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# Gravitational dipole radiations from binary systems

J.-M.Gérard and Y.Wiaux*

Institut de Physique Théorique
Université catholique de Louvain
B-1348 Louvain-la-Neuve, Belgium



**Abstract**

We investigate the possibility of generating sizeable dipole radiations in relativistic theories of gravity. Optimal parameters to observe their effects through the orbital period decay of binary star systems are discussed. Constraints on gravitational couplings beyond general relativity are derived.

## 1 Introduction

General relativity predicts dominant quadrupole gravitational radiations. However, alternatives to Einstein's theory actually extend the concept of gravitational charge beyond the mass, leading, in principle, to dipole radiations.

In this paper, we wish to investigate the possibility of generating sizeable dipole radiations, in the framework of two minimal, but conceptually important, extensions of general relativity. First, we emphasize that a violation of the equivalence principle provides us with a rather powerful indicator for gravitational dipole radiations. In section 3, we contemplate the dominance of dipole radiations in the Brans-Dicke scalar-tensor theory where the additional gravitational charge is related to the gravitational binding energy of compact bodies. Section 4 is devoted to a Kaluza-inspired vector-tensor theory where the new gravitational charge is proportional to the extra dimensional velocity. In the last section, we consider binary pulsars as indirect searching tools for gravitational dipole radiations. Monitoring the orbital period decrease rate of binary systems leads to limits on either the scalar or vector charges involved. In particular, we advocate realistic tight constraints through the analysis of existing neutron star - white dwarf binary pulsars. The resulting bounds on a scalar gravitational coupling might actually compete with the expected level of precision of future satellite experiments dedicated to probing this coupling.

*FNRS Research Fellow (email: ywiaux@fyma.ucl.ac.be)

## 2 Gravitational charges and the equivalence principle

In electromagnetism, the classical Larmor result [1, 2] for the dipole radiation rate due to cyclic motions is given by

$$-\frac{dE}{dt} = \frac{1}{6\pi c^3}\ddot{D}^i\ddot{D}^i\,, \tag{1}$$

with $D^i = \sum_a q_a r_a^i$, the dipole moment of the source and $q$, the electric charge. In Einstein's tensor theory, the analogous gravitational charge is the mass $m$ itself. Consequently, the conservation of momentum in the non-relativistic limit implies that dipole radiations are forbidden in general relativity and the $v^2/c^2$-suppressed quadrupole radiations dominate, $v$ being the average velocity in the (binary) system under scrutiny. The possibility to generate dipole radiations in relativistic theories of gravity beyond general relativity is therefore challenging from a phenomenological point of view.

An intuitive way to ensure such a dipole radiation is to immerse Einstein's theory of gravity in a five dimensional spacetime[1]: a projection of the quadrupole motion on our 4-dimensional spacetime corresponds then to a dipole one (see Fig.1).

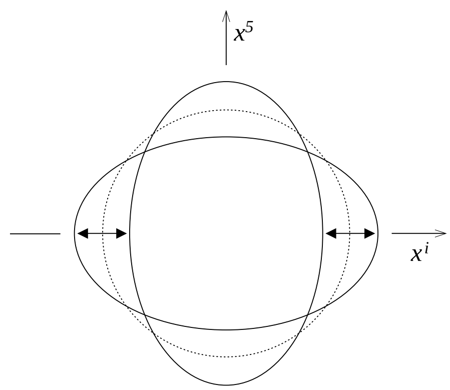


Figure 1: Dipole as projection of extra dimensional quadrupole.

Indeed, the action for the geodesic motion of a test particle in five dimensions,

$$S = -c\int m\, ds^{(5)}\,, \tag{2}$$

reduces to

$$S \simeq -\int \{mc + (m\frac{u^5}{c})\, u^\mu g_{5\mu}\}\, ds \tag{3}$$

in the weak field approximation, if $ds = cd\tau$ is the 4-dimensional infinitesimal length ($u^\mu = dx^\mu/d\tau$ and $u^5 = dx^5/d\tau$) and for $u^5 < c$. The second term in this action corresponds to a vector gravitational interaction similar to electromagnetism. The appearance of a second gravitational charge ($mu^5/c$) associated with the 4-dimensional vector field $g_{5\mu}(\vec{x}, t)$ circumvents the momentum conservation law. It allows for dipole radiations if particles

[1] Our convention for the metric signature is as follows: $(+, -, -, -...)$.

propagate in extra dimensions with constant intrinsic velocities. Such a possibility will be considered in details in section 4. For the time being, we simply notice that the reduced action implies the following equations of motion

$$m\dot{u}^{\alpha} + m\Gamma^{\alpha}_{\mu\nu}u^{\mu}u^{\nu} \simeq c^2(m\frac{u^5}{c})\,(g_5^{\rho,\alpha} - g_5^{\alpha,\rho})\frac{u_{\rho}}{c}\,. \tag{4}$$

Consequently, the acceleration of a body in a freely falling frame is non zero since we obtain

$$\vec{a} - \vec{g} \simeq -c^2(\frac{v^5}{c})\vec{\nabla}g_{50}\,, \tag{5}$$

in the non-relativistic limit. The right-hand side term being of gravitational nature, we face here a violation of the equivalence principle, i.e., a violation of the universality of free fall. We therefore have established a connection between the existence of dipole radiations and the violation of the equivalence principle, thus generalizing a conjecture made in [3].

