cutting parameter = $2Q/(\sigma_y h)$	[]
B_{PB}	Bite per blade	[m]
B_{SC}	Normalized scratch cutting parameter = $2cQ/(A_H \sigma_y)$	[]
c_{cut}	Length of the cross section of the scratch groove	[m]
C_{el}	Elastic hardness proportionality factor	[]
C_{elpl}	Elastoplastic hardness proportionality factor	[]
C_{pl}	Rigid perfectly plastic hardness proportionality factor	[]
C_{plr}	Elastoplastic with strain hardening hardness proportionality factor	[]
d_0	Initial gas path clearance	[m]
d_1	Post-rub gas path clearance	[m]
D_{el}	Elastic sink-in proportionality factor	[]
D_{pl}	Plastic pile-up proportionality factor	[]
D_{plr}	Elastoplastic with strain hardening pile-up proportionality factor	[]
E	Young's modulus	[N/m ²]
E_t	Young's modulus of the tip	[N/m ²]
E_{eq}	Reduced Young's modulus	[N/m ²]
E_{secant}	Secant modulus	[N/m ²]
E_{Unload}	Modulus measured during the unloading in a tensile test	[N/m ²]
f	Non-dimensional indentation parameter of elastoplastic material	[]
f_V	Vertical force of indentation per unit length (for 2D tips)	[N/m]

f_{velpl}	Vertical force of indentation per unit length (for 2D tips) in the case of elastoplastic materials	[N/m]
$f_{Vunload}$	Vertical force of indentation per unit length (for 2D tips) during unloading	[N/m]
F_{fr}	Friction force between the chip and the cutting tool	[N]
F_H	Horizontal force on the tip/cutting tool	[N]
F_{Hel}	Horizontal friction force due to the elastic recovery force	[N]
F_{Helpl}	Horizontal force on the tip in the case of elastoplastic materials	[N]
F_{Hpl}	Horizontal force on the tip in the case of perfectly-plastic materials	[N]
$F_{sh//}$	Shear force component parallel to the shear plane in cutting	[N]
$F_{sh\perp}$	Shear force component perpendicular to the shear plane in cutting	[N]
F_V	Vertical force on the tip/cutting tool	[N]
F_{Vel}	Vertical elastic on the tip indenting an ideal elastic material	[N]
F_{Velpl}	Vertical force on the tip indenting an elastoplastic material	[N]
F_{Vload}	Vertical force on the tip during the first loading	[N]
$F_{Vreload}$	Vertical force on the tip during the second loading	[N]
$F_{VUnload}$	Vertical force on the tip during unloading	[N]
g^{FW}	Term used in the rectangular flat punch plowing model	[m ⁻¹]
G_{Ic}	Plane strain energy release rate in mode one	[J/m ²]
G_a	Adhesion toughness due to chips sticking to the tool	[J/m ²]
h	Depth of penetration of the tip/cutting tool	[m]
h_c	Real height of contact	[m]
h_{el}	Recoverable elastic depth in indentation	[m]

h_{pl}	Residual depth after unloading in indentation	[m]
$h_{pile-up}$	Height of the pile-up	[m]
h_{res}	Height of the cross section of the residual imprint after unloading	[m]
$h_{Sink-In}$	Depth of the sink-in	[m]
h_{wear}	The height of wear (residual depth after contact)	[m]
H	Hardness, mean vertical pressure under the tip	[N/m ²]
H_{el}	Hardness in ideal elastic material	[N/m ²]
H_{elpl}	Hardness in elastic perfectly plastic material	[N/m ²]
H_{pl}	Hardness in rigid perfectly plastic material	[N/m ²]
$I_1(n)$	First polynomial fitting of the height of contact	[]
$I_2(n)$	Second polynomial fitting of the height of contact	[]
$I_3(n)$	Third polynomial fitting of the height of contact	[]
J_{IC}	Mode 1 plane strain fracture toughness characterized by J-integral	[J/m ²]
k_1	Equivalent stiffness of the upper part of the system	[N/m]
k_2	Equivalent stiffness of the lower part of the system	[N/m]
k_T	Equivalent stiffness of the entire system	[N/m]
K_A	Archard's wear rate efficiency constant	[]
K_F	Finnie's erosion rate efficiency constant	[]
K_{Ic}	Plane strain fracture toughness in mode one	[N/m ^{1.5}]
L	Length of the scratch, cutting or spalling groove	[m]
$L_{element}$	Size of the smallest elements in the finite element simulations	[m]
m	Mass of an erosive particle	[kg]

n	Strain hardening exponent	[]
N_b	Number of blades	[]
O	Constant used in the erosion cutting model = Z^2+1	[]
P	Constant used in the erosion cutting model = $hB_{clpl} (Z+1)$	[]
Q	Specific work of surface formation in cutting/scratching	[J/m ²]
r	Horizontal distance measured from the center of the tip	[m]
rp	Radius of the plastic zone	[m]
R	Radius of the tip/cutting tool	[m]
R_f	Image analysis median filter radius	[pixel]
R_r	Radius of the rotor	[m]
S	Stiffness of contact, slope of the unloading curve = dF_V/dh	[N/m]
t	Time	[s]
T	Thickness of the tip/cutting tool	[m]
v_H	Horizontal speed of the tip/cutting tool (parallel to the surface)	[m/s]
v_V	Vertical speed of the tip/cutting tool (perpendicular to the surface)	[m/s]
u_V	Vertical displacement imposed by the tribometer or the indenter	[m]
U_{crack}	Energy consumed in cracking	[J]
U_{cut}	Energy consumed during cutting	[J]
U_{el}	Energy consumed in elastic deformation	[J]
U_{fric}	Energy consumed in friction	[J]
U_{ind}	Energy consumed during indentation	[J]
U_{pl}	Energy consumed in plastic deformation	[J]

U_{scr}	Energy consumed during scratching	[J]
v_H	Horizontal speed of scratch or cutting	[m/s]
v_V	Vertical speed during indentation, scratch or cutting	[m/s]
V	Wear volume	[m ³]
V_{er}	Erosion wear volume	[m ³]
w_e	Essential work of fracture	[J/m ²]
W	Width of the tip/cutting tool	[m]
Z	Friction parameter = $\tan(\lambda - \alpha) = (\mu - \tan \alpha)/(1 + \mu \tan \alpha)$	[]

List of acronyms

AlSi	Aluminum alloy with 12% in weight of silicon
AlSi-44 μm	Fine Al _{12%} Si powder (>44 μm)
AlSi-44 μm	Coarse Al _{12%} Si powder (>150 μm)
CES	Cambridge engineering selector
CoNiCrAlY	Cobalt nickel chromium aluminum yttrium alloy
hBN	Hexagonal boron nitride
LCP	Liquid crystal polymer
LEFM	Linear elastic fracture mechanics
NiCrFe	Nickel chromium iron alloy
NiCrAl	Nickel Chromium aluminum alloy
NiCrAlY	Nickel Chromium aluminum yttrium alloy
PAC-904	Powder Alloy Corporation 904 powder (full LCP Polyester)

PAC-905	Powder Alloy Corporation 905-3 powder (mix of Al ₁₂ %Si and LCP Polyester)
rpm	Rotation per minute
SEM	Scanning electron microscope
SENB	Single edge notched bend
SiC	Silicon carbide
TBC	Thermal barrier coating
TiA6V	Alpha – beta titanium alloy with 6% aluminum and 4% vanadium
YSZ	Yttria stabilized zirconia

Chapter 1 Introduction

1.1 General context

Between the years 1970 and 2016, the number of passengers traveling by aircraft increases from 0.3 to 3.7 billion per year [WBG 2016]. This considerable growth comes with a lot of technical challenges. Indeed, the aeronautic industry is continuously challenged to improve the performance of aircraft including reducing weight, fuel consumption, and pollution as well as improving safety. Aircraft manufacturers are therefore adapting their designs and their materials to make planes more efficient. One of these improvements concerns the sealing system in the engines. These seals are exposed to extreme environments. Indeed, inside an aircraft engine, pressure and temperature gradients are combined with moving parts going at up to 600 meters per second and situated less than a millimeter away from the static parts. This environment operates safely owing to an ingenious assembly of sealing systems including the abradable materials. Abradable seals maintain a close gap between the tip of the blade and the casing of the engine. These materials were introduced in the aviation in the late 1960s and early 1970s [Chupp 2006]. They are designed to be easily worn in case of a contact with the blades. The environment inside an aircraft engine is quite aggressive for an abradable material. It includes erosion, corrosion, oxidation and temperature cycling as well as occasional contact with the blades. In practice, abradable seals should survive long enough before being eventually replaced during overhaul maintenance. A large spacing between the tip of the blade and the casing will decrease the efficiency of the engine. Maintaining a close gap is therefore essential. Improving the performance of abradable materials includes (i) increasing their erosion resistance, (ii) reducing contact force with the blades and (iii) the amount of material removed in case of a contact. However, the development of these materials is still empirical and based on trial and error approaches. In addition, obtaining the certification for a new material to be used in aeronautics is a long and expensive process that involves a lot of validation tests. The aim of this thesis is to contribute to the design of better abradable materials using a scientific approach based on a rational material selection method [Ashby 2011]. Therefore, this work is centered on the

following question: “How to design a good abradable material?” The preliminary question is: “What is a good abradable material?”

1.2 Technical challenges and scientific strategy

Using a material selection method is not straightforward. Indeed, nowadays, there are thousands of different materials available. Most of these materials are well characterized. Large databases containing their mechanical resistance and thermal properties are available. However, the real challenge in selecting a material for a specific application is to identify how the properties inside these databases (Young’s modulus, yield strength, fracture toughness...) affect the final performance of the product in service. Therefore, this thesis aims at translating the performance of an abradable material in physical parameters. The present research focuses mainly on abradable materials used in the low-pressure booster of the engine. Our starting point is a reference material, a hybrid material made of polyester and aluminum-silicon alloy deposited by thermal spraying. This material is the most widely used abradable material in low-pressure compressors. It presents a very particular combination of microstructure and mechanical properties. The research starts with this material and is then extended to other materials using a five-step approach described hereunder.

- (i) Model materials are designed based on the reference material to vary the microstructure and the mechanical properties of the reference materials (stiffness, strength and fracture toughness).
- (ii) The wear behavior of these model materials is examined using nanoscratch tests, erosion tests, pin-on-disc and abradability tests. In addition, the mechanical properties of the model materials are measured.
- (iii) A physics-based model that includes multiple wear mechanisms (plowing, cutting and spalling) is developed to make the link between the mechanical properties of a material and his wear behavior.
- (iv) This model is used to identify the right combination of properties that controls the abradability.

(v) Materials databases are combined with the result of the model to propose potential future abradable material candidates.

1.3 Outline of the thesis

The thesis is divided in 8 chapters, including the present short introduction chapter. This section describes the main content of these chapters as well as the interconnections between them. The diagram in Figure 1.1 gives a summary of the organization of the thesis.

Chapter 2 is a state of the art dedicated to abradable materials in the booster. The purpose of this chapter is to answer the following questions: What are those materials made off? What are their mechanical properties? What are their tribological behaviors? How are these materials tested and validated?

Chapter 3 focuses on the relationship between mechanical properties and tribological behaviors. It describes a state of the art of the mechanical modelling of contact and wear. One of the key messages of Chapter 3 is that present modelling approaches are adapted to idealized materials. For instance, most models focus on rigid-perfectly-plastic materials or ideal elastic materials.

Chapter 4 proposes several adjustments to some of these models in order to include the effect of elastoplasticity and strain hardening.

Chapter 5 contains a detailed description of the processing of the model materials. It also describes the testing protocols and the characterization methods.

Chapter 6 presents the results of the tests and compares them with the predictions of the models.

Chapter 7 focuses on the interpretation of the results and on the comparison between the model predictions and the experiments. It also describes the search for an optimal equilibrium between the different requirements of an abradable material. This chapter ends with a confrontation between what is expected from an optimal abradable material and the real world of materials using the Cambridge engineering selector (CES) materials database [CES2016].

Chapter 8 concludes this study with a summary of the key aspects of each chapter. This chapter also presents some perspectives to the present work.

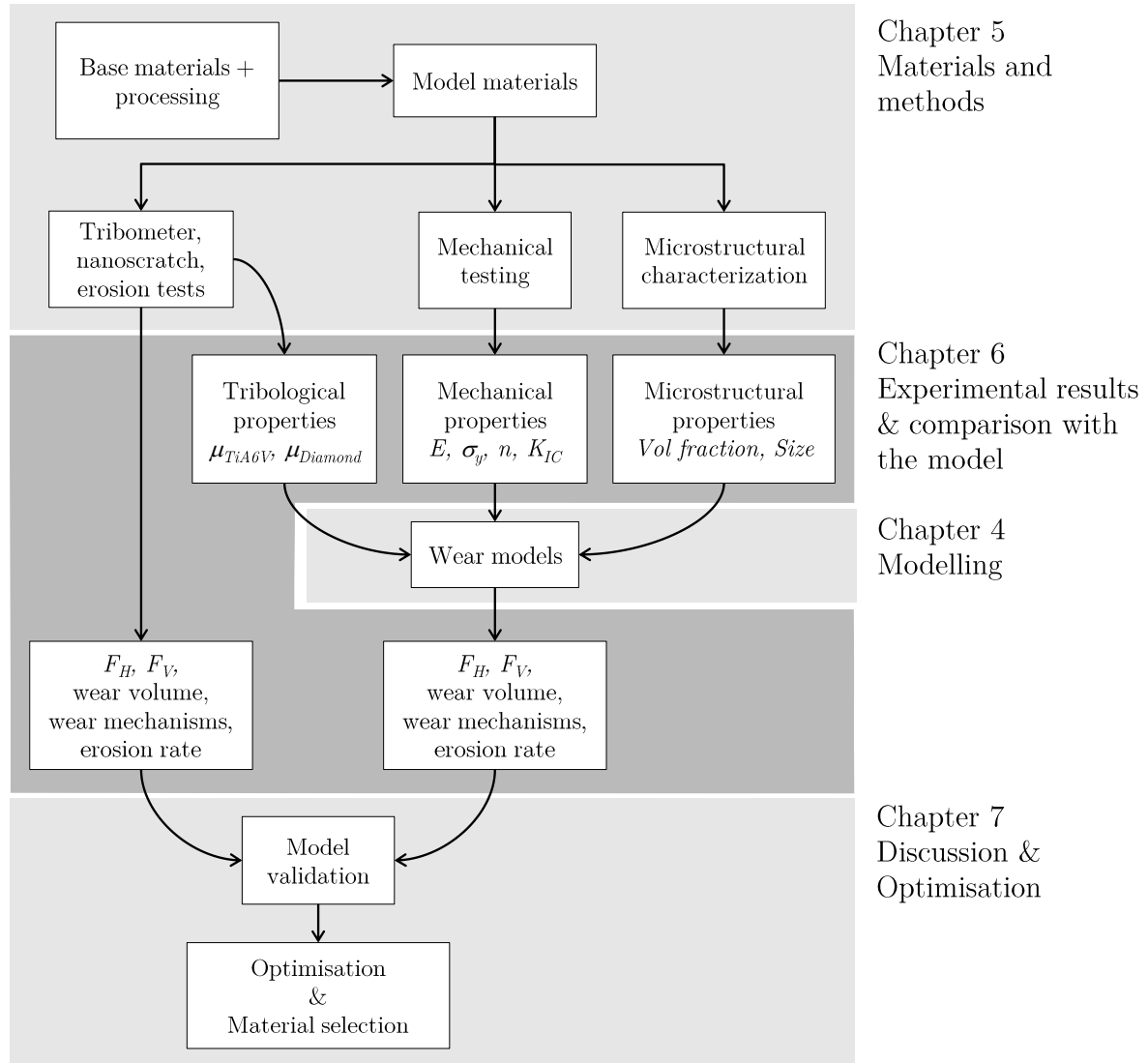


Figure 1.1: Diagram presenting the strategy of this study with an outline by chapter

Chapter 2 State of the art on abradable materials

The previous chapter stated the objectives of this study. The present chapter aims at giving a better understanding of the abradable materials from the perspective of the scientific and industrial literature. This bibliographic survey is constructed on one question: How are these abradable materials conceived, nowadays, from an engineering point of view? This question will decline in different steps, starting with (i) the design specifications, (ii) the strategies used to fulfill these specifications, (iii) the techniques used to evaluate their performances and finally, (iv) the validation of these materials through numerical simulations and bench testing.

2.1 Aircraft engines and abradable materials

The story of most abradable materials starts inside aircraft jet engines. These engines transform fuel into thrust and heat. The most common design uses a double air flux system. Once the air enters the engine, some of it is dragged in the secondary flux and is just pushed around the engine by the fan blades that act as giant ventilators. The rest of airflow is compressed in the compressor, then mixed with fuel and burnt in the combustion chamber. After that, it is expanded in the turbine before being released (see Figure 2.1).

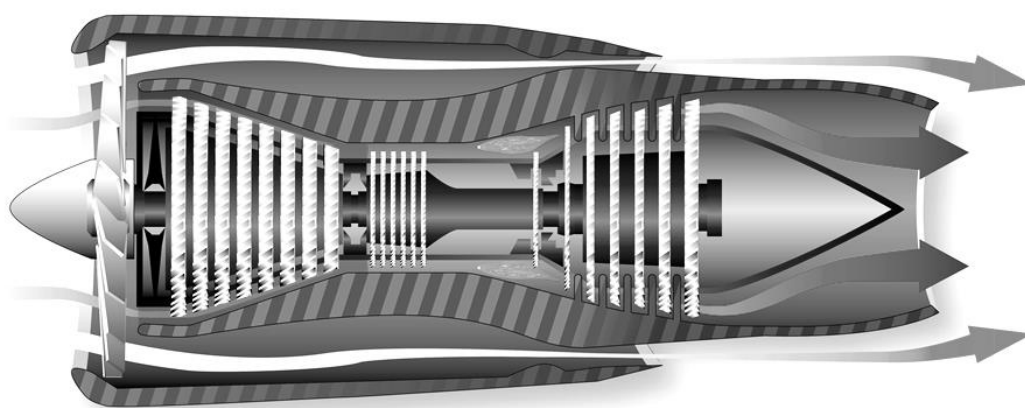


Figure 2.1: Schematic representation of a section of a double flux aircraft engine. Picture from [FAA 2012]

In the first stage of the engine, the temperature and the pressure of the gas that follows the primary airflow stream increases. This creates a gradient of pressure and temperature (Figure 2.2). The compressed air will tend to leak through any opening, including through the spacing between the tip of the blade and the casing. This phenomenon is avoided/limited using an ingenious combination of sealing systems, including labyrinth seals, contact seals, and abradable seals. For a comprehensive review on these sealing systems used in aircraft engines, see [Chupp 2006].

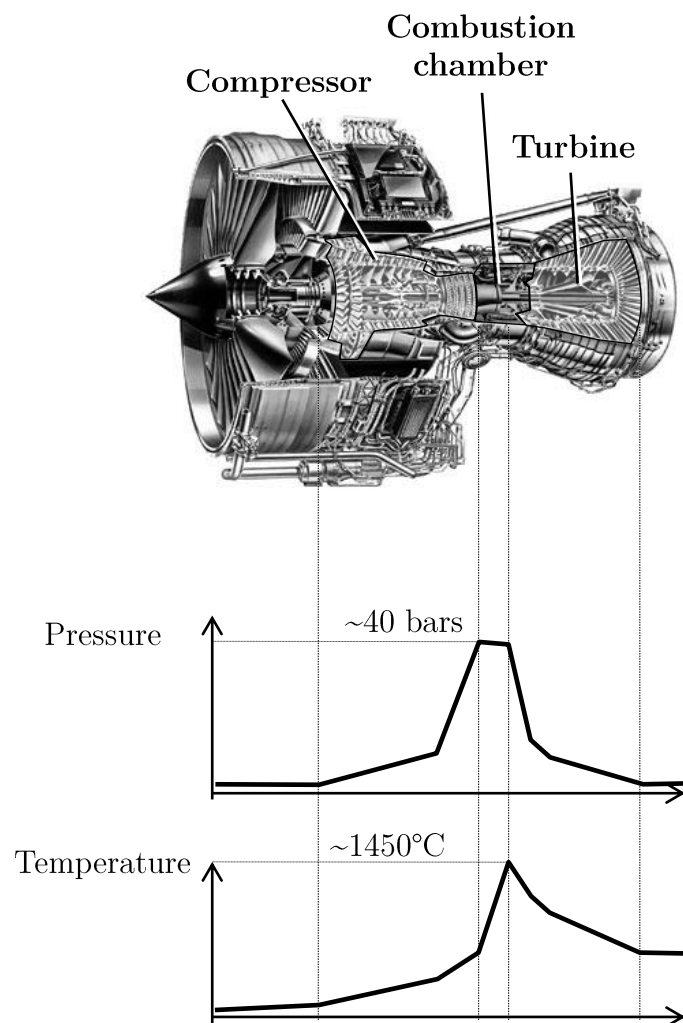


Figure 2.2: Temperature and pressure evolution in the primary flow of a Trent 800 Rolls-Royce Engine. Picture from [Chupp 2006].

The lateral speed of the tip of a blade can go up to 650 meters per second [Faraoun 2005]. However, above 150 m/s, conventional contacting seals melt due to the high friction [Schmidt 1997]. Abradable materials are non-contacting sealing systems used between the tip of the blades and the casing where the relative motion speed is the

highest (Figure 2.3). They act as sacrificial layers with a thickness ranging from 1 to 4 mm [Schmidt 1997]. These seals are therefore designed to be worn out by the blade in case of a contact. These abradable layers are deposited slightly thicker than necessary. During the first rotations of the engine, the blade tip scrapes the abradable material and forms a trench with a minimal clearance [Flitney 2014]. The smaller the clearance, the better the sealing [Chupp 2006]. Each time the blade comes into contact with the abradable layer, the latter will be worn out.

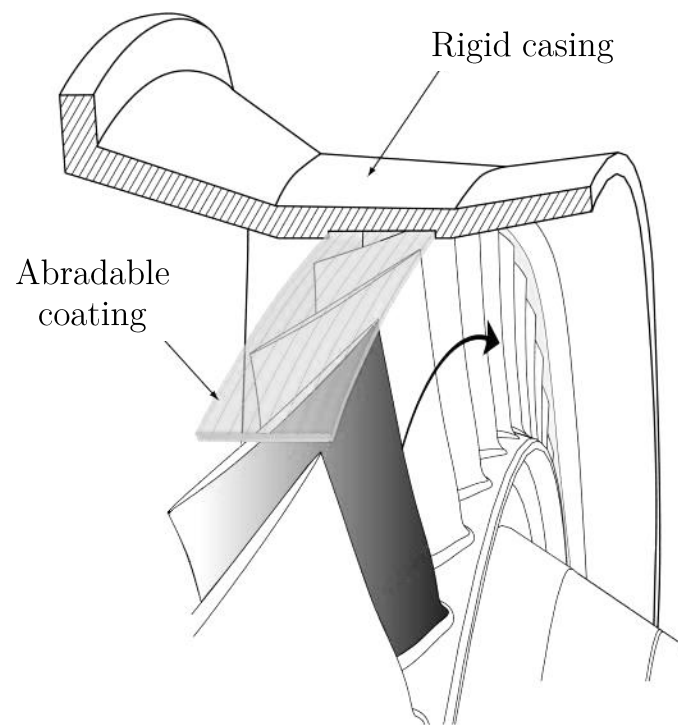


Figure 2.3: Schematic representation of an abradable seal. Picture from [Legrand 2011]

These layers are placed on the stator and not on the tip of the blade to avoid a clearance opening on the entire circumference of the engine in case of a contact (Figure 2.4). Most of these contacts are due to thermal expansion and elastic deformations of the engine during transient situations (take off, landing, acceleration...). For a more detailed description of this phenomenon, see [Lattime 2002].

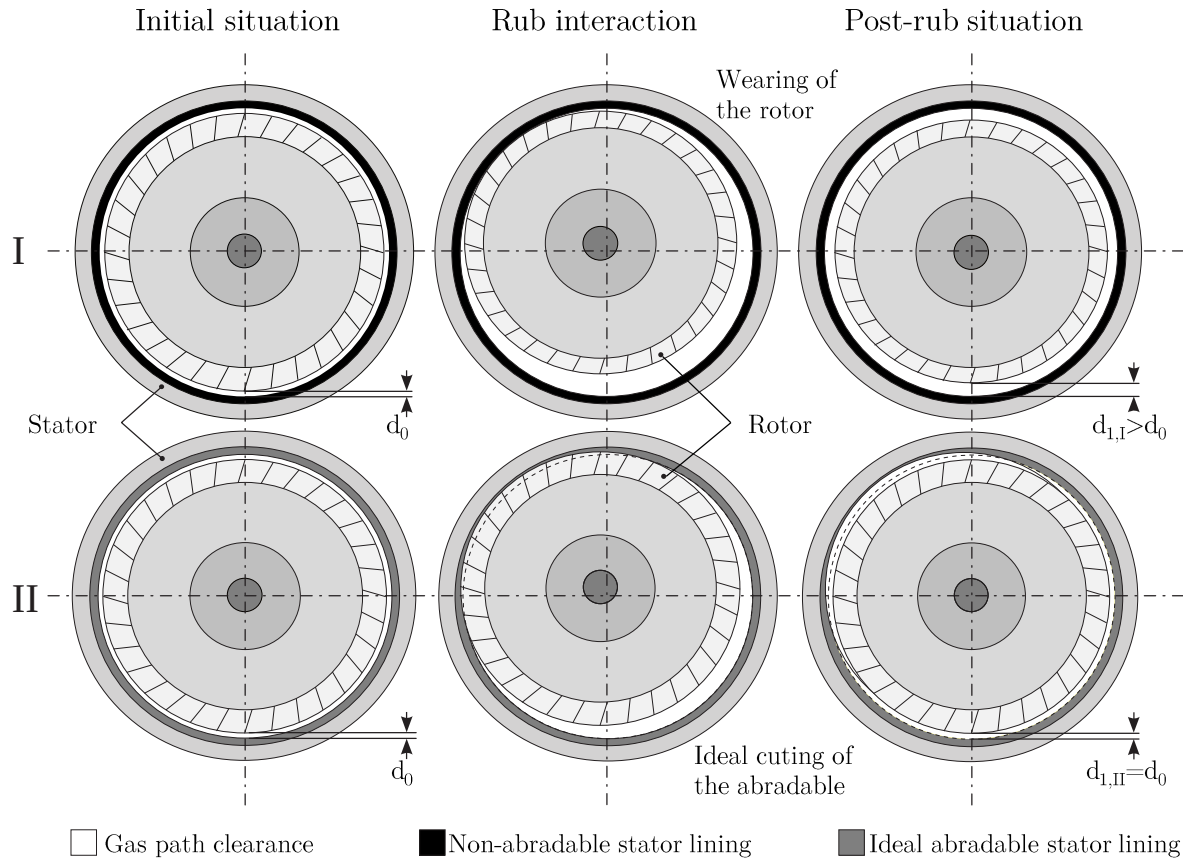


Figure 2.4: The abrasible is placed on the casing to allow a localized clearance opening in case of contact (in dark grey) and avoid a global circumferential opening abrasible (in black). Picture from [Oerlikon 2014]

2.2 Design requirements for abrasible materials

Designing a material for an abrasible seal coating is quite challenging since these sealing systems must satisfy a very restrictive list of specifications. These abrasible seals often operate in aggressive environments, yet, they have to fulfill their function for long periods of time before being replaced (up to 10.000 hours of flight) [Schmidt 1997]. The following specification list gives a better understanding of the constraints imposed on these materials.

- *Abradability*

The abrasible coating must have a good abrasibility. This quality englobes two different things. Firstly, it describes the ease to remove the abrasible material in case of a contact between the blade and the abrasible [Schmidt 1997]. Secondly, it implies

a non-abrasive behavior against the other rubbing surface [Baiz 2011]. Any damage due to an elevation of the temperature or a rise of the stress on the rotating components is thus avoided. It also ensures that the surface of the blade to not wear during an incursion.

- *Safe debris*

In case of a rub event, the debris of abradable materials should not interfere negatively with the components that lie deeper in the primary air flux. These negative interactions include, impact damages, erosive wear, plugging, sticking deposits and abrupt energy release during the combustion [Schmidt 1997, Zhang 2013].

- *Erosion resistance*

A running engine swallows a lot of air, along with the solid particles in it, including sand, dust, water droplet and ice. All these particles can impact and wear out the abradable material, therefore opening the clearance [Borel 1989]. The abradable should resist erosion to ensure a long-standing efficiency of the engine [Baiz 2011, Zhang 2013].

- *Temperature resistance*

Each stage of the engine operates in a given temperature range. The abradables have to withstand these temperatures to avoid any thermal degradation [Schmidt 1997] [Zhang 2013].

- *Temperature cycling*

The temperature inside the engine fluctuates a lot between the take-off, acceleration phase, cruise and landing. The abradable materials must therefore resist the mechanical deterioration due to thermal cycling [Schmidt 1997].

- *Corrosion resistance*

Abradable materials are also exposed to corrosive compounds, especially in marine environment: salty water, salt fog [Xu 2011], pollutants dissolved in rain water and sulfur compounds coming from fuel combustion products [Agard 1994]. In addition, the

abradable materials are sometimes exposed to cleaning detergents, oils and fuels. Moreover, metallic abradable materials can also suffer from galvanic corrosion created between the casing, the bond coating and the abradable material itself [Lei 2014]. Corrosion can increase the blade gap and therefore reducing the engine efficiency [Borel 1989].

- *Oxidation resistance*

Abradable seals must be resistant to oxidation. Indeed, the high oxygen partial pressure in the compressor combined with the energy generated during a rub event should not generate the ignition of the abradable material or the blade [Schmidt 1997].

- *Others*

In addition, the abradable material should be impermeable with an aerodynamically smooth surface in order to reduce aerodynamic losses [Hendricks 2004, Borel 1989]. The processing of abradable material should be cheap and reproducible. Finally, it should be possible to remove the abradable layer and replaced it for reparation [Schmidt 1997].

2.3 Design strategies used to improve abrasability

Most of the known materials do not exhibit a good abrasability. However, many strategies can be used to design materials with a satisfying level of abrasability. Sometimes, these strategies are combined. In theory, the best abrasable material is the one that deforms only elastically. In that spirit, rub tolerant materials were introduced. Rub tolerance is the ability of a layer to deform elastically during a contact without wearing or damaging the seal or the rotor. Some polymers deposited in the cold part of the engine are rub-tolerant [Chupp 2006, Flitney 2014]. However, most of these polymers can only be used in the first stage of the engine because of their limited temperature resistance. Some abrasable materials are made weaker using low shear strength materials to ensure that, if plastic deformation occurs, it happens with the lowest stress level possible [Steinke 2010, Sporer 2007, Wilson 2012, Zhang 2013]. Lowering the resistance to crack propagation is another strategy used in abrasable design. The idea is to ease the release of small debris by brittle fracture in case of a contact. It is done by introducing small defects in the material. These defects can be porosity or a second phase that presents a weak interface with the matrix or a brittle phase like graphite [Schmidt 1997, Zhang 2013]. This second phase is often called a “lubricant” phase.

2.4 Commercial abrasable materials

Many materials are designed with these strategies and are now commercially available [Oerlikon 2015]. Most of them are designed for a specific temperature range. Therefore the choice of the abrasable material is mostly limited by the temperature and the pressure corresponding to a position in the engine as well as the wear resistance of the blade tip (Figure 2.5). At low temperature, polymer materials like rubbers, thermoplastics or epoxy are used. They are used in labyrinth seals or against the fan blades [Schmidt 1997, Chupp 2006, Flitney 2014]. In the compressor stage, hybrid materials made of a metallic matrix mixed with a lubricant phase are used. The metallic phase is made of aluminum alloys, silver, zinc, nickel, bronze or copper alloys. The lubricant phase is either a polymer or a brittle material like graphite, hexagonal boron nitride, bentonite clay or diatomite. At higher temperatures, refractory alloys are used, sometimes mixed with polymers that will be burned after the deposition to

leave porosity. In some cases, they are also mixed with a lubricant phase. In the highest temperature, ceramics are used, mainly yttria-stabilized zirconia (YSZ) (Table 2.1). In some cases, a wear-resistant coating made of alumina or zirconia is added on the tip of the blades to protect the blade during a rubbing event against the abradable [Faraoun 2005]. For a review, see [Schmidt 1997] and [Chupp 2006].

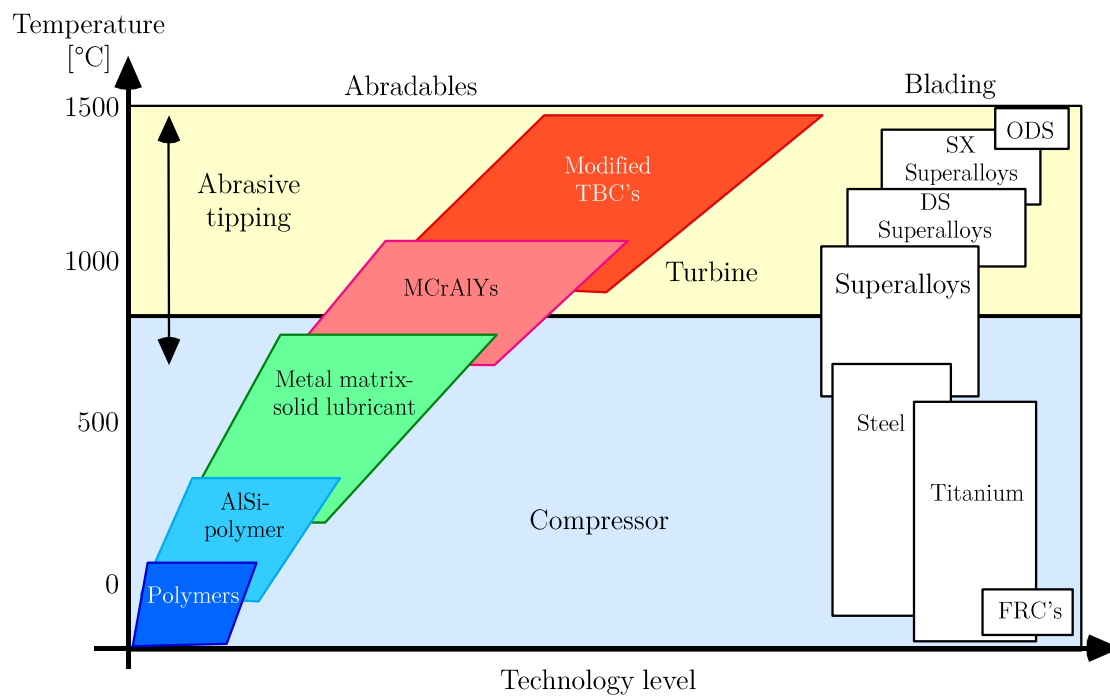


Figure 2.5: Families of abradable materials presented as a function of the operating temperature and as a function of their technological processing complexity. On the right, the corresponding blade tip materials. Picture from [Schmidt 1997]

Table 2.1: List of some abradable materials used in the engine with their temperature range. Data from [Zhang 2013].

Name	Temperature range [C°]	Applications
Polymers	<325	Compressor + Fan blades
AlSi-polymer	<350	Compressor
AlSi-graphite	315 - 415	
AlSi-hBN	<480	
Ni-graphite	<480	
NiCrFe-hBN	<480	
Bronze-polymer	<650	
Bronze	<650	
NiCrAl-diatomite	<815	Hot Sections
NiCrAl-diatomite-hBN	<750	
CoNiCrAlY-polymer	<850	
CoNiCrAlY	<850	
NiCrAlY-polymer	<1000	
NiCrAlY-Al ₂ O ₃	<1000	
NiCrAlY-graphite	<1100	
YSZ-polymer	<1250	
YSZ	<1250	

2.5 Performance evaluation of abradable materials

Assessing the performance of an abradable material is not straightforward. It requires many tests to satisfy and validate the design specifications. Some of these specifications, like the resistance to temperature, are simple to evaluate and easy to interpret. However, others, like the abradability are harder to quantify and requires complex dedicated test rigs. Some of these test rigs have been used for decades. Even though, the results of these tests are often accompanied with a lot of scientific questions and investigations. The present section focuses on these unusual testing machines combined with an overview of some of the research done to interpret the results of these tests.

2.5.1 Abradability test rigs

Before entering into service, abradable materials are tested on high-speed abrasion test rigs in which they are rubbed against an instrumented blade. The aim of such tests is to understand the behavior of the abradable material under rubbing similar to the conditions in the engine. These test rigs decline into three possible configurations. The “classical” configuration reproduces a simplified version of the engine including a rotating disc with blades and a sample holder covered with an abradable material. During the test, the abradable seal is forced radially against the disc (Table 2.2). The incursion speed and the rubbing speed are the two main adaptable parameters. In some cases, these rigs are equipped with a heating device added to increase the temperature of the sample (Table 2.3). The second design is called “inverted”. It uses a rotating disc covered with abradable material. In that design, the blade holder is pushed radially against the rotating disc (Table 2.4). The third one is the “linear” configuration, in which the abradable material is attached to a projectile. The projectile is pushed by a gas under pressure inside a vacuum tube in which it accelerates before entering into contact with a blade. The projectile covered with the abradable rubs against the blade and finishes its course in a shock absorber (Table 2.5). In this configuration, the two changeable parameters are the incursion depth and the rubbing speed.

Table 2.2: Room temperature abrasability test rigs operating in the classical configuration

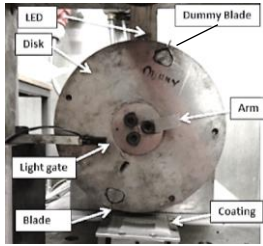
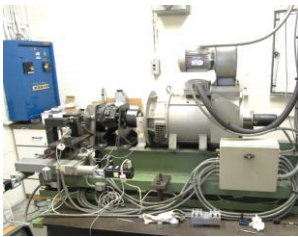
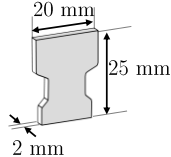
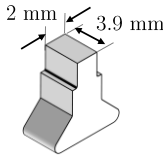
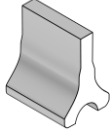
Laboratory name	University of Sheffield	IMAP – UCLouvain	IPEK & Alstom
Picture			
Nb of blades	1 – 2	1	1 – 24
Incursion speed [$\mu\text{m/s}$]	0.1 – 2000	2.5 – 250	1 – 7000
Rubbing speed [m/s]	200	315	25 – 400
Disk diameter [mm]	187	175	347
Temperature [$^{\circ}\text{C}$]	Room T°	Room T°	Room T°
Blade geometry			
References	[Fois 2013, Fois 2016]	[See Chapter 5]	[Rathmann 2007, IPEK 2014]

Table 2.3: High temperature abrasability test rigs with the classical configuration. These rigs are equipped with a torch that can fire a flame on the abradable to increase the temperature.

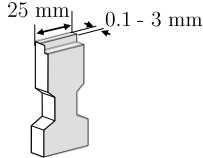
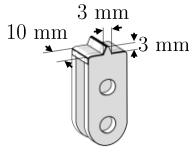
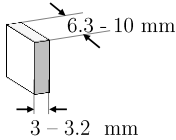
Laboratory name	Oerlikon Metco	BGRIMM - Beijing	NASA - AVRADCOM
Picture			
Nb of blades	1 – 4	1 – 2	1 – 12
Incursion speed [$\mu\text{m/s}$]	1 – 3000	5 – 2000	2.5 – 254
Rubbing speed [m/s]	100 - 500	520	320
Disk diameter [mm]	600	470	191
Temperature [$^{\circ}\text{C}$]	<1200 $^{\circ}\text{C}$	<1200 $^{\circ}\text{C}$	<1100 $^{\circ}\text{C}$
Blade geometry			
References	[Schmidt 1997, Lu 2013]	[Xuan 2015]	[Shiembob 1977, Bill 1978]

Table 2.4: Abradability test rig with an inverted configuration. The test rig from Onera (on the left) is equipped with an induction heating device.

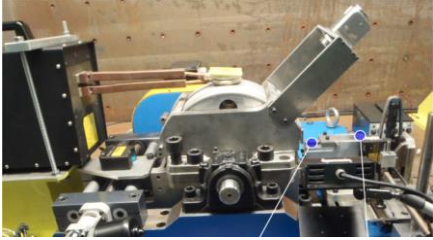
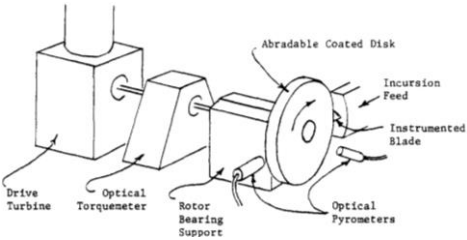
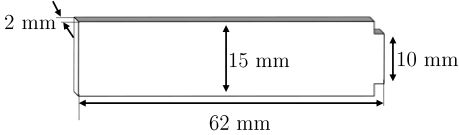
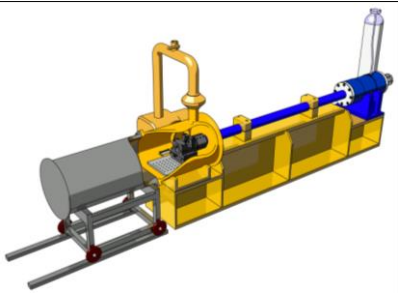
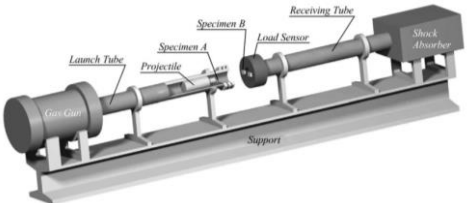
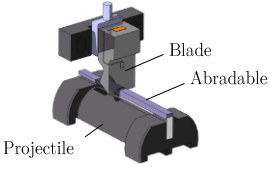
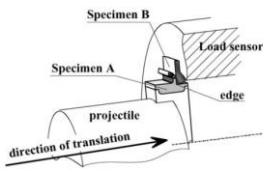
Laboratory name	Onera - Lille	Pratt & Whitney Aircraft
Picture		
Nb of blades	1	1
Incursion speed [$\mu\text{m/s}$]	15000	2.5 – 250
Rubbing speed [m/s]	95	213
Disk diameter [mm]	300	127
Temperature [$^{\circ}\text{C}$]	<300	Room T°
Blade geometry		
References	[Baiz 2013, Mandard 2015]	[Lavery 1982]

Table 2.5: Linear abrader test rigs.

Laboratory name	La.B.P.S. - E.N.I.M.	LEM3 – Lorraine University
Picture		
Nb of blades	1	2
Incursion depth [μm]	130 - 350	50 - 500
Rubbing speed [m/s]	60-500	5 - 120
Temperature [$^{\circ}\text{C}$]	Room T°	Room T°
Blade geometry		
References	[Cuny 2012, Cuny 2014]	[Sutter 2006, Cuny 2012]

2.5.2 Wear mechanisms occurring in abradable materials

After being tested on abrasability test rigs, the abradable materials are examined to identify the dominant wear mechanism. Some mechanisms are more desirable since they lead to a smooth surface, with no excessive material removal and no contamination of the blade or the abradable [Borel 1989, Shmidt 1997, Faraoun 2005]. Some of these wear mechanisms encountered during an abrasability test are described hereunder. They are sorted by order of desirability.

- *Cutting*

In the field of abrasability, cutting corresponds to the separation of the surface of the abradable in a brittle manner that leaves a smooth and shiny appearance with little or no deformation of the subsurface [Borel 1989].

- *Smearing*¹

This mechanism is due to an extensive plastic flow of the coating, leading to the smearing of one or more component of the abradable. If one of the phases is more brittle, the ductile phase can be mixed with small fractured chunk of the brittle phase [Borel 1989]. This mechanism is less desirable since it is often accompanied by grooving and transfer.

- *Grooving with transfer*²

Grooving is associated with large plastic deformations of the abradable material combined with adhesive transfer of material from the abradable to the blade. The transferred material can build up on the blade tip, creating a toothed surface [Schmidt 1997]. The surface of the abradable is therefore marked with grooves. The width and the height of the grooves can vary from micrometers to millimeters [Borel 1989].

¹ « Etatement » in French.

² « Rainurage » in French.

- *Densification*

Abradable materials can present high porosity content (up to 40%) [Shmidt 1997]. During an interaction with the blade, the plastic deformation can lead to the densification of the abrasible material by crushing the porosities. This mechanism is unwanted since densification leads to a loss of abrasibility.

- *Melting wear with transfer*

During the interaction, the friction heat and the plastic deformation can lead to material transfer from the abrasible to the tip and vice versa. This transfer can lead to chemical exchanges and to the formation of new compounds on the blade or on the abrasible material. These compounds can have a lower melting temperature. They can melt and after solidification, forming a hard and dense layer on the abrasible material. Eventually this hard layer can lead to blade damage if a second contact occurs [Borel 1989].

- *Micro-rupture*

Micro-rupture is a brittle wear mechanism in which small cracks propagate in the abrasible material and form micrometer size chunks that are ejected from the abrasible surface [Shmidt 1997].

- *Rupture*

Rupture is a detrimental wear mechanism in which cracks propagate in the abrasible material releasing large debris [Shmidt 1997].

- *Thermal shock*

Thermal shock appends in brittle abrasible materials when there is a step temperature rise during rubbing. This heating can generate a fracture of the coating due to an excessive localized thermal distortion [Schmidt 1997].

2.5.3 Abradability wear maps

Wear mechanism analyses give important tribological informations, essential to the evaluation of an abradable material and to the modelling of the interaction between a blade and an abradable coating. However, for a given abradable material, the dominant wear mechanisms can be affected by the input parameters programmed in the abradability test rig. Abradability wear maps are then used to synthesize visually the results of many abradability tests. These maps present the dominant wear mechanisms as a function of two contact parameters [Lim 1987, Ashby 1990]. These parameters can be the contact pressure, sliding velocity, penetration depth, incursion speed or temperature. In the field of abradability, the wear maps are often drawn at a given temperature with the sliding speed as primary variable versus the incursion speed (Figure 2.6) or versus the incursion depth (Figure 2.7) [Schmidt 1997, Cuny 2012]. These maps help visualizing the range of speed and of temperature in which an abradable is going to perform well. Ultimately, wear maps are used to compare and select adequate abradable material candidates.

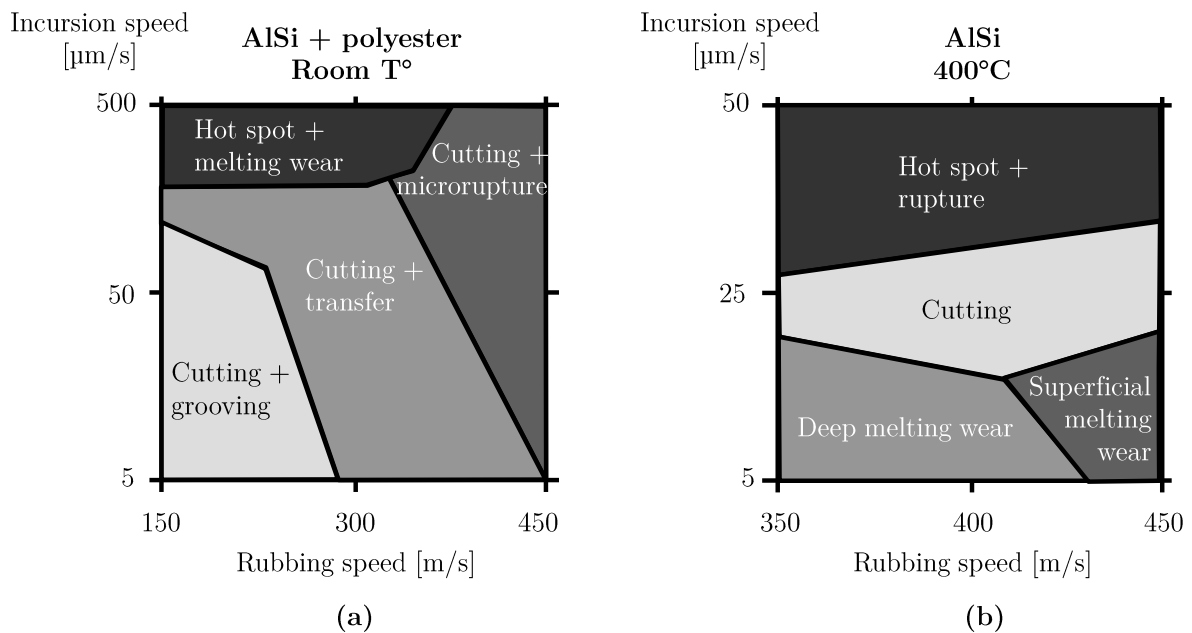


Figure 2.6: Wear maps measured with the Oerlikon Metco classical abradability test rig; (a) AlSi + polyester abradable material at room temperature; (b) dense AlSi abradable material at 400°C. Picture from [Schmidt 1997]

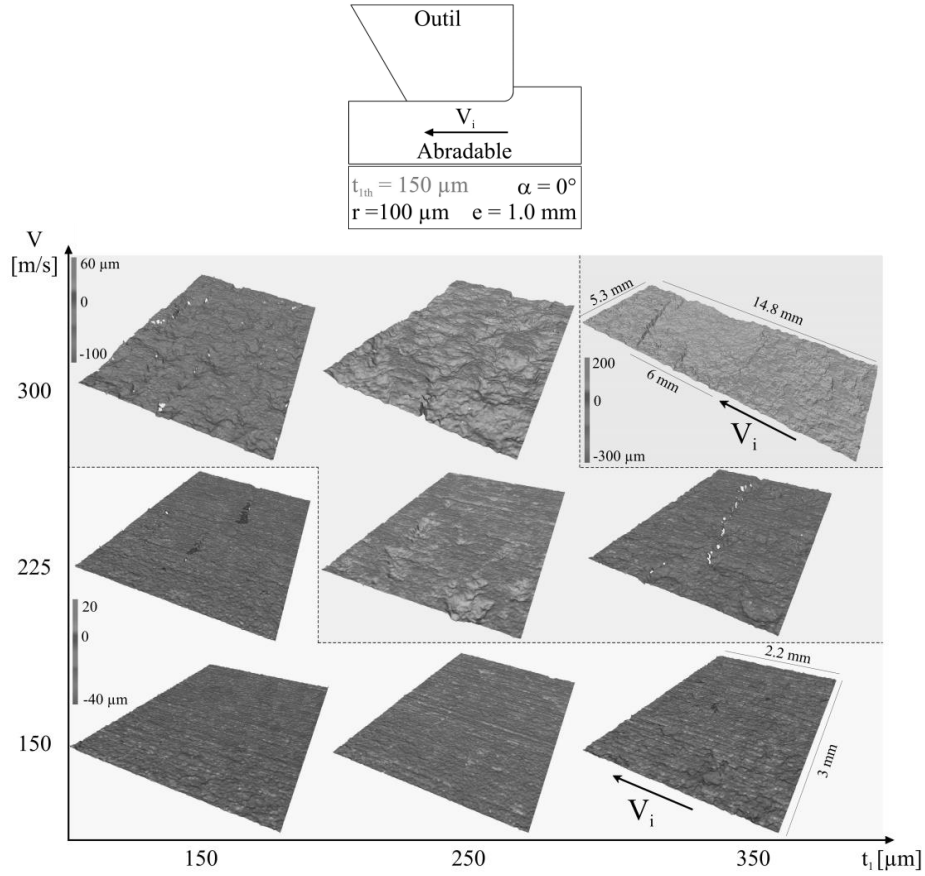


Figure 2.7: Wear map obtained with the La.B.P.S. – E.N.I.M. linear abrasability test rig on the AlSi and polyester abrasible material at room temperature. The colored map shows the level of roughness of the surface of the sample. Picture from [Cuny 2012]

2.5.4 High speed abrasible blade interactions

In addition to the observation of wear mechanisms, some researchers investigated the interactions between the tip of the blade and the abrasible in order to explain the origin of some wear mechanisms. Schmidt suggested that, at rubbing speeds higher than 100 m/s, most abrasible materials are pushed under the tip and release in the rear of the tip [Schmidt 1997]. This mechanism was supported by experiments using blades of different thicknesses. The rebound of the abraded particles under the tip damages more the thicker blades than the thinner ones (Figure 2.8).

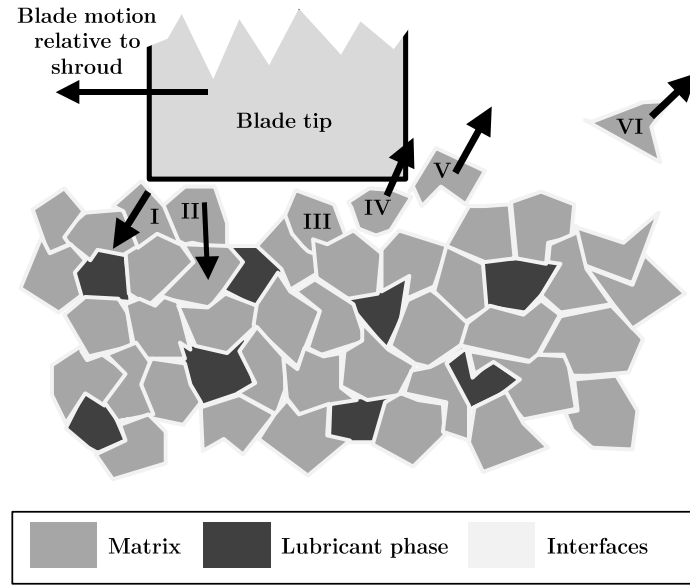


Figure 2.8: Schmidt description of the phenomenology of interaction between the tip of a blade and any abradable materials. The abradable particles are pushed under the blade and released in the back of the tip. Picture from [Schmidt 1997].

In their investigations on the aluminum-silicon polyester abradable material, Cuny et al. suggested that during a contact with the blade, part of the abradable material would form a chip, or debris in front of the blade and the rest would be pushed under the blade. She observed that using a thicker blade creates a longer surface of friction with the abradable which in turn increased the smearing of the abradable surface. If the radius of the tip of the blade increased, so does the quantity of abradable material pushed under the blade along with the amount of detrimental wear by fracture of the coating [Cuny 2012].

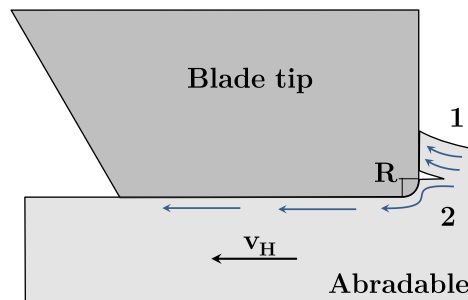


Figure 2.9: Phenomenological description of the accommodation of the blade tip incursion by the abradable coating. Picture from [Cuny 2012].

Studying the same family of abradable materials, Mandard et al. proposed an interaction scenario based on four mechanisms: debris release, surface shearing,

compaction and elastic recovery (Figure 2.10) [Mandard 2015]. They noted that a significant part of the deformation was accommodated by densification of the abradable. Using 3D X-ray tomography combined with image analysis, they showed that there is less porosity near the surface of the abradable after a contact with the blade. Concerning the microstructural evolution of the abradable, their observations showed that both inside the chip and underneath the surface, the polymer phase exhibits large plastic deformations. The metallic phase fragments and gets dragged with the polymer forming a third body that can either be released as a debris, transfers to the tip or remains on the wear track surface.

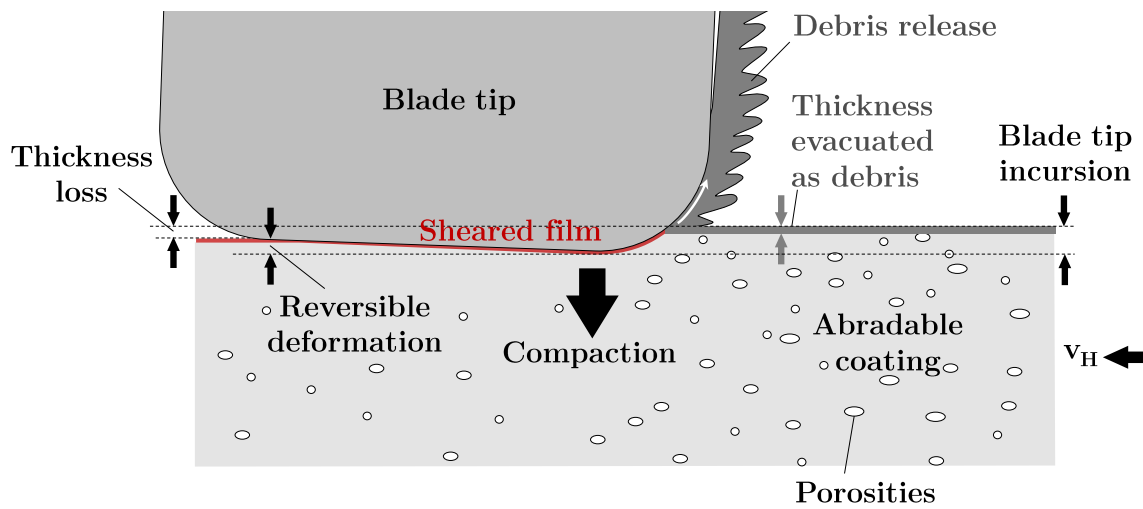


Figure 2.10: Schematic representation of the accommodation of the blade tip incursion by the abradable coating. Picture from [Mandard 2015].

- *Numerical modelling of the blade casing interactions*

Once the wear mechanisms are highlighted on test rigs, only a part of the validation work is done. The remaining work consists mainly in predicting and simulating the behavior of the abradable material inside an engine with real operating conditions. The purpose of these simulations is to take into account the effect of the contact on the abradable and on the rest of the engine. Indeed, due to the high rotational speed and the flexibility of the blades, interactions between a rotor blade and a casing give rise to complex phenomena involving heat generation, deformation, debris, thermal expansions and vibrations in the rotor and in the stator part. Many numerical simulations were developed to predict the vibrational state of the engine after one or many contacts and also to estimate the amount of abradable removed by the blade.

Due to the complexity of such numerical simulations, in most cases, only simplified wear mechanisms are implemented such as plastic compression [Legrand 2009], elastoplastic compression with viscoplasticity [Batailly 2013] as well as cutting and fragmentation of the abrasable [Berthoul 2015]. For a detailed review, see [Ma 2016]. These simulations present some advantages over tests on real engines. Indeed, they are more cost effective and less time consuming. However, these simulations require specific inputs such as the mechanical and thermal properties of the abrasable material which are not always easy to measure.

2.5.5 Mechanical properties of abrasable materials

Given the brittle nature and the heterogeneity of most abrasable materials, measuring their mechanical properties can be quite difficult. Ma and Matthews performed bending, tensile tests and micro-hardness measurements on systems composed of the abrasable on a steel substrate [Ma 09, Ma 07]. The effect of the substrate is estimated based on some hypotheses and then subtracted from the measurements to isolate the abrasable properties (Table 2.6).

Table 2.6: Mechanical properties of abrasable materials measured by Ma and Matthews [Ma 09, Ma 07]

Abrasable material	Young's Modulus [GPa]	Ultimate tensile strength [MPa]	Vickers microhardness [MPa]
AlSi-Polyester	2.08	8.6	20.7
AlSi-Graphite	3.11	23.1	22.2
Ni-Graphite	3.95	48.6	33.9

Johnston and Evans developed a method to produce free standing abrasable specimens using a dissolvable substrate that resists the thermal spraying process. After machining the sample, they performed uniaxial tensile tests on two different abrasable materials (Table 2.7). Their results show that these metallic abrasable materials exhibit very low deformations at break (less than 1%). Additionally, the stress-strain curves present a non-linear behavior (Figure 2.11). The variability of their results is attributed to the randomly distributed shape and size of the defects in the microstructure [Johnston 2007].

Table 2.7: Mechanical properties of abradable materials measured by Johnston [Johnston 2011]

Abradable material	Young's Modulus [GPa]	Ultimate tensile strength [MPa]	Uniform deformation [%]
AlSi-hBN	17.2 ± 1	29.5 ± 6.3	0.24 ± 0.07
NiCrAl-Bentonite	1.4 ± 0.2	3.2 ± 0.14	0.68 ± 0.25

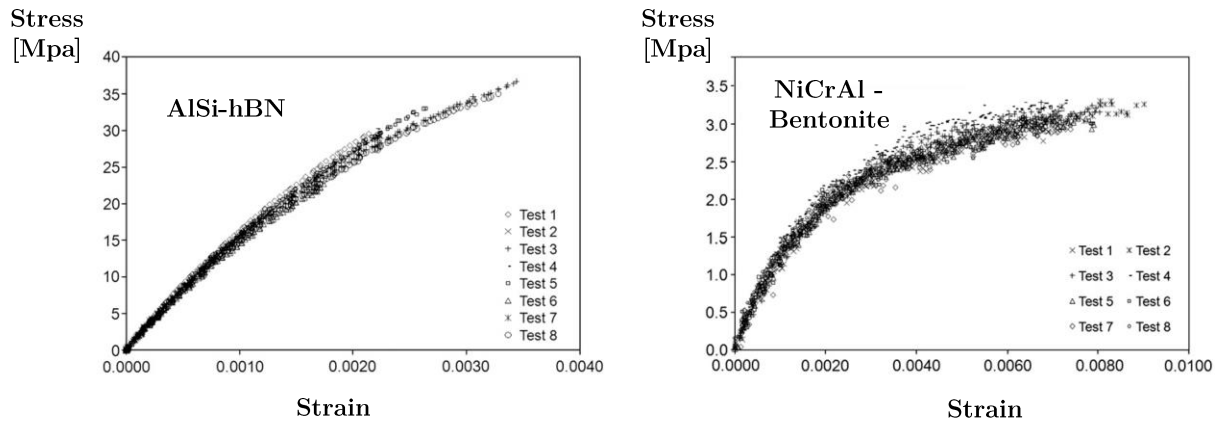


Figure 2.11: Tensile stress-strain curves obtained from free-standing abradable specimens. Picture from [Johnston 2011].

Baiz et al. also studied the mechanical properties of free-standing AlSi-hBN abradable specimens. The samples are separated from the substrate by abrasive wire cutting. Samples are then subjected to tensile, compression tests as well as Brazilian tests. Their results showed that the material accommodates the deformation by compaction under compressive stress and by crack initiation and propagation in area subjected to tensile stress. In addition, cyclic tensile tests reveal that, even though the deformations at break are very low, these materials exhibit some degree of plasticity before breaking, (Figure 2.12) [Baiz 2011].

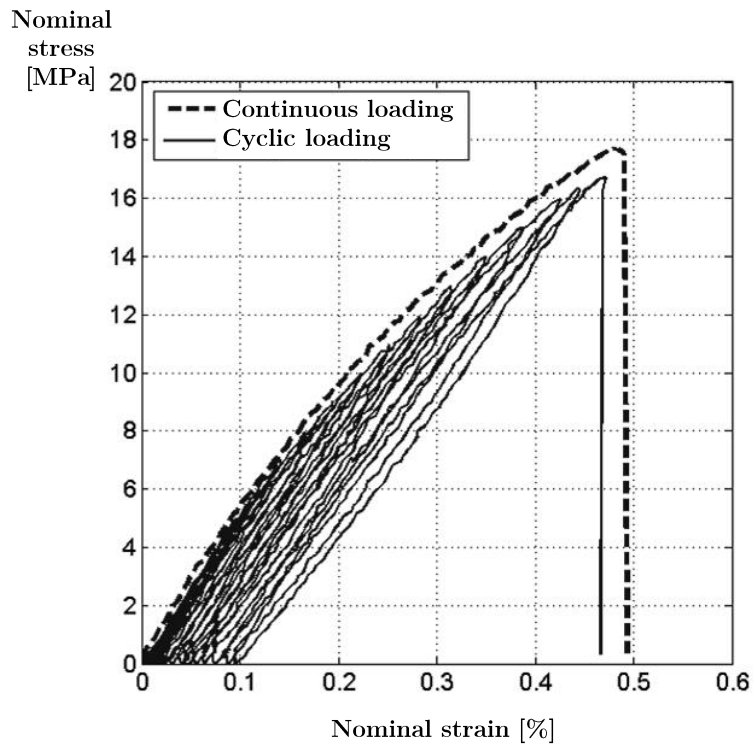


Figure 2.12: Tensile stress-strain curves obtained from a free-standing AlSi-hBN abradable specimen. Picture from [Baiz 2011].

2.5.6 Erosion resistance

In addition to their abrasability, abradable materials are supposed to be resistant to erosion. Indeed, as stated in Section 2.2, suspended particles are ingested in the airstream of the engines. Therefore, many components of the engine, including the abradable materials are subjected to solid particles erosion [Hamed 2006, Tabakoff 1992]. Achieving a satisfactory level of abrasability and erosion resistance is often considered paradoxical as one requires the material to be weak and the other one needs it to be resistant. The conflicting properties of combining a good erosion resistance and abrasability is often mentioned [Lavery 1982, Colomban 1999, Hendricks 2004, Ma 2007, Bardi 2008, Dowson 2008, Johnston 2011, Delebarre 2014, Fois 2015] but rarely studied. Chappel et al. studied the effect of the tensile strength of an abradable material made of felted and sintered metal fibers on the erosion resistance as well as on the abrasability [Chappel 2001]. By varying the processing conditions, they showed that, for this particular abradable material, increasing the tensile strength would

improve the abrasability but lowering the erosion resistance. Their results suggest that a compromise between the two properties can be found (Figure 2.13).

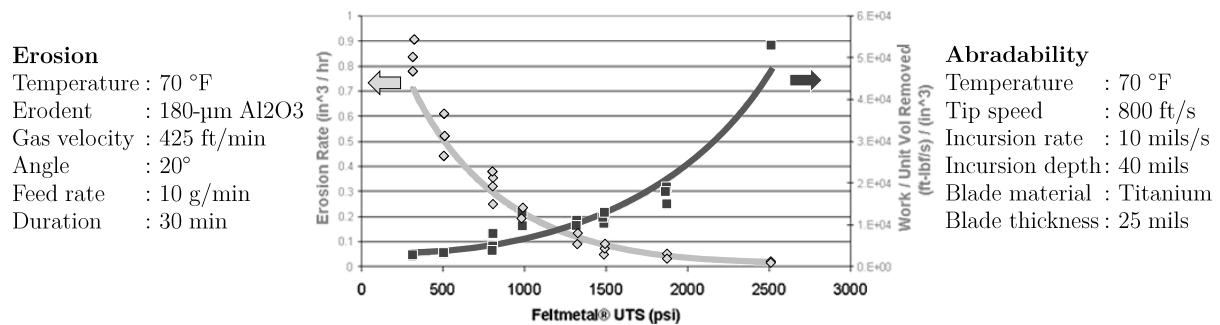


Figure 2.13: Evolution of the erosion rate and the abrasability as a function of the ultimate tensile strength (UTS) of different Feltmetal® abrasable materials. Picture from [Hendricks 2004].

Yi et al. studied the erosion resistance of a variety of thermally sprayed abrasable materials [Yi 2002]. Their study offers a phenomenological description of the erosion wear mechanisms depending on the angle of impact with the surface. They observed that, at 90° impact angle, the erosion happens via indentation, extrusion of lips and cutting of the lips as well as by debonding of metallic phase by repeated impact. At 30° angle, the erosion wear is produced by cutting, plowing and tunneling via the weaker grain phase (Figure 2.14 and Figure 2.15).

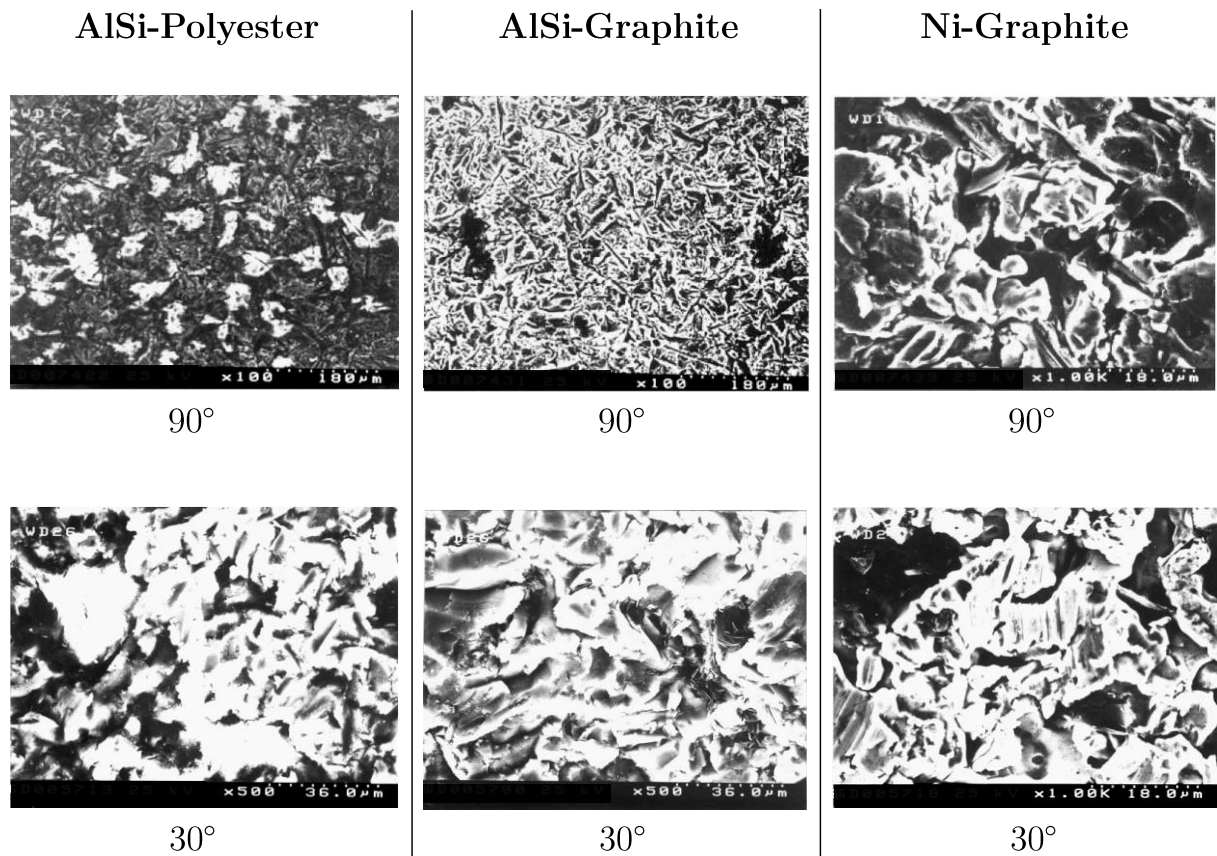


Figure 2.14: Scanning electron microscopy of the surface of different abrasable materials after erosion tests with 100 mesh Al_2O_3 solid particles at 50 m/s. Picture from [Yi 2002]

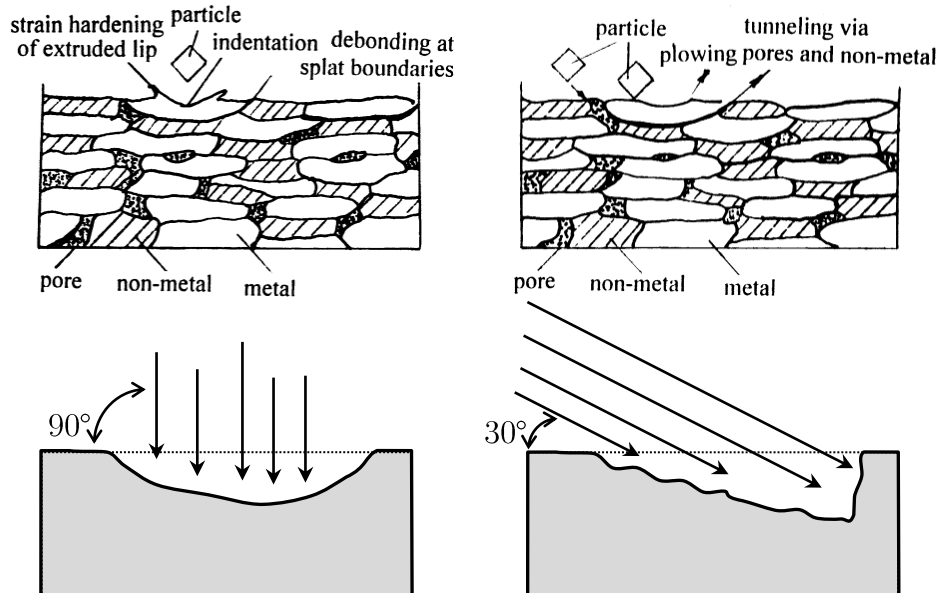


Figure 2.15: Schematic representation of solid particles wear mechanisms in thermally sprayed metal matrix abrasable material. Picture from [Yi 2002]

2.6 Summary

Abradable materials are used to improve the performances of turbo engines by acting as a non-contacting sealing system. These materials have to comply with a lot of design specifications. Many abradable materials are commercially available and the choice of a material is mainly dictated by the position in the engine. The abradability of these materials comes from their high elasticity, low yield strength or from low fracture toughness. Dedicated test rigs are used to evaluate the performance of abradable materials. The complexity of these tests is increased by the number of wear mechanisms and interaction scenarios between the abradable material and the blade. Abradables material should also be resistant to solid particles erosion. In addition to be a conflicting property with the abradability, erosion also involves a large panel of specific wear mechanisms.

Chapter 3 State of the art on contact and wear modelling

The previous chapter showed that interactions between a blade and an abradable material lead to a panel of different wear mechanisms and also that modelling these interactions require the knowledge of the mechanical properties of the abradable material. This chapter explores the literature with three main questions: How do mechanical properties relate to contact forces or to a wear volume? Is it possible to model many wear mechanisms with a single model? In which case will a wear mechanism be the dominant one? In order to answer these questions, some important physically based models of contact, wear, cutting and erosion are presented. Understanding these models is important as they are the starting point of more advanced models described in Chapter 4 and used during the optimization in Chapter 7. Before entering in the physical description of these models, the basic wear mechanisms are described from the perspective of contact mechanics and tribology (Figure 3.1). These mechanisms present some common points and some differences with the ones described in the abradable literature (see Section 2.5.2).

3.1 Wear mechanisms

In tribology, wear mechanisms are the description of the transformation experienced by a surface before and after the contact. The following list represents some of the mechanisms having the closest similarities with the mechanisms encountered in abradable materials.

- *Plowing*³

Plowing is the elastoplastic displacement of one material around the tip or around the contact between two asperities in the surfaces. Plowing can produce wear by a combination of work hardening, lips formation and finally tearing of highly deformed surface volume elements upon repeated passage over the same wear track [Gahr 1998].

³ « Labourage » in French.

- *Cutting*

Cutting is the separation of a part of the surface of a material by the formation of a chip. The chip is formed in the front of the tip or the front of an asperity in the contact area [Gahr 1998]. The chip can be continuous or discontinuous [Merchant 1945 a].

- *Adhesive wear*

Adhesive wear happens when a strong interface is created between two surfaces in motion one relatively to the other. If the interface is tougher than the bulk material beneath the surface, a part of the material can be teared off by the other surface. The adhesion between the two surfaces can be generated due to the combination of the friction heat, the normal load and the plastic deformation. This combination can create a mechanical anchorage, a hot sticking of melted surface or the welding of one of the surface on the other.

- *Delamination wear*

In the field of contact mechanics, delamination wear relates to the loss of material due to subsurface planar crack propagation [Stachowiak 2005]. This delamination can occur at the interface between the matrix and reinforcement [Hong 2011], between a coating and a substrate or by ductile fracture of a thin layer of material at a finite distance below the surface [Jahanmir 1977].

- *Spalling* ⁴

Spalling is the formation of chunks of material by brittle fracture under and around the surface at the contact points [Stachowiak 2005]. The main difference with delamination is the non-directionality of the crack.

- *Fatigue wear*

Fatigue wear happens after a progressive deterioration of the surface of a material by repeated contact [Stachowiak 2005]. Each interaction propagates subsurface cracks

⁴ « Ecaillage » in French

deeper by fatigue. After a certain number of interactions, the crack is long enough to join the surface and form a chunk of material.

- *Melting wear*

Rubbing between two surfaces can locally generated a lot of heat by friction and by plastic deformations. Melting wear occurs when the temperature rise above the melting point of one of the two materials in contact [Stachowiak 2005].

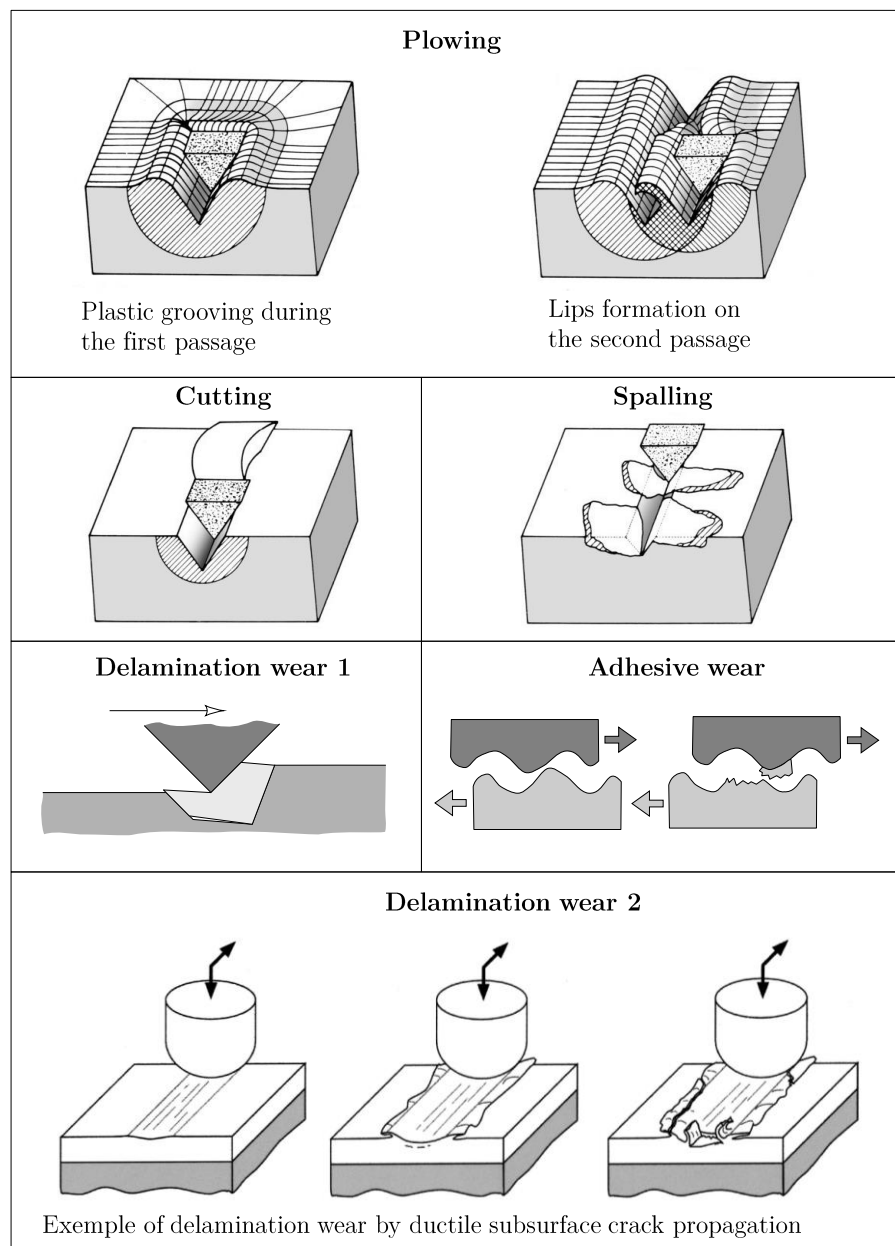


Figure 3.1: Schematic representation of fundamental wear mechanisms. Picture from [Wang 2003, Stachowiak 2005, Gahr 1998]

3.2 Indentation models

Indentation consists in pushing vertically one material against another. From a mechanical point of view, indentation is associated with many basic questions: How are the indentation forces related to the mechanical properties of the base material? How does the geometry of the tip affect the response of the material? What is the relationship between the penetration depth and the forces? The present section explores these questions through a description of some physics-based indentation models.

3.2.1 The hardness

In order to simplify the indentation problems as much as possible, it is often assumed that the tip is rigid. In addition, the base material is generally considered to be an infinite half-space. In reality, the finite dimension of the sample can have an effect on the indentation response. However, as long as the depth and width of the contact are smaller than 10 times the size of the sample, this hypothesis gives satisfactory results. In the field of contact mechanics, the contact forces are generally related to a material property known as the hardness (H). H represents the mean vertical pressure under the tip. It is calculated by dividing the vertical force of contact (F_V) by the projected area of contact (A) in the form

$$H = \frac{F_V}{A}. \quad \text{Eq. 3.1}$$

The hardness and the area of contact are both affected by the mechanical properties of the base material. Many models are used to describe the effect of the mechanical properties on the indentation response. Some of the models are briefly described hereunder.

3.2.2 Ideal elastic models

Indentation of ideally elastic materials has been widely studied. Hertz, Sneddon, Boussinesq, Brothers, Flamant and many others developed solutions in elastic contacts [Johnson 1985, Sneddon 1951, Brothers 1977]. A summary of some important elastic indentation models can be found in Table 3.1. The hardness of elastic material is

denoted H_{el} . In that table, both the mean pressure under the tip and the contact area, including the sink-in of the surface, can be estimated for simple geometries of tip. In these models, the base material is considered to obey Hooke's law of linear elasticity. Consequently, the indentation is considered completely reversible. After unloading, the recovered depth (h_{el}) is equal to the depth of indentation (h).

3.2.3 Rigid perfectly-plastic models

Rigid perfectly plastic materials are theoretical materials in which deformation occurs only if the stress exceeds the yield strength. Once exceeded, the material deforms plastically with no strain hardening. Perfectly plastic indentation modelling has been mostly addressed using the slip line field theory [Hill 1947, Lockett 1963, Shield 1955, Druyanov 1994] and later through finite element simulations [Kermouche 2007]. Equations for the contact area and the mean pressure are available for simple tip geometries. Although not proposed in a closed form solution, these models can be fitted with a polynomial regression to give an approximated continuous solution [Yu 1996] (Table 3.1). In these models, the indentation is considered fully irreversible. After unloading, the residual depth (h_{pl}) is the same as the indentation depth (h).

3.2.4 Elastic perfectly-plastic models

The behavior of elastic perfectly-plastic materials depends on the level of stress. If the stress is lower than the yield strength, the material behaves like an ideal elastic material. When the yield strength is reached, the material deforms plastically with no strain hardening. After deformation, if the stress goes back to a value lower than σ_y the material behaves again like an ideal elastic material. The modelling of the indentation of elastoplastic material was approached through the spherical and cylindrical cavity expansion models of Johnson [Johnson 1970]. In this model, the indenting part of the tip is inscribed in a hydrostatic spherical core surrounded by an elastoplastic material. This core is forced to increase in size during the indentation. This approach simplifies the problem to a spherical cavity pushing on an elastoplastic surrounding (Figure 3.2). Yu and Blanchard also proposed a methodology to derive the hardness of elastoplastic material based on a combination of the ideal elastic model and the perfectly plastic model [Yu 1996]. In their model, the pressure distribution

under the tip is a combination of an elastic area that follows an ideal elastic model solution and a plastic area that follows the rigid perfectly plastic solution. The size of the plastic zone is defined by the intersection between the ideal elastic and the perfectly plastic pressure distribution (Figure 3.3).

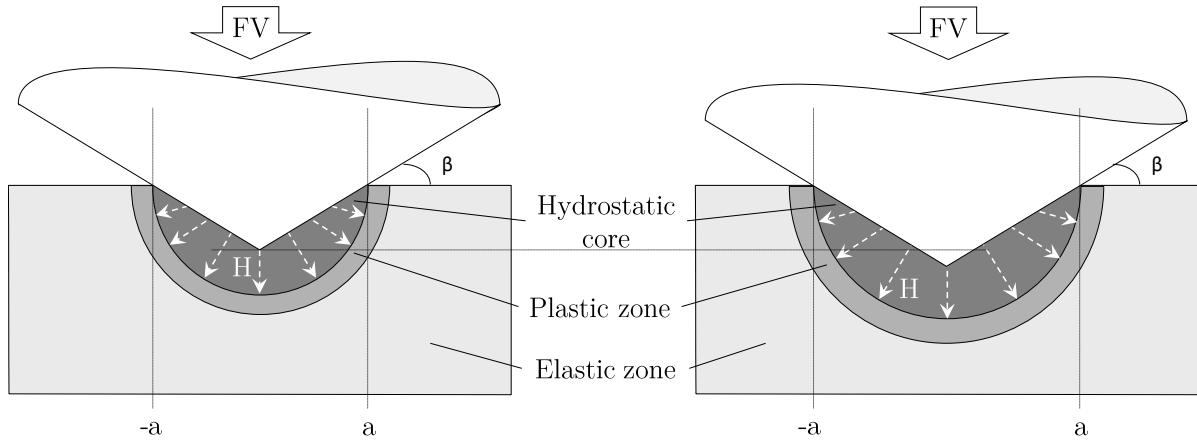


Figure 3.2: Schematic representation of the expansion cavity model of Johnson [Johnson 1970]. The indenting tip is associated to a hydrostatic spherical core increasing in size during the indentation in an elastoplastic material.

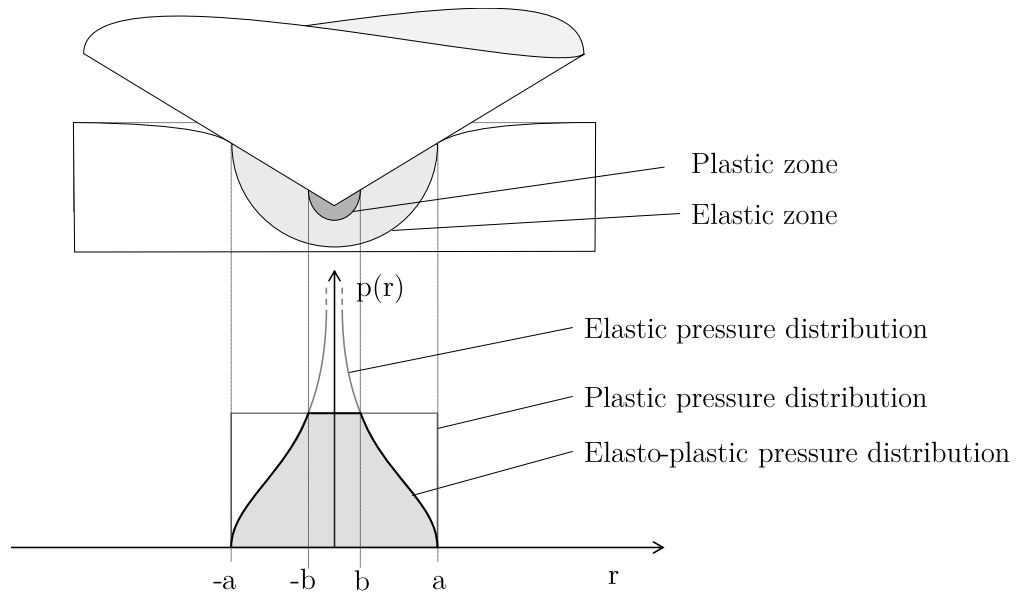


Figure 3.3: Schematic representation of the elastoplastic localized pressure yielding model of Yu and Blanchard. The normal pressure under the tip follows the elastic model up to the yield strength where the perfectly plastic model is followed.

These two elastoplastic approaches give a relationship between the hardness, the Young's modulus and the yield strength for different tip geometries. The cavity

expansion model is simpler yet less accurate in the case of materials with either very high or very low values of σ_y/E , whereas Yu and Blanchard model shows a continuous transition between the elastic and the plastic model (Figure 3.4 and Figure 3.5).

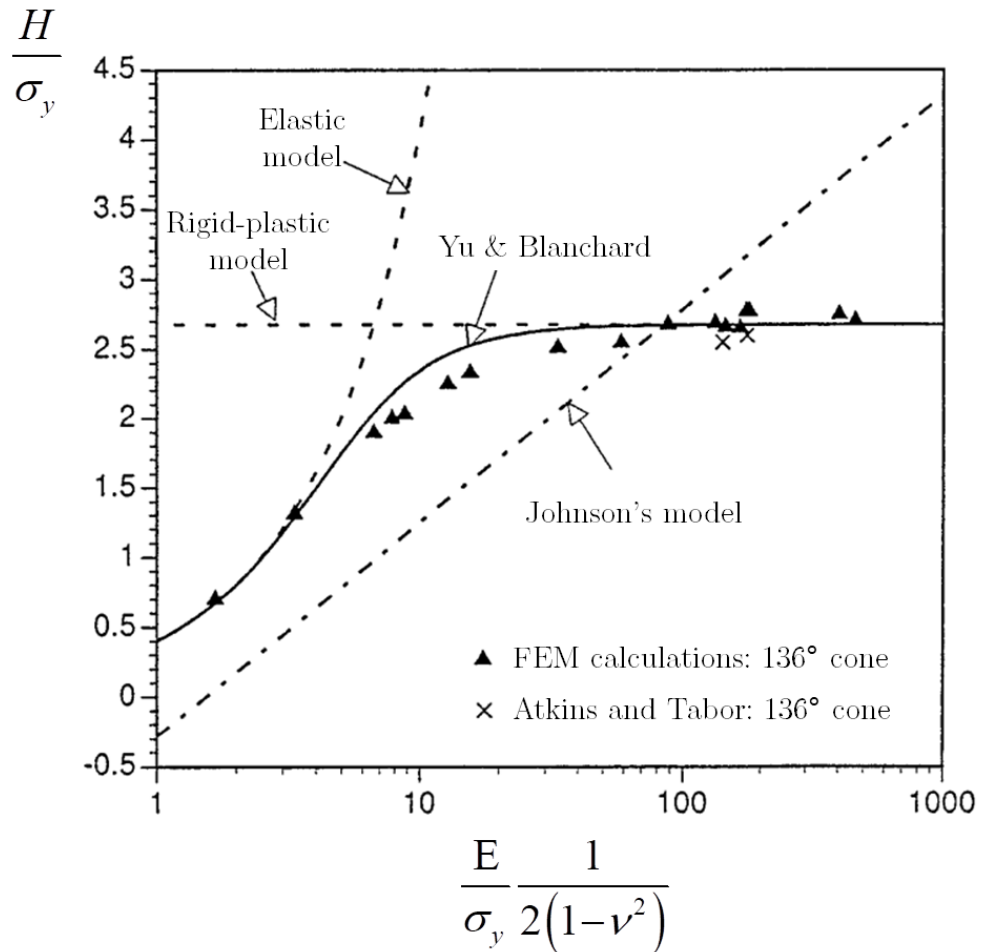
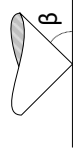
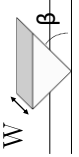
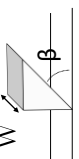
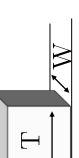
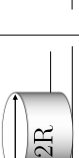
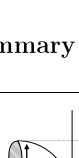
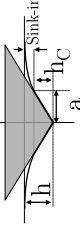
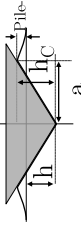
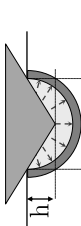


Figure 3.4: Comparison of the elastic, plastic and the elastoplastic model of Johnson and Yu and Blanchard with finite element simulations in the case of conical indentations. From [Yu 1996].

Table 3.1: Summary of elementary contact mechanics models for simple geometries of tip.

General case $F_V = HA$	Cone 	Wedge 	Half wedge 	Flat wedge 	Cylinder 	Sphere 
						
Ideal Elastic	Rigid-plastic	Elasto-plastic	Cavity expansion	Yu and Blanchard		
$A = \pi a^2$	$A = \pi a^2$	$A = 2aW$	$A = aW$	$A = 2aW$	$A = \pi a^2$	$A = \pi a^2$
$H_d = C_d E \tan \beta / (1 - \nu^2)$ $C_d = 1/2$ $a_d = D_d h \cot \beta$ $h_{\text{sink-in}} = h(1 - D_d)$ $D_d = 2/\pi$	$H_d = C_d E \tan \beta / (1 - \nu^2)$ $C_d = 1/2$ $a_d = D_d h \cot \beta$ $h_{\text{sink-in}} = h(1 - D_d)$ $D_d = 2/(1 + \ln 2)$	$H_d = C_d E \tan \beta / (1 - \nu^2)$ $C_d = \eta(2.57 - 1.318\beta)$ $0 < \beta < 0.65$ $a_d = D_d h \cot \beta$ $h_{\text{pile-up}} = h(D_d - 1)$ $D_d \approx 1.282$	$H_d = C_d E \tan \beta / (1 - \nu^2)$ $C_d = \eta(2.52 - 1.47\beta + 2.52\beta^2)$ $a_d = D_d h \cot \beta$ $h_{\text{pile-up}} = h(D_d - 1)$ $D_d \approx 1.46$	$H_d = C_d \frac{E}{a_d} \frac{1}{(1 - \nu^2)}$ $C_d = \frac{\pi}{4 \ln 2}$ $a_d = T/2$ $h_{\text{sink-in}} = h$	$H_d = C_d \frac{E}{a_d} \frac{1}{(1 - \nu^2)}$ $C_d = \frac{2}{\pi}$ $a_d = R$ $h_{\text{sink-in}} = h$	$H_d = C_d \frac{E}{a_d} \frac{1}{(1 - \nu^2)}$ $C_d = \frac{4}{3\pi}$ $a_d = \sqrt{hR}$ $h_{\text{sink-in}} = h + \sqrt{R^2 - hR} - R$
$H_p = C_p \sigma_y$ $C_p \approx \eta(2.845 - 1.3612\beta)$ $0 < \beta < 0.65$ $a_p = D_p h \cot \beta$ $h_{\text{pile-up}} = h(D_p - 1)$ $D_p \approx 1.282$	$H_p = C_p \sigma_y$ $C_p = \eta(2.57 - 1.318\beta)$ $0 < \beta < 0.52$ $a_p = D_p h \cot \beta$ $h_{\text{pile-up}} = h(D_p - 1)$ $D_p \approx 1.4614$	$H_p = C_p \sigma_y$ $C_p = \eta(2.57 - 1.318\beta)$ $0 < \beta < 0.52$ $a_p = D_p h \cot \beta$ $h_{\text{pile-up}} = h(D_p - 1)$ $D_p \approx 1.4614$	$H_p = C_p \sigma_y$ $C_p = \eta(2.52 - 1.47\beta + 2.52\beta^2)$ $a_p = D_p h \cot \beta$ $h_{\text{pile-up}} = h(D_p - 1)$ $D_p \approx 1.46$	$H_p = C_p \sigma_y$ $C_p = \eta(1 + \frac{\pi}{2})$ $a_p = T/2$	$H_p = C_p \sigma_y$ $C_p = \eta(2.845 - 0.4921a/R)$ $a_p = \sqrt{D_p R h}$ $h_{\text{pile-up}} = R - \sqrt{R^2 - D_p R h} - h$ $D_p = 2.7$	$H_p = C_p \sigma_y$ $C_p = \eta(2.845 - 0.4921a/R)$ $a_p = \sqrt{D_p R h}$ $h_{\text{pile-up}} = R - \sqrt{R^2 - D_p R h} - h$ $D_p = 2.7$
$H_{dpl} = C_{dpl} E \tan \beta / (1 - \nu^2)$ $C_{dpl} = \frac{2\sigma_y(1 - \nu^2)}{3E \tan \beta} \left[1 + \ln \left(\frac{E \tan \beta}{3\sigma_y} \right) \right]$	$H_{dpl} = C_{dpl} E \tan \beta / (1 - \nu^2)$ $C_{dpl} = \frac{2\sigma_y(1 - \nu^2)}{3E \tan \beta} \left[1 + \ln \left(\frac{E \tan \beta}{3\sigma_y} \right) \right]$	$H_{dpl} = C_{dpl} E \tan \beta / (1 - \nu^2)$ $C_{dpl} = \frac{\sigma_y(1 - \nu^2)}{\sqrt{3} E \tan \beta} \left[1 + \ln \left(\frac{4E \tan \beta}{3\sigma_y} \right) \right]$	$H_{dpl} = C_{dpl} E \tan \beta / (1 - \nu^2)$ $C_{dpl} = \frac{\pi}{4} \frac{1}{2} \arctan \left(\frac{E \tan \beta}{2C_p \sigma_y (1 - \nu^2)} \right)$ $C_p = \eta(2.52 - 1.47\beta + 2.52\beta^2)$	$H_{dpl} = C_{dpl} \frac{h}{a_{dpl}} \frac{E}{(1 - \nu^2)}$ $C_{dpl} = \frac{1}{2 \ln 2} \left[\arcsin(f) + \frac{1}{f} (1 - \sqrt{1 - f^2}) \right]$ $f = \frac{a_{dpl}}{2 \ln 2} \frac{1}{E} \frac{1}{C_p \sigma_y}$ $C_p = \eta(1 + \pi/2)$ $a_{dpl} = T/2$	$H_{dpl} = C_{dpl} \frac{h}{a_{dpl}} \frac{E}{(1 - \nu^2)}$ $C_{dpl} = \frac{(2 - f)/\pi}{1/(\pi f)}$ $f = \frac{1}{\pi} \frac{h}{a_{dpl}} \frac{1}{C_p \sigma_y}$ $C_p = \eta(2.845)$ $a_{dpl} = R$	$H_{dpl} = C_{dpl} \frac{E}{R} \frac{1}{(1 - \nu^2)}$ $C_{dpl} = \frac{4/3\pi}{3\pi \left(\frac{6}{f} - \frac{2}{f^3} \right)}$ $f = \frac{2}{\pi} \frac{a_{dpl}}{R} \frac{E}{C_p \sigma_y} \frac{1}{(1 - \nu^2)}$ $C_p = \eta(2.845 - 0.4921a/R)$
$H_{dpl} = C_{dpl} E \tan \beta / (1 - \nu^2)$ $C_{dpl} = \frac{1}{2} \tanh \left(\frac{2C_p \sigma_y (1 - \nu^2)}{E \tan \beta} \right)$ $C_p \approx \eta(2.845 - 1.3612\beta)$	$H_{dpl} = C_{dpl} E \tan \beta / (1 - \nu^2)$ $C_{dpl} = \frac{1}{2} \tanh \left(\frac{2C_p \sigma_y (1 - \nu^2)}{E \tan \beta} \right)$ $C_p \approx \eta(2.845 - 1.3612\beta)$	$H_{dpl} = C_{dpl} E \tan \beta / (1 - \nu^2)$ $C_{dpl} = 1 - (2/\pi) \arcsin(1/f)$ $f = \cosh \left(\frac{\pi C_p \sigma_y (1 - \nu^2)}{E \tan \beta} \right)$ $C_p = \eta(2.845 - 1.3178\beta)$	$H_{dpl} = C_{dpl} E \tan \beta / (1 - \nu^2)$ $C_{dpl} = \frac{\pi}{4} \frac{1}{2} \arctan \left(\frac{E \tan \beta}{2C_p \sigma_y (1 - \nu^2)} \right)$ $C_p = \eta(2.52 - 1.47\beta + 2.52\beta^2)$	$H_{dpl} = C_{dpl} \frac{h}{a_{dpl}} \frac{E}{(1 - \nu^2)}$ $C_{dpl} = \frac{1}{2 \ln 2} \left[\arcsin(f) + \frac{1}{f} (1 - \sqrt{1 - f^2}) \right]$ $f = \frac{a_{dpl}}{2 \ln 2} \frac{1}{E} \frac{1}{C_p \sigma_y}$ $C_p = \eta(1 + \pi/2)$ $a_{dpl} = T/2$	$H_{dpl} = C_{dpl} \frac{h}{a_{dpl}} \frac{E}{(1 - \nu^2)}$ $C_{dpl} = \frac{(2 - f)/\pi}{1/(\pi f)}$ $f = \frac{1}{\pi} \frac{h}{a_{dpl}} \frac{1}{C_p \sigma_y}$ $C_p = \eta(2.845)$ $a_{dpl} = R$	$H_{dpl} = C_{dpl} \frac{E}{R} \frac{1}{(1 - \nu^2)}$ $C_{dpl} = \frac{4/3\pi}{3\pi \left(\frac{6}{f} - \frac{2}{f^3} \right)}$ $f = \frac{2}{\pi} \frac{a_{dpl}}{R} \frac{E}{C_p \sigma_y} \frac{1}{(1 - \nu^2)}$ $C_p = \eta(2.845 - 0.4921a/R)$

$$\eta = 1(\text{Tresca}), \eta = \frac{2}{\sqrt{3}}(\text{von Mises}), (\beta \text{ in radian})$$

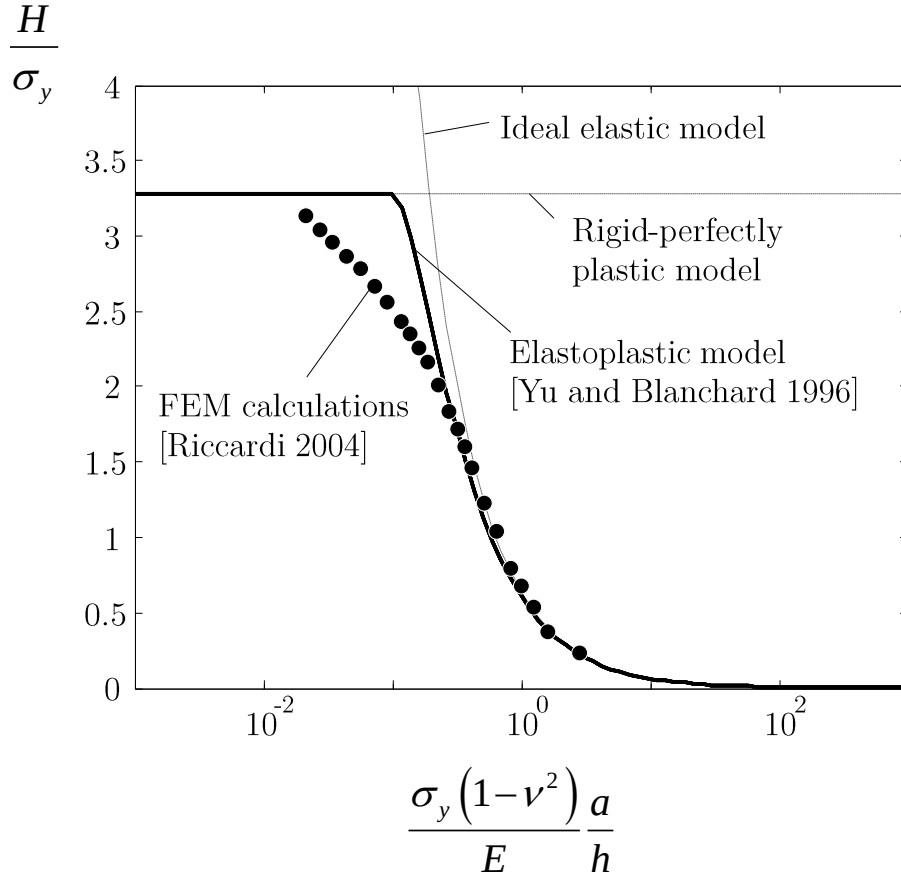


Figure 3.5: Comparison of the elastic, plastic and the elastoplastic model of Yu and Blanchard [Yu 1996] with finite element simulations in the case of a cylindrical indentation on the flat side of the cylinder from [Riccardi 2004]. Here, the hardness is also a function of the radius of the cylinder (a) and of the depth of indentation (h).

These two elastoplastic models focus mainly on modelling the hardness. However, they both omit the calculation of the contact area (A). Yet A is essential to calculate the indentation response (Eq. 3.1). These models are therefore very useful if the tip is flat and offers a constant contact area during indentation. However, if the tip is sharp or rounded, the contact area is affected by pile-up and sink-in effects which make the relationship between h and A harder to model precisely. Indeed, pile-up is a deformation of the surface of the base material going upward against the side of the indenter and resulting in an increase of the contact area at a given depth. [McElhaney 1998, Oliver 2004]. On the other hand, sink-in is a downward deformation of the surface of the base material that reduces the contact area (Figure 3.6).

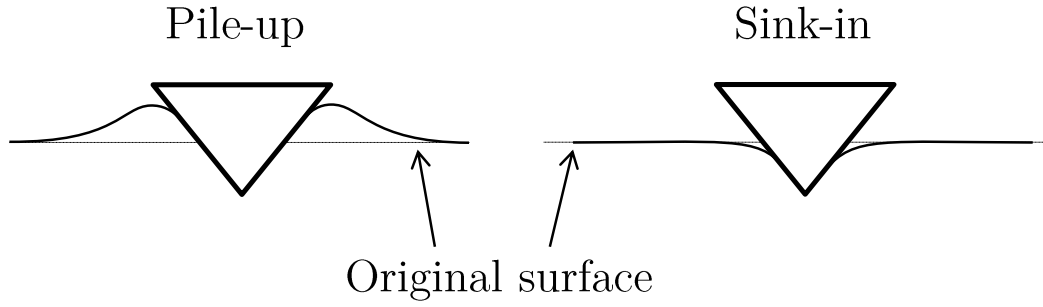


Figure 3.6: Schematic representation of pile-up and sink-in effects.

3.2.5 Elastoplastic models including strain hardening

The effect of the strain hardening was studied by following two distinct approaches: with finite element modelling (FEM) and through analytical modelling. Regarding the analytical route, a lot of discoveries are attributed to Tabor. Tabor proposed a series of relationships between the hardness, Young's modulus, yield strength and strain hardening exponent. The relationship,

$$H = C_{pl} \sigma_y, \quad \text{Eq. 3.2}$$

where $C_{pl} = 2.6 - 3$, was proposed by Tabor based on experimentations and was inspired from the slip line field solutions of Hencky and Ishlinsky [Tabor 1948, Hencky 1923]. Supporting the elastoplastic model of [Johnson 1970], Tabor observed that the hardness can vary with the ratio σ_y/E . In materials where this ratio is large, the hardness does not follow Eq. 3.2. Instead, the constant C_{pl} linking σ_y to H can be as small as 1.5. Tabor also described the effect of the strain hardening on the pile up and on the sink-in. He postulated that a low strain hardening ability brings an important pile-up while a sink-in can occur in a high strain hardening material due to a redistribution of the plastic flow away from the indenter [Tabor 1970] (Figure 3.7).

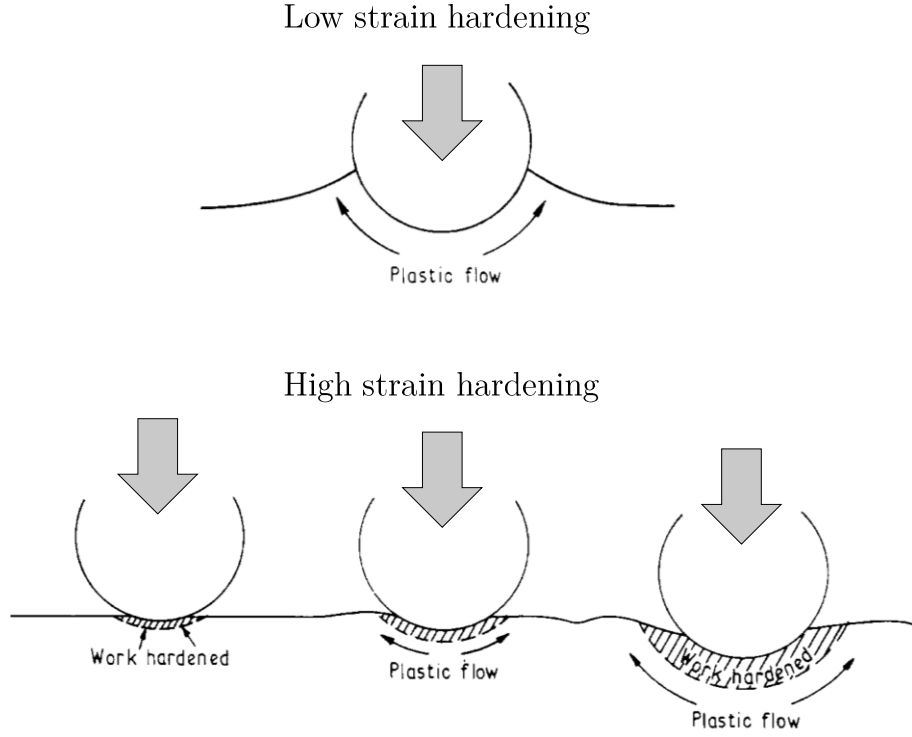


Figure 3.7: Schematic representation of the effect of the strain hardening ability of a rigid plastic material on the plastic flow of the surface during indentation. Indentation in a material with a low strain hardening creates pile-up while a high strain hardening brings sink-in. Picture from [Tabor 1970].

