A geometrical approach to evaluating constitutive elements of structural robustness

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Summary

The approach proposed here is linked to Maxwell's reciprocal representation of force and geometry for structural modelling. It is based on the approach of Fivet & Zastavni [2014] of modelling interactive constraint-based structural equilibriums in which geometrical regions are computed to assess a domain of solutions. An examination is undertaken to establish whether the integral of relevant characterising domains can represent an interactive measure of the level of robustness. The approach is applied to case studies, one of which is the Ponte della Musica in Rome, Italy (by the architect Kit Powell-Williams and engineers C. Lotti & Associati and BuroHappold). Structures are analysed in terms of their strength for withstanding different load combinations and degrees of damage. Allowable geometric areas for thrust line(s) are calculated, synthesising the strengths and dimensional constraints, as well as the redistribution of internal forces.

Keywords: structural robustness, geometrical approach, graphical methods, structural design, engineering tools, structural reliability and efficiency, computer modelling.

1. Introduction

Most methods proposed today for assessing the robustness of structures are based on probabilistic approaches [1]. Of the few that have adopted a deterministic formulation, all provide a type of survey that is based on an in-depth analysis of the structure once it has been designed, according to specific scenarios. A central challenge in structural design should involve managing the issue of robustness earlier on in the design process, or even being able to interact with a model of the future structure to adjust the features of robustness. This paper contains an overview of a geometrical approach for evaluating constitutive elements of structural robustness. This research is linked to modelling methods and the analysis and refinement of structural designs using geometrical tools almost exclusively, even if they are implemented by means of computers and dynamic geometry. This is a way of simplifying analyses and making them more visual, enabling the designer to interact with the structure during the early stages of its design.

The paper commences with a literature review of methods to assess robustness issues and explains how they have been interpreted in the context of the geometrical approaches taken. It then introduces the geometrical methods that provide the origin of this geometrical approach to robustness [2]. A characterisation of two case studies is then presented to highlight the major features of the geometrical results when considering undamaged and damaged structures. Elements of deterministic and energetic approaches are consequently resorted to that are compared with the geometrical assessment. The paper finally provides conclusions and future recommendations about the advocate approach.

2. The issue of robustness in literature

A series of methods are proposed in literature to characterise robustness [1,3]. Four major approaches can be identified: risk-based, probabilistic, deterministic and energetic approaches. Risk-based approaches and probabilistic approaches are adopted by specialists and require very specific methods. In short, probabilistic approaches can be evaluated by a reliability-based index linked to redundancy that compares the probability of ruptures as in:

$$RI = \frac{P_{f\,(damaged)} - P_{f\,(intact)}}{P_{f\,(intact)}} \quad [4]$$

Risk-based approaches are based on a comparison of direct and indirect risks:

$$I_{rob} = \frac{R_{Dir}}{R_{Dir} + R_{Ind}} \quad [5]$$

They are said to be of limited practical interest [3]. This is certainly the case at the design stage. Deterministic and energetic approaches provide indicators produced by structural analyses. They will be used below as references for the assessment of the two case studies presented here.

2.1 Deterministic approaches

A deterministic approach is proposed by Frangopol & Curley[4] as the application of a *reserve strength factor* based on the *Residual Influence Factor* used in the offshore industry. It compares the structural capacity of intact and damaged structures where an element has been completely damaged.

A simple way of appropriating this approach is to compare the load capacity of damaged and intact structures according to chosen scenarios:

$$R = \frac{L_{intact}}{L_{intact} - L_{damaged}}$$

2.2 Energetic approaches

Energetic approaches classically consist of calculating the deformation energy (work of failure) of a structure led to failure [6]. It consists of integrating the space below the curve that characterises the stain-stress relation of the structure up to the collapse point.