Let us take advantage of this link to single out another class of relevant metric theories, in which the mass of particles would explicitly depend on an additional scalar gravitational field $\phi$:

$$S = -c\int m(\phi)\,ds\,. \tag{6}$$

In the weak field approximation, this action for matter particles becomes

$$S \simeq -c\int \{m + (ms)\,\phi\}\;ds\,, \tag{7}$$

with $s = (d\ln m/d\phi)_{|0}$, and the inferred equations of motion are

$$m\dot{u}^{\alpha} + m\Gamma^{\alpha}_{\mu\nu}u^{\mu}u^{\nu} \simeq c^2(ms)\,\phi_{,\mu}(g^{\mu\alpha} - u^{\mu}u^{\alpha}/c^2)\,. \tag{8}$$

In the non-relativistic limit,

$$\vec{a} - \vec{g} \simeq -c^2 s\vec{\nabla}\phi\,, \tag{9}$$

and, once again, the existence of the second gravitational charge $(ms)$ implies both a violation of the equivalence principle (see Eq.(9)) and dipole radiations (see Eq.(7)).

Additional scalar and vector gravitational fields seem to be unavoidable in quantum unifications of the four known fundamental interactions. In the next two sections, we will focus on popular prototypes of scalar-tensor and vector-tensor models of gravity resulting from String Theory and Supergravity, respectively.

## 3 Brans-Dicke theories

The Brans-Dicke theory [4] is the minimal extension of general relativity including a second gravitational field. It is however already sophisticated enough, as we indicated in the previous section, to encounter an equivalence principle violation and to allow for large dipole radiations.

### 3.1 Scalar-tensor theory for compact bodies

In the context of scalar-tensor theories, the mass of extended bodies will naturally depend on the scalar field's background value. Indeed, considering a freely falling frame (hence switching off the metric field), the scalar interaction survives. The internal structure of a body depends on it, and therefore its overall mass too. As proposed by Eardley [5], we will treat a compact (i.e. self-gravitating) body in this framework as being poinlike, though with an explicit scalar dependence in its mass. The corresponding action (including now dynamics for the gravitational fields) of Brans-Dicke theory reads

$$S^{bd} = -\frac{c^3}{16\pi}\int \sqrt{g}d^4x\,\left(\Phi R - \frac{\omega}{\Phi}\Phi^{,\alpha}\Phi_{,\alpha}\right) - c\int m(\Phi)\,ds\,, \tag{10}$$

where $g$ is the absolute value of the determinant of the metric $g_{\mu\nu}$, $\Phi$ is the massless scalar field and $\omega$, a dimensionless parameter. We are however only interested here in the linearized version of this action. The constant $\Phi_0$ standing for the cosmological background value of the scalar field, let us define the metric and scalar perturbations as follows:

$$\begin{aligned} g_{\mu\nu} &= \eta_{\mu\nu} + \sqrt{2c\kappa'}(h_{\mu\nu} - \alpha\eta_{\mu\nu}\varphi) \\ \frac{\Phi}{\Phi_0} &= 1 + \alpha\sqrt{2c\kappa'}\varphi\,. \end{aligned} \tag{11}$$

The decoupling and canonical normalization[2] of the corresponding kinetic terms in the resulting theory,

$$\begin{aligned} S^{grav}_{(bd)} = \int d^4x\,\Bigg[&\left(\frac{1}{4}\partial_\mu h^{\alpha\beta}\partial^\mu h_{\alpha\beta} - \frac{1}{4}\partial_\mu h\partial^\mu h + \frac{1}{2}\partial^\mu h\partial^\nu h_{\mu\nu} - \frac{1}{2}\partial_\nu h^{\mu\nu}\partial^\rho h_{\mu\rho}\right) \\ &+ \left(\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi\right)\Bigg]\,, \end{aligned} \tag{12}$$

are implemented for $\kappa' \equiv 8\pi G'/c^4$, ($G' \equiv 1/\Phi_0$ being the bare tensor coupling constant) and

$$\alpha^2 = \frac{1}{2\omega+3}\,. \tag{13}$$

With such a normalization, the tensor and scalar propagators are given by

$$G^{\mu\nu,\alpha\beta}(q) = \frac{\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha} - \eta^{\mu\nu}\eta^{\alpha\beta}}{q^2 + i\varepsilon} \tag{14}$$

[2]Notice that the somewhat unusual additional $\sqrt{c}$ factor in the normalization of the gravitational fields allows us to transfer the whole dependence on the light velocity into the couplings. The $1/c$ counting will then be obvious when evaluating the intensity of radiation.

and

$$\Delta_F(q) = \frac{1}{q^2 + i\varepsilon}\,, \tag{15}$$

respectively, for a transfer momentum $q$. Notice that the tensor propagator is the same as in general relativity.

The linearized action for matter (assuming non-relativistic motion) gives the following interaction terms:

$$S^{int}_{(bd)} = -\sqrt{\frac{\kappa'}{2c}} \int d^4x \; \left[ h^{\mu\nu} T_{\mu\nu} - \alpha \varphi T \,(1 - 2s) \right] \,, \tag{16}$$

where $T^{\mu\nu} = \rho\, v^\mu v^\nu$ $(\rho \equiv m\, \delta^3(\vec{x} - \vec{x}(t)))$ is the non-relativistic expression of the energy-momentum tensor for ordinary matter and $T = \rho c^2$, its trace. The constant $\alpha$ is identified with the scalar coupling to matter and, consequently, we recover general relativity in the limit $\omega \to \infty$ (see Eq.(13)). The new gravitational charge $s$ is the sensitivity of the variable mass with respect to the scalar field:

$$s \equiv \frac{\partial \ln m}{\partial \ln \Phi}_{|\Phi_0}\,. \tag{17}$$

### 3.2 Strong equivalence principle violation

The effective gravitational coupling $G_{12}$ between two compact bodies (labeled by 1 and 2) here follows from the virtual exchange of a tensor ($T$) and a scalar ($S$) gravitational fields. In the non-relativistic limit, the interaction potential is given by the scattering amplitude $\mathcal{M} \equiv \mathcal{M}^T + \mathcal{M}^S$ of the two particles. In the limit of static bodies ($T^{\mu\nu} \equiv \rho c^2 \delta^\mu_0 \delta^\nu_0$), we get from Eqs.(14-16),

$$\mathcal{M} = \mathcal{N} c^4 \; \left[ 1 + \alpha^2 (1 - 2s_1)(1 - 2s_2) \right] G' \frac{m_1 m_2}{-\vec{q}^2}\,, \tag{18}$$

where $\mathcal{N}$ is a standard normalization factor, and $\vec{q}$, the spatial component of the transfer momentum. The corresponding interaction potential is therefore the Newton potential with

$$G_{12} = \left[ 1 + \alpha^2 (1 - 2s_1)(1 - 2s_2) \right] G'\,. \tag{19}$$

In the case of test particles, this effective coupling constant reduces to

$$G = (1 + \alpha^2) G'\,, \tag{20}$$

which defines the universal Newton coupling constant $G$ in terms of the bare coupling $G'$. This relation allows us to interpret the charge $s$ as the ratio of the internal gravitational binding energy $\Omega_{gr}$ of the body to its mass $m$:

$$\begin{aligned} s &= -\frac{\partial \ln m}{\partial \ln G} \\ &= \frac{|\Omega_{gr}|}{mc^2}\,. \end{aligned} \tag{21}$$

Before tackling the issue of dipole radiations, we would like to emphasize that, here we have in fact a violation of the strong equivalence principle, i.e., the violation of the universal geodesic motion for self-gravitating bodies only. Indeed, the free fall of a compact body towards a non compact source ($s_1 = 0$) is slowed down proportionally to its own sensitivity ($s_2$): from Eqs.(19) and (20), we obtain

$$G_{12} = (1 - \xi s_2) G\,, \tag{22}$$

with

$$\xi = \frac{2\alpha^2}{(1+\alpha^2)}\,, \tag{23}$$

the effective scalar coupling responsible for what is called the "Nordtvedt effect" [6, 3]. This effect, anomalous with respect to general relativity, is responsible for a polarization towards the Sun of the Moon's orbit around the Earth. The Lunar Laser Ranging experiment, monitoring the motion of the Moon with an impressive current precision of the order of one centimeter, puts a tight bound on the amplitude of such a polarization. Working in the restrictive limit of the Brans-Dicke theory, this amounts to constraining the effective scalar coupling to extremely small values [7]:

$$\xi \leq 6\,10^{-4}\,. \tag{24}$$

### 3.3 Scalar dipole

As already noticed in the first paragraph of this section, (see Eq.(16)), the source of the scalar field is $m(1-2s)$ rather than the mass $m$ itself. Consequently, monopole radiations are still forbidden, but dipole ones (induced by the charge $s$) will appear in the same way as in electromagnetism. Let us therefore make the identification of the scalar Brans-Dicke interaction proportional to the sensitivity $s$ with the vector interaction present in the Maxwell action

$$S^{em} = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{c^{3/2}} A_\mu j^\mu \right]\,, \tag{25}$$

where $j^\mu = q\delta^3(\vec{x} - \vec{x}(t))\, v^\mu$. If we perform the substitution

$$\frac{q_a}{m_a} = \sqrt{16\pi G'}\,\alpha s_a \tag{26}$$

in our basic Eq.(1), taking into account a polarization correction factor 1/2 when identifying $A_\mu v^\mu/c$ with $\varphi$, we readily obtain the scalar dipole radiation rate

$$-\frac{dE_S^{dip}}{dt} = \frac{2}{3}\xi\frac{G}{c^3}\ddot{D}^i_{(s)}\ddot{D}^i_{(s)}\,, \tag{27}$$

with $D^i_{(s)} \equiv \int d^3\vec{x}'\,\rho s\, x'^i$. This is precisely the result of the more standard procedure of developping the radiative solution for $\varphi$ in multipole moments. In the particular case of a binary source in Keplerian motion, the dipole moment is given by $\vec{D}_{(s)} = m_1 s_1 \vec{r}_1 + m_2 s_2 \vec{r}_2$ and we get, to first significant order in the scalar coupling $\xi$ , the average rate

$$\frac{1}{T}\int_0^T \left(-\frac{dE_S^{dip}}{dt}\right) dt \equiv \left\langle -\frac{dE_S^{dip}}{dt}\right\rangle_K = \frac{2}{3}\xi\frac{G^3}{c^3}\frac{\mu^2 M^2}{a^4}\left(s_1 - s_2\right)^2 g(e)\,, \tag{28}$$

with $\mu$ and $M$, the reduced and total masses respectively, and $a$, the semi major axis of the classical orbit. The enhancement factor $g(e) = \left(1 + e^2/2\right) / \left(1 - e^2\right)^{5/2}$ is a function of the orbital eccentricity $e$.

The second kind of emission[3] coming out in the context of these theories is the standard quadrupole tensor radiation [2, 8, 9] of general relativity (with again $G'$ substituted for $G$):

$$-\frac{dE_T^{quad}}{dt} = \frac{G'}{5c^5}\dddot{D}^{ij}\dddot{D}_{ij}\,, \tag{29}$$

with $D^{ij} \equiv \int d^3\vec{x}'\,\rho\,(x'^i x'^j - \frac{1}{3}\delta^{ij} r'^2)$. The Keplerian motion of a two-body system leads to[4] the average rate

$$\left\langle -\frac{dE_T^{quad}}{dt}\right\rangle_K = \frac{32}{5}\frac{G^4}{c^5}\frac{\mu^2 M^3}{a^5} f(e)\,, \tag{30}$$

with the enhancement factor $f(e) = \left(1 + 73/24e^2 + 37/96e^4\right) / \left(1 - e^2\right)^{7/2}$.