Tabor extended the relationship given in Eq. 3.2 to include the effect of strain hardening with the introduction of the concept of the representative strain under the tip (ϵ_{r-ind}) [Tabor 1948]. In the case of a sphere, the relationship becomes

$$H = C_r \sigma_{r-ind} (\epsilon_{r-ind}), \quad \text{Eq. 3.3}$$

where σ_{r-ind} is a representative indentation stress and

$$\epsilon_{r-ind} [\%] \approx 20h/R, \quad \text{Eq. 3.4}$$

where R is the radius of the tip, h is the depth of indentation and C_r is a constant in the range 2.6 - 3. Concerning pyramidal and conical indenters, because of the geometrical similarity, ϵ_{r-ind} is independent of h . In order to fit with experimental results, Tabor proposed the value

$$\epsilon_{r-ind} \approx 8-10\%. \quad \text{Eq. 3.5}$$

In both cases, the relationship between ε_{r-ind} and σ_{r-ind} can be identified with a stress-strain curve of the base material from a tensile or a compression test. Atkins and Tabor also showed that the opening angle of the tip affects the representative strain [Atkins 1965]. Later on, Johnson proposed the following relationship to take into account the effect of the angle,

$$\varepsilon_{r-ind} = 0.2 \tan \beta, \quad \text{Eq. 3.6}$$

where β is the angle between the surface and the lateral surface of the conical tip [Johnson 1970]. Many researchers then proposed different values for ε_{r-ind} and for C_r to fit experimental or finite elements simulations results (Table 3.2). Surprisingly, these values of C_r and ε_{r-ind} vary over up to one order of magnitude.

Table 3.2: List of values for C_r and ε_{r-ind} in the case of indentation by a cone with a half-opening angle of 70.3° , equivalent to the Vickers and the Berkovich pyramidal tip.

C_r	References	ε_{r-ind}	References
2.7	[Mata 2003]	0.03	[Dao 2001]
3	[Atkins 1965]	0.08 – 0.1	[Tabor 1948, Atkins 1965]
2.8 - 11.5	[Chaudhri 1998]	0.1	[Mata 2003]
		0.25 - 0.36	[Chaudhri 1998]
		0.3	[Giannakopoulos 1999]

The variability of those two parameters was explained later by Hernot [Hernot 2014]. Hernot et al. used finite element simulations to study the indentation of elastoplastic materials with strain hardening. In their simulations, the tip is a cone with a half-opening angle of 70.3° . Concerning the base material, the elastic part obeys the Hooke's law and the plastic part follows a Hollomon's law. This combination gives

$$\begin{cases} \sigma = E\varepsilon & , \varepsilon < \sigma_y/E \\ \sigma = E^n \sigma_y^{1-n} \varepsilon^n & , \varepsilon > \sigma_y/E \end{cases} \quad \text{Eq. 3.7}$$

where n is the strain hardening coefficient. Hernot studied the effect of the adimensional parameter σ_y/E and n on the representative indentation strain and on C_r (Eq. 3.3). His study shows that ε_{r-ind} and C_r are not constant parameters and can vary in a large way with n and with σ_y/E (Figure 3.8 and Figure 3.9).

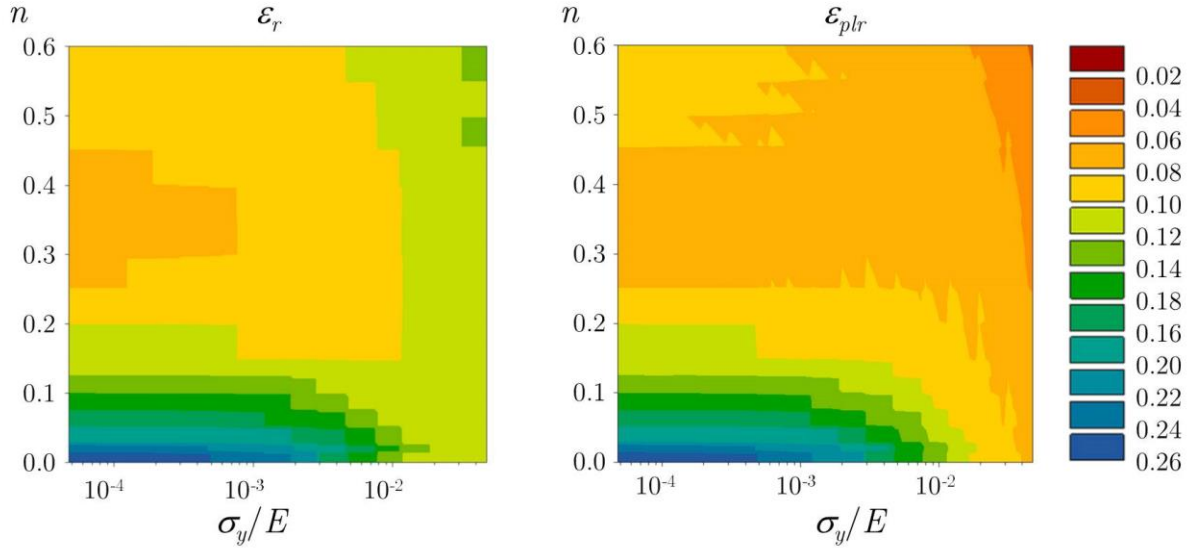


Figure 3.8: Colored map showing the influence of σ_y/E and of n on the total representative strain (ϵ_{r-ind}) and on the plastic representative strain ($\epsilon_{r-pl} = \epsilon_{r-ind} - \sigma_y/E$) during an indentation of an elastoplastic material with a cone of 70.3° of half-opening angle. Picture from [Hernot 2014].

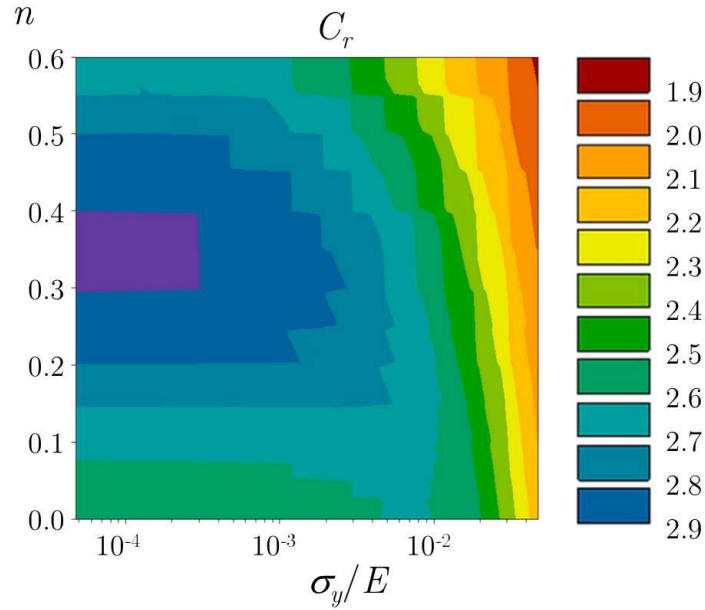


Figure 3.9: Evolution of the parameter C_r with n and with σ_y/E . Picture from [Hernot 2014].

In the same idea, Cheng and Mata also used finite element simulations to study the effect of the two parameters σ_y/E and n but this time, with a deeper focus on the contact area during indentation. Cheng and Cheng used a cone with a half-opening angle of 68° while Mata choose an angle of 70.3° [Cheng 1998, Mata 2002]. Their study showed the evolution of the real height of contact (h_c) with σ_y/E and n (Figure 3.10).

They confirmed the observation of Tabor concerning the effect of the strain hardening coefficient on the pile-up and on the sink-in [Tabor 1970]. They also showed that there is a progressive transition from pile-up to sink-in as the parameter σ_y/E increases.

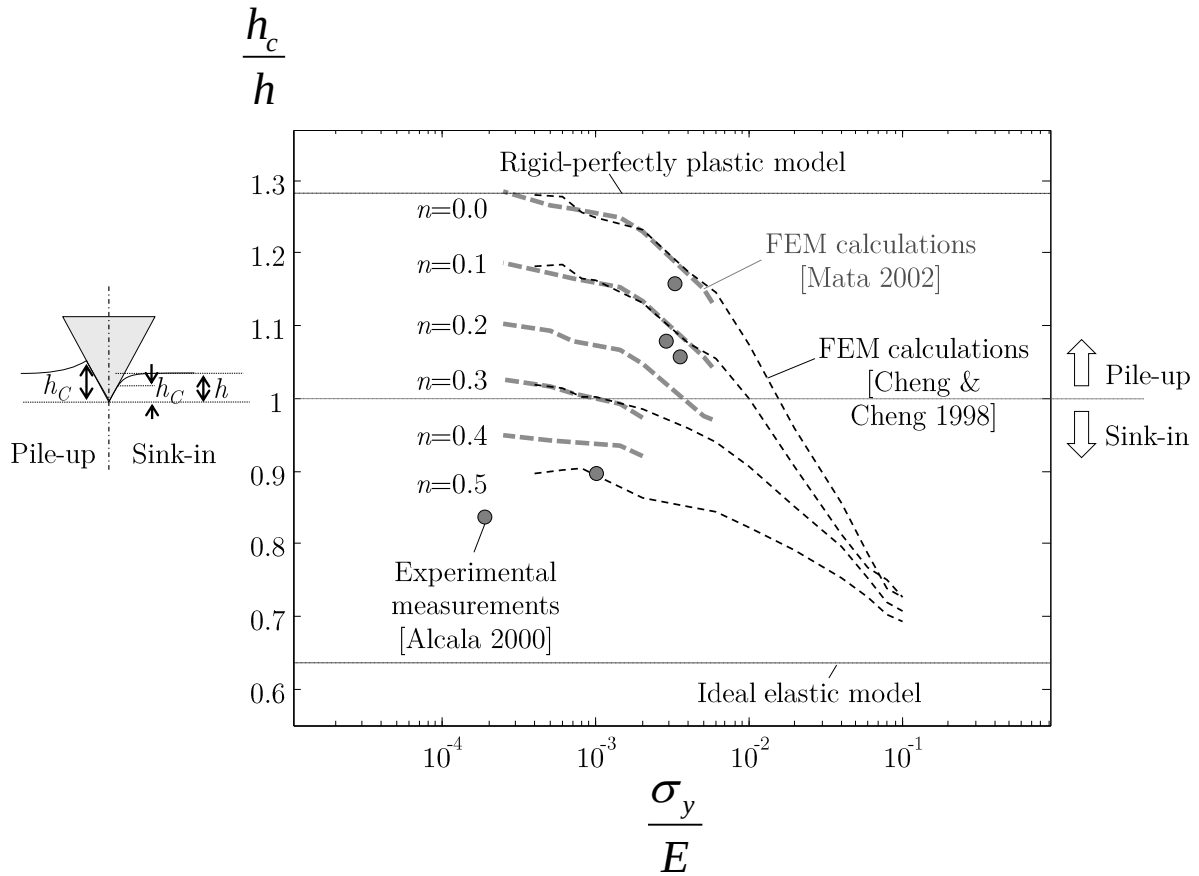


Figure 3.10: Evolution of the contact height (h_c) normalized by the depth of indentation (h) as a function of σ_y/E for different values of n . When h_c/h is bigger than 1, there is some pile-up. When it is smaller than one, the surface sinks-in. These FEM calculations consider indentations with a conical tip on power law strain hardening elastoplastic materials.

Mata and Alcala [Mata 2003] used numerical curve fitting methods on these results to propose an expression for h_c as function of σ_y/E and n giving

$$\begin{cases} \frac{h_c}{h} = I_1(n) + I_2(n) e^{-\frac{\sigma_y}{E} \frac{1}{I_3(n)}} \\ I_1(n) = 8.7760n^3 - 6.7180n^2 + 1.0478n + 0.8148 \\ I_2(n) = -7.7892n^3 + 6.8690n^2 - 2.0273n + 0.4790 \\ I_3(n) = -0.35359n^3 + 0.40038n^2 - 0.05926n + 0.01426 \end{cases} \quad \text{Eq. 3.8}$$

3.2.6 The effect of the indenter elasticity

Indenting a material brings a deformation both in the indenter and in the base material. In some models, the tip is considered as an elastic material. In that case, the deformation of the tip can affect the indentation response. In order to take this deformation into account, the concept of reduced modulus was proposed. In order to visualize the concept, an analogy can be made between a tip indenting an elastic material and two springs being pushed against each other. These two springs in series can be approximated by one equivalent spring (Figure 3.11). This equivalent spring exhibit a lower stiffness than the stiffness of the two springs. The equivalent stiffness follows

$$k_T = \left(\frac{1}{k_1} + \frac{1}{k_2} \right)^{-1}, \quad \text{Eq. 3.9}$$

where k_1 is the stiffness of the upper spring, k_2 is the stiffness of the lower spring and k_T is the equivalent stiffness.

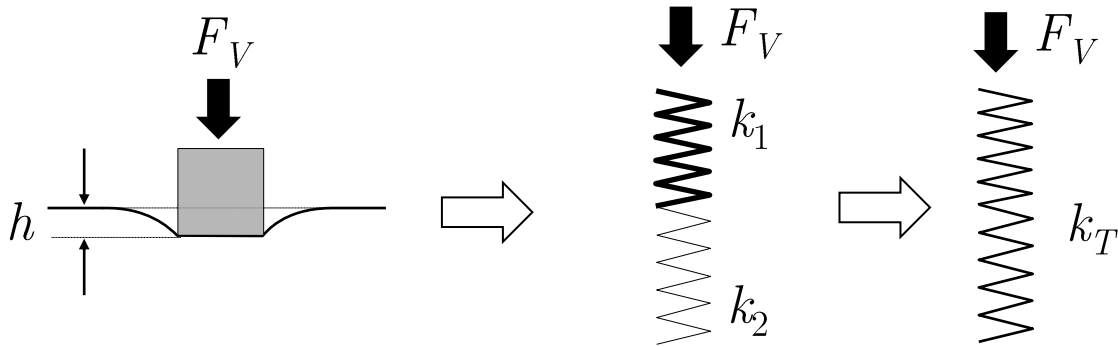


Figure 3.11: Schematic representation of the elastic indentation between a tip and a material represented by two springs being pushed against each other.

The reduced modulus is related to the Young's modulus of the material and the tip through

$$E_{eq} = \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu^2}{E} \right)^{-1}, \quad \text{Eq. 3.10}$$

where E_{eq} is the reduced modulus, E_1 is the Young's modulus of the tip and ν_1 is the Poisson ratio of the tip. E_{eq} can be used instead of the Young's modulus of the base material in indentation models to take into account the elastic deformation of the tip.

3.2.7 Friction

Friction is defined by Coulomb as a force that is opposed to the sliding motion of two surfaces in contact with one another. In his memoir, Coulomb adds that the physical origin of friction comes from either (i) the mechanical anchorage of the asperities of the two surfaces that undergoes deformations when they meet and interact with one another, or (ii), from the molecules of the two surfaces that develop a mutual bonding due to their proximity. That bonding needs to be overcome to produce the movement [Coulomb 1821]. Amontons stated that the friction force is proportional to the force normal to the two surfaces in motion [Popova 2015]. The coefficient of proportionality between the normal force and the friction force is called the friction coefficient (μ). The law of Amontons, also called the law of Coulomb, is a first order approximation of the friction force [Popov 2010]. In the following part of this manuscript, the Coulomb's law is used to model the friction force between two surfaces sliding on one another.

3.2.8 Effect of the friction coefficient on indentation

The indentation models presented previously are based on the hypothesis of a frictionless contact between the tip and the base material. The effect of the friction coefficient (μ) was studied by Ponthot and his colleagues in the case of conical indentations of elastoplastic metallic materials [Ponthot 2009]. They performed FEM calculations with friction coefficients ranging from 0 to 0.15. They showed that increasing the friction coefficient translates into an increase of the hardness of up to 5% and also into a reduction of the pile-up.

3.2.9 Extracting properties from indentation measurements

Contact mechanics models are used extensively to extract properties from indentation measurements. For instance, Oliver and Pharr developed a model to extract the Young's modulus from indentation measurements on a large variety of materials including metals, polymer and ceramics with a reliable level of precision [Oliver 1992]. Given the complexity of the indentation problems, how did they manage to take into account the effect of pile-up and sink-in on the contact area?

Their technique requires the measurement of three quantities: the indentation force, the slope of the unloading curve (S) and the penetration depth (h). They consider that the unloading is fully elastic and can be analyzed by an elastic indentation model (Table 3.1). First, they showed that if the tip geometry is a solid of revolution, there is a direct relationship between E , the contact area and S [Pharr 1992] that follows

$$\frac{dF_{vel}}{dh} = S = \frac{2}{\sqrt{\pi}} \sqrt{A} \frac{E}{1-\nu^2}, \quad \text{Eq. 3.11}$$

where F_{vel} is the unloading force. Secondly, assuming that there is no pile-up during the first loading, they found another relationship between the recovered elastic depth (h_{el}), the vertical force (F_V) and S writing

$$h_{el} = \frac{2F_V}{S}. \quad \text{Eq. 3.12}$$

With the same “no pile-up” assumption, they proposed a relationship between h_{el} and the real height of contact (h_c) through

$$h_c = h_{el} D_{el}, \quad \text{Eq. 3.13}$$

where D_{el} is a constant that depends only on the shape of the tip (Table 3.1). If the tip is a cone, $D_{el} = (2/\pi)$. In the case of a cylindrical tip, D_{el} equals 1.

Knowing h_c , the contact area can be calculated from the geometry of the tip. However, in practice, the tip involves geometrical imperfections. The relationship between A and h_c is therefore obtained by indentation of a reference material with known mechanical properties (usually, silica). Combining the last three equations gives

$$h_c = \frac{2F_V}{S} D_{el}, \quad \text{Eq. 3.14}$$

and

$$\sqrt{\frac{\pi}{A(h_c)}} \frac{S}{2} = \frac{E}{1-\nu^2}. \quad \text{Eq. 3.15}$$

This model is therefore useful to link the Young’s modulus of a material with the indentation unloading force.

Using a combination of analytical modelling, finite element simulations and experiments Poon et al. [Poon 2008] studied the validity of the model of Oliver and Pharr in the case of an indenter with an ideal conical geometry. More precisely, they evaluated the applicability of the elastic conical indentation solution of Sneddon (Table 3.1) combined with an equivalent shape of the indenter to predict the entire unloading curve. Their model is based on some assumptions: (i) the indenter tip is perfectly sharp, (ii) there are no radial displacement recovery during the unloading, (iii) the loading stage of the indentation is considered elastoplastic, (iv) the unloading stage or any reloading up to the initial loading force is assumed to be only elastic and (v) the reloading of the residual imprint is modelled by an indentation of an elastic flat surface with an indenter having an equivalent geometry. This “equivalent geometry” is supposed to generate the same load and the same contact area as the reloading of the residual imprint. In their analysis, due to the hypothesis (ii) the residual imprint cross section is considered to have no curvature which translates into a sharp equivalent indenter geometry (Figure 3.12).

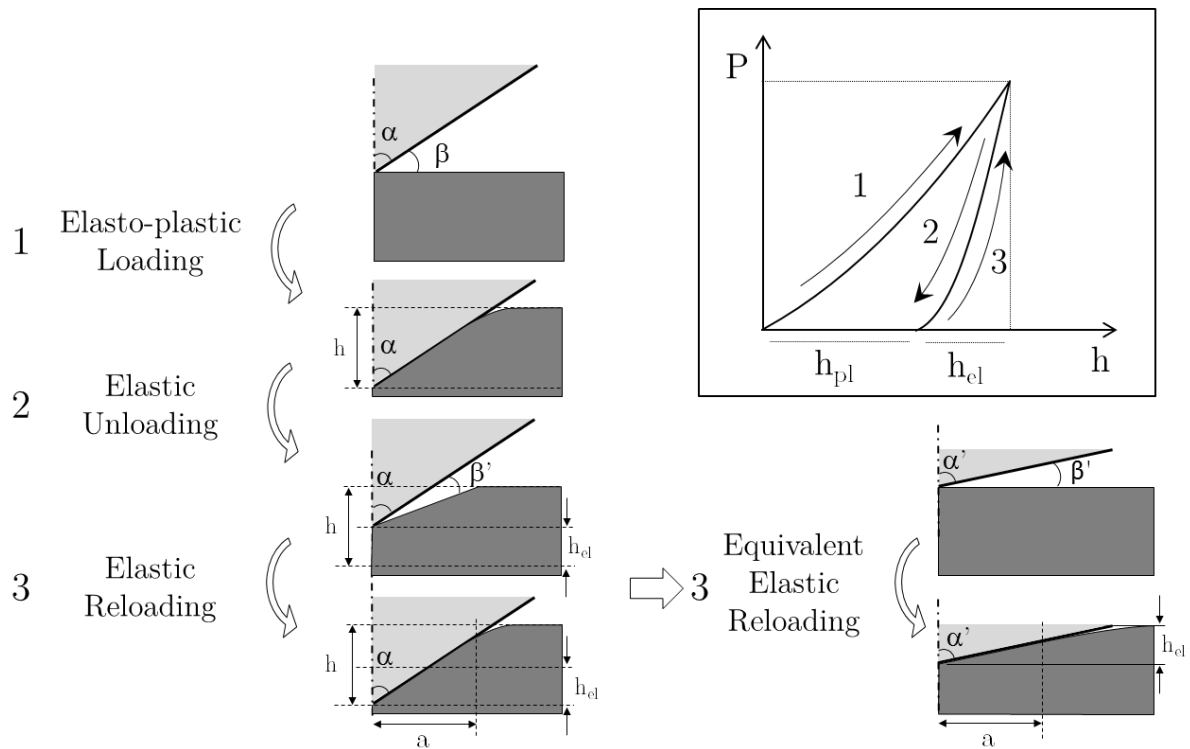


Figure 3.12: Schematic representation of a sharp equivalent shape of an indenter.

Using this method, Poon reported some errors in the prediction of the Young' modulus ranging between 10 and 45%. They conclude that the error originated from two effects:

on the one hand, from the estimation of the projected contact area which can be affected by pile-up effects and by the imperfections in the tip geometry that are not taken into account in their model. They also concluded that the error can originate from residual stresses that tend to curve the cross section of the residual imprint making the sharp equivalent shape not as appropriate as a smooth equivalent shape (Figure 3.13).

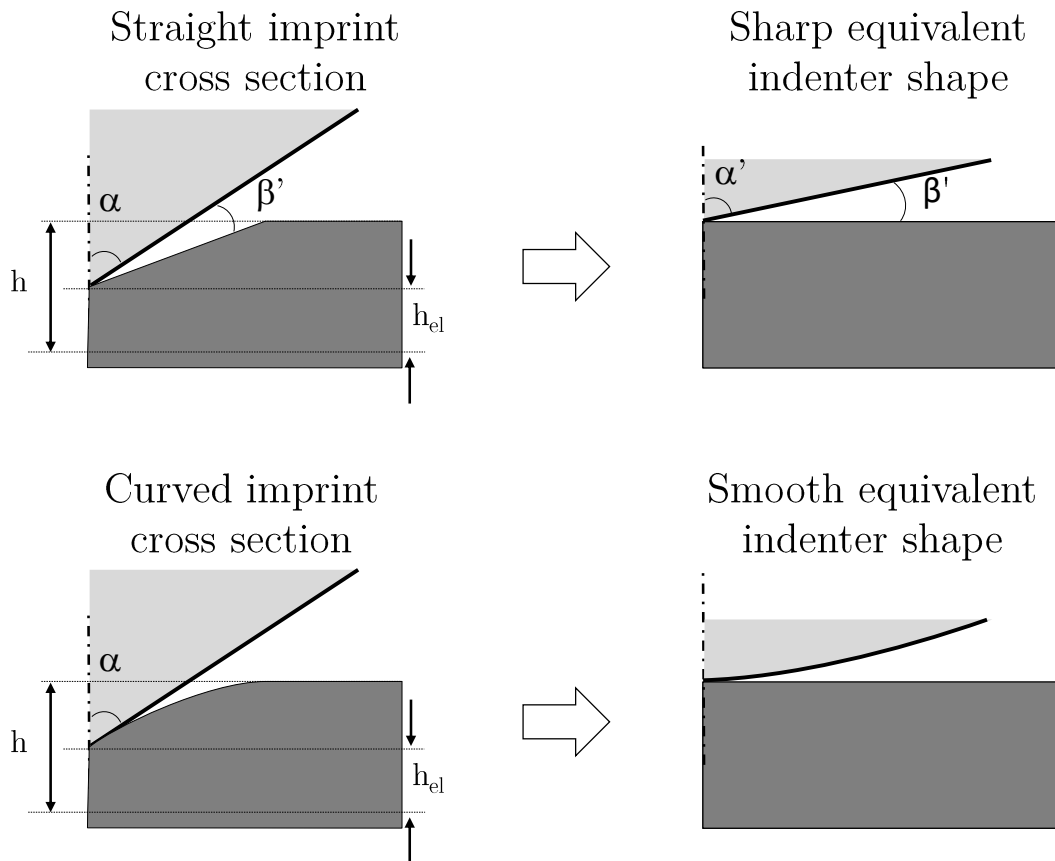


Figure 3.13: Schematic representation of the difference between a sharp and a smooth equivalent shape of an indenter.

3.3 Plowing models

The previous section presented an overview of models used to apprehend the vertical contact between a tip and a flat surface. If additionally, there is a horizontal displacement of this tip over the surface of the base material, a new set of mechanisms can take place, including plowing. If plowing occurs, the surface of the base material is deformed and pushed under the tip as well as on the side the tip (see Figure 3.1). This phenomenon is more complicated to model than indentation. However, many models were developed to understand this phenomenon from a mechanical point of view. This section focuses on three simple physics-based models used to predict the wear volume and the horizontal sliding force.

Archard proposed a simple equation to model the sliding wear between a hard surface and a soft surface. The physics behind his equation is based on the observation that the real contact between two flat surfaces can be approximated by a series of localized contacts. These localized contacts happen between the adjacent asperities created by the roughness of the two surfaces in contact. Archard considered that the macroscopic wear is a sum of microscopic wear in the contact between asperities. In this model, each asperity is modelled by a hemisphere. The surface of contact between two asperities is assumed to be a disc of radius a [Mellor 2006]. Archard assumed that the softer surface asperities would all be displaced and removed by the asperities of the harder surface (Figure 3.14). Displacing the harder asperity over a distance $2a$ removes one hemisphere of soft material so that the volume removed over a sliding distance L can be expressed as

$$V = \frac{L}{2a} \frac{4}{3} \frac{\pi a^3}{2} = \frac{L}{3} \pi a^2, \quad \text{Eq. 3.16}$$

where V is the wear volume. Using the definition of the hardness (Eq. 3.1) and inserting it into the previous equation gives

$$V = \frac{L}{3} \pi a^2 \frac{F_v}{AH} = L \frac{F_v}{3H}. \quad \text{Eq. 3.17}$$

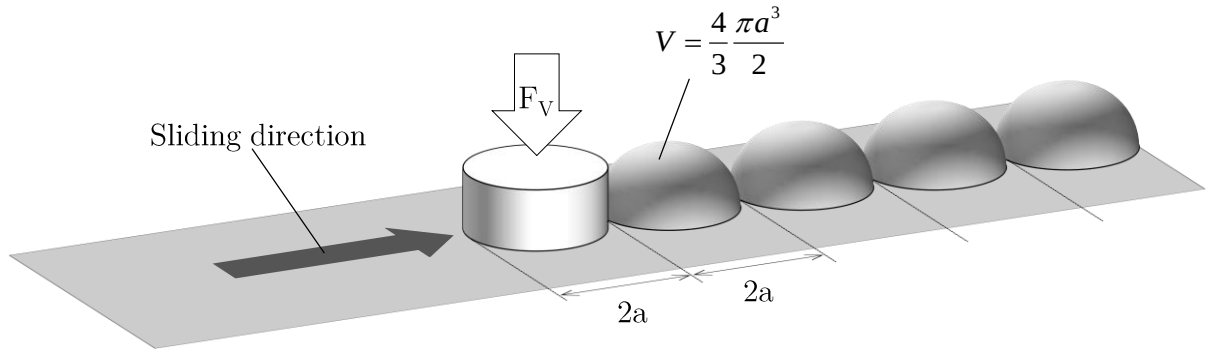


Figure 3.14: Schematic representation of the Archard model. The cylinder represents the harder asperity contact zone sliding on the softer surface.

Archard also assumed that each asperity would wear the material with a given efficiency. A proportionality constant was added to the equation to take this phenomenon into account. The Archard law states that the wear volume (V) is a function of the hardness (H), of the vertical force (F_V) and the distance of sliding (L) as

$$V = L \frac{K_A F_V}{H}. \quad \text{Eq. 3.18}$$

In this equation, K_A is a constant that depends on the material and on the geometry of the abrasive in contact with the material [Kato 2001].

Using a different approach, Childs [Childs 1970] proposed a plowing model based on the slip line field theory. This model predicts the sliding force of a rigid conical indenter scratching a rigid perfectly plastic material at a constant depth (Figure 3.15). Usually, slip line field solutions are given in the form of discrete numerical tables of numbers (Table 3.3).

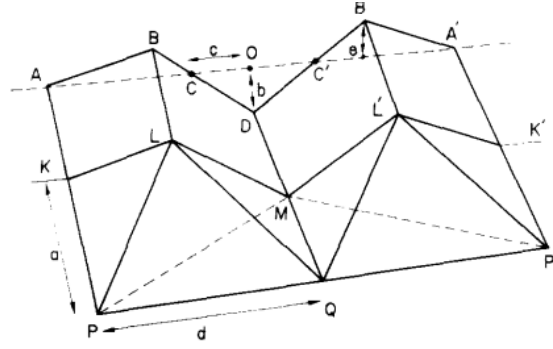


Figure 3.15: Schematic representation of the three-dimensional model of scratching of Childs. In this model, the material is plastically displaced laterally and vertically by the tip to create the residual groove with the lateral pile-up. Picture from [Childs 1970]

Table 3.3: Solution of the Childs model presented in a tabulated form. In this table, k is the shear yield stress. Table from [Childs 1970]

c/b	a/b	d/b	F/kb^2	Cone semi-angle
0.20	0.65	1.0	1.99	11°
0.50	1.05	1.65	4.64	25°
0.60	1.05	1.80	5.70	30°
0.80	1.30	2.25	8.09	40°
1.00	1.50	2.55	10.9	45°
1.40	1.80	3.60	17.8	55°
2.10	2.25	5.00	34.3	65°
3.70	2.95	8.90	93.6	75°
11.4	4.4	27.8	795.0	85°
50.0	9.5	120.0	14,600	89°

These tabulated data contain the normalized lateral force ($2F_H/\sigma_y h^2$). This force can be expressed as a continuous function of the angle between the cone and the surface of the specimen (β) using curve fitting (Figure 3.16).

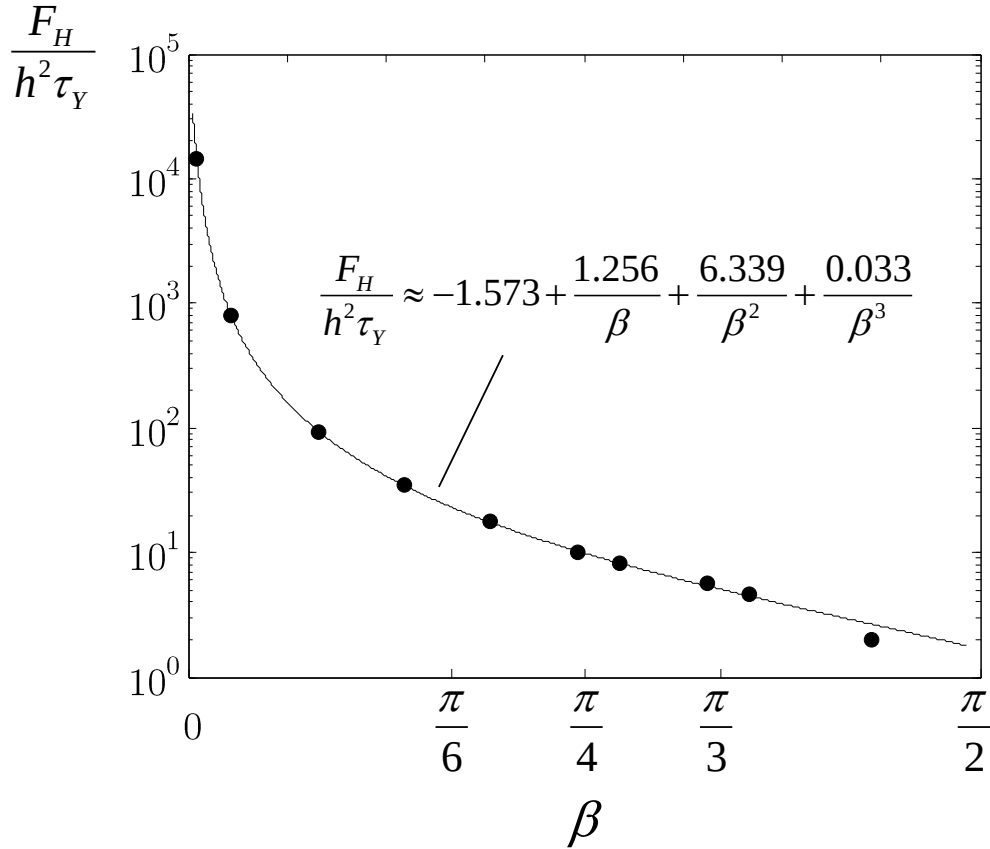


Figure 3.16: Polynomial curve fitting on the data calculated by Childs. β is expressed in radians.

Flores [Flores 2008] developed an analytical model of scratching of a rigid perfectly plastic material with a sphere. This model gives a lower bound estimate of the scratch force. The base material is divided into slices of infinitesimal thickness. The scratching process is decomposed as a succession of indentation and friction sliding of each slice of the material. Concerning the indentation part, the problem is simplified in two dimensions. The scratching tool, here a sphere, is approximated by a cylinder with the same radius. The energy required to plastically indent a two-dimensional slice of the material to a depth h is calculated by integrating the vertical force on the cylinder over the indentation depth (h) (Figure 3.17). The friction energy dissipated between the surface of the material and the sphere is modelled by a Coulomb law. Since the scratch happened at constant depth, globally, the force normal to the surface does not generate any mechanical work, only the lateral force does. Therefore, the lateral force is calculated by dividing the total mechanical work (plastic and frictional) by the sliding distance through

$$F_H = \frac{dU_{pl}}{dx} + \frac{dU_{fric}}{dx} = \int_0^h f_V^{Cylinder}(h) dh + \mu F_V^{Sphere}(h), \quad \text{Eq. 3.19}$$

where W is the width of the tool, f_V is the indentation force acting on a cylinder per unit length, μ is the friction coefficient, F_V is the indentation force acting on a sphere and h is the depth of the scratch. Solving this equation requires knowing f_V as well as F_V . Flores used the rigid perfectly plastic solution for the sphere (Table 3.1) and for the cylinder to find these two forces. Ultimately, the model gives a relationship between the normal force and the lateral force during a scratch at constant depth in the form

$$\frac{F_H}{F_V} = \mu + 0.05 \sqrt{\frac{F_V(h)}{R^2 \sigma_y}}, \quad \text{Eq. 3.20}$$

Where R is the radius of the sphere.

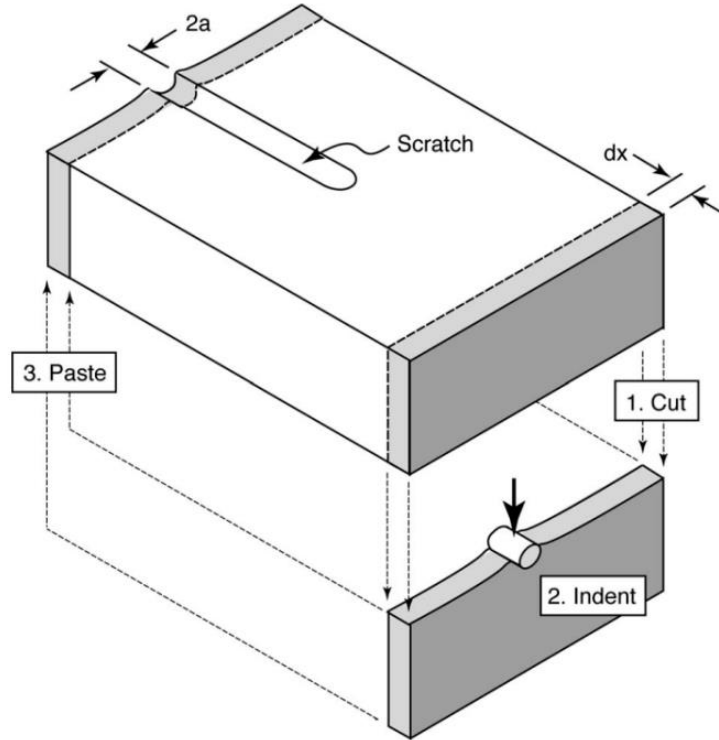


Figure 3.17: Schematic representation of the steady-state scratching model of Flores. A slice of the base material is indented by a cylinder (the 2D projection of a sphere), rubbed to include the effect of friction and then this slice is pasted in the continuity of the scratch track. Picture from [Flores 2008].

3.4 Cutting models

In contrast with plowing, cutting involves the detachment of a part of the base material surface in the form of a chip that is removed by the cutting tool. Again, many physics-based models exist to predict the forces acting on the cutting tool as a function of the depth of penetration of the tool and of the geometry of the tool. Most of them are based on Merchant model. The present section describes some of these models focusing on the physical hypotheses and on the final expression of the vertical and the horizontal cutting forces.

Merchant analytical model focuses on the orthogonal cutting at constant depth in the case of a continuous chip [Merchant 1945 a, Merchant 1945 b]. The model is expressed in terms of material properties and of the tool's geometry. This model gives an approximation of several physical quantities, including the shearing strain in the chip (ε_{shear}), the velocity of chip flow, the forces (lateral and normal to the surface) and the energy dissipated in friction and in plastic deformations. The main assumptions are that (i) the base material is rigid perfectly plastic, (ii) the tool is a rigid body, (iii) the shear zone is infinitely small, (iv) the chip is continuous, (v) the cutting edge is perfectly sharp and (vi) the chip is incompressible. From there, the analysis starts with a kinematic simplification of the chip formation based on Pijspannen deck of cards [Piispanen 1948]. The chip formation is seen as a series of cards sliding on one another to accommodate the deformation. The corresponding plastic shear strain is calculated as a function of the cutting angle (α) and the shear angle (ϕ). The cutting angle is the angle between the frontal face of the tool and a vertical plane, ϕ is the unknown of the problem (Figure 3.18). The friction force between the cutting tool and the chip is modelled with a Coulomb law. The cutting force (F_H) and the force normal to the surface (F_V) are obtained by balancing the forces necessary to deform plastically the chip and to overcome the friction forces. The shear angle is finally obtained by energy minimization. The result of this model is a system of equation presented hereunder

$$\varepsilon_{shear} = \cot(\phi) + \tan(\phi - \alpha), \quad \text{Eq. 3.21}$$

$$F_V = F_H \tan(\lambda - \alpha), \quad \text{Eq. 3.22}$$

$$F_H = \frac{\sigma_y h W}{2 \sin \phi} \frac{1}{\cos(\lambda - \alpha + \phi)} \cos(\lambda - \alpha), \quad \text{Eq. 3.23}$$

$$\phi = \frac{\pi}{4} + \frac{\alpha}{2} - \frac{\lambda}{2}, \quad \text{Eq. 3.24}$$

$$\lambda = \arctan(\mu), \quad \text{Eq. 3.25}$$

where μ is the friction coefficient between the tool and the base material.

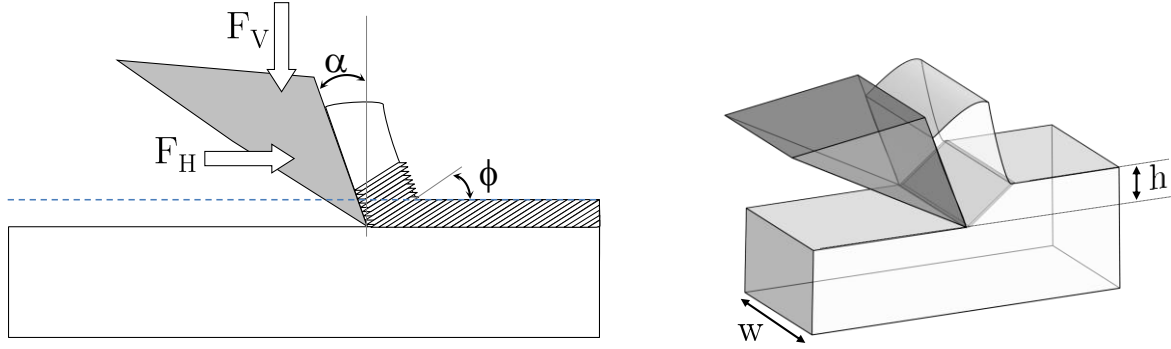


Figure 3.18: Schematic representation of the merchant model with the deck of cards approximation with the cutting angle (α) and the shear angle (ϕ).

Atkins proposed a modification of the Merchant model with the introduction of an energy of separation between the base material and the newly formed chip surface [Atkins 2003]. The idea defended by Atkins is that the energy needed to separate the chip from the surface is equivalent to the energy required to propagate a crack between the base material and the chip. The specific work of surface formation (Q) is added to the total energy needed during the cutting process. This approach gives

$$U_{cut} = U_{plast} + U_{frict} + U_{crack} = F_H L, \quad \text{Eq. 3.26}$$

$$F_V = F_H \tan(\lambda - \alpha), \quad \text{Eq. 3.27}$$

$$F_H = \left(\frac{\sigma_y h W}{2} \frac{\cos \alpha}{\sin \phi \cos(\phi - \alpha)} + QW \right) \left(1 - \frac{\sin \lambda \sin \phi}{\cos(\lambda - \alpha) \cos(\alpha - \phi)} \right)^{-1}. \quad \text{Eq. 3.28}$$

Where L is the length of cut, W is the width of the cutting tool, λ is the friction angle defined by Eq. 3.25 and Q is an energy per unit surface [J/m^2]. If the linear elastic fracture mechanics (LEFM) applies, the magnitude of Q is considered to be close to the plane strain critical energy release rate (G_{IC}). Using an energy minimization criterion, Atkins derived an equation relating ϕ and the material properties Q and σ_y combines into an adimensional orthogonal cutting parameter (B_{OC}) through

$$\left[\frac{\cos(\lambda - \alpha)}{\sin \lambda} - \frac{\sin \phi}{\cos(\phi - \alpha)} \right] \left[1 - \frac{\cos^2(\phi - \alpha)}{\sin^2 \phi} \right] =, \quad \text{Eq. 3.29}$$

$$\left[\cot \phi + \tan(\phi - \alpha) + B_{OC} \right] \cos \alpha$$

where $B_{OC} = \frac{2Q}{\sigma_y h}$. Eq. 3.30

Although the equation is not proposed in a closed form solution, Atkins used numerical methods to expose the effect of B_{OC} , α and μ on the shear angle and on the cutting force (Figure 3.19).

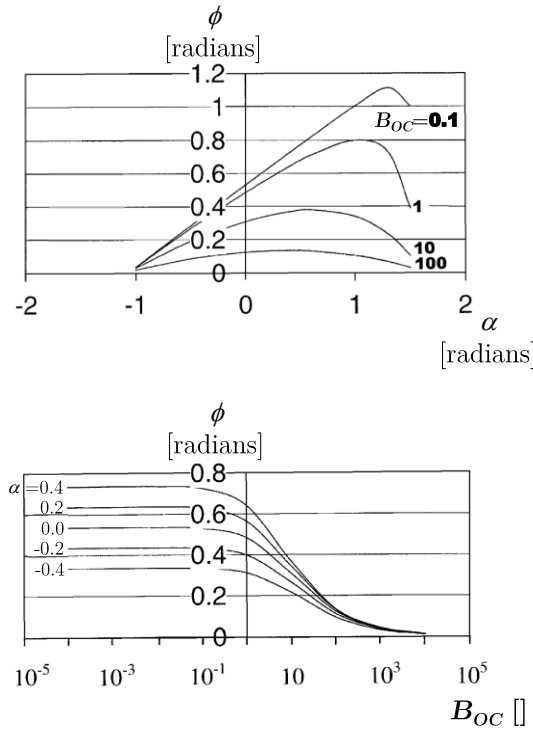


Figure 3.19: Effect of B_{OC} and α on the shear cutting angle in Atkins model with $\mu = 0.54$. Picture from [Atkins 2003]

This model was also adapted to cutting with a 3D pyramidal cutting tool [Atkins 2007] (Figure 3.20). In this case, the cutting forces incorporate the angle (β) between the lateral surface of the pyramid and the surface of the material, the cross section of the cutted groove (A_{cut}) and the length of the cross section of the cutted groove (c_{cut}) combined in an adimensional scratch cutting parameter (B_{SC}) through

$$F_V = F_H \tan(\lambda - \alpha), \quad \text{Eq. 3.31}$$

$$F_H = \frac{\sigma_y h^2}{2} \left[\frac{\tan \beta}{\sin \phi \cos(\phi - \alpha)} + B_{SC} \right] \left[1 - \frac{\sin \lambda \sin \phi}{\cos(\lambda - \alpha) \cos(\phi - \alpha)} \right]^{-1}, \quad \text{Eq. 3.32}$$

$$B_{SC} = \frac{2Q}{\sigma_y} \frac{c_{cut}}{A_{cut}}, \quad \text{Eq. 3.33}$$

$$c_{cut} = \frac{2h}{\sin \beta}, \quad \text{Eq. 3.34}$$

$$A_{cut} = h^2 \cot \beta. \quad \text{Eq. 3.35}$$

The shear angle (ϕ) can be obtained numerically by minimization of the energy through the calculation of the derivative of F_H with respect to ϕ .

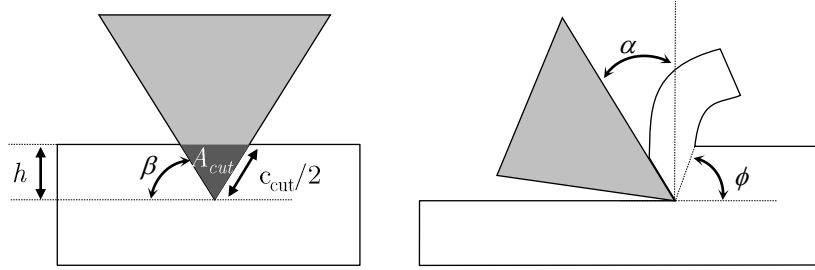
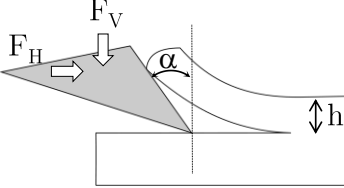
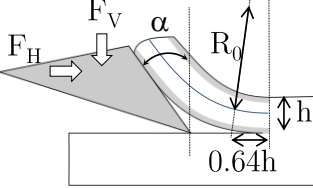
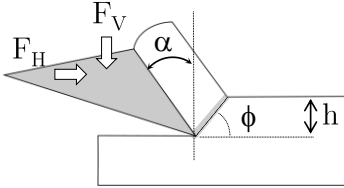


Figure 3.20: Schematic representation of the geometry of cutting with a pyramidal tip.

Williams, Patel and Blackman also analyzed the cutting process through a combination of plasticity, friction and fracture mechanics [Williams 2010]. They included a term of adhesion toughness (G_a) between the chip and the cutting tool to model an eventual chip sticking to the rake face of the tool. In their analysis, they derived a closed form solutions for three cutting scenarios. The two first ones are based on beam theory and consist of elastic bending and elastic-plastic bending in the chip. The third scenario is a perfectly plastic yielding of the chip similar the model of Atkins with a closed form solution for ϕ . However, in their model, the shear angle does not depend on Q (Table 3.4) which is in contradiction with the results of Atkins [Atkins 2003].

Table 3.4: Description of the three cutting models of Williams, Patel and Blackman.

Elastic bending	Elasto-plastic bending	Shear yielding
 $\frac{F_H}{W} = \frac{G_a + Q(1 + \mu \tan \alpha)}{1 - \mu \left(\frac{1 - \sin \alpha}{\cos \alpha} \right)}$ $\frac{F_V}{W} = \frac{F_H}{W} Z + \frac{G_a}{\cos \alpha + \mu \sin \alpha}$	 $\frac{F_V}{W} = \frac{\left(1 - \frac{4}{3} \frac{\sigma_y R_0}{h E} \right) \frac{\sigma_y^2 h}{2 E} - Q}{0.64 \frac{h}{R_0}}$ $\frac{F_H}{W} = \frac{1}{Z} \frac{F_V}{W} - \frac{1}{Z} \frac{G_a}{\cos \alpha + \mu \sin \alpha}$	 $\frac{F_H}{W} = Q + \sigma_y h \cot \phi$ $\cot \phi = Z + \sqrt{1 + Z^2 + \frac{2 G_a}{\sigma_y h} \frac{1}{(\cos \alpha + \mu \sin \alpha)}}$ $\frac{F_V}{W} = \frac{F_H}{W} Z + \frac{G_a}{\cos \alpha + \mu \sin \alpha}$

In this table, Z is an adimensional parameter related to the friction coefficient and the cutting angle through

$$Z = \frac{\mu - \tan \alpha}{1 + \mu \tan \alpha}. \quad \text{Eq. 3.36}$$

This model was validated by [Patel 2009] through experimental measurements of fracture toughness by orthogonal cutting. Their model was also adapted and validated experimentally in the case of scratch cutting with a 3D pyramidal cutting tool [Blackman 2016]. In the scratch cutting case, a relationship between the horizontal force and the vertical force is

$$\frac{F_H}{c_{cut}} - \frac{F_N}{c_{cut}} \tan \phi = \frac{\sigma_y}{2} \frac{A_{cut}}{c_{cut}} \left(\tan \phi + \frac{1}{\tan \phi} \right) + Q. \quad \text{Eq. 3.37}$$

This equation can be used to estimate the yield strength and the fracture toughness of a material from the vertical force and the horizontal force of a scratching test. However, this equation alone cannot be used in the other way around to estimate the scratching forces knowing the mechanical properties σ_y and G_{IC} . These models succeeded in predicting the plane strain fracture toughness of three polymer materials. However, the prediction of the yield stress is three times larger than the experimental measurements obtained by uniaxial compression tests [Blackman 2016, Patel 2009].

Islam et al. developed a mechanical model for the nanoscratch test in order to measure the fracture toughness of bones [Islam 2012]. The interest in this model lies in the methodology proposed to take into account the elastic recovery in the back of the tool.

Their model assumes that the energy involved in the cutting process can be divided into four terms. Like in the model of Atkins, the first three terms are the friction between the chip and the tool, the plastic shear strain in the chip and the surface separation energy. The fourth term is the friction at the back of the tool due to the elastic recovery of the base material (Figure 3.21). Islam proposed a method to measure the friction force in the back of the cutting tool in four steps. (i) The friction coefficient (μ) is measured using a post scratch surface scanning. (ii) The elastic recovery depth (h_{el}) is measured by a post scan of the surface after the scratch. (iii) The relationship between F_{Vel} and h_{el} is obtained by indentation using the unloading curve. (iv) The relationship between F_{Vel} and the friction force at the back of the tool (F_{Hel}) is obtained using the friction coefficient in a Coulomb law. The friction coefficient in the Coulomb law is obtained experimentally using a low force scratch in the residual groove of the scratch.

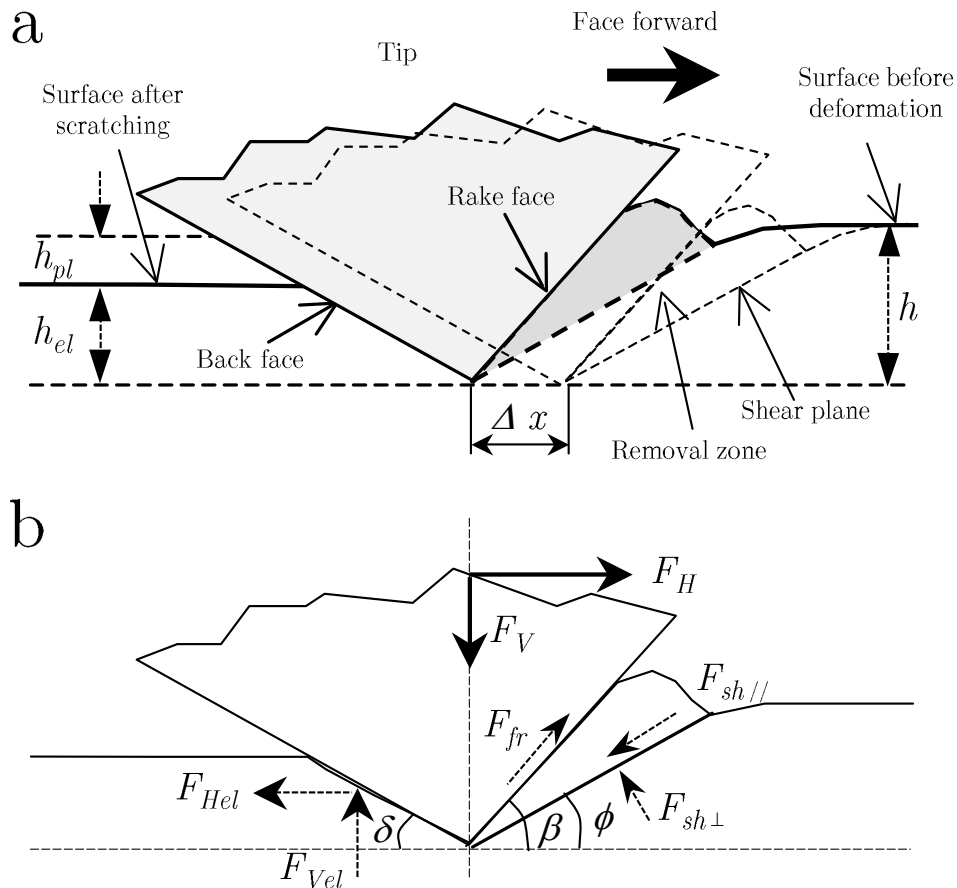
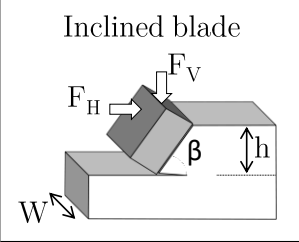
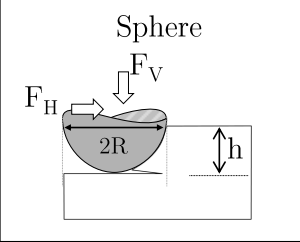
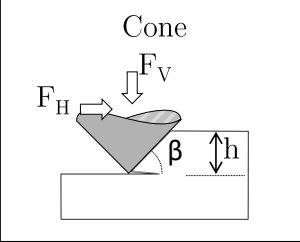
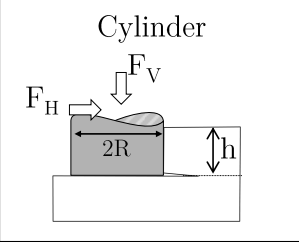


Figure 3.21: Schematic representation of the different forces acting on the tip in Islam model. $F_{sh//}$ and $F_{sh\perp}$ are the forces acting parallel and perpendicular to the shear plane, F_{fr} is the friction force between the chip and the tip. F_{Vel} and F_{Hel} are the results of the elastic recovery forces and the friction that it generates in the back of the tool. Picture from [Islam 2012].

Focusing more on the brittle aspect of cutting and scratching, Akono et al. proposed a scratch model that uses the J-integral method to extract the fracture toughness (K_C) from a scratch test, relying exclusively on fracture mechanics [Akono 2011 a, Akono 2014]. The plastic deformation and the friction are not considered in the model. The contour of the J-integral is chosen so that only the stress field at the interface between the tool and the base material contribute to the J-integral. Akono used FEM calculations and showed that the model remains valid with large plastic deformations in the front of the tip. The final expression of this model gives a relationship between the geometry of the tool, the horizontal force, the vertical force, the depth of the tool and the fracture toughness [Akono 2011 b]. Akono adapted her model to four different geometries, an inclined blade with negative rack angle [Akono 2011 a, Akono 2011 b], a cone, a sphere and a cylinder [Akono 2012 a, Akono 2012 b] (Table 3.5).

Table 3.5: Description of the scratching models of Akono for the four geometries of tip. F_C is the critical force of scratching/cutting.

Inclined blade 	Sphere 	Cone 	Cylinder 
$K_C = \frac{F_C}{W\sqrt{2h}} \left(1 + 2\frac{h}{W}\right)^{-\frac{1}{2}}$ $F_C \geq F_{eq} = \begin{cases} F_H & , \beta = \frac{\pi}{2} \\ \sqrt{F_H^2 + \frac{3}{5}F_V^2} & , \beta < \frac{\pi}{2} \end{cases}$	$K_C = \frac{F_C}{4h} \sqrt{\frac{3}{R\xi}}$ $F_C \geq F_{eq} = \sqrt{F_H^2 + \frac{665}{1188}F_V^2}$ $\xi = \sqrt{1 + 2\frac{h}{R}} + \sqrt{\frac{R}{2h}} \arcsin\left(\sqrt{2\frac{h}{R}}\right)$	$K_C = \frac{\sin \beta}{\sqrt{\cos \beta}} \frac{F_C h^{\frac{3}{2}}}{2}$ $F_C \geq F_{eq} = \sqrt{F_H^2 + \frac{3}{5}F_V^2}$	$F_C = 2R\sqrt{2h\left(1 + \frac{h}{R}\right)}K_C$ $F_C \geq F_H$

3.5 Solid particles erosion modelling

In addition to direct contact with the blades, abradable materials are exposed to erosion. Erosion is a general term that represents the wear of a solid surface by a fluid or by solid particles [Blanco 2010]. Scientists have also proposed models to predict the erosion resistance of materials. This section focuses mainly on *solid particles erosion* which can be defined as a “progressive loss of material from a solid surface by repeated impacts from fluid-borne solid substances-particles” [Budinski 2014]. From a physical point of view, during the impact, the solid particles transfer a part or all of their kinetic energy to the surface of the material. In reaction, the material surface deforms or cracks. The succession of impacts eventually wears out the surface. Depending on the base material and on the impact angle, the erosion impacts generate different wear mechanisms (see Figure 3.22).

Different models can be used to predict the erosion rate depending on the dominant erosion wear mechanisms and on material properties. Most of these models are based on a mechanical energy balance calculation between the impacting particles and the base material. For instance, Finnie developed an erosion wear model base on an energy transfer between the impacting particles and the base materials [Finnie 1960]. His model is based on a series of assumptions: (i) the surface of the base material is considered perfectly flat, (ii) the pressure flow between the particle and the material is considered to be constant, (iii) the rotation of the particles is negligible, (iv) the volume displaced by the particle is removed from the surface, (v) the impacting particle is rigid and does not fracture, (vi) the vertical and the horizontal force acting on the particle are linked together with a Coulomb law and (vii) the lateral surface of contact is affected by a pile up approximated by twice the theoretical lateral surface of contact.

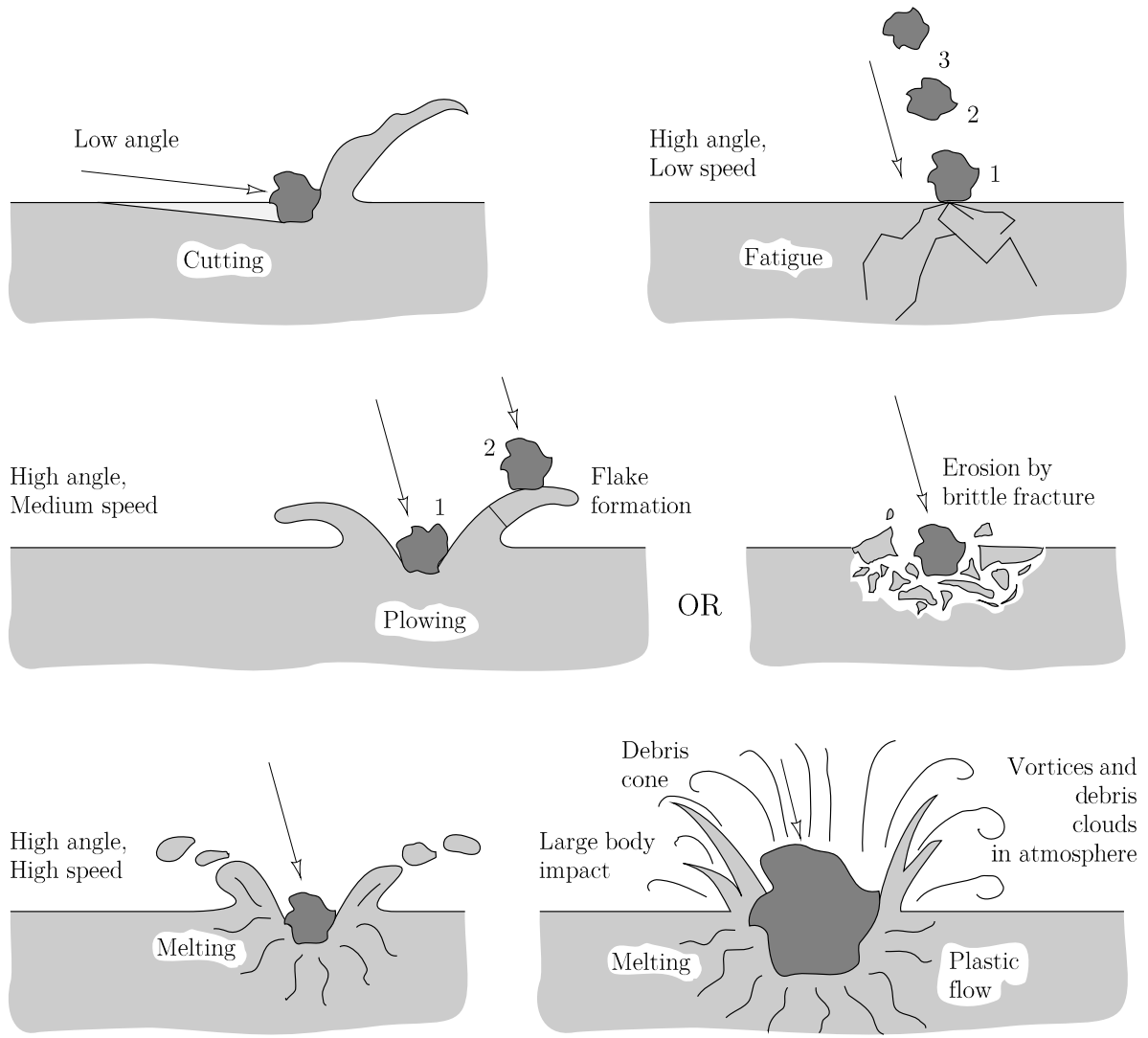


Figure 3.22: Schematic representation of the effects of the impact angle and the speed of the impacting particles on the erosion wear mechanisms. Picture from [Stachowiak 2005].

Finnie then solved two equations of motion of the particle. One of them concerns small impacting angle, for which the particle scratches the surface of the materials and rebounds thereby keeping part of its kinetic energy. The second one considers that the particle transfers all its kinetic energy to the material. These two scenarios generate a volume loss described by

$$V = \frac{\mu K_F m v_H^2}{H_H} \left(\sin 2\omega - \frac{6}{\mu} \sin^2 \omega \right), \text{ if } \tan \omega \leq \frac{\mu}{6}, \quad \text{Eq. 3.38}$$

$$V = \frac{K_F m v_H^2}{H_H} (\cos^2 \omega), \quad \text{if } \tan \omega > \frac{\mu}{6}, \quad \text{Eq. 3.39}$$

where V is the eroded volume, m is the mass of the particle, K_F is an erosion rate efficiency constant, H_H is the plastic flow pressure, μ is the friction coefficient and ω is the angle of impact. Regarding the other approaches in erosion wear modelling, a detailed review of solid particles erosion wear models can be found in [Parsi 2014].

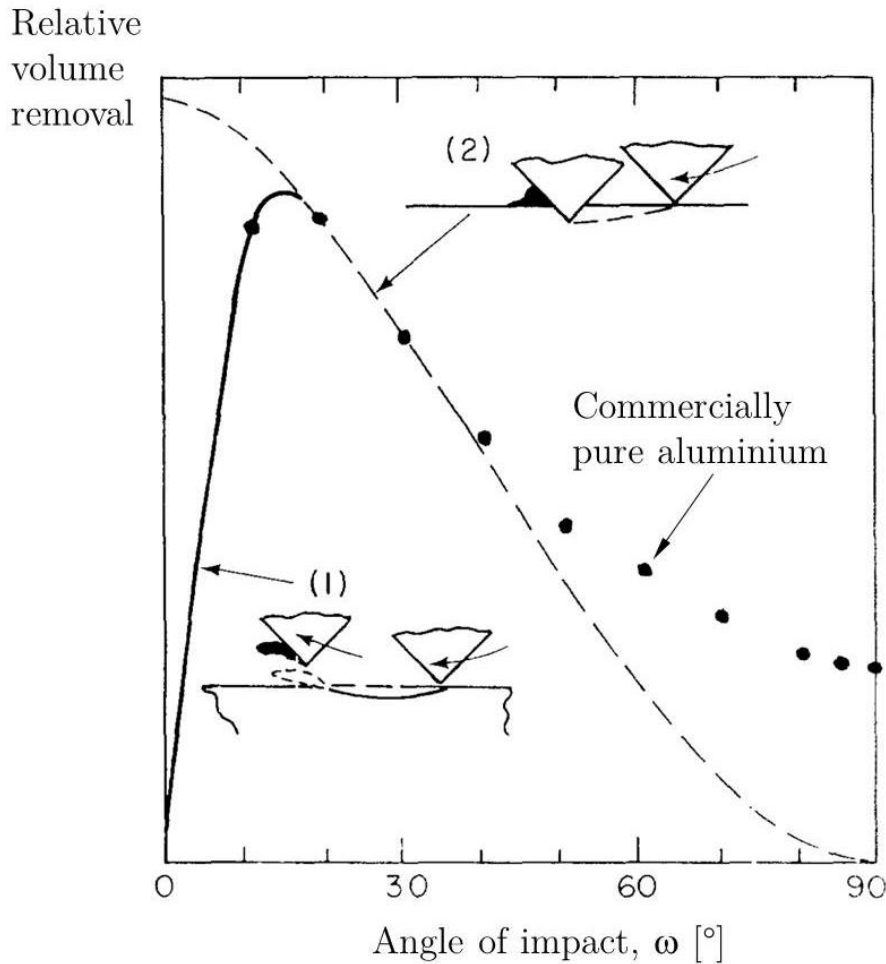


Figure 3.23: Graph showing the evolution of the relative volume removal rate as a function of the impact angle. The two lines represent the results of the two equations of Finnie. Picture from [Finnie 1995].

3.6 Transition between wear mechanisms

In the previous sections, several wear mechanisms were described and modelled. As stated in Chapter 2, Abradable materials can experience several wear mechanisms depending on the incursion speed of the blade (see section 2.5.3). Therefore, how to predict which wear mechanism is the dominant one?

Capturing the transition between one wear mechanism and another can be done through a criterion of energy minimization. The idea is that the dominant wear mechanism is the one that consumes the least amount of energy to occur. This strategy requires at least two models, one by mechanism. Indeed, if two wear mechanisms can be properly modelled, it is possible to calculate the energy cost of each mechanism. From there, it is possible determine which one is more likely to happen in a given contact situation. For instance, Atkins et al. studied the transition between cutting and plowing in the case of a pyramidal tool scratching a surface [Atkins 2007]. They showed that depending on (i) the opening angle of the pyramidal tool, (ii) on the non-dimensional parameter B_{OC} as well as (iii) on the friction coefficient, the dominant wear mechanism could be predicted (Figure 3.24).

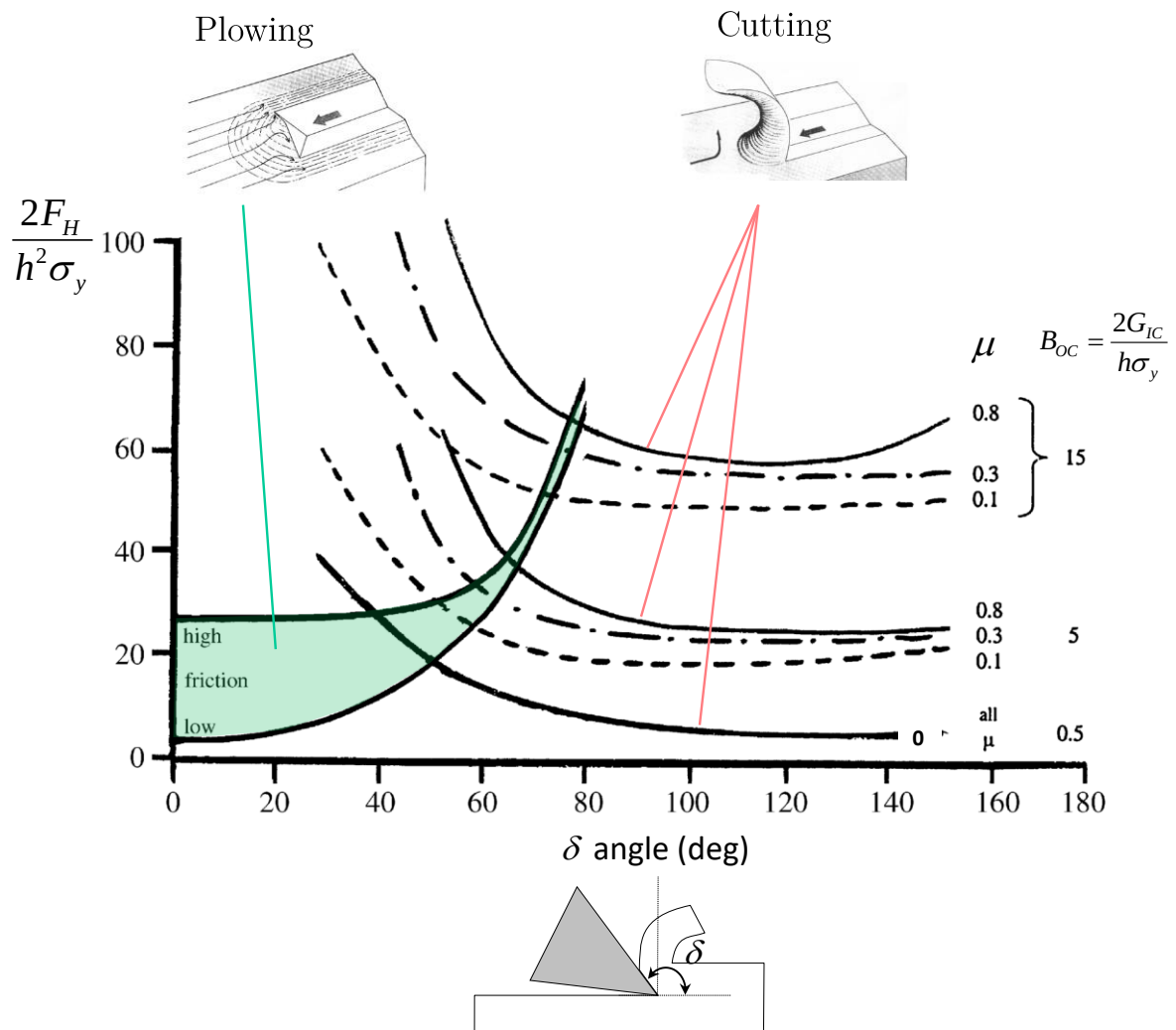


Figure 3.24: Normalized cutting and plowing force as a function of δ angle for various frictional conditions and values of B_{OC} . The dominant wear mechanism presents the smallest normalized force. Picture from [Atkins 2007]

3.7 Summary

This chapter started with three questions. First, how do mechanical properties relate to contact forces or to a wear volume? The present chapter shows that it depends on the wear mechanism. Some models offer equations that make this connection based on a series of assumptions on the base material and on the geometry of the contact. The second question was: is it possible to model many wear mechanisms with a single model? The answer is no. Each model focuses only on one mechanism and on one geometry of contact at a time. Regarding the third question, in which case will a wear mechanism be the dominant one? Following the approach of the weakest link, the mechanism that consumes the least energy dominates the others.

Chapter 4 Model extensions and validations

The previous chapter described several wear models as well as a strategy to predict the dominant wear mechanism. However, most of these models consider the base material as rigid perfectly plastic. This assumption is quite limiting since abradable materials are sometimes made of materials involving large elastic strains including polymers and porous materials. Also, the strain hardening is often neglected. This chapter describes some modifications of these models to include elasticity, and strain hardening. The first section focuses on indentation forces and on the elastic recovery in elastoplastic materials. The next three sections focus on three wear mechanisms often encountered in abradable materials: plowing, cutting and spalling. The fifth section analyzes erosion models based on the three mechanisms of plowing, cutting and spalling.

4.1 Indentation modelling

Regarding indentation modelling, elegant solutions exist to predict the hardness of elastoplastic materials (see Section 3.2). However, these models do not offer a simple way to predict the vertical force (F_V) upon loading and unloading as well as the elastic recovery after unloading (h_{el}). In the case of an abradable material, these two values will help to predict the quantity of material being displaced by the blade as well as the forces acting on the blade. This section describes an add-on, based on the indentation models presented in Chapter 3 that allows approximating these quantities. In the following subsections, this add-on model is presented in the case of flat and sharp tips followed by a comparison with FEM calculations and a sensitivity analysis.

4.1.1 Flat tips indentations

Some particular tip geometries maintain a constant surface of contact during indentation. It is the case for flat tips like rectangular and cylindrical punches. As presented in Table 3.1, the hardness of elastic and elastoplastic material pressed by a flat tip take respectively the form

$$H_{el}(h) = \frac{h}{a} C_{el} \frac{E}{(1-\nu^2)} \quad \text{Eq. 4.1}$$

and

$$H_{elpl}(h) = \frac{h}{a} C_{elpl} \frac{E}{(1-\nu^2)}, \quad \text{Eq. 4.2}$$

where C_{el} and C_{elpl} are defined in Table 3.1 for simple tip geometries. From there, a simple model is proposed to predict the elastic recovery after unloading of the tip. The model is constructed based on the following hypotheses:

1. The base material is considered elastic perfectly plastic with Young's modulus E , Poisson ratio ν , and yield stress σ_y .

2. The total depth of penetration can be decomposed into the recoverable depth (h_{el}) and the residual depth (h_{pl}) through

$$h = h_{pl} + h_{el} \quad \text{Eq. 4.3}$$

3. The indenter does not deform significantly during the loading so that the radius of contact (a) is considered to be constant.

4. The unloading is considered perfectly elastic. Residual stress, creep and reverse plasticity are thus considered to be negligible. The unloading and any successive reloading to the same depth h would therefore reach the same load F_V .

5. The reloading step is modelled by an ideal elastic indentation model. Pushing back the tip down to a depth h is thus considered equivalent to an elastic indentation of the same flat tip in a semi-infinite half space to a depth h_{el} . With this set of hypotheses, hypotheses 4 and 5 can be translated in mathematical terms as

$$F_{V_{elpl}}(h) = F_{V_{el}}(h_{el}), \quad \text{Eq. 4.4}$$

And

$$A_{elpl}(h) = A_{el}(h_{el}) = A. \quad \text{Eq. 4.5}$$

Combining Eq. 4.4 and Eq. 4.5 with the definition of the hardness gives

$$H_{elpl}(h) = H_{el}(h_{el}). \quad \text{Eq. 4.6}$$

Substituting Eq. 4.1 and Eq. 4.2 in equation Eq. 4.6 and simplifying gives

$$\frac{h_{el}}{h} = \frac{C_{elpl}(h)}{C_{el}}. \quad \text{Eq. 4.7}$$

From there, the loading and the unloading curves can be constructed based on

$$F_{V_{elpl}}(h) = A H_{elpl}(h), \quad \text{Eq. 4.8}$$

and

$$F_{V_{unload}}(h) = A H_{el}(h). \quad \text{Eq. 4.9}$$

4.1.2 Sharp tips indentation

In modern instrumented indentation techniques, the determination of the area of contact remains the main difficulty. As described in Section 3.2.3, during sharp tip indentations, pile-up and sink-in phenomena can affect the real area of contact. These two behaviors can significantly affect the estimation of the hardness and of the Young's modulus. For instance, if the tip is a cone, the surface of contact is four times larger in a rigid-perfectly plastic material compared to an ideal elastic material. This ratio is equal to 2.7 for a spherical tip and 1.57 for a wedge (see Table 3.1). Therefore, making assumptions on the real surface of contact comes with a significant potential for error with this kind of tip geometries. So far, the approach of Oliver and Pharr succeeded in taking the sink-in into account but not the pile-up [Cheng 1998].

The present add-on is based on the model of [Oliver 1992] and on the model of [Poon 2008] but it includes the effect of the pile-up and of the sink-in. This part of the model focuses on sharp tips, including cones, wedges and half-wedges. The approach is similar to the flat tip model (see Section 4.1.1). However, it is followed with an additional development to predict the area of contact. This development is based on experimental observations from [Tabor 1947] and [Stilwell 1961] reported by Oliver and Pharr [Oliver 1992]. These observations are that, in the case of conical and spherical indentation, the radius of the impression in the surface does not recover after unloading, only part of the depth recovers. Moreover, in the case of a conical tip, when the elastic recovery is small, the cross section of the imprint stays straight. In addition to these observations, the model is based on six main physical assumptions, two of which are inspired from the literature. (i) The reloading of a residual imprint can be modelled by the indentation of a flat surface with an equivalent indenter geometry

[Oliver 1992]. (ii) The unloading and subsequent reloading load displacement curves coincide [Poon 2008]. (iii) A third, original hypothesis is that the pile-up and the sink-in take place at the same time. One leads to an increase of the contact area, the other to a decrease of it, each with its own amplitude. If these amplitudes are the same, their effects cancel each other. (iv) Sink-in is considered as an elastic phenomenon which recovers during unloading. (v) Pile-up is assumed to be a plastic effect, which remains after unloading. (vi) Another original hypothesis is that the pile-up is considered to be proportional to the residual depth of indentation (h_{pl}). Combining these observations and hypotheses will be the basis for this enhanced model.

As presented in Table 3.1, the hardness of elastic and elastoplastic material indented by a sharp tip take respectively the form

$$H_{el}(\beta) = C_{el} \frac{E \tan \beta}{(1-\nu^2)}, \quad \text{Eq. 4.10}$$

and

$$H_{elpl}(\beta) = C_{elpl} \frac{E \tan \beta}{(1-\nu^2)}, \quad \text{Eq. 4.11}$$

Where β is the angle between the surface of the base material and the side of the tip and C_{el} and C_{elpl} are defined in Table 3.1 for simple geometries. The model is constructed based on the list of hypotheses briefly mentioned above. The detailed list of hypotheses is described hereafter, some of which are commonly used in indentation modelling.

1. The base material is considered elastic perfectly plastic with Young's modulus E , Poisson ratio ν , and yield stress σ_y .
2. The total depth of indentation can be decomposed into the recoverable elastic depth (h_{el}) and the residual depth (h_{pl}), see Eq. 4.3.
3. The indenter does not significantly deform during loading so that the radius of contact (a) is approximated by a function of the shape of the tip and the real height of contact (h_c) through

$$a = h_c \cot \beta. \quad \text{Eq. 4.12}$$

4. After unloading, the side of the cross section of the residual imprint is approximated by a straight line of slope $(\beta - \beta')$.
5. During loading, pile-up and sink-in are considered to occur simultaneously.
6. During unloading, like in the ideal elastic case, the sink-in recovers by elasticity.
7. After unloading, like in the rigid perfectly plastic case, the pile-up remains above the original surface (see picture Figure 4.1). Combining hypothesis 4, 5, 6 and 7 gives

$$\tan(\beta - \beta') = \frac{h_{pl} + h_{PileUp}}{a}. \quad \text{Eq. 4.13}$$

8. The unloading is considered purely elastic. Residual stress, creep and reverse plasticity are thus considered negligible. The unloading and any successive reloading to the same depth h would involve the same load (P) and the same contact radius (a).
9. The reloading step is modelled by an ideal elastic indentation model. A sharp equivalent indenter shape (see section 2.2.2) is used to model the elastic reloading in the already imprinted surface. Pushing the tip again to a depth h is thus considered equivalent to an elastic indentation in a semi-infinite half space to a depth h_{el} with a tip having a half-opening angle α' . The angle α is the yet unknown complementary angle of β' as depicted on Figure 4.1. Translated into equations, hypotheses 8 and 9 become

$$F_{V_{elpl}}(h, \beta) = F_{V_{el}}(h_{el}, \beta'), \quad \text{Eq. 4.14}$$

and

$$A_{elpl}(h, \beta) = A_{el}(h_{el}, \beta') = A. \quad \text{Eq. 4.15}$$

10. Like in perfect plastic material, the pile-up is considered proportional to the residual depth of the imprint (h_{pl}) through

$$h_{pile-up} = h_{pl}(D_{pl} - 1), \quad \text{Eq. 4.16}$$

where D_{pl} is the rigid perfectly plastic pile-up factor that depends on the shape of the indenter. The value of D_{pl} is given for simple geometries in Table 3.1.

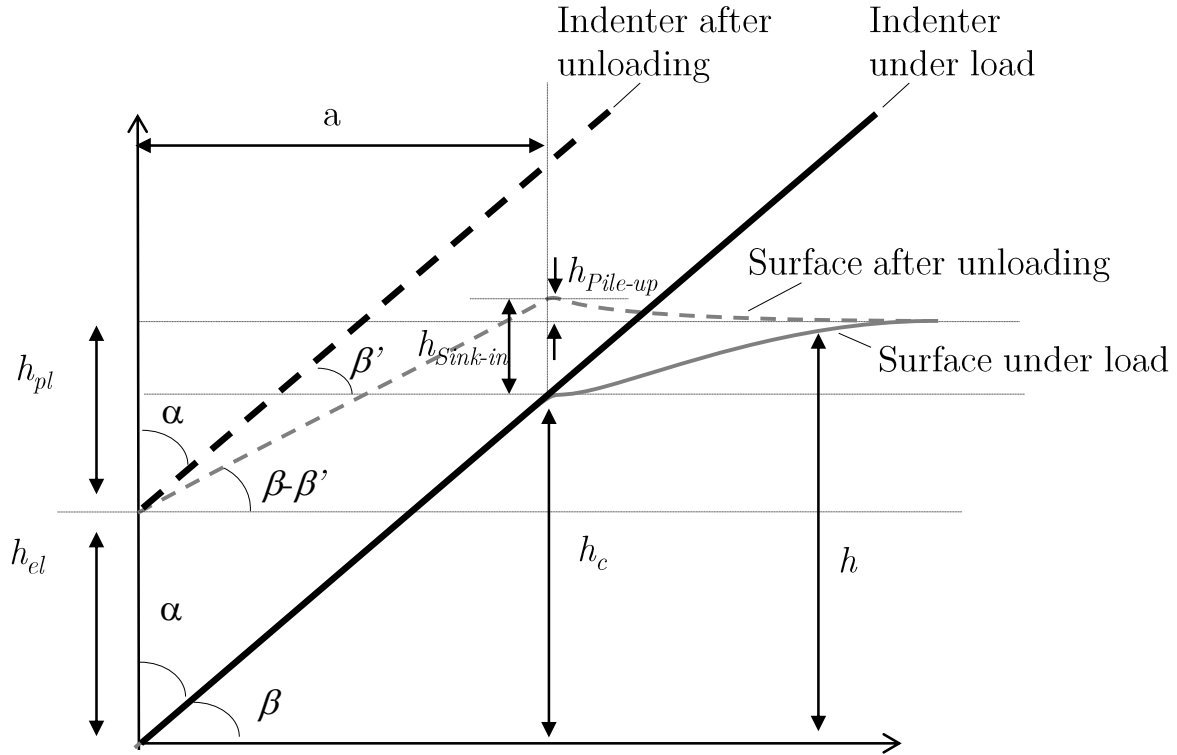


Figure 4.1: Schematic representation of an indentation under a full load and after unloading.

Combining equation Eq. 4.14 with the definition of the hardness gives

$$A_{elpl}(h, \beta) H_{elpl}(\beta) = A_{el}(h_{el}, \beta') H_{el}(\beta'). \quad \text{Eq. 4.17}$$

Eq. 4.17 can be simplified with Eq. 4.15, giving

$$H_{elpl}(\beta) = H_{el}(\beta'). \quad \text{Eq. 4.18}$$

Combining Eq. 4.10, Eq. 4.11 and Eq. 4.18 gives

$$\tan \beta' = \frac{C_{elpl}}{C_{el}} \tan \beta. \quad \text{Eq. 4.19}$$

Eq. 4.15 can be rewritten in terms of the radius of contact through

$$a_{elpl}(h, \beta) = a_{el}(h_{el}, \beta') = a, \quad \text{Eq. 4.20}$$

where a is the radius of contact at a depth h . Using the formulas of the radius of contact in ideal elastic contact (see Table 3.1), Eq. 4.20 becomes

$$a = h_{el} D_{el} \cot \beta', \quad \text{Eq. 4.21}$$

where D_{el} is the ideal elastic sink-in constant that depends on the shape of the indenter and is given for simple geometries in Table 3.1. Combining Eq. 4.19 and Eq. 4.21 gives

$$\frac{h_{el}}{a} = \frac{\tan \beta}{D_{el}} \frac{C_{elpl}}{C_{el}}. \quad \text{Eq. 4.22}$$

Combining Eq. 4.13 and Eq. 4.16 gives

$$\frac{\tan(\beta - \beta')}{D_{pl}} = \frac{h_{pl}}{a}. \quad \text{Eq. 4.23}$$

Combining Eq. 4.19 with Eq. 4.23 gives

$$\frac{h_{pl}}{a} = \frac{1 - \frac{C_{elpl}}{C_{el}}}{1 + \tan^2 \beta \frac{C_{elpl}}{C_{el}}} \frac{\tan \beta}{D_{pl}}. \quad \text{Eq. 4.24}$$

Dividing Eq. 4.3 by the radius of contact gives

$$\frac{h}{a} = \frac{h_{pl}}{a} + \frac{h_{el}}{a}. \quad \text{Eq. 4.25}$$

Combining Eq. 4.22, Eq. 4.24 and Eq. 4.25 provides an expression for the radius of contact through

$$\frac{a}{h} = \left(\frac{1 - \frac{C_{elpl}}{C_{el}}}{1 + \tan^2 \beta \frac{C_{elpl}}{C_{el}}} \frac{\tan \beta}{D_{pl}} + \frac{\tan \beta}{D_{el}} \frac{C_{elpl}}{C_{el}} \right)^{-1}. \quad \text{Eq. 4.26}$$

Substituting Eq. 4.26 in Eq. 4.12 gives the height of contact

$$\frac{h_C}{h} = \left(\frac{1 - \frac{C_{elpl}}{C_{el}}}{1 + \tan^2 \beta \frac{C_{elpl}}{C_{el}}} \frac{1}{D_{pl}} + \frac{C_{elpl}}{D_{el} C_{el}} \right)^{-1}. \quad \text{Eq. 4.27}$$

The area of contact can be calculated based on the expression of the radius of contact in Eq. 4.26. The relationship between the radius of contact and the area of contact can be found in Table 3.1 for different shapes of sharp tips. From there, the loading curves corresponding to an indentation test can be determined from the knowledge of h , σ_y , E , ν and β using

$$F_{V_{elpl}}(h, \beta) = A(a) C_{elpl} \frac{E \tan \beta}{(1 - \nu^2)}, \quad \text{Eq. 4.28}$$

where $A(a)$ is the area of contact for a given radius of contact a (see Table 3.1). Concerning the unloading curves, substituting Eq. 4.26 in Eq. 4.22 gives

$$\frac{h_{el}}{h} = \left(\frac{\frac{C_{el}}{C_{elpl}} - 1}{1 + \tan^2 \beta \frac{C_{elpl}}{C_{el}}} \frac{D_{el}}{D_{pl}} + 1 \right)^{-1}. \quad \text{Eq. 4.29}$$

In order to predict the unloading curve as a function of h , the ideal elastic indentation model with an equivalent angle β' is used. Using the formulas for the radius of contact in ideal elastic contact given in Table 3.1 gives

$$a_{Unload}(h, \beta') = D_{el} h \cot \beta', \quad \text{Eq. 4.30}$$

and

$$H_{el}(\beta') = C_{el} \frac{E \tan \beta'}{(1 - \nu^2)}, \quad \text{Eq. 4.31}$$

where a_{Unload} is the contact area during elastic unloading.

Combining Eq. 2.1, Eq. 4.19, Eq. 4.30 and Eq. 4.31 gives

$$a_{Unload}(h, \beta') = D_{el} h \frac{C_{el}}{C_{elpl}} \cot \beta, \quad \text{Eq. 4.32}$$

$$H_{el}(\beta') = C_{elpl} \frac{E \tan \beta}{(1 - \nu^2)}, \quad \text{Eq. 4.33}$$

and

$$F_{V_{Unload}}(h, \beta') = A(a_{Unload}) C_{elpl} \frac{E \tan \beta}{(1 - \nu^2)}, \quad \text{Eq. 4.34}$$

Where $A(a_{Unload})$ is the area of contact for a given radius of contact a_{Unload} (see Table 3.1)

4.1.3 Sharp tip indentation with strain hardening

Up to now, the model does not take into account the effect of strain hardening. As described in section 3.2.4, strain hardening has an impact on the contact area. Indeed, the pile-up is less important in material with a high strain hardening coefficient. It also affects the average indentation flow stress which is equal to σ_y in perfectly plastic materials. Finally, it affects the constraint factor (C_{pl}), the constant linking σ_y to H in perfectly plastic materials. To sum up, the three terms D_{pl} , σ_y , and C_{pl} are affected by the strain hardening exponent (n). However, the effect of n on these parameters is not quantitatively known. To overcome this problem, a two-step approach is used. First, a rigid plastic model with strain hardening is approximated using curve fitting on FEM results taken from the literature. This model is then combined with the ideal elastic model using Yu and Blanchard approach in order to calculate an elastoplastic model with strain hardening.

The first step consists in identifying the solution of a rigid plastic model with strain hardening. In order to do that, the solution of the model is assumed to be similar to the rigid-perfectly plastic solution except for the three properties D_{pl} , σ_y , and C_{pl} that are affected by n . As stated previously, the effect of n on these parameters is obtained through curve fitting on FEM results from the literature. Most of the FEM calculations coming from the literature are performed on a large panel of elastoplastic materials but, unfortunately, not on the extreme case of a rigid plastic materials with strain hardening. In spite of that, observing the FEM results presented on figure 3.10, one can see that, regardless of the strain hardening coefficient, there seems to be a sigmoidal evolution of the height of contact (h_c/h) with the ratio σ_y/E . Indeed, h_c/h seems to tend asymptotically towards a finite value on both sides of this graph. For high values of σ_y/E , it tends asymptotically towards the ideal elastic solution and for low values of σ_y/E , it tends asymptotically towards a finite value as well. In the case $n=0$, it tends towards the rigid-perfectly plastic solution. This trend also happens in

the case of the representative indentation strain as well as on the constraint factor (Figure 3.8 and Figure 3.9). These trends make sense; indeed, the ratio σ_y/E is the elastic strain (ε_y) of the base material. ε_y is the strain above which the material enters plasticity. If ε_y is small enough, the elastic contribution to the indentation becomes negligible and the materials behave like a rigid plastic material. Therefore, the left side of these graphs corresponds approximatively to the solution of the rigid-plastic model with strain hardening. Therefore, the data corresponding to low values of the ratio σ_y/E are selected for the curve fitting.

Regarding the indentation representative flow stress (σ_{r-ind}), it can be related to the indentation representative strain (ε_{r-ind}) through a Hollomon law, see Eq. 3.7, ε_{r-ind} can be obtained using a polynomial curve fitting on the data from Hernot [Hernot 2004] (see Figure 2.21). Based on these data, the representative indentation strain of materials with low values of σ_y/E can be approximately linked to the strain hardening exponent (n) through

$$\varepsilon_{r-ind} \approx -2.71 n^3 + 3.49 n^2 - 1.39 n + 0.25. \quad \text{Eq. 4.35}$$

Since the FEM calculations from [Hernot 2004] are valid for a conical tip with a half-opening angle of 70.3° only, a more general, yet less precise, solution can be used in the case of other sharp tip geometries based on Tabor approximation in which the representative strain is a function of the opening angle of the tip (see Eq. 3.6).

Using polynomial curve fitting on Hernot data [Hernot 2004] (see Figure 2.22), the representative constraint factor (C_{plr}) of materials with low values of σ_y/E can be linked to the strain hardening exponent through

$$C_{plr} \approx C_{pl} (-1.66 n^3 + 0.22 n^2 - 0.54 n + 1), \quad \text{Eq. 4.36}$$

where C_{pl} is the rigid perfectly plastic constraint factor defined in Table 3.1 for simple geometries.

Using polynomial curve fitting on data from [Mata 2002 and Cheng 1998], the representative pile-up factor (D_{plr}) of materials with low values of σ_y/E can be linked to the strain hardening exponent through

$$D_{plr} \approx D_{pl} (0.265 n^2 - 0.749 n + 1), \quad \text{Eq. 4.37}$$

where D_{pl} is the rigid perfectly plastic pile-up factor defined in Table 3.1 for simple geometries of tips. Finally, the effect of the strain hardening is taken into account by replacing D_{pl} , σ_y , and C_{pl} by their corresponding strain hardening dependent quantities: D_{plr} , σ_r , and C_{plr} in the model presented in Section 4.1.2 .

4.1.4 Assessment of the assumptions using finite element simulations

The hypotheses presented in section 4.1.2 and 4.1.3 are now assessed through finite element simulations. This validation is performed on the well documented problem of a rigid conical indenter penetrating an elastoplastic infinite half space allowing for simple axisymmetric conditions. In the present sub section, some of the key characteristics of these finite element calculations are presented as well as the results.

• *Finite element model*

The commercial finite element software ABAQUS 6.13 is used in the framework of a finite strain setting. The simulations are performed with axisymmetric conditions. The indenter tip is modelled as a perfectly rigid cone with a half-included angle of 70.3° . This geometry is equivalent to Vickers and Berkovich indenters in term of contact area versus depth of penetration. The constitutive model of the base material is the isotropic J_2 plasticity flow theory. The elastic part follows Hooke's law. The hardening law follows the Hollomon power law strain hardening corresponding to Eq. 3.7. The indentation is displacement controlled and includes a loading step to a depth of $1\ \mu\text{m}$ followed by an unloading to a zero depth and a reloading to a depth of $1\ \mu\text{m}$. The specimen size is chosen large enough so that the specimen can be considered as a semi-infinite half space (see Figure 4.2). The height of the specimen is 150 times larger than the depth of contact. The specimen radius is, in all cases, larger than 55 times the radius of contact. The mesh in the region near the indenter is refined and made of square shaped elements (see Figure 4.2). Regarding the boundary conditions, the displacements of the elements situated on the surface in the bottom of the base material as well as in the external lateral surface are blocked in the vertical and radial direction. The contact between the indenter and the specimen is considered frictionless. Regarding the mechanical properties of the indented material, Young's modulus (E) and Poisson ratio (ν) are fixed, equal to 100 GPa and 0.3 respectively. Different

combinations of yield strength (σ_y) and strain hardening exponent (n) are investigated (see Table 4.1). Some of these values can be considered unrealistic, for instance $\sigma_y > 10$ GPa. However these values were chosen to reach ratios of σ_y/E ranging from 10^{-6} to 10^1 in order to investigate a large panel of behaviors ranging from almost perfectly plastic to ideal elastic ones.

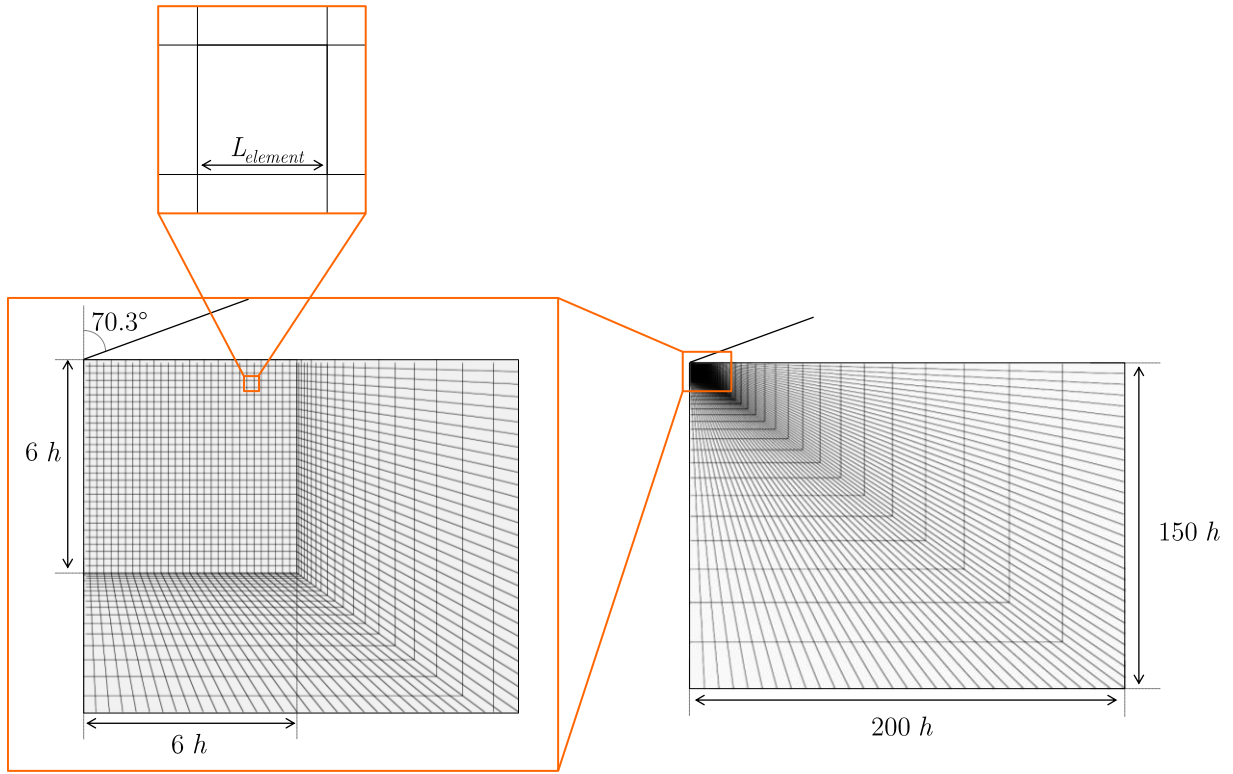


Figure 4.2: Representation of the finite element mesh geometry, including the base material made of axisymmetric elements and the conical indenter with a half-angle of 70.3° . In the refined area, the size of the side of the square element is $L_{element}$.

Table 4.1: List of mechanical properties used in the FEM model ($\nu = 0.3$)

$E[\text{GPa}]$	$\sigma_y [\text{GPa}]$	n	σ_y / E
100	10^{-3}	0	10^{-5}
“ ”	0.01	0	10^{-4}
“ ”	0.1	0	10^{-3}
“ ”	1	0	0.01
“ ”	0.5	0	0.05
“ ”	10	0	0.1
“ ”	25	0	0.25
“ ”	50	0	0.5
“ ”	100	0	1
“ ”	10^3	0	10
“ ”	10^4	0	100
“ ”	10^{-3}	0.1	10^{-5}
“ ”	10^{-3}	0.2	10^{-5}
“ ”	10^{-3}	0.3	10^{-5}
“ ”	10^{-3}	0.5	10^{-5}

• *Determination of optimum mesh size*

A first couple of simulations were performed to select the mesh size in the refined area which ensures that the simulation results are independent of the size of the mesh. Therefore, five sizes of elements in the refined area are tested: 50, 75, 100, 150 and 200 nm. Then, numerical indentation experiments are performed on two materials with extreme mechanical behaviors (i.e. σ_y/E ratios equal to 10^{-5} and 10^2). Two outputs are investigated: the indentation force at maximum indentation depth (F_V) and the radius of contact at maximum indentation depth (a). The radius of contact is measured with the coordinates of the two upper nodes in the last element in contact with the indenter at maximum penetration depth. The vertical position of these two nodes after unloading is considered as $h_{pile-up}$. The results are presented in Figure 4.1. These results suggest that below a mesh size of 50 nm, the vertical force and the radius of contact are no longer affected by the size of the element in the refined area.

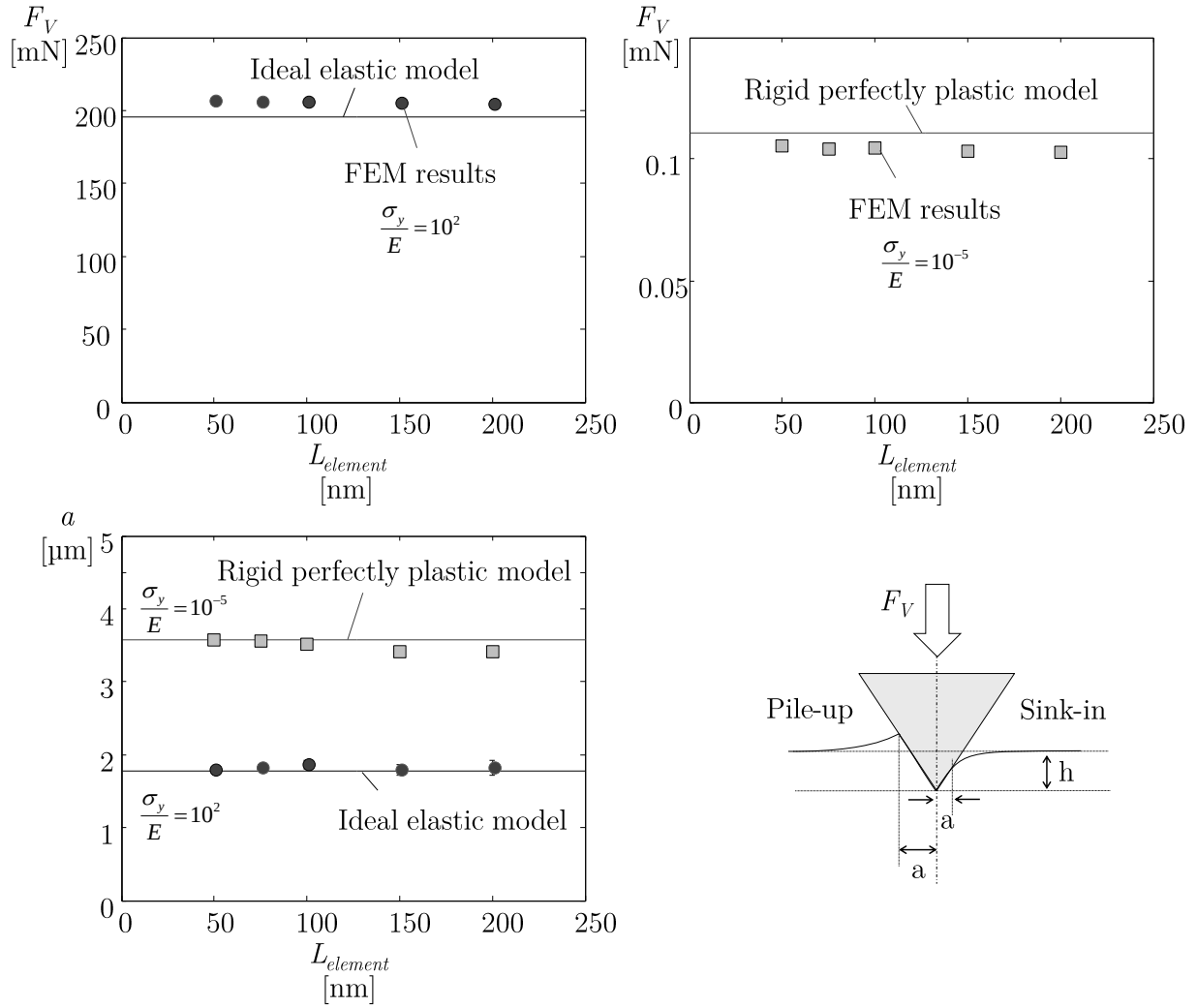


Figure 4.3: Graphs showing the effect of the size of the elements in the refined area on the radius of contact (a) and on the indentation vertical force (F_V). The round dots and the squares represent the FEM results.

• Hypotheses validation

The size of the elements in the refined area is therefore fixed equal to 50 nm. The numerical indentation experiments are performed on the 12 combinations of materials presented in Table 4.1. Approximately 24000 four-noded axisymmetric quadrilateral elements are used in the model. At maximum indentation depth, at least 37 elements are in contact with the indenter. Three hypotheses are tested here: (i) after unloading, the side of the cross section of the residual imprint can be approximated by a straight line. (ii) The unloading is considered purely elastic which implies that the unloading and any successive reloading to the same depth h would involve the same vertical force

(F_V) and the same contact radius (a). (iii) The pile-up ($h_{pile-up}$) is considered proportional to the residual depth of the imprint (h_{pl}).

Regarding the straight line cross section hypothesis (i), the results are presented in Figure 4.4. The FEM calculations shows that this hypothesis is reasonable if the ratio σ_y/E is smaller than 0.05 or bigger than 1, otherwise the cross section of the residual imprint presents a significant non-linearity.

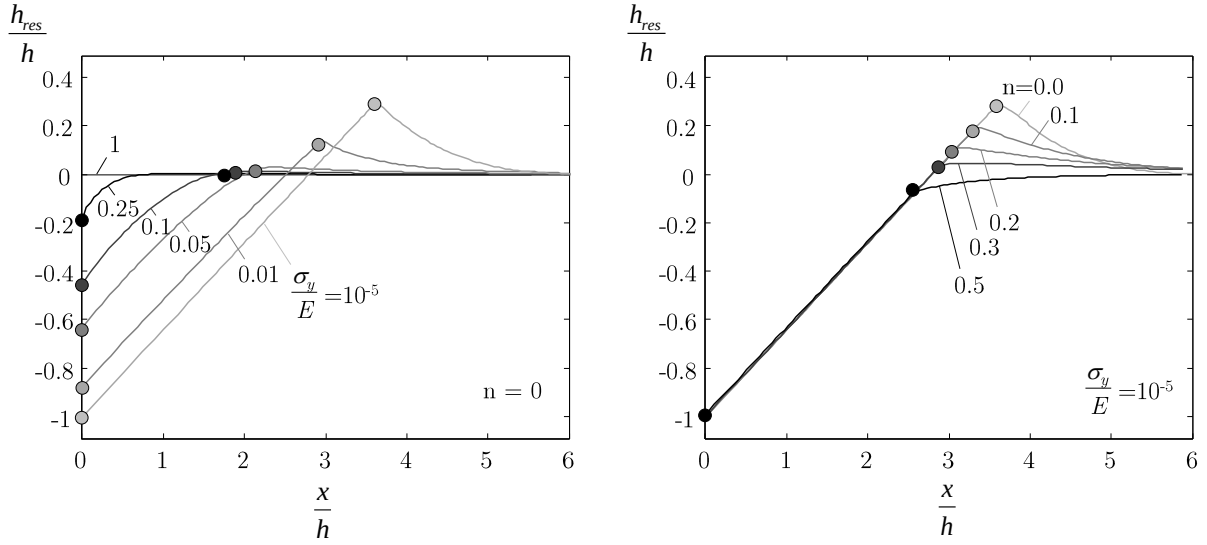


Figure 4.4: Cross section of the residual imprint after unloading. The small round dots represent the last element in contact with the tip as well as the bottom of the imprint. The straight line cross section hypothesis implies that the cross section between these two points is a straight line.

Concerning the elastic reloading hypothesis (ii), the results are presented in Figure 4.5 and in Figure 4.6. The difference between the radius of contact during the first loading (a_{load}) and after the second loading (a_{reload}) remains smaller than 0.03% in all the tested cases. In the case of the vertical indentation force, the difference between the first loading and the second is smaller than 1.5%. Therefore, this hypothesis can be considered valid.

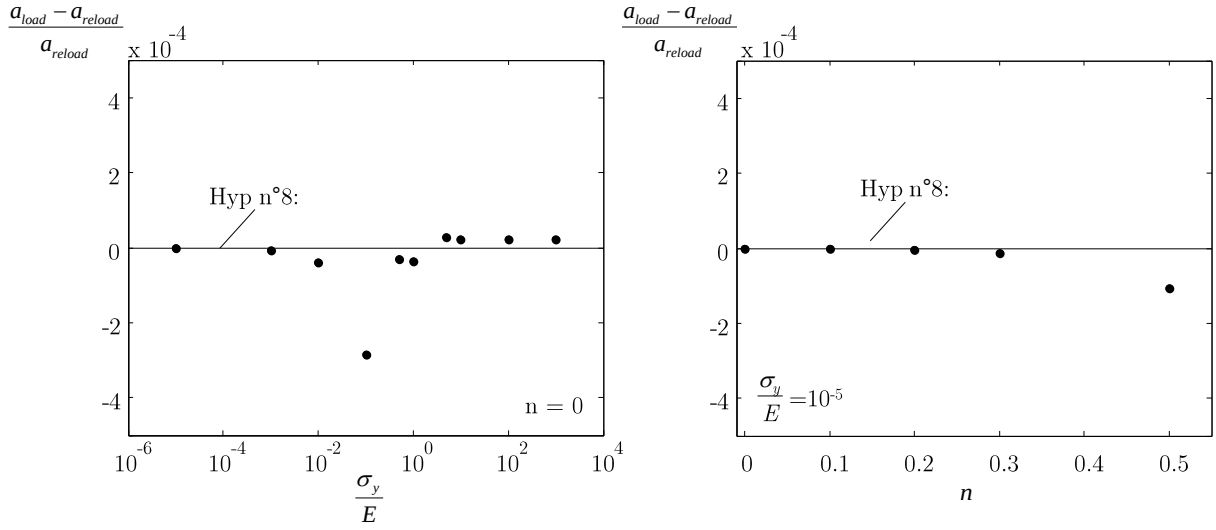


Figure 4.5: Difference between the radius of contact during the first loading (a_{load}) and during the second loading (a_{reload}) as a function of the yield strain (σ_y/E) and as a function of the strain hardening exponent (n). The difference remains smaller than 0.03% in all the tested situations.