The energetic approach considered here is slightly different and is adapted from the deterministic approach of Starossek and Haberland [7,1]

$R = 1 - \max j\ (E_{r,j}/E_{f,k})$

where *Er,j* is the energy released by the initial failure of an element *j* and available for the damage of the next structural element *k* and *Ef,k* is the energy required for the failure of the next structural element *k*. The appropriation of the method in this study consists in dividing the structure into its elements. The most fragile element is researched as being the one that reduces stiffness most. For a model made of bars, the stiffness matrix is calculated and *Kx = f* is solved. The deformation energy of the system is *1/2 x^T K x*. The structural elements *i* considered in scenarios are removed and K_i *x_i = f* is calculated. The difference in deformation energy in each structural element is calculated with the deformation energy before and after the element *i* is removed.

This numerical method has been developed by Jean-François Remacle (IMMC/UCLouvain, Louvain-la-Neuve, Belgium - Jean-Francois.Remacle@UCLouvain.be).

3. Geometrical domains of available equilibriums

The originality of the approach depicted in this paper mainly lies in its use of graphical representations of solution spaces within reciprocal diagrams. The following sub-sections introduce these two concepts.

3.1 Maxwell's reciprocal diagrams

Reciprocal diagrams, as formalised by Maxwell [8], are networks of connected segments satisfying the following rules (Fig. 1): (1) each segment in a diagram is related to one unique "reciprocal" segment in the other diagram such that the difference in angle between them is always the same (a common practice is to choose an angle equal to zero so that both segments of a same pair are parallel); (2) all the segments that connect one point in a diagram have reciprocal segments that form a closed polygon in the other diagram.

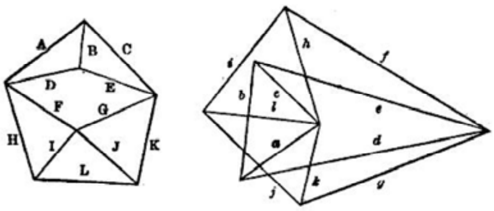

Fig.1 - Two reciprocal diagrams [8]. Pairs of segments are tagged with the same letter

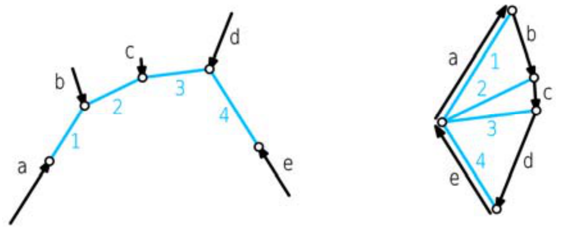

Fig.2 - A form diagram on the left and its reciprocal force diagram on the right

Graphic statics [9,10,11] employs two reciprocal diagrams: a "form diagram" that represents the geometry of a network of bars in compression and tension, and a "force diagram" in which the length of every segment is equal to the force magnitude of its reciprocal segment (a bar or an applied load) in the form diagram (Fig. 2). The existence of reciprocity ensures that the form diagram is in static equilibrium. Indeed, a closed polygon in the force diagram corresponds to every point in the form diagram, meaning that the vectorial sum of the forces acting on each point is zero.

3.2 Graphical solution spaces

Constraint-Based Graphic Statics [2,12] is a recent development in graphic statics. If both diagrams are built parametrically, geometric constraints can be applied on every node in order to control the range of possible equilibriums. For instance, the position of a node in the form diagram can be limited by constraining this point inside a bounding box (Fig. 3, left). Furthermore, the force magnitude of a bar can be limited in the force diagram by compelling one extremity of a segment to remain inside a circle of a given radius (equal to the scaled maximum magnitude) and centred on the other extremity (Fig. 3, circle in the force diagram). Since the only variables are nodes in two planes, these geometric constraints can be computationally propagated to any other node defining its parameterisation. As a result, every node in the form diagram and in the force diagram will be restricted inside a graphical region that is equal to its solution space, *i.e.* the set of all positions for which no constraint applied on the diagram is violated (Fig. 3).