As already emphasized, dipole radiations (of order $O(1/c^3)$) could a priori dominate quadrupole ones (of order $O(1/c^5)$). But, before discussing phenomenological implications, let us turn our attention to the other previously mentioned theoretical frame where dipole radiations are also expected.

## 4 Kaluza theories

We have considered the possibility of dipole radiations arising from the coupling of a *scalar* gravitational field to the *sensitivity* of compact systems. Could this radiation rather come from the coupling of a *vector* field of gravitation to an *extra dimensional velocity*, as already suggested in section 2 ?

[3]We will neglect the scalar quadrupole (as well as a scalar monopole not even encountered by our linearized theory) as it would be of order $O(\xi/c^5)$, hence negligible compared to the tensor quadrupole, of order $O(1/c^5)$.

[4]Here $G'$ and $G_{12}$ are set equal to $G$. Once more, this corresponds to neglecting small corrections of order $O(\xi/c^5)$.

In 1921, Kaluza [10] introduced for the first time the seminal concept of extra dimensions in a superb attempt to unify gravitation and electromagnetism. In this theory, classical matter particles are allowed to propagate in a 5-dimensional spacetime and the momentum ($mv^5$) in the fifth direction is interpreted in terms of the electric charge ($q$)

$$\frac{q}{m} = \sqrt{16\pi G}(\frac{v^5}{c}) \,. \tag{31}$$

However, this identification constrains the electric charge $q$ to irrationally small values for elementary particles, which led to abandon temporarily the concept of extra dimensional spacetime. Here, we would like to preserve the gravitational nature of the induced vector interaction and to analyze the observable consequences of the associated charge for macroscopic phenomena.

### 4.1 Gravity in $4+\delta$ dimensions

The Einstein theory in $4+\delta$ dimensions is given by the action

$$S_{(4+\delta)} = -\frac{1}{2c\kappa_{(4+\delta)}} \int d^{(4+\delta)}x \, \sqrt{\hat{g}}\hat{R} + \int d^{(4+\delta)}x \, \sqrt{\hat{g}}\mathcal{L}_{mat}\left(\Psi, \, \hat{g}_{MN}\right) \,, \tag{32}$$

where $\kappa_{(4+\delta)} \equiv 8\pi G_{(4+\delta)}/c^4$. Here, $G_{(4+\delta)}$ stands for the $(4+\delta)$-dimensional coupling constant and $\hat{g}_{MN}$ is the metric of the $(4+\delta)$-dimensional spacetime[5]. If one defines the metric perturbation as $\hat{g}_{MN} = \eta_{MN} + \sqrt{2c\kappa_{(4+\delta)}}\hat{h}_{MN}$, the corresponding linearized theory (which is the one of interest) reads

$$\begin{aligned} S^{quad}_{(4+\delta)} \;\; = \;\; & \int d^{(4+\delta)}x \, \Bigg[ \bigg( \frac{1}{4}\partial_M \hat{h}^{AB}\partial^M \hat{h}_{AB} - \frac{1}{4}\partial_M \hat{h}\partial^M \hat{h} + \frac{1}{2}\partial^M \hat{h}\partial^N \hat{h}_{MN} \\ & -\frac{1}{2}\partial_N \hat{h}^{MN}\partial^R \hat{h}_{MR} \bigg) - \sqrt{\frac{\kappa_{(4+\delta)}}{2c}}\hat{h}^{MN}T_{MN} \Bigg] \,, \end{aligned} \tag{33}$$

where $T_{MN}$ is the stress tensor for ordinary matter. For simplicity, we will assume that the extra dimensions are compactified. The compactification on a torus (with identical radii $R$) implies that each field will decompose into modes $\hat{h}^{(\vec{n})}_{MN}(x^\mu)$. Only modes with the same $\vec{n}$ will couple to each other. Consequently, the integration over the $\delta$ extra dimensions in the action simply leads to a volume factor $V^\delta$ which will renormalize the gravitational constant when going from $4+\delta$ to 4 dimensions: $G_4 \equiv G_{4+\delta}/V^\delta$ . The structure of the 4-dimensional interaction (fields and couplings) may be derived through the following "metric reduction" [11]:

$$\eta_{MN} + \sqrt{2c\kappa_{(4+\delta)}}\hat{h}_{MN} = \eta_{MN} + \sqrt{2c\kappa_4} \begin{bmatrix} h_{\mu\nu} - \alpha\eta_{\mu\nu}\varphi & A_{\mu n} \\ A_{m\nu} & -2\alpha\varphi_{mn} \end{bmatrix} \,, \tag{34}$$

[5] Throughout this article, we will stick to the following indices conventions: capital indices run over all dimensions ($0 \leq M, N... \leq 4+\delta$), greek indices over the usual spacetime dimensions ($0 \leq \mu, \nu... \leq 3$), latin $ij$-indices run over the usual spatial dimensions ($1 \leq i, j... \leq 3$), while latin $mn$-indices run over extra dimensions only ($4 \leq m, n... \leq 4+\delta$).

where $\kappa_4$ is the 4-dimensional coupling. The scalar $\varphi = \delta^{mn}\varphi_{mn}$ is the trace of the $\delta(\delta+1)/2$ 4-dimensional scalar fields, $A_{\mu n}$ are $\delta$ vector fields. The parameter $\alpha$ is again introduced to normalize the kinetic term for the scalar $\varphi$. This process of "metric reduction" is rather long and awkward, as one has to operate intricate field redefinitions before correctly identifying $\vec{n}=0$ modes with massless mediators and $\vec{n} \neq 0$ modes with infinite (Kaluza-Klein) towers of massive mediators of the interaction.