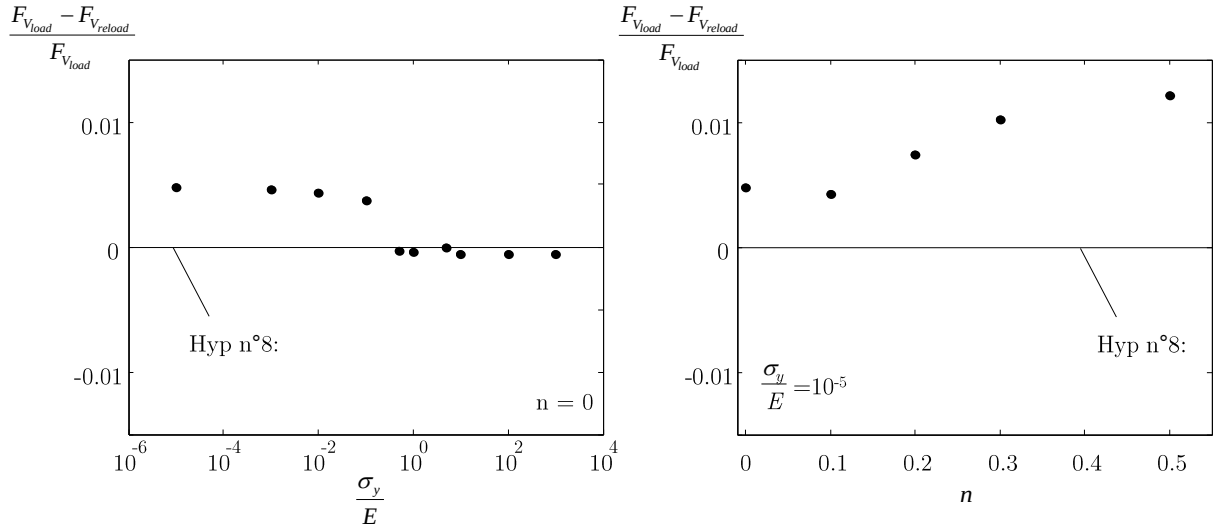


Figure 4.6: Difference between the vertical indentation force during the first loading (F_{Vload}) and during the second loading ($F_{Vreload}$) as a function of σ_y/E and as a function of n . The difference remains smaller than 1.5% in all the tested configurations.

The hypothesis on the proportionality between the pile-up height and the residual depth (iii) is presented in Figure 4.7 as well as the approximation on the representative pile up factor (D_{plr}) in the case of strain hardening materials. The pile-up hypothesis is valid in the case of σ_y/E ratio smaller than 0.001 and larger than 0.5. The pile-up hypothesis can be modified to be more accurate by replacing h_{pl} by $(h_{pl})^\delta$ in Eq. 4.16. Regarding the approximation on D_{plr} , the present FEM results are in good agreement with the results from [Mata 2002 and Cheng 1998].

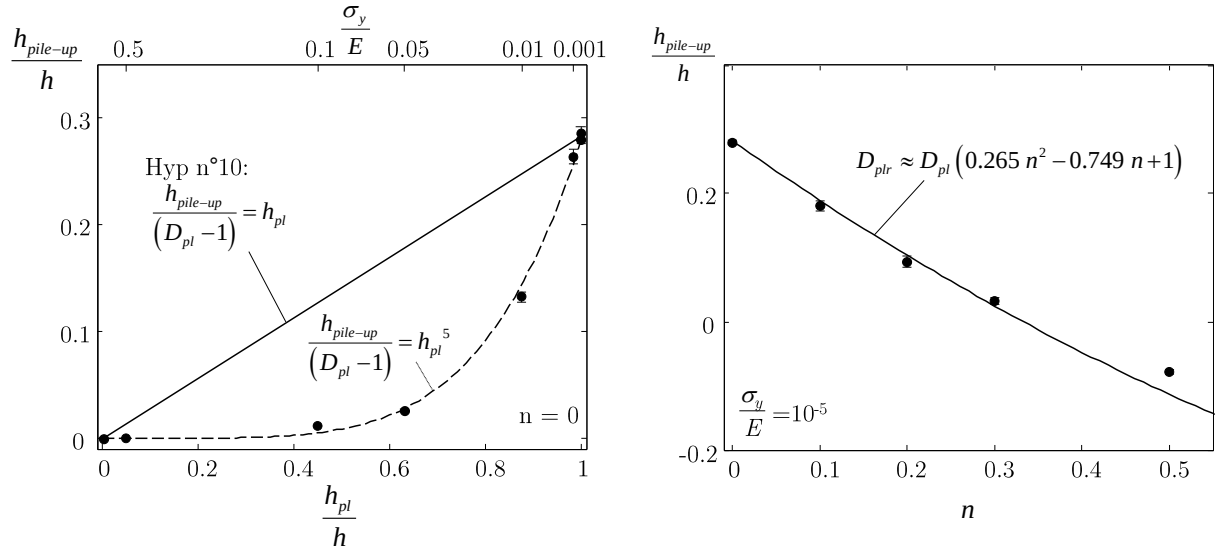


Figure 4.7: On the left, the height of the pile-up ($h_{pile-up}$) is represented as a function of the residual depth of indentation (h_{pl}). The graph on the right represents $h_{pile-up}$ as a function of the strain hardening coefficient (n). The solid lines represent the hypothesis, the round dots are the results of the FEM calculations.

4.1.5 Parametric study and sensitivity analysis

In the present sub section, the results of the indentation models are presented along with a description of the effect of the geometry of the tip and the material properties E , σ_y and n on the indentation force (F_V) as well as on the contact area and on the elastic recovery.

• Flat tip geometries

Concerning the flat punch geometries, the loading part of the curves and the unloading part can be drawn using Eq. 4.8 and Eq. 4.9. A typical solution is presented in Figure 4.8 in the case of a cylindrical flat punch indenting against steel. The main difference with the model of Yu and Blanchard is the presence of the unloading part of the curve. The shape of the curve presented in Figure 4.8 is similar in the case of a rectangular flat punch. The amplitude of the elastic recovery after unloading is shown in Figure 4.9. There is a progressive transition from the elastic solution to the perfectly plastic solution.

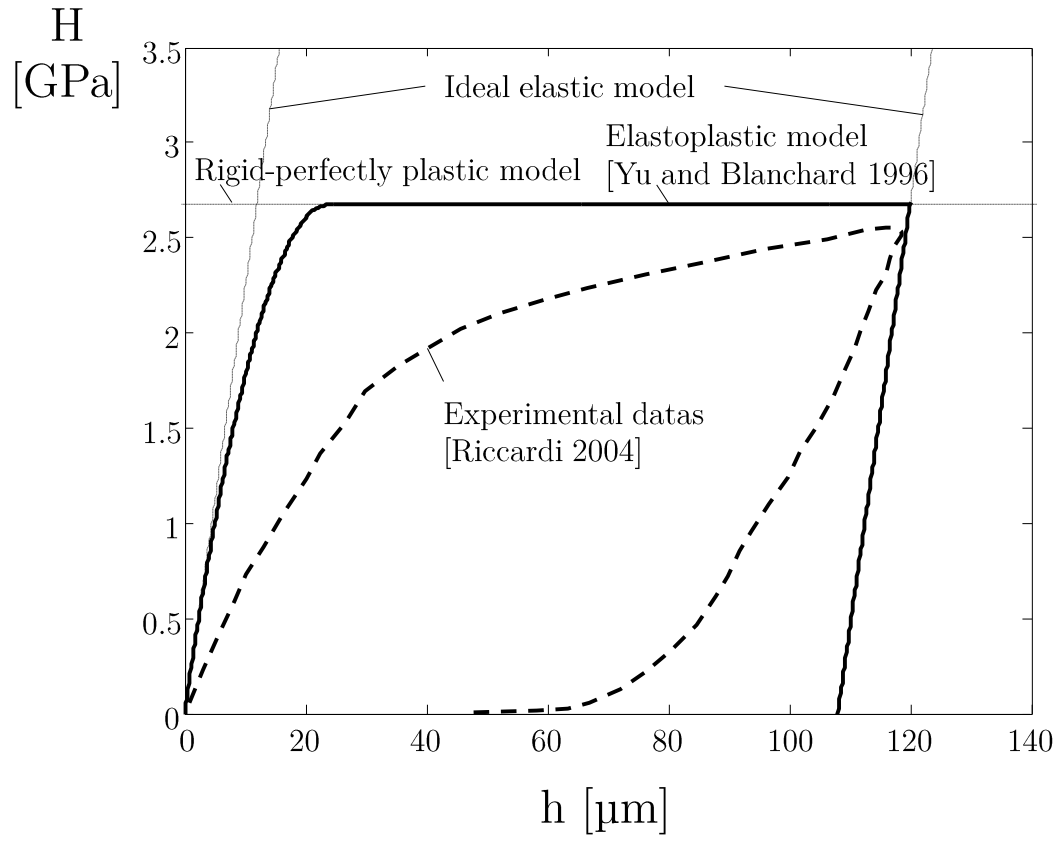


Figure 4.8: Comparison between the model and an indentation experiment performed by [Riccardi 2004] indenting a F82H steel ($\sigma_y = 813$ MPa, $n = 0.07$ and $E_{eq} = 162$ GPa), with a 1 mm diameter cylindrical indenter made of tungsten carbide.

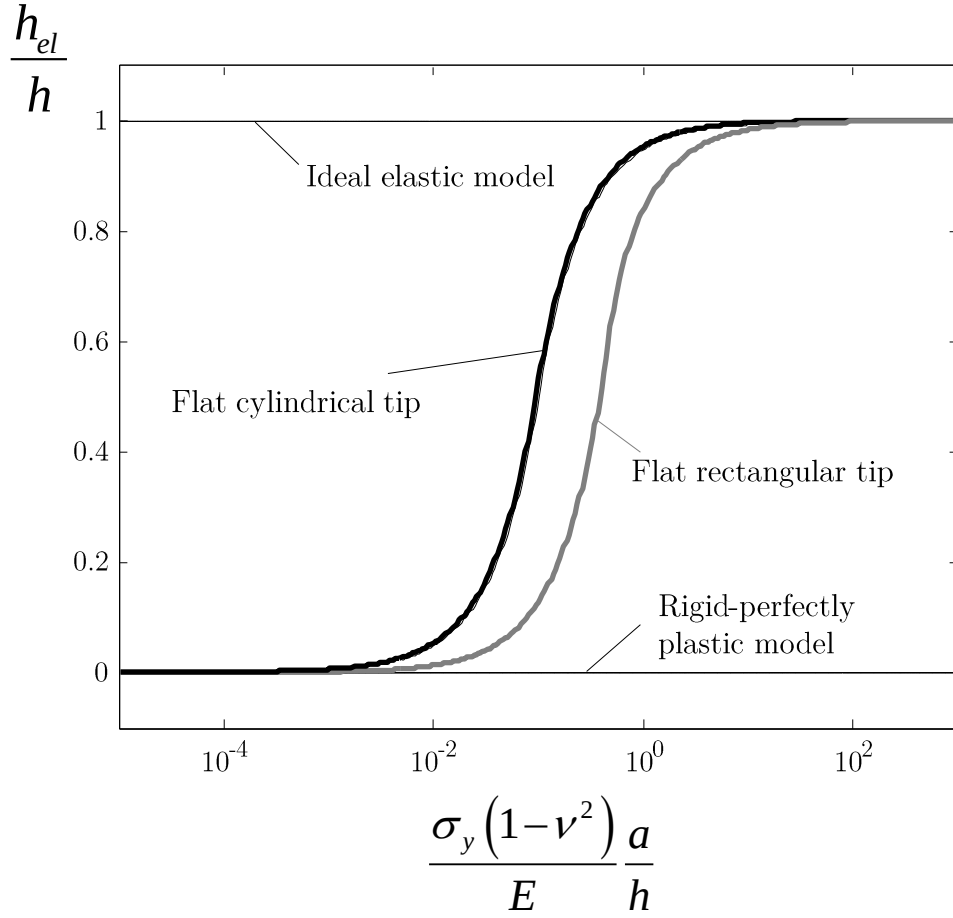


Figure 4.9: Evolution of the elastic recovery normalized by the indentation depth of a cylindrical flat tip and a rectangular one as a function of the yield strain (σ_y/E) multiplied by the tip radius (a) and divided by the depth (h).

- *Sharp tip geometries*

Regarding sharp tip indentations, typical indentation curves computed with the present model are presented in Figure 4.10. The half-wedge and the wedge geometries force-depth relationships are linear during loading and unloading. With the same half-opening angle, the wedge geometry generates a vertical force twice as large as a half wedge when penetrating at the same depth. Regarding the elastic recovery (h_{el}) and the height of contact (h_c) the three sharp tips present the same transition shape from the elastic model to the plastic model (see Figure 4.11 and Figure 4.12). Changing the β angle has a strong effect on the elastoplastic transition (see Figure 4.13) as well as on the real height of contact (see Figure 4.14). Indenting the same material, a sharper tip seems to present a smaller elastic recovery and a larger height of contact than a tip with a more open angle.

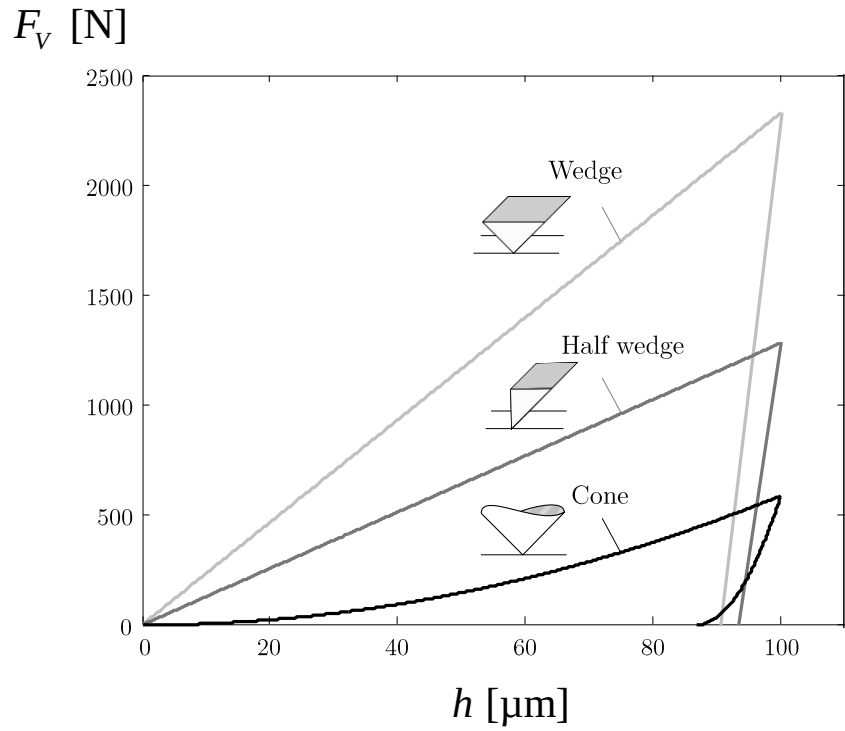


Figure 4.10: Model indentation curves applied on three sharp tips. These curves correspond to an angle β of 19.7° , $W = 2$ mm, $h = 100$ μm , $\nu = 0.3$, $\sigma_y = 600$ MPa, $E_{eq} = 120$ GPa and $n = 0$.

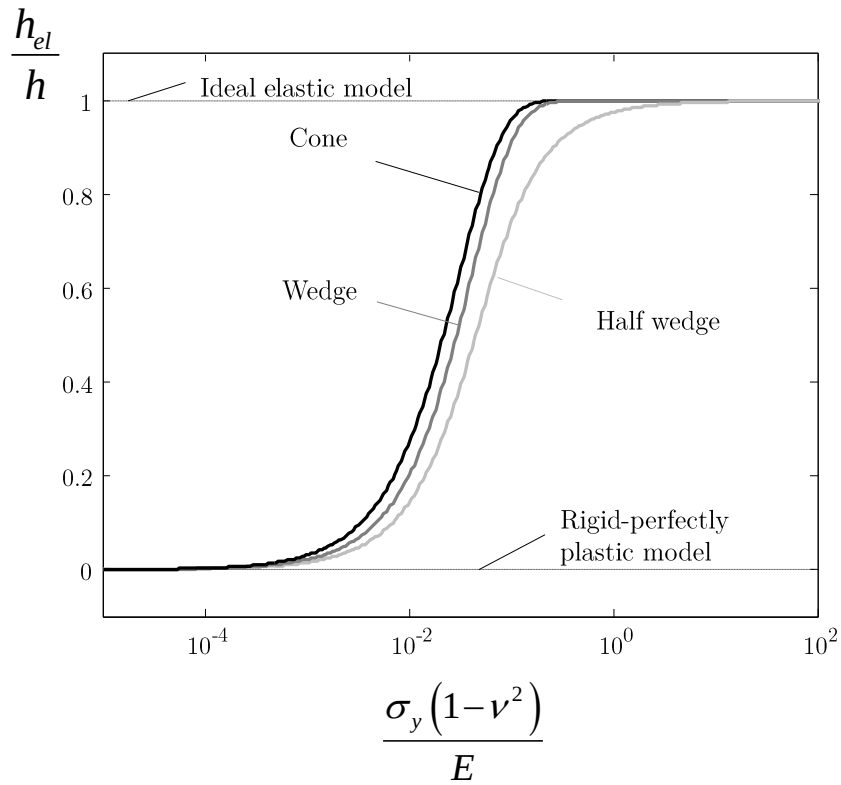


Figure 4.11: Evolution of the elastic recovery normalized by the indentation depth of three sharp tips as a function of the yield strain (σ_y/E). These curves correspond to an angle β of 19.7° .

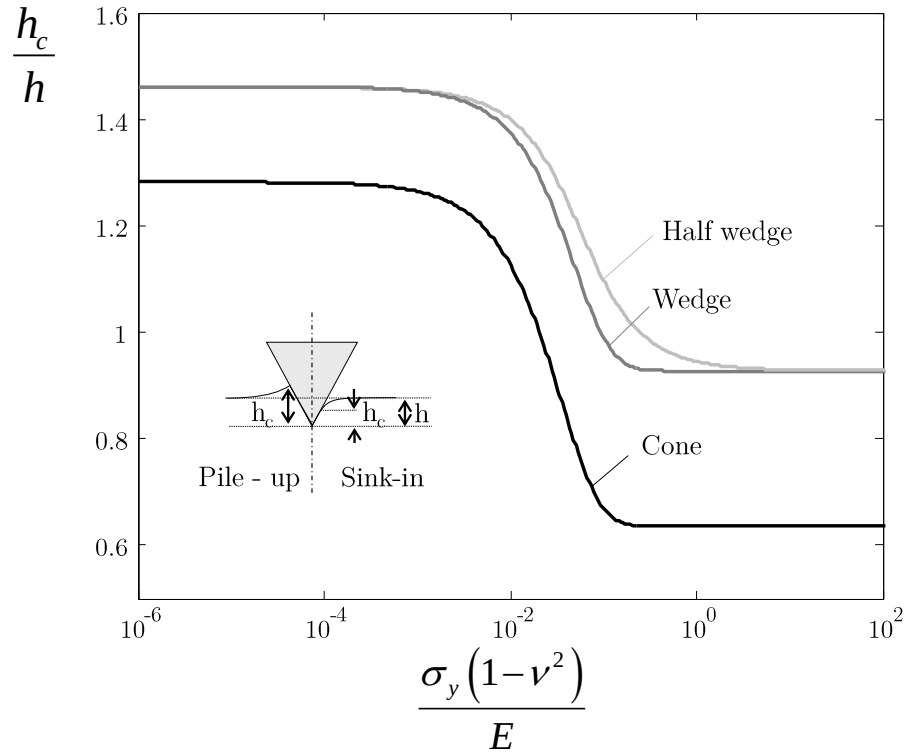


Figure 4.12: Evolution of the real height of contact normalized by the indentation depth of three sharp tips as a function of the yield strain (σ_y/E). These curves correspond to an angle β of 19.7° .

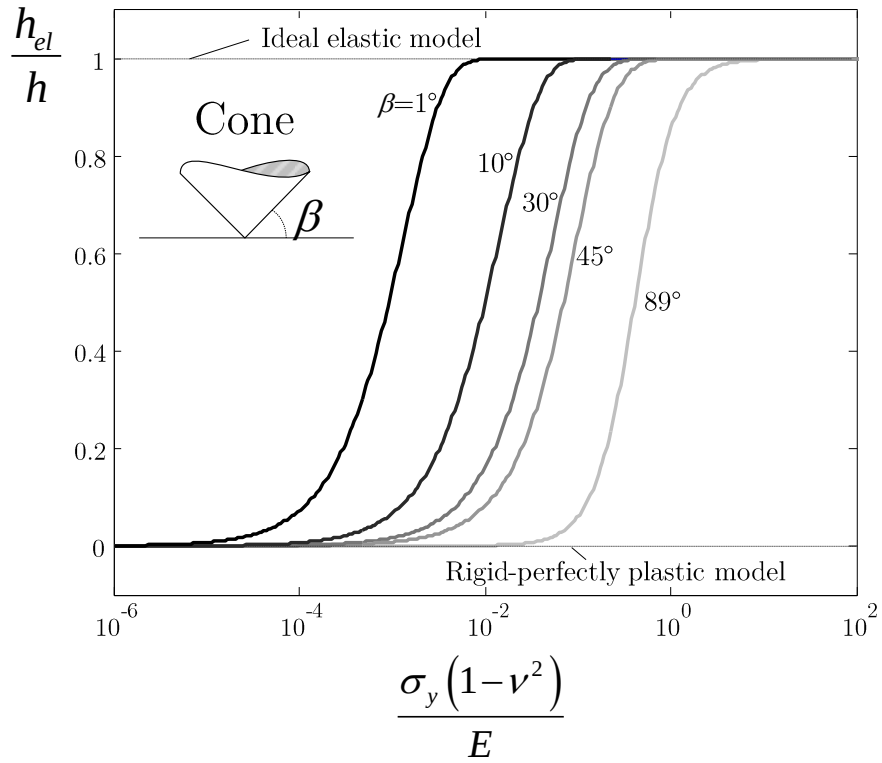


Figure 4.13: Elastic recovery normalized by the indentation depth of a conical tip with different β angles as a function of the yield strain (σ_y/E). The effect of β is similar in the case of the wedge and the half-wedge tip geometries.

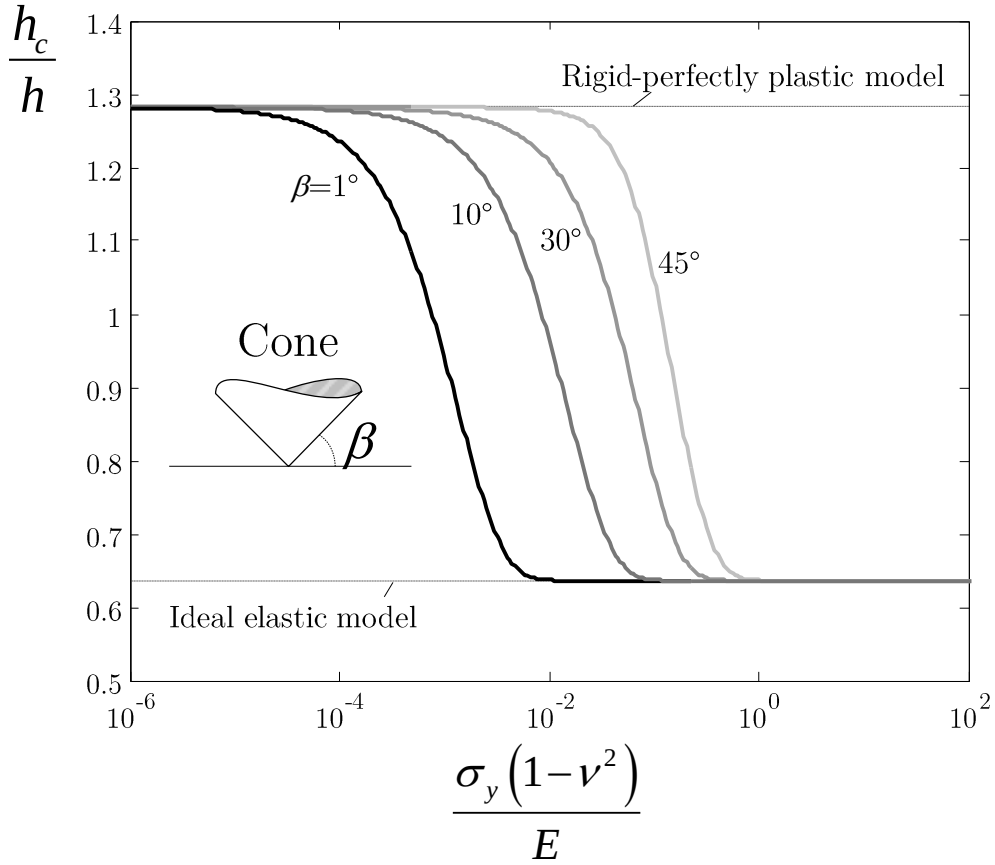


Figure 4.14: Real height of contact (h_c) normalized by the indentation depth of a conical tip with different β angles as a function of the yield strain (σ_y/E). The effect of β is similar in the case of the wedge and the half-wedge tip geometries.

- *Sharp tip with strain hardening materials*

Regarding the effect of strain hardening on the height of contact, as also predicted by FEM calculations [Cheng 1998, Mata 2002], the model predicts a decrease of the pile-up with increasing strain hardening exponent (see Figure 4.15) as well as a progressive transition from rigid plastic to ideal elastic. The elastic recovery is also affected by the strain hardening coefficient. The model predicts that a larger strain hardening exponent will give rise to proportionally more elastic recovery (see Figure 4.16). The model deviates from the FEM calculations from [Bolshakov 1998] and [Poon 2008]. However, the present model agrees well with experimental data points measured by [Poon 2008]. The model predicts a strong effect of the strain hardening on the constraint factor (H/σ_y), especially in the case of materials with a low σ_y/E ratio (see Figure 4.17 and Figure 4.18). In these two figures, experimental values are compared with the model. The position of the real materials varies a lot from one figure to the

other especially in the case of ceramic materials. This can be explained by the choice of the physical definition of σ_y . In the data provided by CES Edupack [CES 2015], σ_y is defined as the fracture strength in tension for ceramics [Ashby 2011]. In the study by [Clausner 2015], regarding ceramics and polymers, σ_y is obtained by calculation using the spherical cavity expansion model of [Johnson 1970]. More precisely, the hardness and the Young's modulus are measured by nanoindentation and by surface acoustic waves measurements then they are introduced in Johnson's equation to calculate σ_y , which gives a way higher value of σ_y . Regardless of these differences, the FEM calculations performed by [Clausner 2015] are slightly different from the present model however, the trends are the same (see Figure 4.18).

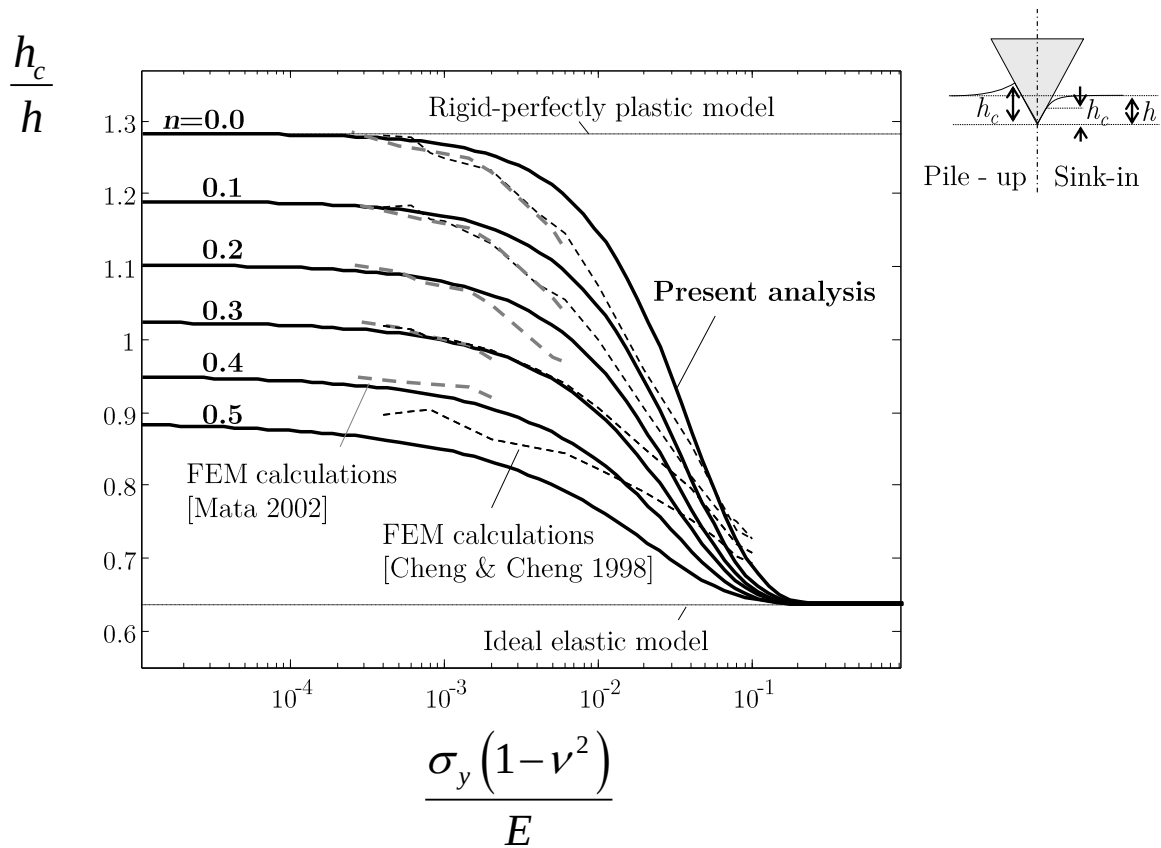


Figure 4.15: Evolution of the real contact height normalized by the indentation depth of a conical tip with an angle β of 19.7° as a function of the yield strain (σ_y/E).

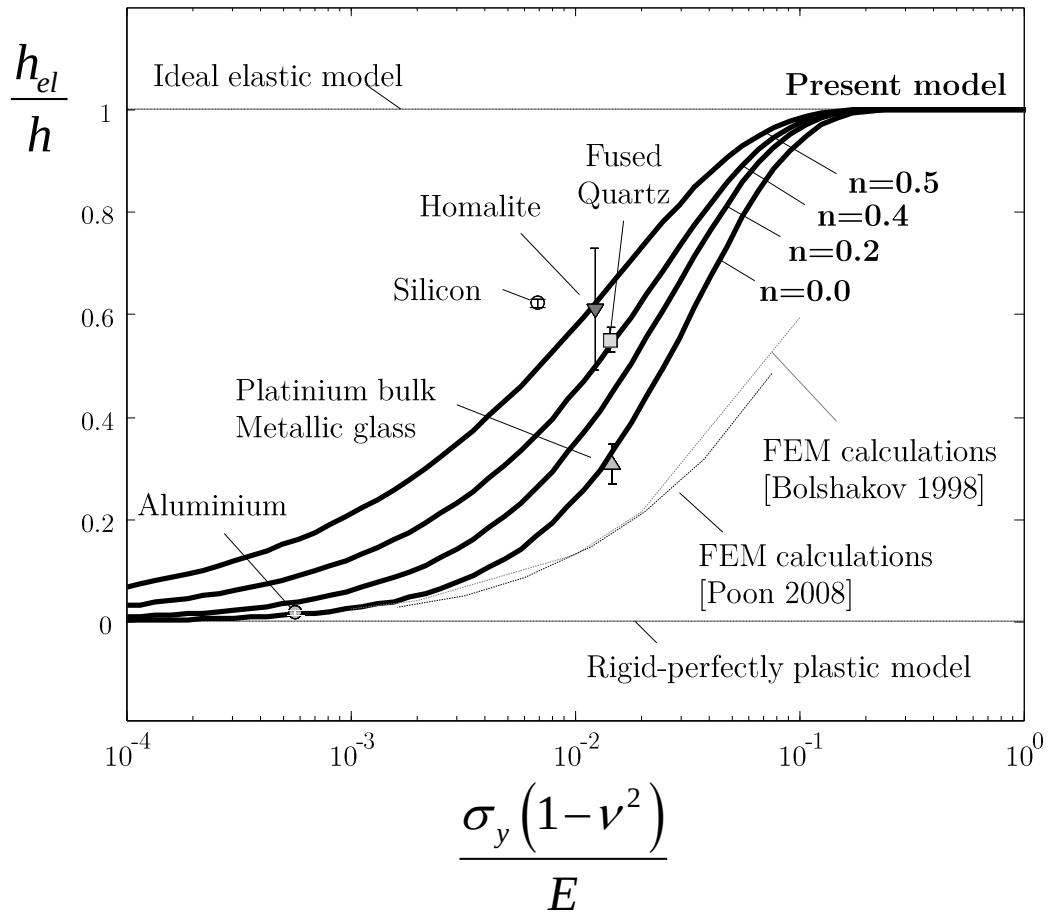


Figure 4.16: normalized elastic recovery of a conical tip with an angle β of 19.7° with different strain hardening exponents as a function of the yield strain (σ_y/E). The two FEM calculations were performed with $n = 0$.

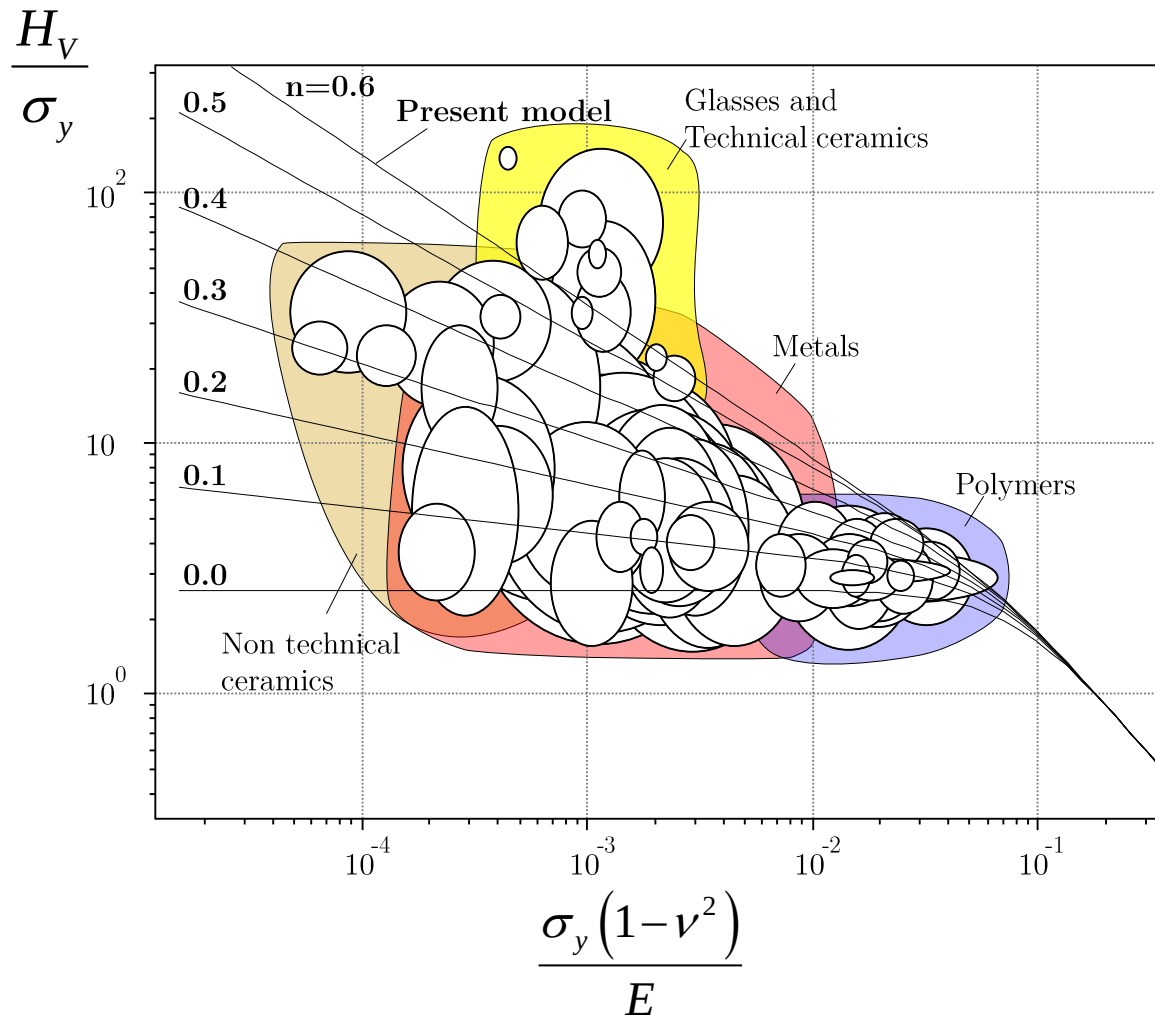


Figure 4.17: Comparison between the present conical indentation model with an angle β of 19.7° and the data base from [CES 2015].

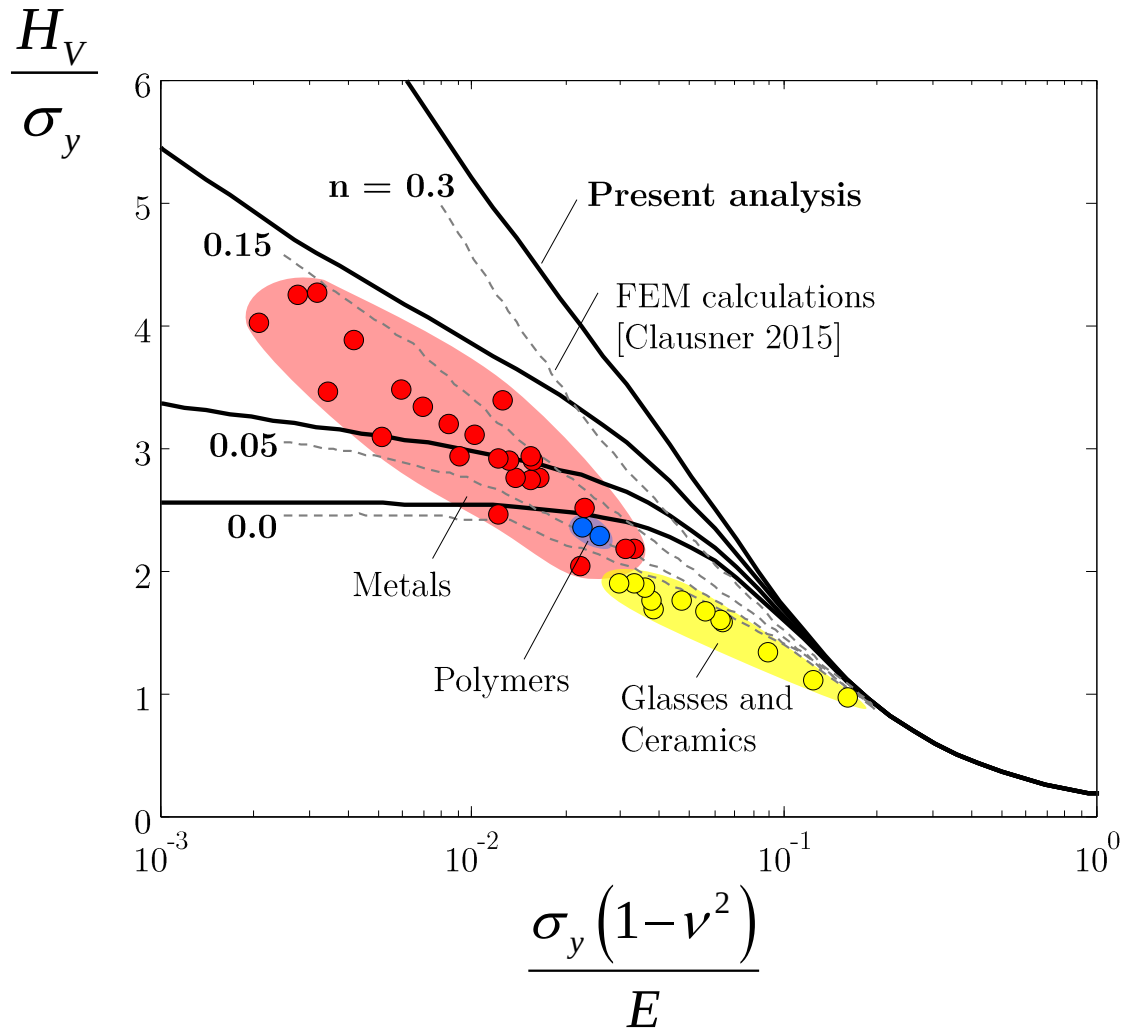


Figure 4.18: Comparison between the present model of conical indentation with an angle β of 19.7° and FEM calculations as well as experimental data from [Clausner 2015].

4.2 Plowing model

Plowing models presented in Section 0 are limited to rigid-perfectly plastic materials. In order to extend these models to elastoplastic materials, the approach developed by [Flores 2008] is combined with the present elastoplastic indentation model to create an elastoplastic plowing solution involving three different shapes of tools: the rectangular flat tip, the orthogonal cutting tool and the cone. As stated in Section 0, at constant depth of plowing, the horizontal force is the only force that produces mechanical work. From an energetic point of view, the present model considers the plowing as the sum of plastic dissipation and friction. In the original model of Flores, the plastic dissipation is calculated based on a series of indentations of two-dimensional slices of the wear track. In a more general elastoplastic framework, the plastic dissipation contribution needs to be separated from the elastic recovery because the later produces no dissipation of energy (see Figure 4.19). Indeed, the elastic energy is stored inside the material during the indentation and recovered during the unloading by the action of the elastic restoring force pushing the tip upward.

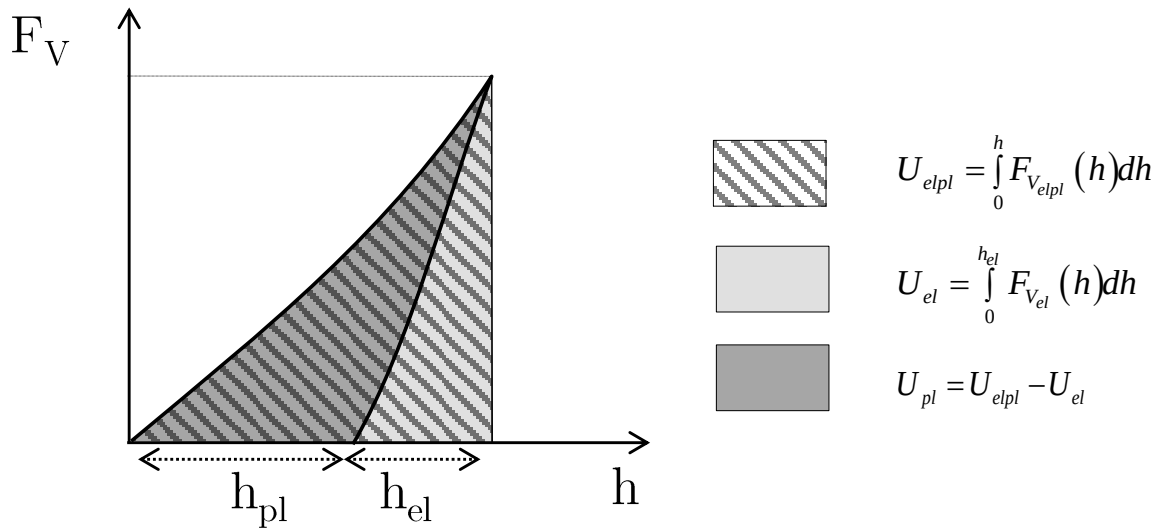


Figure 4.19: The amount of plastic dissipation (U_{pl}) occurring during indentation can be calculated by subtracting the elastic recovery energy (U_{el}) from the total energy of indentation (U_{elpl}).

Using this approach, the elastoplastic energy balance can be linked to the plowing horizontal force through

$$\frac{dW}{dx} = F_{H_{elpl}} = \int_0^h f_{V_{elpl}} dh - \int_0^{h_{el}} f_{V_{unload}} dh + \mu F_{V_{elpl}}, \quad \text{Eq. 4.38}$$

where μ is the friction coefficient, $F_{V_{elpl}}$ is the indentation force during loading of the tip in an elastoplastic material, $f_{V_{elpl}}$ and $f_{V_{unload}}$ are the indentation force and the elastic recovery force acting on a two-dimensional version of the tip per unit width.

4.2.1 Conical tip plowing solution

The conical tip plowing model is represented in Figure 4.20. In this conical configuration, the two-dimensional projection of the cone is a wedge with the same opening angle. Therefore, Eq. 4.38 becomes

$$F_{H_{elpl}}^C = \int_0^h \frac{F_{V_{elpl}}^{W_2}}{W_2} dh - \int_0^{h_{el}} \frac{F_{V_{unload}}^{W_2}}{W_2} dh + \mu F_{V_{elpl}}^C, \quad \text{Eq. 4.39}$$

where W_2 is the width of the wedge, the superscripts ' C ' and ' W_2 ' designate respectively the indentation solutions of a cone and a wedge of width W_2 . Combining Eq. 4.39 with the definition of the hardness in Eq. 2.1 gives

$$F_{H_{elpl}}^C = \int_0^h 2a_{elpl}^{W_2} H_{elpl}^{W_2} dh - \int_0^{h_{el}} 2a_{unload}^{W_2} H_{el}^{W_2} (\beta') dh + \mu (a_{elpl}^C)^2 \pi H_{elpl}^C, \quad \text{Eq. 4.40}$$

which can be rewritten as

$$F_{H_{elpl}}^C = \int_0^h \left(\frac{a_{elpl}^{W_2}}{h} \right) 2h H_{elpl}^{W_2} dh - \int_0^{h_{el}} \left(\frac{a_{unload}^{W_2}}{h} \right) 2h H_{el}^{W_2} (\beta') dh + \mu h^2 \pi H_{elpl}^C \left(\frac{a_{elpl}^C}{h} \right)^2, \quad \text{Eq. 4.41}$$

The terms in brackets are constant and are given in Eq. 4.26 and Eq. 4.30. Integrating Eq. 4.41 and simplifying it (a step-by-step calculation is given in Appendix A.2.1) gives

$$F_{H_{elpl}}^C = F_{V_{elpl}}^C \left[\left(\frac{h}{a_{elpl}^C} \right)^2 \left(\frac{a_{elpl}^{W_2}}{h} \right) \left(\frac{h_{pl}^{W_2}}{h} \right) \frac{H_{elpl}^{W_2}}{H_{elpl}^C} \frac{1}{\pi} + \mu \right]. \quad \text{Eq. 4.42}$$

The same strategy can be followed in the case of an ideal elastic material and in the case of a rigid plastic material giving respectively

$$F_{H_{el}}^C = \mu F_{V_{el}}^C \quad \text{Eq. 4.43}$$

and

$$F_{H_{pl}}^C = F_{V_{pl}}^C \left[\left(\frac{h}{a_{pl}^C} \right)^2 \left(\frac{a_{pl}^{W_2}}{h} \right) \frac{H_{pl}^{W_2}}{H_{pl}^C} \frac{1}{\pi} + \mu \right]. \quad \text{Eq. 4.44}$$

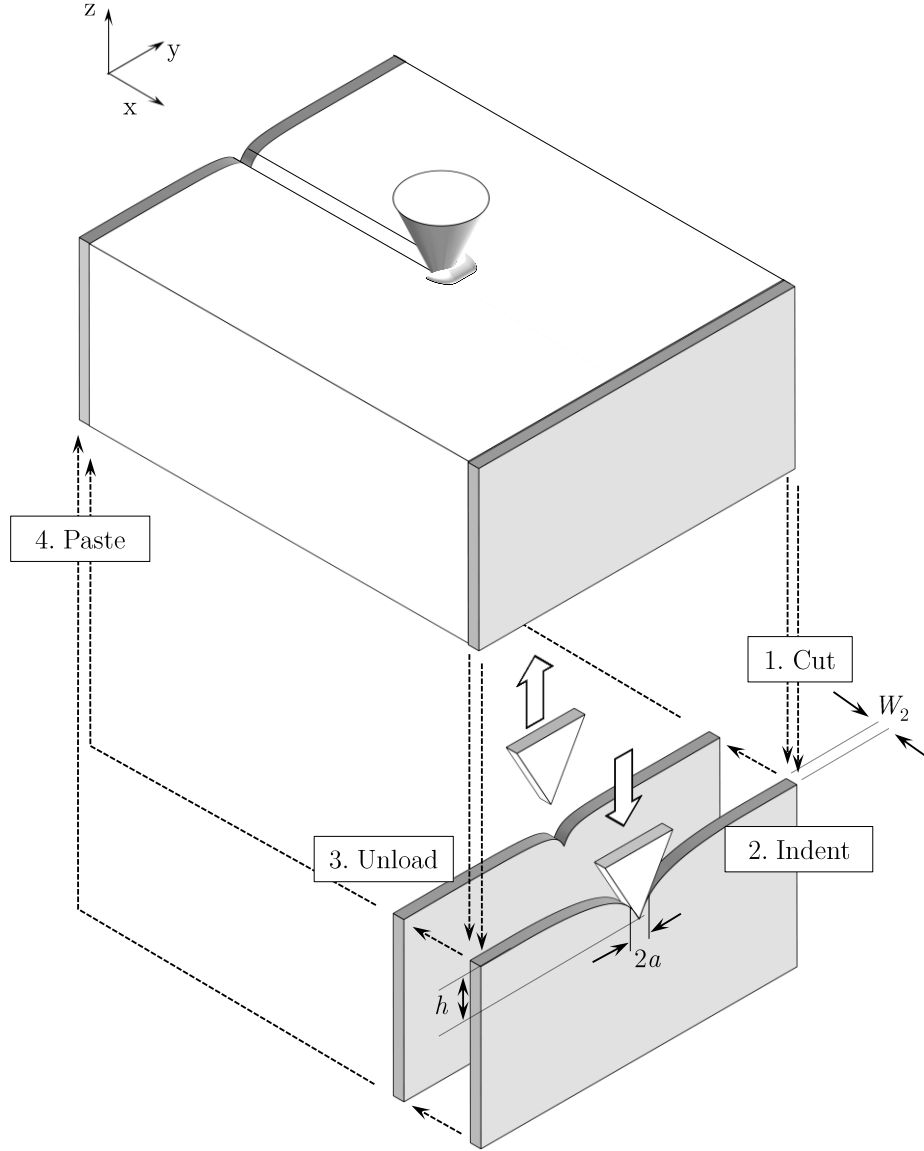


Figure 4.20: Schematic representation of the plowing model in the case of a conical tip. The two-dimensional projection of the cone is a wedge with the same half-opening angle.

The quantity of wear during plowing can be associated to the volume of the residual groove after the elastic recovery. The cross section of the groove is presented in Figure 4.21, the cross section area of the residual groove is

$$A_{wear} = (h_{pl}^C)^2 \cot(\beta - \beta'). \quad \text{Eq. 4.45}$$

Combining Eq. 4.45 with Eq. 4.19 and simplifying gives

$$A_{wear} = h^2 \left(\frac{h_{pl}^C}{h} \right)^2 \cot \beta \frac{1 + \tan^2 \beta \frac{C_{elpl}}{C_{el}}}{1 - \frac{C_{elpl}}{C_{el}}}. \quad \text{Eq. 4.46}$$

The wear volume is equal to the product of the section with the plowing length (L).

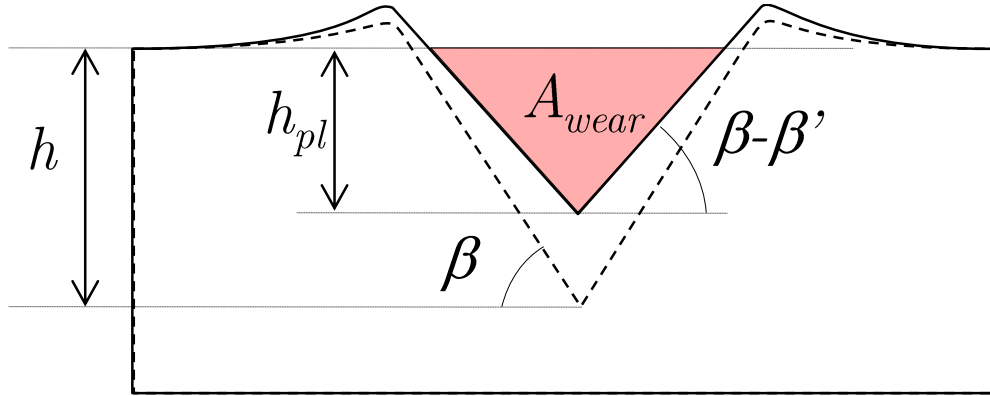


Figure 4.21: Cross section of the groove after elastic recovery in the case of an elastoplastic plowing mechanism with a conical tip. The dotted lines represent the position of the surface under load.

4.2.2 Half-wedge tip plowing solution

The half-wedge tip plowing model scheme is represented in Figure 4.22. In this configuration, a half-wedge with a back angle β and a width W_1 is plowing the material in the x direction. Using Flores approach [Flores 2008], the two-dimensional projection of this half-wedge in the y - z plan is a rectangular flat punch of thickness T_2 and width W_2 and Eq. 4.38 becomes

$$F_{H_{elpl}}^{HW_1} = \int_0^h \frac{F_{V_{elpl}}^{FW_2}(h)}{W_2} dh - \int_0^{h_{el}} \frac{F_{V_{el}}^{FW_2}(h)}{W_2} dh + \mu F_{V_{elpl}}^{HW_1}(h), \quad \text{Eq. 4.47}$$

where the superscript ' HW_1 ' represents the half-wedge indentation solutions with a width W_1 and ' FW_2 ' the rectangular flat punch indentation solutions with a width W_2 . Integrating and simplifying Eq. 4.47 (detailed calculation in appendix A.2.3) gives

$$F_{H_{elpl}}^{HW_1} = F_{V_{elpl}}^{HW_1} \mu + F_{V_{elpl}}^{HW_1} \left(\frac{h}{a_{elpl}^{HW}} \right) \frac{C_{pl}^{FW_2} \sigma_y}{4H_{elpl}^{HW}} \left[-\frac{1}{f^{FW_2}} \sin^{-1} \left(f^{FW_2} \right) - f^{FW_2} \cos^{-1} \left(f^{FW_2} \right) + 3 \frac{H_{elpl}^{FW_2}}{C_{pl}^{FW_2} \sigma_y} + 1 - \left(\frac{H_{elpl}^{FW_2}}{C_{pl}^{FW_2} \sigma_y} \right)^2 \frac{4}{f^{FW_2} \pi} \right]. \quad \text{Eq. 4.48}$$

Regarding the wear volume, the height of wear is

$$h_{wear} = h_{pl}^{HW}. \quad \text{Eq. 4.49}$$

The residual wear cross section can be defined as

$$A_{wear} = W h_{wear}. \quad \text{Eq. 4.50}$$

The same strategy can be followed in the case of an ideal elastic material and in the case of a rigid plastic material giving respectively

$$F_{H_{el}}^{HW_1} = \mu F_{V_{el}}^{HW_1} \quad \text{Eq. 4.51}$$

and

$$F_{H_{pl}}^{HW_1} = F_{V_{pl}}^{HW_1} \left[\mu + \frac{C_{pl}^{FW_2}}{C_{pl}^{HW_1}} \frac{\tan \beta}{1.46} \right]. \quad \text{Eq. 4.52}$$

Again, the wear volume is the product of the residual cross section with the plowing length (L).

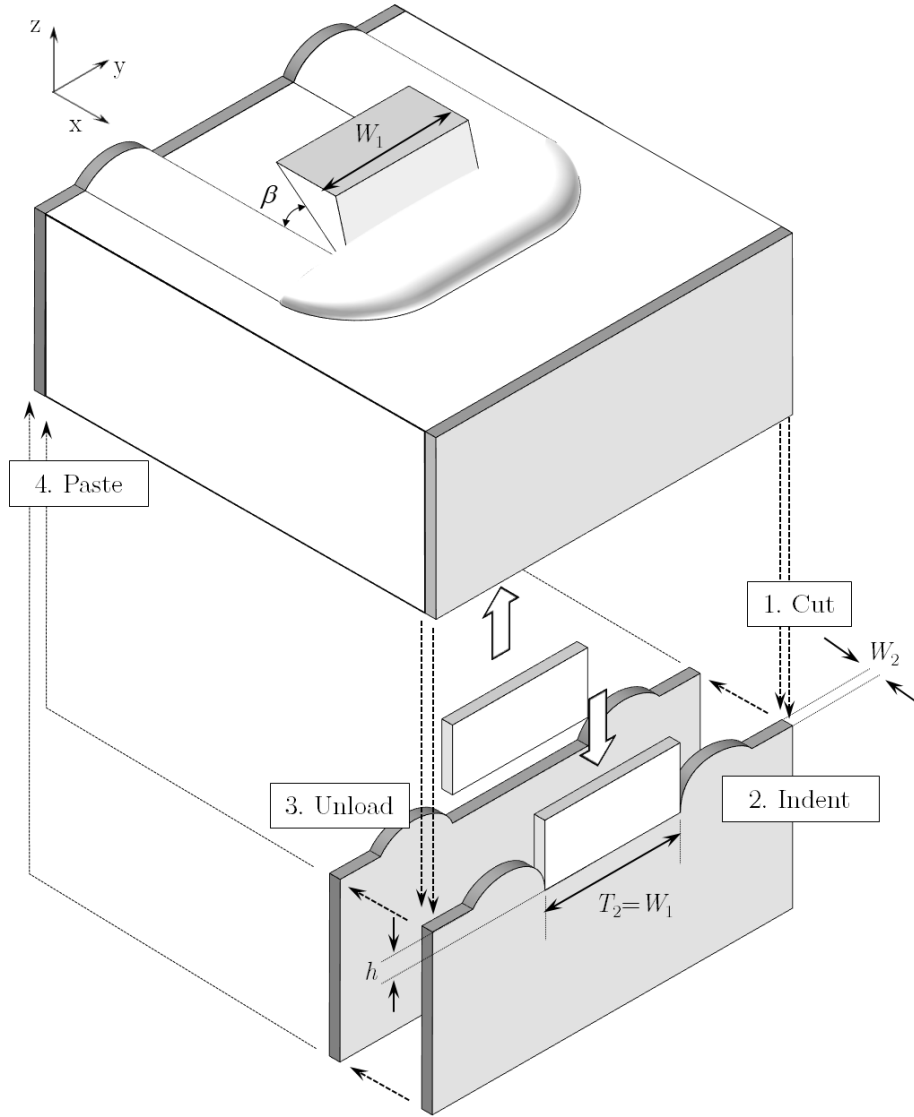


Figure 4.22: Schematic representation of the plowing model in the case of a half-wedge tip geometry.

4.2.3 Rectangular flat wedge plowing solution

The rectangular flat wedge plowing model is represented in Figure 4.23. In the present situation, a rectangular punch of thickness T_1 and width W_1 is plowing the surface of the material in the x direction. Using Flores approach [Flores 2008], the two-dimensional projection of the rectangular punch in the $z y$ plan is a rectangular punch of thickness T_2 and width W_2 (see Figure 4.23). In our present plowing situation, the original rectangular flat wedge is denoted by the superscript FW_1 . The projected flat wedge is denoted by the FW_2 superscript. Using this configuration, Eq. 4.38 becomes

$$F_{H_{elpl}}^{FW_1} = \int_0^h \frac{F_{V_{elpl}}^{FW_2}(h)}{W_2} dh - \int_0^{h_{el}} \frac{F_{V_{el}}^{FW_2}(h)}{W_2} dh + \mu F_{V_{elpl}}^{FW_1}(h) . \quad \text{Eq. 4.53}$$

Combining Eq. 4.53 with the definition of the hardness in Eq. 2.1 gives

$$F_{H_{elpl}}^{FW_1} = \int_0^h T_2 H_{elpl}^{FW_2} dh - \int_0^{h_{el}} T_2 H_{el}^{FW_2} dh + \mu W_1 T_1 H_{elpl}^{FW_1} , \quad \text{Eq. 4.54}$$

which can be rewritten through,

$$F_{H_{elpl}}^{FW_1} = \int_0^h \frac{h}{\ln 2} \frac{E}{(1-\nu^2)} \left[\cos^{-1}(g^{FW_2} h) + \frac{1}{g^{FW_2} h} \left(1 - \sqrt{1 - (g^{FW_2} h)^2} \right) \right] dh \\ - \int_0^{h_{el}} \frac{h\pi}{2 \ln 2} \frac{E}{(1-\nu^2)} dh + \mu W_1 T_1 H_{elpl}^{FW_1} , \quad \text{Eq. 4.55}$$

where

$$g^{FW_2} = \frac{1}{W_1} \frac{1}{\ln 2} \frac{E}{(1-\nu^2)} \frac{1}{C_{pl}^{FW_2} \sigma_y} . \quad \text{Eq. 4.56}$$

Integrating Eq. 4.55 (a step-by-step calculation is given in appendix A.2.4) gives

$$F_{H_{elpl}}^{FW_1} = \mu F_{V_{elpl}}^{FW_1} + F_{V_{elpl}}^{FW_1} \frac{h C_{pl}^{FW_2} \sigma_y}{4 T_1 H_{elpl}^{FW_1}} \\ \left[-\frac{1}{f^{FW_2}} \sin^{-1}(f^{FW_2}) - f^{FW_2} \cos^{-1}(f^{FW_2}) + 3 \frac{H_{elpl}^{FW_2}}{C_{pl}^{FW_2} \sigma_y} + 1 - \left(\frac{H_{elpl}^{FW_2}}{C_{pl}^{FW_2} \sigma_y} \right)^2 \frac{4}{f^{FW_2} \pi} \right] , \quad \text{Eq. 4.57}$$

where

$$f^{FW_2} = \frac{h}{W_1} \frac{1}{\ln 2} \frac{E}{(1-\nu^2)} \frac{1}{C_{pl}^{FW_2} \sigma_y} . \quad \text{Eq. 4.58}$$

Regarding the wear volume, the height of wear is

$$h_{wear} = h_{pl}^{FW} , \quad \text{Eq. 4.59}$$

the residual wear cross section can be defined as

$$A_{wear} = W h_{wear} . \quad \text{Eq. 4.60}$$

The same strategy can be followed in the case of ideal elastic materials and in the case of rigid plastic materials being plowed by a half-wedge, which gives respectively

$$F_{H_{el}}^{FW_1} = \mu F_{V_{elpl}}^{FW_1} \quad \text{Eq. 4.61}$$

and

$$F_{H_{pl}}^{FW_1} = F_{V_{elpl}}^{FW_1} \left(\mu + \frac{h}{T_1} \right). \quad \text{Eq. 4.62}$$

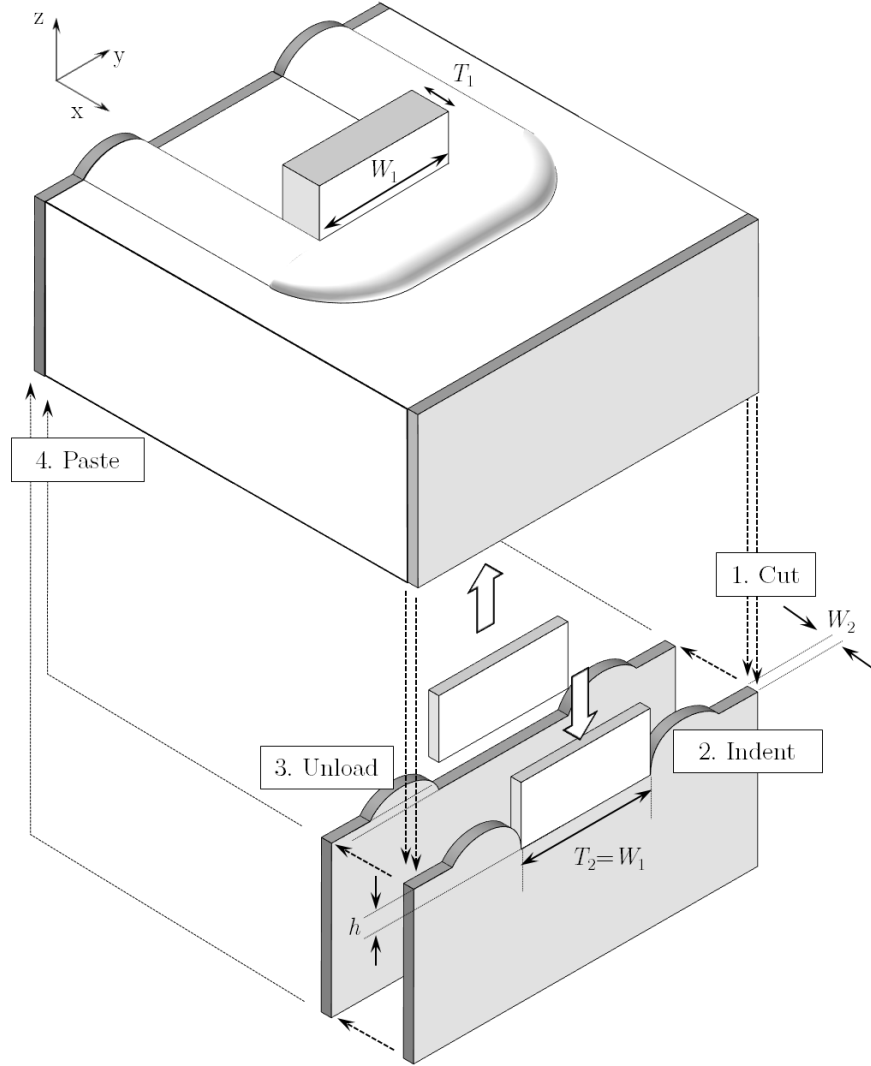


Figure 4.23: Schematic representation of the plowing model in the case of a rectangular tip.

4.2.4 Plowing with strain hardening

Since plowing models developed in this section are strongly based on contact mechanics and on indentation theory, the effect of strain hardening can be incorporated in the plowing models following the same strategy as in Section 4.1.3. However, two problems can be pointed out: (i) the two last plowing models use the flat

wedge indentation solution. The representative strains associated to flat tips indentations are unknown. (ii) [Bucaille 2002] and [Larsson 2006] reported that the representative strain is larger during plowing than during indentation.

- *Sharp tip plowing with strain hardening*

Scratching elastoplastic materials with a conical tip, [Bucaille 2002] suggested that the plowing representative strain follows

$$\varepsilon_{r-plow} = 0.33 \tan \beta, \quad \text{Eq. 4.63}$$

which is 65% higher than the indentation representative strain based on Johnson relationship (see Eq. 3.6). Using Eq. 4.63 and assuming that the base material follows an elastoplastic behavior with a Hollomon power law strain hardening presented in Eq. 2.10, a representative plowing stress flow (σ_{r-plow}) can be calculated and used instead of σ_y . Concerning the effect of strain hardening on the constraint factor and on the pile-up, the same approximation used for indentation modelling (see section 5.1.4) will be used.

4.2.5 Parametric study and sensitivity analysis

These plowing models give an expression of the horizontal force (F_H) as a function of geometrical and material related parameters as well as the friction coefficient. The present section describes the effect of these parameters on F_H . In order to summarize the results, F_H is normalized by F_V . Giving the general form of Eq. 4.44, Eq. 4.48 and Eq. 4.57, the results can be presented in a form that is independent of the friction coefficient by subtracting μ from the ratio F_H/F_V .

Regarding plowing with a conical tip, the two main parameters are β and the ratio σ_y/E (see Figure 4.24). Surprisingly, the model predicts that a sharper tip give rise to a larger normalized F_H . A sharper tip also presents a delayed transition from the rigid plastic behavior to the ideal elastic one.

Concerning the half-wedge tip, the non-dimensional main parameter is the ratio $\sigma_y W/hE$. Here, increasing β also increase the normalized F_H (See Figure 4.25).

In addition to the parameters σ_y , E and μ , the rectangular flat tip plowing model is affected by the thickness (T), the width (W) of the tip and the penetration depth (h). However, the horizontal force can be normalized by the ratio T/h . In that case, the effect of h , T and W on the elastoplastic transition is summarized in Figure 4.26.

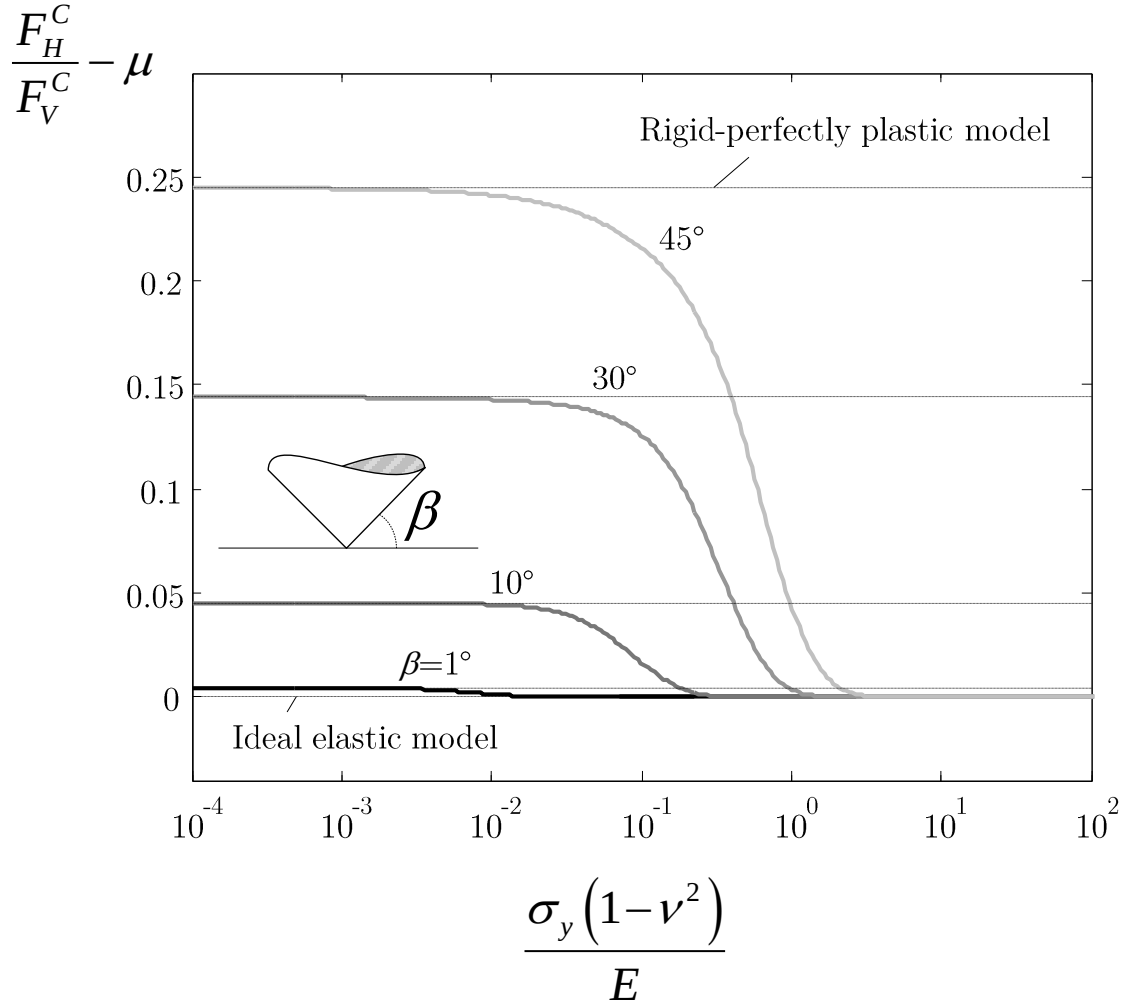


Figure 4.24: Plowing horizontal force normalized by the vertical force of a conical tip with different β angles as a function of the yield strain (σ_y/E).

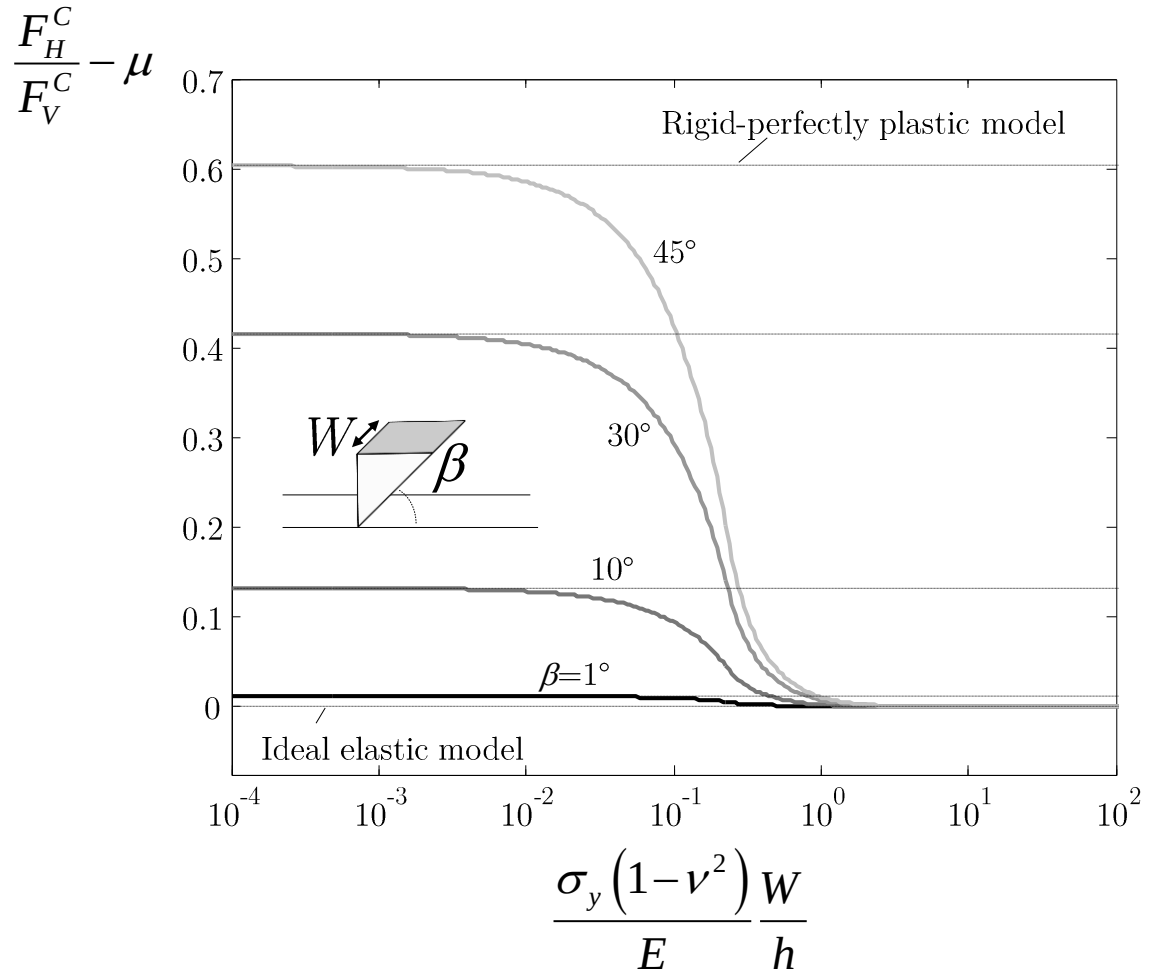


Figure 4.25: Plowing horizontal force normalized by the vertical force of a half-wedge tip with different β angles as a function of the yield strain (σ_y/E) multiplied by the width of the tip and divided by the penetration depth (h). The model is stable as long as $h \leq W/25$, above this value, the model gives aberrant solutions.

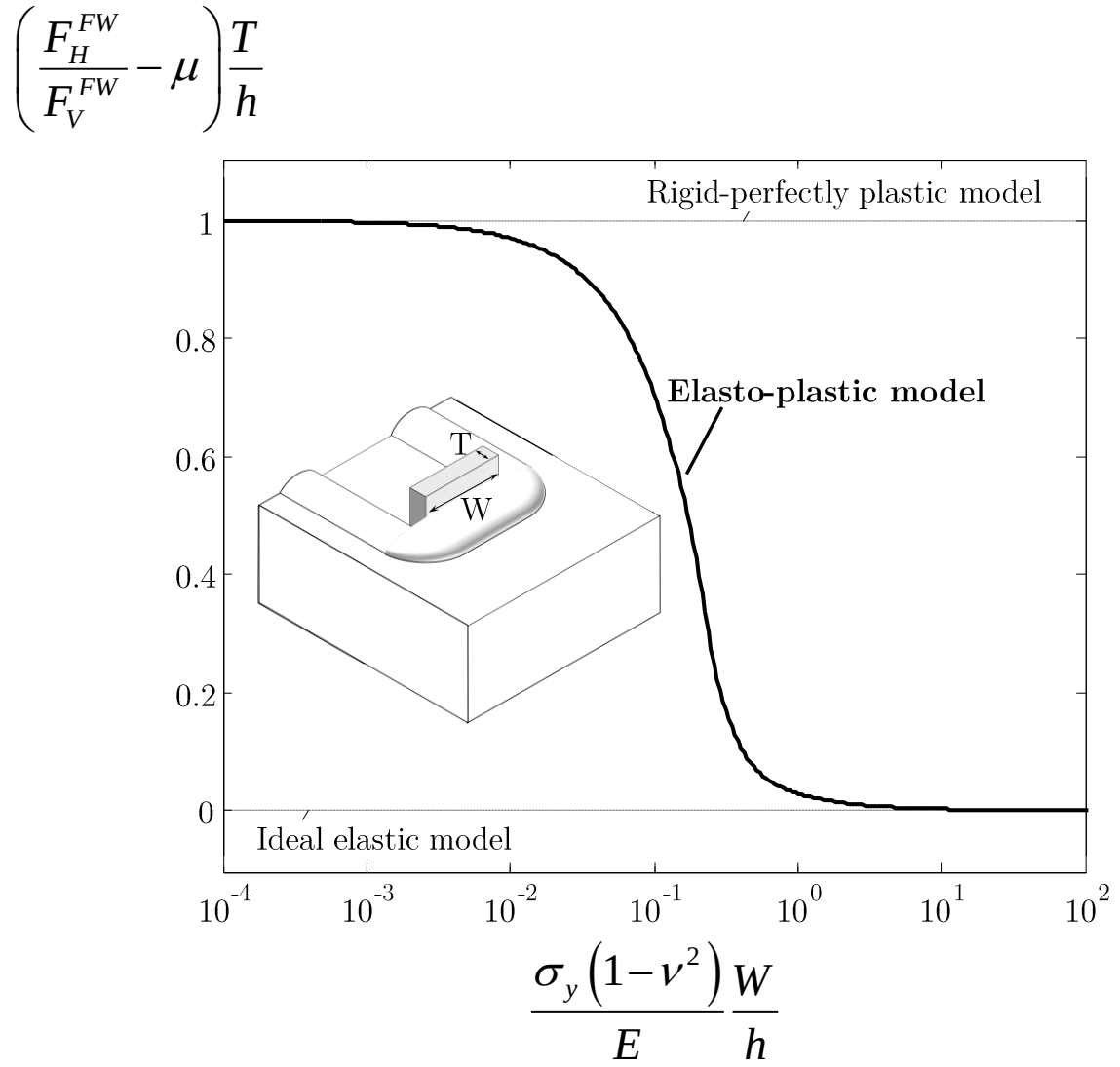


Figure 4.26: Plowing horizontal force normalized by the vertical force in the case of a rectangular flat wedge as a function of the yield strain (σ_y/E) multiplied by the width of the tip and divided by the penetration depth (h). The model is stable as long as $W \geq T$, above this value, the model gives aberrant solutions.

4.3 Cutting model

As stated in Section 3.4 , cutting models focus on rigid-plastic materials. This section is dedicated to the development of cutting models for elastoplastic materials based on the models of Atkins, Williams, Blackman, Patel and Islam described in section 2.2.4. Concerning the modelling, although [William 2010] proposed an orthogonal cutting model with a closed form solution, some modifications are implemented to ensure that the solutions take into account the elastoplasticity of the base material as well as the strain hardening. Also, to ensure that the plane strain fracture toughness of the base material (G_{Ic}) affects the shear angle (ϕ) as shown by [Atkins 2003]. Finally, modifications are added to the pyramidal scratch cutting model to derive a closed form solution of the cutting forces.

Three cutting tool geometries are considered here: the two-dimensional inclined wedge, the rectangle and the pyramidal cutting tool. Because of their two-dimensional appearances, the two first geometries are referenced as “orthogonal cutting” because the cutting edge is perpendicular to the cutting direction. Whereas the pyramidal cutting tool geometry is referenced as “pyramidal scratch cutting”. In the following subsections, rigid plastic cutting models are developed for these three geometries followed by a more general elastoplastic version.

4.3.1 Rigid plastic orthogonal cutting model

Patel and his colleagues derived two base equations in rigid plastic orthogonal cutting [Patel 2009]. The first one is obtained by an energy balance and makes a connection between F_H and F_V through

$$\frac{F_{H_{pl}}^{OC}}{W} - \frac{F_{V_{pl}}^{OC}}{W} \tan \phi = \frac{\sigma_y h}{2} \left(\tan \phi + \frac{1}{\tan \phi} \right) + G_c, \quad \text{Eq. 4.64}$$

where the exponent “OC” stand for “orthogonal cutting”. The second equation is obtained by a force equilibrium on the tool [Patel 2009] and also gives a relationship between F_H and F_V through

$$\frac{F_{V_{pl}}^{OC}}{W} = Z \left(\frac{F_{H_{pl}}^{OC}}{W} - G_c \right) + \frac{G_a}{(\cos \alpha + \mu \sin \alpha)}, \quad \text{Eq. 4.65}$$

where

$$Z = \frac{\mu - \tan \alpha}{1 + \mu \tan \alpha}, \quad \text{Eq. 4.66}$$

and where G_a refers to the adhesion toughness between the chip and the tool. This adhesion occurs if the chip is at high temperature and starts to stick to the tool. Since this last mechanism is not occurring with most materials and cutting conditions, in the following part of this work, the term G_a is considered equal to zero. Here starts the modification of Patel's model. Combining Eq. 4.64 and Eq. 4.65 gives

$$\frac{F_{H_{pl}}^{OC}}{W} = \frac{\frac{\sigma_y h}{2} \left(\tan \phi + \frac{1}{\tan \phi} \right) + G_c (1 + \tan \phi)}{(1 - Z \tan \phi)}. \quad \text{Eq. 4.67}$$

Then, considering that cutting takes place at a constant depth, the horizontal force is the only force that is associated to mechanical work. Using an energy minimization approach on F_H allows identifying the shear angle that minimizes the horizontal cutting force through

$$\frac{\partial}{\partial \tan \phi} \left(\frac{F_H^{OC}}{W} \right) = 0, \quad \text{Eq. 4.68}$$

Solving Eq. 4.68 (a step-by-step solution is given in appendix A.3.1) gives

$$\cot \phi = \sqrt{Z^2 + 1 + B_{pl}^{OC} Z + B_{pl}^{OC}} + Z, \quad \text{Eq. 4.69}$$

where

$$B_{pl}^{OC} = \frac{2G_c}{\sigma_y h}. \quad \text{Eq. 4.70}$$

Substituting Eq. 4.69 into Eq. 4.67 and simplifying gives

$$\frac{F_{H_{pl}}^{OC}}{W} = \frac{\sigma_y h}{2} (2 \cot \phi + B_{pl}^{OC}). \quad \text{Eq. 4.71}$$

4.3.2 Elastoplastic orthogonal cutting model

The orthogonal rigid plastic cutting model can be extended to include an elastoplastic behavior in the base material. This model is inspired from the strategy developed by

[Islam 2012] that takes into account the elastic recovery behind the tool. The hypotheses are that (i) the base material is elastoplastic, (ii) the depth of cut is decomposed into a rigid plastic layer that will undergo chipping by plastic shearing and (iii) an elastoplastic layer that will be deformed by plowing under and around the cutting tool, (iv) The thickness of the layer which is cut is considered equal to the residual indentation depth (h_{pl}) that would result from the indentation of the base material by the same cutting tool.

• *Sharp inclined cutting tool*

If the cutting tool has an inclined wedge geometry (see Figure 4.27), the lower layer undergoes plowing assuming that the cutting tool is associated to a half-wedge geometry. In the following equations, the exponent “HW” refers to “half-wedge”.

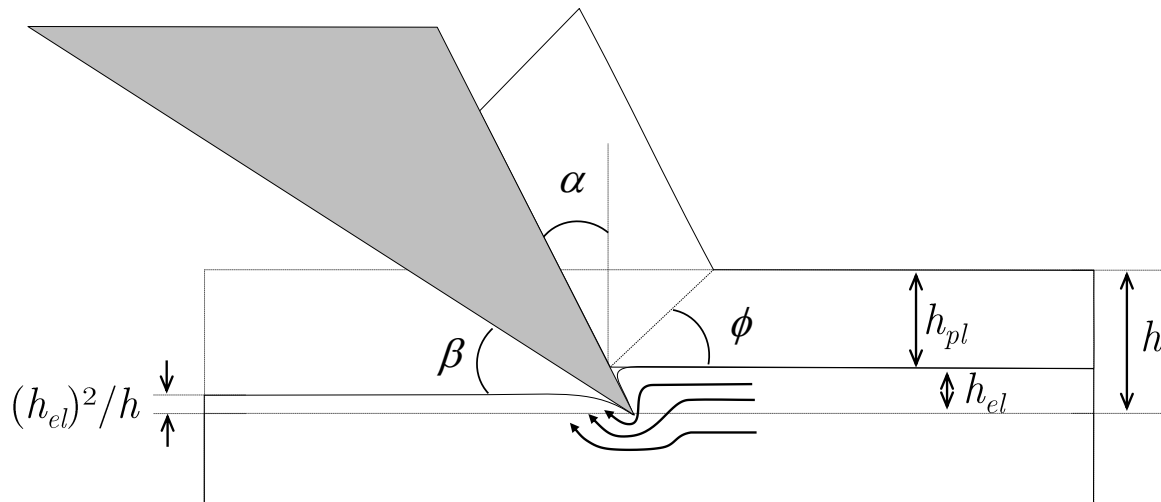


Figure 4.27: Schematic representation of the elastoplastic orthogonal cutting model with an inclined cutting tool.

Here, the top layer undergoes cutting and the lower layer is pushed under the tool by plowing. Both mechanisms are considered independent from each other. As a consequence, the total force on the tool is the sum of the forces generated by each layer. Therefore, Eq. 4.47, Eq. 4.65 and Eq. 4.71 are combined together in the form

$$\frac{F_{H_{elpl}}^{OC}}{W} = \frac{\sigma_y h}{2} \left(\frac{h_{pl}}{h} \right)_{elpl}^{HW} (2 \cot \phi + B_{elpl}^{OC}) + \frac{F_{H_{elpl}}^{HW}(h_{el})}{W}, \quad \text{Eq. 4.72}$$

where

$$B_{elpl}^{OC} = \frac{2G_c}{\sigma_y h} \left(\frac{h}{h_{pl}} \right)_{elpl}^{HW}. \quad \text{Eq. 4.73}$$

Concerning the vertical force, combining Eq. 4.65 and Eq. 4.71 gives

$$\frac{F_{V_{elpl}}^{OC}}{W} = Z \left(\frac{F_{H_{elpl}}^{OC}}{W} - G_c \right) + \frac{F_{V_{elpl}}^{HW}(h_{el})}{W}. \quad \text{Eq. 4.74}$$

Regarding the wear volume, the height of wear is

$$h_{wear} = h - \frac{(h_{el}^{HW})^2}{h}. \quad \text{Eq. 4.75}$$

• *Rectangular cutting tool*

If the cutting tool is a rectangular flat wedge (see Figure 4.28), the lower layer undergoes plowing assuming that the cutting tool is associated to a flat wedge geometry. In the following mathematical expressions, the exponent “FW” refers to “Flat wedge”.

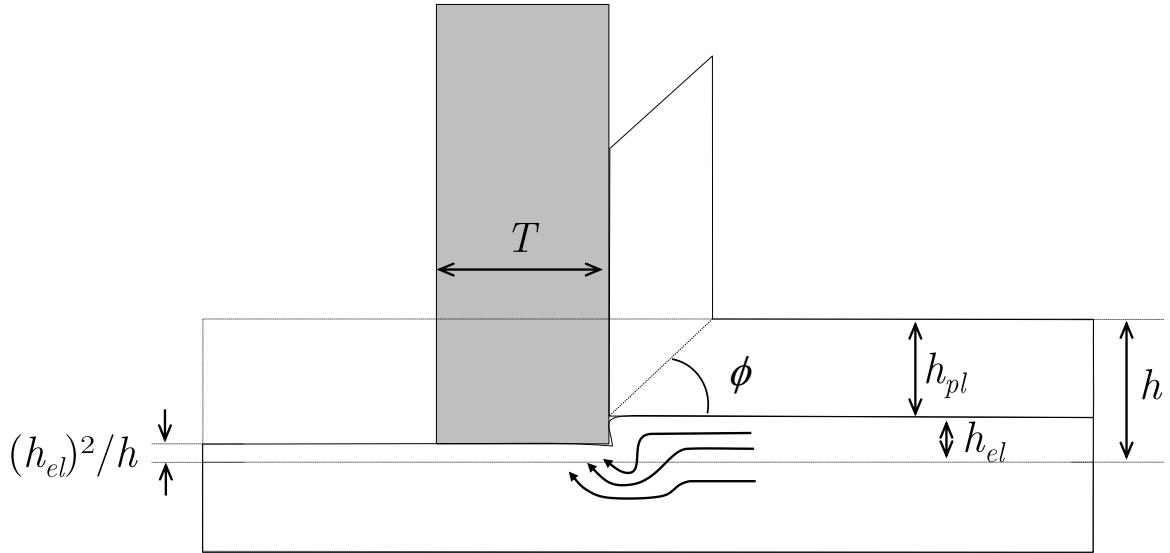


Figure 4.28: Schematic representation of the elastoplastic orthogonal cutting model with a rectangular cutting tool.

Therefore, Eq. 4.72, Eq. 4.73 and Eq. 4.74 becomes

$$\frac{F_{H_{elpl}}^{OC}}{W} = \frac{\sigma_y h}{2} \left(\frac{h_{pl}}{h} \right)_{elpl}^{FW} \left(2 \cot \phi + B_{elpl}^{OC} \right) + \frac{F_{H_{elpl}}^{FW}(h_{el})}{W}, \quad \text{Eq. 4.76}$$

where

$$B_{elpl}^{OC} = \frac{2G_c}{\sigma_y h} \left(\frac{h}{h_{pl}} \right)_{elpl}^{FW}. \quad \text{Eq. 4.77}$$

Concerning the vertical force, combining Eq. 4.65 and Eq. 4.71 gives

$$\frac{F_{V_{elpl}}^{OC}}{W} = Z \left(\frac{F_{H_{elpl}}^{OC}}{W} - G_c \right) + \frac{F_{V_{elpl}}^{FW}(h_{el})}{W}. \quad \text{Eq. 4.78}$$

4.3.3 Rigid plastic pyramidal scratch cutting model

Blackman and his colleagues derived an equation in rigid plastic pyramidal scratch cutting [Blackman 2016]. This equation is based on an energy balance and making a connection between F_H and F_V through

$$\frac{F_{H_{pl}}^{SC}}{c_{cut}} - \frac{F_{V_{pl}}^{SC}}{c_{cut}} \tan \phi = \frac{\sigma_y}{2} \frac{A_{cut}}{c_{cut}} \left(\tan \phi + \frac{1}{\tan \phi} \right) + G_c, \quad \text{Eq. 4.79}$$

where

$$A_{cut} = h^2 \cot \beta \quad \text{Eq. 4.80}$$

Is the cross section of the cutted groove and

$$c_{cut} = \frac{2h}{\sin \beta} \quad \text{Eq. 4.81}$$

is the length of the cross section of the groove (see Figure 4.29). The exponent “*SC*” stands for “scratch cutting”. In contrast with perfectly plastic indentation and plowing, in the case of cutting, the radius of contact is considered to be unaffected by any pile-up. Instead of pilling up on the side of the tip, the material is removed within the chip.

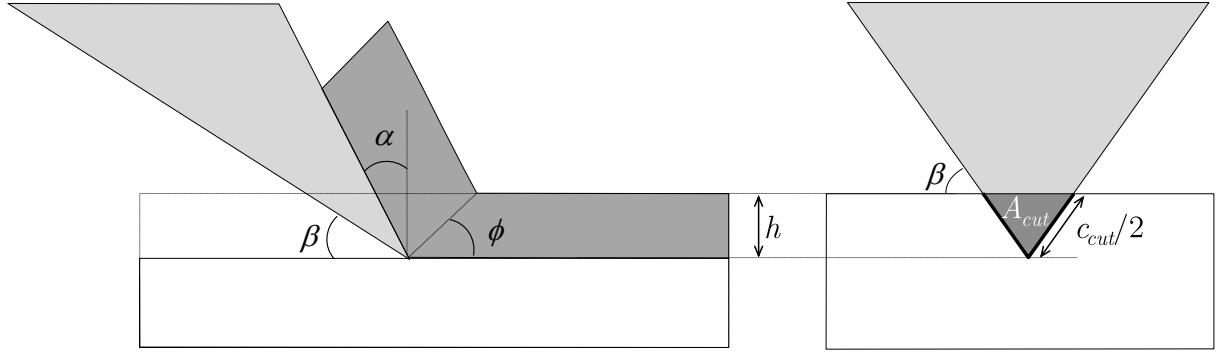


Figure 4.29: Schematic representation of the rigid plastic scratch cutting geometry.

Eq. 4.65 can be adapted to the pyramidal geometry, in which case, the relationship between F_H and F_V becomes

$$\frac{F_{V_{pl}}^{SC}}{c_{cut}} = Z \left(\frac{F_{H_{pl}}^{SC}}{c_{cut}} - G_c \right) + \frac{G_a}{(\cos \alpha + \mu \sin \alpha)}. \quad \text{Eq. 4.82}$$

Again, considering that G_a is equal to zero, and combining Eq. 4.79 and Eq. 4.82 gives

$$\frac{F_{H_{pl}}^{SC}}{c_{cut}} = \frac{\frac{\sigma_y}{2} \frac{A_{cut}}{c_{cut}} \left(\tan \phi + \frac{1}{\tan \phi} \right) + G_c (1 + \tan \phi)}{(1 - Z \tan \phi)}. \quad \text{Eq. 4.83}$$

Deriving Eq. 4.83 with respect to $\tan(\phi)$ and equating to zero (the development is similar to the development in appendix A.3.1) provides the shear angle that minimizes the horizontal cutting force through

$$\cot \phi = \sqrt{Z^2 + 1 + B_{pl}^{SC} Z + B_{pl}^{SC}} + Z, \quad \text{Eq. 4.84}$$

where

$$B_{pl}^{SC} = \frac{2G_c}{\sigma_y} \frac{c_{cut}}{A_{cut}} = \frac{G_c}{\sigma_y h} \frac{4}{\cos \beta} = B_{pl}^{OC} \frac{2}{\cos \beta}. \quad \text{Eq. 4.85}$$

Substituting Eq. 4.84 into Eq. 4.83 and simplifying gives

$$\frac{F_{H_{pl}}^{SC}}{c_{cut}} = \frac{\sigma_y}{2} \frac{A_{cut}}{c_{cut}} (2 \cot \phi + B_{pl}^{OC}). \quad \text{Eq. 4.86}$$

The wear volume can be calculated based on the residual cross section of the groove A_{cut} (Eq. 4.80).

4.3.4 Elastoplastic pyramidal scratch cutting model

The pyramidal rigid plastic scratch cutting model can also be extended to include the elastoplastic response of the base material. The hypotheses are the same as in the orthogonal cutting model. However, instead of decomposing the cutting into two-dimensional layers, here, three-dimensional layers are involved. The upper layer that undergoes cutting has a trapezoidal cross section (see Figure 4.30). The lower layer undergoes plowing, similar to plowing under a conical tip with an angle β between the horizontal surface and the side of the cone.

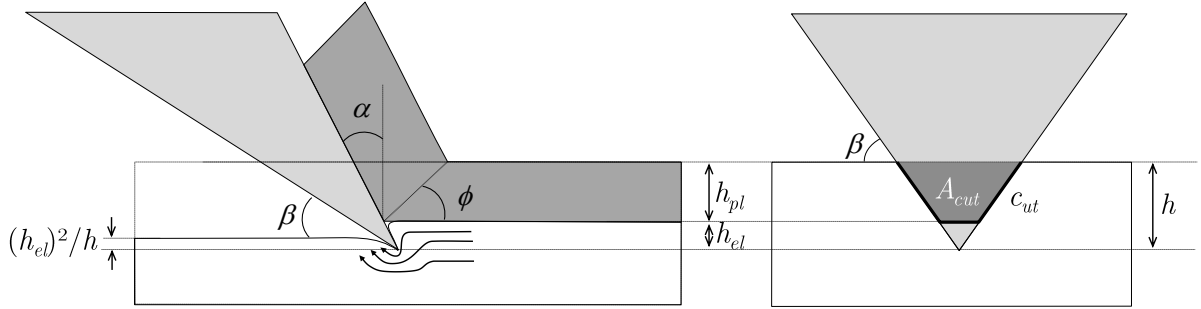


Figure 4.30: Schematic representations of the elastoplastic scratch cutting geometry.

Again, the two layers are considered independent from each other. As a consequence the total force on the tool is the sum of the forces acting on both layers independently. Therefore, Eq. 4.42, Eq. 4.82 and Eq. 4.71 can be combined together in the form

$$F_{H_{elpl}}^{SC} = \frac{\sigma_y}{2} \left(2 \cot \phi + B_{elpl}^{SC} \right) A_{cut} + F_{H_{elpl}}^C(h_{el}), \quad \text{Eq. 4.87}$$

where

$$A_{cut} = h^2 \cot \beta \left(1 - \left(\frac{h_{el}^C}{h} \right)^2 \right), \quad \text{Eq. 4.88}$$

$$c_{cut} = \frac{2h}{\sin \beta} \left(1 - (1 - \cos \beta) \left(\frac{h_{el}^C}{h} \right) \right), \quad \text{Eq. 4.89}$$

and

$$B_{elpl}^{SC} = \frac{2G_c}{\sigma_y} \frac{c_{cut}}{A_{cut}} = \frac{2G_c}{h\sigma_y \cos \beta} \frac{2}{1 - \left(\frac{h_{el}^C}{h}\right)^2} \frac{1 - (1 - \cos \beta) \left(\frac{h_{el}^C}{h}\right)}{1 - \left(\frac{h_{el}^C}{h}\right)^2}. \quad \text{Eq. 4.90}$$

Concerning the vertical force, adding a plowing vertical force to Eq. 4.82 gives

$$F_{V_{elpl}}^{SC} = Z \left(F_{H_{elpl}}^{SC} - c_{cut} G_c \right) + F_{V_{elpl}}^C (h_{el}). \quad \text{Eq. 4.91}$$

The residual groove cross section is presented in Figure 4.31 and can be calculated by adapting Eq. 4.46 to the plowed layer and adding it to Eq. 4.88. The result is

$$A_{wear} = h^2 \cot \beta \left[1 - \left(\frac{h_{el}^C}{h}\right)^2 + \left(\frac{h_{el}^C}{h}\right)^2 \left(\frac{h_{pl}^C}{h}\right)^2 \frac{1 + \tan^2 \beta \frac{C_{elpl}}{C_{el}}}{1 - \frac{C_{elpl}}{C_{el}}} \right]. \quad \text{Eq. 4.92}$$

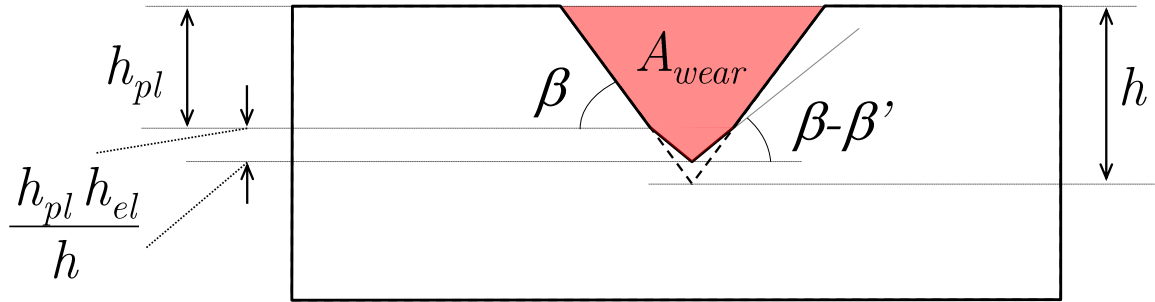


Figure 4.31: Cross section of the groove after elastic recovery in the case of an elastoplastic pyramidal cutting mechanism. The dotted lines represent the position of the surface under load.

4.3.5 Cutting model with strain hardening

Including the effect of strain hardening is quite tricky since the two layers undergo different magnitudes of plastic strain. The effect of the strain hardening in the lower plowed layer is discussed in section 4.2.4 . Regarding the effect of the strain hardening in the upper layer, it depends on the geometry of the tip.

- *Orthogonal inclined wedge cutting tool*

In the case of an inclined wedge cutting tool, the plastic strain in the chip is a function of the shear cutting angle (ϕ). However, ϕ is a function of B^{OC} . The latter depends on the flow stress in the chip (σ_{r-cut}) through the following system of equations:

$$\left\{ \begin{array}{l} B_{elpl}^{OC} = \frac{2G_c}{h\sigma_{r-cut}} \left(\frac{h}{h_{pl}} \right)^{HW} \\ \cot \phi = \sqrt{Z^2 + 1 + B_{elpl}^{OC} (1 + Z)} + Z \\ \varepsilon_{r-cut} = \frac{\cot^2 \phi + 1}{\cot \phi + \tan \alpha} \\ \sigma_{r-cut} = \sigma_y \left(E \varepsilon_{r-cut} / \sigma_y \right)^n \end{array} \right. \quad \text{Eq. 4.93}$$

This system of equation cannot be solved analytically but it still can be solved numerically to find ε_{r-cut} with an iterative procedure. For instance, σ_y can be used instead of σ_{r-cut} to calculate a first B^{OC} . After that, $\cot \phi$ can be calculated which in turn gives ε_{r-cut} and σ_{r-cut} through

$$\left\{ \begin{array}{l} B_{elpl \sim 0}^{OC} = \frac{2G_c}{h\sigma_y} \left(\frac{h}{h_{pl}} \right)^{HW} \\ \cot \phi_{\sim i} = \sqrt{Z^2 + 1 + B_{elpl \sim i-1}^{OC} (1 + Z)} + Z \\ \varepsilon_{r-cut \sim i} = \frac{\cot^2 \phi_{\sim i} + 1}{\cot \phi_{\sim i} + \tan \alpha} \\ \sigma_{r-cut \sim i} = \sigma_y \left(E \varepsilon_{r-cut \sim i} / \sigma_y \right)^n \\ B_{elpl \sim i}^{OC} = \frac{2G_c}{h\sigma_y} \left(\frac{h}{h_{pl}} \right)^{HW} \frac{\sigma_y}{\sigma_{r-cut \sim i}} \end{array} \right. , \quad \text{Eq. 4.94}$$

where i is the number of iterations. Repeating this loop rapidly converges to the solution. With three iterations, the error on the horizontal cutting force gets lower than 12%. The expression for the horizontal and vertical cutting forces becomes

$$\left\{ \begin{array}{l} \frac{F_{H_{elpl}}^{OC}}{\sigma_y h W} = \frac{1}{2} \frac{\sigma_{r-cut \sim i}}{\sigma_y} \left(\frac{h_{pl}}{h} \right)_{elpl}^{HW} \left(2 \cot \phi_{\sim i} + B_{elpl \sim i}^{OC} \right) + \frac{F_{H_{elpl}}^{HW}}{\sigma_y h W} (h_{el}) \\ \frac{F_{V_{elpl}}^{OC}}{\sigma_y h W} = Z \left(\frac{F_{H_{elpl}}^{OC}}{\sigma_y h W} - \frac{B_{pl}^{OC}}{2} \right) + \frac{F_{V_{elpl}}^{HW}}{\sigma_y h W} (h_{el}, \delta) \end{array} \right. \quad \text{Eq. 4.95}$$

- *Orthogonal rectangular cutting tool*

Concerning the rectangular cutting tool, the expression becomes

$$\begin{aligned}
 B_{elpl \sim 0}^{OC} &= \frac{2G_c}{h\sigma_y} \left(\frac{h}{h_{pl}} \right)^{FW} \\
 \left\{ \begin{aligned}
 \cot \phi_{\sim i} &= \sqrt{Z^2 + 1 + B_{elpl \sim i-1}^{OC} (1 + Z)} + Z \\
 \varepsilon_{r_{cut \sim i}} &= \frac{\cot^2 \phi_{\sim i} + 1}{\cot \phi_{\sim i}} \\
 \sigma_{r_{cut \sim i}} &= \sigma_y \left(E \varepsilon_{r_{cut \sim i}} / \sigma_y \right)^n \\
 B_{elpl \sim i}^{OC} &= \frac{2G_c}{h\sigma_y} \left(\frac{h}{h_{pl}} \right)^{FW} \frac{\sigma_y}{\sigma_{r_{cut \sim i}}}
 \end{aligned} \right. .
 \end{aligned} \tag{Eq. 4.96}$$

The expressions for the horizontal and vertical cutting force become

$$\left\{ \begin{aligned}
 \frac{F_{H_{elpl}}^{OC}}{\sigma_y h W} &= \frac{1}{2} \frac{\sigma_{r_{cut \sim i}}}{\sigma_y} \left(\frac{h_{pl}}{h} \right)_{elpl}^{FW} \left(2 \cot \phi_{\sim i} + B_{elpl \sim i}^{OC} \right) + \frac{F_{H_{elpl}}^{FW}}{\sigma_y h W} (h_{el}, T) \\
 \frac{F_{V_{elpl}}^{OC}}{\sigma_y h W} &= Z \left(\frac{F_{H_{elpl}}^{OC}}{\sigma_y h W} - B_{pl}^{OC} \right) + \frac{F_{V_{elpl}}^{FW}}{\sigma_y h W} (h_{el}, T)
 \end{aligned} \right. . \tag{Eq. 4.97}$$

- *Pyramidal scratch cutting tool*

Concerning the pyramidal cutting tool, the system of equations takes the following form

$$\begin{aligned}
 B_{elpl \sim 0}^{SC} &= \frac{G_c}{h\sigma_y} \frac{4}{\cos \beta} \left(1 - (1 - \cos \beta) \left(\frac{h_{el}}{h} \right)_{elpl}^C \right) \left/ \left(1 - \left[\left(\frac{h_{el}}{h} \right)_{elpl}^C \right]^2 \right) \right. \\
 \left\{ \begin{aligned}
 \cot \phi_{\sim i} &= \sqrt{Z^2 + 1 + B_{elpl \sim i-1}^{SC} (1 + Z)} + Z \\
 \varepsilon_{r_{cut \sim i}} &= \frac{\cot^2 \phi_{\sim i} + 1}{\cot \phi_{\sim i} + \tan \alpha} \\
 \sigma_{r_{cut \sim i}} &= \sigma_y \left(E \varepsilon_{r_{cut \sim i}} / \sigma_y \right)^n \\
 B_{elpl \sim i}^{SC} &= \frac{G_c}{h\sigma_y} \frac{4}{\cos \beta} \left(1 - (1 - \cos \beta) \left(\frac{h_{el}}{h} \right)_{elpl}^C \right) \left/ \left(1 - \left[\left(\frac{h_{el}}{h} \right)_{elpl}^C \right]^2 \right) \right. \frac{\sigma_y}{\sigma_{r_{cut \sim i}}}
 \end{aligned} \right. ,
 \end{aligned} \tag{Eq. 4.98}$$

where i is the number of iterations. The final expression for the horizontal and vertical cutting forces becomes

$$\begin{cases} F_{H_{elpl}}^{SC} = \frac{\sigma_{r_{cut \sim i}}}{2} A_{H_{elpl}} \left(2 \cot \phi_{\sim i} + B_{elpl \sim i}^{SC} \right) + F_{H_{elpl}}^C(h_{el}, \delta) \\ F_{V_{elpl}}^{SC} = Z \left(F_{H_{elpl}}^{SC} - c_{cut} G_c \right) + F_{V_{elpl}}^C(h_{el}, \delta) \\ c_{cut} = \frac{2h}{\sin \beta} \left(1 - (1 - \cos \beta) \left(\frac{h_{el}}{h} \right)_{elpl}^C \right) \\ A_{cut} = h^2 \cot \beta \left(1 - \left[\left(\frac{h_{el}}{h} \right)_{elpl}^C \right]^2 \right) \end{cases} \quad \text{Eq. 4.99}$$

4.3.6 Parametric study and sensitivity analysis

The cutting force can be normalized by the product of the yield strength (σ_y) with the depth of cut (h) and with the width of the cutting tool (W) in the case of orthogonal cutting. It can also be normalized by the product $\sigma_y h^2$ in the case of pyramidal scratch cutting. A second non-dimensional variable of interest is $G_{IC}/h\sigma_y$. When this variable is low, the cutting force is dominated by plastic shearing in the chip, whereas when this variable is high, the cutting force is dominated by the energy required to separate the surface between the chip and the base material. The transition between these two regimes will be presented in the present sub section as well as the effect of other material and geometrical parameters.