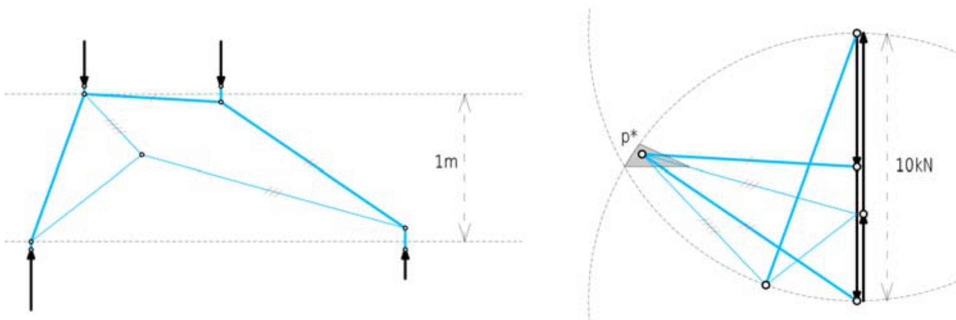

Fig. 3 - The shaded area in the force diagram (right) is the solution space of the node p such that the strut-and-tie network (left) is not higher than 1 metre and the magnitudes are below 10kN.*

4. Geometrical approach to robustness

"Robustness is defined as insensitivity to local failure" [13]. This definition emphasises the capacity of force redistribution in a structure. In other words, it is about the possibility of finding alternative load paths in a structure.

Under certain conditions, the geometrical domains presented above are a convenient tool for exploring the possible redistributions of forces in a strut-and-tie model, and hence for characterising its robustness. The first condition is the necessary aptitude in the structure for developing a plastic redistribution of forces so that the lower bound theorem of plastic design can be implemented. Such behaviour is commonly assumed for steel frames, concrete frames, arches and shear walls, masonry structures, timber with screw or threaded rods *etc*.

4.1 Presentation and methods

According to the analysis of dimensions of robustness proposed by Knoll & Vogel [14,15], five dimensions of robustness are likely to concern the design more directly: strength, second line of defence, multiple load paths and redundancy, stiffness considerations and post-buckling resistance.

These dimensions come from the authors' complete list of elements of robustness, as follows:

1/ Strength – 2/ Structural integrity and solidarisation – 3/ Second line of defence – 4/ Multiple load path or redundancy – 5/ Ductility *versus* brittle failure – 6/ Progressive failure *versus* zipper stopper – 7/ Capacity design and fuse element – 8/ Sacrificial and protective devices – 9/ the knock-out scenario – 10/ Stiffness considerations – 11/ The benefits of strain hardening – 12/ Post-buckling resistance – 13/ Warning, active intervention and rescue – 14/ Testing – 15/ Monitoring, quality control, correction and prevention – 16/ Mechanical devices. These strategies are not all applicable simultaneously. Some of them are related to the ductility of the structure or its constitutive elements (2, 5, 7, 11), making a link with the theorems of plastic design that provide the scope of application of the approach presented in this paper. Others are specific to the erection of the structure, its life and maintenance, or disruptive elements to be implemented.

Implementing the dimensions related to design – geometry and dimensioning – means (inter)acting with the design, with key milestones mainly associated with the designer's experience. Features linked to the resistance and redistribution of forces are likely to be modelled by load paths, struts and ties or thrust lines, close to geometrical thinking.

The key idea is to associate with the structure a load path made of struts, ties and/or thrust lines. In the context of constraint-based graphic statics, allowable stresses and spatial limits are likely to be represented by geometrical constraints applied on this load path. The extent to which a node of this load path is free to move can then be seen as a measure of the model's capacity to redistribute loads. The assumption will be that the integral (in the mathematical meaning of a sum) of relevant characterising domains represents an interactive measure of the level of robustness.