However, one may find out the field content (with correct couplings) much more easily via a "propagator reduction". Let us first consider how the $(4+\delta)$-interaction is mediated and decompose the propagator in a second step. In $4+\delta$ dimensions, there is only one graviton. From Eq.(33), it is straightforward to define its propagator as

$$G^{MN,AB}(q) = \frac{\eta^{MA}\eta^{NB} + \eta^{MB}\eta^{NA} - \frac{2}{2+\delta}\eta^{MN}\eta^{AB}}{q^2_{(4+\delta)} + i\varepsilon}\,, \tag{35}$$

where $q_{(4+\delta)}$ is the momentum transfer in $4+\delta$ dimensions. In order to understand the couplings to the 4-dimensional matter particles, we may once again consider the scattering of two particles by the gravitational interaction. For matter propagating in our 4-dimensional world, the corresponding amplitude reads

$$\begin{aligned} \mathcal{M} &= \mathcal{N} \sum_{q_{(4+\delta)}} \frac{1}{V^\delta} G_{(4+\delta)}\, T^{(1)}_{\mu\nu} G^{\mu\nu,\alpha\beta}(q) T^{(2)}_{\alpha\beta} \\ &= \mathcal{N} \sum_{q_{(4+\delta)}} G_4\, T^{(1)}_{\mu\nu} G^{\mu\nu,\alpha\beta}(q) T^{(2)}_{\alpha\beta}\,, \end{aligned} \tag{36}$$

with $q_4$, a given transfer momentum. From compactification, we know that each mode is associated with a mass: $q^2_{(4+\delta)} = q_4^2 - m_n^2 c^2$, where $m_n c = \frac{|\vec{n}|}{R}$. The total scattering amplitude splits then into one massless and massive terms,

$$\mathcal{M} = \mathcal{M}_0^{ST} + \sum_n \mathcal{M}_n^{ST}\,, \tag{37}$$

each of them corresponding to the interaction between matter fields through the propagation of massless or massive gravitational mediators, respectively.

Neglecting for a while the massive states, we may decompose the original propagator into massless tensor and scalar propagators in 4 dimensions, with fixed relative couplings:

$$\begin{aligned} \mathcal{M}_0^{ST} &= \mathcal{N} \left(\frac{1}{q_4^2 + i\varepsilon}\right) G_4\, T^{(1)}_{\mu\nu} \left(\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha} - \frac{2}{2+\delta}\eta^{\mu\nu}\eta^{\alpha\beta}\right) T^{(2)}_{\alpha\beta} \\ &= \mathcal{N} \Bigg( G_4\, T^{(1)}_{\mu\nu} \frac{\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha} - \eta^{\mu\nu}\eta^{\alpha\beta}}{q_4^2 + i\varepsilon} T^{(2)}_{\alpha\beta} \\ &\qquad\qquad + \left[\frac{\delta}{\delta+2} G_4\right] T^{(1)} \frac{1}{q_4^2 + i\varepsilon} T^{(2)} \Bigg)\,. \end{aligned} \tag{38}$$

The corresponding effective action for gravitation in 4 dimensions is given by

$$S_0^{ST} = S_0^T + S_0^S\,, \tag{39}$$

where the tensor part $S_0^T$ is

$$S_0^T = \int d^4x \left[\left(\frac{1}{4}\partial_\mu h^{\alpha\beta}\partial^\mu h_{\alpha\beta} - \frac{1}{4}\partial_\mu h \partial^\mu h + \frac{1}{2}\partial^\mu h \partial^\nu h_{\mu\nu} - \frac{1}{2}\partial_\nu h^{\mu\nu}\partial^\rho h_{\mu\rho}\right) - \sqrt{\frac{\kappa_4}{2c}} h_{\mu\nu}T^{\mu\nu}\right] , \tag{40}$$

and the scalar one $S_0^S$ reads

$$S_0^S = \int d^4x \left[\left(\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi\right) + \sqrt{\frac{\kappa_4}{2c}}\sqrt{\frac{\delta}{\delta+2}}\varphi T\right] . \tag{41}$$

These are precisely the results obtained through the usual "metric reduction". They correspond to the linearized version of a Brans-Dicke scalar-tensor theory with strong scalar coupling,

$$\alpha^2 = \frac{\delta}{\delta+2} \geq \frac{1}{3} , \tag{42}$$

if $G_4 = G'$ (see Eq.(16) in the limit of test bodies ($s = 0$)).

The massive modes[6] are supposed to be heavy enough to ensure that the corresponding Yukawa contributions do not interfere with the experimentally verified $1/r$ law, above one millimiter. However, the resulting scalar-tensor theory is totally excluded by solar system observations. In fact, the well-known deflection of star light by the Sun is sufficient to rule out a strong scalar coupling. Indeed, the light bending by a heavy body is given by the scattering amplitude of a photon and a massive scalar, through the exchange of gravitational fields. The tracelessness of the photonic energy-momentum tensor ($T^{(\gamma)} = 0$) prevents from any coupling to the massless field $\varphi$ (see Eq.(38)). Consequently, the predicted deflection angle $\Theta_{ST}$ is equal to the angle $\Theta_{GR}$ of general relativity, up to a sizeable rescaling of the effective coupling constant (see Eq.(20)):

$$\Theta_{ST} = \frac{G'}{G}\,\Theta_{GR} = \frac{1}{1+\alpha^2}\,\Theta_{GR} \leq \frac{3}{4}\,\Theta_{GR} . \tag{43}$$

So, we have to invoke some mechanism by which the scalar $\varphi$ also acquires a mass and decouples [13]. This amounts to sending the scalar coupling $\alpha$ to zero, providing therefore an obviously viable model.