- *Orthogonal inclined wedge cutting tool*

In the case of orthogonal inclined wedge cutting tools, the other independent variables are the strain hardening coefficient (n), σ_y/E , the friction coefficient (μ), W/h , the cutting angle (α) and the clearance angle (β). The effect of these variables can be observed in Figure 4.32. In this figure, the two regimes can be easily distinguished. The first plateau corresponds to a cut dominated by shearing, whereas the area with a steady positive slope is dominated by fracture. In addition, increasing n as well as μ increases the cutting force in the shear dominated zone. The β angle and the ratio W/h do not seem to play a significant role on the normalized cutting force. Regarding the effect of σ_y/E , as expected, the normalized cutting force decreases with this ratio.

Indeed, increasing σ_y/E increases the elastic recovery and therefore decreases the thickness of the cut layer. The effect of the cutting angle (α) is also presented. A sharper cutting angle (larger α) leads to smaller cutting forces.

- *Orthogonal rectangular cutting tool*

Regarding the rectangular tool geometry, the independent variables are quite similar except that the cutting angle and the angle in the back of the tool are constant and equal to zero. However, the thickness of the tool normalized by the depth of cut (T/h) is a new variable. The effect of these variables can be seen in Figure 4.33. When T/h is large, the cut is longer dominated by plastic shearing. The effect of T/h is probably due to the plowing as well as the friction under the cut layer. The two contributions consume more energy when T is large.

- *Pyramidal scratch cutting tool*

The results for the pyramidal cutting tool are presented in Figure 4.34. The big difference here is the effect of β . Indeed β plays a triple role. (i) β modifies the plowing forces. The β angle also changes the shape of the cross section of the chip, therefore impacting (ii) the amount of new surfaces created between the chip and the base material. Finally (iii) β is also impacting the section of the sheared layer.

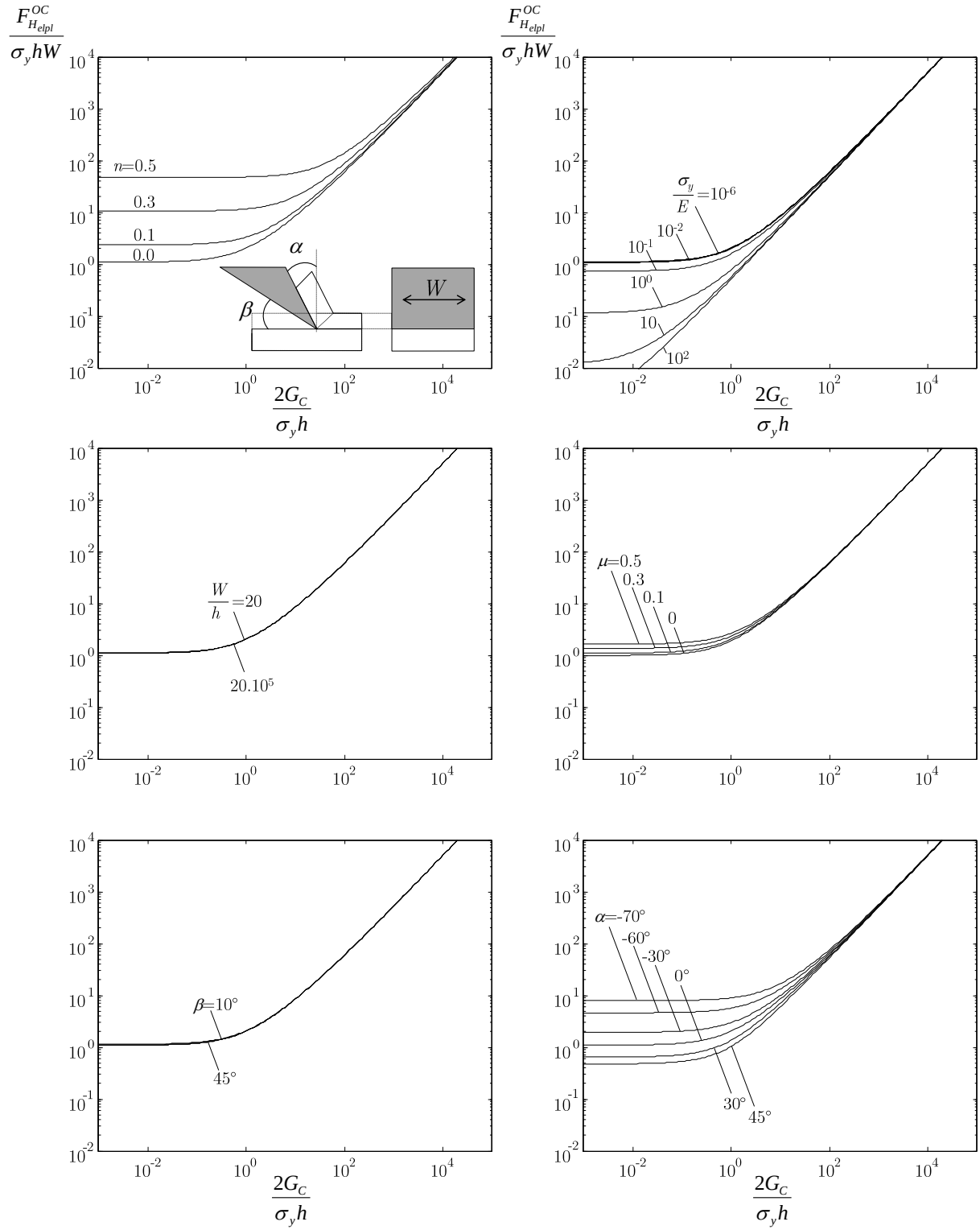


Figure 4.32: Elastoplastic orthogonal cutting with an inclined wedge. In each graph, one parameter is changed at a time to observe its effect on the horizontal cutting force while the other parameters are kept constant with the following values: $n = 0$, $\sigma_y/E = 10^{-3}$, $W/h = 200$, $\mu = 0.1$, $\beta = 30^\circ$ and $\alpha = 30^\circ$.

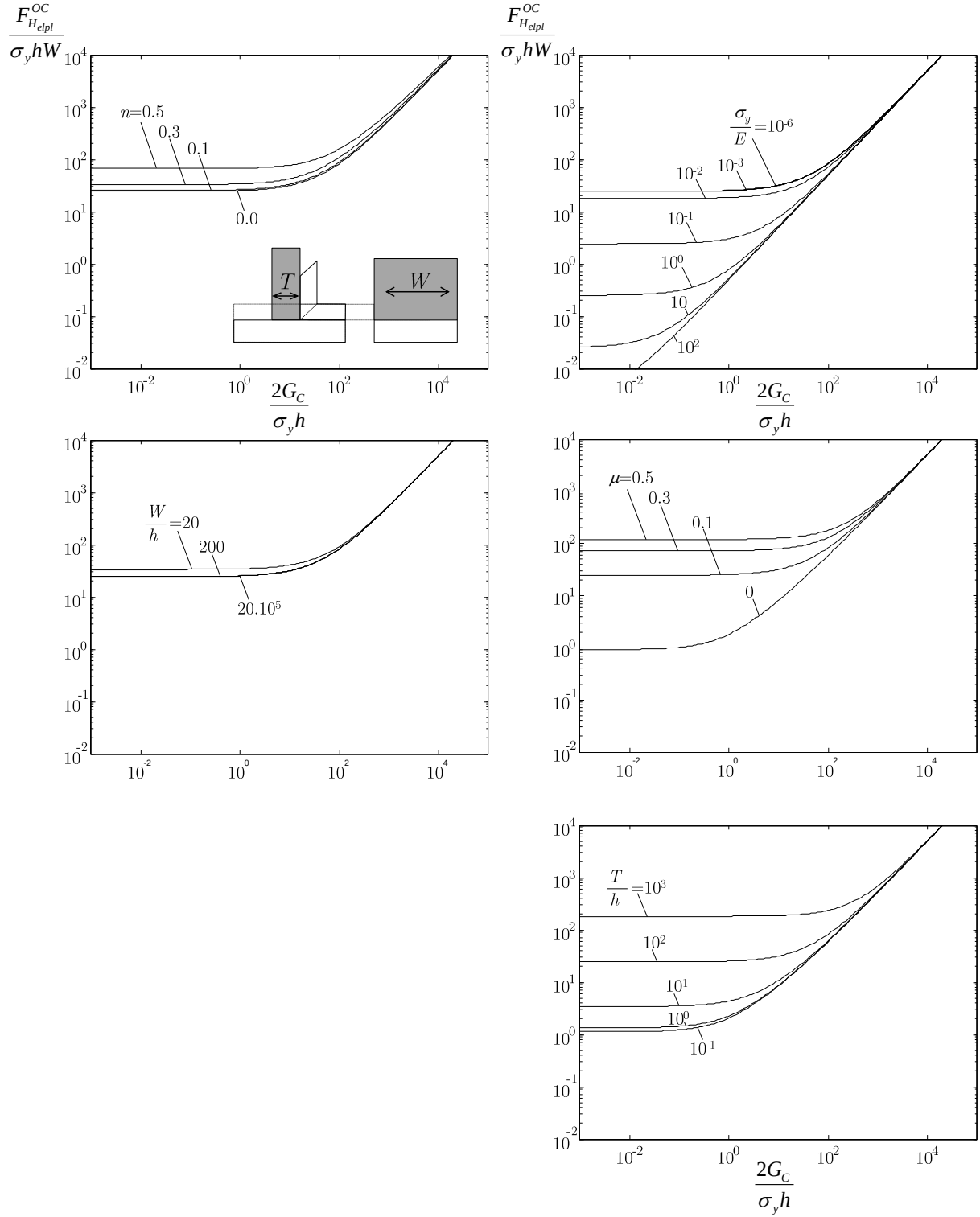


Figure 4.33: Elastoplastic orthogonal cutting with a rectangular cutting tool. In each graph, one parameter is changed to observe its effect on the horizontal cutting force while the other parameters are kept constant with the following values: $n = 0$, $\sigma_y/E = 10^{-3}$, $W/h = 200$, $\mu = 0.1$, $T/h = 10^2$.

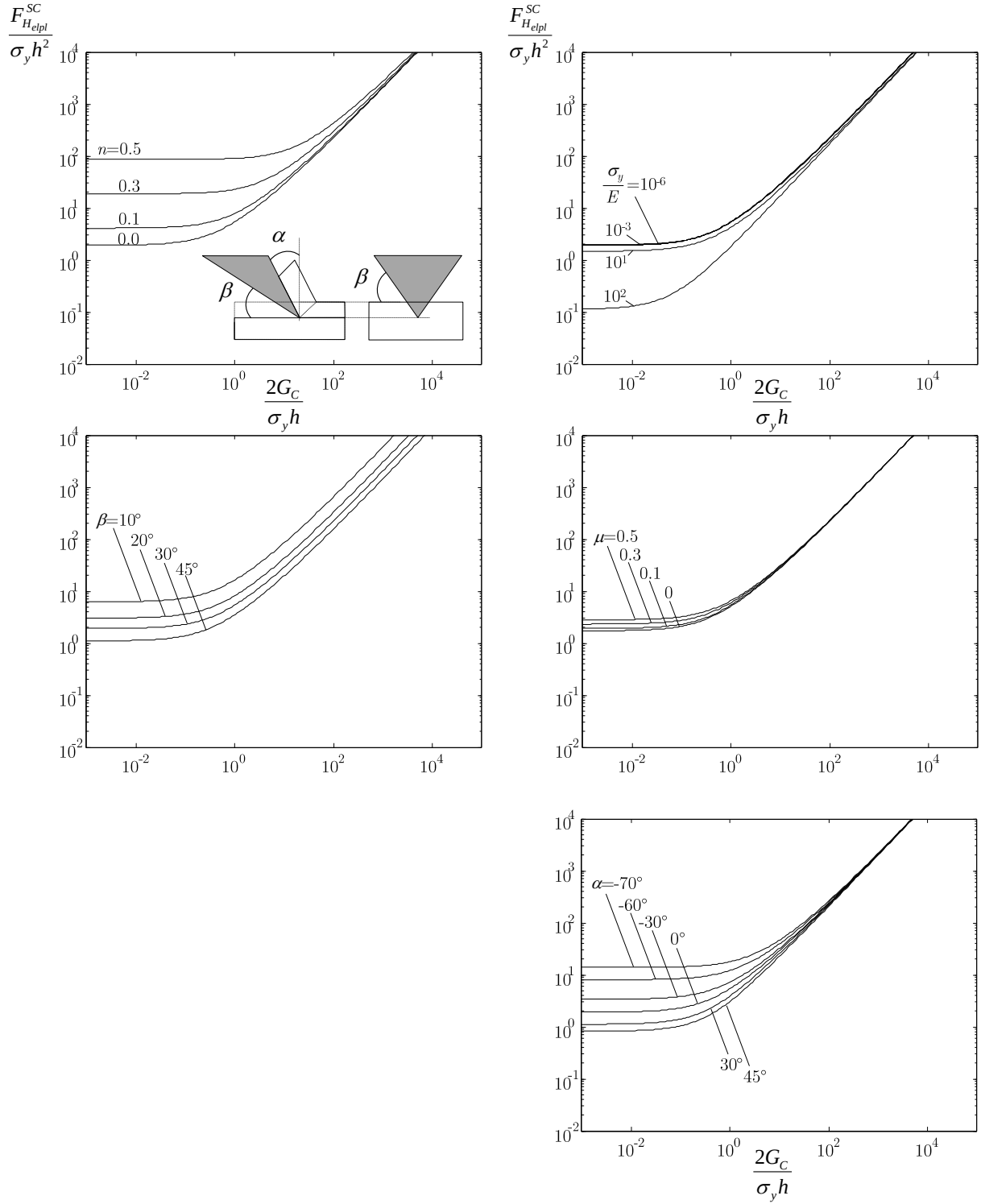


Figure 4.34: Elastoplastic orthogonal cutting with a pyramidal cutting tool. In each graph, one parameter is changed to observe its effect on the horizontal cutting force while the other parameters are kept constant with the following values: $n = 0$, $\sigma_y/E = 10^{-3}$, $\mu = 0.1$, $\beta = 30^\circ$, $\alpha = 30^\circ$.

4.4 Spalling model

Thermally sprayed abrasible materials have a complex microstructure often made of accumulation of splats. These splats originate from the thermal spraying process in which the powder particles are fused into droplets, these droplets eventually impact the substrate and solidify during the manufacturing of the abrasible materials. This kind of microstructure contains a lot of interfaces between the splats. Abrasibles can wear off by a mechanism of micro-rupture which consists in separation of the interface between these splats.

In order to model this mechanism, a few hypotheses are set: (i) the base material is made of a model microstructure composed of small rectangular segments. (ii) Each segment is connected to its neighbors with an interface having a fracture toughness $G_{c_{int}}$. (iii) In case of a contact between a tool and the material, every segment in the trajectory of the tip will be separated from the surface and from his neighbors. (iv) Each segment has an elastoplastic mechanical behavior. (v) The vertical force comes from the resistance of all the segments to indentation. Therefore, the vertical force is the same as the indentation force of a bulk material with the homogenized properties of the segments. (vi) The horizontal force corresponds to the energy consumed in surface separation between the segments as well as the friction between the bottom of the tip and the surface of the material.

4.4.1 Orthogonal spalling with an inclined wedge

The spalling model with an inclined wedge tool is represented in Figure 4.35. After the passage of the tool, this geometry of tool leaves a rectangular trench on the surface of the material. A cross section of the trench is presented in Table 4.2 with the expression of the depth (h_{trench}) and the width of the trench (W_{trench}) as well as the number of segments in contact with the blade ($N_{seg/row}$). These expressions contain the upper bracket “ $\lceil$ ” and the lower bracket “ $\lfloor$ ” symbols which stand for rounding up the number in the bracket to the upper or the lower closer integer respectively. If the blade is centered between two segments or centered in the middle of a segment during the contact, the number of segments impacted will be different. These two geometrical situations are also presented in Table 4.2. From there, the number of segments being removed by the tip in a contact over a distance L follows

$$N_{seg} = \left\lceil \frac{L}{T_{seg}} \right\rceil N_{seg/row} , \quad \text{Eq. 4.100}$$

where T_{seg} is the thickness of a segment. The total surface created during the separation of every segment in the trench is

$$S_{tot} = \left\lceil \frac{L}{T_{seg}} \right\rceil \left[N_{seg/row} (W_{seg} T_{seg} + h_{seg} T_{seg} + h_{seg} W_{seg}) + h_{trench} T_{seg} \right] + N_{seg/row} h_{seg} W_{seg} , \quad \text{Eq. 4.101}$$

where W_{seg} and h_{seg} are the width and the height of a segment respectively. If we consider a steady state spalling, that is, if $L \gg T_{seg}$, the expression of the total surface created becomes

$$S_{tot}^{steady} \approx \frac{L}{T_{seg}} \left[N_{seg/row} (W_{seg} T_{seg} + h_{seg} T_{seg} + h_{seg} W_{seg}) + h_{trench} T_{seg} \right] . \quad \text{Eq. 4.102}$$

In that case, the horizontal and vertical spalling forces are

$$F_{H_{elpl}}^{OS} = G_{c-int} \frac{S_{tot}^{steady}}{L} + \mu F_{V_{elpl}}^{HW} , \quad \text{Eq. 4.103}$$

$$F_{V_{elpl}}^{OS} = F_{V_{elpl}}^{HW} . \quad \text{Eq. 4.104}$$

The spalling forces can take the following non-dimensional form

$$\frac{F_{H_{elpl}}^{OS}}{hW\sigma_y} = \frac{G_{c-int}}{h\sigma_y} \frac{S_{tot}^{steady}}{WL} + \mu \frac{F_{V_{elpl}}^{HW}}{hW\sigma_y} , \quad \text{Eq. 4.105}$$

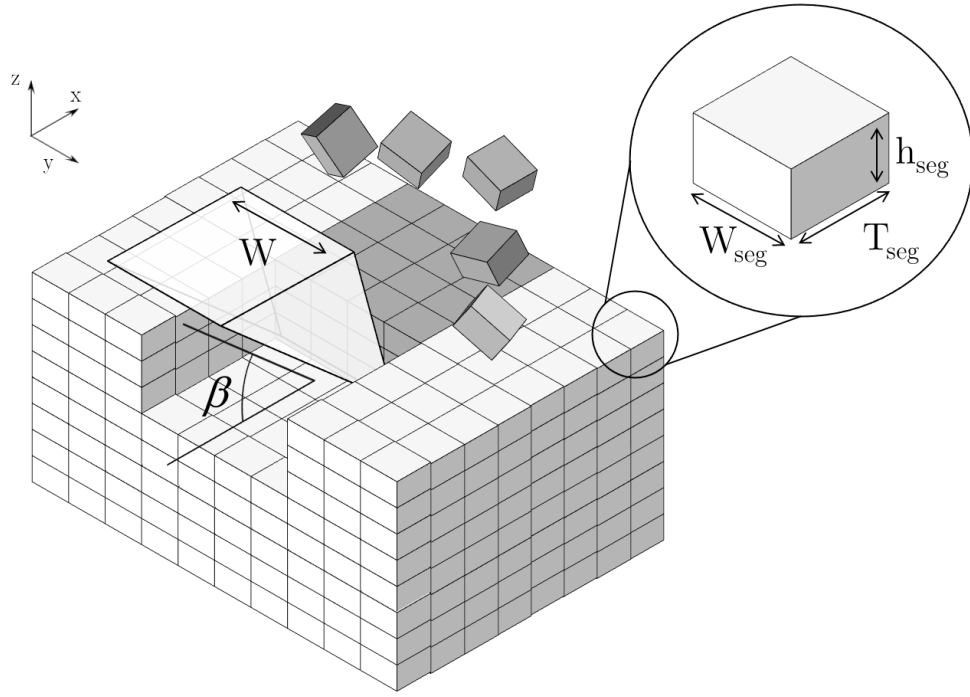


Figure 4.35: Schematic representation of spalling mechanism with an inclined wedge of width W . The back of the wedge makes an angle β with the horizontal surface.

Table 4.2: Schematic representation of the cross section of a segmented material in contact with an inclined wedge tool. The grey segments are in contact with the tool and will be removed.

Blade centered between two segments	$h_{trench} = \left\lceil \frac{h}{h_{seg}} \right\rceil h_{seg}$ $W_{trench} = 2 \left\lceil \frac{W}{2W_{seg}} \right\rceil W_{seg}$ $N_{seg/slice} = \frac{W_{trench}}{W_{seg}} \frac{h_{trench}}{h_{seg}}$
Blade centered in the middle of a segment	$h_{trench} = \left\lceil \frac{h}{h_{seg}} \right\rceil h_{seg}$ $W_{trench} = 2W_{seg} \left\lceil \frac{W}{2W_{seg}} - \frac{1}{2} \right\rceil + W_{seg}$ $N_{seg/slice} = \frac{W_{trench}}{W_{seg}} \frac{h_{trench}}{h_{seg}}$

4.4.2 Orthogonal spalling with a rectangular tool

The spalling model with a rectangular tool is represented in Figure 4.36. After the passage of the tool, this geometry of tool also leaves a rectangular trench on the surface of the material. The expression of h_{trench} , W_{trench} , $N_{seg/row}$ and S_{total} is the same as in the case of the inclined wedge tool (see Table 4.2 and Eq. 4.101). The horizontal and vertical spalling forces are

$$F_{H_{elpl}}^{OS} = G_{c-int} \frac{S_{tot}^{steady}}{L} + \mu F_{V_{elpl}}^{FW}, \quad \text{Eq. 4.106}$$

$$F_{V_{elpl}}^{OS} = F_{V_{elpl}}^{FW}. \quad \text{Eq. 4.107}$$

The spalling forces can take the following non-dimensional form

$$\frac{F_{H_{elpl}}^{OS}}{hW\sigma_y} = \frac{G_{c-int}}{h\sigma_y} \frac{S_{tot}^{steady}}{WL} + \mu \frac{F_{V_{elpl}}^{FW}}{hW\sigma_y}. \quad \text{Eq. 4.108}$$

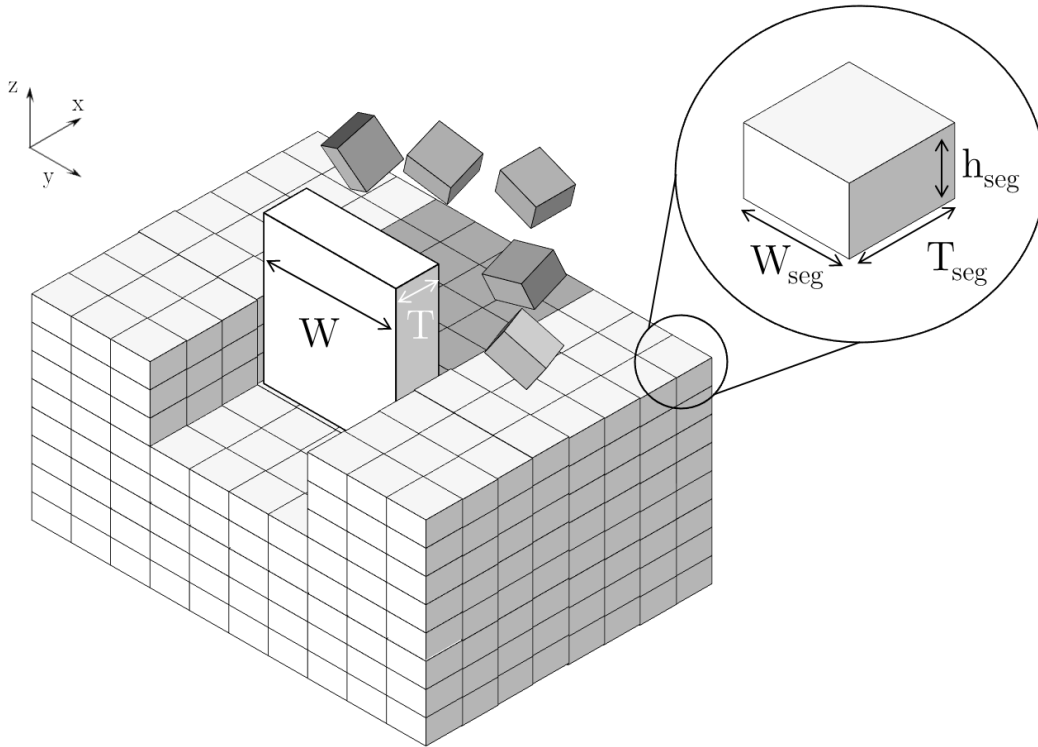


Figure 4.36: Schematic representation of spalling mechanism with a rectangular punch of width W and thickness T .

4.4.3 Pyramidal scratch spalling with a conical tool

The spalling model with a conical tool is represented in Figure 4.37. The passage of this conical tool in the segmented material leaves a triangular trench on the surface of the material. The expressions of h_{trench} , W_{trench} , $N_{seg/row}$ are presented in Table 4.3. The total surface created during the separation of the segments in the trench is given in Eq. 4.101.

$$F_{H_{elpl}}^{SS} = G_{c-int} \frac{S_{tot}^{steady}}{L} + \mu F_{V_{elpl}}^C, \quad \text{Eq. 4.109}$$

and

$$F_{V_{elpl}}^{SS} = F_{V_{elpl}}^C, \quad \text{Eq. 4.110}$$

The spalling forces can take the following non-dimensional form

$$\frac{F_{H_{elpl}}^{SS}}{h^2 \sigma_y} = \frac{G_{c-int}}{h \sigma_y} \frac{S_{tot}^{steady}}{hL} + \mu \frac{F_{V_{elpl}}^C}{h^2 \sigma_y}, \quad \text{Eq. 4.111}$$

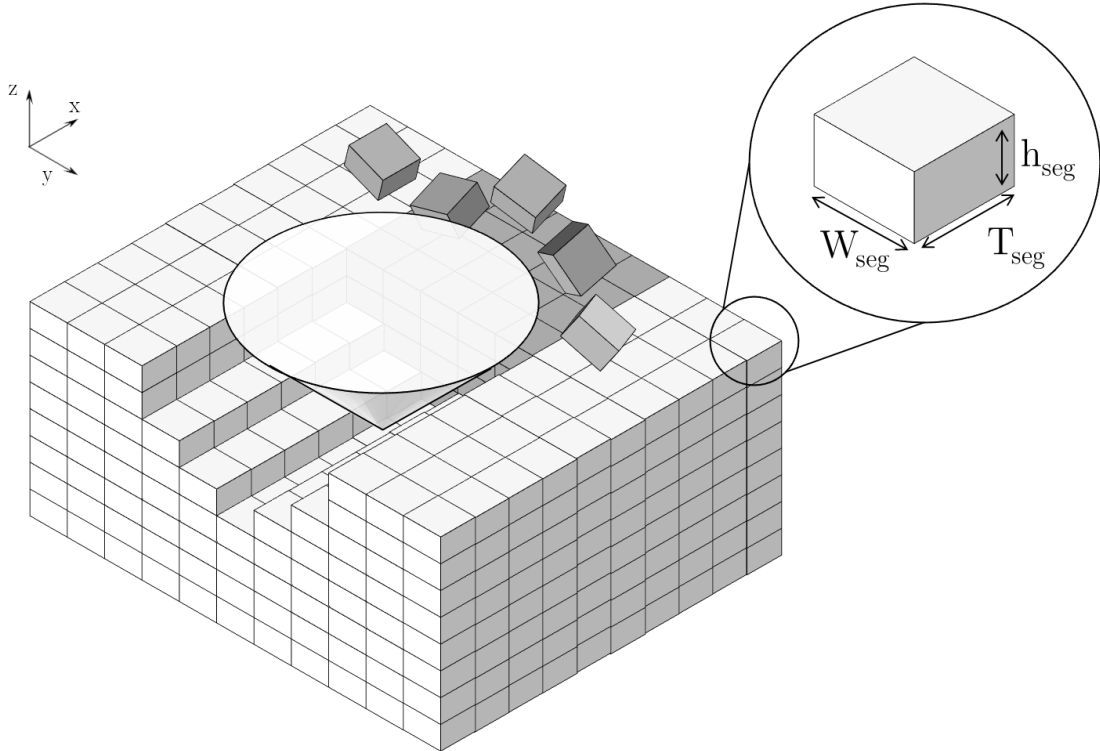
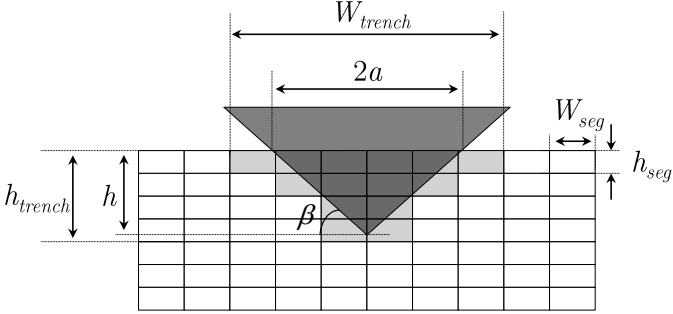
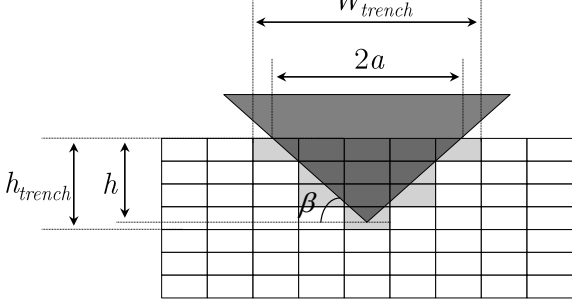


Figure 4.37: Schematic representation of spalling mechanism with a conical tip making an angle β with the horizontal surface.

Table 4.3: Schematic representation of the cross section of a segmented material in contact with a conical tool. The grey segments are in contact with the tool and are removed during the contact.

Blade centered between two segments 	$h_{trench} = \left\lceil \frac{h}{h_{seg}} \right\rceil h_{seg}$ $a = h \cot \beta$ $W_{trench} = 2 \left\lceil \frac{a}{W_{seg}} \right\rceil W_{seg}$ $N_{seg/row} = 2 \sum_{i=0}^{\left\lfloor \frac{W_{trench}}{2W_{seg}} \right\rfloor} \left\lceil \frac{h_{trench}}{h_{seg}} \left(1 - \frac{W_{seg}}{W_{trench}} 2i \right) \right\rceil$
Blade centered in the middle of a segment 	$h_{trench} = \left\lceil \frac{h}{h_{seg}} \right\rceil h_{seg}$ $a = h \cot \beta$ $W_{trench} = 2 \left\lceil \frac{a}{W_{seg}} - \frac{1}{2} \right\rceil W_{seg} + W_{seg}$ $N_{seg/row} = \left\lceil \frac{h}{h_{seg}} \right\rceil +$ $2 \sum_{i=0}^{\left\lfloor \frac{a}{W_{seg}} - \frac{1}{2} \right\rfloor} \left\lceil \frac{h}{h_{seg}} - \frac{W_{seg}}{h_{seg}} \tan \beta \left(\frac{1}{2} + i \right) \right\rceil$

4.4.4 Parametric study and sensitivity analysis

In order to simplify the presentation of the results, the segments are considered cubic, i.e. $h_{seg} = W_{seg} = T_{seg}$. This significantly simplifies the equations and decreases the number of variables in the problem. Another approach could have consisted in assigning aspect ratios $h_{seg} = w_1 W_{seg} = w_2 T_{seg}$, where w_1 and w_2 are the aspect ratios of each segment. The results of the spalling model with an inclined wedge, with a rectangular wedge and with a cone are presented in Figure 4.38, Figure 4.39 and Figure 4.40 respectively. On these figures, a plateau is still observable. This area corresponds to a spalling force dominated by the friction and by the plastic dissipation due to the indentation of the segments. The main difference with the cutting model lies in the effect of the ratio h/h_{seg} . Increasing this ratio is equivalent to removing more segments by unit depth (h) which translates into a faster transition between the plastic dominated zone and the fracture dominated one as well as an increase of the total surface of the interfaces.

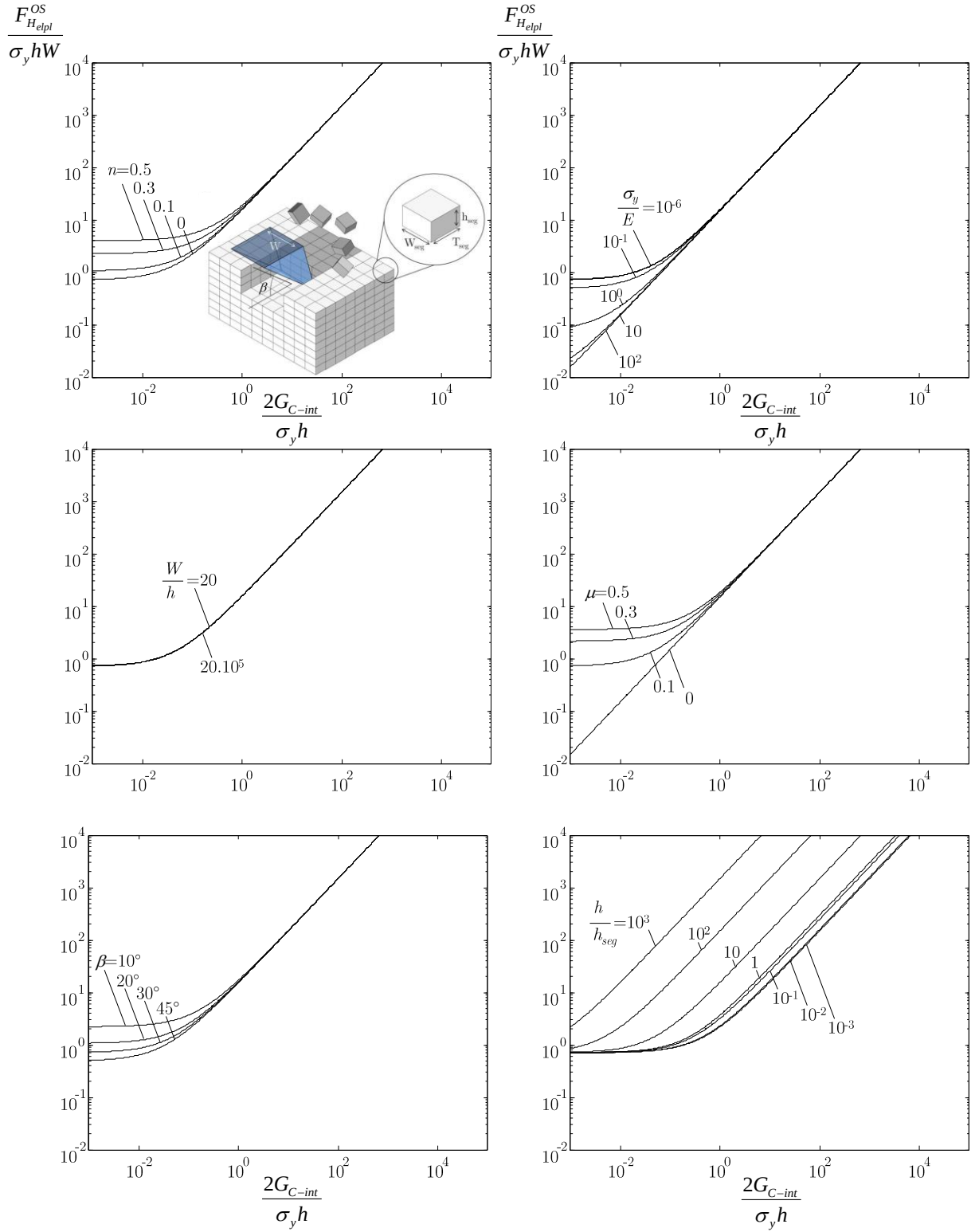


Figure 4.38: Elastoplastic orthogonal spalling with an inclined wedge. The tip is centered in the middle of a segment. In each graph, one parameter is changed to observe its effect on the horizontal spalling force while the other parameters are kept constant with the following values: $n = 0$, $\sigma_y/E = 10^{-3}$, $W/h = 200$, $\mu = 0.1$, $\beta = 30^\circ$ and $h/h_{seg} = 10$.

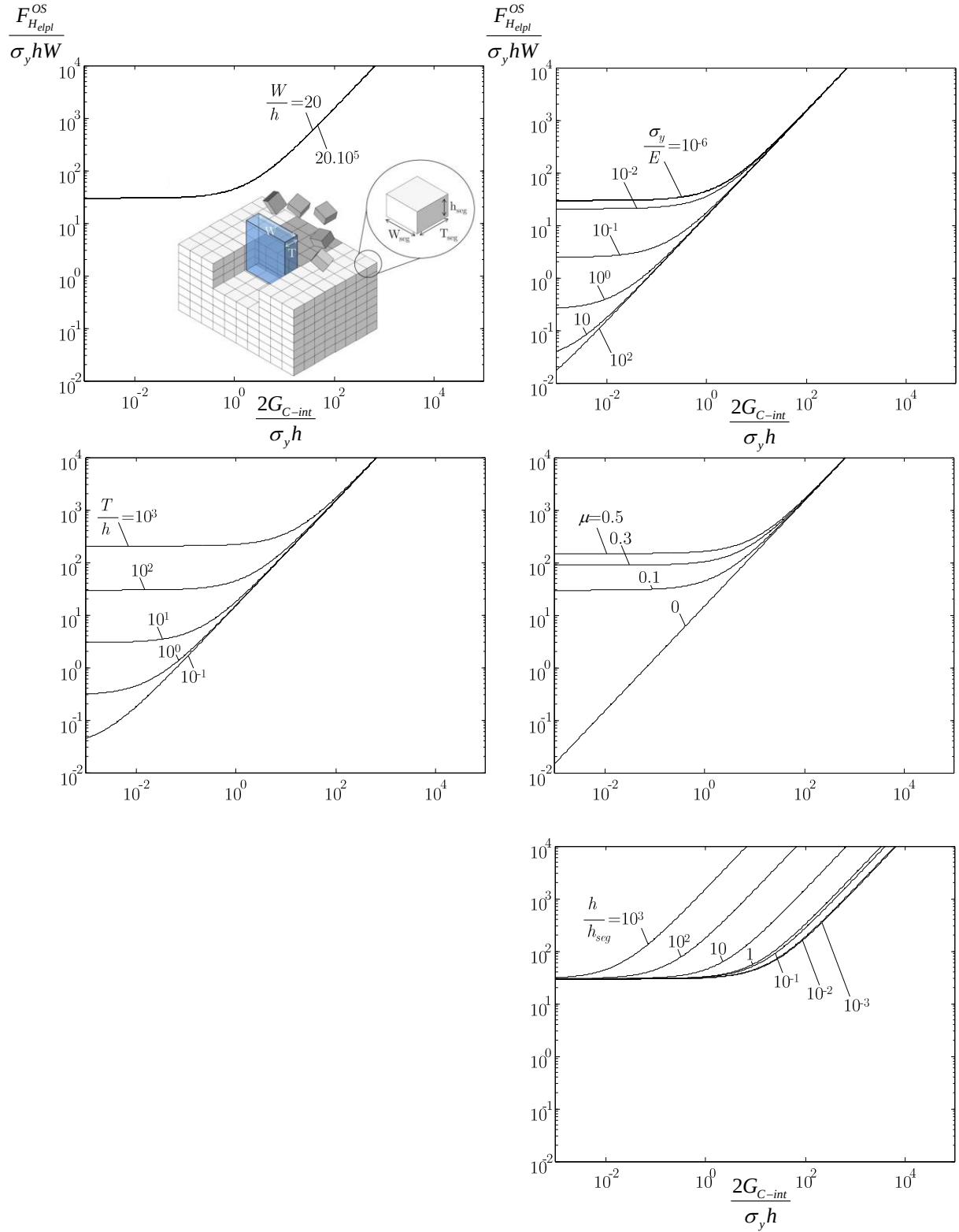


Figure 4.39: Elastoplastic orthogonal spalling with a rectangular wedge. In each graph, one parameter is changed to observe its effect on the horizontal spalling force while the other parameters are kept constant with the following values: $n = 0$, $\sigma_y/E = 10^{-3}$, $W/h = 200$, $T/h = 100$, $\mu = 0.1$, $h/h_{seg} = 10$.

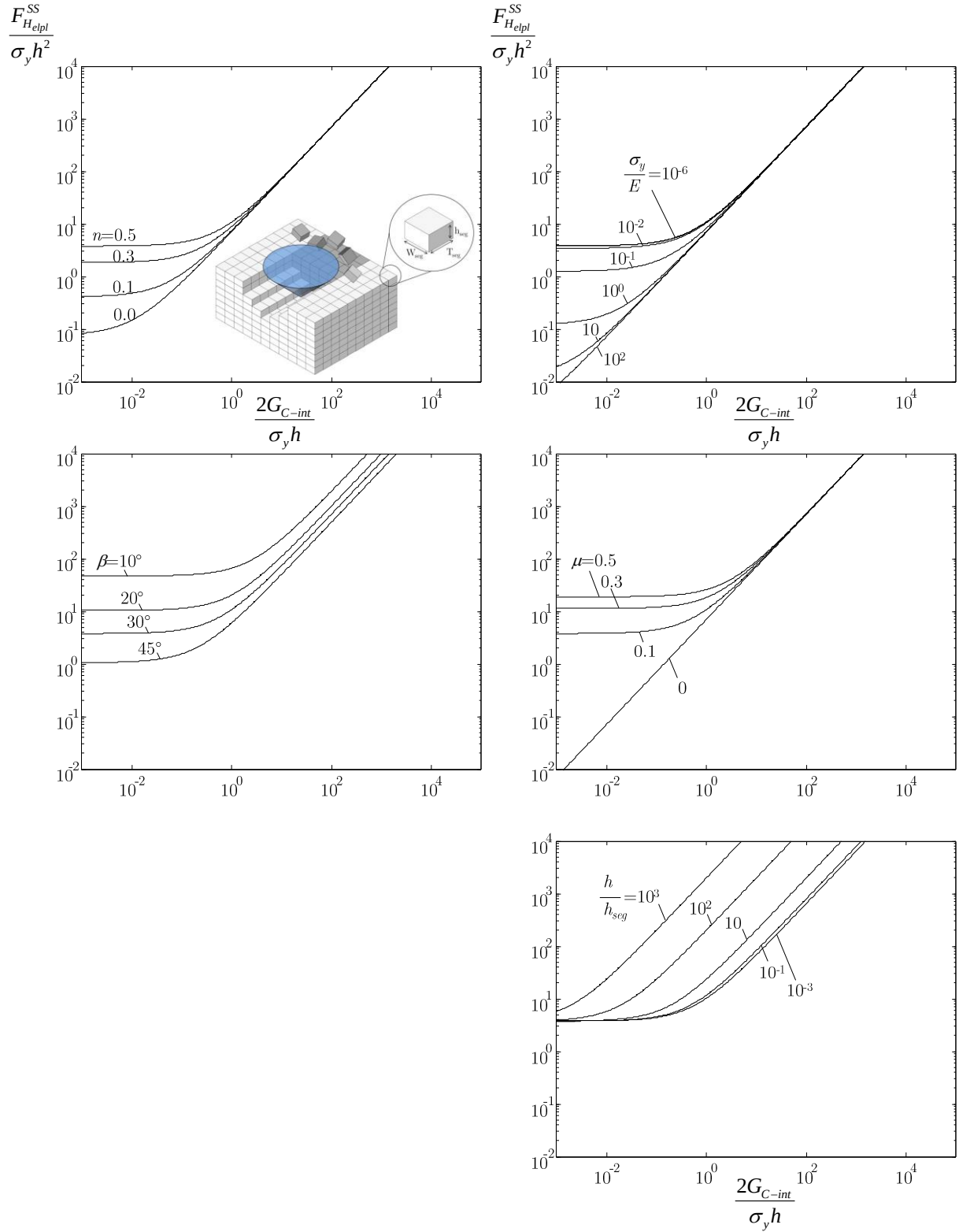


Figure 4.40: Elastoplastic orthogonal spalling with a conical tip. In each graph, one parameter is changed to observe its effect on the horizontal spalling force while the other parameters are kept constant with the following values: $n = 0$, $\sigma_y/E = 10^{-3}$, $\mu = 0.1$, $\beta = 30^\circ$ and $h/h_{seg}=10$.

4.5 Erosion modelling

Erosion modelling is the second main step of this study. Using an approach similar to Finnie [Finnie 1960], erosion is modelled in the general framework of elastoplasticity. The erosive particles are associated to cones with their axis of symmetry perpendicular to the surface of the base material. The three main mechanisms of plowing, cutting and spalling are modelled based on the following hypotheses: (i) an erosive particle impacts the base material with an angle ω and keeps its original trajectory during the penetration in the surface of the base material. (ii) The particle is arrested by the materials surface once all its kinetic energy is transferred into mechanical strain and/or fracture of the base material surface. (iii) The eroded volume corresponds the volume of the residual groove created by the impacting particles in the base material after elastic recovery. (ii) The rotation of the particles during the impact is not taken into account. (iv) The total eroded volume is considered proportional to the eroded volume created by a single impacting particle. The main differences with Finnie model are that (i) there is no assumption on the pile-up, the pile-up depends on the wear mechanism. (ii) The solution is not obtained by solving the equation of the motion of the particles but by an energy balance. (iii) The rebound of the particles in case of low impacting angle is not considered here. (iv) Finnie also takes into account the rotation of the particle in the equation of the motion of the particle, while it is not the case here.

The erosion resistance (e_{res}) is considered here as the amount of kinetic energy (U_{cin}) required to erode a volume (V_{er}). In order to display the erosion resistance of a material in a non-dimensional way, e_{res} is normalized by the yield strength (σ_y). The erosion rate (e_{rate}) is defined here as the inverse of the erosion resistance through

$$e_{res} = \frac{U_{cin}}{\sigma_y V_{er}} = \frac{1}{e_{rate}}. \quad \text{Eq. 4.112}$$

Putting hypothesis (ii) into equation gives

$$U_{cin} = \frac{m \|\vec{v}\|^2}{2}. \quad \text{Eq. 4.113}$$

In the present model, all the energy is transferred. Therefore, the energy balance equation is

$$\int_0^h F_V dh + \int_0^L F_H dx = U_{cin} . \quad \text{Eq. 4.114}$$

From hypothesis (i), the following change of variable can be done:

$$L = h \cot \omega . \quad \text{Eq. 4.115}$$

where L is the length of the groove created by the particle and ω is the impact angle. Combining Eq. 4.114 and Eq. 4.115 gives

$$\int_0^h F_V dh + \int_0^h F_H dh \cot \omega = U_{cin} . \quad \text{Eq. 4.116}$$

The variables V_{er} , F_V , and F_H are dependent on the wear mechanism. Therefore, each erosion mechanism requires a specific analysis.

4.5.1 Erosion by plowing

In the case the erosion mechanism takes place by a plowing mechanism, the erosion wear volume (V_{er}) will be affected by the impacting angle (ω). If ω is small compared to β , V_{er} is the sum of a half cone and a tetrahedron with a side angle ($\beta - \beta'$) (see Figure 4.41).

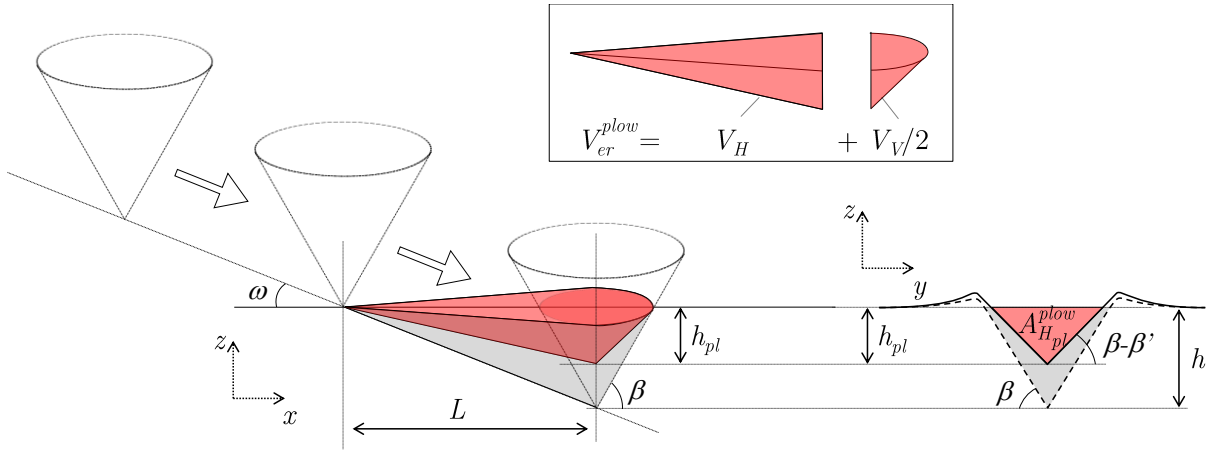


Figure 4.41: Schematic representation of the erosion by plowing when $\omega < \beta$. The volume of the residual groove after elastic recovery (V_{er}) is also represented.

$$V_{er}^{plow}(h) = V_H + \frac{1}{2} V_V , \quad \text{Eq. 4.117}$$

$$V_{er}^{plow}(h) = \int_0^h A_{wear}^{plow} dh \cot \omega + \frac{1}{6} h^3 \left(\frac{h_{pl}^C}{h} \right)^3 \pi \cot^2 \beta \left(\frac{1 + \tan^2 \beta \frac{C_{elpl}}{C_{el}}}{1 - \frac{C_{elpl}}{C_{el}}} \right)^2 \quad \text{Eq. 4.118}$$

Integrating and simplifying gives

$$V_{er}^{plow}(h) = \frac{h^3}{3} \left(\frac{h_{pl}^C}{h} \right)^2 \cot \omega \cot \beta \frac{1 + \tan^2 \beta \frac{C_{elpl}}{C_{el}}}{1 - \frac{C_{elpl}}{C_{el}}} + \frac{1}{6} h^3 \left(\frac{h_{pl}^C}{h} \right)^3 \pi \cot^2 \beta \left(\frac{1 + \tan^2 \beta \frac{C_{elpl}}{C_{el}}}{1 - \frac{C_{elpl}}{C_{el}}} \right)^2. \quad \text{Eq. 4.119}$$

If ω is large compared to β , V_{er} becomes the volume of a cone with a side angle $(\beta - \beta')$ (see Figure 4.42). In that case,

$$V_{er}^{plow}(h) = V_V, \quad \text{Eq. 4.120}$$

which gives

$$V_{er}^{plow}(h) = \frac{1}{3} h^3 \left(\frac{h_{pl}^C}{h} \right)^3 \pi \cot^2 \beta \left(\frac{1 + \tan^2 \beta \frac{C_{elpl}}{C_{el}}}{1 - \frac{C_{elpl}}{C_{el}}} \right)^2. \quad \text{Eq. 4.121}$$

The transition between these two situations happened when those two volumes are equal to each other. It is the case when

$$\omega \geq \text{arccot} \left[\frac{1}{2} \left(\frac{h_{pl}^C}{h} \right) \pi \cot \beta \left(\frac{1 + \tan^2 \beta \frac{C_{elpl}}{C_{el}}}{1 - \frac{C_{elpl}}{C_{el}}} \right) \right]. \quad \text{Eq. 4.122}$$

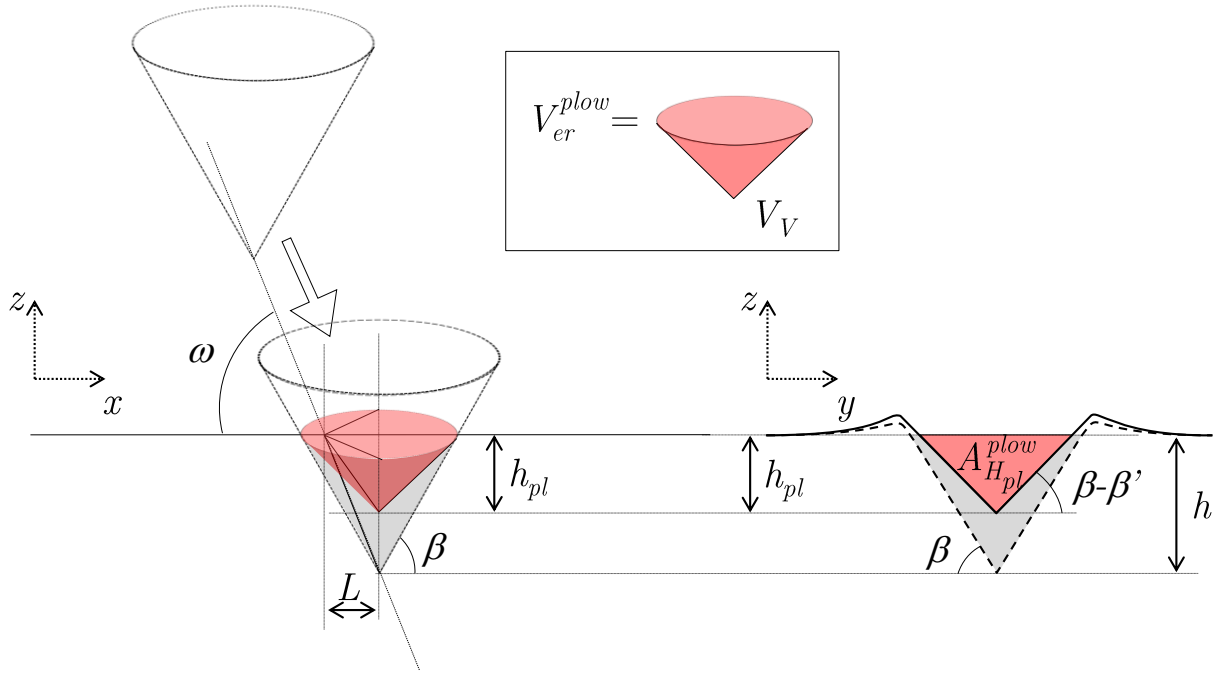


Figure 4.42: Schematic representation of the erosion by plowing when $\omega > \beta$. The volume of the residual groove after elastic recovery (V_{er}) is also represented.

In both cases, the horizontal and vertical energy dissipated in the impact are

$$U_{H_{er}}^{plow} = \int_0^h F_{H_{elpl}}^C dh \cot \omega, \quad \text{Eq. 4.123}$$

and

$$U_{V_{er}}^{plow} = \int_0^h F_{V_{elpl}}^C dx. \quad \text{Eq. 4.124}$$

Integrating and simplifying these two equations gives

$$U_{V_{er}}^{plow} = \frac{h^3}{3} \pi H_{elpl}^C \left(\frac{a_{elpl}^C}{h} \right)^2, \quad \text{Eq. 4.125}$$

and

$$U_{H_{er}}^{plow} = \frac{h^3}{3} \pi H_{elpl}^C \left(\frac{a_{elpl}^C}{h} \right)^2 \left[\left(\frac{h}{a_{elpl}^C} \right)^2 \left(\frac{a_{elpl}^W}{h} \right) \left(\frac{h_{pl}^W}{h} \right) \frac{H_{elpl}^W}{H_{elpl}^C} \frac{1}{\pi} + \mu \right] \cot \omega. \quad \text{Eq. 4.126}$$

4.5.2 Erosion by cutting

Erosion by cutting generates an erosion wear volume that can be described either by Eq. 4.117 or Eq. 4.120 depending if ω is smaller or larger than β (see Figure 4.43). The main difference in the cutting situation is that the residual cross section follows Eq. 4.92. If ω is small compared to β ,

$$V_{er}^{cut}(h) = \int_0^L A_{H_{pl}}^{cut} dx + \frac{1}{2} V_V. \quad \text{Eq. 4.127}$$

Using the same change of variable as in Eq. 4.118 and integrating gives

$$V_{er}^{cut}(h) = \frac{h^3}{3} \cot \beta \cot \omega \left[1 - \left(\frac{h_{el}^C}{h} \right)^2 + \left(\frac{h_{el}^C}{h} \right)^2 \left(\frac{h_{pl}^C}{h} \right)^2 \frac{1 + \tan^2 \beta \frac{C_{elpl}}{C_{el}}}{1 - \frac{C_{elpl}}{C_{el}}} \right] \\ + \frac{1}{6} \pi h^3 \cot^2 \beta \left[1 - \left(\frac{h_{pl}^C}{h} \right)^3 \left(\frac{h_{el}^C}{h} \right)^3 \left(\frac{1 + \tan^2 \beta \frac{C_{elpl}}{C_{el}}}{1 - \frac{C_{elpl}}{C_{el}}} \right)^2 \right]. \quad \text{Eq. 4.128}$$

If ω is large compared to β , V_{er} is the volume given by the revolution of the surface presented in Figure 4.31. In that case,

$$V_{er}^{cut}(h) = \frac{1}{3} \pi h^3 \cot^2 \beta \left[1 - \left(\frac{h_{el}}{h} \right)^3 + \left(\frac{h_{pl}}{h} \right)^3 \left(\frac{h_{el}}{h} \right)^3 \left(\frac{1 + \tan^2 \beta \frac{C_{elpl}}{C_{el}}}{1 - \frac{C_{elpl}}{C_{el}}} \right)^2 \right]. \quad \text{Eq. 4.129}$$

The transition happens when

$$\cot \omega \leq \frac{1}{2} \pi \cot \beta \frac{1 - \left(\frac{h_{pl}^C}{h} \right)^3 \left(\frac{h_{el}^C}{h} \right)^3 \left(\frac{1 + \tan^2 \beta (C_{elpl}/C_{el})}{1 - (C_{elpl}/C_{el})} \right)^2}{1 - \left(\frac{h_{el}^C}{h} \right)^2 + \left(\frac{h_{el}^C}{h} \right)^2 \left(\frac{h_{pl}^C}{h} \right)^2 \frac{1 + \tan^2 \beta (C_{elpl}/C_{el})}{1 - (C_{elpl}/C_{el})}}. \quad \text{Eq. 4.130}$$

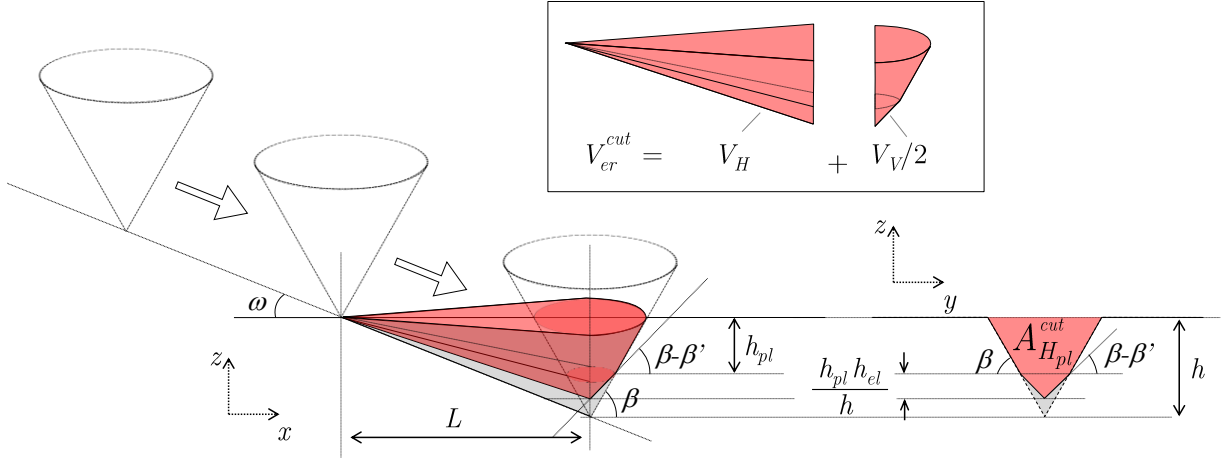


Figure 4.43: Schematic representation of the erosion by cutting when $\omega > \beta$. The chip in front of the particle is not represented.

In both cases, the horizontal and vertical energy dissipated in the impact are

$$U_{H_{er}}^{cut} = \int_0^h F_{H_{elpl}}^{SC} dh \cot \omega, \quad \text{Eq. 4.131}$$

and

$$U_{V_{er}}^{cut} = \int_0^h F_{V_{elpl}}^{SC} dh. \quad \text{Eq. 4.132}$$

Integrating and simplifying these equations gives

$$U_{H_{er}}^{cut} = \sigma_y \cot \omega \cot \beta \left(1 - \left(\frac{h_{el}^C}{h} \right)^2 \right) \left[Z \frac{h^3}{3} + \frac{h^2}{2} \frac{P}{(1+Z)} + M(h) \right] + \left(\frac{h_{el}^C}{h} \right)^3 U_{H_{er}}^{plow} \quad \text{Eq. 4.133}$$

and

$$U_{V_{er}}^{cut} = Z U_{H_{er}}^{cut} \tan \omega - Z G_c \frac{h^2}{\sin \beta} \left[1 - (1 - \cos \beta) \left(\frac{h_{el}^C}{h} \right) \right] + \left(\frac{h_{el}^C}{h} \right)^3 U_{V_{er}}^{plow}, \quad \text{Eq. 4.134}$$

where

$$M(h) \simeq \frac{\sqrt{Oh(Oh+P)}(8O^2h^2 + 2OPh)}{24 O^{5/2}}, \quad \text{Eq. 4.135}$$

in which

$$O = Z^2 + 1, \quad \text{Eq. 4.136}$$

and

$$P = \frac{G_c}{\sigma_y} \frac{4}{\cos \beta} \frac{1 - (1 - \cos \beta) (h_{el}^C / h)}{1 - (h_{el}^C / h)^2} (1 + Z). \quad \text{Eq. 4.137}$$

4.5.3 Erosion by spalling

Regarding erosion by a spalling mechanism, the hypotheses presented in Section 4.4 are applied. The wear volume also depends on ω and β . The first case ($\omega < \beta$) is presented in Figure 4.44. In that case, the number of segments in contact with the impacting particle is

$$N_{seg} = N_{seg/row}(h) + \sum_{i=1}^{\left\lfloor \frac{L}{T_{seg}} \right\rfloor} N_{seg/row}(h_i) + \sum_{j=1}^{\left\lfloor \frac{L_2 + L - \lceil L \rceil}{T_{seg}} \right\rfloor} N_{seg/row}(h_j), \quad \text{Eq. 4.138}$$

where

$$h_i = iT_{seg} \tan \omega, \quad \text{Eq. 4.139}$$

$$h_j = jT_{seg} \tan \beta, \quad \text{Eq. 4.140}$$

$$N_{seg/row}(h) = \left\lceil \frac{h}{h_{seg}} \right\rceil + 2 \sum_{j=0}^{\left\lceil \frac{h \cot \beta - 1}{W_{seg}} \right\rceil} \left\lceil \frac{h}{h_{seg}} - \frac{W_{seg}}{2h_{seg}} \tan \beta \right\rceil - \left\lceil j \frac{W_{seg}}{h_{seg}} \tan \beta \right\rceil, \quad \text{Eq. 4.141}$$

$$L = h \cot \omega, \quad \text{Eq. 4.142}$$

and

$$L_2 = h \cot \beta. \quad \text{Eq. 4.143}$$

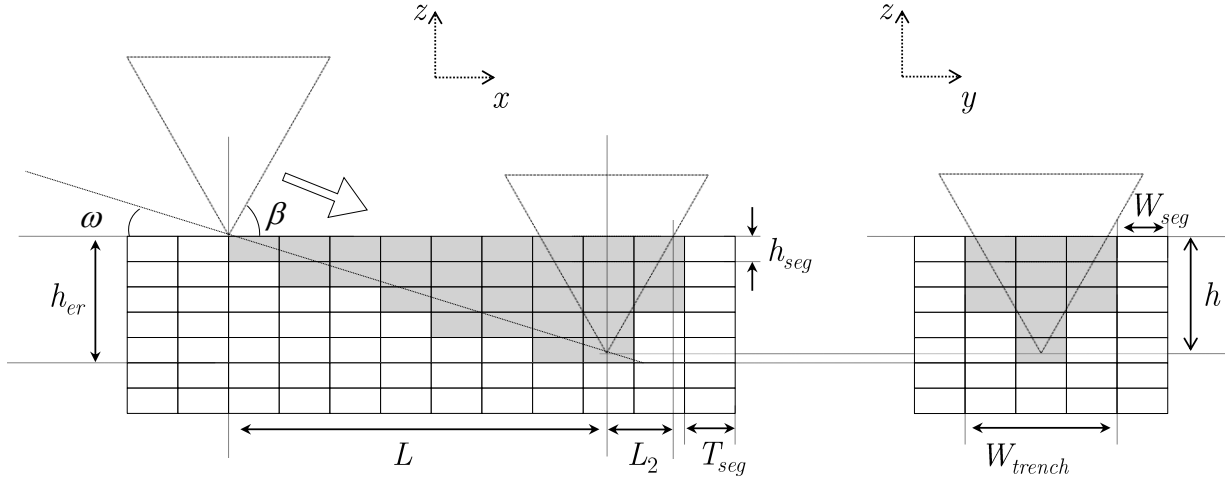


Figure 4.44: Schematic representation of the trajectory of an erosive particle impacting a segmented material. Here, the impacting angle ω is smaller than the side angle of the conical particle β . The segments in grey are removed by the particle.

The expression of the eroded volume is

$$V_{er}^{spal}(h) = N_{seg} h_{seg} T_{seg} W_{seg} . \quad \text{Eq. 4.144}$$

The total surface created during the separation of the segments is

$$S_{tot} = (h_{seg} T_{seg} + h_{seg} W_{seg} + T_{seg} W_{seg}) N_{seg} + h_{seg} W_{seg} N_{seg/row}(h) + S_{lat} , \quad \text{Eq. 4.145}$$

where

$$S_{lat} = \left[\left\lceil \frac{h}{h_{seg}} \right\rceil + \sum_{i=1}^{\left\lfloor \frac{L}{T_{seg}} \right\rfloor} \left\lceil \frac{h_i}{h_{seg}} \right\rceil + \sum_{j=1}^{\left\lfloor \frac{L_2 + L - \lceil L \rceil}{T_{seg}} \right\rfloor} \left\lceil \frac{h_j}{h_{seg}} \right\rceil \right] h_{seg} T_{seg} . \quad \text{Eq. 4.146}$$

The second case ($\omega > \beta$) (Figure 4.45) can be calculated using the same equations with the following changes:

$$h_j = h_i , \quad \text{Eq. 4.147}$$

and

$$L = L_2 . \quad \text{Eq. 4.148}$$

In both cases, regarding the energy dissipated in the process,

$$U_{H_{er}}^{spal} = G_{C-int} S_{tot} + \int_0^h \mu F_{V_{elpl}}^C dh \cot \omega , \quad \text{Eq. 4.149}$$

and

$$U_{Ver}^{spal} = \int_0^h F_{V_{elpl}}^C dh. \quad \text{Eq. 4.150}$$

Integrating and simplifying these equations gives

$$U_{Her}^{spal} = G_{C-int} S_{tot} + \mu \cot \omega \frac{h^3}{3} \pi H_{elpl}^C \left(\frac{a_{elpl}^C}{h} \right)^2, \quad \text{Eq. 4.151}$$

and

$$U_{Ver}^{spal} = \frac{h^3}{3} \pi H_{elpl}^C \left(\frac{a_{elpl}^C}{h} \right)^2. \quad \text{Eq. 4.152}$$

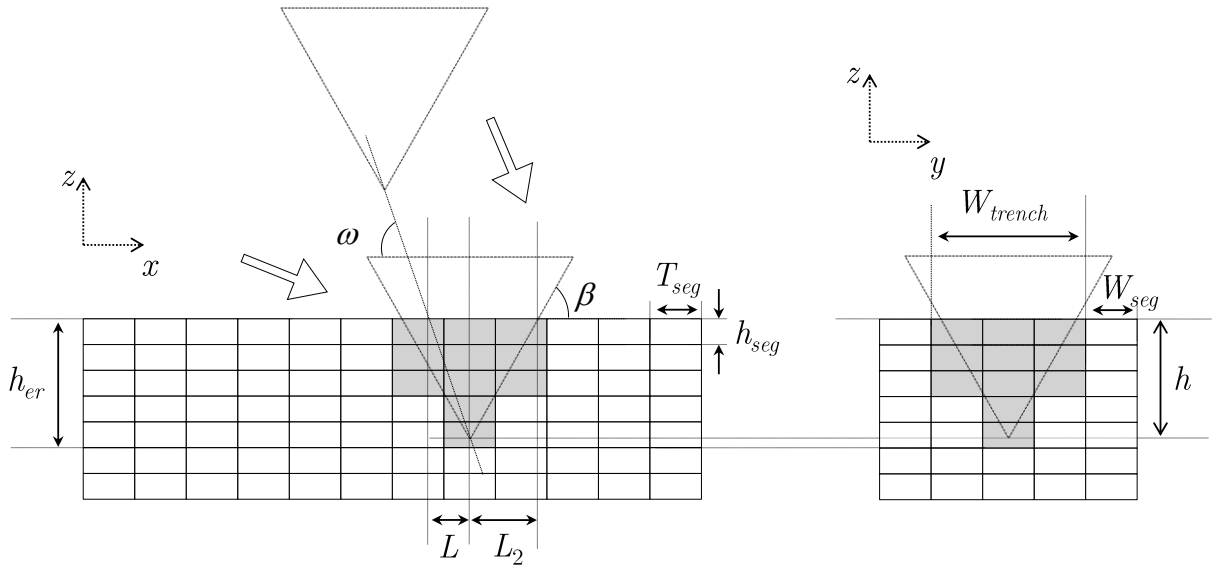


Figure 4.45: Schematic representation of the trajectory of an erosive particle impacting a segmented material with an impacting angle ω wider than the side angle of the conical particle β . The segments in grey are removed by the particle.

4.5.4 Parametric study and sensitivity analysis

The results of the three erosion models (plowing, cutting and spalling) are presented in Figure 4.46, Figure 4.47 and Figure 4.48. On these figures, the erosion rate is defined as the erosion volume (V_{er}) normalized by the kinetic energy of the impacting particle (U_{cin}) and by σ_y (see Eq. 4.112).

- *Erosion by plowing*

Concerning the modelling of erosion by plowing (see Figure 4.46), the model predicts that the erosion rate increases with the angle β . It means that a sharper particle faces less resistance eroding the material by an indentation and plowing mechanism. In consequence, the normalized erosion rate is higher. The friction coefficient (μ) as well as the strain hardening exponent (n) also seems to decrease the erosion rate. Increasing these two parameters seems to increase the quantity of energy dissipated by unit volume in the base material. The effect of the impacting angle (ω) is more nuanced. A transition occurs at an angle that depends on β . This transition corresponds to the change in the shape of the crater (see Figure 4.41 and Figure 4.42). The ratio σ_y/E seems to decrease the erosion rate. Two competing phenomena are in play here. Indeed a higher elastic strain comes with less energy dissipated in plastic deformations. However, it also means a larger elastic recovery which seems to dominate in this case. Finally, the normalized erosion rate is not affected by the ratio U_{cin}/σ_y . It means that the normalized eroded volume is, with this mechanism, always proportional to the kinetic energy of the particle.

- *Erosion by cutting*

Regarding the modelling of erosion by a cutting mechanism (see Figure 4.47), the predictions are mainly the same as in the plowing case. However the model shows that the normalized erosion rate is lower if U_{cin}/σ_y is low. It means that the erosion resistance is bigger if the impacting particles have a low kinetic energy. This phenomenon is due to a cutting size effect described in [Atkins 2009]. In pyramidal cutting, the energy consumed in plastic shearing is proportional to $h^3\sigma_y$ while the separation energy required to produce the chip is proportional to h^2G_{IC} . Therefore at small depth, the separation energy can be way larger than the dissipation by plastic strain. In consequence, the normalized erosion rate is lower.

- *Erosion by spalling*

In order to simplify the presentation of the results, the segments are again considered cubic, i.e. $h_{seg} = W_{seg} = T_{seg}$. Regarding the prediction of the erosion rate by spalling, the effect of the ratio σ_y/E is in opposition with the cutting and plowing models

predictions. Erosion by spalling is assumed to happen with no elastic recovery. In that case, a larger elastic strain leads to less energy dissipation by plastic deformations. Therefore a larger σ_y/E ratio translates into a larger erosion rate. In addition, the effect of the size of the segments is significant. Indeed, with very large segments, an impacting particle might not have enough energy to separate the interface in one strike. In that case, the erosion rate is zero. However, with smaller segments, the separation energy is proportional to many factors, including (i) the surface of each segment, (ii) the number of segments removed and (iii), the fracture toughness. This separation energy is thereby roughly proportional to $N_{seg}h_{seg}^2G_{c-int}$. Since the number of segments by unit volume is equal to h_{seg}^{-3} , the normalized erosion rate is mainly proportional to h_{seg}/G_{c-int} . Therefore smaller segments and higher interface fracture toughness increase the erosion resistance.

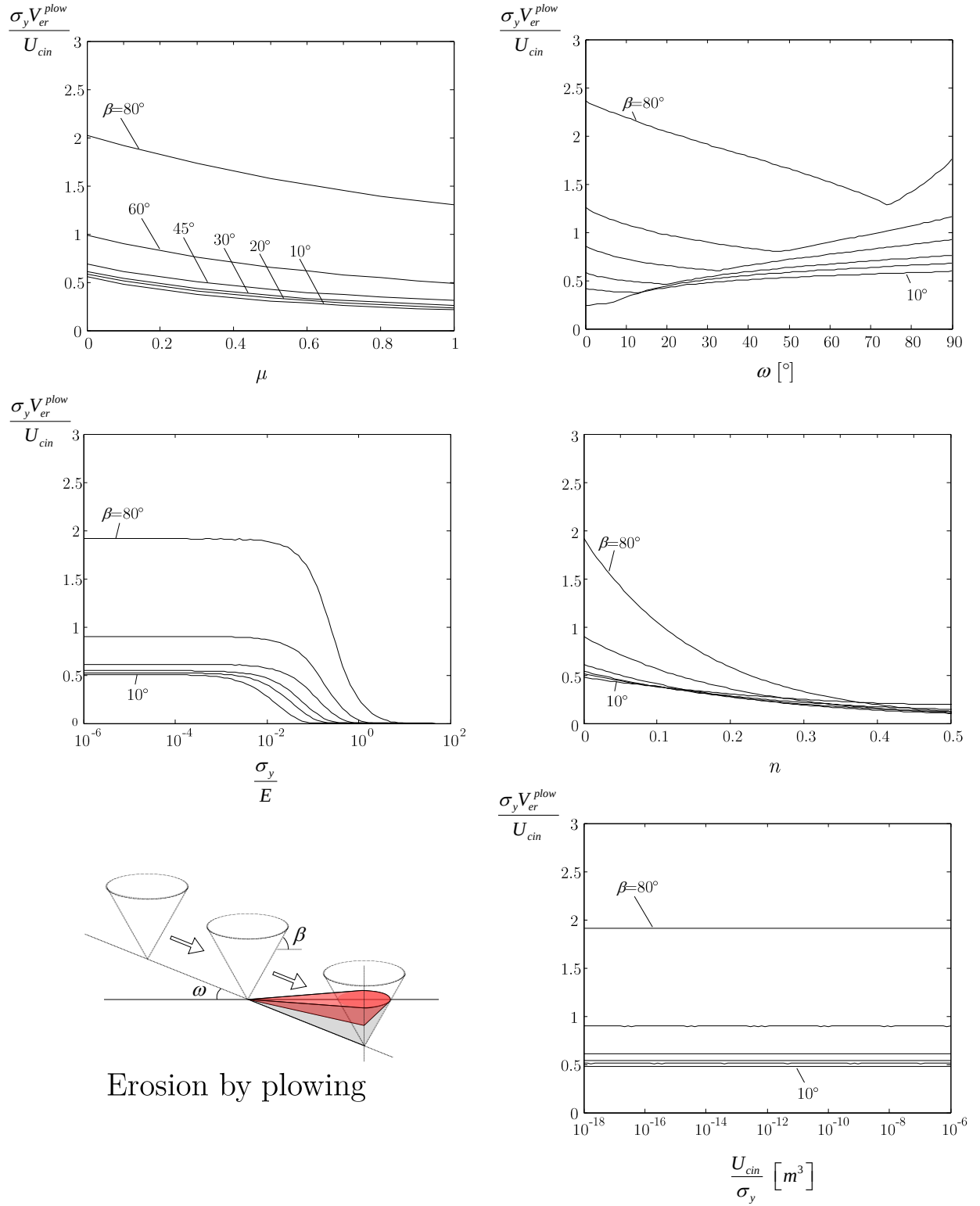


Figure 4.46: Sensitivity analysis in the case of erosion by a plowing mechanism. In each graph, one parameter is changed to observe its effect on the erosion rate while the other parameters are kept constant with the following values: $n = 0$, $\sigma_y/E = 10^{-3}$, $\mu = 0.1$, $U_{cin}/\sigma_y = 10^{-12}$ and $\omega = 30^\circ$.

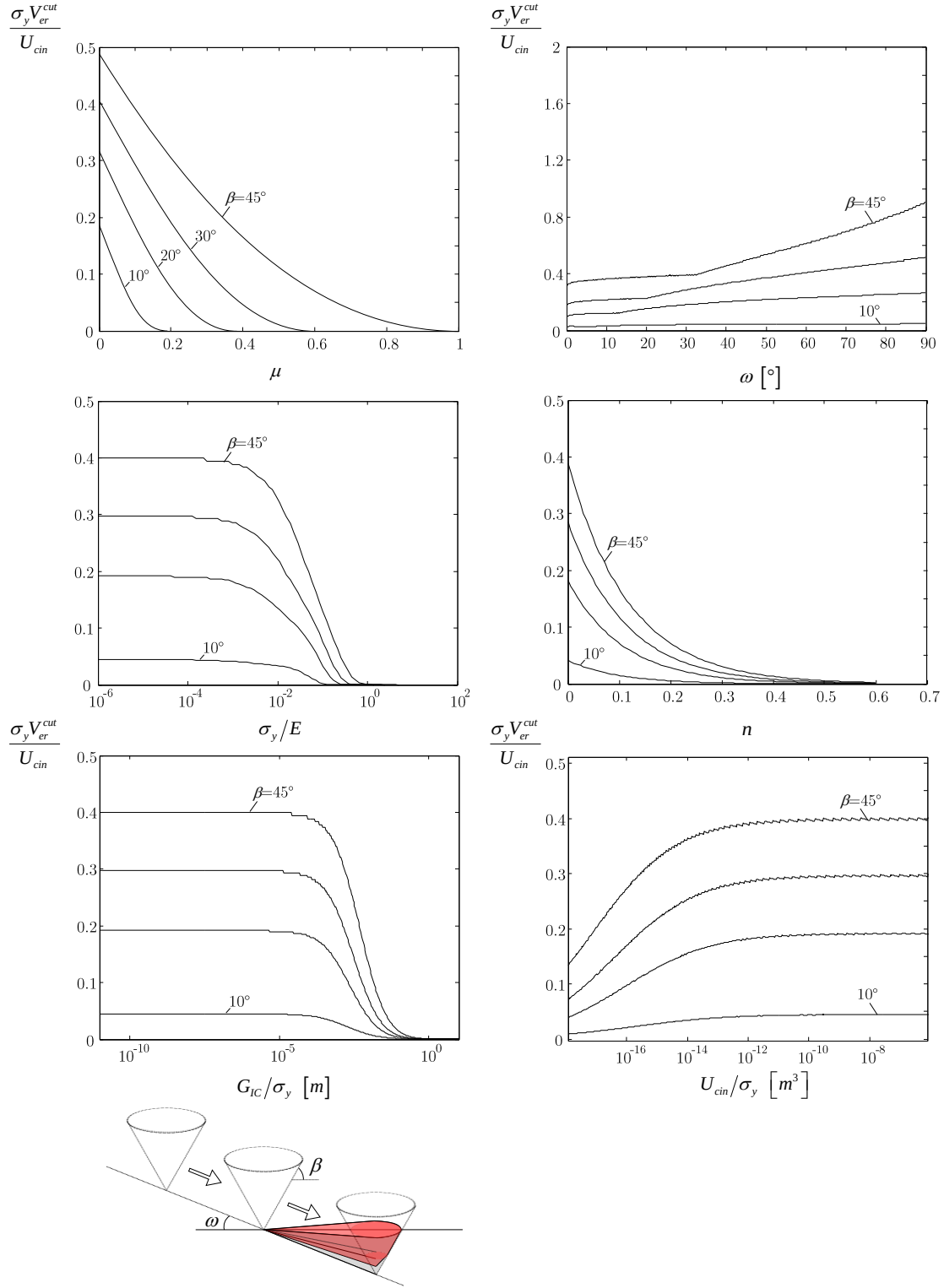


Figure 4.47: Sensitivity analysis in the case of erosion by a cutting mechanism. In each graph, one parameter is changed to observe its effect on the erosion rate while the other parameters are kept constant with the following values: $n = 0$, $\sigma_y/E = 10^{-3}$, $\mu = 0.1$, $U_{cin}/\sigma_y = 10^{-12}$, $G_{IC}/\sigma_y = 10^{-4}$, $\omega = 30^\circ$.

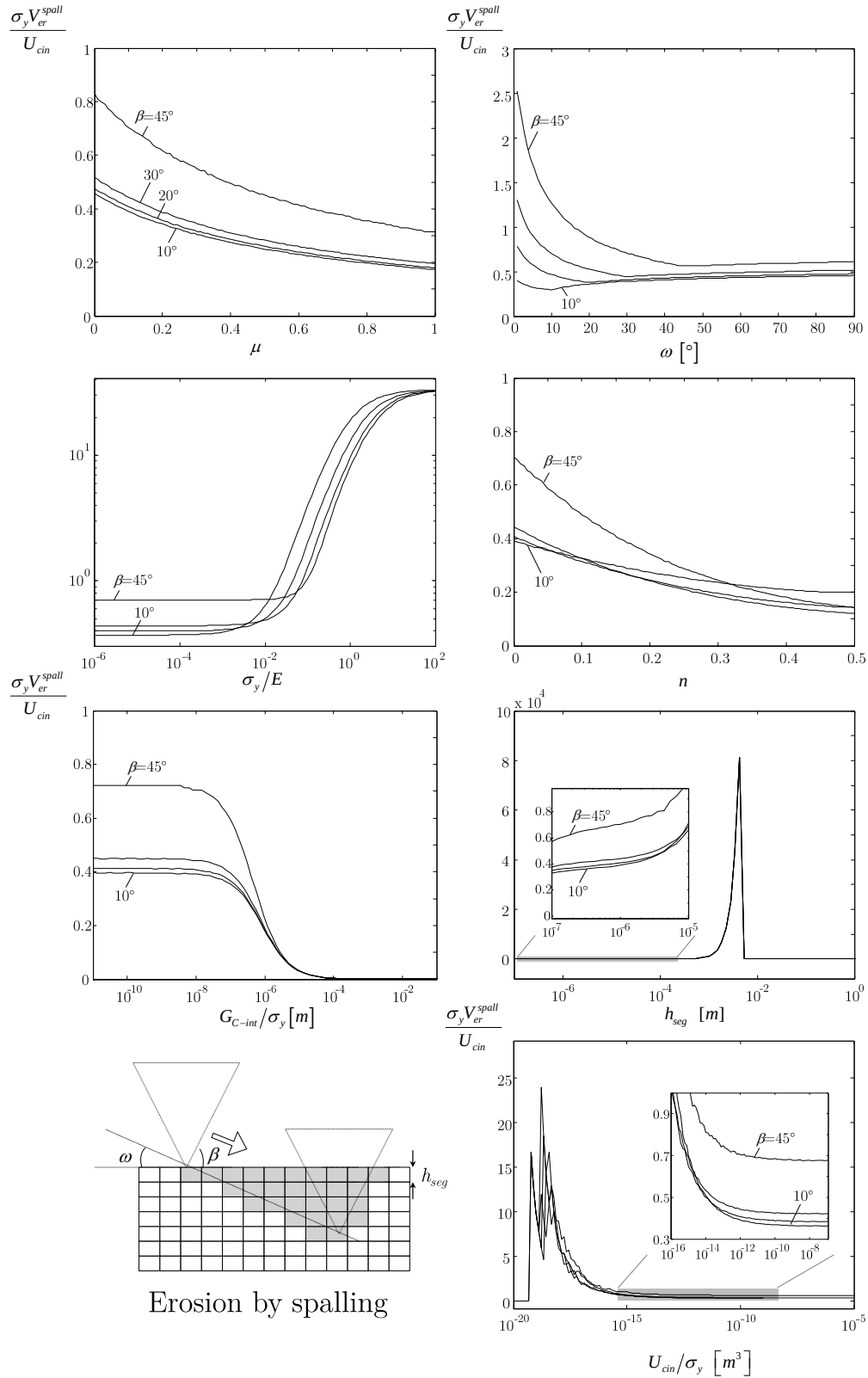


Figure 4.48: Sensitivity analysis in the case of erosion by a spalling mechanism. In each graph, one parameter is changed at a time to observe its effect on the erosion rate while the other parameters are kept constant with the following values: $n = 0$, $\sigma_y/E = 10^{-3}$, $\mu = 0.1$, $U_{cin}/\sigma_y = 10^{-12} [m^3]$, $\omega = 30^\circ$, $h_{seg} = 10^{-6} [m]$ and $G_{c-int}/\sigma_y = 10^{-8} [m]$.

4.6 Summary

In this chapter, an add on model was developed to predict the contact force and the area of contact as well as the elastic recovery in the case of flat and sharp tips. This model is applied to three wear mechanisms: plowing, cutting and spalling. These models predict that the forces of interaction between a blade and an elastoplastic material are affected by the geometry of the blade and by several other parameters. Amongst these parameters some increase the interaction forces: μ , n , σ_y , E , G_{Ic} , G_{c-int} , h and $1/h_{seg}$. The ratio σ_y/E increases the elastic recovery and decreases the interaction forces. Regarding the spalling mechanism, larger segment size leads to a bigger quantity of material being removed.

These three wear models are adapted to predict the resistance to erosion. The models show that erosion resistance increases when the base material is able to dissipate more energy, meaning increasing μ , n , σ_y , G_{Ic} , G_{c-int} and $1/h_{seg}$. The ratio σ_y/E increases the erosion resistance except if the erosion happens via a spalling mechanism. Concerning erosion by cutting, a size effect can occur. The latter increases the erosion resistance, especially in the case of low energy impacting particles. This effect is accentuated with the ratio G_{Ic}/σ_y . In the case of erosion by spalling, the effect of h_{seg} depends on the kinetic energy of the particle. Small segments dissipate more energy per unit volume and therefore offer a better erosion resistance than larger segments. However, if the segments are large enough, a single eroding particle might not have enough energy to detach it. In that case, there is no erosion by spalling.

Chapter 5 Materials and methods

This chapter describes the base materials, processing methods and testing protocols.

5.1 Base materials

The base materials are the elementary ingredients used for processing the model systems on which most of our experimental campaigns were based. For the purpose of keeping the same chemistry and basic constituents of the industrial abradable materials used in the compressor (such as the PAC-905 and the Metco 601-NS), the base materials are made of aluminum alloys and liquid crystal polyester (LCP) or a combination of the two. The idea is essentially to play with the internal architecture of the material.

5.1.1 PAC-905 powder

The Powder Alloy Corporation 905-3 powder (PAC-905) is a mix of aluminum with 12% silicon alloy (AlSi) and LCP polyester. The composition of the mix is 60% in weight of AlSi and 40 percent in weight of polyester (see Figure 5.1). Converted in volume fractions, this mixture is made of 50% in volume of both constituents. The PAC-905 powder is referenced as an equivalent to the Metco® 601-NS powder and the Amdry® 2010 [PAC 2016]. The particle size ranges between 10 and 100 μm . This powder was used to manufacture the industrial reference material.

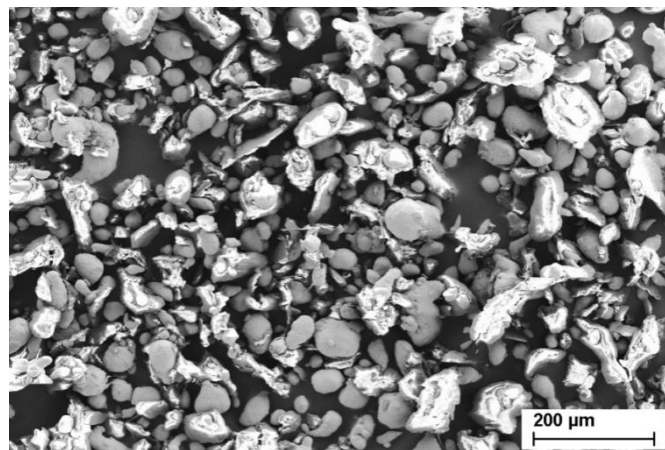


Figure 5.1: SEM micrograph of the PAC-905 powder. The polyester appears in white and the AlSi in grey.

5.1.2 PAC-904 powder

The Powder Alloy Corporation 904 powder (PAC-904) is made of pure LCP polyester. More precisely, a fully aromatic LCP polyester named poly(oxy-1,4-phenylenecarbonyl). The PAC-904 is referenced as an equivalent to the Metco® 600-NS powder [PAC 2016]. The particle size ranges between 44 and 100 μm (see Figure 5.2). This powder is made of the same polymer as the reference PAC-905 powder.

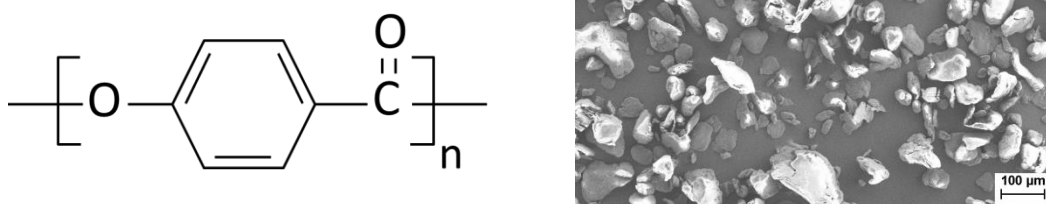


Figure 5.2: Chemical formula of LCP polyester (on the left) and SEM micrograph of the polyester powder (on the right).

5.1.3 AlSi powders

Two batches of AlSi powder were used in this study, a fine one and a coarse one. The fine powder will be referenced as AlSi-44 μm and the coarse powder as AlSi-150 μm .

- *AlSi-44 μm*

The finer powder has been purchased from Alpha Aesar and is a -325 #powder (this terminology means that more than 90% of the material can move through a 325-mesh sieve corresponding to a 44 μm spacing)(see Figure 5.3).

- *AlSi-150 μm*

The coarser powder was purchased from Goodfellow and has a diameter smaller than 150 μm (see Figure 5.3).

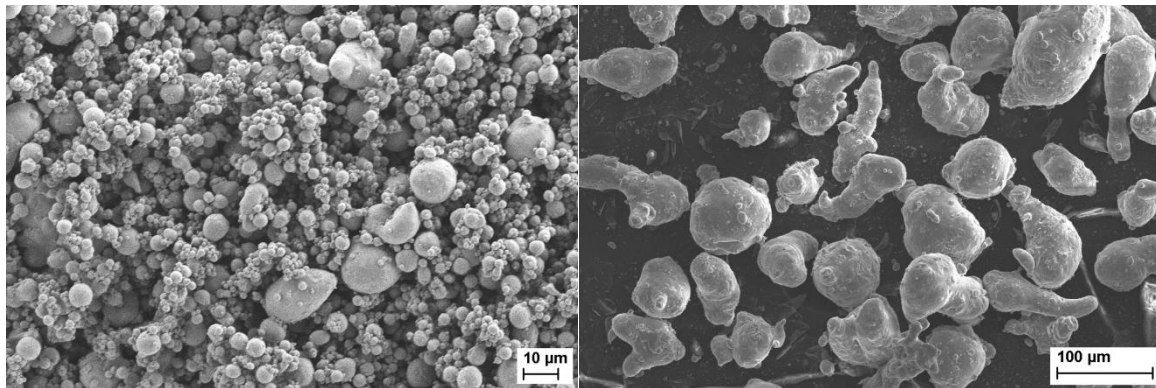


Figure 5.3: SEM micrograph of the AlSi-44 μm powder (on the left) and of the AlSi-150 μm powder (on the right)

5.1.4 AlSi bulk plate

Plates made of bulk AlSi were purchased from Goodfellow. The initial dimensions of the plates are 50 mm x 50 mm x 2 mm (see Figure 5.4).

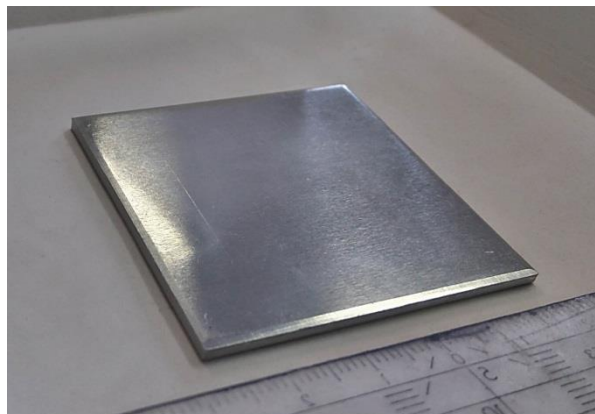


Figure 5.4: Picture of the 2 mm thick AlSi plate

5.1.5 AlSi and AlMgSi foams

Two families of open cell foams are used in this study, involving a dense foam purchased from Exxentis and a lighter foam from ERG aerospace. The microstructures of the foams are described based on three characteristic parameters: the pore size, the bubble size and the relative density. The pore size is the mean diameter of the holes that connect larger cavities between one another. The pore size gives information on the ability of a porous material to act as a filter. The bubble size is the mean diameter of the cavities inside the materials (see Figure 5.5). The relative density is the density

of the foam divided by the density of the bulk material. The relative density is also the volume fraction of solid material inside the foam.

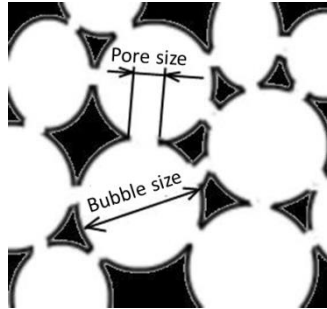


Figure 5.5: Schematic representation of the pore size and the bubble size in an open cell cellular material. Image from [Exxentis 2015].

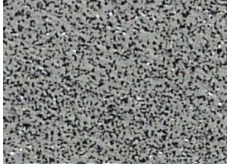
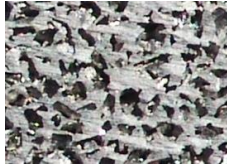
- *Exxentis foams*

The Exxentis® foams are manufactured from an aluminum alloy and NaCl crystals cast together. The NaCl remains solid in the liquid metal. After cooling and solidification of the alloy, the salt is dissolved using water. The salt grain size determines the size of the bubbles in the foam. The aluminum alloy composition is presented in Table 5.1. The size of the raw sample is 0.5 cm x 20 cm x 20 cm. the microstructure and the characteristic size of the three Exxentis foams is summarized in Table 5.2.

Table 5.1: The chemical composition of the aluminum alloy used in the exxentis foams.

Al	Fe	Si	Mn	Ti	Cu	Zr	Mg	Zn
Balance	1.5%	10-13%	<0.5%	<0.1%	<0.6%	<0.1%	<0.1%	<0.3%

Table 5.2: Description and characteristic sizes of the Exxentis foams. The micrographs come from [Exxentis 2015].

Name	Ex-350	Ex-630	Ex-3000
Pictures	 1 cm	 1 cm	 1 cm
Mean pore size [μm]	140	350	630
Mean bubble size [μm]	350	630	3000
Relative density [%]	50-55	50-55	50-55
Pore per inch	180	71	40

• *ERG foams*

The ERG Duocell® foams are produced using polymeric foam as a first mold. The polymeric foam is filled with a heat resistant material. The polymer is removed by a thermal treatment and the remaining negative mold is filled with melted metals. The metal is separated from the mold using pressurized water [Banhart 1998]. The aluminum alloy is a 6101-T6. The aluminum alloy composition is presented in Table 5.3. The microstructure and the characteristic size of the three Exxentis foams is synthesized in Table 5.4. The size of the raw sample is 2.54 cm x 20 cm x 20 cm.

Table 5.3: Chemical composition of the aluminum alloy used in the ERG foams.

Al	Si	Fe	Cu	Mn	Mg	Cr	Zn	Ti	B	Other
Balance	0.3-0.7	<0.5	<0.1	<0.03	0.35-0.8	<0.03	<0.1	-	<0.06	<0.1

Table 5.4: Description and characteristic sizes of the ERG foams.

Name	ERG-1080	ERG-2160	ERG-4320	ERG-8640
Pictures	 1cm	 1cm	 1cm	 1cm
Mean pore size [μm]	635	1270	2540	5080
Mean bubble size [μm]	1080	2160	4320	8640
Relative density [%]	10-12	10-12	10-12	10-12
Nb of pore per inch	40	20	10	5

5.2 Model materials processing

Model materials were designed to gain access to a wide range of microstructures, volume fractions of polyester and AlSi, and interface bonding strength between the constituents. In that spirit, four processing techniques were used, involving (i) thermal spraying, (ii) foams permeation at high temperature, as well as uniaxial pressing (iii) at room temperature and (iv) at high temperature. These techniques are described hereunder.

5.2.1 Thermal spraying

Air plasma spraying is a thermal spraying process in which plasma is created with an electric current produced between a cathode made of tungsten and a water-cooled copper anode. A plasma forming gases is forced between the cathode and the anode, generally made of helium or argon mixed with hydrogen and nitrogen [Davis 2004]. The future coating, in the form of a powder is mixed with the plasma, and transported by the flow of the plasma. The powder particles partially or completely melt in the plasma, then impact the substrate, cool down and solidify (see Figure 5.6).

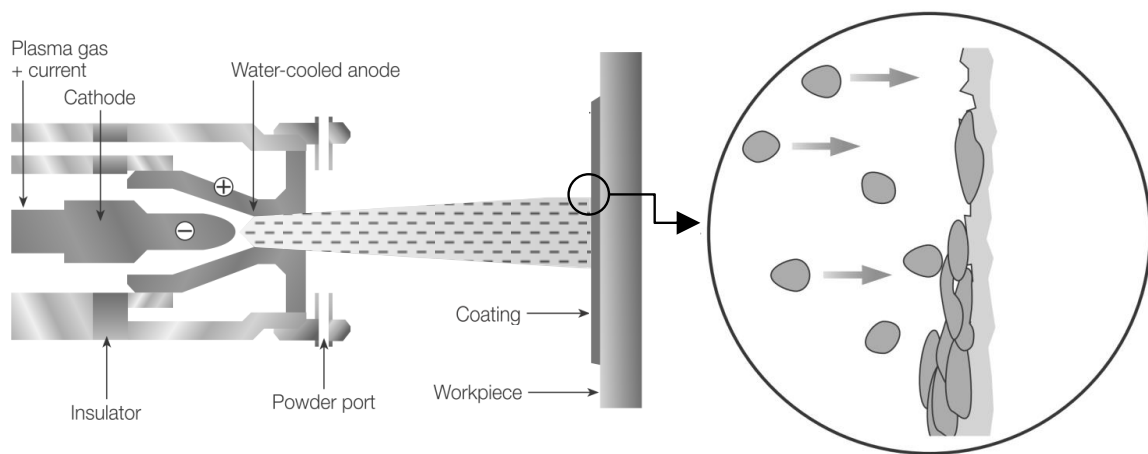


Figure 5.6: Sketch of the working principle of the air plasma spraying. Image from [Oerlikon 2015]

The reference material is produced by air plasma spraying of PAC-905 powder on a 1 mm thick titanium alloy substrate. The deposit is approximately 3 mm thick. After cooling, the industrial material is separated from its substrate by interfacial delamination (see Figure 5.7).

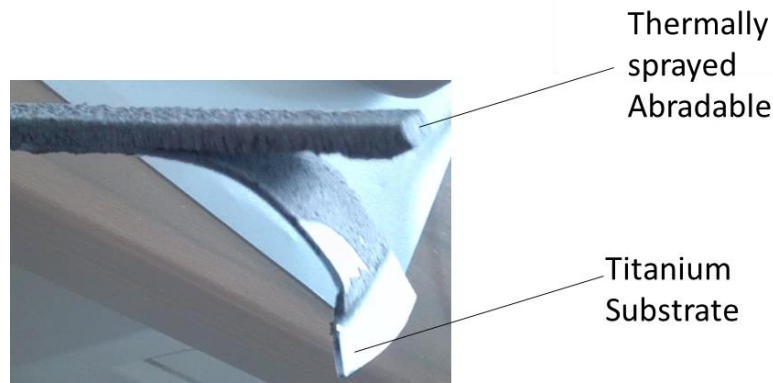


Figure 5.7: The reference abradable material is separated from the substrate by bending the substrate and forcing delamination.

5.2.2 Uniaxial pressing

Model materials were designed by uniaxial pressing of a mixture of AlSi powder and PAC-904 polyester powder. The process is decomposed in two steps consisting of mixing the powders together and then pressing them. Seven different volume fractions of metal and polyester were prepared, see Table 5.5. In this table, a code name is assigned to each model material, with a first symbol “C” or “H” for cold or hot-pressing, respectively and “C” or “F” symbol for coarse or fine AlSi powder as a base material. The numbers that follow the two letters represent the volume fraction of AlSi powder in the mix expressed in percent.

Table 5.5: Code names of the model materials obtained by uniaxial pressing of a mix of AlSi powder and LCP polyester powder. The first letter is either cold (C) or hot (H). The second letter stands for coarse (C) or fine (F) AlSi powder. The number after the 2 letters indicates the volume fraction of AlSi in the mix.

AlSi Volume fraction [%]	0	25	37.5	50	62.5	75	100
Cold-pressed AlSi-44 μm	C0	CF25	CF37	CF50	CF62	CF75	CF100
Cold-pressed AlSi-150 μm		CC25	CC37	CC50	CC62	CC75	CC100
Hot-pressed AlSi-44 μm	H0	HF25	HF37	HF50	HF62	/	/
Hot-pressed AlSi-150 μm		HC25	HC37	HC50	HC62	/	/
AlSi Weight fraction [%]	0	38.6	53	65.3	76	85	100

• Powder mixing

The coarse and the fine AlSi powder (AlSi-150 μm and AlSi-44 μm) were separately mixed with the polyester powder by means of a Framo® magnetic stirrer at 900 rpm

for 6 minutes in a beaker, sealed by an aluminum foil. The powder is then maintained at 80°C in a vacuum drying oven Binder VD 53 ® during 12 hours in order to evaporate any residual water.

- *Cold-pressing*

The cold-pressed materials were produced by uniaxial pressing of the mixed powder at room temperature in a die. A pressure of 400 MPa is maintained during 90 seconds to form a compact in a green state in which the powder particles are only mechanically interlocked.

- *Hot-pressing*

The hot-pressed materials were produced by uniaxial hot-pressing in a stainless steel mold (see Figure 5.8). The mold is covered with a 20 µm aluminum foil to facilitate the separation of the samples from the mold at the end of the process. The mold is preheated in a Carbolite® oven at 500 °C for 3 hours. The hot mold is removed from the oven and filled with the powder mixture. The assembly is then put in a hydraulic press in order to compact the powder mix and to transfer the heat from the mold to the powder in order to melt the polymer phase. The press is not preheated and is originally at room temperature so that the heat comes only from the thermal inertia of the mold. This configuration allows melting the polymer phase while rapidly cooling down the samples to limit the polymer thermal degradation at high temperature. During compaction, the pressure in the press is increased up to 15 MPa followed by three consecutive pressure relaxations to allow for degassing. After degassing, the pressure is maintained at 15 MPa for one hour before removing the samples. The residues of aluminum foils are later removed with a Struers® #80 silicon carbide (SiC) grinding paper.

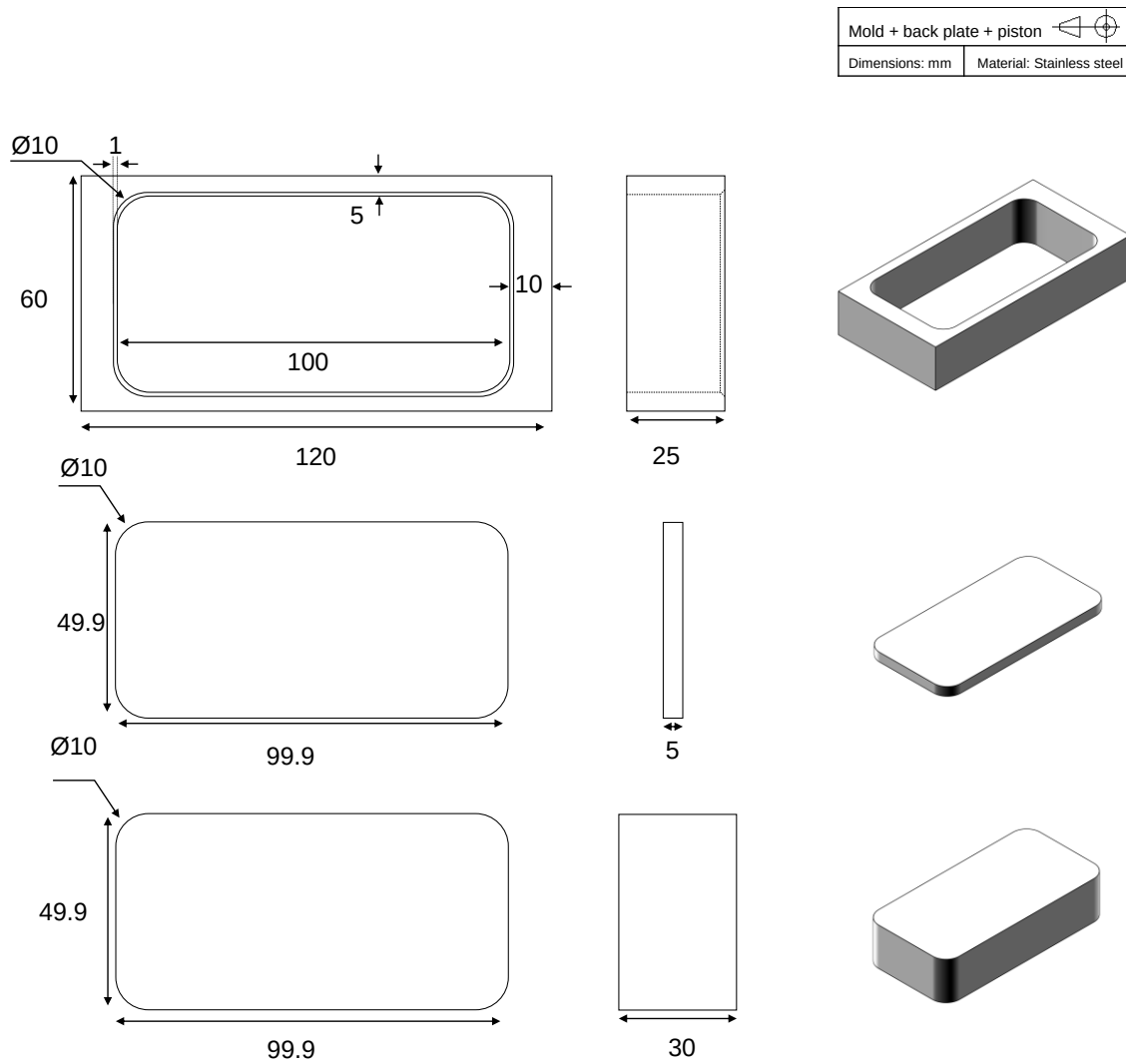


Figure 5.8: Sketch and dimensions of the stainless steel mold, the back plate and the piston.

5.2.3 Foams infiltration

The foam infiltration is performed by melting the polyester and using it to fill the open porosity of the aluminum foams in order to create a hybrid material made of a co-continuous phase of aluminum and polyester. In this work, the infiltration process is performed by uniaxial hot-pressing, in the same stainless steel mold covered with an aluminum foil. The foams described in section 5.1.5 were cut manually in 5 cm x 10 cm rectangle manually with a hacksaw to fit the dimension of the mold. The thickness of the Exxentis foam was kept at 5 mm. The ERG foams thickness was reduced to 8 mm using a water-cooled Struers Accutom 50® cut-off machine. Once cut to dimension, the foams samples are preheated with the mold at 500 °C for 3 hours. The hot mold is removed from the oven and partially filled with the polyester powder. The foam is

placed in the mold on top of the polyester powder then covered with more polyester powder (see Figure 5.9). The next steps are the same as for the hot uniaxial pressing process. The assembly is pressed at room temperature. The pressure is increased up to 15 MPa followed by three consecutive relaxations to allow for degassing and then maintained under 15 MPa for one hour before removing the samples. During the compression step, the ERG foams were densified. The relative density of the metallic phase is increased by a factor 3, reaching a value between 30.4% and 36.6%. This effect was not observed on the Exxentis foams.