5. Application to study cases

The analysis of the geometrical domains characterising two structures has been performed for different scenarios of integrity: the whole intact structure, variations due to damages and variations of design geometry. The first set of scenarios refers to a comparison of the capacity of redistribution between the undamaged structure and damaged structures according to several scenarios. The second series, implementing geometrical variations, shows the influence of design choices on the capacity to redistribute loads. The extent of geometrical domains is understood to be a constitutive dimension of the robustness. The two study cases analyse (1) a concrete shear wall with openings and (2) the Ponte della Musica in Rome, Italy.

5.1 Study case 1: Concrete shear wall

The first study case is adapted from the classic example developed by Schlaich, Schäfer and Jennewein [16]. Struts and ties are modelled inside a shear wall essentially constituting a D-region (Fig. 5). The analysis of the structure as presented here is partial since it only considers one typology out of all the possible strut-and-tie models. However, the analysis of considered points is already representative.

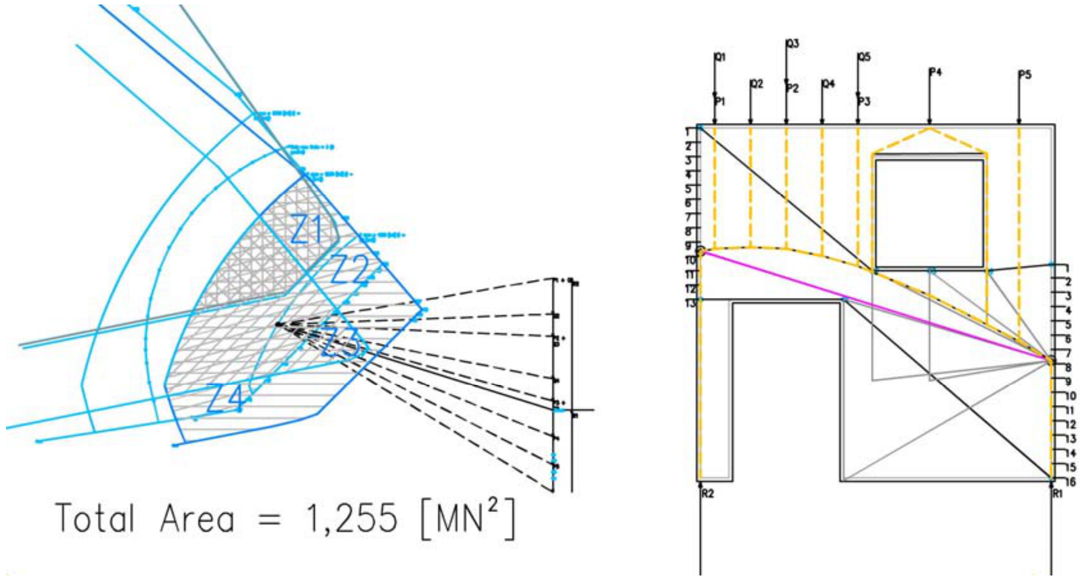

Fig. 4: Shear wall and partial and simplified geometrical domain corresponding to the pole of the arched trajectory of forces. The final geometrical domain is comprised of the sum of domains corresponding to several scenarios which are: damage of the column-support (Z3) or the support to the right of the shear wall (Z1) and displacement of the reactions under the wall in different parts of the wall (Z2, Z4 and the larger zone). The scenario corresponding to the reaction forces as represented in the right-hand figure is linked to Z2 for a domain of 0,817 MN².

Fig. 5: Strut-and-tie modelling of the shear wall

The geometrical analysis shows a sensible reduction of the possible variation of the pole in the case of a damaged support (Z3 only in Fig. 4) corresponding to 0,592 MN² compared to an undamaged arrangement (whole domain in Fig. 4). The geometrical domain shown in grey characterises the possibility of finding variations in the drawing of the load path leading the forces to the support.

If, for instance, the supporting left column disappears, the domain is moved. The redistribution of the load path can be found elsewhere.

Similarly, if the width of the left column is modified (Fig. 6), the possible base geometrical domain of (Fig. 4) is displaced, but it will extend the domain of the base scenario of Z2. It is therefore a better option for a robustness-oriented design. Diagrams may also show the cases where there is no solution according to the design constraints, or where the existence of a solution requires an increase in the magnitude of forces and therefore a revision of the dimensioning.