Such models with "gravity propagating in a $(4+\delta)$-dimensional bulk and ordinary

[6]For these modes, the "propagator reduction" method leads to:

$$\begin{aligned}\sum_{\vec{n}} \mathcal{M}_{\vec{n}} &= \mathcal{N}\left(\sum_{\vec{n}} \frac{1}{q_4^2 - m_n^2c^2 + i\varepsilon}\right) G_4\, T^{(1)}_{\mu\nu}\left(\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha} - \frac{2}{2+\delta}\eta^{\mu\nu}\eta^{\alpha\beta}\right) T^{(2)}_{\alpha\beta} \\ &= \mathcal{N}\left[\sum_{\vec{n}} G_4\, T^{(1)}_{\mu\nu}\frac{\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha} - \frac{2}{3}\eta^{\mu\nu}\eta^{\alpha\beta}}{q_4^2 - m_n^2c^2 + i\varepsilon} T^{(2)}_{\alpha\beta} + \sum_{\vec{n}} \left[\beta^2 G_4\right] T^{(1)}\frac{1}{q_4^2 - m_n^2c^2 + i\varepsilon} T^{(2)}\right] ,\end{aligned}$$

with $\beta^2 = (2/3)\,[(\delta-1)/(\delta+2)]$. The corresponding 4-dimensional theory [11, 12] encounters one massive graviton (whose tensor structure is known to be slightly different from the massless case given in Eq.(14)) and one massive scalar for each mode $\vec{n}$.

matter confined on our 4-dimensional brane world" have recently been introduced to solve the hierarchy problem of elementary particle physics [14, 11, 12]. In the original models, the $\delta$ extra dimensions are compactified, as we have chosen to do it here. Experimental constraints still allow these dimensions to be large, as long as they are smaller than one millimiter (see [14] and references therein).

Back to our concern, looking for potential sources of dipole radiations, we abandon the so-called Large Extra Dimensional framework and allow for matter particles themselves to propagate also in the extra dimensions. Treating the extra dimensional velocity as a new gravitational charge (see Eq.(3)), we wonder whether its existence may lead to observable consequences in terms of dipole radiations.

### 4.2 Equivalence principle violation

For simplicity, we confine the motion of bodies to one extra direction, say $x^5$. The unique vector interaction ($V$) easily comes out from the "propagator reduction" when considering once more the dominant scattering amplitude for two particles (1 and 2). Thanks to the extra dimensional coupling, this massless amplitude now reads

$$\mathcal{M}_0 = \mathcal{M}_0^T + \mathcal{M}_0^V \tag{44}$$

where $\mathcal{M}_0^T$ is the tensor part of $\mathcal{M}_0^{ST}$ given in Eq.(38), and

$$\begin{aligned} \mathcal{M}_0^V &= \mathcal{N}\, 4G\, T^{(1)}_{\mu i} \frac{\eta^{\mu\alpha}\eta^{ij} + \eta^{\mu j}\eta^{\alpha i} - \frac{2}{2+\delta}\eta^{\mu i}\eta^{\alpha j}}{q_4^2 + i\varepsilon} T^{(2)}_{\alpha j} \\ &= \mathcal{N} c^2\, 4G\, j^{(1)}_{(v)\mu} \frac{-\eta^{\mu\alpha}}{q_4^2 + i\varepsilon} j^{(2)}_{(v)\alpha}\,, \end{aligned} \tag{45}$$

where $j^\mu_{(v)} = (\rho v^5/c)\, v^\mu$ defines the vector current[7] . In the non-relativistic limit, and considering static sources ($T^{\mu\nu} \equiv \rho c^2 \delta^\mu_0 \delta^\nu_0$), the scattering amplitude is then given by

$$\mathcal{M}_0 = \mathcal{N} c^4\, G(1 - 4\frac{v_1^5 v_2^5}{c^2})\frac{m_1 m_2}{-\vec{q}^2}\,. \tag{46}$$

It corresponds to a Newtonian interaction, with a renormalized gravitational constant

$$G_{12} \equiv G(1 - 4\frac{v_1^5 v_2^5}{c^2})\,. \tag{47}$$

This is the mathematical expression of the equivalence principle violation in extra dimensional (vector-tensor) theories. Notice here some fundamental differences with the case of the Brans-Dicke (scalar-tensor) theory. On one hand, we have no clue of how the extra dimensional charge ($v^5/c$) may a priori be linked to the nature of the body considered. In particular, there is no reason whatsoever to focus on compact bodies only. Eötvös-type experiments should also be considered in this context. They could tightly constrain the

[7] The conservation of this current readily follows from the linearized conservation equations in $4+\delta$ dimensions ($T^{MN}{}_{,N} = 0$), after integration over the whole compactified extra dimensional volume.

extra dimensional (vector) charge [15] of laboratory-size objects. On the other hand, just as in the case of the strong equivalence principle violation in the Brans-Dicke theory, an anomalous acceleration is also expected for planetary-size bodies, though with two major conceptual differences. First, the two bodies (1 and 2) considered in this case need now to be charged (see Eq.(47)). And secondly, the additional interaction being of vector nature, this anomalous motion may either be slowed down or accelerated. On the contrary, the analytical expressions for scalar and vector dipole radiation rates turn out to be very similar.