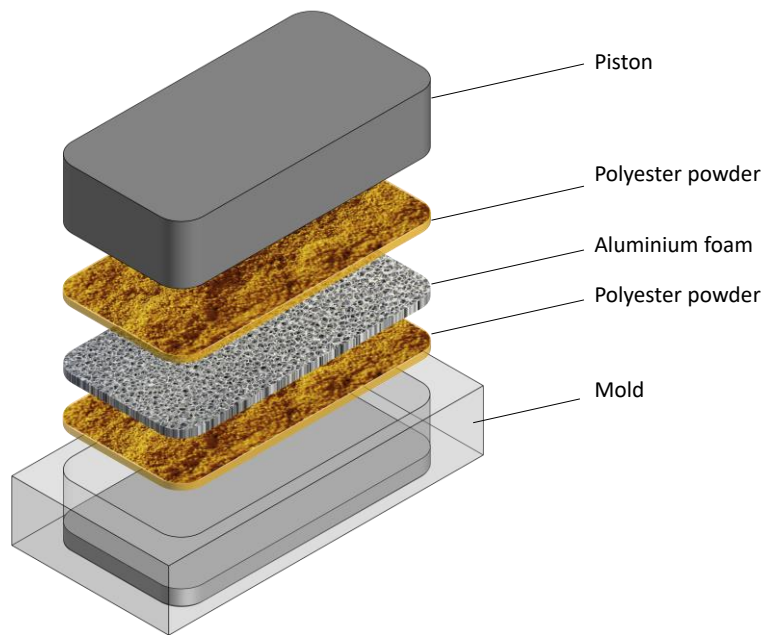


Figure 5.9: Sketch of the foam infiltration steps, the metallic foam is placed between two layers of polyester powder before compression.

5.3 Methods

This section describes all the tests that were performed on the model materials. Each method has a given purpose, either for extracting an intrinsic property, identifying a microstructure or a mechanism after a test or recording a set of data describing the behavior of a given material.

5.3.1 Surface preparation

All the model material samples were subjected to a special surface preparation before testing and observation. Depending on the nature of the test, the complexity of the

surface treatment was adapted. Table 5.6 describes the type of surface treatment for each test. Three levels of surface preparation are used and listed hereunder.

- *Coarse grinding*

Coarse grinding was performed on every sample and consists of smoothing and flattening the surface. The grinding is performed in a water-cooled grinder-polisher Metaserv® 2000 first with a #80 followed by a #320 SiC grinding paper purchase from Struers®.

- *Fine grinding*

The fine grinding starts with a coarse grinding step followed by a finer grinding with #800 and #1200 SiC paper.

- *Polishing*

The polishing is performed after a fine grinding step in a Struers Pedemin DAP-7 polishing machine on polishing cloths with diamond Struers DP-Spray P powder of 9 microns, 3 microns and 1 micron suspended in a lubricant.

Table 5.6: List of the surface treatments depending on the test

Test name	Surface preparation level
Tensile	Coarse grinding
Compression	“ ”
Fracture toughness	“ ”
Density	“ ”
X-ray CT	“ ”
Pin-on-disc	Fine grinding
Abradability	“ ”
Erosion	“ ”
SEM	Polishing
Nanoindentation	“ ”
Nanoscratch	“ ”

5.3.2 Characterization of the microstructures and wear mechanisms

In this subsection, the tools used to characterize the microstructures and the mechanisms are briefly described.

- *Scanning electron microscopy*

A scanning electron microscope (SEM) is used for the examination of microstructure morphology and, with the EDS add-on, chemical composition characterization [Zhou 2006]. In a SEM, an electron beam scans the surface of a material. The interaction between the electrons and the material liberates a large number of electrons and photons. Some of these emitted particles can be captured in dedicated receptors. In this work, three particle receptors were used. Secondary electrons (SE), back scattered electrons (BSE), and X-photons. Depending on the nature and the energy of the captured particles, the microscope can display different images of the surface of the material, mostly in grey scale.

SEs are released from ionized atoms from a few nanometers under the surface of the sample. With SE detector, the image contrast reveals information mainly on the topography of the surface. BSEs are the result of elastic collisions between the electron beam and the specimen atomic nuclei. These electrons go through one or many scattering events before escaping the surface of the sample and hitting the receptor. The amount of BSEs liberated by the sample surface increases with the atomic number. Therefore, using a BSE detector reveals both compositional contrast and topographic contrast. The X-photons are produced by the atoms of the surface of the sample. The electron beam excites the electrons of the surface. The excited electrons can release the excessive energy by liberating an X-ray photon. The energy content of the X-rays is dependent on the atomic number of the surface atoms. The X-rays collected in the detector form a spectrum. The spectrum of X-rays is analyzed with the so-called energy-dispersive X-ray spectroscopy (EDS) technique. The EDS technique can be used in combination with the SE or the BSE to produce localized chemical analysis or colored chemical cartography of a surface.

In this study, the SEM is a Zeiss FEG-SEM Ultra-55 with an accelerating voltage of 15 kV. Prior to observation, a carbon coating was deposited on every material containing polyester to avoid surface charging. The carbon coater is a SCD 030 Blazers Union sputter-coater used with 2.5 mm diameter and 1.55g/m carbon fibers purchase from Agar Scientific.

- *X-ray computed tomography*

X-ray computed tomography (CT) is a tool used for the observation and the analysis of 3D morphologies of an object. The 3D object is analyzed slice by slice. Each slice is analyzed by sending X-ray towards the material and using a X-ray detector on the other side and record images at different angles [Hsieh 2009] (see Figure 5.10). An image reconstruction software is used to recalculate the X-ray absorbance cartography of one section of the object. Repeated over many sections, the 3Dabsorbance cartography of an object can be made.

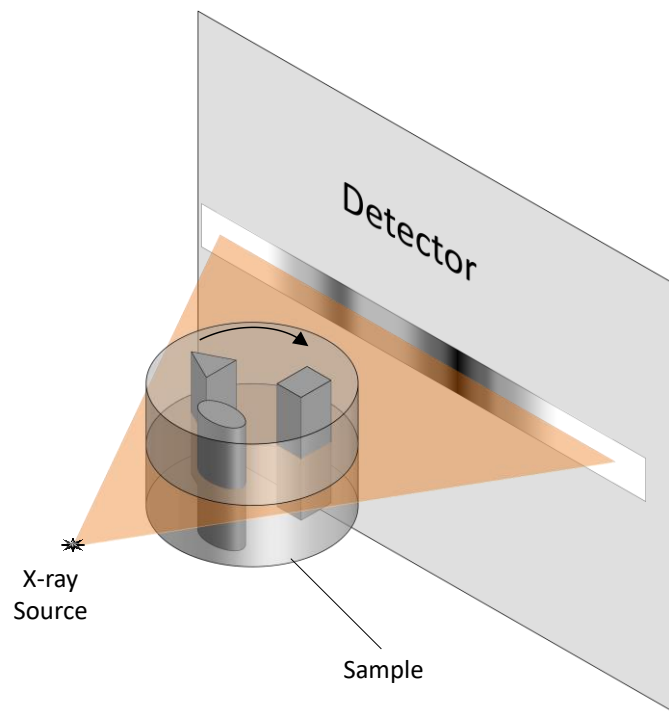


Figure 5.10: Schematic representation of the computed tomography working principle. The sample is incrementally rotated and exposed to X-ray. A reconstruction software recreate a cartography of the X-ray absorbance of one slice of the material. Repeated over many slices, the 3D morphology of the sample can be obtained.

In this study, only the thermally sprayed reference material is analyzed by CT. A sample with a square section of about 1 mm x 1 mm x 10 mm is mounted inside a CT scanner Phoenix NanoTom S with a 180kV / 15 W high performance nanofocus X-ray tube and a 2304 x 2304 pixel Hamamatsu detector. A voltage of 80 kV and 180 A were applied. A tungsten target was used and a 0.3 mm copper filter was installed. An exposure time of 500 ms was applied. Scanning reconstruction was completed using the Phoenix datos|x 2.0 reconstruction software. The resulting images had an isotropic voxel size of 1.25 μm . The output is a pile of 2D images representing a section of the

material in grey levels from which analysis can be used to determine the proportion of phases.

- *Image analysis*

Image analysis represents a set of techniques used for the quantitative study of an image in 2D or in 3D [Ohser 2009]. Among other things, it can be used to estimate the size of particles, their spatial distribution, the area covered by each phase and the mean free path between particles. The analysis can be performed manually or with the help of a computer. The real challenge using computer software for image analysis lies in the detection or the segmentation of the different phase [Russ 2000]. In this study, the ImageJ software [Schneider 2012] was used to analyze the porosity, the polyester domains and the AlSi domains by grey level segmentation on the tomography image. Prior to the process, the images were cropped to remove the background noise. Then, following the technique proposed by [Mandar 2015 b], the images were filtered using a median filter of radius of 3 pixels. This step reduces the noise in each image and eases the grey level segmentation. The grey level segmentation was performed by an empirical thresholding at a grey level comprised between 0 and 79 for the porosity, between 80 and 131 for the polyester and between 132 and 255 for the AlSi.

5.3.3 Density measurements

Density measurements are performed by the Archimedean method to evaluate the level of porosity of the different materials following the EN623-2 standard [CEN 1993]. In this technique, three consecutive weight measurements are performed on the sample. A first one with a dry sample, a second immersed in water and a third in air while still soaked with the liquid. From these three measurements, the density and the porosity are obtained by calculation based on the theoretical density of the material's constituents. The weighting is performed using a Mettler AE200® precision scale with a density determination kit. The immersion liquid is distilled water. The impregnation is performed with a Struers Epovac® vacuum impregnation chamber unit.

5.3.4 Mechanical testing

This subsection describes the tests performed on the model materials in order to evaluate their mechanical properties.

- *Uniaxial tensile testing*

Uniaxial tensile testing is the simplest load configuration used to measure mechanical properties of a material. During this test, a machine pulls on one side of a dog bone sample while the other side is fixed. A pulling force is generated due to the imposed displacement leading to a unidirectional tensile state of stress in the central portion of the sample. The machine records the displacement and the force. From these two values, the stress and the tensile strain can be calculated. The results are used to draw a stress strain curve. From these data, it is possible to determine the Young's modulus (E), the yield stress at 0.2% strain ($\sigma_{y0.2}$), the ultimate true tensile stress (σ_u) and the true uniform strain (ε_u).

In this study, the tensile tests are performed on flat dog-bone shaped specimens made of the model materials. Concerning the cold-pressed materials, the elongation is measured with 5 mm MM® strain gauges. The elongation of the other model materials was measured with convex knife edge extensometers. The cold-pressed materials are pressed in molds directly with the dog bone shape. The other materials are machined into dog bone specimens (see Figure 5.11). The tests were performed using a Zwick Z250 with a load cell of 10 kN in a displacement controlled mode at a constant rate of 1 mm/min. Five loading and unloading cycles are performed when the nominal deformation reaches 0.1, 0.2, 0.5, 1 and 2%. The unloading is stopped when the force reaches 0 N followed by the next reloading (see Figure 5.12).

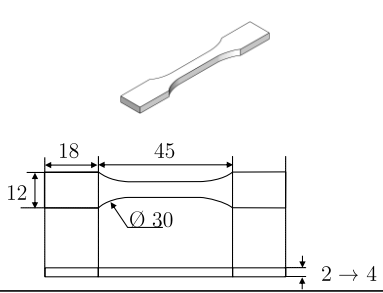
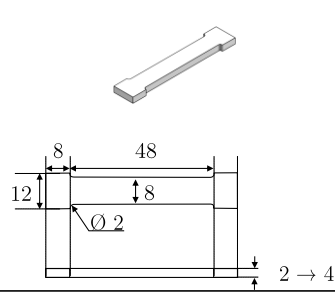
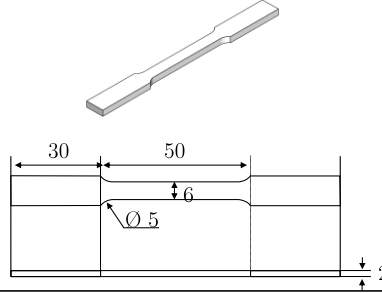
Cold pressed material	Hot pressed materials & infiltrated metallic foams	Thermally sprayed reference material
		
Dimension in mm 		

Figure 5.11: Schematic representation of the tensile specimen geometries.

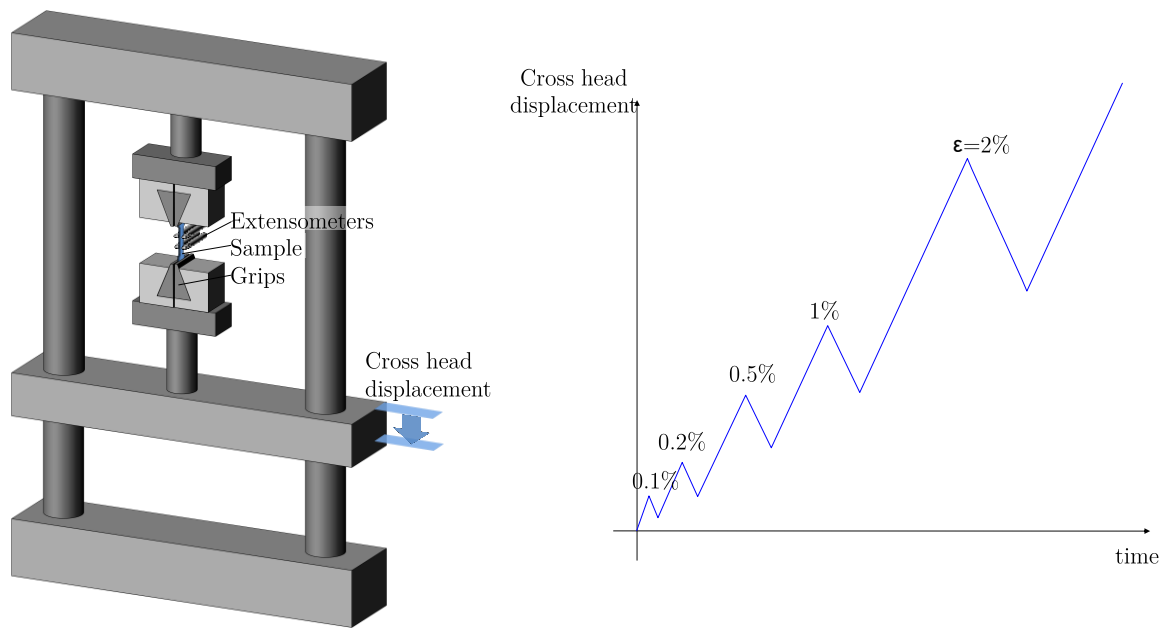


Figure 5.12: On the left, a schematic representation of the tensile testing machine and of the extensometers. On the right, a representation of the evolution of the cross-head displacement during a cyclic tensile test.

- *Uniaxial compression test*

Because of the brittleness of some of the model material, compression tests are also performed in order to identify the mechanical behavior, including strain hardening at higher deformations. Compression tests are performed on rectangular specimens made of the model materials (see Figure 5.13). In order to avoid excessive friction between the platens and the specimen, PTFE films (Teflon) of 70 μm thickness are placed on each side of the specimen in contact with the platens. The deformation in the samples is measured with a linear variational displacement transducer (LVDT). The tests are performed using a Zwick Z250 with a load cell of 10 kN in a displacement controlled mode at a constant rate of 1 mm/min. One loading and unloading cycle is performed when the nominal deformation reaches 1%. The unloading is stopped when the force reaches 0 N followed by the next reloading.

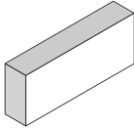
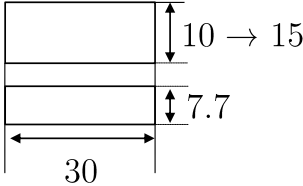
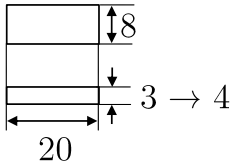
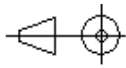
Uniaxial compression specimen geometry of the cold pressed material	Uniaxial compression specimen geometry of the other model materials
 	 
Dimension in mm 	

Figure 5.13: Schematic representation of the uniaxial compression specimen geometries.

- *Fracture toughness*

Fracture toughness characterizes the ability of a material to resist the initiation of a pre-existing crack under a load. Since some tribological wear mechanisms involve the propagation of a crack as the dominant mechanism, it was considered important to measure the fracture toughness of the model materials.

In this study, the plane strain fracture toughness (K_{Ic}) is measured using single edge notched bend (SENB) method. This method consists of a three point bending test on pre-cracked samples. The test standard ASTM D5045-99 is followed [ASTM 1999]. In this test method, the linear elastic fracture mechanics (LEFM) framework is used. The radius of the plastic zone (r_p) ahead of the crack tip is calculated after the test. The test is valid if r_p is smaller than a known fraction of the smallest dimension of the specimen. The cold-pressed materials are pressed in molds with a rectangular shape. A central notch is cut with a water-cooled Struers Accutom 50® cut-off machine. The other materials are machined into rectangular shape with a central notch (see Figure 5.14). A fine slot is carved in the center of the notch by pushing and sliding a stainless steel razor blade from American Safety Razor Company. The tests are performed using a Zwick Z250 with a load cell of 10 kN in a displacement controlled mode at a constant rate of 1 mm/min.

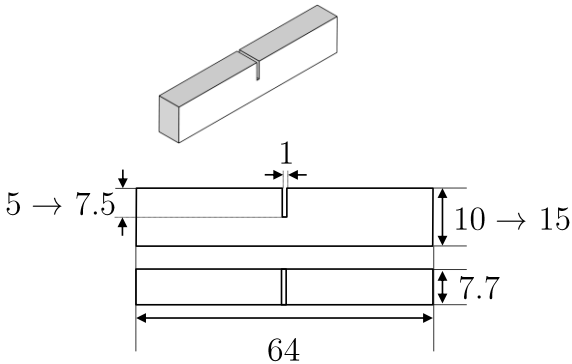
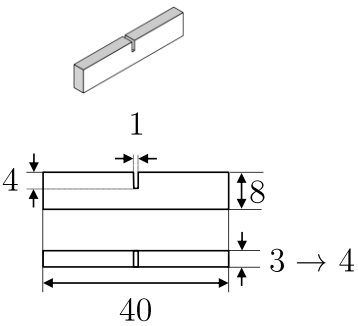
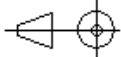
SENB specimen geometry of the cold pressed material	SENB specimen geometry of the other model materials
	
Dimension in mm 	

Figure 5.14: Schematic representation of the SENB specimen geometry.

- *Nanoindentation*

Indentation constitutes a family of techniques used for probing the mechanical properties of materials [Oliver 1992]. The methods consist of penetrating the material with a tip made of a hard material (diamonds, tungsten carbide, martensitic steel...). The geometry of the tip and the mechanical properties of the tip material are known. In nanoindentation, the length scale of the penetration is on the order of a few nanometers up to several micrometers. The vertical force and the penetration depth of the tip are measured. From these measurements, the indentation curves can be used to calculate several mechanical properties. Hardness, Young's modulus, strain hardening exponent, viscoelastic properties and, for brittle materials, the fracture toughness are all examples of material properties that can be estimated, more or less precisely, by instrumented indentation [Fischer-Cripps 2011].

In this study, an Agilent G200 nanoindenter is used, with a standard XP indentation head on which a Berkovich tip is mounted (see Figure 5.15). The indentation proceeded in a progressive increase of the load from zero to 200 mN with 5 unloading cycles at 12.5, 25, 50, 100 and 200 mN. Before each unloading, the force is maintained during 10 seconds. During the unloading, the force goes down to 99% of the initial load

before going up again. Each force plateau is reached in 15 seconds (see Figure 5.16). As a consequence, the rate of loading and unloading is different at each of the five cycles and is respectively 0.83, 1.67, 3.33, 6.67 and 13.33 mN/s.

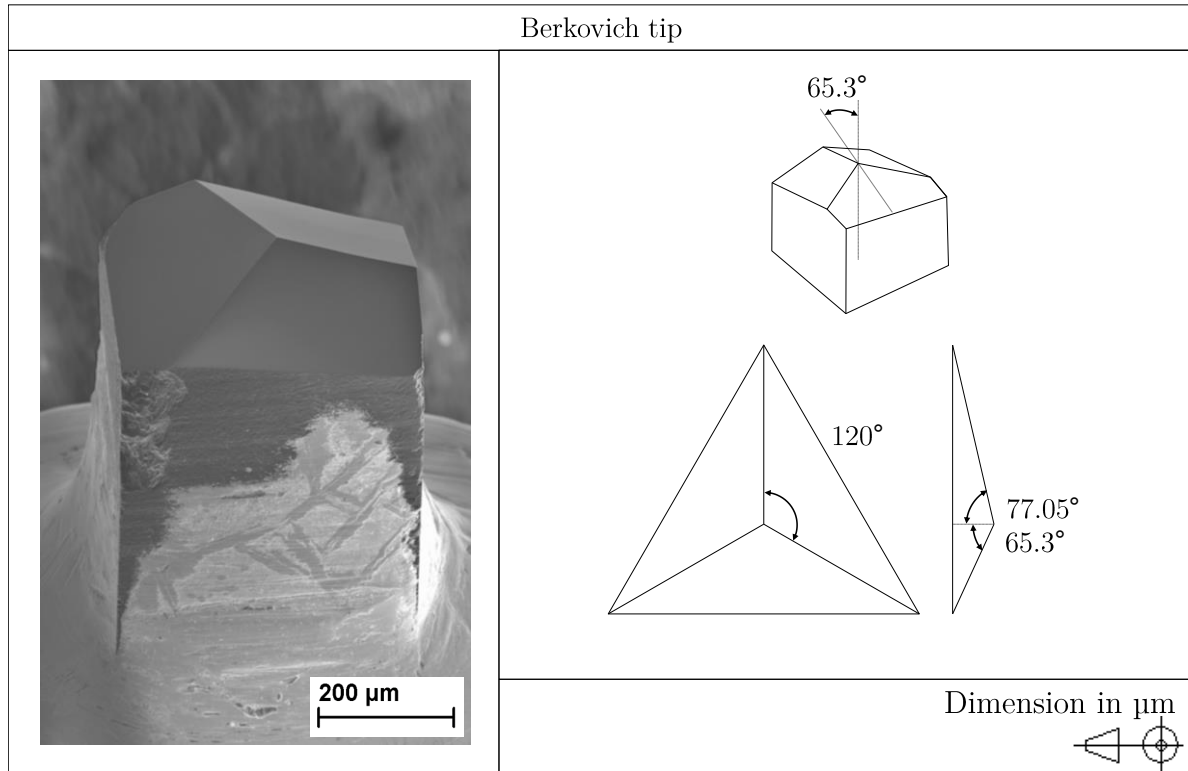


Figure 5.15: On the left, a SEM micrograph of the diamond Berkovich. On the right, a schematic representation of the tip with the angles of interest.

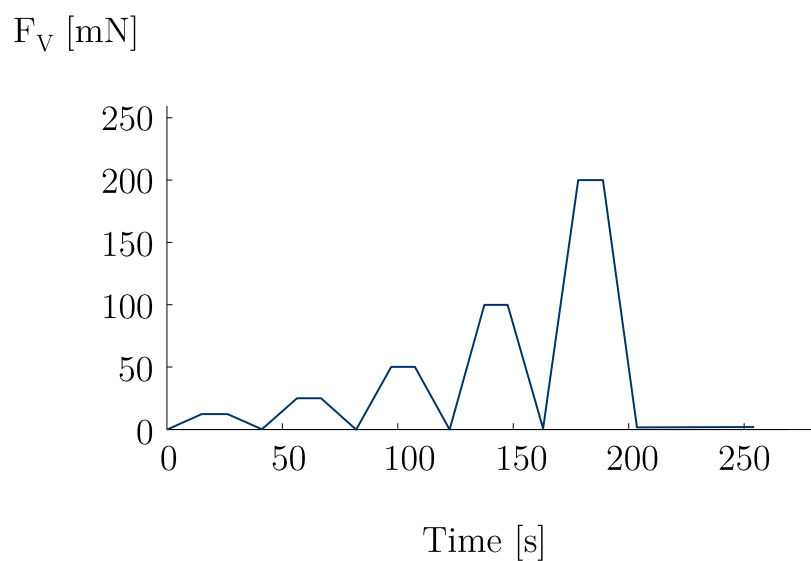


Figure 5.16: Evolution of the vertical force with the time during the nanoindentation tests.

5.3.5 Tribological testing

In order to evaluate the resistance of the model materials to wear, different techniques and equipment are used. These tribological tests are described in the following subsections.

- *Pin-on-disc tests*

A pin-on-disc test is a wear testing method during which a pin is displaced vertically against a horizontal rotating flat disc. In the classical configuration, the pin is pressed against the surface by a dead weight mass. The pin is often harder than the disc and generates a circular wear path on the disc [Bhushan 2001]. This kind of test is used to measure the resistance of a material to wear by sliding. In this study, the pin on disc test is performed with a Bruker UMT3 tribometer. This system works with a vertical displacement system regulated by a vertical load sensor. The disc is placed on a rotary drive S35ME. The vertical and the lateral forces are measured with a bidirectional load sensor DHF-50. The pin holder is attached to a double suspension blade in order to reduce the stiffness of the system and smooth the regulation of the vertical position during the test (see Figure 5.17).

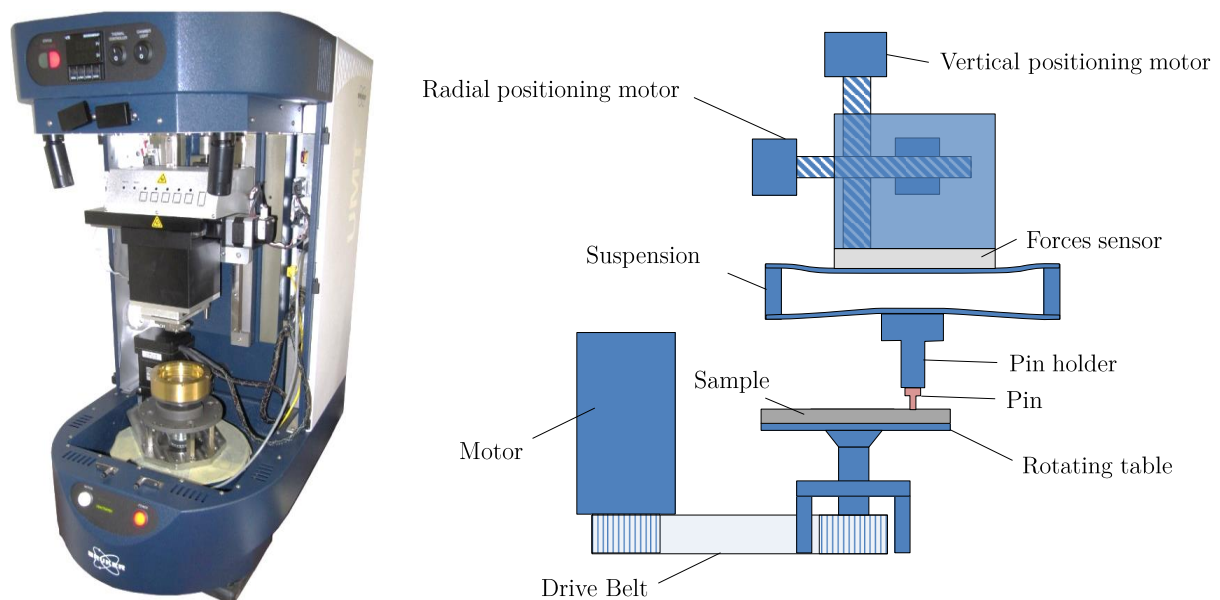


Figure 5.17: On the left, a picture of the UMT 3 bruker tribometer. On the right, a schematic representation of the working principle of the tribometer.

The pin is a 2 mm width cutting tool with an attack angle of 0° and an angle of retreat of 30° made of a titanium alloy with 6% aluminum and 4% vanadium (TiA6V) shaped by electrical discharge machining (see Figure 5.18).

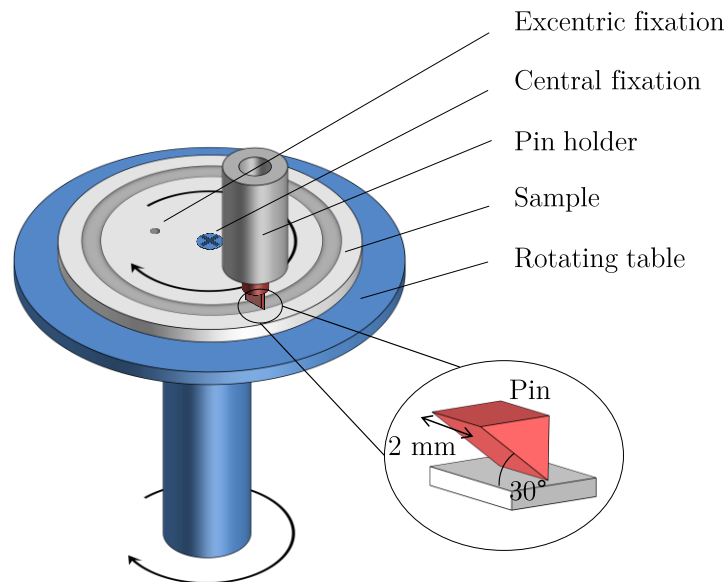


Figure 5.18: Sketch of the interaction between the TiA6V pin and the sample.

Regarding the disc samples, the cold-pressed materials are pressed in molds directly with the flat disc shape. The other materials are cut into 5 cm x 5 cm flat rectangle using a water-cooled Struers Accutom 50® cut-off machine. All the materials are drilled in the center and two centimeters away from the center to allow the fixation on the tribometer (see Figure 5.19).

Pin geometry	Disk of the cold pressed materials	Disk of the other materials
Dimension in mm 		

Figure 5.19: The geometry and the dimensions of the pin and of the discs.

Concerning the test protocol, the tribometer is used in vertical force control mode. In this mode, the user chooses a vertical force and a range of tolerance around this force. During the rotation of the disc, the system adjusts the vertical position of the pin to maintain the force inside the tolerance force level. This regulation works with three parametrizable vertical speeds. The pretouch speed is active below a touch force. Above that force, the system switches to a lower speed called “the touch speed”. Finally, the tracking speed starts once the vertical force reaches the force set by the user and is used to maintain the force within the tolerance bounds (see Figure 5.20). In this study, the set of force regulation parameters is described in Table 5.7.

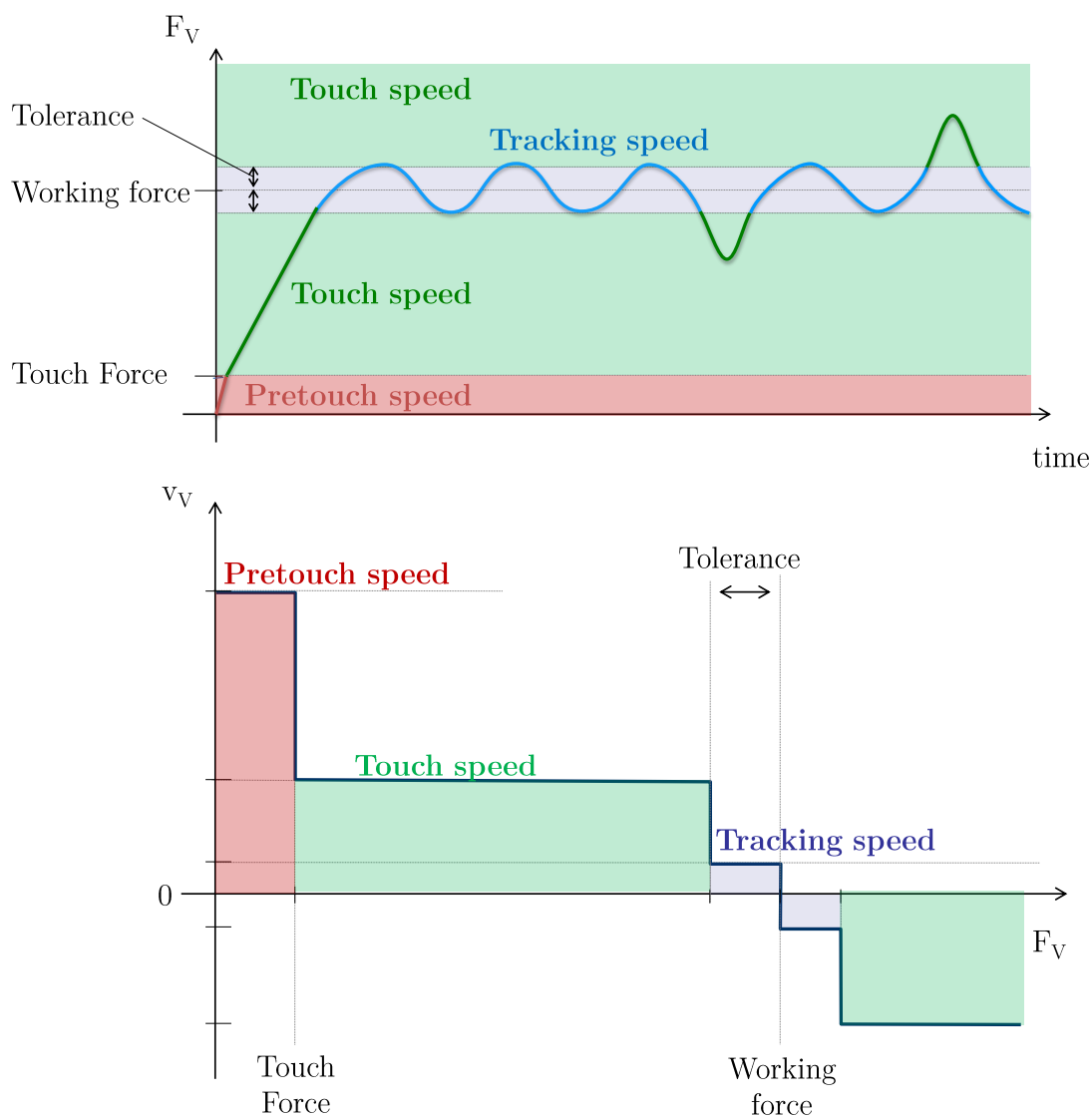


Figure 5.20: The Force control mode use three levels of vertical speed to reach and to maintain the selected force. The pretouch speed, the touch speed and the tracking speed.

Table 5.7: List of the force regulation parameters used on the tribometer.

Touch force	0.2 N
Tolerance	0.5 N
Pretouch speed	0.1 mm/s
Touch speed	0.06 mm/s
Tracking speed	0.02 mm/s

The complete wear test takes place in four stages. The contact initiation is followed by a first scan of the surface at a force of 2 N. The third step is the wear step followed by a post scan of the surface at a 2 N force. The two scans are used to measure the height of the surface of the wear track before and after the test. The wear volume is calculated with these two scans. The two scans are performed in the counterclockwise direction while the wear step is performed clockwise (see Figure 5.21).

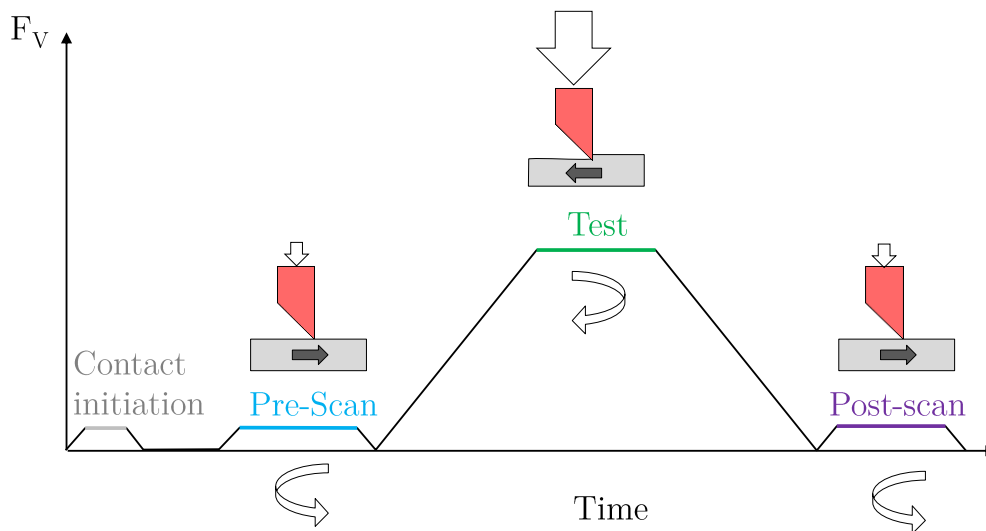


Figure 5.21: Representation of the four characteristic steps in the constant force mode. During the pre-scan and the post scan, the force is maintained at a low level and the rotation is imposed in the direction opposite to the one during the test.

During the test, the system records the angular position of the disc, the vertical and the lateral position of the pin, as well as the horizontal and vertical forces. The data are post-treated using Matlab. Vertical displacement data are corrected to take into account the elastic deformations of the assembly between the motors and the pin (see Figure 5.22). The machine compliance is measured with a simple indentation test on a 4 mm thick hard plate made of alumina. The alumina plate is considered infinitely

rigid compared to the assembly. Because the elastic deformations of the machine are nonlinear, a third order polynomial regression is used to fit the relationship between the vertical force and the elastic deformation of the assembly. This elastic deformation of the machine is subtracted to the vertical displacement data of the pre-scan, the post-scan and the wear step.

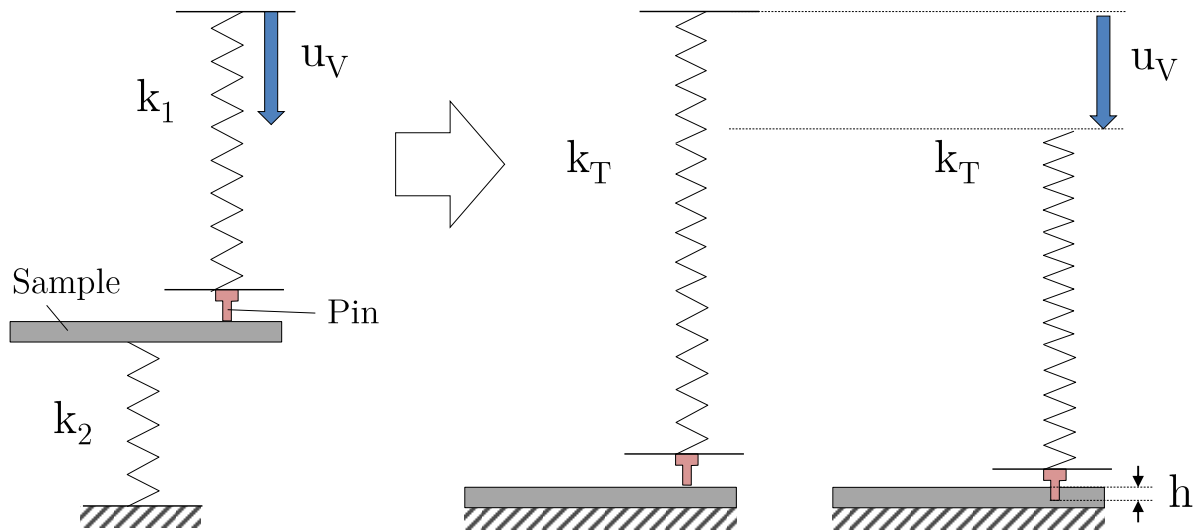


Figure 5.22: Schematic representation of the vertical compliance of the tribometer. The assembly above and under the sample are simplified by springs. The total vertical displacement of the system (u_v) can be decomposed into the depth of penetration (h) and the elastic deformation of an equivalent spring with a stiffness k_T .

The two scan data are subjected to a second correction. During the scans, the surface is vertically deformed by the pin. Since a low force is used, the deformation is considered purely elastic. The elastic deformation of the sample is also subtracted to the vertical displacement via a measurement of the sample compliance. The sample compliance is calculated by indentation during the contact initiation step. A first order polynomial regression is used to fit the relationship between the vertical force and the elastic deformation of the sample. However, due to the intrinsic drift of the force sensor, the noise in the force signal and the 0.2 N resolution threshold of the force sensor, a special data treatment is necessary to identify the real contact initiation point between the pin and the sample surface during the contact initiation step.

The contact detection algorithm uses one parameter, the noise threshold, which is defined by the user. Before the contact, the signal fluctuates around a mean value. After the contact, the signal starts to increase with the vertical displacement. All the

force data points between the lowest value and the lowest value plus the noise threshold are used to calculate a mean background force. A linear regression is used on the rest of the data to evaluate the sample compliance. The contact point is approximated by the intersection between the linear regression line and the mean background force (see Figure 5.23). In this case, the noise threshold is fixed at 1 N.

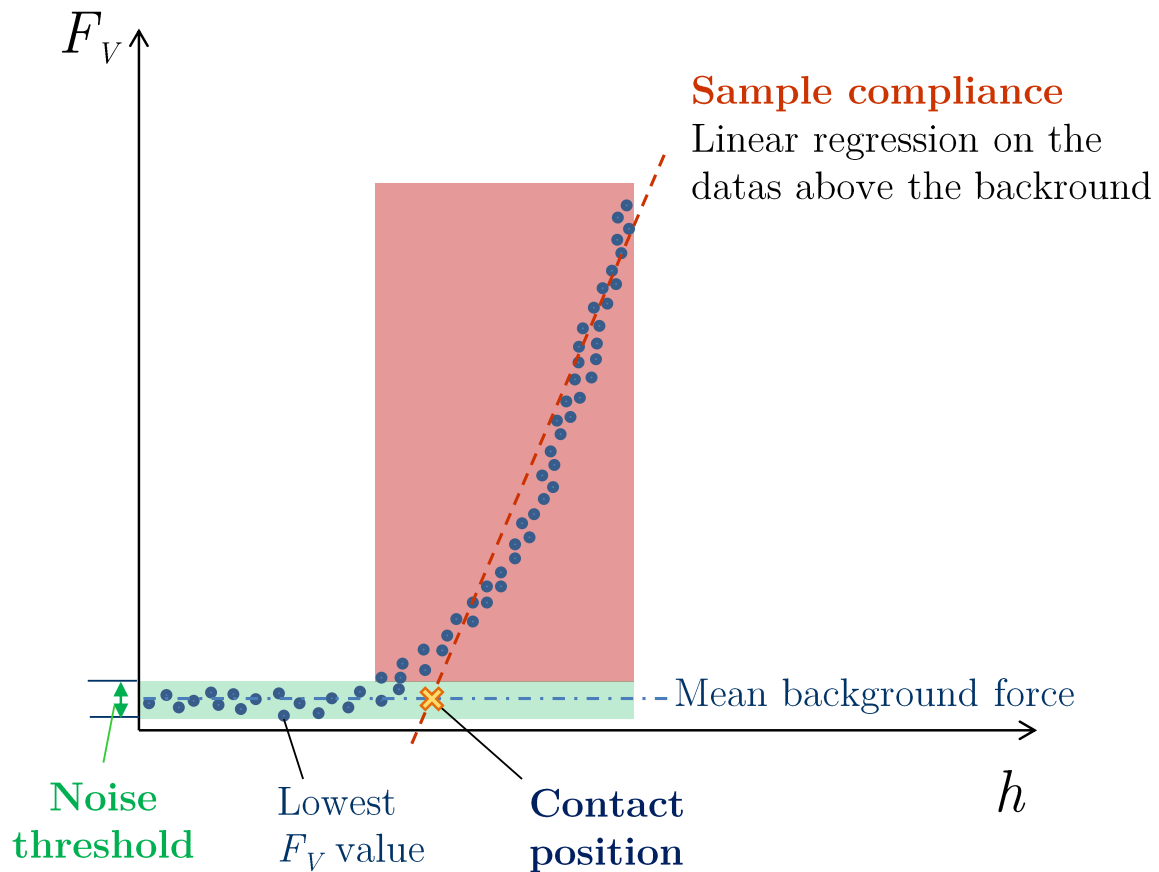


Figure 5.23: The sample compliance and the contact detection were approximated via a linear regression and an intersection with an average background force.

At the end of a test, all these corrections allow visualizing the results in terms of effective penetration of the tools in the surface and in terms of the quantity of material removed (see Figure 5.24).

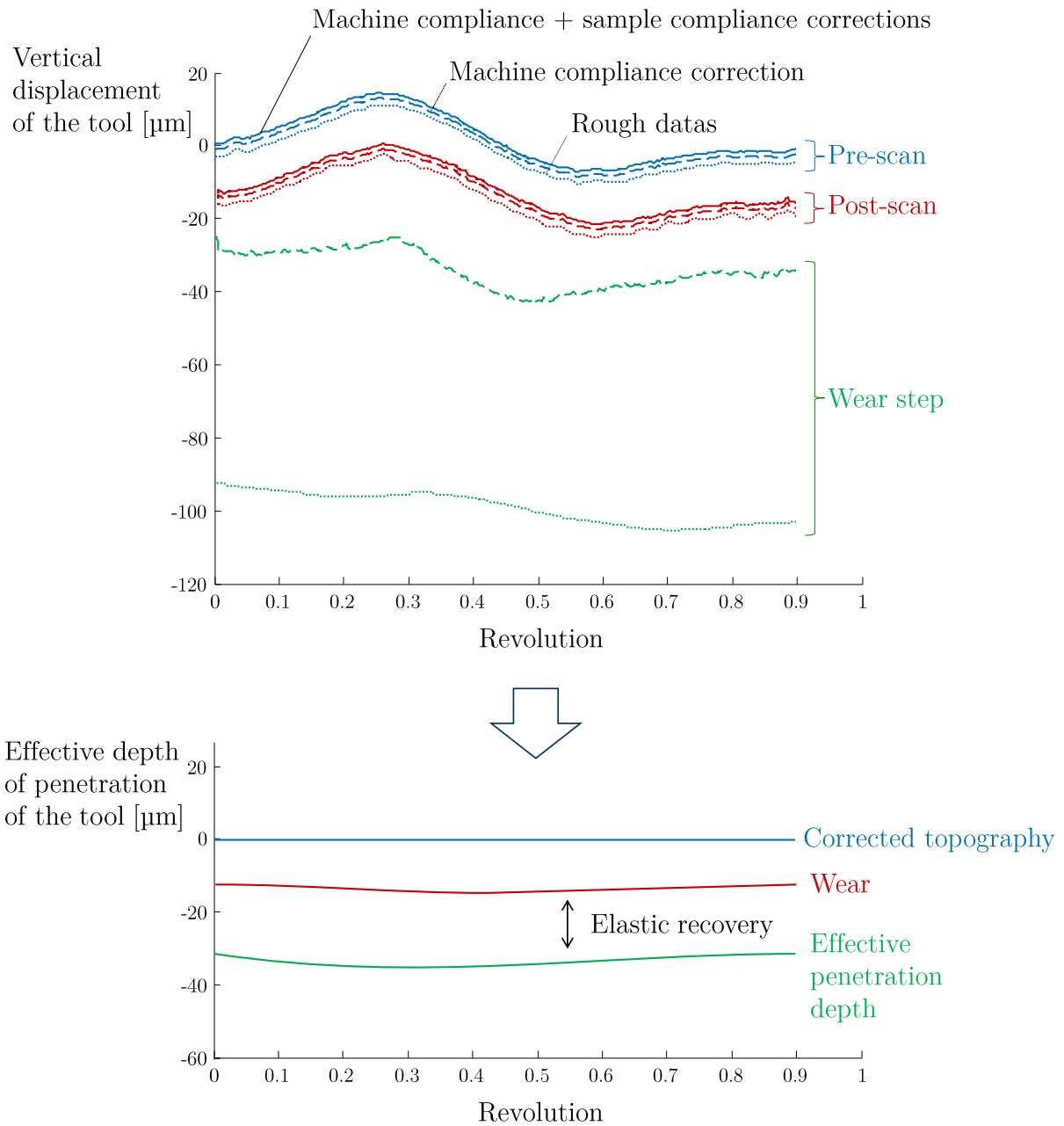


Figure 5.24: On the top, an example of the data recorded by the tribometer and the effect of the correction of machine compliance and sample compliance corrections. On the bottom, the same example with a correction of the initial topography gives the effective depth of penetration of the tool in the surface.

In addition to the wear test, the friction coefficient (μ) of each sample is measured using a 6 mm diameter TiA6V sphere in a controlled force mode with a 2 N vertical force during 10 turns. The first rotation is not taken into account in the calculation of μ as it might involve some elastoplastic energy dissipation due to the formation of a permanent wear track on the samples surfaces. Once the groove track is formed, the

interaction between the TiA6V sphere and the surface of the samples is considered only elastic.

- *Abradability tests*

The abrasability test is a method designed to reproduce the contact between a blade and an abradable material with a very high rubbing speed. In this study, the abrasability test rig is composed of a rotating engine followed by a gearbox connected to a disc. The disc contains a rectangular protruding TiA6V blade. At full speed, the rotating speed of the disc goes up to 20.000 rotations per minute (rpm). Close to the disc, a flat abradable sample is attached to a sample holder. The sample holder is approached with two stepper motors and forced against the disc at a constant incursion rate to a desired depth (see Figure 5.25). The disc contains only one blade and the radius of the disc plus the blade is 9 cm (see Figure 5.26).

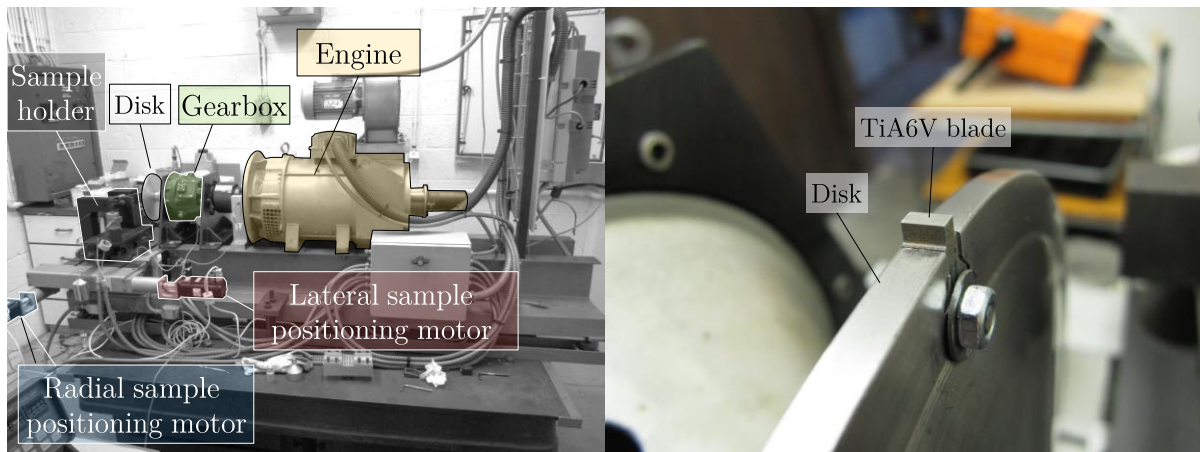


Figure 5.25: On the left, a picture of the abrasability test rig with the principal components. On the right, a picture of the disc with the rectangular blade.

In this configuration, the blade touches the abradable many times before reaching the final depth. At each pass, the blade displaces/removes a given quantity of material known as the bite per blade [Schmidt 1997]. The bite per blade can be related to the number of blades (N_b), the blade tip velocity (v_H), the radius of the disc (R_d) and the incursion rate (v_v) through

$$B_{pb} = \frac{2\pi R_d v_v}{N_b v_H}. \quad \text{Eq. 5.1}$$

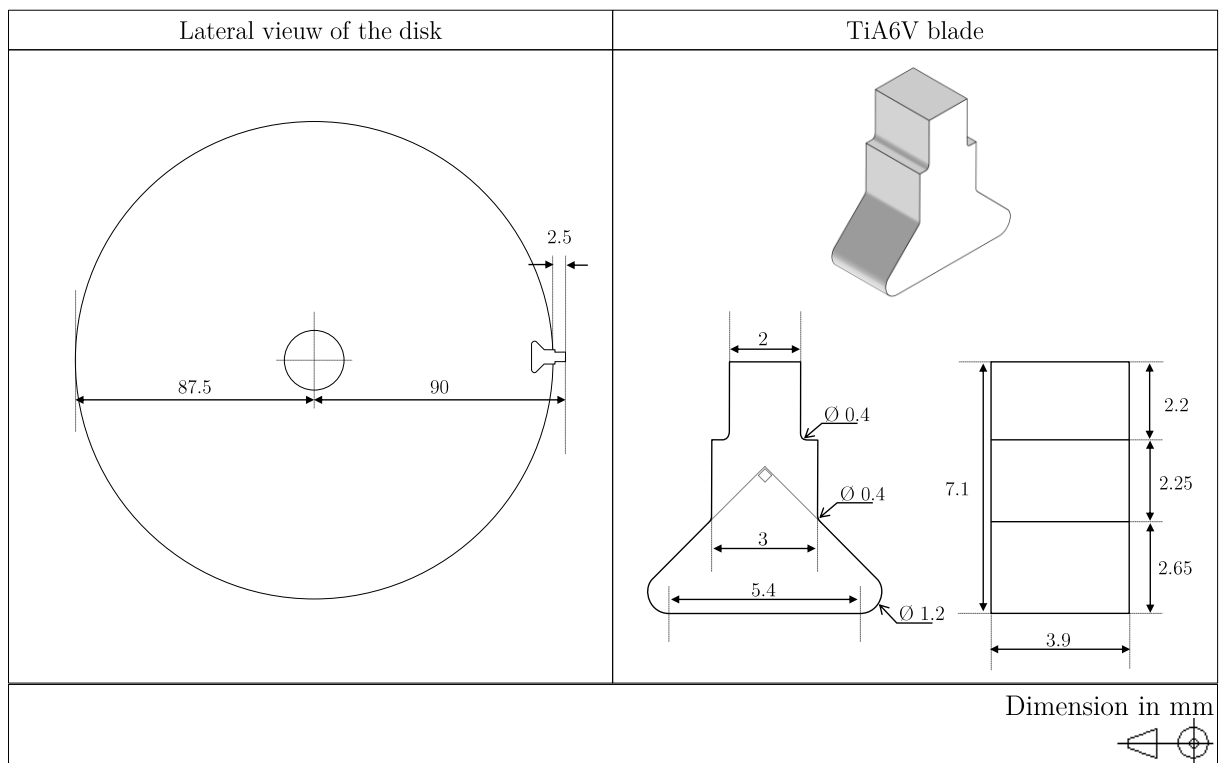


Figure 5.26: Schematic representation of the disc and the blade.

The sample holder is equipped with two HBM Q11 coaxial capacitor load cells. One of the load cells is directed parallel and one perpendicular to the surface of the sample (see Figure 5.27). However, in the present configuration, the force sensors are affected by the vibrations of the machine. In consequence, it was not possible to extract any useful information from the force signals. Even without a force signal, the abrasability test rig remains a useful tool to characterize the wear mechanisms at high speed. Four model materials were tested. The PAC-905, the CF50, the H0 and the ERG-4320 foam filled with polyester. A new blade is used for each material. The height of the blade is measured before and after the test to calculate the wear of the blade. Three operating conditions are tested on those four materials (see Table 5.8). The final incursion depth is 500 μm . After the test, the samples are observed macroscopically and in a SEM to identify the wear mechanisms.

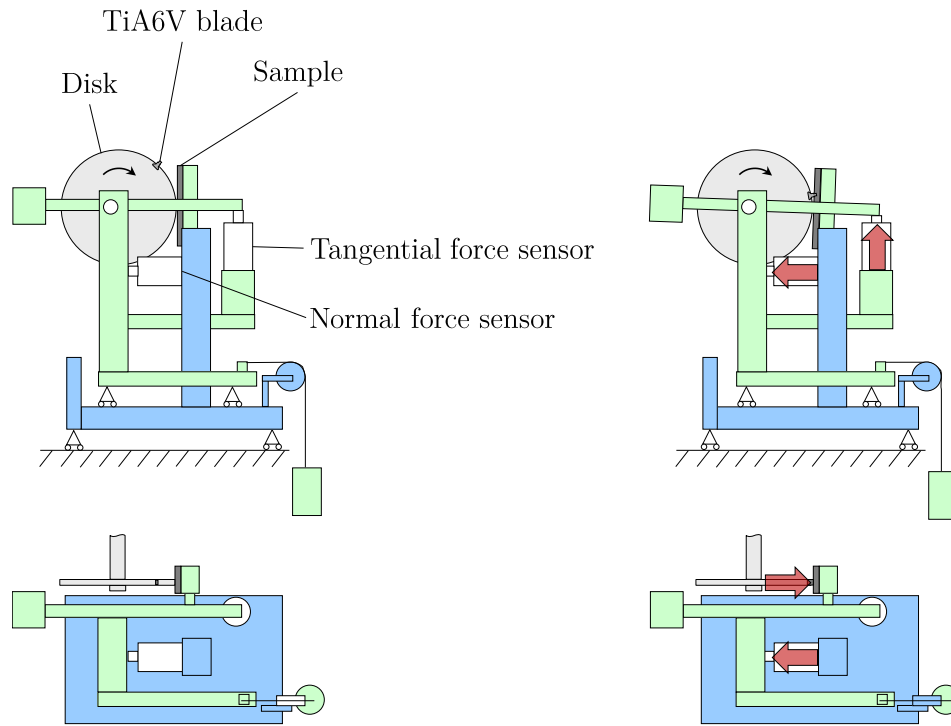


Figure 5.27: The sample holder is designed to record the normal and the tangential force during the tests with two force sensors. The blue part is displaced by the stepper motors. The green part has some degrees of freedom to activate the force sensor upon contact.

Table 5.8: Description of the operating conditions used with the abrasability test rig.

Operating condition number	Blade tip velocity [m/s]	Incursion rate [$\mu\text{m/s}$]	Bite per blade [μm]
1	200	10	0.028
2	200	50	0.142
3	23	50	1.230

• Erosion tests

Erosion resistance of an abradable material is an essential characteristic to ensure a sufficient life time. Before entering in service, every new abradable material candidate is tested and validated on erosion test rig. In this study, the erosion resistance of the model materials are tested on an erosion test rig in Safran Aero Booster © facilities. The machine is an air sand blaster gun combined with a continuous abrasive powder delivery system. An air compressor supplies the airgun with 3.5 bars compressed air. Inside the gun, portion of the main air pipe has a smaller section that creates locally a depression zone due to a Ventury effect. The powder inlet tube is connected to this depression zone. The powder is initially stored in a reservoir with a funnel in the

bottom. The reservoir stands above a rotating wheel that displaces the powder laterally to a suction tube. The aspiration is driven by the depression created by the Ventury effect in the center of the gun. Inside the gun, the abrasive sand is mixed with the compressed air and projected on the sample. The sample holder is attached to an adjustable pivot. The pivot enables the user to choose the angle between the impacting particles and the surface of the sample (see Figure 5.28).

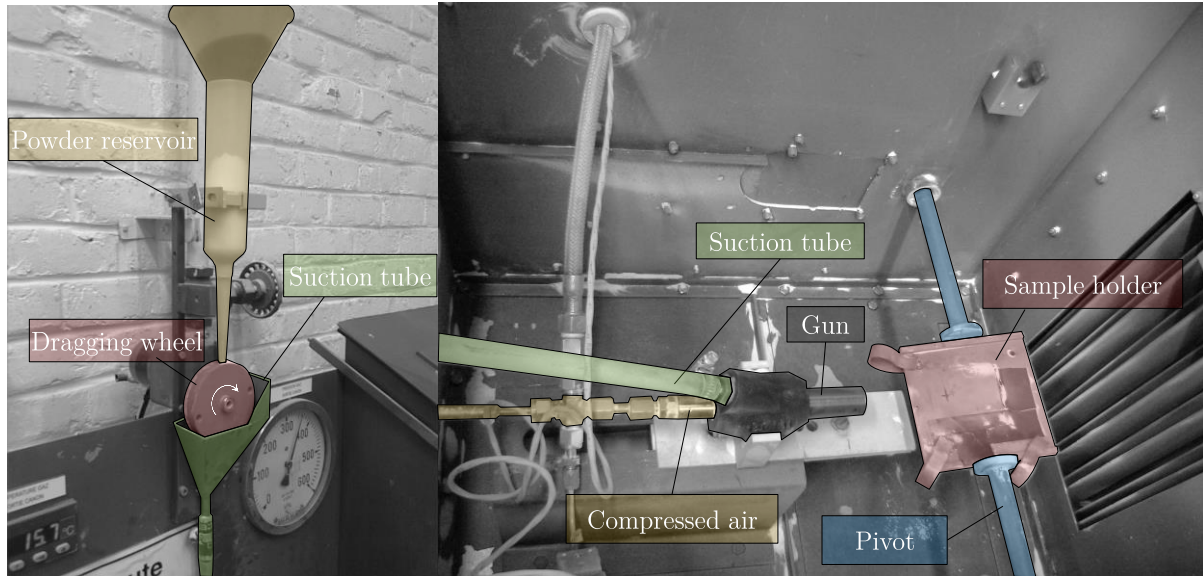


Figure 5.28: On the left, a picture of the powder delivery system. On the right, a picture of the sand-blasting gun and the sample holder with the pivot.

An impact angle of 20° is chosen. 400 grams of sharp edges #230 alumina abrasive powder is loaded in the powder reservoir (see Figure 5.29). The rotation speed of the dragging wheel is chosen to empty the reservoir in 10 to 11 minutes per test. A mask made of a 0.6 mm steel foil with a 13 mm diameter hole in the middle is attached on the surface of every sample to concentrate the erosion area to a small fraction of the surface of the samples. The mask is changed every four tests to avoid an excessive opening of the central hole due to the erosion of the mask. The tests are repeated twice on each material. The center of the sample is located 10 cm away from the gun. Before and after testing, each specimen is weighted and the thickness is measured with an Oditest caliper gauge (see Figure 5.30).

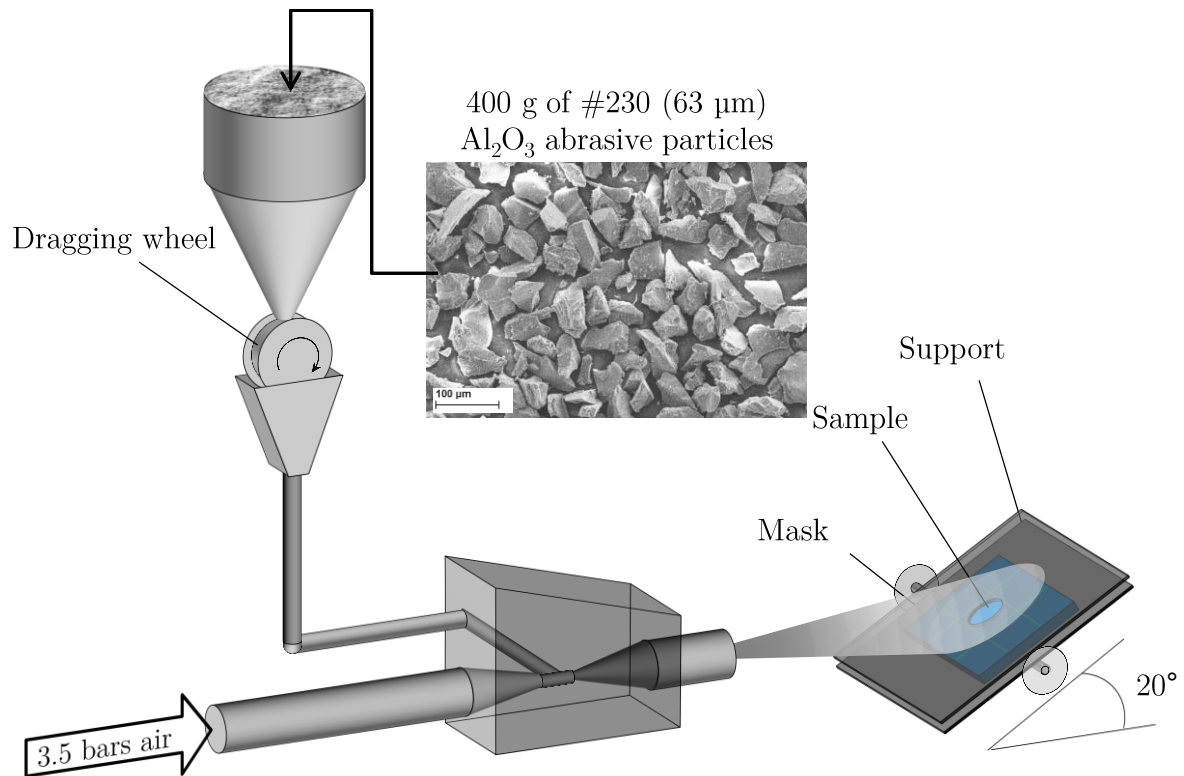


Figure 5.29: Schematic representation of the erosion rig components with a SEM micrograph of the alumina abrasive powder.



Figure 5.30: Picture of the caliper gauge used to measure the thickness of the erosion samples before and after the test.

- *Nanoscratch*

Scratch tests constitute a family of methods designed to study the resistance of a surface or a coating to grooving. It consists of a vertical indentation superposed with a lateral displacement of the tip in the surface of the material. The length scale of the penetration of the tip is on the order of a few nanometers up to several micrometers. In this study, the nanoscratch tests are performed using an Agilent G200 with a

standard XP indentation head. The scratching measurement involves three steps. First, a scanning of the surface is performed with the tip at very low force ($20\text{ }\mu\text{N}$). At the end of the scan, the vertical force on the tip is increased and the real scratch step takes place. After the scratching, a second scan is performed to make a second mapping of the surface. These two scans were used to measure the initial surface topography and the amount of elastic recovery of the surface after the scratch. The effective penetration depth and the residual depth are extracted from these measurements (see Figure 5.31).

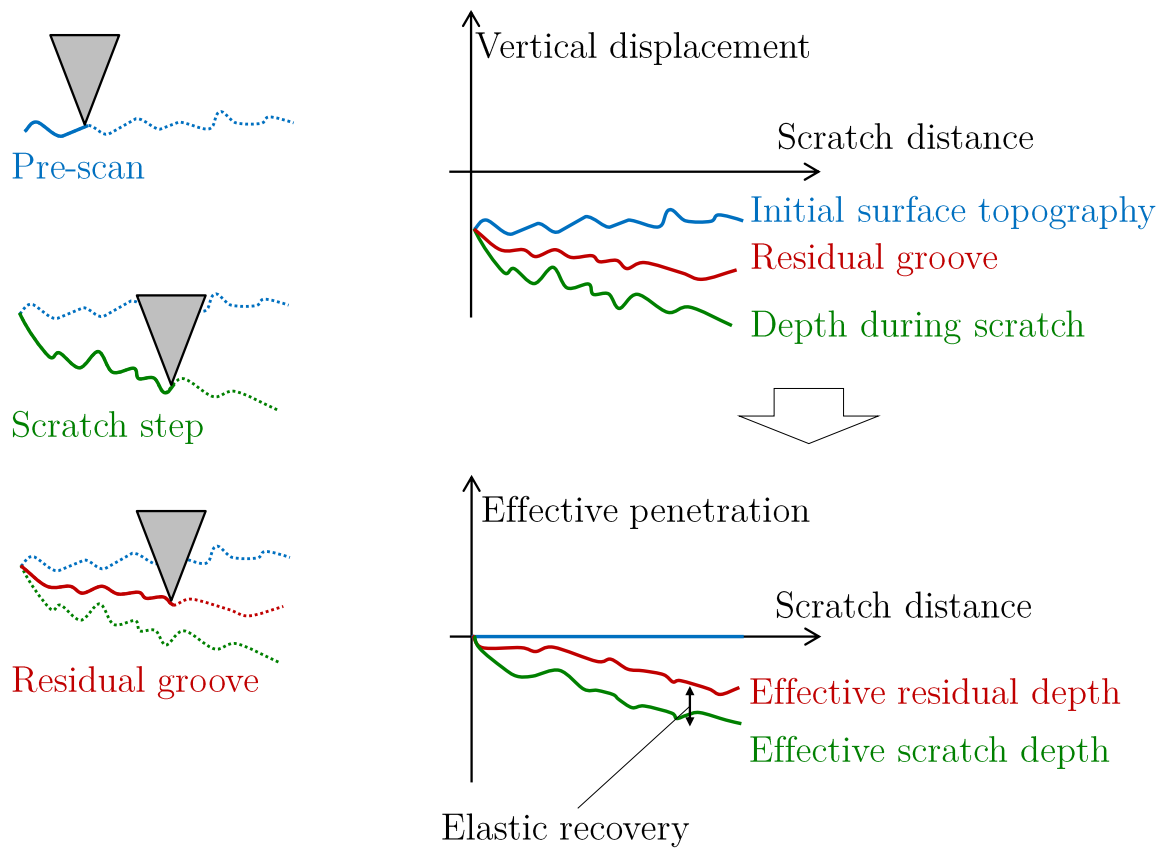


Figure 5.31: Schematic representation of the surface topography correction procedure with two scans, one before and one after the scratch step. Image from [Favache 2013]

Concerning the vertical force, two modes are used, a constant force mode and a load ramp mode. The constant force mode applies a user defined constant force during the scratch step. The load ramp force is a progressive linear increase of the vertical force during the scratch step. The user also defines the length of the scratch, the maximum force and the horizontal speed of the tip. The scratches are performed with two

different geometries of diamond tips. A sphero-conical tip with an opening angle of 120° and a radius of $200\text{ }\mu\text{m}$ (see Figure 5.32) and a Berkovich tip are used.

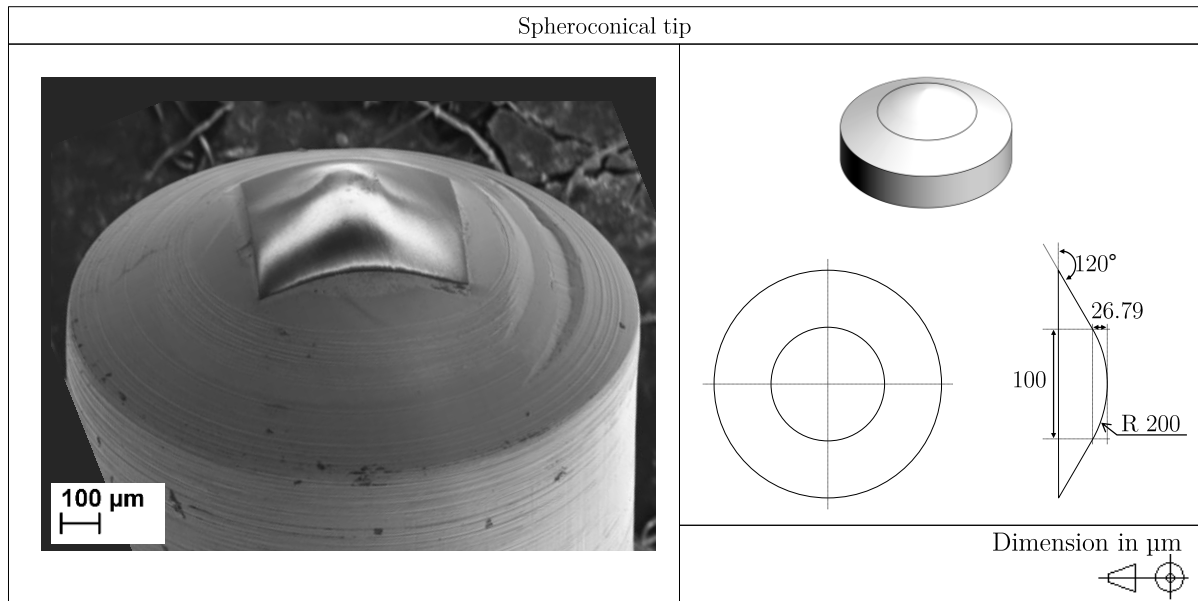


Figure 5.32: On the left, a SEM micrograph of the sphero-conical scratch tip. On the right, a schematic representation of the tip with the dimensions of interest.

Since the Berkovich tip is not an axisymmetric solid, the scratching step is performed in the face forward direction. In this configuration, one of the three faces of the pyramidal tip is displaced perpendicularly to the lateral displacement direction (see Figure 5.33).

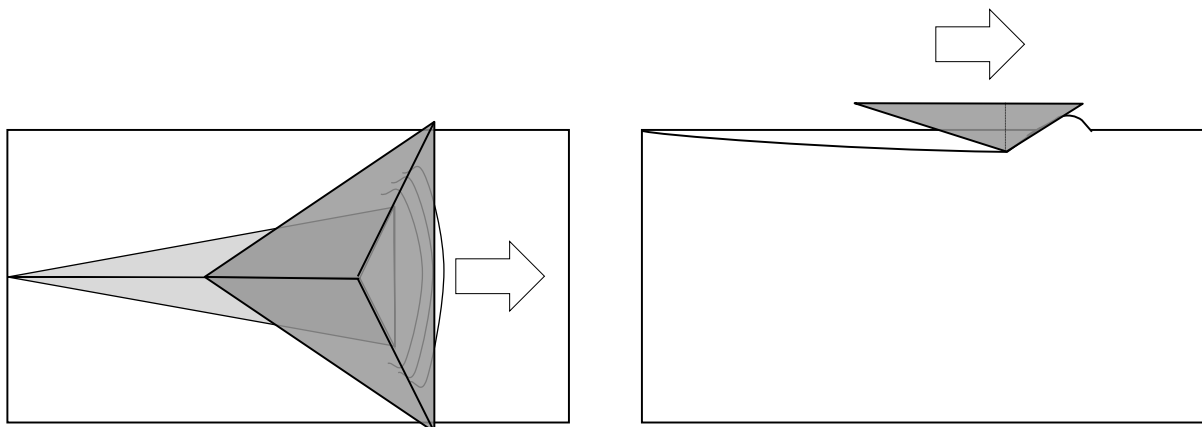


Figure 5.33: Schematic representation of the face forward direction of scratching used with the Berkovich tip.

All the scratches are performed with a constant lateral scratch speed of 40 $\mu\text{m/s}$. The scratches are repeated 5 times. The different scratch configurations are detailed in Table 5.9.

Table 5.9: Description of the scratch test parameters for the different scratch configurations.

Code name	Tip geometry	Load mode	Normal force [mN]	Scratch length [μm]	Scan velocity [$\mu\text{m/s}$]
BC-40	Berkovich	Constant	40	300	50
BC-80	Berkovich	Constant	80	300	50
BC-120	Berkovich	Constant	120	300	50
BR-400	Berkovich	Ramp	400	400	30
SR-400	Spheroconical	Ramp	400	400	20

Chapter 6 Experimental results and comparison with model predictions

In the present chapter, the experimental results are presented, starting with the microstructural analysis, the mechanical testing, and the tribological tests. Tribological results are compared to the model predictions described in Chapter 4.

6.1 Microstructure analysis

This section has two purposes. First, it describes the microstructures from a qualitative point of view with a focus on the main differences between the model materials. Then, it focuses on the extraction of parameters to describe quantitatively these microstructures for further used in the wear models.

6.1.1 General microstructural description

The microstructure reference material is presented in Figure 6.1. As a reminder, the reference material is processed by thermal spraying. The SEM micrograph reveals a complex microstructure in which the polyester phase appears in the form of disconnected clouds inside a continuous AlSi matrix. The 3D tomography reconstruction images show that the polyester phase and the AlSi phase are intermingled and continuous accompanied by small round porosities (Figure 6.2). The model materials obtained by cold-pressing of a mixture of AlSi and polyester powders are shown in Figure 6.3. In this figure, the interfaces between the powder grains are clearly observed, along with holes in the material surface. These holes are grains of powder that were removed during the polishing of the surface.

Materials obtained by pressing at 500°C are presented in Figure 6.4. Here, the AlSi particles are surrounded by a continuous matrix of polyester. In the case of fine AlSi powder, areas without AlSi are detected. They correspond to unmelted or partially melted polyester particles. The metallic foams infiltrated with polyester are shown in Figure 6.5. The metallic phase appears continuous or discontinuous depending on the metallic volume fraction of the foam prior to infiltration. Due to the open cell

morphology of the metallic foam, the final microstructure is a co-continuous dual phase material made of polyester and aluminum alloy.

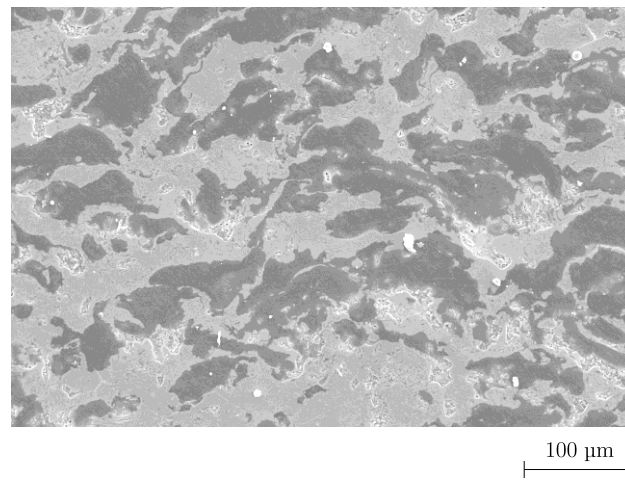


Figure 6.1: SEM micrograph of the reference thermally sprayed abrasible material. The aluminum-silicon phase (AlSi) appears in light grey, the polyester phase is in dark grey.

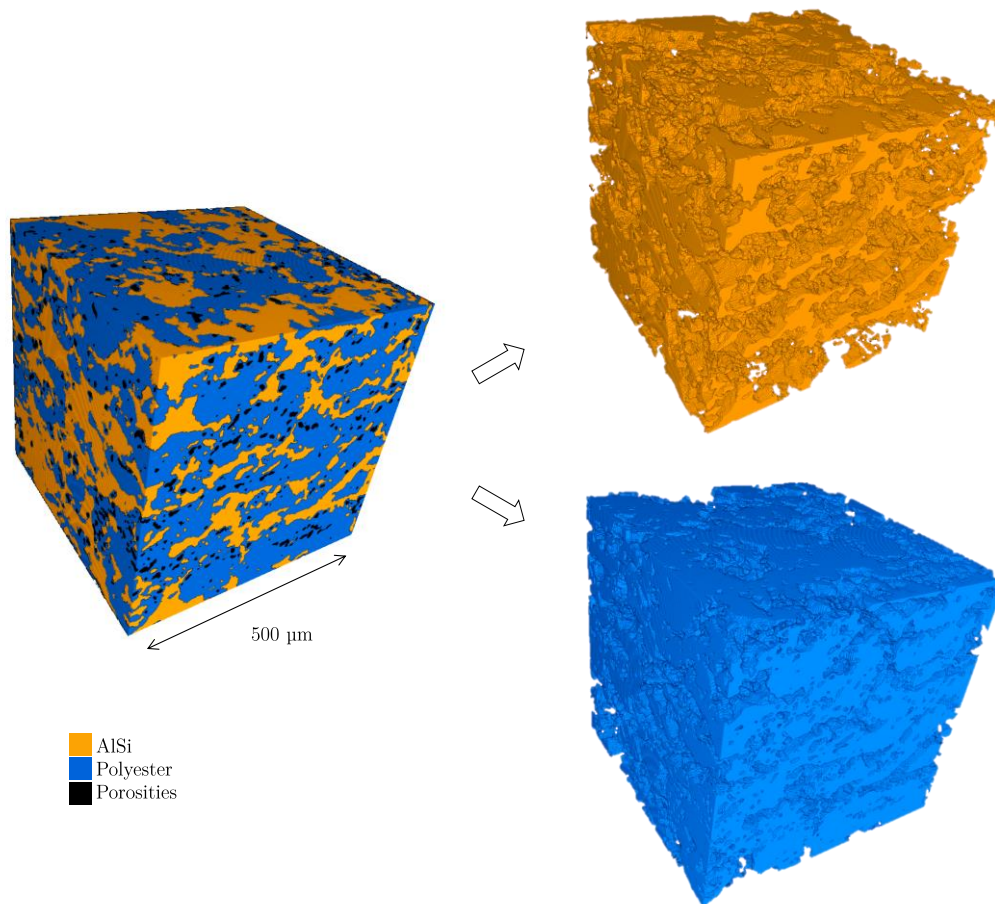


Figure 6.2: 3D reconstruction from the computer tomography images of the reference thermally sprayed abrasible material.

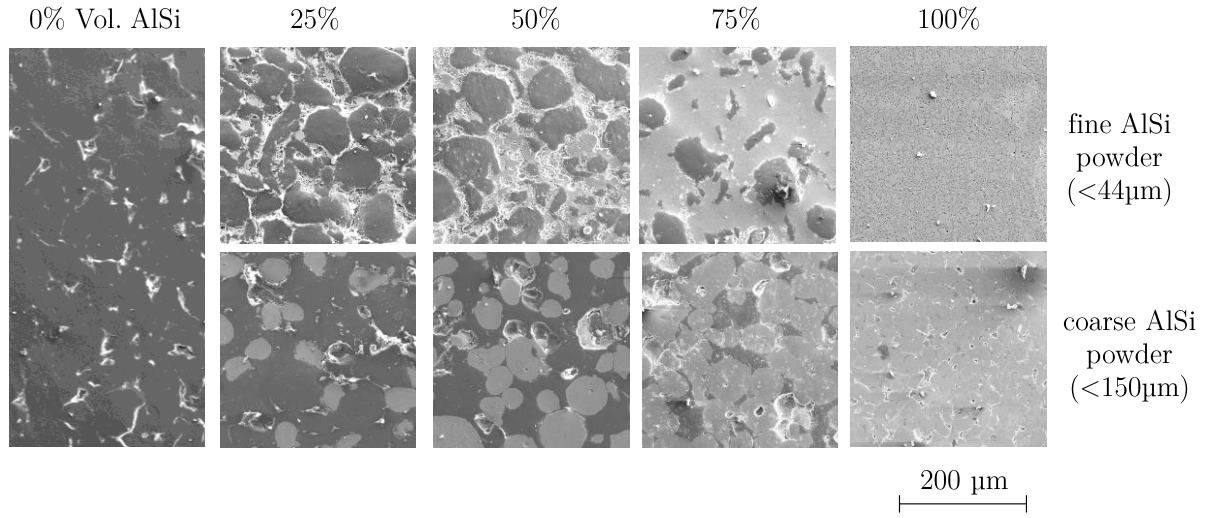


Figure 6.3: SEM micrographs of the model materials obtained by cold-pressing at room temperature under 400 MPa pressure. The AlSi phase appears in light grey and the polyester phase is in dark grey.

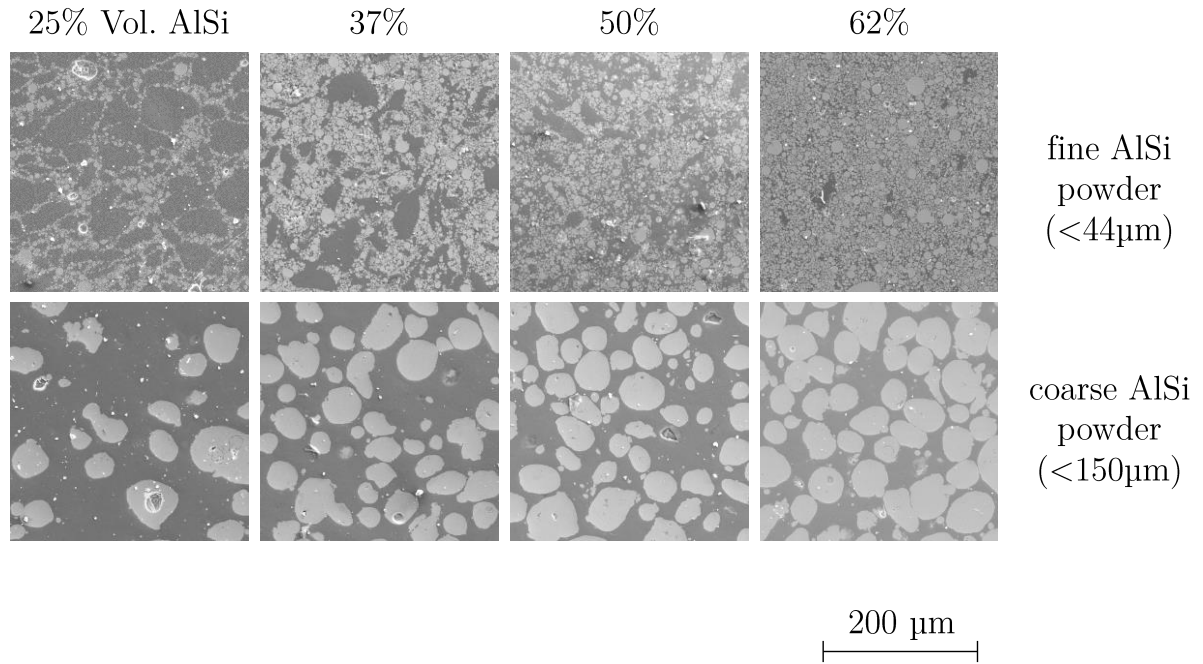


Figure 6.4: SEM micrographs of the model materials obtained by hot-pressing at 500°C under 15 MPa pressure. The AlSi phase appears in light grey while the polyester phase is in dark grey.

The microstructure of the model materials is expected to impact their wear behavior. In Chapter 4, the size of the segments (h_{seg}) is the only microstructure feature taken into account in the spalling models. However, the spalling model involves a pre-existing segmentation in the material, which is not always the case. The following subsection describes how to determine the size of these segments for each model.

ERG-8640

Ex-350



Figure 6.5: Pictures of two metallic foams infiltrated with polyester by hot uniaxial pressing. The metallic phase appears in grey, the polyester in yellow.

6.1.2 Extraction of a characteristic microstructure size (h_{seg})

In order to characterize the microstructure of the model materials with a segment size, a few simplifications are made depending on the type of model material. In the case of cold-pressed materials, the size of the powder grains is used. More precisely, the size distributions of the powders (AlSi and polyester) before compaction are measured by image analysis. The result is fitted with a Weibull probability function. A mean radius is then calculated for each powder (Figure 6.6). A rule of mixture is used based on (i) the average radius of each phase and based on (ii) the volume fraction of AlSi (f_{VAlSi}) in the powder blend to calculate the average radius of the mixture. This averaged radius is then used to calculate the volume of each segment (Figure 6.7). A detailed description of the calculations is given in Appendix B.1. Using this approach, the characteristic size of the segment follows as

$$h_{seg} = r_{mean_{mix}} \sqrt[3]{4\pi/3}, \quad \text{Eq. 6.1}$$

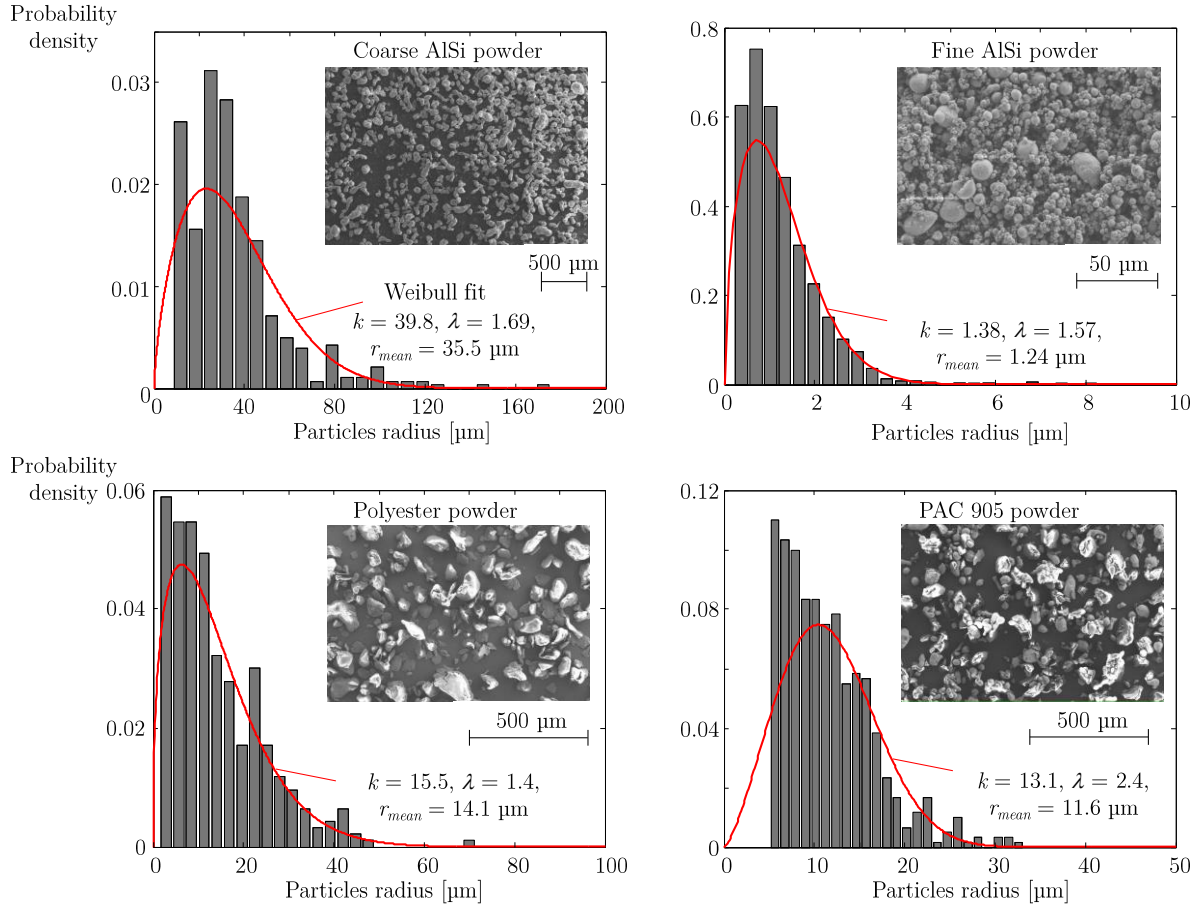
where

$$r_{mean_{mix}} = r_{AlSi} f_{n_{AlSi}} + r_{PO} (1 - f_{n_{AlSi}}), \quad \text{Eq. 6.2}$$

is the mean radius of the powder mixture, and

$$f_{n_{AlSi}} = \left[1 + \left(\frac{1}{f_{V_{AlSi}}} - 1 \right) \frac{r_{AlSi}^3}{r_{PO}^3} \right]^{-1}, \quad \text{Eq. 6.3}$$

is the number of AlSi particles divided by the total number of particles in the mix. In this last equation, $f_{V_{AlSi}}$ is the volume fraction of AlSi particles, r_{AlSi} and r_{PO} correspond to the average radius of the AlSi and the polyester powder.



Weibull probability
density function

$$f(r, k, \lambda) = \frac{k}{\lambda} \left(\frac{r}{\lambda} \right)^{k-1} e^{-(r/\lambda)^k}$$

Figure 6.6: Size distribution of the powder used to determine the characteristic size of the model materials. These measurements are obtained by image analysis. The mean radius is calculated based on a curve fitting with a Weibull law.

Regarding the hot-pressed materials, the microstructure is made of AlSi spheres in a continuous polyester matrix. Therefore, there are no discontinuities or weak spots in this microstructure where a crack could easily propagate through. Even if the interfaces between the polyester and the metal are weak, the crack would still need to

propagate inside the polymer phase to extract a metallic particle out of the continuous polymer matrix. In the case of metallic foam infiltrated with polyester, both phases are continuous. The segment size in these model materials is therefore considered infinitely large.

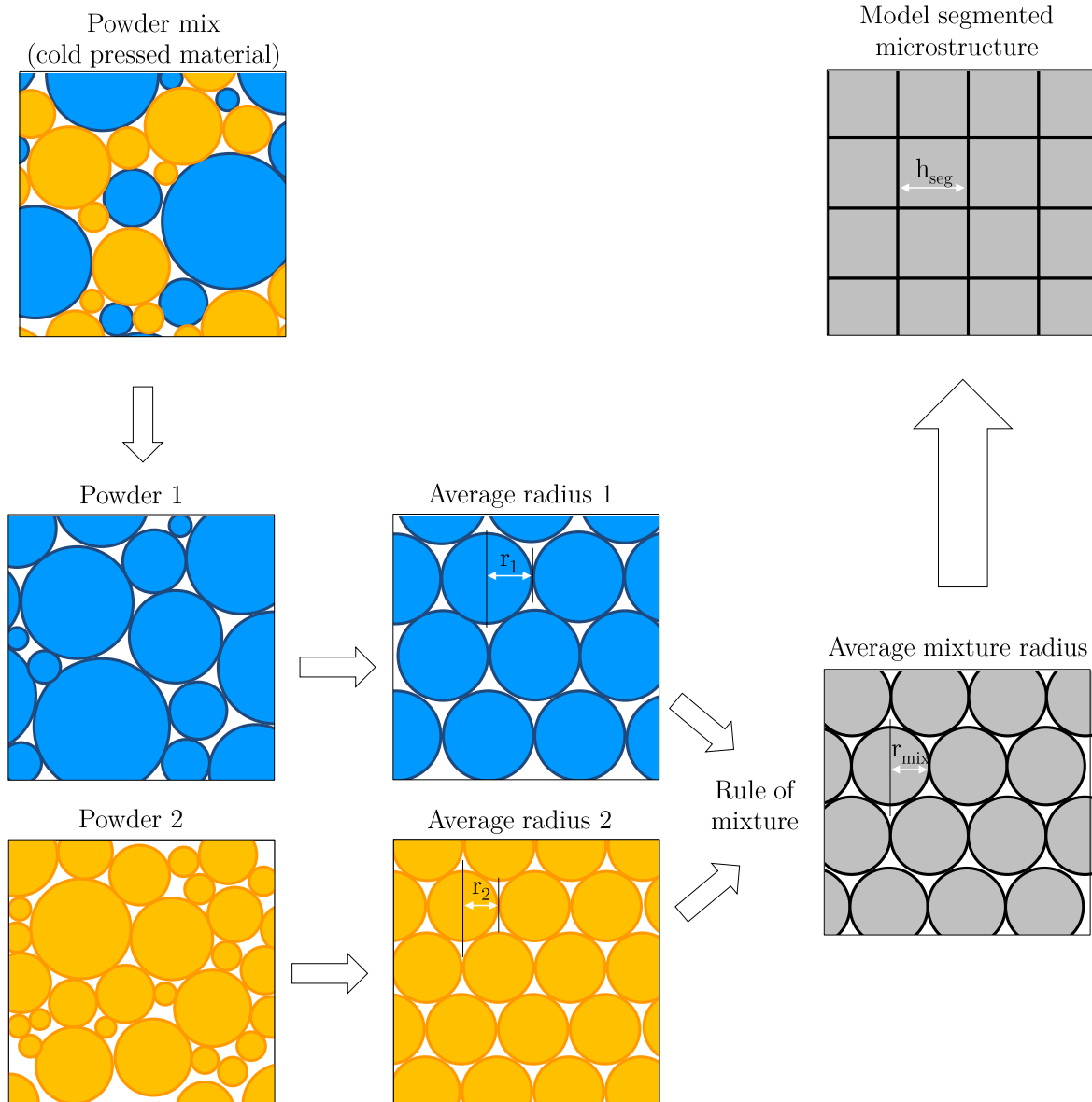


Figure 6.7: Schematic representation of the cold-pressed material microstructure in terms of an equivalent cubic segmented microstructure.

The microstructure of the thermally sprayed material (PAC905) is, in appearance, also co-continuous, however, the thermal spray process can give rise to different levels of

adhesion between the sprayed particles. Therefore, the size of the PAC905 segments is approximated by the mean size of the powder grains before thermal projection (Figure 6.6). The result of these rules of mixture is presented in Figure 6.8 and in Table 6.1.

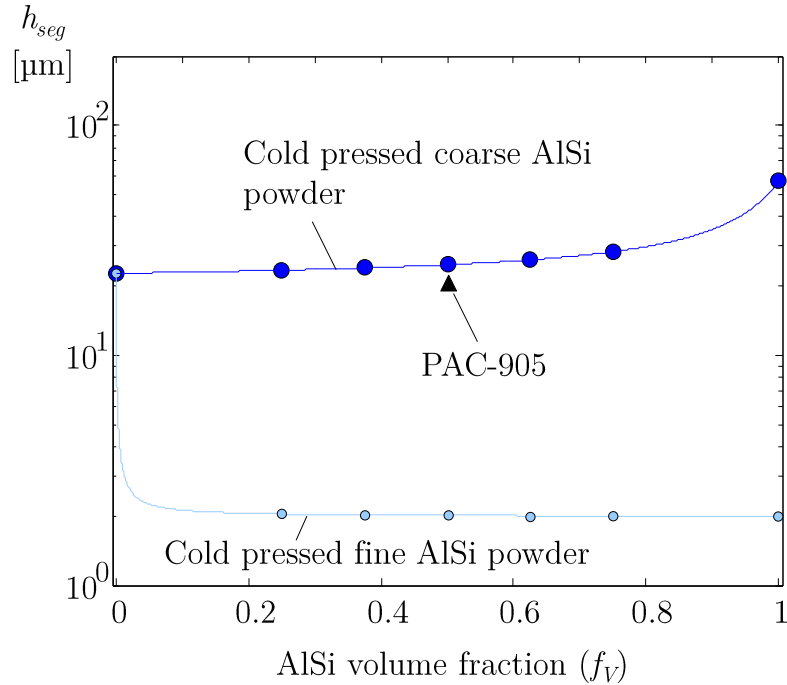


Figure 6.8: Variation of the characteristic size of the cubic segment (h_{seg}) used in the spalling model as a function of the AlSi volume fraction.

Table 6.1: Summary of the characteristic microstructure size of the model materials

Model materials	Name	h_{seg} [μm]
<i>Thermally sprayed</i>	PAC905	18.70
<i>Cold-pressed (fine)</i>	C0	22.73
	CF25	2.041
	CF37	2.022
	CF50	2.013
	CF62	2.007
	CF75	2.004
	CF100	1.999
<i>Cold-pressed (coarse)</i>	CC25	23.44
	CC37	23.98
	CC50	24.76
	CC62	25.99
	CC75	28.19
	CC100	57.23

6.2 Density measurements

The results of the density measurements are summed up in Figure 6.9 and compared to theoretical rules of mixture. These two graphs show that some model materials are less dense than expected and some are denser. The first situation is explained by the presence of porosity inside the material. The second occurs with some metallic foams that probably densify during hot-pressing. In some cases, the porosity content is quite high (more than 15%). However, the density measurement technique can lead to large overestimations of the porosity during the weighting of the soaked sample. Indeed, a thin water layer on the sample can significantly affect the results. For instance, the computerized tomography measurement on the industrial abrasible material (PAC-905) showed porosity content lower than 4%, compared to 17% by density measurements. These results should therefore be used as indicators of general trends. In particular, (i) cold-pressed materials exhibit a larger porosity than hot-pressed materials. (ii) Infiltrated metallic foams are subject to densification during the process.

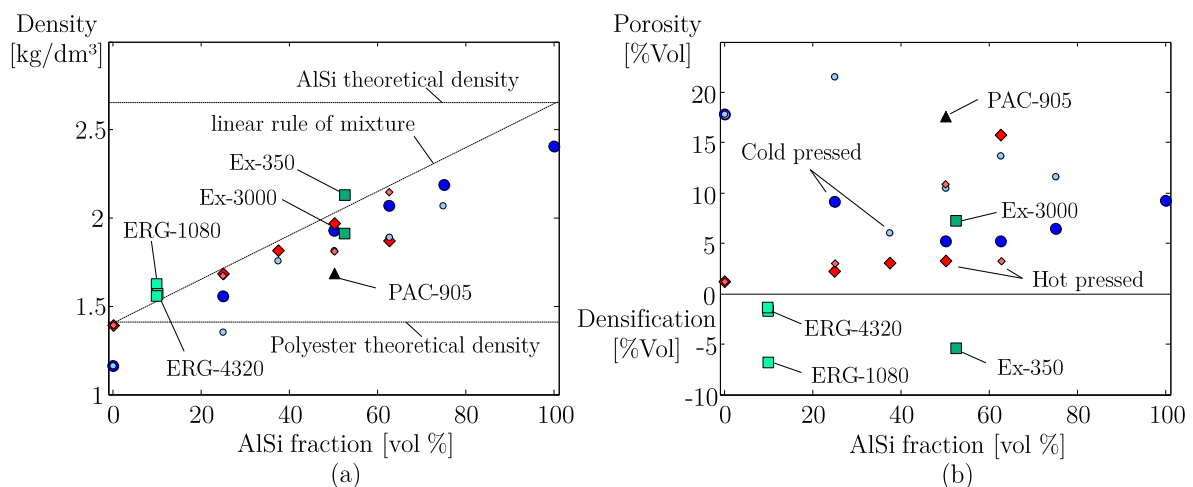


Figure 6.9: a) evolution of the density with the volume fraction of AlSi in the model materials; b) calculated porosity and densification of the model materials. The blue dots represent the cold-pressed materials, the red diamonds represent the hot-pressed ones. Small markers correspond to the fine AlSi powder, larger markers correspond to the coarse AlSi powder.

6.3 Mechanical properties

This section focuses on the results of the macroscopic mechanical measurements using uniaxial tension, compression and three points bending tests. Regarding the tensile tests, a classical stress-strain curve is presented in Figure 6.10. In most test model materials, some non-linearity occurs upon loading and unloading in the elastic part of the curves. This non-linear behavior is often observed in polymer materials as a sign of viscoelasticity, creep, back stress and/or kinematic hardening. The Young's modulus is therefore measured using two methods (see Figure 6.10). (i) The secant modulus (E_{secant}) is defined here as the slope of a line connecting the unloading point and the following loading point on the stress-strain curve. (ii) The unloading modulus (E_{unload}) is measured by linear regression in the beginning of the unloading curve. When subjected to a tensile test, most of the model materials present small elongations at break (less than 1% for most model materials) and most of the stress strain curves do not present a clear transition between the elastic part and the plastic part.

In metallic materials, the yield stress at 0.2% ($\sigma_{y0.2}$) is often used as a reference stress to characterize this transition between elastic and plastic state. $\sigma_{y0.2}$ is measured at the intersection between the stress strain curve and a straight line with a slope equal to the Young's modulus and translated by +0.2% on the strain axis (see Figure 6.10). In the present study, some model materials break before reaching 0.2% of plastic strain. In that case, $\sigma_{y0.2}$ is considered equal to the ultimate tensile strength (σ_{u-t}). All of the cold-pressed materials behave in a brittle way, i.e., elastic deformation interrupted by a sudden breaking. The other materials present some amount of plastic strain after unloading.

The tensile results are summarized in Figure 6.11. In this figure, it appears that E_{unload} and σ_{u-t} increase with the AlSi content except for the cold-pressed materials where there is no clear effect of the AlSi volume fraction on σ_{u-t} .

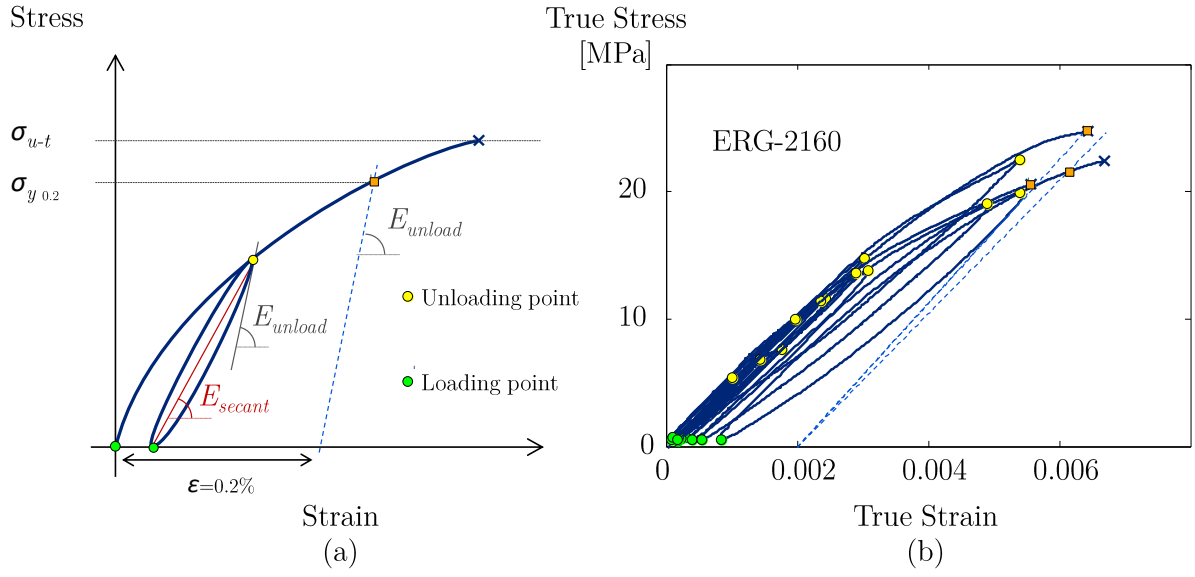


Figure 6.10: a) schematic representation of a tensile result with the measured parameters; b) example of a typical tensile test curves with an ERG metallic foams infiltrated with polyester.

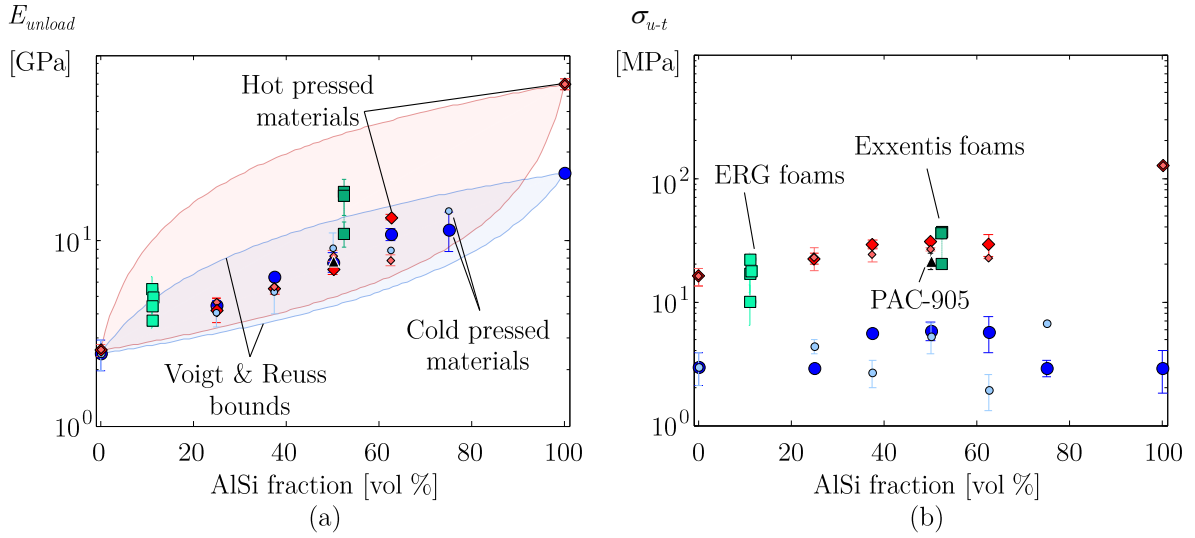


Figure 6.11: Effect of the AlSi content on tensile mechanical properties of the model materials expressed in terms of a) E_{unload} and b) σ_{u-t} .

Regarding compression tests results, for most of the test materials, there is a curvature on the stress strain curve at the beginning of the first loading. This behavior is the sign of an imperfect alignment between the upper surface of the sample and the compression plate. The first few points of the curve therefore correspond to compression of a part of the sample until the entire surface of the sample is in contact with the plates. This part of the curve is supposed to be linear, so the foot of the curve is replaced by a linear regression of the upper part of the curve (see Figure 6.12). As in

the tensile tests, most of the stress strain curves do not present a clear transition between the elastic part and the plastic part. The yield strength σ_y is therefore measured at 1% strain ($\sigma_{y1\%}$) using an approach similar to metals but at a larger strain. $\sigma_{y1\%}$ is typically used as a reference stress to characterize the yield strength in polymer materials [Ashby 2011]. The strain hardening exponent is measured by fitting the stress strain curve above $\sigma_{y1\%}$ with a theoretical elastoplastic behavior that follows the linear elastic part and a Hollomon law in the plastic part. This curve fitting is guided using E_{unload} as the Young's modulus and $\sigma_{y1\%}$ as the yield strength. The compression test results are summarized in Figure 6.13. The ultimate compressive strength (σ_{u-c}) of the model materials increases with the AlSi content. $\sigma_{y1\%}$ also increases with the AlSi content except for the cold-pressed materials. Table 6.3 shows the comparison between the mechanical properties measured through tensile and compression tests. On average, cold pressed materials have a σ_y 4 times larger and a σ_u 12 times larger in compression than in tension. Regarding the hot pressed materials this ratio becomes respectively 1.6 for σ_y and 2.6 σ_u . These large differences between tensile and compressive properties highlight the brittle nature of most of the model materials, especially the cold pressed materials that do break before reaching 0.2% strain in tension.