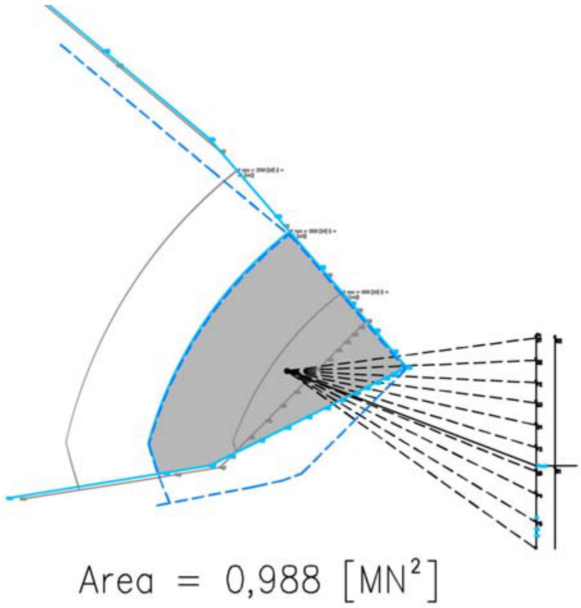

Fig.6: Shear wall and geometrical domain corresponding to the pole of the discharging arch when a variant in the design is taken with a thin column in Fig. 5

In summary, the correlation can be shown between the aptitude of the structure to redistribute forces – comprised as an indicator of some constitutive dimensions of the robustness – and a geometrical characterisation of the admitted variations of the position of nodes constituting a strut-and-tie modelling of the structure.

5.2 Study case 2: Ponte della Musica

The Ponte della Musica in Rome (Fig. 7), built in 2011 in Italy by the architect Kit Powell-Williams and engineers C. Lotti & Associati and BuroHappold, is a hybrid typology between a steel arch bridge and a bow-string bridge with a clear span of 130m. The hangers are made of rigid steel profiles moving forwards towards the longitudinal central symmetrical axis of the structure. The bridge is used as a footbridge, but is likely to be implemented as a bridge carrying buses and trams as well. The exercise consists in simulating the possible redistributions of forces in the arch according to different support conditions. Finally it compares these redistributions with those allowed when some of the hangers sustaining the deck are damaged.

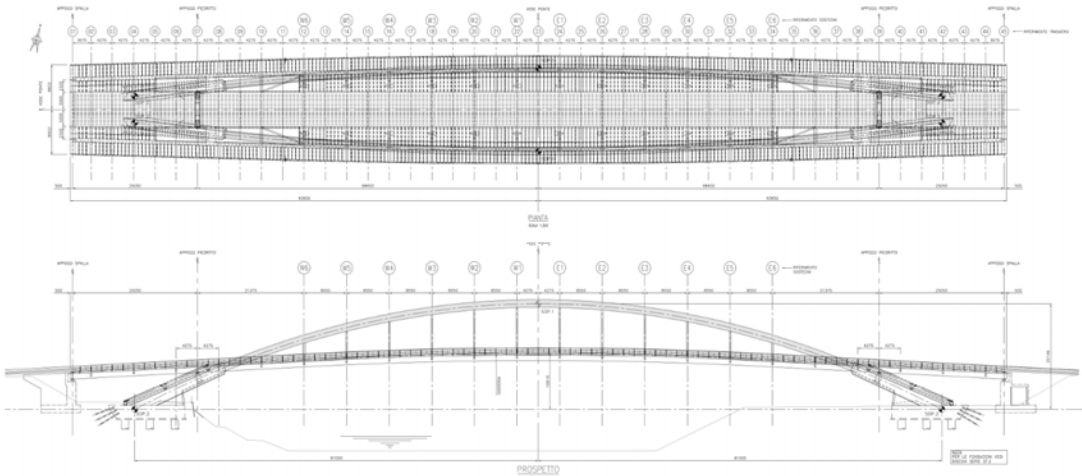

Fig.7 - Ponte della Musica, Rome 2011, Powell-Williams, Lotti & Associati and Buro Happold,