### 4.3 Vector dipole

The effective vector-tensor action corresponding to the propagator reduction given in Eq.(45) reads

$$S_0^{VT} = S_0^T + S_0^V\,, \tag{48}$$

with

$$S_0^V = \int d^4x \left[ -\left( \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right) - \frac{1}{c^{3/2}} \sqrt{16\pi G} A_\mu j^\mu_{(v)} \right] . \tag{49}$$

Consequently, the dipole formula readily comes out from the identification of the extra dimensional velocity with the electromagnetic charge (see Eq.(31)) in our basic Eq.(1). The vector dipole radiation rate therefore reads

$$-\frac{dE_V^{dip}}{dt} = \frac{8}{3}\frac{G}{c^3} \ddot{D}^i_{(v)} \ddot{D}^i_{(v)}\,, \tag{50}$$

where the dipole moment is now $D^i \equiv \int d^3x\, (\rho v^5/c)\, x^i$. To first significant order in the velocities ($G_{12} \to G$), the vector dipole radiations due to a Keplerian motion of a binary system is therefore given by

$$\left\langle -\frac{dE_V^{dip}}{dt} \right\rangle_K = \frac{8}{3}\frac{G^3}{c^3}\frac{\mu^2 M^2}{a^4} \left( \frac{v_1^5}{c} - \frac{v_2^5}{c} \right)^2 g(e)\,, \tag{51}$$

where $g(e)$ is the enhancement factor already defined for the scalar dipole (see Eq.(28)).

## 5 Constraints from binary pulsars

Binary pulsars are ideal laboratories for testing relativistic gravity, thanks to the extreme precision of their radio pulse. The most famous among them is the double neutron star system $PSR\,B1913+16$, discovered by Hulse and Taylor in 1974. The analysis of radio pulse arrival times provides us with a measurement of the decrease rate of the system's orbital period, in agreement with the general relativity prediction for gravitational radiations to within 0.4% [16]. Until now, the $PSR\,B1534+12$ is the second and last example of a double neutron star system for which the measurement of the orbital period decrease rate was found [17] to be in agreement with general relativity, though with a less impressive precision[8] of about 15%. However, in the last few years, there have been numerous discoveries [19, 20, 21] of neutron star - white dwarf systems. Notice that the birth rate of these binaries is, according to the most recent models, at least as high as for double neutron star binaries. Prospects for relativistic parameter measurements in these systems, including orbital decay, are very encouraging, particularly in the case of $PSR\,J1141-6545$ considered here below.

Before coming up with more phenomenological considerations, we wish here to put forward the optimal conditions on the system's parameters under which gravitational dipole radiations dominate. The scalar (or vector) to tensor ratio of energy losses turns out to be simply expressed in terms of the model-dependent gravitational charges, and of the observable eccentricity and periastron shift (the latter including in fact the standard $v^2/c^2$-quadrupole suppression factor). From an astrophysical point of view, it is more convenient to introduce the corresponding ratio of the orbital period decrease rates induced by gravitational radiations. In the Keplerian approximation, these ratios are identical. For Brans-Dicke theories, Eqs.(30) and (28), lead, to first order in $\xi$, to

$$\left\langle \frac{\dot{T}^{ST}}{\dot{T}^{RG}} \right\rangle_K = \left\langle \frac{\dot{E}^{ST}}{\dot{E}^{RG}} \right\rangle_K = 1 + \frac{5\pi}{8}\xi\,(s_1 - s_2)^2 \left(\frac{h(e)}{\Delta\omega}\right) , \tag{52}$$

while for Kaluza-inspired theories, Eqs.(30) and (51), lead, to first order in the vector charges, to

$$\left\langle \frac{\dot{T}^{VT}}{\dot{T}^{RG}} \right\rangle_K = \left\langle \frac{\dot{E}^{VT}}{\dot{E}^{RG}} \right\rangle_K = 1 + \frac{5\pi}{2}\left(\frac{v_1^5}{c} - \frac{v_2^5}{c}\right)^2 \left(\frac{h(e)}{\Delta\omega}\right) . \tag{53}$$

The parameter

$$\Delta\omega = \frac{6\pi GM}{ac^2(1-e^2)} \tag{54}$$

stands for the general relativistic periastron precession in radians per revolution, while

$$h(e) \equiv \frac{1+e^2/2}{1+73/24e^2+37/96e^4} \tag{55}$$

is the reduction factor for eccentric orbits. The crucial parameter is therefore $h(e)/\Delta\omega$: the more circular ($h(e)\to 1$) and the less relativistic the orbit ($\Delta\omega \to 0$), the more important the dipole radiations (relative to the quadrupole ones). However, for maximum eccentricity ($e=1$), the reduction factor $h(e)$ is roughly 1/3 such that its impact on the question of dipole dominance is not dramatic. On the other hand, too small a relativistic precession

[8]More recently, a new test of relativistic gravity was achieved [18] in the closest and brightest of the known binary pulsars in our galaxy, the neutron star - white dwarf $PSR\,J0437-4715$. However, this test is not provided by the analysis of radiation effects, but through the measurement of the (range and shape of) time delay undergone by radio pulses periodically passing close to the companion star. A similar measurement has also been achieved for $PSR\,B1534+12$ leading to test the theory to within 1% [17], that is, a far better precision than what orbital decay analysis provides.

would correspond to a vanishing and therefore unobservable orbital decay. For further reference, the high eccentricity ($e \simeq 0.62$) and relatively large periastron angular shift ($\Delta\omega \simeq 6.5\,10^{-5}$ or $\dot{\omega} \simeq 4.2\,^{\circ}yr^{-1}$) of $PSR\,B1913+16$ [16] are not considered as optimal parameters for dipole dominance.