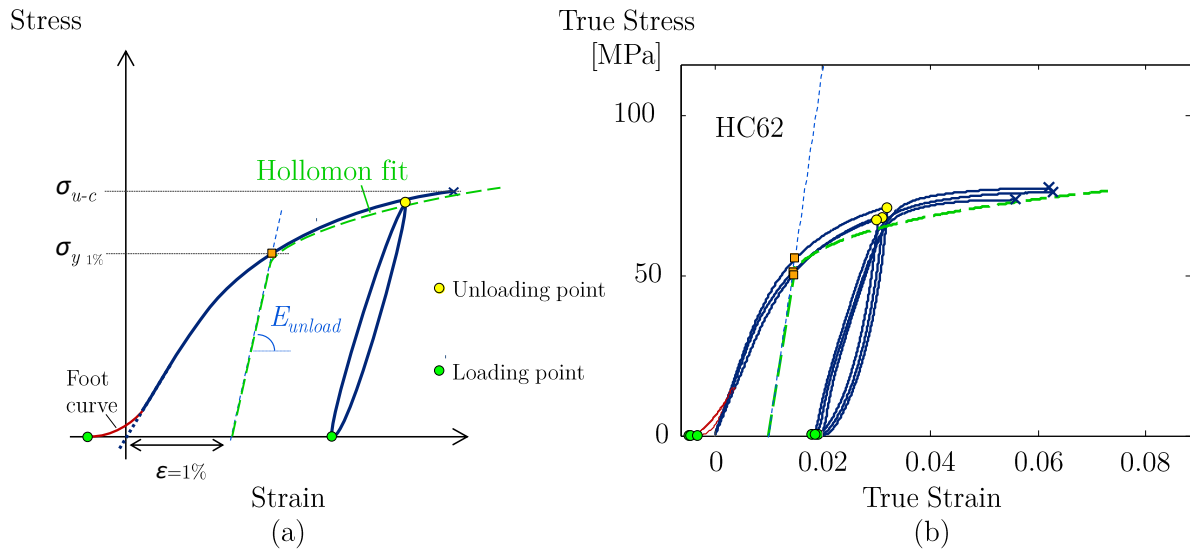


Figure 6.12: a) schematic representation of a compression result with the measured parameters; b) example of typical compression test curves with a hot-pressed powder mix made of polyester and 62% volume fraction of AlSi.

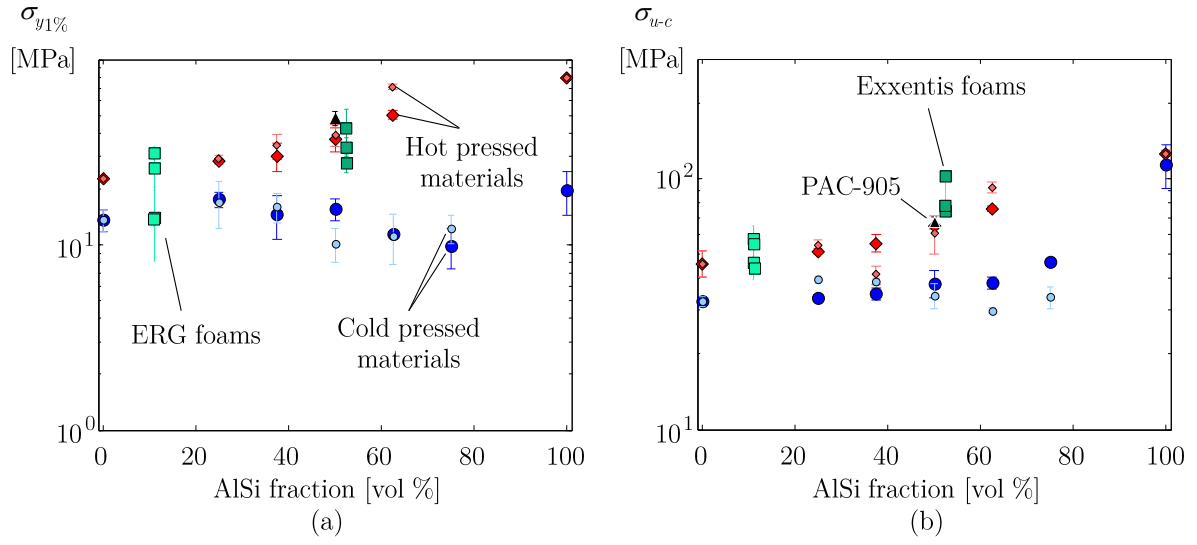


Figure 6.13: Effect of the AlSi content on mechanical properties of the model materials measured by compression tests expressed in terms of a) $\sigma_{y1\%}$ and b) σ_{u-c} .

Concerning the fracture toughness experiments, as stated in Chapter 5, the measurements are performed by bending of pre-cracked samples. During these experiments, the force applied by the machine as well as the cross-head displacements are used to calculate K_{Ic} using the ASTM procedure [ASTM 1999]. The ASTM standard involves the identification of a linear area on the force-displacement curve and measuring its slope (S). It also involves drawing a line with a 5% smaller slope and measuring the intersection between this line and the force-displacement curve. This intersection gives a force (F_Q) that is used to determine K_{Ic} (Figure 6.14). Using the values of K_{Ic} , G_{Ic} is calculated using the following formula for plane strain conditions:

$$G_{Ic} = \frac{K_{Ic}^2}{E(1-\nu^2)}, \quad \text{Eq. 6.4}$$

where E is the Young's modulus and ν is the Poisson ratio. In this study, G_{Ic} is measured considering that E_{unload} is the Young's modulus of the model materials and that ν is equal to 0.3. According to CES Edupack data base [CES 2015], $\nu_{AlSi} = 0.33$ and $\nu_{Polyester} = 0.41$.

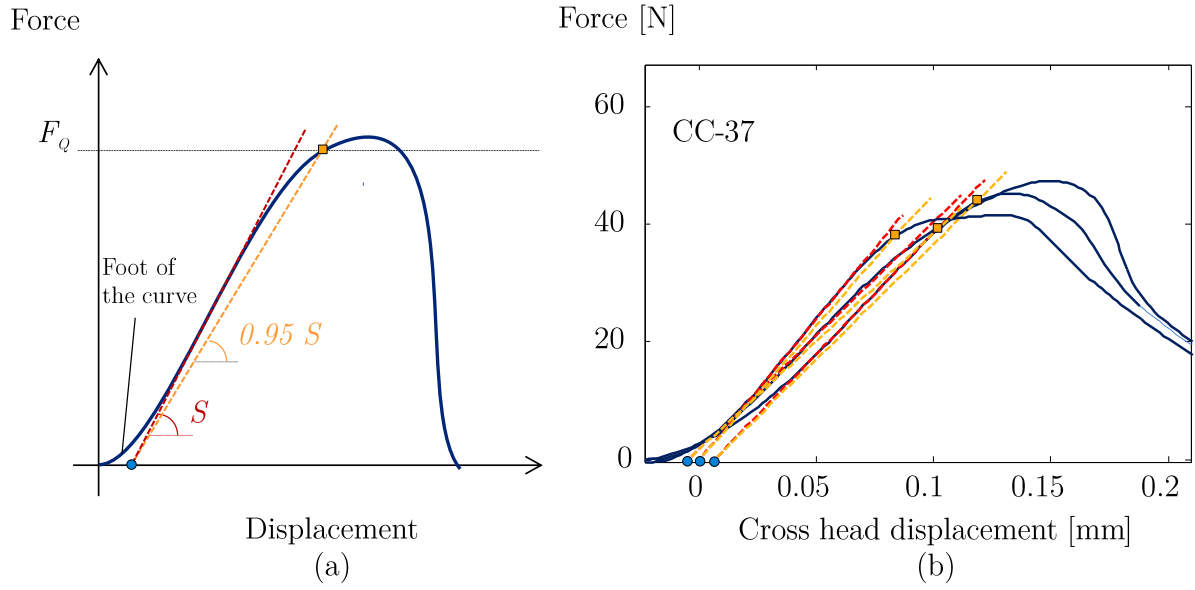


Figure 6.14: a) schematic representation of a three point bending test result with the measured parameters; b) examples of typical three points bending test results for a cold-pressed model material made of 37.5% of coarse AlSi powder mixed with polyester.

Values of K_{Ic} and G_{Ic} of the model material are given in Figure 6.15 and Table 6.2. The fracture toughness (K_{Ic}) increases with increasing AlSi volume fraction in the case of hot-pressed materials. Regarding the cold-pressed materials, only the 100% AlSi exhibit a slightly higher K_{Ic} . The AlSi content has a positive effect on G_{Ic} for the hot-pressed and a negative effect for the cold-pressed.

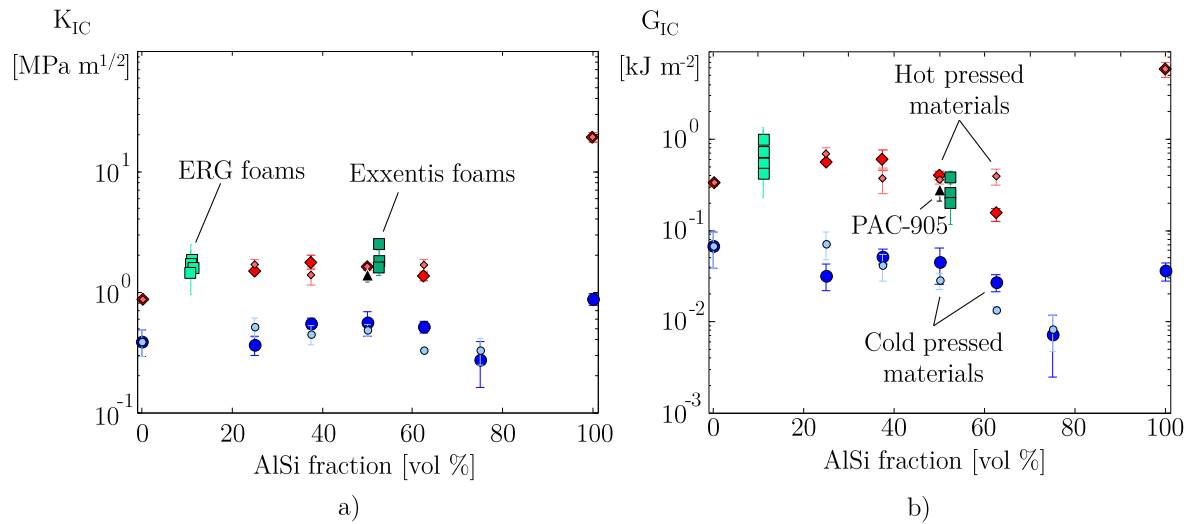


Figure 6.15: Effect of the AlSi content on the fracture toughness of the model materials expressed in terms of a) K_{Ic} and b) G_{Ic} .

Table 6.2: Mechanical properties of the model materials measured through tensile tests, uniaxial compression and three points bending tests. The values marked with asterisks (*) are from [CES 2015].

Model materials	Name	Uniaxial tension 				Uniaxial compression 			3 points bending 	
		E_{secant} [GPa]	E_{unload} [GPa]	$\sigma_{0.2\%}$ [MPa]	σ_{br} [MPa]	$\sigma_{y1\%}$ [MPa]	σ_{br} [MPa]	n	K_{IC} [MPa m ^{1/2}]	G_{IC} [J/m ²]
Bulk	Polyester	2.3 ± 0.1	2.6 ± 0.2	16.0 ± 2.5	16.0 ± 2.5	22.7 ± 0.4	45.9 ± 5.5	0.25	0.87 ± 0.03	237 ± 23
	AlSi	60.3 ± 0.4	69.7 ± 0.5	54.0 ± 8.5	126.4 ± 3.0	79.8 ± 1.6	126.4 ± 3.0	0.09	23.7 ± 1.8 (*)	5843 ± 1025
Thermally sprayed	PAC905	7.4 ± 0.5	7.6 ± 0.5	21.3 ± 3.1	21.4 ± 3.2	47.9 ± 4.2	65.9 ± 4.6	0.18	1.38 ± 0.2	273 ± 63
Cold pressed (fine)	C0	2.5 ± 0.5		3.0 ± 0.9	3.0 ± 0.9	13.6 ± 1.9	32.4 ± 1.6	0.32	0.39 ± 0.10	67.8 ± 29.4
	CF25	3.9 ± 0.5	4.1 ± 0.6	4.4 ± 0.6	4.4 ± 0.6	17.1 ± 4.9	39.6 ± 1.3	0.28	0.51 ± 0.09	71.6 ± 23.9
	CF37	4.9 ± 2.4	6.0 ± 1.5	3.4 ± 1.4	3.4 ± 1.4	16.1 ± 2.9	38.9 ± 1.2	0.29	0.44 ± 0.08	41.1 ± 13.6
	CF50	9.0 ± 1.9	9.1 ± 1.8	5.2 ± 1.5	5.2 ± 1.5	10.2 ± 2.1	34.1 ± 3.9	0.34	0.48 ± 0.05	28.2 ± 5.7
	CF62	9.3 ± 1.4	8.8 ± 0.1	1.9 ± 0.6	1.9 ± 0.6	11.2 ± 3.4	29.8 ± 0.7	0.26	0.33 ± 0.01	13.3 ± 1.1
	CF75	14.1 ± 0.6	14.4 ± 0.2	6.7 ± 0.0	6.7 ± 0.0	12.3 ± 2.1	33.7 ± 3.4	0.27	0.33 ± 0.08	8.3 ± 3.6
Cold pressed (coarse)	CC25	4.4 ± 0.2	4.5 ± 0.3	2.9 ± 0.1	2.9 ± 0.1	17.6 ± 1.7	33.5 ± 0.9	0.23	0.36 ± 0.06	32.2 ± 10.4
	CC37	6.2 ± 0.1	6.4 ± 0.1	5.6 ± 0.2	5.6 ± 0.2	14.6 ± 3.9	34.8 ± 2.0	0.25	0.55 ± 0.05	52.3 ± 9.4
	CC50	7.8 ± 1.3	7.7 ± 1.0	5.9 ± 1.0	5.9 ± 1.0	15.6 ± 2.17	38.3 ± 4.7	0.26	0.56 ± 0.13	44.6 ± 18.7
	CC62	11.0 ± 1.2	10.8 ± 0.8	5.7 ± 1.9	5.7 ± 1.9	11.5 ± 0.26	38.4 ± 2.1	0.29	0.52 ± 0.06	26.9 ± 5.8
	CC75	10.7 ± 2.8	11.5 ± 2.8	2.9 ± 0.5	2.9 ± 0.5	9.8 ± 2.4	46.6 ± 1.2	0.34	0.27 ± 0.11	7.2 ± 4.7
	CC100	22.7 ± 0.9	23.2 ± 0.5	2.9 ± 1.1	2.9 ± 1.1	19.6 ± 5.1	114 ± 22.7	0.42	0.87 ± 0.10	36.1 ± 8.0
Hot pressed (fine)	HF25	4.1 ± 0.2	4.7 ± 0.3	20.9 ± 2.6	22.7 ± 5.0	29.4 ± 8.1	54.0 ± 2.9	0.23	1.7 ± 0.1	701 ± 11
	HF37	5.2 ± 0.6	5.6 ± 0.4	23.2 ± 4.6	24.3 ± 3.3	34.2 ± 2.8	41.2 ± 3.5	0.14	1.4 ± 0.2	369 ± 11
	HF50	7.9 ± 0.6	8.3 ± 0.5	26.8 ± 0.8	26.8 ± 0.8	39.1 ± 2.6	60.8 ± 10.6	0.19	1.6 ± 0.1	358 ± 3.5
	HF62	6.7 ± 0.5	7.9 ± 0.6	22.5 ± 0.0	2.25 ± 0.0	71.3 ± 4.13	92.4 ± 4.5	0.15	1.7 ± 0.2	390 ± 7.7
Hot pressed (coarse)	HC25	4.1 ± 0.4	4.3 ± 0.6	21.1 ± 1.4	22.3 ± 2.2	28.4 ± 0.9	51.3 ± 1.1	0.22	1.5 ± 0.03	569 ± 23
	HC37	4.8 ± 0.2	5.6 ± 0.4	24.3 ± 1.0	29.4 ± 2.4	30.1 ± 5.4	55.4 ± 4.3	0.23	1.8 ± 0.2	617 ± 154
	HC50	6.0 ± 0.3	7.0 ± 0.5	25.7 ± 0.5	30.6 ± 1.1	37.2 ± 5.3	64.1 ± 0.9	0.2	1.6 ± 0.1	400 ± 42
	HC62	11.4 ± 1.0	13.3 ± 0.5	28.9 ± 5.8	29.0 ± 5.8	50.5 ± 2.7	75.8 ± 1.9	0.15	1.4 ± 0.1	150 ± 25
Exxentis foams	Ex-350	17.2 ± 0.7	17.6 ± 3.8	34.0 ± 2.9	36.6 ± 2.9	42.6 ± 11.6	102 ± 4.0	0.2	2.5 ± 0.2	383 ± 63
	Ex-630	18.0 ± 1.2	18.7 ± 0.4	27.6 ± 1.3	37.0 ± 4.8	27.3 ± 2.7	77.6 ± 1.1	0.26	1.8 ± 0.4	197 ± 81
	Ex-3000	10.1 ± 1.1	11.0 ± 1.6	17.1 ± 1.0	20.0 ± 1.3	33.4 ± 4.5	73.9 ± 1.5	0.21	1.6 ± 0.1	235 ± 16
ERG foams	ERG-1080	4.1 ± 0.1	5.0 ± 0.3	15.2 ± 3.3	17.4 ± 5.5	31.4 ± 2.6	54.6 ± 1.2	0.20	1.6 ± 0.1	542 ± 85
	ERG-2160	4.9 ± 0.3	5.5 ± 0.9	22.3 ± 2.2	22.5 ± 2.1	25.8 ± 5.2	57.0 ± 8.19	0.24	1.5 ± 0.1	420 ± 30
	ERG-4320	4.0 ± 0.4	4.5 ± 0.6	16.5 ± 0.5	17.3 ± 14.4	14.0 ± 5.8	46.3 ± 0.1	0.32	1.7 ± 0.8	732 ± 505
	ERG-8640	3.4 ± 0.4	3.7 ± 0.3	9.4 ± 2.7	10.0 ± 3.7	13.8 ± 1.2	43.9 ± 4.3	0.33	1.8 ± 0.4	1001 ± 345

Table 6.3 : Comparison between the yield stress (σ_y) and ultimate stress (σ_u) measured during tensile and compression tests.

Model materials	Name	$\sigma_{yT\%} / \sigma_{y0.2\%}$	$\sigma_{tc} / \sigma_{tt}$
Bulk	Polyester AlSi	1.4	2.9
		1.5	1.0
Thermally sprayed	PAC905	2.2	3.1
Cold pressed (fine)	C0	4.5	10.8
	CF25	3.9	9.0
	CF37	4.7	11.4
	CF50	2.0	6.6
	CF62	5.9	15.7
	CF75	1.8	5.0
Cold pressed (coarse)	CC25	6.1	11.6
	CC37	2.6	6.2
	CC50	2.6	6.5
	CC62	2.0	6.7
	CC75	3.4	16.1
	CC100	6.8	39.3

Model materials	Name	$\sigma_{yT\%} / \sigma_{y0.2\%}$	$\sigma_{tc} / \sigma_{tt}$
Hot pressed (fine)	HF25	1.4	2.4
	HF37	1.5	1.7
	HF50	1.5	2.3
	HF62	3.2	4.1
Hot pressed (coarse)	HC25	1.3	2.3
	HC37	1.2	1.9
	HC50	1.4	2.1
	HC62	1.7	2.6
Exxentis foams	Ex-350	1.3	2.8
	Ex-630	1.0	2.1
	Ex-3000	2.0	3.7
ERG foams	ERG-1080	2.1	3.1
	ERG-2160	1.2	2.5
	ERG-4320	0.8	2.7
	ERG-8640	1.5	4.4

6.4 Tribological behavior

In the following section, the tribological tests results are analyzed and compared to the predictions produced by the models described in Chapter 4. These models predict the tribological response without any fitting constants. Therefore, in some cases, differences between the model and the experiments are observed. These differences are discussed in Chapter 7. The indentation experiments are presented followed by nanoscratch, pin-on-disc tests, abrasability and erosion tests. Regarding the parameters used in the models, the unloading modulus (E_{unload}) measured by tensile tests is used as the Young's modulus. The yield stress measured at 1% of plastic strain ($\sigma_{y1\%}$) by compression tests is used as the yield stress (σ_y). The strain hardening exponent extracted from the compression tests results (see Table 6.2) is considered as the strain hardening exponent (n).

6.4.1 Nanoindentation

Nanoindentation with a Berkovich tip is modelled by an equivalent conical tip with a half-opening angle of 70.3° . The nanoindentation force is modelled using the sharp tip indentation model with strain hardening (see Section 4.1.3). The equations used to model the indentation forces are summarized in Table 6.4. Given the geometry of the tip, the model predicts an evolution of the indentation force proportional to the square of the indentation depth. This trend is illustrated in Figure 6.16 in the case of a cold-pressed material. The general shapes of the curves are the same for all tested materials. The model also predicts that the relative residual depth (h_{pl}/h) is not affected by the indentation depth. The experiments confirm this prediction.

Table 6.4 : Summary of the model used to predict the nanoindentation forces

<i>Sharp tip indentation with strain hardening (Section 4.1.3)</i>	
Variable name	Equation reference
Indentation force (F_V)	Eq. 3.1
Hardness (H_{clpl})	Eq. 4.12 & Table 3.1
Radius of contact (a)	Eq. 4.26 & Table 3.1
Indentation flow stress (σ_{rind})	Eq. 3.7
Indentation representative strain (ε_r)	Eq. 4.35
Constrain factor (C_{plr})	Eq. 4.36 & Table 3.1
Representative pile-up factor (D_{plr})	Eq. 4.37 & Table 3.1
h_{pl}/h	Eq. 4.3 & Eq. 4.29

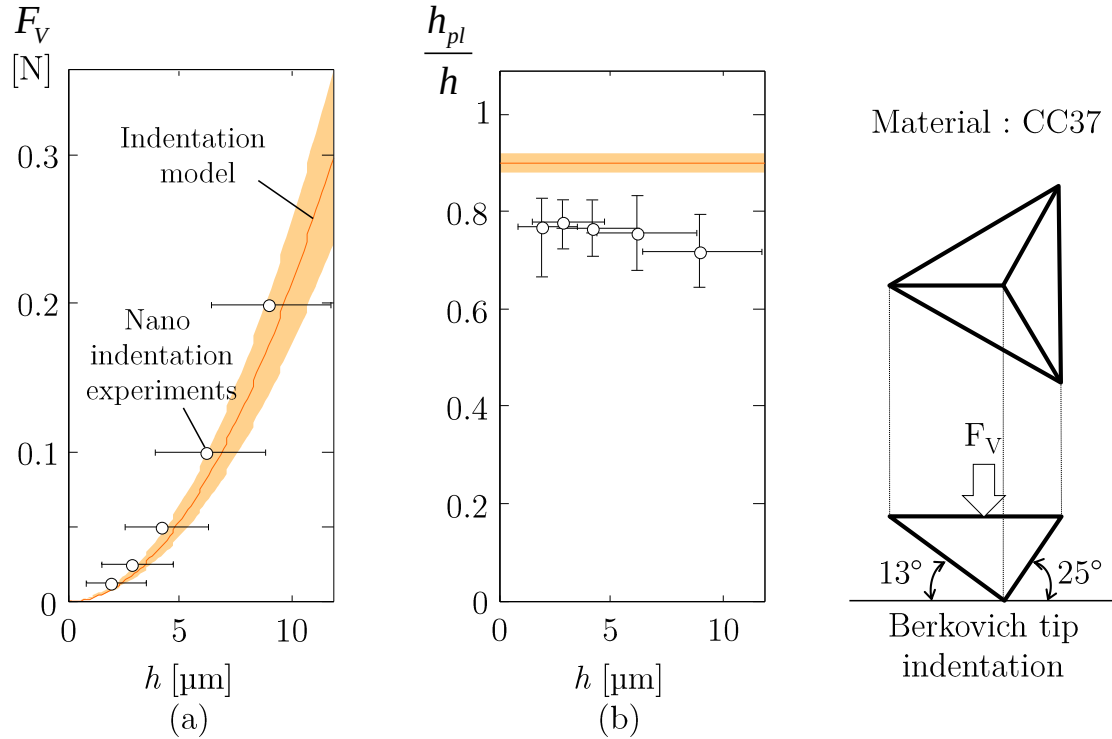


Figure 6.16: Comparison between the indentation model and the nanoindentation results on a cold-pressed model material (CC37); a) the vertical force versus the penetration depth; b) the relative residual depth. The error bar represents the maximum and minimum measured values.

Regarding the experimental results, the hardness of the model materials is calculated using the following formula:

$$H = \frac{F_V}{A_V},$$

where A_V is the projected contact area approximated by

$$A_V = \pi [h \tan(70.3^\circ)]^2.$$

The hardness of the model materials as well as the relative residual depth are presented in Figure 6.17. Except for one cold-pressed material, H increases with the AlSi content. h_{pl}/h increases with the AlSi content as well. These trends make sense since σ_y increases with the AlSi content while the ratio σ_y/E decreases with it. H is proportional to σ_y and the elastic recovery is proportional to σ_y/E .

The nanoindentation results are compared with the model in Figure 6.18 using Eq. 4.3, Eq. 4.28 and Eq. 4.29. On this figure, each experimental value of F_V and h_{pl}/h are compared with the model by dividing the value predicted by the model with the corresponding experimental result. This ratio shows if the model is over or under predicting values and for which materials. This ratio shows the accuracy of the model (see Figure 6.18). This ratio lies between 0.4 and 2. Figure 6.18 also shows that the response of the cold-pressed materials is generally predicted with less accuracy than the hot-pressed ones. The model slightly overestimates h_{pl}/h , especially in the case of cold-pressed materials. The results are summarized in Table 6.5. In this table, the precision of the model is based on the ratio between the prediction of the model and the experimental result. In the case of the vertical force for instance, the precision is defined as

$$\text{Precision}(F_V) = 10^{-\left| \log \left(\frac{F_{V_{model}}}{F_{V_{exp}}} \right) \right|}, \quad \text{Eq. 6.5}$$

where $F_{V_{model}}$ is the prediction of the model and $F_{V_{exp}}$ is the experimental result. With this definition, the value of the precision always lies between 0 and 1 (see Figure 6.19). For instance, a precision of 0.5 means that there is a factor 2 between the model and the experiment. A precision of 0.1 means a factor 10. This definition has one advantage compared to the simple ratio between model and experiment. It allows calculating the global accuracy of the model by averaging the precision over all the different tested materials without having overestimations compensating underestimations. In the case of nanoindentation, the precision of the model, averaged over all the test materials, is 76% for F_V and 88% for h_{pl}/h .

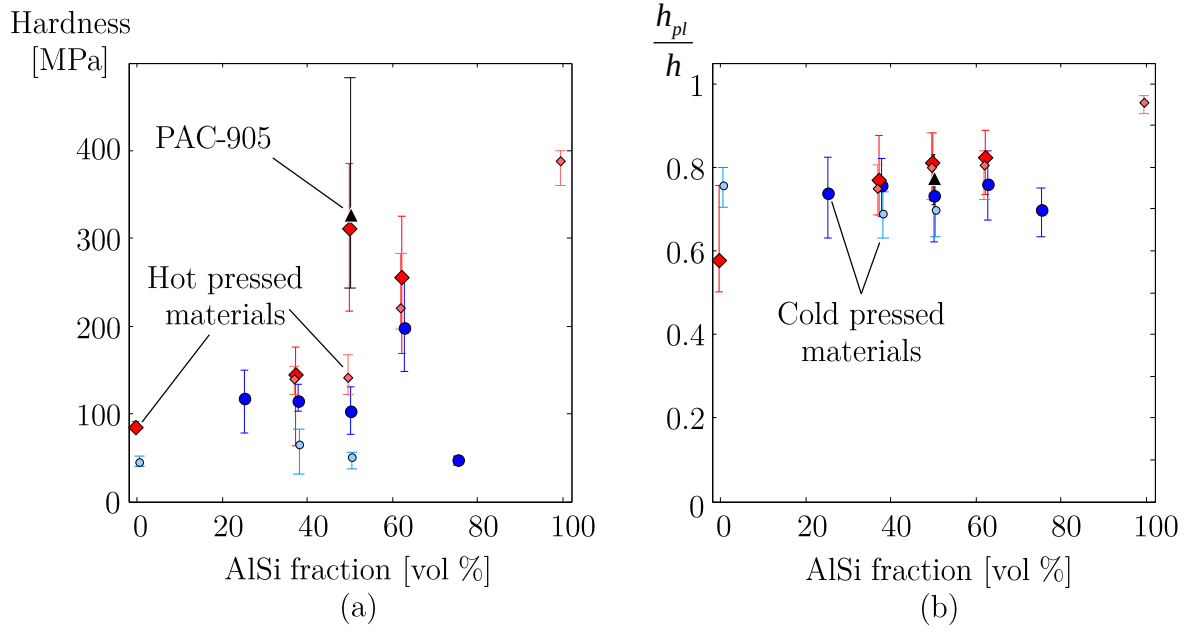


Figure 6.17: Variation of a) the hardness and b) of the relative residual depth measured by nanoindentation as a function of the AlSi volume fraction of the model materials.

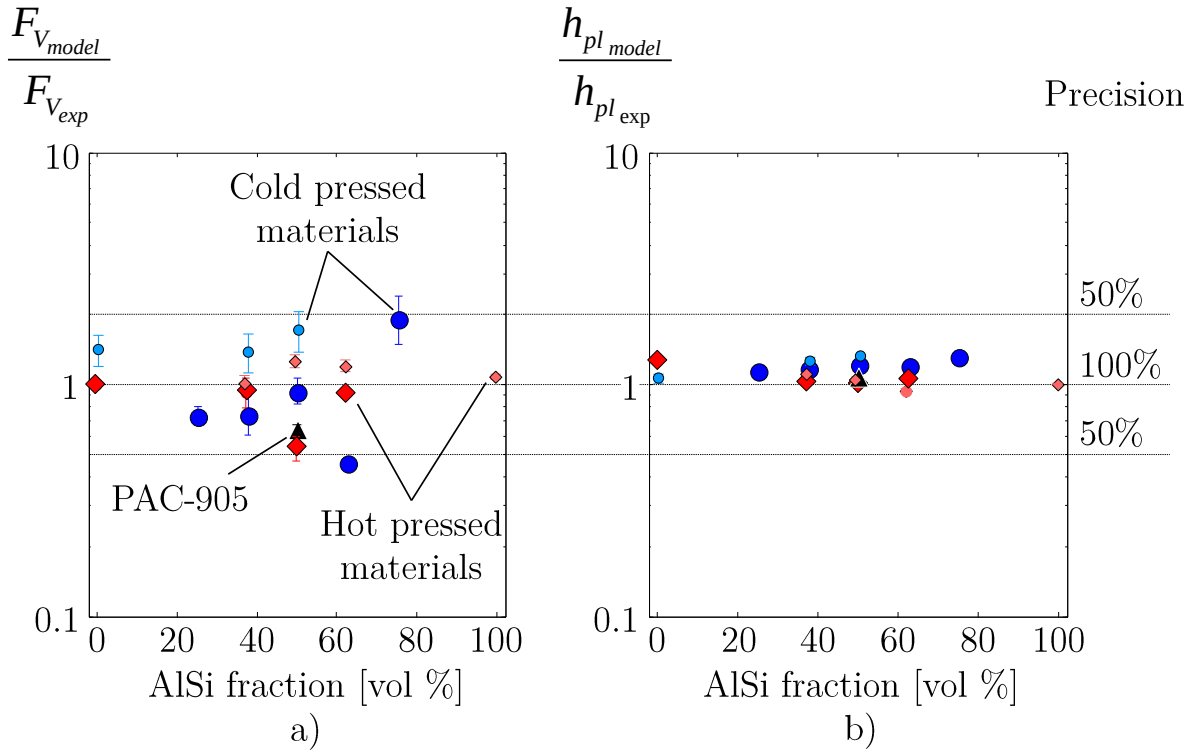


Figure 6.18: Effect of AlSi content on the ratio between the model predictions and nanoindentation measurements expressed in terms of a) indentation force and b) residual depth. The error bars represent the error of the model divided by the mean experimental value.

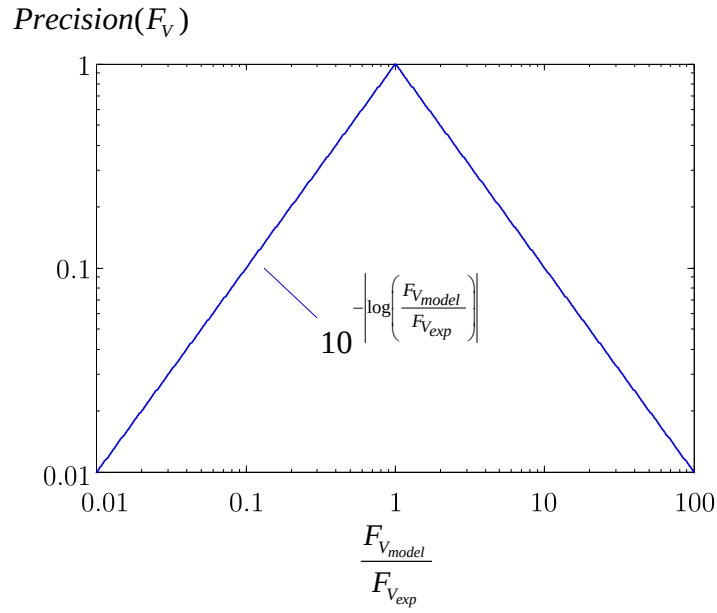
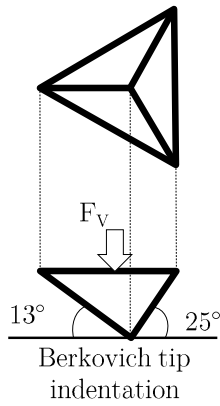


Figure 6.19: Variation of the precision of the model as a function of the ratio between the model prediction and the experimental results. The precision always lies between 0 and 1.

Table 6.5: Summary of the comparison between the model predictions and the experimental nanoindentation results.



Model materials	Name	$\frac{F_{V_{model}}}{F_{V_{exp}}}$	Precision $F_{V_{model}}$	$\frac{h_{pl_{model}}}{h_{pl_{exp}}}$	Precision $h_{pl_{model}}$
Bulk	Polyester	1.00	1.00	1.30	0.77
	AlSi	1.08	0.93	1.00	1.00
Thermally sprayed	PAC905	0.62	0.62	1.05	0.95
Cold pressed (fine)	C0	1.41	0.71	1.06	0.94
	CF37	1.39	0.72	1.26	0.79
	CF50	1.72	0.58	1.33	0.75
Cold pressed (coarse)	CC25	0.74	0.74	1.16	0.86
	CC37	0.75	0.75	1.19	0.84
	CC50	0.95	0.95	1.24	0.81
	CC62	0.46	0.46	1.23	0.81
	CC75	1.95	0.51	1.34	0.75
Hot pressed (fine)	HF37	1.01	0.99	1.10	0.91
	HF50	1.26	0.79	1.06	0.94
	HF62	1.21	0.83	0.94	1.06
Hot pressed (coarse)	HC37	0.93	0.93	1.07	0.93
	HC50	0.53	0.53	1.02	0.98
	HC62	0.93	0.93	1.06	0.94
Mean values		1.06	0.76	1.15	0.88

6.4.2 Macroindentation

Macroindentation with the TiA6V cutting tool is modelled with a half-wedge having a 30° angle between the horizontal and the inclined face. The macroindentation force is modelled using the sharp tip indentation model with strain hardening (see Section 4.1.3). The equations used to model the indentation forces are summarized in Table 6.4. With this tip geometry, the model predicts a linear relationship between the indentation force and the indentation depth. This trend is illustrated in Figure 6.20 in the case of a cold-pressed material. The general shapes of the curves are the same for all the test materials. The model also predicts that the relative residual depth (h_{pl}/h) is not affected by the indentation depth. This prediction is confirmed by all the experiments. The hardness of the model materials is calculated using the following formula:

$$H = \frac{F_V}{A_V},$$

where A_V is the projected contact area approximated by

$$A_V = W[h \cot(30^\circ)],$$

where W is the width of the tip. The hardness of the model materials as well as the relative residual depth (h_{pl}/h) is presented in Figure 6.21. In this figure, H increases with AlSi content. On the same figure, h_{pl}/h decreases with the AlSi content which is surprising since the σ_y/E ratio decreases with the AlSi volume fraction.

The macroindentation results are compared with the model in Figure 6.22 using the same method described for the nanoindentation results. Concerning the indentation force (F_V), Figure 6.22 shows that the response for half model materials is overestimated and half is underestimated. The model overestimates h_{pl}/h . This overestimation increases with the AlSi content. The results of the macroindentations are summarized in Table 6.6. The precision of the model, averaged over all the test materials is 75% for F_V and 41% for h_{pl}/h .

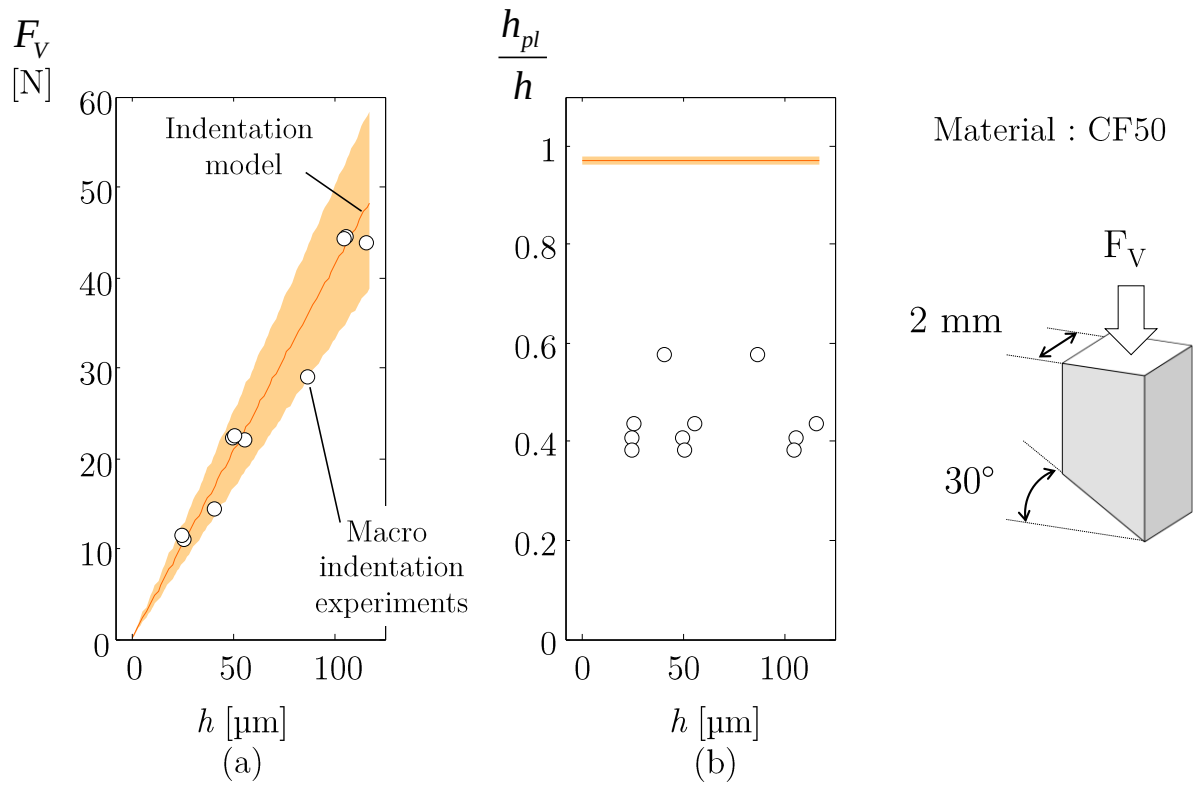


Figure 6.20: Comparison between the indentation model and the macroindentation results on a cold-pressed model material (CF50); a) the vertical force versus the penetration depth; b) the relative residual depth.

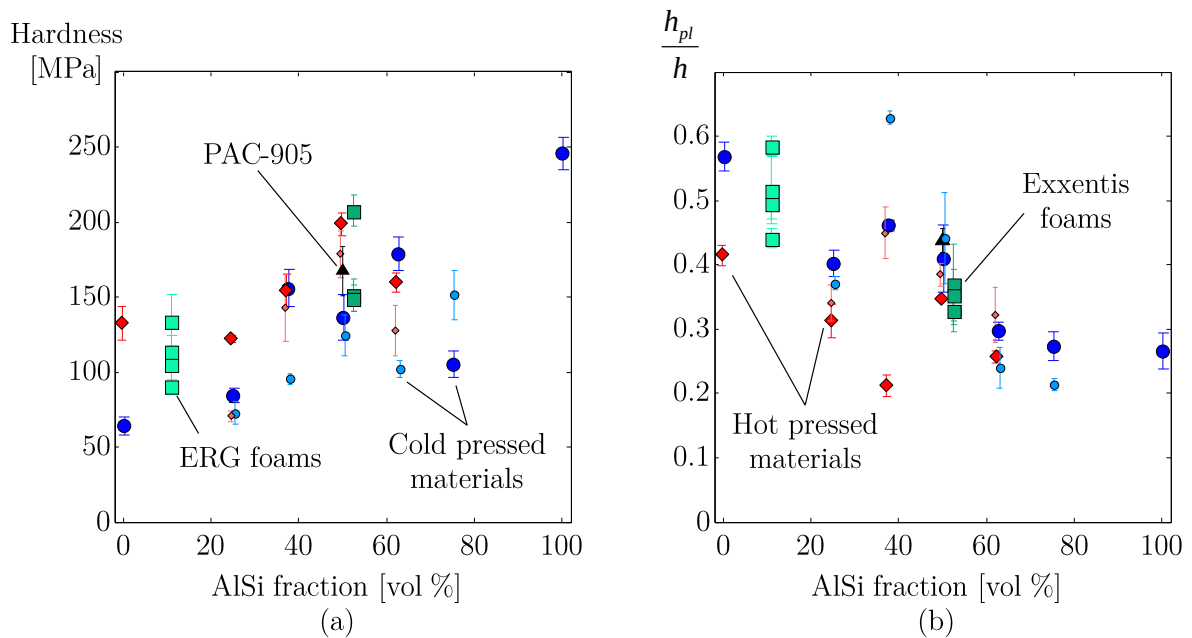


Figure 6.21: Variation of a) hardness and b) relative residual depth measured by macroindentation as a function of the AlSi volume fraction of the model materials.

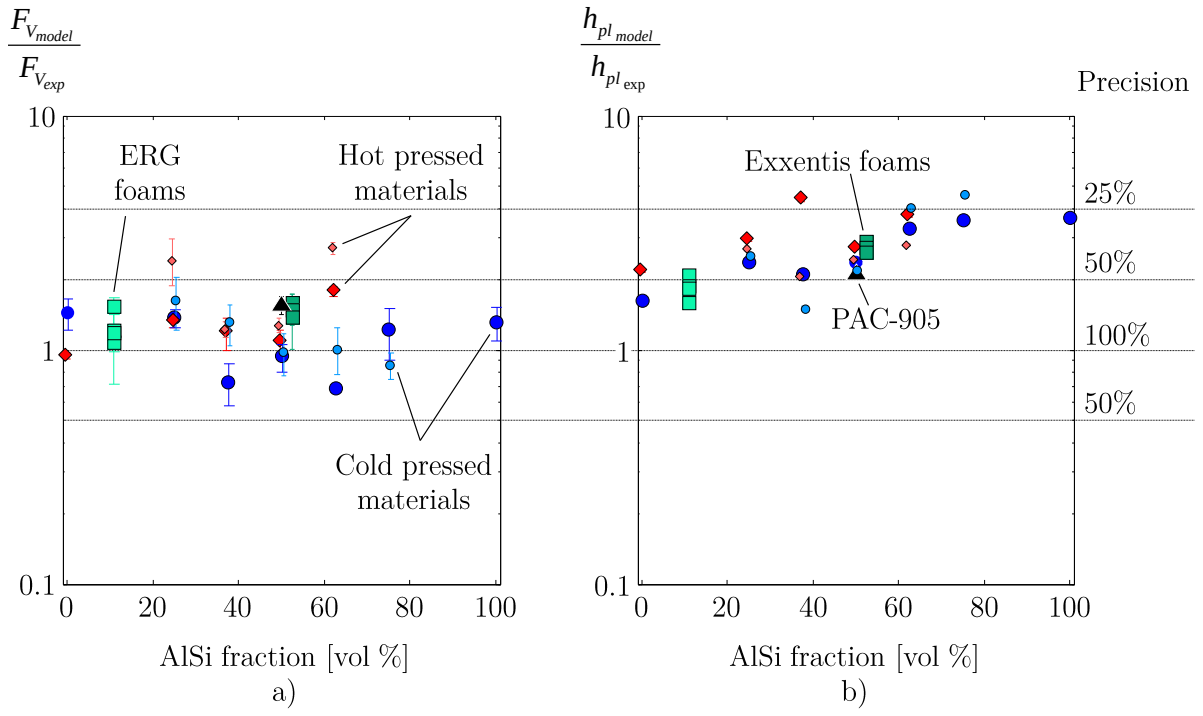
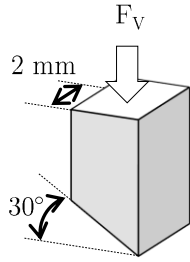


Figure 6.22: Effect of AlSi content on the ratio between the model predictions and macroindentation measurements expressed in terms of a) indentation force and b) residual depth. The error bars represent the error of the model divided by the mean experimental value.

Table 6.6: Summary of the comparison between the model predictions and the experimental macroindentation results.



Model materials	Name	$\frac{F_{V_{model}}}{F_{V_{exp}}}$	Precision $F_{V_{model}}$	$\frac{h_{pl_{model}}}{h_{pl_{exp}}}$	Precision $h_{pl_{model}}$
Bulk	Polyester	0.94	0.94	2.16	0.46
Thermally sprayed	PAC905	1.55	0.64	2.12	0.47
Cold pressed (fine)	C0	1.44	0.69	1.61	0.62
	CF25	1.63	0.61	2.52	0.40
	CF37	1.31	0.76	1.51	0.66
	CF50	0.98	0.98	2.20	0.45
	CF62	1.01	0.99	4.06	0.25
	CF75	0.86	0.86	4.59	0.22
Cold pressed (coarse)	CC25	1.36	0.73	2.35	0.43
	CC37	0.72	0.72	2.08	0.48
	CC50	0.93	0.93	2.35	0.42
	CC62	0.68	0.68	3.28	0.31
	CC75	1.22	0.82	3.57	0.28
	CC100	1.30	0.77	3.65	0.27
Hot pressed (fine)	HF25	2.42	0.41	2.70	0.37
	HF37	1.23	0.81	2.07	0.48
	HF50	1.28	0.78	2.44	0.41
	HF62	2.73	0.37	2.81	0.36
Hot pressed (coarse)	HC25	1.32	0.76	2.93	0.34
	HC37	1.18	0.85	4.39	0.23
	HC50	1.08	0.92	2.70	0.37
	HC62	1.78	0.56	3.72	0.27
Exxentis foams	Ex-350	1.38	0.72	2.73	0.37
	Ex-630	1.59	0.63	2.62	0.38
	Ex-3000	1.48	0.68	2.90	0.35
ERG foams	ERG-1080	1.55	0.65	2.09	0.48
	ERG-2160	1.22	0.82	1.88	0.53
	ERG-4320	1.08	0.93	1.83	0.55
	ERG-8640	1.19	0.84	1.60	0.63
Mean values		1.33	0.75	2.67	0.41

6.4.3 Friction coefficient

The results of the measurements of the friction coefficients are shown in Figure 6.23 and summarized in Table 6.7. The friction coefficient between the diamond round tip and the model materials ($\mu_{Diamond}$) ranges between 0.1 and 0.33. It decreases progressively with the AlSi content. On the other hand, μ_{TiA6V} increases with the AlSi content and ranges from 0.1 to 0.8.

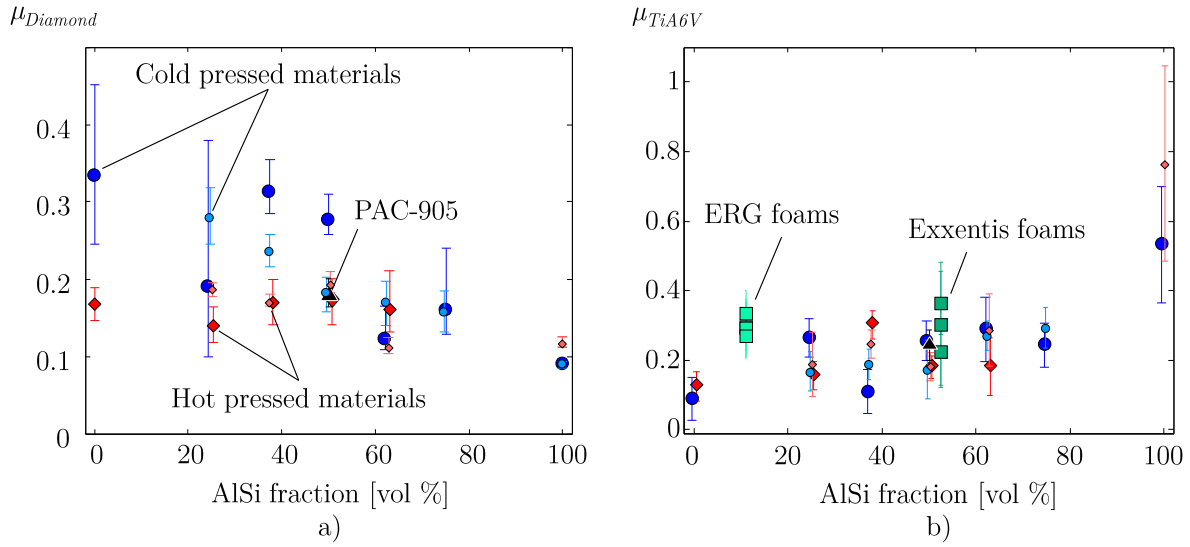


Figure 6.23: a) the friction coefficients of the model materials measured by nanoscratch with a diamond spherical tip; b) the friction coefficients of the model materials measured through pin-on-disc test with a TiA6V sphere.

Table 6.7: Summary of the friction coefficients of the model materials against TiA6V and diamond surfaces.

Model materials	Name	$\mu_{diamond}$	μ_{TiA6V}
Bulk	Polyester	0.17 ± 0.02	0.12 ± 0.04
	AlSi	0.12 ± 0.01	0.76 ± 0.28
Thermally sprayed	PAC905	0.18 ± 0.02	0.24 ± 0.04
Cold pressed (fine)	C0	0.34 ± 0.10	0.09 ± 0.06
	CF25	0.28 ± 0.04	0.17 ± 0.06
	CF37	0.24 ± 0.02	0.19 ± 0.05
	CF50	0.18 ± 0.02	0.18 ± 0.08
	CF62	0.17 ± 0.03	0.27 ± 0.04
	CF75	0.16 ± 0.03	0.29 ± 0.06
	CF100	0.09 ± 0.04	0.31 ± 0.08
Cold pressed (coarse)	CC25	0.19 ± 0.14	0.26 ± 0.05
	CC37	0.32 ± 0.03	0.11 ± 0.06
	CC50	0.28 ± 0.03	0.26 ± 0.06
	CC62	0.13 ± 0.03	0.29 ± 0.09
	CC75	0.16 ± 0.06	0.24 ± 0.06
	CC100	0.09 ± 0.003	0.53 ± 0.17
Hot pressed (fine)	HF25	0.19 ± 0.09	0.19 ± 0.09
	HF37	0.17 ± 0.01	0.25 ± 0.04
	HF50	0.19 ± 0.01	0.18 ± 0.04
	HF62	0.11 ± 0.01	0.29 ± 0.10
Hot pressed (coarse)	HC25	0.14 ± 0.02	0.16 ± 0.04
	HC37	0.17 ± 0.03	0.30 ± 0.04
	HC50	0.18 ± 0.03	0.18 ± 0.03
	HC62	0.16 ± 0.04	0.18 ± 0.08
Exxentis foams	Ex-350		0.36 ± 0.09
	Ex-630		0.22 ± 0.10
	Ex-3000		0.30 ± 0.18
ERG foams	ERG-1080		0.33 ± 0.07
	ERG-2160		0.30 ± 0.07
	ERG-4320		0.29 ± 0.08
	ERG-8640		0.27 ± 0.07

6.4.4 Nanoscratch

Nanoscratch with a Berkovich tip is modelled by an equivalent conical tip with a half-opening angle of 70.3° . In this case, the three models of plowing, cutting and spalling are competing with one another. The mechanism that minimizes the horizontal force (F_H) is considered as the dominant one. This dominant mechanism can change depending on the material and on the penetration depth (h). The plowing forces are modelled using the elastoplastic conical plowing model with strain hardening (see Section 4.2.4). The cutting forces are modelled using the elastoplastic pyramidal scratch cutting model with strain hardening (see Section 4.3.5). The spalling forces are modelled using the pyramidal scratch spalling model with a conical tool (see Section 4.4.3). The equations used to model the plowing, cutting and spalling forces are summarized in Table 6.8. In the case of a conical tip geometry, the cutting and the plowing models predict an evolution of F_V and F_H proportional to h^2 . This tendency is illustrated in Figure 6.16 in the case of a cold-pressed material.

In the case of the ramp loading scratch tests, h , F_V and F_H are continuously changing during the test. Therefore, in order to ease the representation, the results are presented at one given vertical force, i.e. 50 mN (see Figure 6.25). On this figure, h decreases with the AlSi content, which is logical since the hardness increases with the AlSi volume fraction. h_{wear} also decreases with the AlSi content, however, from a relative point of view, the polyester recovers 50% of the penetration depth whereas the bulk AlSi exhibits almost no elastic recovery.

A ramp load scratch test presents the advantages of covering a continuous range of vertical force and if a change in the wear mechanism occurs, the transition can be observed after testing by microscopic observations. The wear tracks are presented in Figure 6.26 and in Figure 6.27. The hot-pressed materials and the bulk materials present wear surface dominated by a plowing mechanism. The cold-pressed materials and the thermally sprayed PAC-905 present plowing and evidence of spalling with cracks and separation of fragments except in the case of 100% aluminum alloy powder. In that last case, the connection between the grains seems to be strong enough to prevent separation between the segments.

The ramp loading nanoscratch results are compared with the model predictions in Figure 6.28 using the same method as the one followed for the nanoindentation results.

Concerning the vertical force (F_V), approximately half of the model materials are overestimated and half are underestimated, also the cold-pressed materials are generally estimated with less accuracy than the hot-pressed ones. The horizontal forces (F_H) are underestimated for most of the model materials. In contrast, the relative wear depth (h_{wear}/h) is overestimated by the model and the prediction is worse in the case cold-pressed materials, especially for the coarse powder.

The results of the nanoscratch performed with a progressive ramp loading mode are summarized in Table 6.9. The prediction of the dominant wear mechanism is in good agreement with the SEM observations for most of the model materials. The precision of the model, averaged over all the test materials is 68% for F_V , 38% for F_H and 39% for h_{wear}/h .

Table 6.8 : Summary of the model used to predict the nanoscratch forces

Variable name	Conical <i>plowing</i> model (Section 4.2.1)	Pyramidal <i>cutting</i> model (Section 4.3.54.1.3)	Pyramidal <i>spalling</i> model (Section 4.4.34.1.3)
Vertical force (F_V)	Eq. 3.1	Eq. 4.99	Eq. 3.1
Horizontal force (F_H)	Eq. 4.42	Eq. 4.99	Eq. 4.109
Hardness (H_{cpl})	Eq. 4.12 & Table 3.1		Eq. 4.12 & Table 3.1
Radius of contact (a)	Eq. 4.26 & Table 3.1		Eq. 4.26 & Table 3.1
Representative flow stress (σ_r)	Eq. 3.7	Eq. 4.99	Eq. 3.7
Representative strain (ϵ_r)	Eq. 4.63	Eq. 4.99 (cutted layer) & Eq. 4.63 (plowed layer)	Eq. 4.63
Constrain factor (C_{plr})	Eq. 4.36 & Table 3.1		Eq. 4.36 & Table 3.1
Representative pile-up factor (D_{plr})	Eq. 4.37 & Table 3.1		Eq. 4.37 & Table 3.1
Height of wear (h_{wear})	Eq. 4.3 & Eq. 4.29	Eq. 4.75	Table 4.3
Cross section area of the residual groove (A_{wear})	Eq. 4.46	Eq. 4.92	Table 4.3

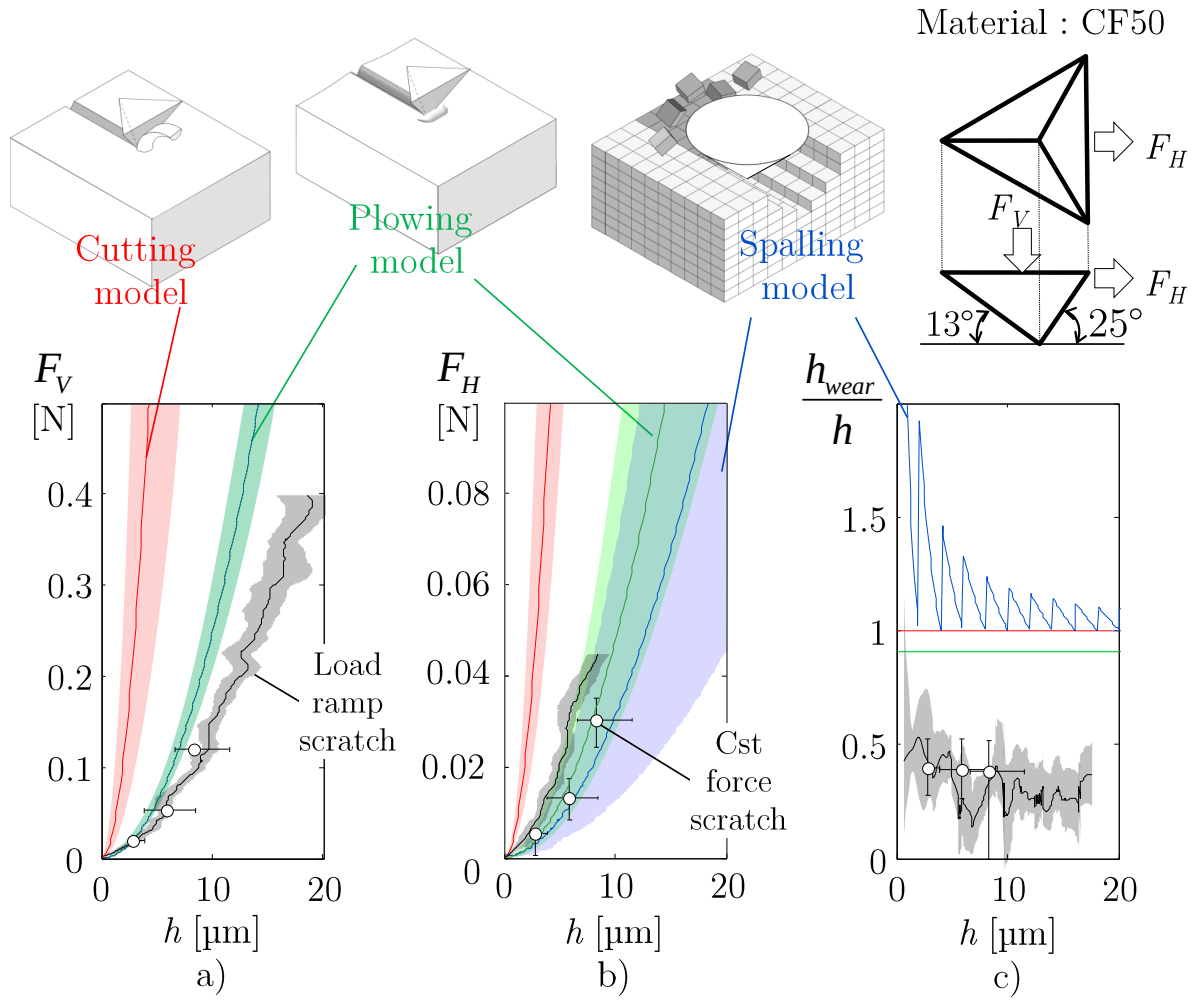


Figure 6.24: Comparison between the plowing, cutting and spalling model predictions and the nanoscratch results for a cold-pressed model material (CF50); a) the vertical force versus the penetration depth; b) the horizontal force and in c) the relative residual scratch depth. The two test configurations are presented, including the constant load test with white round dots and the ramp load tests with a black line surrounded with a grey envelope.

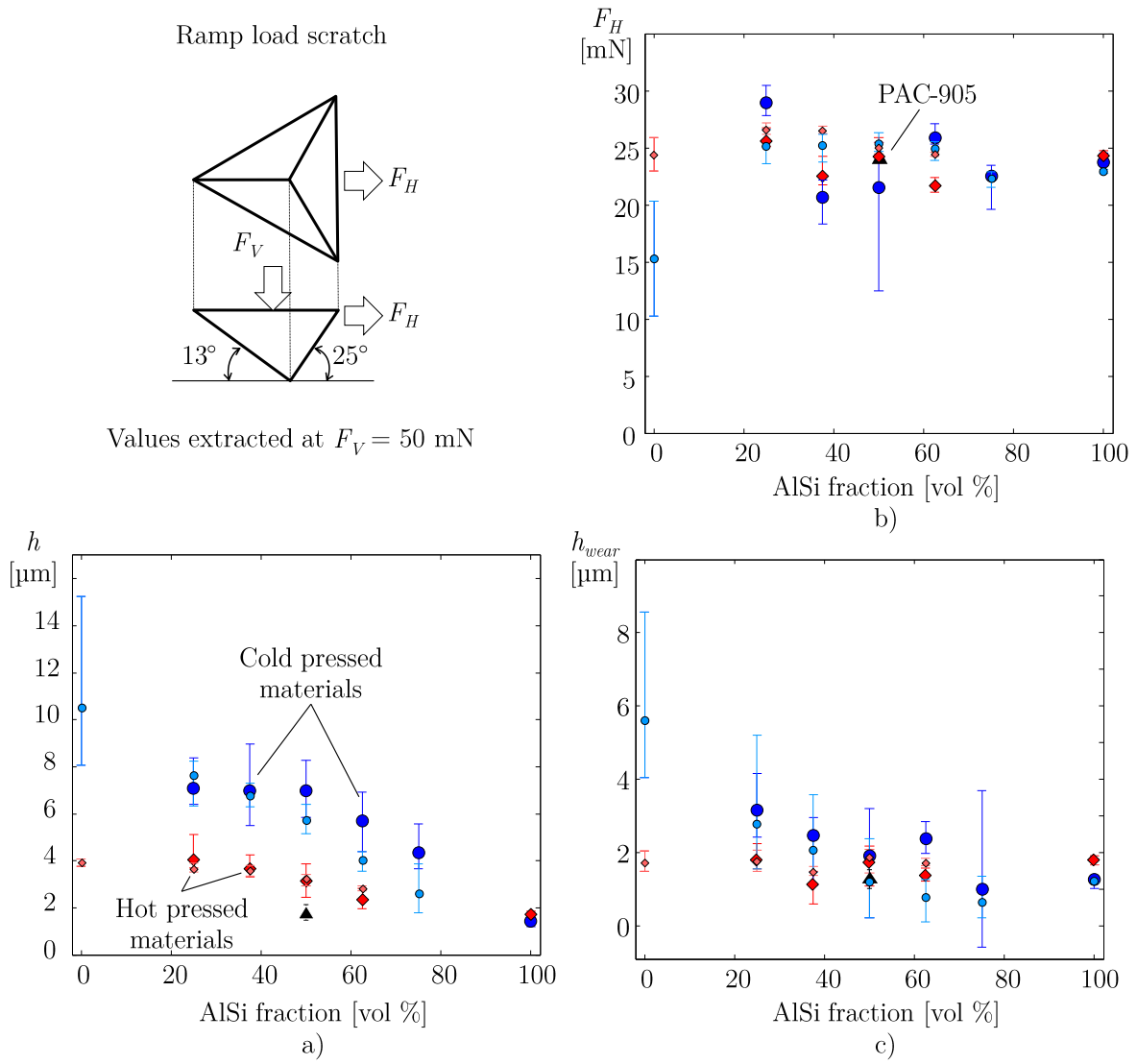


Figure 6.25: Effect of the AlSi content on a) the penetration depth, b) the horizontal force, and c) the residual depth measured by nanoscratch in the ramp load configuration.

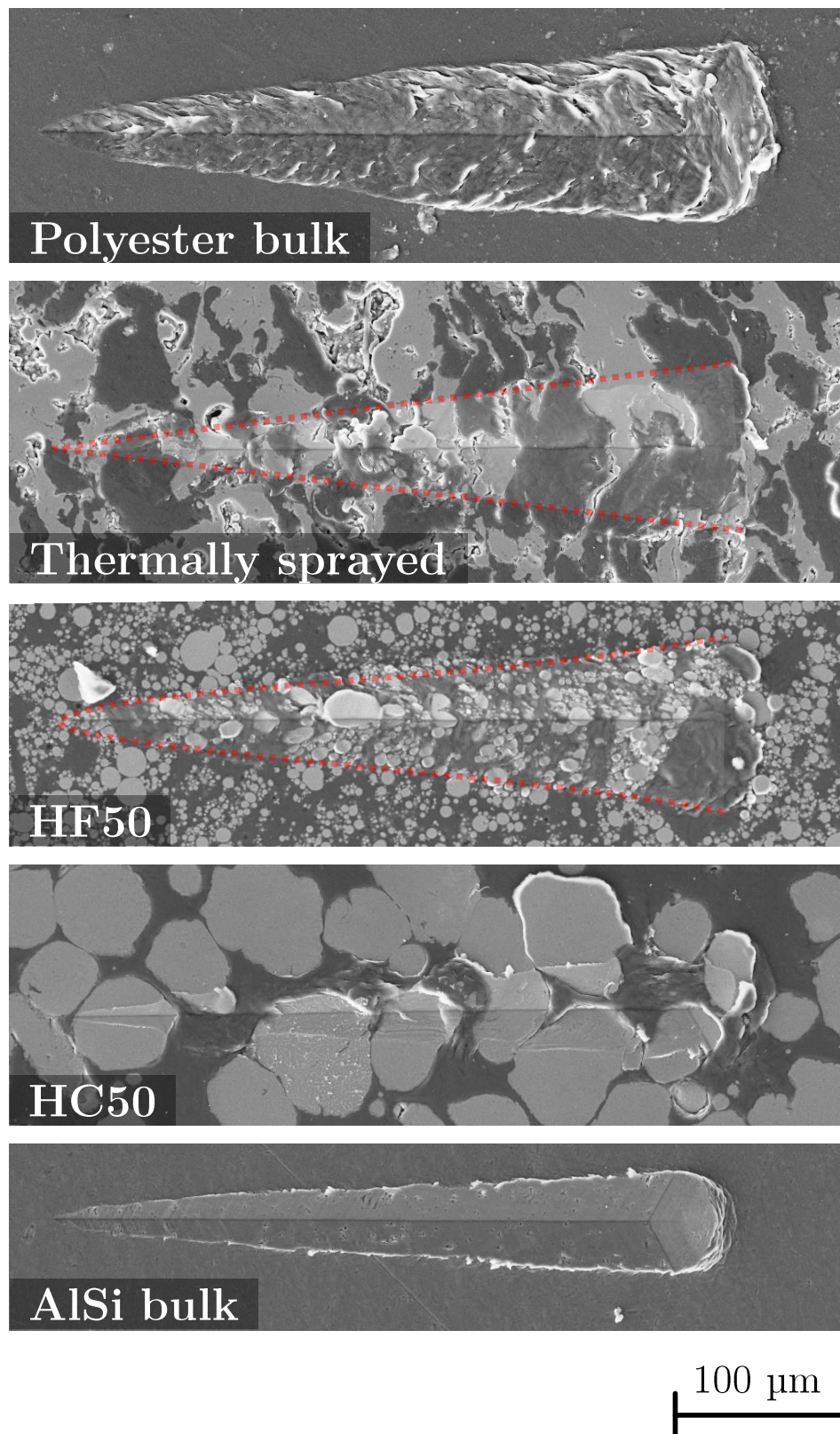


Figure 6.26: SEM micrographs of five model materials after a ramp load nanoscratch test. These materials include bulk polyester, bulk AlSi, the PAC-905, and two hot-pressed materials. The dominant wear mechanism is plowing.

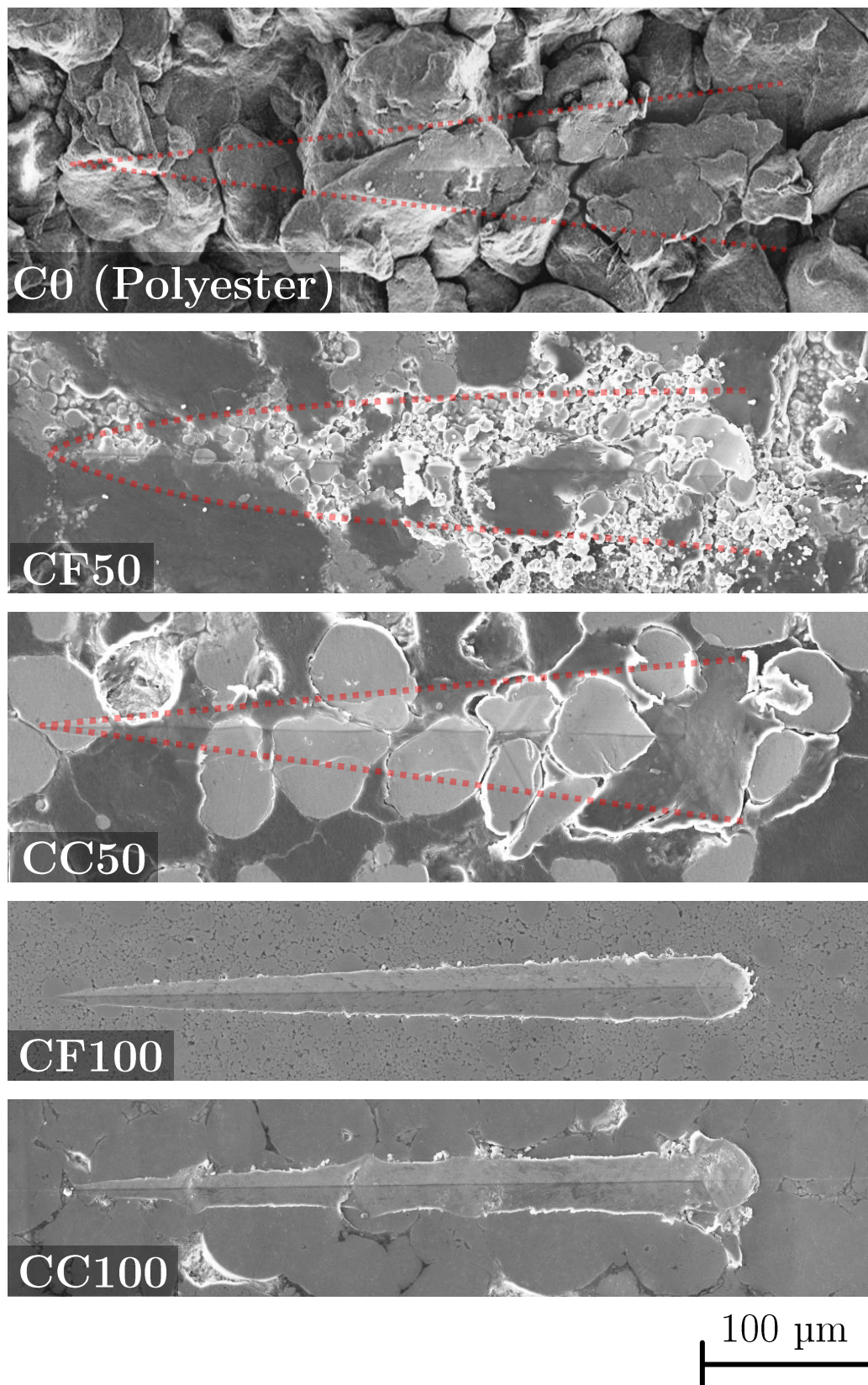


Figure 6.27: SEM micrographs of four cold-pressed materials after a ramp load nanoscratch test. The dominant wear mechanism is spalling except for the two last ones in which plowing dominates.

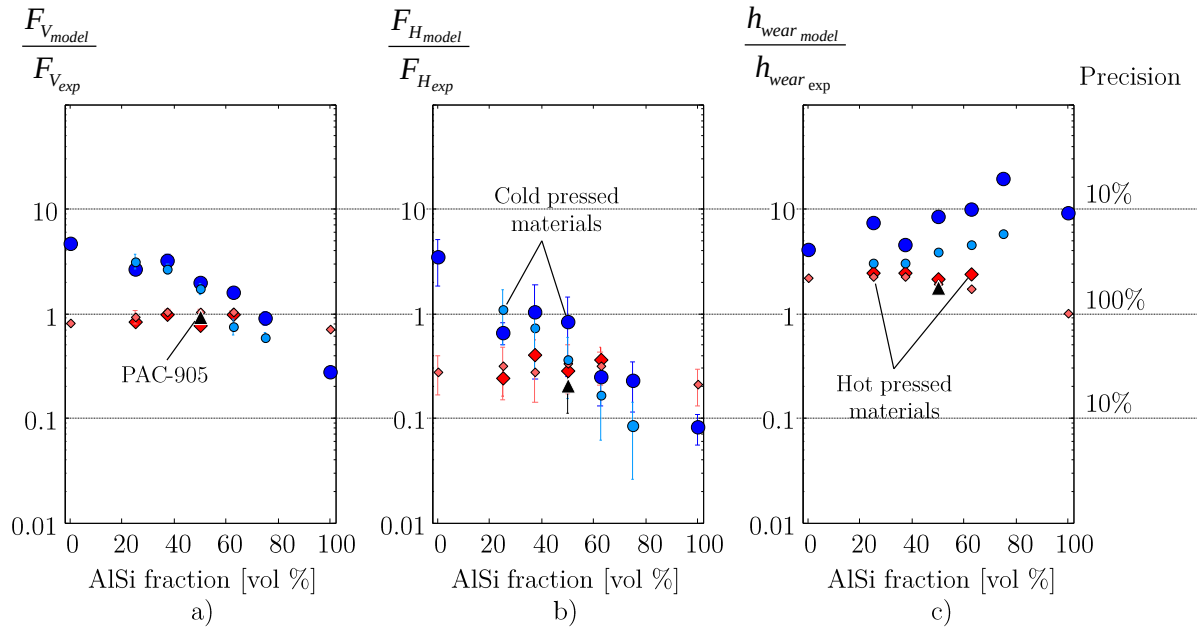
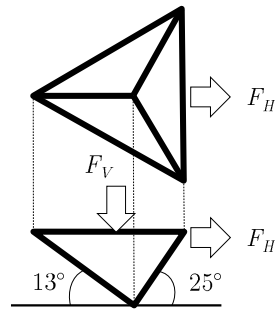


Figure 6.28: Effect of AlSi content on the ratio between the model predictions and ramp load nanoscratch measurements expressed in terms of a) vertical force, b) horizontal forces and c) residual depth. The error bars represent the error of the model divided by the mean experimental value.

Table 6.9: Summary of the comparison between the model predictions and the results from nanoscratch tests in the ramp-loading configuration.

Model materials	Name	$\frac{F_{V_{model}}}{F_{V_{exp}}}$	Precision $F_{V_{model}}$	$\frac{F_{H_{model}}}{F_{H_{exp}}}$	Precision $F_{H_{model}}$	$\frac{h_{wear_{model}}}{h_{wear_{exp}}}$	Precision $h_{wear_{model}}$	Wear Mechanism (Model)	Wear Mechanism (exp)
Bulk	Polyester	0.82	0.82	0.28	0.28	1.71	0.58	Plowing	Plowing
	AlSi	0.71	0.71	0.21	0.21	0.98	0.98	Plowing	Plowing
Thermally sprayed	PAC905	0.95	0.95	0.21	0.21	1.59	0.63	Plow/Spall	Plow/Spall
Cold pressed (fine)	C0	4.66	0.21	3.32	0.30	4.04	0.25	Plow/Spall	Plow/Spall
	CF25	3.18	0.31	1.1	0.91	2.62	0.38	" "	" "
	CF37	2.68	0.37	0.72	0.72	2.97	0.34	" "	" "
	CF50	1.72	0.58	0.37	0.37	3.92	0.26	" "	" "
	CF62	0.76	0.76	0.17	0.17	4.60	0.22	" "	" "
	CF75	0.59	0.59	0.08	0.08	5.85	0.17	" "	" "
Cold pressed (coarse)	CC25	2.64	0.38	0.66	0.66	7.51	0.13	Plow/Spall	Plow/Spall
	CC37	3.22	0.31	1.05	0.95	4.51	0.22	" "	" "
	CC50	1.99	0.50	0.85	0.85	8.49	0.12	" "	" "
	CC62	1.60	0.63	0.25	0.25	9.90	0.10	" "	" "
	CC75	0.90	0.90	0.23	0.23	19.75	0.05	" "	" "
	CC100	0.28	0.28	0.08	0.08	9.04	0.11	" "	Plowing
Hot pressed (fine)	HF25	0.94	0.94	0.32	0.32	1.86	0.54	Plowing	Plowing
	HF37	1.04	0.97	0.28	0.28	1.90	0.53	" "	" "
	HF50	1.06	0.95	0.34	0.34	1.64	0.61	" "	" "
	HF62	1.04	0.96	0.32	0.32	1.38	0.72	" "	" "
Hot pressed (coarse)	HC25	0.83	0.83	0.25	0.25	1.99	0.50	Plowing	Plowing
	HC37	0.98	0.98	0.40	0.40	2.05	0.49	" "	" "
	HC50	0.77	0.77	0.29	0.29	1.84	0.54	" "	" "
	HC62	0.98	0.98	0.36	0.36	2.14	0.47	" "	" "
Mean values		1.49	0.68	0.53	0.38	4.45	0.39		



Ramp load scratch

In the case of the constant load scratch tests, F_V is constant during the test. The results are presented in Figure 6.25 and correspond to a vertical force equal to 80 mN. The observations are the same as for the ramp load tests except for some of the cold-pressed material where the depth of penetration increases with the AlSi content which is surprising since the hardness increases with the AlSi content. Micrographs of the wear tracks are presented in Figure 6.30 and Figure 6.31. The observations are the same as in the ramp-loading scratch results.

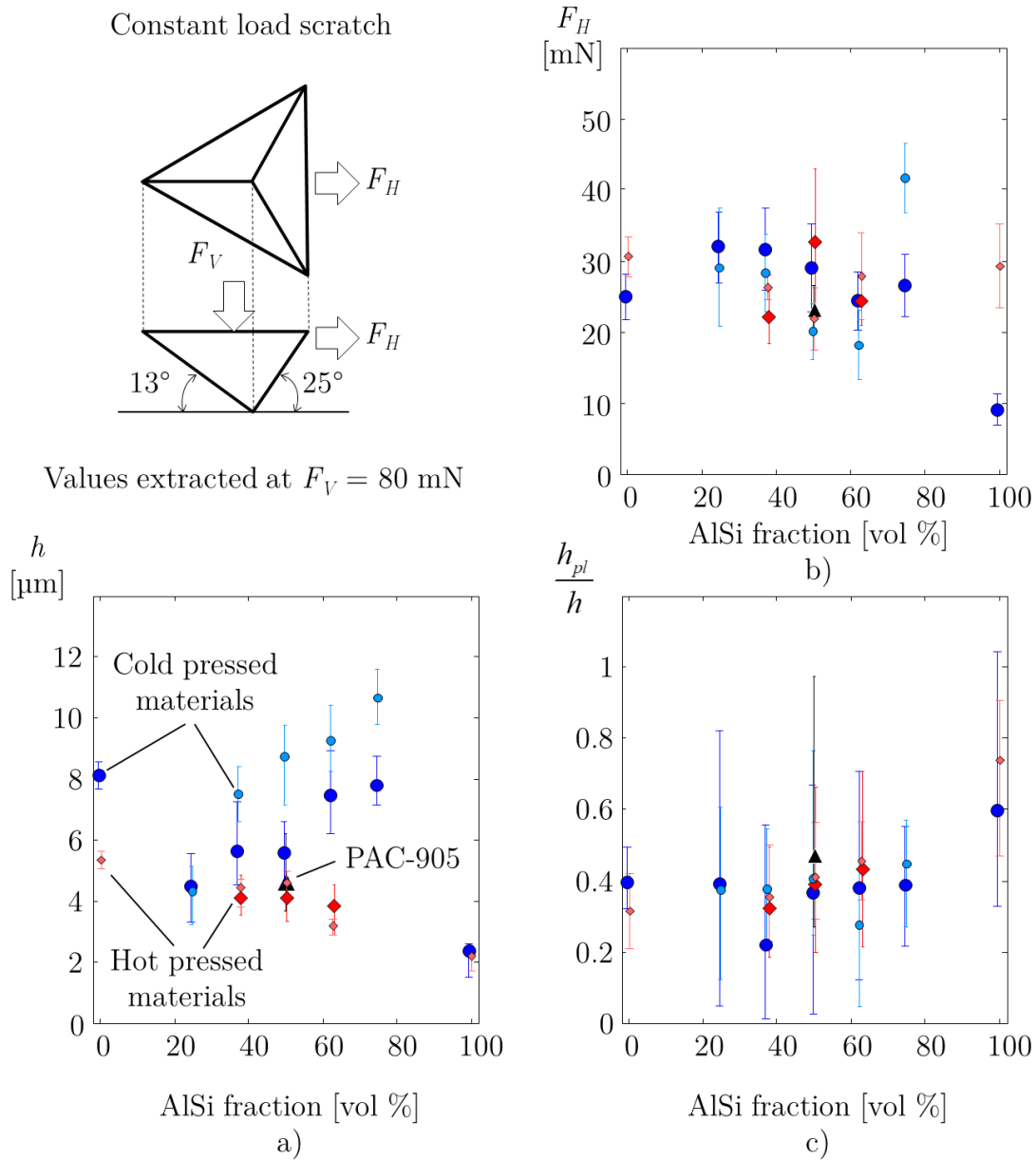


Figure 6.29: a) the penetration depth, b) the horizontal force and c) residual depth measured by nanoscratch in the constant load configuration. The results are presented as a function of the AlSi volume fraction of the model materials.

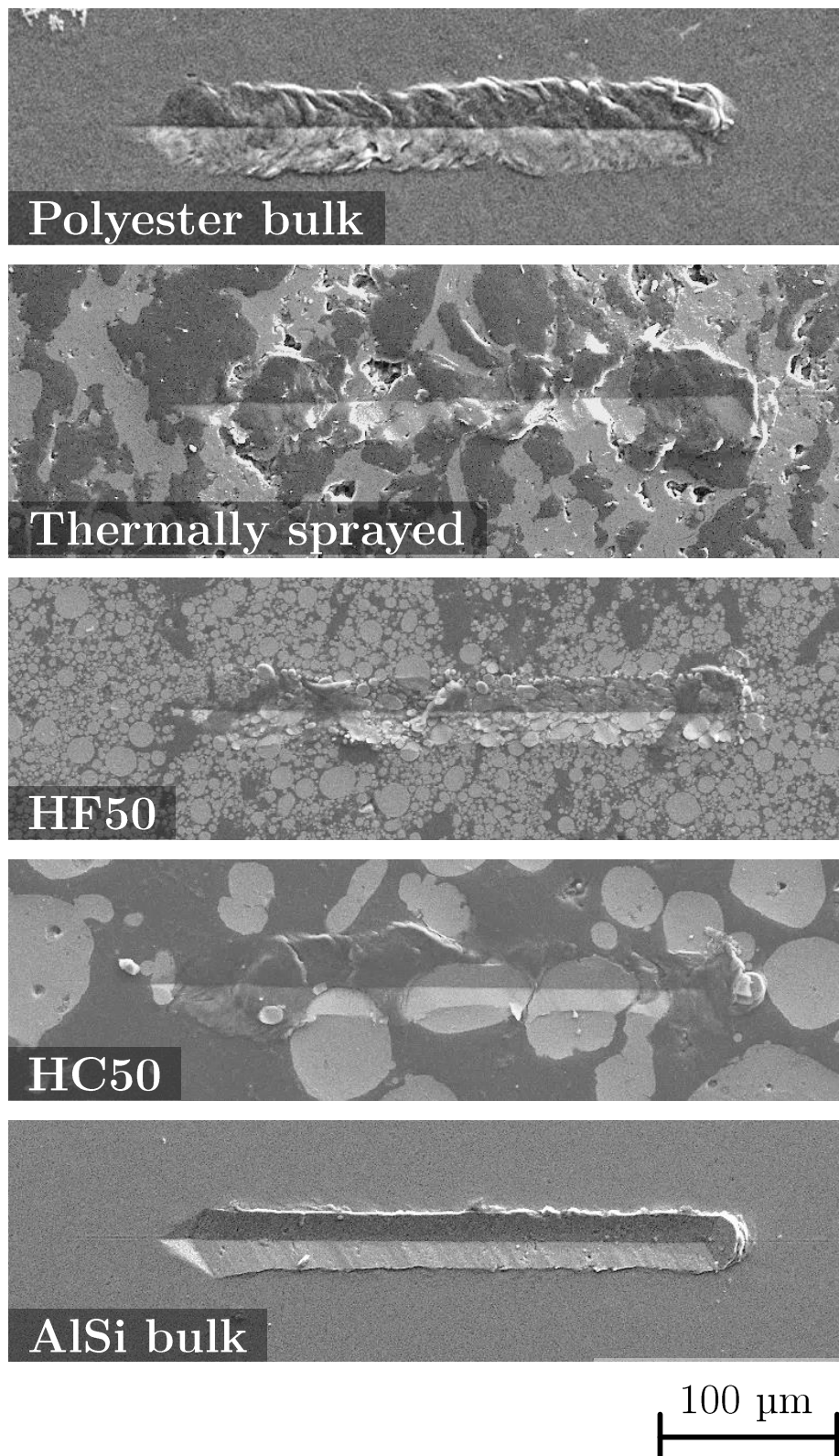


Figure 6.30: SEM micrographs of five model materials after a constant load nanoscratch test. These materials include bulk polyester, bulk AlSi, the PAC-905, and two hot-pressed materials. The dominant wear mechanism is plowing.

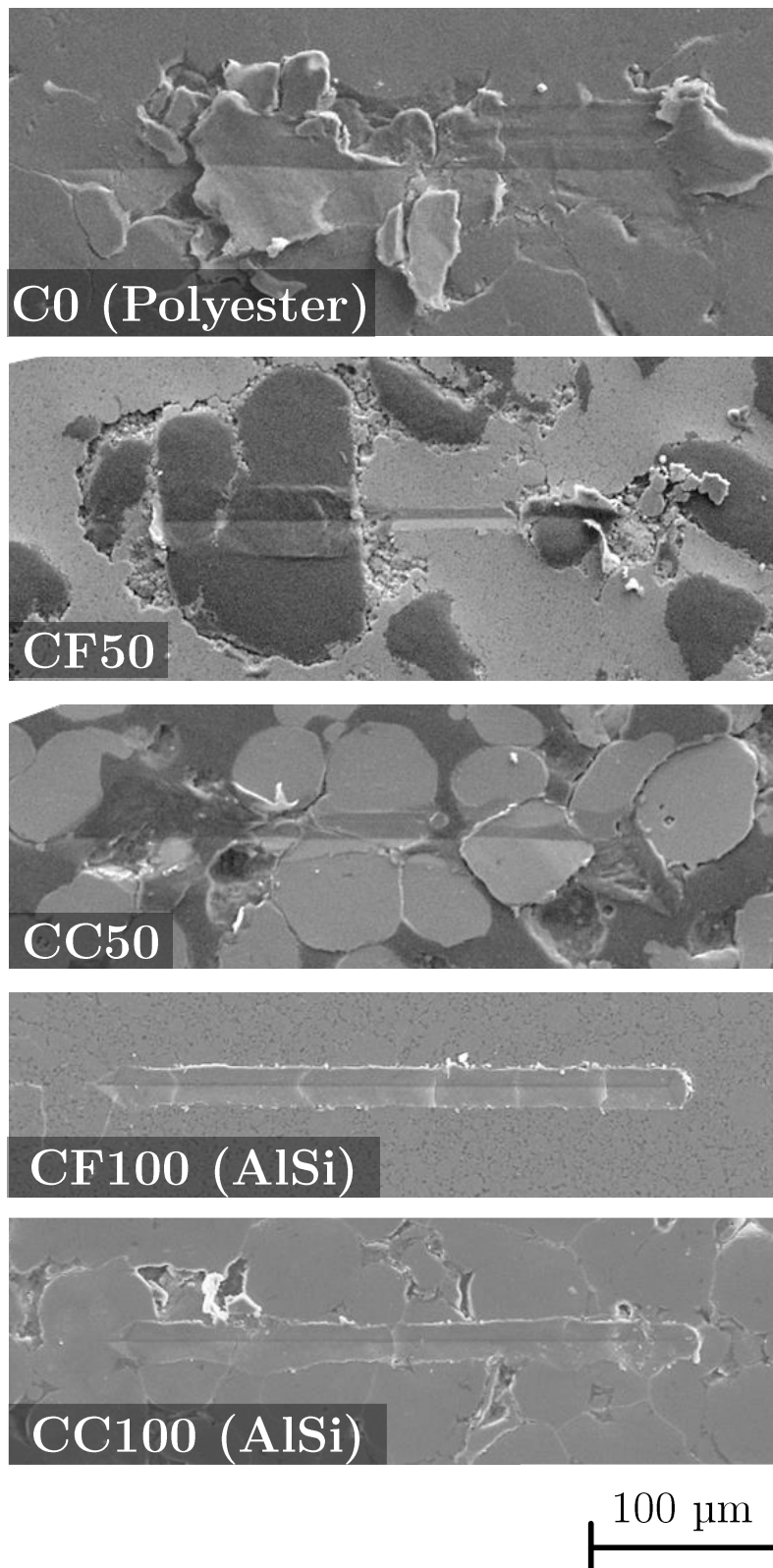


Figure 6.31: SEM micrographs of four cold-pressed materials after a ramp load nanoscratch test. The dominant wear mechanism is spalling except for the two last ones in which plowing dominates.

The constant load nanoscratch results are compared with the model in Figure 6.32 using the same method described in the nanoindentation results. The general trend of the results is similar to the ramp loading scratch tests: The model under predict F_H for most of the model materials and h_{wear}/h is over predicted by the model, particularly in the case cold-pressed materials with a coarse powder. The results of the nanoscratch performed under constant loads are summarized in Table 6.10. The prediction of the dominant wear mechanism is in good agreement with the SEM observations for most of the model materials. The precision of the model, averaged over all the test materials is 69% for F_V , 52% for F_H and 40% for h_{wear}/h .

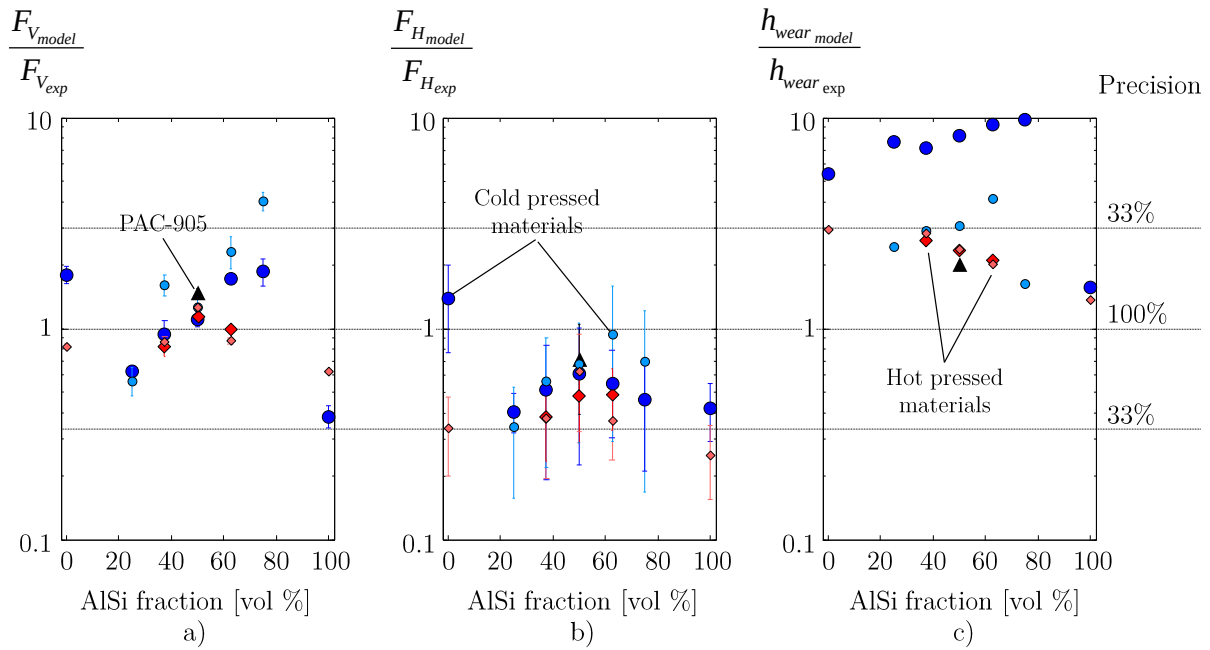
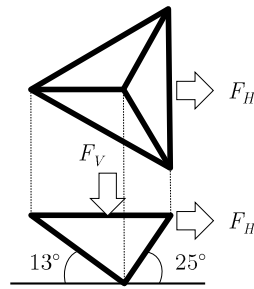


Figure 6.32: Ratios between the model predictions and the results from nanoscratch tests in the constant load configuration; a) the vertical force, b) the horizontal force and c) the relative residual depth as a function of the AlSi volume fraction of the model materials. The error bars represent the error of the model divided by the mean experimental value.

Table 6.10: Summary of the comparison between the model prediction and the results from nanoscratch tests in the constant load configuration.

Model materials	Name	$\frac{F_{V_{model}}}{F_{V_{exp}}}$	Precision $F_{V_{model}}$	$\frac{F_{H_{model}}}{F_{H_{exp}}}$	Precision $F_{H_{model}}$	$\frac{h_{wear_{model}}}{h_{wear_{exp}}}$	Precision $h_{wear_{model}}$	Wear Mechanism (Model)	Wear Mechanism (exp)
Bulk	Polyester	0.83	0.83	0.34	0.34	2.32	0.43	Plowing	Plowing
	AlSi	0.63	0.63	0.25	0.25	1.31	0.76	" "	" "
Thermally sprayed	PAC905	1.48	0.68	0.72	0.72	1.69	0.59	Plowing	Plowing
Cold pressed (fine)	C0	1.80	0.55	1.39	0.72	5.35	0.19	Plow/Spall	Plow/Spall
	CF25	0.57	0.57	0.34	0.34	2.07	0.48	" "	" "
	CF37	1.62	0.62	0.56	0.56	2.91	0.34	" "	" "
	CF50	1.27	0.78	0.68	0.68	3.06	0.33	" "	" "
	CF62	2.33	0.43	0.94	0.94	4.16	0.24	" "	" "
	CF75	4.04	0.25	0.70	0.70	1.64	0.61	" "	" "
Cold pressed (coarse)	CC25	0.63	0.63	0.40	0.40	7.64	0.13	Plow/Spall	Plow/Spall
	CC37	0.94	0.94	0.52	0.52	7.05	0.14	" "	" "
	CC50	1.11	0.90	0.62	0.62	8.20	0.12	" "	" "
	CC62	1.74	0.58	0.54	0.54	9.24	0.11	Plowing	" "
	CC75	1.87	0.53	0.46	0.46	9.85	0.10	" "	" "
	CC100	0.38	0.38	0.42	0.42	1.43	0.70	Plow/Spall	Plowing
Hot pressed (fine)	HF37	0.87	0.87	0.38	0.38	2.40	0.42	Plowing	Plowing
	HF50	1.26	0.79	0.63	0.63	2.03	0.49	Plowing	Plowing
	HF62	0.87	0.87	0.37	0.37	1.68	0.60	Plowing	Plowing
Hot pressed (coarse)	HC37	0.82	0.82	0.38	0.38	2.18	0.46	Plowing	Plowing
	HC50	1.16	0.87	0.48	0.48	1.99	0.50	" "	" "
	HC62	1.00	1.00	0.49	0.49	1.79	0.56	" "	" "
Mean values		1.30	0.69	0.55	0.52	3.97	0.40		



Constant load scratch

6.4.5 Pin-on-disc

The pin-on-disc test with a cutting tool is modelled with a half-wedge having a 30° angle between the horizontal and the inclined face. Once more, the three models of plowing, cutting and spalling are competing with one another. The *plowing* forces are modelled using the elastoplastic half-wedge plowing model with strain hardening (see Section 4.2.2 and Section 4.2.4). The *cutting* forces are modelled using the elastoplastic •rthogonal inclined wedge cutting model with strain hardening (see Section 4.3.5). The *spalling* forces are modelled using the orthogonal spalling with an model with an inclined wedge (see Section 4.4.1). The equations used to model the plowing, cutting and spalling forces are summarized in Table 6.11. In the case of the half-wedge tip geometry, the cutting and the plowing models predict an evolution of F_V and F_H proportional to h . This trend is illustrated in Figure 6.33 in the case of a cold-pressed material. In this figure, the model predictions indicate that the dominant wear mechanism should be spalling.

Table 6.11: Summary of the model used to predict the pin-on-disc test forces

Variable name	Half wedge <i>plowing</i> model (Section 4.2.2)	Half wedge <i>cutting</i> model (Section 4.3.5)	Half wedge <i>spalling</i> model (Section 4.4.1)
Vertical force (F_V)	Eq. 3.1	Eq. 4.95	Eq. 3.1
Horizontal force (F_H)	Eq. 4.48	Eq. 4.95	Eq. 4.109
Hardness (H_{cpl})	Eq. 4.12 & Table 3.1		Eq. 4.12 & Table 3.1
Radius of contact (a)	Eq. 4.26 & Table 3.1		Eq. 4.26 & Table 3.1
Representative flow stress (σ_r)	Eq. 3.7	Eq. 4.94	Eq. 3.7
Representative strain (ϵ_r)	Eq. 4.63	Eq. 4.94 (cutted layer) & Eq. 4.63 (plowed layer)	Eq. 4.63
Constrain factor (C_{plr})	Eq. 4.36 & Table 3.1		Eq. 4.36 & Table 3.1
Representative pile-up factor (D_{plr})	Eq. 4.37 & Table 3.1		Eq. 4.37 & Table 3.1
Height of wear (h_{wear})	Eq. 4.49	Eq. 4.75	Table 4.2
Cross section area of the residual groove (A_{wear})	Eq. 4.50	Eq. 4.50	Table 4.2

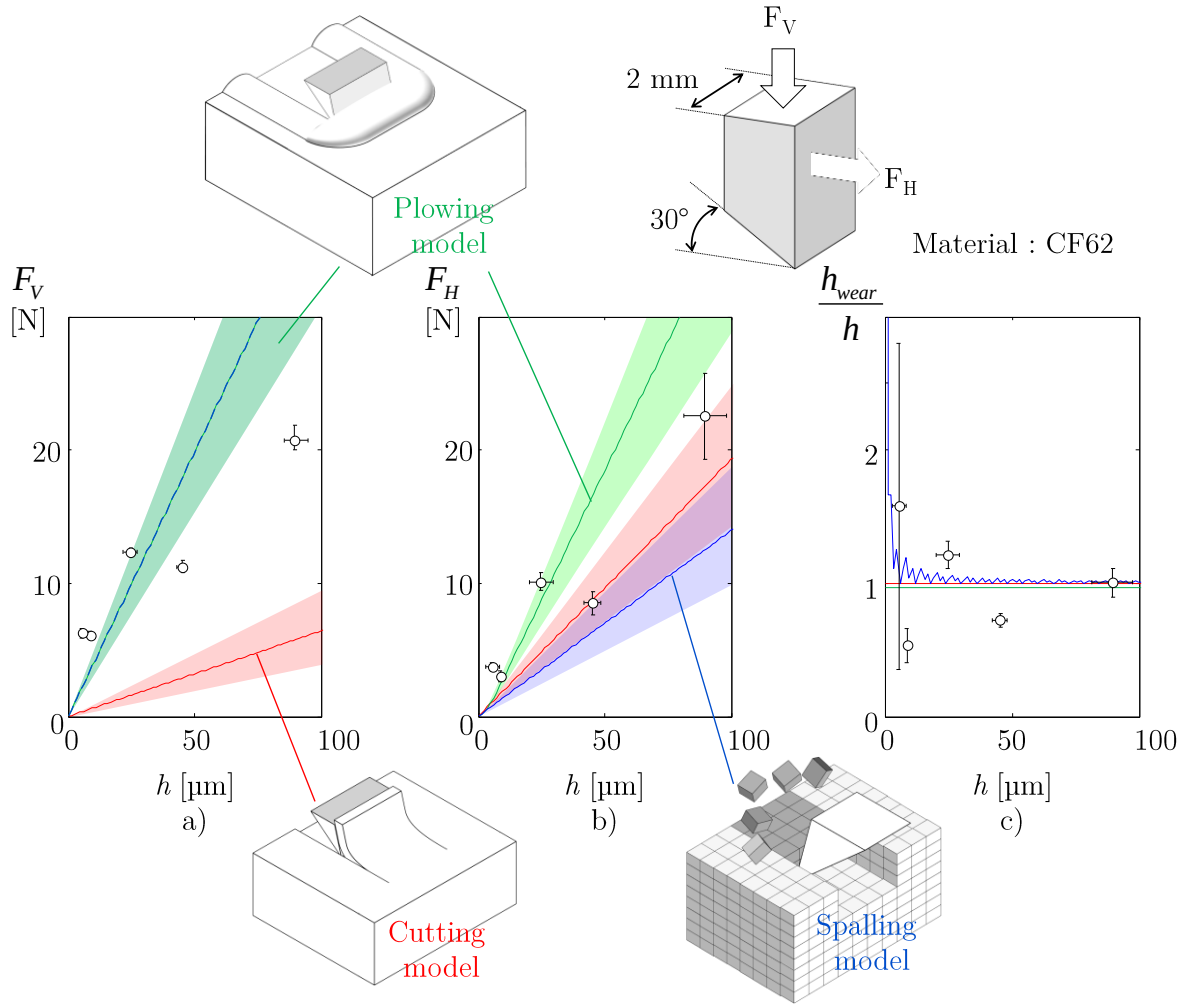


Figure 6.33: Comparison between the plowing, cutting and spalling model versus the pin-on-disc experimental results on a cold-pressed model material (CF62). The white round dots are the experimental results. The left graph shows the vertical force versus the penetration depth. In the center, the horizontal force is presented and, the left graph shows the relative residual scratch depth.

Regarding pin-on-disc results, F_V , F_H , and h_{wear} increase proportionally to the penetration depth. In order to compare the experimental results of all model materials, a linear regression is applied on the results. F_V , F_H , and h_{wear} are then extracted, using this linear regression at a penetration depth of 200 μm (see Figure 6.34).

F_V , F_H , and h_{wear}/h increase with the AlSi content which makes sense since material involving a higher AlSi content are both harder and more rigid. In addition, hot-pressed materials exhibit larger F_V and F_H than the cold-pressed ones. The metallic foams infiltrated with polyester exhibit even larger vertical and horizontal forces.

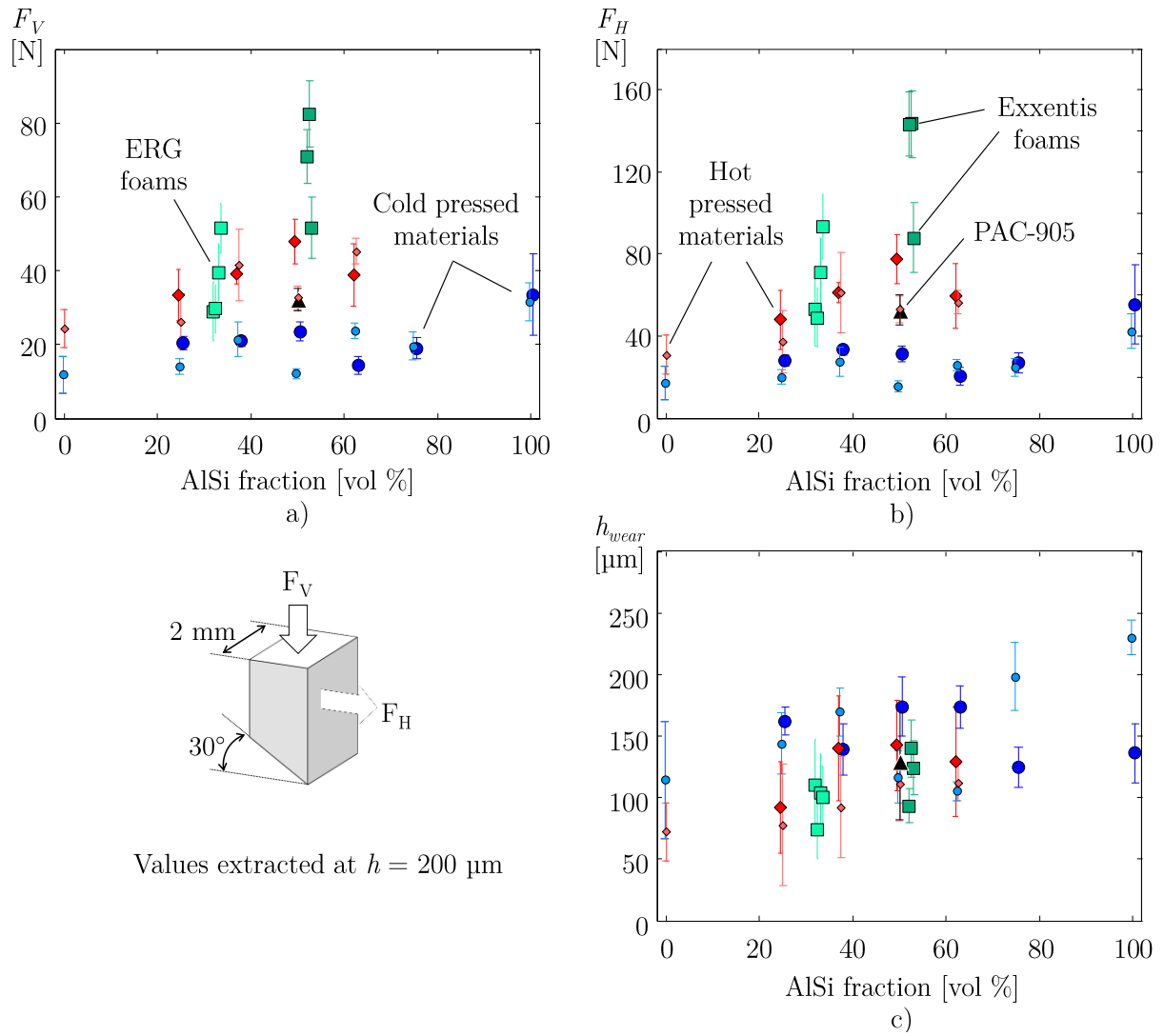


Figure 6.34: Variation of a) vertical force, b) horizontal force and c) residual depth measured by pin-on-disc tests as a function of the AlSi volume fraction of the model materials. These values are measured at a depth of $200 \mu\text{m}$ by extrapolation of a linear regression on the experimental results. The error bars are the error associated with the linear regression.

The appearances of the surfaces after the tests are shown in Figure 6.35. Mainly two wear mechanisms are primarily present including *spalling* in the case of cold-pressed materials and *cutting* for the other ones with continuous and discontinuous chip.

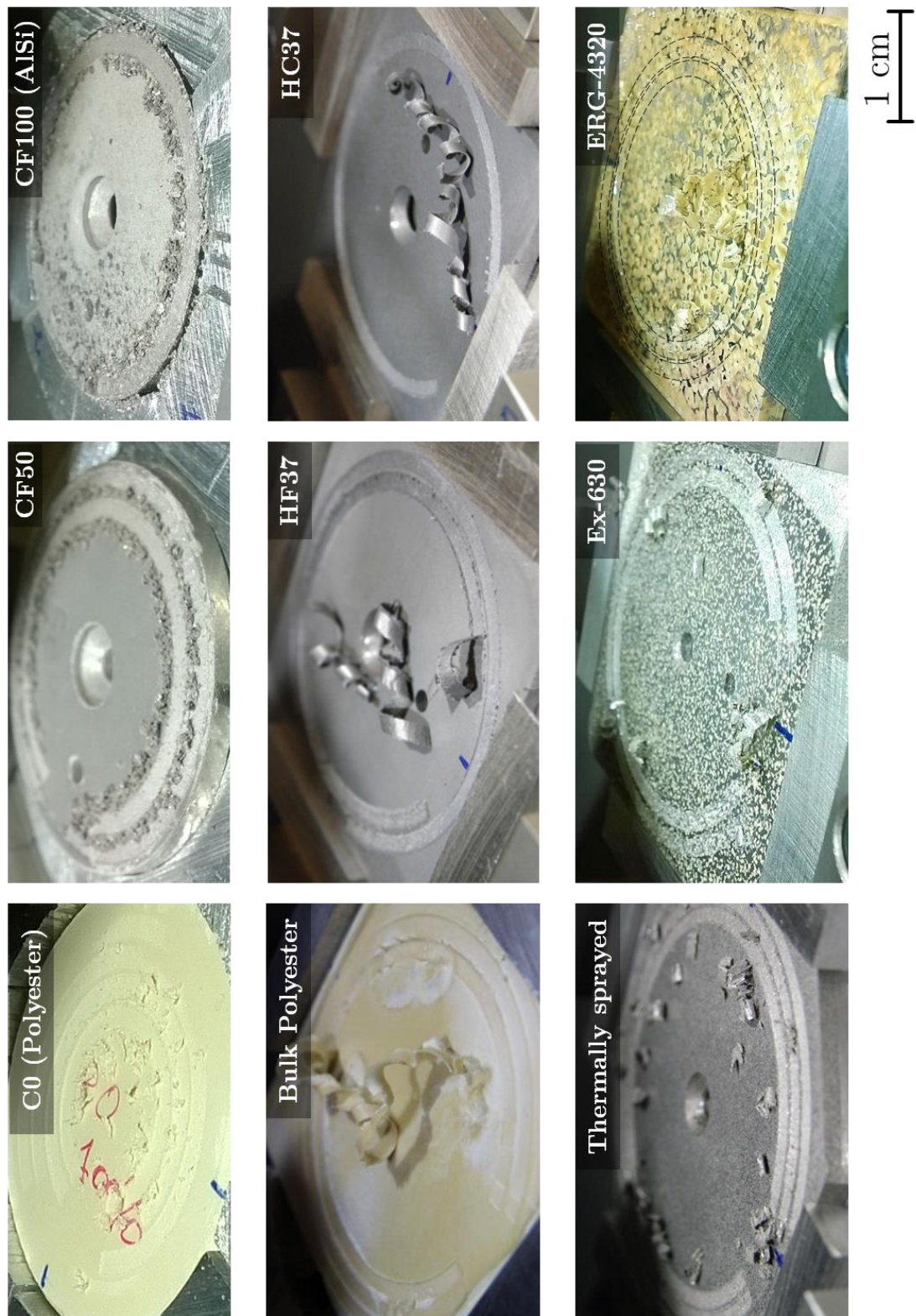


Figure 6.35: Pictures of the pin-on-disc tests samples after the test. The cold-pressed materials exhibit a spalling wear mechanism whereas the other materials show evidence of cutting with continuous and discontinuous chipping.