Firstly, the bridge's bending resistance is analysed. Bending forces are modelled as thrust lines, the off-centring of which is related to the magnitude of the axial compression forces (Fig. 8). Using graphic statics, the geometry of this thrust line is actually defined by a single point in the force diagram. The domain of this point consequently informs all the possible configurations of bending resistance. Other domains are then generated for altered structures in which hangers are damaged. In the first instance, the structural collaboration between hangers and the arch are neglected.

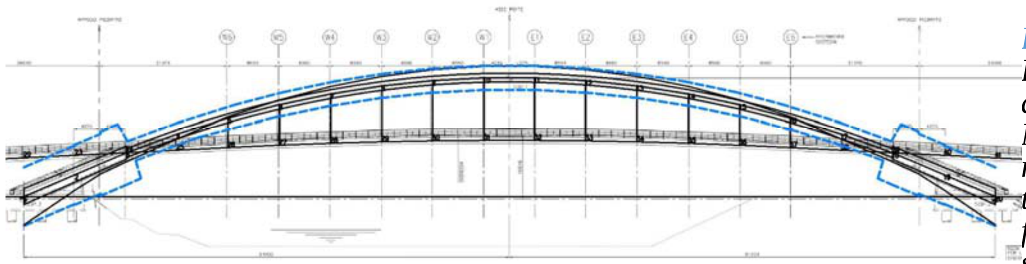

Fig. 8 - Ponte della Musica: resistance to bending forces shown in blue as off-centring, with a possible thrust line in the case of limited bending forces in the support.

The result of the analysis (Fig. 9) shows a domain (1) of **18,8 MN²** for the pole of the thrust line defining the extent of possible geometries for the load path in the case of symmetrical loading. In the case of damage to four central hangers (2), this domain is reduced by 24 % to **14,4 MN²** but still allows multiple load paths. Under asymmetrical loadings, the structure has an aptitude to redistribute load paths that equals **16,4 MN²** (3), *i.e.* slightly less than the reference maximum symmetrical loading. The slightly different position of this domain demonstrates the ability of the structure to redistribute bending forces on both sides of the arch.