Notice that these theoretical criteria selecting which binaries could give the best constraints on dipole radiations are analogous to those defined for testing explicitly the strong equivalence principle in the strong field regime. It has been shown indeed [22] that small-eccentricity long-orbital-period binary pulsars are also the best laboratories in which a polarized motion (towards the center of our galaxy) due to the strong equivalence principle violation can be measured. However, in the light of the suggested alternative between scalar and vector gravitational interactions, any constraint resulting from such a direct analysis of the anomalous motion due to an equivalence principle violation is in principle ambiguous. Indeed, the weakness of the orbital polarization of a binary system (say, pulsar or Earth-Moon) might be attributed to an accidental cancellation between the scalar and vector perturbations rather than to the smallness of both the scalar and vector charges. Such an ambiguity is absent from an indirect analysis of dipole radiations since, in this case, the scalar and vector emission rates simply add. Hence, any bound on dipole radiations from binary pulsars provides a limit on $(v_1^5/c - v_2^5/c)^2$ and $\xi(s_1 - s_2)^2$.

### 5.1 Vector dipole in binary pulsars

As already noticed, we don't know what really determines the vector charge $(v^5/c)$ of individual stars. However, its origin can be understood as follows. The extra dimensional momentum of elementary particles propagating in the compactified space is quantized. The allowed values for this "transverse" momentum actually define the masses of the associated Kaluza-Klein excitations in the effective 4-dimensional theory $(q^2_{(4+\delta)} = q^2_4 - m^2_n c^2)$. Hence, one might envisage the possibility that such excitations be trapped into stars through gravitational accretion, just as WIMP's ought to be. The vector charge for celestial bodies would then be defined by the fraction of the total mass $m$ due to these Kaluza-Klein particles $(v^5/c = \sum m_{KK}/m)$. But a dynamical model is needed in order to estimate such a fraction in neutron stars.

### 5.2 Scalar dipole in neutron star - white dwarf binaries

The sensitivity $(s)$ of each body being related to its gravitational binding energy, the nature of the companion star is crucial for scalar dipole radiations. In particular, the fact that neutron stars have essentially identical sensitivities $(s_{ns} \simeq 0.2)$ leaves little hope for discriminating between general relativity and Brans-Dicke with double neutron star binaries. Let us therefore turn to another class of binaries.

Neutron star - white dwarf binaries are particularly interesting systems when searching for dipole radiation effects since a white dwarf is much less compact $(s_{wd} \simeq 10^{-3})$ than a neutron star. Moreover, their orbital eccentricity is expected to be much smaller if the observed neutron star is the first-born star and was recycled by mass transfer from its companion [19]. So, the determination of the orbital period decrease rate of a neutron star - white dwarf binary pulsar might strongly constrain the scalar coupling of gravitation. For illustration, we focus on the particular case of a recently discovered system, $PSR\,J1141-6545$, which seems to be promising in terms of relativistic orbital parameter measurements. Given [19] its eccentricity ($e \simeq 0.17$) and periastron angular shift ($\Delta\omega \simeq 5.0\,10^{-5}$ or $\dot{\omega} \simeq 5.3\,{}^{\circ}yr^{-1}$), Eq.(52) implies

$$\left\langle \frac{\dot{T}^{ST}}{\dot{T}^{RG}} \right\rangle_{|(1141-6545)} \simeq 1 + 1.4\,10^{3}\xi\,. \tag{56}$$

An agreement to within 10% with the general relativity prediction[9] would already give a constraint on the effective scalar coupling $\xi$, of the same order of magnitude as the best current limits given by solar system experiments. A more optimistic precision of 1% would provide us with the limit

$$\xi \leq 10^{-5} \quad (\omega \geq 10^{5})\,, \tag{57}$$

far better than the best current $\xi \leq 1.5\,10^{-4}$ constraint [7] and reaching the precision level expected from future satellite experiments dedicated to probing scalar gravitational couplings. Needless to say that an indirect measurement of dipole radiations would establish the existence of new gravitational charges.

## 6 Conclusion

Until now, the observed rates of relativistic orbital decay for binary pulsars are consistent with a pure quadrupole emission of gravitational waves, as predicted by general relativity. However, the appearance of dipole radiations is quite generic in effective models inferred from quantum theories that unify gravity with the other fundamental interactions. In a scalar-tensor theory of gravitation, a natural source for such radiations is the sensitivity ($s$) associated with compact bodies. We have shown how more precise measurements of the orbital parameters for peculiar binary pulsars consisting of a neutron star and a white dwarf would strongly constrain the scalar gravitational coupling ($\xi$) of the Brans-Dicke model. Nevertheless, if orbital decay rates ever suggest a non-vanishing anomalous coupling, we will never be able to disentangle the minimal scalar-tensor extension of Einstein's theory from a Kaluza-inspired vector-tensor one characterized by an extra dimensional charge ($v^5/c$). The following interchange between the hypothetical gravitational charges

$$\sqrt{\xi}\frac{s}{2} \leftrightarrow \frac{v^5}{c}$$

[9] The total mass of the system is $M \simeq 2.300\,M_\odot$. The mass function gives the bounds $m_p \lesssim 1.348\,M_\odot$ and $m_c \gtrsim 0.968\,M_\odot$ for the mass of the pulsar (which actually corresponds to the statistical value for neutron star masses in known binary pulsars) and of the companion respectively. Those figures lead to a prediction for the orbital period decrease rate in general relativity of order $\dot{T}^{RG} \simeq -3.881\,10^{-13}\,s/s$.

leads indeed to identical dipole rates. Only a direct detection of gravitational waves could distinguish between longitudinal (scalar) and transverse (vector) polarizations, and would consequently fix the nature of the new charge involved.

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