SEM micrographs of the wear tracks are presented in Figure 6.36, Figure 6.37 and Figure 6.38. These figures show that, in the case of cold-pressed materials, the spalling is sometimes accompanied by plastic deformations of the underneath surface.

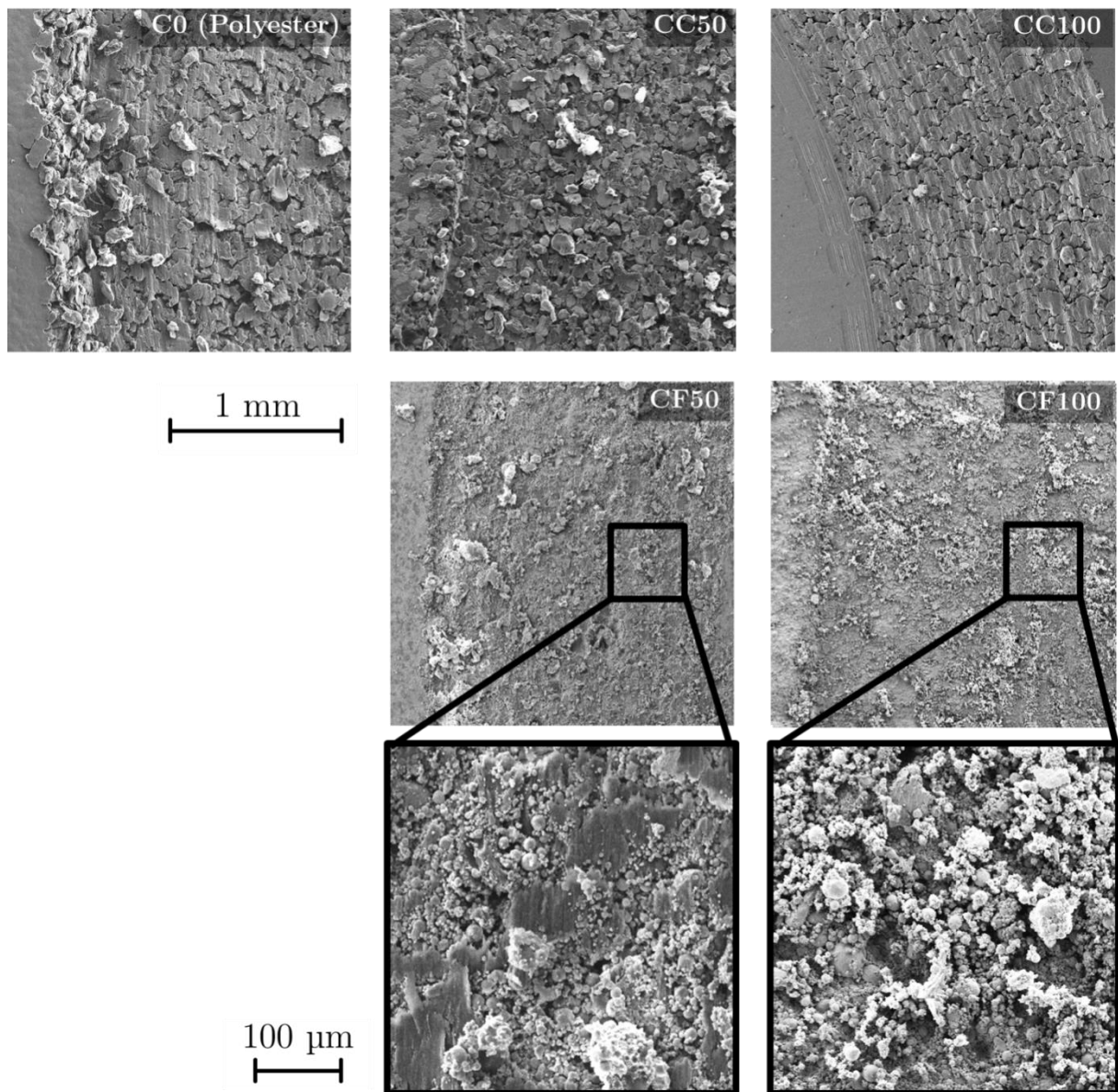


Figure 6.36: SEM micrographs showing a wear track after a pin-on-disc test on of three cold-pressed materials.

Regarding the other model materials, cutting is the dominant mechanism. However, in some cases, cutting is accompanied with large plastic deformations of the underneath surface or with cracking and detachments of fragments of materials.

A micrograph of a chip from the thermally sprayed PAC-905 material is presented in Figure 6.39. It appears that the chip is non-continuous and covered with cracks. This last figure suggests that, in some situations, the real wear mechanism can be a combination of cutting and spalling.

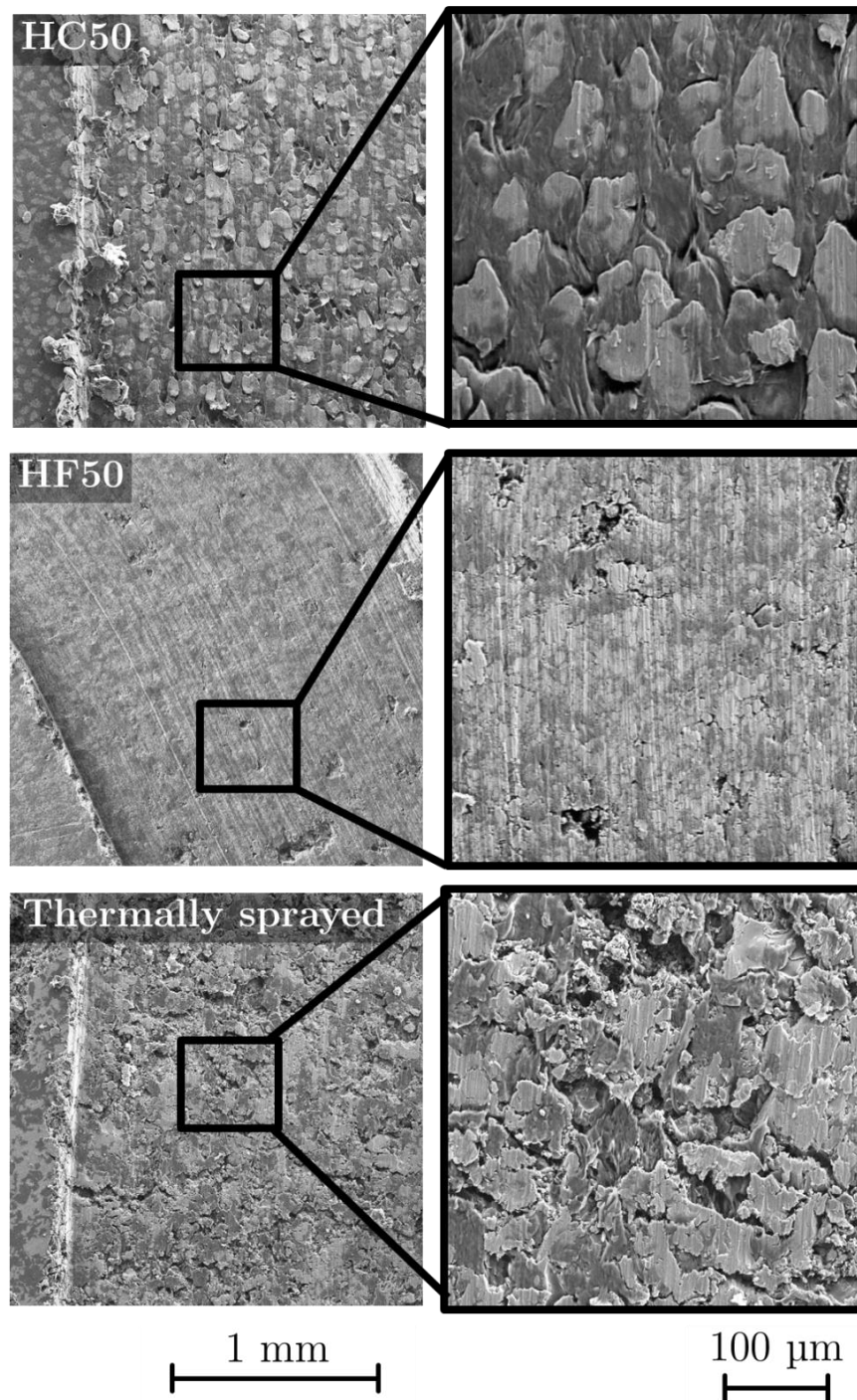


Figure 6.37: SEM micrographs showing a wear track after a pin-on-disc test on hot-pressed model materials and the thermally sprayed PAC-905 material.

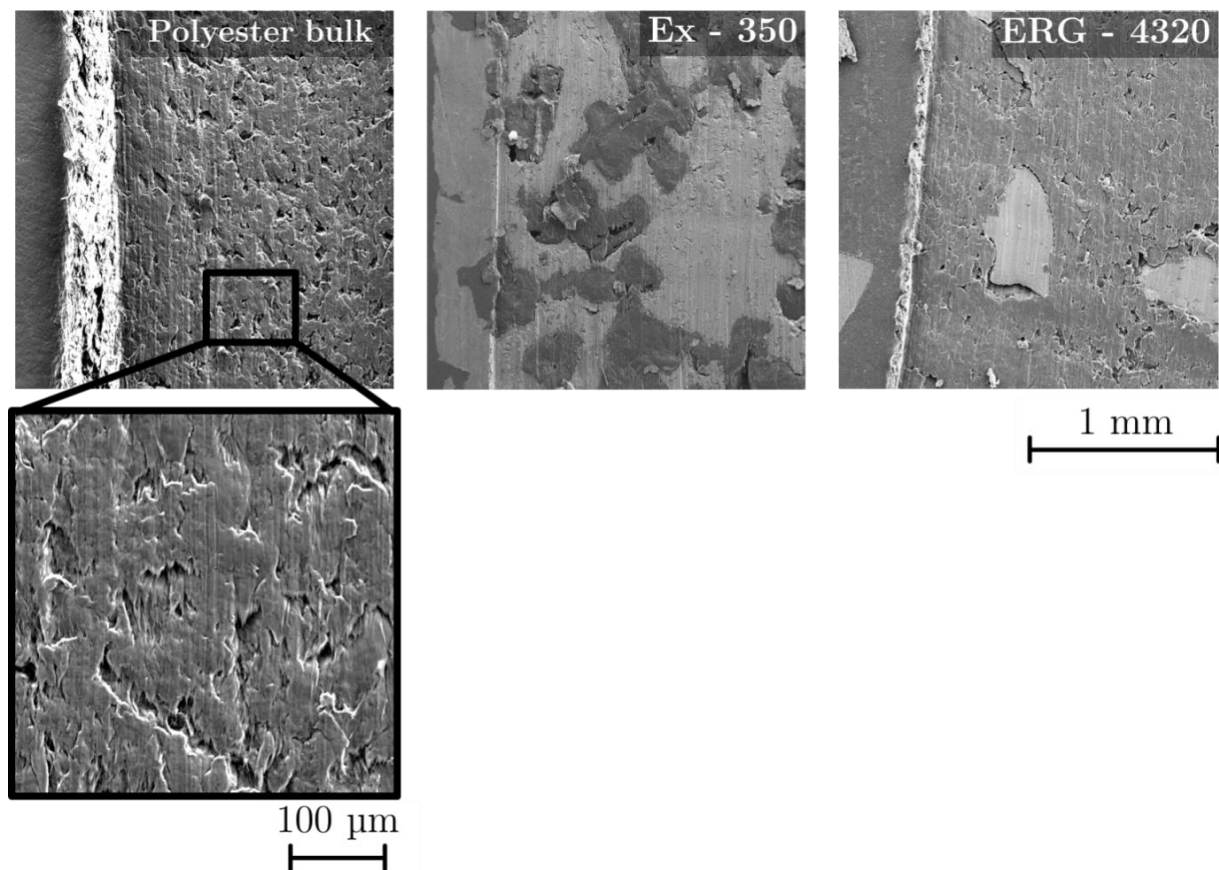


Figure 6.38: SEM micrographs showing a wear track after a pin-on-disc test on bulk polyester and two foams infiltrated with polyester.

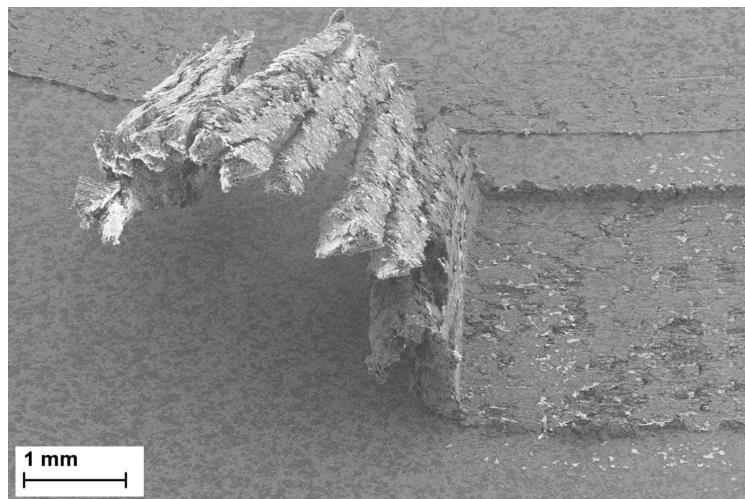


Figure 6.39: SEM micrograph of the thermally sprayed PAC-905 material after a test on the tribometer. The chip is discontinuous and the wear surface presents both rough and smoothly cut areas.

The pin-on-disc test results are compared with model predictions in Figure 6.40 using the same method described in the nanoindentation results. The model overestimates F_V in the case of cold-pressed materials. Concerning the other materials, F_V is slightly underestimated by the model. h_{wear}/h is overestimated by the model for most of the model materials.

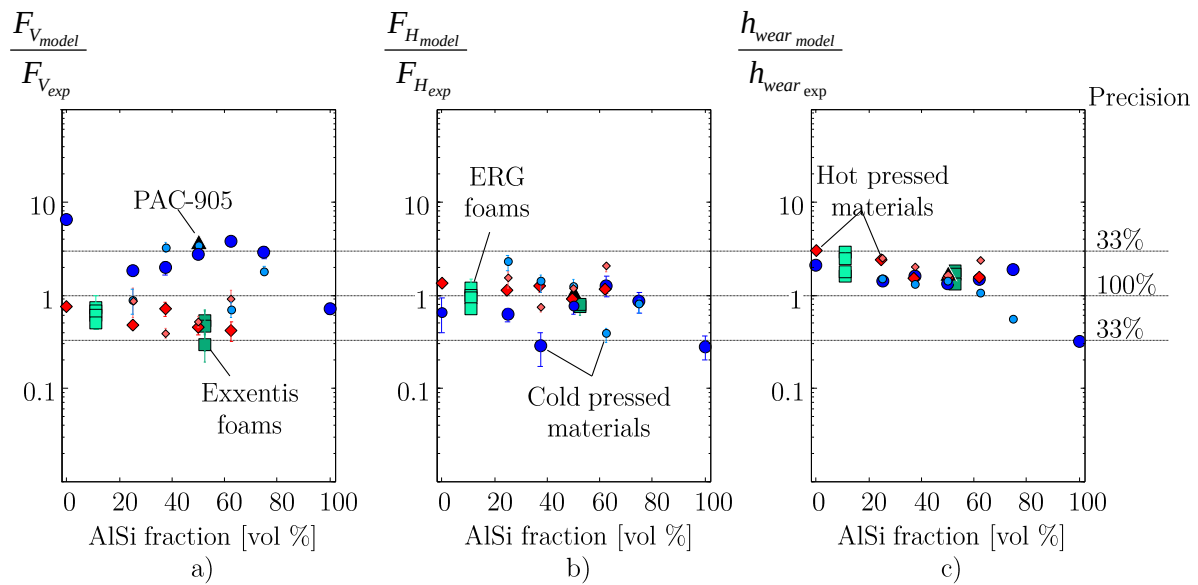
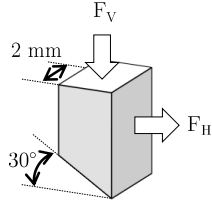


Figure 6.40: Effect of AlSi content on the ratios between the model predictions and the pin-on-disc tests results expressed in terms of a) vertical force, b) horizontal forces and c) residual depth. The error bars represent the error of the model divided by the mean experimental value.

The results are summarized in Table 6.12. The prediction of the dominant wear mechanism is in good agreement with the SEM observations for most of the model materials. The precision of the model, averaged over all the test materials is 52% for F_V , 75% for F_H and 60% for h_{wear}/h .

Table 6.12: Comparisons between the model predictions and the pin-on-disc test results.



Type of model materials	Name	$\frac{F_{V_{model}}}{F_{V_{exp}}}$	Precision $F_{V_{model}}$	$\frac{F_{H_{model}}}{F_{H_{exp}}}$	Precision $F_{H_{model}}$	$\frac{h_{wear_{model}}}{h_{wear_{exp}}}$	Precision $h_{wear_{model}}$	Wear Mechanism (Model)	Wear Mechanism (exp)
Bulk	Polyester	0.75	0.75	1.37	0.73	3.12	0.32	Cutting	Cutting
	AlSi								
Thermally sprayed	PAC905	3.84	0.26	1.11	0.90	1.72	0.58	Spalling	Cut / Spall
Cold pressed (fine)	C0	6.58	0.15	0.67	0.67	2.27	0.44	Spalling	Spalling
	CF25	0.89	0.89	2.24	0.45	1.52	0.66	Cutting	" "
	CF37	3.31	0.30	1.35	0.74	1.25	0.80	Spalling	" "
	CF50	3.50	0.29	1.19	0.84	1.37	0.73	" "	" "
	CF62	0.71	0.71	0.38	0.38	1.06	0.95	" "	" "
	CF75	2.20	0.45	0.94	0.94	1.00	1.00	" "	" "
Cold pressed (coarse)	CC25	1.82	0.55	0.62	0.62	1.45	0.69	Spalling	Spalling
	CC37	1.97	0.51	0.28	0.28	1.59	0.63	" "	" "
	CC50	2.77	0.36	0.77	0.77	1.31	0.76	" "	" "
	CC62	3.80	0.26	1.27	0.78	1.47	0.68	" "	" "
	CC75	3.77	0.27	0.94	0.94	2.19	0.46	" "	" "
	CC100	1.64	0.61	0.65	0.65	0.32	0.32	Plow / Spall	Spalling
Hot pressed (fine)	HF25	0.89	0.89	1.50	0.67	2.42	0.41	Cutting	Cutting
	HF37	0.40	0.40	0.71	0.71	1.92	0.52	Plow / Cutt	" "
	HF50	0.52	0.52	1.18	0.85	1.65	0.61	Plow / Cutt	" "
	HF62	0.93	0.93	1.98	0.50	2.29	0.44	Cutting	" "
Hot pressed (coarse)	HC25	0.48	0.48	1.09	0.92	2.42	0.41	Cutting	Cutting
	HC37	0.72	0.72	1.21	0.82	1.49	0.67	" "	" "
	HC50	0.45	0.45	0.88	0.88	1.58	0.63	" "	" "
	HC62	0.42	0.42	1.14	0.88	1.52	0.66	" "	" "
Exxentis foams	Ex-350	0.53	0.53	0.75	0.75	1.80	0.56	Cutting	Cutting
	Ex-630	0.29	0.29	0.76	0.76	1.42	0.71	" "	" "
	Ex-3000	0.46	0.46	0.80	0.80	1.96	0.51	" "	" "
ERG foams	ERG-1080	0.50	0.50	0.72	0.72	2.49	0.40	Cutting	Cutting
	ERG-2160	0.61	0.61	0.92	0.92	1.77	0.56	" "	" "
	ERG-4320	0.74	0.74	1.20	0.83	2.93	0.34	" "	" "
	ERG-8640	0.70	0.70	1.07	0.93	1.62	0.62	" "	" "
Mean values		1.59	0.52	1.02	0.75	1.76	0.60		

6.4.6 Abradability

Wear tracks produced during abradability tests are presented in Figure 6.42, Figure 6.43, Figure 6.41 and Figure 6.44. These figures show that, with the three tested conditions, the dominant wear mechanism is cutting. In some cases, cutting is combined with spalling, smearing of the polymer phase and grooving. On these figures, *BPB* refers to the bite per blade.

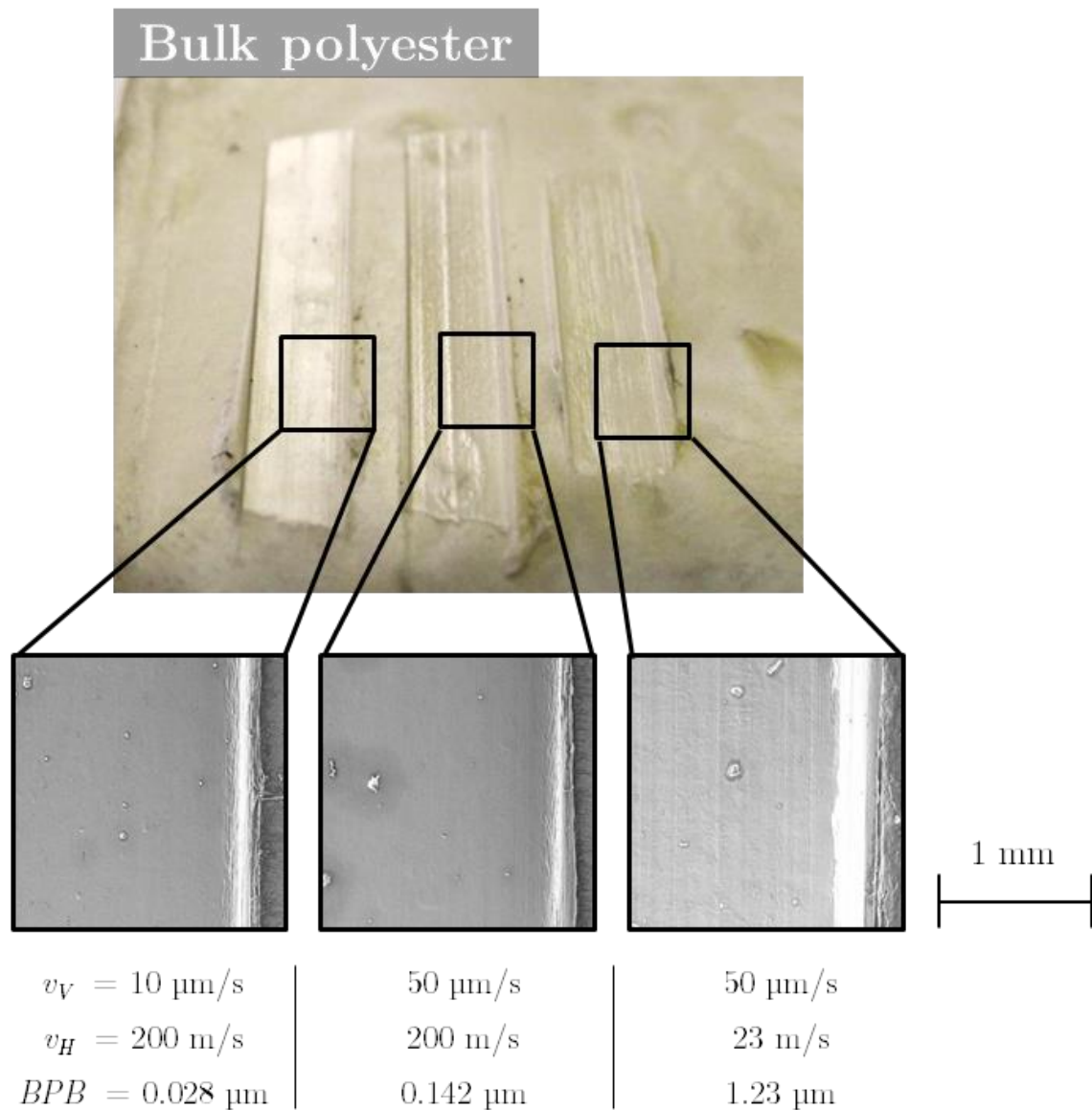


Figure 6.41: Macro pictures and SEM micrographs showing three wear tracks corresponding to the bulk polyester after a test performed with the abradability test rig. The dominant wear mechanism is cutting sometimes combined with grooving depending on the test conditions.

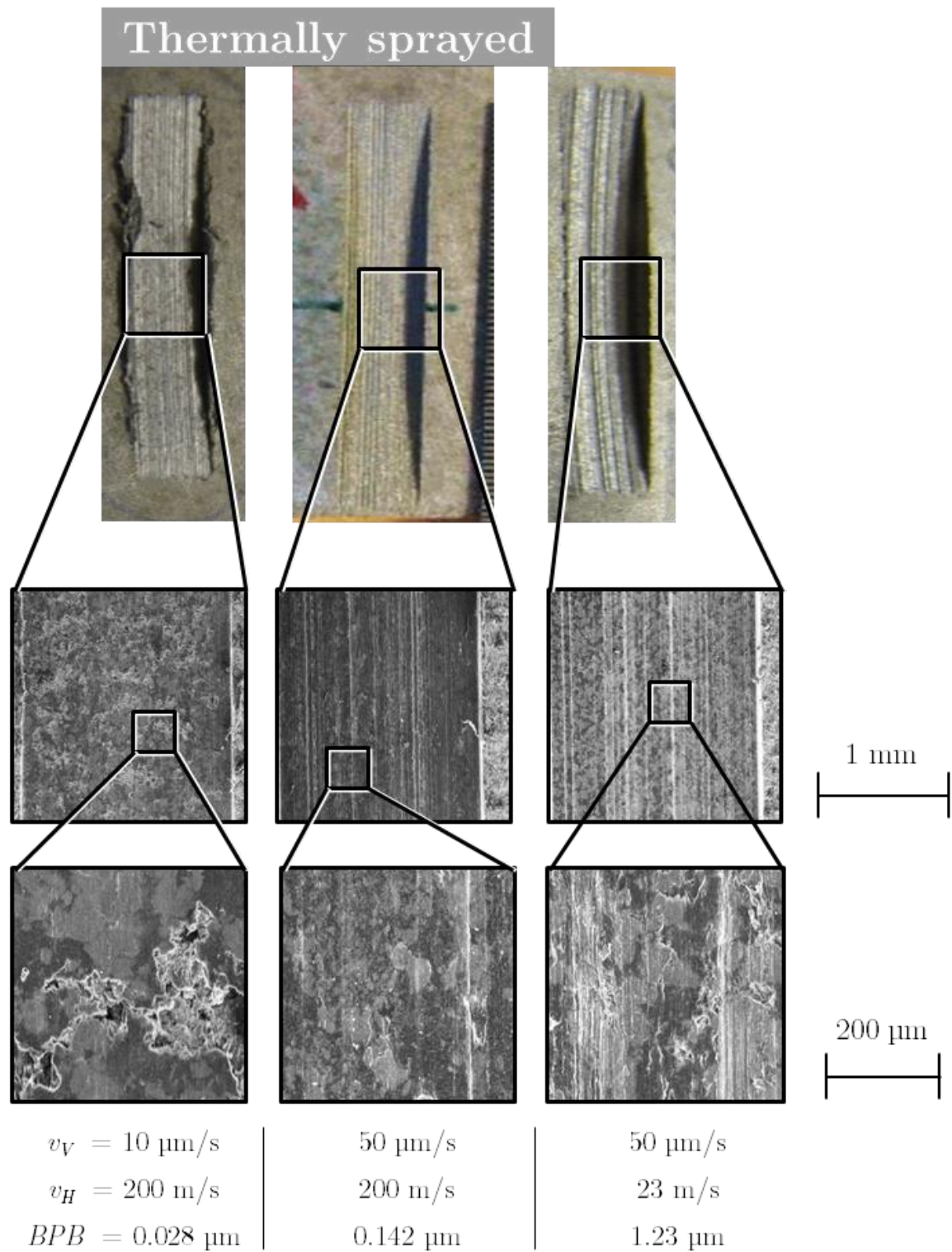


Figure 6.42: Macro pictures and SEM micrographs showing three wear tracks corresponding to the thermally sprayed PAC-905 material after a test performed with the abrasability test rig. The dominant wear mechanism is cutting combined with spalling, smearing of the polymer phase and grooving depending on the test conditions.

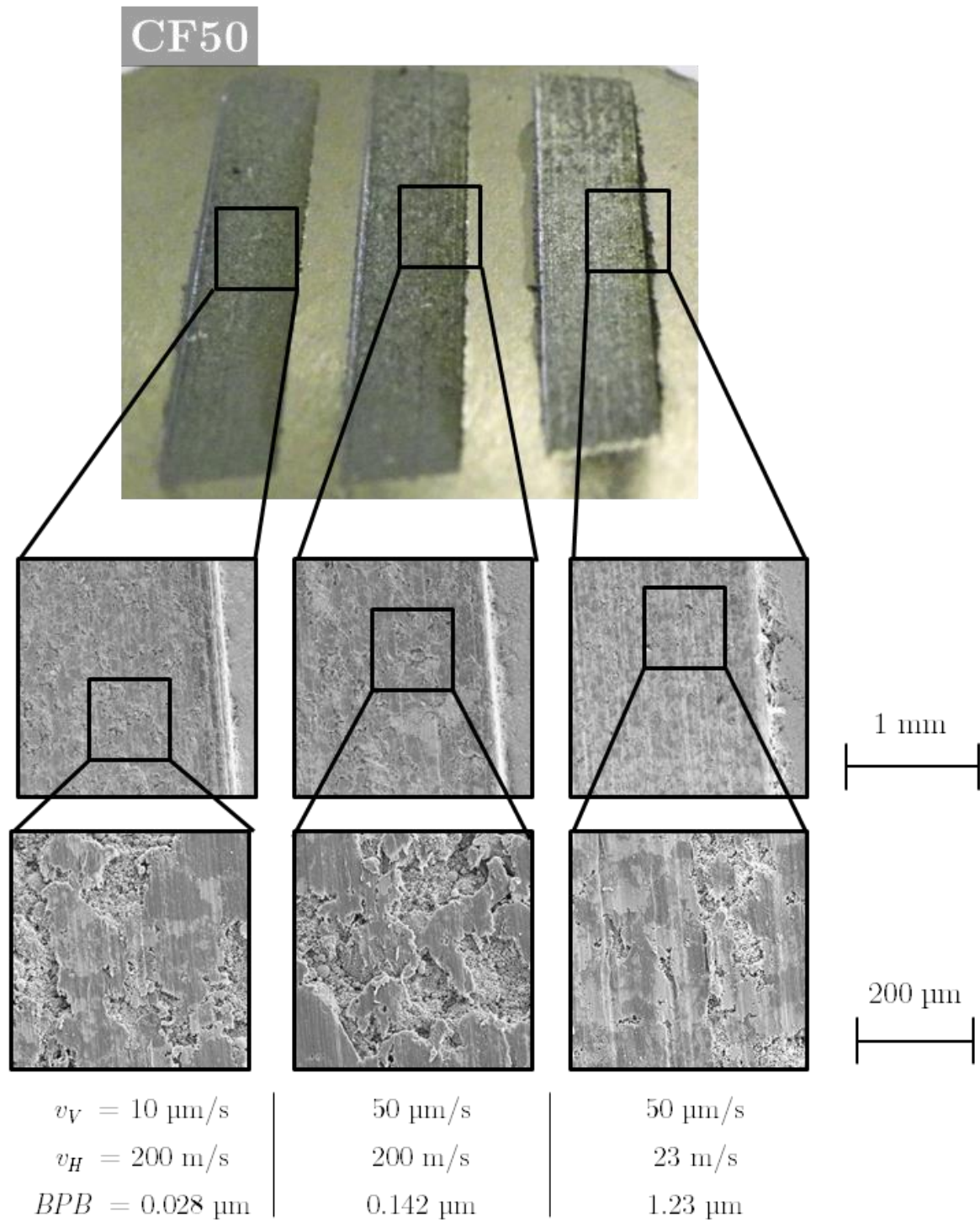


Figure 6.43: Macro pictures and SEM micrographs showing three wear tracks corresponding to the cold-pressed material (CF50) after a test performed with the abrasability test rig. The dominant wear mechanism is cutting combined with more or less spalling depending on the test conditions.

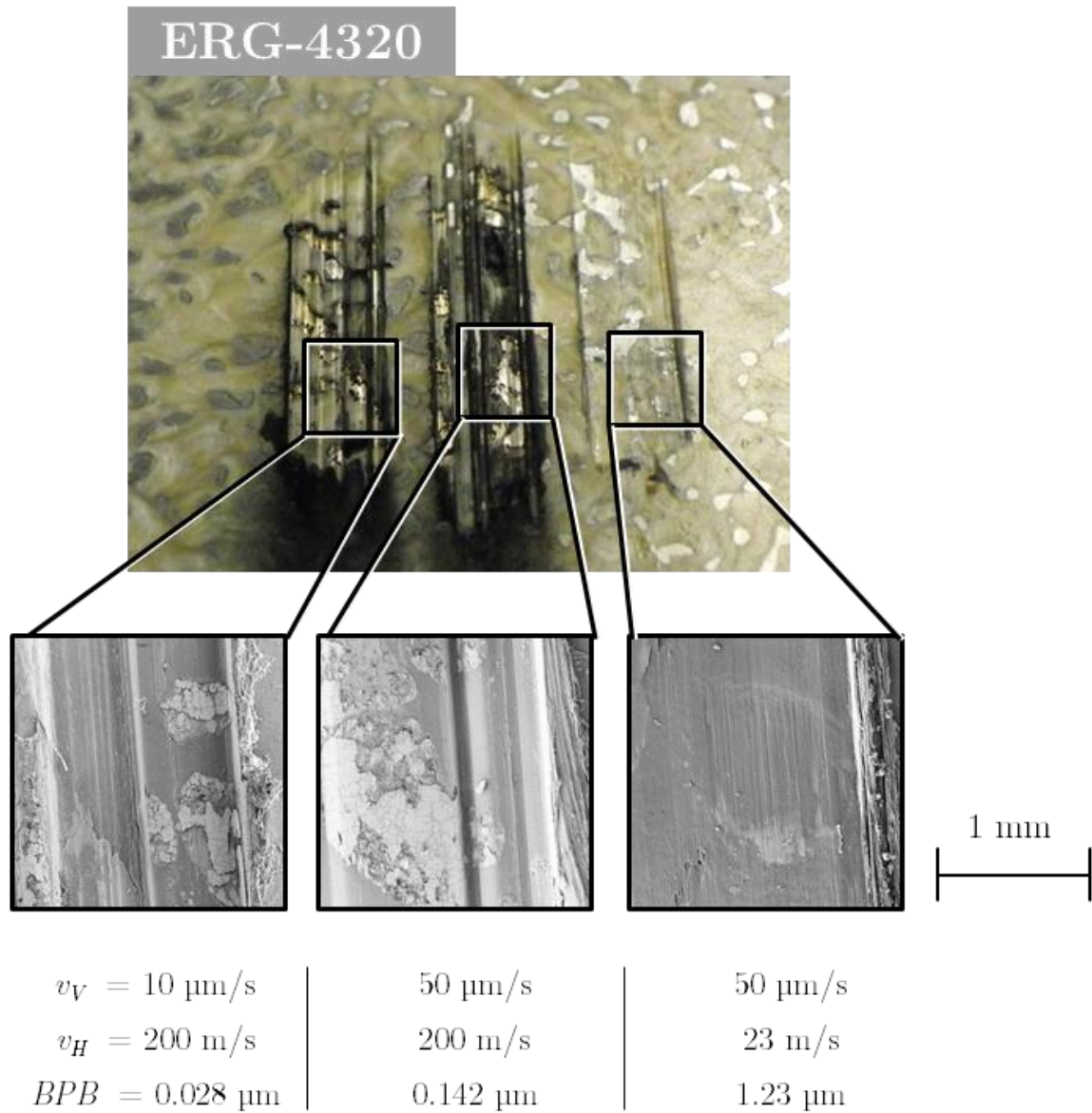


Figure 6.44: Macro pictures and SEM micrographs showing wear tracks corresponding to the ERG foam infiltrated with polyester after a test performed with the abrasability test rig. The dominant wear mechanism is cutting combined with more or less grooving depending on the test conditions. Some wear tracks present a dark trail which is the result of transfers from the blade to the material.

The TiA6V tip used during these tests is a rectangle. The abrasability test is modelled by a rectangular wedge with a width of 3.9 mm and a thickness of 2 mm. The three models of plowing, cutting and spalling are modelled. The *plowing* forces are modelled using the elastoplastic rectangular flat wedge plowing model with strain hardening (see Section 4.2.3 and Section 4.2.4). The *cutting* forces are modelled using the

elastoplastic orthogonal rectangular cutting model with strain hardening (see Section 4.3.5). The *spalling* forces are modelled using the orthogonal spalling model with an rectangular tool (see Section 4.4.2). The equations used to model the plowing, cutting and spalling forces are summarized in Table 6.13. The model prediction in this configuration is presented in Figure 6.45 in the case of the thermally sprayed PAC-905. The model predicts that the three bite per blade (BBP) used during the tests activate preferably a plowing mechanism. The model also predicts that the dominant mechanism switches to cutting at a depth larger than 2 μm and it switch to a spalling mechanism around 110 μm . Finally, the model predicts that with the BBP tested, the elastic recovery is larger than 98% which suggest that the first interactions between the tip and the material are essentially elastic. The deeper the blade goes, the less elastic recovery occurs and the more material is removed by the blade per rotation.

Table 6.13: Summary of the model used to predict the abrasability test forces

Variable name	Flat wedge <i>plowing</i> model (Section 4.2.3)	Flat wedge <i>cutting</i> model (Section 4.3.5)	flat wedge <i>spalling</i> model (Section 4.4.2)
Vertical force (F_V)	Eq. 3.1	Eq. 4.97	Eq. 3.1
Horizontal force (F_H)	Eq. 4.57	Eq. 4.97	Eq. 4.106
Hardness (H_{clpl})	Eq. 4.2 & Table 3.1		Eq. 4.2 & Table 3.1
Surface of contact (A)	Table 3.1		Table 3.1
Representative flow stress (σ_r)	σ_y	Eq. 4.96	σ_y
Representative strain (ε_r)		Eq. 4.96 (cutted layer) & is equal to 0 (plowed layer)	
Height of wear (h_{wear})	Eq. 4.59	Eq. 4.75	Table 4.2
Cross section area of the residual groove (A_{wear})	Eq. 4.60	$A_{wear} = W h_{wear}$	Table 4.2

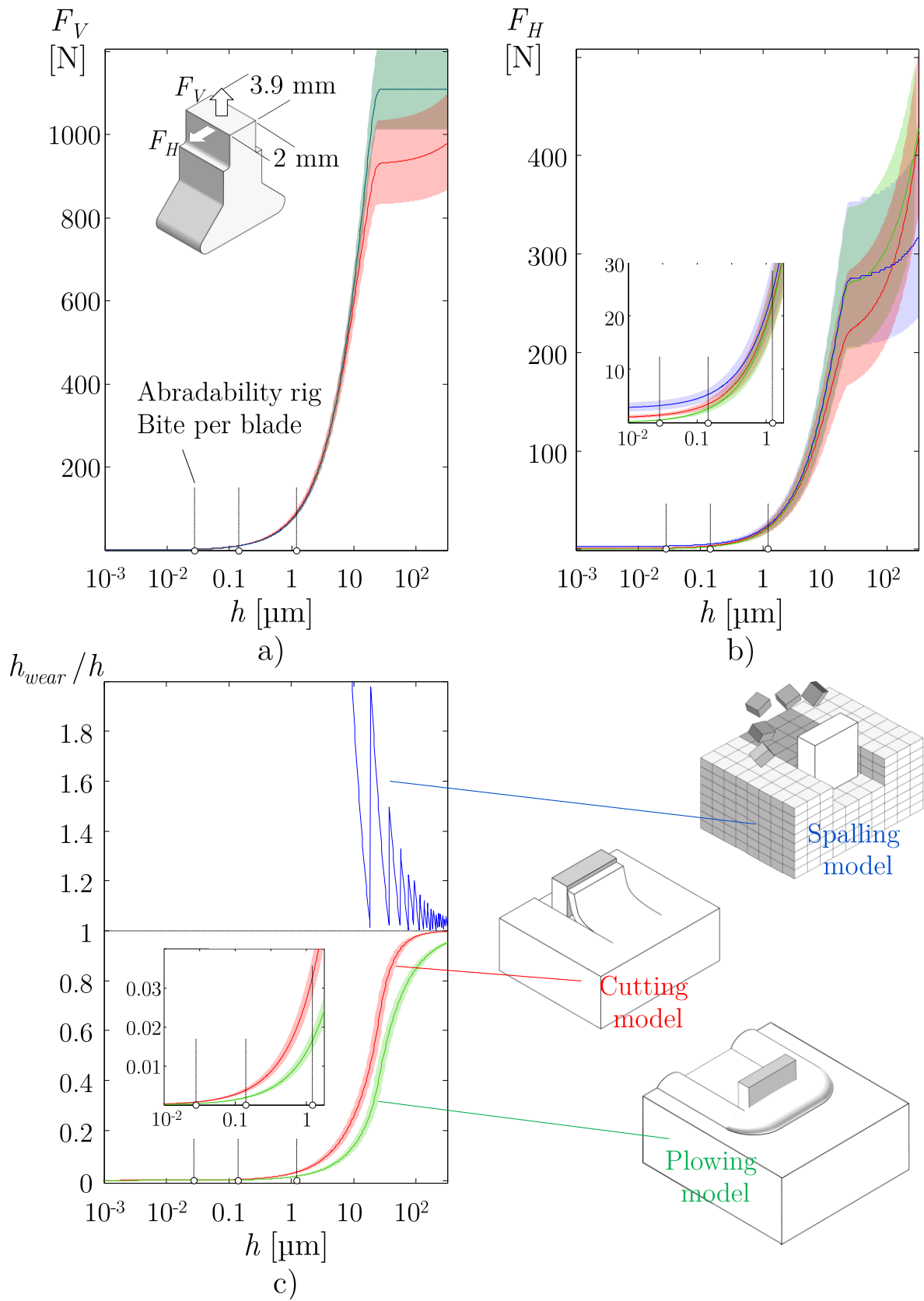


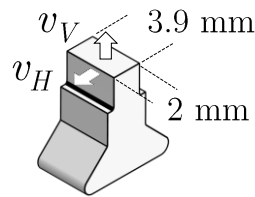
Figure 6.45: Comparison between the plowing, cutting and spalling model of the abrasability tests on the thermally sprayed PAC-905 material. The white round dots represent the three levels of bite

per blade experimentally tested. The upper left graph shows the vertical force versus the penetration depth. The upper right graph shows the horizontal force, the lower right graph shows the relative residual scratch depth.

The abrasability test results are compared with the model in Table 6.14. The model predicts that plowing is the dominant mechanism for most test materials. The model also predicts that the elastic recovery is larger than 90% for the four test materials.

Table 6.14: Summary of the comparison between the model prediction and the abrasability rig tests results.

Type of model materials	Name	Incursion speed (v_V) [$\mu\text{m/s}$]	Tangential Speed (v_H) [m/s]	Bite per blade (BPB) [μm]	h_{wear}/h (model)	Wear mechanism (exp)	Wear mechanism (Model)
Bulk	Polyester	10	200	0.028	2.7×10^{-4}	Cutting	Plowing
		50	200	0.142	1.4×10^{-3}	Cut / Grooving	" "
		50	23	1.230	1.2×10^{-2}	" "	" "
Thermally sprayed	PAC905	10	200	0.028	3.8×10^{-4}	Cut / Spall	Plowing
		50	200	0.142	1.9×10^{-3}	Cut / Smearing	" "
		50	23	1.230	1.7×10^{-2}	Cut / Grooving	" "
Cold pressed (fine)	CF50	10	200	0.028	2.2×10^{-3}	Cut / Spall	Plowing
		50	200	0.142	1.1×10^{-2}	" "	" "
		50	23	1.230	9.7×10^{-2}	" "	Cutting
ERG foams	ERG-4320	10	200	0.028	8.6×10^{-4}	Cut / Grooving	Plowing
		50	200	0.142	4.4×10^{-3}	" "	" "
		50	23	1.230	3.8×10^{-2}	" "	" "



6.4.7 Erosion

Before presenting the erosion results and the comparison with the model, some modelling details need to be described. Indeed, the model requires a kinetic energy as input, as well as a half-opening angle for the eroding particle. The two parameters are identified using the following approach. The size of the eroding particles is analyzed by SEM combined with image analysis (see Figure 6.46), the mass of one particle is calculated based on the average radius of the particles combined with the theoretical

density of Al_2O_3 (3.97 g/cm^3). The kinetic energy of a particle is estimated assuming that all the particles have the same speed as the airstream measured 10 cm away from the airgun ($\sim 100 \text{ m/s}$). The eroding particles are considered to have a conical shape with a half-opening angle of 42.28° which is equivalent to the half-opening angle of the corner of a cube. Models presented in Section 4.5 are used to model the erosion.

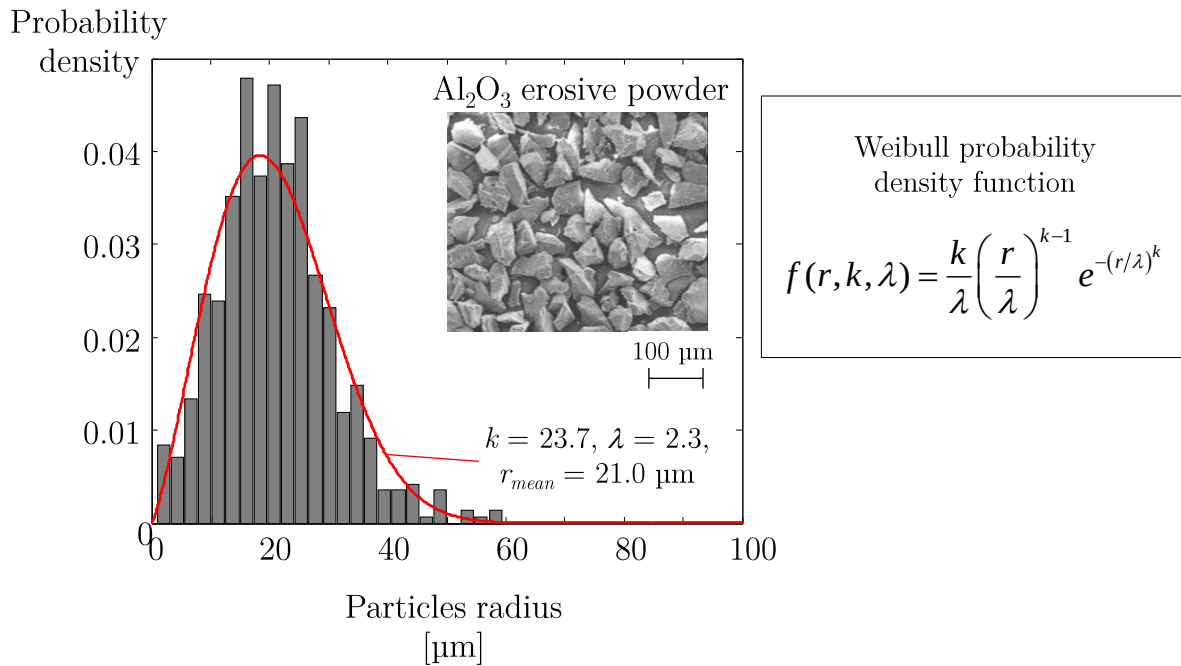


Figure 6.46: Size distribution of the Al_2O_3 erosive powder used during the erosion tests. These measurements are obtained by image analysis. The mean radius is calculated based on a curve fitting with a Weibull law.

The erosion test results as well as the model predictions are presented in Figure 6.47. The experiments show that hot-pressed materials resist erosion better than the cold-pressed ones. The erosion rate of the hot-pressed materials decreases with the AlSi content except for the bulk polyester. The cold-pressed materials wear off faster when the AlSi content is low. Surprisingly, this last trend is not captured by the model. In order to compare the model prediction with the erosion results, the erosion rate of the model and the experiment are normalized by the erosion rate of the thermally sprayed PAC-905 material. The ratio between the normalized model prediction and the normalized experimental results is presented in Figure 6.48. In this figure the worst prediction concerns the bulk polyester, and the cold-pressed materials that are underestimated in the case of fine powder and overestimated in the case of coarse powder.

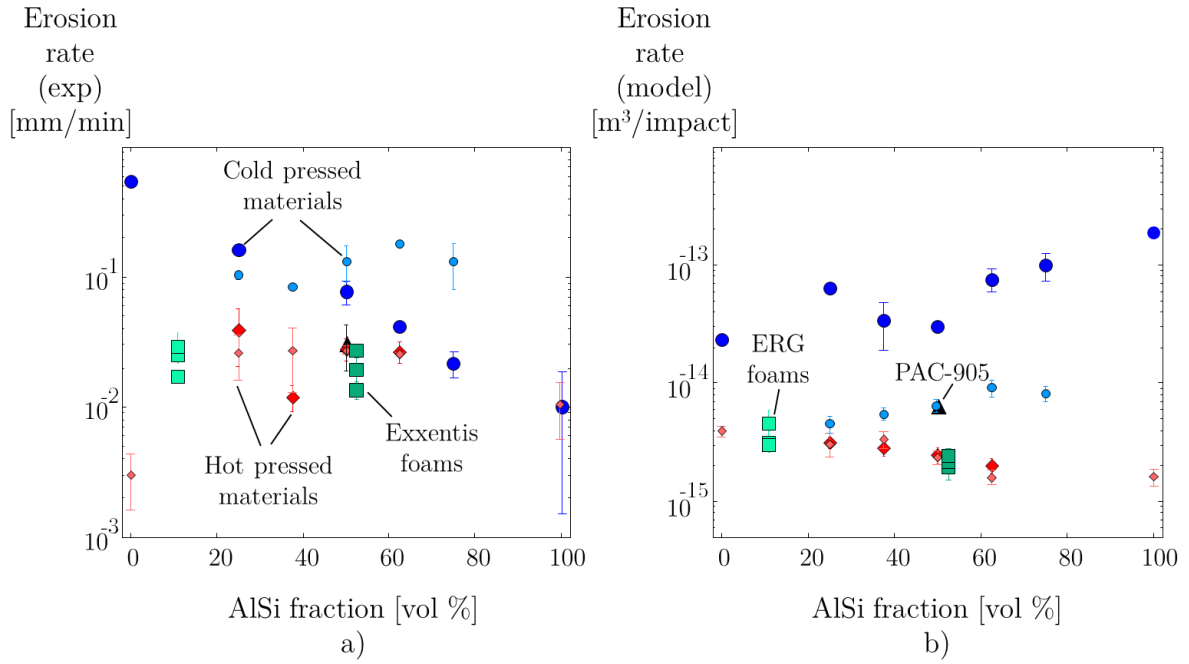


Figure 6.47: a) the results of the erosion test and b) the model predictions. The results are presented as a function of the AlSi volume fraction of the model materials.

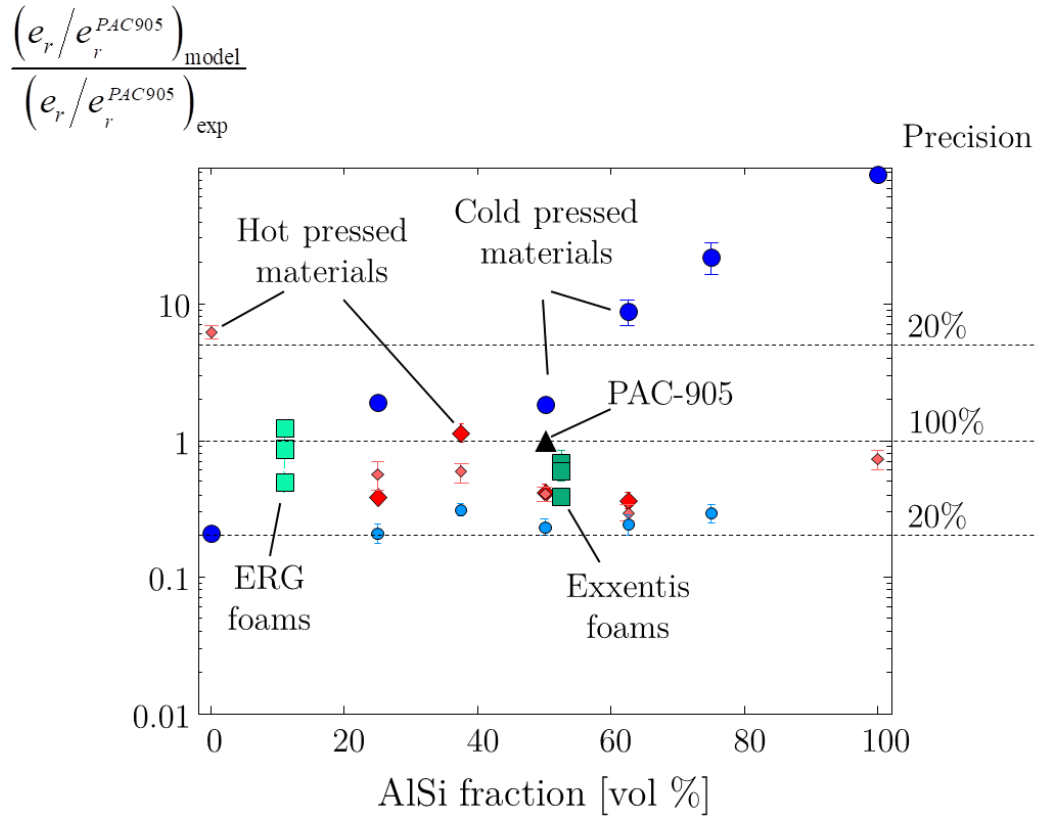


Figure 6.48: Ratios between the normalized model predictions and the normalized erosion tests results as a function of the AlSi volume fraction of the model materials. The error bars represent the total error of the model divided by the mean experimental value.

Erosion test surfaces are presented in Figure 6.49, Figure 6.50 and Figure 6.51. These figures show that the cold-pressed materials wear off mainly via a spalling mechanism. However, in the case of 100% cold-pressed materials, spalling is accompanied with plowing. The others model materials exhibit wear by plowing.

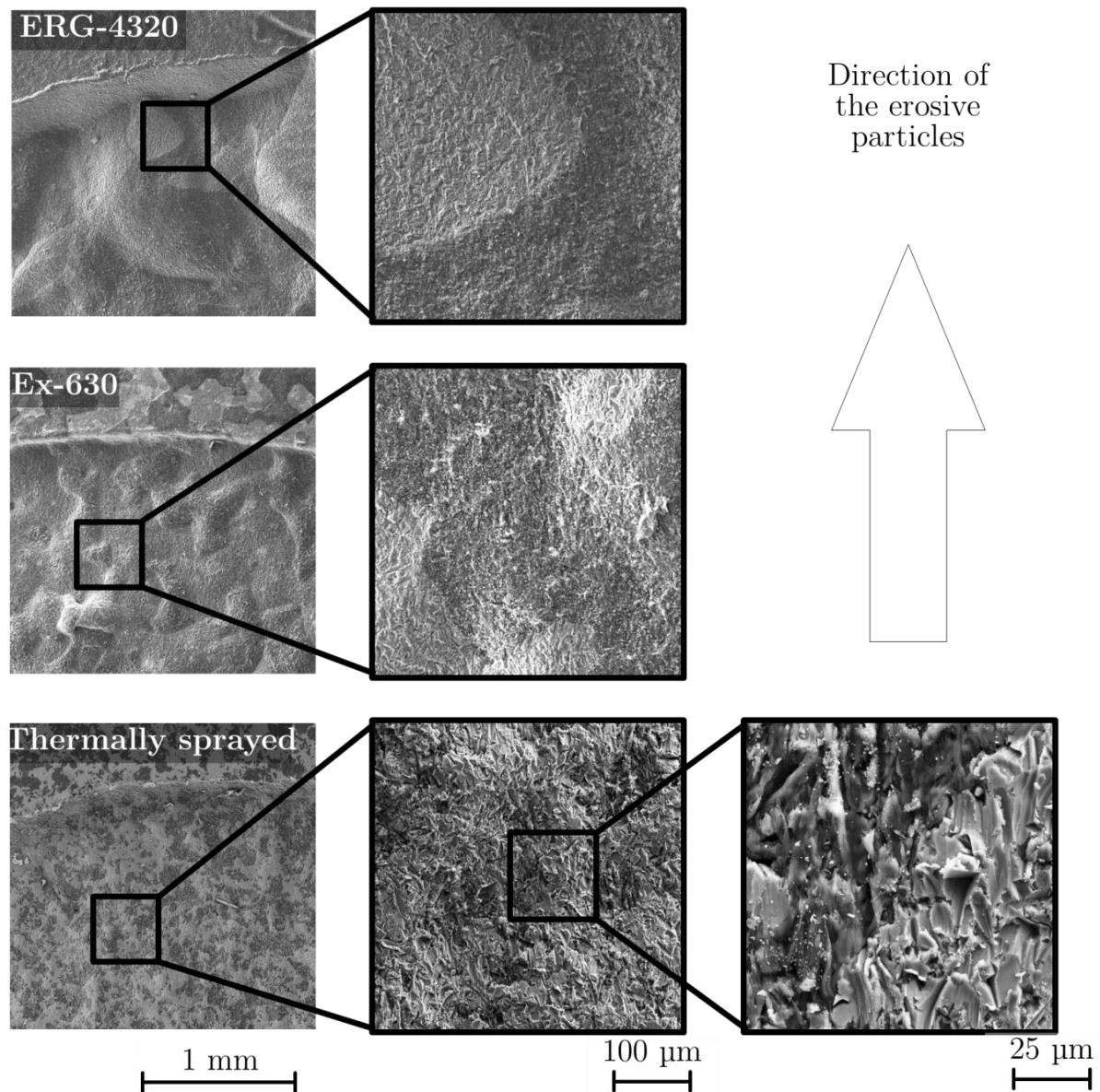


Figure 6.49: SEM micrographs showing a wear track after an erosion test using a 20° impact angle on two foams infiltrated with polyester and the thermally sprayed PAC-905. The dominant wear mechanism is plowing.

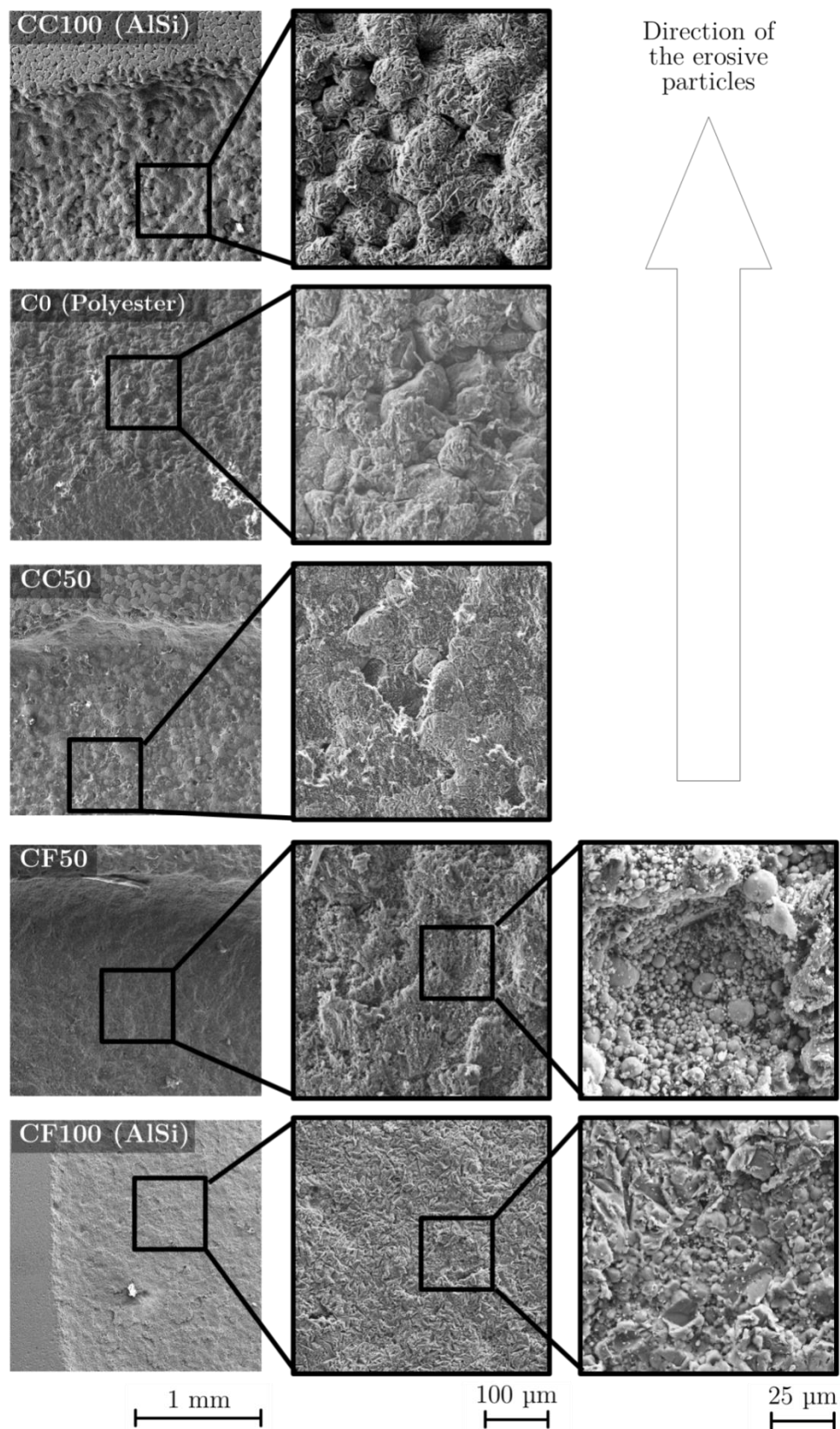


Figure 6.50: SEM micrographs of four cold-pressed materials after an erosion test using a 20° impact angle. The dominant wear mechanism is spalling and, in some case, there are evidences of plowing.

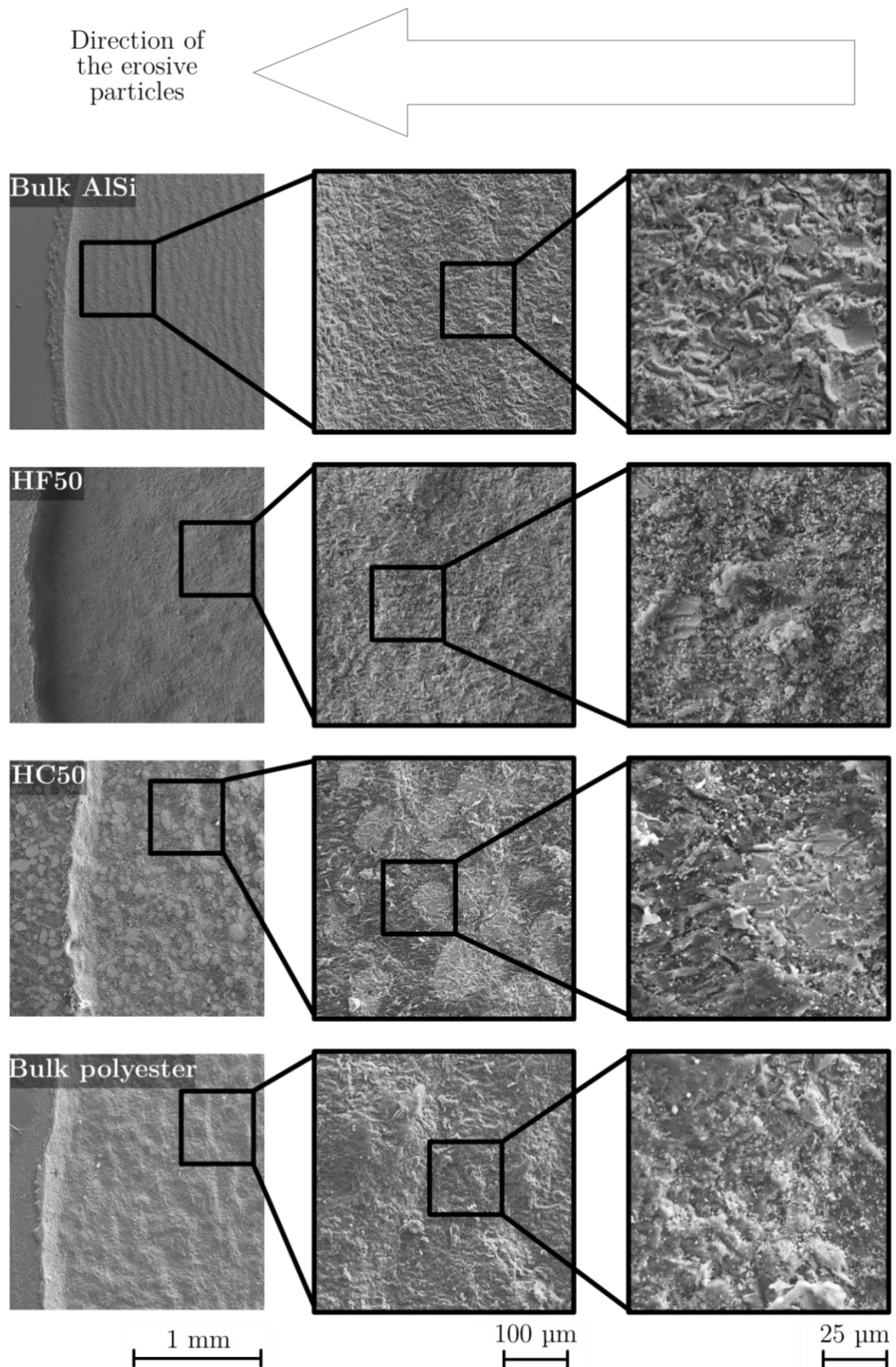


Figure 6.51: SEM micrographs of five model materials after an erosion test using a 20° impact angle. These materials include the bulk AlSi, two hot-pressed materials and the bulk polyester. The dominant wear mechanism is plowing.

The erosion tests results are compared with the model in Table 6.15. The prediction of the dominant wear mechanism is in good agreement with the SEM observations for most of the model materials. The precision of the model, averaged over all the test materials is 44%.

Table 6.15: Summary of the comparison between the model prediction and the erosion tests results.

Type of model materials	Name	Erosion rate (exp) [mm/min]	Erosion rate (model) [m ³ /impact]	Wear mechanism (exp)	Wear mechanism (Model)	$\frac{(e_r/e_r^{PAC905})_{model}}{(e_r/e_r^{PAC905})_{exp}}$	Precision of the model
Bulk	Polyester	0.003 ± 0.001	3.92 ± 0.04 10 ⁻¹⁵	Plowing	Plowing	6.18	0.16
	AlSi	0.011 ± 0.005	1.61 ± 0.26 10 ⁻¹⁵	" "	Cutting	0.73	0.73
Thermally sprayed	PAC905	0.031 ± 0.012	6.54 ± 0.00 10 ⁻¹⁵	Plowing	Spalling	1.00	1.00
Cold pressed (fine)	C0	0.545 ± 0.013	23.5 ± 0.00 10 ⁻¹⁵	Spalling	Spalling	0.20	0.20
	CF25	0.103 ± 0.007	4.55 ± 0.73 10 ⁻¹⁵	" "	Plow/Spall	0.21	0.21
	CF37	0.084 ± 0.004	5.56 ± 0.65 10 ⁻¹⁵	" "	Spalling	0.31	0.31
	CF50	0.132 ± 0.040	6.42 ± 0.85 10 ⁻¹⁵	" "	" "	0.23	0.23
	CF62	0.179 ± 0.011	9.10 ± 1.56 10 ⁻¹⁵	" "	" "	0.24	0.24
	CF75	0.131 ± 0.050	8.20 ± 1.23 10 ⁻¹⁵	" "	" "	0.30	0.30
	CF100	0.013 ± 0.007		Plow/Spall			
Cold pressed (coarse)	CC25	0.163 ± 0.001	64.4 ± 0.00 10 ⁻¹⁵	Spalling	Spalling	1.87	0.53
	CC37		33.7 ± 1.48 10 ⁻¹⁵		" "		
	CC50	0.077 ± 0.016	30.4 ± 0.00 10 ⁻¹⁵	" "	" "	1.86	0.54
	CC62	0.042 ± 0.002	76.1 ± 1.67 10 ⁻¹⁵	" "	" "	8.69	0.12
	CC75	0.022 ± 0.005	99.6 ± 2.63 10 ⁻¹⁵	" "	" "	21.95	0.05
	CC100	0.010 ± 0.009	187.4 ± 0.00 10 ⁻¹⁵	Plow/Spall	" "	88.86	0.01
Hot pressed (fine)	HF25	0.026 ± 0.010	3.09 ± 0.72 10 ⁻¹⁵	Plowing	Plowing	0.56	0.56
	HF37	0.027 ± 0.014	3.35 ± 0.53 10 ⁻¹⁵	" "	Plow/Cutt	0.59	0.59
	HF50	0.028 ± 0.002	2.36 ± 0.29 10 ⁻¹⁵	" "	Plowing	0.41	0.41
	HF62	0.026 ± 0.001	1.60 ± 0.22 10 ⁻¹⁵	" "	Plow/Cutt	0.30	0.30
Hot pressed (coarse)	HC25	0.039 ± 0.018	3.17 ± 0.35 10 ⁻¹⁵	Plowing	Plowing	0.39	0.39
	HC37	0.012 ± 0.003	2.86 ± 0.46 10 ⁻¹⁵	" "	" "	1.13	0.88
	HC50	0.028 ± 0.006	2.48 ± 0.36 10 ⁻¹⁵	" "	" "	0.42	0.42
	HC62	0.027 ± 0.005	2.03 ± 0.27 10 ⁻¹⁵	" "	Plow/Cutt	0.36	0.36
Exxentis foams	Ex-350	0.013 ± 0.002	1.95 ± 0.43 10 ⁻¹⁵	Plowing	Plowing	0.68	0.68
	Ex-630	0.027 ± 0.003	2.17 ± 0.26 10 ⁻¹⁵	" "	" "	0.38	0.38
	Ex-3000	0.020 ± 0.005	2.43 ± 0.35 10 ⁻¹⁵	" "	" "	0.59	0.59
ERG foams	ERG-1080	0.029 ± 0.007	2.98 ± 0.38 10 ⁻¹⁵	Plowing	Plowing	0.49	0.49
	ERG-2160	0.030 ± 0.008	3.15 ± 0.57 10 ⁻¹⁵	" "	" "	0.51	0.51
	ERG-4320	0.025 ± 0.001	4.58 ± 1.27 10 ⁻¹⁵	" "	" "	0.87	0.87
	ERG-8640	0.017 ± 0.001	4.43 ± 0.55 10 ⁻¹⁵	" "	" "	1.24	0.81
Average Value						4.88	0.44

6.5 Summary

Model materials were successfully processed with a large panel of microstructure configurations and sizes as well as different level of cohesion between the metallic phase and the polymeric phases.

The mechanical measurements show that the Young's modulus of these materials increases with the AlSi content. This is also true for the yield strength and the fracture toughness, except for the cold-pressed materials where σ_y and K_{Ic} seems to be dominated by the interface between the powder grains. Most model materials present a different behavior in tension and in compression. In some cases, there is more than a factor 10 in fracture strain between the two modes.

Regarding the tribological responses,

- The shape of the tip and the penetration depth affect the wear mechanisms as well as the contact forces and the elastic recovery. Model materials can exhibit ductility in some contact situations and brittleness in others.
- Model materials present a significant elastic recovery, both during indentation, scratch test and pin-on-disc test.
- Materials with weak interfaces between segments tend to wear off in a brittle manner and offer less erosion resistance.
- The amount of AlSi in the model materials has a strong effect on the tribological response. It increases the hardness, decreases elastic recovery and, in the case of cold-pressed materials, decreases the erosion rate.
- The size of the microstructure is important, especially in cold-pressed materials where it affects both the contact forces and the wear volume.

Comparison between the tribological experiments and the model depends on the type of test:

- In the case of indentations, the model gives satisfactory results except in macro indentation where the prediction of h_{pl} is overestimated.
- Regarding scratching experiments, the model estimates F_V properly, underestimates F_H and overestimates h_{wear} .
- In the pin-on-disc tests, F_V and F_H are reasonably well predicted except in the case of cold-pressed materials.

- With the erosion tests, again, the precision of the model is lower in the case of cold-pressed materials.
- Concerning the dominant wear mechanism, the weak link approach gives very good predictions over a large variety of contact geometry and depth, except in the case of the abrasability tests where the model systematically predicts plowing instead of cutting.

Chapter 7 Discussion, optimization and material selection

This chapter is dedicated to the interpretation of the results, to the comparison between the model predictions and the experiments and, finally, to the determination of the optimal set of properties leading to an efficient abradable material. The first section discusses the differences between the experimental results and the model predictions. The second section focuses on the combination of erosion resistance and abradability. The third section is dedicated to the material selection for an abradable material to be used in the low-pressure compressor of an engine.

7.1 Modelling and experimental confrontation

In this section, some of the main differences between the predictions of the model and the experiments are discussed. The aim of this discussion is to understand the origin of these differences and, in some cases, to propose improvements for future modelling and experiments. This discussion focuses on the main parameters described in the previous chapter, including h_{pl} , h_{wear} , F_H , F_V as well as the wear mechanisms.

7.1.1 Why are h_{pl} and h_{wear} overestimated?

In the previous chapter, the most recurrent discrepancy between the model predictions and experimental values is that the wear depth (h_{wear}) and the residual depth of indentation (h_{pl}) are overestimated in almost every case. Regarding nanoindentation and nanoscratch, the problem could originate in the model itself. However, comparison with FEM calculations shows that h_{pl}/h is underestimated by the model (see Figure 4.16). Therefore the problem most likely results from a phenomenon that is not taken into account both in the model and in the FEM calculations. It could be the result of (i) viscous effects, (ii) residual stresses from the process, (iii) bluntness of the tip, (iv) errors in the calibration of the machine compliance or from a (v) composite effect: compared to the size of the indent, the model materials are highly heterogeneous in nature. Indenting in the polymer matrix or having porosity in the proximity of the indent can locally lower the stiffness of the material. However, these composite effects

should not play a role at deeper penetration depth when the size of the area mechanically deformed by the tip is large enough compared to the microstructure size.

Regarding macroindentation and pin-on-disc experiments, except for the composite effects, the same justifications can be used. In addition, the overestimation of h_{pl}/h can be attributed to (vi) a misalignment between the tip and the sample surface or to (vii) an effect of the proximity of the support under the sample due to insufficient thickness of the sample with respect to the penetration depth. Finally, there can be an effect of (viii) the sample compliance. Indeed, the surfaces of the samples are not necessarily perfectly straight. The samples are attached by four external grips that clamp the sample with a lateral force. If the sample surface is curved, a part of the vertical displacement of the tip can be attributed to the elastic deflection of the sample (see Figure 7.1).

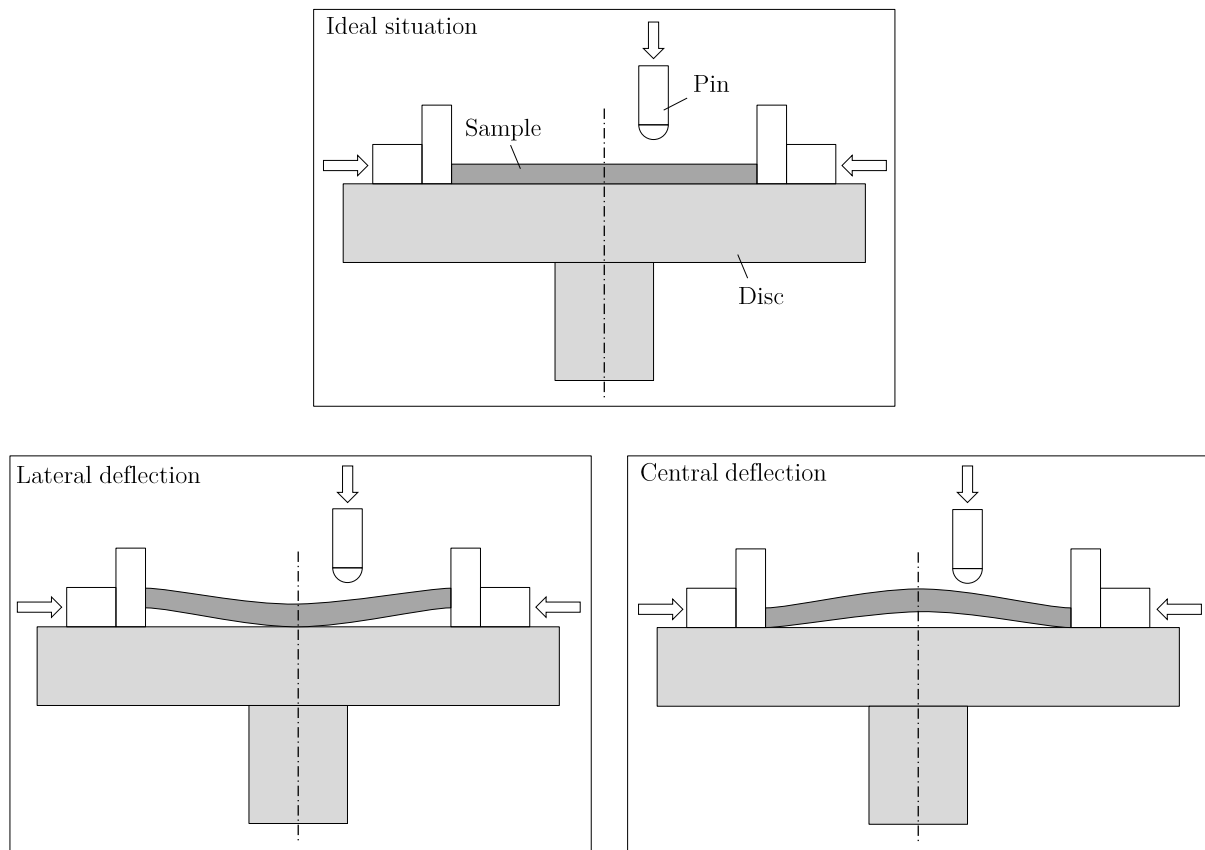
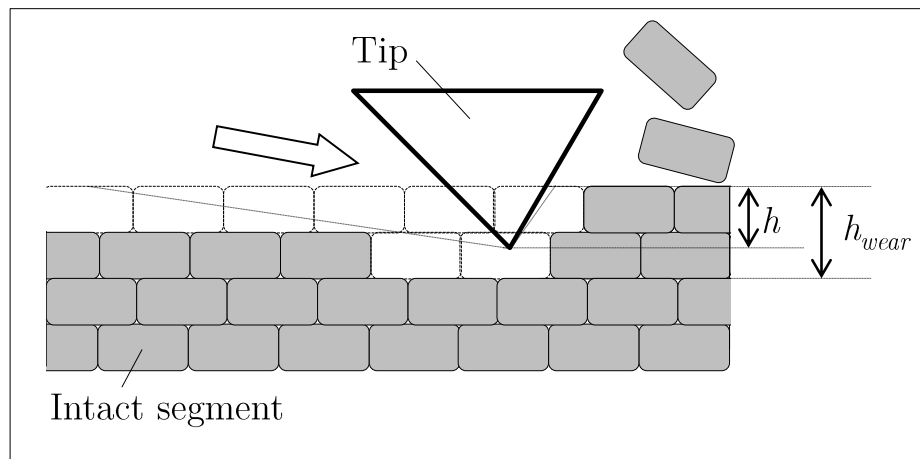


Figure 7.1: A deflection of the sample before a pin-on-disc test can lead to an overestimation of the elastic recovery of the surface of the sample during the experiment.

The value of h_{wear}/h is particularly overestimated by the model in the case of cold-pressed materials. Most of these materials exhibit wear by spalling. In the model, each segment in contact with the tip is removed. On the other hand, the SEM observations showed that even if decohesion occurs, most of the particles remain in the bottom of the wear track (see Figure 7.2). The wear depth (h_{wear}) is measured by a post scan of the surface by the tip. The particles lying in the bottom of the wear track can influence the measurement of h_{wear} . This phenomenon gives rise to an important mismatch between the model and the experiment, especially if the size of the segment is large compared to the penetration depth (h).

Predicted



Experience

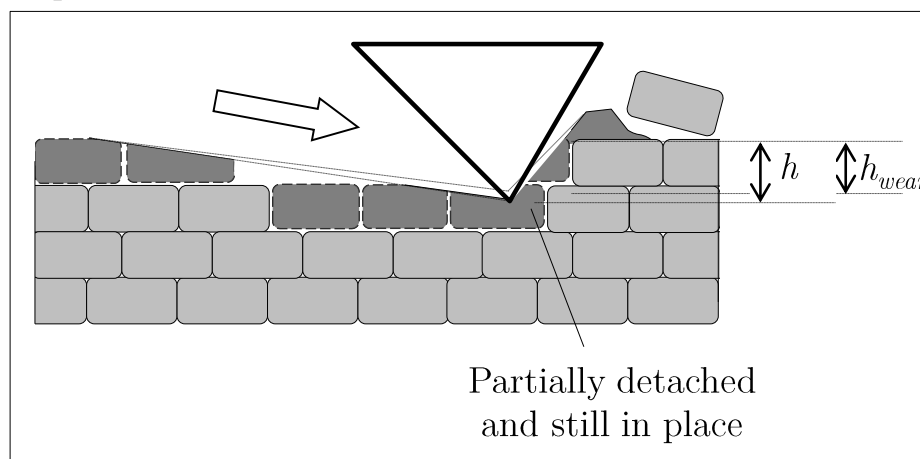


Figure 7.2: A spalling mechanism can be accompanied with partial decohesion of the segments. In that case, the measured residual depth will be lower than the prediction.

7.1.2 In the nanoscratch test, why is F_H underestimated?

In the case of the nanoscratch experiments, the horizontal force is significantly underestimated by the model. During these tests, most of the materials exhibit wear by plowing. The model underestimated the horizontal force, probably because it focuses mainly on the energy required to displace matter on the side of the wear track. There are at least two energetic contributions that could be added to the model: the shearing of the surface beneath the tip and the displacement of the pile-up material in front of the tip. Adding these two contributions would probably lead to a better prediction of F_H .

7.1.3 In the pin-on-disc test, why is F_V overestimated?

The vertical force is overestimated in the pin-on-disc experiment particularly with cold-pressed materials. This is not the case for the nanoscratch experiments. The SEM observations showed that during the pin-on-disc test, an intermediate mechanism occurs, partly consisting in cutting and partly in spalling. The present spalling model is built on the hypothesis that the vertical force is the same as the plowing vertical force. This hypothesis gives satisfactory results when the real mechanism is a combination of plowing and spalling as in the nanoscratch experiment. However, when the mechanism is between cutting and spalling, it fails to predict the right vertical force. This poor prediction probably happens because most of the material is being removed in front of the tip in the form of a fragmented chip instead of being pushed under the tip. This problem could be tackled by dividing the spalling mechanism into two mechanisms. One of the two would be the actual one « Plowing-spalling » and the second one would be « Cutting-spalling ». In the latter, the same general approach based on the energy needed to separate fragments as the tip progresses into the material should be used. However, this time, the vertical force should be modelled as the result of the friction between these fragments and the cutting tool, a little bit like in a classical cutting model.

7.1.4 In the abrasability tests, are the results consistent with other studies?

The abrasability tests are used to study the wear mechanisms under conditions close to real engine conditions. The analysis of the wear tracks showed that the most

dominant wear mechanism is cutting, sometimes combined with grooving, transfer and micro-rupture. The results on the thermally sprayed material can be compared with the results from the study of [Schmidt 1997]. Schmidt reported the same wear mechanisms with similar positioning in the wear map. However, in his study, the corresponding wear mechanisms do not occur in the same range of velocities. There is indeed a shift between our results and the wear map from [Schmidt 1997] (see Figure 7.3). There can be several reasons for that. First, (i) the material is not exactly the same. Indeed, Schmidt tested the Metco-601, compared to the PAC-905 in this study. In addition, (ii) the thermal spray parameters might be different. Therefore, even if the base powders were the same, the material after projection can be different in terms of AlSi volume fraction and porosity. Another explanation is (iii) that the test rig itself is different in geometry of the disc and of the blade. The Oerlikon Metco test rig uses a disc of 620 mm diameter (the blade project 10 mm out the disc), compared to 180 mm in this study. Working with the same incursion speed (v_V) and rubbing speed (v_H) gives rise to a different bite per blade (BBP) and a different frequency of contact. Translating our results in terms of the BPB and the frequency of contact shows a better compatibility between our results and Schmidt results (see Figure 7.3). Therefore, the frequency of contact and the BPB might be important parameters to take into account while comparing results from different test rigs.

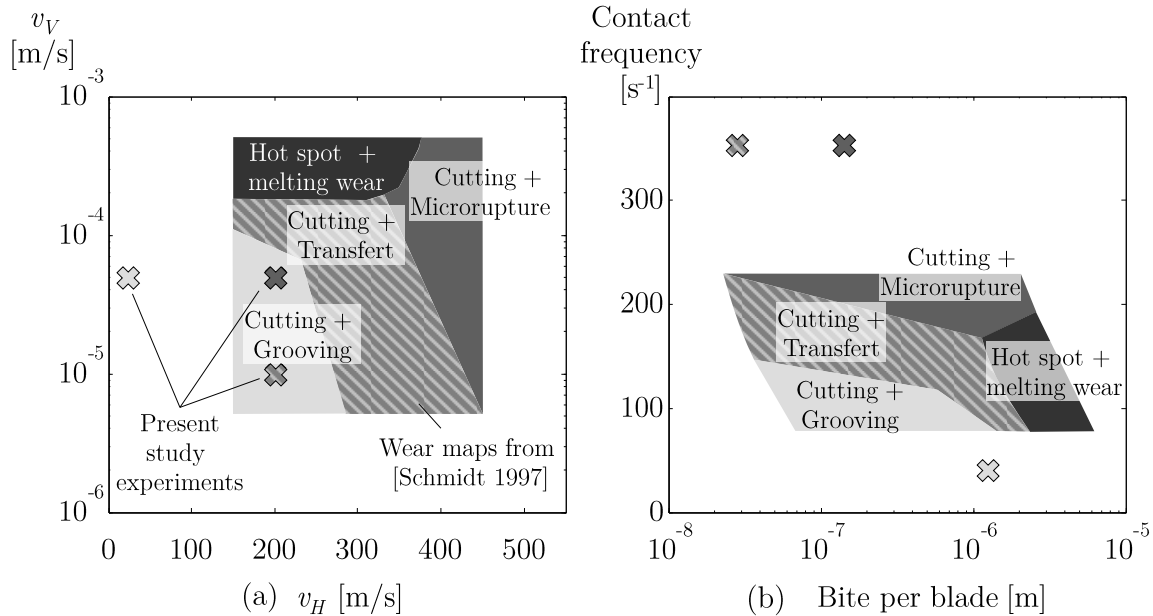


Figure 7.3: a) superposition of the wear maps from [Schmidt 1997] and the present test results on the abrasability rig. Each color represents a specific wear mechanism; b) a transformation of these wear maps in terms of the BPB and the frequency of contact.

7.1.5 Why do we predict wrong wear mechanisms in our abrasability tests?

Continuing with the thermally sprayed PAC-905 material, the model predicted four features: (i) with a penetration depth (h) smaller than $\sim 1\text{ }\mu\text{m}$, the dominant wear mechanism is a plowing mechanism, (ii) Between $\sim 1\text{ }\mu\text{m}$ and $\sim 100\text{ }\mu\text{m}$ wear happens by cutting, (iii) above $100\text{ }\mu\text{m}$, a spalling mechanism dominates. The model also predicted that (iv) the relative wear depth (h_{wear}/h) increases with the BPB, starting at 0 and ending at a value close to 1.1.

Experiments showed that the dominant wear mechanism is cutting. Using the hypothesis that the bite per blade is representative of the penetration depth, the model exclusively predicted a plowing mechanism. Therefore, something must be wrong with the model. The solution probably lies in the comparison between h and the BPB. Indeed the BPB represents the final penetration depth divided by the number of rotations and divided by the number of blades on the disc. After every rotation, the blade is going one BPB deeper in the material. The BPB can be equal to the penetration depth of the blade at each rotation (h) only if the material presents no elastic recovery. If there is some elastic recovery, then h will eventually increase during the first few rotations. In order to illustrate this point, let us assume that the ratio between the elastic recovery depth and the penetration depth (h_{el}/h) is constant and equal 50%. In that case, the penetration depth of the first rotation is equal to the BPB. On the second rotation, h is equal to the BPB plus the elastic recovery from the previous interaction. This phenomenon goes on and on, increasing the penetration depth until the quantity of material removed per rotation (h_{wear}) becomes equal to the BPB (Figure 7.4). At that moment, the system reaches a stable regime where h is equal to two times the BPB and h_{wear} is equal to one BPB. Therefore, h might change during an abrasability test and become way larger than the BPB. For the same reason, the wear mechanism may also change during the test, starting with plowing and switching to cutting once h is large enough.

This hypothesis was tested using the three wear models in a loop that calculated the dominant wear mechanism as well as h and h_{wear} at each blade revolution for different bites per blades and for different final penetrations of the blade (see Figure 7.5). The model shows that the dominant wear mechanism depends both on the final depth and on the BPB. The probability to activate a cutting mechanism increases with the BPB.

In some case, the wear mechanism can alternate between plowing and cutting. This algorithm was adapted to the geometry of the abrasability test rig from the present study and to the one from Oerlikon Metco [Schmidt 1997] (see Figure 7.6). One can see that, with the parameters from the present study and the ones from [Schmidt 1997], the model predicts a cutting mechanism for two thirds of the cases. This last prediction is still not perfect. However, it is closer to the reality. The model also predicts that spalling is rarely activated even for a large BPB.

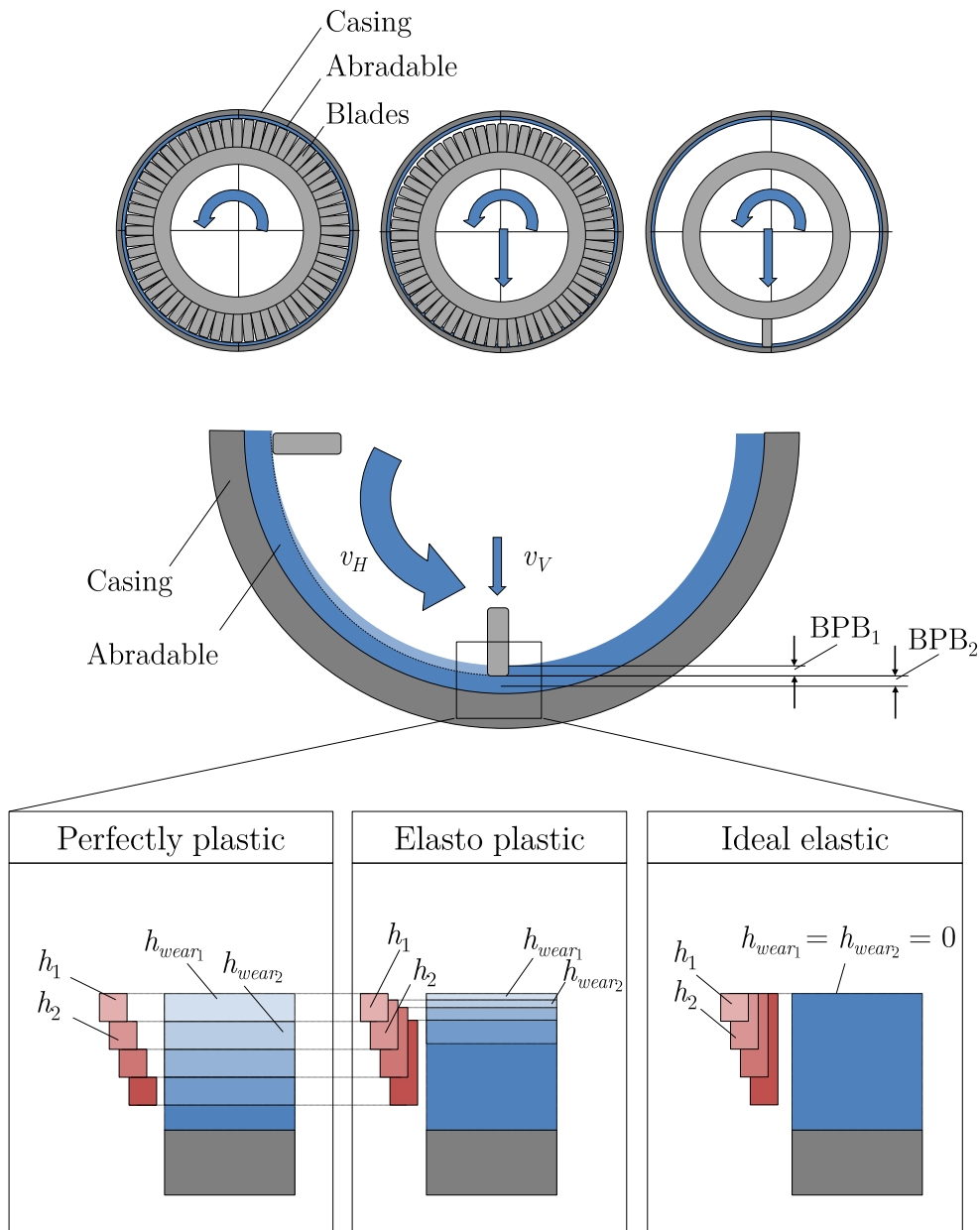


Figure 7.4: Schematic representation of the evolution of h as a function of the number of contact between the blade and the abrasible. Three cases are represented: perfectly plastic material, elastic material and elastoplastic material.

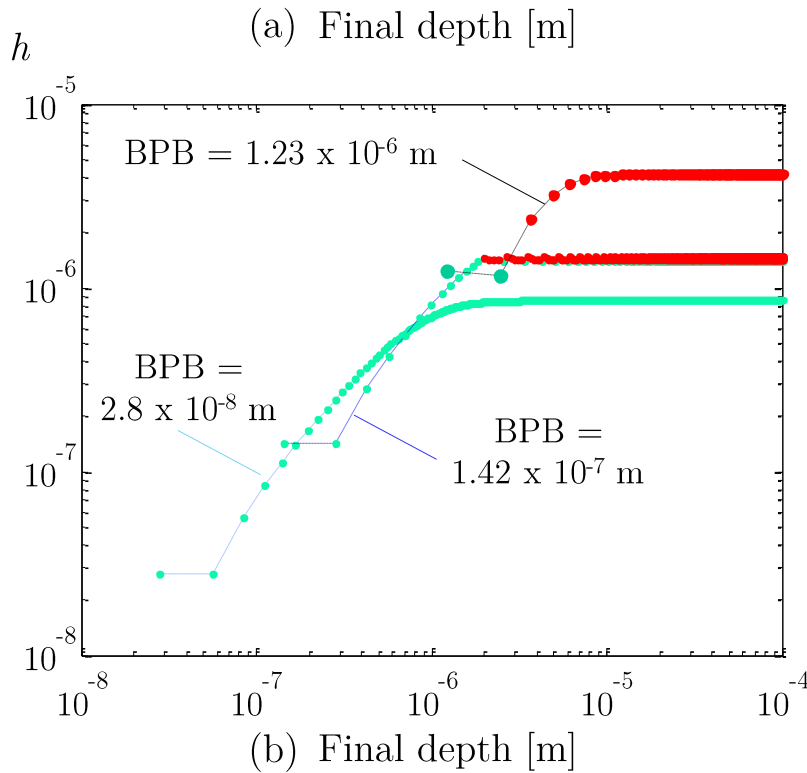
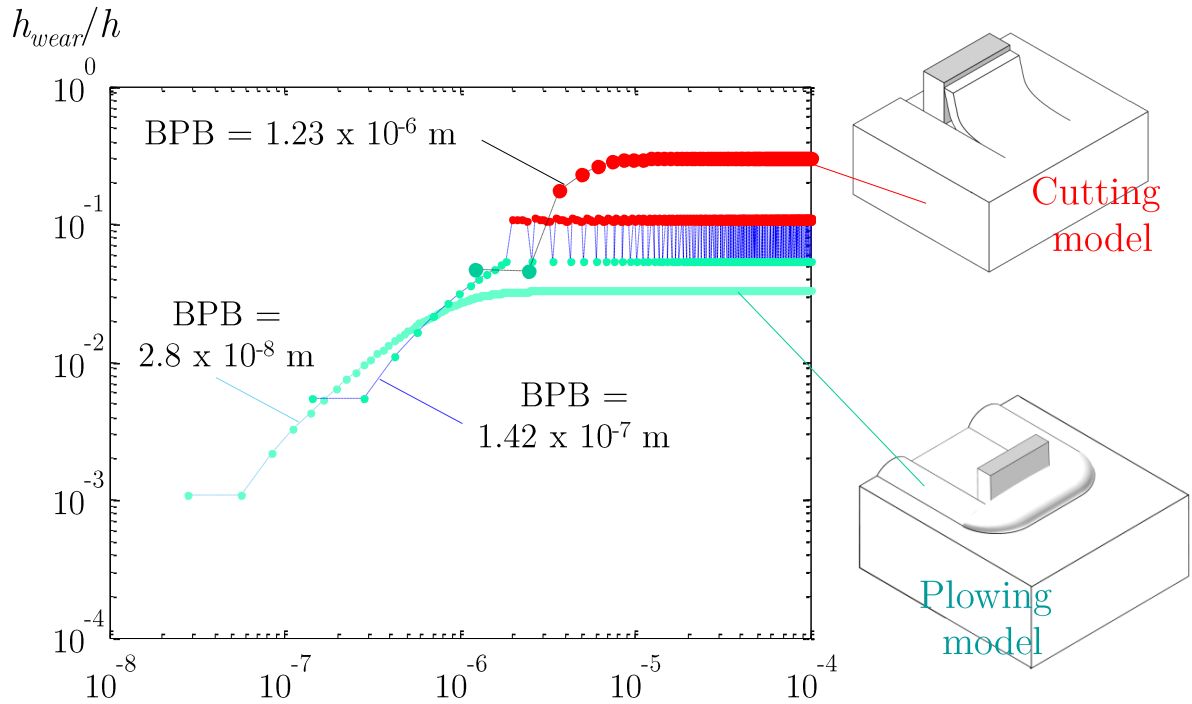


Figure 7.5: a) evolution of the relative residual depth and, b) evolution of the penetration depth as a function of the final depth of the blade. Three different bites per blade are considered. Each BPB were tested on the abrasability test rig. Each round dot represents one rotation of the blade, the color represents the wear mechanism.

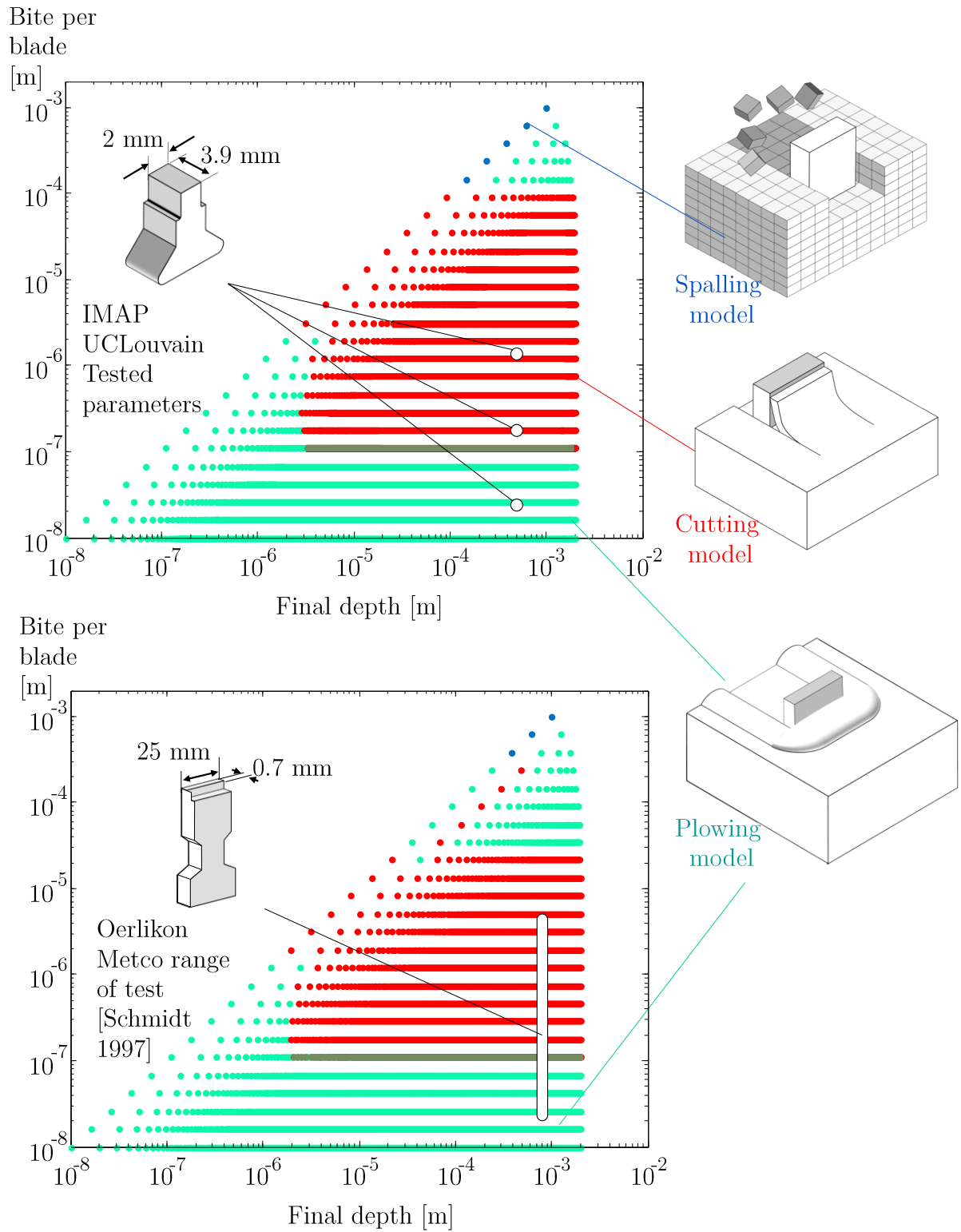


Figure 7.6: Wear maps showing the effect of the bite per blade and the final depth of the blade on the wear mechanism. Each round dot represents one rotation of the blade, the color represents the wear mechanism.

7.1.6 Regarding the erosion test, why is there a poor prediction in the case of cold-pressed materials ?

In the erosion tests, the model overestimates the erosion rate of cold-pressed materials with a coarse grain size. Regarding SEM observations, it appears that some impacts are strong enough to detach a segment of material and some are not. In the spalling model, every segment impacted with sufficient energy disappears. In reality, some impacts are probably being absorbed by plastic deformation in the segment. To improve the model, instead of considering one kinetic energy for every particle, it might be more realistic to consider a distribution of kinetic energy. In that case, only a fraction of the impacts will lead to spalling. Regarding the cold-pressed materials with smaller grains, the prediction is lower than the experiments. One possibility is that, instead of removing the segments by separating all the interfaces between one another, one cluster of segments can be detached in one chunk during an impact without complete fragmentation. In that case, more volume can be removed with less energy.

7.2 Combining abrasability and erosion resistance

Abradable materials are exposed to paradoxical design requirements of being easy to wear out in case of a contact with the blade and being resistant to erosion. In this section, this problem is explored from an experimental point of view. So far, the erosion resistance and the abrasability have been analyzed independently. One key aspect in abrasability is the ease of removal of material in case of a contact in order to avoid any deterioration due to an elevation of the temperature or a damaging stress in the blade. In addition, a high elastic recovery is also appreciated in order to maintain a close gap between the tip of the blade and the casing after the contact. Using the pin-on-disc results, and the erosion test results, one way to identify a good abrasable material is to look for materials that minimize the contact forces while presenting a low wear depth (h_{wear}) and a low erosion rate. Figure 7.7 compares the results obtained in the pin-on-disc test with the erosion results. This figure shows that F_H and F_V are proportional for every model materials. Metallic foams infiltrated with polyester present a high erosion resistance combined with a high contact forces. Some model materials exhibit a high erosion resistance and a low F_H during the pin-on-disc test, especially the hot-pressed and cold-pressed materials. Residual wear depth (h_{wear}) is

higher with cold-pressed materials. In terms of forces, erosion rate and elastic recovery, some cold-pressed and hot-pressed model materials behave better than the thermally sprayed material.

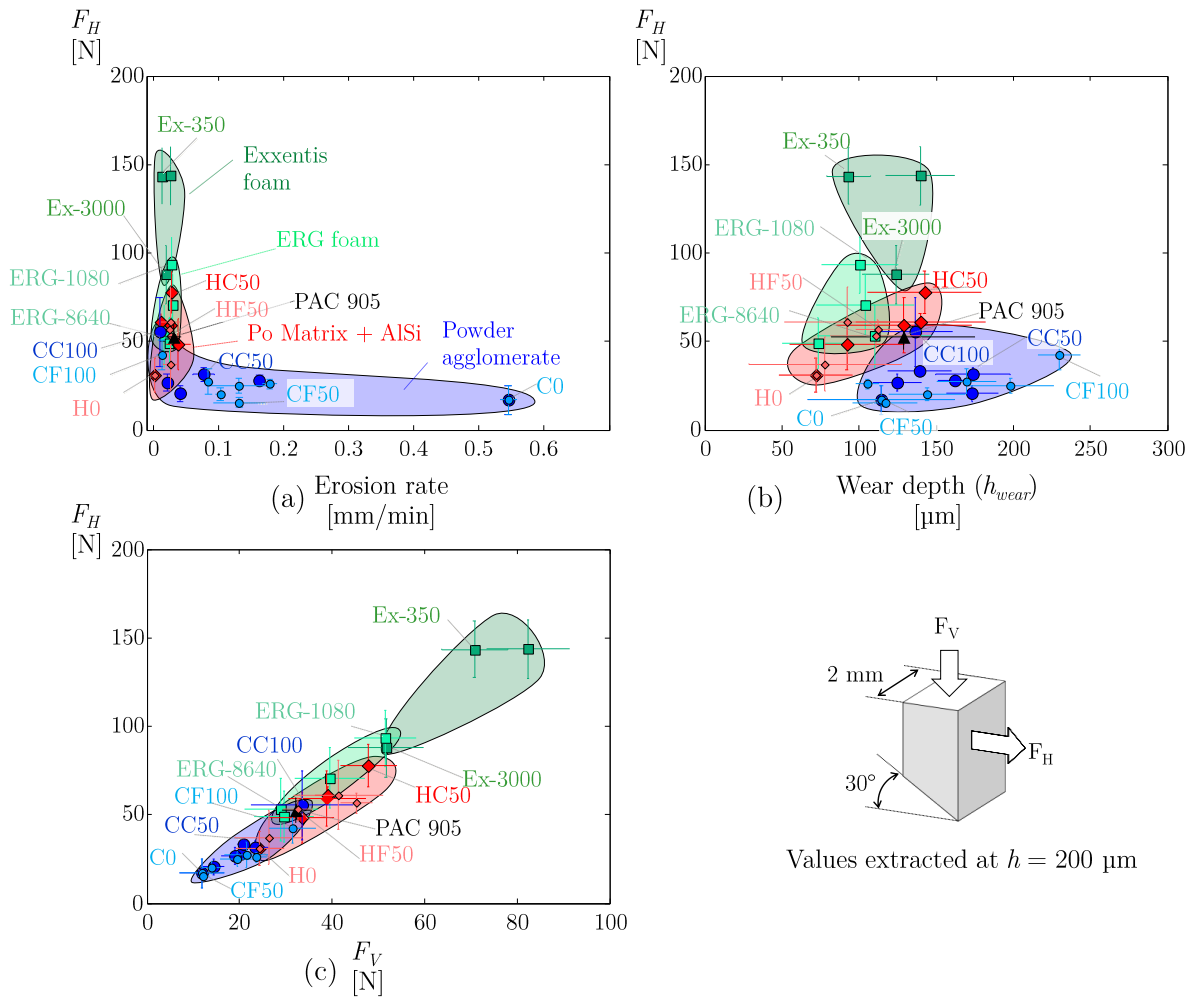


Figure 7.7: Superposition of the results from the pin-on-disc tests including F_H , F_V and h_{wear} with the erosion tests results.