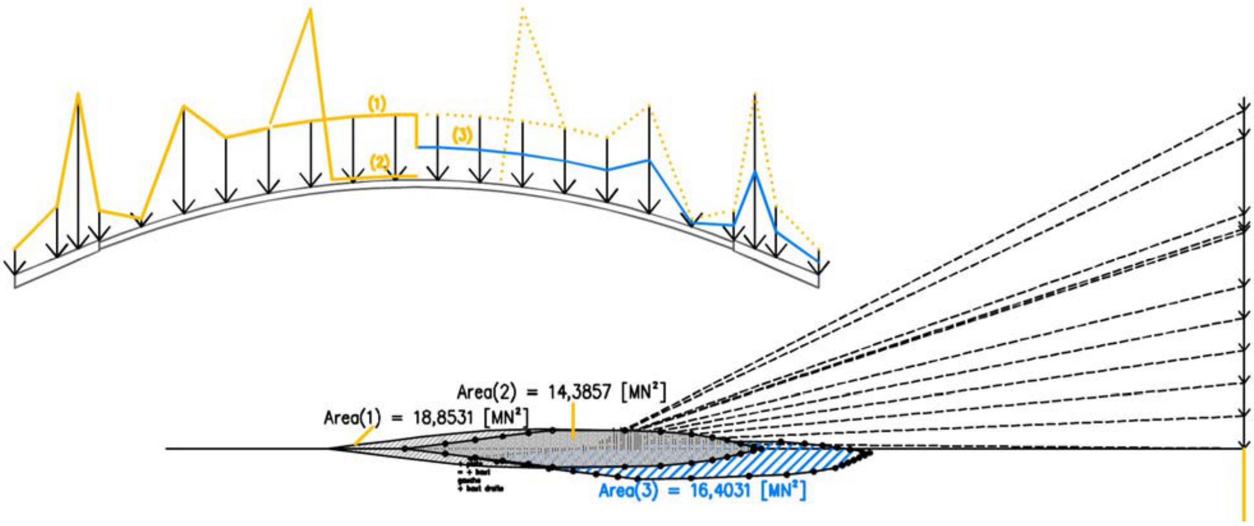

Fig. 9 - Ponte della Musica: domains corresponding to the characteristic point (pole) of the curve of the thrust line (load path in the arch) for: (1) symmetrical loading, (2) symmetrical loading in the case of four central hangers being damaged, (3) asymmetrical loadings in the undamaged structure.

6. Comparison with deterministic and energetic approaches to robustness

6.1 Deterministic approach: Ponte della Musica

A simulation was undertaken for different cases for the factor multiplying service loadings to lead to the failure. The structure was modelled as an arch in the software for maximum bending resistance of 29,6 MN.m (considering the axial compression force in the arch) in steel tubes and 146 MN.m in concrete bases of the arch, with an elastic embedding in the foundation of 60 MN.m/degree enabling a plastic redistribution of forces. The results are given below:

- symmetrical loading, undamaged structure: **23,8** * the usual service load
- asymmetrical loading, undamaged structure: **2,49** * the usual service load
- symmetrical loading, structure with 4 central hangers damaged: **6,5** * the usual service load
- asymmetrical loading, structure with 4 central hangers damaged: **1,84** * the usual service load.

The factor for the symmetrical loading is quite large, leading to a complex comparison between the different cases. If the 74 % reduction between the damaged and undamaged structure in the asymmetrical case can be taken as a temporary reference, the reduction was 77 % in the geometrical approach (symmetrical loads). However without more investigation the comparison cannot be sustained further between these different indices.

6.2 Energetic approach: Ponte della Musica

The energetic approach used here is a *stiffness-based* measure of robustness expressed in energies computed as described above in paragraph 2.2. The cases and the energies obtained for services loading | yielding loading (as computed in 6.1) are:

- symmetrical loading, undamaged structure: 3,08*10^2 kJ | 1,75*10^5 kJ
- asymmetrical loading, undamaged structure: 2,54*10^2 kJ | 1,58*10^3 kJ
- symmetrical loading, structure with 4 central hangers damaged: 3,69*10^2 kJ | 1,56*10^3 kJ
- asymmetrical loading, structure with 4 central hangers damaged: 3,21*10^2 kJ | 1,09*10^3 kJ

During the service phase, the total asymmetric loading is half the symmetric total loading, so for a similar total loading, energies in the asymmetric case will be greater. As has been seen in 6.1, the damage occurs earlier if loading is asymmetrical, so the work is reduced. A comparison between the undamaged and damaged bridge shows that the energy of deformation is greater in the latter case, since there will be larger deformations between the start and the point when the failure occurs.

Again, it was observed that both measures – the geometrical domain approach and the energetic approach – were not directly correlated, as they were not correlated to the deterministic approach shown in 6.1. Nevertheless, a similar evolution between different measures is observed, but at variable scales. The impact of the asymmetrical loading was observed for the two approaches, but to a rather different extent. Indeed, one analysis referred to the complete structure (energetic approach) and the other to the extent of availabilities for redistributing load paths. The parallel evolution of the indicators of both approaches cannot be sustained further without more extensive research and an attempt to correlate these measures more closely.

7. Conclusions and perspectives

This contribution presents a geometrical approach to evaluating constitutive elements of structural robustness and compares it to other indices from literature. The geometrical approach proves of interest during the design phase since it provides a qualitative summary of the possible load path redistributions. However, further research must be carried out to correlate quantitative geometric results with deterministic and energetic indices of robustness if these comparisons prove to be meaningful.

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