This figure shows that erosion resistance and abrasability are not antagonist properties. This observation confirms the results from [Chappel 2001] but on different abrasable materials. Unfortunately, these graphs represent the wear behavior corresponding to only one penetration depth, with one geometry of tip, in a single contact. However, these results bring the following questions: “Is there an optimum in terms of material properties and microstructure that gives the best combination of abrasability and erosion resistance? How is this optimum affected by the shape of the blade, the bite per blade or by the impact angle of the erosive particles?”

7.3 Optimization and materials selection

Is it possible to combine erosion resistance and abrasability in one material? From an experimental point of view, the results presented in the previous section suggest that it is. Indeed, some model material performed better than the actual industrial material in terms of erosion and abrasability. The previous section also showed that the wear behavior of different materials can be modelled with a reliable level of accuracy in terms of force, residual wear and erosion resistance. In this section, the models presented in Chapter 4 are used to guide the search for an optimal set of microstructure and mechanical properties that improve the abrasability of a material as well as the erosion resistance. This search for an optimum starts with a description of the strategy followed by the hypotheses, a definition of the performance indices, a list of constraints on the materials and the results.

7.3.1 Optimization hypotheses

There are many ways to conduct an optimization. In this case, a very simple strategy is used to rank materials by performance in a given contact situation. Then, depending on the contact geometry, the best candidates are identified and selected. As stated in Chapter 3 and Chapter 4, many parameters can affect the wear behavior of a material, including the geometry of the tip, the microstructure of the material and the mechanical properties of the base material. Therefore, in order to simplify the problem, the blade is considered as a rigid rectangle of width W and thickness T . The depth of penetration of the blade into the abrasable (h) is considered to be constant. The friction coefficient is also considered to be a constant equal to 0.1. The Poisson ratio of the abrasable material is assumed to be constant and equal to 0.3. The abrasable materials are considered elastoplastic materials without strain hardening ($n = 0$). Regarding erosion, the impacting particles are considered as sharp cone with a half-opening angle of 42.28° which is the equivalent half opening angle of a cube corner tip.

7.3.2 Performance indices for abrasable materials

As mentioned in Chapter 2, an abrasable main function is to protect the blade from damage in case of a contact. These damages can come from temperature elevations and from stresses due to contact. The thermal aspects of the problem are not considered

here, only the mechanical stresses. Therefore, the first performance indices to be minimized are the forces acting on the blade during a penetration in the abradable (M_{FH} and M_{FV}), where

$$M_{FH} = F_H \quad \text{Eq. 7.1}$$

and

$$M_{FV} = F_V. \quad \text{Eq. 7.2}$$

A second important function of an abradable layer is to maintain a close gap between the tip of the blades and the casing. In consequence, the quantity of material removed by a blade in case of a contact should also be minimized (M_{wear}), where

$$M_{wear} = \frac{h_{wear}}{h}. \quad \text{Eq. 7.3}$$

A second consequence is that the erosion rate should be as low as possible to maintain a close gap. The last performance index is therefore the volume removed by an erosive particle impacting the material with a given kinetic energy at a given angle:

$$M_{er} = e_{rate}. \quad \text{Eq. 7.4}$$

These four indices will form the base of the evaluation of an abradable material performance. In order to manipulate these different performance indices easily, each index is normalized (divided) by the corresponding performance of the actual industrial material. Therefore, the normalized indices take the following form:

$$\left\{ \begin{array}{l} M_{FH}^n = F_H / F_{H_0} \\ M_{FV}^n = F_V / F_{V_0} \\ M_{er}^n = e_{rate} / e_{rate_0} \\ M_{wear}^n = \frac{h_{wear}}{h_{wear_0}} \end{array} \right. \quad \text{Eq. 7.5}$$

These performances indices are then combined in two indices (M^{n*} and M^{n+}) that are respectively the product and the addition of these four normalized indices

$$M^{n*} = M_{FH}^n M_{FV}^n M_{er}^n M_{wear}^n \quad \text{Eq. 7.6}$$

and

$$M^{n+} = M_{FH}^n + M_{FV}^n + M_{er}^n + M_{wear}^n . \quad \text{Eq. 7.7}$$

The best materials present the smallest M^{n*} and M^{n+} .

7.3.3 Optimization technique

In the present case, the simplest algorithm is used, consisting in calculating the value of M^{n*} and M^{n+} in the case of a finite number of predefined set of material properties, blade geometries and erosion parameters (see Table 7.1). The best candidates present the smallest M^{n*} and M^{n+} . More precisely, five variables are considered: E , σ_y , K_{Ic} , h_{seg} and G_{Ic-int} . Each variable takes the form of a vector with a finite length, limited to a realistic range of values. For instance, the Young's modulus varies between 0.01 MPa and 1000 GPa. A large matrix of dimension 5 is then created containing every possible combinations of E , σ_y , K_{Ic} , h_{seg} and G_{Ic-int} . This matrix is used to calculate the performance indices in the case of a contact with a blade of thickness T and width W with a penetration depth h and in the case of erosion with an impact of energy U_{cin} and an impact angle ω . The 3 best combinations of properties that minimize M^{n*} and M^{n+} are selected. This procedure is repeated for different values of T , W , h , U_{cin} and ω .

Table 7.1: List of the predefined set of material properties, blade geometries and erosion parameters used in the optimization. In addition, the material properties of the thermally sprayed PAC-905 abradable material are also included.

Property type	Name	Values	Unit
Blade geometry	T	0.25, 0.5, 1, 2, 4	[mm]
	W	1, 2, 4, 8, 16	[cm]
	h	10^{-10} , 10^{-9} , ..., 10^{-3}	[m]
Erosion	U_{cin}	10^{-12} , 10^{-11} , ..., 10^{-1}	[J]
	ω	10, 30, 60, 90	[°]
Material properties	E	10^4 , 10^5 , ..., 10^{12}	[Pa]
	σ_y	10^3 , 10^4 , ..., 10^{10}	[Pa]
	K_{Ic}	10^3 , 10^4 , ..., 10^9	[Pa m ^{1/2}]
	G_{Ic-int}	10^{-4} , 10^{-3} , ..., 10^6	[J/m ²]
	h_{seg}	10^{-9} , 10^{-8} , ..., 10^{-1}	[m]

Regarding the range of kinetic energy of the eroding particles, these values correspond to the kinetic energy of a spherical particle made of alumina traveling at a speed of 100 m/s with a radius ranging from 100 nm up to 1 mm. This range of size of particles includes many families of powder including clays, silts and sands (see Figure 7.8).

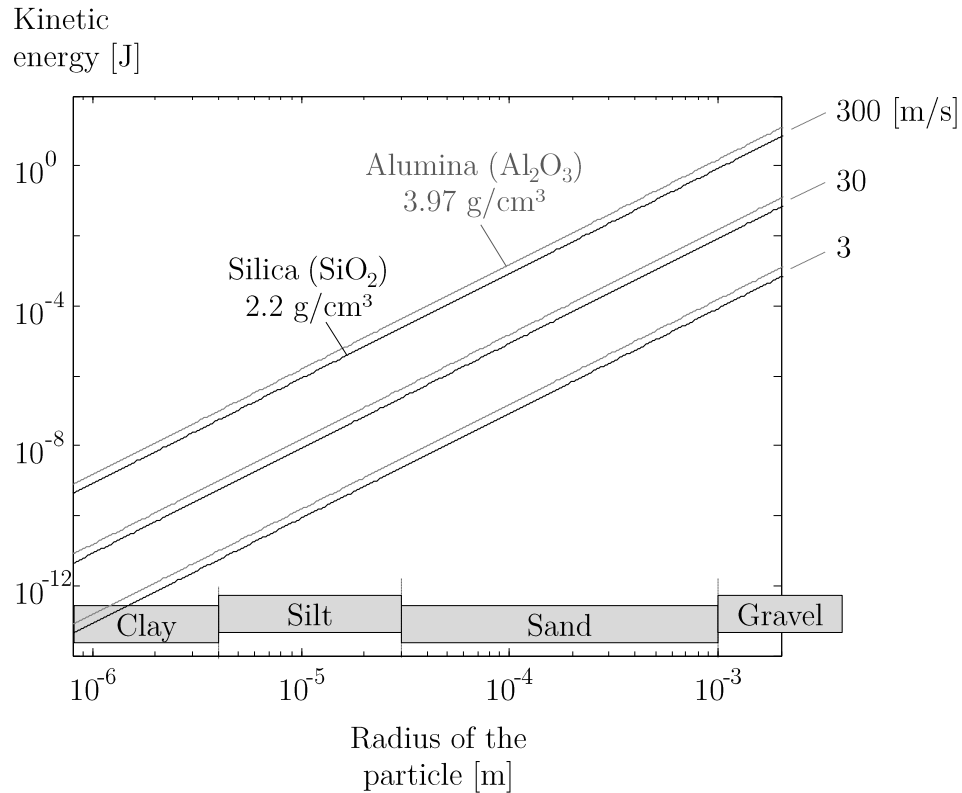


Figure 7.8: Evolution of the kinetic energy of spherical particles as a function of its size, speed and density. The grey intervals represent the name of the family of powder as a function of its size.

Concerning Table 7.1, some combinations of these material properties are impossible. For instance, there are no materials with $\sigma_y = 10^{10}$ Pa and with $E = 10^4$ Pa. In addition, a realistic material selection includes a series of *constraints* to ensure that the material with the best performances will survive in the engine long enough. Indeed, if the temperature in the engine is too high for a material, no matter how good an abrasible it is, it will eventually burn or melt before fulfilling its function.

7.3.4 Material constraints

Regarding the minimal service temperature, let us assume that, even in winter conditions, the engine is exposed to temperature above -50°C . Regarding the maximum service temperature, it depends on the position in the engine. Four cases are therefore

considered: (i) abradable materials in contact with the fan blades at the entry of the engine are exposed to temperatures up to 50°C. (ii) In the case of the first stages of the low-pressure compressor, the maximal temperature is set at 150°C, (iii) in the last stages of the low-pressure compressor it goes to 300°C and (iv) in the high-pressure compressor, up to 600°C. In practice, these temperatures can change from one engine to another. The main idea is to show the effect of the temperature on the selection for an abradable material. In addition, the blade must not be worn out in case of a contact. Therefore, the hardness of the abradable must be lower than the hardness of the blade made of TiA6V (330 HV). In case of friction at high speed, the temperature may increase locally. In any case, the temperature in the friction area must remain beneath the melting temperature of the blade. Therefore, abradable materials presenting a melting temperature larger than the melting temperature of the blade ($\sim 1200^{\circ}\text{C}$) are also rejected. Regarding the *objectives* of the selection, they are the same as the ones covered in the optimization: (i) minimizing the contact force on the blade, (ii) minimizing the wear in case of a contact with the blade and (iii) minimizing the erosion rate. The *free variable* is the choice of the material. These design requirements are summarized in Table 7.2.

The constraints listed in Table 7.2 reduce the number of available materials for the selection. These constraints are implemented using CES Edupack 2016 software [CES2016] with the level 3 material database. Depending on the maximum service temperature, the remaining materials present a limited range of mechanical properties (see Figure 7.9 and Figure 7.10). These remaining property ranges are summarized in Table 7.3. The combinations of mechanical properties that are lying outside these boundaries are removed from the optimization.

Table 7.2: Design requirements for the selection of an abradable material. Four cases are presented. Each case corresponds to a different position in the engine.

Function	Abradable materials
Constraints	Minimum service $T^\circ < -50^\circ\text{C}$ Maximum service T° $> 50^\circ\text{C}$ (case 1: abradable in contact with the fan blades) $>150^\circ\text{C}$ (case 2: entry of the low-pressure compressor) $>300^\circ\text{C}$ (case 3: end of the low-pressure compressor) $>600^\circ\text{C}$ (case 4: high-pressure compressor) Melting $T^\circ < 1200^\circ\text{C}$ Hardness $< 330 \text{ HV}$
Objectives	Minimizing the contact force on the blade Minimizing the wear in case of a contact with the blade Minimizing the erosion rate
Free variable	Choice of material

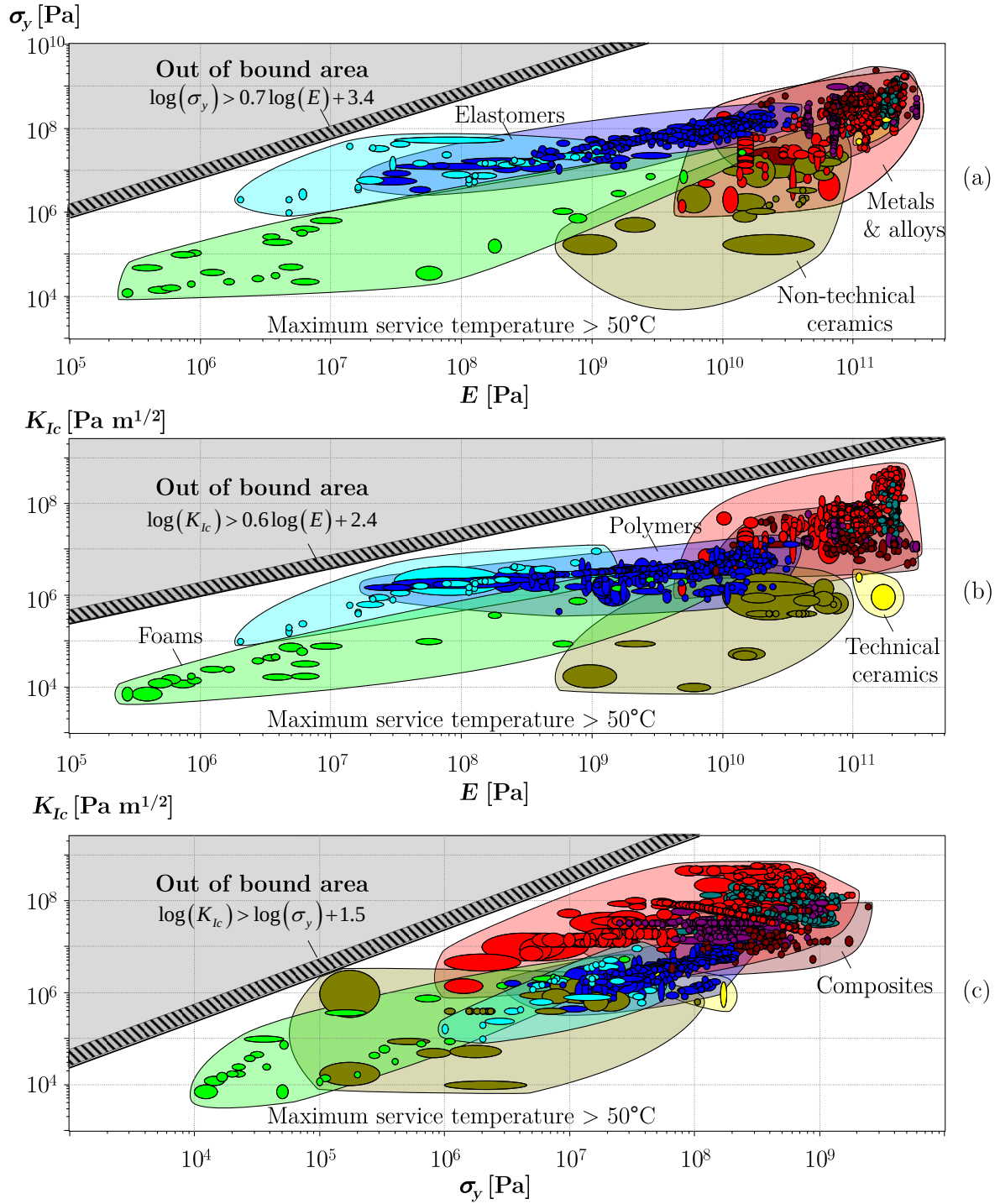


Figure 7.9: Material charts extracted from CES Edupack data base level 3 [CES 2016]; a) the yield strength versus the Young's modulus chart; b) the fracture toughness versus E charts; c) the fracture toughness versus the yield strength. The constraints corresponding to an abrasible with a maximum service temperature of 50°C are applied. Only the materials that fulfill the constraints are shown. The inaccessible material properties combinations correspond to the grey area.

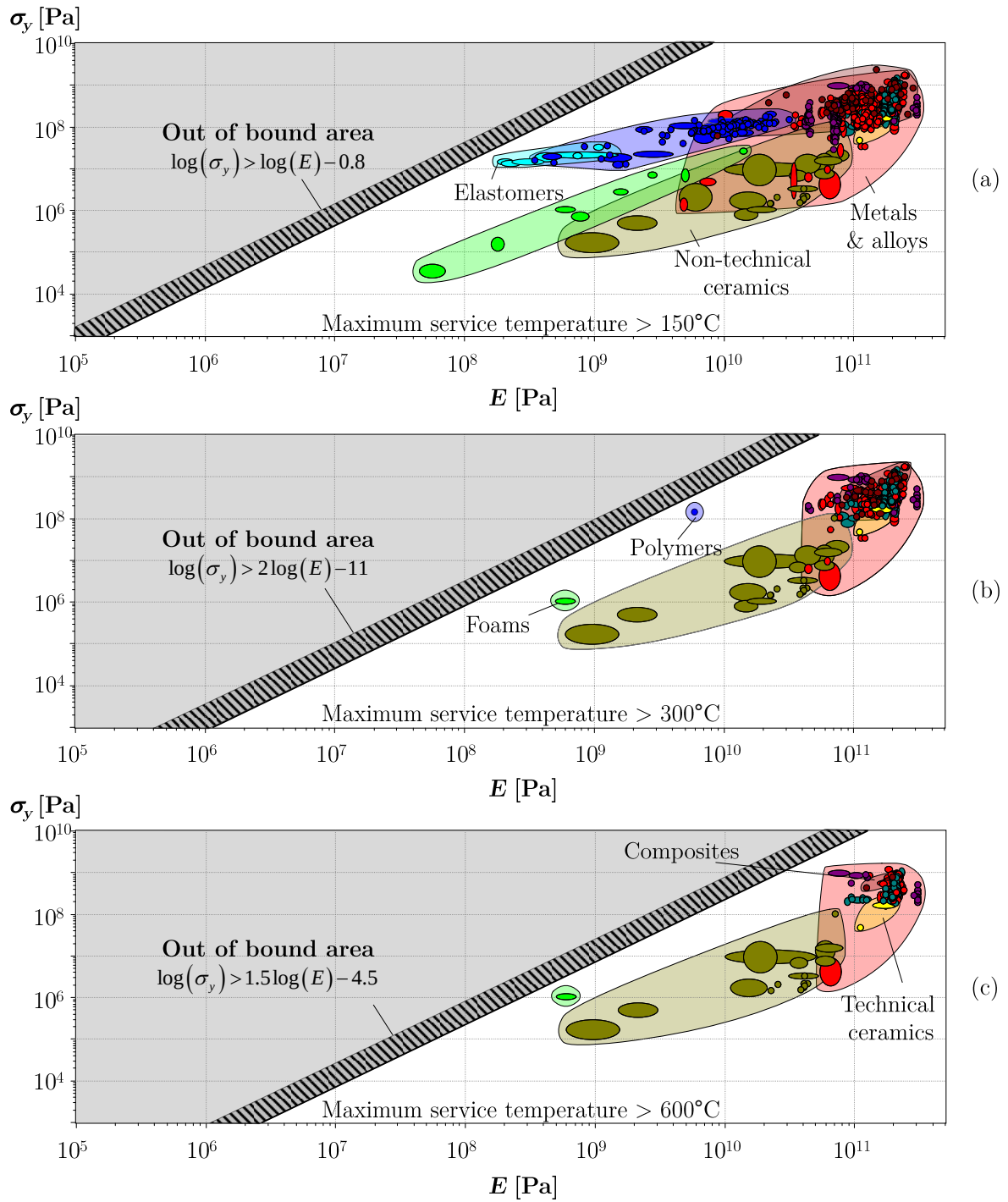


Figure 7.10: Material charts extracted from CES Edupack data base level 3 [CES 2016]. The constraints corresponding to three different maximum service temperatures are applied; a) maximum service temperature $> 150^\circ\text{C}$, b) $> 300^\circ\text{C}$ and c) $> 600^\circ\text{C}$.

Table 7.3: Boundaries of the mechanical properties of the remaining materials after applying the constraints using CES Edupack 2016 database as a function of the maximal service temperature.

Maximal service $T^\circ > 50^\circ\text{C}$	Maximal service $T^\circ > 150^\circ\text{C}$
$E > 2 \cdot 10^5 \text{ [Pa]}$ $\sigma_y > 10^4 \text{ [Pa]}$ $\log(\sigma_y) < \log(E) + 0.72$ $\log(K_{Ic}) < 0.7 \log(E) + 1.65$ $\log(K_{Ic}) > \log(\sigma_y) + 1.73$	$E > 3 \cdot 10^7 \text{ [Pa]}$ $\sigma_y > 2 \cdot 10^4 \text{ [Pa]}$ $\log(\sigma_y) < \log(E) - 0.8$ $\log(K_{Ic}) < \log(E) - 1.45$ $\log(K_{Ic}) > 1.3 \log(\sigma_y) - 0.45$
Maximal service $T^\circ > 300^\circ\text{C}$	Maximal service $T^\circ > 600^\circ\text{C}$
$E > 3 \cdot 10^8 \text{ [Pa]}$ $\sigma_y > 10^5 \text{ [Pa]}$ $\log(\sigma_y) < 2 \log(E) - 11$ $\log(K_{Ic}) < 1.5 \log(E) - 7.5$ $\log(K_{Ic}) > 2 \log(\sigma_y) - 5$	$E > 3 \cdot 10^8 \text{ [Pa]}$ $\sigma_y > 10^5 \text{ [Pa]}$ $\log(\sigma_y) < 1.5 \log(E) - 6.5$ $\log(K_{Ic}) < 1.5 \log(E) - 7.5$ $\log(K_{Ic}) > 2 \log(\sigma_y) - 5$

7.3.5 Results of the optimization

An example of the optimization results in the case of $h = 1 \text{ }\mu\text{m}$, $W = 4 \text{ cm}$, $T = 1 \text{ mm}$, $U_{cin} = 10^{-7} \text{ J}$ and $\omega = 10^\circ$ with a maximum service temperature of 50°C is presented in Figure 7.11. This figure shows that, in that particular contact situation, F_H and F_V are proportional to each other. It also shows that, in terms of wear mechanisms, the best candidates wear off via a plowing mechanism when impacted by the blades and also via a plowing mechanism when impacted by an erosive particle. Also, M^{n*} and M^{n+} point toward the same material. The best candidates depend on h , W , T , U_{cin} and ω . In order to present the results of the optimization in the simplest way, only one variable is changed at a time, with the others kept constant. For instance, the effect of varying the bite per blade (h) on the optimum set of material properties are summarized in Figure 7.12. This figure shows that the optimum systematically corresponds to a plowing mechanism in case of a contact with the blades and also via a plowing mechanism in case of erosion. This figure also shows that the best set of properties correspond to a Young's modulus of 10 MPa, a yield strength of 48 MPa, a fracture toughness of 1 MPa. Regarding the interface, the optimum interface toughness

is 10^5 J/m² with a segment size of 1 μ m. This set of optimal material properties remains the same when varying W , T and the impacting angle (ω). However, a significant change in the optimum size of the segments (h_{seg}) is observed when varying the kinetic energy of the eroding particles (see Figure 7.13).

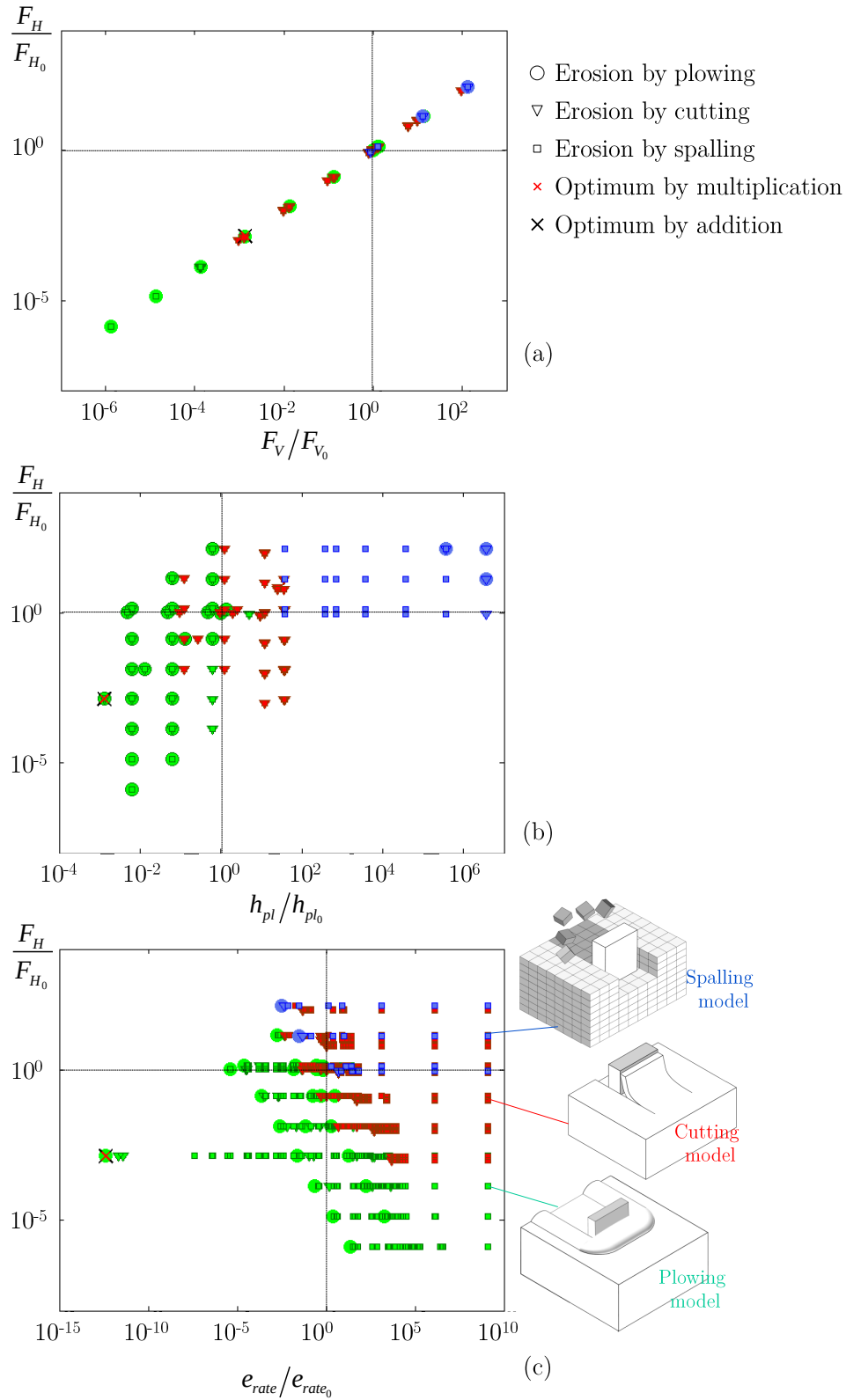


Figure 7.11: Example of optimization results corresponding to $h = 1 \mu\text{m}$, $W = 4 \text{ cm}$, $T = 1 \text{ mm}$, $U_{cin} = 10^{-7} \text{ J}$, $\omega = 10^\circ$ and a maximum service temperature of 50°C . The best candidates are in the bottom left part of each graph. The optimum properties are represented with a cross.

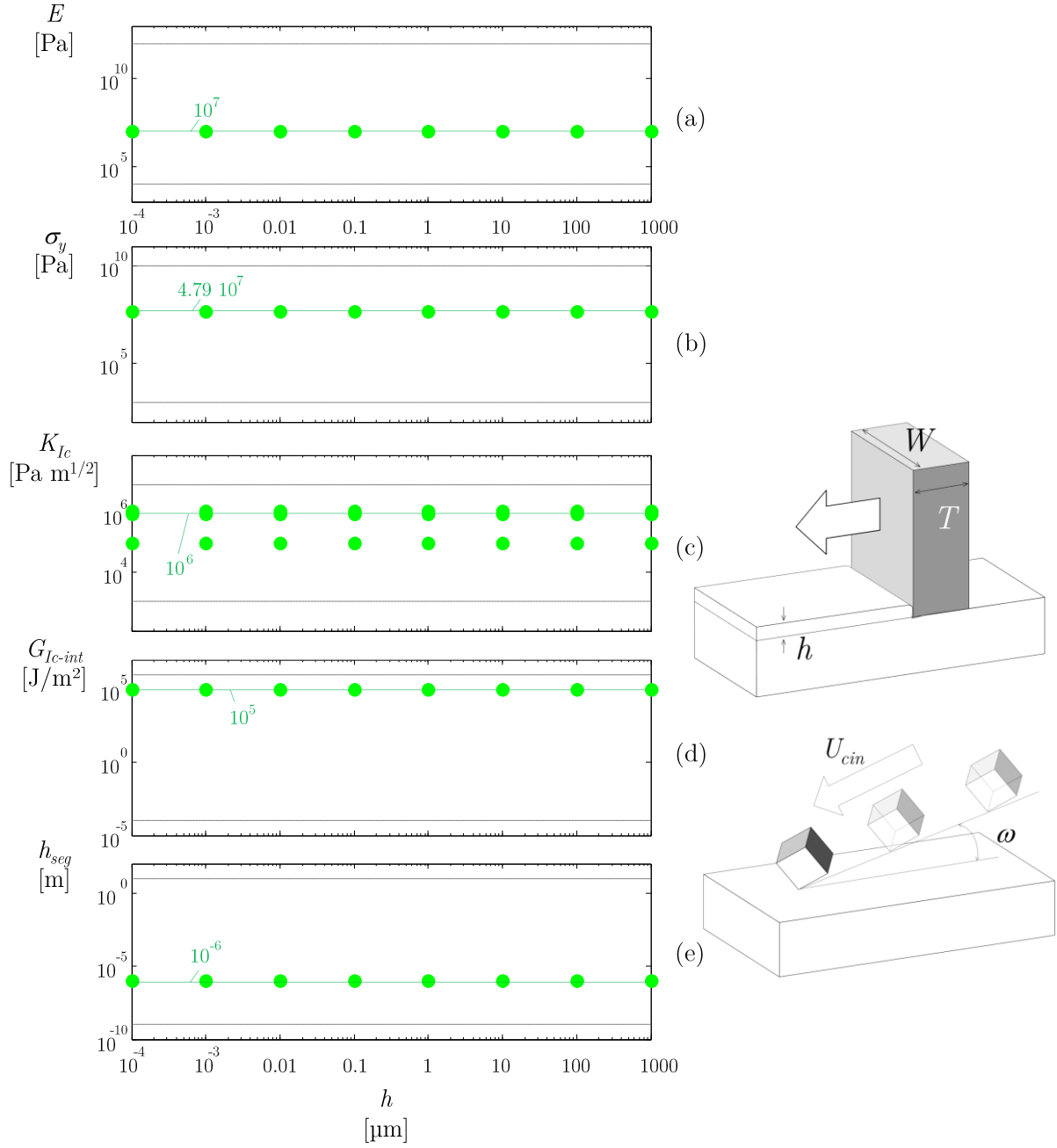


Figure 7.12: Evolution of the material properties of the three best candidates as a function of the bite per blade (h); a) the Young's modulus; b) the yield strength; c) the fracture toughness; d) the toughness of the interface between the segments; e) the size of the segments. In this graph, $W = 4$ cm, $T = 1$ mm, $U_{cin} = 10^{-7}$ J, $\omega = 10^\circ$ and the maximum service temperature is 50°C .

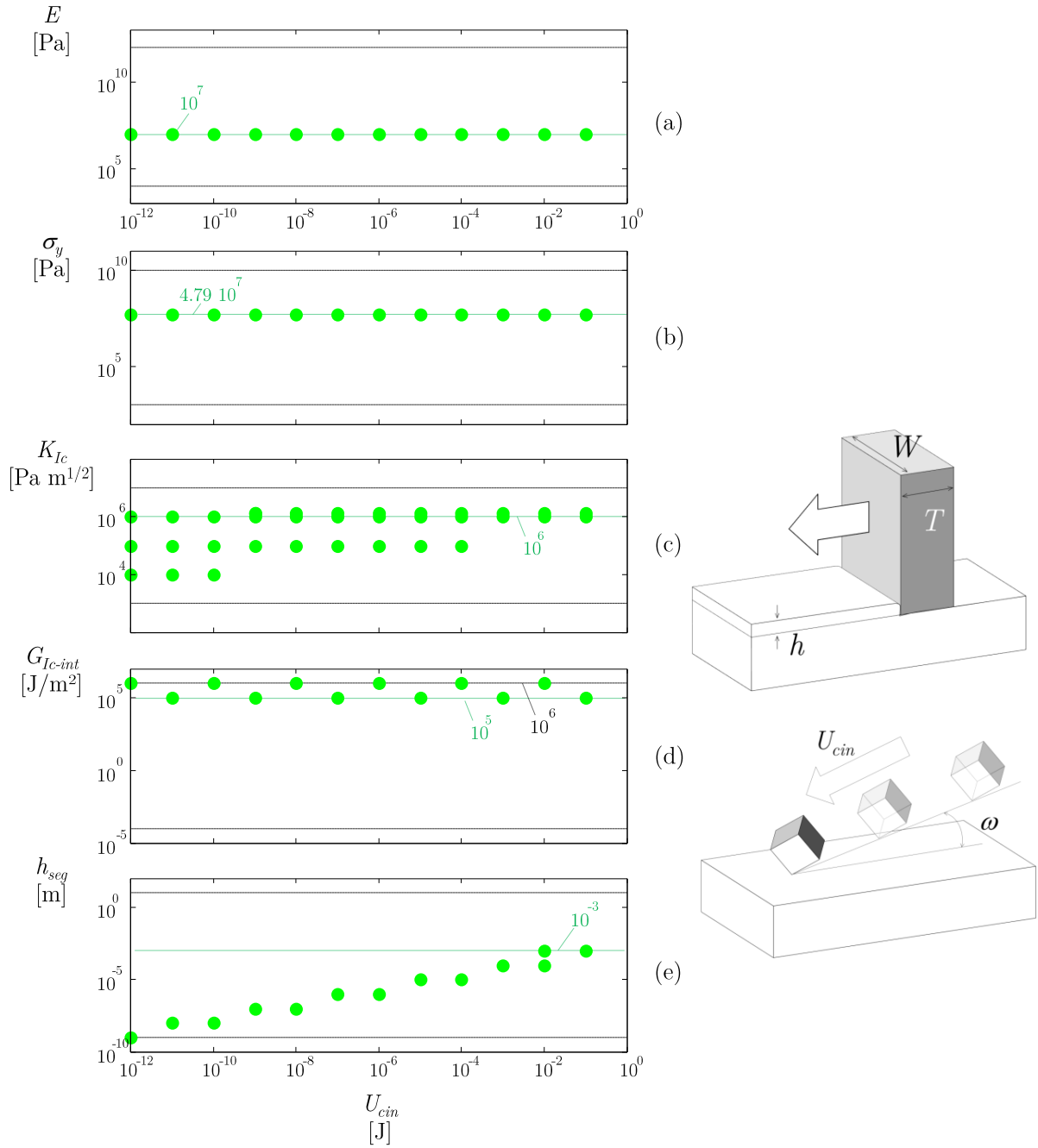


Figure 7.13: Evolution of the material properties of the three best candidates as a function of the kinetic energy of the impacting eroding particle; a) the Young's modulus; b) the yield strength; c) the fracture toughness; d) the toughness of the interface between the segments; e) the size of the segments.. In this graph, $h = 1 \mu\text{m}$, $W = 4 \text{ cm}$, $T = 1 \text{ mm}$, $\omega = 10^\circ$ and the maximum service temperature is 50°C .

At higher temperature, the optimum properties follow the same trends, except for G_{Ic-int} that seems to be lower at higher maximum service temperature (see Table 7.4). Regarding the wear mechanisms, below 300°C, the optimum materials wear off via a plowing mechanism both in case of a contact with the blade and in case of erosion. Above 300°C, a contact with the blade often results in plowing except in case of extreme incursion depth (larger than 1 mm). Regarding the erosion wear mechanisms, plowing dominates and switches to cutting when the impact energy is larger than 10⁴J which corresponds to large sand particles impacting the abradable at high speed (see Figure 7.8).

Table 7.4: List of the optimal material properties of the best candidates as well as the associated wear mechanism.

Material properties	<i>Maximum service temperature</i>			
	<i>50°C</i>	<i>150°C</i>	<i>300°C</i>	<i>600°C</i>
E [Pa]	10 ⁷	10 ⁹	10 ¹⁰	7.6 10 ⁹
σ_y [Pa]	4.79 10 ⁷	10 ⁸	10 ⁹	10 ⁸
σ_y/E []	4.79	0.1	0.1	0.13
K_{Ic} [Pa m ^{1/2}]	10 ⁶ -> 10 ⁷	10 ⁶ -> 10 ⁷	10 ⁶ -> 10 ⁷	10 ⁶ -> 10 ⁷
G_{Ic} [J/m ²]	10 ⁵ -> 10 ⁷	10 ³ -> 10 ⁵	10 ² -> 10 ⁴	10 ² -> 10 ⁴
G_{Ic-int} [J/m ²]	10 ⁵ -> 10 ⁶	10 ⁻¹ -> 10	10 ⁻⁴ -> 100	10 ⁻⁴ -> 10 ⁻¹
h_{seg} [m]	10 ⁻⁹ -> 10 ⁻³	10 ⁻⁹ -> 10 ⁻³	10 ⁻⁹ -> 10 ⁻³	10 ⁻⁹ -> 10 ⁻³
Wear mechanism				
Blade contact	Plowing	Plowing	Plowing/spalling	Plowing/spalling
Erosion	Plowing	Plowing	Plowing/cutting	Plowing/cutting

In this table, E and h_{seg} are quite low, the ratio σ_y/E as well as G_{Ic} are very large. With these sets of properties, the optimization seems to choose materials with large elastic recovery, and a large toughness to favor plowing instead of spalling and cutting. The Young's modulus is low, probably because when the ratio σ_y/E is large, materials behave like ideal elastic materials. In the case of elastic materials, the hardness, as well

as the forces acting on the blade are proportional to E . These sets of optimum mechanical properties are compared with material charts extracted from CES Edupack 2016 data base (see Figure 7.14). On this figure, the best materials for an abradable depend on the temperature. Below 50°C, elastomers are the closest to the optimum. At 150°C, the remaining elastomers accompany by other polymers are close to the optimum. At 300°C, one polymer is remaining and competes with magnesium alloy and sandstone around the optimum. Above 600°C, the close proximity of the optimum is left vacant surrounded by sandstone, porcelain, titanium alloys and cordierite foam. Regarding the model materials, their chemistry allows them to operate at up to 300°C. At low temperature, the model materials are far away from the optimum. However, at 300°C, they are amongst the best candidates with a close proximity to the optimum.

At high temperature, many materials compete with each other. How to discriminate between these competing candidates? In the previous section, the results of the optimization shown that the best abradable material seems to be the result of (i) maximizing σ_y/E in order to increase elastic recovery which is good regarding the residual wear in case of a contact with the blade and with erosive particles. (ii) Maintaining a large value of G_{Ic} to avoid cutting and (iii), reducing E to maintain low contact forces with the blades in the elastic regime. Therefore, selecting the best material involves a trade-off between two performance indices σ_y/E and E . Figure 7.15 presents the two indicators: E/σ_y and E in the case of a maximum service temperature larger than 300°C. In this figure, the first indicator is the inverse of σ_y/E . This choice is more convenient from a graphical point of view. Indeed, in that case, the best materials are the ones that minimize both indicators. This figure shows which candidates to select in which conditions. If the contact forces with the blades are more important than the elastic recovery, the cordierite foam is better than the polymer (PBI). If the elastic recovery is considered more important than the contact force, the PBI is the best candidates. Intermediate solutions could consist in using a porous polymer but with a particular attention to maintain a high G_{Ic} to remain in a plowing mechanism. The next step in material selection consists in acquiring additional documentation on these materials candidates to ensure that they are compliant with additional constraints described in Section 2.2 , including corrosion and oxidation resistance. A ranking of the materials as a function of the ratio σ_y/E is presented in Table 7.5 and Table 7.6.

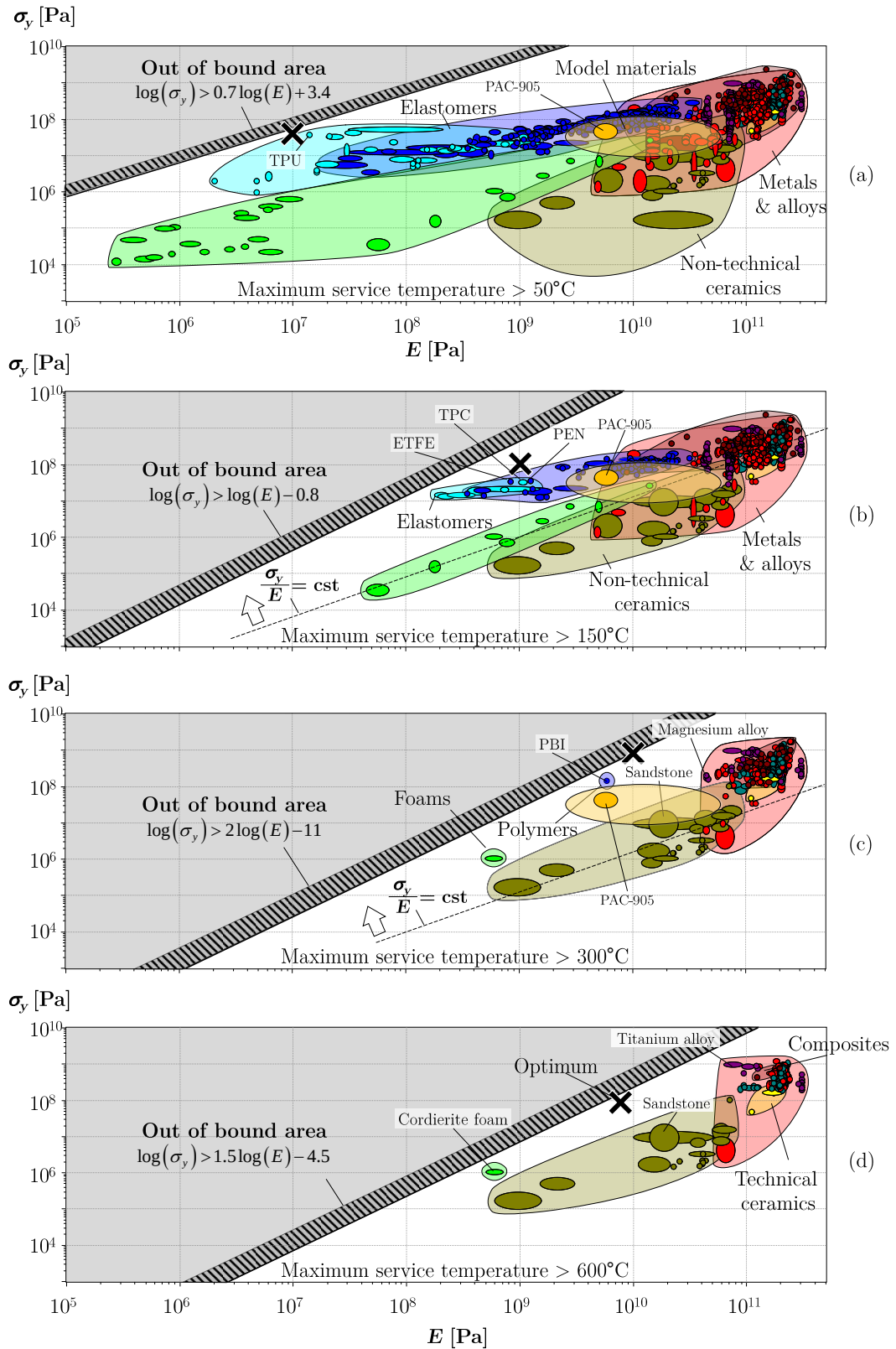


Figure 7.14: Material charts of the Young's modulus versus the yield strength as a function of the maximal service temperature of the materials; a) $T^\circ > 50^\circ\text{C}$, b) $T^\circ > 150^\circ\text{C}$, c) $T^\circ > 300^\circ\text{C}$ and d) $T^\circ > 600^\circ\text{C}$. The optimum set of properties is represented with a black cross.

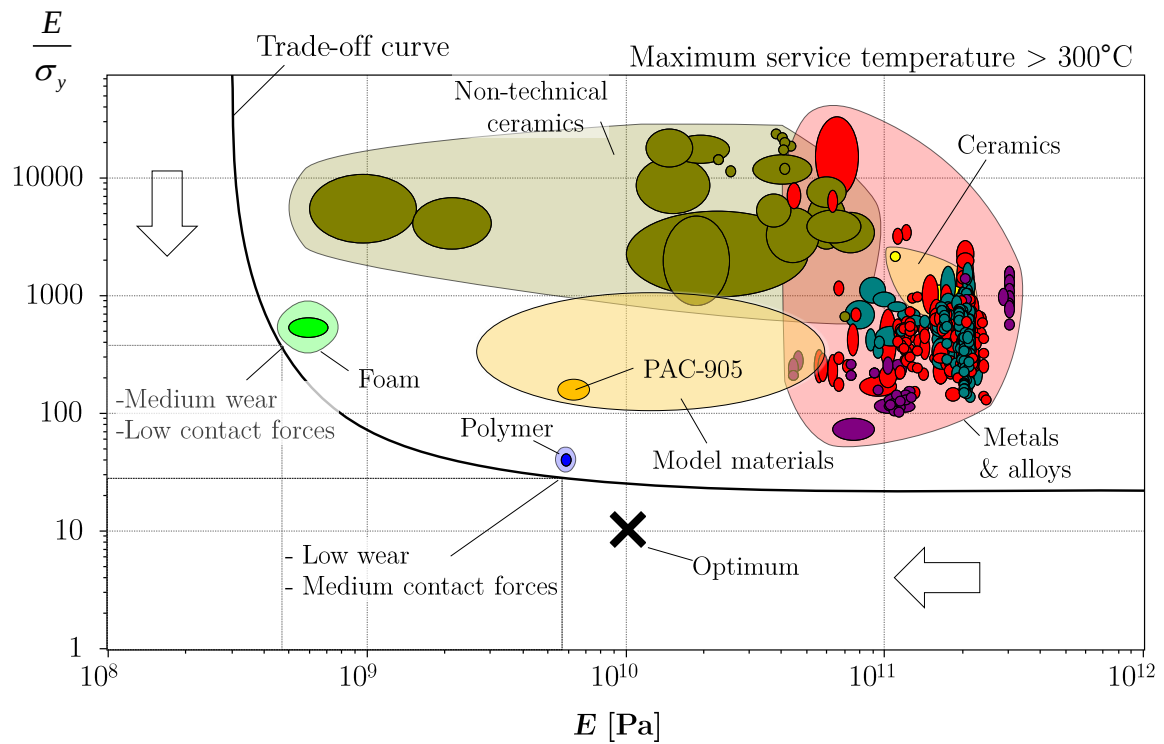


Figure 7.15: Material charts extracted from CES Edupack data base in the case of a maximum service temperature higher than 300°C. Only the materials that fulfill the constraints are shown. The trade-off curve represents the boundary of non-dominated solutions. The best materials lie in the bottom left of the graph.

Table 7.5: List of materials that fulfill the constraints, ranked as a function of the ratio σ_y/E (part 1 on 2).

Material family	Name	$\frac{\sigma_y}{E}$	Maximum service temperature [$^{\circ}C$]
Elastomers	TPU (Thermoplastic Polyurethane Elastomer)	2.7	93 - 107
	TPO (Blend of PP + EPM + EPDM copolymer)	$7.4 \cdot 10^{-2}$	135 - 150
	TPC (Block copolymer of PBT and polyether)	$6.2 \cdot 10^{-2}$	150 - 170
Polymers	FEP (Fluorinated ethylene propylene)	$4.6 \cdot 10^{-2}$	200
	PTFE (Poly-tetra-fluoro-ethylene)	$4.4 \cdot 10^{-2}$	260
	ETFE (Ethylene-tetra-fluoro-ethylene)	$4.3 \cdot 10^{-2}$	150
	PI (Polyimide)	$3.7 \cdot 10^{-2}$	230
	PEN (Poly-ethylene naphthalate)	$3.4 \cdot 10^{-2}$	170
	PFA (Perfluoro-alkoxyethylene)	$3.0 \cdot 10^{-2}$	260
	PCTFE (Polychlorotrifluoroethylene)	$3.0 \cdot 10^{-2}$	190
	PBI (Poly-benzimidazole)	$2.5 \cdot 10^{-2}$	300
	PEEK (Poly-ether-ether-kethone)	$2.4 \cdot 10^{-2}$	250
	PPS (Poly-phenylene Sulfide)	$2.0 \cdot 10^{-2}$	260
	PMP (Poly-methyl-pentene)	$1.3 \cdot 10^{-2}$	205
	LCP (Liquid Crystal Polyester)	$8.0 \cdot 10^{-3}$	190 - 300
Metals and alloys (part 1)	Selenium	$1.9 \cdot 10^{-2}$	190 - 200
	Titanium alloys	$0.4 - 1.4 \cdot 10^{-2}$	400 - 500
	Magnesium alloys	$1.5 - 8.8 \cdot 10^{-3}$	130 - 200
	Gold	$0.4 - 7.7 \cdot 10^{-3}$	100 - 330
	Cobalt alloys	$1.5 - 7.4 \cdot 10^{-3}$	260 - 1100
	Copper alloys	$0.7 - 7.0 \cdot 10^{-3}$	160 - 350
	Nickel alloys	$0.4 - 6.8 \cdot 10^{-3}$	800 - 1000
	Low alloys steel	$1.3 - 6.7 \cdot 10^{-3}$	500
	Bronze	$0.9 - 6.6 \cdot 10^{-3}$	180 - 260
	Stainless steel	$0.9 - 6.1 \cdot 10^{-3}$	750 - 800
	Aluminium alloys	$0.4 - 6.0 \cdot 10^{-3}$	130 - 230

Table 7.6: List of materials that fulfill the constraints, ranked as a function of the ratio σ_y/E (part 2 on 2).

Material family	Name	$\frac{\sigma_y}{E}$	Maximum service temperature [°C]
Metals and alloys (part 2)	Brass	$0.8 - 4.9 \cdot 10^{-3}$	200
	Cast iron	$0.8 - 3.6 \cdot 10^{-3}$	450 - 800
	Carbon steel	$1.1 - 2.9 \cdot 10^{-3}$	350
	Silver	$0.7 - 2.3 \cdot 10^{-3}$	100 - 190
	Beryllium alloys	$0.7 - 1.7 \cdot 10^{-3}$	255 - 830
	Lithium	$2.9 \cdot 10^{-4}$	150
	Bismuth	$1.5 \cdot 10^{-4}$	245
	Antimony	$6.5 \cdot 10^{-5}$	600
Foams	Aluminium + Silicon carbide foam	$0.6 - 2.6 \cdot 10^{-3}$	140 - 200
	Aluminium alloy foam	$1.4 - 2.0 \cdot 10^{-3}$	140 - 200
	Cordierite foam	$1.9 \cdot 10^{-3}$	1100 - 1200
Non-technical ceramics	Porcelain	$1.5 \cdot 10^{-3}$	700
	Sandstone	$5.0 \cdot 10^{-4}$	400 - 600
	Brick	$2.4 - 4.4 \cdot 10^{-4}$	700 - 900
	Plaster	$3.5 \cdot 10^{-4}$	110 - 180
	Limestone	$3.0 \cdot 10^{-4}$	350
	Slate	$2.9 \cdot 10^{-4}$	350 - 500
	Granite	$2.3 \cdot 10^{-4}$	600 - 700
	Concrete	$0.6 - 1.9 \cdot 10^{-4}$	200 - 300
	Marble	$1.3 \cdot 10^{-4}$	350
	Cement	$4.4 - 8.4 \cdot 10^{-5}$	600 - 850
Technical ceramics	Mica	$1.0 \cdot 10^{-3}$	500 - 600
	Cordierite	$4.5 \cdot 10^{-4}$	1000 - 1200

7.4 Summary

This chapter can be summarized with four key messages:

- The observed differences between the model predictions and the experimental results are attributed in some cases, to the model and in other cases to experimental setup imperfections.
- During an abrasability test, the repetition of contacts combined with the elastic recovery of the abradable material can lead to nonlinear wear. Indeed, the model predicts that the penetration depth and the wear mechanism can evolve from one blade revolution to another.

- Experimental results show that the erosion resistance and the abrasability are not necessarily antagonist properties.
- The optimization showed that the best abrasable candidates exhibit wear by plowing and present a high ratio σ_y/E combined with a low E and a high G_{Ic} .
- The model materials, including the thermally sprayed PAC-905 are amongst the best candidates at temperatures larger than 300°C.

Chapter 8 Conclusions and perspectives

The goal of this thesis was centered on the following question: How to design a good abradable material? In order to answer this question, a material selection was followed. This material selection involved several steps. The design requirements were identified through a literature survey on abradable materials. An assesement of the literature on contact mechanics and wear modelling gave the basic ingredients to model the interaction between a blade and an abradable material. These models were modified to include elastoplasticity and used to translate the design requirements into equations. These models were calibrated and validated owing to a large set of experimental results that includes the processing of model materials as well as mechanical and tribological measurements on these materials. Given the complexity of the equations of the models, and the large number of parameters, an optimization was performed to select the best abradable material candidates. Materials that did not fulfill the constraints imposed by the environment of the engine were excluded. The remaining materials were selected based on the minimization of (i) the forces acting on the blade in case of a contact, (ii) the wear after contact and (iii) the erosion wear. The material selection pointed towards materials with high toughness (G_{Ic}), with low Young's modulus (E) and with high ratio σ_y/E .

The state of the art on abradable materials was covered in Chapter 2. This chapter brought (i) a definition of abrasability, (ii) a description of the wear mechanisms and (iii) an overview of the testing methods as well as (iv) the strategies that are used to improve the abrasability of materials including using elastic materials, materials with low yield strength or favoring fragmentations by means of defects in the microstructure.

Chapter 3 was centered on a state of the art on contact mechanics and wear modelling. This chapter offered a summary of indentation models as well as a general overview of wear modelling. In particular, this chapter showed that physics-based wear models follow a general pattern that always includes: (i) the geometry of contact, (ii) a simplified representation of the deformations of the surface of the base material as well as (iii) a simple mechanical behavior for the base material, usually rigid perfectly plastic or ideal elastic. In the case of cutting models, some of these models also include

fracture mechanics. Regarding the modelling of multiple wear mechanisms, the dominant wear mechanism can be identified based on an energy minimization criterion.

In Chapter 4, wear models were adapted to include elastoplastic materials. The motivation was to quantify the elastic recovery after contact. Three contact geometries were modelled including orthogonal cutting tools, rectangular wedge and cones. In addition to the modelling of indentation, three wear mechanisms observed in abradable materials were modelled (plowing, cutting and spalling). These models were analyzed in the case of a contact at a given penetration depth and in the case of a contact with erosive particles impacting the material with a given kinetic energy. One key result of these models is that the behavior of elastoplastic materials is governed by the ratio σ_y/E which defines the extend of elasticity in these materials.

Chapter 5 presented the process of the model material and the testing methods. Foam infiltration, cold-pressing and hot-pressing processes were successfully used to create model materials with a large panel of microstructures and mechanical properties. These model materials include powder aggregates, polyester matrix with spherical metallic reinforcements as well as co-continuous metallic and polymer phases. The testing protocols were designed to study the tribological response of the model materials from the microscale to the macroscale in addition to high speed testing on abrasability test rig and erosion test rig.

Chapter 6 presented the experimental results as well as a comparison with the predictions of the models. As a consequence of the large variety of model materials and contact geometries, the three wear mechanisms of plowing, cutting and spalling were observed and predicted successfully by the model. The multi-mechanisms approach followed in this work showed that the criteria based on the weakest link to determine the dominant wear mechanism works. The amount of AlSi in the model materials has a strong effect on the tribological response. It increases the hardness, decreases elastic recovery and, in the case of cold-pressed materials, increases the erosion resistance. The geometry of contact and the penetration depth did also affect the tribological response of the model materials. Indeed, nanoscratch tests and erosion tests resulted in wear by plowing and spalling whereas pin-on-disc tests showed cutting sometimes combined with spalling. Abrasability test results also showed cutting, sometimes combined with smearing and grooving. Regarding the prediction of the model from a

quantitative point of view, the accuracy of the models is promising yet perfectible, especially in the case of materials that exhibit wear by spalling.

Chapter 7 started by offering explanations for the discrepancies between the model predictions and the experimental results. Regarding the mechanisms observed on the abrasability test rig, a novel explanation is proposed, based on the combination of elastic recovery and on the repetitions of contacts that can lead to nonlinear wear. The comparison between erosion and pin-on-disc test results showed that the erosion resistance and the abrasability are not antagonist properties. Indeed, some model materials exhibited a better erosion resistance with lower contact forces and lower residual wear than the industrial material.

An optimization was performed to identify the optimal set of properties for an abrasable material in order to combine erosion resistance, low contact forces with the blade and low wear in case of a contact. The optimization showed that the best abrasable candidates exhibit wear by plowing. In the case of abrasable materials used at low temperatures, the best candidates present a high ratio σ_y/E , a high G_{Ic} and a low E . Therefore, elastomers and polymers appear as the best candidates. The optimization also showed that the model materials, including the thermally sprayed PAC-905 are amongst the best candidates at temperatures larger than 300°C.

Regarding the perspectives, in order to validate the hypotheses on the nonlinear wear happening during an abrasability test, it would be interesting to observe the wear mechanisms in situ using high-speed camera during an abrasability test. Indeed, current observations are based on post-mortem analyses that are performed after a very large number of interactions between the blade and the abrasable material.

In order to validate the optimization results, it would be useful to assess the performance of elastomers and polymers on abrasability test rigs and on erosion tests. These tests will help to understand the limits of these potential abrasable material candidates.

From a modelling point of view, the present multi-mechanism approach is promising. Taking this method to the next level includes:

- Taking into account the strain rate sensitivity in the models as well as the effect of temperature.

- Including additional wear mechanisms, like melting wear, brittle cracking and thermal shock.
- Extending the models to additional tip geometries, including sphere and cylinder.
- Investigating the effect of geometrical imperfections like the bluntness of the tip or the surface roughness of the base material. Indeed, in some contact situations, the roughness of the tip or the roughness of the base material is larger than the penetration depth. In addition, some wear mechanisms like *grooving* seems to be the results of an effect of the surface roughness.
- Finally, a real blade presents a complex curved geometry. In this work, blades are modelled as rectangles. It might be interesting to investigate the effect of the real shape of the blade in terms of contact forces, elastic recovery and wear mechanisms.

Appendix A Additional calculations

This appendix is a compilation of detailed calculations that are considered too voluminous to enter the main text.

A.1 Indentation calculations

The following calculations complete the models presented in Chapter 3 and Chapter 4.

A.1.1 Elastic contact with a half-wedge tip

The ideal elastic solution of the half-wedge tip presented in Table 3.1 derives from calculations presented hereunder. The geometry of contact is presented in Figure A.1. Following the methods proposed by [Johnson 1985] in his book section 2.4, the elastic displacements of the surface can be related to the normal pressure $p(x)$ and the lateral pressure $q(x)$ through:

$$\begin{cases} \frac{\partial \bar{u}_x}{\partial x} = -\frac{2(1-\nu^2)}{\pi E} \int_{-b}^b \frac{q(s)}{x-s} ds + \frac{(1-2\nu)(1+\nu)}{E} p(x) \\ \frac{\partial \bar{u}_z}{\partial x} = \frac{2(1-\nu^2)}{\pi E} \int_{-b}^b \frac{p(s)}{x-s} ds + \frac{(1-2\nu)(1+\nu)}{E} q(x) \end{cases} \quad \text{Eq. A.1}$$

Here, there is no lateral displacement of the tip, therefore, $q(x) = 0$.

$$\begin{cases} \frac{\partial \bar{u}_x}{\partial x} = \frac{(1-2\nu)(1+\nu)}{E} p(x) \\ \frac{\partial \bar{u}_z}{\partial x} = \frac{2(1-\nu^2)}{\pi E} \int_{-b}^b \frac{p(s)}{x-s} ds \end{cases} \quad \text{Eq. A.2}$$

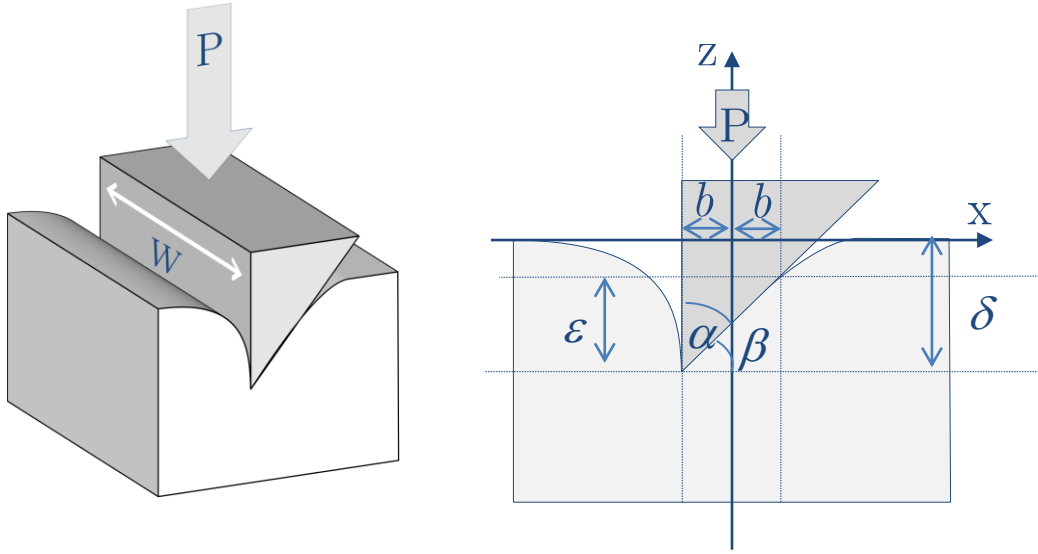


Figure A.1: Schematic representation of elastic the indentation of an infinite half space by an half-wedge tip of width W at a depth δ .

From the geometry of the problem it can be shown that

$$\begin{aligned}
 &\begin{cases} 2b = a = R \sin \alpha \\ \varepsilon = R \cos \alpha \end{cases} \\
 &\frac{2b}{\varepsilon} = \tan \alpha \\
 &\Leftrightarrow b = \frac{\varepsilon}{2} \tan \alpha \\
 &\Leftrightarrow \varepsilon = 2b \cot \alpha \\
 &\Leftrightarrow \frac{\partial \bar{u}_z}{\partial x} = \cot \alpha
 \end{aligned}
 \tag{Eq. A.3}$$

Combining Eq A.2 with Eq A.3 gives

$$\begin{aligned}
&\Leftrightarrow \frac{\partial \bar{u}_z}{\partial x} = \frac{2(1-\nu^2)}{\pi E} \int_{-b}^b \frac{p(s)}{x-s} ds = \cot \alpha \\
&\Leftrightarrow \int_{-b}^b \frac{p(s)}{x-s} ds = g = \frac{\pi E \cot \alpha}{2(1-\nu^2)} \\
&\Leftrightarrow p(x) = \frac{1}{\pi^2 \sqrt{b^2 - x^2}} \int_{-b}^b \frac{g \sqrt{b^2 - s^2}}{x-s} ds + \frac{C}{\pi^2 \sqrt{b^2 - x^2}} \quad [\text{Eq. 2.41 Johnson 1985}] \\
&\Leftrightarrow p(x) = \frac{g}{\pi^2 \sqrt{b^2 - x^2}} \int_{-b}^b \frac{\sqrt{b^2 - s^2}}{x-s} ds + \frac{C}{\pi^2 \sqrt{b^2 - x^2}} \quad [\text{Change of Variable : } x=aX, s=aS] \\
&\Leftrightarrow p(X) = \frac{g b}{\pi^2 \sqrt{b^2 - b^2 X^2}} \int_{-1}^1 \frac{\sqrt{b^2 - b^2 S^2}}{aX - aS} dS + \frac{C}{\pi^2 \sqrt{b^2 - b^2 X^2}} \\
&\Leftrightarrow p(X) = \frac{g b}{\pi^2 b \sqrt{1 - X^2}} \int_{-1}^1 \frac{\sqrt{1 - S^2}}{X - S} dS + \frac{C}{\pi^2 b \sqrt{1 - X^2}} \\
&\Leftrightarrow p(X) = \frac{g}{\pi^2 \sqrt{1 - X^2}} \pi X + \frac{C}{\pi^2 b \sqrt{1 - X^2}} \quad [\text{Appendix 1 - (4) Johnson 1985}] \\
&\Leftrightarrow p(x) = \frac{g}{\pi^2 \sqrt{1 - \frac{x^2}{b^2}}} \pi \frac{x}{b} + \frac{C}{\pi^2 \sqrt{b^2 - x^2}} \\
&\Leftrightarrow p(x) = \frac{1}{\pi^2 \sqrt{b^2 - x^2}} (\pi g x + C) \\
&\Leftrightarrow p(x) = \frac{1}{\pi \sqrt{b^2 - x^2}} (g x + P) \quad [\text{Eq. 1.10 \& 2.42 Johnson 1985}] \\
&\Leftrightarrow p(x) = \frac{1}{\pi \sqrt{b^2 - x^2}} \left(\frac{\pi E \cot \alpha}{2(1-\nu^2)} x + P \right)
\end{aligned}$$

Eq. A.4

Using boundary conditions,

$$\begin{aligned}
&p(b) = 0, \\
&\Leftrightarrow p(b) = \frac{1}{\pi \sqrt{b^2 - b^2}} \left(\frac{b \pi E \cot \alpha}{2(1-\nu^2)} + P \right) = 0 \\
&\Leftrightarrow P = -\frac{b \pi E \cot \alpha}{2(1-\nu^2)}
\end{aligned}$$

Eq. A.5

Therefore,

$$\begin{aligned}
&p(x) = \frac{1}{\pi \sqrt{b^2 - x^2}} \left(\frac{x \pi E \cot \alpha}{2(1-\nu^2)} - \frac{b \pi E \cot \alpha}{2(1-\nu^2)} \right) \\
&\Leftrightarrow p(x) = \frac{E \cot \alpha}{2(1-\nu^2) \sqrt{b^2 - x^2}} (x - b) \\
&\Leftrightarrow p(x) = \frac{-E \cot \alpha \sqrt{(b-x)} \sqrt{(b-x)}}{2(1-\nu^2) \sqrt{(b-x)} \sqrt{(b+x)}} \\
&\Leftrightarrow p(x) = \frac{-E \cot \alpha}{2(1-\nu^2)} \sqrt{\frac{b-x}{b+x}}
\end{aligned}$$

Eq. A.6

Replacing b by $a/2$ and translating the half-wedge tip on the right from $a/2$ gives (see Figure A.2)

$$\Leftrightarrow p(x) = \frac{-E \cot \alpha}{2(1-\nu^2)} \sqrt{\frac{\frac{a}{2} - \left(x - \frac{a}{2}\right)}{\frac{a}{2} + \left(x - \frac{a}{2}\right)}} \quad \text{Eq. A.7}$$

$$\Leftrightarrow p(x) = \frac{-E \cot \alpha}{2(1-\nu^2)} \sqrt{\frac{a}{x} - 1}$$

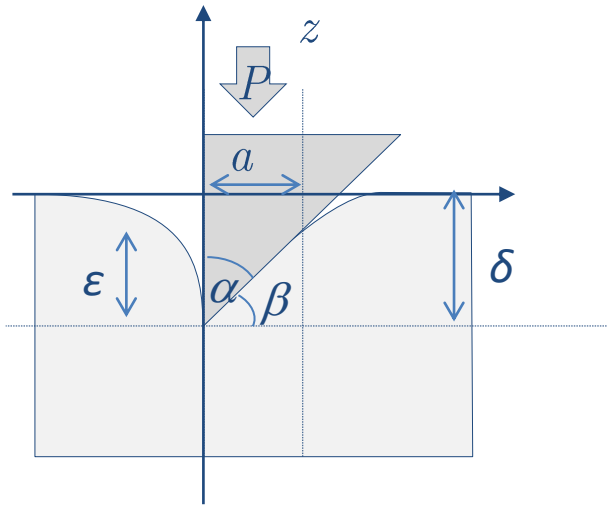


Figure A.2: Schematic representation of the elastic indentation of an infinite half space by a half-wedge tip of width W at a depth δ translated on the right by a factor $b/2$.

The mean pressure under the tip can be calculated using

$$\begin{aligned}
p_m &= \int_0^a p(x) dx \\
\Leftrightarrow p_m &= \frac{E \cot \alpha}{2(1-\nu^2)} \int_0^a \sqrt{\frac{a}{x}-1} dx \\
\Leftrightarrow p_m &= x \sqrt{\frac{a}{x}-1} - \frac{1}{2} \tan^{-1} \left(\frac{(a-2x) \sqrt{\frac{a}{x}-1}}{2(a-x)} \right) \Bigg|_0^a \frac{E \cot \alpha}{2(1-\nu^2)} \\
\Leftrightarrow p_m &= -\frac{1}{2} \tan^{-1}(+\infty) - \left(-\frac{1}{2} \tan^{-1}(-\infty) \right) \Bigg|_0^a \frac{E \cot \alpha}{2(1-\nu^2)} \\
\Leftrightarrow p_m &= \left(-\frac{\pi}{4} - \frac{\pi}{4} \right) \frac{E \cot \alpha}{2(1-\nu^2)} \\
\Leftrightarrow p_m &= \frac{\pi}{4} \frac{E \cot \alpha}{(1-\nu^2)} \\
\Leftrightarrow \frac{P}{W} &= a \frac{\pi}{4} \frac{E \cot \alpha}{(1-\nu^2)}
\end{aligned} \tag{Eq. A.8}$$

A.1.2 Elastoplastic contact with a half-wedge tip

In order to derive the elastoplastic contact equation of the half-wedge, the rigid perfectly plastic solution is combined with the ideal elastic solution (see Table 3.1) using Yu and Blanchart approach [Yu 1996]. The idea is that part of surface under the tip enters plasticity and follows the rigid plastic model, the rest of the contact surface is elastic and follows the ideal elastic model. The intersection between the two correspond a distance b from the center of the tip which follows

$$\begin{aligned}
p_{el}(b) &= p_{pl} \\
\Leftrightarrow \frac{E \tan \beta}{2(1-\nu^2)} \sqrt{\frac{a}{b}-1} &= C \sigma_y \\
\Leftrightarrow \frac{a}{b}-1 &= \left(C \sigma_y \frac{2(1-\nu^2)}{E \tan \beta} \right)^2 \\
\Leftrightarrow \frac{a}{b} &= \left(C \sigma_y \frac{2(1-\nu^2)}{E \tan \beta} \right)^2 + 1 = \frac{1}{f}
\end{aligned} \tag{Eq. A.9}$$

From there, the force acting on the tip is the sum of the pressure acting on the elastic part and on the plastic part integrated over their respective surfaces.

$$\begin{aligned}
P &= \int_S p(x) ds \quad , \quad ds = W dr \\
\frac{P}{W} &= \int_b^a p_{el}(x) dr + \int_0^b p_{pl}(x) dr \\
\Leftrightarrow \frac{P}{W} &= \int_b^a \frac{E \tan \beta}{2(1-\nu^2)} \frac{1}{\sqrt{r}} \sqrt{a-r} dr + \int_0^b C \sigma_y dr \\
\Leftrightarrow \frac{P}{W} &= \frac{E \tan \beta}{2(1-\nu^2)} \left(\sqrt{r} \sqrt{a-r} + a \tan^{-1} \left(\frac{\sqrt{r}}{\sqrt{a-r}} \right) \right) \Big|_b^a + C \sigma_y b \\
\Leftrightarrow \frac{P}{W} &= \frac{E \tan \beta}{2(1-\nu^2)} \left[\left(\sqrt{a} \sqrt{a-a} + a \tan^{-1} \left(\frac{\sqrt{a}}{\sqrt{a-a}} \right) \right) - \left(\sqrt{b} \sqrt{a-b} + a \tan^{-1} \left(\frac{\sqrt{b}}{\sqrt{a-b}} \right) \right) \right] + C \sigma_y b \\
\Leftrightarrow \frac{P}{W} &= \frac{E \tan \beta}{2(1-\nu^2)} \left[a \frac{\pi}{2} - \left(\sqrt{b} \sqrt{a-b} + a \tan^{-1} \left(\frac{\sqrt{b}}{\sqrt{a-b}} \right) \right) \right] + C \sigma_y b \\
\Leftrightarrow \frac{P}{W} &= \frac{E \tan \beta}{2(1-\nu^2)} \left[a \frac{\pi}{2} - \left(\sqrt{a} \sqrt{a} \frac{\sqrt{b}}{\sqrt{a}} \frac{\sqrt{a-b}}{\sqrt{a}} + a \tan^{-1} \left(\frac{\sqrt{a} \sqrt{b/a}}{\sqrt{a} \sqrt{1-(b/a)}} \right) \right) \right] + C \sigma_y b \\
\Leftrightarrow \frac{P}{W} &= \frac{E \tan \beta}{2(1-\nu^2)} \left[a \frac{\pi}{2} - \left(a \sqrt{f} \sqrt{1-(b/a)} + a \tan^{-1} \left(\frac{\sqrt{b/a}}{\sqrt{1-(b/a)}} \right) \right) \right] + a C \sigma_y \frac{b}{a} \\
\Leftrightarrow p_m &= \frac{P}{aW} = \frac{E \tan \beta}{2(1-\nu^2)} \left[\frac{\pi}{2} - \left(\sqrt{f} \sqrt{1-f} + \tan^{-1} \left(\frac{\sqrt{f}}{\sqrt{1-f}} \right) \right) \right] + C \sigma_y f \\
\Leftrightarrow p_m &= \frac{P}{aW} = \frac{E \tan \beta}{2(1-\nu^2)} \left[\frac{\pi}{2} - \tan^{-1} \left(\frac{\sqrt{f}}{\sqrt{1-f}} \right) \right] \\
\Leftrightarrow p_m &= \frac{P}{aW} = \frac{E \tan \beta}{2(1-\nu^2)} \left[\frac{\pi}{2} - \tan^{-1} \left(\frac{1}{\sqrt{(1/f)-1}} \right) \right] \\
\Leftrightarrow p_m &= \frac{P}{aW} = \frac{E \tan \beta}{2(1-\nu^2)} \left[\frac{\pi}{2} - \tan^{-1} \left(\frac{E \tan \beta}{C \sigma_y 2(1-\nu^2)} \right) \right]
\end{aligned}$$

Eq. A.10

A.1.3 Elastoplastic contact with a flat-wedge tip

Regarding the elastoplastic solution of the rectangular flat wedge tip, the same strategy is followed. The intersection between the two correspond a distance b from the center of the tip which follows

$$\begin{aligned}
p_{el}(b) &= p_{m_{pl}} \\
\Leftrightarrow \frac{1}{2} \frac{\delta}{\ln 2} \frac{E}{(1-\nu^2)} \frac{1}{\sqrt{a^2-b^2}} &= C\sigma_y \\
\Leftrightarrow \frac{1}{2} \frac{\delta}{\ln 2} \frac{E}{(1-\nu^2)} \frac{1}{C\sigma_y} &= \sqrt{a^2-b^2} \\
\Leftrightarrow a^2-b^2 &= \left(\frac{1}{2} \frac{\delta}{\ln 2} \frac{E}{(1-\nu^2)} \frac{1}{C\sigma_y} \right)^2 \\
\Leftrightarrow b^2 &= a^2 - \left(\frac{1}{2} \frac{\delta}{\ln 2} \frac{E}{(1-\nu^2)} \frac{1}{C\sigma_y} \right)^2 \\
\Leftrightarrow \frac{b^2}{a^2} &= 1 - \left(\frac{1}{2a} \frac{\delta}{\ln 2} \frac{E}{(1-\nu^2)} \frac{1}{C\sigma_y} \right)^2 \\
\Leftrightarrow 1 - \frac{b}{a} &= 1 - \sqrt{1 - \left(\frac{1}{2a} \frac{\delta}{\ln 2} \frac{E}{(1-\nu^2)} \frac{1}{C\sigma_y} \right)^2} = f = 1 - \sqrt{1-N^2} \\
\text{with } N &= \frac{1}{2a} \frac{\delta}{\ln 2} \frac{E}{(1-\nu^2)} \frac{1}{C\sigma_y}
\end{aligned}
\tag{Eq. A.11}$$

From there, the force acting on the tip is the sum of the pressure acting on the elastic part and on the plastic part integrated over their respective surfaces.

$$\begin{aligned}
P &= \int_s p(r) ds \quad , \quad ds = 2Wdr \\
\frac{P}{W} &= \int_0^b p_{el}(r) 2dr + \int_b^a p_{pl}(r) 2dr \\
\Leftrightarrow \frac{P}{W} &= \int_0^b \frac{1}{2} \frac{\delta}{a \ln 2} \frac{E}{(1-\nu^2)} \frac{1}{\sqrt{1-r^2/a^2}} 2dr + \int_b^a C\sigma_y 2dr \\
\Leftrightarrow \frac{P}{W} &= \frac{\delta}{\ln 2} \frac{E}{(1-\nu^2)} \left(\sin^{-1} \left(\frac{r}{a} \right) \right) \Big|_0^b + (2C\sigma_y [a-b]) \\
\Leftrightarrow \frac{P}{W} &= \frac{\delta}{\ln 2} \frac{E}{(1-\nu^2)} \left(\sin^{-1} \left(\frac{b}{a} \right) - \sin^{-1} \left(\frac{0}{a} \right) \right) + (2C\sigma_y (a-b)) \\
\Leftrightarrow \frac{P}{W} &= \frac{\delta}{\ln 2} \frac{E}{(1-\nu^2)} \sin^{-1} \left(\frac{b}{a} \right) + 2C\sigma_y a \left(1 - \frac{b}{a} \right) \\
\Leftrightarrow \frac{P}{W} &= \frac{\delta}{\ln 2} \frac{E}{(1-\nu^2)} \sin^{-1} (1-f) + 2C\sigma_y af \\
\Leftrightarrow p_m &= \frac{P}{2aW} = \frac{\delta}{2a \ln 2} \frac{E}{(1-\nu^2)} \sin^{-1} (1-f) + C\sigma_y f \\
\Leftrightarrow p_m &= \frac{h}{2a \ln 2} \frac{E}{(1-\nu^2)} \sin^{-1} \left(\sqrt{1-N^2} \right) + C\sigma_y \left(1 - \sqrt{1-N^2} \right) \\
\Leftrightarrow p_m &= C\sigma_y \left[\frac{h}{2a \ln 2} \frac{E}{C\sigma_y (1-\nu^2)} \cos^{-1} (N) + 1 - \sqrt{1-(N)^2} \right] \\
\Leftrightarrow p_m &= \frac{h}{2a \ln 2} \frac{E}{(1-\nu^2)} \left[\cos^{-1} (N) + \frac{1}{N} \left(1 - \sqrt{1-N^2} \right) \right]
\end{aligned}$$

Eq. A.12

where

$$N = \frac{1}{2a} \frac{h}{\ln 2} \frac{E}{(1-\nu^2)} \frac{1}{C\sigma_y}$$

A.2 Plowing calculations

A.2.1 Conical tip plowing solution (elastoplastic)

Integration of Eq. 4.41 is described step by step hereunder

$$\begin{aligned}
F_{H_{elpl}}^C &= \int_0^h \left(\frac{a_{elpl}^W}{h} \right) 2h H_{elpl}^{W_2} dh - \int_0^{h_{el}} \left(\frac{a_{Unload}^W}{h} \right) 2h H_{el}^W (\beta') dh + \mu h^2 \pi H_{elpl}^C \left(\frac{a_{elpl}^C}{h} \right)^2 \\
&= \int_0^h 2h \left(\frac{a_{elpl}^W}{h} \right) H_{elpl}^{W_2} dh - \int_0^{h_{el}} 2h H_{elpl}^W \left(\frac{h}{h_{el}^W} \right) \left(\frac{a_{elpl}^W}{h} \right) dh + \mu h^2 \pi H_{elpl}^C \left(\frac{a_{elpl}^C}{h} \right)^2 \\
&= h^2 H_{elpl}^W \left(\frac{a_{elpl}^W}{h} \right) - H_{elpl}^W \left(\frac{h}{h_{el}^W} \right) \left(\frac{a_{elpl}^W}{h} \right) (h_{el}^W)^2 + \mu h^2 \pi H_{elpl}^C \left(\frac{a_{elpl}^C}{h} \right)^2 \\
&= h^2 H_{elpl}^W \left(\frac{a_{elpl}^W}{h} \right) - h^2 H_{elpl}^W \left(\frac{a_{elpl}^W}{h} \right) \left(\frac{h_{el}^W}{h} \right) + \mu h^2 \pi H_{elpl}^C \left(\frac{a_{elpl}^C}{h} \right)^2 \\
&= h^2 H_{elpl}^W \left(\frac{a_{elpl}^W}{h} \right) \left[1 - \left(\frac{h_{el}^W}{h} \right) \right] + \mu h^2 \pi H_{elpl}^C \left(\frac{a_{elpl}^C}{h} \right)^2 \\
&= h^2 H_{elpl}^W \left(\frac{a_{elpl}^W}{h} \right) \left(\frac{h_{pl}^W}{h} \right) + \mu h^2 \pi H_{elpl}^C \left(\frac{a_{elpl}^C}{h} \right)^2 \\
&= h^2 \pi H_{elpl}^C \left(\frac{a_{elpl}^C}{h} \right)^2 \left[\left(\frac{h}{a_{elpl}^C} \right)^2 \left(\frac{a_{elpl}^W}{h} \right) \left(\frac{h_{pl}^W}{h} \right) \frac{H_{elpl}^W}{H_{elpl}^C} \frac{1}{\pi} + \mu \right] \\
F_{H_{elpl}}^C &= F_{V_{elpl}}^C \left[\left(\frac{h}{a_{elpl}^C} \right)^2 \left(\frac{a_{elpl}^W}{h} \right) \left(\frac{h_{pl}^W}{h} \right) \frac{H_{elpl}^W}{H_{elpl}^C} \frac{1}{\pi} + \mu \right]
\end{aligned} \tag{Eq. A.13}$$

A.2.2 Conical tip plowing solution (rigid perfectly plastic)

Integration of Eq. 4.44 is described step by step hereunder

$$\begin{aligned}
F_{H_{pl}}^C &= \int_0^h \frac{P_{pl}^W(h)}{W} dh + \mu F_{V_{pl}}^C \\
&= \int_0^h 2h \left(\frac{a_{pl}^W}{h} \right) H_{pl}^W dh + \mu h^2 \left(\frac{a_{pl}^C}{h} \right)^2 \pi H_{pl}^C \\
&= h^2 \left(\frac{a_{pl}^W}{h} \right) H_{pl}^W + \mu h^2 \left(\frac{a_{pl}^C}{h} \right)^2 \pi H_{pl}^C \\
&= h^2 \left(\frac{a_{pl}^C}{h} \right)^2 \pi H_{pl}^C \left[\left(\frac{a_{pl}^W}{h} \right) \left(\frac{h}{a_{pl}^C} \right)^2 \frac{H_{pl}^W}{\pi H_{pl}^C} + \mu \right] \\
F_{H_{pl}}^C &= F_{V_{pl}}^C \left[\left(\frac{h}{a_{pl}^C} \right)^2 \left(\frac{a_{pl}^W}{h} \right) \frac{H_{pl}^W}{H_{pl}^C} \frac{1}{\pi} + \mu \right]
\end{aligned} \tag{Eq. A.14}$$

A.2.3 Half-wedge tip plowing solution (elastoplastic)

Integration of Eq. 4.47 is described step by step hereunder

$$\begin{aligned}
F_{H_{elpl}}^{HW_1} &= \int_0^h \frac{F_{V_{elpl}}^{FW_2}(h)}{W_2} dh - \int_0^{h_{el}} \frac{F_{V_{el}}^{FW_2}(h)}{W_2} dh + \mu F_{V_{elpl}}^{HW_1}(h) \\
&= \int_0^h \frac{WT}{T} \frac{1}{\ln 2} \frac{h}{W} \frac{E}{(1-\nu^2)} \left[\cos^{-1}(f^{FW_2}) + \frac{1}{f_{FW-2}} \left(1 - \sqrt{1 - (f^{FW_2})^2} \right) \right] dh \\
&\quad - \int_0^{h_{el}} \frac{WT}{T} \frac{\pi}{2 \ln 2} \frac{h}{W} \frac{E}{(1-\nu^2)} dh + \mu h W \left(\frac{a_{elpl}^{HW}}{h} \right) H_{elpl}^{HW} \\
&= \int_0^h \frac{h}{\ln 2} \frac{E}{(1-\nu^2)} \left[\cos^{-1}(f^{FW_2}) + \frac{1}{f_{FW-2}} \left(1 - \sqrt{1 - (f^{FW_2})^2} \right) \right] dh \\
&\quad - \int_0^{h_{el}} \frac{h\pi}{2 \ln 2} \frac{E}{(1-\nu^2)} dh + \mu h W \left(\frac{a_{elpl}^{HW}}{h} \right) H_{elpl}^{HW} \\
&= \int_0^h \frac{h}{\ln 2} \frac{E}{(1-\nu^2)} \left[\cos^{-1}(g^{FW_2} h) + \frac{1}{g^{FW_2} h} \left(1 - \sqrt{1 - (g^{FW_2} h)^2} \right) \right] dh \\
&\quad - \int_0^{h_{el}} \frac{h\pi}{2 \ln 2} \frac{E}{(1-\nu^2)} dh + \mu h W \left(\frac{a_{elpl}^{HW}}{h} \right) H_{elpl}^{HW} \\
&= \int_0^h \frac{h}{\ln 2} \frac{E}{(1-\nu^2)} \cos^{-1}(g^{FW_2} h) dh + \int_0^h W C_{pl}^{FW_2} \sigma_y \left(1 - \sqrt{1 - (g^{FW_2} h)^2} \right) dh \\
&\quad - \int_0^{h_{el}} \frac{h\pi}{2 \ln 2} \frac{E}{(1-\nu^2)} dh + \mu h W \left(\frac{a_{elpl}^{HW}}{h} \right) H_{elpl}^{HW} \\
&= \frac{1}{\ln 2} \frac{E}{(1-\nu^2)} \int_0^{h_{tot}} h \sin^{-1} \left(\sqrt{1 - (g^{FW_2} h)^2} \right) dh + W C_{pl}^{FW_2} \sigma_y \int_0^{h_{tot}} \left(1 - \sqrt{1 - (g^{FW_2} h)^2} \right) dh \\
&\quad - \int_0^{h_{el}} h \frac{\pi}{2 \ln 2} \frac{E}{(1-\nu^2)} dh + \mu h W \left(\frac{a_{elpl}^{HW}}{h} \right) H_{elpl}^{HW}
\end{aligned} \tag{Eq. A.15}$$

Integrating gives

$$\begin{aligned}
&= W g^{FW_2} C_{pl}^{FW_2} \sigma_y \left[\frac{1}{4(g^{FW_2})^2} \sin^{-1}(g^{FW_2} h) + \frac{h^2}{2} \cos^{-1}(g^{FW_2} h) - \frac{h}{4g^{FW_2}} \sqrt{1-(g^{FW_2} h)^2} \right]_0^h \\
&\quad + W C_{pl}^{FW_2} \sigma_y h \left[h - \frac{1}{2g^{FW_2}} \sin^{-1}(g^{FW_2} h) - \frac{1}{2} h \sqrt{1-(g^{FW_2} h)^2} \right]_0^h - \left[\frac{h_{el}^2 \pi}{4 \ln 2} \frac{E}{(1-\nu^2)} \right] + \mu h W \left(\frac{a_{elpl}^{HW}}{h} \right) H_{elpl}^{HW} \\
&= W C_{pl}^{FW_2} \sigma_y \left[\frac{1}{4g^{FW_2}} \sin^{-1}(g^{FW_2} h) + \frac{g^{FW_2} h^2}{2} \cos^{-1}(g^{FW_2} h) - \frac{h}{4} \sqrt{1-(g^{FW_2} h)^2} + h \right] \\
&\quad + W C_{pl}^{FW_2} \sigma_y \left[-\frac{2}{4g^{FW_2}} \sin^{-1}(g^{FW_2} h) - \frac{2}{4} h \sqrt{1-(g^{FW_2} h)^2} \right] - \left[\frac{h_{el}^2 \pi}{4 \ln 2} \frac{E}{(1-\nu^2)} \right] + \mu h W \left(\frac{a_{elpl}^{HW}}{h} \right) H_{elpl}^{HW} \\
&= W C_{pl}^{FW_2} \sigma_y \left[-\frac{1}{4g^{FW_2}} \sin^{-1}(f^{FW_2}) + \frac{g^{FW_2} h^2}{2} \cos^{-1}(f^{FW_2}) - \frac{3h}{4} \sqrt{1-(f^{FW_2})^2} + h \right] \\
&\quad - \left[\frac{(W H_{elpl}^{FW_2})^2 \ln 2}{\pi} \frac{(1-\nu^2)}{E} \right] + \mu h W \left(\frac{a_{elpl}^{HW}}{h} \right) H_{elpl}^{HW} \\
&= W C_{pl}^{FW_2} \sigma_y \left[-\frac{1}{4g^{FW_2}} \sin^{-1}(f^{FW_2}) + \frac{g^{FW_2} h^2}{2} \cos^{-1}(f^{FW_2}) - \frac{3h}{4} \sqrt{1-(f^{FW_2})^2} + h \right] \\
&\quad - W \left[\left(H_{elpl}^{FW_2} \right)^2 \frac{1}{\pi} \frac{1}{g^{FW_2} C_{FW-pl} \sigma_y} \right] + \mu h W \left(\frac{a_{elpl}^{HW}}{h} \right) H_{elpl}^{HW} \\
&= \mu h W \left(\frac{a_{elpl}^{HW}}{h} \right) H_{elpl}^{HW} + W C_{pl}^{FW_2} \sigma_y \left[-\frac{1}{4g^{FW_2}} \sin^{-1}(f^{FW_2}) + \frac{g^{FW_2} h^2}{2} \cos^{-1}(f^{FW_2}) - \frac{3h}{4} \sqrt{1-(f^{FW_2})^2} + h - \left(\frac{H_{elpl}^{FW_2}}{C_{pl}^{FW_2} \sigma_y} \right)^2 \frac{1}{\pi} \frac{1}{g^{FW_2}} \right] \\
&= \mu h W \left(\frac{a_{elpl}^{HW}}{h} \right) H_{elpl}^{HW} + \frac{W C_{pl}^{FW_2} \sigma_y}{g^{FW_2}} \left[-\frac{1}{4} \sin^{-1}(f^{FW_2}) + \frac{(f^{FW_2})^2}{2} \cos^{-1}(f^{FW_2}) - f^{FW_2} \frac{3}{4} \sqrt{1-(f^{FW_2})^2} + f^{FW_2} - \left(\frac{H_{elpl}^{FW_2}}{C_{pl}^{FW_2} \sigma_y} \right)^2 \frac{1}{\pi} \right] \\
&\Leftrightarrow \frac{F_L}{F_N} = \mu + \left(\frac{h}{a_{elpl}^{HW}} \right) \frac{C_{pl}^{FW_2} \sigma_y}{H_{elpl}^{HW}} \left[-\frac{1}{4} \sin^{-1}(f^{FW_2}) + \frac{(f^{FW_2})^2}{2} \cos^{-1}(f^{FW_2}) - f^{FW_2} \frac{3}{4} \sqrt{1-(f^{FW_2})^2} + f^{FW_2} - \left(\frac{H_{elpl}^{FW_2}}{C_{pl}^{FW_2} \sigma_y} \right)^2 \frac{1}{\pi} \right] \\
&\Leftrightarrow \frac{F_L}{F_N} = \mu + \left(\frac{h}{a_{elpl}^{HW}} \right) \frac{C_{pl}^{FW_2} \sigma_y}{H_{elpl}^{HW}} \left[-\frac{1}{4 f^{FW_2}} \sin^{-1}(f^{FW_2}) + \frac{f^{FW_2}}{2} \cos^{-1}(f^{FW_2}) - \frac{3}{4} \sqrt{1-(f^{FW_2})^2} + 1 - \left(\frac{H_{elpl}^{FW_2}}{C_{pl}^{FW_2} \sigma_y} \right)^2 \frac{1}{f^{FW_2} \pi} \right] \\
&\Leftrightarrow \frac{F_L}{F_N} = \mu + \left(\frac{h}{a_{elpl}^{HW}} \right) \frac{C_{pl}^{FW_2} \sigma_y}{4 H_{elpl}^{HW}} \left[-\frac{1}{f^{FW_2}} \sin^{-1}(f^{FW_2}) + 2 f^{FW_2} \cos^{-1}(f^{FW_2}) - 3 \sqrt{1-(f^{FW_2})^2} + 4 - \left(\frac{H_{elpl}^{FW_2}}{C_{pl}^{FW_2} \sigma_y} \right)^2 \frac{4}{f^{FW_2} \pi} \right] \\
&F_{H_{elpl}}^{HW_1} = F_{V_{elpl}}^{HW_1} \mu + F_{V_{elpl}}^{HW_1} \left(\frac{h}{a_{elpl}^{HW}} \right) \frac{C_{pl}^{FW_2} \sigma_y}{4 H_{elpl}^{HW}} \left[-\frac{1}{f^{FW_2}} \sin^{-1}(f^{FW_2}) - f^{FW_2} \cos^{-1}(f^{FW_2}) + 3 \frac{H_{elpl}^{FW_2}}{C_{pl}^{FW_2} \sigma_y} + 1 - \left(\frac{H_{elpl}^{FW_2}}{C_{pl}^{FW_2} \sigma_y} \right)^2 \frac{4}{f^{FW_2} \pi} \right]
\end{aligned}$$

where

$$\begin{aligned}
H_{elpl}^{HW} &= \frac{E \tan \delta}{2(1-\nu^2)} \left[\frac{\pi}{2} - \tan^{-1} \left(\frac{E \tan \delta}{C_{pl}^{HW} \sigma_y 2(1-\nu^2)} \right) \right] \\
H_{elpl}^{FW_2} &= C_{pl}^{FW_2} \sigma_y \left[f^{FW_2} \cos^{-1}(f^{FW_2}) + 1 - \sqrt{1-(f^{FW_2})^2} \right] \\
f^{FW_2} &= \frac{1}{W} \frac{h}{\ln 2} \frac{E}{(1-\nu^2)} \frac{1}{C_{pl}^{FW_2} \sigma_y} \\
g^{FW_2} &= \frac{f^{FW_2}}{h}
\end{aligned}$$

Eq. A.16

A.2.4 Flat wedge tip plowing solution (elastoplastic)

Integration of Eq. 4.55 is described step by step hereunder

$$\begin{aligned}
F_{H_{elpl}}^{FW_1} &= \int_0^h \frac{h}{\ln 2} \frac{E}{(1-\nu^2)} \left[\cos^{-1}(g^{FW_2} h) + \frac{1}{g^{FW_2} h} \left(1 - \sqrt{1 - (g^{FW_2} h)^2} \right) \right] dh - \int_0^{h_d} \frac{h\pi}{2 \ln 2} \frac{E}{(1-\nu^2)} dh + \mu W_1 T_1 H_{elpl}^{FW_1} \\
&= W g^{FW_2} C_{pl}^{FW_2} \sigma_y \left[\frac{1}{4 (g^{FW_2})^2} \sin^{-1}(g^{FW_2} h) + \frac{h^2}{2} \cos^{-1}(g^{FW_2} h) - \frac{h}{4 g^{FW_2}} \sqrt{1 - (g^{FW_2})^2 h^2} \right]_0^h \\
&\quad + W C_{pl}^{FW_2} \sigma_y h \left[h - \frac{1}{2 g^{FW_2}} \sin^{-1}((g^{FW_2})^2 h) - \frac{1}{2} h \sqrt{1 - (g^{FW_2})^2 h^2} \right]_0^h - \left[\frac{h_{el}^2 \pi}{4 \ln 2} \frac{E}{(1-\nu^2)} \right] + \mu H_{elpl}^{FW_1} T W \\
&= W C_{pl}^{FW_2} \sigma_y \left[\frac{1}{4 g^{FW_2}} \sin^{-1}(g^{FW_2} h) + \frac{g^{FW_2} h^2}{2} \cos^{-1}(g^{FW_2} h) - \frac{h}{4} \sqrt{1 - (g^{FW_2})^2 h^2} + h - \frac{2}{4 g^{FW_2}} \sin^{-1}(g^{FW_2} h) - \frac{2}{4} h \sqrt{1 - (g^{FW_2})^2 h^2} \right] \\
&\quad - \left[\frac{h_{el}^2 \pi}{4 \ln 2} \frac{E}{(1-\nu^2)} \right] + \mu H_{elpl}^{FW_1} T W \\
&= W C_{pl}^{FW_2} \sigma_y \left[-\frac{1}{4 g^{FW_2}} \sin^{-1}(g^{FW_2} h) + \frac{g^{FW_2} h^2}{2} \cos^{-1}(g^{FW_2} h) - \frac{3h}{4} \sqrt{1 - (g^{FW_2})^2 h^2} + h \right] \\
&\quad - \left[\frac{(W H_{elpl}^{FW_2})^2 \ln 2 (1-\nu^2)}{\pi E} \right] + \mu H_{elpl}^{FW_1} T W \\
&= W C_{pl}^{FW_2} \sigma_y \left[-\frac{1}{4 g^{FW_2}} \sin^{-1}(g^{FW_2} h) + \frac{g^{FW_2} h^2}{2} \cos^{-1}(g^{FW_2} h) - \frac{3h}{4} \sqrt{1 - (g^{FW_2})^2 h^2} + h \right] \\
&\quad - W \left[\left(H_{elpl}^{FW_2} \right)^2 \frac{1}{\pi} \frac{1}{g^{FW_2} C_{pl}^{FW_2} \sigma_y} \right] + \mu H_{elpl}^{FW_1} T W \\
&= W C_{pl}^{FW_2} \sigma_y \left[-\frac{1}{4 g^{FW_2}} \sin^{-1}(g^{FW_2} h) + \frac{g^{FW_2} h^2}{2} \cos^{-1}(g^{FW_2} h) - \frac{3h}{4} \sqrt{1 - (g^{FW_2})^2 h^2} + h - \left(\frac{H_{elpl}^{FW_2}}{C_{pl}^{FW_2} \sigma_y} \right)^2 \frac{1}{\pi} \frac{1}{g^{FW_2}} \right] + \mu H_{elpl}^{FW_1} T W \\
&= \frac{W C_{pl}^{FW_2} \sigma_y}{g^{FW_2}} \left[-\frac{1}{4} \sin^{-1}(g^{FW_2} h) + \frac{(f^{FW_2})^2}{2} \cos^{-1}(g^{FW_2} h) - g^{FW_2} h \frac{3}{4} \sqrt{1 - (f^{FW_2})^2} + g^{FW_2} h - \left(\frac{H_{elpl}^{FW_2}}{C_{pl}^{FW_2} \sigma_y} \right)^2 \frac{1}{\pi} \right] + \mu H_{elpl}^{FW_1} T W \\
&= \mu F_{V_{elpl}}^{FW_1} + F_{V_{elpl}}^{FW_1} \frac{C_{pl}^{FW_2} \sigma_y}{T H_{elpl}^{FW_1} g^{FW_2}} \left[-\frac{1}{4} \sin^{-1}(f^{FW_2}) + \frac{f^{FW_2-2}}{2} \cos^{-1}(f^{FW_2}) - f^{FW_2} \frac{3}{4} \sqrt{1 - (f^{FW_2})^2} + g^{FW_2} h - \left(\frac{H_{elpl}^{FW_2}}{C_{pl}^{FW_2} \sigma_y} \right)^2 \frac{1}{\pi} \right] \\
&= \mu F_{V_{elpl}}^{FW_1} + F_{V_{elpl}}^{FW_1} \frac{h C_{pl}^{FW_2} \sigma_y}{4 T H_{elpl}^{FW_1}} \left[-\frac{1}{f^{FW_2}} \sin^{-1}(f^{FW_2}) + 2 f^{FW_2} \cos^{-1}(f^{FW_2}) - 3 \sqrt{1 - (f^{FW_2})^2} + 4 - \left(\frac{H_{elpl}^{FW_2}}{C_{pl}^{FW_2} \sigma_y} \right)^2 \frac{4}{f^{FW_2} \pi} \right] \\
&= \mu F_{V_{elpl}}^{FW_1} + F_{V_{elpl}}^{FW_1} \frac{h C_{pl}^{FW_2} \sigma_y}{4 T H_{elpl}^{FW_1}} \left[-\frac{1}{f^{FW_2}} \sin^{-1}(f^{FW_2}) - f^{FW_2} \cos^{-1}(f^{FW_2}) + 3 \frac{H_{elpl}^{FW_2}}{C_{pl}^{FW_2} \sigma_y} + 1 - \left(\frac{H_{elpl}^{FW_2}}{C_{pl}^{FW_2} \sigma_y} \right)^2 \frac{4}{f^{FW_2} \pi} \right]
\end{aligned}$$

where

$$g^{FW_2} = \frac{1}{W_1} \frac{1}{\ln 2} \frac{E}{(1-\nu^2)} \frac{1}{C_{pl}^{FW_2} \sigma_y}$$

$$f^{FW_2} = \frac{1}{W} \frac{h}{\ln 2} \frac{E}{(1-\nu^2)} \frac{1}{C_{pl}^{FW_2} \sigma_y}$$

$$H_{elpl}^{FW_2} = C_{pl}^{FW_2} \sigma_y \left[f^{FW_2} \cos^{-1}(f^{FW_2}) + 1 - \sqrt{1 - (f^{FW_2})^2} \right]$$

Eq. A.17

A.3 Cutting calculations

A.3.1 Orthogonal cutting (perfectly plastic)

Integration of Eq. 4.68 is described step by step hereunder

$$\begin{aligned}
\frac{\partial}{\partial \tan \phi} \frac{F_{H_{pl}}^{OC}}{W} &= 0 \\
\Leftrightarrow \frac{\frac{\sigma_y h}{2} (2Z \tan \phi + \tan^2 \phi - 1) + G_c (Z + 1) \tan^2 \phi}{\tan^2 \phi (1 - Z \tan \phi)^2} &= 0 \\
\Leftrightarrow \tan \phi &= \frac{\pm \sqrt{\left(\frac{\sigma_y h}{2} Z\right)^2 + \left(\frac{\sigma_y h}{2}\right)^2 + \frac{\sigma_y h}{2} Z G_c + \frac{\sigma_y h}{2} G_c - \frac{\sigma_y h}{2} Z}}{\frac{\sigma_y h}{2} + Z G_c + G_c} \\
\Leftrightarrow \tan \phi &= \frac{\pm \sqrt{(Z)^2 + 1 + \frac{2}{\sigma_y h} Z G_c + \frac{2}{\sigma_y h} G_c - Z}}{1 + \frac{2}{\sigma_y h} Z G_c + \frac{2}{\sigma_y h} G_c} \\
\Leftrightarrow \tan \phi &= \frac{\sqrt{Z^2 + 1 + B_{pl}^{OC} Z + B_{pl}^{OC}} - Z}{1 + B_{pl}^{OC} Z + B_{pl}^{OC}} \\
\Leftrightarrow \tan \phi &= \frac{\left(\sqrt{Z^2 + 1 + B_{pl}^{OC} Z + B_{pl}^{OC}} - Z\right) \left(\sqrt{Z^2 + 1 + B_{pl}^{OC} Z + B_{pl}^{OC}} + Z\right)}{\left(1 + B_{pl}^{OC} Z + B_{pl}^{OC}\right) \left(\sqrt{Z^2 + 1 + B_{pl}^{OC} Z + B_{pl}^{OC}} + Z\right)} \\
\Leftrightarrow \tan \phi &= \frac{Z^2 + 1 + B_{pl}^{OC} Z + B_{pl}^{OC} - Z^2}{\left(1 + B_{pl}^{OC} Z + B_{pl}^{OC}\right) \left(\sqrt{Z^2 + 1 + B_{pl}^{OC} Z + B_{pl}^{OC}} + Z\right)} \\
\Leftrightarrow \tan \phi &= \frac{1}{\sqrt{Z^2 + 1 + B_{pl}^{OC} Z + B_{pl}^{OC}} + Z} \\
\Leftrightarrow \cot \phi &= \sqrt{Z^2 + 1 + B_{pl}^{OC} Z + B_{pl}^{OC}} + Z
\end{aligned}$$

Eq. A.18

From there,

$$\begin{aligned}
& \begin{cases} F_{H_{pl}}^{OC} = \frac{W\sigma_y h \cot^2 \varphi + 1 + B_{OC} (\cot \varphi + 1)}{2 (\cot \varphi - Z)} \\ \cot \varphi = \sqrt{Z^2 + 1 + B_{OC} Z + B_{OC}} + Z \end{cases} \\
& \Leftrightarrow F_{H_{pl}}^{OC} = \frac{W\sigma_y h}{2} \frac{Z^2 + 1 + B_{OC} Z + B_{OC} + 2Z\sqrt{Z^2 + 1 + B_{OC} Z + B_{OC}} + Z^2 + 1 + B_{OC}\sqrt{Z^2 + 1 + B_{OC} Z + B_{OC}} + B_{OC} Z + B_{OC}}{\left(\sqrt{Z^2 + 1 + B_{OC} Z + B_{OC}}\right)} \\
& \Leftrightarrow F_{H_{pl}}^{OC} = \frac{W\sigma_y h}{2} \frac{2Z^2 + 2 + 2B_{OC} Z + 2B_{OC} + 2Z\sqrt{Z^2 + 1 + B_{OC} Z + B_{OC}} + B_{OC}\sqrt{Z^2 + 1 + B_{OC} Z + B_{OC}}}{\left(\sqrt{Z^2 + 1 + B_{OC} Z + B_{OC}}\right)} \\
& \Leftrightarrow F_{H_{pl}}^{OC} = \frac{W\sigma_y h}{2} \left[2 \frac{Z^2 + 1 + B_{OC} Z + B_{OC}}{\left(\sqrt{Z^2 + 1 + B_{OC} Z + B_{OC}}\right)} + 2Z + B_{OC} \right] \\
& \Leftrightarrow F_H = \frac{W\sigma_y h}{2} \left(2\sqrt{Z^2 + 1 + B_{OC} Z + B_{OC}} + 2Z + B_{OC} \right) \\
& \Leftrightarrow F_H = \frac{W\sigma_y h}{2} (2 \cot \varphi + B_{OC})
\end{aligned}$$

where

$$\begin{aligned}
B_{pl}^{OC} &= \frac{2G_c}{\sigma_y h} \\
Z &= \frac{\mu - \tan \alpha}{1 + \mu \tan \alpha}
\end{aligned}$$

Eq. A.19

Appendix B Characteristic size (h_{seg})

In a volume that contains spherical particles A and B, the number of spherical particles A and B is given by

$$\begin{cases} n_A = \frac{3V_A}{4\pi r_A^3} \\ n_B = \frac{3V_B}{4\pi r_B^3} \end{cases} \quad \text{Eq. B.1}$$

$$\Leftrightarrow \frac{n_B}{n_A} = \frac{V_B/r_B^3}{V_A/r_A^3} = \frac{V_B}{V_A} \frac{r_A^3}{r_B^3}$$

is the volume fraction of A in the mix(f_{VA}) can be expressed through

$$\begin{aligned} f_{VA} &= \frac{V_A}{V_A + V_B} \\ \Leftrightarrow f_{VA} &= \frac{1}{1 + \frac{V_B}{V_A}} \\ \Leftrightarrow f_{VA} \left(1 + \frac{V_B}{V_A} \right) &= 1 \\ \Leftrightarrow f_{VA} \frac{V_B}{V_A} &= 1 - f_{VA} \\ \Leftrightarrow \frac{V_B}{V_A} &= \frac{1 - f_{VA}}{f_{VA}} \end{aligned} \quad \text{Eq. B.2}$$

The number fraction of particles A in the mix is given by.

$$\begin{aligned}
f_{n_A} &= \frac{n_A}{n_A + n_B} \\
\Leftrightarrow f_{n_A} &= \frac{1}{1 + \frac{n_B}{n_A}} \\
\Leftrightarrow f_{n_A} &= \frac{1}{1 + \frac{V_B}{V_A} \frac{r_A^3}{r_B^3}} \\
\Leftrightarrow f_{n_A} &= \frac{1}{\frac{3(1 - f_{V_A})}{\frac{4\pi r_B^3}{3f_{V_A}}}} \\
\Leftrightarrow f_{n_A} &= \frac{1}{1 + \frac{(1 - f_{V_A}) r_A^3}{f_{V_A} r_B^3}} \\
f_{n_A} &= \left[1 + \left(\frac{1}{f_{V_A}} - 1 \right) \frac{r_A^3}{r_B^3} \right]^{-1}
\end{aligned}$$

Eq. B.3

From there, the mean radius can be found by a simple rule of mixture

$$r_{mean_{mix}} = r_A f_{n_A} + r_B (1 - f_{n_A})$$

Eq. B.4

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